Computer Algebra
Independent Integration Tests

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4-Trig-functions/4-Miscellaneous/257-4.3

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3.226 $\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$ . . . 2044
3.227 $\int (c+dx)^m\csc(a+bx)\sec(a+bx)\,dx$ . . . 2052
3.228 $\int (c+dx)^4\csc(a+bx)\sec(a+bx)\,dx$ . . . 2057
3.229 $\int (c+dx)^3\csc(a+bx)\sec(a+bx)\,dx$ . . . 2068
3.230 $\int (c+dx)^2\csc(a+bx)\sec(a+bx)\,dx$ . . . 2077
3.231 $\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx$ . . . 2085

3.232 $\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$ . . . 2092
3.233 $\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$ . . . 2097
3.234 $\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx$ . . . 2102
3.235 $\int (c+dx)^3 \csc^2(a+bx)\sec(a+bx)\,dx$ . . . 2107
3.236 $\int (c+dx)^2 \csc^2(a+bx)\sec(a+bx)\,dx$ . . . 2116
3.237 $\int (c+dx) \csc^2(a+bx)\sec(a+bx)\,dx$ . . . 2125
3.238 $\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$ . . . 2132
3.239 $\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$ . . . 2137
3.240 $\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx$ . . . 2143
3.241 $\int (c+dx)^3 \csc^3(a+bx)\sec(a+bx)\,dx$ . . . 2148
3.242 $\int (c+dx)^2 \csc^3(a+bx)\sec(a+bx)\,dx$ . . . 2156
3.243 $\int (c+dx) \csc^3(a+bx)\sec(a+bx)\,dx$ . . . 2164
3.244 $\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$ . . . 2171
3.245 $\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$ . . . 2177
3.246 $\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx$ . . . 2182
3.247 $\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx$ . . . 2187
3.248 $\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx$ . . . 2197
3.249 $\int (c+dx)^2 \sec(a+bx)\tan(a+bx)\,dx$ . . . 2206
3.250 $\int (c+dx) \sec(a+bx)\tan(a+bx)\,dx$ . . . 2213
3.251 $\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx$ . . . 2220
3.252 $\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$ . . . 2225
3.253 $\int (c+dx)^m \tan^2(a+bx)\,dx$ . . . 2231
3.254 $\int (c+dx)^3 \tan^2(a+bx)\,dx$ . . . 2237
3.255 $\int (c+dx)^2 \tan^2(a+bx)\,dx$ . . . 2246
3.256 $\int (c+dx) \tan^2(a+bx)\,dx$ . . . 2254
3.257 $\int \frac{\tan^2(a+bx)}{c+dx}\,dx$ . . . 2260
3.258 $\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx$ . . . 2266
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3.262 $\int (c+dx) \sin(a+bx)\tan^2(a+bx)\,dx$ . . . 2299
3.263 $\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$ . . . 2307
3.264 $\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$ . . . 2314
3.265 $\int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx$ . . . 2322
3.266 $\int (c+dx)^4 \csc(a+bx)\sec^2(a+bx)\,dx$ . . . 2327
3.267 $\int (c+dx)^3 \csc(a+bx)\sec^2(a+bx)\,dx$ . . . 2337
3.268 $\int (c+dx)^2 \csc(a+bx)\sec^2(a+bx)\,dx$ . . . 2346
3.269 $\int (c+dx) \csc(a+bx)\sec^2(a+bx)\,dx$ . . . 2354

3.270 $\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$ . . . 2361
3.271 $\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$ . . . 2366
3.272 $\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx$ . . . 2371
3.273 $\int (c+dx)^3 \csc^2(a+bx)\sec^2(a+bx)\,dx$ . . . 2376
3.274 $\int (c+dx)^2 \csc^2(a+bx)\sec^2(a+bx)\,dx$ . . . 2386
3.275 $\int (c+dx) \csc^2(a+bx)\sec^2(a+bx)\,dx$ . . . 2395
3.276 $\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$ . . . 2402
3.277 $\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$ . . . 2408
3.278 $\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx$ . . . 2414
3.279 $\int (c+dx)^3 \csc^3(a+bx)\sec^2(a+bx)\,dx$ . . . 2419
3.280 $\int (c+dx)^2 \csc^3(a+bx)\sec^2(a+bx)\,dx$ . . . 2429
3.281 $\int (c+dx) \csc^3(a+bx)\sec^2(a+bx)\,dx$ . . . 2438
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3.286 $\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx$ . . . 2471
3.287 $\int x \csc^3(a+bx)\sec^2(a+bx)\,dx$ . . . 2480
3.288 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx$ . . . 2487
3.289 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$ . . . 2493
3.290 $\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx$ . . . 2499
3.291 $\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx$ . . . 2504
3.292 $\int (c+dx)^3 \sec^2(a+bx)\tan(a+bx)\,dx$ . . . 2515
3.293 $\int (c+dx)^2 \sec^2(a+bx)\tan(a+bx)\,dx$ . . . 2524
3.294 $\int (c+dx) \sec^2(a+bx)\tan(a+bx)\,dx$ . . . 2532
3.295 $\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$ . . . 2538
3.296 $\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$ . . . 2544
3.297 $\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx$ . . . 2550
3.298 $\int (c+dx)^3 \sec(a+bx)\tan^2(a+bx)\,dx$ . . . 2555
3.299 $\int (c+dx)^2 \sec(a+bx)\tan^2(a+bx)\,dx$ . . . 2569
3.300 $\int (c+dx) \sec(a+bx)\tan^2(a+bx)\,dx$ . . . 2581
3.301 $\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx$ . . . 2590
3.302 $\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$ . . . 2596
3.303 $\int (c+dx)^m \tan^3(a+bx)\,dx$ . . . 2602
3.304 $\int (c+dx)^3 \tan^3(a+bx)\,dx$ . . . 2608
3.305 $\int (c+dx)^2 \tan^3(a+bx)\,dx$ . . . 2621
3.306 $\int (c+dx) \tan^3(a+bx)\,dx$ . . . 2631

3.307 $\int \frac{\tan^3(a+bx)}{c+dx}\,dx$ . . . 2639
3.308 $\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx$ . . . 2645
3.309 $\int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx$ . . . 2651
3.310 $\int (c+dx)^4 \csc(a+bx)\sec^3(a+bx)\,dx$ . . . 2656
3.311 $\int (c+dx)^3 \csc(a+bx)\sec^3(a+bx)\,dx$ . . . 2665
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3.313 $\int (c+dx) \csc(a+bx)\sec^3(a+bx)\,dx$ . . . 2683
3.314 $\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$ . . . 2691
3.315 $\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$ . . . 2697
3.316 $\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx$ . . . 2702
3.317 $\int (c+dx)^3 \csc^2(a+bx)\sec^3(a+bx)\,dx$ . . . 2707
3.318 $\int (c+dx)^2 \csc^2(a+bx)\sec^3(a+bx)\,dx$ . . . 2718
3.319 $\int (c+dx) \csc^2(a+bx)\sec^3(a+bx)\,dx$ . . . 2729
3.320 $\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$ . . . 2736
3.321 $\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$ . . . 2742
3.322 $\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx$ . . . 2747
3.323 $\int (c+dx)^3 \csc^3(a+bx)\sec^3(a+bx)\,dx$ . . . 2752
3.324 $\int (c+dx)^2 \csc^3(a+bx)\sec^3(a+bx)\,dx$ . . . 2762
3.325 $\int (c+dx) \csc^3(a+bx)\sec^3(a+bx)\,dx$ . . . 2772
3.326 $\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$ . . . 2781
3.327 $\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$ . . . 2787
3.328 $\int x\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx$ . . . 2793
3.329 $\int x\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx$ . . . 2799
3.330 $\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx$ . . . 2804
3.331 $\int \frac{x\sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx$ . . . 2809
3.332 $\int \frac{x\sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx$ . . . 2814
3.333 $\int \frac{x\sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx$ . . . 2819
3.334 $\int \frac{x\sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx$ . . . 2824
3.335 $\int \frac{x\sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx$ . . . 2829
3.336 $\int x\sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx$ . . . 2835
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3.338 $\int x\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx$ . . . 2847
3.339 $\int x\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx$ . . . 2853
3.340 $\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx$ . . . 2858
3.341 $\int \frac{x\sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx$ . . . 2864

3.342 $\int \frac{x \sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx$ . . . 2870
3.343 $\int \frac{x \sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx$ . . . 2876
3.344 $\int x \cos(a+bx) \sin^{\frac{5}{2}}(a+bx)\,dx$ . . . 2882
3.345 $\int x \cos(a+bx) \sin^{\frac{3}{2}}(a+bx)\,dx$ . . . 2888
3.346 $\int x \cos(a+bx) \sqrt{\sin(a+bx)}\,dx$ . . . 2893
3.347 $\int \frac{x \cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx$ . . . 2898
3.348 $\int \frac{x \cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$ . . . 2903
3.349 $\int \frac{x \cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx$ . . . 2908
3.350 $\int \frac{x \cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx$ . . . 2913
3.351 $\int \frac{x \cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx$ . . . 2918
3.352 $\int x \cos(a+bx) \csc^{\frac{9}{2}}(a+bx)\,dx$ . . . 2924
3.353 $\int x \cos(a+bx) \csc^{\frac{7}{2}}(a+bx)\,dx$ . . . 2930
3.354 $\int x \cos(a+bx) \csc^{\frac{5}{2}}(a+bx)\,dx$ . . . 2936
3.355 $\int x \cos(a+bx) \csc^{\frac{3}{2}}(a+bx)\,dx$ . . . 2942
3.356 $\int x \cos(a+bx) \sqrt{\csc(a+bx)}\,dx$ . . . 2947
3.357 $\int \frac{x \cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$ . . . 2953
3.358 $\int \frac{x \cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx$ . . . 2959
3.359 $\int \frac{x \cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx$ . . . 2965
3.360 $\int x \csc(x) \sin(3x)\,dx$ . . . 2971
3.361 $\int (c+dx)^4 \csc(x) \sin(3x)\,dx$ . . . 2976
3.362 $\int (c+dx)^3 \csc(x) \sin(3x)\,dx$ . . . 2983
3.363 $\int (c+dx)^2 \csc(x) \sin(3x)\,dx$ . . . 2989
3.364 $\int (c+dx) \csc(x) \sin(3x)\,dx$ . . . 2995
3.365 $\int \frac{\csc(x) \sin(3x)}{c+dx}\,dx$ . . . 3000
3.366 $\int \frac{\csc(x) \sin(3x)}{(c+dx)^2}\,dx$ . . . 3005
3.367 $\int \frac{\csc(x) \sin(3x)}{(c+dx)^3}\,dx$ . . . 3011
3.368 $\int (c+dx)^4 \csc(a+bx) \sin(3a+3bx)\,dx$ . . . 3017
3.369 $\int (c+dx)^3 \csc(a+bx) \sin(3a+3bx)\,dx$ . . . 3024
3.370 $\int (c+dx)^2 \csc(a+bx) \sin(3a+3bx)\,dx$ . . . 3031
3.371 $\int (c+dx) \csc(a+bx) \sin(3a+3bx)\,dx$ . . . 3037
3.372 $\int \frac{\csc(a+bx) \sin(3a+3bx)}{c+dx}\,dx$ . . . 3042
3.373 $\int \frac{\csc(a+bx) \sin(3a+3bx)}{(c+dx)^2}\,dx$ . . . 3047
3.374 $\int \frac{\csc(a+bx) \sin(3a+3bx)}{(c+dx)^3}\,dx$ . . . 3053
3.375 $\int \frac{\csc(a+bx) \sin(3a+3bx)}{(c+dx)^4}\,dx$ . . . 3060

3.376 $\int (c+dx)^3 \csc^2(a+bx)\sin(3a+3bx)\,dx$ . . . 3067
3.377 $\int (c+dx)^2 \csc^2(a+bx)\sin(3a+3bx)\,dx$ . . . 3075
3.378 $\int (c+dx) \csc^2(a+bx)\sin(3a+3bx)\,dx$ . . . 3082
3.379 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$ . . . 3088
3.380 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$ . . . 3094
3.381 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$ . . . 3100
3.382 $\int (c+dx)^4 \sec(a+bx)\sin(3a+3bx)\,dx$ . . . 3106
3.383 $\int (c+dx)^3 \sec(a+bx)\sin(3a+3bx)\,dx$ . . . 3115
3.384 $\int (c+dx)^2 \sec(a+bx)\sin(3a+3bx)\,dx$ . . . 3123
3.385 $\int (c+dx) \sec(a+bx)\sin(3a+3bx)\,dx$ . . . 3130
3.386 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx$ . . . 3136
3.387 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$ . . . 3142
3.388 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$ . . . 3148
3.389 $\int (c+dx)^3 \sec^2(a+bx)\sin(3a+3bx)\,dx$ . . . 3154
3.390 $\int (c+dx)^2 \sec^2(a+bx)\sin(3a+3bx)\,dx$ . . . 3162
3.391 $\int (c+dx) \sec^2(a+bx)\sin(3a+3bx)\,dx$ . . . 3170
3.392 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$ . . . 3177
3.393 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$ . . . 3183
3.394 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$ . . . 3189
3.395 $\int x\cos(2x)\sec(x)\,dx$ . . . 3195
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4 Appendix **3212**
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4.2 Links to plain text integration problems used in this report for each CAS 3230

CHAPTER 1

Introduction

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**397**]. This is test number [257].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (397)	0.00 (0)
Rubi	99.50 (395)	0.50 (2)
Fricas	91.94 (365)	8.06 (32)
Maple	90.43 (359)	9.57 (38)
Maxima	85.89 (341)	14.11 (56)
Giac	61.46 (244)	38.54 (153)
Mupad	39.04 (155)	60.96 (242)
Reduce	35.52 (141)	64.48 (256)
Sympy	30.98 (123)	69.02 (274)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

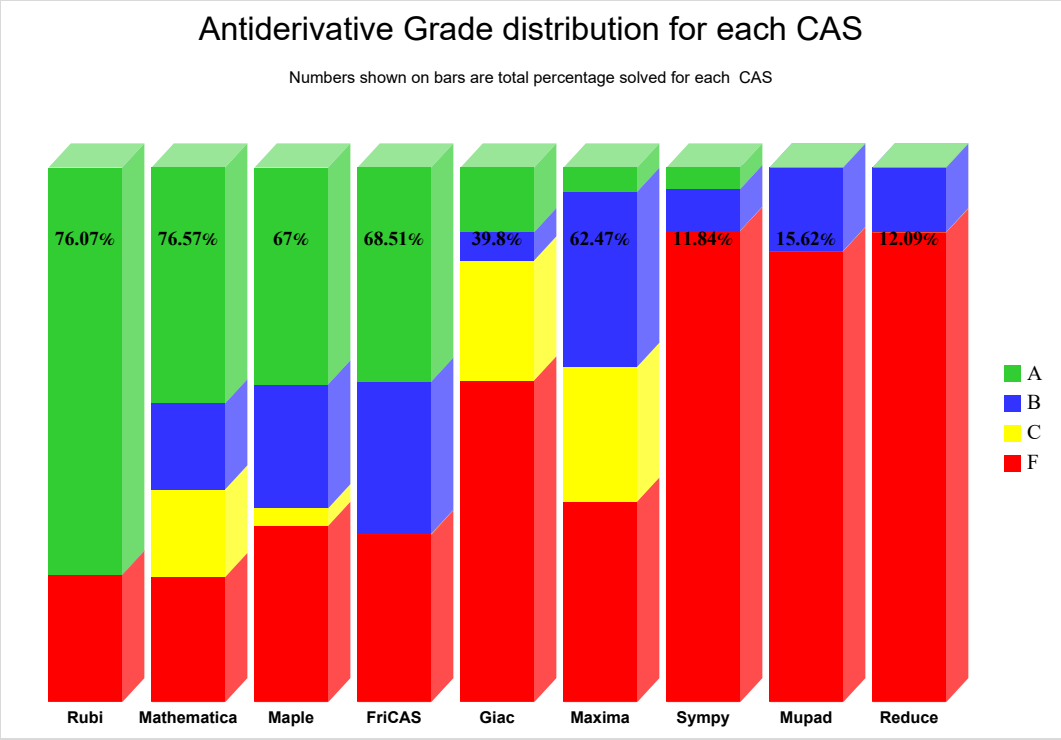
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

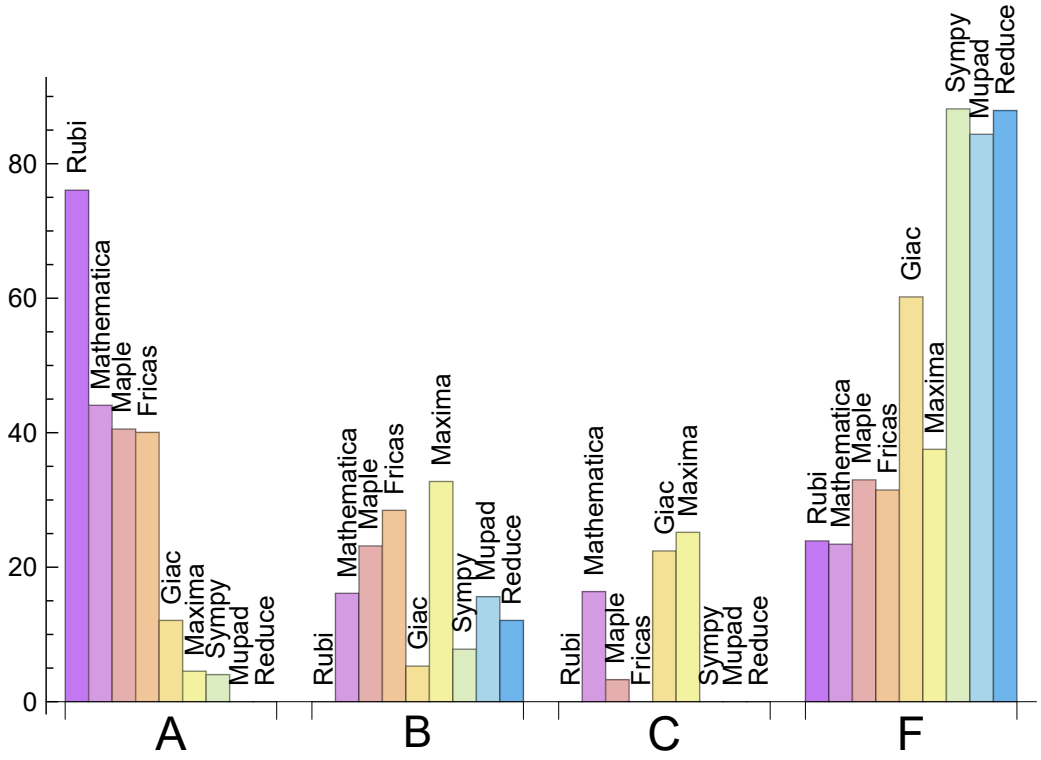
System	% A grade	% B grade	% C grade	% F grade
Rubi	76.071	0.000	0.000	23.929
Mathematica	44.081	16.121	16.373	23.426
Maple	40.554	23.174	3.275	32.997
Fricas	40.050	28.463	0.000	31.486
Giac	12.091	5.290	22.418	60.202
Maxima	4.534	32.746	25.189	37.531
Sympy	4.030	7.809	0.000	88.161
Mupad	0.000	15.617	0.000	84.383
Reduce	0.000	12.091	0.000	87.909

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Rubi	2	100.00	0.00	0.00
Fricas	32	0.00	0.00	100.00
Maple	38	100.00	0.00	0.00
Maxima	56	94.64	0.00	5.36
Giac	153	93.46	3.92	2.61
Mupad	242	0.00	100.00	0.00
Reduce	256	100.00	0.00	0.00
Sympy	274	70.44	27.37	2.19

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.10
Rubi	0.61
Maxima	0.81
Maple	1.19
Mathematica	3.76
Giac	3.96
Reduce	4.53
Sympy	4.80
Mupad	17.01

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	99.61	1.37	26.00	1.17
Rubi	164.07	1.06	126.00	1.00
Sympy	165.90	1.71	22.00	1.00
Reduce	171.14	4.06	33.00	1.40
Mathematica	287.02	1.55	127.00	1.09
Maple	294.44	1.75	187.00	1.15
Fricas	398.21	2.26	213.00	1.38
Maxima	1089.23	15.08	499.00	2.78
Giac	40643.64	136.83	209.00	1.43

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

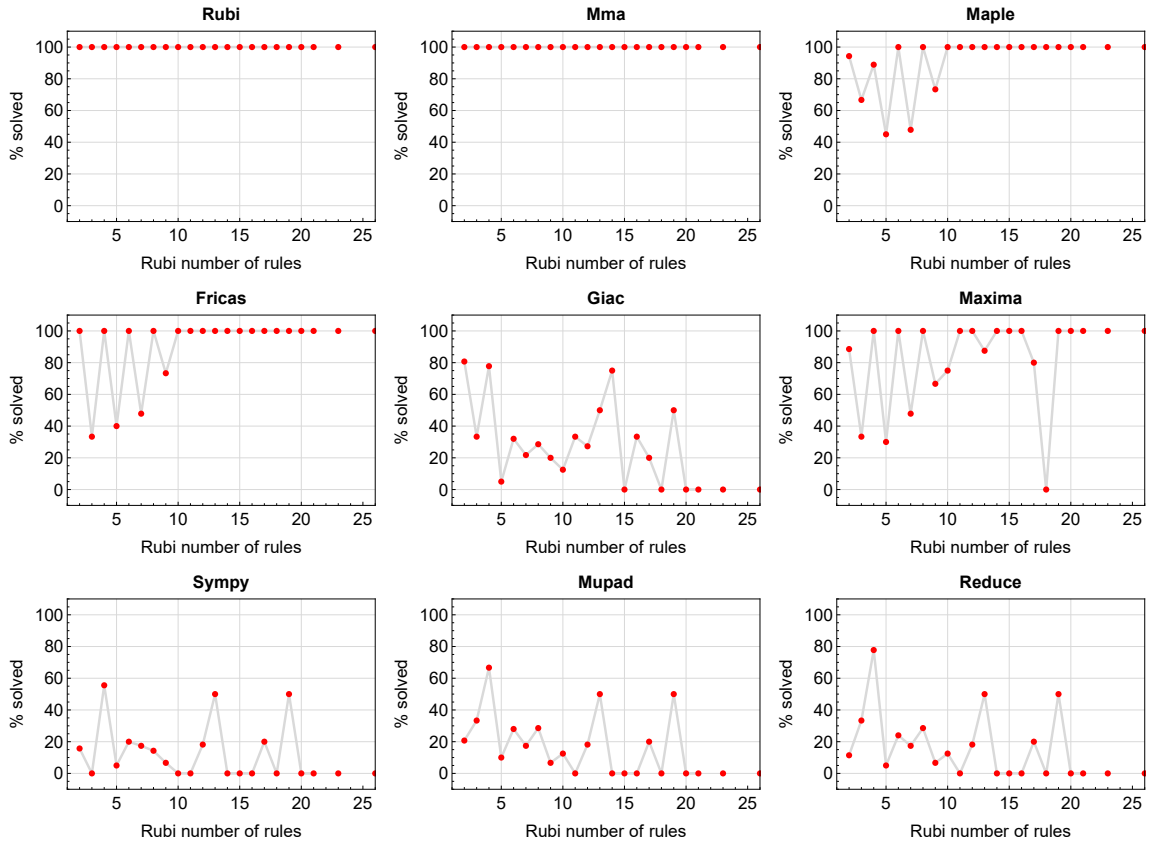


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

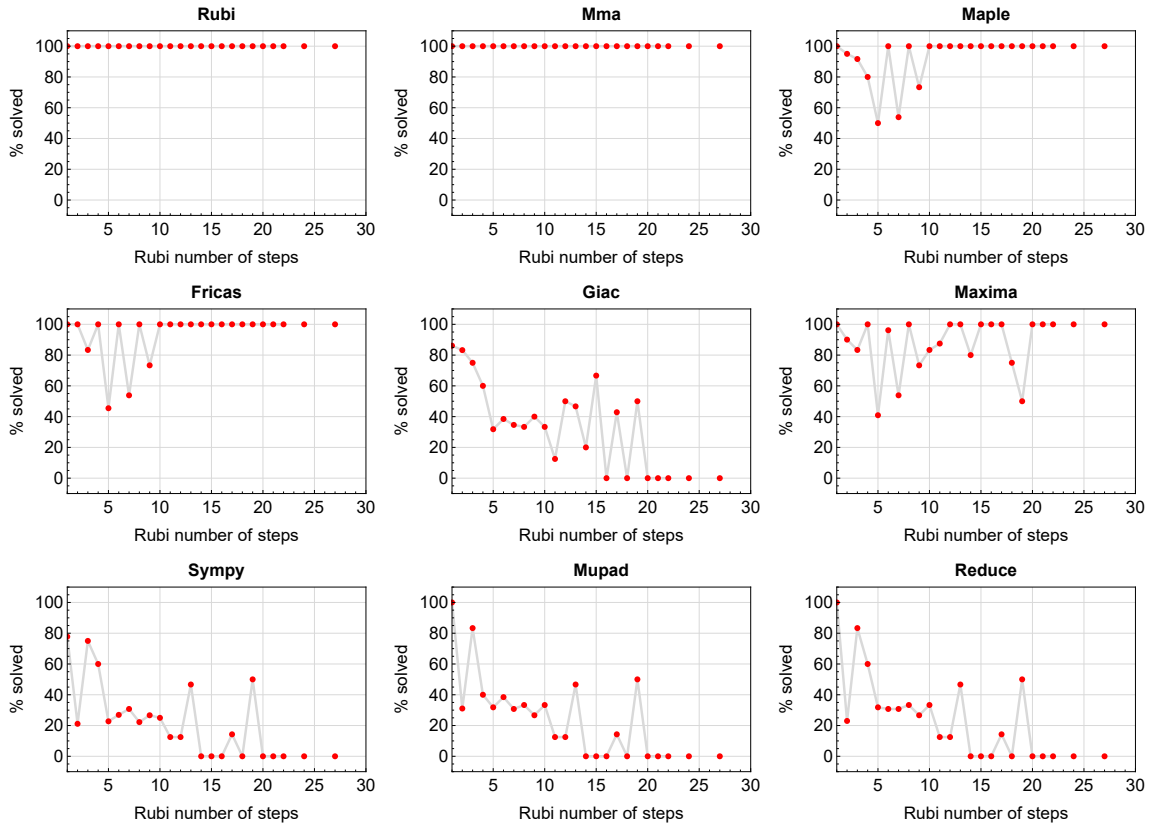


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

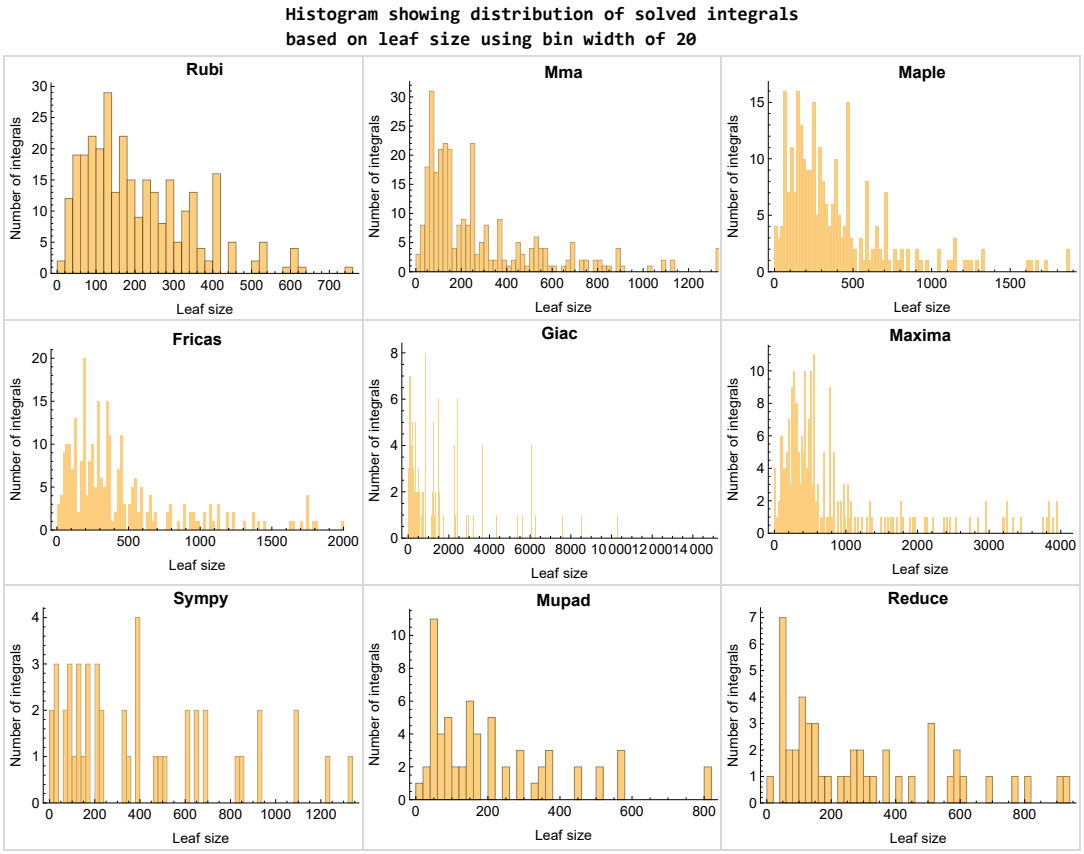


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

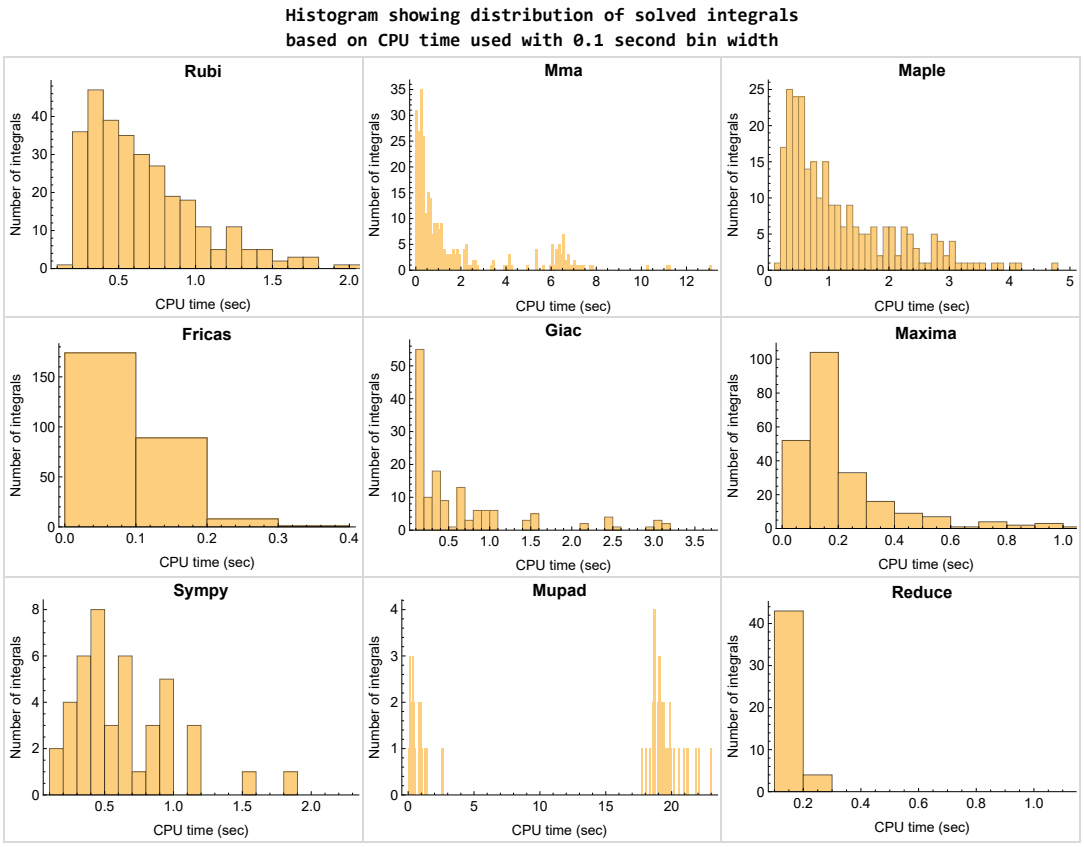


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

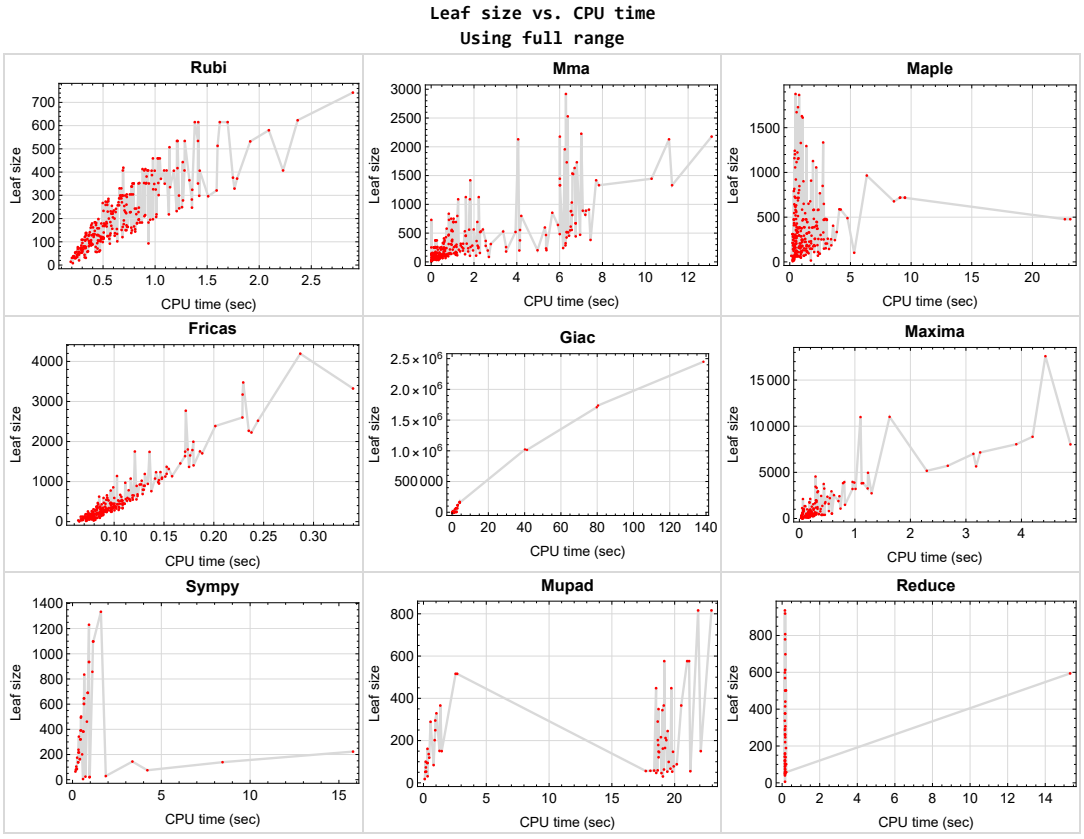


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{31, 36, 37, 38, 43, 44, 45, 50, 51, 97, 102, 103, 104, 109, 110, 111, 116, 117, 163, 168, 169, 170, 175, 176, 177, 182, 183, 208, 213, 214, 215, 219, 220, 221, 225, 226, 227, 232, 233, 234, 238, 239, 240, 244, 245, 246, 251, 252, 253, 257, 258, 259, 263, 264, 265, 270, 271, 272, 276, 277, 278, 282, 283, 284, 288, 289, 290, 295, 296, 297, 301, 302, 303, 307, 308, 309, 314, 315, 316, 320, 321, 322, 326, 327, 379, 380, 381, 386, 387, 388, 392, 393, 394}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {32, 33, 34, 35, 41, 46, 47, 105, 106, 107, 164, 165, 166, 173, 178, 179, 180, 181, 196, 197, 200, 201, 222, 223, 236, 237, 241, 242, 249, 254, 261, 268, 280, 285, 286, 291, 292, 300, 304, 305, 306, 310, 311, 312, 318, 319, 382, 383, 384, 385, 390}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
```

```
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
```

```
  if x is None:
    return 1
  else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

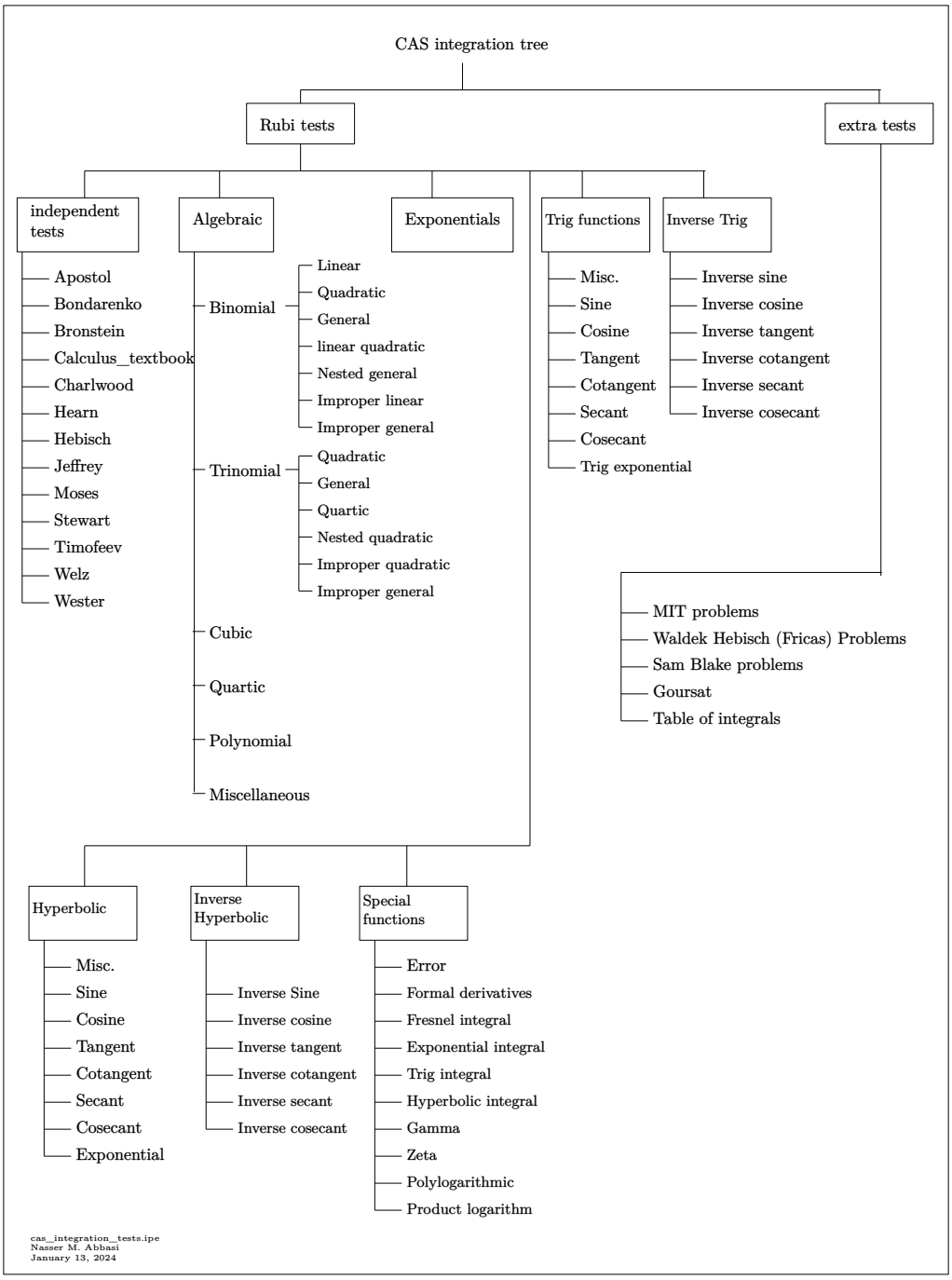
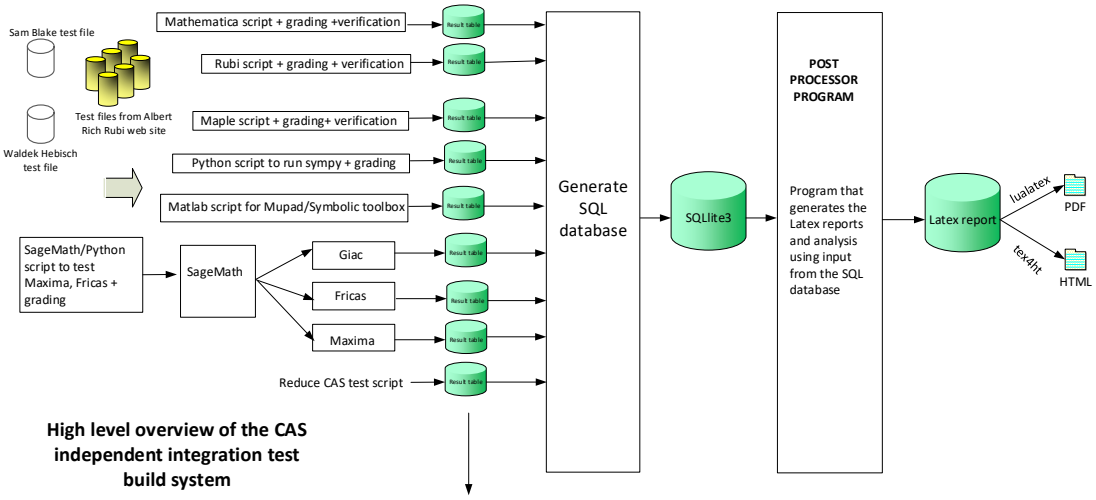


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
January 13, 2024
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

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2.1 List of integrals sorted by grade for each CAS

Rubi 34
Mma 35
Maple 35
Fricas 36
Maxima 37
Giac 37
Mupad 38
Sympy 39
Reduce 39

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 39, 40, 41, 42, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 105, 106, 107, 108, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 171, 172, 173, 174, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 207, 209, 210, 211, 212, 216, 217, 218, 222, 223, 224, 228, 229, 230, 231, 235, 236, 237, 241, 242, 243, 247, 248, 249, 250, 254, 255, 256, 260, 261, 262, 266, 267, 268, 269, 273, 274, 275, 279, 280, 281, 285, 286, 287, 291, 292, 293, 294, 298, 299, 300, 304, 305, 306, 310, 311, 312, 313, 317, 318, 319, 323, 324, 325, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 382, 383, 384, 385, 389, 390, 391, 395, 396, 397 }

B grade { }

C grade { }

F normal fail { 205, 206 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 39, 47, 49, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 113, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 167, 174, 202, 203, 204, 205, 206, 207, 209, 210, 211, 212, 217, 224, 229, 230, 231, 235, 243, 247, 248, 249, 256, 262, 266, 267, 268, 269, 275, 279, 285, 293, 294, 298, 313, 317, 323, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 341, 343, 344, 346, 348, 349, 350, 351, 352, 353, 354, 355, 357, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 391, 395, 396 }

B grade { 32, 33, 34, 35, 40, 41, 42, 46, 98, 105, 106, 107, 112, 114, 115, 164, 165, 166, 171, 172, 173, 178, 179, 180, 181, 216, 218, 222, 223, 228, 236, 241, 242, 250, 254, 255, 260, 261, 273, 274, 280, 281, 286, 287, 291, 292, 299, 300, 304, 305, 306, 310, 311, 312, 318, 324, 325, 382, 383, 384, 385, 389, 390, 397 }

C grade { 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 108, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 237, 319, 331, 340, 342, 345, 347, 356, 358 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 108, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 218, 224, 237, 256, 269, 281, 294, 306, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 374, 375, 385, 391, 395, 396 }

B grade { 32, 33, 34, 35, 39, 40, 41, 42, 46, 47, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 164, 165, 166, 167, 171, 172, 173, 178, 179, 180, 181, 209, 210, 211, 212, 216, 217, 222, 223, 228, 229, 230, 231, 235, 236, 241, 242, 243, 247, 248, 249, 250, 254, 255, 260, 261, 266,

267, 268, 273, 274, 279, 280, 286, 287, 291, 292, 298, 299, 300, 304, 305, 310, 311, 312, 313, 317, 318, 319, 323, 324, 325, 376, 377, 378, 382, 383, 384, 389, 390, 397 }

C grade { 48, 84, 174, 204, 262, 275, 293, 331, 340, 347, 356, 372, 373 }

F normal fail { 1, 13, 22, 70, 79, 88, 136, 145, 154, 285, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 357, 358, 359 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 157, 158, 159, 160, 161, 162, 174, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 256, 262, 293, 294, 306, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 391, 396 }

B grade { 32, 33, 34, 35, 39, 40, 41, 42, 46, 47, 80, 81, 82, 87, 98, 99, 100, 101, 105, 106, 107, 108, 112, 113, 114, 115, 155, 156, 164, 165, 166, 167, 171, 172, 173, 178, 179, 180, 181, 202, 203, 205, 206, 207, 209, 210, 211, 212, 216, 217, 218, 222, 223, 224, 228, 229, 230, 231, 235, 236, 237, 241, 242, 243, 247, 248, 249, 250, 254, 255, 260, 261, 266, 267, 268, 269, 273, 274, 275, 279, 280, 281, 285, 286, 287, 291, 292, 298, 299, 300, 304, 305, 310, 311, 312, 313, 317, 318, 319, 323, 324, 325, 376, 377, 378, 382, 383, 384, 385, 389, 390, 395, 397 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359 }

Maxima

A grade { 5, 17, 26, 74, 92, 140, 149, 158, 224, 360, 361, 362, 363, 364, 368, 369, 370, 371 }

B grade { 2, 3, 4, 14, 15, 16, 23, 24, 25, 32, 33, 34, 35, 39, 40, 41, 42, 46, 47, 48, 49, 71, 72, 73, 80, 81, 82, 83, 89, 90, 91, 98, 99, 100, 101, 105, 106, 107, 108, 112, 113, 114, 115, 137, 138, 139, 146, 147, 148, 155, 156, 157, 164, 165, 166, 167, 171, 172, 173, 174, 178, 179, 180, 181, 205, 206, 207, 209, 210, 211, 212, 216, 217, 222, 223, 228, 229, 230, 231, 235, 236, 241, 242, 243, 247, 248, 250, 254, 255, 256, 260, 262, 266, 267, 268, 269, 273, 274, 275, 279, 280, 281, 285, 286, 287, 291, 292, 293, 294, 298, 299, 304, 305, 306, 310, 311, 312, 313, 317, 318, 323, 324, 325, 376, 377, 382, 383, 384, 391, 396 }

C grade { 6, 7, 8, 9, 10, 11, 12, 18, 19, 20, 21, 27, 28, 29, 30, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 75, 76, 77, 78, 84, 85, 86, 87, 93, 94, 95, 96, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 365, 366, 367, 372, 373, 374, 375 }

F normal fail { 1, 13, 22, 70, 79, 88, 136, 145, 154, 218, 237, 249, 261, 300, 319, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 378, 385, 389, 390, 395, 397 }

F(-1) timedout fail { }

F(-2) exception fail { 202, 203, 204 }

Giac

A grade { 2, 3, 4, 5, 10, 11, 12, 14, 15, 16, 17, 23, 24, 25, 26, 71, 72, 73, 74, 80, 81, 82, 83, 89, 90, 91, 92, 137, 138, 139, 140, 146, 147, 148, 149, 155, 156, 157, 158, 360, 361, 362, 363, 364, 365, 366, 371, 372 }

B grade { 42, 48, 49, 108, 174, 204, 250, 256, 262, 275, 293, 294, 367, 368, 369, 370, 373, 374, 375, 391, 396 }

C grade { 6, 7, 8, 9, 18, 19, 20, 21, 27, 28, 29, 30, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 75, 76, 77, 78, 84, 85, 86, 87, 93, 94, 95, 96, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 159, 160, 161, 162, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201 }

F normal fail { 1, 13, 22, 32, 33, 34, 35, 39, 40, 41, 46, 47, 70, 79, 88, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 136, 145, 154, 164, 165, 166, 167, 171, 172, 173, 178, 179, 180,

181, 202, 203, 205, 206, 207, 209, 210, 211, 212, 216, 217, 218, 222, 223, 224, 228, 229, 230, 231, 235, 237, 241, 242, 243, 247, 248, 249, 254, 255, 260, 261, 266, 267, 269, 273, 274, 279, 280, 281, 285, 286, 287, 291, 292, 298, 299, 300, 304, 305, 306, 310, 311, 312, 313, 317, 318, 319, 323, 324, 325, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 376, 377, 378, 382, 383, 384, 385, 389, 390, 395, 397 }

F(-1) timedout fail { 153, 245, 283, 315, 321, 327 }

F(-2) exception fail { 236, 268, 271, 326 }

Mupad

A grade { }

B grade { 2, 3, 4, 5, 14, 15, 16, 17, 23, 24, 25, 26, 42, 48, 49, 71, 72, 73, 74, 80, 81, 82, 83, 89, 90, 91, 92, 108, 137, 138, 139, 140, 146, 147, 148, 149, 155, 156, 157, 158, 174, 204, 212, 250, 256, 262, 275, 293, 294, 360, 361, 362, 363, 364, 368, 369, 370, 371, 391, 395, 396, 397 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 6, 7, 8, 9, 10, 11, 12, 13, 18, 19, 20, 21, 22, 27, 28, 29, 30, 32, 33, 34, 35, 39, 40, 41, 46, 47, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 75, 76, 77, 78, 79, 84, 85, 86, 87, 88, 93, 94, 95, 96, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 141, 142, 143, 144, 145, 150, 151, 152, 153, 154, 159, 160, 161, 162, 164, 165, 166, 167, 171, 172, 173, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 209, 210, 211, 216, 217, 218, 222, 223, 224, 228, 229, 230, 231, 235, 236, 237, 241, 242, 243, 247, 248, 249, 254, 255, 260, 261, 266, 267, 268, 269, 273, 274, 279, 280, 281, 285, 286, 287, 291, 292, 298, 299, 300, 304, 305, 306, 310, 311, 312, 313, 317, 318, 319, 323, 324, 325, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 365, 366, 367, 372, 373, 374, 375, 376, 377, 378, 382, 383, 384, 385, 389, 390 }

F(-2) exception fail { }

Sympy

A grade { 5, 10, 11, 12, 17, 26, 74, 92, 140, 149, 256, 360, 361, 362, 363, 364 }

B grade { 2, 3, 4, 14, 15, 16, 23, 24, 25, 71, 72, 73, 80, 81, 82, 83, 89, 90, 91, 108, 137, 138, 139, 146, 147, 148, 155, 156, 157, 158, 396 }

C grade { }

F normal fail { 1, 6, 7, 8, 9, 13, 18, 19, 20, 21, 27, 28, 29, 30, 32, 33, 34, 35, 39, 40, 41, 42, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 65, 66, 67, 68, 70, 75, 76, 77, 78, 79, 84, 85, 86, 87, 88, 93, 94, 95, 96, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 119, 120, 121, 122, 125, 126, 127, 128, 131, 132, 133, 134, 136, 141, 142, 143, 144, 145, 150, 151, 152, 153, 154, 159, 160, 161, 164, 165, 166, 167, 171, 172, 173, 174, 178, 179, 180, 181, 185, 186, 187, 188, 191, 192, 193, 194, 198, 199, 202, 203, 204, 205, 206, 207, 209, 210, 211, 212, 216, 217, 218, 222, 223, 224, 228, 229, 230, 231, 235, 236, 237, 242, 243, 247, 248, 249, 250, 254, 255, 260, 261, 262, 267, 268, 269, 274, 275, 281, 286, 287, 291, 292, 293, 294, 298, 299, 300, 304, 305, 306, 312, 313, 319, 330, 331, 332, 340, 341, 342, 343, 345, 346, 347, 348, 349, 350, 356, 357, 358, 359, 365, 366, 367, 371, 372, 395, 397 }

F(-1) timedout fail { 58, 63, 64, 69, 118, 123, 124, 129, 130, 135, 162, 184, 189, 190, 195, 196, 197, 200, 201, 234, 240, 241, 266, 272, 273, 278, 279, 280, 284, 285, 309, 310, 311, 316, 317, 318, 322, 323, 324, 325, 328, 329, 333, 334, 335, 336, 337, 338, 339, 344, 351, 352, 353, 354, 355, 368, 369, 370, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 388, 389, 390, 394 }

F(-2) exception fail { 22, 386, 387, 391, 392, 393 }

Reduce

A grade { }

B grade { 2, 3, 4, 5, 10, 14, 15, 16, 17, 23, 24, 25, 26, 42, 48, 49, 71, 72, 73, 74, 80, 81, 82, 83, 89, 90, 91, 92, 108, 137, 138, 139, 140, 146, 147, 148, 149, 155, 156, 157, 158, 174, 204, 250, 256, 262, 275, 294 }

C grade { }

F normal fail { 1, 6, 7, 8, 9, 11, 12, 13, 18, 19, 20, 21, 22, 27, 28, 29, 30, 32, 33, 34, 35, 39, 40, 41, 46, 47, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 75, 76, 77, 78, 79, 84, 85, 86, 87, 88, 93, 94, 95, 96, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 141, 142, 143, 144, 145, 150, 151, 152, 153, 154, 159, 160, 161, 162, 164, 165, 166, 167, 171,

172, 173, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 209, 210, 211, 212, 216, 217, 218, 222, 223, 224, 228, 229, 230, 231, 235, 236, 237, 241, 242, 243, 247, 248, 249, 254, 255, 260, 261, 266, 267, 268, 269, 273, 274, 279, 280, 281, 285, 286, 287, 291, 292, 293, 298, 299, 300, 304, 305, 306, 310, 311, 312, 313, 317, 318, 319, 323, 324, 325, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 382, 383, 384, 385, 389, 390, 391, 395, 396, 397 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	141	138	0	0	96	0	0	22	0
N.S.	1	1.03	1.01	0.00	0.00	0.70	0.00	0.00	0.16	0.00
time (sec)	N/A	0.372	0.215	0.000	0.000	0.075	0.000	0.000	0.185	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	158	86	109	586	255	502	181	415	245
N.S.	1	1.07	0.58	0.74	3.96	1.72	3.39	1.22	2.80	1.66
time (sec)	N/A	0.446	0.276	0.738	0.063	0.074	0.482	0.119	0.161	19.442

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	128	71	85	342	166	342	121	264	165
N.S.	1	1.07	0.59	0.71	2.85	1.38	2.85	1.01	2.20	1.38
time (sec)	N/A	0.390	0.166	0.576	0.049	0.073	0.358	0.149	0.158	19.232

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	50	64	171	92	175	73	139	100
N.S.	1	1.05	0.62	0.80	2.14	1.15	2.19	0.91	1.74	1.25
time (sec)	N/A	0.292	0.131	0.475	0.041	0.069	0.261	0.113	0.180	0.174

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	54	34	36	65	42	80	38	57	47
N.S.	1	1.08	0.68	0.72	1.30	0.84	1.60	0.76	1.14	0.94
time (sec)	N/A	0.233	0.103	0.369	0.035	0.069	0.188	0.125	0.159	18.532

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	63	60	82	143	64	0	569	22	0
N.S.	1	0.97	0.92	1.26	2.20	0.98	0.00	8.75	0.34	0.00
time (sec)	N/A	0.471	0.065	0.408	0.087	0.067	0.000	0.148	0.159	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	92	80	123	166	105	0	2870	33	0
N.S.	1	1.08	0.94	1.45	1.95	1.24	0.00	33.76	0.39	0.00
time (sec)	N/A	0.566	0.221	0.538	0.104	0.070	0.000	0.385	0.158	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	120	102	160	201	187	0	5398	44	0
N.S.	1	1.05	0.89	1.40	1.76	1.64	0.00	47.35	0.39	0.00
time (sec)	N/A	0.686	0.699	0.729	0.157	0.072	0.000	0.308	0.163	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	154	164	220	251	263	0	7592	55	0
N.S.	1	1.07	1.14	1.53	1.74	1.83	0.00	52.72	0.38	0.00
time (sec)	N/A	0.808	0.384	1.002	0.244	0.078	0.000	0.369	0.203	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	13	6	5	6	6	0
N.S.	1	1.00	1.00	0.88	1.62	0.75	0.62	0.75	0.75	0.00
time (sec)	N/A	0.210	0.004	0.238	0.057	0.065	0.591	0.113	0.155	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	20	16	15	15	17	22	19	10	0
N.S.	1	1.25	1.00	0.94	0.94	1.06	1.38	1.19	0.62	0.00
time (sec)	N/A	0.270	0.005	0.339	0.057	0.065	0.967	0.136	0.176	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	31	29	26	15	30	24	26	10	0
N.S.	1	1.07	1.00	0.90	0.52	1.03	0.83	0.90	0.34	0.00
time (sec)	N/A	0.335	0.006	0.355	0.059	0.064	0.727	0.114	0.167	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	237	0	0	188	0	0	24	0
N.S.	1	1.00	0.86	0.00	0.00	0.68	0.00	0.00	0.09	0.00
time (sec)	N/A	0.551	0.667	0.000	0.000	0.087	0.000	0.000	0.192	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	249	385	173	880	352	646	350	501	448
N.S.	1	1.21	1.88	0.84	4.29	1.72	3.15	1.71	2.44	2.19
time (sec)	N/A	1.259	0.985	1.526	0.072	0.083	0.643	0.135	0.183	18.522

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	164	121	224	499	227	391	231	307	289
N.S.	1	1.09	0.80	1.48	3.30	1.50	2.59	1.53	2.03	1.91
time (sec)	N/A	0.738	0.642	1.221	0.062	0.078	0.452	0.119	0.172	0.561

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	106	93	128	240	130	216	137	156	161
N.S.	1	1.03	0.90	1.24	2.33	1.26	2.10	1.33	1.51	1.56
time (sec)	N/A	0.451	0.389	1.071	0.038	0.071	0.330	0.123	0.152	0.317

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	48	44	56	85	59	85	69	59	59
N.S.	1	0.94	0.86	1.10	1.67	1.16	1.67	1.35	1.16	1.16
time (sec)	N/A	0.264	0.124	0.837	0.038	0.078	0.241	0.125	0.152	18.649

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	102	171	276	122	0	6059	24	0
N.S.	1	1.00	0.84	1.41	2.28	1.01	0.00	50.07	0.20	0.00
time (sec)	N/A	0.479	0.212	0.802	0.101	0.070	0.000	0.331	0.185	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	139	247	305	186	0	67350	35	0
N.S.	1	1.00	0.83	1.47	1.82	1.11	0.00	400.89	0.21	0.00
time (sec)	N/A	0.513	0.947	1.015	0.145	0.076	0.000	2.105	0.154	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	183	316	340	312	0	114422	46	0
N.S.	1	1.00	0.83	1.43	1.54	1.41	0.00	517.75	0.21	0.00
time (sec)	N/A	0.593	1.351	1.407	0.236	0.088	0.000	3.146	0.160	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	298	389	390	450	0	168646	0	0
N.S.	1	1.00	1.10	1.44	1.44	1.67	0.00	624.61	0.00	0.00
time (sec)	N/A	0.681	0.960	2.035	0.397	0.100	0.000	4.107	0.464	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	246	0	0	190	0	0	24	0
N.S.	1	1.00	0.91	0.00	0.00	0.70	0.00	0.00	0.09	0.00
time (sec)	N/A	0.540	0.406	0.000	0.000	0.087	0.000	0.000	0.168	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	326	158	187	967	434	935	361	613	576
N.S.	1	1.29	0.63	0.74	3.84	1.72	3.71	1.43	2.43	2.29
time (sec)	N/A	0.982	1.149	2.003	0.076	0.084	0.929	0.125	0.164	20.998

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	243	135	148	549	283	602	241	376	366
N.S.	1	1.24	0.69	0.76	2.80	1.44	3.07	1.23	1.92	1.87
time (sec)	N/A	0.806	0.591	1.664	0.052	0.081	0.641	0.123	0.163	20.524

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	133	91	111	263	159	320	145	189	202
N.S.	1	1.06	0.73	0.89	2.10	1.27	2.56	1.16	1.51	1.62
time (sec)	N/A	0.408	0.302	1.415	0.039	0.078	0.435	0.126	0.204	19.338

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	80	75	69	92	76	138	75	76	94
N.S.	1	1.11	1.04	0.96	1.28	1.06	1.92	1.04	1.06	1.31
time (sec)	N/A	0.314	0.077	1.144	0.037	0.069	0.313	0.125	0.155	0.286

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	110	178	281	126	0	6046	24	0
N.S.	1	1.00	0.85	1.38	2.18	0.98	0.00	46.87	0.19	0.00
time (sec)	N/A	0.446	0.219	1.030	0.106	0.068	0.000	0.343	0.153	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	151	256	308	194	0	63510	35	0
N.S.	1	1.00	0.84	1.43	1.72	1.08	0.00	354.80	0.20	0.00
time (sec)	N/A	0.510	0.844	1.253	0.184	0.079	0.000	2.443	0.158	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	199	329	343	334	0	111694	46	0
N.S.	1	1.00	0.87	1.44	1.50	1.46	0.00	487.75	0.20	0.00
time (sec)	N/A	0.592	1.956	1.550	0.272	0.092	0.000	3.051	0.164	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	316	404	393	474	0	157526	0	0
N.S.	1	1.00	1.10	1.41	1.37	1.65	0.00	548.87	0.00	0.00
time (sec)	N/A	0.677	1.381	2.171	0.459	0.097	0.000	4.187	0.364	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	14	16	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	1.00	1.14	1.14	1.14
time (sec)	N/A	0.195	1.695	0.082	0.201	0.064	0.683	0.127	0.160	19.376

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	215	799	1159	1281	776	0	0	112	0
N.S.	1	1.42	5.29	7.68	8.48	5.14	0.00	0.00	0.74	0.00
time (sec)	N/A	0.904	4.208	0.383	0.224	0.097	0.000	0.000	0.175	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	171	560	792	759	548	0	0	91	0
N.S.	1	1.35	4.41	6.24	5.98	4.31	0.00	0.00	0.72	0.00
time (sec)	N/A	0.725	1.774	0.334	0.172	0.094	0.000	0.000	0.154	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	125	356	477	411	350	0	0	70	0
N.S.	1	1.34	3.83	5.13	4.42	3.76	0.00	0.00	0.75	0.00
time (sec)	N/A	0.549	0.977	0.328	0.126	0.086	0.000	0.000	0.184	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	81	180	204	190	196	0	0	47	0
N.S.	1	1.25	2.77	3.14	2.92	3.02	0.00	0.00	0.72	0.00
time (sec)	N/A	0.363	3.500	0.306	0.117	0.081	0.000	0.000	0.160	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	12	16	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	0.86	1.14	1.14	1.14
time (sec)	N/A	0.210	2.943	0.125	0.216	0.060	0.385	0.124	0.156	19.175

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	27	14	16	27	16
N.S.	1	1.00	1.14	1.00	1.14	1.93	1.00	1.14	1.93	1.14
time (sec)	N/A	0.211	5.493	0.132	0.484	0.060	0.558	1.718	0.152	19.640

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	20	22	22	24
N.S.	1	1.00	1.10	1.00	1.10	1.10	1.00	1.10	1.10	1.20
time (sec)	N/A	0.333	1.775	0.087	0.473	0.066	2.589	0.153	0.153	19.324

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	228	308	716	2948	1021	0	0	377	0
N.S.	1	1.10	1.48	3.44	14.17	4.91	0.00	0.00	1.81	0.00
time (sec)	N/A	0.841	0.928	0.460	0.337	0.128	0.000	0.000	0.225	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	156	311	433	1770	669	0	0	350	0
N.S.	1	1.07	2.13	2.97	12.12	4.58	0.00	0.00	2.40	0.00
time (sec)	N/A	0.589	0.818	0.352	0.180	0.106	0.000	0.000	0.168	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	92	234	212	553	375	0	0	210	0
N.S.	1	1.02	2.60	2.36	6.14	4.17	0.00	0.00	2.33	0.00
time (sec)	N/A	0.369	1.504	0.220	0.156	0.091	0.000	0.000	0.158	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	131	68	259	62	0	697	40	88
N.S.	1	1.00	4.37	2.27	8.63	2.07	0.00	23.23	1.33	2.93
time (sec)	N/A	0.239	0.061	0.125	0.040	0.077	0.000	0.348	0.152	20.173

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	508	22	19	22	22	24
N.S.	1	1.00	1.10	1.00	25.40	1.10	0.95	1.10	1.10	1.20
time (sec)	N/A	0.271	11.257	0.149	0.356	0.070	0.592	0.593	0.150	19.467

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	745	33	20	22	33	24
N.S.	1	1.00	1.10	1.00	37.25	1.65	1.00	1.10	1.65	1.20
time (sec)	N/A	0.287	14.463	0.150	1.021	0.075	1.188	2.667	0.172	18.293

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	22	24	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	1.00	1.09	1.09	1.09
time (sec)	N/A	0.341	4.451	0.095	0.459	0.071	8.364	0.135	0.164	18.384

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	179	512	716	4549	1075	0	0	532	0
N.S.	1	1.31	3.74	5.23	33.20	7.85	0.00	0.00	3.88	0.00
time (sec)	N/A	0.826	6.403	0.627	0.291	0.117	0.000	0.000	0.149	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	137	202	409	1044	591	0	0	298	0
N.S.	1	1.19	1.76	3.56	9.08	5.14	0.00	0.00	2.59	0.00
time (sec)	N/A	0.615	4.967	0.506	0.327	0.100	0.000	0.000	0.144	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	59	94	148	1129	102	0	2978	142	147
N.S.	1	1.09	1.74	2.74	20.91	1.89	0.00	55.15	2.63	2.72
time (sec)	N/A	0.356	0.666	0.328	0.050	0.082	0.000	1.025	0.138	19.870

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	48	61	287	44	0	526	49	53
N.S.	1	1.00	1.37	1.74	8.20	1.26	0.00	15.03	1.40	1.51
time (sec)	N/A	0.252	0.196	0.247	0.036	0.068	0.000	0.201	0.140	19.416

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	1664	24	20	24	24	24
N.S.	1	1.00	1.09	1.00	75.64	1.09	0.91	1.09	1.09	1.09
time (sec)	N/A	0.266	8.474	0.152	0.720	0.074	0.646	0.155	0.174	18.722

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	2153	35	22	24	35	24
N.S.	1	1.00	1.09	1.00	97.86	1.59	1.00	1.09	1.59	1.09
time (sec)	N/A	0.292	8.014	0.155	2.248	0.076	1.296	2.321	0.153	19.413

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	218	144	234	277	222	0	1192	80	0
N.S.	1	1.11	0.73	1.19	1.41	1.13	0.00	6.08	0.41	0.00
time (sec)	N/A	1.132	0.418	0.517	0.138	0.088	0.000	0.433	0.191	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	183	128	187	256	167	0	732	48	0
N.S.	1	1.09	0.76	1.11	1.52	0.99	0.00	4.36	0.29	0.00
time (sec)	N/A	0.874	0.374	0.387	0.146	0.088	0.000	0.306	0.178	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	150	145	142	209	125	0	406	21	0
N.S.	1	1.06	1.02	1.00	1.47	0.88	0.00	2.86	0.15	0.00
time (sec)	N/A	0.716	0.251	0.382	0.137	0.080	0.000	0.222	0.179	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	150	145	142	209	125	0	406	21	0
N.S.	1	1.06	1.02	1.00	1.47	0.88	0.00	2.86	0.15	0.00
time (sec)	N/A	0.725	0.005	0.265	0.141	0.079	0.000	0.220	0.189	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	183	128	187	256	167	0	732	48	0
N.S.	1	1.09	0.76	1.11	1.52	0.99	0.00	4.36	0.29	0.00
time (sec)	N/A	0.855	0.010	0.293	0.154	0.082	0.000	0.335	0.181	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	218	144	234	277	222	0	1192	80	0
N.S.	1	1.11	0.73	1.19	1.41	1.13	0.00	6.08	0.41	0.00
time (sec)	N/A	1.013	0.060	0.440	0.140	0.084	0.000	0.419	0.202	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	1418	474	547	370	0	2453	86	0
N.S.	1	1.00	3.49	1.17	1.35	0.91	0.00	6.04	0.21	0.00
time (sec)	N/A	1.429	7.705	1.336	0.165	0.099	0.000	0.817	0.202	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	730	386	497	298	0	1520	52	0
N.S.	1	1.00	2.07	1.09	1.41	0.84	0.00	4.31	0.15	0.00
time (sec)	N/A	1.031	0.831	0.803	0.163	0.092	0.000	0.674	0.179	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	251	294	424	245	0	848	23	0
N.S.	1	1.00	0.83	0.97	1.39	0.81	0.00	2.79	0.08	0.00
time (sec)	N/A	0.788	0.152	0.750	0.153	0.092	0.000	0.378	0.164	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	251	294	424	245	0	848	23	0
N.S.	1	1.00	0.83	0.97	1.39	0.81	0.00	2.79	0.08	0.00
time (sec)	N/A	0.772	0.005	0.440	0.152	0.088	0.000	0.387	0.153	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	730	386	497	298	0	1520	52	0
N.S.	1	1.00	2.07	1.09	1.41	0.84	0.00	4.31	0.15	0.00
time (sec)	N/A	0.894	0.007	0.560	0.154	0.089	0.000	0.675	0.182	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	1418	474	547	370	0	2453	86	0
N.S.	1	1.00	3.49	1.17	1.35	0.91	0.00	6.04	0.21	0.00
time (sec)	N/A	0.996	1.827	1.113	0.167	0.101	0.000	0.811	0.202	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	1332	470	551	406	0	2412	86	0
N.S.	1	1.00	3.27	1.15	1.35	1.00	0.00	5.93	0.21	0.00
time (sec)	N/A	1.282	11.251	3.056	0.165	0.097	0.000	0.986	0.218	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	693	376	503	316	0	1490	52	0
N.S.	1	1.00	1.97	1.07	1.43	0.90	0.00	4.25	0.15	0.00
time (sec)	N/A	0.940	1.156	0.970	0.165	0.094	0.000	0.676	0.219	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	252	286	429	244	0	826	23	0
N.S.	1	1.00	0.84	0.96	1.43	0.82	0.00	2.76	0.08	0.00
time (sec)	N/A	0.712	0.248	0.909	0.147	0.088	0.000	0.457	0.176	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	252	286	429	244	0	826	23	0
N.S.	1	1.00	0.84	0.96	1.43	0.82	0.00	2.76	0.08	0.00
time (sec)	N/A	0.670	0.135	0.551	0.143	0.089	0.000	0.449	0.167	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	693	376	503	316	0	1490	52	0
N.S.	1	1.00	1.97	1.07	1.43	0.90	0.00	4.25	0.15	0.00
time (sec)	N/A	0.795	1.027	0.764	0.155	0.094	0.000	0.709	0.194	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	1332	470	551	406	0	2412	86	0
N.S.	1	1.00	3.27	1.15	1.35	1.00	0.00	5.93	0.21	0.00
time (sec)	N/A	0.926	6.016	2.793	0.159	0.100	0.000	0.976	0.250	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	250	0	0	186	0	0	24	0
N.S.	1	1.00	0.94	0.00	0.00	0.70	0.00	0.00	0.09	0.00
time (sec)	N/A	0.497	0.476	0.000	0.000	0.083	0.000	0.000	0.197	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	247	150	181	889	294	646	350	594	448
N.S.	1	1.20	0.73	0.88	4.34	1.43	3.15	1.71	2.90	2.19
time (sec)	N/A	1.354	1.072	1.613	0.079	0.083	0.648	0.138	15.322	19.734

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	165	127	224	505	183	391	231	377	290
N.S.	1	1.09	0.84	1.48	3.34	1.21	2.59	1.53	2.50	1.92
time (sec)	N/A	0.731	0.530	1.315	0.054	0.075	0.483	0.135	0.153	18.651

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	105	86	128	243	100	216	137	221	145
N.S.	1	1.02	0.83	1.24	2.36	0.97	2.10	1.33	2.15	1.41
time (sec)	N/A	0.469	0.298	1.271	0.044	0.073	0.363	0.120	0.161	18.763

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	50	71	64	86	46	85	69	92	58
N.S.	1	0.98	1.39	1.25	1.69	0.90	1.67	1.35	1.80	1.14
time (sec)	N/A	0.263	0.087	0.882	0.033	0.068	0.244	0.123	0.163	18.365

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	100	172	275	120	0	6279	24	0
N.S.	1	1.00	0.83	1.42	2.27	0.99	0.00	51.89	0.20	0.00
time (sec)	N/A	0.410	0.194	0.777	0.108	0.070	0.000	0.339	0.155	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	139	245	302	182	0	66726	35	0
N.S.	1	1.00	0.83	1.46	1.80	1.08	0.00	397.18	0.21	0.00
time (sec)	N/A	0.499	0.758	1.062	0.157	0.078	0.000	2.111	0.209	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	181	318	337	306	0	118262	46	0
N.S.	1	1.00	0.82	1.44	1.52	1.38	0.00	535.12	0.21	0.00
time (sec)	N/A	0.579	1.727	1.391	0.264	0.086	0.000	3.160	0.161	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	300	386	387	444	0	166374	0	0
N.S.	1	1.00	1.11	1.43	1.43	1.64	0.00	616.20	0.00	0.00
time (sec)	N/A	0.649	1.185	1.908	0.406	0.091	0.000	4.248	0.273	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	155	0	0	136	0	0	26	0
N.S.	1	1.00	0.96	0.00	0.00	0.84	0.00	0.00	0.16	0.00
time (sec)	N/A	0.404	2.276	0.000	0.000	0.085	0.000	0.000	0.161	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	132	146	735	466	1231	224	779	349
N.S.	1	1.00	1.01	1.11	5.61	3.56	9.40	1.71	5.95	2.66
time (sec)	N/A	0.368	0.770	1.832	0.067	0.084	0.927	0.129	0.158	18.714

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	106	121	442	308	835	153	502	329
N.S.	1	1.00	1.01	1.15	4.21	2.93	7.95	1.46	4.78	3.13
time (sec)	N/A	0.329	0.384	1.418	0.048	0.079	0.657	0.137	0.205	1.028

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	77	79	232	180	493	94	287	179
N.S.	1	1.00	0.97	1.00	2.94	2.28	6.24	1.19	3.63	2.27
time (sec)	N/A	0.317	0.249	1.308	0.039	0.078	0.471	0.128	0.152	18.693

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	46	96	85	238	48	120	57
N.S.	1	1.00	1.02	0.87	1.81	1.60	4.49	0.91	2.26	1.08
time (sec)	N/A	0.233	0.365	0.918	0.033	0.072	0.335	0.122	0.152	18.068

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	65	107	162	73	0	669	26	0
N.S.	1	1.00	0.83	1.37	2.08	0.94	0.00	8.58	0.33	0.00
time (sec)	N/A	0.333	0.106	0.972	0.078	0.073	0.000	0.152	0.160	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	81	151	171	112	0	3218	37	0
N.S.	1	1.00	0.78	1.45	1.64	1.08	0.00	30.94	0.36	0.00
time (sec)	N/A	0.364	0.296	1.176	0.103	0.077	0.000	0.553	0.160	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	105	190	206	213	0	5600	48	0
N.S.	1	1.00	0.83	1.50	1.62	1.68	0.00	44.09	0.38	0.00
time (sec)	N/A	0.411	0.597	1.398	0.152	0.083	0.000	0.302	0.169	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	B	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	123	231	258	349	0	8508	59	0
N.S.	1	1.00	0.78	1.46	1.63	2.21	0.00	53.85	0.37	0.00
time (sec)	N/A	0.460	1.148	1.750	0.242	0.089	0.000	0.386	0.208	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	376	0	0	280	0	0	26	0
N.S.	1	1.00	0.92	0.00	0.00	0.69	0.00	0.00	0.06	0.00
time (sec)	N/A	0.690	0.628	0.000	0.000	0.093	0.000	0.000	0.153	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	238	259	1339	471	1098	531	936	816
N.S.	1	1.00	0.72	0.78	4.06	1.43	3.33	1.61	2.84	2.47
time (sec)	N/A	0.659	2.202	3.131	0.094	0.090	1.171	0.160	0.150	22.932

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	369	205	766	296	690	351	599	516
N.S.	1	1.00	1.42	0.79	2.96	1.14	2.66	1.36	2.31	1.99
time (sec)	N/A	0.541	0.966	2.954	0.069	0.081	0.850	0.144	0.147	2.681

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	127	152	375	166	382	209	337	249
N.S.	1	1.00	0.69	0.83	2.04	0.90	2.08	1.14	1.83	1.35
time (sec)	N/A	0.421	0.520	2.965	0.047	0.077	0.595	0.142	0.145	0.937

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	94	92	139	76	163	106	140	99
N.S.	1	1.00	0.86	0.84	1.28	0.70	1.50	0.97	1.28	0.91
time (sec)	N/A	0.305	0.178	2.476	0.034	0.072	0.409	0.123	0.159	19.528

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	154	258	414	182	0	46675	26	0
N.S.	1	1.00	0.83	1.39	2.24	0.98	0.00	252.30	0.14	0.00
time (sec)	N/A	0.570	0.271	2.084	0.134	0.071	0.000	1.570	0.156	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	213	370	445	272	0	1014406	37	0
N.S.	1	1.00	0.83	1.44	1.73	1.06	0.00	3947.11	0.14	0.00
time (sec)	N/A	0.654	1.007	2.247	0.210	0.090	0.000	41.303	0.189	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	279	480	480	456	0	1737414	48	0
N.S.	1	1.00	0.83	1.42	1.42	1.35	0.00	5140.28	0.14	0.00
time (sec)	N/A	0.777	2.548	2.896	0.350	0.109	0.000	80.473	0.190	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	457	585	530	653	0	2449286	0	0
N.S.	1	1.00	1.11	1.42	1.28	1.58	0.00	5930.47	0.00	0.00
time (sec)	N/A	0.907	1.617	4.197	0.585	0.120	0.000	138.554	0.376	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	20	22	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	1.00	1.10	1.10	1.10
time (sec)	N/A	0.474	9.837	0.110	0.481	0.068	7.024	0.141	0.160	19.136

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	376	837	1295	1548	1367	0	0	132	0
N.S.	1	1.13	2.51	3.89	4.65	4.11	0.00	0.00	0.40	0.00
time (sec)	N/A	1.748	0.824	1.365	0.250	0.153	0.000	0.000	0.182	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	278	330	847	929	925	0	0	105	0
N.S.	1	1.09	1.30	3.33	3.66	3.64	0.00	0.00	0.41	0.00
time (sec)	N/A	1.268	0.546	0.793	0.181	0.126	0.000	0.000	0.219	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	181	221	479	513	562	0	0	78	0
N.S.	1	1.06	1.29	2.80	3.00	3.29	0.00	0.00	0.46	0.00
time (sec)	N/A	0.827	0.528	0.520	0.121	0.112	0.000	0.000	0.168	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	176	177	200	277	0	0	47	0
N.S.	1	1.00	1.87	1.88	2.13	2.95	0.00	0.00	0.50	0.00
time (sec)	N/A	0.444	0.201	0.406	0.111	0.096	0.000	0.000	0.177	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	228	22	19	22	22	22
N.S.	1	1.00	1.10	1.00	11.40	1.10	0.95	1.10	1.10	1.10
time (sec)	N/A	0.535	6.218	0.308	0.271	0.075	0.474	0.329	0.153	19.152

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	340	33	20	22	33	22
N.S.	1	1.00	1.10	1.00	17.00	1.65	1.00	1.10	1.65	1.10
time (sec)	N/A	0.671	3.002	0.298	0.601	0.073	0.710	2.456	0.153	19.559

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12	1.12
time (sec)	N/A	0.201	0.831	0.104	0.399	0.068	1.440	0.175	0.171	18.594

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	210	598	921	3242	856	0	0	677	0
N.S.	1	1.35	3.86	5.94	20.92	5.52	0.00	0.00	4.37	0.00
time (sec)	N/A	0.897	5.311	0.498	0.448	0.099	0.000	0.000	0.206	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	164	374	581	1953	599	0	0	502	0
N.S.	1	1.29	2.94	4.57	15.38	4.72	0.00	0.00	3.95	0.00
time (sec)	N/A	0.689	4.147	0.470	0.303	0.093	0.000	0.000	0.167	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	120	198	305	645	384	0	0	331	0
N.S.	1	1.24	2.04	3.14	6.65	3.96	0.00	0.00	3.41	0.00
time (sec)	N/A	0.498	4.118	0.445	0.220	0.083	0.000	0.000	0.175	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	45	82	60	292	97	104	1151	105	67
N.S.	1	1.05	1.91	1.40	6.79	2.26	2.42	26.77	2.44	1.56
time (sec)	N/A	0.270	0.380	0.385	0.117	0.073	0.209	0.349	0.155	19.665

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	581	18	14	18	18	18
N.S.	1	1.00	1.12	1.00	36.31	1.12	0.88	1.12	1.12	1.12
time (sec)	N/A	0.205	4.820	0.246	0.339	0.062	0.399	0.155	0.156	19.079

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	813	29	15	18	29	18
N.S.	1	1.00	1.12	1.00	50.81	1.81	0.94	1.12	1.81	1.12
time (sec)	N/A	0.212	1.635	0.251	0.973	0.063	0.637	2.248	0.157	18.955

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	22	24	24	26
N.S.	1	1.00	1.09	1.00	1.09	1.09	1.00	1.09	1.09	1.18
time (sec)	N/A	0.309	24.449	0.117	0.962	0.071	5.664	0.158	0.196	18.899

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	742	854	1673	7004	2770	0	0	187	0
N.S.	1	1.78	2.05	4.02	16.84	6.66	0.00	0.00	0.45	0.00
time (sec)	N/A	2.900	5.656	0.562	3.134	0.172	0.000	0.000	0.200	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	532	528	1056	3887	1742	0	0	150	0
N.S.	1	1.73	1.71	3.43	12.62	5.66	0.00	0.00	0.49	0.00
time (sec)	N/A	1.914	3.362	0.443	0.991	0.136	0.000	0.000	0.182	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	324	471	546	1938	970	0	0	113	0
N.S.	1	1.81	2.63	3.05	10.83	5.42	0.00	0.00	0.63	0.00
time (sec)	N/A	1.357	6.957	0.361	0.309	0.111	0.000	0.000	0.173	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	176	260	246	762	454	0	0	74	0
N.S.	1	1.63	2.41	2.28	7.06	4.20	0.00	0.00	0.69	0.00
time (sec)	N/A	0.699	1.183	0.273	0.169	0.096	0.000	0.000	0.188	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	1793	24	20	24	24	26
N.S.	1	1.00	1.09	1.00	81.50	1.09	0.91	1.09	1.09	1.18
time (sec)	N/A	0.301	29.933	0.174	2.544	0.082	1.047	4.681	0.163	18.747

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	2289	35	22	24	35	26
N.S.	1	1.00	1.09	1.00	104.05	1.59	1.00	1.09	1.59	1.18
time (sec)	N/A	0.308	34.167	0.181	8.296	0.086	4.069	16.943	0.201	18.426

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	1445	476	547	341	0	2445	86	0
N.S.	1	1.00	3.56	1.17	1.35	0.84	0.00	6.02	0.21	0.00
time (sec)	N/A	1.180	10.293	1.357	0.164	0.098	0.000	0.831	0.244	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	751	384	499	280	0	1515	52	0
N.S.	1	1.00	2.13	1.09	1.41	0.79	0.00	4.29	0.15	0.00
time (sec)	N/A	0.916	1.047	0.819	0.157	0.094	0.000	0.712	0.206	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	251	296	424	235	0	844	23	0
N.S.	1	1.00	0.83	0.97	1.39	0.77	0.00	2.78	0.08	0.00
time (sec)	N/A	0.757	0.153	0.782	0.158	0.086	0.000	0.392	0.170	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	251	296	424	235	0	844	23	0
N.S.	1	1.00	0.83	0.97	1.39	0.77	0.00	2.78	0.08	0.00
time (sec)	N/A	0.728	0.144	0.456	0.160	0.086	0.000	0.379	0.204	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	751	384	499	280	0	1515	52	0
N.S.	1	1.00	2.13	1.09	1.41	0.79	0.00	4.29	0.15	0.00
time (sec)	N/A	0.849	0.935	0.661	0.164	0.090	0.000	0.684	0.258	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	1445	476	547	341	0	2445	86	0
N.S.	1	1.00	3.56	1.17	1.35	0.84	0.00	6.02	0.21	0.00
time (sec)	N/A	0.966	6.009	1.109	0.170	0.095	0.000	0.834	0.229	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	140	251	285	347	0	1358	92	0
N.S.	1	1.00	0.61	1.10	1.25	1.52	0.00	5.96	0.40	0.00
time (sec)	N/A	0.715	0.873	2.796	0.138	0.098	0.000	0.888	0.228	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	140	206	264	249	0	833	56	0
N.S.	1	1.00	0.70	1.03	1.32	1.24	0.00	4.16	0.28	0.00
time (sec)	N/A	0.610	0.623	0.812	0.135	0.090	0.000	0.624	0.214	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	120	159	219	175	0	460	25	0
N.S.	1	1.00	0.69	0.91	1.26	1.01	0.00	2.64	0.14	0.00
time (sec)	N/A	0.521	0.554	0.741	0.149	0.089	0.000	0.417	0.167	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	120	159	219	175	0	460	25	0
N.S.	1	1.00	0.69	0.91	1.26	1.01	0.00	2.64	0.14	0.00
time (sec)	N/A	0.517	0.012	0.481	0.135	0.088	0.000	0.403	0.167	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	140	206	264	249	0	833	56	0
N.S.	1	1.00	0.70	1.03	1.32	1.24	0.00	4.16	0.28	0.00
time (sec)	N/A	0.556	0.031	0.553	0.141	0.098	0.000	0.637	0.238	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	140	251	285	347	0	1358	92	0
N.S.	1	1.00	0.61	1.10	1.25	1.52	0.00	5.96	0.40	0.00
time (sec)	N/A	0.621	0.089	2.608	0.145	0.097	0.000	0.965	0.255	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	615	2177	719	826	521	0	3673	92	0
N.S.	1	1.00	3.54	1.17	1.34	0.85	0.00	5.97	0.15	0.00
time (sec)	N/A	1.621	13.098	9.487	0.196	0.113	0.000	1.583	0.271	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	1126	580	760	427	0	2279	56	0
N.S.	1	1.00	2.11	1.09	1.42	0.80	0.00	4.27	0.10	0.00
time (sec)	N/A	1.285	2.223	2.312	0.189	0.108	0.000	1.039	0.226	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	372	447	680	356	0	1270	25	0
N.S.	1	1.00	0.81	0.97	1.48	0.78	0.00	2.77	0.05	0.00
time (sec)	N/A	1.031	0.219	2.203	0.171	0.101	0.000	0.629	0.192	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	372	447	680	356	0	1270	25	0
N.S.	1	1.00	0.81	0.97	1.48	0.78	0.00	2.77	0.05	0.00
time (sec)	N/A	0.979	0.202	1.431	0.174	0.100	0.000	0.648	0.171	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	1126	580	760	427	0	2279	56	0
N.S.	1	1.00	2.11	1.09	1.42	0.80	0.00	4.27	0.10	0.00
time (sec)	N/A	1.210	1.619	1.940	0.182	0.109	0.000	1.036	0.261	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	615	2177	719	826	521	0	3673	92	0
N.S.	1	1.00	3.54	1.17	1.34	0.85	0.00	5.97	0.15	0.00
time (sec)	N/A	1.381	6.011	9.107	0.193	0.116	0.000	1.548	0.272	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	245	0	0	188	0	0	24	0
N.S.	1	1.00	0.90	0.00	0.00	0.69	0.00	0.00	0.09	0.00
time (sec)	N/A	0.525	0.176	0.000	0.000	0.085	0.000	0.000	0.173	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	325	158	187	967	378	935	361	698	576
N.S.	1	1.29	0.63	0.74	3.84	1.50	3.71	1.43	2.77	2.29
time (sec)	N/A	1.006	1.194	2.280	0.079	0.086	0.927	0.144	0.174	19.172

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	243	135	148	549	238	602	241	441	366
N.S.	1	1.24	0.69	0.76	2.80	1.21	3.07	1.23	2.25	1.87
time (sec)	N/A	0.818	0.591	1.741	0.053	0.079	0.645	0.138	0.170	19.134

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	133	89	109	263	130	320	145	247	202
N.S.	1	1.06	0.71	0.87	2.10	1.04	2.56	1.16	1.98	1.62
time (sec)	N/A	0.419	0.308	1.457	0.042	0.074	0.457	0.131	0.167	18.663

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	80	75	68	92	58	138	75	101	94
N.S.	1	1.11	1.04	0.94	1.28	0.81	1.92	1.04	1.40	1.31
time (sec)	N/A	0.327	0.090	1.172	0.038	0.069	0.319	0.147	0.215	0.329

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	110	178	281	124	0	6046	24	0
N.S.	1	1.00	0.85	1.38	2.18	0.96	0.00	46.87	0.19	0.00
time (sec)	N/A	0.442	0.159	1.030	0.113	0.067	0.000	0.352	0.170	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	151	256	308	183	0	63510	35	0
N.S.	1	1.00	0.84	1.43	1.72	1.02	0.00	354.80	0.20	0.00
time (sec)	N/A	0.483	1.115	1.269	0.157	0.082	0.000	2.555	0.170	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	197	329	343	309	0	111694	46	0
N.S.	1	1.00	0.85	1.42	1.48	1.34	0.00	483.52	0.20	0.00
time (sec)	N/A	0.574	2.645	1.602	0.256	0.088	0.000	3.039	0.192	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	316	404	393	455	0	157526	0	0
N.S.	1	1.00	1.10	1.41	1.37	1.59	0.00	548.87	0.00	0.00
time (sec)	N/A	0.655	1.493	2.237	0.430	0.097	0.000	4.179	0.351	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	409	0	0	280	0	0	26	0
N.S.	1	1.00	0.98	0.00	0.00	0.67	0.00	0.00	0.06	0.00
time (sec)	N/A	0.695	0.598	0.000	0.000	0.090	0.000	0.000	0.196	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	563	253	1339	527	1098	531	807	816
N.S.	1	1.00	1.71	0.77	4.06	1.60	3.33	1.61	2.45	2.47
time (sec)	N/A	0.675	2.415	3.398	0.107	0.094	1.152	0.165	0.164	21.869

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	195	205	766	342	690	351	501	516
N.S.	1	1.00	0.75	0.79	2.96	1.32	2.66	1.36	1.93	1.99
time (sec)	N/A	0.525	1.481	3.069	0.076	0.083	0.862	0.140	0.161	2.559

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	252	144	375	193	382	209	260	295
N.S.	1	1.00	1.37	0.78	2.04	1.05	2.08	1.14	1.41	1.60
time (sec)	N/A	0.423	0.597	3.233	0.051	0.075	0.615	0.154	0.160	0.938

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	110	91	139	91	163	106	102	119
N.S.	1	1.00	1.01	0.83	1.28	0.83	1.50	0.97	0.94	1.09
time (sec)	N/A	0.295	0.232	2.430	0.041	0.072	0.415	0.145	0.158	0.458

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	154	257	413	182	0	44961	26	0
N.S.	1	1.00	0.83	1.39	2.23	0.98	0.00	243.03	0.14	0.00
time (sec)	N/A	0.532	0.262	2.053	0.151	0.074	0.000	1.499	0.156	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	212	372	446	264	0	1022022	37	0
N.S.	1	1.00	0.82	1.45	1.74	1.03	0.00	3976.74	0.14	0.00
time (sec)	N/A	0.641	1.290	2.288	0.227	0.092	0.000	40.171	0.160	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	283	478	481	438	0	1708998	48	0
N.S.	1	1.00	0.84	1.41	1.42	1.30	0.00	5056.21	0.14	0.00
time (sec)	N/A	0.773	1.920	3.008	0.349	0.099	0.000	79.752	0.203	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	451	588	531	640	0	0	59	0
N.S.	1	1.00	1.09	1.42	1.29	1.55	0.00	0.00	0.14	0.00
time (sec)	N/A	0.879	1.612	4.087	0.587	0.124	0.000	0.000	0.172	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	255	0	0	190	0	0	26	0
N.S.	1	1.00	0.89	0.00	0.00	0.67	0.00	0.00	0.09	0.00
time (sec)	N/A	0.552	1.442	0.000	0.000	0.089	0.000	0.000	0.157	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	153	185	1033	546	1334	359	919	576
N.S.	1	1.00	0.66	0.79	4.43	2.34	5.73	1.54	3.94	2.47
time (sec)	N/A	0.504	0.892	3.424	0.076	0.092	1.597	0.196	0.158	21.147

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	132	148	602	349	857	241	569	366
N.S.	1	1.00	0.73	0.82	3.33	1.93	4.73	1.33	3.14	2.02
time (sec)	N/A	0.434	1.582	2.818	0.060	0.083	1.118	0.185	0.155	1.339

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	91	111	303	194	461	145	293	202
N.S.	1	1.00	0.71	0.86	2.35	1.50	3.57	1.12	2.27	1.57
time (sec)	N/A	0.340	0.355	2.330	0.040	0.076	0.808	0.149	0.159	0.892

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	63	69	119	87	201	75	118	84
N.S.	1	1.00	0.82	0.90	1.55	1.13	2.61	0.97	1.53	1.09
time (sec)	N/A	0.258	0.319	2.319	0.033	0.075	0.541	0.135	0.149	0.805

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	110	178	281	126	0	6046	26	0
N.S.	1	1.00	0.85	1.38	2.18	0.98	0.00	46.87	0.20	0.00
time (sec)	N/A	0.454	0.187	2.076	0.109	0.073	0.000	0.341	0.182	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	189	256	308	198	0	63798	37	0
N.S.	1	1.00	1.06	1.43	1.72	1.11	0.00	356.41	0.21	0.00
time (sec)	N/A	0.536	0.648	2.354	0.160	0.086	0.000	2.989	0.151	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	239	329	343	348	0	111694	48	0
N.S.	1	1.00	1.02	1.40	1.46	1.48	0.00	475.29	0.20	0.00
time (sec)	N/A	0.604	0.716	2.765	0.257	0.099	0.000	3.021	0.153	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	554	404	393	524	0	157526	0	0
N.S.	1	1.00	1.93	1.41	1.37	1.83	0.00	548.87	0.00	0.00
time (sec)	N/A	0.663	4.150	3.570	0.410	0.108	0.000	4.291	0.445	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	22	24	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	1.00	1.09	1.09	1.09
time (sec)	N/A	0.581	7.834	0.236	0.487	0.071	25.952	0.126	0.154	18.232

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	372	2918	1335	1654	1453	0	0	160	0
N.S.	1	1.24	9.76	4.46	5.53	4.86	0.00	0.00	0.54	0.00
time (sec)	N/A	1.787	6.293	2.751	0.281	0.166	0.000	0.000	0.222	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	298	1956	908	979	984	0	0	131	0
N.S.	1	1.21	7.95	3.69	3.98	4.00	0.00	0.00	0.53	0.00
time (sec)	N/A	1.424	6.242	1.757	0.200	0.144	0.000	0.000	0.171	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	207	564	544	529	594	0	0	102	0
N.S.	1	1.20	3.28	3.16	3.08	3.45	0.00	0.00	0.59	0.00
time (sec)	N/A	1.010	1.987	1.266	0.141	0.118	0.000	0.000	0.161	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	134	131	249	223	292	0	0	69	0
N.S.	1	1.18	1.15	2.18	1.96	2.56	0.00	0.00	0.61	0.00
time (sec)	N/A	0.635	0.453	0.988	0.144	0.105	0.000	0.000	0.160	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	231	24	20	24	24	24
N.S.	1	1.00	1.09	1.00	10.50	1.09	0.91	1.09	1.09	1.09
time (sec)	N/A	0.637	0.539	0.366	0.350	0.072	0.962	0.136	0.142	18.823

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	343	35	22	24	35	24
N.S.	1	1.00	1.09	1.00	15.59	1.59	1.00	1.09	1.59	1.09
time (sec)	N/A	0.758	1.805	0.378	0.723	0.071	1.255	2.415	0.168	19.895

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	22	24	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	1.00	1.09	1.09	1.09
time (sec)	N/A	0.633	8.258	0.118	0.992	0.072	20.707	0.141	0.174	19.515

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	329	798	1056	17589	1233	0	0	805	0
N.S.	1	1.10	2.67	3.53	58.83	4.12	0.00	0.00	2.69	0.00
time (sec)	N/A	1.763	1.223	2.178	4.432	0.142	0.000	0.000	1.471	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	232	539	649	10994	797	0	0	544	0
N.S.	1	1.07	2.50	3.00	50.90	3.69	0.00	0.00	2.52	0.00
time (sec)	N/A	1.207	0.950	1.144	1.101	0.129	0.000	0.000	0.194	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	144	310	332	3199	448	0	0	295	0
N.S.	1	1.04	2.23	2.39	23.01	3.22	0.00	0.00	2.12	0.00
time (sec)	N/A	0.809	2.786	0.686	0.958	0.099	0.000	0.000	0.172	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	104	124	2110	95	0	1759	88	162
N.S.	1	1.00	1.79	2.14	36.38	1.64	0.00	30.33	1.52	2.79
time (sec)	N/A	0.463	0.689	0.507	0.065	0.083	0.000	0.662	0.151	19.058

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	1438	24	20	24	24	24
N.S.	1	1.00	1.09	1.00	65.36	1.09	0.91	1.09	1.09	1.09
time (sec)	N/A	0.700	2.479	0.180	0.479	0.074	0.661	0.902	0.153	18.170

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	1678	35	22	24	35	24
N.S.	1	1.00	1.09	1.00	76.27	1.59	1.00	1.09	1.59	1.09
time (sec)	N/A	0.860	2.597	0.197	1.208	0.072	0.929	6.214	0.185	18.730

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12	1.12
time (sec)	N/A	0.219	9.571	0.112	0.524	0.069	1.963	0.139	0.187	18.077

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	407	1632	1877	7158	1751	0	0	196	0
N.S.	1	1.35	5.40	6.22	23.70	5.80	0.00	0.00	0.65	0.00
time (sec)	N/A	2.229	6.700	0.473	3.256	0.121	0.000	0.000	0.156	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	321	1032	1203	3958	1139	0	0	165	0
N.S.	1	1.25	4.03	4.70	15.46	4.45	0.00	0.00	0.64	0.00
time (sec)	N/A	1.591	6.569	0.428	0.959	0.103	0.000	0.000	0.153	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	197	548	644	1966	657	0	0	134	0
N.S.	1	1.22	3.40	4.00	12.21	4.08	0.00	0.00	0.83	0.00
time (sec)	N/A	0.979	6.490	0.374	0.298	0.093	0.000	0.000	0.167	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	124	234	281	830	339	0	0	97	0
N.S.	1	1.14	2.15	2.58	7.61	3.11	0.00	0.00	0.89	0.00
time (sec)	N/A	0.542	5.368	0.257	0.188	0.083	0.000	0.000	0.154	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	1636	18	14	18	18	18
N.S.	1	1.00	1.12	1.00	102.25	1.12	0.88	1.12	1.12	1.12
time (sec)	N/A	0.207	6.114	0.151	2.684	0.072	0.424	0.190	0.151	17.970

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	2124	29	15	18	29	18
N.S.	1	1.00	1.12	1.00	132.75	1.81	0.94	1.12	1.81	1.12
time (sec)	N/A	0.211	7.504	0.162	11.378	0.067	0.670	2.001	0.200	18.167

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	1332	470	551	376	0	2412	86	0
N.S.	1	1.00	3.27	1.15	1.35	0.92	0.00	5.93	0.21	0.00
time (sec)	N/A	1.087	7.837	3.038	0.175	0.100	0.000	0.919	0.269	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	693	376	503	294	0	1490	52	0
N.S.	1	1.00	1.97	1.07	1.43	0.84	0.00	4.25	0.15	0.00
time (sec)	N/A	0.840	1.049	0.959	0.166	0.097	0.000	0.632	0.227	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	252	286	429	233	0	826	23	0
N.S.	1	1.00	0.84	0.96	1.43	0.78	0.00	2.76	0.08	0.00
time (sec)	N/A	0.685	0.248	0.957	0.154	0.087	0.000	0.411	0.183	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	252	286	429	233	0	826	23	0
N.S.	1	1.00	0.84	0.96	1.43	0.78	0.00	2.76	0.08	0.00
time (sec)	N/A	0.688	0.109	0.574	0.150	0.088	0.000	0.419	0.180	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	693	376	503	294	0	1490	52	0
N.S.	1	1.00	1.97	1.07	1.43	0.84	0.00	4.25	0.15	0.00
time (sec)	N/A	0.793	1.039	0.695	0.154	0.097	0.000	0.636	0.269	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	1332	470	551	376	0	2412	86	0
N.S.	1	1.00	3.27	1.15	1.35	0.92	0.00	5.93	0.21	0.00
time (sec)	N/A	0.896	6.015	2.827	0.164	0.102	0.000	0.924	0.283	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	615	2130	716	826	548	0	3677	92	0
N.S.	1	1.00	3.46	1.16	1.34	0.89	0.00	5.98	0.15	0.00
time (sec)	N/A	1.697	11.109	9.493	0.188	0.119	0.000	1.487	0.272	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	1088	583	760	446	0	2278	56	0
N.S.	1	1.00	2.04	1.09	1.42	0.84	0.00	4.27	0.10	0.00
time (sec)	N/A	1.412	1.809	2.365	0.173	0.111	0.000	1.007	0.219	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	374	444	680	365	0	1270	25	0
N.S.	1	1.00	0.81	0.97	1.48	0.80	0.00	2.77	0.05	0.00
time (sec)	N/A	1.045	0.266	2.239	0.166	0.102	0.000	0.611	0.186	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	374	444	680	365	0	1270	25	0
N.S.	1	1.00	0.81	0.97	1.48	0.80	0.00	2.77	0.05	0.00
time (sec)	N/A	1.024	0.198	1.443	0.166	0.102	0.000	0.622	0.254	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	1088	583	760	446	0	2278	56	0
N.S.	1	1.00	2.04	1.09	1.42	0.84	0.00	4.27	0.10	0.00
time (sec)	N/A	1.216	1.261	1.992	0.182	0.108	0.000	1.012	0.256	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F(-1)	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	615	2130	716	826	548	0	3677	92	0
N.S.	1	1.00	3.46	1.16	1.34	0.89	0.00	5.98	0.15	0.00
time (sec)	N/A	1.415	4.063	9.066	0.191	0.116	0.000	1.475	0.293	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F(-1)	C	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	257	477	561	445	0	2411	92	0
N.S.	1	1.00	0.63	1.17	1.38	1.09	0.00	5.92	0.23	0.00
time (sec)	N/A	1.110	2.208	23.117	0.159	0.110	0.000	2.444	0.302	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F(-1)	C	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	259	383	511	326	0	1489	56	0
N.S.	1	1.00	0.74	1.09	1.46	0.93	0.00	4.24	0.16	0.00
time (sec)	N/A	0.911	1.840	1.841	0.164	0.102	0.000	1.599	0.269	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	256	293	439	242	0	826	25	0
N.S.	1	1.00	0.86	0.98	1.47	0.81	0.00	2.76	0.08	0.00
time (sec)	N/A	0.688	1.063	1.792	0.164	0.093	0.000	0.941	0.198	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	256	293	439	242	0	826	25	0
N.S.	1	1.00	0.86	0.98	1.47	0.81	0.00	2.76	0.08	0.00
time (sec)	N/A	0.664	0.181	1.168	0.155	0.092	0.000	1.025	0.186	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F(-1)	C	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	259	383	511	326	0	1489	56	0
N.S.	1	1.00	0.74	1.09	1.46	0.93	0.00	4.24	0.16	0.00
time (sec)	N/A	0.783	0.295	1.598	0.180	0.099	0.000	1.576	0.245	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	F(-1)	C	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	257	477	561	445	0	2411	92	0
N.S.	1	1.00	0.63	1.17	1.38	1.09	0.00	5.92	0.23	0.00
time (sec)	N/A	0.934	0.199	22.674	0.179	0.106	0.000	2.440	0.293	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	138	104	150	0	244	0	0	14	0
N.S.	1	1.23	0.93	1.34	0.00	2.18	0.00	0.00	0.12	0.00
time (sec)	N/A	0.771	0.145	0.881	0.000	0.097	0.000	0.000	0.199	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	102	72	112	0	162	0	0	14	0
N.S.	1	1.23	0.87	1.35	0.00	1.95	0.00	0.00	0.17	0.00
time (sec)	N/A	0.637	0.078	0.641	0.000	0.089	0.000	0.000	0.279	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	60	0	45	0	206	58	56
N.S.	1	1.00	1.00	1.82	0.00	1.36	0.00	6.24	1.76	1.70
time (sec)	N/A	0.324	0.020	0.581	0.000	0.081	0.000	0.146	0.221	17.701

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	B	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	0	159	240	3740	508	0	0	14	0
N.S.	1	0.00	0.88	1.33	20.78	2.82	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	0.285	0.990	0.439	0.105	0.000	0.000	0.188	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	B	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	0	108	170	2842	370	0	0	14	0
N.S.	1	0.00	1.02	1.60	26.81	3.49	0.00	0.00	0.13	0.00
time (sec)	N/A	0.000	0.232	0.957	0.325	0.105	0.000	0.000	0.171	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	93	62	109	1718	203	0	0	12	0
N.S.	1	1.27	0.85	1.49	23.53	2.78	0.00	0.00	0.16	0.00
time (sec)	N/A	0.936	0.084	0.937	0.184	0.088	0.000	0.000	0.168	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	14	16	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	1.00	1.14	1.14	1.14
time (sec)	N/A	0.189	1.645	0.100	0.178	0.067	0.654	0.148	0.181	17.601

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	195	157	625	773	624	0	0	96	0
N.S.	1	1.23	0.99	3.96	4.89	3.95	0.00	0.00	0.61	0.00
time (sec)	N/A	0.841	0.053	0.423	0.222	0.085	0.000	0.000	0.218	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	155	126	432	474	432	0	0	75	0
N.S.	1	1.17	0.95	3.27	3.59	3.27	0.00	0.00	0.57	0.00
time (sec)	N/A	0.633	0.061	0.373	0.202	0.088	0.000	0.000	0.168	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	113	100	266	267	270	0	0	54	0
N.S.	1	1.18	1.04	2.77	2.78	2.81	0.00	0.00	0.56	0.00
time (sec)	N/A	0.505	0.031	0.343	0.177	0.081	0.000	0.000	0.166	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	73	70	123	115	136	0	0	33	148
N.S.	1	1.11	1.06	1.86	1.74	2.06	0.00	0.00	0.50	2.24
time (sec)	N/A	0.333	0.013	0.313	0.160	0.076	0.000	0.000	0.177	18.717

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	12	16	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	0.86	1.14	1.14	1.14
time (sec)	N/A	0.186	2.609	0.138	0.228	0.060	0.346	0.158	0.181	18.193

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	27	14	16	27	16
N.S.	1	1.00	1.14	1.00	1.14	1.93	1.00	1.14	1.93	1.14
time (sec)	N/A	0.186	4.125	0.142	0.344	0.067	0.535	3.860	0.181	18.868

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	20	22	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	1.00	1.10	1.10	1.10
time (sec)	N/A	0.470	8.640	0.115	0.449	0.069	13.403	0.173	0.227	17.920

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	299	557	901	934	1075	0	0	120	0
N.S.	1	1.09	2.03	3.28	3.40	3.91	0.00	0.00	0.44	0.00
time (sec)	N/A	1.229	0.963	1.125	0.258	0.141	0.000	0.000	0.192	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	197	315	512	516	656	0	0	93	0
N.S.	1	1.06	1.69	2.75	2.77	3.53	0.00	0.00	0.50	0.00
time (sec)	N/A	0.844	0.583	0.674	0.204	0.118	0.000	0.000	0.190	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	213	180	0	331	0	0	62	0
N.S.	1	1.00	2.07	1.75	0.00	3.21	0.00	0.00	0.60	0.00
time (sec)	N/A	0.456	0.314	0.415	0.000	0.108	0.000	0.000	0.185	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	210	22	19	22	22	22
N.S.	1	1.00	1.10	1.00	10.50	1.10	0.95	1.10	1.10	1.10
time (sec)	N/A	0.569	4.035	0.301	0.358	0.072	0.498	0.314	0.181	18.339

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	270	33	20	22	33	22
N.S.	1	1.00	1.10	1.00	13.50	1.65	1.00	1.10	1.65	1.10
time (sec)	N/A	0.665	4.564	0.305	0.618	0.072	0.814	8.721	0.187	18.058

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	27	22	24	1558	24
N.S.	1	1.00	1.09	1.00	1.09	1.23	1.00	1.09	70.82	1.09
time (sec)	N/A	0.557	9.099	0.280	0.438	0.072	41.651	0.373	0.250	18.124

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	282	1731	650	692	1134	0	0	646	0
N.S.	1	1.12	6.90	2.59	2.76	4.52	0.00	0.00	2.57	0.00
time (sec)	N/A	1.353	6.804	1.543	0.215	0.158	0.000	0.000	0.227	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	195	518	388	383	688	0	0	410	0
N.S.	1	1.11	2.96	2.22	2.19	3.93	0.00	0.00	2.34	0.00
time (sec)	N/A	0.953	6.339	1.173	0.189	0.129	0.000	0.000	0.193	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	126	134	179	146	346	0	0	182	0
N.S.	1	1.10	1.17	1.56	1.27	3.01	0.00	0.00	1.58	0.00
time (sec)	N/A	0.612	0.383	0.895	0.173	0.108	0.000	0.000	0.183	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	182	27	20	24	24	24
N.S.	1	1.00	1.09	1.00	8.27	1.23	0.91	1.09	1.09	1.09
time (sec)	N/A	0.637	0.499	0.362	0.296	0.077	0.997	0.190	0.165	19.619

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	242	38	22	24	35	24
N.S.	1	1.00	1.09	1.00	11.00	1.73	1.00	1.09	1.59	1.09
time (sec)	N/A	0.764	1.834	0.339	0.526	0.073	1.441	7.530	0.171	18.868

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	20	22	22	26
N.S.	1	1.00	1.10	1.00	1.10	1.10	1.00	1.10	1.10	1.30
time (sec)	N/A	0.307	5.686	0.118	0.215	0.065	35.281	0.129	0.174	18.513

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	301	578	1242	1794	2600	0	0	151	0
N.S.	1	1.22	2.34	5.03	7.26	10.53	0.00	0.00	0.61	0.00
time (sec)	N/A	1.034	0.725	0.413	0.310	0.229	0.000	0.000	0.243	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	223	350	816	1078	1786	0	0	124	0
N.S.	1	1.13	1.78	4.14	5.47	9.07	0.00	0.00	0.63	0.00
time (sec)	N/A	0.723	0.618	0.374	0.257	0.179	0.000	0.000	0.188	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	140	213	469	599	1098	0	0	97	0
N.S.	1	1.10	1.68	3.69	4.72	8.65	0.00	0.00	0.76	0.00
time (sec)	N/A	0.506	0.454	0.335	0.288	0.148	0.000	0.000	0.182	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	73	132	208	269	554	0	0	66	0
N.S.	1	1.03	1.86	2.93	3.79	7.80	0.00	0.00	0.93	0.00
time (sec)	N/A	0.324	0.097	0.284	0.265	0.122	0.000	0.000	0.170	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	19	22	22	26
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.10	1.30
time (sec)	N/A	0.249	2.987	0.237	0.531	0.072	0.633	0.222	0.164	19.342

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	33	20	22	33	26
N.S.	1	1.00	1.10	1.00	1.10	1.65	1.00	1.10	1.65	1.30
time (sec)	N/A	0.248	3.770	0.243	1.407	0.072	1.029	3.506	0.188	19.681

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	0	24	24	26
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.00	1.09	1.09	1.18
time (sec)	N/A	0.322	14.347	0.129	0.524	0.072	0.000	0.138	0.194	19.209

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	387	642	1158	3257	1753	0	0	271	0
N.S.	1	1.11	1.83	3.31	9.31	5.01	0.00	0.00	0.77	0.00
time (sec)	N/A	0.970	5.921	0.722	1.230	0.186	0.000	0.000	0.214	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	260	467	556	1638	1067	0	0	220	0
N.S.	1	1.15	2.07	2.46	7.25	4.72	0.00	0.00	0.97	0.00
time (sec)	N/A	0.725	5.363	0.533	0.462	0.148	0.000	0.000	0.183	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	128	517	235	0	434	0	0	177	0
N.S.	1	0.98	3.95	1.79	0.00	3.31	0.00	0.00	1.35	0.00
time (sec)	N/A	0.334	2.285	0.386	0.000	0.103	0.000	0.000	0.164	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	677	24	20	24	24	26
N.S.	1	1.00	1.09	1.00	30.77	1.09	0.91	1.09	1.09	1.18
time (sec)	N/A	0.296	17.638	0.229	0.904	0.081	1.134	5.158	0.161	18.540

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	1008	35	22	24	35	26
N.S.	1	1.00	1.09	1.00	45.82	1.59	1.00	1.09	1.59	1.18
time (sec)	N/A	0.321	17.019	0.237	1.867	0.077	2.048	35.411	0.174	18.530

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	0	24	24	26
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.00	1.09	1.09	1.18
time (sec)	N/A	0.383	18.134	0.132	0.895	0.076	0.000	0.160	0.179	18.360

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	350	1526	1223	5165	3475	0	0	206	0
N.S.	1	1.08	4.70	3.76	15.89	10.69	0.00	0.00	0.63	0.00
time (sec)	N/A	1.092	6.630	0.628	2.295	0.229	0.000	0.000	0.229	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	228	880	632	2528	1995	0	0	169	0
N.S.	1	1.18	4.54	3.26	13.03	10.28	0.00	0.00	0.87	0.00
time (sec)	N/A	0.765	6.524	0.530	0.623	0.179	0.000	0.000	0.183	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	138	216	270	1028	942	0	0	124	0
N.S.	1	0.98	1.53	1.91	7.29	6.68	0.00	0.00	0.88	0.00
time (sec)	N/A	0.340	0.877	0.383	0.384	0.134	0.000	0.000	0.177	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	2040	24	20	24	24	26
N.S.	1	1.00	1.09	1.00	92.73	1.09	0.91	1.09	1.09	1.18
time (sec)	N/A	0.304	11.203	0.249	3.819	0.125	2.653	0.557	0.165	19.936

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	2702	35	22	0	35	26
N.S.	1	1.00	1.09	1.00	122.82	1.59	1.00	0.00	1.59	1.18
time (sec)	N/A	0.324	10.921	0.246	14.712	0.173	5.189	0.000	0.173	19.674

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	20	22	22	24
N.S.	1	1.00	1.10	1.00	1.10	1.10	1.00	1.10	1.10	1.20
time (sec)	N/A	0.301	1.856	0.105	0.858	0.068	4.190	0.163	0.171	19.245

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	247	428	767	2944	1190	0	0	299	0
N.S.	1	1.09	1.89	3.38	12.97	5.24	0.00	0.00	1.32	0.00
time (sec)	N/A	0.844	0.690	2.483	0.578	0.131	0.000	0.000	0.161	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	169	256	463	1770	783	0	0	222	0
N.S.	1	1.06	1.61	2.91	11.13	4.92	0.00	0.00	1.40	0.00
time (sec)	N/A	0.579	0.475	1.043	0.469	0.115	0.000	0.000	0.191	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	99	174	227	0	446	0	0	145	0
N.S.	1	1.02	1.79	2.34	0.00	4.60	0.00	0.00	1.49	0.00
time (sec)	N/A	0.382	1.147	0.517	0.000	0.097	0.000	0.000	0.157	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	93	67	259	60	0	1267	71	78
N.S.	1	1.00	3.21	2.31	8.93	2.07	0.00	43.69	2.45	2.69
time (sec)	N/A	0.231	0.053	0.267	0.132	0.079	0.000	0.417	0.162	19.923

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	349	22	19	22	22	24
N.S.	1	1.00	1.10	1.00	17.45	1.10	0.95	1.10	1.10	1.20
time (sec)	N/A	0.249	7.445	0.180	0.478	0.072	0.413	0.738	0.157	18.801

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	494	33	20	22	745	24
N.S.	1	1.00	1.10	1.00	24.70	1.65	1.00	1.10	37.25	1.20
time (sec)	N/A	0.271	16.613	0.181	1.328	0.074	0.608	8.391	0.199	18.810

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	127	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	7.94	1.12
time (sec)	N/A	0.212	1.803	0.107	0.525	0.068	0.992	0.295	0.154	18.388

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	151	424	356	1363	373	0	0	159	0
N.S.	1	1.18	3.31	2.78	10.65	2.91	0.00	0.00	1.24	0.00
time (sec)	N/A	0.678	6.324	0.503	0.414	0.082	0.000	0.000	0.153	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	111	203	199	418	210	0	0	106	0
N.S.	1	1.16	2.11	2.07	4.35	2.19	0.00	0.00	1.10	0.00
time (sec)	N/A	0.480	5.384	0.491	0.303	0.074	0.000	0.000	0.159	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	76	52	237	53	65	203	57	52
N.S.	1	1.00	1.81	1.24	5.64	1.26	1.55	4.83	1.36	1.24
time (sec)	N/A	0.253	0.246	0.365	0.136	0.072	0.164	0.212	0.195	0.143

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	388	18	14	18	18	18
N.S.	1	1.00	1.12	1.00	24.25	1.12	0.88	1.12	1.12	1.12
time (sec)	N/A	0.205	4.968	0.247	0.705	0.062	0.344	0.242	0.149	18.369

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	523	29	15	18	29	18
N.S.	1	1.00	1.12	1.00	32.69	1.81	0.94	1.12	1.81	1.12
time (sec)	N/A	0.212	3.785	0.256	1.378	0.060	0.519	7.390	0.148	19.072

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	22	24	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	1.00	1.09	1.09	1.09
time (sec)	N/A	0.588	16.619	0.120	1.710	0.069	36.687	0.330	0.159	19.825

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	244	532	677	11010	896	0	0	740	0
N.S.	1	1.07	2.33	2.97	48.29	3.93	0.00	0.00	3.25	0.00
time (sec)	N/A	1.232	1.006	1.684	1.625	0.131	0.000	0.000	0.165	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	149	362	345	0	511	0	0	389	0
N.S.	1	1.03	2.50	2.38	0.00	3.52	0.00	0.00	2.68	0.00
time (sec)	N/A	0.779	2.122	0.905	0.000	0.108	0.000	0.000	0.156	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	107	123	2123	93	0	2330	123	151
N.S.	1	1.00	1.91	2.20	37.91	1.66	0.00	41.61	2.20	2.70
time (sec)	N/A	0.440	0.474	0.573	0.196	0.083	0.000	0.778	0.151	1.276

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	1268	24	20	24	24	24
N.S.	1	1.00	1.09	1.00	57.64	1.09	0.91	1.09	1.09	1.09
time (sec)	N/A	0.655	2.646	0.181	0.968	0.077	0.675	0.917	0.206	19.571

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	1412	35	22	24	35	24
N.S.	1	1.00	1.09	1.00	64.18	1.59	1.00	1.09	1.59	1.09
time (sec)	N/A	0.785	2.803	0.204	1.316	0.074	1.112	16.833	0.160	20.123

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	22	24	24	26
N.S.	1	1.00	1.09	1.00	1.09	1.09	1.00	1.09	1.09	1.18
time (sec)	N/A	0.355	14.817	0.128	0.623	0.071	116.658	0.139	0.167	19.341

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	469	507	694	1866	5709	2519	0	0	177	0
N.S.	1	1.08	1.48	3.98	12.17	5.37	0.00	0.00	0.38	0.00
time (sec)	N/A	1.138	2.156	0.769	2.673	0.244	0.000	0.000	0.203	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	381	473	1152	3202	1705	0	0	142	0
N.S.	1	1.11	1.38	3.36	9.34	4.97	0.00	0.00	0.41	0.00
time (sec)	N/A	0.918	0.822	0.674	1.012	0.188	0.000	0.000	0.182	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	254	317	568	1590	1035	0	0	107	0
N.S.	1	1.16	1.45	2.59	7.26	4.73	0.00	0.00	0.49	0.00
time (sec)	N/A	0.704	1.660	0.522	0.528	0.145	0.000	0.000	0.174	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	121	212	160	800	366	0	0	68	0
N.S.	1	1.07	1.88	1.42	7.08	3.24	0.00	0.00	0.60	0.00
time (sec)	N/A	0.325	0.624	0.351	0.532	0.099	0.000	0.000	0.159	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	603	24	20	24	24	26
N.S.	1	1.00	1.09	1.00	27.41	1.09	0.91	1.09	1.09	1.18
time (sec)	N/A	0.278	16.143	0.232	0.937	0.078	1.120	5.244	0.200	19.850

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	931	35	22	0	35	26
N.S.	1	1.00	1.09	1.00	42.32	1.59	1.00	0.00	1.59	1.18
time (sec)	N/A	0.309	16.656	0.224	1.587	0.077	1.969	0.000	0.155	19.412

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	0	26	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08	1.08
time (sec)	N/A	0.363	1.854	0.131	0.473	0.067	0.000	0.161	0.161	18.855

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	161	285	687	2360	1635	0	0	1171	0
N.S.	1	1.36	2.42	5.82	20.00	13.86	0.00	0.00	9.92	0.00
time (sec)	N/A	0.771	1.380	0.673	0.505	0.171	0.000	0.000	0.191	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	113	277	351	772	950	0	0	568	0
N.S.	1	1.28	3.15	3.99	8.77	10.80	0.00	0.00	6.45	0.00
time (sec)	N/A	0.561	1.129	0.603	0.378	0.145	0.000	0.000	0.162	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	44	32	72	308	75	0	10271	160	55
N.S.	1	1.26	0.91	2.06	8.80	2.14	0.00	293.46	4.57	1.57
time (sec)	N/A	0.322	0.345	0.618	0.162	0.079	0.000	2.418	0.153	21.250

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	687	26	22	26	26	26
N.S.	1	1.00	1.08	1.00	28.62	1.08	0.92	1.08	1.08	1.08
time (sec)	N/A	0.285	5.740	0.240	0.935	0.073	2.633	0.142	0.153	19.713

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	1024	37	24	26	37	26
N.S.	1	1.00	1.08	1.00	42.67	1.54	1.00	1.08	1.54	1.08
time (sec)	N/A	0.282	5.943	0.255	3.307	0.073	5.105	1.321	0.192	20.176

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	0	26	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08	1.08
time (sec)	N/A	0.358	17.012	0.135	1.142	0.074	0.000	0.163	0.167	19.777

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	601	623	907	1613	8032	3173	0	0	214	0
N.S.	1	1.04	1.51	2.68	13.36	5.28	0.00	0.00	0.36	0.00
time (sec)	N/A	2.369	7.368	1.063	4.883	0.229	0.000	0.000	0.189	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	335	889	802	3814	1801	0	0	169	0
N.S.	1	1.10	2.91	2.63	12.50	5.90	0.00	0.00	0.55	0.00
time (sec)	N/A	1.144	7.226	0.773	1.123	0.174	0.000	0.000	0.176	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	171	520	267	1481	621	0	0	118	0
N.S.	1	1.11	3.38	1.73	9.62	4.03	0.00	0.00	0.77	0.00
time (sec)	N/A	0.386	3.940	0.552	0.822	0.107	0.000	0.000	0.165	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	3697	26	22	26	26	26
N.S.	1	1.00	1.08	1.00	154.04	1.08	0.92	1.08	1.08	1.08
time (sec)	N/A	0.315	15.028	0.258	7.899	0.104	7.419	120.904	0.157	20.563

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	4757	37	24	0	37	26
N.S.	1	1.00	1.08	1.00	198.21	1.54	1.00	0.00	1.54	1.08
time (sec)	N/A	0.361	19.071	0.219	21.662	0.139	14.778	0.000	0.160	20.648

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	0	22	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.00	1.10	1.10	1.10
time (sec)	N/A	0.853	26.615	0.107	0.985	0.071	0.000	0.174	0.155	20.150

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	387	417	672	0	3940	1747	0	0	22	0
N.S.	1	1.08	1.74	0.00	10.18	4.51	0.00	0.00	0.06	0.00
time (sec)	N/A	1.215	6.744	0.000	0.809	0.171	0.000	0.000	0.200	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	613	429	2205	1237	0	0	22	0
N.S.	1	1.00	2.61	1.83	9.38	5.26	0.00	0.00	0.09	0.00
time (sec)	N/A	0.706	6.518	0.585	0.549	0.146	0.000	0.000	0.162	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	282	519	1173	531	0	0	20	0
N.S.	1	1.00	2.24	4.12	9.31	4.21	0.00	0.00	0.16	0.00
time (sec)	N/A	0.362	2.169	0.476	0.298	0.104	0.000	0.000	0.159	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	1872	22	19	22	22	22
N.S.	1	1.00	1.10	1.00	93.60	1.10	0.95	1.10	1.10	1.10
time (sec)	N/A	0.558	34.825	0.191	0.833	0.075	8.634	0.996	0.160	20.007

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	1884	22	20	22	22	22
N.S.	1	1.00	1.10	1.00	94.20	1.10	1.00	1.10	1.10	1.10
time (sec)	N/A	0.597	17.122	0.187	1.007	0.071	11.311	1.493	0.158	19.812

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	22	24	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	1.00	1.09	1.09	1.09
time (sec)	N/A	0.337	4.035	0.112	0.475	0.073	9.817	0.198	0.170	19.680

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	166	324	489	3446	892	0	0	0	0
N.S.	1	1.19	2.33	3.52	24.79	6.42	0.00	0.00	0.00	0.00
time (sec)	N/A	0.798	6.279	4.753	0.291	0.123	0.000	0.000	0.202	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	128	286	301	668	540	0	0	0	0
N.S.	1	1.11	2.49	2.62	5.81	4.70	0.00	0.00	0.00	0.00
time (sec)	N/A	0.568	6.272	2.539	0.278	0.108	0.000	0.000	0.240	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	57	66	149	988	86	0	4331	0	150
N.S.	1	1.04	1.20	2.71	17.96	1.56	0.00	78.75	0.00	2.73
time (sec)	N/A	0.349	0.392	1.357	0.140	0.081	0.000	0.857	0.180	22.080

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	48	61	283	36	0	571	49	53
N.S.	1	1.00	1.37	1.74	8.09	1.03	0.00	16.31	1.40	1.51
time (sec)	N/A	0.245	0.192	0.679	0.043	0.067	0.000	0.223	0.169	19.808

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	1090	24	20	24	24	24
N.S.	1	1.00	1.09	1.00	49.55	1.09	0.91	1.09	1.09	1.09
time (sec)	N/A	0.273	5.108	0.215	0.587	0.079	0.438	0.264	0.169	19.733

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	1396	35	22	24	5187	24
N.S.	1	1.00	1.09	1.00	63.45	1.59	1.00	1.09	235.77	1.09
time (sec)	N/A	0.298	7.561	0.177	1.606	0.078	0.692	6.178	0.380	21.053

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	22	24	24	26
N.S.	1	1.00	1.09	1.00	1.09	1.09	1.00	1.09	1.09	1.18
time (sec)	N/A	0.301	34.767	0.118	0.968	0.070	9.506	0.245	0.174	20.531

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	580	530	1127	3831	1315	0	0	291	0
N.S.	1	1.72	1.57	3.34	11.37	3.90	0.00	0.00	0.86	0.00
time (sec)	N/A	2.093	2.326	1.911	1.149	0.155	0.000	0.000	0.243	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	351	443	584	1891	795	0	0	225	0
N.S.	1	1.82	2.30	3.03	9.80	4.12	0.00	0.00	1.17	0.00
time (sec)	N/A	1.412	6.751	1.015	0.703	0.130	0.000	0.000	0.182	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	192	555	267	0	435	0	0	153	0
N.S.	1	1.64	4.74	2.28	0.00	3.72	0.00	0.00	1.31	0.00
time (sec)	N/A	0.700	6.565	0.525	0.000	0.104	0.000	0.000	0.174	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	1331	24	20	24	24	26
N.S.	1	1.00	1.09	1.00	60.50	1.09	0.91	1.09	1.09	1.18
time (sec)	N/A	0.310	24.971	0.231	1.773	0.085	0.498	4.785	0.169	19.770

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	1645	35	22	24	35	26
N.S.	1	1.00	1.09	1.00	74.77	1.59	1.00	1.09	1.59	1.18
time (sec)	N/A	0.306	27.296	0.247	5.211	0.091	0.754	27.577	0.174	20.323

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12	1.12
time (sec)	N/A	0.199	6.803	0.120	0.575	0.068	1.725	0.458	0.165	20.695

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	296	814	729	2413	592	0	0	94	0
N.S.	1	1.14	3.14	2.81	9.32	2.29	0.00	0.00	0.36	0.00
time (sec)	N/A	1.510	6.591	0.404	0.722	0.089	0.000	0.000	0.168	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	183	454	409	1228	354	0	0	71	0
N.S.	1	1.12	2.79	2.51	7.53	2.17	0.00	0.00	0.44	0.00
time (sec)	N/A	0.936	6.440	0.343	0.327	0.086	0.000	0.000	0.221	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	115	240	183	517	168	0	0	46	0
N.S.	1	1.06	2.22	1.69	4.79	1.56	0.00	0.00	0.43	0.00
time (sec)	N/A	0.502	6.164	0.266	0.241	0.080	0.000	0.000	0.169	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	1077	18	14	18	18	18
N.S.	1	1.00	1.12	1.00	67.31	1.12	0.88	1.12	1.12	1.12
time (sec)	N/A	0.216	5.385	0.166	1.426	0.068	0.379	0.264	0.167	21.360

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	1386	29	15	18	29	18
N.S.	1	1.00	1.12	1.00	86.62	1.81	0.94	1.12	1.81	1.12
time (sec)	N/A	0.210	5.103	0.165	6.629	0.067	0.553	11.516	0.163	20.633

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	0	24	24	26
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.00	1.09	1.09	1.18
time (sec)	N/A	0.388	14.670	0.136	0.387	0.077	0.000	0.155	0.180	19.943

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	443	2227	1729	8853	3324	0	0	399	0
N.S.	1	1.11	5.58	4.33	22.19	8.33	0.00	0.00	1.00	0.00
time (sec)	N/A	1.266	7.012	0.677	4.200	0.340	0.000	0.000	0.210	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	349	1535	1115	4954	2268	0	0	333	0
N.S.	1	1.07	4.72	3.43	15.24	6.98	0.00	0.00	1.02	0.00
time (sec)	N/A	0.992	6.595	0.556	1.236	0.235	0.000	0.000	0.197	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	227	883	614	2447	1404	0	0	267	0
N.S.	1	1.16	4.53	3.15	12.55	7.20	0.00	0.00	1.37	0.00
time (sec)	N/A	0.759	6.536	0.472	0.392	0.180	0.000	0.000	0.231	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	138	216	270	1028	760	0	0	191	0
N.S.	1	0.99	1.55	1.94	7.40	5.47	0.00	0.00	1.37	0.00
time (sec)	N/A	0.334	0.600	0.362	0.255	0.137	0.000	0.000	0.179	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	1865	24	20	24	24	26
N.S.	1	1.00	1.09	1.00	84.77	1.09	0.91	1.09	1.09	1.18
time (sec)	N/A	0.317	6.748	0.245	1.533	0.112	2.766	0.601	0.167	20.839

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	2516	35	22	0	35	26
N.S.	1	1.00	1.09	1.00	114.36	1.59	1.00	0.00	1.59	1.18
time (sec)	N/A	0.330	4.913	0.252	5.332	0.187	5.246	0.000	0.169	21.399

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	0	26	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08	1.08
time (sec)	N/A	0.396	21.629	0.143	0.999	0.076	0.000	0.174	0.182	22.038

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	486	513	819	1629	8046	2226	0	0	0	0
N.S.	1	1.06	1.69	3.35	16.56	4.58	0.00	0.00	0.00	0.00
time (sec)	N/A	1.600	7.200	0.998	3.907	0.238	0.000	0.000	1.506	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	371	889	770	3828	1366	0	0	1412	0
N.S.	1	1.09	2.61	2.26	11.23	4.01	0.00	0.00	4.14	0.00
time (sec)	N/A	1.056	7.081	0.714	0.787	0.175	0.000	0.000	0.196	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	179	669	344	0	592	0	0	545	0
N.S.	1	1.10	4.13	2.12	0.00	3.65	0.00	0.00	3.36	0.00
time (sec)	N/A	0.394	6.750	0.566	0.000	0.113	0.000	0.000	0.179	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	3695	26	22	26	26	26
N.S.	1	1.00	1.08	1.00	153.96	1.08	0.92	1.08	1.08	1.08
time (sec)	N/A	0.349	12.844	0.284	2.620	0.103	7.709	115.577	0.170	21.495

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	4747	37	24	0	37	26
N.S.	1	1.00	1.08	1.00	197.79	1.54	1.00	0.00	1.54	1.08
time (sec)	N/A	0.365	16.338	0.234	7.231	0.138	15.368	0.000	0.224	20.933

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	0	26	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.00	1.08	1.08	1.08
time (sec)	N/A	0.389	26.229	0.150	0.533	0.075	0.000	0.212	0.185	20.205

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	365	483	1329	5646	4193	0	0	417	0
N.S.	1	1.15	1.52	4.18	17.75	13.19	0.00	0.00	1.31	0.00
time (sec)	N/A	1.330	6.006	0.907	3.183	0.287	0.000	0.000	0.201	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	219	381	716	2728	2387	0	0	339	0
N.S.	1	1.15	2.01	3.77	14.36	12.56	0.00	0.00	1.78	0.00
time (sec)	N/A	0.820	7.443	0.728	1.301	0.201	0.000	0.000	0.187	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	125	236	325	1070	1193	0	0	247	0
N.S.	1	1.14	2.15	2.95	9.73	10.85	0.00	0.00	2.25	0.00
time (sec)	N/A	0.466	1.730	0.603	0.741	0.153	0.000	0.000	0.172	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	2410	26	22	0	26	26
N.S.	1	1.00	1.08	1.00	100.42	1.08	0.92	0.00	1.08	1.08
time (sec)	N/A	0.286	20.475	0.283	6.959	0.108	20.239	0.000	0.165	20.453

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	3083	37	24	0	37	26
N.S.	1	1.00	1.08	1.00	128.46	1.54	1.00	0.00	1.54	1.08
time (sec)	N/A	0.286	23.435	0.253	20.788	0.139	40.318	0.000	0.173	20.614

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	96	73	0	0	0	0	0	25	0
N.S.	1	1.16	0.88	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.369	0.390	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	68	51	0	0	0	0	0	23	0
N.S.	1	1.13	0.85	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.299	0.393	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	68	52	0	0	0	0	0	17	0
N.S.	1	1.13	0.87	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.294	0.209	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	66	310	0	0	0	0	25	0
N.S.	1	1.00	2.00	9.39	0.00	0.00	0.00	0.00	0.76	0.00
time (sec)	N/A	0.218	0.671	0.234	0.000	0.000	0.000	0.000	0.150	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	0	0	0	0	0	49	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.48	0.00
time (sec)	N/A	0.218	0.249	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	64	54	0	0	0	0	0	51	0
N.S.	1	1.07	0.90	0.00	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.295	0.277	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	68	53	0	0	0	0	0	51	0
N.S.	1	1.13	0.88	0.00	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.287	0.309	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	92	65	0	0	0	0	0	51	0
N.S.	1	1.11	0.78	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.383	0.332	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	112	65	0	0	0	0	0	25	0
N.S.	1	1.09	0.63	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.487	0.345	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	88	61	0	0	0	0	0	25	0
N.S.	1	1.10	0.76	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.396	0.284	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	54	0	0	0	0	0	25	0
N.S.	1	1.05	0.68	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.386	0.255	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	42	0	0	0	0	0	23	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	0.297	0.212	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	66	310	0	0	0	0	17	0
N.S.	1	1.00	1.25	5.85	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	0.289	0.537	0.221	0.000	0.000	0.000	0.000	0.158	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	88	63	0	0	0	0	0	25	0
N.S.	1	1.10	0.79	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.378	0.333	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	88	97	0	0	0	0	0	25	0
N.S.	1	1.10	1.21	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.396	0.799	0.000	0.000	0.000	0.000	0.000	0.144	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	116	89	0	0	0	0	0	25	0
N.S.	1	1.13	0.86	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.487	0.406	0.000	0.000	0.000	0.000	0.000	0.151	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	101	67	0	0	0	0	0	25	0
N.S.	1	1.15	0.76	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.378	0.523	0.000	0.000	0.000	0.000	0.000	0.145	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	73	108	0	0	0	0	0	23	0
N.S.	1	1.12	1.66	0.00	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	0.297	2.100	0.000	0.000	0.000	0.000	0.000	0.144	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	73	56	0	0	0	0	0	17	0
N.S.	1	1.12	0.86	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.284	0.256	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	86	308	0	0	0	0	25	0
N.S.	1	1.00	2.26	8.11	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.214	2.697	0.252	0.000	0.000	0.000	0.000	0.144	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	0	0	0	0	0	49	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	1.29	0.00
time (sec)	N/A	0.219	0.304	0.000	0.000	0.000	0.000	0.000	0.143	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	69	56	0	0	0	0	0	51	0
N.S.	1	1.06	0.86	0.00	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.287	0.268	0.000	0.000	0.000	0.000	0.000	0.146	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	73	57	0	0	0	0	0	51	0
N.S.	1	1.12	0.88	0.00	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.285	0.300	0.000	0.000	0.000	0.000	0.000	0.145	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	97	73	0	0	0	0	0	51	0
N.S.	1	1.10	0.83	0.00	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.383	0.342	0.000	0.000	0.000	0.000	0.000	0.142	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	117	73	0	0	0	0	0	25	0
N.S.	1	1.08	0.68	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.506	0.392	0.000	0.000	0.000	0.000	0.000	0.144	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	93	65	0	0	0	0	0	25	0
N.S.	1	1.09	0.76	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.388	0.373	0.000	0.000	0.000	0.000	0.000	0.143	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	89	56	0	0	0	0	0	25	0
N.S.	1	1.05	0.66	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.399	0.277	0.000	0.000	0.000	0.000	0.000	0.146	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	46	0	0	0	0	0	23	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	0.298	0.254	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	106	308	0	0	0	0	17	0
N.S.	1	1.00	1.83	5.31	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.301	1.812	0.274	0.000	0.000	0.000	0.000	0.143	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	93	65	0	0	0	0	0	25	0
N.S.	1	1.09	0.76	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.380	0.356	0.000	0.000	0.000	0.000	0.000	0.144	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	93	114	0	0	0	0	0	25	0
N.S.	1	1.09	1.34	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.376	1.217	0.000	0.000	0.000	0.000	0.000	0.150	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	121	93	0	0	0	0	0	25	0
N.S.	1	1.12	0.86	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.487	0.492	0.000	0.000	0.000	0.000	0.000	0.147	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	19	18	17	19	18	10	18
N.S.	1	1.00	0.71	0.61	0.58	0.55	0.61	0.58	0.32	0.58
time (sec)	N/A	0.202	0.013	0.223	0.050	0.072	0.953	0.124	0.144	0.099

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	154	174	146	200	223	167	84	212
N.S.	1	1.00	1.18	1.33	1.11	1.53	1.70	1.27	0.64	1.62
time (sec)	N/A	0.402	0.135	1.931	0.070	0.078	15.794	0.119	0.261	19.272

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	109	119	101	127	139	112	64	136
N.S.	1	1.00	1.01	1.10	0.94	1.18	1.29	1.04	0.59	1.26
time (sec)	N/A	0.331	0.111	0.937	0.054	0.076	8.447	0.119	0.168	0.415

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	70	60	70	75	64	44	73
N.S.	1	1.00	0.82	0.96	0.82	0.96	1.03	0.88	0.60	1.00
time (sec)	N/A	0.286	0.075	0.511	0.067	0.076	4.214	0.125	0.177	0.190

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	34	28	27	27	29	27	24	30
N.S.	1	1.00	0.77	0.64	0.61	0.61	0.66	0.61	0.55	0.68
time (sec)	N/A	0.248	0.050	0.262	0.053	0.073	1.874	0.128	0.437	19.063

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	61	97	51	0	51	16	0
N.S.	1	1.00	0.86	1.07	1.70	0.89	0.00	0.89	0.28	0.00
time (sec)	N/A	0.429	0.048	0.471	0.124	0.075	0.000	0.109	0.144	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	61	87	330	76	0	111	16	0
N.S.	1	1.00	0.78	1.12	4.23	0.97	0.00	1.42	0.21	0.00
time (sec)	N/A	0.429	0.103	0.914	0.130	0.077	0.000	0.112	200.022	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	77	110	365	128	0	201	16	0
N.S.	1	1.00	0.78	1.11	3.69	1.29	0.00	2.03	0.16	0.00
time (sec)	N/A	0.497	0.176	2.012	0.129	0.078	0.000	0.117	200.022	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	128	226	244	283	0	1255	129	344
N.S.	1	1.00	0.65	1.14	1.23	1.43	0.00	6.34	0.65	1.74
time (sec)	N/A	0.455	0.421	1.603	0.112	0.081	0.000	0.270	0.355	19.024

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	105	156	173	188	0	682	100	216
N.S.	1	1.00	0.65	0.97	1.07	1.17	0.00	4.24	0.62	1.34
time (sec)	N/A	0.382	0.247	0.790	0.108	0.079	0.000	0.219	0.208	18.906

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	73	95	108	111	0	313	71	121
N.S.	1	1.00	0.65	0.85	0.96	0.99	0.00	2.79	0.63	1.08
time (sec)	N/A	0.349	0.257	0.497	0.074	0.074	0.000	0.184	0.194	18.717

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	46	44	55	54	0	106	42	53
N.S.	1	1.00	0.67	0.64	0.80	0.78	0.00	1.54	0.61	0.77
time (sec)	N/A	0.257	0.125	0.266	0.063	0.074	0.000	0.158	0.464	0.215

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	63	106	119	71	0	102	25	0
N.S.	1	1.00	0.89	1.49	1.68	1.00	0.00	1.44	0.35	0.00
time (sec)	N/A	0.447	0.137	0.452	0.144	0.077	0.000	0.147	9.527	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	81	155	120	106	0	308	25	0
N.S.	1	1.00	0.79	1.52	1.18	1.04	0.00	3.02	0.25	0.00
time (sec)	N/A	0.476	0.411	0.839	0.168	0.080	0.000	0.168	200.020	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	104	207	132	182	0	704	25	0
N.S.	1	1.00	0.76	1.52	0.97	1.34	0.00	5.18	0.18	0.00
time (sec)	N/A	0.572	0.714	1.763	0.163	0.087	0.000	0.200	200.018	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	125	243	143	286	0	1305	25	0
N.S.	1	1.00	0.61	1.19	0.70	1.40	0.00	6.37	0.12	0.00
time (sec)	N/A	0.595	0.748	3.717	0.192	0.087	0.000	0.228	200.024	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	459	849	606	929	0	0	108	0
N.S.	1	1.00	1.80	3.33	2.38	3.64	0.00	0.00	0.42	0.00
time (sec)	N/A	0.585	0.808	2.710	0.278	0.127	0.000	0.000	0.209	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	223	481	413	566	0	0	77	0
N.S.	1	1.00	1.30	2.80	2.40	3.29	0.00	0.00	0.45	0.00
time (sec)	N/A	0.432	0.592	1.981	0.213	0.115	0.000	0.000	0.188	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	171	205	0	281	0	0	46	0
N.S.	1	1.00	1.80	2.16	0.00	2.96	0.00	0.00	0.48	0.00
time (sec)	N/A	0.302	0.575	0.886	0.000	0.100	0.000	0.000	0.197	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	230	27	0	27	27	27
N.S.	1	1.00	1.08	1.00	9.20	1.08	0.00	1.08	1.08	1.08
time (sec)	N/A	0.421	4.009	0.415	0.248	0.075	0.000	0.810	200.023	23.624

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	342	38	0	27	27	27
N.S.	1	1.00	1.08	1.00	13.68	1.52	0.00	1.08	1.08	1.08
time (sec)	N/A	0.465	4.343	0.419	0.297	0.072	0.000	1.726	200.021	24.838

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	463	49	0	27	27	27
N.S.	1	1.00	1.08	1.00	18.52	1.96	0.00	1.08	1.08	1.08
time (sec)	N/A	0.530	5.150	0.423	0.502	0.076	0.000	3.433	200.020	26.445

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	2529	965	610	1652	0	0	703	0
N.S.	1	1.00	8.63	3.29	2.08	5.64	0.00	0.00	2.40	0.00
time (sec)	N/A	0.728	6.379	6.348	0.310	0.176	0.000	0.000	0.195	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	1730	648	444	1126	0	0	1303	0
N.S.	1	1.00	7.15	2.68	1.83	4.65	0.00	0.00	5.38	0.00
time (sec)	N/A	0.624	6.332	2.807	0.252	0.151	0.000	0.000	0.201	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	516	386	303	681	0	0	339	0
N.S.	1	1.00	3.11	2.33	1.83	4.10	0.00	0.00	2.04	0.00
time (sec)	N/A	0.535	6.324	1.215	0.217	0.123	0.000	0.000	0.185	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	257	177	0	340	0	0	66	0
N.S.	1	1.00	2.40	1.65	0.00	3.18	0.00	0.00	0.62	0.00
time (sec)	N/A	0.385	3.498	1.365	0.000	0.105	0.000	0.000	0.159	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	182	25	0	25	25	27
N.S.	1	1.00	1.09	1.00	7.91	1.09	0.00	1.09	1.09	1.17
time (sec)	N/A	0.473	2.438	0.428	0.217	0.077	0.000	0.155	0.159	20.259

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	242	36	0	25	36	27
N.S.	1	1.00	1.09	1.00	10.52	1.57	0.00	1.09	1.57	1.17
time (sec)	N/A	0.514	3.159	0.488	0.232	0.072	0.000	0.204	0.167	21.258

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	308	47	0	25	47	27
N.S.	1	1.00	1.09	1.00	13.39	2.04	0.00	1.09	2.04	1.17
time (sec)	N/A	0.582	4.248	0.451	0.268	0.075	0.000	0.239	0.169	22.147

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	532	677	0	896	0	0	0	0
N.S.	1	1.00	2.31	2.94	0.00	3.90	0.00	0.00	0.00	0.00
time (sec)	N/A	0.545	1.575	8.585	0.000	0.130	0.000	0.000	0.685	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	364	334	0	513	0	0	0	0
N.S.	1	1.00	2.48	2.27	0.00	3.49	0.00	0.00	0.00	0.00
time (sec)	N/A	0.420	2.548	3.872	0.000	0.107	0.000	0.000	0.332	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	105	96	3330	93	0	365	1100	150
N.S.	1	1.00	1.84	1.68	58.42	1.63	0.00	6.40	19.30	2.63
time (sec)	N/A	0.269	0.640	1.549	0.311	0.086	0.000	0.210	0.234	1.450

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	1275	27	0	27	27	27
N.S.	1	1.00	1.08	1.00	51.00	1.08	0.00	1.08	1.08	1.08
time (sec)	N/A	0.479	9.643	0.329	0.430	0.074	0.000	1.705	0.168	22.621

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	1419	38	0	27	38	27
N.S.	1	1.00	1.08	1.00	56.76	1.52	0.00	1.08	1.52	1.08
time (sec)	N/A	0.536	15.105	0.347	0.517	0.077	0.000	3.202	0.203	24.116

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	1563	49	0	27	49	27
N.S.	1	1.00	1.08	1.00	62.52	1.96	0.00	1.08	1.96	1.08
time (sec)	N/A	0.629	16.243	0.443	0.451	0.078	0.000	9.490	0.334	25.362

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	77	66	0	106	0	0	86	46
N.S.	1	1.00	1.35	1.16	0.00	1.86	0.00	0.00	1.51	0.81
time (sec)	N/A	0.246	0.025	0.723	0.000	0.086	0.000	0.000	0.170	18.951

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	111	26	144	118	118	31
N.S.	1	1.00	1.00	1.07	7.93	1.86	10.29	8.43	8.43	2.21
time (sec)	N/A	0.190	0.021	1.740	0.157	0.080	3.372	0.130	0.179	0.326

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	146	102	0	144	0	0	972	63
N.S.	1	1.00	2.18	1.52	0.00	2.15	0.00	0.00	14.51	0.94
time (sec)	N/A	0.308	0.213	5.309	0.000	0.087	0.000	0.000	0.200	19.148

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [207] had the largest ratio of [2.6000000000000009]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.03	20	0.250
2	A	7	7	1.07	20	0.350
3	A	7	7	1.07	20	0.350
4	A	4	4	1.05	20	0.200
5	A	4	4	1.08	18	0.222
6	A	7	7	0.97	20	0.350
7	A	9	9	1.08	20	0.450
8	A	12	12	1.05	20	0.600
9	A	14	14	1.07	20	0.700
10	A	4	4	1.00	8	0.500
11	A	6	6	1.25	8	0.750
12	A	9	9	1.07	8	1.125
13	A	2	2	1.00	22	0.091
14	A	17	17	1.21	22	0.773
15	A	13	12	1.09	22	0.545
16	A	7	7	1.03	22	0.318
17	A	5	4	0.94	20	0.200
18	A	2	2	1.00	22	0.091
19	A	2	2	1.00	22	0.091
20	A	2	2	1.00	22	0.091
21	A	2	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.00	22	0.091
23	A	13	13	1.29	22	0.591
24	A	13	13	1.24	22	0.591
25	A	6	6	1.06	22	0.273
26	A	6	6	1.11	20	0.300
27	A	2	2	1.00	22	0.091
28	A	2	2	1.00	22	0.091
29	A	2	2	1.00	22	0.091
30	A	2	2	1.00	22	0.091
31	N/A	3	0	1.00	14	0.000
32	A	10	9	1.42	14	0.643
33	A	9	8	1.35	14	0.571
34	A	8	7	1.34	14	0.500
35	A	7	6	1.25	12	0.500
36	N/A	3	0	1.00	14	0.000
37	N/A	3	0	1.00	14	0.000
38	N/A	1	0	1.00	20	0.000
39	A	8	7	1.10	20	0.350
40	A	7	6	1.07	20	0.300
41	A	6	5	1.02	20	0.250
42	A	3	3	1.00	18	0.167
43	N/A	1	0	1.00	20	0.000
44	N/A	1	0	1.00	20	0.000
45	N/A	1	0	1.00	22	0.000
46	A	11	10	1.31	22	0.455
47	A	10	9	1.19	22	0.409
48	A	6	6	1.09	22	0.273
49	A	5	4	1.00	20	0.200
50	N/A	1	0	1.00	22	0.000
51	N/A	1	0	1.00	22	0.000
52	A	17	16	1.11	22	0.727

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
53	A	15	14	1.09	22	0.636
54	A	12	11	1.06	22	0.500
55	A	12	11	1.06	22	0.500
56	A	15	14	1.09	22	0.636
57	A	17	16	1.11	22	0.727
58	A	2	2	1.00	24	0.083
59	A	2	2	1.00	24	0.083
60	A	2	2	1.00	24	0.083
61	A	2	2	1.00	24	0.083
62	A	2	2	1.00	24	0.083
63	A	2	2	1.00	24	0.083
64	A	2	2	1.00	24	0.083
65	A	2	2	1.00	24	0.083
66	A	2	2	1.00	24	0.083
67	A	2	2	1.00	24	0.083
68	A	2	2	1.00	24	0.083
69	A	2	2	1.00	24	0.083
70	A	2	2	1.00	22	0.091
71	A	19	19	1.20	22	0.864
72	A	13	12	1.09	22	0.545
73	A	8	8	1.02	22	0.364
74	A	5	4	0.98	20	0.200
75	A	2	2	1.00	22	0.091
76	A	2	2	1.00	22	0.091
77	A	2	2	1.00	22	0.091
78	A	2	2	1.00	22	0.091
79	A	2	2	1.00	24	0.083
80	A	2	2	1.00	24	0.083
81	A	2	2	1.00	24	0.083
82	A	2	2	1.00	24	0.083
83	A	2	2	1.00	22	0.091
84	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
85	A	2	2	1.00	24	0.083
86	A	2	2	1.00	24	0.083
87	A	2	2	1.00	24	0.083
88	A	2	2	1.00	24	0.083
89	A	2	2	1.00	24	0.083
90	A	2	2	1.00	24	0.083
91	A	2	2	1.00	24	0.083
92	A	2	2	1.00	22	0.091
93	A	2	2	1.00	24	0.083
94	A	2	2	1.00	24	0.083
95	A	2	2	1.00	24	0.083
96	A	2	2	1.00	24	0.083
97	N/A	5	0	1.00	20	0.000
98	A	20	19	1.13	20	0.950
99	A	16	15	1.09	20	0.750
100	A	13	12	1.06	20	0.600
101	A	9	8	1.00	18	0.444
102	N/A	7	0	1.00	20	0.000
103	N/A	9	0	1.00	20	0.000
104	N/A	2	0	1.00	16	0.000
105	A	13	12	1.35	16	0.750
106	A	12	11	1.29	16	0.688
107	A	11	10	1.24	16	0.625
108	A	7	7	1.05	14	0.500
109	N/A	2	0	1.00	16	0.000
110	N/A	2	0	1.00	16	0.000
111	N/A	3	0	1.00	22	0.000
112	A	15	14	1.78	22	0.636
113	A	14	13	1.73	22	0.591
114	A	13	12	1.81	22	0.545
115	A	11	10	1.63	20	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
116	N/A	3	0	1.00	22	0.000
117	N/A	3	0	1.00	22	0.000
118	A	2	2	1.00	24	0.083
119	A	2	2	1.00	24	0.083
120	A	2	2	1.00	24	0.083
121	A	2	2	1.00	24	0.083
122	A	2	2	1.00	24	0.083
123	A	2	2	1.00	24	0.083
124	A	2	2	1.00	26	0.077
125	A	2	2	1.00	26	0.077
126	A	2	2	1.00	26	0.077
127	A	2	2	1.00	26	0.077
128	A	2	2	1.00	26	0.077
129	A	2	2	1.00	26	0.077
130	A	2	2	1.00	26	0.077
131	A	2	2	1.00	26	0.077
132	A	2	2	1.00	26	0.077
133	A	2	2	1.00	26	0.077
134	A	2	2	1.00	26	0.077
135	A	2	2	1.00	26	0.077
136	A	2	2	1.00	22	0.091
137	A	13	13	1.29	22	0.591
138	A	13	13	1.24	22	0.591
139	A	6	6	1.06	22	0.273
140	A	6	6	1.11	20	0.300
141	A	2	2	1.00	22	0.091
142	A	2	2	1.00	22	0.091
143	A	2	2	1.00	22	0.091
144	A	2	2	1.00	22	0.091
145	A	2	2	1.00	24	0.083
146	A	2	2	1.00	24	0.083
147	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
148	A	2	2	1.00	24	0.083
149	A	2	2	1.00	22	0.091
150	A	2	2	1.00	24	0.083
151	A	2	2	1.00	24	0.083
152	A	2	2	1.00	24	0.083
153	A	2	2	1.00	24	0.083
154	A	2	2	1.00	24	0.083
155	A	2	2	1.00	24	0.083
156	A	2	2	1.00	24	0.083
157	A	2	2	1.00	24	0.083
158	A	2	2	1.00	22	0.091
159	A	2	2	1.00	24	0.083
160	A	2	2	1.00	24	0.083
161	A	2	2	1.00	24	0.083
162	A	2	2	1.00	24	0.083
163	N/A	9	0	1.00	22	0.000
164	A	18	17	1.24	22	0.773
165	A	17	16	1.21	22	0.727
166	A	13	12	1.20	22	0.545
167	A	12	11	1.18	20	0.550
168	N/A	11	0	1.00	22	0.000
169	N/A	13	0	1.00	22	0.000
170	N/A	6	0	1.00	22	0.000
171	A	21	20	1.10	22	0.909
172	A	18	17	1.07	22	0.773
173	A	14	13	1.04	22	0.591
174	A	9	9	1.00	20	0.450
175	N/A	7	0	1.00	22	0.000
176	N/A	10	0	1.00	22	0.000
177	N/A	3	0	1.00	16	0.000
178	A	24	23	1.35	16	1.438

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
179	A	22	21	1.25	16	1.312
180	A	18	17	1.22	16	1.062
181	A	13	12	1.14	14	0.857
182	N/A	3	0	1.00	16	0.000
183	N/A	3	0	1.00	16	0.000
184	A	2	2	1.00	24	0.083
185	A	2	2	1.00	24	0.083
186	A	2	2	1.00	24	0.083
187	A	2	2	1.00	24	0.083
188	A	2	2	1.00	24	0.083
189	A	2	2	1.00	24	0.083
190	A	2	2	1.00	26	0.077
191	A	2	2	1.00	26	0.077
192	A	2	2	1.00	26	0.077
193	A	2	2	1.00	26	0.077
194	A	2	2	1.00	26	0.077
195	A	2	2	1.00	26	0.077
196	A	2	2	1.00	26	0.077
197	A	2	2	1.00	26	0.077
198	A	2	2	1.00	26	0.077
199	A	2	2	1.00	26	0.077
200	A	2	2	1.00	26	0.077
201	A	2	2	1.00	26	0.077
202	A	19	18	1.23	12	1.500
203	A	18	17	1.23	12	1.417
204	A	10	10	1.00	10	1.000
205	F	0	0	N/A	0.000	N/A
206	F	0	0	N/A	0.000	N/A
207	A	27	26	1.27	10	2.600
208	N/A	2	0	1.00	14	0.000
209	A	9	8	1.23	14	0.571
210	A	8	7	1.17	14	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
211	A	7	6	1.18	14	0.429
212	A	6	5	1.11	12	0.417
213	N/A	2	0	1.00	14	0.000
214	N/A	2	0	1.00	14	0.000
215	N/A	6	0	1.00	20	0.000
216	A	17	16	1.09	20	0.800
217	A	13	12	1.06	20	0.600
218	A	10	9	1.00	18	0.500
219	N/A	7	0	1.00	20	0.000
220	N/A	10	0	1.00	20	0.000
221	N/A	8	0	1.00	22	0.000
222	A	16	15	1.12	22	0.682
223	A	12	11	1.11	22	0.500
224	A	11	10	1.10	20	0.500
225	N/A	10	0	1.00	22	0.000
226	N/A	12	0	1.00	22	0.000
227	N/A	1	0	1.00	20	0.000
228	A	9	8	1.22	20	0.400
229	A	8	7	1.13	20	0.350
230	A	7	6	1.10	20	0.300
231	A	6	5	1.03	18	0.278
232	N/A	3	0	1.00	20	0.000
233	N/A	3	0	1.00	20	0.000
234	N/A	1	0	1.00	22	0.000
235	A	5	5	1.11	22	0.227
236	A	5	5	1.15	22	0.227
237	A	2	2	0.98	20	0.100
238	N/A	1	0	1.00	22	0.000
239	N/A	1	0	1.00	22	0.000
240	N/A	1	0	1.00	22	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
241	A	6	6	1.08	22	0.273
242	A	6	6	1.18	22	0.273
243	A	2	2	0.98	20	0.100
244	N/A	1	0	1.00	22	0.000
245	N/A	1	0	1.00	22	0.000
246	N/A	1	0	1.00	20	0.000
247	A	8	7	1.09	20	0.350
248	A	7	6	1.06	20	0.300
249	A	6	5	1.02	20	0.250
250	A	3	3	1.00	18	0.167
251	N/A	1	0	1.00	20	0.000
252	N/A	1	0	1.00	20	0.000
253	N/A	2	0	1.00	16	0.000
254	A	10	9	1.18	16	0.562
255	A	9	8	1.16	16	0.500
256	A	5	5	1.00	14	0.357
257	N/A	2	0	1.00	16	0.000
258	N/A	2	0	1.00	16	0.000
259	N/A	5	0	1.00	22	0.000
260	A	17	16	1.07	22	0.727
261	A	14	13	1.03	22	0.591
262	A	8	8	1.00	20	0.400
263	N/A	7	0	1.00	22	0.000
264	N/A	9	0	1.00	22	0.000
265	N/A	1	0	1.00	22	0.000
266	A	6	6	1.08	22	0.273
267	A	6	6	1.11	22	0.273
268	A	6	6	1.16	22	0.273
269	A	2	2	1.07	20	0.100
270	N/A	1	0	1.00	22	0.000
271	N/A	1	0	1.00	22	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
272	N/A	1	0	1.00	24	0.000
273	A	11	10	1.36	24	0.417
274	A	10	9	1.28	24	0.375
275	A	6	6	1.26	22	0.273
276	N/A	3	0	1.00	24	0.000
277	N/A	3	0	1.00	24	0.000
278	N/A	1	0	1.00	24	0.000
279	A	6	6	1.04	24	0.250
280	A	6	6	1.10	24	0.250
281	A	2	2	1.11	22	0.091
282	N/A	1	0	1.00	24	0.000
283	N/A	1	0	1.00	24	0.000
284	N/A	1	0	1.00	20	0.000
285	A	4	4	1.08	20	0.200
286	A	4	4	1.00	20	0.200
287	A	2	2	1.00	18	0.111
288	N/A	1	0	1.00	20	0.000
289	N/A	1	0	1.00	20	0.000
290	N/A	1	0	1.00	22	0.000
291	A	11	10	1.19	22	0.455
292	A	10	9	1.11	22	0.409
293	A	6	6	1.04	22	0.273
294	A	5	4	1.00	20	0.200
295	N/A	1	0	1.00	22	0.000
296	N/A	1	0	1.00	22	0.000
297	N/A	3	0	1.00	22	0.000
298	A	14	13	1.72	22	0.591
299	A	13	12	1.82	22	0.545
300	A	11	10	1.64	20	0.500
301	N/A	3	0	1.00	22	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
302	N/A	3	0	1.00	22	0.000
303	N/A	2	0	1.00	16	0.000
304	A	17	16	1.14	16	1.000
305	A	13	12	1.12	16	0.750
306	A	10	9	1.06	14	0.643
307	N/A	2	0	1.00	16	0.000
308	N/A	2	0	1.00	16	0.000
309	N/A	1	0	1.00	22	0.000
310	A	6	6	1.11	22	0.273
311	A	6	6	1.07	22	0.273
312	A	6	6	1.16	22	0.273
313	A	2	2	0.99	20	0.100
314	N/A	1	0	1.00	22	0.000
315	N/A	1	0	1.00	22	0.000
316	N/A	1	0	1.00	24	0.000
317	A	6	6	1.06	24	0.250
318	A	6	6	1.09	24	0.250
319	A	2	2	1.10	22	0.091
320	N/A	1	0	1.00	24	0.000
321	N/A	1	0	1.00	24	0.000
322	N/A	1	0	1.00	24	0.000
323	A	12	11	1.15	24	0.458
324	A	10	9	1.15	24	0.375
325	A	8	7	1.14	22	0.318
326	N/A	3	0	1.00	24	0.000
327	N/A	3	0	1.00	24	0.000
328	A	7	7	1.16	18	0.389
329	A	5	5	1.13	18	0.278
330	A	5	5	1.13	18	0.278
331	A	3	3	1.00	18	0.167
332	A	3	3	1.00	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
333	A	5	5	1.07	18	0.278
334	A	5	5	1.13	18	0.278
335	A	7	7	1.11	18	0.389
336	A	9	9	1.09	18	0.500
337	A	7	7	1.10	18	0.389
338	A	7	7	1.05	18	0.389
339	A	5	5	1.00	18	0.278
340	A	5	5	1.00	18	0.278
341	A	7	7	1.10	18	0.389
342	A	7	7	1.10	18	0.389
343	A	9	9	1.13	18	0.500
344	A	7	7	1.15	18	0.389
345	A	5	5	1.12	18	0.278
346	A	5	5	1.12	18	0.278
347	A	3	3	1.00	18	0.167
348	A	3	3	1.00	18	0.167
349	A	5	5	1.06	18	0.278
350	A	5	5	1.12	18	0.278
351	A	7	7	1.10	18	0.389
352	A	9	9	1.08	18	0.500
353	A	7	7	1.09	18	0.389
354	A	7	7	1.05	18	0.389
355	A	5	5	1.00	18	0.278
356	A	5	5	1.00	18	0.278
357	A	7	7	1.09	18	0.389
358	A	7	7	1.09	18	0.389
359	A	9	9	1.12	18	0.500
360	A	2	2	1.00	8	0.250
361	A	2	2	1.00	14	0.143
362	A	2	2	1.00	14	0.143
363	A	2	2	1.00	14	0.143
364	A	2	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
365	A	2	2	1.00	14	0.143
366	A	2	2	1.00	14	0.143
367	A	2	2	1.00	14	0.143
368	A	2	2	1.00	23	0.087
369	A	2	2	1.00	23	0.087
370	A	2	2	1.00	23	0.087
371	A	2	2	1.00	21	0.095
372	A	2	2	1.00	23	0.087
373	A	2	2	1.00	23	0.087
374	A	2	2	1.00	23	0.087
375	A	2	2	1.00	23	0.087
376	A	2	2	1.00	25	0.080
377	A	2	2	1.00	25	0.080
378	A	2	2	1.00	23	0.087
379	N/A	2	0	1.00	25	0.000
380	N/A	2	0	1.00	25	0.000
381	N/A	2	0	1.00	25	0.000
382	A	2	2	1.00	23	0.087
383	A	2	2	1.00	23	0.087
384	A	2	2	1.00	23	0.087
385	A	2	2	1.00	21	0.095
386	N/A	2	0	1.00	23	0.000
387	N/A	2	0	1.00	23	0.000
388	N/A	2	0	1.00	23	0.000
389	A	2	2	1.00	25	0.080
390	A	2	2	1.00	25	0.080
391	A	2	2	1.00	23	0.087
392	N/A	2	0	1.00	25	0.000
393	N/A	2	0	1.00	25	0.000
394	N/A	2	0	1.00	25	0.000
395	A	2	2	1.00	8	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
396	A	2	2	1.00	10	0.200
397	A	2	2	1.00	10	0.200

CHAPTER 3

Listing of integrals

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3.205 $\int x^3 \cos^2(x)\cot^3(x)\,dx$ . . . 1867
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3.208 $\int (c+dx)^m \tan(a+bx)\,dx$ . . . 1901
3.209 $\int (c+dx)^4 \tan(a+bx)\,dx$ . . . 1907
3.210 $\int (c+dx)^3 \tan(a+bx)\,dx$ . . . 1916
3.211 $\int (c+dx)^2 \tan(a+bx)\,dx$ . . . 1924
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3.221 $\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx$ . . . 1999
3.222 $\int (c+dx)^3 \sin^2(a+bx)\tan(a+bx)\,dx$ . . . 2006
3.223 $\int (c+dx)^2 \sin^2(a+bx)\tan(a+bx)\,dx$ . . . 2019
3.224 $\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx$ . . . 2029
3.225 $\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx$ . . . 2037
3.226 $\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$ . . . 2044
3.227 $\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$ . . . 2052
3.228 $\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx$ . . . 2057
3.229 $\int (c+dx)^3 \csc(a+bx)\sec(a+bx)\,dx$ . . . 2068
3.230 $\int (c+dx)^2 \csc(a+bx)\sec(a+bx)\,dx$ . . . 2077
3.231 $\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx$ . . . 2085
3.232 $\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$ . . . 2092
3.233 $\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$ . . . 2097
3.234 $\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx$ . . . 2102
3.235 $\int (c+dx)^3 \csc^2(a+bx)\sec(a+bx)\,dx$ . . . 2107
3.236 $\int (c+dx)^2 \csc^2(a+bx)\sec(a+bx)\,dx$ . . . 2116
3.237 $\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx$ . . . 2125
3.238 $\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$ . . . 2132
3.239 $\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$ . . . 2137
3.240 $\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx$ . . . 2143
3.241 $\int (c+dx)^3 \csc^3(a+bx)\sec(a+bx)\,dx$ . . . 2148
3.242 $\int (c+dx)^2 \csc^3(a+bx)\sec(a+bx)\,dx$ . . . 2156
3.243 $\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx$ . . . 2164
3.244 $\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$ . . . 2171
3.245 $\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$ . . . 2177
3.246 $\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx$ . . . 2182
3.247 $\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx$ . . . 2187
3.248 $\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx$ . . . 2197
3.249 $\int (c+dx)^2 \sec(a+bx)\tan(a+bx)\,dx$ . . . 2206
3.250 $\int (c+dx)\sec(a+bx)\tan(a+bx)\,dx$ . . . 2213

3.251 $\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx$ . . . 2220
3.252 $\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$ . . . 2225
3.253 $\int (c+dx)^m \tan^2(a+bx)\,dx$ . . . 2231
3.254 $\int (c+dx)^3 \tan^2(a+bx)\,dx$ . . . 2237
3.255 $\int (c+dx)^2 \tan^2(a+bx)\,dx$ . . . 2246
3.256 $\int (c+dx) \tan^2(a+bx)\,dx$ . . . 2254
3.257 $\int \frac{\tan^2(a+bx)}{c+dx}\,dx$ . . . 2260
3.258 $\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx$ . . . 2266
3.259 $\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx$ . . . 2272
3.260 $\int (c+dx)^3 \sin(a+bx)\tan^2(a+bx)\,dx$ . . . 2278
3.261 $\int (c+dx)^2 \sin(a+bx)\tan^2(a+bx)\,dx$ . . . 2290
3.262 $\int (c+dx) \sin(a+bx)\tan^2(a+bx)\,dx$ . . . 2299
3.263 $\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$ . . . 2307
3.264 $\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$ . . . 2314
3.265 $\int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx$ . . . 2322
3.266 $\int (c+dx)^4 \csc(a+bx)\sec^2(a+bx)\,dx$ . . . 2327
3.267 $\int (c+dx)^3 \csc(a+bx)\sec^2(a+bx)\,dx$ . . . 2337
3.268 $\int (c+dx)^2 \csc(a+bx)\sec^2(a+bx)\,dx$ . . . 2346
3.269 $\int (c+dx) \csc(a+bx)\sec^2(a+bx)\,dx$ . . . 2354
3.270 $\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$ . . . 2361
3.271 $\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$ . . . 2366
3.272 $\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx$ . . . 2371
3.273 $\int (c+dx)^3 \csc^2(a+bx)\sec^2(a+bx)\,dx$ . . . 2376
3.274 $\int (c+dx)^2 \csc^2(a+bx)\sec^2(a+bx)\,dx$ . . . 2386
3.275 $\int (c+dx) \csc^2(a+bx)\sec^2(a+bx)\,dx$ . . . 2395
3.276 $\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$ . . . 2402
3.277 $\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$ . . . 2408
3.278 $\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx$ . . . 2414
3.279 $\int (c+dx)^3 \csc^3(a+bx)\sec^2(a+bx)\,dx$ . . . 2419
3.280 $\int (c+dx)^2 \csc^3(a+bx)\sec^2(a+bx)\,dx$ . . . 2429
3.281 $\int (c+dx) \csc^3(a+bx)\sec^2(a+bx)\,dx$ . . . 2438
3.282 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$ . . . 2446
3.283 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$ . . . 2452
3.284 $\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx$ . . . 2457
3.285 $\int x^3 \csc^3(a+bx)\sec^2(a+bx)\,dx$ . . . 2462
3.286 $\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx$ . . . 2471
3.287 $\int x \csc^3(a+bx)\sec^2(a+bx)\,dx$ . . . 2480

3.288 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx$ . . . 2487
3.289 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$ . . . 2493
3.290 $\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx$ . . . 2499
3.291 $\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx$ . . . 2504
3.292 $\int (c+dx)^3 \sec^2(a+bx)\tan(a+bx)\,dx$ . . . 2515
3.293 $\int (c+dx)^2 \sec^2(a+bx)\tan(a+bx)\,dx$ . . . 2524
3.294 $\int (c+dx) \sec^2(a+bx)\tan(a+bx)\,dx$ . . . 2532
3.295 $\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$ . . . 2538
3.296 $\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$ . . . 2544
3.297 $\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx$ . . . 2550
3.298 $\int (c+dx)^3 \sec(a+bx)\tan^2(a+bx)\,dx$ . . . 2555
3.299 $\int (c+dx)^2 \sec(a+bx)\tan^2(a+bx)\,dx$ . . . 2569
3.300 $\int (c+dx) \sec(a+bx)\tan^2(a+bx)\,dx$ . . . 2581
3.301 $\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx$ . . . 2590
3.302 $\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$ . . . 2596
3.303 $\int (c+dx)^m \tan^3(a+bx)\,dx$ . . . 2602
3.304 $\int (c+dx)^3 \tan^3(a+bx)\,dx$ . . . 2608
3.305 $\int (c+dx)^2 \tan^3(a+bx)\,dx$ . . . 2621
3.306 $\int (c+dx) \tan^3(a+bx)\,dx$ . . . 2631
3.307 $\int \frac{\tan^3(a+bx)}{c+dx}\,dx$ . . . 2639
3.308 $\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx$ . . . 2645
3.309 $\int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx$ . . . 2651
3.310 $\int (c+dx)^4 \csc(a+bx)\sec^3(a+bx)\,dx$ . . . 2656
3.311 $\int (c+dx)^3 \csc(a+bx)\sec^3(a+bx)\,dx$ . . . 2665
3.312 $\int (c+dx)^2 \csc(a+bx)\sec^3(a+bx)\,dx$ . . . 2674
3.313 $\int (c+dx) \csc(a+bx)\sec^3(a+bx)\,dx$ . . . 2683
3.314 $\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$ . . . 2691
3.315 $\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$ . . . 2697
3.316 $\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx$ . . . 2702
3.317 $\int (c+dx)^3 \csc^2(a+bx)\sec^3(a+bx)\,dx$ . . . 2707
3.318 $\int (c+dx)^2 \csc^2(a+bx)\sec^3(a+bx)\,dx$ . . . 2718
3.319 $\int (c+dx) \csc^2(a+bx)\sec^3(a+bx)\,dx$ . . . 2729
3.320 $\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$ . . . 2736
3.321 $\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$ . . . 2742
3.322 $\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx$ . . . 2747
3.323 $\int (c+dx)^3 \csc^3(a+bx)\sec^3(a+bx)\,dx$ . . . 2752
3.324 $\int (c+dx)^2 \csc^3(a+bx)\sec^3(a+bx)\,dx$ . . . 2762

3.325 $\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx$ . . . 2772
3.326 $\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$ . . . 2781
3.327 $\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$ . . . 2787
3.328 $\int x\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx$ . . . 2793
3.329 $\int x\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx$ . . . 2799
3.330 $\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx$ . . . 2804
3.331 $\int \frac{x\sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx$ . . . 2809
3.332 $\int \frac{x\sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx$ . . . 2814
3.333 $\int \frac{x\sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx$ . . . 2819
3.334 $\int \frac{x\sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx$ . . . 2824
3.335 $\int \frac{x\sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx$ . . . 2829
3.336 $\int x\sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx$ . . . 2835
3.337 $\int x\sec^{\frac{7}{2}}(a+bx)\sin(a+bx)\,dx$ . . . 2841
3.338 $\int x\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx$ . . . 2847
3.339 $\int x\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx$ . . . 2853
3.340 $\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx$ . . . 2858
3.341 $\int \frac{x\sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx$ . . . 2864
3.342 $\int \frac{x\sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx$ . . . 2870
3.343 $\int \frac{x\sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx$ . . . 2876
3.344 $\int x\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)\,dx$ . . . 2882
3.345 $\int x\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)\,dx$ . . . 2888
3.346 $\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx$ . . . 2893
3.347 $\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx$ . . . 2898
3.348 $\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$ . . . 2903
3.349 $\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx$ . . . 2908
3.350 $\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx$ . . . 2913
3.351 $\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx$ . . . 2918
3.352 $\int x\cos(a+bx)\csc^{\frac{9}{2}}(a+bx)\,dx$ . . . 2924
3.353 $\int x\cos(a+bx)\csc^{\frac{7}{2}}(a+bx)\,dx$ . . . 2930
3.354 $\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx$ . . . 2936
3.355 $\int x\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)\,dx$ . . . 2942
3.356 $\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx$ . . . 2947
3.357 $\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$ . . . 2953

3.358 $\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx$ . . . 2959
3.359 $\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx$ . . . 2965
3.360 $\int x\csc(x)\sin(3x)\,dx$ . . . 2971
3.361 $\int (c+dx)^4\csc(x)\sin(3x)\,dx$ . . . 2976
3.362 $\int (c+dx)^3\csc(x)\sin(3x)\,dx$ . . . 2983
3.363 $\int (c+dx)^2\csc(x)\sin(3x)\,dx$ . . . 2989
3.364 $\int (c+dx)\csc(x)\sin(3x)\,dx$ . . . 2995
3.365 $\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx$ . . . 3000
3.366 $\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx$ . . . 3005
3.367 $\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx$ . . . 3011
3.368 $\int (c+dx)^4\csc(a+bx)\sin(3a+3bx)\,dx$ . . . 3017
3.369 $\int (c+dx)^3\csc(a+bx)\sin(3a+3bx)\,dx$ . . . 3024
3.370 $\int (c+dx)^2\csc(a+bx)\sin(3a+3bx)\,dx$ . . . 3031
3.371 $\int (c+dx)\csc(a+bx)\sin(3a+3bx)\,dx$ . . . 3037
3.372 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx$ . . . 3042
3.373 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$ . . . 3047
3.374 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$ . . . 3053
3.375 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx$ . . . 3060
3.376 $\int (c+dx)^3\csc^2(a+bx)\sin(3a+3bx)\,dx$ . . . 3067
3.377 $\int (c+dx)^2\csc^2(a+bx)\sin(3a+3bx)\,dx$ . . . 3075
3.378 $\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx$ . . . 3082
3.379 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$ . . . 3088
3.380 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$ . . . 3094
3.381 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$ . . . 3100
3.382 $\int (c+dx)^4\sec(a+bx)\sin(3a+3bx)\,dx$ . . . 3106
3.383 $\int (c+dx)^3\sec(a+bx)\sin(3a+3bx)\,dx$ . . . 3115
3.384 $\int (c+dx)^2\sec(a+bx)\sin(3a+3bx)\,dx$ . . . 3123
3.385 $\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx$ . . . 3130
3.386 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx$ . . . 3136
3.387 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$ . . . 3142
3.388 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$ . . . 3148
3.389 $\int (c+dx)^3\sec^2(a+bx)\sin(3a+3bx)\,dx$ . . . 3154
3.390 $\int (c+dx)^2\sec^2(a+bx)\sin(3a+3bx)\,dx$ . . . 3162
3.391 $\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx$ . . . 3170
3.392 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$ . . . 3177
3.393 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$ . . . 3183

3.394 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$ 3189
3.395 $\int x\cos(2x)\sec(x)\,dx$ 3195
3.396 $\int x\cos(2x)\sec^2(x)\,dx$ 3200
3.397 $\int x\cos(2x)\sec^3(x)\,dx$ 3206

3.1 $\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 137

$$\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx$$
$$= -\frac{2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}$$
$$-\frac{2^{-3-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2ib(c+dx)}{d}\right)}{b}$$

output

```
-2^(-3-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*
b*(d*x+c)/d)^m)-2^(-3-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a
-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

$$\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{2^{-3-m}e^{-\frac{2i(bc+ad)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(e^{4ia}\left(\frac{ib(c+dx)}{d}\right)^m\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)+e^{\frac{4ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m\Gamma\right.}{b}$$

input
```
Integrate[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
-((2^(-3 - m)*(c + d*x)^m*(E^((4*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m,
((-2*I)*b*(c + d*x))/d] + E^(((4*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma
[1 + m, ((2*I)*b*(c + d*x))/d]))/(b*E^(((2*I)*(b*c + a*d))/d)*((b^2*(c + d
*x)^2)/d^2)^m))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4906, 27, 3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cos(a+bx)(c+dx)^m\,dx$$

$$\downarrow 4906$$

$$\int \frac{1}{2}\sin(2a+2bx)(c+dx)^m dx$$

$$\downarrow 27$$

$$\frac{1}{2}\int (c+dx)^m \sin(2a+2bx)dx$$

$$\downarrow 3042$$

$$\frac{1}{2}\int (c+dx)^m \sin(2a+2bx)dx$$

$$\downarrow 3789$$

$$\frac{1}{2}\left(\frac{1}{2}i\int e^{-2i(a+bx)}(c+dx)^m dx - \frac{1}{2}i\int e^{2i(a+bx)}(c+dx)^m dx\right)$$

$$\downarrow 2612$$

$$\frac{1}{2}\left(-\frac{2^{-m-2}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-m-2}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-}}{b}\right.$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-((2^(-2 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*
(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)) - (2^(-2 - m)*(c + d*x)^m*Gam
ma[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*
x))/d)^m))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [F]

$$\int (dx+c)^m \cos(bx+a) \sin(bx+a)\, dx$$

input

```
int((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x)
```

output

```
int((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.70

$$\int (c+dx)^m \cos(a+bx) \sin(a+bx)\, dx = -\frac{e^{\left(-\frac{dm \log\left(-\frac{2ib}{d}\right)+2ibc-2iad}{d}\right)} \Gamma\left(m+1, -\frac{2(ibdx+ibc)}{d}\right) + e^{\left(-\frac{dm \log\left(\frac{2ib}{d}\right)-2ibc+2iad}{d}\right)} \Gamma\left(m+1, -\frac{2(-ibdx-ibc)}{d}\right)}{8b}$$

input

```
integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output

```
-1/8*(e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d)*gamma(m + 1, -2*(I*b*
d*x + I*b*c)/d) + e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a*d)/d)*gamma(m +
1, -2*(-I*b*d*x - I*b*c)/d))/b
```

Sympy [F]

$$\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx = \int (c+dx)^m \sin(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cos(b*x+a)*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**m*sin(a + b*x)*cos(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\sin(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a), x)
```

Giac [F]

$$\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\sin(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)\,(c+d\,x)^m\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\sin(bx+a)\,dx$$

input
```
int((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x)
```

output
```
int((c + d*x)**m*cos(a + b*x)*sin(a + b*x),x)
```

3.2 $\int (c+dx)^4 \cos(a+bx)\sin(a+bx)\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 148

$$\begin{aligned}\int (c+dx)^4 \cos(a+bx)\sin(a+bx)\,dx = {} & \frac{3d^2(c+dx)^2}{4b^3} - \frac{(c+dx)^4}{4b} \\ & - \frac{3d^3(c+dx)\cos(a+bx)\sin(a+bx)}{2b^4} \\ & + \frac{d(c+dx)^3\cos(a+bx)\sin(a+bx)}{b^2} \\ & + \frac{3d^4\sin^2(a+bx)}{4b^5} - \frac{3d^2(c+dx)^2\sin^2(a+bx)}{2b^3} \\ & + \frac{(c+dx)^4\sin^2(a+bx)}{2b}\end{aligned}$$

output

```
3/4*d^2*(d*x+c)^2/b^3-1/4*(d*x+c)^4/b-3/2*d^3*(d*x+c)*cos(b*x+a)*sin(b*x+a
)/b^4+d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2+3/4*d^4*sin(b*x+a)^2/b^5-3/2*d
^2*(d*x+c)^2*sin(b*x+a)^2/b^3+1/2*(d*x+c)^4*sin(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.58

$$\int (c+dx)^4 \cos(a+bx)\sin(a+bx)\,dx$$
$$= \frac{-2(3d^4 - 6b^2d^2(c+dx)^2 + 2b^4(c+dx)^4)\cos(2(a+bx)) + 4bd(c+dx)\left(-3d^2 + 2b^2(c+dx)^2\right)\sin(2(a+$$
$$16b^5$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-2*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)] +
 4*b*d*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(16*b^5)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {4904, 3042, 3792, 17, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sin(a+bx)\cos(a+bx)\,dx$$
$$\downarrow 4904$$
$$\frac{(c+dx)^4\sin^2(a+bx)}{2b} - \frac{2d\int (c+dx)^3\sin^2(a+bx)dx}{b}$$
$$\downarrow 3042$$
$$\frac{(c+dx)^4\sin^2(a+bx)}{2b} - \frac{2d\int (c+dx)^3\sin(a+bx)^2dx}{b}$$
$$\downarrow 3792$$
$$\frac{(c+dx)^4\sin^2(a+bx)}{2b} -$$
$$\frac{2d\left(-\frac{3d^2\int (c+dx)\sin^2(a+bx)dx}{2b^2} + \frac{1}{2}\int (c+dx)^3dx + \frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2} - \frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}\right)}{b}$$

$$\downarrow 17$$

$$\frac{(c+dx)^4\sin^2(a+bx)}{2b} - \frac{2d\left(-\frac{3d^2\int(c+dx)\sin^2(a+bx)dx}{2b^2} + \frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2} - \frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b} + \frac{(c+dx)^4}{8d}\right)}{b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^4\sin^2(a+bx)}{2b} - \frac{2d\left(-\frac{3d^2\int(c+dx)\sin(a+bx)^2dx}{2b^2} + \frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2} - \frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b} + \frac{(c+dx)^4}{8d}\right)}{b}$$

$$\downarrow 3791$$

$$\frac{(c+dx)^4\sin^2(a+bx)}{2b} - \frac{2d\left(-\frac{3d^2\left(\frac{1}{2}\int(c+dx)dx + \frac{d\sin^2(a+bx)}{4b^2} - \frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2} + \frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2} - \frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b} + \frac{(c+dx)^4}{8d}\right.}{b}$$

$$\downarrow 17$$

$$\frac{(c+dx)^4\sin^2(a+bx)}{2b} - \frac{2d\left(-\frac{3d^2\left(\frac{d\sin^2(a+bx)}{4b^2} - \frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b} + \frac{(c+dx)^2}{4d}\right)}{2b^2} + \frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2} - \frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b} + \frac{(c+dx)^4}{8d}\right)}{b}$$

input
```
Int[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
((c + d*x)^4*Sin[a + b*x]^2)/(2*b) - (2*d*((c + d*x)^4/(8*d) - ((c + d*x)^
3*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (3*d*(c + d*x)^2*Sin[a + b*x]^2)/(4*b
^2) - (3*d^2*((c + d*x)^2/(4*d) - ((c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(2
*b) + (d*Sin[a + b*x]^2)/(4*b^2)))/(2*b^2)))/b
```

Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4904

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{\left(-2(dx+c)^4b^4+6d^2(dx+c)^2b^2-3d^4\right)\cos(2bx+2a)+4d\left((dx+c)^2b^2-\frac{3d^2}{2}\right)(dx+c)b\sin(2bx+2a)+2b^4c^4-6b^2c^2d^2+3d^4}{8b^5}$
risch	$-\frac{\left(2d^4x^4b^4+8b^4cd^3x^3+12b^4c^2d^2x^2+8b^4c^3dx+2b^4c^4-6b^2d^4x^2-12b^2cd^3x-6b^2c^2d^2+3d^4\right)\cos(2bx+2a)}{8b^5}+\frac{d(2b^2d^3$
orering	$\frac{d\left(4d^4x^4b^4+16b^4cd^3x^3+24b^4c^2d^2x^2+16b^4c^3dx+4b^4c^4-9b^2d^4x^2-18b^2cd^3x-9b^2c^2d^2+3d^4\right)\cos(bx+a)\sin(bx+a)}{2b^6(dx+c)}-$
norman	$\frac{\left(2b^4c^4-6b^2c^2d^2+3d^4\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{b^5}+\frac{cd\left(2b^2c^2-3d^2\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{b^4}-\frac{d^4x^4}{4b}-\frac{cd^3x^3}{b}-\frac{3d^2\left(2b^2c^2-d^2\right)x^2}{4b^3}+\frac{2d^4x^3\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{b^2}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/8*((-2*(d*x+c)^4*b^4+6*d^2*(d*x+c)^2*b^2-3*d^4)*cos(2*b*x+2*a)+4*d*((d*x
+c)^2*b^2-3/2*d^2)*(d*x+c)*b*sin(2*b*x+2*a)+2*b^4*c^4-6*b^2*c^2*d^2+3*d^4)
/b^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.72

$$\int (c+dx)^4\cos(a+bx)\sin(a+bx)\,dx$$
$$=\frac{b^4d^4x^4+4\,b^4cd^3x^3+3\,(2\,b^4c^2d^2-b^2d^4)x^2-(2\,b^4d^4x^4+8\,b^4cd^3x^3+2\,b^4c^4-6\,b^2c^2d^2+3\,d^4+6\,(2\,b^4c^2d$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 3*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 - (2*
b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b
^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^
2 + 2*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^
3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)*sin(b*x + a) + 2*(2*b^4*c^3*d - 3*b^2*c
*d^3)*x)/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(143) = 286$.

Time = 0.48 (sec) , antiderivative size = 502, normalized size of antiderivative = 3.39

$$\int (c+dx)^4 \cos(a+bx)\sin(a+bx)\,dx$$
$$= \begin{cases} \frac{c^4 \sin^2(a+bx)}{2b} + \frac{c^3 dx \sin^2(a+bx)}{b} - \frac{c^3 dx \cos^2(a+bx)}{b} + \frac{3c^2d^2x^2\sin^2(a+bx)}{2b} - \frac{3c^2d^2x^2\cos^2(a+bx)}{2b} + \frac{cd^3x^3\sin^2(a+bx)}{b} - \frac{cd^3x} \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\sin(a)\cos(a) \end{cases}$$

input
```
integrate((d*x+c)**4*cos(b*x+a)*sin(b*x+a),x)
```

output
```
Piecewise((c**4*sin(a + b*x)**2/(2*b) + c**3*d*x*sin(a + b*x)**2/b - c**3*
d*x*cos(a + b*x)**2/b + 3*c**2*d**2*x**2*sin(a + b*x)**2/(2*b) - 3*c**2*d*
*2*x**2*cos(a + b*x)**2/(2*b) + c*d**3*x**3*sin(a + b*x)**2/b - c*d**3*x**
3*cos(a + b*x)**2/b + d**4*x**4*sin(a + b*x)**2/(4*b) - d**4*x**4*cos(a +
b*x)**2/(4*b) + c**3*d*sin(a + b*x)*cos(a + b*x)/b**2 + 3*c**2*d**2*x*sin(
a + b*x)*cos(a + b*x)/b**2 + 3*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)/b**2
+ d**4*x**3*sin(a + b*x)*cos(a + b*x)/b**2 - 3*c**2*d**2*sin(a + b*x)**2/(
2*b**3) - 3*c*d**3*x*sin(a + b*x)**2/(2*b**3) + 3*c*d**3*x*cos(a + b*x)**2
/(2*b**3) - 3*d**4*x**2*sin(a + b*x)**2/(4*b**3) + 3*d**4*x**2*cos(a + b*x
)**2/(4*b**3) - 3*c*d**3*sin(a + b*x)*cos(a + b*x)/(2*b**4) - 3*d**4*x*sin
(a + b*x)*cos(a + b*x)/(2*b**4) + 3*d**4*sin(a + b*x)**2/(4*b**5), Ne(b, 0
)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/
5)*sin(a)*cos(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. 2(136) = 272.

Time = 0.06 (sec) , antiderivative size = 586, normalized size of antiderivative = 3.96

$$\int (c+dx)^4 \cos(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/8*(4*c^4*cos(b*x + a)^2 - 16*a*c^3*d*cos(b*x + a)^2/b + 24*a^2*c^2*d^2*
cos(b*x + a)^2/b^2 - 16*a^3*c*d^3*cos(b*x + a)^2/b^3 + 4*a^4*d^4*cos(b*x +
 a)^2/b^4 + 4*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*c^3*d/b -
12*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a*c^2*d^2/b^2 + 12*(2
*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a^2*c*d^3/b^3 - 4*(2*(b*x
+ a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a^3*d^4/b^4 + 6*((2*(b*x + a)^2
- 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 - 12*((2
*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3
/b^3 + 6*((2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2
*a))*a^2*d^4/b^4 + 2*(2*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3
*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3 - 2*(2*(2*(b*x + a)^3 - 3
*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a*d
^4/b^4 + ((2*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*cos(2*b*x + 2*a) - 2*(2*(b*x
 + a)^3 - 3*b*x - 3*a)*sin(2*b*x + 2*a))*d^4/b^4)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.22

$$\int (c+dx)^4 \cos(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{\left(2\,b^4d^4x^4+8\,b^4cd^3x^3+12\,b^4c^2d^2x^2+8\,b^4c^3dx+2\,b^4c^4-6\,b^2d^4x^2-12\,b^2cd^3x-6\,b^2c^2d^2+3\,d^4\right)\cos\left(2\right.}{8\,b^5}$$
$$+\frac{\left(2\,b^3d^4x^3+6\,b^3cd^3x^2+6\,b^3c^2d^2x+2\,b^3c^3d-3\,bd^4x-3\,bcd^3\right)\sin\left(2\,bx+2\,a\right)}{4\,b^5}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/8*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x
 + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3*x - 6*b^2*c^2*d^2 + 3*d^4)*cos
(2*b*x + 2*a)/b^5 + 1/4*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x
 + 2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

Mupad [B] (verification not implemented)

Time = 19.44 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.66

$$\int (c+dx)^4 \cos(a+bx)\sin(a+bx)\,dx$$
$$= \frac{3x^2\cos(2a+2bx)\,(d^4-2b^2c^2d^2)}{4b^3} - \frac{\cos(2a+2bx)\left(\frac{b^4c^4}{2}-\frac{3b^2c^2d^2}{2}+\frac{3d^4}{4}\right)}{2b^5}$$
$$- \frac{3x\sin(2a+2bx)\,(d^4-2b^2c^2d^2)}{4b^4} - \frac{d^4x^4\cos(2a+2bx)}{4b}$$
$$- \frac{\sin(2a+2bx)\,(3cd^3-2b^2c^3d)}{4b^4} + \frac{x\cos(2a+2bx)\,(3cd^3-2b^2c^3d)}{2b^3}$$
$$+ \frac{d^4x^3\sin(2a+2bx)}{2b^2} - \frac{cd^3x^3\cos(2a+2bx)}{b} + \frac{3cd^3x^2\sin(2a+2bx)}{2b^2}$$

input
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^4,x)
```

output
```
(3*x^2*cos(2*a + 2*b*x)*(d^4 - 2*b^2*c^2*d^2))/(4*b^3) - (cos(2*a + 2*b*x)
*((3*d^4)/4 + (b^4*c^4)/2 - (3*b^2*c^2*d^2)/2))/(2*b^5) - (3*x*sin(2*a + 2
*b*x)*(d^4 - 2*b^2*c^2*d^2))/(4*b^4) - (d^4*x^4*cos(2*a + 2*b*x))/(4*b) -
(sin(2*a + 2*b*x)*(3*c*d^3 - 2*b^2*c^3*d))/(4*b^4) + (x*cos(2*a + 2*b*x)*(
3*c*d^3 - 2*b^2*c^3*d))/(2*b^3) + (d^4*x^3*sin(2*a + 2*b*x))/(2*b^2) - (c*
d^3*x^3*cos(2*a + 2*b*x))/b + (3*c*d^3*x^2*sin(2*a + 2*b*x))/(2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.80

$$\int (c+dx)^4 \cos(a+bx)\sin(a+bx)\,dx$$
$$= \frac{-2\cos(bx+a)^2 b^4c^4 - 4\cos(bx+a)^2 b^4c^3dx - 6\cos(bx+a)^2 b^4c^2d^2x^2 - 4\cos(bx+a)^2 b^4c\,d^3x^3 - \cos($$

input
```
int((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x)
```

output
```
( - 2*cos(a + b*x)**2*b**4*c**4 - 4*cos(a + b*x)**2*b**4*c**3*d*x - 6*cos(
a + b*x)**2*b**4*c**2*d**2*x**2 - 4*cos(a + b*x)**2*b**4*c*d**3*x**3 - cos
(a + b*x)**2*b**4*d**4*x**4 + 6*cos(a + b*x)**2*b**2*c**2*d**2 + 6*cos(a +
 b*x)**2*b**2*c*d**3*x + 3*cos(a + b*x)**2*b**2*d**4*x**2 - 3*cos(a + b*x)
**2*d**4 + 4*cos(a + b*x)*sin(a + b*x)*b**3*c**3*d + 12*cos(a + b*x)*sin(a
 + b*x)*b**3*c**2*d**2*x + 12*cos(a + b*x)*sin(a + b*x)*b**3*c*d**3*x**2 +
 4*cos(a + b*x)*sin(a + b*x)*b**3*d**4*x**3 - 6*cos(a + b*x)*sin(a + b*x)*
b*c*d**3 - 6*cos(a + b*x)*sin(a + b*x)*b*d**4*x + 4*sin(a + b*x)**2*b**4*c
**3*d*x + 6*sin(a + b*x)**2*b**4*c**2*d**2*x**2 + 4*sin(a + b*x)**2*b**4*c
*d**3*x**3 + sin(a + b*x)**2*b**4*d**4*x**4 - 6*sin(a + b*x)**2*b**2*c*d**
3*x - 3*sin(a + b*x)**2*b**2*d**4*x**2)/(4*b**5)
```

3.3 $\int (c+dx)^3 \cos(a+bx)\sin(a+bx)\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 120

$$\int (c+dx)^3 \cos(a+bx)\sin(a+bx)\,dx = \frac{3d^3x}{8b^3} - \frac{(c+dx)^3}{4b} - \frac{3d^3\cos(a+bx)\sin(a+bx)}{8b^4} + \frac{3d(c+dx)^2\cos(a+bx)\sin(a+bx)}{4b^2} - \frac{3d^2(c+dx)\sin^2(a+bx)}{4b^3} + \frac{(c+dx)^3\sin^2(a+bx)}{2b}$$

output

```
3/8*d^3*x/b^3-1/4*(d*x+c)^3/b-3/8*d^3*cos(b*x+a)*sin(b*x+a)/b^4+3/4*d*(d*x
+c)^2*cos(b*x+a)*sin(b*x+a)/b^2-3/4*d^2*(d*x+c)*sin(b*x+a)^2/b^3+1/2*(d*x+
c)^3*sin(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.59

$$\int (c+dx)^3 \cos(a+bx)\sin(a+bx)\,dx$$
$$= \frac{-2b(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\cos(2(a+bx)) + 3d\left(-d^2+2b^2(c+dx)^2\right)\sin(2(a+bx))}{16b^4}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-2*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 3*d*(-d^2
+ 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(16*b^4)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {4904, 3042, 3792, 17, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin(a+bx)\cos(a+bx)\,dx$$
$$\downarrow 4904$$
$$\frac{(c+dx)^3\sin^2(a+bx)}{2b} - \frac{3d\int (c+dx)^2\sin^2(a+bx)dx}{2b}$$
$$\downarrow 3042$$
$$\frac{(c+dx)^3\sin^2(a+bx)}{2b} - \frac{3d\int (c+dx)^2\sin(a+bx)^2dx}{2b}$$
$$\downarrow 3792$$
$$\frac{(c+dx)^3\sin^2(a+bx)}{2b} - \frac{3d\left(-\frac{d^2\int \sin^2(a+bx)dx}{2b^2} + \frac{1}{2}\int (c+dx)^2dx + \frac{d(c+dx)\sin^2(a+bx)}{2b^2} - \frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b}$$

$$\downarrow 17$$

$$\frac{(c+dx)^3\sin^2(a+bx)}{2b} - \frac{3d\left(-\frac{d^2\int\sin^2(a+bx)dx}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^3\sin^2(a+bx)}{2b} - \frac{3d\left(-\frac{d^2\int\sin(a+bx)^2dx}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}$$

$$\downarrow 3115$$

$$\frac{(c+dx)^3\sin^2(a+bx)}{2b} - \frac{3d\left(-\frac{d^2\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}$$

$$\downarrow 24$$

$$\frac{(c+dx)^3\sin^2(a+bx)}{2b} - \frac{3d\left(\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}$$

input

```
Int[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x],x]
```

output

```
((c + d*x)^3*Sin[a + b*x]^2)/(2*b) - (3*d*((c + d*x)^3/(6*d) - ((c + d*x)^
2*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (d*(c + d*x)*Sin[a + b*x]^2)/(2*b^2)
- (d^2*(x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)))/(2*b^2)))/(2*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)  Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)  Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))  Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))  Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

method	result
parallelrisch	$\frac{-2\left((dx+c)^2b^2-\frac{3d^2}{2}\right)(dx+c)b\cos(2bx+2a)+3d\left((dx+c)^2b^2-\frac{d^2}{2}\right)\sin(2bx+2a)+2b^3c^3-3cd^2b}{8b^4}$
risch	$-\frac{(2b^2d^3x^3+6b^2cd^2x^2+6b^2c^2dx+2b^2c^3-3d^3x-3cd^2)\cos(2bx+2a)}{8b^3}+\frac{3d(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2)\sin(2bx+2a)}{16b^4}$
orering	$\frac{3d(x^2d^2b^2+2b^2cdx+b^2c^2-d^2)\cos(bx+a)\sin(bx+a)}{2b^4}-\frac{(2x^2d^2b^2+4b^2cdx+2b^2c^2-3d^2)\left(3(dx+c)^2\cos(bx+a)\sin(bx\right.}{8(dx+c)^2}$
norman	$\frac{\left(2b^2c^3-3cd^2\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{b^3}-\frac{d^3x^3}{4b}-\frac{3cd^2x^2}{4b}+\frac{3d\left(2b^2c^2-d^2\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{4b^4}-\frac{3d\left(2b^2c^2-d^2\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3}{4b^4}-\frac{3d\left(2b^2c^2-d^2\right)x}{8b^3}$
derivativedivides	$\frac{a^3d^3\cos(bx+a)^2}{2b^3}-\frac{3a^2cd^2\cos(bx+a)^2}{2b^2}+\frac{3a^2d^3\left(-\frac{(bx+a)\cos(bx+a)^2}{2}+\frac{\sin(bx+a)\cos(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}\right)}{b^3}+\frac{3ac^2d\cos(bx+a)^2}{2b}-\frac{6a}{}$
default	$\frac{a^3d^3\cos(bx+a)^2}{2b^3}-\frac{3a^2cd^2\cos(bx+a)^2}{2b^2}+\frac{3a^2d^3\left(-\frac{(bx+a)\cos(bx+a)^2}{2}+\frac{\sin(bx+a)\cos(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}\right)}{b^3}+\frac{3ac^2d\cos(bx+a)^2}{2b}-\frac{6a}{}$

input
```
int((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/8*(-2*((d*x+c)^2*b^2-3/2*d^2)*(d*x+c)*b*cos(2*b*x+2*a)+3*d*((d*x+c)^2*b^
2-1/2*d^2)*sin(2*b*x+2*a)+2*b^3*c^3-3*c*d^2*b)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.38

$$\int (c+dx)^3\cos(a+bx)\sin(a+bx)\,dx$$
$$=\frac{2\,b^3d^3x^3+6\,b^3cd^2x^2-2\,(2\,b^3d^3x^3+6\,b^3cd^2x^2+2\,b^3c^3-3\,bcd^2+3\,(2\,b^3c^2d-bd^3)x)\cos(bx+a)^2+3\,(2}{8\,b^4}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2
+ 2*b^3*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2
*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a
) + 3*(2*b^3*c^2*d - b*d^3)*x)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(116) = 232$.

Time $= 0.36$ (sec) , antiderivative size $= 342$, normalized size of antiderivative $= 2.85$

$$\int (c+dx)^3 \cos(a+bx)\sin(a+bx)\,dx$$
$$= \begin{cases} \frac{c^3 \sin^2(a+bx)}{2b} + \frac{3c^2 dx \sin^2(a+bx)}{4b} - \frac{3c^2 dx \cos^2(a+bx)}{4b} + \frac{3cd^2x^2 \sin^2(a+bx)}{4b} - \frac{3cd^2x^2\cos^2(a+bx)}{4b} + \frac{d^3x^3\sin^2(a+bx)}{4b} - \frac{d^3x^3}{4b} \\ \left(c^3 x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin(a)\cos(a) \end{cases}$$

input
```
integrate((d*x+c)**3*cos(b*x+a)*sin(b*x+a),x)
```

output
```
Piecewise((c**3*sin(a + b*x)**2/(2*b) + 3*c**2*d*x*sin(a + b*x)**2/(4*b) -
 3*c**2*d*x*cos(a + b*x)**2/(4*b) + 3*c*d**2*x**2*sin(a + b*x)**2/(4*b) -
3*c*d**2*x**2*cos(a + b*x)**2/(4*b) + d**3*x**3*sin(a + b*x)**2/(4*b) - d*
*3*x**3*cos(a + b*x)**2/(4*b) + 3*c**2*d*sin(a + b*x)*cos(a + b*x)/(4*b**2
) + 3*c*d**2*x*sin(a + b*x)*cos(a + b*x)/(2*b**2) + 3*d**3*x**2*sin(a + b*
x)*cos(a + b*x)/(4*b**2) - 3*c*d**2*sin(a + b*x)**2/(4*b**3) - 3*d**3*x*si
n(a + b*x)**2/(8*b**3) + 3*d**3*x*cos(a + b*x)**2/(8*b**3) - 3*d**3*sin(a
+ b*x)*cos(a + b*x)/(8*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d*
*2*x**3 + d**3*x**4/4)*sin(a)*cos(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(108) = 216.

Time = 0.05 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.85

$$\int (c+dx)^3 \cos(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{8c^3\cos(bx+a)^2 - \frac{24ac^2d\cos(bx+a)^2}{b} + \frac{24a^2cd^2\cos(bx+a)^2}{b^2} - \frac{8a^3d^3\cos(bx+a)^2}{b^3} + \frac{6(2(bx+a)\cos(2bx+2a) - \sin(2bx+2a}{b}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/16*(8*c^3*cos(b*x + a)^2 - 24*a*c^2*d*cos(b*x + a)^2/b + 24*a^2*c*d^2*c
os(b*x + a)^2/b^2 - 8*a^3*d^3*cos(b*x + a)^2/b^3 + 6*(2*(b*x + a)*cos(2*b*
x + 2*a) - sin(2*b*x + 2*a))*c^2*d/b - 12*(2*(b*x + a)*cos(2*b*x + 2*a) -
sin(2*b*x + 2*a))*a*c*d^2/b^2 + 6*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*
x + 2*a))*a^2*d^3/b^3 + 6*((2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x +
 a)*sin(2*b*x + 2*a))*c*d^2/b^2 - 6*((2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a)
- 2*(b*x + a)*sin(2*b*x + 2*a))*a*d^3/b^3 + (2*(2*(b*x + a)^3 - 3*b*x - 3*
a)*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*d^3/b^3)/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.01

$$\int (c+dx)^3 \cos(a+bx)\sin(a+bx)\,dx$$
$$= -\frac{(2b^3d^3x^3 + 6b^3cd^2x^2 + 6b^3c^2dx + 2b^3c^3 - 3bd^3x - 3bcd^2)\cos(2bx+2a)}{8b^4}$$
$$+ \frac{3(2b^2d^3x^2 + 4b^2cd^2x + 2b^2c^2d - d^3)\sin(2bx+2a)}{16b^4}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^
3*x - 3*b*c*d^2)*cos(2*b*x + 2*a)/b^4 + 3/16*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*
x + 2*b^2*c^2*d - d^3)*sin(2*b*x + 2*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 19.23 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.38

$$\begin{aligned}\int (c+dx)^3 \cos(a+bx)\sin(a+bx)\,dx = {} & \frac{\cos(2a+2bx)\left(\frac{3cd^2}{4}-\frac{b^2c^3}{2}\right)}{2b^3} \\ & - \frac{3\sin(2a+2bx)\,(d^3-2b^2c^2d)}{16b^4} \\ & - \frac{d^3x^3\cos(2a+2bx)}{4b} \\ & + \frac{3d^3x^2\sin(2a+2bx)}{8b^2} \\ & + \frac{3x\cos(2a+2bx)\,(d^3-2b^2c^2d)}{8b^3} \\ & + \frac{3cd^2x\sin(2a+2bx)}{4b^2} \\ & - \frac{3cd^2x^2\cos(2a+2bx)}{4b}\end{aligned}$$

input
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^3,x)
```

output
```
(cos(2*a + 2*b*x)*((3*c*d^2)/4 - (b^2*c^3)/2))/(2*b^3) - (3*sin(2*a + 2*b*
x)*(d^3 - 2*b^2*c^2*d))/(16*b^4) - (d^3*x^3*cos(2*a + 2*b*x))/(4*b) + (3*d
^3*x^2*sin(2*a + 2*b*x))/(8*b^2) + (3*x*cos(2*a + 2*b*x)*(d^3 - 2*b^2*c^2*
d))/(8*b^3) + (3*c*d^2*x*sin(2*a + 2*b*x))/(4*b^2) - (3*c*d^2*x^2*cos(2*a
+ 2*b*x))/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.20

$$\int (c+dx)^3 \cos(a+bx)\sin(a+bx)\,dx$$
$$= \frac{-4\cos(bx+a)^2 b^3c^3 - 6\cos(bx+a)^2 b^3c^2dx - 6\cos(bx+a)^2 b^3cd^2x^2 - 2\cos(bx+a)^2 b^3d^3x^3 + 6\cos($$

input
```
int((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x)
```

output

```
( - 4*cos(a + b*x)**2*b**3*c**3 - 6*cos(a + b*x)**2*b**3*c**2*d*x - 6*cos(
a + b*x)**2*b**3*c*d**2*x**2 - 2*cos(a + b*x)**2*b**3*d**3*x**3 + 6*cos(a
+ b*x)**2*b*c*d**2 + 3*cos(a + b*x)**2*b*d**3*x + 6*cos(a + b*x)*sin(a + b
*x)*b**2*c**2*d + 12*cos(a + b*x)*sin(a + b*x)*b**2*c*d**2*x + 6*cos(a + b
*x)*sin(a + b*x)*b**2*d**3*x**2 - 3*cos(a + b*x)*sin(a + b*x)*d**3 + 6*sin
(a + b*x)**2*b**3*c**2*d*x + 6*sin(a + b*x)**2*b**3*c*d**2*x**2 + 2*sin(a
+ b*x)**2*b**3*d**3*x**3 - 3*sin(a + b*x)**2*b*d**3*x)/(8*b**4)
```

3.4 $\int (c+dx)^2 \cos(a+bx)\sin(a+bx)\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 80

$$\int (c+dx)^2 \cos(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^2}{4b} + \frac{d(c+dx)\cos(a+bx)\sin(a+bx)}{2b^2} - \frac{d^2\sin^2(a+bx)}{4b^3} + \frac{(c+dx)^2\sin^2(a+bx)}{2b}$$

output
```
-1/4*(d*x+c)^2/b+1/2*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2-1/4*d^2*sin(b*x+a
)^2/b^3+1/2*(d*x+c)^2*sin(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int (c+dx)^2 \cos(a+bx)\sin(a+bx)\,dx = \frac{(d^2 - 2b^2(c+dx)^2)\cos(2(a+bx)) + 2bd(c+dx)\sin(2(a+bx))}{8b^3}$$

input
```
Integrate[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
((d^2 - 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 2*b*d*(c + d*x)*Sin[2*(a + b
*x)])/(8*b^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4904, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin(a+bx)\cos(a+bx)\,dx$$

$$\downarrow 4904$$

$$\frac{(c+dx)^2\sin^2(a+bx)}{2b} - \frac{d\int (c+dx)\sin^2(a+bx)dx}{b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^2\sin^2(a+bx)}{2b} - \frac{d\int (c+dx)\sin(a+bx)^2dx}{b}$$

$$\downarrow 3791$$

$$\frac{(c+dx)^2\sin^2(a+bx)}{2b} - \frac{d\left(\frac{1}{2}\int (c+dx)dx + \frac{d\sin^2(a+bx)}{4b^2} - \frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right)}{b}$$

$$\downarrow 17$$

$$\frac{(c+dx)^2\sin^2(a+bx)}{2b} - \frac{d\left(\frac{d\sin^2(a+bx)}{4b^2} - \frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b} + \frac{(c+dx)^2}{4d}\right)}{b}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
((c + d*x)^2*Sin[a + b*x]^2)/(2*b) - (d*((c + d*x)^2/(4*d) - ((c + d*x)*Co
s[a + b*x]*Sin[a + b*x])/(2*b) + (d*Sin[a + b*x]^2)/(4*b^2)))/b
```

Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4904

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

method	result
parallelrisch	$\frac{\left(-2(dx+c)^2b^2+d^2\right)\cos(2bx+2a)+2bd(dx+c)\sin(2bx+2a)+2b^2c^2-d^2}{8b^3}$
risch	$-\frac{\left(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2\right)\cos(2bx+2a)}{8b^3}+\frac{d(dx+c)\sin(2bx+2a)}{4b^2}$
orering	$\frac{d\left(4x^2d^2b^2+8b^2cdx+4b^2c^2-d^2\right)\cos(bx+a)\sin(bx+a)}{4b^4(dx+c)}-\frac{\left(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2\right)\left(2(dx+c)\cos(bx+a)\sin(bx+$
derivativedivides	$\frac{-\frac{a^2d^2\cos(bx+a)^2}{2b^2}+\frac{acd\cos(bx+a)^2}{b}-\frac{2a\,d^2\left(-\frac{(bx+a)\cos(bx+a)^2}{2}+\frac{\sin(bx+a)\cos(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}\right)}{b^2}-\frac{c^2\cos(bx+a)^2}{2}+\frac{2cd\left(-\frac{(b}{}\right.}{}$
default	$\frac{-\frac{a^2d^2\cos(bx+a)^2}{2b^2}+\frac{acd\cos(bx+a)^2}{b}-\frac{2a\,d^2\left(-\frac{(bx+a)\cos(bx+a)^2}{2}+\frac{\sin(bx+a)\cos(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}\right)}{b^2}-\frac{c^2\cos(bx+a)^2}{2}+\frac{2cd\left(-\frac{(b}{}\right.}{}$
norman	$\frac{\left(2b^2c^2-d^2\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{b^3}+\frac{cd\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{b^2}+\frac{d^2x\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{b^2}-\frac{d^2x^2}{4b}-\frac{cd\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3}{b^2}-\frac{cdx}{2b}+\frac{3d^2x^2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{2b}-\frac{d^2x^2\,t}{}$ $\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)^2$

input
```
int((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/8*((-2*(d*x+c)^2*b^2+d^2)*cos(2*b*x+2*a)+2*b*d*(d*x+c)*sin(2*b*x+2*a)+2*
b^2*c^2-d^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.15

$$\int (c+dx)^2\cos(a+bx)\sin(a+bx)\,dx$$
$$=\frac{b^2d^2x^2+2\,b^2cdx-\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2\right)\cos\left(bx+a\right)^2+2\left(bd^2x+bcd\right)\cos\left(bx+a\right)\sin\left(bx+\right.}{4\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2
- d^2)*cos(b*x + a)^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(70) = 140.

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.19

$$\int (c+dx)^2 \cos(a+bx)\sin(a+bx)\,dx$$

$$= \begin{cases} \frac{c^2 \sin^2(a+bx)}{2b} + \frac{cdx\sin^2(a+bx)}{2b} - \frac{cdx\cos^2(a+bx)}{2b} + \frac{d^2x^2\sin^2(a+bx)}{4b} - \frac{d^2x^2\cos^2(a+bx)}{4b} + \frac{cd\sin(a+bx)\cos(a+bx)}{2b^2} + \frac{d^2x\sin} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin(a)\cos(a) \end{cases}$$

input
```
integrate((d*x+c)**2*cos(b*x+a)*sin(b*x+a),x)
```

output
```
Piecewise((c**2*sin(a + b*x)**2/(2*b) + c*d*x*sin(a + b*x)**2/(2*b) - c*d*
x*cos(a + b*x)**2/(2*b) + d**2*x**2*sin(a + b*x)**2/(4*b) - d**2*x**2*cos(
a + b*x)**2/(4*b) + c*d*sin(a + b*x)*cos(a + b*x)/(2*b**2) + d**2*x*sin(a
+ b*x)*cos(a + b*x)/(2*b**2) - d**2*sin(a + b*x)**2/(4*b**3), Ne(b, 0)), (
(c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)*cos(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(72) = 144.

Time = 0.04 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.14

$$\int (c+dx)^2 \cos(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{4c^2\cos(bx+a)^2 - \frac{8acd\cos(bx+a)^2}{b} + \frac{4a^2d^2\cos(bx+a)^2}{b^2} + \frac{2(2(bx+a)\cos(2bx+2a) - \sin(2bx+2a))cd}{b} - \frac{2(2(bx+a)\cos(2b}{8b}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/8*(4*c^2*cos(b*x + a)^2 - 8*a*c*d*cos(b*x + a)^2/b + 4*a^2*d^2*cos(b*x
+ a)^2/b^2 + 2*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*c*d/b - 2
*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a*d^2/b^2 + ((2*(b*x +
a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*d^2/b^2)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int (c+dx)^2 \cos(a+bx)\sin(a+bx)\,dx = -\frac{(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2)\cos(2\,bx+2\,a)}{8\,b^3}+\frac{(bd^2x+bcd)\sin(2\,bx+2\,a)}{4\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/8*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a)/b^3
+ 1/4*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

$$\int (c+dx)^2 \cos(a+bx)\sin(a+bx)\,dx = \frac{\cos(2\,a+2\,b\,x)\left(\frac{d^2}{4}-\frac{b^2c^2}{2}\right)}{2\,b^3}+\frac{d^2\,x\,\sin(2\,a+2\,b\,x)}{4\,b^2}-\frac{d^2\,x^2\,\cos(2\,a+2\,b\,x)}{4\,b}+\frac{c\,d\,\sin(2\,a+2\,b\,x)}{4\,b^2}-\frac{c\,d\,x\,\cos(2\,a+2\,b\,x)}{2\,b}$$

input
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^2,x)
```

output
```
(cos(2*a + 2*b*x)*(d^2/4 - (b^2*c^2)/2))/(2*b^3) + (d^2*x*sin(2*a + 2*b*x)
)/(4*b^2) - (d^2*x^2*cos(2*a + 2*b*x))/(4*b) + (c*d*sin(2*a + 2*b*x))/(4*b
^2) - (c*d*x*cos(2*a + 2*b*x))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.74

$$\int (c+dx)^2 \cos(a+bx)\sin(a+bx)\,dx$$
$$= \frac{-2\cos(bx+a)^2 b^2c^2 - 2\cos(bx+a)^2 b^2cdx - \cos(bx+a)^2 b^2d^2x^2 + \cos(bx+a)^2 d^2 + 2\cos(bx+a)\sin}{4b^3}$$

input
```
int((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x)
```

output
```
( - 2*cos(a + b*x)**2*b**2*c**2 - 2*cos(a + b*x)**2*b**2*c*d*x - cos(a + b
*x)**2*b**2*d**2*x**2 + cos(a + b*x)**2*d**2 + 2*cos(a + b*x)*sin(a + b*x)
*b*c*d + 2*cos(a + b*x)*sin(a + b*x)*b*d**2*x + 2*sin(a + b*x)**2*b**2*c*d
*x + sin(a + b*x)**2*b**2*d**2*x**2)/(4*b**3)
```

3.5 $\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx$

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Maple [A] (verified) 200
Fricas [A] (verification not implemented) 201
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Maxima [A] (verification not implemented) 202
Giac [A] (verification not implemented) 202
Mupad [B] (verification not implemented) 202
Reduce [B] (verification not implemented) 203

Optimal result

Integrand size = 18, antiderivative size = 50

$$\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx = -\frac{dx}{4b} + \frac{d\cos(a+bx)\sin(a+bx)}{4b^2} + \frac{(c+dx)\sin^2(a+bx)}{2b}$$

output
```
-1/4*d*x/b+1/4*d*cos(b*x+a)*sin(b*x+a)/b^2+1/2*(d*x+c)*sin(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx = \frac{-2b(c+dx)\cos(2(a+bx)) + d\sin(2(a+bx))}{8b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-2*b*(c + d*x)*Cos[2*(a + b*x)] + d*Sin[2*(a + b*x)])/(8*b^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4904, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin(a+bx)\cos(a+bx)\,dx$$

$$\downarrow 4904$$

$$\frac{(c+dx)\sin^2(a+bx)}{2b} - \frac{d\int \sin^2(a+bx)dx}{2b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)\sin^2(a+bx)}{2b} - \frac{d\int \sin(a+bx)^2 dx}{2b}$$

$$\downarrow 3115$$

$$\frac{(c+dx)\sin^2(a+bx)}{2b} - \frac{d\left(\frac{\int 1dx}{2} - \frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b}$$

$$\downarrow 24$$

$$\frac{(c+dx)\sin^2(a+bx)}{2b} - \frac{d\left(\frac{x}{2} - \frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b}$$

input
```
Int[(c + d*x)*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
((c + d*x)*Sin[a + b*x]^2)/(2*b) - (d*(x/2 - (Cos[a + b*x]*Sin[a + b*x])/(
2*b)))/(2*b)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{(dx+c)\cos(2bx+2a)}{4b}+\frac{d\sin(2bx+2a)}{8b^2}$	36
parallelrisch	$\frac{-2b(dx+c)\cos(2bx+2a)+2cb+d\sin(2bx+2a)}{8b^2}$	39
orering	$\frac{d\cos(bx+a)\sin(bx+a)}{2b^2}-\frac{\cos(bx+a)\sin(bx+a)d-(dx+c)b\sin(bx+a)^2+(dx+c)\cos(bx+a)^2b}{4b^2}$	71
derivativedivides	$\frac{\frac{da\cos(bx+a)^2}{2b}-\frac{c\cos(bx+a)^2}{2}+\frac{d\left(-\frac{(bx+a)\cos(bx+a)^2}{2}+\frac{\sin(bx+a)\cos(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}\right)}{b}}{b}$	74
default	$\frac{\frac{da\cos(bx+a)^2}{2b}-\frac{c\cos(bx+a)^2}{2}+\frac{d\left(-\frac{(bx+a)\cos(bx+a)^2}{2}+\frac{\sin(bx+a)\cos(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}\right)}{b}}{b}$	74
norman	$\frac{\frac{d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{2b^2}-\frac{d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3}{2b^2}-\frac{dx}{4b}+\frac{2c\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{b}+\frac{3dx\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{2b}-\frac{dx\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4}{4b}}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)^2}$	110

input
```
int((d*x+c)*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(d*x+c)*cos(2*b*x+2*a)/b+1/8*d*sin(2*b*x+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx = \frac{bdx - 2\,(bdx+bc)\cos(bx+a)^2 + d\cos(bx+a)\sin(bx+a)}{4\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a))
/b^2
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.60

$$\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx = \begin{cases} \frac{c\sin^2(a+bx)}{2b} + \frac{dx\sin^2(a+bx)}{4b} - \frac{dx\cos^2(a+bx)}{4b} + \frac{d\sin(a+bx)\cos(a+bx)}{4b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\sin(a)\cos(a) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x)
```

output
```
Piecewise((c*sin(a + b*x)**2/(2*b) + d*x*sin(a + b*x)**2/(4*b) - d*x*cos(a
 + b*x)**2/(4*b) + d*sin(a + b*x)*cos(a + b*x)/(4*b**2), Ne(b, 0)), ((c*x
+ d*x**2/2)*sin(a)*cos(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx = -\frac{4\,c\cos(bx+a)^2 - \frac{4\,ad\cos(bx+a)^2}{b} + \frac{(2\,(bx+a)\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a))d}{b}}{8\,b}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/8*(4*c*cos(b*x + a)^2 - 4*a*d*cos(b*x + a)^2/b + (2*(b*x + a)*cos(2*b*x
 + 2*a) - sin(2*b*x + 2*a))*d/b)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx = -\frac{(bdx+bc)\cos(2\,bx+2\,a)}{4\,b^2} + \frac{d\sin(2\,bx+2\,a)}{8\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/4*(b*d*x + b*c)*cos(2*b*x + 2*a)/b^2 + 1/8*d*sin(2*b*x + 2*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 18.53 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx = \frac{d\,\sin(2\,a+2\,b\,x)}{8\,b^2} - \frac{c\,\cos(2\,a+2\,b\,x)}{4\,b} - \frac{d\,x\,\cos(2\,a+2\,b\,x)}{4\,b}$$

input
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x),x)
```

output
```
(d*sin(2*a + 2*b*x))/(8*b^2) - (c*cos(2*a + 2*b*x))/(4*b) - (d*x*cos(2*a +
 2*b*x))/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14

$$\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx$$
$$= \frac{-2\cos(bx+a)^2 bc - \cos(bx+a)^2 bdx + \cos(bx+a)\sin(bx+a)\,d + \sin(bx+a)^2 bdx}{4b^2}$$

input
```
int((d*x+c)*cos(b*x+a)*sin(b*x+a),x)
```

output
```
( - 2*cos(a + b*x)**2*b*c - cos(a + b*x)**2*b*d*x + cos(a + b*x)*sin(a + b
*x)*d + sin(a + b*x)**2*b*d*x)/(4*b**2)
```

3.6 $\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 65

$$\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx = \frac{\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d} + \frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

output

```
1/2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d+1/2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2
*b*x)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx = \frac{\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)+\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

input

```
Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x),x]
```

output
```
(CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d] + Cos[2*a - (2*b*c)/d
]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {4906, 27, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos(a+bx)}{c+dx}\,dx$$

$$\downarrow 4906$$

$$\int \frac{\sin(2a+2bx)}{2(c+dx)}dx$$

$$\downarrow 27$$

$$\frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}dx$$

$$\downarrow 3042$$

$$\frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}dx$$

$$\downarrow 3784$$

$$\frac{1}{2}\left(\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx+\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx+\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)$$

$$\downarrow 3780$$

$$\frac{1}{2}\left(\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)$$

$$\downarrow 3783$$

$$\frac{1}{2}\left(\frac{\sin\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x),x]
```

output
```
((CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/d + (Cos[2*a - (2*b
*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

method	result	siz
derivativedivides	$\frac{-\mathrm{Si}\left(-2bx-2a+\frac{2ad-2cb}{d}\right)\cos\left(\frac{2ad-2cb}{d}\right)+\mathrm{Ci}\left(2bx+2a-\frac{2(ad-cb)}{d}\right)\sin\left(\frac{2ad-2cb}{d}\right)}{2d}$	82
default	$\frac{-\mathrm{Si}\left(-2bx-2a+\frac{2ad-2cb}{d}\right)\cos\left(\frac{2ad-2cb}{d}\right)+\mathrm{Ci}\left(2bx+2a-\frac{2(ad-cb)}{d}\right)\sin\left(\frac{2ad-2cb}{d}\right)}{2d}$	82
risch	$-\frac{ie^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{4d}+\frac{ie^{\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(-2ibx-2ia-\frac{2(-iad+icb)}{d}\right)}{4d}$	98

input

```
int(cos(b*x+a)*sin(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output

```
1/2/d*(-Si(-2*b*x-2*a+2*(a*d-b*c)/d)*cos(2*(a*d-b*c)/d)+Ci(2*b*x+2*a-2*(a*
d-b*c)/d)*sin(2*(a*d-b*c)/d))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx = \frac{\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)}{2\,d}$$

input

```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output

```
1/2*(cos_integral(2*(b*d*x + b*c)/d)*sin(-2*(b*c - a*d)/d) + cos(-2*(b*c -
 a*d)/d)*sin_integral(2*(b*d*x + b*c)/d))/d
```

Sympy [F]

$$\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx = \int \frac{\sin(a+bx)\cos(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.20

$$\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx = -\frac{b\left(-i\,E_1\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_1\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b\left(E_1\left(\frac{2\,(-i\,bc-i\,(bx+a)}{d}\right.\right.}{4\,bd}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
-1/4*(b*(-I*exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*ex
p_integral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)
/d) + b*(exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_int
egral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d))/
(b*d)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 569, normalized size of antiderivative = 8.75

$$\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
1/4*(imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - imag
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*sin_integr
al(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(2*b
*x + 2*b*c/d))*tan(a)^2*tan(b*c/d) + 2*real_part(cos_integral(-2*b*x - 2*b
*c/d))*tan(a)^2*tan(b*c/d) - 2*real_part(cos_integral(2*b*x + 2*b*c/d))*ta
n(a)*tan(b*c/d)^2 - 2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan
(b*c/d)^2 - imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 + imag_part(
cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - 2*sin_integral(2*(b*d*x + b*c)/
d)*tan(a)^2 + 4*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)
 - 4*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 8*sin_i
ntegral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d) - imag_part(cos_integral(2*b*
x + 2*b*c/d))*tan(b*c/d)^2 + imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(b*c/d)^2 - 2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 2*real_part(c
os_integral(2*b*x + 2*b*c/d))*tan(a) + 2*real_part(cos_integral(-2*b*x - 2
*b*c/d))*tan(a) - 2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) -
2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + imag_part(cos_int
egral(2*b*x + 2*b*c/d)) - imag_part(cos_integral(-2*b*x - 2*b*c/d)) + 2*si
n_integral(2*(b*d*x + b*c)/d))/(d*tan(a)^2*tan(b*c/d)^2 + d*tan(a)^2 + d*t
an(b*c/d)^2 + d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+b\,x)\,\sin(a+b\,x)}{c+d\,x}\,dx$$

input
```
int((cos(a + b*x)*sin(a + b*x))/(c + d*x),x)
```

output
```
int((cos(a + b*x)*sin(a + b*x))/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)\sin(bx+a)}{dx+c}dx$$

input
```
int(cos(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

output
```
int((cos(a + b*x)*sin(a + b*x))/(c + d*x),x)
```

3.7 $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$

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Reduce [F] . . . 218

Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \frac{b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2} - \frac{\sin(2a+2bx)}{2d(c+dx)} - \frac{b\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}$$

output

```
b*cos(2*a-2*b*c/d)*Ci(2*b*c/d+2*b*x)/d^2-1/2*sin(2*b*x+2*a)/d/(d*x+c)-b*si
n(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^2
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \frac{2b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) - \frac{d\sin(2(a+bx))}{c+dx} - 2b\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b(c+dx)}{d}\right)}{2d^2}$$

input

```
Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^2,x]
```

output
```
(2*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - (d*Sin[2*(a + b
*x)])/(c + d*x) - 2*b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])
/(2*d^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {4906, 27, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos(a+bx)}{(c+dx)^2}\,dx$$

$$\downarrow 4906$$

$$\int \frac{\sin(2a+2bx)}{2(c+dx)^2}dx$$

$$\downarrow 27$$

$$\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}dx$$

$$\downarrow 3042$$

$$\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}dx$$

$$\downarrow 3778$$

$$\frac{1}{2}\left(\frac{2b\int \frac{\cos(2a+2bx)}{c+dx}dx}{d} - \frac{\sin(2a+2bx)}{d(c+dx)}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{2b\int \frac{\sin\left(2a+2bx+\frac{\pi}{2}\right)}{c+dx}dx}{d} - \frac{\sin(2a+2bx)}{d(c+dx)}\right)$$

$$\downarrow 3784$$

$$\frac{1}{2}\left(\frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}-\frac{\sin(2a+2bx)}{d(c+dx)}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}-\frac{\sin(2a+2bx)}{d(c+dx)}\right)$$

$$\downarrow 3780$$

$$\frac{1}{2}\left(\frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx-\frac{\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}-\frac{\sin(2a+2bx)}{d(c+dx)}\right)$$

$$\downarrow 3783$$

$$\frac{1}{2}\left(\frac{2b\left(\frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}-\frac{\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}-\frac{\sin(2a+2bx)}{d(c+dx)}\right)$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^2,x]
```

output
```
(-(Sin[2*a + 2*b*x]/(d*(c + d*x))) + (2*b*((Cos[2*a - (2*b*c)/d]*CosIntegr
al[(2*b*c)/d + 2*b*x])/d - (Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2
*b*x])/d))/d)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.45

method	result
derivativedivides	$\frac{\sin(2bx+2a)b^2}{2d(ad-cb-d(bx+a))}+\frac{b^2\left(\mathrm{Si}\left(-2bx-2a+\frac{2ad-2cb}{d}\right)\sin\left(\frac{2ad-2cb}{d}\right)+\mathrm{Ci}\left(2bx+2a-\frac{2(ad-cb)}{d}\right)\cos\left(\frac{2ad-2cb}{d}\right)\right)}{d^2}}{b}$
default	$\frac{\sin(2bx+2a)b^2}{2d(ad-cb-d(bx+a))}+\frac{b^2\left(\mathrm{Si}\left(-2bx-2a+\frac{2ad-2cb}{d}\right)\sin\left(\frac{2ad-2cb}{d}\right)+\mathrm{Ci}\left(2bx+2a-\frac{2(ad-cb)}{d}\right)\cos\left(\frac{2ad-2cb}{d}\right)\right)}{d^2}}{b}$
risch	$-\frac{b\,\mathrm{e}^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{2d^2}-\frac{b\,\mathrm{e}^{\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(-2ibx-2ia-\frac{2(-iad+icb)}{d}\right)}{2d^2}-($

input
```
int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*sin(2*b*x+2*a)*b^2/d/(a*d-c*b-d*(b*x+a))+b^2/d^2*(Si(-2*b*x-2*a+2
*(a*d-b*c)/d)*sin(2*(a*d-b*c)/d)+Ci(2*b*x+2*a-2*(a*d-b*c)/d)*cos(2*(a*d-b*
c)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.24

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{(bdx+bc)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)-d\cos\left(bx+a\right)\sin\left(bx+a\right)-(bdx+bc)\sin\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left($$
$$d^3x+cd^2$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
((b*d*x + b*c)*cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*d*x + b*c)/d) - d*c
os(b*x + a)*sin(b*x + a) - (b*d*x + b*c)*sin(-2*(b*c - a*d)/d)*sin_integra
l(2*(b*d*x + b*c)/d))/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(a+bx)\cos(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.95

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = -\frac{b^2\left(-i\,E_2\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_2\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^2\left(E_2\left(\frac{2\,(-i\,bc-i\,(bx+}{d}\right.\right.}{4\,(bcd+(bx+a)d^2-ad^2)b}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/4*(b^2*(-I*exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*
exp_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*
d)/d) + b^2*(exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp
_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/
d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 2870, normalized size of antiderivative = 33.76

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/2*(b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*ta
n(b*c/d)^2 + b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*ta
n(a)^2*tan(b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan
(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 4*b*d*x*sin_integral(2*(b*d*x + b*c
)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b*d*x*imag_part(cos_integral(2*b*x
 + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b*d*x*imag_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 4*b*d*x*sin_integra
l(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b*c*real_part(cos_in
tegral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + b*c*real_part(
cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b*d*x*r
eal_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - b*d*x*real_p
art(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 4*b*d*x*real_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 4*b*d*x*re
al_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 2*b
*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)
 + 2*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan
(b*c/d) - 4*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*
c/d) - b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d
)^2 - b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(a+b\,x)\,\sin(a+b\,x)}{(c+d\,x)^2}\,dx$$

input
```
int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^2,x)
```

output
```
int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(bx+a)\sin(bx+a)}{d^2x^2+2cdx+c^2}dx$$

input
```
int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x)
```

output
```
int((cos(a + b*x)*sin(a + b*x))/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.8 $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 114

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = -\frac{b\cos(2a+2bx)}{2d^2(c+dx)} - \frac{b^2\,\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^3} - \frac{\sin(2a+2bx)}{4d(c+dx)^2} - \frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}$$

output

```
-1/2*b*cos(2*b*x+2*a)/d^2/(d*x+c)-b^2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d
^3-1/4*sin(2*b*x+2*a)/d/(d*x+c)^2-b^2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d
^3
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = -\frac{4b^2\,\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right) + \frac{d(2b(c+dx)\cos(2(a+bx))+d\sin(2(a+bx)))}{(c+dx)^2} + 4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(}{}\right.}{4d^3}$$

input
```
Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^3,x]
```

output
```
-1/4*(4*b^2*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*(2*b*
(c + d*x)*Cos[2*(a + b*x)] + d*Sin[2*(a + b*x)]))/(c + d*x)^2 + 4*b^2*Cos[
2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d^3
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4906, 27, 3042, 3778, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos(a+bx)}{(c+dx)^3}\,dx$$

$$\downarrow 4906$$

$$\int \frac{\sin(2a+2bx)}{2(c+dx)^3}\,dx$$

$$\downarrow 27$$

$$\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx$$

$$\downarrow 3042$$

$$\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx$$

$$\downarrow 3778$$

$$\frac{1}{2}\left(\frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^2}\,dx}{d} - \frac{\sin(2a+2bx)}{2d(c+dx)^2}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{b\int \frac{\sin\left(2a+2bx+\frac{\pi}{2}\right)}{(c+dx)^2}\,dx}{d} - \frac{\sin(2a+2bx)}{2d(c+dx)^2}\right)$$

$$\downarrow 3778$$

$$\frac{1}{2}\left(\frac{b\left(\frac{2b\int -\frac{\sin(2a+2bx)}{c+dx}dx}{d}-\frac{\cos(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\sin(2a+2bx)}{2d(c+dx)^2}\right)$$

$$\downarrow 25$$

$$\frac{1}{2}\left(\frac{b\left(-\frac{2b\int \frac{\sin(2a+2bx)}{c+dx}dx}{d}-\frac{\cos(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\sin(2a+2bx)}{2d(c+dx)^2}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{b\left(-\frac{2b\int \frac{\sin(2a+2bx)}{c+dx}dx}{d}-\frac{\cos(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\sin(2a+2bx)}{2d(c+dx)^2}\right)$$

$$\downarrow 3784$$

$$\frac{1}{2}\left(\frac{b\left(-\frac{2b\left(\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx+\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}-\frac{\cos(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\sin(2a+2bx)}{2d(c+dx)^2}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{b\left(-\frac{2b\left(\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx+\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}-\frac{\cos(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\sin(2a+2bx)}{2d(c+dx)^2}\right)$$

$$\downarrow 3780$$

$$\frac{1}{2}\left(\frac{b\left(-\frac{2b\left(\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}-\frac{\cos(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\sin(2a+2bx)}{2d(c+dx)^2}\right)$$

$$\downarrow 3783$$

$$\frac{1}{2}\left(\frac{b\left(-\frac{2b\left(\frac{\sin\left(2a-\frac{2bc}{d}\right)\,\mathrm{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}-\frac{\cos(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\sin(2a+2bx)}{2d(c+dx)^2}\right)$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^3,x]
```

output
```
(-1/2*Sin[2*a + 2*b*x]/(d*(c + d*x)^2) + (b*(-(Cos[2*a + 2*b*x]/(d*(c + d*
x))) - (2*b*((CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/d + (Co
s[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d))/d))/d)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{-\frac{\sin(2bx+2a)b^3}{4d(ad-cb-d(bx+a))^2}+\frac{\cos(2bx+2a)b^3}{2d^2(ad-cb-d(bx+a))}-\frac{b^3\left(-\mathrm{Si}\left(-2bx-2a+\frac{2ad-2cb}{d}\right)\cos\left(\frac{2ad-2cb}{d}\right)+\mathrm{Ci}\left(2bx+2a-\frac{2(ad-cb)}{d}\right)\sin\left(\frac{2}{}\right.\right.}{d^3}}{b}$
default	$\frac{-\frac{\sin(2bx+2a)b^3}{4d(ad-cb-d(bx+a))^2}+\frac{\cos(2bx+2a)b^3}{2d^2(ad-cb-d(bx+a))}-\frac{b^3\left(-\mathrm{Si}\left(-2bx-2a+\frac{2ad-2cb}{d}\right)\cos\left(\frac{2ad-2cb}{d}\right)+\mathrm{Ci}\left(2bx+2a-\frac{2(ad-cb)}{d}\right)\sin\left(\frac{2}{}\right.\right.}{d^3}}{b}$
risch	$\frac{ib^2\mathrm{e}^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{2d^3}-\frac{ib^2\mathrm{e}^{\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(-2ibx-2ia-\frac{2(-iad+icb)}{d}\right)}{2d^3}+\frac{i}{}$

input
```
int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4*sin(2*b*x+2*a)*b^3/d/(a*d-c*b-d*(b*x+a))^2+1/2*cos(2*b*x+2*a)*b^
3/d^2/(a*d-c*b-d*(b*x+a))-b^3/d^3*(-Si(-2*b*x-2*a+2*(a*d-b*c)/d)*cos(2*(a*
d-b*c)/d)+Ci(2*b*x+2*a-2*(a*d-b*c)/d)*sin(2*(a*d-b*c)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.64

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{bd^2x - d^2\cos(bx+a)\sin(bx+a) + bcd - 2\,(bd^2x+bcd)\cos(bx+a)^2 - 2\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\,\mathrm{Ci}}{2\,(d^5x^2+2\,cd^4x+c^2}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(b*d^2*x - d^2*cos(b*x + a)*sin(b*x + a) + b*c*d - 2*(b*d^2*x + b*c*d)
*cos(b*x + a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(2*(
b*d*x + b*c)/d)*sin(-2*(b*c - a*d)/d) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d))/(d^5*x^2 + 2*
c*d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin(a+bx)\cos(a+bx)}{(c+dx)^3}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.76

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = -\frac{b^3\left(-i\,E_3\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_3\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^3\left(E_3\left(\frac{2\,(-i\,bc-i\,(bx+}{4\left(b^2c^2d-2\,abcd^2+(bx+a)^2d^3+a^2d^3+2\,(bcd^2-ad^3)\right.}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*(b^3*(-I*exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*
exp_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*
d)/d) + b^3*(exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp
_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/
d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a
*d^3)*(b*x + a))*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.31 (sec) , antiderivative size = 5398, normalized size of antiderivative = 47.35

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output

```
-1/2*(b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(
a)^2*tan(b*c/d)^2 - b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*
tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*sin_integral(2*(b*d*x + b
*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*real_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*d^2*x^2*real
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b
^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(
b*c/d)^2 - 2*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x
)^2*tan(a)*tan(b*c/d)^2 + 2*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c
/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 2*b^2*c*d*x*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 4*b^2*c*d*x*sin_int
egral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b^2*d^2*x^2*im
ag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 + b^2*d^2*x^2*i
mag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 2*b^2*d^2*x
^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2 + 4*b^2*d^2*x^2*ima
g_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 4*b^2
*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b
*c/d) + 8*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*ta
n(b*c/d) + 4*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2
*tan(a)^2*tan(b*c/d) + 4*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos(a+b\,x)\,\sin(a+b\,x)}{(c+d\,x)^3}\,dx$$

input

```
int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^3,x)
```

output

```
int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos(bx+a)\sin(bx+a)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx$$

input
```
int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x)
```

output
```
int((cos(a + b*x)*sin(a + b*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*
x**3),x)
```

3.9 $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 144

$$\begin{aligned}\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = &-\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2}\\ &-\frac{2b^3\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}\\ &-\frac{\sin(2a+2bx)}{6d(c+dx)^3}+\frac{b^2\sin(2a+2bx)}{3d^3(c+dx)}\\ &+\frac{2b^3\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}\end{aligned}$$

output

```
-1/6*b*cos(2*b*x+2*a)/d^2/(d*x+c)^2-2/3*b^3*cos(2*a-2*b*c/d)*Ci(2*b*c/d+2*
b*x)/d^4-1/6*sin(2*b*x+2*a)/d/(d*x+c)^3+1/3*b^2*sin(2*b*x+2*a)/d^3/(d*x+c)
+2/3*b^3*sin(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^4
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.14

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$$
$$= \frac{-d\cos(2bx)\,(bd(c+dx)\cos(2a)+(d^2-2b^2(c+dx)^2)\sin(2a))+d((-d^2+2b^2(c+dx)^2)\cos(2a)+bd(c}{6d^4(}$$

input
```
Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^4,x]
```

output
```
(-(d*Cos[2*b*x]*(b*d*(c + d*x)*Cos[2*a] + (d^2 - 2*b^2*(c + d*x)^2)*Sin[2*
a])) + d*((-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*a] + b*d*(c + d*x)*Sin[2*a])*Si
n[2*b*x] - 4*b^3*(c + d*x)^3*(Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d
*x))/d] - Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d]))/(6*d^4*(c
+ d*x)^3)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {4906, 27, 3042, 3778, 3042, 3778, 25, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos(a+bx)}{(c+dx)^4}\,dx$$
$$\downarrow 4906$$
$$\int \frac{\sin(2a+2bx)}{2(c+dx)^4}\,dx$$
$$\downarrow 27$$
$$\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^4}\,dx$$
$$\downarrow 3042$$

$$\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^4}dx$$

$$\downarrow 3778$$

$$\frac{1}{2}\left(\frac{2b\int \frac{\cos(2a+2bx)}{(c+dx)^3}dx}{3d} - \frac{\sin(2a+2bx)}{3d(c+dx)^3}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{2b\int \frac{\sin\left(2a+2bx+\frac{\pi}{2}\right)}{(c+dx)^3}dx}{3d} - \frac{\sin(2a+2bx)}{3d(c+dx)^3}\right)$$

$$\downarrow 3778$$

$$\frac{1}{2}\left(\frac{2b\left(\frac{b\int -\frac{\sin(2a+2bx)}{(c+dx)^2}dx}{d} - \frac{\cos(2a+2bx)}{2d(c+dx)^2}\right)}{3d} - \frac{\sin(2a+2bx)}{3d(c+dx)^3}\right)$$

$$\downarrow 25$$

$$\frac{1}{2}\left(\frac{2b\left(-\frac{b\int \frac{\sin(2a+2bx)}{(c+dx)^2}dx}{d} - \frac{\cos(2a+2bx)}{2d(c+dx)^2}\right)}{3d} - \frac{\sin(2a+2bx)}{3d(c+dx)^3}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{2b\left(-\frac{b\int \frac{\sin(2a+2bx)}{(c+dx)^2}dx}{d} - \frac{\cos(2a+2bx)}{2d(c+dx)^2}\right)}{3d} - \frac{\sin(2a+2bx)}{3d(c+dx)^3}\right)$$

$$\downarrow 3778$$

$$\frac{1}{2}\left(\frac{2b\left(-\frac{b\left(\frac{2b\int \frac{\cos(2a+2bx)}{c+dx}dx}{d} - \frac{\sin(2a+2bx)}{d(c+dx)}\right)}{d} - \frac{\cos(2a+2bx)}{2d(c+dx)^2}\right)}{3d} - \frac{\sin(2a+2bx)}{3d(c+dx)^3}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{2b\left(-\frac{b\left(\frac{2b\int\frac{\sin\left(2a+2bx+\frac{\pi}{2}\right)}{c+dx}dx}{d}-\frac{\sin(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\cos(2a+2bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\sin(2a+2bx)}{3d(c+dx)^3}\right)$$

$$\downarrow 3784$$

$$\frac{1}{2}\left(\frac{2b\left(-\frac{b\left(\frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}-\frac{\sin(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\cos(2a+2bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\sin(2a+2bx)}{3d(c+dx)^3}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{2b\left(-\frac{b\left(\frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}-\frac{\sin(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\cos(2a+2bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\sin(2a+2bx)}{3d(c+dx)^3}\right)$$

$$\downarrow 3780$$

$$\frac{1}{2}\left(\frac{2b\left(-\frac{b\left(\frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx-\frac{\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}-\frac{\sin(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\cos(2a+2bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\sin(2a+2bx)}{3d(c+dx)^3}\right)$$

$$\downarrow 3783$$

$$\frac{1}{2}\left(\frac{2b\left(-\frac{b\left(\frac{2b\left(\frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}-\frac{\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}-\frac{\sin(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\cos(2a+2bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\sin(2a+2bx)}{3d(c+dx)^3}\right)$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^4,x]
```

output
```
(-1/3*Sin[2*a + 2*b*x]/(d*(c + d*x)^3) + (2*b*(-1/2*Cos[2*a + 2*b*x]/(d*(c
 + d*x)^2) - (b*(-(Sin[2*a + 2*b*x]/(d*(c + d*x))) + (2*b*((Cos[2*a - (2*b
*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d - (Sin[2*a - (2*b*c)/d]*SinIntegr
al[(2*b*c)/d + 2*b*x])/d))/d))/d))/(3*d))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.53

method	result
derivativedivides	$\frac{-\frac{\sin(2bx+2a)b^4\left(2d^2a^2-4cdab-4a\,d^2(bx+a)+2b^2c^2+4bcd(bx+a)+2(bx+a)^2d^2-d^2\right)}{6d^3(ad-cb-d(bx+a))^3}-\frac{\cos(2bx+2a)b^4}{6d^2(ad-cb-d(bx+a))^2}-\frac{2b^4\left(\mathrm{Si}\left(-2bx-2\right.\right.}{b}$
default	$\frac{-\frac{\sin(2bx+2a)b^4\left(2d^2a^2-4cdab-4a\,d^2(bx+a)+2b^2c^2+4bcd(bx+a)+2(bx+a)^2d^2-d^2\right)}{6d^3(ad-cb-d(bx+a))^3}-\frac{\cos(2bx+2a)b^4}{6d^2(ad-cb-d(bx+a))^2}-\frac{2b^4\left(\mathrm{Si}\left(-2bx-2\right.\right.}{b}$
risch	$\frac{b^3\mathrm{e}^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{3d^4}+\frac{b^3\mathrm{e}^{\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(-2ibx-2ia-\frac{2(-iad+icb)}{d}\right)}{3d^4}+\frac{i(2}{}$

input
```
int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/6*sin(2*b*x+2*a)*b^4*(2*d^2*a^2-4*c*d*a*b-4*a*d^2*(b*x+a)+2*b^2*c^
2+4*b*c*d*(b*x+a)+2*(b*x+a)^2*d^2-d^2)/d^3/(a*d-c*b-d*(b*x+a))^3-1/6*cos(2
*b*x+2*a)/d^2*b^4/(a*d-c*b-d*(b*x+a))^2-2/3*b^4/d^4*(Si(-2*b*x-2*a+2*(a*d-
b*c)/d)*sin(2*(a*d-b*c)/d)+Ci(2*b*x+2*a-2*(a*d-b*c)/d)*cos(2*(a*d-b*c)/d))
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.83

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$$
$$= \frac{bd^3x+bcd^2-2\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^2-4\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}}{}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/6*(b*d^3*x + b*c*d^2 - 2*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 - 4*(b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-2*(b*c - a*d)/d)*co
s_integral(2*(b*d*x + b*c)/d) + 2*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c
^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
 3*b^3*c^2*d*x + b^3*c^3)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*
c)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F]

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sin(a+bx)\cos(a+bx)}{(c+dx)^4}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)**4,x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x)**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.74

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx =$$
$$-\frac{b^4\left(-i\,E_4\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_4\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^4\left(E_4\left(\frac{2\,(-i\,bc-i\,(bx+}{4\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3+(bx+a)^3d^4-a^3d^4+3\,(bcd^3-ad^4)(bx+a)^2+3\right.}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

output

```
-1/4*(b^4*(-I*exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*
exp_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*
d)/d) + b^4*(exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp
_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/
d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*
d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2
*d^4)*(b*x + a))*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 7592, normalized size of antiderivative = 52.72

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input

```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

output

```
-1/6*(2*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*ta
n(a)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 4*b^3*d^3*x^3*imag_part(cos_integra
l(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 4*b^3*d^3*x^3*imag_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 8*b^3*
d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 4
*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*ta
n(b*c/d)^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b
*x)^2*tan(a)*tan(b*c/d)^2 + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*
tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(2*
b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 2*b^
3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 2
*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2
 + 8*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a
)*tan(b*c/d) + 8*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(b*x)^2*tan(a)*tan(b*c/d) - 12*b^3*c*d^2*x^2*imag_part(cos_integral(2*b*x
+ 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 12*b^3*c*d^2*x^2*imag_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 24*b^3*c*d^
2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos(a+b\,x)\,\sin(a+b\,x)}{(c+d\,x)^4}\,dx$$

input

```
int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^4,x)
```

output

```
int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^4, x)
```

Reduce [F]

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos(bx+a)\sin(bx+a)}{d^4x^4+4cd^3x^3+6c^2d^2x^2+4c^3dx+c^4}\,dx$$

input
```
int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x)
```

output
```
int((cos(a + b*x)*sin(a + b*x))/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*
c*d**3*x**3 + d**4*x**4),x)
```

3.10 $\int \frac{\cos(x)\sin(x)}{x}\,dx$

Optimal result 239
Mathematica [A] (verified) 239
Rubi [A] (verified) 240
Maple [A] (verified) 241
Fricas [A] (verification not implemented) 242
Sympy [A] (verification not implemented) 242
Maxima [C] (verification not implemented) 242
Giac [A] (verification not implemented) 243
Mupad [F(-1)] 243
Reduce [B] (verification not implemented) 243

Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{\cos(x)\sin(x)}{x}\,dx = \frac{\mathrm{Si}(2x)}{2}$$

output

```
1/2*Si(2*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)\sin(x)}{x}\,dx = \frac{\mathrm{Si}(2x)}{2}$$

input

```
Integrate[(Cos[x]*Sin[x])/x,x]
```

output

```
SinIntegral[2*x]/2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4906, 27, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)\cos(x)}{x}\,dx$$
$$\downarrow 4906$$
$$\int \frac{\sin(2x)}{2x}\,dx$$
$$\downarrow 27$$
$$\frac{1}{2}\int \frac{\sin(2x)}{x}\,dx$$
$$\downarrow 3042$$
$$\frac{1}{2}\int \frac{\sin(2x)}{x}\,dx$$
$$\downarrow 3780$$
$$\frac{\text{Si}(2x)}{2}$$

input
```
Int[(Cos[x]*Sin[x])/x,x]
```

output
```
SinIntegral[2*x]/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\mathrm{Si}(2x)}{2}$	7
meijerg	$\frac{\mathrm{Si}(2x)}{2}$	7
risch	$-\frac{\pi\ \mathrm{csgn}(x)}{4}+\frac{\mathrm{Si}(2x)}{2}$	13

input
```
int(cos(x)*sin(x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*Si(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cos(x)\sin(x)}{x}\,dx = \frac{1}{2}\,\mathrm{Si}\,(2\,x)$$

input
```
integrate(cos(x)*sin(x)/x,x, algorithm="fricas")
```

output
```
1/2*sin_integral(2*x)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{\cos(x)\sin(x)}{x}\,dx = \frac{\mathrm{Si}\,(2x)}{2}$$

input
```
integrate(cos(x)*sin(x)/x,x)
```

output
```
Si(2*x)/2
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.62

$$\int \frac{\cos(x)\sin(x)}{x}\,dx = -\frac{1}{4}i\,\mathrm{Ei}(2i\,x) + \frac{1}{4}i\,\mathrm{Ei}(-2i\,x)$$

input
```
integrate(cos(x)*sin(x)/x,x, algorithm="maxima")
```

output
```
-1/4*I*Ei(2*I*x) + 1/4*I*Ei(-2*I*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cos(x)\sin(x)}{x}\,dx = \frac{1}{2}\,\mathrm{Si}\,(2\,x)$$

input
```
integrate(cos(x)*sin(x)/x,x, algorithm="giac")
```

output
```
1/2*sin_integral(2*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)\sin(x)}{x}\,dx = \int \frac{\cos\,(x)\,\sin\,(x)}{x}\,dx$$

input
```
int((cos(x)*sin(x))/x,x)
```

output
```
int((cos(x)*sin(x))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cos(x)\sin(x)}{x}\,dx = \frac{si(2x)}{2}$$

input
```
int(cos(x)*sin(x)/x,x)
```

output
```
si(2*x)/2
```

3.11 $\int \frac{\cos(x)\sin(x)}{x^2}\,dx$

Optimal result 244
Mathematica [A] (verified) 244
Rubi [A] (verified) 245
Maple [A] (verified) 246
Fricas [A] (verification not implemented) 247
Sympy [A] (verification not implemented) 247
Maxima [C] (verification not implemented) 247
Giac [A] (verification not implemented) 248
Mupad [F(-1)] 248
Reduce [F] 248

Optimal result

Integrand size = 8, antiderivative size = 16

$$\int \frac{\cos(x)\sin(x)}{x^2}\,dx = \text{CosIntegral}(2x) - \frac{\sin(2x)}{2x}$$

output
```
Ci(2*x)-1/2*sin(2*x)/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)\sin(x)}{x^2}\,dx = \text{CosIntegral}(2x) - \frac{\sin(2x)}{2x}$$

input
```
Integrate[(Cos[x]*Sin[x])/x^2,x]
```

output
```
CosIntegral[2*x] - Sin[2*x]/(2*x)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {4906, 27, 3042, 3778, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)\cos(x)}{x^2}\,dx$$

$$\downarrow 4906$$

$$\int \frac{\sin(2x)}{2x^2}\,dx$$

$$\downarrow 27$$

$$\frac{1}{2}\int \frac{\sin(2x)}{x^2}\,dx$$

$$\downarrow 3042$$

$$\frac{1}{2}\int \frac{\sin(2x)}{x^2}\,dx$$

$$\downarrow 3778$$

$$\frac{1}{2}\left(2\int \frac{\cos(2x)}{x}\,dx - \frac{\sin(2x)}{x}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(2\int \frac{\sin\left(2x+\frac{\pi}{2}\right)}{x}\,dx - \frac{\sin(2x)}{x}\right)$$

$$\downarrow 3783$$

$$\frac{1}{2}\left(2\,\text{CosIntegral}(2x) - \frac{\sin(2x)}{x}\right)$$

input
```
Int[(Cos[x]*Sin[x])/x^2,x]
```

output
```
(2*CosIntegral[2*x] - Sin[2*x]/x)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\operatorname{Ci}(2x) - \frac{\sin(2x)}{2x}$	15
risch	$\operatorname{Ci}(2x) - \frac{i\pi \operatorname{csgn}(ix)\operatorname{csgn}(x)}{2} + \frac{i\pi \operatorname{csgn}(ix)}{2} - \frac{\sin(2x)}{2x}$	35
meijerg	$\frac{\sqrt{\pi}\left(\frac{4\gamma-4+4\ln(2)+4\ln(x)}{\sqrt{\pi}} + \frac{4}{\sqrt{\pi}} - \frac{4\gamma}{\sqrt{\pi}} - \frac{4\ln(2)}{\sqrt{\pi}} - \frac{4\ln(x)}{\sqrt{\pi}} - \frac{2\sin(2x)}{\sqrt{\pi}\,x} + \frac{4\operatorname{Ci}(2x)}{\sqrt{\pi}}\right)}{4}$	71

input
```
int(cos(x)*sin(x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
Ci(2*x)-1/2*sin(2*x)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\cos(x)\sin(x)}{x^2}\,dx = \frac{x\,\mathrm{Ci}\left(2\,x\right) - \cos\left(x\right)\sin\left(x\right)}{x}$$

input
```
integrate(cos(x)*sin(x)/x^2,x, algorithm="fricas")
```

output
```
(x*cos_integral(2*x) - cos(x)*sin(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\cos(x)\sin(x)}{x^2}\,dx = -\log\left(x\right) + \frac{\log\left(x^2\right)}{2} + \mathrm{Ci}\left(2x\right) - \frac{\sin\left(2x\right)}{2x}$$

input
```
integrate(cos(x)*sin(x)/x**2,x)
```

output
```
-log(x) + log(x**2)/2 + Ci(2*x) - sin(2*x)/(2*x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\cos(x)\sin(x)}{x^2}\,dx = \frac{1}{2}\,\Gamma(-1, 2i\,x) + \frac{1}{2}\,\Gamma(-1, -2i\,x)$$

input
```
integrate(cos(x)*sin(x)/x^2,x, algorithm="maxima")
```

output
```
1/2*gamma(-1, 2*I*x) + 1/2*gamma(-1, -2*I*x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{\cos(x)\sin(x)}{x^2}\,dx = \frac{2\,x\,\mathrm{Ci}\,(2\,x) - \sin(2\,x)}{2\,x}$$

input
```
integrate(cos(x)*sin(x)/x^2,x, algorithm="giac")
```

output
```
1/2*(2*x*cos_integral(2*x) - sin(2*x))/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)\sin(x)}{x^2}\,dx = \int \frac{\cos(x)\,\sin(x)}{x^2}\,dx$$

input
```
int((cos(x)*sin(x))/x^2,x)
```

output
```
int((cos(x)*sin(x))/x^2, x)
```

Reduce [F]

$$\int \frac{\cos(x)\sin(x)}{x^2}\,dx = \int \frac{\cos(x)\sin(x)}{x^2}\,dx$$

input
```
int(cos(x)*sin(x)/x^2,x)
```

output
```
int((cos(x)*sin(x))/x**2,x)
```

3.12 $\int \frac{\cos(x)\sin(x)}{x^3}\,dx$

Optimal result 249
Mathematica [A] (verified) 249
Rubi [A] (verified) 250
Maple [A] (verified) 252
Fricas [A] (verification not implemented) 252
Sympy [A] (verification not implemented) 253
Maxima [C] (verification not implemented) 253
Giac [A] (verification not implemented) 253
Mupad [F(-1)] 254
Reduce [F] 254

Optimal result

Integrand size = 8, antiderivative size = 29

$$\int \frac{\cos(x)\sin(x)}{x^3}\,dx = -\frac{\cos(2x)}{2x} - \frac{\sin(2x)}{4x^2} - \operatorname{Si}(2x)$$

output
```
-1/2*cos(2*x)/x-1/4*sin(2*x)/x^2-Si(2*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)\sin(x)}{x^3}\,dx = -\frac{\cos(2x)}{2x} - \frac{\sin(2x)}{4x^2} - \operatorname{Si}(2x)$$

input
```
Integrate[(Cos[x]*Sin[x])/x^3,x]
```

output
```
-1/2*Cos[2*x]/x - Sin[2*x]/(4*x^2) - SinIntegral[2*x]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 1.125$, Rules used = {4906, 27, 3042, 3778, 3042, 3778, 25, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)\cos(x)}{x^3}\,dx$$

$$\downarrow 4906$$

$$\int \frac{\sin(2x)}{2x^3}dx$$

$$\downarrow 27$$

$$\frac{1}{2}\int \frac{\sin(2x)}{x^3}dx$$

$$\downarrow 3042$$

$$\frac{1}{2}\int \frac{\sin(2x)}{x^3}dx$$

$$\downarrow 3778$$

$$\frac{1}{2}\left(\int \frac{\cos(2x)}{x^2}dx - \frac{\sin(2x)}{2x^2}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\int \frac{\sin\left(2x+\frac{\pi}{2}\right)}{x^2}dx - \frac{\sin(2x)}{2x^2}\right)$$

$$\downarrow 3778$$

$$\frac{1}{2}\left(2\int -\frac{\sin(2x)}{x}dx - \frac{\sin(2x)}{2x^2} - \frac{\cos(2x)}{x}\right)$$

$$\downarrow 25$$

$$\frac{1}{2}\left(-2\int \frac{\sin(2x)}{x}dx - \frac{\sin(2x)}{2x^2} - \frac{\cos(2x)}{x}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(-2\int\frac{\sin(2x)}{x}dx-\frac{\sin(2x)}{2x^2}-\frac{\cos(2x)}{x}\right)$$

$$\downarrow 3780$$

$$\frac{1}{2}\left(-2\text{Si}(2x)-\frac{\sin(2x)}{2x^2}-\frac{\cos(2x)}{x}\right)$$

input
```
Int[(Cos[x]*Sin[x])/x^3,x]
```

output
```
(-(Cos[2*x]/x) - Sin[2*x]/(2*x^2) - 2*SinIntegral[2*x])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{\cos(2x)}{2x}-\frac{\sin(2x)}{4x^2}-\mathrm{Si}\left(2x\right)$	26
risch	$\frac{\pi\ \mathrm{csgn}(x)}{2}-\mathrm{Si}\left(2x\right)-\frac{\cos(2x)}{2x}-\frac{\sin(2x)}{4x^2}$	31
meijerg	$\frac{\sqrt{\pi}\left(-\frac{2\cos(2x)}{x\sqrt{\pi}}-\frac{\sin(2x)}{x^2\sqrt{\pi}}-\frac{4\ \mathrm{Si}(2x)}{\sqrt{\pi}}\right)}{4}$	40

input

```
int(cos(x)*sin(x)/x^3,x,method=_RETURNVERBOSE)
```

output

```
-1/2*cos(2*x)/x-1/4*sin(2*x)/x^2-Si(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int \frac{\cos(x)\sin(x)}{x^3}\,dx = -\frac{2\,x\cos\left(x\right)^2+2\,x^2\,\mathrm{Si}\left(2\,x\right)+\cos\left(x\right)\sin\left(x\right)-x}{2\,x^2}$$

input

```
integrate(cos(x)*sin(x)/x^3,x, algorithm="fricas")
```

output

```
-1/2*(2*x*cos(x)^2 + 2*x^2*sin_integral(2*x) + cos(x)*sin(x) - x)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{\cos(x)\sin(x)}{x^3}\,dx = -\operatorname{Si}(2x) - \frac{\cos(2x)}{2x} - \frac{\sin(2x)}{4x^2}$$

input
```
integrate(cos(x)*sin(x)/x**3,x)
```

output
```
-Si(2*x) - cos(2*x)/(2*x) - sin(2*x)/(4*x**2)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.52

$$\int \frac{\cos(x)\sin(x)}{x^3}\,dx = i\,\Gamma(-2, 2i\,x) - i\,\Gamma(-2, -2i\,x)$$

input
```
integrate(cos(x)*sin(x)/x^3,x, algorithm="maxima")
```

output
```
I*gamma(-2, 2*I*x) - I*gamma(-2, -2*I*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{\cos(x)\sin(x)}{x^3}\,dx = -\frac{4\,x^2\operatorname{Si}(2\,x) + 2\,x\cos(2\,x) + \sin(2\,x)}{4\,x^2}$$

input
```
integrate(cos(x)*sin(x)/x^3,x, algorithm="giac")
```

output
```
-1/4*(4*x^2*sin_integral(2*x) + 2*x*cos(2*x) + sin(2*x))/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)\sin(x)}{x^3}\,dx = \int \frac{\cos(x)\,\sin(x)}{x^3}\,dx$$

input
```
int((cos(x)*sin(x))/x^3,x)
```

output
```
int((cos(x)*sin(x))/x^3, x)
```

Reduce [F]

$$\int \frac{\cos(x)\sin(x)}{x^3}\,dx = \int \frac{\cos(x)\sin(x)}{x^3}dx$$

input
```
int(cos(x)*sin(x)/x^3,x)
```

output
```
int((cos(x)*sin(x))/x**3,x)
```

3.13 $\int (c+dx)^m \cos(a+bx) \sin^2(a+bx)\, dx$

Optimal result . . . 255
Mathematica [A] (verified) . . . 256
Rubi [A] (verified) . . . 256
Maple [F] . . . 258
Fricas [A] (verification not implemented) . . . 258
Sympy [F] . . . 258
Maxima [F] . . . 259
Giac [F] . . . 259
Mupad [F(-1)] . . . 259
Reduce [F] . . . 260

Optimal result

Integrand size = 22, antiderivative size = 275

$$\begin{aligned}
&\int (c+dx)^m \cos(a+bx) \sin^2(a+bx)\, dx \\
&= -\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{ib(c+dx)}{d}\right)}{8b} \\
&\quad + \frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, \frac{ib(c+dx)}{d}\right)}{8b} \\
&\quad + \frac{i3^{-1-m}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{3ib(c+dx)}{d}\right)}{8b} \\
&\quad - \frac{i3^{-1-m}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, \frac{3ib(c+dx)}{d}\right)}{8b}
\end{aligned}$$

output

```
-1/8*I*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+
c)/d)^m)+1/8*I*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b
*(d*x+c)/d)^m)+1/8*I*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*
b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)-1/8*I*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I
*b*(d*x+c)/d)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.86

$$\int (c+dx)^m \cos(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{ie^{-\frac{3i(bc+ad)}{d}}(c+dx)^m\left(3e^{2i\left(2a+\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)+\left(\frac{ib(c+dx)}{d}\right)^{-m}\left(-3e^{2ia+\frac{4ibc}{d}}\Gamma\left(\right.\right.\right.}{}$$

input

```
Integrate[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output

```
((-1/24*I)*(c + d*x)^m*((3*E^((2*I)*(2*a + (b*c)/d))*Gamma[1 + m, ((-I)*b*
(c + d*x))/d])/(((-I)*b*(c + d*x))/d)^m + (-3*E^((2*I)*a + ((4*I)*b*c)/d)*
Gamma[1 + m, (I*b*(c + d*x))/d] + (-((E^((6*I)*a)*((I*b*(c + d*x))/d)^(2*m
)*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/((b^2*(c + d*x)^2)/d^2)^m) + E^(((
6*I)*b*c)/d)*Gamma[1 + m, ((3*I)*b*(c + d*x))/d])/3^m)/((I*b*(c + d*x))/d)
^m))/(b*E^(((3*I)*(b*c + a*d))/d))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos(a+bx)(c+dx)^m\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}\cos(a+bx)(c+dx)^m - \frac{1}{4}\cos(3a+3bx)(c+dx)^m\right)dx$$

$$\downarrow 2009$$

$$\frac{-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}+\frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{8b}+\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{8b}-\frac{i3^{-m-1}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{3ib(c+dx)}{d}\right)}{8b}}{8b}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
((-1/8*I)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/
d])/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/8)*(c + d*x)^m*Gamma[1 + m, (I*b*(c
 + d*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + ((I/8)*3^(-1
- m)*E^((3*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))
/d])/(b*(((-I)*b*(c + d*x))/d)^m) - ((I/8)*3^(-1 - m)*(c + d*x)^m*Gamma[1
+ m, ((3*I)*b*(c + d*x))/d])/(b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d
)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [F]

$$\int (dx+c)^m \cos(bx+a) \sin(bx+a)^2 \, dx$$

input
```
int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.68

$$\int (c+dx)^m \cos(a+bx) \sin^2(a+bx) \, dx$$
$$= \frac{3i\, e^{\left(-\frac{dm \log\left(\frac{ib}{d}\right) - ibc + iad}{d}\right)} \Gamma\left(m+1, \frac{ibdx+ibc}{d}\right) + i\, e^{\left(-\frac{dm \log\left(-\frac{3ib}{d}\right) + 3ibc - 3iad}{d}\right)} \Gamma\left(m+1, -\frac{3(ibdx+ibc)}{d}\right) - 3i\, e^{\left(-\frac{dm}{}\right.}}{24\, b}$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/24*(3*I*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x +
I*b*c)/d) + I*e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d)*gamma(m + 1,
-3*(I*b*d*x + I*b*c)/d) - 3*I*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gam
ma(m + 1, (-I*b*d*x - I*b*c)/d) - I*e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*
a*d)/d)*gamma(m + 1, -3*(-I*b*d*x - I*b*c)/d))/b
```

Sympy [F]

$$\int (c+dx)^m \cos(a+bx) \sin^2(a+bx) \, dx = \int (c+dx)^m \sin^2(a+bx) \cos(a+bx) \, dx$$

input
```
integrate((d*x+c)**m*cos(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*sin(a + b*x)**2*cos(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)^m \cos(a+bx) \sin^2(a+bx)\, dx = \int (dx+c)^m \cos(bx+a) \sin(bx+a)^2\, dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^2, x)
```

Giac [F]

$$\int (c+dx)^m \cos(a+bx) \sin^2(a+bx)\, dx = \int (dx+c)^m \cos(bx+a) \sin(bx+a)^2\, dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos(a+bx) \sin^2(a+bx)\, dx = \int \cos(a+b\,x)\, \sin(a+b\,x)^2\, (c+d\,x)^m\, dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \cos(a+bx) \sin^2(a+bx)\, dx = \int (dx+c)^m \cos(bx+a) \sin(bx+a)^2\, dx$$

input
```
int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x)
```

output
```
int((c + d*x)**m*cos(a + b*x)*sin(a + b*x)**2,x)
```

3.14 $\int (c+dx)^4 \cos(a+bx) \sin^2(a+bx)\, dx$

Optimal result . . . 261
Mathematica [A] (verified) . . . 262
Rubi [A] (verified) . . . 262
Maple [A] (verified) . . . 267
Fricas [A] (verification not implemented) . . . 268
Sympy [B] (verification not implemented) . . . 268
Maxima [B] (verification not implemented) . . . 269
Giac [A] (verification not implemented) . . . 270
Mupad [B] (verification not implemented) . . . 271
Reduce [B] (verification not implemented) . . . 272

Optimal result

Integrand size = 22, antiderivative size = 205

$$\begin{aligned}\int (c+dx)^4 \cos(a+bx) \sin^2(a+bx)\, dx = &-\frac{160 d^3 (c+dx)\cos(a+bx)}{27b^4} \\ &+\frac{8d(c+dx)^3\cos(a+bx)}{9b^2}+\frac{160d^4\sin(a+bx)}{27b^5} \\ &-\frac{8d^2(c+dx)^2\sin(a+bx)}{3b^3} \\ &-\frac{8d^3(c+dx)\cos(a+bx)\sin^2(a+bx)}{27b^4} \\ &+\frac{4d(c+dx)^3\cos(a+bx)\sin^2(a+bx)}{9b^2} \\ &+\frac{8d^4\sin^3(a+bx)}{81b^5} \\ &-\frac{4d^2(c+dx)^2\sin^3(a+bx)}{9b^3} \\ &+\frac{(c+dx)^4\sin^3(a+bx)}{3b}\end{aligned}$$

output

```
-160/27*d^3*(d*x+c)*cos(b*x+a)/b^4+8/9*d*(d*x+c)^3*cos(b*x+a)/b^2+160/27*d
^4*sin(b*x+a)/b^5-8/3*d^2*(d*x+c)^2*sin(b*x+a)/b^3-8/27*d^3*(d*x+c)*cos(b*
x+a)*sin(b*x+a)^2/b^4+4/9*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2/b^2+8/81*d^4
*sin(b*x+a)^3/b^5-4/9*d^2*(d*x+c)^2*sin(b*x+a)^3/b^3+1/3*(d*x+c)^4*sin(b*x
+a)^3/b
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.88

$$\int (c+dx)^4 \cos(a+bx) \sin^2(a+bx)\, dx$$
$$= \frac{324bd(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\cos(a+bx) - 12bd(c+dx)\left(-2d^2+3b^2(c+dx)^2\right)\cos(3(a+bx)) +$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
(324*b*d*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 12*b*d*(c + d
*x)*(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 81*b^4*c^4*Sin[a + b*x
] - 972*b^2*c^2*d^2*Sin[a + b*x] + 1944*d^4*Sin[a + b*x] + 324*b^4*c^3*d*x
*Sin[a + b*x] - 1944*b^2*c*d^3*x*Sin[a + b*x] + 486*b^4*c^2*d^2*x^2*Sin[a
+ b*x] - 972*b^2*d^4*x^2*Sin[a + b*x] + 324*b^4*c*d^3*x^3*Sin[a + b*x] + 8
1*b^4*d^4*x^4*Sin[a + b*x] - 27*b^4*c^4*Sin[3*(a + b*x)] + 36*b^2*c^2*d^2*
Sin[3*(a + b*x)] - 8*d^4*Sin[3*(a + b*x)] - 108*b^4*c^3*d*x*Sin[3*(a + b*x
)] + 72*b^2*c*d^3*x*Sin[3*(a + b*x)] - 162*b^4*c^2*d^2*x^2*Sin[3*(a + b*x)
] + 36*b^2*d^4*x^2*Sin[3*(a + b*x)] - 108*b^4*c*d^3*x^3*Sin[3*(a + b*x)] -
 27*b^4*d^4*x^4*Sin[3*(a + b*x)])/(324*b^5)
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.21, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {4904, 3042, 3792, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117, 3791, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sin^2(a+bx)\cos(a+bx)\, dx$$
$$\downarrow 4904$$
$$\frac{(c+dx)^4 \sin^3(a+bx)}{3b} - \frac{4d\int (c+dx)^3 \sin^3(a+bx) dx}{3b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\int(c+dx)^3\sin(a+bx)^3dx}{3b}$$

$$\downarrow 3792$$

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\left(-\frac{2d^2\int(c+dx)\sin^3(a+bx)dx}{3b^2}+\frac{2}{3}\int(c+dx)^3\sin(a+bx)dx+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}-\frac{(c+dx)^3\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\left(-\frac{2d^2\int(c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\int(c+dx)^3\sin(a+bx)dx+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}-\frac{(c+dx)^3\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b}$$

$$\downarrow 3777$$

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\left(-\frac{2d^2\int(c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{3d\int(c+dx)^2\cos(a+bx)dx}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}-\frac{(c+dx)^3\sin^2(a+bx)}{3b}\right)}{3b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\left(-\frac{2d^2\int(c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{3d\int(c+dx)^2\sin(a+bx+\frac{\pi}{2})dx}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}-\frac{(c+dx)^3\sin^2(a+}{3b}\right)}{3b}$$

$$\downarrow 3777$$

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\left(-\frac{2d^2\int(c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{2d\int-((c+dx)\sin(a+bx))dx}{b}+\frac{(c+dx)^2\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)+\frac{d(c+dx)^2\sin^3(a+b}{3b^2}\right)}{3b}$$

$$\downarrow 25$$

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\left(-\frac{2d^2\int(c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int(c+dx)\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}\right)}{3b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b} - \frac{4d\left(-\frac{2d^2\int(c+dx)\sin(a+bx)^3dx}{3b^2} + \frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b} - \frac{2d\int(c+dx)\sin(a+bx)dx}{b}\right)}{b} - \frac{(c+dx)^3\cos(a+bx)}{b}\right) + \frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}\right)}{3b}$$

$$\downarrow 3777$$

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b} - \frac{4d\left(-\frac{2d^2\int(c+dx)\sin(a+bx)^3dx}{3b^2} + \frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b} - \frac{2d\left(\frac{d\int\cos(a+bx)dx}{b} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b} - \frac{(c+dx)^3\cos(a+bx)}{b}\right) + \frac{d(c+}{}\right)}{3b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b} - \frac{4d\left(-\frac{2d^2\int(c+dx)\sin(a+bx)^3dx}{3b^2} + \frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b} - \frac{2d\left(\frac{d\int\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b} - \frac{(c+dx)^3\cos(a+bx)}{b}\right) + \frac{c}{}\right)}{3b}$$

$$\downarrow 3117$$

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b} - \frac{4d\left(-\frac{2d^2\int(c+dx)\sin(a+bx)^3dx}{3b^2} + \frac{d(c+dx)^2\sin^3(a+bx)}{3b^2} + \frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b} - \frac{2d\left(\frac{d\sin(a+bx)}{b^2} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b} - \frac{(c+dx)^3c}{b}\right)\right)}{3b}$$

$$\downarrow 3791$$

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b} -$$

$$4d\left(-\frac{2d^2\left(\frac{2}{3}\int(c+dx)\sin(a+bx)dx+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}+\frac{2}{3}\left(3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\right.\right.\right.\right.$$

$$\Big\downarrow 3042$$

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b} -$$

$$4d\left(-\frac{2d^2\left(\frac{2}{3}\int(c+dx)\sin(a+bx)dx+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}+\frac{2}{3}\left(3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\right.\right.\right.\right.$$

$$\Big\downarrow 3777$$

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b} -$$

$$4d\left(-\frac{2d^2\left(\frac{2}{3}\left(\frac{d\int\cos(a+bx)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}+\frac{2}{3}\left(3d\left(\frac{(c+dx)^2\,si\right.\right.\right.\right.$$

$$\Big\downarrow 3042$$

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b} -$$

$$4d\left(-\frac{2d^2\left(\frac{2}{3}\left(\frac{d\int\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}+\frac{2}{3}\left(3d\left(\frac{(c+\right.\right.\right.\right.$$

$$\Big\downarrow 3117$$

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b} -$$

$$4d\left(-\frac{2d^2\left(\frac{2}{3}\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}+\frac{2}{3}\left(3d\left(\frac{(c+dx)^2\sin(a+\right.\right.\right.\right.$$

input
```
Int[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
((c + d*x)^4*Sin[a + b*x]^3)/(3*b) - (4*d*(-1/3*((c + d*x)^3*Cos[a + b*x]*
Sin[a + b*x]^2)/b + (d*(c + d*x)^2*Sin[a + b*x]^3)/(3*b^2) - (2*d^2*(-1/3*
((c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2)/b + (d*Sin[a + b*x]^3)/(9*b^2) + (
2*(-(((c + d*x)*Cos[a + b*x])/b) + (d*Sin[a + b*x])/b^2))/3))/(3*b^2) + (2
*(-(((c + d*x)^3*Cos[a + b*x])/b) + (3*d*(((c + d*x)^2*Sin[a + b*x])/b - (
2*d*(-(((c + d*x)*Cos[a + b*x])/b) + (d*Sin[a + b*x])/b^2))/b))/b))/3))/(3
*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)  Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))  Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4904

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))  Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.84

method	result
parallelrisch	$\frac{\left(-27(dx+c)^4b^4+36d^2(dx+c)^2b^2-8d^4\right)\sin(3bx+3a)-36d\left((dx+c)^2b^2-\frac{2d^2}{3}\right)(dx+c)b\cos(3bx+3a)+81\left((dx+c)^4b^4-\right.}{324b^5}$
risch	$\frac{d\left(b^2d^3x^3+3b^2cd^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6cd^2\right)\cos(bx+a)}{b^4}+\frac{\left(d^4x^4b^4+4b^4cd^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+b^4c^4-1\right.}{4b}$
derivativedivides	Expression too large to display
default	Expression too large to display
orering	$\frac{16d\left(135b^6d^6x^6+810b^6cd^5x^5+2025b^6c^2d^4x^4+2700b^6c^3d^3x^3+2025b^6c^4d^2x^2-891b^4d^6x^4+810b^6c^5dx-3564b^4cd^5x^3+\right.}{243}$

input

```
int((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
1/324*((-27*(d*x+c)^4*b^4+36*d^2*(d*x+c)^2*b^2-8*d^4)*sin(3*b*x+3*a)-36*d*
((d*x+c)^2*b^2-2/3*d^2)*(d*x+c)*b*cos(3*b*x+3*a)+81*((d*x+c)^4*b^4-12*d^2*
(d*x+c)^2*b^2+24*d^4)*sin(b*x+a)+324*d*(((d*x+c)^2*b^2-6*d^2)*(d*x+c)*cos(
b*x+a)+8/9*b^2*c^3-160/27*c*d^2)*b)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.72

$$\int (c+dx)^4 \cos(a+bx) \sin^2(a+bx)\, dx =$$

$$-\frac{12\left(3\,b^3d^4x^3+9\,b^3cd^3x^2+3\,b^3c^3d-2\,bcd^3+(9\,b^3c^2d^2-2\,bd^4)x\right)\cos\left(bx+a\right)^3-36\left(3\,b^3d^4x^3+9\,b^3cd\right.}{}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/81*(12*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d - 2*b*c*d^3 + (9*
b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^3 - 36*(3*b^3*d^4*x^3 + 9*b^3*c*d^3
*x^2 + 3*b^3*c^3*d - 14*b*c*d^3 + (9*b^3*c^2*d^2 - 14*b*d^4)*x)*cos(b*x +
a) - (27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 - 252*b^2*c^2*d^2 +
488*d^4 + 18*(9*b^4*c^2*d^2 - 14*b^2*d^4)*x^2 - (27*b^4*d^4*x^4 + 108*b^4*
c*d^3*x^3 + 27*b^4*c^4 - 36*b^2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 - 2*b^
2*d^4)*x^2 + 36*(3*b^4*c^3*d - 2*b^2*c*d^3)*x)*cos(b*x + a)^2 + 36*(3*b^4*
c^3*d - 14*b^2*c*d^3)*x)*sin(b*x + a))/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. 2(207) = 414.

Time = 0.64 (sec) , antiderivative size = 646, normalized size of antiderivative = 3.15

$$\int (c+dx)^4 \cos(a+bx) \sin^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**4*cos(b*x+a)*sin(b*x+a)**2,x)
```

output

```
Piecewise((c**4*sin(a + b*x)**3/(3*b) + 4*c**3*d*x*sin(a + b*x)**3/(3*b) +
 2*c**2*d**2*x**2*sin(a + b*x)**3/b + 4*c*d**3*x**3*sin(a + b*x)**3/(3*b)
+ d**4*x**4*sin(a + b*x)**3/(3*b) + 4*c**3*d*sin(a + b*x)**2*cos(a + b*x)/
(3*b**2) + 8*c**3*d*cos(a + b*x)**3/(9*b**2) + 4*c**2*d**2*x*sin(a + b*x)*
*2*cos(a + b*x)/b**2 + 8*c**2*d**2*x*cos(a + b*x)**3/(3*b**2) + 4*c*d**3*x
**2*sin(a + b*x)**2*cos(a + b*x)/b**2 + 8*c*d**3*x**2*cos(a + b*x)**3/(3*b
**2) + 4*d**4*x**3*sin(a + b*x)**2*cos(a + b*x)/(3*b**2) + 8*d**4*x**3*cos
(a + b*x)**3/(9*b**2) - 28*c**2*d**2*sin(a + b*x)**3/(9*b**3) - 8*c**2*d**
2*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 56*c*d**3*x*sin(a + b*x)**3/(9*b
**3) - 16*c*d**3*x*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 28*d**4*x**2*si
n(a + b*x)**3/(9*b**3) - 8*d**4*x**2*sin(a + b*x)*cos(a + b*x)**2/(3*b**3)
 - 56*c*d**3*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 160*c*d**3*cos(a + b*
x)**3/(27*b**4) - 56*d**4*x*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 160*d*
*4*x*cos(a + b*x)**3/(27*b**4) + 488*d**4*sin(a + b*x)**3/(81*b**5) + 160*
d**4*sin(a + b*x)*cos(a + b*x)**2/(27*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*
d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**2*cos(a), T
rue))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 880 vs. $2(187) = 374$.

Time $= 0.07$ (sec) , antiderivative size $= 880$, normalized size of antiderivative $= 4.29$

$$\int (c+dx)^4 \cos(a+bx) \sin^2(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output

```
1/324*(108*c^4*sin(b*x + a)^3 - 432*a*c^3*d*sin(b*x + a)^3/b + 648*a^2*c^2
*d^2*sin(b*x + a)^3/b^2 - 432*a^3*c*d^3*sin(b*x + a)^3/b^3 + 108*a^4*d^4*s
in(b*x + a)^3/b^4 - 36*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x
 + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*c^3*d/b + 108*(3*(b*x + a)*sin(
3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a
))*a*c^2*d^2/b^2 - 108*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x
 + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*a^2*c*d^3/b^3 + 36*(3*(b*x + a)
*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*
x + a))*a^3*d^4/b^4 - 18*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(
b*x + a) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin
(b*x + a))*c^2*d^2/b^2 + 36*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*c
os(b*x + a) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*
sin(b*x + a))*a*c*d^3/b^3 - 18*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a
)*cos(b*x + a) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 -
2)*sin(b*x + a))*a^2*d^4/b^4 - 12*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) -
81*((b*x + a)^2 - 2)*cos(b*x + a) + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*
b*x + 3*a) - 27*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*c*d^3/b^3 + 12*(
(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a) +
 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) - 27*((b*x + a)^3 - 6*b*
x - 6*a)*sin(b*x + a))*a*d^4/b^4 - (12*(3*(b*x + a)^3 - 2*b*x - 2*a)*co...
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.71

$$
\begin{aligned}
&\int (c+dx)^4 \cos(a+bx) \sin^2(a+bx)\, dx \\
&= -\frac{\left(3\,b^3d^4x^3 + 9\,b^3cd^3x^2 + 9\,b^3c^2d^2x + 3\,b^3c^3d - 2\,bd^4x - 2\,bcd^3\right)\cos\left(3\,bx + 3\,a\right)}{27\,b^5} \\
&\quad + \frac{\left(b^3d^4x^3 + 3\,b^3cd^3x^2 + 3\,b^3c^2d^2x + b^3c^3d - 6\,bd^4x - 6\,bcd^3\right)\cos\left(bx + a\right)}{b^5} \\
&\quad - \frac{\left(27\,b^4d^4x^4 + 108\,b^4cd^3x^3 + 162\,b^4c^2d^2x^2 + 108\,b^4c^3dx + 27\,b^4c^4 - 36\,b^2d^4x^2 - 72\,b^2cd^3x - 36\,b^2c^2d^2 + \right.}{324\,b^5} \\
&\quad + \frac{\left(b^4d^4x^4 + 4\,b^4cd^3x^3 + 6\,b^4c^2d^2x^2 + 4\,b^4c^3dx + b^4c^4 - 12\,b^2d^4x^2 - 24\,b^2cd^3x - 12\,b^2c^2d^2 + 24\,d^4\right)\sin\left(bx\right.}{4\,b^5}
\end{aligned}
$$

input

```
integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output

```
-1/27*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2
*b*d^4*x - 2*b*c*d^3)*cos(3*b*x + 3*a)/b^5 + (b^3*d^4*x^3 + 3*b^3*c*d^3*x^
2 + 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*cos(b*x + a)/b^5
 - 1/324*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^
4*c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2
 + 8*d^4)*sin(3*b*x + 3*a)/b^5 + 1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4
*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x -
 12*b^2*c^2*d^2 + 24*d^4)*sin(b*x + a)/b^5
```

Mupad [B] (verification not implemented)

Time = 18.52 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.19

$$
\begin{aligned}
&\int (c+dx)^4 \cos(a+bx)\sin^2(a+bx)\,dx \\
&= \frac{\sin(a+bx)^3\left(27b^4c^4-252b^2c^2d^2+488d^4\right)}{81b^5} - \frac{8\cos(a+bx)^3\left(20cd^3-3b^2c^3d\right)}{27b^4} \\
&\quad + \frac{8\cos(a+bx)^2\sin(a+bx)\left(20d^4-9b^2c^2d^2\right)}{27b^5} \\
&\quad - \frac{4\cos(a+bx)\sin(a+bx)^2\left(14cd^3-3b^2c^3d\right)}{9b^4} \\
&\quad + \frac{8d^4x^3\cos(a+bx)^3}{9b^2} - \frac{8x\cos(a+bx)^3\left(20d^4-9b^2c^2d^2\right)}{27b^4} \\
&\quad + \frac{d^4x^4\sin(a+bx)^3}{3b} - \frac{4x\sin(a+bx)^3\left(14cd^3-3b^2c^3d\right)}{9b^3} \\
&\quad - \frac{2x^2\sin(a+bx)^3\left(14d^4-9b^2c^2d^2\right)}{9b^3} + \frac{8cd^3x^2\cos(a+bx)^3}{3b^2} \\
&\quad + \frac{4d^4x^3\cos(a+bx)\sin(a+bx)^2}{3b^2} - \frac{8d^4x^2\cos(a+bx)^2\sin(a+bx)}{3b^3} \\
&\quad + \frac{4cd^3x^3\sin(a+bx)^3}{3b} - \frac{4x\cos(a+bx)\sin(a+bx)^2\left(14d^4-9b^2c^2d^2\right)}{9b^4} \\
&\quad + \frac{4cd^3x^2\cos(a+bx)\sin(a+bx)^2}{b^2} - \frac{16cd^3x\cos(a+bx)^2\sin(a+bx)}{3b^3}
\end{aligned}
$$

input

```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^4,x)
```

output
```
(sin(a + b*x)^3*(488*d^4 + 27*b^4*c^4 - 252*b^2*c^2*d^2))/(81*b^5) - (8*co
s(a + b*x)^3*(20*c*d^3 - 3*b^2*c^3*d))/(27*b^4) + (8*cos(a + b*x)^2*sin(a
+ b*x)*(20*d^4 - 9*b^2*c^2*d^2))/(27*b^5) - (4*cos(a + b*x)*sin(a + b*x)^2
*(14*c*d^3 - 3*b^2*c^3*d))/(9*b^4) + (8*d^4*x^3*cos(a + b*x)^3)/(9*b^2) -
(8*x*cos(a + b*x)^3*(20*d^4 - 9*b^2*c^2*d^2))/(27*b^4) + (d^4*x^4*sin(a +
b*x)^3)/(3*b) - (4*x*sin(a + b*x)^3*(14*c*d^3 - 3*b^2*c^3*d))/(9*b^3) - (2
*x^2*sin(a + b*x)^3*(14*d^4 - 9*b^2*c^2*d^2))/(9*b^3) + (8*c*d^3*x^2*cos(a
 + b*x)^3)/(3*b^2) + (4*d^4*x^3*cos(a + b*x)*sin(a + b*x)^2)/(3*b^2) - (8*
d^4*x^2*cos(a + b*x)^2*sin(a + b*x))/(3*b^3) + (4*c*d^3*x^3*sin(a + b*x)^3
)/(3*b) - (4*x*cos(a + b*x)*sin(a + b*x)^2*(14*d^4 - 9*b^2*c^2*d^2))/(9*b^
4) + (4*c*d^3*x^2*cos(a + b*x)*sin(a + b*x)^2)/b^2 - (16*c*d^3*x*cos(a + b
*x)^2*sin(a + b*x))/(3*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 501, normalized size of antiderivative = 2.44

$$\int (c+dx)^4 \cos(a+bx) \sin^2(a+bx)\, dx$$
$$= \frac{-480\cos(bx+a)\, b\, d^4x + 27\sin(bx+a)^3\, b^4d^4x^4 - 36\sin(bx+a)^3\, b^2c^2d^2 - 36\sin(bx+a)^3\, b^2d^4x^2 - 21(}{}$$

input
```
int((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x)
```

output
```
(36*cos(a + b*x)*sin(a + b*x)**2*b**3*c**3*d + 108*cos(a + b*x)*sin(a + b*
x)**2*b**3*c**2*d**2*x + 108*cos(a + b*x)*sin(a + b*x)**2*b**3*c*d**3*x**2
 + 36*cos(a + b*x)*sin(a + b*x)**2*b**3*d**4*x**3 - 24*cos(a + b*x)*sin(a
+ b*x)**2*b*c*d**3 - 24*cos(a + b*x)*sin(a + b*x)**2*b*d**4*x + 72*cos(a +
 b*x)*b**3*c**3*d + 216*cos(a + b*x)*b**3*c**2*d**2*x + 216*cos(a + b*x)*b
**3*c*d**3*x**2 + 72*cos(a + b*x)*b**3*d**4*x**3 - 480*cos(a + b*x)*b*c*d*
*3 - 480*cos(a + b*x)*b*d**4*x + 27*sin(a + b*x)**3*b**4*c**4 + 108*sin(a
+ b*x)**3*b**4*c**3*d*x + 162*sin(a + b*x)**3*b**4*c**2*d**2*x**2 + 108*si
n(a + b*x)**3*b**4*c*d**3*x**3 + 27*sin(a + b*x)**3*b**4*d**4*x**4 - 36*si
n(a + b*x)**3*b**2*c**2*d**2 - 72*sin(a + b*x)**3*b**2*c*d**3*x - 36*sin(a
 + b*x)**3*b**2*d**4*x**2 + 8*sin(a + b*x)**3*d**4 - 216*sin(a + b*x)*b**2
*c**2*d**2 - 432*sin(a + b*x)*b**2*c*d**3*x - 216*sin(a + b*x)*b**2*d**4*x
**2 + 480*sin(a + b*x)*d**4 - 216*a*b**2*c**2*d**2 + 192*a*d**4 + 72*b**3*
c**3*d - 192*b*c*d**3)/(81*b**5)
```

3.15 $\int (c+dx)^3 \cos(a+bx) \sin^2(a+bx)\, dx$

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Optimal result

Integrand size = 22, antiderivative size = 151

$$\begin{aligned}\int (c+dx)^3 \cos(a+bx) \sin^2(a+bx)\, dx = &-\frac{14d^3 \cos(a+bx)}{9b^4} + \frac{2d(c+dx)^2 \cos(a+bx)}{3b^2} \\ &+ \frac{2d^3 \cos^3(a+bx)}{27b^4} - \frac{4d^2(c+dx)\sin(a+bx)}{3b^3} \\ &+ \frac{d(c+dx)^2 \cos(a+bx)\sin^2(a+bx)}{3b^2} \\ &- \frac{2d^2(c+dx)\sin^3(a+bx)}{9b^3} \\ &+ \frac{(c+dx)^3 \sin^3(a+bx)}{3b}\end{aligned}$$

output

```
-14/9*d^3*cos(b*x+a)/b^4+2/3*d*(d*x+c)^2*cos(b*x+a)/b^2+2/27*d^3*cos(b*x+a
)^3/b^4-4/3*d^2*(d*x+c)*sin(b*x+a)/b^3+1/3*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+
a)^2/b^2-2/9*d^2*(d*x+c)*sin(b*x+a)^3/b^3+1/3*(d*x+c)^3*sin(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.80

$$\int (c+dx)^3 \cos(a+bx) \sin^2(a+bx)\, dx =$$
$$-\frac{-81d(-2d^2+b^2(c+dx)^2)\cos(a+bx)+d(-2d^2+9b^2(c+dx)^2)\cos(3(a+bx))+6b(c+dx)\,(26d^2-}{108b^4}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
-1/108*(-81*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + d*(-2*d^2 + 9*b^2*
(c + d*x)^2)*Cos[3*(a + b*x)] + 6*b*(c + d*x)*(26*d^2 - 3*b^2*(c + d*x)^2
+ (-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/b^4
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {4904, 3042, 3792, 3042, 3113, 2009, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin^2(a+bx)\cos(a+bx)\, dx$$
$$\downarrow 4904$$
$$\frac{(c+dx)^3\sin^3(a+bx)}{3b} - \frac{d\int (c+dx)^2\sin^3(a+bx)dx}{b}$$
$$\downarrow 3042$$
$$\frac{(c+dx)^3\sin^3(a+bx)}{3b} - \frac{d\int (c+dx)^2\sin(a+bx)^3dx}{b}$$
$$\downarrow 3792$$

$$\frac{(c+dx)^3\sin^3(a+bx)}{3b} - \frac{d\left(-\frac{2d^2\int\sin^3(a+bx)dx}{9b^2}+\frac{2}{3}\int(c+dx)^2\sin(a+bx)dx+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^3\sin^3(a+bx)}{3b} - \frac{d\left(-\frac{2d^2\int\sin(a+bx)^3dx}{9b^2}+\frac{2}{3}\int(c+dx)^2\sin(a+bx)dx+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{b}$$

$$\downarrow 3113$$

$$\frac{(c+dx)^3\sin^3(a+bx)}{3b} - \frac{d\left(\frac{2d^2\int(1-\cos^2(a+bx))d\cos(a+bx)}{9b^3}+\frac{2}{3}\int(c+dx)^2\sin(a+bx)dx+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{b}$$

$$\downarrow 2009$$

$$\frac{(c+dx)^3\sin^3(a+bx)}{3b} - \frac{d\left(\frac{2}{3}\int(c+dx)^2\sin(a+bx)dx+\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{b}$$

$$\downarrow 3777$$

$$\frac{(c+dx)^3\sin^3(a+bx)}{3b} - \frac{d\left(\frac{2}{3}\left(\frac{2d\int(c+dx)\cos(a+bx)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)+\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)}{3b}\right.}{b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^3\sin^3(a+bx)}{3b} - \frac{d\left(\frac{2}{3}\left(\frac{2d\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)+\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+}{3b}\right.}{b}$$

$$\downarrow 3777$$

$$\frac{(c+dx)^3\sin^3(a+bx)}{3b} - \frac{d\left(\frac{2}{3}\left(\frac{2d\left(\frac{d\int-\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)+\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+}{3b}\right.}{b}$$

$$\downarrow 25$$

$$\frac{(c+dx)^3\sin^3(a+bx)}{3b} - \frac{d\left(\frac{2}{3}\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b} - \frac{d\int\sin(a+bx)dx}{b}\right)}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right) + \frac{2d^2\left(\cos(a+bx) - \frac{1}{3}\cos^3(a+bx)\right)}{9b^3} + \frac{2d(c+dx)\sin^3(a+bx)}{9b^2} - \frac{(c+dx}{}\right.}{b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^3\sin^3(a+bx)}{3b} - \frac{d\left(\frac{2}{3}\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b} - \frac{d\int\sin(a+bx)dx}{b}\right)}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right) + \frac{2d^2\left(\cos(a+bx) - \frac{1}{3}\cos^3(a+bx)\right)}{9b^3} + \frac{2d(c+dx)\sin^3(a+bx)}{9b^2} - \frac{(c+dx}{}\right.}{b}$$

$$\downarrow 3118$$

$$\frac{(c+dx)^3\sin^3(a+bx)}{3b} - \frac{d\left(\frac{2d^2\left(\cos(a+bx) - \frac{1}{3}\cos^3(a+bx)\right)}{9b^3} + \frac{2d(c+dx)\sin^3(a+bx)}{9b^2} + \frac{2}{3}\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2} + \frac{(c+dx)\sin(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right) - \frac{(c+dx)^2}{}\right.}{b}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
((c + d*x)^3*Sin[a + b*x]^3)/(3*b) - (d*((2*d^2*(Cos[a + b*x] - Cos[a + b*
x]^3/3))/(9*b^3) - ((c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2)/(3*b) + (2*d*
(c + d*x)*Sin[a + b*x]^3)/(9*b^2) + (2*(-(((c + d*x)^2*Cos[a + b*x])/b) +
(2*d*((d*Cos[a + b*x])/b^2 + ((c + d*x)*Sin[a + b*x])/b))/b))/3))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.48

method	result
risch	$\frac{3d(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\cos(bx+a)}{4b^4}+\frac{(b^2d^3x^3+3b^2cd^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6cd^2)\sin(bx+a)}{4b^3}-\frac{d(9x}{}$
parallelrisch	$\frac{-36d^2x\left(\frac{dx}{2}+c\right)b^2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^6-72bd^2(dx+c)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^5+((-54x^2b^2-72)d^3-108b^2cd^2x)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4+72((}{}$
norman	$\frac{36b^2c^2d-80d^3}{27b^4}+\frac{2d^3x^2}{3b^2}-\frac{8d^3\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4}{3b^4}+\frac{\left(36b^2c^2d-56d^3\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{9b^4}-\frac{8cd^2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{3b^3}-\frac{8cd^2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^5}{3b^3}+\frac{4cd^2}{3b^2}$
derivativedivides	$-\frac{a^3d^3\sin(bx+a)^3}{3b^3}+\frac{a^2cd^2\sin(bx+a)^3}{b^2}+\frac{3a^2d^3\left(\frac{(bx+a)\sin(bx+a)^3}{3}+\frac{\left(2+\sin(bx+a)^2\right)\cos(bx+a)}{9}\right)}{b^3}-\frac{ac^2d\sin(bx+a)^3}{b}-\frac{6acd^2}{}$
default	$-\frac{a^3d^3\sin(bx+a)^3}{3b^3}+\frac{a^2cd^2\sin(bx+a)^3}{b^2}+\frac{3a^2d^3\left(\frac{(bx+a)\sin(bx+a)^3}{3}+\frac{\left(2+\sin(bx+a)^2\right)\cos(bx+a)}{9}\right)}{b^3}-\frac{ac^2d\sin(bx+a)^3}{b}-\frac{6acd^2}{}$
orering	$\frac{20d(9d^4x^4b^4+36b^4cd^3x^3+54b^4c^2d^2x^2+36b^4c^3dx+9b^4c^4-22b^2d^4x^2-44b^2cd^3x-22b^2c^2d^2-72d^4)\cos(bx+a)\sin(bx+}{27b^6(dx+c)^2}$

input
```
int((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3/4*d*(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2-2*d^2)/b^4*cos(b*x+a)+1/4/b^3*(b^2*
d^3*x^3+3*b^2*c*d^2*x^2+3*b^2*c^2*d*x+b^2*c^3-6*d^3*x-6*c*d^2)*sin(b*x+a)-
1/108*d*(9*b^2*d^2*x^2+18*b^2*c*d*x+9*b^2*c^2-2*d^2)/b^4*cos(3*b*x+3*a)-1/
36/b^3*(3*b^2*d^3*x^3+9*b^2*c*d^2*x^2+9*b^2*c^2*d*x+3*b^2*c^3-2*d^3*x-2*c*
d^2)*sin(3*b*x+3*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.50

$$\int (c+dx)^3\cos(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3)\cos(bx+a)^3-3\,(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-14\,d^3)\cos(}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/27*((9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a)
^3 - 3*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 14*d^3)*cos(b*x + a
) - 3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 14*b*c*d^2 - (3*b^3*d
^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 2*b*c*d^2 + (9*b^3*c^2*d - 2*b*d^3)
*x)*cos(b*x + a)^2 + (9*b^3*c^2*d - 14*b*d^3)*x)*sin(b*x + a))/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(150) = 300$.

Time $= 0.45$ (sec) , antiderivative size $= 391$, normalized size of antiderivative $= 2.59$

$$\int (c+dx)^3 \cos(a+bx) \sin^2(a+bx)\, dx$$
$$= \begin{cases} \frac{c^3 \sin^3(a+bx)}{3b} + \frac{c^2 dx \sin^3(a+bx)}{b} + \frac{cd^2x^2 \sin^3(a+bx)}{b} + \frac{d^3x^3 \sin^3(a+bx)}{3b} + \frac{c^2 d \sin^2(a+bx)\cos(a+bx)}{b^2} + \frac{2c^2 d \cos^3(a+bx)}{3b^2} + \frac{2c} \\ \left(c^3 x + \frac{3c^2 dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) \sin^2(a) \cos(a) \end{cases}$$

input
```
integrate((d*x+c)**3*cos(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Piecewise((c**3*sin(a + b*x)**3/(3*b) + c**2*d*x*sin(a + b*x)**3/b + c*d**
2*x**2*sin(a + b*x)**3/b + d**3*x**3*sin(a + b*x)**3/(3*b) + c**2*d*sin(a
+ b*x)**2*cos(a + b*x)/b**2 + 2*c**2*d*cos(a + b*x)**3/(3*b**2) + 2*c*d**2
*x*sin(a + b*x)**2*cos(a + b*x)/b**2 + 4*c*d**2*x*cos(a + b*x)**3/(3*b**2)
 + d**3*x**2*sin(a + b*x)**2*cos(a + b*x)/b**2 + 2*d**3*x**2*cos(a + b*x)*
*3/(3*b**2) - 14*c*d**2*sin(a + b*x)**3/(9*b**3) - 4*c*d**2*sin(a + b*x)*c
os(a + b*x)**2/(3*b**3) - 14*d**3*x*sin(a + b*x)**3/(9*b**3) - 4*d**3*x*si
n(a + b*x)*cos(a + b*x)**2/(3*b**3) - 14*d**3*sin(a + b*x)**2*cos(a + b*x)
/(9*b**4) - 40*d**3*cos(a + b*x)**3/(27*b**4), Ne(b, 0)), ((c**3*x + 3*c**
2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**2*cos(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(137) = 274$.

Time = 0.06 (sec) , antiderivative size = 499, normalized size of antiderivative = 3.30

$$\int (c+dx)^3 \cos(a+bx) \sin^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/108*(36*c^3*sin(b*x + a)^3 - 108*a*c^2*d*sin(b*x + a)^3/b + 108*a^2*c*d^
2*sin(b*x + a)^3/b^2 - 36*a^3*d^3*sin(b*x + a)^3/b^3 - 9*(3*(b*x + a)*sin(
3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a
))*c^2*d/b + 18*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) +
 cos(3*b*x + 3*a) - 9*cos(b*x + a))*a*c*d^2/b^2 - 9*(3*(b*x + a)*sin(3*b*x
 + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*a^
2*d^3/b^3 - 3*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a) +
(9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))*
c*d^2/b^2 + 3*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a) +
(9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))*
a*d^3/b^3 - ((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*c
os(b*x + a) + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) - 27*((b*x
+ a)^3 - 6*b*x - 6*a)*sin(b*x + a))*d^3/b^3)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.53

$$\begin{aligned}
&\int (c+dx)^3 \cos(a+bx) \sin^2(a+bx)\, dx \\
&= -\frac{(9\,b^2d^3x^2 + 18\,b^2cd^2x + 9\,b^2c^2d - 2\,d^3)\cos(3\,bx+3\,a)}{108\,b^4} \\
&\quad + \frac{3\,(b^2d^3x^2 + 2\,b^2cd^2x + b^2c^2d - 2\,d^3)\cos(bx+a)}{4\,b^4} \\
&\quad - \frac{(3\,b^3d^3x^3 + 9\,b^3cd^2x^2 + 9\,b^3c^2dx + 3\,b^3c^3 - 2\,bd^3x - 2\,bcd^2)\sin(3\,bx+3\,a)}{36\,b^4} \\
&\quad + \frac{(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3 - 6\,bd^3x - 6\,bcd^2)\sin(bx+a)}{4\,b^4}
\end{aligned}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/108*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(3*b*x +
3*a)/b^4 + 3/4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x +
 a)/b^4 - 1/36*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^
3 - 2*b*d^3*x - 2*b*c*d^2)*sin(3*b*x + 3*a)/b^4 + 1/4*(b^3*d^3*x^3 + 3*b^3
*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(b*x + a)
/b^4
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.91

$$
\begin{aligned}
&\int (c+dx)^3 \cos(a+bx) \sin^2(a+bx)\, dx \\
&= \frac{2\,d^3\,x^2 \cos\left(a+b\,x\right)^3}{3\,b^2} - \frac{\sin\left(a+b\,x\right)^3 \left(14\,c\,d^2 - 3\,b^2\,c^3\right)}{9\,b^3} \\
&\quad - \frac{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2 \left(14\,d^3 - 9\,b^2\,c^2\,d\right)}{9\,b^4} - \frac{x \sin\left(a+b\,x\right)^3 \left(14\,d^3 - 9\,b^2\,c^2\,d\right)}{9\,b^3} \\
&\quad - \frac{2\cos\left(a+b\,x\right)^3 \left(20\,d^3 - 9\,b^2\,c^2\,d\right)}{27\,b^4} + \frac{d^3\,x^3 \sin\left(a+b\,x\right)^3}{3\,b} \\
&\quad - \frac{4\,c\,d^2 \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{3\,b^3} + \frac{4\,c\,d^2\,x \cos\left(a+b\,x\right)^3}{3\,b^2} \\
&\quad - \frac{4\,d^3\,x \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{3\,b^3} + \frac{d^3\,x^2\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2}{b^2} \\
&\quad + \frac{c\,d^2\,x^2 \sin\left(a+b\,x\right)^3}{b} + \frac{2\,c\,d^2\,x\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2}{b^2}
\end{aligned}
$$

input
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^3,x)
```

output
```
(2*d^3*x^2*cos(a + b*x)^3)/(3*b^2) - (sin(a + b*x)^3*(14*c*d^2 - 3*b^2*c^3
))/(9*b^3) - (cos(a + b*x)*sin(a + b*x)^2*(14*d^3 - 9*b^2*c^2*d))/(9*b^4)
- (x*sin(a + b*x)^3*(14*d^3 - 9*b^2*c^2*d))/(9*b^3) - (2*cos(a + b*x)^3*(2
0*d^3 - 9*b^2*c^2*d))/(27*b^4) + (d^3*x^3*sin(a + b*x)^3)/(3*b) - (4*c*d^2
*cos(a + b*x)^2*sin(a + b*x))/(3*b^3) + (4*c*d^2*x*cos(a + b*x)^3)/(3*b^2)
 - (4*d^3*x*cos(a + b*x)^2*sin(a + b*x))/(3*b^3) + (d^3*x^2*cos(a + b*x)*s
in(a + b*x)^2)/b^2 + (c*d^2*x^2*sin(a + b*x)^3)/b + (2*c*d^2*x*cos(a + b*x
)*sin(a + b*x)^2)/b^2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.03

$$\int (c+dx)^3 \cos(a+bx) \sin^2(a+bx)\, dx$$
$$= \frac{9\cos(bx+a)\sin(bx+a)^2 b^2c^2d + 18\cos(bx+a)\sin(bx+a)^2 b^2c\,d^2x + 9\cos(bx+a)\sin(bx+a)^2 b^2d^3}$$

input
```
int((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x)
```

output
```
(9*cos(a + b*x)*sin(a + b*x)**2*b**2*c**2*d + 18*cos(a + b*x)*sin(a + b*x)
**2*b**2*c*d**2*x + 9*cos(a + b*x)*sin(a + b*x)**2*b**2*d**3*x**2 - 2*cos(
a + b*x)*sin(a + b*x)**2*d**3 + 18*cos(a + b*x)*b**2*c**2*d + 36*cos(a + b
*x)*b**2*c*d**2*x + 18*cos(a + b*x)*b**2*d**3*x**2 - 40*cos(a + b*x)*d**3
+ 9*sin(a + b*x)**3*b**3*c**3 + 27*sin(a + b*x)**3*b**3*c**2*d*x + 27*sin(
a + b*x)**3*b**3*c*d**2*x**2 + 9*sin(a + b*x)**3*b**3*d**3*x**3 - 6*sin(a
+ b*x)**3*b*c*d**2 - 6*sin(a + b*x)**3*b*d**3*x - 36*sin(a + b*x)*b*c*d**2
 - 36*sin(a + b*x)*b*d**3*x - 36*a*b*c*d**2 + 18*b**2*c**2*d - 16*d**3)/(2
7*b**4)
```

3.16 $\int (c+dx)^2 \cos(a+bx) \sin^2(a+bx)\, dx$

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Optimal result

Integrand size = 22, antiderivative size = 103

$$\int (c+dx)^2 \cos(a+bx) \sin^2(a+bx)\, dx = \frac{4d(c+dx)\cos(a+bx)}{9b^2} - \frac{4d^2 \sin(a+bx)}{9b^3} + \frac{2d(c+dx)\cos(a+bx)\sin^2(a+bx)}{9b^2} - \frac{2d^2 \sin^3(a+bx)}{27b^3} + \frac{(c+dx)^2 \sin^3(a+bx)}{3b}$$

output

```
4/9*d*(d*x+c)*cos(b*x+a)/b^2-4/9*d^2*sin(b*x+a)/b^3+2/9*d*(d*x+c)*cos(b*x+
a)*sin(b*x+a)^2/b^2-2/27*d^2*sin(b*x+a)^3/b^3+1/3*(d*x+c)^2*sin(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.90

$$\int (c+dx)^2 \cos(a+bx) \sin^2(a+bx)\, dx = \frac{54bd(c+dx)\cos(a+bx) - 6bd(c+dx)\cos(3(a+bx)) - 2(26d^2 - 9b^2(c+dx)^2 + (-2d^2 + 9b^2(c+dx)^2}{108b^3}$$

input

```
Integrate[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output

```
(54*b*d*(c + d*x)*Cos[a + b*x] - 6*b*d*(c + d*x)*Cos[3*(a + b*x)] - 2*(26*
d^2 - 9*b^2*(c + d*x)^2 + (-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*S
in[a + b*x])/(108*b^3)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {4904, 3042, 3791, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin^2(a+bx)\cos(a+bx)\,dx$$

$$\downarrow 4904$$

$$\frac{(c+dx)^2\sin^3(a+bx)}{3b} - \frac{2d\int (c+dx)\sin^3(a+bx)dx}{3b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^2\sin^3(a+bx)}{3b} - \frac{2d\int (c+dx)\sin(a+bx)^3dx}{3b}$$

$$\downarrow 3791$$

$$\frac{(c+dx)^2\sin^3(a+bx)}{3b} - \frac{2d\left(\frac{2}{3}\int (c+dx)\sin(a+bx)dx + \frac{d\sin^3(a+bx)}{9b^2} - \frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^2\sin^3(a+bx)}{3b} - \frac{2d\left(\frac{2}{3}\int (c+dx)\sin(a+bx)dx + \frac{d\sin^3(a+bx)}{9b^2} - \frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b}$$

$$\downarrow 3777$$

$$\frac{(c+dx)^2\sin^3(a+bx)}{3b} - \frac{2d\left(\frac{2}{3}\left(\frac{d\int\cos(a+bx)dx}{b} - \frac{(c+dx)\cos(a+bx)}{b}\right) + \frac{d\sin^3(a+bx)}{9b^2} - \frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^2\sin^3(a+bx)}{3b} - \frac{2d\left(\frac{2}{3}\left(\frac{d\int\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b} - \frac{(c+dx)\cos(a+bx)}{b}\right) + \frac{d\sin^3(a+bx)}{9b^2} - \frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b}$$

$$\downarrow 3117$$

$$\frac{(c+dx)^2\sin^3(a+bx)}{3b} - \frac{2d\left(\frac{2}{3}\left(\frac{d\sin(a+bx)}{b^2} - \frac{(c+dx)\cos(a+bx)}{b}\right) + \frac{d\sin^3(a+bx)}{9b^2} - \frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
((c + d*x)^2*Sin[a + b*x]^3)/(3*b) - (2*d*(-1/3*((c + d*x)*Cos[a + b*x]*Si
n[a + b*x]^2)/b + (d*Sin[a + b*x]^3)/(9*b^2) + (2*(-(((c + d*x)*Cos[a + b*
x])/b) + (d*Sin[a + b*x])/b^2))/3))/(3*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)  Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4904

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))  Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.24

method	result
risch	$\frac{d(dx+c)\cos(bx+a)}{2b^2}+\frac{(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\sin(bx+a)}{4b^3}-\frac{d(dx+c)\cos(3bx+3a)}{18b^2}-\frac{(9x^2d^2b^2+18b^2cdx+9}{10}$
parallelrisch	$\frac{-12d^2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^6xb-24d^2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^5-36d^2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4xb+\left((72x^2b^2-64)d^2+144b^2cdx+72b^2c^2\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right.}{27b^3\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)^3}$
derivativedivides	$\frac{a^2d^2\sin(bx+a)^3}{3b^2}-\frac{2acd\sin(bx+a)^3}{3b}-\frac{2a\,d^2\left(\frac{(bx+a)\sin(bx+a)^3}{3}+\frac{\left(2+\sin(bx+a)^2\right)\cos(bx+a)}{9}\right)}{b^2}+\frac{c^2\sin(bx+a)^3}{3}+\frac{2cd\left(\frac{(bx+a)\sin}{}\right.}{}$
default	$\frac{a^2d^2\sin(bx+a)^3}{3b^2}-\frac{2acd\sin(bx+a)^3}{3b}-\frac{2a\,d^2\left(\frac{(bx+a)\sin(bx+a)^3}{3}+\frac{\left(2+\sin(bx+a)^2\right)\cos(bx+a)}{9}\right)}{b^2}+\frac{c^2\sin(bx+a)^3}{3}+\frac{2cd\left(\frac{(bx+a)\sin}{}\right.}{}$
norman	$\frac{\frac{8cd}{9b^2}-\frac{8d^2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{9b^3}-\frac{8d^2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^5}{9b^3}+\frac{4d^2x}{9b^2}+\frac{8\left(9b^2c^2-8d^2\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3}{27b^3}+\frac{8cd\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{3b^2}+\frac{8d^2x^2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3}{3b}+}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)^3}$
orering	$\frac{40d\left(9d^4x^4b^4+36b^4cd^3x^3+54b^4c^2d^2x^2+36b^4c^3dx+9b^4c^4-b^2d^4x^2-2b^2cd^3x-b^2c^2d^2-12d^4\right)\cos(bx+a)\sin(bx+a)^2}{81b^6(dx+c)^3}-$

input

```
int((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*d*(d*x+c)*cos(b*x+a)/b^2+1/4*(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2-2*d^2)/b
^3*sin(b*x+a)-1/18*d*(d*x+c)*cos(3*b*x+3*a)/b^2-1/108*(9*b^2*d^2*x^2+18*b^
2*c*d*x+9*b^2*c^2-2*d^2)/b^3*sin(3*b*x+3*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.26

$$\int (c+dx)^2 \cos(a+bx) \sin^2(a+bx)\, dx =$$
$$-\frac{6\left(bd^2x+bcd\right)\cos\left(bx+a\right)^3 - 18\left(bd^2x+bcd\right)\cos\left(bx+a\right) - \left(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-(9\,b^2d^2x^2\right.}{27\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/27*(6*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 18*(b*d^2*x + b*c*d)*cos(b*x +
 a) - (9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - (9*b^2*d^2*x^2 + 18*b^2*
c*d*x + 9*b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - 14*d^2)*sin(b*x + a))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(102) = 204.

Time = 0.33 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.10

$$\int (c+dx)^2 \cos(a+bx) \sin^2(a+bx)\, dx$$
$$= \begin{cases} \frac{c^2 \sin^3(a+bx)}{3b} + \frac{2cdx \sin^3(a+bx)}{3b} + \frac{d^2x^2 \sin^3(a+bx)}{3b} + \frac{2cd \sin^2(a+bx)\cos(a+bx)}{3b^2} + \frac{4cd\cos^3(a+bx)}{9b^2} + \frac{2d^2x \sin^2(a+bx)\cos(a+b}{3b^2} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin^2(a)\cos(a) \end{cases}$$

input
```
integrate((d*x+c)**2*cos(b*x+a)*sin(b*x+a)**2,x)
```

output

```
Piecewise((c**2*sin(a + b*x)**3/(3*b) + 2*c*d*x*sin(a + b*x)**3/(3*b) + d*
*2*x**2*sin(a + b*x)**3/(3*b) + 2*c*d*sin(a + b*x)**2*cos(a + b*x)/(3*b**2
) + 4*c*d*cos(a + b*x)**3/(9*b**2) + 2*d**2*x*sin(a + b*x)**2*cos(a + b*x)
/(3*b**2) + 4*d**2*x*cos(a + b*x)**3/(9*b**2) - 14*d**2*sin(a + b*x)**3/(2
7*b**3) - 4*d**2*sin(a + b*x)*cos(a + b*x)**2/(9*b**3), Ne(b, 0)), ((c**2*
x + c*d*x**2 + d**2*x**3/3)*sin(a)**2*cos(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(93) = 186$.

Time $= 0.04$ (sec) , antiderivative size $= 240$, normalized size of antiderivative $= 2.33$

$$\int (c+dx)^2 \cos(a+bx) \sin^2(a+bx)\, dx$$

$$= \frac{36\, c^2 \sin(bx+a)^3 - \frac{72\, acd \sin(bx+a)^3}{b} + \frac{36\, a^2 d^2 \sin(bx+a)^3}{b^2} - \frac{6\,(3\,(bx+a)\sin(3\,bx+3\,a) - 9\,(bx+a)\sin(bx+a) + \cos(3\,bx+3\,a) - 9\,\cos}{b}$$

input

```
integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output

```
1/108*(36*c^2*sin(b*x + a)^3 - 72*a*c*d*sin(b*x + a)^3/b + 36*a^2*d^2*sin(
b*x + a)^3/b^2 - 6*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a
) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*c*d/b + 6*(3*(b*x + a)*sin(3*b*x +
3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*a*d^2
/b^2 - (6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a) + (9*(b*x
 + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))*d^2/b^2
)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.33

$$\begin{aligned}
&\int (c+dx)^2 \cos(a+bx) \sin^2(a+bx)\, dx \\
&= -\frac{(bd^2x+bcd)\cos(3bx+3a)}{18b^3} + \frac{(bd^2x+bcd)\cos(bx+a)}{2b^3} \\
&\quad - \frac{(9b^2d^2x^2+18b^2cdx+9b^2c^2-2d^2)\sin(3bx+3a)}{108b^3} \\
&\quad + \frac{(b^2d^2x^2+2b^2cdx+b^2c^2-2d^2)\sin(bx+a)}{4b^3}
\end{aligned}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/18*(b*d^2*x + b*c*d)*cos(3*b*x + 3*a)/b^3 + 1/2*(b*d^2*x + b*c*d)*cos(b
*x + a)/b^3 - 1/108*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*sin
(3*b*x + 3*a)/b^3 + 1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(
b*x + a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.56

$$\begin{aligned}
&\int (c+dx)^2 \cos(a+bx) \sin^2(a+bx)\, dx \\
&= \frac{4d^2 x \cos(a+bx)^3}{9b^2} - \frac{4d^2 \cos(a+bx)^2 \sin(a+bx)}{9b^3} - \frac{\sin(a+bx)^3 (14d^2-9b^2c^2)}{27b^3} \\
&\quad + \frac{d^2 x^2 \sin(a+bx)^3}{3b} + \frac{4cd\cos(a+bx)^3}{9b^2} + \frac{2cd\cos(a+bx)\sin(a+bx)^2}{3b^2} \\
&\quad + \frac{2cdx\sin(a+bx)^3}{3b} + \frac{2d^2 x\cos(a+bx)\sin(a+bx)^2}{3b^2}
\end{aligned}$$

input
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^2,x)
```

output
```
(4*d^2*x*cos(a + b*x)^3)/(9*b^2) - (4*d^2*cos(a + b*x)^2*sin(a + b*x))/(9*
b^3) - (sin(a + b*x)^3*(14*d^2 - 9*b^2*c^2))/(27*b^3) + (d^2*x^2*sin(a + b
*x)^3)/(3*b) + (4*c*d*cos(a + b*x)^3)/(9*b^2) + (2*c*d*cos(a + b*x)*sin(a
+ b*x)^2)/(3*b^2) + (2*c*d*x*sin(a + b*x)^3)/(3*b) + (2*d^2*x*cos(a + b*x)
*sin(a + b*x)^2)/(3*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.51

$$\int (c+dx)^2 \cos(a+bx) \sin^2(a+bx)\, dx$$
$$= \frac{6\cos(bx+a)\sin(bx+a)^2 bcd + 6\cos(bx+a)\sin(bx+a)^2 b\,d^2x + 12\cos(bx+a)\,bcd + 12\cos(bx+a)}$$

input
```
int((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x)
```

output
```
(6*cos(a + b*x)*sin(a + b*x)**2*b*c*d + 6*cos(a + b*x)*sin(a + b*x)**2*b*d
**2*x + 12*cos(a + b*x)*b*c*d + 12*cos(a + b*x)*b*d**2*x + 9*sin(a + b*x)*
*3*b**2*c**2 + 18*sin(a + b*x)**3*b**2*c*d*x + 9*sin(a + b*x)**3*b**2*d**2
*x**2 - 2*sin(a + b*x)**3*d**2 - 12*sin(a + b*x)*d**2 - 12*a*d**2 + 12*b*c
*d)/(27*b**3)
```

3.17 $\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal result . . . 291
Mathematica [A] (verified) . . . 291
Rubi [A] (verified) . . . 292
Maple [A] (verified) . . . 293
Fricas [A] (verification not implemented) . . . 294
Sympy [A] (verification not implemented) . . . 294
Maxima [A] (verification not implemented) . . . 295
Giac [A] (verification not implemented) . . . 295
Mupad [B] (verification not implemented) . . . 296
Reduce [B] (verification not implemented) . . . 296

Optimal result

Integrand size = 20, antiderivative size = 51

$$\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx = \frac{d\cos(a+bx)}{3b^2} - \frac{d\cos^3(a+bx)}{9b^2} + \frac{(c+dx)\sin^3(a+bx)}{3b}$$

output
```
1/3*d*cos(b*x+a)/b^2-1/9*d*cos(b*x+a)^3/b^2+1/3*(d*x+c)*sin(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx = \frac{9d\cos(a+bx) - d\cos(3(a+bx)) + 12b(c+dx)\sin^3(a+bx)}{36b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
(9*d*Cos[a + b*x] - d*Cos[3*(a + b*x)] + 12*b*(c + d*x)*Sin[a + b*x]^3)/(3
6*b^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4904, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin^2(a+bx)\cos(a+bx)\,dx$$

$$\downarrow 4904$$

$$\frac{(c+dx)\sin^3(a+bx)}{3b} - \frac{d\int \sin^3(a+bx)dx}{3b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)\sin^3(a+bx)}{3b} - \frac{d\int \sin(a+bx)^3dx}{3b}$$

$$\downarrow 3113$$

$$\frac{d\int \left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{3b^2} + \frac{(c+dx)\sin^3(a+bx)}{3b}$$

$$\downarrow 2009$$

$$\frac{d\left(\cos(a+bx) - \frac{1}{3}\cos^3(a+bx)\right)}{3b^2} + \frac{(c+dx)\sin^3(a+bx)}{3b}$$

input
```
Int[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
(d*(Cos[a + b*x] - Cos[a + b*x]^3/3))/(3*b^2) + ((c + d*x)*Sin[a + b*x]^3)
/(3*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

method	result
parallelrisch	$\frac{\frac{8b(dx+c)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3}{3}+\frac{4\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2 d}{3}+\frac{4d}{9}}{b^2\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)^3}$
risch	$\frac{d\cos(bx+a)}{4b^2}+\frac{(dx+c)\sin(bx+a)}{4b}-\frac{d\cos(3bx+3a)}{36b^2}-\frac{(dx+c)\sin(3bx+3a)}{12b}$
derivativedivides	$\frac{-\frac{da\sin(bx+a)^3}{3b}+\frac{c\sin(bx+a)^3}{3}+\frac{d\left(\frac{(bx+a)\sin(bx+a)^3}{3}+\frac{\left(2+\sin(bx+a)^2\right)\cos(bx+a)}{9}\right)}{b}}{b}$
default	$\frac{-\frac{da\sin(bx+a)^3}{3b}+\frac{c\sin(bx+a)^3}{3}+\frac{d\left(\frac{(bx+a)\sin(bx+a)^3}{3}+\frac{\left(2+\sin(bx+a)^2\right)\cos(bx+a)}{9}\right)}{b}}{b}$
norman	$\frac{\frac{4d}{9b^2}+\frac{8c\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3}{3b}+\frac{4d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{3b^2}+\frac{8dx\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3}{3b}}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)^3}$
orering	$\frac{4d\left(5x^2d^2b^2+10b^2cdx+5b^2c^2+2d^2\right)\cos(bx+a)\sin(bx+a)^2}{9b^4(dx+c)^2}-\frac{2\left(5x^2d^2b^2+10b^2cdx+5b^2c^2+4d^2\right)\left(d\cos(bx+a)\sin(bx$

input
```
int((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
4/9*(6*b*(d*x+c)*tan(1/2*a+1/2*b*x)^3+3*tan(1/2*a+1/2*b*x)^2*d+d)/b^2/(1+t
an(1/2*a+1/2*b*x)^2)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{d\cos(bx+a)^3 - 3\,d\cos(bx+a) - 3\left(bdx - (bdx+bc)\cos(bx+a)^2 + bc\right)\sin(bx+a)}{9\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/9*(d*cos(b*x + a)^3 - 3*d*cos(b*x + a) - 3*(b*d*x - (b*d*x + b*c)*cos(b
*x + a)^2 + b*c)*sin(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.67

$$\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx$$
$$= \begin{cases} \frac{c\sin^3(a+bx)}{3b} + \frac{dx\sin^3(a+bx)}{3b} + \frac{d\sin^2(a+bx)\cos(a+bx)}{3b^2} + \frac{2d\cos^3(a+bx)}{9b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\sin^2(a)\cos(a) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Piecewise((c*sin(a + b*x)**3/(3*b) + d*x*sin(a + b*x)**3/(3*b) + d*sin(a +
 b*x)**2*cos(a + b*x)/(3*b**2) + 2*d*cos(a + b*x)**3/(9*b**2), Ne(b, 0)),
((c*x + d*x**2/2)*sin(a)**2*cos(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.67

$$\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{12\,c\sin(bx+a)^3 - \frac{12\,ad\sin(bx+a)^3}{b} - \frac{(3\,(bx+a)\sin(3\,bx+3\,a) - 9\,(bx+a)\sin(bx+a) + \cos(3\,bx+3\,a) - 9\,\cos(bx+a))d}{b}}{36\,b}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/36*(12*c*sin(b*x + a)^3 - 12*a*d*sin(b*x + a)^3/b - (3*(b*x + a)*sin(3*b
*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*
d/b)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.35

$$\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx = -\frac{d\cos(3\,bx+3\,a)}{36\,b^2} + \frac{d\cos(bx+a)}{4\,b^2} - \frac{(bdx+bc)\sin(3\,bx+3\,a)}{12\,b^2} + \frac{(bdx+bc)\sin(bx+a)}{4\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/36*d*cos(3*b*x + 3*a)/b^2 + 1/4*d*cos(b*x + a)/b^2 - 1/12*(b*d*x + b*c)
*sin(3*b*x + 3*a)/b^2 + 1/4*(b*d*x + b*c)*sin(b*x + a)/b^2
```

Mupad [B] (verification not implemented)

Time = 18.65 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx = \frac{\frac{2\,d\cos(a+b\,x)^3}{9}+b\left(\frac{c\sin(a+b\,x)^3}{3}+\frac{d\,x\sin(a+b\,x)^3}{3}\right)+\frac{d\,\cos(a+b\,x)\sin(a+b\,x)^2}{3}}{b^2}$$

input
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x),x)
```

output
```
((2*d*cos(a + b*x)^3)/9 + b*((c*sin(a + b*x)^3)/3 + (d*x*sin(a + b*x)^3)/3
) + (d*cos(a + b*x)*sin(a + b*x)^2)/3)/b^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx = \frac{\cos(bx+a)\sin(bx+a)^2 d+2\cos(bx+a)d+3\sin(bx+a)^3 bc+3\sin(bx+a)^3 bdx+2d}{9b^2}$$

input
```
int((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x)
```

output
```
(cos(a + b*x)*sin(a + b*x)**2*d + 2*cos(a + b*x)*d + 3*sin(a + b*x)**3*b*c
 + 3*sin(a + b*x)**3*b*d*x + 2*d)/(9*b**2)
```

3.18 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx$

Optimal result . . . 297
Mathematica [A] (verified) . . . 298
Rubi [A] (verified) . . . 298
Maple [A] (verified) . . . 299
Fricas [A] (verification not implemented) . . . 300
Sympy [F] . . . 300
Maxima [C] (verification not implemented) . . . 300
Giac [C] (verification not implemented) . . . 301
Mupad [F(-1)] . . . 302
Reduce [F] . . . 303

Optimal result

Integrand size = 22, antiderivative size = 121

$$\begin{aligned}\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx = &\frac{\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}\\ &-\frac{\cos\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}\\ &-\frac{\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{4d}\\ &+\frac{\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}\end{aligned}$$

output

```
1/4*cos(a-b*c/d)*Ci(b*c/d+b*x)/d-1/4*cos(3*a-3*b*c/d)*Ci(3*b*c/d+3*b*x)/d-
1/4*sin(a-b*c/d)*Si(b*c/d+b*x)/d+1/4*sin(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx$$
$$= \frac{\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right) - \cos\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3b(c+dx)}{d}\right) - \sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(b\left(\frac{c}{d}+x\right)\right)}{4d}$$

input
```
Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x),x]
```

output
```
(Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] - Cos[3*a - (3*b*c)/d]*CosInteg
ral[(3*b*(c + d*x))/d] - Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Sin[3
*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos(a+bx)}{c+dx}\,dx$$
$$\downarrow 4906$$
$$\int \left(\frac{\cos(a+bx)}{4(c+dx)} - \frac{\cos(3a+3bx)}{4(c+dx)}\right)dx$$
$$\downarrow 2009$$
$$\frac{\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d} - \frac{\cos\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d} - \frac{\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{4d} + \frac{\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x),x]
```

output
```
(Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d) - (Cos[3*a - (3*b*c)/d
]*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d) - (Sin[a - (b*c)/d]*SinIntegral[(b
*c)/d + b*x])/(4*d) + (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x]
)/(4*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{b\left(-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(bx+a+\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}\right)}{4}-\frac{b\left(-\frac{3\ \mathrm{Si}\left(-3bx-3a-\frac{3(-ad+cb)}{d}\right)\sin\left(\frac{-3ad}{d}\right.\right.}{d}}{b}$
default	$\frac{b\left(-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(bx+a+\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}\right)}{4}-\frac{b\left(-\frac{3\ \mathrm{Si}\left(-3bx-3a-\frac{3(-ad+cb)}{d}\right)\sin\left(\frac{-3ad}{d}\right.\right.}{d}}{b}$
risch	$\frac{e^{-\frac{3i(ad-cb)}{d}}\ \mathrm{expIntegral}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{8d}-\frac{e^{-\frac{i(ad-cb)}{d}}\ \mathrm{expIntegral}_1\left(ibx+ia-\frac{i(ad-cb)}{d}\right)}{8d}-\frac{e^{\frac{i(ad-cb)}{d}}\ \mathrm{exp}}{}$

input
```
int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4*b*(-Si(-b*x-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*
c)/d)*cos((-a*d+b*c)/d)/d)-1/12*b*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*sin(3*
(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.01

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx =$$
$$-\frac{\cos\left(-\frac{3\,(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)-\cos\left(-\frac{bc-ad}{d}\right)\operatorname{Ci}\left(\frac{bdx+bc}{d}\right)-\sin\left(-\frac{3\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{3\,(bdx+bc)}{d}\right)+\sin\left(-\frac{bc-}{}\right.}{4\,d}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
-1/4*(cos(-3*(b*c - a*d)/d)*cos_integral(3*(b*d*x + b*c)/d) - cos(-(b*c -
a*d)/d)*cos_integral((b*d*x + b*c)/d) - sin(-3*(b*c - a*d)/d)*sin_integral
(3*(b*d*x + b*c)/d) + sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

Sympy [F]

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \int \frac{\sin^2(a+bx)\cos(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.28

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx =$$
$$-\frac{b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b\left(E_1\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_1\left(-\right.\right.}{}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
-1/8*(b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integr
al_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b*(exp_
integral_e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1, -3
*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) + b*(-I*exp_in
tegral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e(1, -(I*b
*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(-I*exp_integral_e
(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integral_e(1, -3*(-I*b*c
 - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/(b*d)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.33 (sec) , antiderivative size = 6059, normalized size of antiderivative = 50.07

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output

```
-1/8*(real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(b*x + b*c/d))*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_
integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2
*b*c/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2
*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(b*x + b
*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_
part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d) + 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)
^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(cos_integral(3*b*x + 3*b*
c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_p
art(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/
d)*tan(1/2*b*c/d)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1
/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(b*x + b
*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*imag_
part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*
tan(1/2*b*c/d)^2 - 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(3*b*x + 3*b*
c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*imag_p
art(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+b\,x)\,\sin(a+b\,x)^2}{c+d\,x}\,dx$$

input

```
int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x),x)
```

output

```
int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \int \frac{\cos\left(bx+a\right)\sin\left(bx+a\right)^2}{dx+c}dx$$

input
```
int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x)
```

output
```
int((cos(a + b*x)*sin(a + b*x)**2)/(c + d*x),x)
```

3.19 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$

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Optimal result

Integrand size = 22, antiderivative size = 168

$$\begin{aligned}\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = &-\frac{\cos(a+bx)}{4d(c+dx)}+\frac{\cos(3a+3bx)}{4d(c+dx)}\\&+\frac{3b\,\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d^2}\\&-\frac{b\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{4d^2}\\&-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}\\&+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d^2}\end{aligned}$$

output

```
-1/4*cos(b*x+a)/d/(d*x+c)+1/4*cos(3*b*x+3*a)/d/(d*x+c)+3/4*b*Ci(3*b*c/d+3*
b*x)*sin(3*a-3*b*c/d)/d^2-1/4*b*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^2-1/4*b*cos(a
-b*c/d)*Si(b*c/d+b*x)/d^2+3/4*b*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^2
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.83

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{\frac{d\cos(a+bx)}{c+dx} - \frac{d\cos(3(a+bx))}{c+dx} - 3b\,\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right) + b\,\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\right.}{4d^2}$$

input
```
Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
-1/4*((d*Cos[a + b*x])/(c + d*x) - (d*Cos[3*(a + b*x)])/(c + d*x) - 3*b*Co
sIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + b*CosIntegral[b*(c/d +
 x)]*Sin[a - (b*c)/d] + b*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 3*b*
Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/d^2
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos(a+bx)}{(c+dx)^2}\,dx$$
$$\downarrow 4906$$
$$\int \left(\frac{\cos(a+bx)}{4(c+dx)^2} - \frac{\cos(3a+3bx)}{4(c+dx)^2}\right)dx$$
$$\downarrow 2009$$
$$\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2} - \frac{b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2} -$$
$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2} + \frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d^2} - \frac{\cos(a+bx)}{4d(c+dx)} + \frac{\cos(3a+3bx)}{4d(c+dx)}$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
-1/4*Cos[a + b*x]/(d*(c + d*x)) + Cos[3*a + 3*b*x]/(4*d*(c + d*x)) + (3*b*
CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(4*d^2) - (b*CosInteg
ral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(4*d^2) - (b*Cos[a - (b*c)/d]*SinInte
gral[(b*c)/d + b*x])/(4*d^2) + (3*b*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*
c)/d + 3*b*x])/(4*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.47

method	result
derivativedivides	$\frac{b^2\left(-\frac{\cos(bx+a)}{(-ad+cb+d(bx+a))d}-\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}}{d}\right)}{4}-\frac{b^2\left(-\frac{3\cos(3b}{(-ad+cb+d}\right.}{b}$
default	$\frac{b^2\left(-\frac{\cos(bx+a)}{(-ad+cb+d(bx+a))d}-\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}}{d}\right)}{4}-\frac{b^2\left(-\frac{3\cos(3b}{(-ad+cb+d}\right.}{b}$
risch	$-\frac{3ib\,\mathrm{e}^{-\frac{3i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{8d^2}+\frac{ib\,\mathrm{e}^{-\frac{i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(ibx+ia-\frac{i(ad-cb)}{d}\right)}{8d^2}-\frac{ib\,\mathrm{e}^{\frac{i(}{}}}{}$

input
```
int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4*b^2*(-cos(b*x+a)/(-a*d+c*b+d*(b*x+a))/d-(-Si(-b*x-a-(-a*d+b*c)/d)
*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)-1/12*b
^2*(-3*cos(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d-3*(-3*Si(-3*b*x-3*a-3*(-a*d+b
*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*
c)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.11

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{4\,d\cos(bx+a)^3 - (bdx+bc)\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)\sin\left(-\frac{bc-ad}{d}\right) + 3\,(bdx+bc)\,\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right) + 3\,(}{4\,(d^3x+c}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/4*(4*d*cos(b*x + a)^3 - (b*d*x + b*c)*cos_integral((b*d*x + b*c)/d)*sin(
-(b*c - a*d)/d) + 3*(b*d*x + b*c)*cos_integral(3*(b*d*x + b*c)/d)*sin(-3*(
b*c - a*d)/d) + 3*(b*d*x + b*c)*cos(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*
x + b*c)/d) - (b*d*x + b*c)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)
/d) - 4*d*cos(b*x + a))/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin^2(a+bx)\cos(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.82

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^2\left(E_2\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_2($$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/8*(b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_inte
gral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b^2*(
exp_integral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(2
, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b^2*(I*e
xp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(2,
-(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^2*(I*exp_inte
gral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(2, -3*(
-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b*c*d + (b*x
+ a)*d^2 - a*d^2)*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.11 (sec) , antiderivative size = 67350, normalized size of antiderivative = 400.89

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output

```
1/8*(3*b*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - b*
d*x*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan
(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + b*d*x*imag_part
(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b*d*x*imag_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b*d*x*sin_integral(3*(b*d*x + b
*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c
/d)^2*tan(1/2*b*c/d)^2 - 2*b*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 - 2*b*d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*b*d*
x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(
3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b*d*x*real_part(
cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 6*b*d*x*real_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*b*d*x*real_part(cos_integral(b*x
+ b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(a+b\,x)\,\sin(a+b\,x)^2}{(c+d\,x)^2}\,dx$$

input

```
int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^2,x)
```

output

```
int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(bx+a\right)\sin\left(bx+a\right)^2}{d^2x^2+2cdx+c^2}dx$$

input
```
int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int((cos(a + b*x)*sin(a + b*x)**2)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.20 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$

Optimal result . . . 311
Mathematica [A] (verified) . . . 312
Rubi [A] (verified) . . . 312
Maple [A] (verified) . . . 314
Fricas [A] (verification not implemented) . . . 314
Sympy [F] . . . 315
Maxima [C] (verification not implemented) . . . 315
Giac [C] (verification not implemented) . . . 316
Mupad [F(-1)] . . . 317
Reduce [F] . . . 318

Optimal result

Integrand size = 22, antiderivative size = 221

$$\begin{aligned}\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = &-\frac{\cos(a+bx)}{8d(c+dx)^2}+\frac{\cos(3a+3bx)}{8d(c+dx)^2}\\ &-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}\\ &+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}\\ &+\frac{b\sin(a+bx)}{8d^2(c+dx)}-\frac{3b\sin(3a+3bx)}{8d^2(c+dx)}\\ &+\frac{b^2\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}\\ &-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^3}\end{aligned}$$

output

```
-1/8*cos(b*x+a)/d/(d*x+c)^2+1/8*cos(3*b*x+3*a)/d/(d*x+c)^2-1/8*b^2*cos(a-b
*c/d)*Ci(b*c/d+b*x)/d^3+9/8*b^2*cos(3*a-3*b*c/d)*Ci(3*b*c/d+3*b*x)/d^3+1/8
*b*sin(b*x+a)/d^2/(d*x+c)-3/8*b*sin(3*b*x+3*a)/d^2/(d*x+c)+1/8*b^2*sin(a-b
*c/d)*Si(b*c/d+b*x)/d^3-9/8*b^2*sin(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^3
```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.83

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{-b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)+\frac{d(-d\cos(a+bx)+b(c+d}{(c+dx)^2}$$

input
```
Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

output
```
(-(b^2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)]) + 9*b^2*Cos[3*a - (3*b*c
)/d]*CosIntegral[(3*b*(c + d*x))/d] + (d*(-(d*Cos[a + b*x]) + b*(c + d*x)*
Sin[a + b*x]))/(c + d*x)^2 + (d*(d*Cos[3*(a + b*x)] - 3*b*(c + d*x)*Sin[3*
(a + b*x)]))/(c + d*x)^2 + b^2*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] -
 9*b^2*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(8*d^3)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos(a+bx)}{(c+dx)^3}\,dx$$
$$\downarrow 4906$$
$$\int \left(\frac{\cos(a+bx)}{4(c+dx)^3}-\frac{\cos(3a+3bx)}{4(c+dx)^3}\right)dx$$
$$\downarrow 2009$$

$$-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}+$$
$$\frac{b^2\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^3}+\frac{b\sin(a+bx)}{8d^2(c+dx)}-$$
$$\frac{3b\sin(3a+3bx)}{8d^2(c+dx)}-\frac{\cos(a+bx)}{8d(c+dx)^2}+\frac{\cos(3a+3bx)}{8d(c+dx)^2}$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

output
```
-1/8*Cos[a + b*x]/(d*(c + d*x)^2) + Cos[3*a + 3*b*x]/(8*d*(c + d*x)^2) - (
b^2*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d^3) + (9*b^2*Cos[3*a
- (3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(8*d^3) + (b*Sin[a + b*x])/(8
*d^2*(c + d*x)) - (3*b*Sin[3*a + 3*b*x])/(8*d^2*(c + d*x)) + (b^2*Sin[a -
(b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^3) - (9*b^2*Sin[3*a - (3*b*c)/d]
*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{b^3\left(-\frac{\cos(bx+a)}{2(-ad+cb+d(bx+a))^2d}-\frac{-\frac{\sin(bx+a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(bx+a+\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+}{d}\right.}{d}}{2d}\right.}{4}$
default	$\frac{b^3\left(-\frac{\cos(bx+a)}{2(-ad+cb+d(bx+a))^2d}-\frac{-\frac{\sin(bx+a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(bx+a+\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+}{d}\right.}{d}}{2d}\right.}{4}$
risch	$-\frac{9b^2\mathrm{e}^{-\frac{3i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{16d^3}+\frac{b^2\mathrm{e}^{-\frac{i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(ibx+ia-\frac{i(ad-cb)}{d}\right)}{16d^3}+\frac{b^2\mathrm{e}^{\frac{i(}{}}}{}$

input
```
int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4*b^3*(-1/2*cos(b*x+a)/(-a*d+c*b+d*(b*x+a))^2/d-1/2*(-sin(b*x+a)/(-
a*d+c*b+d*(b*x+a))/d+(-Si(-b*x-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+
a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)-1/12*b^3*(-3/2*cos(3*b*x+3*a)/(
-a*d+c*b+d*(b*x+a))^2/d-3/2*(-3*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d+3*(-
3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a
*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.41

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$$

$$= \frac{4\,d^2\cos(bx+a)^3 - 4\,d^2\cos(bx+a) + 9\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right) - (b^2d^2}{}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/8*(4*d^2*cos(b*x + a)^3 - 4*d^2*cos(b*x + a) + 9*(b^2*d^2*x^2 + 2*b^2*c*
d*x + b^2*c^2)*cos(-3*(b*c - a*d)/d)*cos_integral(3*(b*d*x + b*c)/d) - (b^
2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-(b*c - a*d)/d)*cos_integral((b*d*x
 + b*c)/d) - 9*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-3*(b*c - a*d)/d)
*sin_integral(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*s
in(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 4*(b*d^2*x + b*c*d - 3*
(b*d^2*x + b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2
*d^3)
```

Sympy [F]

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin^2(a+bx)\cos(a+bx)}{(c+dx)^3}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.54

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx =$$
$$-\frac{b^3\left(E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^3\left(E_3\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_3($$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

output

```
-1/8*(b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_inte
gral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b^3*(
exp_integral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(3
, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b^3*(I*e
xp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(3,
-(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^3*(I*exp_inte
gral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(3, -3*(
-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^2*c^2*d - 2
*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.15 (sec) , antiderivative size = 114422, normalized size of antiderivative = 517.75

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input

```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

output

```
1/16*(9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^
2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
^2 - b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - b^
2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^
2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^
2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*t
an(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*b^2*d^2*x^2*i
mag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2
*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 4*b^2*d^2*x^2*sin_int
egral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 18*b^2*d^2*x^2*imag_part(cos_integr
al(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)
^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integral
(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^
2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*d^2*x^2*sin_integral(3*(b*d*x +
 b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b
*c/d)*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos(a+b\,x)\,\sin(a+b\,x)^2}{(c+d\,x)^3}\,dx$$

input

```
int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^3,x)
```

output

```
int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos(bx+a)\sin(bx+a)^2}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx$$

input
```
int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x)
```

output
```
int((cos(a + b*x)*sin(a + b*x)**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d*
**3*x**3),x)
```

3.21 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$

Optimal result . . . 319
Mathematica [A] (verified) . . . 320
Rubi [A] (verified) . . . 320
Maple [A] (verified) . . . 322
Fricas [A] (verification not implemented) . . . 323
Sympy [F] . . . 323
Maxima [C] (verification not implemented) . . . 324
Giac [C] (verification not implemented) . . . 324
Mupad [F(-1)] . . . 325
Reduce [F] . . . 326

Optimal result

Integrand size = 22, antiderivative size = 270

$$\begin{aligned}\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = &-\frac{\cos(a+bx)}{12d(c+dx)^3}+\frac{b^2\cos(a+bx)}{24d^3(c+dx)}\\ &+\frac{\cos(3a+3bx)}{12d(c+dx)^3}-\frac{3b^2\cos(3a+3bx)}{8d^3(c+dx)}\\ &-\frac{9b^3\,\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{8d^4}\\ &+\frac{b^3\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{24d^4}+\frac{b\sin(a+bx)}{24d^2(c+dx)^2}\\ &-\frac{b\sin(3a+3bx)}{8d^2(c+dx)^2}+\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{24d^4}\\ &-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^4}\end{aligned}$$

output

```
-1/12*cos(b*x+a)/d/(d*x+c)^3+1/24*b^2*cos(b*x+a)/d^3/(d*x+c)+1/12*cos(3*b*
x+3*a)/d/(d*x+c)^3-3/8*b^2*cos(3*b*x+3*a)/d^3/(d*x+c)-9/8*b^3*Ci(3*b*c/d+3
*b*x)*sin(3*a-3*b*c/d)/d^4+1/24*b^3*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^4+1/24*b*
sin(b*x+a)/d^2/(d*x+c)^2-1/8*b*sin(3*b*x+3*a)/d^2/(d*x+c)^2+1/24*b^3*cos(a
-b*c/d)*Si(b*c/d+b*x)/d^4-9/8*b^3*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^4
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.10

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$$
$$= \frac{d\cos(bx)\left(\left(-2d^2+b^2(c+dx)^2\right)\cos(a)+bd(c+dx)\sin(a)\right)-d\cos(3bx)\left(\left(-2d^2+9b^2(c+dx)^2\right)\cos(3a)\right.}{}$$

input
```
Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

output
```
(d*Cos[b*x]*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*d*(c + d*x)*Sin[a]) - d
*Cos[3*b*x]*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*x)*Sin[3
*a]) + d*(b*d*(c + d*x)*Cos[a] - (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*
x] - d*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)*Sin[3*a])*
Sin[3*b*x] + b^3*(c + d*x)^3*(CosIntegral[b*(c/d + x)]*Sin[a - (b*c)/d] +
Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)]) - 27*b^3*(c + d*x)^3*(CosIntegr
al[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + Cos[3*a - (3*b*c)/d]*SinInteg
ral[(3*b*(c + d*x))/d]))/(24*d^4*(c + d*x)^3)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos(a+bx)}{(c+dx)^4}\,dx$$
$$\downarrow 4906$$
$$\int \left(\frac{\cos(a+bx)}{4(c+dx)^4}-\frac{\cos(3a+3bx)}{4(c+dx)^4}\right)dx$$
$$\downarrow 2009$$

$$-\frac{9b^3 \sin\left(3a-\frac{3bc}{d}\right) \text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^3 \sin\left(a-\frac{bc}{d}\right) \text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{24d^4}+$$
$$\frac{b^3 \cos\left(a-\frac{bc}{d}\right) \text{Si}\left(\frac{bc}{d}+bx\right)}{24d^4}-\frac{9b^3 \cos\left(3a-\frac{3bc}{d}\right) \text{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^2 \cos(a+bx)}{24d^3(c+dx)}-$$
$$\frac{3b^2 \cos(3a+3bx)}{8d^3(c+dx)}+\frac{b \sin(a+bx)}{24d^2(c+dx)^2}-\frac{b \sin(3a+3bx)}{8d^2(c+dx)^2}-\frac{\cos(a+bx)}{12d(c+dx)^3}+\frac{\cos(3a+3bx)}{12d(c+dx)^3}$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

output
```
-1/12*Cos[a + b*x]/(d*(c + d*x)^3) + (b^2*Cos[a + b*x])/(24*d^3*(c + d*x))
 + Cos[3*a + 3*b*x]/(12*d*(c + d*x)^3) - (3*b^2*Cos[3*a + 3*b*x])/(8*d^3*(
c + d*x)) - (9*b^3*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(8
*d^4) + (b^3*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(24*d^4) + (b*Si
n[a + b*x])/(24*d^2*(c + d*x)^2) - (b*Sin[3*a + 3*b*x])/(8*d^2*(c + d*x)^2
) + (b^3*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(24*d^4) - (9*b^3*Co
s[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{b^4\left(-\frac{\cos(bx+a)}{3(-ad+cb+d(bx+a))^3d}-\frac{-\frac{\sin(bx+a)}{2(-ad+cb+d(bx+a))^2d}+\frac{-\frac{\cos(bx+a)}{(-ad+cb+d(bx+a))d}-\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}-\frac{\mathrm{Ci}}{d}}{2d}}{3d}\right)}{4}$
default	$\frac{b^4\left(-\frac{\cos(bx+a)}{3(-ad+cb+d(bx+a))^3d}-\frac{-\frac{\sin(bx+a)}{2(-ad+cb+d(bx+a))^2d}+\frac{-\frac{\cos(bx+a)}{(-ad+cb+d(bx+a))d}-\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}-\frac{\mathrm{Ci}}{d}}{2d}}{3d}\right)}{4}$
risch	$\frac{9ib^3\mathrm{e}^{-\frac{3i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{16d^4}-\frac{ib^3\mathrm{e}^{-\frac{i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(ibx+ia-\frac{i(ad-cb)}{d}\right)}{48d^4}+\frac{ib^3\mathrm{e}^{\frac{i(}{}}}{}$

input
```
int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4*b^4*(-1/3*cos(b*x+a)/(-a*d+c*b+d*(b*x+a))^3/d-1/3*(-1/2*sin(b*x+a
)/(-a*d+c*b+d*(b*x+a))^2/d+1/2*(-cos(b*x+a)/(-a*d+c*b+d*(b*x+a))/d-(-Si(-b
*x-a-(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*
c)/d)/d)/d)/d)/d)-1/12*b^4*(-cos(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))^3/d-(-3/2
*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))^2/d+3/2*(-3*cos(3*b*x+3*a)/(-a*d+c*b+
d*(b*x+a))/d-3*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*C
i(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.67

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx =$$

$$-\frac{4\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)^3-\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\mathrm{Ci}\left(\frac{bdx}{}\right.}{}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/24*(4*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x +
a)^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integ
ral((b*d*x + b*c)/d)*sin(-(b*c - a*d)/d) + 27*(b^3*d^3*x^3 + 3*b^3*c*d^2*x
^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(3*(b*d*x + b*c)/d)*sin(-3*(b*c
- a*d)/d) + 27*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*c
os(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - (b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-(b*c - a*d)/d)*sin_integral((b
*d*x + b*c)/d) - 4*(7*b^2*d^3*x^2 + 14*b^2*c*d^2*x + 7*b^2*c^2*d - 2*d^3)*
cos(b*x + a) - 4*(b*d^3*x + b*c*d^2 - 3*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2
)*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F]

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sin^2\left(a+bx\right)\cos\left(a+bx\right)}{\left(c+dx\right)^4}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c)**4,x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x)**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.44

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx =$$
$$-\frac{b^4\left(E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^4\left(E_4\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_4(\right.}{8\left(b^3c^3d-3\,ab\right.}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/8*(b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_inte
gral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b^4*(
exp_integral_e(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(4
, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b^4*(I*e
xp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(4,
-(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^4*(I*exp_inte
gral_e(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(4, -3*(
-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^3*c^3*d - 3
*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 -
a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.11 (sec) , antiderivative size = 168646, normalized size of antiderivative = 624.61

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

output

```
-1/48*(27*b^3*d^3*x^3*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 - b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan
(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 +
b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b
*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 27*b^3
*d^3*x^3*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 54*b^
3*d^3*x^3*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*
sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*t
an(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*real_part(co
s_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/
2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*b^3*d^3*x^3*real_part(cos_integ
ral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 54*b^3*d^3*x^3*real_part(cos_integral(3
*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*t
an(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 54*b^3*d^3*x^3*real_part(cos_integral(-3*
b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(b*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos(a+b\,x)\,\sin(a+b\,x)^2}{(c+d\,x)^4}\,dx$$

input

```
int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^4,x)
```

output

```
int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^4, x)
```

Reduce [F]

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \text{too large to display}$$

input

```
int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x)
```

output

```
( - 164*cos(a + b*x)*sin(a + b*x)**2*c*d**4 + 1350*cos(a + b*x)*sin(a + b*
x)*b**3*c**4*d + 2970*cos(a + b*x)*sin(a + b*x)*b**3*c**3*d**2*x + 1620*co
s(a + b*x)*sin(a + b*x)*b**3*c**2*d**3*x**2 - 702*cos(a + b*x)*sin(a + b*x
)*b*c**2*d**3 - 702*cos(a + b*x)*sin(a + b*x)*b*c*d**4*x - 4320*cos(a + b*
x)*b**2*c**3*d**2 - 6480*cos(a + b*x)*b**2*c**2*d**3*x + 2336*cos(a + b*x)
*c*d**4 - 432*int(cos(a + b*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c
*d**3*x**3 + d**4*x**4),x)*b**4*c**8*d - 1296*int(cos(a + b*x)/(c**4 + 4*c
**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*b**4*c**7*d**2*
x - 1296*int(cos(a + b*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3
*x**3 + d**4*x**4),x)*b**4*c**6*d**3*x**2 - 432*int(cos(a + b*x)/(c**4 + 4
*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*b**4*c**5*d**
4*x**3 + 504*int(cos(a + b*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*
d**3*x**3 + d**4*x**4),x)*b**2*c**6*d**3 + 1512*int(cos(a + b*x)/(c**4 + 4
*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*b**2*c**5*d**
4*x + 1512*int(cos(a + b*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d*
*3*x**3 + d**4*x**4),x)*b**2*c**4*d**5*x**2 + 504*int(cos(a + b*x)/(c**4 +
 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*b**2*c**3*d
**6*x**3 + 15648*int(cos(a + b*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 +
4*c*d**3*x**3 + d**4*x**4),x)*c**4*d**5 + 46944*int(cos(a + b*x)/(c**4 + 4
*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*c**3*d**6*...
```

3.22 $\int (c+dx)^m \cos(a+bx) \sin^3(a+bx)\, dx$

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Optimal result

Integrand size = 22, antiderivative size = 271

$$\begin{aligned}
&\int (c+dx)^m \cos(a+bx) \sin^3(a+bx)\, dx \\
&= -\frac{2^{-4-m} e^{2i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{2ib(c+dx)}{d}\right)}{b} \\
&\quad - \frac{2^{-4-m} e^{-2i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, \frac{2ib(c+dx)}{d}\right)}{b} \\
&\quad + \frac{2^{-2(3+m)} e^{4i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{4ib(c+dx)}{d}\right)}{b} \\
&\quad + \frac{2^{-2(3+m)} e^{-4i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, \frac{4ib(c+dx)}{d}\right)}{b}
\end{aligned}$$

output

```
-2^(-4-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*
b*(d*x+c)/d)^m)-2^(-4-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a
-b*c/d))/((I*b*(d*x+c)/d)^m)+exp(4*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-4*I*b
*(d*x+c)/d)/(2^(6+2*m))/b/((-I*b*(d*x+c)/d)^m)+(d*x+c)^m*GAMMA(1+m,4*I*b*(
d*x+c)/d)/(2^(6+2*m))/b/exp(4*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.91

$$\int (c+dx)^m \cos(a+bx)\sin^3(a+bx)\,dx$$

$$= \frac{4^{-3-m}e^{-\frac{4i(bc+ad)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-2^{2+m}e^{2i\left(3a+\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^m\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)-2^{2+m}e^{2i\left(\right.}\right.}{}$$

input

```
Integrate[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output

```
(4^(-3 - m)*(c + d*x)^m*(-(2^(2 + m)*E^((2*I)*(3*a + (b*c)/d))*((I*b*(c +
d*x))/d)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d]) - 2^(2 + m)*E^((2*I)*(a +
 (3*b*c)/d))*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d]
+ E^((8*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-4*I)*b*(c + d*x))/d] +
 E^(((8*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((4*I)*b*(c + d*x
))/d]))/(b*E^(((4*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\cos(a+bx)(c+dx)^m\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}\sin(2a+2bx)(c+dx)^m - \frac{1}{8}\sin(4a+4bx)(c+dx)^m\right)dx$$

$$\downarrow 2009$$

$$-\frac{2^{-m-4}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{4ib(c+dx)}{d}\right)}{b}-\frac{2^{-m-4}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-2(m+3)}e^{-4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{4ib(c+dx)}{d}\right)}{b}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
-((2^(-4 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(
c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)) - (2^(-4 - m)*(c + d*x)^m*Gamm
a[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x
))/d)^m) + (E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-4*I)*b*(c
+ d*x))/d])/(2^(2*(3 + m))*b*(((-I)*b*(c + d*x))/d)^m) + ((c + d*x)^m*Gamm
a[1 + m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*E^((4*I)*(a - (b*c)/d))*
((I*b*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [F]

$$\int (dx+c)^m \cos(bx+a) \sin(bx+a)^3 \, dx$$

input
```
int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x)
```

output
```
int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.70

$$\int (c+dx)^m \cos(a+bx) \sin^3(a+bx) \, dx =$$

$$-\frac{4\, e^{\left(-\frac{dm \log\left(-\frac{2ib}{d}\right)+2ibc-2iad}{d}\right)} \Gamma\left(m+1, -\frac{2\,(ibdx+ibc)}{d}\right) - e^{\left(-\frac{dm \log\left(-\frac{4ib}{d}\right)+4ibc-4iad}{d}\right)} \Gamma\left(m+1, -\frac{4\,(ibdx+ibc)}{d}\right) +}{64}$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/64*(4*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d)*gamma(m + 1, -2*(I
*b*d*x + I*b*c)/d) - e^(-(d*m*log(-4*I*b/d) + 4*I*b*c - 4*I*a*d)/d)*gamma(
m + 1, -4*(I*b*d*x + I*b*c)/d) + 4*e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a
*d)/d)*gamma(m + 1, -2*(-I*b*d*x - I*b*c)/d) - e^(-(d*m*log(4*I*b/d) - 4*I
*b*c + 4*I*a*d)/d)*gamma(m + 1, -4*(-I*b*d*x - I*b*c)/d))/b
```

Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m \cos(a+bx) \sin^3(a+bx) \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d*x+c)**m*cos(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (c+dx)^m \cos(a+bx) \sin^3(a+bx)\, dx = \int (dx+c)^m \cos(bx+a) \sin(bx+a)^3\, dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^3, x)
```

Giac [F]

$$\int (c+dx)^m \cos(a+bx) \sin^3(a+bx)\, dx = \int (dx+c)^m \cos(bx+a) \sin(bx+a)^3\, dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos(a+bx) \sin^3(a+bx)\, dx = \int \cos(a+b\,x)\, \sin(a+b\,x)^3\, (c+d\,x)^m\, dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \cos(a+bx) \sin^3(a+bx)\, dx = \int (dx+c)^m \cos(bx+a) \sin(bx+a)^3\, dx$$

input
```
int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x)
```

output
```
int((c + d*x)**m*cos(a + b*x)*sin(a + b*x)**3,x)
```

3.23 $\int (c+dx)^4 \cos(a+bx) \sin^3(a+bx)\, dx$

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Optimal result

Integrand size = 22, antiderivative size = 252

$$
\begin{aligned}
&\int (c+dx)^4 \cos(a+bx) \sin^3(a+bx)\, dx \\
&= \frac{45d^2(c+dx)^2}{128b^3} - \frac{3(c+dx)^4}{32b} - \frac{45d^3(c+dx)\cos(a+bx)\sin(a+bx)}{64b^4} \\
&\quad + \frac{3d(c+dx)^3\cos(a+bx)\sin(a+bx)}{8b^2} + \frac{45d^4\sin^2(a+bx)}{128b^5} \\
&\quad - \frac{9d^2(c+dx)^2\sin^2(a+bx)}{16b^3} - \frac{3d^3(c+dx)\cos(a+bx)\sin^3(a+bx)}{32b^4} \\
&\quad + \frac{d(c+dx)^3\cos(a+bx)\sin^3(a+bx)}{4b^2} + \frac{3d^4\sin^4(a+bx)}{128b^5} \\
&\quad - \frac{3d^2(c+dx)^2\sin^4(a+bx)}{16b^3} + \frac{(c+dx)^4\sin^4(a+bx)}{4b}
\end{aligned}
$$

output

```
45/128*d^2*(d*x+c)^2/b^3-3/32*(d*x+c)^4/b-45/64*d^3*(d*x+c)*cos(b*x+a)*sin
(b*x+a)/b^4+3/8*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2+45/128*d^4*sin(b*x+a
)^2/b^5-9/16*d^2*(d*x+c)^2*sin(b*x+a)^2/b^3-3/32*d^3*(d*x+c)*cos(b*x+a)*si
n(b*x+a)^3/b^4+1/4*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3/b^2+3/128*d^4*sin(b
*x+a)^4/b^5-3/16*d^2*(d*x+c)^2*sin(b*x+a)^4/b^3+1/4*(d*x+c)^4*sin(b*x+a)^4
/b
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.63

$$\int (c+dx)^4 \cos(a+bx) \sin^3(a+bx)\, dx$$
$$= \frac{-64(3d^4 - 6b^2d^2(c+dx)^2 + 2b^4(c+dx)^4)\cos(2(a+bx)) + (3d^4 - 24b^2d^2(c+dx)^2 + 32b^4(c+dx)^4)\cos$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
(-64*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)]
+ (3*d^4 - 24*b^2*d^2*(c + d*x)^2 + 32*b^4*(c + d*x)^4)*Cos[4*(a + b*x)] -
 8*b*d*(c + d*x)*(-16*(-3*d^2 + 2*b^2*(c + d*x)^2) + (-3*d^2 + 8*b^2*(c +
d*x)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)])/(1024*b^5)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.29, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {4904, 3042, 3792, 3042, 3791, 3042, 3791, 17, 3792, 17, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sin^3(a+bx) \cos(a+bx)\, dx$$
$$\downarrow 4904$$
$$\frac{(c+dx)^4 \sin^4(a+bx)}{4b} - \frac{d\int (c+dx)^3 \sin^4(a+bx) dx}{b}$$
$$\downarrow 3042$$
$$\frac{(c+dx)^4 \sin^4(a+bx)}{4b} - \frac{d\int (c+dx)^3 \sin(a+bx)^4 dx}{b}$$
$$\downarrow 3792$$

$$\frac{\frac{(c+dx)^4\sin^4(a+bx)}{4b} - d\left(-\frac{3d^2\int(c+dx)\sin^4(a+bx)dx}{8b^2} + \frac{3}{4}\int(c+dx)^3\sin^2(a+bx)dx + \frac{3d(c+dx)^2\sin^4(a+bx)}{16b^2} - \frac{(c+dx)^3\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{b}$$

$$\downarrow 3042$$

$$\frac{\frac{(c+dx)^4\sin^4(a+bx)}{4b} - d\left(-\frac{3d^2\int(c+dx)\sin(a+bx)^4dx}{8b^2} + \frac{3}{4}\int(c+dx)^3\sin(a+bx)^2dx + \frac{3d(c+dx)^2\sin^4(a+bx)}{16b^2} - \frac{(c+dx)^3\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{b}$$

$$\downarrow 3791$$

$$\frac{\frac{(c+dx)^4\sin^4(a+bx)}{4b} - d\left(-\frac{3d^2\left(\frac{3}{4}\int(c+dx)\sin^2(a+bx)dx + \frac{d\sin^4(a+bx)}{16b^2} - \frac{(c+dx)\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{8b^2} + \frac{3}{4}\int(c+dx)^3\sin(a+bx)^2dx + \frac{3d(c+dx)^2\sin^4(a+\right.}{b}$$

$$\downarrow 3042$$

$$\frac{\frac{(c+dx)^4\sin^4(a+bx)}{4b} - d\left(-\frac{3d^2\left(\frac{3}{4}\int(c+dx)\sin(a+bx)^2dx + \frac{d\sin^4(a+bx)}{16b^2} - \frac{(c+dx)\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{8b^2} + \frac{3}{4}\int(c+dx)^3\sin(a+bx)^2dx + \frac{3d(c+dx)^2\sin^4(a+\right.}{b}$$

$$\downarrow 3791$$

$$\frac{\frac{(c+dx)^4\sin^4(a+bx)}{4b} - d\left(-\frac{3d^2\left(\frac{3}{4}\left(\frac{1}{2}\int(c+dx)dx + \frac{d\sin^2(a+bx)}{4b^2} - \frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right) + \frac{d\sin^4(a+bx)}{16b^2} - \frac{(c+dx)\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{8b^2} + \frac{3}{4}\int(c+dx)^3\sin(a\right.}{b}$$

$$\downarrow 17$$

$$\frac{\frac{(c+dx)^4\sin^4(a+bx)}{4b} - d\left(\frac{3}{4}\int(c+dx)^3\sin(a+bx)^2dx - \frac{3d^2\left(\frac{3}{4}\left(\frac{d\sin^2(a+bx)}{4b^2} - \frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b} + \frac{(c+dx)^2}{4d}\right) + \frac{d\sin^4(a+bx)}{16b^2} - \frac{(c+dx)\sin^3(a+bx)\cos(a+\right.}{8b^2}}{b}$$

$$\downarrow 3792$$

$$\frac{(c+dx)^4\sin^4(a+bx)}{4b} - d\left(\frac{3}{4}\left(-\frac{3d^2\int(c+dx)\sin^2(a+bx)dx}{2b^2}+\frac{1}{2}\int(c+dx)^3dx+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}\right)-\frac{3d^2\left(\frac{3}{4}\left(\frac{d\sin^2(a}{4b^2}\right.\right.\right.$$

$$\downarrow 17$$

$$\frac{(c+dx)^4\sin^4(a+bx)}{4b} - \frac{d\left(\frac{3}{4}\left(-\frac{3d^2\int(c+dx)\sin^2(a+bx)dx}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)-\frac{3d^2\left(\frac{3}{4}\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+d}{}\right.\right.\right.}{b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^4\sin^4(a+bx)}{4b} - \frac{d\left(\frac{3}{4}\left(-\frac{3d^2\int(c+dx)\sin(a+bx)^2dx}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)-\frac{3d^2\left(\frac{3}{4}\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+d}{}\right.\right.\right.}{b}$$

$$\downarrow 3791$$

$$\frac{(c+dx)^4\sin^4(a+bx)}{4b} - d\left(\frac{3}{4}\left(-\frac{3d^2\left(\frac{1}{2}\int(c+dx)dx+\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx}{8d}\right.\right.$$

$$\downarrow 17$$

$$\frac{(c+dx)^4\sin^4(a+bx)}{4b} - d\left(\frac{3}{4}\left(-\frac{3d^2\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)\right)$$

input
```
Int[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
((c + d*x)^4*Sin[a + b*x]^4)/(4*b) - (d*(-1/4*((c + d*x)^3*Cos[a + b*x]*Si
n[a + b*x]^3)/b + (3*d*(c + d*x)^2*Sin[a + b*x]^4)/(16*b^2) - (3*d^2*(-1/4
*((c + d*x)*Cos[a + b*x]*Sin[a + b*x]^3)/b + (d*Sin[a + b*x]^4)/(16*b^2) +
 (3*((c + d*x)^2/(4*d) - ((c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (d*
Sin[a + b*x]^2)/(4*b^2)))/4))/(8*b^2) + (3*((c + d*x)^4/(8*d) - ((c + d*x)
^3*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (3*d*(c + d*x)^2*Sin[a + b*x]^2)/(4*
b^2) - (3*d^2*((c + d*x)^2/(4*d) - ((c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(
2*b) + (d*Sin[a + b*x]^2)/(4*b^2)))/(2*b^2)))/4))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)  Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)  Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))  Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))  Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{\left(-128(dx+c)^4b^4+384d^2(dx+c)^2b^2-192d^4\right)\cos(2bx+2a)+\left(32(dx+c)^4b^4-24d^2(dx+c)^2b^2+3d^4\right)\cos(4bx+4a)+256c}{1024}$
risch	$\frac{(32d^4x^4b^4+128b^4cd^3x^3+192b^4c^2d^2x^2+128b^4c^3dx+32b^4c^4-24b^2d^4x^2-48b^2cd^3x-24b^2c^2d^2+3d^4)\cos(4bx+4a)}{1024b^5}$ —
orering	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/1024*((-128*(d*x+c)^4*b^4+384*d^2*(d*x+c)^2*b^2-192*d^4)*cos(2*b*x+2*a)+
(32*(d*x+c)^4*b^4-24*d^2*(d*x+c)^2*b^2+3*d^4)*cos(4*b*x+4*a)+256*d*((d*x+c
)^2*b^2-3/2*d^2)*(d*x+c)*b*sin(2*b*x+2*a)-32*d*((d*x+c)^2*b^2-3/8*d^2)*(d*
x+c)*b*sin(4*b*x+4*a)+96*b^4*c^4-360*b^2*c^2*d^2+189*d^4)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.72

$$\int (c+dx)^4\cos(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{20\,b^4d^4x^4+80\,b^4cd^3x^3+(32\,b^4d^4x^4+128\,b^4cd^3x^3+32\,b^4c^4-24\,b^2c^2d^2+3\,d^4+24\,(8\,b^4c^2d^2-b^2d^4)x^2+}{}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output

```
1/128*(20*b^4*d^4*x^4 + 80*b^4*c*d^3*x^3 + (32*b^4*d^4*x^4 + 128*b^4*c*d^3
*x^3 + 32*b^4*c^4 - 24*b^2*c^2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*
x^2 + 16*(8*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^4 + 3*(40*b^4*c^2*d^2
 - 17*b^2*d^4)*x^2 - (64*b^4*d^4*x^4 + 256*b^4*c*d^3*x^3 + 64*b^4*c^4 - 12
0*b^2*c^2*d^2 + 51*d^4 + 24*(16*b^4*c^2*d^2 - 5*b^2*d^4)*x^2 + 16*(16*b^4*
c^3*d - 15*b^2*c*d^3)*x)*cos(b*x + a)^2 + 2*(40*b^4*c^3*d - 51*b^2*c*d^3)*
x - 2*(2*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(
8*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^3 - (40*b^3*d^4*x^3 + 120*b^3*c*d^3
*x^2 + 40*b^3*c^3*d - 51*b*c*d^3 + 3*(40*b^3*c^2*d^2 - 17*b*d^4)*x)*cos(b*
x + a))*sin(b*x + a))/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(252) = 504$.

Time $= 0.93$ (sec) , antiderivative size $= 935$, normalized size of antiderivative $= 3.71$

$$\int (c+dx)^4 \cos(a+bx) \sin^3(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)**4*cos(b*x+a)*sin(b*x+a)**3,x)
```

output

```
Piecewise((c**4*sin(a + b*x)**4/(4*b) + 5*c**3*d*x*sin(a + b*x)**4/(8*b) -
 3*c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(4*b) - 3*c**3*d*x*cos(a + b*x
)**4/(8*b) + 15*c**2*d**2*x**2*sin(a + b*x)**4/(16*b) - 9*c**2*d**2*x**2*s
in(a + b*x)**2*cos(a + b*x)**2/(8*b) - 9*c**2*d**2*x**2*cos(a + b*x)**4/(1
6*b) + 5*c*d**3*x**3*sin(a + b*x)**4/(8*b) - 3*c*d**3*x**3*sin(a + b*x)**2
*cos(a + b*x)**2/(4*b) - 3*c*d**3*x**3*cos(a + b*x)**4/(8*b) + 5*d**4*x**4
*sin(a + b*x)**4/(32*b) - 3*d**4*x**4*sin(a + b*x)**2*cos(a + b*x)**2/(16*
b) - 3*d**4*x**4*cos(a + b*x)**4/(32*b) + 5*c**3*d*sin(a + b*x)**3*cos(a +
 b*x)/(8*b**2) + 3*c**3*d*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 15*c**2*
d**2*x*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 9*c**2*d**2*x*sin(a + b*x)*
cos(a + b*x)**3/(8*b**2) + 15*c*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)/(8*
b**2) + 9*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 5*d**4*x**3*
sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 3*d**4*x**3*sin(a + b*x)*cos(a + b
*x)**3/(8*b**2) - 15*c**2*d**2*sin(a + b*x)**4/(32*b**3) + 9*c**2*d**2*cos
(a + b*x)**4/(32*b**3) - 51*c*d**3*x*sin(a + b*x)**4/(64*b**3) + 9*c*d**3*
x*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**3) + 45*c*d**3*x*cos(a + b*x)**4/
(64*b**3) - 51*d**4*x**2*sin(a + b*x)**4/(128*b**3) + 9*d**4*x**2*sin(a +
b*x)**2*cos(a + b*x)**2/(64*b**3) + 45*d**4*x**2*cos(a + b*x)**4/(128*b**3
) - 51*c*d**3*sin(a + b*x)**3*cos(a + b*x)/(64*b**4) - 45*c*d**3*sin(a + b
*x)*cos(a + b*x)**3/(64*b**4) - 51*d**4*x*sin(a + b*x)**3*cos(a + b*x)/...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 967 vs. $2(230) = 460$.

Time $= 0.08$ (sec) , antiderivative size $= 967$, normalized size of antiderivative $= 3.84$

$$\int (c + dx)^4 \cos(a + bx) \sin^3(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output

```
1/1024*(256*c^4*sin(b*x + a)^4 - 1024*a*c^3*d*sin(b*x + a)^4/b + 1536*a^2*
c^2*d^2*sin(b*x + a)^4/b^2 - 1024*a^3*c*d^3*sin(b*x + a)^4/b^3 + 256*a^4*d
^4*sin(b*x + a)^4/b^4 + 32*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*co
s(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*c^3*d/b - 96*(4*(b
*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a
) + 8*sin(2*b*x + 2*a))*a*c^2*d^2/b^2 + 96*(4*(b*x + a)*cos(4*b*x + 4*a) -
 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*a^
2*c*d^3/b^3 - 32*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x +
2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*a^3*d^4/b^4 + 24*((8*(b*x +
a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(
b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 - 4
8*((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x
 + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*a*
c*d^3/b^3 + 24*((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 -
 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b
*x + 2*a))*a^2*d^4/b^4 + 4*(4*(8*(b*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*
a) - 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2
- 1)*sin(4*b*x + 4*a) + 96*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3
 - 4*(4*(8*(b*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) - 64*(2*(b*x + a)^3
 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*...
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.43

$$
\begin{aligned}
&\int (c+dx)^4 \cos(a+bx) \sin^3(a+bx)\, dx \\
&= \frac{(32\,b^4d^4x^4 + 128\,b^4cd^3x^3 + 192\,b^4c^2d^2x^2 + 128\,b^4c^3dx + 32\,b^4c^4 - 24\,b^2d^4x^2 - 48\,b^2cd^3x - 24\,b^2c^2d^2 + 3\,d^4)\cos(4\,bx+4\,a)}{1024\,b^5} \\
&\quad - \frac{(2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 12\,b^4c^2d^2x^2 + 8\,b^4c^3dx + 2\,b^4c^4 - 6\,b^2d^4x^2 - 12\,b^2cd^3x - 6\,b^2c^2d^2 + 3\,d^4)\cos(2
}{16\,b^5} \\
&\quad - \frac{(8\,b^3d^4x^3 + 24\,b^3cd^3x^2 + 24\,b^3c^2d^2x + 8\,b^3c^3d - 3\,bd^4x - 3\,bcd^3)\sin(4\,bx+4\,a)}{256\,b^5} \\
&\quad + \frac{(2\,b^3d^4x^3 + 6\,b^3cd^3x^2 + 6\,b^3c^2d^2x + 2\,b^3c^3d - 3\,bd^4x - 3\,bcd^3)\sin(2\,bx+2\,a)}{8\,b^5}
\end{aligned}
$$

input

```
integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output

```
1/1024*(32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 192*b^4*c^2*d^2*x^2 + 128*b^4
*c^3*d*x + 32*b^4*c^4 - 24*b^2*d^4*x^2 - 48*b^2*c*d^3*x - 24*b^2*c^2*d^2 +
 3*d^4)*cos(4*b*x + 4*a)/b^5 - 1/16*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*
b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3
*x - 6*b^2*c^2*d^2 + 3*d^4)*cos(2*b*x + 2*a)/b^5 - 1/256*(8*b^3*d^4*x^3 +
24*b^3*c*d^3*x^2 + 24*b^3*c^2*d^2*x + 8*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)
*sin(4*b*x + 4*a)/b^5 + 1/8*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d
^2*x + 2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

Mupad [B] (verification not implemented)

Time = 21.00 (sec) , antiderivative size = 576, normalized size of antiderivative = 2.29

$$\int (c+dx)^4 \cos(a+bx) \sin^3(a+bx)\, dx =$$

$$-\frac{192\, d^4 \cos(2\,a+2\,b\,x) - 3\, d^4 \cos(4\,a+4\,b\,x) + 128\, b^4\, c^4 \cos(2\,a+2\,b\,x) - 32\, b^4\, c^4 \cos(4\,a+4\,b\,x) -$$

input

```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^4,x)
```

output

```
-(192*d^4*cos(2*a + 2*b*x) - 3*d^4*cos(4*a + 4*b*x) + 128*b^4*c^4*cos(2*a
+ 2*b*x) - 32*b^4*c^4*cos(4*a + 4*b*x) - 256*b^3*c^3*d*sin(2*a + 2*b*x) +
32*b^3*c^3*d*sin(4*a + 4*b*x) - 384*b^2*c^2*d^2*cos(2*a + 2*b*x) + 24*b^2*
c^2*d^2*cos(4*a + 4*b*x) - 384*b^2*d^4*x^2*cos(2*a + 2*b*x) + 24*b^2*d^4*x
^2*cos(4*a + 4*b*x) + 128*b^4*d^4*x^4*cos(2*a + 2*b*x) - 32*b^4*d^4*x^4*co
s(4*a + 4*b*x) - 256*b^3*d^4*x^3*sin(2*a + 2*b*x) + 32*b^3*d^4*x^3*sin(4*a
 + 4*b*x) + 384*b*c*d^3*sin(2*a + 2*b*x) - 12*b*c*d^3*sin(4*a + 4*b*x) + 3
84*b*d^4*x*sin(2*a + 2*b*x) - 12*b*d^4*x*sin(4*a + 4*b*x) + 768*b^4*c^2*d^
2*x^2*cos(2*a + 2*b*x) - 192*b^4*c^2*d^2*x^2*cos(4*a + 4*b*x) - 768*b^2*c*
d^3*x*cos(2*a + 2*b*x) + 512*b^4*c^3*d*x*cos(2*a + 2*b*x) + 48*b^2*c*d^3*x
*cos(4*a + 4*b*x) - 128*b^4*c^3*d*x*cos(4*a + 4*b*x) + 512*b^4*c*d^3*x^3*c
os(2*a + 2*b*x) - 128*b^4*c*d^3*x^3*cos(4*a + 4*b*x) - 768*b^3*c^2*d^2*x*s
in(2*a + 2*b*x) - 768*b^3*c*d^3*x^2*sin(2*a + 2*b*x) + 96*b^3*c^2*d^2*x*si
n(4*a + 4*b*x) + 96*b^3*c*d^3*x^2*sin(4*a + 4*b*x))/(1024*b^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 613, normalized size of antiderivative = 2.43

$$\int (c+dx)^4 \cos(a+bx) \sin^3(a+bx)\, dx$$

$$= \frac{-72 \sin(bx+a)^2 b^2 d^4 x^2 + 48 \cos(bx+a) \sin(bx+a) b^3 c^3 d + 48 \cos(bx+a) \sin(bx+a) b^3 d^4 x^3 - 90 \co$$

input

```
int((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x)
```

output

```
(32*cos(a + b*x)*sin(a + b*x)**3*b**3*c**3*d + 96*cos(a + b*x)*sin(a + b*x
)**3*b**3*c**2*d**2*x + 96*cos(a + b*x)*sin(a + b*x)**3*b**3*c*d**3*x**2 +
 32*cos(a + b*x)*sin(a + b*x)**3*b**3*d**4*x**3 - 12*cos(a + b*x)*sin(a +
b*x)**3*b*c*d**3 - 12*cos(a + b*x)*sin(a + b*x)**3*b*d**4*x + 48*cos(a + b
*x)*sin(a + b*x)*b**3*c**3*d + 144*cos(a + b*x)*sin(a + b*x)*b**3*c**2*d**
2*x + 144*cos(a + b*x)*sin(a + b*x)*b**3*c*d**3*x**2 + 48*cos(a + b*x)*sin
(a + b*x)*b**3*d**4*x**3 - 90*cos(a + b*x)*sin(a + b*x)*b*c*d**3 - 90*cos(
a + b*x)*sin(a + b*x)*b*d**4*x + 32*sin(a + b*x)**4*b**4*c**4 + 128*sin(a
+ b*x)**4*b**4*c**3*d*x + 192*sin(a + b*x)**4*b**4*c**2*d**2*x**2 + 128*si
n(a + b*x)**4*b**4*c*d**3*x**3 + 32*sin(a + b*x)**4*b**4*d**4*x**4 - 24*si
n(a + b*x)**4*b**2*c**2*d**2 - 48*sin(a + b*x)**4*b**2*c*d**3*x - 24*sin(a
 + b*x)**4*b**2*d**4*x**2 + 3*sin(a + b*x)**4*d**4 - 72*sin(a + b*x)**2*b*
*2*c**2*d**2 - 144*sin(a + b*x)**2*b**2*c*d**3*x - 72*sin(a + b*x)**2*b**2
*d**4*x**2 + 45*sin(a + b*x)**2*d**4 - 48*a*b**3*c**3*d - 54*a*b*c*d**3 -
48*b**4*c**3*d*x - 72*b**4*c**2*d**2*x**2 - 48*b**4*c*d**3*x**3 - 12*b**4*
d**4*x**4 + 72*b**2*c**2*d**2 + 90*b**2*c*d**3*x + 45*b**2*d**4*x**2 - 45*
d**4)/(128*b**5)
```

3.24 $\int (c+dx)^3 \cos(a+bx) \sin^3(a+bx)\, dx$

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Optimal result

Integrand size = 22, antiderivative size = 196

$$\begin{aligned}
&\int (c+dx)^3 \cos(a+bx) \sin^3(a+bx)\, dx \\
&= \frac{45d^3x}{256b^3} - \frac{3(c+dx)^3}{32b} - \frac{45d^3 \cos(a+bx)\sin(a+bx)}{256b^4} \\
&\quad + \frac{9d(c+dx)^2 \cos(a+bx)\sin(a+bx)}{32b^2} - \frac{9d^2(c+dx)\sin^2(a+bx)}{32b^3} \\
&\quad - \frac{3d^3 \cos(a+bx)\sin^3(a+bx)}{128b^4} + \frac{3d(c+dx)^2 \cos(a+bx)\sin^3(a+bx)}{16b^2} \\
&\quad - \frac{3d^2(c+dx)\sin^4(a+bx)}{32b^3} + \frac{(c+dx)^3 \sin^4(a+bx)}{4b}
\end{aligned}$$

output

```
45/256*d^3*x/b^3-3/32*(d*x+c)^3/b-45/256*d^3*cos(b*x+a)*sin(b*x+a)/b^4+9/3
2*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^2-9/32*d^2*(d*x+c)*sin(b*x+a)^2/b^3-
3/128*d^3*cos(b*x+a)*sin(b*x+a)^3/b^4+3/16*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+
a)^3/b^2-3/32*d^2*(d*x+c)*sin(b*x+a)^4/b^3+1/4*(d*x+c)^3*sin(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.69

$$\int (c+dx)^3 \cos(a+bx) \sin^3(a+bx)\, dx$$
$$= \frac{-64b(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\cos(2(a+bx)) + 4b(c+dx)\left(-3d^2+8b^2(c+dx)^2\right)\cos(4(a+bx)) -}{1024b^4}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
(-64*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 4*b*(c +
d*x)*(-3*d^2 + 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] - 6*d*(-16*(-d^2 + 2*b^
2*(c + d*x)^2) + (-d^2 + 8*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b
*x)])/(1024*b^4)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.24, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {4904, 3042, 3792, 3042, 3115, 3042, 3115, 24, 3792, 17, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin^3(a+bx)\cos(a+bx)\, dx$$
$$\downarrow 4904$$
$$\frac{(c+dx)^3\sin^4(a+bx)}{4b} - \frac{3d\int (c+dx)^2\sin^4(a+bx)dx}{4b}$$
$$\downarrow 3042$$
$$\frac{(c+dx)^3\sin^4(a+bx)}{4b} - \frac{3d\int (c+dx)^2\sin(a+bx)^4 dx}{4b}$$
$$\downarrow 3792$$

$$\frac{(c+dx)^3\sin^4(a+bx)}{4b} - \frac{3d\left(-\frac{d^2\int\sin^4(a+bx)dx}{8b^2}+\frac{3}{4}\int(c+dx)^2\sin^2(a+bx)dx+\frac{d(c+dx)\sin^4(a+bx)}{8b^2}-\frac{(c+dx)^2\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{4b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^3\sin^4(a+bx)}{4b} - \frac{3d\left(-\frac{d^2\int\sin(a+bx)^4dx}{8b^2}+\frac{3}{4}\int(c+dx)^2\sin(a+bx)^2dx+\frac{d(c+dx)\sin^4(a+bx)}{8b^2}-\frac{(c+dx)^2\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{4b}$$

$$\downarrow 3115$$

$$\frac{(c+dx)^3\sin^4(a+bx)}{4b} - \frac{3d\left(-\frac{d^2\left(\frac{3}{4}\int\sin^2(a+bx)dx-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int(c+dx)^2\sin(a+bx)^2dx+\frac{d(c+dx)\sin^4(a+bx)}{8b^2}-\frac{(c+dx)^2\sin^3(a+bx)\,c}{4b}\right)}{4b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^3\sin^4(a+bx)}{4b} - \frac{3d\left(-\frac{d^2\left(\frac{3}{4}\int\sin(a+bx)^2dx-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int(c+dx)^2\sin(a+bx)^2dx+\frac{d(c+dx)\sin^4(a+bx)}{8b^2}-\frac{(c+dx)^2\sin^3(a+bx)\,c}{4b}\right)}{4b}$$

$$\downarrow 3115$$

$$\frac{(c+dx)^3\sin^4(a+bx)}{4b} - \frac{3d\left(-\frac{d^2\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int(c+dx)^2\sin(a+bx)^2dx+\frac{d(c+dx)\sin^4(a+bx)}{8b^2}-\frac{(c+dx)^2}{}\right)}{4b}$$

$$\downarrow 24$$

$$\frac{(c+dx)^3\sin^4(a+bx)}{4b} - \frac{3d\left(\frac{3}{4}\int(c+dx)^2\sin(a+bx)^2dx+\frac{d(c+dx)\sin^4(a+bx)}{8b^2}-\frac{d^2\left(\frac{3}{4}\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{8b^2}-\frac{(c+dx)^2\sin^3(a}{}\right)}{4b}$$

$$\downarrow 3792$$

$$\frac{(c+dx)^3\sin^4(a+bx)}{4b} - \frac{3d\left(\frac{3}{4}\left(-\frac{d^2\int\sin^2(a+bx)dx}{2b^2}+\frac{1}{2}\int(c+dx)^2dx+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{d(c+dx)\sin^4(a+bx)}{8b^2}-\frac{d}{4b}\right.}{4b}$$

$$\downarrow 17$$

$$\frac{(c+dx)^3\sin^4(a+bx)}{4b} - \frac{3d\left(\frac{3}{4}\left(-\frac{d^2\int\sin^2(a+bx)dx}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)+\frac{d(c+dx)\sin^4(a+bx)}{8b^2}-\frac{d^2\left(\frac{3}{4}\left(\frac{x}{2}-\right.\right.\right.}{4b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^3\sin^4(a+bx)}{4b} - \frac{3d\left(\frac{3}{4}\left(-\frac{d^2\int\sin(a+bx)^2dx}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)+\frac{d(c+dx)\sin^4(a+bx)}{8b^2}-\frac{d^2\left(\frac{3}{4}\left(\frac{x}{2}-\right.\right.\right.}{4b}$$

$$\downarrow 3115$$

$$\frac{(c+dx)^3\sin^4(a+bx)}{4b} - \frac{3d\left(\frac{3}{4}\left(-\frac{d^2\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)+\frac{d(c+dx)\sin^4(a+bx)}{8b^2}-\right.}{4b}$$

$$\downarrow 24$$

$$\frac{(c+dx)^3\sin^4(a+bx)}{4b} - \frac{3d\left(\frac{3}{4}\left(\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)+\frac{d(c+dx)\sin^4(a+bx)}{8b^2}-\frac{d^2\left(\right.\right.}{4b}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
((c + d*x)^3*Sin[a + b*x]^4)/(4*b) - (3*d*(-1/4*((c + d*x)^2*Cos[a + b*x]*
Sin[a + b*x]^3)/b + (d*(c + d*x)*Sin[a + b*x]^4)/(8*b^2) - (d^2*(-1/4*(Cos
[a + b*x]*Sin[a + b*x]^3)/b + (3*(x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b))
)/4))/(8*b^2) + (3*((c + d*x)^3/(6*d) - ((c + d*x)^2*Cos[a + b*x]*Sin[a +
b*x])/(2*b) + (d*(c + d*x)*Sin[a + b*x]^2)/(2*b^2) - (d^2*(x/2 - (Cos[a +
b*x]*Sin[a + b*x])/(2*b)))/(2*b^2)))/4))/(4*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.76

method	result
parallelrisch	$\frac{-32\left((dx+c)^2b^2-\frac{3d^2}{2}\right)(dx+c)b\cos(2bx+2a)+8\left((dx+c)^2b^2-\frac{3d^2}{8}\right)(dx+c)b\cos(4bx+4a)+48d\left((dx+c)^2b^2-\frac{d^2}{2}\right)\sin(2$
risch	$\frac{(8b^2d^3x^3+24b^2cd^2x^2+24b^2c^2dx+8b^2c^3-3d^3x-3cd^2)\cos(4bx+4a)}{256b^3}-\frac{3d(8x^2d^2b^2+16b^2cdx+8b^2c^2-d^2)\sin(4bx+$
derivativedivides	$-\frac{a^3d^3\sin(bx+a)^4}{4b^3}+\frac{3a^2cd^2\sin(bx+a)^4}{4b^2}+\frac{3a^2d^3\left(\frac{(bx+a)\sin(bx+a)^4}{4}+\frac{\left(\sin(bx+a)^3+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{16}-\frac{3bx}{32}-\frac{3a}{32}\right)}{b^3}-\frac{3a}$
default	$-\frac{a^3d^3\sin(bx+a)^4}{4b^3}+\frac{3a^2cd^2\sin(bx+a)^4}{4b^2}+\frac{3a^2d^3\left(\frac{(bx+a)\sin(bx+a)^4}{4}+\frac{\left(\sin(bx+a)^3+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{16}-\frac{3bx}{32}-\frac{3a}{32}\right)}{b^3}-\frac{3a}$
orering	$\frac{15d(8d^4x^4b^4+32b^4cd^3x^3+48b^4c^2d^2x^2+32b^4c^3dx+8b^4c^4-2b^2d^4x^2-4b^2cd^3x-2b^2c^2d^2-9d^4)\cos(bx+a)\sin(bx+a)^3}{64b^6(dx+c)^2}$

input
```
int((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/256*(-32*((d*x+c)^2*b^2-3/2*d^2)*(d*x+c)*b*cos(2*b*x+2*a)+8*((d*x+c)^2*b
^2-3/8*d^2)*(d*x+c)*b*cos(4*b*x+4*a)+48*d*((d*x+c)^2*b^2-1/2*d^2)*sin(2*b*
x+2*a)-6*((d*x+c)^2*b^2-1/8*d^2)*d*sin(4*b*x+4*a)+24*b^3*c^3-45*c*d^2*b)/b
^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.44

$$\int (c+dx)^3\cos(a+bx)\sin^3(a+bx)\,dx$$
$$=\frac{40\,b^3d^3x^3+120\,b^3cd^2x^2+8\left(8\,b^3d^3x^3+24\,b^3cd^2x^2+8\,b^3c^3-3\,bcd^2+3\left(8\,b^3c^2d-bd^3\right)x\right)\cos\left(bx+a\right)^4-$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/256*(40*b^3*d^3*x^3 + 120*b^3*c*d^2*x^2 + 8*(8*b^3*d^3*x^3 + 24*b^3*c*d^
2*x^2 + 8*b^3*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^4
- 8*(16*b^3*d^3*x^3 + 48*b^3*c*d^2*x^2 + 16*b^3*c^3 - 15*b*c*d^2 + 3*(16*b
^3*c^2*d - 5*b*d^3)*x)*cos(b*x + a)^2 + 3*(40*b^3*c^2*d - 17*b*d^3)*x - 3*
(2*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^3 - (
40*b^2*d^3*x^2 + 80*b^2*c*d^2*x + 40*b^2*c^2*d - 17*d^3)*cos(b*x + a))*sin
(b*x + a))/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(197) = 394$.

Time $= 0.64$ (sec) , antiderivative size $= 602$, normalized size of antiderivative $= 3.07$

$$\int (c+dx)^3 \cos(a+bx) \sin^3(a+bx)\, dx$$

$$= \begin{cases} \frac{c^3 \sin^4(a+bx)}{4b} + \frac{15c^2 dx \sin^4(a+bx)}{32b} - \frac{9c^2 dx \sin^2(a+bx)\cos^2(a+bx)}{16b} - \frac{9c^2 dx \cos^4(a+bx)}{32b} + \frac{15cd^2x^2 \sin^4(a+bx)}{32b} - \frac{9cd^2x^2 \sin^2(a \\ \left(c^3 x + \frac{3c^2 dx^2}{2} + cd^2 x^3 + \frac{d^3 x^4}{4}\right) \sin^3(a) \cos(a) \end{cases}$$

input
```
integrate((d*x+c)**3*cos(b*x+a)*sin(b*x+a)**3,x)
```

output

```
Piecewise((c**3*sin(a + b*x)**4/(4*b) + 15*c**2*d*x*sin(a + b*x)**4/(32*b)
 - 9*c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 9*c**2*d*x*cos(a +
b*x)**4/(32*b) + 15*c*d**2*x**2*sin(a + b*x)**4/(32*b) - 9*c*d**2*x**2*sin
(a + b*x)**2*cos(a + b*x)**2/(16*b) - 9*c*d**2*x**2*cos(a + b*x)**4/(32*b)
 + 5*d**3*x**3*sin(a + b*x)**4/(32*b) - 3*d**3*x**3*sin(a + b*x)**2*cos(a
+ b*x)**2/(16*b) - 3*d**3*x**3*cos(a + b*x)**4/(32*b) + 15*c**2*d*sin(a +
b*x)**3*cos(a + b*x)/(32*b**2) + 9*c**2*d*sin(a + b*x)*cos(a + b*x)**3/(32
*b**2) + 15*c*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 9*c*d**2*x*s
in(a + b*x)*cos(a + b*x)**3/(16*b**2) + 15*d**3*x**2*sin(a + b*x)**3*cos(a
 + b*x)/(32*b**2) + 9*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(32*b**2) - 1
5*c*d**2*sin(a + b*x)**4/(64*b**3) + 9*c*d**2*cos(a + b*x)**4/(64*b**3) -
51*d**3*x*sin(a + b*x)**4/(256*b**3) + 9*d**3*x*sin(a + b*x)**2*cos(a + b*
x)**2/(128*b**3) + 45*d**3*x*cos(a + b*x)**4/(256*b**3) - 51*d**3*sin(a +
b*x)**3*cos(a + b*x)/(256*b**4) - 45*d**3*sin(a + b*x)*cos(a + b*x)**3/(25
6*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4
)*sin(a)**3*cos(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(178) = 356$.

Time $= 0.05$ (sec) , antiderivative size $= 549$, normalized size of antiderivative $= 2.80$

$$\int (c + dx)^3 \cos(a + bx) \sin^3(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/1024*(256*c^3*sin(b*x + a)^4 - 768*a*c^2*d*sin(b*x + a)^4/b + 768*a^2*c*
d^2*sin(b*x + a)^4/b^2 - 256*a^3*d^3*sin(b*x + a)^4/b^3 + 24*(4*(b*x + a)*
cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*si
n(2*b*x + 2*a))*c^2*d/b - 48*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*
cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*a*c*d^2/b^2 + 24
*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x
 + 4*a) + 8*sin(2*b*x + 2*a))*a^2*d^3/b^3 + 12*((8*(b*x + a)^2 - 1)*cos(4*
b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b
*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*c*d^2/b^2 - 12*((8*(b*x + a)^2
 - 1)*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x +
 a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*a*d^3/b^3 + (4*(8*(b
*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) - 64*(2*(b*x + a)^3 - 3*b*x - 3*
a)*cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) + 96*(2*(b*x
 + a)^2 - 1)*sin(2*b*x + 2*a))*d^3/b^3)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.23

$$\begin{aligned}
&\int (c+dx)^3 \cos(a+bx) \sin^3(a+bx)\, dx \\
&= \frac{(8\,b^3d^3x^3 + 24\,b^3cd^2x^2 + 24\,b^3c^2dx + 8\,b^3c^3 - 3\,bd^3x - 3\,bcd^2)\cos(4\,bx+4\,a)}{256\,b^4} \\
&\quad - \frac{(2\,b^3d^3x^3 + 6\,b^3cd^2x^2 + 6\,b^3c^2dx + 2\,b^3c^3 - 3\,bd^3x - 3\,bcd^2)\cos(2\,bx+2\,a)}{16\,b^4} \\
&\quad - \frac{3\,(8\,b^2d^3x^2 + 16\,b^2cd^2x + 8\,b^2c^2d - d^3)\sin(4\,bx+4\,a)}{1024\,b^4} \\
&\quad + \frac{3\,(2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d - d^3)\sin(2\,bx+2\,a)}{32\,b^4}
\end{aligned}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/256*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 3*b
*d^3*x - 3*b*c*d^2)*cos(4*b*x + 4*a)/b^4 - 1/16*(2*b^3*d^3*x^3 + 6*b^3*c*d
^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(2*b*x + 2*
a)/b^4 - 3/1024*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*sin(4
*b*x + 4*a)/b^4 + 3/32*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)
*sin(2*b*x + 2*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 20.52 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.87

$$\int (c+dx)^3 \cos(a+bx) \sin^3(a+bx)\, dx =$$

$$-\frac{24\,d^3\,\sin(2\,a+2\,b\,x) - \frac{3\,d^3\,\sin(4\,a+4\,b\,x)}{4} + 32\,b^3\,c^3\,\cos(2\,a+2\,b\,x) - 8\,b^3\,c^3\,\cos(4\,a+4\,b\,x) - 48\,b^2\,c^2\,d$$

input
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^3,x)
```

output
```
-(24*d^3*sin(2*a + 2*b*x) - (3*d^3*sin(4*a + 4*b*x))/4 + 32*b^3*c^3*cos(2*
a + 2*b*x) - 8*b^3*c^3*cos(4*a + 4*b*x) - 48*b^2*c^2*d*sin(2*a + 2*b*x) +
6*b^2*c^2*d*sin(4*a + 4*b*x) + 32*b^3*d^3*x^3*cos(2*a + 2*b*x) - 8*b^3*d^3
*x^3*cos(4*a + 4*b*x) - 48*b^2*d^3*x^2*sin(2*a + 2*b*x) + 6*b^2*d^3*x^2*si
n(4*a + 4*b*x) - 48*b*c*d^2*cos(2*a + 2*b*x) + 3*b*c*d^2*cos(4*a + 4*b*x)
- 48*b*d^3*x*cos(2*a + 2*b*x) + 3*b*d^3*x*cos(4*a + 4*b*x) + 96*b^3*c^2*d*
x*cos(2*a + 2*b*x) - 24*b^3*c^2*d*x*cos(4*a + 4*b*x) - 96*b^2*c*d^2*x*sin(
2*a + 2*b*x) + 12*b^2*c*d^2*x*sin(4*a + 4*b*x) + 96*b^3*c*d^2*x^2*cos(2*a
+ 2*b*x) - 24*b^3*c*d^2*x^2*cos(4*a + 4*b*x))/(256*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.92

$$\int (c+dx)^3 \cos(a+bx) \sin^3(a+bx)\, dx$$

$$= \frac{48\cos(bx+a)\sin(bx+a)^3\,b^2c^2d + 96\cos(bx+a)\sin(bx+a)^3\,b^2c\,d^2x + 48\cos(bx+a)\sin(bx+a)^3\,b^2}$$

input
```
int((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x)
```

output

```
(48*cos(a + b*x)*sin(a + b*x)**3*b**2*c**2*d + 96*cos(a + b*x)*sin(a + b*x
)**3*b**2*c*d**2*x + 48*cos(a + b*x)*sin(a + b*x)**3*b**2*d**3*x**2 - 6*co
s(a + b*x)*sin(a + b*x)**3*d**3 + 72*cos(a + b*x)*sin(a + b*x)*b**2*c**2*d
 + 144*cos(a + b*x)*sin(a + b*x)*b**2*c*d**2*x + 72*cos(a + b*x)*sin(a + b
*x)*b**2*d**3*x**2 - 45*cos(a + b*x)*sin(a + b*x)*d**3 + 64*sin(a + b*x)**
4*b**3*c**3 + 192*sin(a + b*x)**4*b**3*c**2*d*x + 192*sin(a + b*x)**4*b**3
*c*d**2*x**2 + 64*sin(a + b*x)**4*b**3*d**3*x**3 - 24*sin(a + b*x)**4*b*c*
d**2 - 24*sin(a + b*x)**4*b*d**3*x - 72*sin(a + b*x)**2*b*c*d**2 - 72*sin(
a + b*x)**2*b*d**3*x - 72*a*b**2*c**2*d - 27*a*d**3 - 72*b**3*c**2*d*x - 7
2*b**3*c*d**2*x**2 - 24*b**3*d**3*x**3 + 72*b*c*d**2 + 45*b*d**3*x)/(256*b
**4)
```

3.25 $\int (c+dx)^2 \cos(a+bx) \sin^3(a+bx)\, dx$

Optimal result 355
Mathematica [A] (verified) 355
Rubi [A] (verified) 356
Maple [A] (verified) 358
Fricas [A] (verification not implemented) 359
Sympy [B] (verification not implemented) 359
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Mupad [B] (verification not implemented) 361
Reduce [B] (verification not implemented) 362

Optimal result

Integrand size = 22, antiderivative size = 125

$$\begin{aligned}
&\int (c+dx)^2 \cos(a+bx) \sin^3(a+bx)\, dx \\
&= -\frac{3(c+dx)^2}{32b} + \frac{3d(c+dx)\cos(a+bx)\sin(a+bx)}{16b^2} - \frac{3d^2 \sin^2(a+bx)}{32b^3} \\
&\quad + \frac{d(c+dx)\cos(a+bx)\sin^3(a+bx)}{8b^2} - \frac{d^2 \sin^4(a+bx)}{32b^3} + \frac{(c+dx)^2 \sin^4(a+bx)}{4b}
\end{aligned}$$

output

```
-3/32*(d*x+c)^2/b+3/16*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2-3/32*d^2*sin(b*
x+a)^2/b^3+1/8*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)^3/b^2-1/32*d^2*sin(b*x+a)^4
/b^3+1/4*(d*x+c)^2*sin(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.73

$$\begin{aligned}
&\int (c+dx)^2 \cos(a+bx) \sin^3(a+bx)\, dx \\
&= \frac{-16(-d^2+2b^2(c+dx)^2)\cos(2(a+bx)) + (-d^2+8b^2(c+dx)^2)\cos(4(a+bx)) - 4bd(c+dx)(-8\sin(2
\end{aligned}$$

$$\frac{}{256b^3}$$

input

```
Integrate[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output

```
(-16*(-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + (-d^2 + 8*b^2*(c + d*x)
^2)*Cos[4*(a + b*x)] - 4*b*d*(c + d*x)*(-8*Sin[2*(a + b*x)] + Sin[4*(a + b
*x)]))/(256*b^3)
```

Rubi [A] (verified)

Time $= 0.41$ (sec) , antiderivative size $= 133$, normalized size of antiderivative $= 1.06$, number of steps used $= 6$, number of rules used $= 6$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used $= \{4904, 3042, 3791, 3042, 3791, 17\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin^3(a+bx)\cos(a+bx)\,dx$$

$$\downarrow 4904$$

$$\frac{(c+dx)^2\sin^4(a+bx)}{4b} - \frac{d\int (c+dx)\sin^4(a+bx)dx}{2b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^2\sin^4(a+bx)}{4b} - \frac{d\int (c+dx)\sin(a+bx)^4dx}{2b}$$

$$\downarrow 3791$$

$$\frac{(c+dx)^2\sin^4(a+bx)}{4b} - \frac{d\left(\frac{3}{4}\int (c+dx)\sin^2(a+bx)dx + \frac{d\sin^4(a+bx)}{16b^2} - \frac{(c+dx)\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{2b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^2\sin^4(a+bx)}{4b} - \frac{d\left(\frac{3}{4}\int (c+dx)\sin(a+bx)^2dx + \frac{d\sin^4(a+bx)}{16b^2} - \frac{(c+dx)\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{2b}$$

$$\downarrow 3791$$

$$\frac{(c+dx)^2\sin^4(a+bx)}{4b} - \frac{d\left(\frac{3}{4}\left(\frac{1}{2}\int(c+dx)dx + \frac{d\sin^2(a+bx)}{4b^2} - \frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right) + \frac{d\sin^4(a+bx)}{16b^2} - \frac{(c+dx)\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{2b}$$

$$\downarrow 17$$

$$\frac{(c+dx)^2\sin^4(a+bx)}{4b} - \frac{d\left(\frac{3}{4}\left(\frac{d\sin^2(a+bx)}{4b^2} - \frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b} + \frac{(c+dx)^2}{4d}\right) + \frac{d\sin^4(a+bx)}{16b^2} - \frac{(c+dx)\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{2b}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
((c + d*x)^2*Sin[a + b*x]^4)/(4*b) - (d*(-1/4*((c + d*x)*Cos[a + b*x]*Sin[
a + b*x]^3)/b + (d*Sin[a + b*x]^4)/(16*b^2) + (3*((c + d*x)^2/(4*d) - ((c
+ d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (d*Sin[a + b*x]^2)/(4*b^2)))/4))
/(2*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)  Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4904

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.89

method	result
parallelrisch	$\frac{16\left(-2(dx+c)^2b^2+d^2\right)\cos(2bx+2a)+\left(8(dx+c)^2b^2-d^2\right)\cos(4bx+4a)+32bd(dx+c)\sin(2bx+2a)-4bd(dx+c)\sin(4bx+$
risch	$\frac{(8x^2d^2b^2+16b^2cdx+8b^2c^2-d^2)\cos(4bx+4a)}{256b^3} - \frac{d(dx+c)\sin(4bx+4a)}{64b^2} - \frac{(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2)\cos(2bx+2}{16b^3}$
derivativedivides	$\frac{a^2d^2\sin(bx+a)^4}{4b^2} - \frac{acd\sin(bx+a)^4}{2b} - \frac{2ad^2\left(\frac{(bx+a)\sin(bx+a)^4}{4}+\frac{\left(\sin(bx+a)^3+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{16}-\frac{3bx}{32}-\frac{3a}{32}\right)}{b^2}+\frac{c^2\sin(bx+}{4}$
default	$\frac{a^2d^2\sin(bx+a)^4}{4b^2} - \frac{acd\sin(bx+a)^4}{2b} - \frac{2ad^2\left(\frac{(bx+a)\sin(bx+a)^4}{4}+\frac{\left(\sin(bx+a)^3+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{16}-\frac{3bx}{32}-\frac{3a}{32}\right)}{b^2}+\frac{c^2\sin(bx+}{4}$
norman	$-\frac{3d^2x^2}{32b} - \frac{3d^2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{8b^3} - \frac{3d^2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^6}{8b^3} + \frac{\left(16b^2c^2-5d^2\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4}{4b^3} + \frac{3cd\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b^2} + \frac{11cd\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3}{8b^2} - \frac{11cd}{}$
orering	$\frac{5d(16d^4x^4b^4+64b^4cd^3x^3+96b^4c^2d^2x^2+64b^4c^3dx+16b^4c^4+3b^2d^4x^2+6b^2cd^3x+3b^2c^2d^2-3d^4)\cos(bx+a)\sin(bx+a)^3}{64b^6(dx+c)^3}$

input

```
int((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output

```
1/256*(16*(-2*(d*x+c)^2*b^2+d^2)*cos(2*b*x+2*a)+(8*(d*x+c)^2*b^2-d^2)*cos(
4*b*x+4*a)+32*b*d*(d*x+c)*sin(2*b*x+2*a)-4*b*d*(d*x+c)*sin(4*b*x+4*a)+24*b
^2*c^2-15*d^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.27

$$\int (c+dx)^2 \cos(a+bx) \sin^3(a+bx)\, dx$$
$$= \frac{5\,b^2d^2x^2 + 10\,b^2cdx + (8\,b^2d^2x^2 + 16\,b^2cdx + 8\,b^2c^2 - d^2)\cos(bx+a)^4 - (16\,b^2d^2x^2 + 32\,b^2cdx + 16\,b^2c^2}{32\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/32*(5*b^2*d^2*x^2 + 10*b^2*c*d*x + (8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2
*c^2 - d^2)*cos(b*x + a)^4 - (16*b^2*d^2*x^2 + 32*b^2*c*d*x + 16*b^2*c^2 -
 5*d^2)*cos(b*x + a)^2 - 2*(2*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 5*(b*d^2*
x + b*c*d)*cos(b*x + a))*sin(b*x + a))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(119) = 238.

Time = 0.43 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.56

$$\int (c+dx)^2 \cos(a+bx) \sin^3(a+bx)\, dx$$
$$= \begin{cases} \frac{c^2 \sin^4(a+bx)}{4b} + \frac{5cdx \sin^4(a+bx)}{16b} - \frac{3cdx \sin^2(a+bx)\cos^2(a+bx)}{8b} - \frac{3cdx \cos^4(a+bx)}{16b} + \frac{5d^2x^2 \sin^4(a+bx)}{32b} - \frac{3d^2x^2 \sin^2(a+bx)\cos}{16b} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \sin^3(a) \cos(a) \end{cases}$$

input
```
integrate((d*x+c)**2*cos(b*x+a)*sin(b*x+a)**3,x)
```

output

```
Piecewise((c**2*sin(a + b*x)**4/(4*b) + 5*c*d*x*sin(a + b*x)**4/(16*b) - 3
*c*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(8*b) - 3*c*d*x*cos(a + b*x)**4/(16
*b) + 5*d**2*x**2*sin(a + b*x)**4/(32*b) - 3*d**2*x**2*sin(a + b*x)**2*cos
(a + b*x)**2/(16*b) - 3*d**2*x**2*cos(a + b*x)**4/(32*b) + 5*c*d*sin(a + b
*x)**3*cos(a + b*x)/(16*b**2) + 3*c*d*sin(a + b*x)*cos(a + b*x)**3/(16*b**
2) + 5*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 3*d**2*x*sin(a + b*
x)*cos(a + b*x)**3/(16*b**2) - 5*d**2*sin(a + b*x)**4/(64*b**3) + 3*d**2*c
os(a + b*x)**4/(64*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*si
n(a)**3*cos(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(113) = 226$.

Time $= 0.04$ (sec) , antiderivative size $= 263$, normalized size of antiderivative $= 2.10$

$$\int (c+dx)^2 \cos(a+bx) \sin^3(a+bx)\, dx$$

$$= \frac{64\,c^2 \sin(bx+a)^4 - \frac{128\,acd \sin(bx+a)^4}{b} + \frac{64\,a^2 d^2 \sin(bx+a)^4}{b^2} + \frac{4\,(4\,(bx+a)\cos(4\,bx+4\,a) - 16\,(bx+a)\cos(2\,bx+2\,a) - \sin(4\,bx+4\,a)}{b}$$

input

```
integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output

```
1/256*(64*c^2*sin(b*x + a)^4 - 128*a*c*d*sin(b*x + a)^4/b + 64*a^2*d^2*sin
(b*x + a)^4/b^2 + 4*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x
 + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*c*d/b - 4*(4*(b*x + a)*co
s(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(
2*b*x + 2*a))*a*d^2/b^2 + ((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b
*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x +
 a)*sin(2*b*x + 2*a))*d^2/b^2)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.16

$$\begin{aligned}&\int (c+dx)^2\cos(a+bx)\sin^3(a+bx)\,dx\\&=\frac{(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2)\cos(4\,bx+4\,a)}{256\,b^3}\\&\quad-\frac{(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2)\cos(2\,bx+2\,a)}{16\,b^3}\\&\quad-\frac{(bd^2x+bcd)\sin(4\,bx+4\,a)}{64\,b^3}+\frac{(bd^2x+bcd)\sin(2\,bx+2\,a)}{8\,b^3}\end{aligned}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/256*(8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(4*b*x + 4*a)/b^
3 - 1/16*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a)/
b^3 - 1/64*(b*d^2*x + b*c*d)*sin(4*b*x + 4*a)/b^3 + 1/8*(b*d^2*x + b*c*d)*
sin(2*b*x + 2*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 19.34 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.62

$$\begin{aligned}&\int (c+dx)^2\cos(a+bx)\sin^3(a+bx)\,dx\\&=\frac{8\,d^2\,\cos(2\,a+2\,b\,x)-\frac{d^2\,\cos(4\,a+4\,b\,x)}{2}-16\,b^2\,c^2\,\cos(2\,a+2\,b\,x)+4\,b^2\,c^2\,\cos(4\,a+4\,b\,x)+16\,b\,c\,d\,\sin(2\end{aligned}$$

input
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^2,x)
```

output
```
(8*d^2*cos(2*a + 2*b*x) - (d^2*cos(4*a + 4*b*x))/2 - 16*b^2*c^2*cos(2*a +
2*b*x) + 4*b^2*c^2*cos(4*a + 4*b*x) + 16*b*c*d*sin(2*a + 2*b*x) - 2*b*c*d*
sin(4*a + 4*b*x) - 16*b^2*d^2*x^2*cos(2*a + 2*b*x) + 4*b^2*d^2*x^2*cos(4*a
 + 4*b*x) + 16*b*d^2*x*sin(2*a + 2*b*x) - 2*b*d^2*x*sin(4*a + 4*b*x) - 32*
b^2*c*d*x*cos(2*a + 2*b*x) + 8*b^2*c*d*x*cos(4*a + 4*b*x))/(128*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.51

$$\int (c+dx)^2 \cos(a+bx) \sin^3(a+bx)\, dx$$

$$= \frac{4\cos(bx+a)\sin(bx+a)^3 bcd + 4\cos(bx+a)\sin(bx+a)^3 b\, d^2x + 6\cos(bx+a)\sin(bx+a) bcd + 6\,\mathrm{co}$$

input

```
int((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x)
```

output

```
(4*cos(a + b*x)*sin(a + b*x)**3*b*c*d + 4*cos(a + b*x)*sin(a + b*x)**3*b*d
**2*x + 6*cos(a + b*x)*sin(a + b*x)*b*c*d + 6*cos(a + b*x)*sin(a + b*x)*b*
d**2*x + 8*sin(a + b*x)**4*b**2*c**2 + 16*sin(a + b*x)**4*b**2*c*d*x + 8*s
in(a + b*x)**4*b**2*d**2*x**2 - sin(a + b*x)**4*d**2 - 3*sin(a + b*x)**2*d
**2 - 6*a*b*c*d - 6*b**2*c*d*x - 3*b**2*d**2*x**2 + 3*d**2)/(32*b**3)
```

3.26 $\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal result . . . 363
Mathematica [A] (verified) . . . 363
Rubi [A] (verified) . . . 364
Maple [A] (verified) . . . 366
Fricas [A] (verification not implemented) . . . 366
Sympy [A] (verification not implemented) . . . 367
Maxima [A] (verification not implemented) . . . 367
Giac [A] (verification not implemented) . . . 368
Mupad [B] (verification not implemented) . . . 368
Reduce [B] (verification not implemented) . . . 369

Optimal result

Integrand size = 20, antiderivative size = 72

$$\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx = -\frac{3dx}{32b} + \frac{3d\cos(a+bx)\sin(a+bx)}{32b^2} + \frac{d\cos(a+bx)\sin^3(a+bx)}{16b^2} + \frac{(c+dx)\sin^4(a+bx)}{4b}$$

output

```
-3/32*d*x/b+3/32*d*cos(b*x+a)*sin(b*x+a)/b^2+1/16*d*cos(b*x+a)*sin(b*x+a)^
3/b^2+1/4*(d*x+c)*sin(b*x+a)^4/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx = \frac{c\sin^4(a+bx)}{4b} + \frac{d(-2bx\cos(2(a+bx)) + \sin(2(a+bx)))}{16b^2} - \frac{d(-4bx\cos(4(a+bx)) + \sin(4(a+bx)))}{128b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
(c*Sin[a + b*x]^4)/(4*b) + (d*(-2*b*x*Cos[2*(a + b*x)] + Sin[2*(a + b*x)])
)/(16*b^2) - (d*(-4*b*x*Cos[4*(a + b*x)] + Sin[4*(a + b*x)]))/(128*b^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4904, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin^3(a+bx)\cos(a+bx)\,dx$$

$$\downarrow 4904$$

$$\frac{(c+dx)\sin^4(a+bx)}{4b} - \frac{d\int \sin^4(a+bx)dx}{4b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)\sin^4(a+bx)}{4b} - \frac{d\int \sin(a+bx)^4 dx}{4b}$$

$$\downarrow 3115$$

$$\frac{(c+dx)\sin^4(a+bx)}{4b} - \frac{d\left(\frac{3}{4}\int \sin^2(a+bx)dx - \frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{4b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)\sin^4(a+bx)}{4b} - \frac{d\left(\frac{3}{4}\int \sin(a+bx)^2 dx - \frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{4b}$$

$$\downarrow 3115$$

$$\frac{(c+dx)\sin^4(a+bx)}{4b} - \frac{d\left(\frac{3}{4}\left(\frac{\int 1dx}{2} - \frac{\sin(a+bx)\cos(a+bx)}{2b}\right) - \frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{4b}$$

$$\downarrow 24$$

$$\frac{(c+dx)\sin^4(a+bx)}{4b} - \frac{d\left(\frac{3}{4}\left(\frac{x}{2} - \frac{\sin(a+bx)\cos(a+bx)}{2b}\right) - \frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{4b}$$

input
```
Int[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
((c + d*x)*Sin[a + b*x]^4)/(4*b) - (d*(-1/4*(Cos[a + b*x]*Sin[a + b*x]^3)/
b + (3*(x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)))/4))/(4*b)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

method	result
parallelrisch	$\frac{-16b(dx+c)\cos(2bx+2a)+4b(dx+c)\cos(4bx+4a)+12cb+8d\sin(2bx+2a)-d\sin(4bx+4a)}{128b^2}$
risch	$\frac{(dx+c)\cos(4bx+4a)}{32b}-\frac{d\sin(4bx+4a)}{128b^2}-\frac{(dx+c)\cos(2bx+2a)}{8b}+\frac{d\sin(2bx+2a)}{16b^2}$
derivativedivides	$\frac{-\frac{da\sin(bx+a)^4}{4b}+\frac{c\sin(bx+a)^4}{4}+d\left(\frac{(bx+a)\sin(bx+a)^4}{4}+\frac{\left(\sin(bx+a)^3+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{16}-\frac{3bx}{32}-\frac{3a}{32}\right)}{b}$
default	$\frac{-\frac{da\sin(bx+a)^4}{4b}+\frac{c\sin(bx+a)^4}{4}+d\left(\frac{(bx+a)\sin(bx+a)^4}{4}+\frac{\left(\sin(bx+a)^3+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{16}-\frac{3bx}{32}-\frac{3a}{32}\right)}{b}$
norman	$\frac{\frac{3d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{16b^2}+\frac{11d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3}{16b^2}-\frac{11d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^5}{16b^2}-\frac{3d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^7}{16b^2}-\frac{3dx}{32b}+\frac{4c\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4}{b}-\frac{3dx\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{8b}+\frac{55dx}{}$
orering	$\frac{d\left(5x^2d^2b^2+10b^2cdx+5b^2c^2+d^2\right)\cos(bx+a)\sin(bx+a)^3}{8b^4(dx+c)^2}-\frac{\left(5x^2d^2b^2+10b^2cdx+5b^2c^2+2d^2\right)\left(d\cos(bx+a)\sin(bx+a)\right.}{16b^4(}$

input
```
int((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/128*(-16*b*(d*x+c)*cos(2*b*x+2*a)+4*b*(d*x+c)*cos(4*b*x+4*a)+12*c*b+8*d*
sin(2*b*x+2*a)-d*sin(4*b*x+4*a))/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx$$
$$=\frac{8\,(bdx+bc)\cos(bx+a)^4+5\,bdx-16\,(bdx+bc)\cos(bx+a)^2-\left(2\,d\cos(bx+a)^3-5\,d\cos(bx+a)\right)\sin}{32\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/32*(8*(b*d*x + b*c)*cos(b*x + a)^4 + 5*b*d*x - 16*(b*d*x + b*c)*cos(b*x
+ a)^2 - (2*d*cos(b*x + a)^3 - 5*d*cos(b*x + a))*sin(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.92

$$\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx$$
$$= \begin{cases} \frac{c\sin^4(a+bx)}{4b} + \frac{5dx\sin^4(a+bx)}{32b} - \frac{3dx\sin^2(a+bx)\cos^2(a+bx)}{16b} - \frac{3dx\cos^4(a+bx)}{32b} + \frac{5d\sin^3(a+bx)\cos(a+bx)}{32b^2} + \frac{3d\sin(a+bx)\cos^3}{32b^2} \\ \left(cx + \frac{dx^2}{2}\right)\sin^3(a)\cos(a) \end{cases}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Piecewise((c*sin(a + b*x)**4/(4*b) + 5*d*x*sin(a + b*x)**4/(32*b) - 3*d*x*
sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 3*d*x*cos(a + b*x)**4/(32*b) + 5*
d*sin(a + b*x)**3*cos(a + b*x)/(32*b**2) + 3*d*sin(a + b*x)*cos(a + b*x)**
3/(32*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*sin(a)**3*cos(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

$$\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{32\,c\sin(bx+a)^4 - \frac{32\,ad\sin(bx+a)^4}{b} + \frac{(4(bx+a)\cos(4bx+4a) - 16(bx+a)\cos(2bx+2a) - \sin(4bx+4a) + 8\sin(2bx+2a))d}{b}}{128\,b}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/128*(32*c*sin(b*x + a)^4 - 32*a*d*sin(b*x + a)^4/b + (4*(b*x + a)*cos(4*
b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*
x + 2*a))*d/b)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx = \frac{(bdx+bc)\cos(4\,bx+4\,a)}{32\,b^2} - \frac{(bdx+bc)\cos(2\,bx+2\,a)}{8\,b^2} - \frac{d\sin(4\,bx+4\,a)}{128\,b^2} + \frac{d\sin(2\,bx+2\,a)}{16\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/32*(b*d*x + b*c)*cos(4*b*x + 4*a)/b^2 - 1/8*(b*d*x + b*c)*cos(2*b*x + 2*
a)/b^2 - 1/128*d*sin(4*b*x + 4*a)/b^2 + 1/16*d*sin(2*b*x + 2*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.31

$$\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx = \frac{2\,d\,\sin(2\,a+2\,b\,x) - \frac{d\,\sin(4\,a+4\,b\,x)}{4} - 2\,b\,c\sin(2\,a+2\,b\,x)^2 + 8\,b\,c\sin(a+b\,x)^2 + 4\,b\,d\,x\left(2\sin(a+b\,x)^2 -}{32\,b^2}$$

input
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x),x)
```

output
```
(2*d*sin(2*a + 2*b*x) - (d*sin(4*a + 4*b*x))/4 - 2*b*c*sin(2*a + 2*b*x)^2
+ 8*b*c*sin(a + b*x)^2 + 4*b*d*x*(2*sin(a + b*x)^2 - 1) - b*d*x*(2*sin(2*a
 + 2*b*x)^2 - 1))/(32*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{6\cos(bx+a)\sin(bx+a)^3 d + 9\cos(bx+a)\sin(bx+a) d + 24\sin(bx+a)^4 bc + 24\sin(bx+a)^4 bdx -}{96b^2}$$

input
```
int((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x)
```

output
```
(6*cos(a + b*x)*sin(a + b*x)**3*d + 9*cos(a + b*x)*sin(a + b*x)*d + 24*sin
(a + b*x)**4*b*c + 24*sin(a + b*x)**4*b*d*x - 9*a*d - 64*b*c - 9*b*d*x)/(9
6*b**2)
```

3.27 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx$

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Optimal result

Integrand size = 22, antiderivative size = 129

$$\begin{aligned}\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx = &-\frac{\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{8d}\\ &+\frac{\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{4d}\\ &+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{4d}\\ &-\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}\end{aligned}$$

output

```
-1/8*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d+1/4*Ci(2*b*c/d+2*b*x)*sin(2*a-2*
b*c/d)/d+1/4*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d-1/8*cos(4*a-4*b*c/d)*Si(
4*b*c/d+4*b*x)/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.85

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx = -\frac{\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)\sin\left(4a-\frac{4bc}{d}\right)-2\,\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right)-2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2}{}\right.}{8d}$$

input
```
Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x),x]
```

output
```
-1/8*(CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] - 2*CosIntegral[
(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - 2*Cos[2*a - (2*b*c)/d]*SinIntegr
al[(2*b*(c + d*x))/d] + Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d
])/d
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos(a+bx)}{c+dx}\,dx$$
$$\downarrow 4906$$
$$\int \left(\frac{\sin(2a+2bx)}{4(c+dx)}-\frac{\sin(4a+4bx)}{8(c+dx)}\right)dx$$
$$\downarrow 2009$$
$$-\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{4d}+$$
$$\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{4d}-\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x),x]
```

output
```
-1/8*(CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/d + (CosIntegra
l[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(4*d) + (Cos[2*a - (2*b*c)/d]*S
inIntegral[(2*b*c)/d + 2*b*x])/(4*d) - (Cos[4*a - (4*b*c)/d]*SinIntegral[(
4*b*c)/d + 4*b*x])/(8*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{b\left(-\frac{2\,\mathrm{Si}\left(-2bx-2a-\frac{2(-ad+cb)}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2ad+2cb}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}\right)}{8}-\frac{b\left(-\frac{4\,\mathrm{Si}\left(-4bx-4a-\frac{4(-a}{}\right.\right.}{}$
default	$\frac{b\left(-\frac{2\,\mathrm{Si}\left(-2bx-2a-\frac{2(-ad+cb)}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2ad+2cb}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}\right)}{8}-\frac{b\left(-\frac{4\,\mathrm{Si}\left(-4bx-4a-\frac{4(-a}{}\right.\right.}{}$
risch	$-\frac{ie^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{8d}+\frac{ie^{-\frac{4i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{16d}+\frac{ie^{\frac{2i(}{}}}{}$

input
```
int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*b*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*
b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)-1/32*b*(-4*Si(-4*b*x-4*a-4*
(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(
-a*d+b*c)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx$$
$$= \frac{2\ \mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right) - \mathrm{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right) - \cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{4\,(bdx+bc)}{d}\right) + 2\ \mathrm{c}}{8\,d}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
1/8*(2*cos_integral(2*(b*d*x + b*c)/d)*sin(-2*(b*c - a*d)/d) - cos_integra
l(4*(b*d*x + b*c)/d)*sin(-4*(b*c - a*d)/d) - cos(-4*(b*c - a*d)/d)*sin_int
egral(4*(b*d*x + b*c)/d) + 2*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x +
 b*c)/d))/d
```

Sympy [F]

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \int \frac{\sin^3(a+bx)\cos(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.18

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx =$$
$$-\frac{2\,b\left(-i\,E_1\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_1\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b\left(-i\,E_1\left(\frac{4\,(-i\,bc-i\,(}{}\right.\right.$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
-1/16*(2*b*(-I*exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I
*exp_integral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a
*d)/d) - b*(-I*exp_integral_e(1, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I
*exp_integral_e(1, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a
*d)/d) + 2*b*(exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + ex
p_integral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)
/d) - b*(exp_integral_e(1, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_int
egral_e(1, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d)/d))/
(b*d)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 6046, normalized size of antiderivative = 46.87

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output

```
-1/16*(imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*
b*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a
)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(c
os_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/
d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^
2*tan(b*c/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan
(2*b*c/d)^2*tan(b*c/d)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(
2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 4*real_part(cos_integral(-2*b*
x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 2*real_part(
cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)
^2 + 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2
*b*c/d)*tan(b*c/d)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)
^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*real_part(cos_integral(-2*b*x -
2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_
integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 -
 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d
)^2*tan(b*c/d)^2 + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan
(a)^2*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)
^2*tan(a)^2*tan(2*b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+b\,x)\,\sin(a+b\,x)^3}{c+d\,x}\,dx$$

input

```
int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x),x)
```

output

```
int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)\sin(bx+a)^3}{dx+c}dx$$

input
```
int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x)
```

output
```
int((cos(a + b*x)*sin(a + b*x)**3)/(c + d*x),x)
```

3.28 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 377
Mathematica [A] (verified) . . . 378
Rubi [A] (verified) . . . 378
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Optimal result

Integrand size = 22, antiderivative size = 179

$$\begin{aligned}\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = &\frac{b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^2}\\ &-\frac{b\cos\left(4a-\frac{4bc}{d}\right)\operatorname{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}\\ &-\frac{\sin(2a+2bx)}{4d(c+dx)}+\frac{\sin(4a+4bx)}{8d(c+dx)}\\ &-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^2}\\ &+\frac{b\sin\left(4a-\frac{4bc}{d}\right)\operatorname{Si}\left(\frac{4bc}{d}+4bx\right)}{2d^2}\end{aligned}$$

output

```
1/2*b*cos(2*a-2*b*c/d)*Ci(2*b*c/d+2*b*x)/d^2-1/2*b*cos(4*a-4*b*c/d)*Ci(4*b
*c/d+4*b*x)/d^2-1/4*sin(2*b*x+2*a)/d/(d*x+c)+1/8*sin(4*b*x+4*a)/d/(d*x+c)-
1/2*b*sin(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^2+1/2*b*sin(4*a-4*b*c/d)*Si(4*b
*c/d+4*b*x)/d^2
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.84

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{4b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) - 4b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right) - \frac{2d\sin(2(a+bx))}{c+dx} + \frac{d\sin(}{}}{8d^2}$$

input
```
Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
(4*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - 4*b*Cos[4*a - (
4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] - (2*d*Sin[2*(a + b*x)])/(c + d*x
) + (d*Sin[4*(a + b*x)])/(c + d*x) - 4*b*Sin[2*a - (2*b*c)/d]*SinIntegral[
(2*b*(c + d*x))/d] + 4*b*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/
d])/(8*d^2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos(a+bx)}{(c+dx)^2}\,dx$$
$$\downarrow 4906$$
$$\int \left(\frac{\sin(2a+2bx)}{4(c+dx)^2} - \frac{\sin(4a+4bx)}{8(c+dx)^2}\right)dx$$
$$\downarrow 2009$$
$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^2} - \frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2} -$$
$$\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^2} + \frac{b\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{2d^2} - \frac{\sin(2a+2bx)}{4d(c+dx)} + \frac{\sin(4a+4bx)}{8d(c+dx)}$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
(b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) - (b*Cos[4
*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(2*d^2) - Sin[2*a + 2*b*x]
/(4*d*(c + d*x)) + Sin[4*a + 4*b*x]/(8*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)
/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) + (b*Sin[4*a - (4*b*c)/d]*SinI
ntegral[(4*b*c)/d + 4*b*x])/(2*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{b^2\left(-\frac{2\sin(2bx+2a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{4\,\mathrm{Si}\left(-2bx-2a-\frac{2(-ad+cb)}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(2bx+2a+\frac{-2ad+2cb}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}}{d}\right)}{8}-$
default	$\frac{b^2\left(-\frac{2\sin(2bx+2a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{4\,\mathrm{Si}\left(-2bx-2a-\frac{2(-ad+cb)}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(2bx+2a+\frac{-2ad+2cb}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}}{d}\right)}{8}-$
risch	$-\frac{b\,\mathrm{e}^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{4d^2}+\frac{b\,\mathrm{e}^{-\frac{4i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{4d^2}-\frac{b\,\mathrm{e}^{2}}{}$

input
```
int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*b^2*(-2*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2*(-2*Si(-2*b*x-2*a
-2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(
2*(-a*d+b*c)/d)/d)/d)-1/32*b^2*(-4*sin(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))/d+4
*(-4*Si(-4*b*x-4*a-4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*b*x+4*a+4*
(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.08

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{(bdx+bc)\cos\left(-\frac{4\,(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)-(bdx+bc)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)-(bdx+bc)\sin$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*((b*d*x + b*c)*cos(-4*(b*c - a*d)/d)*cos_integral(4*(b*d*x + b*c)/d)
- (b*d*x + b*c)*cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*d*x + b*c)/d) - (b
*d*x + b*c)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + (b*d*x
 + b*c)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - 2*(d*cos(b
*x + a)^3 - d*cos(b*x + a))*sin(b*x + a))/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin^3(a+bx)\cos(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.72

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{2b^2\left(-i\,E_2\left(\frac{2(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_2\left(-\frac{2(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)-b^2\left(-i\,E_2\left(\frac{4(-i\,bc-}\right.}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/16*(2*b^2*(-I*exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 I*exp_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c -
 a*d)/d) - b^2*(-I*exp_integral_e(2, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d)
 + I*exp_integral_e(2, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c
 - a*d)/d) + 2*b^2*(exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d
) + exp_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c
- a*d)/d) - b^2*(exp_integral_e(2, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 exp_integral_e(2, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a
*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.44 (sec) , antiderivative size = 63510, normalized size of antiderivative = 354.80

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output

```
-1/4*(b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)
^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - b*d*x*real_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(
2*b*c/d)^2*tan(b*c/d)^2 - b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*
tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 +
b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*ta
n(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*b*d*x*imag_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b
*c/d)^2*tan(b*c/d) - 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 4*b*d
*x*sin_integral(2*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(
a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 2*b*d*x*imag_part(cos_integral(4*b*x + 4*
b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d
)^2 + 2*b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b
*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 4*b*d*x*sin_integral
(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/
d)*tan(b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b
*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*b*d*x*i
mag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^
2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*b*d*x*sin_integral(2*(b*d*x + ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(a+b\,x)\,\sin(a+b\,x)^3}{(c+d\,x)^2}\,dx$$

input

```
int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^2,x)
```

output

```
int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(bx+a\right)\sin\left(bx+a\right)^3}{d^2x^2+2cdx+c^2}dx$$

input
```
int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int((cos(a + b*x)*sin(a + b*x)**3)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.29 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$

Optimal result . . . 384
Mathematica [A] (verified) . . . 385
Rubi [A] (verified) . . . 385
Maple [A] (verified) . . . 387
Fricas [A] (verification not implemented) . . . 387
Sympy [F] . . . 388
Maxima [C] (verification not implemented) . . . 388
Giac [C] (verification not implemented) . . . 389
Mupad [F(-1)] . . . 390
Reduce [F] . . . 391

Optimal result

Integrand size = 22, antiderivative size = 229

$$\begin{aligned}\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = &-\frac{b\cos(2a+2bx)}{4d^2(c+dx)}+\frac{b\cos(4a+4bx)}{4d^2(c+dx)}\\&+\frac{b^2\,\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{d^3}\\&-\frac{b^2\,\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d^3}\\&-\frac{\sin(2a+2bx)}{8d(c+dx)^2}+\frac{\sin(4a+4bx)}{16d(c+dx)^2}\\&-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^3}\\&+\frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{d^3}\end{aligned}$$

output

```
-1/4*b*cos(2*b*x+2*a)/d^2/(d*x+c)+1/4*b*cos(4*b*x+4*a)/d^2/(d*x+c)+b^2*Ci(
4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^3-1/2*b^2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*
c/d)/d^3-1/8*sin(2*b*x+2*a)/d/(d*x+c)^2+1/16*sin(4*b*x+4*a)/d/(d*x+c)^2-1/
2*b^2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^3+b^2*cos(4*a-4*b*c/d)*Si(4*b*c
/d+4*b*x)/d^3
```

Mathematica [A] (verified)

Time = 1.96 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.87

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{16b^2\,\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)\sin\left(4a-\frac{4bc}{d}\right)+\frac{d(4b(c+dx)\cos(4(a+bx))+d\sin(4(a+bx)))}{(c+dx)^2}-2\left(4b^2\,\text{CosIntegral}\left(\frac{2b(c+d}{d}\right.\right.$$

input
```
Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

output
```
(16*b^2*CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + (d*(4*b*(c +
 d*x)*Cos[4*(a + b*x)] + d*Sin[4*(a + b*x)]))/(c + d*x)^2 - 2*(4*b^2*CosIn
tegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*(2*b*(c + d*x)*Cos[2*(
a + b*x)] + d*Sin[2*(a + b*x)]))/(c + d*x)^2 + 4*b^2*Cos[2*a - (2*b*c)/d]*
SinIntegral[(2*b*(c + d*x))/d]) + 16*b^2*Cos[4*a - (4*b*c)/d]*SinIntegral[
(4*b*(c + d*x))/d])/(16*d^3)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos(a+bx)}{(c+dx)^3}\,dx$$
$$\downarrow 4906$$
$$\int \left(\frac{\sin(2a+2bx)}{4(c+dx)^3}-\frac{\sin(4a+4bx)}{8(c+dx)^3}\right)dx$$
$$\downarrow 2009$$

$$\frac{b^2 \sin\left(4a-\frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3} - \frac{b^2 \sin\left(2a-\frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^3} - \frac{b^2 \cos\left(2a-\frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^3} + \frac{b^2 \cos\left(4a-\frac{4bc}{d}\right) \text{Si}\left(\frac{4bc}{d}+4bx\right)}{d^3} - \frac{b\cos(2a+2bx)}{4d^2(c+dx)} + \frac{b\cos(4a+4bx)}{4d^2(c+dx)} - \frac{\sin(2a+2bx)}{8d(c+dx)^2} + \frac{\sin(4a+4bx)}{16d(c+dx)^2}$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

output
```
-1/4*(b*Cos[2*a + 2*b*x])/(d^2*(c + d*x)) + (b*Cos[4*a + 4*b*x])/(4*d^2*(c
 + d*x)) + (b^2*CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/d^3 -
 (b^2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d^3) - Sin[2
*a + 2*b*x]/(8*d*(c + d*x)^2) + Sin[4*a + 4*b*x]/(16*d*(c + d*x)^2) - (b^2
*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^3) + (b^2*Cos[4
*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{b^3\left(-\frac{\sin(2bx+2a)}{(-ad+cb+d(bx+a))^2 d}+\frac{-\frac{2\cos(2bx+2a)}{(-ad+cb+d(bx+a))d}-\frac{2\left(-\frac{2\,\mathrm{Si}\left(-2bx-2a-\frac{2(-ad+cb)}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2}{d}\right.}{d}\right.}{d}\right.}{8}$
default	$\frac{b^3\left(-\frac{\sin(2bx+2a)}{(-ad+cb+d(bx+a))^2 d}+\frac{-\frac{2\cos(2bx+2a)}{(-ad+cb+d(bx+a))d}-\frac{2\left(-\frac{2\,\mathrm{Si}\left(-2bx-2a-\frac{2(-ad+cb)}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2a}{d}\right.}{d}\right.}{d}\right.}{8}$
risch	$\frac{ib^2 e^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{4d^3}-\frac{ib^2 e^{-\frac{4i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{2d^3}-\frac{ib^2}{}$

input
```
int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*b^3*(-sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2/d+(-2*cos(2*b*x+2*a)/
(-a*d+c*b+d*(b*x+a))/d-2*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c
)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)-1/32*b^
3*(-2*sin(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))^2/d+2*(-4*cos(4*b*x+4*a)/(-a*d+c
*b+d*(b*x+a))/d-4*(-4*Si(-4*b*x-4*a-4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-
4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.46

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$$

$$= \frac{4\left(bd^2x+bcd\right)\cos\left(bx+a\right)^4+bd^2x+bcd-5\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2-\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{Ci}\left(\right.}{}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(4*(b*d^2*x + b*c*d)*cos(b*x + a)^4 + b*d^2*x + b*c*d - 5*(b*d^2*x + b
*c*d)*cos(b*x + a)^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(
2*(b*d*x + b*c)/d)*sin(-2*(b*c - a*d)/d) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*cos_integral(4*(b*d*x + b*c)/d)*sin(-4*(b*c - a*d)/d) + 2*(b^2*d^
2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x
 + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-2*(b*c - a*d)/d)*s
in_integral(2*(b*d*x + b*c)/d) + (d^2*cos(b*x + a)^3 - d^2*cos(b*x + a))*s
in(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin^3(a+bx)\cos(a+bx)}{(c+dx)^3}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.50

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx =$$
$$-\frac{2\,b^3\left(-i\,E_3\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_3\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^3\left(-i\,E_3\left(\frac{4\,(-i\,bc-}{}\right.\right.$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/16*(2*b^3*(-I*exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 I*exp_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c -
 a*d)/d) - b^3*(-I*exp_integral_e(3, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d)
 + I*exp_integral_e(3, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c
 - a*d)/d) + 2*b^3*(exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d
) + exp_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c
- a*d)/d) - b^3*(exp_integral_e(3, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 exp_integral_e(3, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a
*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2
 - a*d^3)*(b*x + a))*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.05 (sec) , antiderivative size = 111694, normalized size of antiderivative = 487.75

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

output

```
1/8*(4*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*t
an(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*b^2*d^2*x^2*
imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^
2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*imag_part(cos_integ
ral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b
*c/d)^2*tan(b*c/d)^2 - 4*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c
/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)
^2 + 8*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2
*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*b^2*d^2*x^2*sin_integ
ral(2*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b
*c/d)^2*tan(b*c/d)^2 - 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/
d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)
- 4*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan
(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 8*b^2*d^2*x^2*real
_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*ta
n(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 8*b^2*d^2*x^2*real_part(cos_integral(-4
*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*
tan(b*c/d)^2 + 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(
2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*b^2*
d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos(a+b\,x)\,\sin(a+b\,x)^3}{(c+d\,x)^3}\,dx$$

input

```
int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^3,x)
```

output

```
int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos(bx+a)\sin(bx+a)^3}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx$$

input
```
int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x)
```

output
```
int((cos(a + b*x)*sin(a + b*x)**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d*
**3*x**3),x)
```

3.30 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$

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Optimal result

Integrand size = 22, antiderivative size = 287

$$\begin{aligned}\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = &-\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}+\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}\\ &-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}\\ &+\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}\\ &-\frac{\sin(2a+2bx)}{12d(c+dx)^3}+\frac{b^2\sin(2a+2bx)}{6d^3(c+dx)}+\frac{\sin(4a+4bx)}{24d(c+dx)^3}\\ &-\frac{b^2\sin(4a+4bx)}{3d^3(c+dx)}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}\\ &-\frac{4b^3\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{3d^4}\end{aligned}$$

output

```
-1/12*b*cos(2*b*x+2*a)/d^2/(d*x+c)^2+1/12*b*cos(4*b*x+4*a)/d^2/(d*x+c)^2-1
/3*b^3*cos(2*a-2*b*c/d)*Ci(2*b*c/d+2*b*x)/d^4+4/3*b^3*cos(4*a-4*b*c/d)*Ci(
4*b*c/d+4*b*x)/d^4-1/12*sin(2*b*x+2*a)/d/(d*x+c)^3+1/6*b^2*sin(2*b*x+2*a)/
d^3/(d*x+c)+1/24*sin(4*b*x+4*a)/d/(d*x+c)^3-1/3*b^2*sin(4*b*x+4*a)/d^3/(d*
x+c)+1/3*b^3*sin(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^4-4/3*b^3*sin(4*a-4*b*c/
d)*Si(4*b*c/d+4*b*x)/d^4
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.10

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$$
$$= \frac{-2d\cos(2bx)\left(bd(c+dx)\cos(2a)+(d^2-2b^2(c+dx)^2)\sin(2a)\right)+d\cos(4bx)\left(2bd(c+dx)\cos(4a)+(d^2\right.}{}$$

input
```
Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

output
```
(-2*d*Cos[2*b*x]*(b*d*(c + d*x)*Cos[2*a] + (d^2 - 2*b^2*(c + d*x)^2)*Sin[2
*a]) + d*Cos[4*b*x]*(2*b*d*(c + d*x)*Cos[4*a] + (d^2 - 8*b^2*(c + d*x)^2)*
Sin[4*a]) + 2*d*((-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*a] + b*d*(c + d*x)*Sin[2
*a])*Sin[2*b*x] - d*((-d^2 + 8*b^2*(c + d*x)^2)*Cos[4*a] + 2*b*d*(c + d*x)
*Sin[4*a])*Sin[4*b*x] - 8*b^3*(c + d*x)^3*(Cos[2*a - (2*b*c)/d]*CosIntegra
l[(2*b*(c + d*x))/d] - Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d]
) + 32*b^3*(c + d*x)^3*(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d
] - Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d]))/(24*d^4*(c + d*x
)^3)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos(a+bx)}{(c+dx)^4}\,dx$$
$$\downarrow 4906$$
$$\int \left(\frac{\sin(2a+2bx)}{4(c+dx)^4}-\frac{\sin(4a+4bx)}{8(c+dx)^4}\right)dx$$
$$\downarrow 2009$$

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+$$
$$\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{4b^3\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+\frac{b^2\sin(2a+2bx)}{6d^3(c+dx)}-$$
$$\frac{b^2\sin(4a+4bx)}{3d^3(c+dx)}-\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}+\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}-\frac{\sin(2a+2bx)}{12d(c+dx)^3}+\frac{\sin(4a+4bx)}{24d(c+dx)^3}$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

output
```
-1/12*(b*Cos[2*a + 2*b*x])/(d^2*(c + d*x)^2) + (b*Cos[4*a + 4*b*x])/(12*d^
2*(c + d*x)^2) - (b^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])
/(3*d^4) + (4*b^3*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(3*
d^4) - Sin[2*a + 2*b*x]/(12*d*(c + d*x)^3) + (b^2*Sin[2*a + 2*b*x])/(6*d^3
*(c + d*x)) + Sin[4*a + 4*b*x]/(24*d*(c + d*x)^3) - (b^2*Sin[4*a + 4*b*x])
/(3*d^3*(c + d*x)) + (b^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b
*x])/(3*d^4) - (4*b^3*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])
/(3*d^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{b^4\left(-\frac{2\sin(2bx+2a)}{3(-ad+cb+d(bx+a))^3d}+\frac{-\frac{2\cos(2bx+2a)}{3(-ad+cb+d(bx+a))^2d}-\frac{2\left(-\frac{2\sin(2bx+2a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{4\,\mathrm{Si}\left(-2bx-2a-\frac{2(-ad+cb)}{d}\right)\sin\left(\frac{-2a}{}\right.}{d}}{3d}\right.}{d}\right.}{8}$
default	$\frac{b^4\left(-\frac{2\sin(2bx+2a)}{3(-ad+cb+d(bx+a))^3d}+\frac{-\frac{2\cos(2bx+2a)}{3(-ad+cb+d(bx+a))^2d}-\frac{2\left(-\frac{2\sin(2bx+2a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{4\,\mathrm{Si}\left(-2bx-2a-\frac{2(-ad+cb)}{d}\right)\sin\left(\frac{-2a}{}\right.}{d}}{3d}\right.}{d}\right.}{8}$
risch	$\frac{b^3\mathrm{e}^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{6d^4}-\frac{2b^3\mathrm{e}^{-\frac{4i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{3d^4}+\frac{b^3\mathrm{e}}{}$

input
```
int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*b^4*(-2/3*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^3/d+2/3*(-cos(2*b*x
+2*a)/(-a*d+c*b+d*(b*x+a))^2/d-(-2*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2
*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*
(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)/d)-1/32*b^4*(-4/3*sin(4*b*x+4*a
)/(-a*d+c*b+d*(b*x+a))^3/d+4/3*(-2*cos(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))^2/d
-2*(-4*sin(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))/d+4*(-4*Si(-4*b*x-4*a-4*(-a*d+b
*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*
c)/d)/d)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.65

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$$

$$= \frac{bd^3x + 4\left(bd^3x + bcd^2\right)\cos\left(bx+a\right)^4 + bcd^2 - 5\left(bd^3x + bcd^2\right)\cos\left(bx+a\right)^2 + 8\left(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^{3}\right.}{}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/6*(b*d^3*x + 4*(b*d^3*x + b*c*d^2)*cos(b*x + a)^4 + b*c*d^2 - 5*(b*d^3*x
 + b*c*d^2)*cos(b*x + a)^2 + 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*
d*x + b^3*c^3)*cos(-4*(b*c - a*d)/d)*cos_integral(4*(b*d*x + b*c)/d) - 2*(
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-2*(b*c - a*d
)/d)*cos_integral(2*(b*d*x + b*c)/d) - 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
3*b^3*c^2*d*x + b^3*c^3)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c
)/d) + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-2*
(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - 2*((8*b^2*d^3*x^2 + 16*b^
2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^3 - (5*b^2*d^3*x^2 + 10*b^2*c*
d^2*x + 5*b^2*c^2*d - d^3)*cos(b*x + a))*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*
x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F]

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sin^3\left(a+bx\right)\cos\left(a+bx\right)}{\left(c+dx\right)^4}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c)**4,x)
```

output
```
Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x)**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.37

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = -\frac{2\,b^4\left(-i\,E_4\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_4\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^4\left(-i\,E_4\left(\frac{4\,(-i\,bc-}{1}\right.\right.$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/16*(2*b^4*(-I*exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 I*exp_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c -
 a*d)/d) - b^4*(-I*exp_integral_e(4, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d)
 + I*exp_integral_e(4, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c
 - a*d)/d) + 2*b^4*(exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d
) + exp_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c
- a*d)/d) - b^4*(exp_integral_e(4, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 exp_integral_e(4, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a
*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 -
a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 +
 a^2*d^4)*(b*x + a))*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.19 (sec) , antiderivative size = 157526, normalized size of antiderivative = 548.87

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")
```

output

```
1/12*(8*b^3*d^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*
tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*b^3*d^3*x^3
*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)
^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*b^3*d^3*x^3*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*
b*c/d)^2*tan(b*c/d)^2 + 8*b^3*d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*
c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d
)^2 + 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*
tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 4*b^3*d^3*x^3*i
mag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^
2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x
 + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(
b*c/d) - 16*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x
)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 16*b^3*d^3*
x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(
2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 32*b^3*d^3*x^3*sin_integral(4*
(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*
tan(b*c/d)^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(
2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*b^3*
d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos(a+b\,x)\,\sin(a+b\,x)^3}{(c+d\,x)^4}\,dx$$

input

```
int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^4,x)
```

output

```
int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^4, x)
```

Reduce [F]

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \text{too large to display}$$

input

```
int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x)
```

output

```
( - 36*cos(a + b*x)*sin(a + b*x)**3*b**2*c**2*d - 72*cos(a + b*x)*sin(a +
b*x)**3*b**2*c*d**2*x - 36*cos(a + b*x)*sin(a + b*x)**3*b**2*d**3*x**2 - 9
8*cos(a + b*x)*sin(a + b*x)**3*d**3 + 64*cos(a + b*x)*sin(a + b*x)**2*b*c*
d**2 + 64*cos(a + b*x)*sin(a + b*x)**2*b*d**3*x + 522*cos(a + b*x)*sin(a +
 b*x)*b**2*c**2*d + 1044*cos(a + b*x)*sin(a + b*x)*b**2*c*d**2*x + 522*cos
(a + b*x)*sin(a + b*x)*b**2*d**3*x**2 + 1232*cos(a + b*x)*sin(a + b*x)*d**
3 - 1024*cos(a + b*x)*b*c*d**2 - 1024*cos(a + b*x)*b*d**3*x + 20160*int(ta
n((a + b*x)/2)**3/(tan((a + b*x)/2)**8*c**4 + 4*tan((a + b*x)/2)**8*c**3*d
*x + 6*tan((a + b*x)/2)**8*c**2*d**2*x**2 + 4*tan((a + b*x)/2)**8*c*d**3*x
**3 + tan((a + b*x)/2)**8*d**4*x**4 + 4*tan((a + b*x)/2)**6*c**4 + 16*tan(
(a + b*x)/2)**6*c**3*d*x + 24*tan((a + b*x)/2)**6*c**2*d**2*x**2 + 16*tan(
(a + b*x)/2)**6*c*d**3*x**3 + 4*tan((a + b*x)/2)**6*d**4*x**4 + 6*tan((a +
 b*x)/2)**4*c**4 + 24*tan((a + b*x)/2)**4*c**3*d*x + 36*tan((a + b*x)/2)**
4*c**2*d**2*x**2 + 24*tan((a + b*x)/2)**4*c*d**3*x**3 + 6*tan((a + b*x)/2)
**4*d**4*x**4 + 4*tan((a + b*x)/2)**2*c**4 + 16*tan((a + b*x)/2)**2*c**3*d
*x + 24*tan((a + b*x)/2)**2*c**2*d**2*x**2 + 16*tan((a + b*x)/2)**2*c*d**3
*x**3 + 4*tan((a + b*x)/2)**2*d**4*x**4 + c**4 + 4*c**3*d*x + 6*c**2*d**2*
x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*c**3*d**4 + 60480*int(tan((a + b*x)/2
)**3/(tan((a + b*x)/2)**8*c**4 + 4*tan((a + b*x)/2)**8*c**3*d*x + 6*tan((a
 + b*x)/2)**8*c**2*d**2*x**2 + 4*tan((a + b*x)/2)**8*c*d**3*x**3 + tan(...
```

3.31 $\int (c+dx)^m \cot(a+bx)\,dx$

Optimal result . . . 400
Mathematica [N/A] . . . 400
Rubi [N/A] . . . 401
Maple [N/A] . . . 402
Fricas [N/A] . . . 402
Sympy [N/A] . . . 403
Maxima [N/A] . . . 403
Giac [N/A] . . . 403
Mupad [N/A] . . . 404
Reduce [N/A] . . . 404

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (c+dx)^m \cot(a+bx)\,dx = \text{Int}((c+dx)^m \cot(a+bx), x)$$

output
```
Defer(Int)((d*x+c)^m*cot(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 1.69 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \cot(a+bx)\,dx = \int (c+dx)^m \cot(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Cot[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Cot[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 25, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(a+bx)(c+dx)^m\,dx$$

$$\downarrow 3042$$

$$\int \tan\left(a+bx+\frac{\pi}{2}\right)(-(c+dx)^m)\,dx$$

$$\downarrow 25$$

$$-\int (c+dx)^m \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\downarrow 4222$$

$$-\int (c+dx)^m \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

input
```
Int[(c + d*x)^m*Cot[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \cot(bx+a)\,dx$$

input

```
int((d*x+c)^m*cot(b*x+a),x)
```

output

```
int((d*x+c)^m*cot(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \cot(a+bx)\,dx = \int (dx+c)^m \cot(bx+a)\,dx$$

input

```
integrate((d*x+c)^m*cot(b*x+a),x, algorithm="fricas")
```

output

```
integral((d*x + c)^m*cot(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \cot(a+bx)\,dx = \int (c+dx)^m \cot(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cot(b*x+a),x)
```

output
```
Integral((c + d*x)**m*cot(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \cot(a+bx)\,dx = \int (dx+c)^m \cot(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cot(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \cot(a+bx)\,dx = \int (dx+c)^m \cot(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cot(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 19.38 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \cot(a+bx)\,dx = \int \cot(a+b\,x)\,(c+d\,x)^m\,dx$$

input
```
int(cot(a + b*x)*(c + d*x)^m,x)
```

output
```
int(cot(a + b*x)*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \cot(a+bx)\,dx = \int (dx+c)^m \cot(bx+a)\,dx$$

input
```
int((d*x+c)^m*cot(b*x+a),x)
```

output
```
int((c + d*x)**m*cot(a + b*x),x)
```

3.32 $\int (c+dx)^4 \cot(a+bx)\,dx$

Optimal result . . . 405
Mathematica [B] (warning: unable to verify) . . . 406
Rubi [A] (verified) . . . 407
Maple [B] (verified) . . . 411
Fricas [B] (verification not implemented) . . . 412
Sympy [F] . . . 413
Maxima [B] (verification not implemented) . . . 414
Giac [F] . . . 415
Mupad [F(-1)] . . . 415
Reduce [F] . . . 415

Optimal result

Integrand size = 14, antiderivative size = 151

$$\begin{aligned}\int (c+dx)^4 \cot(a+bx)\,dx = &-\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4 \log\left(1-e^{2i(a+bx)}\right)}{b} \\ &- \frac{2id(c+dx)^3 \operatorname{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^2} \\ &+ \frac{3d^2(c+dx)^2 \operatorname{PolyLog}\left(3, e^{2i(a+bx)}\right)}{b^3} \\ &+ \frac{3id^3(c+dx) \operatorname{PolyLog}\left(4, e^{2i(a+bx)}\right)}{b^4} \\ &- \frac{3d^4 \operatorname{PolyLog}\left(5, e^{2i(a+bx)}\right)}{2b^5}\end{aligned}$$

output

```
-1/5*I*(d*x+c)^5/d+(d*x+c)^4*ln(1-exp(2*I*(b*x+a)))/b-2*I*d*(d*x+c)^3*poly
log(2,exp(2*I*(b*x+a)))/b^2+3*d^2*(d*x+c)^2*polylog(3,exp(2*I*(b*x+a)))/b^
3+3*I*d^3*(d*x+c)*polylog(4,exp(2*I*(b*x+a)))/b^4-3/2*d^4*polylog(5,exp(2*
I*(b*x+a)))/b^5
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 799 vs. $2(151) = 302$.

Time = 4.21 (sec) , antiderivative size = 799, normalized size of antiderivative = 5.29

$$\int (c+dx)^4 \cot(a+bx)\,dx = \text{Too large to display}$$

input

```
Integrate[(c + d*x)^4*Cot[a + b*x],x]
```

output

```
((2*I)*c^3*d*Pi*x)/b + (2*I)*c^2*d^2*x^3 + I*c*d^3*x^4 + (I/5)*d^4*x^5 - (
(4*I)*c^3*d*x*ArcTan[Tan[a]])/b + 2*c^3*d*x^2*Cot[a] + (2*c^3*d*Pi*Log[1 +
 E^((-2*I)*b*x)])/b^2 + (6*c^2*d^2*x^2*Log[1 - E^((-I)*(a + b*x))])/b + (4
*c*d^3*x^3*Log[1 - E^((-I)*(a + b*x))])/b + (d^4*x^4*Log[1 - E^((-I)*(a +
b*x))])/b + (6*c^2*d^2*x^2*Log[1 + E^((-I)*(a + b*x))])/b + (4*c*d^3*x^3*L
og[1 + E^((-I)*(a + b*x))])/b + (d^4*x^4*Log[1 + E^((-I)*(a + b*x))])/b +
(4*c^3*d*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))])/b + (4*c^3*d*ArcTan[
Tan[a]]*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))])/b^2 - (2*c^3*d*Pi*Log[C
os[b*x]])/b^2 + (c^4*Log[Sin[a + b*x]])/b - (4*c^3*d*ArcTan[Tan[a]]*Log[Si
n[b*x + ArcTan[Tan[a]]]])/b^2 + ((4*I)*d^2*x*(3*c^2 + 3*c*d*x + d^2*x^2)*P
olyLog[2, -E^((-I)*(a + b*x))])/b^2 + ((4*I)*d^2*x*(3*c^2 + 3*c*d*x + d^2*
x^2)*PolyLog[2, E^((-I)*(a + b*x))])/b^2 - ((2*I)*c^3*d*PolyLog[2, E^((2*I
)*(b*x + ArcTan[Tan[a]]))])/b^2 + (12*c^2*d^2*PolyLog[3, -E^((-I)*(a + b*x
))])/b^3 + (24*c*d^3*x*PolyLog[3, -E^((-I)*(a + b*x))])/b^3 + (12*d^4*x^2*
PolyLog[3, -E^((-I)*(a + b*x))])/b^3 + (12*c^2*d^2*PolyLog[3, E^((-I)*(a +
 b*x))])/b^3 + (24*c*d^3*x*PolyLog[3, E^((-I)*(a + b*x))])/b^3 + (12*d^4*x
^2*PolyLog[3, E^((-I)*(a + b*x))])/b^3 - ((24*I)*c*d^3*PolyLog[4, -E^((-I)
*(a + b*x))])/b^4 - ((24*I)*d^4*x*PolyLog[4, -E^((-I)*(a + b*x))])/b^4 - (
(24*I)*c*d^3*PolyLog[4, E^((-I)*(a + b*x))])/b^4 - ((24*I)*d^4*x*PolyLog[4
, E^((-I)*(a + b*x))])/b^4 - (24*d^4*PolyLog[5, -E^((-I)*(a + b*x))])/b...
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.42, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3042, 25, 4202, 2620, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \cot(a+bx)\,dx$$

$$\downarrow 3042$$

$$\int -(c+dx)^4 \tan\left(a+bx+\frac{\pi}{2}\right) dx$$

$$\downarrow 25$$

$$-\int (c+dx)^4 \tan\left(\frac{1}{2}(2a+\pi)+bx\right) dx$$

$$\downarrow 4202$$

$$2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)^4}{1+e^{i(2a+2bx+\pi)}}dx - \frac{i(c+dx)^5}{5d}$$

$$\downarrow 2620$$

$$2i\left(\frac{2id\int (c+dx)^3 \log\left(1+e^{i(2a+2bx+\pi)}\right) dx}{b} - \frac{i(c+dx)^4 \log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right) - \frac{i(c+dx)^5}{5d}$$

$$\downarrow 3011$$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3 \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b} - \frac{3id\int (c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b} - \frac{i(c+dx)^4 \log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$

$$\frac{i(c+dx)^5}{5d}$$

$$\downarrow 7163$$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\left(\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)dx}{b}-\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{2b}\right)}{b}-$$

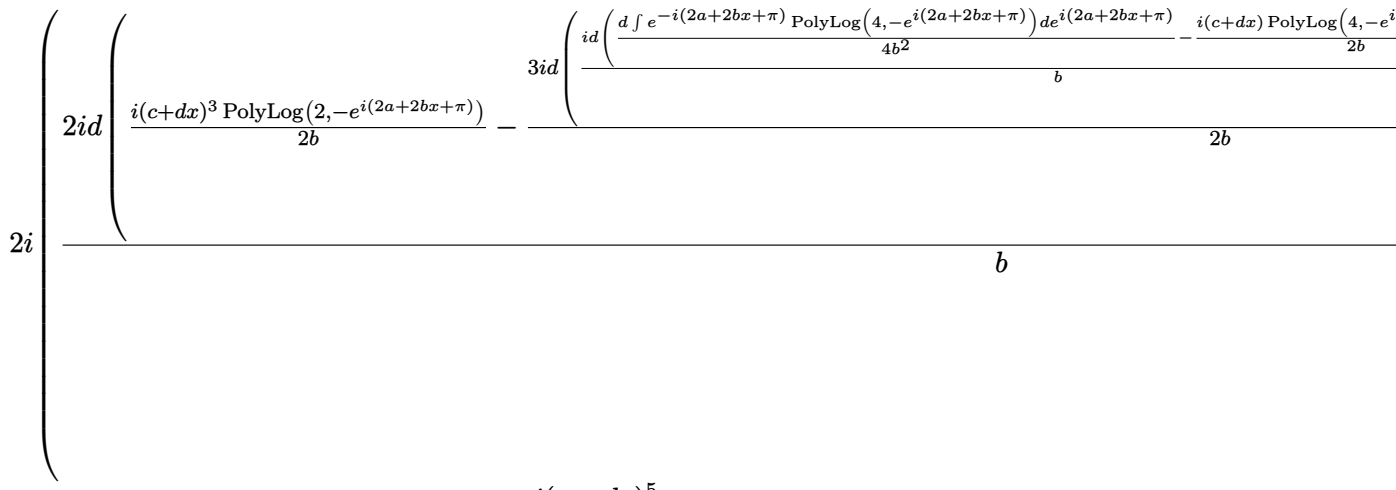
$$\frac{i(c+dx)^5}{5d}$$

$$\downarrow 7163$$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\left(\frac{id\left(\frac{id\int\mathrm{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}-\frac{i(c+dx)^2\,\mathrm{P}}{2b}\right)}{2b}\right)}{b}$$

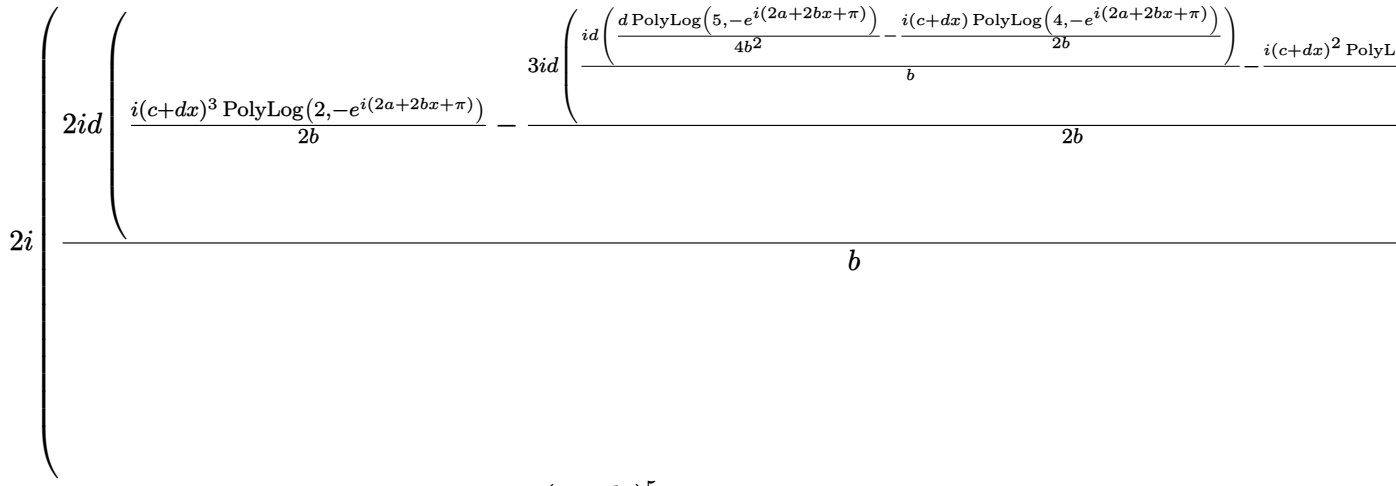
$$\frac{i(c+dx)^5}{5d}$$

$$\downarrow 2720$$



$$\frac{i(c+dx)^5}{5d}$$

$$\downarrow 7143$$



$$\frac{i(c+dx)^5}{5d}$$

input

```
Int[(c + d*x)^4*Cot[a + b*x],x]
```

output
```
((-1/5*I)*(c + d*x)^5)/d + (2*I)*(((-1/2*I)*(c + d*x)^4*Log[1 + E^(I*(2*a
+ Pi + 2*b*x))])/b + ((2*I)*d*(((I/2)*(c + d*x)^3*PolyLog[2, -E^(I*(2*a +
Pi + 2*b*x))])/b - (((3*I)/2)*d*(((-1/2*I)*(c + d*x)^2*PolyLog[3, -E^(I*(2
*a + Pi + 2*b*x))])/b + (I*d*(((-1/2*I)*(c + d*x)*PolyLog[4, -E^(I*(2*a +
Pi + 2*b*x))])/b + (d*PolyLog[5, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)))/b))
/b))/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1158 vs. $2(134) = 268$.

Time = 0.38 (sec) , antiderivative size = 1159, normalized size of antiderivative = 7.68

method	result	size
risch	Expression too large to display	1159

input

```
int((d*x+c)^4*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
-12*I/b^2*d^3*c*polylog(2,-exp(I*(b*x+a)))*x^2+4/b*d^3*c*ln(1-exp(I*(b*x+a
)))*x^3+24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x+8*I/b^3*d^2*c^2*a^3-4*I/b
^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3-4*I/b^2*d^4*polylog(2,exp(I*(b*x+a))
)*x^3+2*I/b^4*d^4*a^4*x+24/b^3*d^3*c*polylog(3,-exp(I*(b*x+a)))*x+24/b^3*d
^3*c*polylog(3,exp(I*(b*x+a)))*x+4/b^4*d^3*c*ln(1-exp(I*(b*x+a)))*a^3+4/b*
d^3*c*ln(exp(I*(b*x+a))+1)*x^3+8/b^2*c^3*d*a*ln(exp(I*(b*x+a)))-4/b^2*c^3*
d*a*ln(exp(I*(b*x+a))-1)-6/b^3*d^2*c^2*ln(1-exp(I*(b*x+a)))*a^2+8/b^4*c*d^
3*a^3*ln(exp(I*(b*x+a)))-4/b^4*c*d^3*a^3*ln(exp(I*(b*x+a))-1)-12/b^3*c^2*d
^2*a^2*ln(exp(I*(b*x+a)))+6/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a))-1)+4/b^2*d*c
^3*ln(1-exp(I*(b*x+a)))*a+6/b*d^2*c^2*ln(exp(I*(b*x+a))+1)*x^2+6/b*d^2*c^2
*ln(1-exp(I*(b*x+a)))*x^2+4/b*d*c^3*ln(exp(I*(b*x+a))+1)*x+4/b*d*c^3*ln(1-
exp(I*(b*x+a)))*x+24*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)))*x-2/b^5*d^4*a^4*
ln(exp(I*(b*x+a)))+1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)+12/b^3*d^2*c^2*polyl
og(3,-exp(I*(b*x+a)))+12/b^3*d^2*c^2*polylog(3,exp(I*(b*x+a)))+12/b^3*d^4*
polylog(3,-exp(I*(b*x+a)))*x^2-1/b^5*d^4*ln(1-exp(I*(b*x+a)))*a^4+12/b^3*d
^4*polylog(3,exp(I*(b*x+a)))*x^2+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4+1/b*d^4*
ln(exp(I*(b*x+a))+1)*x^4+8/5*I/b^5*d^4*a^5-I*d^3*c*x^4-2*I*d^2*c^2*x^3-2*I
*d*c^3*x^2-12*I/b^2*d^3*c*polylog(2,exp(I*(b*x+a)))*x^2+12*I/b^2*d^2*c^2*a
^2*x-8*I/b^3*d^3*c*a^3*x-12*I/b^2*d^2*c^2*polylog(2,-exp(I*(b*x+a)))*x-12*
I/b^2*d^2*c^2*polylog(2,exp(I*(b*x+a)))*x-8*I/b*d*c^3*a*x+24*I/b^4*d^3*...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 776 vs. $2(130) = 260$.

Time $= 0.10$ (sec) , antiderivative size $= 776$, normalized size of antiderivative $= 5.14$

$$\int (c + dx)^4 \cot(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^4*cot(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(3*d^4*polylog(5, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + 3*d^4*poly
log(5, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 4*(I*b^3*d^4*x^3 + 3*I*b^3
*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(cos(2*b*x + 2*a) + I*s
in(2*b*x + 2*a)) + 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2
*x - I*b^3*c^3*d)*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) - 2*(b^4*c^
4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-1/2*
cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) - 2*(b^4*c^4 - 4*a*b^3*c^
3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-1/2*cos(2*b*x + 2*
a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) - 2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*
b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^
3*b*c*d^3 - a^4*d^4)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1) - 2*(
b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^
3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-cos(2*b*x + 2*
a) - I*sin(2*b*x + 2*a) + 1) + 6*(-I*b*d^4*x - I*b*c*d^3)*polylog(4, cos(2
*b*x + 2*a) + I*sin(2*b*x + 2*a)) + 6*(I*b*d^4*x + I*b*c*d^3)*polylog(4, c
os(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b
^2*c^2*d^2)*polylog(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) - 6*(b^2*d^4
*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, cos(2*b*x + 2*a) - I*sin(2*
b*x + 2*a)))/b^5
```

Sympy [F]

$$\int (c+dx)^4 \cot(a+bx)\,dx = \int (c+dx)^4 \cot\left(a+bx\right)\,dx$$

input
```
integrate((d*x+c)**4*cot(b*x+a),x)
```

output
```
Integral((c + d*x)**4*cot(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1281 vs. $2(130) = 260$.

Time = 0.22 (sec) , antiderivative size = 1281, normalized size of antiderivative = 8.48

$$\int (c+dx)^4 \cot(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^4*cot(b*x+a),x, algorithm="maxima")
```

output

```
1/10*(10*c^4*log(sin(b*x + a)) - 40*a*c^3*d*log(sin(b*x + a))/b + 60*a^2*c
^2*d^2*log(sin(b*x + a))/b^2 - 40*a^3*c*d^3*log(sin(b*x + a))/b^3 + 10*a^4
*d^4*log(sin(b*x + a))/b^4 + (-2*I*(b*x + a)^5*d^4 - 10*(I*b*c*d^3 - I*a*d
^4)*(b*x + a)^4 - 240*d^4*polylog(5, -e^(I*b*x + I*a)) - 240*d^4*polylog(5
, e^(I*b*x + I*a)) - 20*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x +
 a)^3 - 20*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 - I*a^3*d^4)
*(b*x + a)^2 - 10*(-I*(b*x + a)^4*d^4 + 4*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)
^3 + 6*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a)^2 + 4*(-I*b^
3*c^3*d + 3*I*a*b^2*c^2*d^2 - 3*I*a^2*b*c*d^3 + I*a^3*d^4)*(b*x + a))*arct
an2(sin(b*x + a), cos(b*x + a) + 1) - 10*(I*(b*x + a)^4*d^4 + 4*(I*b*c*d^3
 - I*a*d^4)*(b*x + a)^3 + 6*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b
*x + a)^2 + 4*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 - I*a^3*d
^4)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 40*(I*b^3*c^3*d
- 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 + I*(b*x + a)^3*d^4 - I*a^3*d^4 + 3*
(I*b*c*d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a
^2*d^4)*(b*x + a))*dilog(-e^(I*b*x + I*a)) - 40*(I*b^3*c^3*d - 3*I*a*b^2*c
^2*d^2 + 3*I*a^2*b*c*d^3 + I*(b*x + a)^3*d^4 - I*a^3*d^4 + 3*(I*b*c*d^3 -
I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x
+ a))*dilog(e^(I*b*x + I*a)) + 5*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b
*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^...
```

Giac [F]

$$\int (c+dx)^4 \cot(a+bx)\,dx = \int (dx+c)^4 \cot(bx+a)\,dx$$

input
```
integrate((d*x+c)^4*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*cot(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \cot(a+bx)\,dx = \int \cot(a+b\,x)\,(c+d\,x)^4\,dx$$

input
```
int(cot(a + b*x)*(c + d*x)^4,x)
```

output
```
int(cot(a + b*x)*(c + d*x)^4, x)
```

Reduce [F]

$$\int (c+dx)^4 \cot(a+bx)\,dx$$
$$= \frac{\left(\int \cot(bx+a)\,x^4dx\right) b\,d^4 + 4\left(\int \cot(bx+a)\,x^3dx\right) bc\,d^3 + 6\left(\int \cot(bx+a)\,x^2dx\right) b\,c^2d^2 + 4\left(\int \cot(bx+\right.}{b}$$

input
```
int((d*x+c)^4*cot(b*x+a),x)
```

output
```
(int(cot(a + b*x)*x**4,x)*b*d**4 + 4*int(cot(a + b*x)*x**3,x)*b*c*d**3 + 6
*int(cot(a + b*x)*x**2,x)*b*c**2*d**2 + 4*int(cot(a + b*x)*x,x)*b*c**3*d -
 log(tan((a + b*x)/2)**2 + 1)*c**4 + log(tan((a + b*x)/2))*c**4)/b
```

3.33 $\int (c+dx)^3 \cot(a+bx)\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 127

$$\begin{aligned}\int (c+dx)^3 \cot(a+bx)\,dx = &-\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3 \log\left(1-e^{2i(a+bx)}\right)}{b} \\ &- \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2} \\ &+ \frac{3d^2(c+dx) \operatorname{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3} \\ &+ \frac{3id^3 \operatorname{PolyLog}\left(4, e^{2i(a+bx)}\right)}{4b^4}\end{aligned}$$

output

```
-1/4*I*(d*x+c)^4/d+(d*x+c)^3*ln(1-exp(2*I*(b*x+a)))/b-3/2*I*d*(d*x+c)^2*po
lylog(2,exp(2*I*(b*x+a)))/b^2+3/2*d^2*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/
b^3+3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 560 vs. $2(127) = 254$.

Time = 1.77 (sec) , antiderivative size = 560, normalized size of antiderivative = 4.41

$$\int (c+dx)^3 \cot(a+bx)\,dx$$

$$= \frac{6ib^3c^2d\pi x + 4ib^4cd^2x^3 + ib^4d^3x^4 - 12ib^3c^2dx \arctan(\tan(a)) + 6b^4c^2dx^2 \cot(a) + 6b^2c^2d\pi \log\left(1+e^{-2ibx}\right.}{}$$

input
```
Integrate[(c + d*x)^3*Cot[a + b*x],x]
```

output
```
((6*I)*b^3*c^2*d*Pi*x + (4*I)*b^4*c*d^2*x^3 + I*b^4*d^3*x^4 - (12*I)*b^3*c
^2*d*x*ArcTan[Tan[a]] + 6*b^4*c^2*d*x^2*Cot[a] + 6*b^2*c^2*d*Pi*Log[1 + E^
((-2*I)*b*x)] + 12*b^3*c*d^2*x^2*Log[1 - E^((-I)*(a + b*x))] + 4*b^3*d^3*x
^3*Log[1 - E^((-I)*(a + b*x))] + 12*b^3*c*d^2*x^2*Log[1 + E^((-I)*(a + b*x
))] + 4*b^3*d^3*x^3*Log[1 + E^((-I)*(a + b*x))] + 12*b^3*c^2*d*x*Log[1 - E
^((2*I)*(b*x + ArcTan[Tan[a]]))] + 12*b^2*c^2*d*ArcTan[Tan[a]]*Log[1 - E^(
(2*I)*(b*x + ArcTan[Tan[a]]))] - 6*b^2*c^2*d*Pi*Log[Cos[b*x]] + 4*b^3*c^3*
Log[Sin[a + b*x]] - 12*b^2*c^2*d*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a
]]]] + (12*I)*b^2*d^2*x*(2*c + d*x)*PolyLog[2, -E^((-I)*(a + b*x))] + (12*
I)*b^2*d^2*x*(2*c + d*x)*PolyLog[2, E^((-I)*(a + b*x))] - (6*I)*b^2*c^2*d*
PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))] + 24*b*c*d^2*PolyLog[3, -E^((
-I)*(a + b*x))] + 24*b*d^3*x*PolyLog[3, -E^((-I)*(a + b*x))] + 24*b*c*d^2*
PolyLog[3, E^((-I)*(a + b*x))] + 24*b*d^3*x*PolyLog[3, E^((-I)*(a + b*x))]
 - (24*I)*d^3*PolyLog[4, -E^((-I)*(a + b*x))] - (24*I)*d^3*PolyLog[4, E^((
-I)*(a + b*x))] - 6*b^4*c^2*d*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^
2])/(4*b^4)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.35, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 25, 4202, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \cot(a+bx)\,dx$$

$$\downarrow 3042$$

$$\int -(c+dx)^3 \tan\left(a+bx+\frac{\pi}{2}\right) dx$$

$$\downarrow 25$$

$$-\int (c+dx)^3 \tan\left(\frac{1}{2}(2a+\pi)+bx\right) dx$$

$$\downarrow 4202$$

$$2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)^3}{1+e^{i(2a+2bx+\pi)}}dx - \frac{i(c+dx)^4}{4d}$$

$$\downarrow 2620$$

$$2i\left(\frac{3id\int (c+dx)^2 \log\left(1+e^{i(2a+2bx+\pi)}\right) dx}{2b} - \frac{i(c+dx)^3 \log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right) - \frac{i(c+dx)^4}{4d}$$

$$\downarrow 3011$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b} - \frac{id\int (c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b} - \frac{i(c+dx)^3 \log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$

$$\frac{i(c+dx)^4}{4d}$$

$$\downarrow 7163$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b} - \frac{id\left(\frac{id\int \operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)dx}{2b} - \frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}\right)}{2b} - \frac{i(c+d}{}\right.$$

$$\left.\frac{i(c+dx)^4}{4d}\right)$$

$$\downarrow 2720$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\left(\frac{d\int e^{-i(2a+2bx+\pi)}\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(2a+2}\right.\right.\right.}{b}\right.}{2b}$$

$$\frac{i(c+dx)^4}{4d}$$

$$\downarrow 7143$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\left(\frac{d\,\mathrm{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(c+dx)^3}{}\right.$$

$$\frac{i(c+dx)^4}{4d}$$

input
```
Int[(c + d*x)^3*Cot[a + b*x],x]
```

output
```
((-1/4*I)*(c + d*x)^4)/d + (2*I)*(((-1/2*I)*(c + d*x)^3*Log[1 + E^(I*(2*a
+ Pi + 2*b*x))])/b + (((3*I)/2)*d*(((I/2)*(c + d*x)^2*PolyLog[2, -E^(I*(2*
a + Pi + 2*b*x))])/b - (I*d*(((-1/2*I)*(c + d*x)*PolyLog[3, -E^(I*(2*a + P
i + 2*b*x))])/b + (d*PolyLog[4, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)))/b))/
b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(108) = 216$.

Time = 0.33 (sec) , antiderivative size = 792, normalized size of antiderivative = 6.24

method	result
risch	$\frac{6d^3\,\mathrm{polylog}\left(3,-e^{i(bx+a)}\right)x}{b^3}+\frac{2d^3a^3\ln\left(e^{i(bx+a)}\right)}{b^4}-\frac{d^3a^3\ln\left(e^{i(bx+a)}-1\right)}{b^4}+\frac{6id^3\,\mathrm{polylog}\left(4,-e^{i(bx+a)}\right)}{b^4}-\frac{3id^3a^4}{2b^4}+\frac{6id^3\,\mathrm{po}}{}$

input

```
int((d*x+c)^3*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
-2/b*c^3*ln(exp(I*(b*x+a)))+1/b*c^3*ln(exp(I*(b*x+a))-1)+1/b*c^3*ln(exp(I*
(b*x+a))+1)-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x-6*I/b*d*c^2*a*x-3/b^
3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2-6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a)))-3*I/b^
2*d*c^2*a^2-2*I/b^3*d^3*a^3*x+4*I/b^3*c*d^2*a^3-3*I/b^2*d^3*polylog(2,exp(
I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-3*I/b^2*d*c^2*p
olylog(2,-exp(I*(b*x+a)))-3*I/b^2*d*c^2*polylog(2,exp(I*(b*x+a)))-6*I/b^2*
c*d^2*polylog(2,-exp(I*(b*x+a)))*x+6*I/b^2*c*d^2*a^2*x-I*d^2*c*x^3-3/2*I*d
*c^2*x^2-1/4*I*d^3*x^4+1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+1/b*d^3*ln(1-exp(I
*(b*x+a)))*x^3+6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(
3,exp(I*(b*x+a)))+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+6/b^3*d^3*polylog(3,e
xp(I*(b*x+a)))*x+6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+2/b^4*d^3*a^3*ln(e
xp(I*(b*x+a)))-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+6*I/b^4*d^3*polylog(4,-e
xp(I*(b*x+a)))-3/2*I/b^4*d^3*a^4+3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)+6/b^
2*c^2*d*a*ln(exp(I*(b*x+a)))-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)+3/b^2*d*c^
2*ln(1-exp(I*(b*x+a)))*a+3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b*c*d^2*ln(1
-exp(I*(b*x+a)))*x^2+3/b*d*c^2*ln(exp(I*(b*x+a))+1)*x+3/b*d*c^2*ln(1-exp(I
*(b*x+a)))*x+I*c^3*x+1/4*I/d*c^4+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(104) = 208$.

Time $= 0.09$ (sec) , antiderivative size $= 548$, normalized size of antiderivative $= 4.31$

$$\int (c+dx)^3 \cot(a+bx)\,dx$$

$$= \frac{3i\,d^3\text{polylog}(4, \cos(2\,bx+2\,a)+i\,\sin(2\,bx+2\,a)) - 3i\,d^3\text{polylog}(4, \cos(2\,bx+2\,a)-i\,\sin(2\,bx+2\,a))}{}$$

input
```
integrate((d*x+c)^3*cot(b*x+a),x, algorithm="fricas")
```

output
```
1/8*(3*I*d^3*polylog(4, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) - 3*I*d^3*p
olylog(4, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) - 6*(I*b^2*d^3*x^2 + 2*I*
b^2*c*d^2*x + I*b^2*c^2*d)*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) -
6*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(cos(2*b*x + 2*a)
- I*sin(2*b*x + 2*a)) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 4*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(2*b*x + 2*a) - 1/
2*I*sin(2*b*x + 2*a) + 1/2) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2
*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(2*b*x + 2*a) + I*
sin(2*b*x + 2*a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x +
 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(2*b*x + 2*a) - I*sin(2*
b*x + 2*a) + 1) + 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(2*b*x + 2*a) + I*si
n(2*b*x + 2*a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(2*b*x + 2*a) - I*si
n(2*b*x + 2*a)))/b^4
```

Sympy [F]

$$\int (c+dx)^3 \cot(a+bx)\,dx = \int (c+dx)^3 \cot(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*cot(b*x+a),x)
```

output
```
Integral((c + d*x)**3*cot(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 759 vs. $2(104) = 208$.

Time = 0.17 (sec) , antiderivative size = 759, normalized size of antiderivative = 5.98

$$\int (c+dx)^3 \cot(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*cot(b*x+a),x, algorithm="maxima")
```

output

```
1/4*(4*c^3*log(sin(b*x + a)) - 12*a*c^2*d*log(sin(b*x + a))/b + 12*a^2*c*d
^2*log(sin(b*x + a))/b^2 - 4*a^3*d^3*log(sin(b*x + a))/b^3 + (-I*(b*x + a)
^4*d^3 - 4*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^3 + 24*I*d^3*polylog(4, -e^(I*b
*x + I*a)) + 24*I*d^3*polylog(4, e^(I*b*x + I*a)) - 6*(I*b^2*c^2*d - 2*I*a
*b*c*d^2 + I*a^2*d^3)*(b*x + a)^2 - 4*(-I*(b*x + a)^3*d^3 + 3*(-I*b*c*d^2
+ I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*(b*x
 + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 4*(I*(b*x + a)^3*d^3 + 3*
(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2
*d^3)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 12*(I*b^2*c^2*
d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3
)*(b*x + a))*dilog(-e^(I*b*x + I*a)) - 12*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I
*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*dilog(e^
(I*b*x + I*a)) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*
(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*
x + a)^2 + 2*cos(b*x + a) + 1) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*
(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x
 + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 24*(b*c*d^2 + (b*x + a)*d
^3 - a*d^3)*polylog(3, -e^(I*b*x + I*a)) + 24*(b*c*d^2 + (b*x + a)*d^3 - a
*d^3)*polylog(3, e^(I*b*x + I*a)))/b^3)/b
```

Giac [F]

$$\int (c+dx)^3 \cot(a+bx)\,dx = \int (dx+c)^3 \cot(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*cot(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \cot(a+bx)\,dx = \int \cot(a+b\,x)\,(c+d\,x)^3\,dx$$

input
```
int(cot(a + b*x)*(c + d*x)^3,x)
```

output
```
int(cot(a + b*x)*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \cot(a+bx)\,dx$$
$$= \frac{\left(\int \cot(bx+a)\,x^3dx\right) b\,d^3 + 3\left(\int \cot(bx+a)\,x^2dx\right) bc\,d^2 + 3\left(\int \cot(bx+a)\,xdx\right) b\,c^2d - \log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}{b}$$

input
```
int((d*x+c)^3*cot(b*x+a),x)
```

output
```
(int(cot(a + b*x)*x**3,x)*b*d**3 + 3*int(cot(a + b*x)*x**2,x)*b*c*d**2 + 3
*int(cot(a + b*x)*x,x)*b*c**2*d - log(tan((a + b*x)/2)**2 + 1)*c**3 + log(
tan((a + b*x)/2))*c**3)/b
```

3.34 $\int (c+dx)^2 \cot(a+bx)\,dx$

Optimal result . . . 425
Mathematica [B] (warning: unable to verify) . . . 425
Rubi [A] (verified) . . . 426
Maple [B] (verified) . . . 429
Fricas [B] (verification not implemented) . . . 429
Sympy [F] . . . 430
Maxima [B] (verification not implemented) . . . 430
Giac [F] . . . 431
Mupad [F(-1)] . . . 431
Reduce [F] . . . 432

Optimal result

Integrand size = 14, antiderivative size = 93

$$\int (c+dx)^2 \cot(a+bx)\,dx = -\frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2 \log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id(c+dx)\,\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2} + \frac{d^2\,\mathrm{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}$$

output

```
-1/3*I*(d*x+c)^3/d+(d*x+c)^2*ln(1-exp(2*I*(b*x+a)))/b-I*d*(d*x+c)*polylog(
2,exp(2*I*(b*x+a)))/b^2+1/2*d^2*polylog(3,exp(2*I*(b*x+a)))/b^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 356 vs. 2(93) = 186.

Time = 0.98 (sec) , antiderivative size = 356, normalized size of antiderivative = 3.83

$$\int (c+dx)^2 \cot(a+bx)\,dx$$

$$= \frac{3ib^2cd\pi x + ib^3d^2x^3 - 6ib^2cdx\arctan(\tan(a)) + 3b^3cdx^2\cot(a) + 3bcd\pi\log\left(1+e^{-2ibx}\right) + 3b^2d^2x^2\log\left(1\right.}{}$$

input
```
Integrate[(c + d*x)^2*Cot[a + b*x],x]
```

output
```
((3*I)*b^2*c*d*Pi*x + I*b^3*d^2*x^3 - (6*I)*b^2*c*d*x*ArcTan[Tan[a]] + 3*b
^3*c*d*x^2*Cot[a] + 3*b*c*d*Pi*Log[1 + E^((-2*I)*b*x)] + 3*b^2*d^2*x^2*Log
[1 - E^((-I)*(a + b*x))] + 3*b^2*d^2*x^2*Log[1 + E^((-I)*(a + b*x))] + 6*b
^2*c*d*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + 6*b*c*d*ArcTan[Tan[a]
]*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] - 3*b*c*d*Pi*Log[Cos[b*x]] + 3
*b^2*c^2*Log[Sin[a + b*x]] - 6*b*c*d*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[T
an[a]]]] + (6*I)*b*d^2*x*PolyLog[2, -E^((-I)*(a + b*x))] + (6*I)*b*d^2*x*P
olyLog[2, E^((-I)*(a + b*x))] - (3*I)*b*c*d*PolyLog[2, E^((2*I)*(b*x + Arc
Tan[Tan[a]]))] + 6*d^2*PolyLog[3, -E^((-I)*(a + b*x))] + 6*d^2*PolyLog[3,
E^((-I)*(a + b*x))] - 3*b^3*c*d*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a
]^2])/(3*b^3)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.34, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 25, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \cot(a+bx)\,dx$$

$$\downarrow 3042$$

$$\int -(c+dx)^2 \tan\left(a+bx+\frac{\pi}{2}\right) dx$$

$$\downarrow 25$$

$$-\int (c+dx)^2 \tan\left(\frac{1}{2}(2a+\pi)+bx\right) dx$$

$$\downarrow 4202$$

$$2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)^2}{1+e^{i(2a+2bx+\pi)}}dx - \frac{i(c+dx)^3}{3d}$$

$$\downarrow 2620$$

$$2i\left(\frac{id\int(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{i(c+dx)^3}{3d}$$

$$\downarrow 3011$$

$$2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{i(c+dx)^3}{3d}$$

$$\downarrow 2720$$

$$2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i}\right.}{2b}\right)-\frac{i(c+dx)^3}{3d}$$

$$\downarrow 7143$$

$$2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{i(c+dx)^3}{3d}$$

input
```
Int[(c + d*x)^2*Cot[a + b*x],x]
```

output
```
((-1/3*I)*(c + d*x)^3)/d + (2*I)*(((-1/2*I)*(c + d*x)^2*Log[1 + E^(I*(2*a
+ Pi + 2*b*x))])/b + (I*d*(((I/2)*(c + d*x)*PolyLog[2, -E^(I*(2*a + Pi + 2
*b*x))])/b - (d*PolyLog[3, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)))/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(81) = 162$.

Time = 0.33 (sec) , antiderivative size = 477, normalized size of antiderivative = 5.13

method	result
risch	$\frac{2dc\ln(1-e^{i(bx+a)})x}{b}+\frac{2dc\ln(1-e^{i(bx+a)})a}{b^2}-\frac{4idcax}{b}+\frac{2dc\ln(e^{i(bx+a)}+1)x}{b}-\frac{2id^2\,\mathrm{polylog}(2,e^{i(bx+a)})x}{b^2}+\frac{d^2\ln(e^{i(bx+}$

input
```
int((d*x+c)^2*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I*d^2*x^3+2/b*d*c*ln(exp(I*(b*x+a))+1)*x+2/b*d*c*ln(1-exp(I*(b*x+a)))
*x+2/b^2*d*c*ln(1-exp(I*(b*x+a)))*a+4/b^2*c*d*a*ln(exp(I*(b*x+a)))-2/b^2*c
*d*a*ln(exp(I*(b*x+a))-1)-2*I/b^2*d*c*a^2-2*I/b^2*d*c*polylog(2,-exp(I*(b*
x+a)))-2*I/b^2*d*c*polylog(2,exp(I*(b*x+a)))+2*I/b^2*d^2*a^2*x-2*I/b^2*d^2
*polylog(2,-exp(I*(b*x+a)))*x-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-4*I/
b*d*c*a*x+1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^
2-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2-2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))+1/b^
3*d^2*a^2*ln(exp(I*(b*x+a))-1)+4/3*I/b^3*d^2*a^3-I*d*c*x^2+I*c^2*x-2/b*c^2
*ln(exp(I*(b*x+a)))+1/b*c^2*ln(exp(I*(b*x+a))-1)+1/b*c^2*ln(exp(I*(b*x+a))
+1)+1/3*I/d*c^3+2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I
*(b*x+a)))/b^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(78) = 156$.

Time = 0.09 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.76

$$\int (c+dx)^2\cot(a+bx)\,dx$$
$$=\frac{d^2\mathrm{polylog}(3,\cos(2\,bx+2\,a)+i\,\sin(2\,bx+2\,a))+d^2\mathrm{polylog}(3,\cos(2\,bx+2\,a)-i\,\sin(2\,bx+2\,a))-2}{}$$

input
```
integrate((d*x+c)^2*cot(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(d^2*polylog(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + d^2*polylog(3
, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(c
os(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog(cos
(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2
) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(2*b*x + 2
*a) + I*sin(2*b*x + 2*a) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d -
 a^2*d^2)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1))/b^3
```

Sympy [F]

$$\int (c+dx)^2 \cot(a+bx)\,dx = \int (c+dx)^2 \cot(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*cot(b*x+a),x)
```

output
```
Integral((c + d*x)**2*cot(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(78) = 156$.

Time $= 0.13$ (sec) , antiderivative size $= 411$, normalized size of antiderivative $= 4.42$

$$\int (c+dx)^2 \cot(a+bx)\,dx$$

$$= \frac{6\,c^2 \log\left(\sin\left(bx+a\right)\right) - \frac{12\,acd\log(\sin(bx+a))}{b} + \frac{6\,a^2d^2\log(\sin(bx+a))}{b^2} + \frac{-2i\,(bx+a)^3d^2 - 6\,(i\,bcd - i\,ad^2)(bx+a)^2 + 12\,d^2\mathrm{Li}_3(-e^{(i\,b}}{}$$

input
```
integrate((d*x+c)^2*cot(b*x+a),x, algorithm="maxima")
```

output
```
1/6*(6*c^2*log(sin(b*x + a)) - 12*a*c*d*log(sin(b*x + a))/b + 6*a^2*d^2*lo
g(sin(b*x + a))/b^2 + (-2*I*(b*x + a)^3*d^2 - 6*(I*b*c*d - I*a*d^2)*(b*x +
 a)^2 + 12*d^2*polylog(3, -e^(I*b*x + I*a)) + 12*d^2*polylog(3, e^(I*b*x +
 I*a)) - 6*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan2
(sin(b*x + a), cos(b*x + a) + 1) - 6*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a
*d^2)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 12*(I*b*c*d +
I*(b*x + a)*d^2 - I*a*d^2)*dilog(-e^(I*b*x + I*a)) - 12*(I*b*c*d + I*(b*x
+ a)*d^2 - I*a*d^2)*dilog(e^(I*b*x + I*a)) + 3*((b*x + a)^2*d^2 + 2*(b*c*d
 - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a)
+ 1) + 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^
2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1))/b^2)/b
```

Giac [F]

$$\int (c+dx)^2 \cot(a+bx)\,dx = \int (dx+c)^2 \cot(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*cot(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \cot(a+bx)\,dx = \int \cot(a+b\,x)\,(c+d\,x)^2\,dx$$

input
```
int(cot(a + b*x)*(c + d*x)^2,x)
```

output
```
int(cot(a + b*x)*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \cot(a+bx)\,dx$$

$$= \frac{\left(\int \cot(bx+a)\,x^2 dx\right) b\,d^2 + 2\left(\int \cot(bx+a)\,x dx\right) bcd - \log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right) c^2 + \log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}$$

input
```
int((d*x+c)^2*cot(b*x+a),x)
```

output
```
(int(cot(a + b*x)*x**2,x)*b*d**2 + 2*int(cot(a + b*x)*x,x)*b*c*d - log(tan
((a + b*x)/2)**2 + 1)*c**2 + log(tan((a + b*x)/2))*c**2)/b
```

3.35 $\int (c+dx)\cot(a+bx)\,dx$

Optimal result . . . 433
Mathematica [B] (warning: unable to verify) . . . 433
Rubi [A] (verified) . . . 434
Maple [B] (verified) . . . 436
Fricas [B] (verification not implemented) . . . 437
Sympy [F] . . . 437
Maxima [B] (verification not implemented) . . . 438
Giac [F] . . . 438
Mupad [F(-1)] . . . 439
Reduce [F] . . . 439

Optimal result

Integrand size = 12, antiderivative size = 65

$$\int (c+dx)\cot(a+bx)\,dx = -\frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id\,\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}$$

output

```
-1/2*I*(d*x+c)^2/d+(d*x+c)*ln(1-exp(2*I*(b*x+a)))/b-1/2*I*d*polylog(2,exp(
2*I*(b*x+a)))/b^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 180 vs. 2(65) = 130.

Time = 3.50 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.77

$$\int (c+dx)\cot(a+bx)\,dx = \frac{1}{2}dx^2\cot(a) + \frac{c\log(\sin(a+bx))}{b} - \frac{d\csc(a)\sec(a)\left(b^2e^{i\arctan(\tan(a))}x^2 + \frac{(ibx(-\pi+2\arctan(\tan(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx+\arctan(\tan(a)))\log\left(1-e^{2i(bx+\arcta}}{2b^2\sqrt{\sec^2(a)\left(\cos^2(a)+\right.}}$$

input
```
Integrate[(c + d*x)*Cot[a + b*x],x]
```

output
```
(d*x^2*Cot[a])/2 + (c*Log[Sin[a + b*x]])/b - (d*Csc[a]*Sec[a]*(b^2*E^(I*Ar
cTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I
)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]])
)] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] +
I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]
))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Rubi [A] (verified)

Time $= 0.36$ (sec) , antiderivative size $= 81$, normalized size of antiderivative $= 1.25$, number of steps used $= 7$, number of rules used $= 6$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used $= \{3042, 25, 4202, 2620, 2715, 2838\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cot(a+bx)\,dx$$

$$\downarrow 3042$$

$$\int -\left((c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)\right)dx$$

$$\downarrow 25$$

$$-\int (c+dx)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\downarrow 4202$$

$$2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)}{1+e^{i(2a+2bx+\pi)}}dx - \frac{i(c+dx)^2}{2d}$$

$$\downarrow 2620$$

$$2i\left(\frac{id\int \log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{2b} - \frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right) - \frac{i(c+dx)^2}{2d}$$

$$\downarrow 2715$$

$$2i\left(\frac{d\int e^{-i(2a+2bx+\pi)}\log\left(1+e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{i(c+dx)^2}{2d}$$

$$\downarrow 2838$$

$$2i\left(-\frac{d\,\text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{i(c+dx)^2}{2d}$$

input
```
Int[(c + d*x)*Cot[a + b*x],x]
```

output
```
((-1/2*I)*(c + d*x)^2)/d + (2*I)*(((-1/2*I)*(c + d*x)*Log[1 + E^(I*(2*a +
Pi + 2*b*x))])/b - (d*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(55) = 110$.

Time = 0.31 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.14

method	result
parts	$-\frac{\ln\left(\cot(bx+a)^2+1\right)dx}{2b} - \frac{\ln\left(\cot(bx+a)^2+1\right)c}{2b} + \frac{d\left(\frac{i\left(\ln(\cot(bx+a)-i)\ln\left(\cot(bx+a)^2+1\right)-\mathrm{dilog}\left(-\frac{i(\cot(bx+a)+i)}{2}\right)-\ln(\cot(bx+a)\right.}{2}$
risch	$-\frac{idx^2}{2} + ixc - \frac{2c\ln\left(e^{i(bx+a)}\right)}{b} + \frac{c\ln\left(e^{i(bx+a)}-1\right)}{b} + \frac{c\ln\left(e^{i(bx+a)}+1\right)}{b} - \frac{2idax}{b} - \frac{ida^2}{b^2} + \frac{d\ln\left(e^{i(bx+a)}+1\right)x}{b} - \frac{id\,p}{}$

input

```
int((d*x+c)*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
-1/2/b*ln(cot(b*x+a)^2+1)*d*x-1/2/b*ln(cot(b*x+a)^2+1)*c+1/2*d/b^2*(1/2*I*
(ln(cot(b*x+a)-I)*ln(cot(b*x+a)^2+1)-dilog(-1/2*I*(cot(b*x+a)+I))-ln(cot(b
*x+a)-I)*ln(-1/2*I*(cot(b*x+a)+I))-1/2*ln(cot(b*x+a)-I)^2)-1/2*I*(ln(cot(b
*x+a)+I)*ln(cot(b*x+a)^2+1)-dilog(1/2*I*(cot(b*x+a)-I))-ln(cot(b*x+a)+I)*l
n(1/2*I*(cot(b*x+a)-I))-1/2*ln(cot(b*x+a)+I)^2))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(52) = 104$.

Time = 0.08 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.02

$$\int (c+dx)\cot(a+bx)\,dx$$
$$= \frac{-i\,d\mathrm{Li}_2(\cos(2\,bx+2\,a)+i\,\sin(2\,bx+2\,a))+i\,d\mathrm{Li}_2(\cos(2\,bx+2\,a)-i\,\sin(2\,bx+2\,a))+2\,(bc-ad)\,\mathrm{lo}}{}$$

input
```
integrate((d*x+c)*cot(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(-I*d*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + I*d*dilog(cos(2*b
*x + 2*a) - I*sin(2*b*x + 2*a)) + 2*(b*c - a*d)*log(-1/2*cos(2*b*x + 2*a)
+ 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b*c - a*d)*log(-1/2*cos(2*b*x + 2*a)
- 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b*d*x + a*d)*log(-cos(2*b*x + 2*a) +
I*sin(2*b*x + 2*a) + 1) + 2*(b*d*x + a*d)*log(-cos(2*b*x + 2*a) - I*sin(2*
b*x + 2*a) + 1))/b^2
```

Sympy [F]

$$\int (c+dx)\cot(a+bx)\,dx = \int (c+dx)\cot(a+bx)\,dx$$

input
```
integrate((d*x+c)*cot(b*x+a),x)
```

output
```
Integral((c + d*x)*cot(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(52) = 104$.

Time = 0.12 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.92

$$\int (c+dx)\cot(a+bx)\,dx$$
$$= \frac{-i\,b^2dx^2 - 2i\,b^2cx - 2i\,bdx\arctan\left(\sin\left(bx+a\right), -\cos\left(bx+a\right)+1\right) + 2i\,bc\arctan\left(\sin\left(bx+a\right), \cos\left(bx\right.\right.}{}$$

input
```
integrate((d*x+c)*cot(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(-I*b^2*d*x^2 - 2*I*b^2*c*x - 2*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x
 + a) + 1) + 2*I*b*c*arctan2(sin(b*x + a), cos(b*x + a) - 1) - 2*(-I*b*d*x
 - I*b*c)*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*I*d*dilog(-e^(I*b*x
+ I*a)) - 2*I*d*dilog(e^(I*b*x + I*a)) + (b*d*x + b*c)*log(cos(b*x + a)^2
+ sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (b*d*x + b*c)*log(cos(b*x + a)^2
+ sin(b*x + a)^2 - 2*cos(b*x + a) + 1))/b^2
```

Giac [F]

$$\int (c+dx)\cot(a+bx)\,dx = \int (dx+c)\cot\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*cot(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\cot(a+bx)\,dx = \int \cot(a+b\,x)\ (c+d\,x)\ dx$$

input
```
int(cot(a + b*x)*(c + d*x),x)
```

output
```
int(cot(a + b*x)*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)\cot(a+bx)\,dx$$
$$= \frac{\left(\int \cot(bx+a)\,x dx\right) bd - \log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right) c + \log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right) c}{b}$$

input
```
int((d*x+c)*cot(b*x+a),x)
```

output
```
(int(cot(a + b*x)*x,x)*b*d - log(tan((a + b*x)/2)**2 + 1)*c + log(tan((a +
 b*x)/2))*c)/b
```

3.36 $\int \frac{\cot(a+bx)}{c+dx}\,dx$

Optimal result 440
Mathematica [N/A] 440
Rubi [N/A] 441
Maple [N/A] 442
Fricas [N/A] 442
Sympy [N/A] 443
Maxima [N/A] 443
Giac [N/A] 443
Mupad [N/A] 444
Reduce [N/A] 444

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\cot(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\cot(a+bx)}{c+dx}, x\right)$$

output
```
Defer(Int)(cot(b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 2.94 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\cot(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+bx)}{c+dx}\,dx$$

input
```
Integrate[Cot[a + b*x]/(c + d*x),x]
```

output
```
Integrate[Cot[a + b*x]/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 25, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(a+bx)}{c+dx}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{\tan\left(a+bx+\frac{\pi}{2}\right)}{c+dx}\,dx$$
$$\downarrow 25$$
$$-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}\,dx$$
$$\downarrow 4222$$
$$-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}\,dx$$

input
```
Int[Cot[a + b*x]/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\cot(bx+a)}{dx+c}dx$$

input

```
int(cot(b*x+a)/(d*x+c),x)
```

output

```
int(cot(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\cot(a+bx)}{c+dx}\,dx = \int \frac{\cot(bx+a)}{dx+c}\,dx$$

input

```
integrate(cot(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output

```
integral(cot(b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\cot(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+bx)}{c+dx}\,dx$$

input
```
integrate(cot(b*x+a)/(d*x+c),x)
```

output
```
Integral(cot(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\cot(a+bx)}{c+dx}\,dx = \int \frac{\cot(bx+a)}{dx+c}\,dx$$

input
```
integrate(cot(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(cot(b*x + a)/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\cot(a+bx)}{c+dx}\,dx = \int \frac{\cot(bx+a)}{dx+c}\,dx$$

input
```
integrate(cot(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(cot(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 19.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\cot(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+b\,x)}{c+d\,x}\,dx$$

input
```
int(cot(a + b*x)/(c + d*x),x)
```

output
```
int(cot(a + b*x)/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\cot(a+bx)}{c+dx}\,dx = \int \frac{\cot\left(bx+a\right)}{dx+c}dx$$

input
```
int(cot(b*x+a)/(d*x+c),x)
```

output
```
int(cot(a + b*x)/(c + d*x),x)
```

3.37 $\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 445
Mathematica [N/A] . . . 445
Rubi [N/A] . . . 446
Maple [N/A] . . . 447
Fricas [N/A] . . . 447
Sympy [N/A] . . . 448
Maxima [N/A] . . . 448
Giac [N/A] . . . 448
Mupad [N/A] . . . 449
Reduce [N/A] . . . 449

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\cot(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(cot(b*x+a)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 5.49 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[Cot[a + b*x]/(c + d*x)^2,x]
```

output
```
Integrate[Cot[a + b*x]/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 25, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{\tan\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx$$
$$\downarrow 25$$
$$-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$
$$\downarrow 4222$$
$$-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$

input
```
Int[Cot[a + b*x]/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\cot(bx+a)}{(dx+c)^2}\,dx$$

input

```
int(cot(b*x+a)/(d*x+c)^2,x)
```

output

```
int(cot(b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(bx+a)}{(dx+c)^2}\,dx$$

input

```
integrate(cot(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output

```
integral(cot(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(cot(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(cot(a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(cot(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(cot(b*x + a)/(d*x + c)^2, x)
```

Giac [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(cot(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(cot(b*x + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 19.64 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(a+b\,x)}{(c+d\,x)^2}\,dx$$

input
```
int(cot(a + b*x)/(c + d*x)^2,x)
```

output
```
int(cot(a + b*x)/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(bx+a)}{d^2x^2+2cdx+c^2}dx$$

input
```
int(cot(b*x+a)/(d*x+c)^2,x)
```

output
```
int(cot(a + b*x)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.38 $\int (c+dx)^m \cot(a+bx) \csc(a+bx)\, dx$

Optimal result 450
Mathematica [N/A] 450
Rubi [N/A] 451
Maple [N/A] 451
Fricas [N/A] 452
Sympy [N/A] 452
Maxima [N/A] 453
Giac [N/A] 453
Mupad [N/A] 453
Reduce [N/A] 454

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^m \cot(a+bx) \csc(a+bx)\, dx = \text{Int}((c+dx)^m \cot(a+bx) \csc(a+bx), x)$$

output
```
Defer(Int)((d*x+c)^m*cot(b*x+a)*csc(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \cot(a+bx) \csc(a+bx)\, dx = \int (c+dx)^m \cot(a+bx) \csc(a+bx)\, dx$$

input
```
Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(a+bx)\csc(a+bx)(c+dx)^m\,dx$$

$$\downarrow 7299$$

$$\int \cot(a+bx)\csc(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \cot(bx+a)\csc(bx+a)\,dx$$

input
```
int((d*x+c)^m*cot(b*x+a)*csc(b*x+a),x)
```

output
```
int((d*x+c)^m*cot(b*x+a)*csc(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \cot(a+bx) \csc(a+bx)\, dx = \int (dx+c)^m \cot(bx+a) \csc(bx+a)\, dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*cot(b*x + a)*csc(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 2.59 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \cot(a+bx) \csc(a+bx)\, dx = \int (c+dx)^m \cot(a+bx) \csc(a+bx)\, dx$$

input
```
integrate((d*x+c)**m*cot(b*x+a)*csc(b*x+a),x)
```

output
```
Integral((c + d*x)**m*cot(a + b*x)*csc(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \cot(a+bx) \csc(a+bx)\, dx = \int (dx+c)^m \cot(bx+a) \csc(bx+a)\, dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cot(b*x + a)*csc(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \cot(a+bx) \csc(a+bx)\, dx = \int (dx+c)^m \cot(bx+a) \csc(bx+a)\, dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cot(b*x + a)*csc(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 19.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (c+dx)^m \cot(a+bx) \csc(a+bx)\, dx = \int \frac{\cot(a+b\,x)\ (c+d\,x)^m}{\sin(a+b\,x)}\, dx$$

input
```
int((cot(a + b*x)*(c + d*x)^m)/sin(a + b*x),x)
```

output
```
int((cot(a + b*x)*(c + d*x)^m)/sin(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \cot(a+bx) \csc(a+bx)\, dx = \int (dx+c)^m \cot(bx+a) \csc(bx+a)\, dx$$

input
```
int((d*x+c)^m*cot(b*x+a)*csc(b*x+a),x)
```

output
```
int((c + d*x)**m*cot(a + b*x)*csc(a + b*x),x)
```

3.39 $\int (c+dx)^4 \cot(a+bx) \csc(a+bx)\, dx$

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Optimal result

Integrand size = 20, antiderivative size = 208

$$\begin{aligned}\int (c+dx)^4 \cot(a+bx) \csc(a+bx)\, dx = &-\frac{8d(c+dx)^3 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b^2} \\ &- \frac{(c+dx)^4 \csc(a+bx)}{b} \\ &+ \frac{12id^2(c+dx)^2 \operatorname{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} \\ &- \frac{12id^2(c+dx)^2 \operatorname{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3} \\ &- \frac{24d^3(c+dx) \operatorname{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^4} \\ &+ \frac{24d^3(c+dx) \operatorname{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^4} \\ &- \frac{24id^4 \operatorname{PolyLog}\left(4, -e^{i(a+bx)}\right)}{b^5} \\ &+ \frac{24id^4 \operatorname{PolyLog}\left(4, e^{i(a+bx)}\right)}{b^5}\end{aligned}$$

output
```
-8*d*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b^2-(d*x+c)^4*csc(b*x+a)/b+12*I*d^2
*(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^3-12*I*d^2*(d*x+c)^2*polylog(2,exp
(I*(b*x+a)))/b^3-24*d^3*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/b^4+24*d^3*(d*x
+c)*polylog(3,exp(I*(b*x+a)))/b^4-24*I*d^4*polylog(4,-exp(I*(b*x+a)))/b^5+
24*I*d^4*polylog(4,exp(I*(b*x+a)))/b^5
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.48

$$\int (c+dx)^4 \cot(a+bx)\csc(a+bx)\,dx$$
$$= \frac{-2b(c+dx)^4\csc(a) + 8id\Big(2i(c+dx)^3\operatorname{arctanh}(\cos(a+bx)+i\sin(a+bx)) + \frac{3d(b^2(c+dx)^2\operatorname{PolyLog}(2,-\cos(a+}$$

input
```
Integrate[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x],x]
```

output
```
(-2*b*(c + d*x)^4*Csc[a] + (8*I)*d*((2*I)*(c + d*x)^3*ArcTanh[Cos[a + b*x]
 + I*Sin[a + b*x]] + (3*d*(b^2*(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Si
n[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x
]] - 2*d^2*PolyLog[4, -Cos[a + b*x] - I*Sin[a + b*x]]))/b^3 - (3*d*(b^2*(c
 + d*x)^2*PolyLog[2, Cos[a + b*x] + I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*
PolyLog[3, Cos[a + b*x] + I*Sin[a + b*x]] - 2*d^2*PolyLog[4, Cos[a + b*x]
 + I*Sin[a + b*x]]))/b^3) + b*(c + d*x)^4*Csc[a/2]*Csc[(a + b*x)/2]*Sin[(b*
x)/2] - b*(c + d*x)^4*Sec[a/2]*Sec[(a + b*x)/2]*Sin[(b*x)/2])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {4910, 3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \cot(a+bx)\csc(a+bx)\,dx$$

$$\downarrow 4910$$

$$\frac{4d\int (c+dx)^3 \csc(a+bx)dx}{b} - \frac{(c+dx)^4\csc(a+bx)}{b}$$

$$\downarrow 3042$$

$$\frac{4d\int (c+dx)^3 \csc(a+bx)dx}{b} - \frac{(c+dx)^4\csc(a+bx)}{b}$$

$$\downarrow 4671$$

$$-\frac{(c+dx)^4\csc(a+bx)}{b} + \frac{4d\left(-\frac{3d\int (c+dx)^2\log\left(1-e^{i(a+bx)}\right)dx}{b} + \frac{3d\int (c+dx)^2\log\left(1+e^{i(a+bx)}\right)dx}{b} - \frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)}{b}$$

$$\downarrow 3011$$

$$-\frac{(c+dx)^4\csc(a+bx)}{b} + \frac{4d\left(\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b} - \frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b} - \frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b} - \frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}\right)}{b}$$

$$\downarrow 7163$$

$$-\frac{(c+dx)^4\csc(a+bx)}{b} + \frac{4d\left(\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b} - \frac{2id\left(\frac{id\int \operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)dx}{b} - \frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b} - \frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b} - \cdots\right)}{b}\right)}{b}$$

$$\downarrow 2720$$

$$4d\left(\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)dx}{b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2}{}\right.}{}$$

$$-\frac{(c+dx)^4\csc(a+bx)}{b}+$$

$$\downarrow 7143$$

$$4d\left(-\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\,\mathrm{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\right.$$

$$-\frac{(c+dx)^4\csc(a+bx)}{b}+$$

input
```
Int[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x],x]
```

output
```
-(((c + d*x)^4*Csc[a + b*x])/b) + (4*d*((-2*(c + d*x)^3*ArcTanh[E^(I*(a +
b*x))])/b + (3*d*((I*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b - ((2*I)*
d*((-I)*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b + (d*PolyLog[4, -E^(I*(
a + b*x))])/b^2))/b))/b - (3*d*((I*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))]
)/b - ((2*I)*d*((-I)*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b + (d*PolyLo
g[4, E^(I*(a + b*x))])/b^2))/b))/b))/b
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4910

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 715 vs. $2(190) = 380$.

Time = 0.46 (sec) , antiderivative size = 716, normalized size of antiderivative = 3.44

method	result
risch	$\frac{4d^4 \ln\left(1-e^{i(bx+a)}\right)x^3}{b^2} - \frac{8d\,c^3 \operatorname{arctanh}\left(e^{i(bx+a)}\right)}{b^2} - \frac{24d^3 c \operatorname{polylog}\left(3,-e^{i(bx+a)}\right)}{b^4} + \frac{24d^3 c \operatorname{polylog}\left(3,e^{i(bx+a)}\right)}{b^4} - \frac{24d^4 \operatorname{polylog}$

input
```
int((d*x+c)^4*cot(b*x+a)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2*I*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*x^2+4*c^3*d*x+c^4)*exp(I*(b*x+a))/b/(e
xp(2*I*(b*x+a))-1)-12*d^2/b^3*c^2*ln(exp(I*(b*x+a))+1)*a+12*d^2/b^3*c^2*ln
(1-exp(I*(b*x+a)))*a+12*d^3/b^4*c*ln(exp(I*(b*x+a))+1)*a^2-12*d^3/b^4*c*ln
(1-exp(I*(b*x+a)))*a^2-12*d^3/b^2*c*ln(exp(I*(b*x+a))+1)*x^2+12*d^3/b^2*c*
ln(1-exp(I*(b*x+a)))*x^2-12*d^2/b^2*c^2*ln(exp(I*(b*x+a))+1)*x+12*d^2/b^2*
c^2*ln(1-exp(I*(b*x+a)))*x-24*d^3/b^4*c*a^2*arctanh(exp(I*(b*x+a)))+24*d^2
/b^3*c^2*a*arctanh(exp(I*(b*x+a)))+12*I*d^2/b^3*c^2*polylog(2,-exp(I*(b*x+
a)))-12*I*d^2/b^3*c^2*polylog(2,exp(I*(b*x+a)))-12*I*d^4/b^3*polylog(2,exp
(I*(b*x+a)))*x^2+4*d^4/b^2*ln(1-exp(I*(b*x+a)))*x^3-8*d/b^2*c^3*arctanh(ex
p(I*(b*x+a)))-24*d^3/b^4*c*polylog(3,-exp(I*(b*x+a)))+24*d^3/b^4*c*polylog
(3,exp(I*(b*x+a)))-24*d^4/b^4*polylog(3,-exp(I*(b*x+a)))*x+24*d^4/b^4*poly
log(3,exp(I*(b*x+a)))*x-4*d^4/b^5*ln(exp(I*(b*x+a))+1)*a^3+4*d^4/b^5*ln(1-
exp(I*(b*x+a)))*a^3+8*d^4/b^5*a^3*arctanh(exp(I*(b*x+a)))-4*d^4/b^2*ln(exp
(I*(b*x+a))+1)*x^3+12*I*d^4/b^3*polylog(2,-exp(I*(b*x+a)))*x^2+24*I*d^3/b^
3*c*polylog(2,-exp(I*(b*x+a)))*x-24*I*d^3/b^3*c*polylog(2,exp(I*(b*x+a)))*
x-24*I*d^4*polylog(4,-exp(I*(b*x+a)))/b^5+24*I*d^4*polylog(4,exp(I*(b*x+a)
))/b^5
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1021 vs. $2(184) = 368$.

Time = 0.13 (sec) , antiderivative size = 1021, normalized size of antiderivative = 4.91

$$\int (c+dx)^4 \cot(a+bx) \csc(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cot(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

output
```
-(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*
c^4 - 12*I*d^4*polylog(4, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12
*I*d^4*polylog(4, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - 12*I*d^4*p
olylog(4, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I*d^4*polylog(
4, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*(I*b^2*d^4*x^2 + 2*I*b
^2*c*d^3*x + I*b^2*c^2*d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x +
 a) + 6*(-I*b^2*d^4*x^2 - 2*I*b^2*c*d^3*x - I*b^2*c^2*d^2)*dilog(cos(b*x +
 a) - I*sin(b*x + a))*sin(b*x + a) + 6*(I*b^2*d^4*x^2 + 2*I*b^2*c*d^3*x +
I*b^2*c^2*d^2)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*(-I*
b^2*d^4*x^2 - 2*I*b^2*c*d^3*x - I*b^2*c^2*d^2)*dilog(-cos(b*x + a) - I*sin
(b*x + a))*sin(b*x + a) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2
*x + b^3*c^3*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 2*(b
^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*log(cos(b*x +
a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3
*a^2*b*c*d^3 - a^3*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*
sin(b*x + a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*l
og(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 2*(b^3*d^4
*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3
 + a^3*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*
d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b...
```

Sympy [F]

$$\int (c+dx)^4 \cot(a+bx) \csc(a+bx)\, dx = \int (c+dx)^4 \cot(a+bx) \csc(a+bx)\, dx$$

input
```
integrate((d*x+c)**4*cot(b*x+a)*csc(b*x+a),x)
```

output
```
Integral((c + d*x)**4*cot(a + b*x)*csc(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2948 vs. $2(184) = 368$.

Time = 0.34 (sec) , antiderivative size = 2948, normalized size of antiderivative = 14.17

$$\int (c+dx)^4 \cot(a+bx) \csc(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^4*cot(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

output

```
-(2*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x + 2
*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x
+ 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (c
os(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b
*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a
))*c^3*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) +
1)*b) - 6*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b
*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(
2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1
) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log
(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b
*x + a))*a*c^2*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x
 + 2*a) + 1)*b^2) + 6*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x
+ a)*cos(2*b*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a
)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(
b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2
*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*
x + a)*sin(b*x + a))*a^2*c*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 -
 2*cos(2*b*x + 2*a) + 1)*b^3) - 2*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*
a) - 4*(b*x + a)*cos(2*b*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + ...
```

Giac [F]

$$\int (c+dx)^4 \cot(a+bx) \csc(a+bx)\, dx = \int (dx+c)^4 \cot(bx+a) \csc(bx+a)\, dx$$

input
```
integrate((d*x+c)^4*cot(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*cot(b*x + a)*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \cot(a+bx) \csc(a+bx)\, dx = \int \frac{\cot(a+b\,x)\,(c+d\,x)^4}{\sin(a+b\,x)}\, dx$$

input
```
int((cot(a + b*x)*(c + d*x)^4)/sin(a + b*x),x)
```

output
```
int((cot(a + b*x)*(c + d*x)^4)/sin(a + b*x), x)
```

Reduce [F]

$$\int (c+dx)^4 \cot(a+bx) \csc(a+bx)\, dx$$
$$= \frac{4\left(\int \frac{x^3}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)} dx\right) \tan\left(\frac{bx}{2}+\frac{a}{2}\right) b\, d^4 + 12\left(\int \frac{x^2}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)} dx\right) \tan\left(\frac{bx}{2}+\frac{a}{2}\right) bc\, d^3 + 12\left(\int \frac{x}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)} dx\right) \tan$$

input
```
int((d*x+c)^4*cot(b*x+a)*csc(b*x+a),x)
```

output

```
(4*int(x**3/tan((a + b*x)/2),x)*tan((a + b*x)/2)*b*d**4 + 12*int(x**2/tan(
(a + b*x)/2),x)*tan((a + b*x)/2)*b*c*d**3 + 12*int(x/tan((a + b*x)/2),x)*t
an((a + b*x)/2)*b*c**2*d**2 + 4*int(tan((a + b*x)/2)*x**3,x)*tan((a + b*x)
/2)*b*d**4 + 12*int(tan((a + b*x)/2)*x**2,x)*tan((a + b*x)/2)*b*c*d**3 + 1
2*int(tan((a + b*x)/2)*x,x)*tan((a + b*x)/2)*b*c**2*d**2 + 8*log(tan((a +
b*x)/2))*tan((a + b*x)/2)*c**3*d - tan((a + b*x)/2)**2*b*c**4 - 4*tan((a +
 b*x)/2)**2*b*c**3*d*x - 6*tan((a + b*x)/2)**2*b*c**2*d**2*x**2 - 4*tan((a
 + b*x)/2)**2*b*c*d**3*x**3 - tan((a + b*x)/2)**2*b*d**4*x**4 - b*c**4 - 4
*b*c**3*d*x - 6*b*c**2*d**2*x**2 - 4*b*c*d**3*x**3 - b*d**4*x**4)/(2*tan((
a + b*x)/2)*b**2)
```

3.40 $\int (c+dx)^3 \cot(a+bx) \csc(a+bx)\, dx$

Optimal result . . . 465
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Optimal result

Integrand size = 20, antiderivative size = 146

$$\begin{aligned}\int (c+dx)^3 \cot(a+bx) \csc(a+bx)\, dx = &-\frac{6d(c+dx)^2 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b^2} \\ &- \frac{(c+dx)^3 \csc(a+bx)}{b} \\ &+ \frac{6id^2(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3} \\ &- \frac{6id^2(c+dx)\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3} \\ &- \frac{6d^3\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4} \\ &+ \frac{6d^3\,\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^4}\end{aligned}$$

output

```
-6*d*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b^2-(d*x+c)^3*csc(b*x+a)/b+6*I*d^2*
(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^3-6*I*d^2*(d*x+c)*polylog(2,exp(I*(b*
x+a)))/b^3-6*d^3*polylog(3,-exp(I*(b*x+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x
+a)))/b^4
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 311 vs. $2(146) = 292$.

Time $= 0.82$ (sec) , antiderivative size $= 311$, normalized size of antiderivative $= 2.13$

$$\int (c+dx)^3 \cot(a+bx)\csc(a+bx)\,dx =$$

$$-\frac{b^3c^3\csc(a+bx) + 3b^3c^2dx\csc(a+bx) + 3b^3cd^2x^2\csc(a+bx) + b^3d^3x^3\csc(a+bx) - 3b^2c^2d\log\left(1-\right.}{}$$

input
```
Integrate[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x],x]
```

output
```
-((b^3*c^3*Csc[a + b*x] + 3*b^3*c^2*d*x*Csc[a + b*x] + 3*b^3*c*d^2*x^2*Csc
[a + b*x] + b^3*d^3*x^3*Csc[a + b*x] - 3*b^2*c^2*d*Log[1 - E^(I*(a + b*x))
] - 6*b^2*c*d^2*x*Log[1 - E^(I*(a + b*x))] - 3*b^2*d^3*x^2*Log[1 - E^(I*(a
 + b*x))] + 3*b^2*c^2*d*Log[1 + E^(I*(a + b*x))] + 6*b^2*c*d^2*x*Log[1 + E
^(I*(a + b*x))] + 3*b^2*d^3*x^2*Log[1 + E^(I*(a + b*x))] - (6*I)*b*d^2*(c
+ d*x)*PolyLog[2, -E^(I*(a + b*x))] + (6*I)*b*d^2*(c + d*x)*PolyLog[2, E^(
I*(a + b*x))] + 6*d^3*PolyLog[3, -E^(I*(a + b*x))] - 6*d^3*PolyLog[3, E^(I
*(a + b*x))])/b^4)
```

Rubi [A] (verified)

Time $= 0.59$ (sec) , antiderivative size $= 156$, normalized size of antiderivative $= 1.07$, number of steps used $= 7$, number of rules used $= 6$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used $= \{4910, 3042, 4671, 3011, 2720, 7143\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \cot(a+bx)\csc(a+bx)\,dx$$

$$\downarrow 4910$$

$$\frac{3d\int (c+dx)^2\csc(a+bx)dx}{b} - \frac{(c+dx)^3\csc(a+bx)}{b}$$

$$\downarrow 3042$$

$$\frac{3d\int (c+dx)^2\csc(a+bx)dx}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

$$\downarrow 4671$$

$$-\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{3d\left(-\frac{2d\int (c+dx)\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{2d\int (c+dx)\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)}{b}$$

$$\downarrow 3011$$

$$-\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{3d\left(\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{id\int \operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{id\int \operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2(c}{b}$$

$$\downarrow 2720$$

$$-\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{3d\left(\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyL}}{b}\right.}{b}$$

$$\downarrow 7143$$

$$-\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{3d\left(-\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\,\mathrm{P}}{b}\right.}{b}$$

input
```
Int[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x],x]
```

output
```
-(((c + d*x)^3*Csc[a + b*x])/b) + (3*d*((-2*(c + d*x)^2*ArcTanh[E^(I*(a +
b*x))])/b + (2*d*((I*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b - (d*PolyLo
g[3, -E^(I*(a + b*x))])/b^2))/b - (2*d*((I*(c + d*x)*PolyLog[2, E^(I*(a +
b*x))])/b - (d*PolyLog[3, E^(I*(a + b*x))])/b^2))/b))/b
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(134) = 268$.

Time = 0.35 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.97

method	result
risch	$\frac{6id^2c\,\mathrm{polylog}\left(2,-\mathrm{e}^{i(bx+a)}\right)}{b^3} - \frac{6d^2c\ln\left(\mathrm{e}^{i(bx+a)}+1\right)a}{b^3} - \frac{2i\left(d^3x^3+3cd^2x^2+3c^2dx+c^3\right)\mathrm{e}^{i(bx+a)}}{b\left(\mathrm{e}^{2i(bx+a)}-1\right)} + \frac{6d^2c\ln\left(1-\mathrm{e}^{i(bx+a)}\right)a}{b^3} - \frac{6i}{}$

input

```
int((d*x+c)^3*cot(b*x+a)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
6*I*d^2/b^3*c*polylog(2,-exp(I*(b*x+a)))-6*d^2/b^3*c*ln(exp(I*(b*x+a))+1)*
a-2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a
))-1)+6*d^2/b^3*c*ln(1-exp(I*(b*x+a)))*a-6*I*d^2/b^3*c*polylog(2,exp(I*(b*
x+a)))-6*d/b^2*c^2*arctanh(exp(I*(b*x+a)))-6*d^3/b^4*a^2*arctanh(exp(I*(b*
x+a)))-6*d^2/b^2*c*ln(exp(I*(b*x+a))+1)*x+6*d^2/b^2*c*ln(1-exp(I*(b*x+a)))
*x+3*d^3/b^4*ln(exp(I*(b*x+a))+1)*a^2-3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a^2+1
2*d^2/b^3*c*a*arctanh(exp(I*(b*x+a)))-6*I*d^3/b^3*polylog(2,exp(I*(b*x+a))
)*x+6*I*d^3/b^3*polylog(2,-exp(I*(b*x+a)))*x-3*d^3/b^2*ln(exp(I*(b*x+a))+1
)*x^2-6*d^3*polylog(3,-exp(I*(b*x+a)))/b^4+3*d^3/b^2*ln(1-exp(I*(b*x+a)))*
x^2+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 669 vs. $2(130) = 260$.

Time = 0.11 (sec) , antiderivative size = 669, normalized size of antiderivative = 4.58

$$\int (c+dx)^3 \cot(a+bx)\csc(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cot(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 6*d^3*
polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 6*d^3*polylog(3,
cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x +
a) + I*sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a) - I*sin
(b*x + a))*sin(b*x + a) + 6*(I*b*d^3*x + I*b*c*d^2)*dilog(cos(b*x + a) + I
*sin(b*x + a))*sin(b*x + a) + 6*(-I*b*d^3*x - I*b*c*d^2)*dilog(cos(b*x + a
) - I*sin(b*x + a))*sin(b*x + a) + 6*(I*b*d^3*x + I*b*c*d^2)*dilog(-cos(b*
x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*(-I*b*d^3*x - I*b*c*d^2)*dilog(-
cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2
*x + b^2*c^2*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b
^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) - I*sin(b*x + a)
+ 1)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x
 + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^
2 + a^2*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a
) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x +
 a) + I*sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x +
2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a
))/(b^4*sin(b*x + a))
```

Sympy [F]

$$\int (c+dx)^3 \cot(a+bx) \csc(a+bx)\, dx = \int (c+dx)^3 \cot(a+bx) \csc(a+bx)\, dx$$

input
```
integrate((d*x+c)**3*cot(b*x+a)*csc(b*x+a),x)
```

output
```
Integral((c + d*x)**3*cot(a + b*x)*csc(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1770 vs. $2(130) = 260$.

Time = 0.18 (sec) , antiderivative size = 1770, normalized size of antiderivative = 12.12

$$\int (c+dx)^3 \cot(a+bx) \csc(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*cot(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

output

```
-1/2*(3*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x
 + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*
b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(c
os(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x
 + a))*c^2*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a
) + 1)*b) - 6*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos
(2*b*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*
cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a)
 + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)
*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*s
in(b*x + a))*a*c*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b
*x + 2*a) + 1)*b^2) + 3*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*
x + a)*cos(2*b*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2
*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*co
s(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x +
 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(
b*x + a)*sin(b*x + a))*a^2*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 -
 2*cos(2*b*x + 2*a) + 1)*b^3) + 2*c^3/sin(b*x + a) - 6*a*c^2*d/(b*sin(b*x
+ a)) + 6*a^2*c*d^2/(b^2*sin(b*x + a)) - 2*a^3*d^3/(b^3*sin(b*x + a)) -...
```

Giac [F]

$$\int (c+dx)^3 \cot(a+bx) \csc(a+bx)\, dx = \int (dx+c)^3 \cot(bx+a) \csc(bx+a)\, dx$$

input
```
integrate((d*x+c)^3*cot(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*cot(b*x + a)*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \cot(a+bx) \csc(a+bx)\, dx = \int \frac{\cot(a+b\,x)\,(c+d\,x)^3}{\sin(a+b\,x)}\, dx$$

input
```
int((cot(a + b*x)*(c + d*x)^3)/sin(a + b*x),x)
```

output
```
int((cot(a + b*x)*(c + d*x)^3)/sin(a + b*x), x)
```

Reduce [F]

$$\int (c+dx)^3 \cot(a+bx) \csc(a+bx)\, dx$$

$$= \frac{3\cos(bx+a)\tan\left(\frac{bx}{2}+\frac{a}{2}\right) bc\,d^2x^2 + \cos(bx+a)\tan\left(\frac{bx}{2}+\frac{a}{2}\right) b\,d^3x^3 + 3\left(\int \frac{x^2}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}dx\right)\sin(bx+a)\,\text{ta}}{}$$

input
```
int((d*x+c)^3*cot(b*x+a)*csc(b*x+a),x)
```

output

```
(3*cos(a + b*x)*tan((a + b*x)/2)*b*c*d**2*x**2 + cos(a + b*x)*tan((a + b*x
)/2)*b*d**3*x**3 + 3*int(x**2/tan((a + b*x)/2),x)*sin(a + b*x)*tan((a + b*
x)/2)*b*d**3 + 6*int(x/tan((a + b*x)/2),x)*sin(a + b*x)*tan((a + b*x)/2)*b
*c*d**2 + 3*int(tan((a + b*x)/2)*x**2,x)*sin(a + b*x)*tan((a + b*x)/2)*b*d
**3 + 6*int(tan((a + b*x)/2)*x,x)*sin(a + b*x)*tan((a + b*x)/2)*b*c*d**2 +
 6*log(tan((a + b*x)/2))*sin(a + b*x)*tan((a + b*x)/2)*c**2*d - 3*sin(a +
b*x)*b*c*d**2*x**2 - sin(a + b*x)*b*d**3*x**3 - 2*tan((a + b*x)/2)*b*c**3
- 6*tan((a + b*x)/2)*b*c**2*d*x - 3*tan((a + b*x)/2)*b*c*d**2*x**2 - tan((
a + b*x)/2)*b*d**3*x**3)/(2*sin(a + b*x)*tan((a + b*x)/2)*b**2)
```

3.41 $\int (c+dx)^2 \cot(a+bx) \csc(a+bx)\, dx$

Optimal result . . . 474
Mathematica [B] (warning: unable to verify) . . . 474
Rubi [A] (verified) . . . 475
Maple [B] (verified) . . . 477
Fricas [B] (verification not implemented) . . . 478
Sympy [F] . . . 478
Maxima [B] (verification not implemented) . . . 479
Giac [F] . . . 479
Mupad [F(-1)] . . . 480
Reduce [F] . . . 480

Optimal result

Integrand size = 20, antiderivative size = 90

$$\int (c+dx)^2 \cot(a+bx) \csc(a+bx)\, dx = -\frac{4d(c+dx)\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2 \csc(a+bx)}{b} + \frac{2id^2 \operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3} - \frac{2id^2 \operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}$$

output

```
-4*d*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^2-(d*x+c)^2*csc(b*x+a)/b+2*I*d^2*po
lylog(2,-exp(I*(b*x+a)))/b^3-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 234 vs. 2(90) = 180.

Time = 1.50 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.60

$$\int (c+dx)^2 \cot(a+bx)\csc(a+bx)\,dx$$

$$= \frac{-8bcd\operatorname{arctanh}\left(\cos(a)-\sin(a)\tan\left(\frac{bx}{2}\right)\right) - 2b^2(c+dx)^2\csc(a) + 4d^2\Big(2\arctan(\tan(a))\operatorname{arctanh}\left(\cos(a)-$$

input
```
Integrate[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x],x]
```

output
```
(-8*b*c*d*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] - 2*b^2*(c + d*x)^2*Csc[a]
 + 4*d^2*(2*ArcTan[Tan[a]]*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] + (((b*x
+ ArcTan[Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[Tan[a]]))] - Log[1 + E^(I*(b
*x + ArcTan[Tan[a]]))]) + I*PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]]))] - I*
PolyLog[2, E^(I*(b*x + ArcTan[Tan[a]]))])*Sec[a])/Sqrt[Sec[a]^2]) + b^2*(c
 + d*x)^2*Csc[a/2]*Csc[(a + b*x)/2]*Sin[(b*x)/2] - b^2*(c + d*x)^2*Sec[a/2
]*Sec[(a + b*x)/2]*Sin[(b*x)/2])/(2*b^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4910, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \cot(a+bx)\csc(a+bx)\,dx$$

$$\downarrow 4910$$

$$\frac{2d\int (c+dx)\csc(a+bx)dx}{b} - \frac{(c+dx)^2\csc(a+bx)}{b}$$

$$\downarrow 3042$$

$$\frac{2d\int (c+dx)\csc(a+bx)dx}{b} - \frac{(c+dx)^2\csc(a+bx)}{b}$$

$$\downarrow 4671$$

$$\frac{-\frac{(c+dx)^2\csc(a+bx)}{b}+\frac{2d\left(-\frac{d\int\log(1-e^{i(a+bx)})dx}{b}+\frac{d\int\log(1+e^{i(a+bx)})dx}{b}-\frac{2(c+dx)\text{arctanh}(e^{i(a+bx)})}{b}\right)}{b}}{b}$$

$$\downarrow 2715$$

$$\frac{-\frac{(c+dx)^2\csc(a+bx)}{b}+\frac{2d\left(\frac{id\int e^{-i(a+bx)}\log(1-e^{i(a+bx)})de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log(1+e^{i(a+bx)})de^{i(a+bx)}}{b^2}-\frac{2(c+dx)\text{arctanh}(e^{i(a+bx)})}{b}\right)}{b}}{b}$$

$$\downarrow 2838$$

$$\frac{-\frac{(c+dx)^2\csc(a+bx)}{b}+\frac{2d\left(-\frac{2(c+dx)\text{arctanh}(e^{i(a+bx)})}{b}+\frac{id\,\text{PolyLog}(2,-e^{i(a+bx)})}{b^2}-\frac{id\,\text{PolyLog}(2,e^{i(a+bx)})}{b^2}\right)}{b}}{b}$$

input
```
Int[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x],x]
```

output
```
-(((c + d*x)^2*Csc[a + b*x])/b) + (2*d*((-2*(c + d*x)*ArcTanh[E^(I*(a + b*
x))])/b + (I*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^(I*(a
 + b*x))])/b^2))/b
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4910

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(82) = 164$.

Time = 0.22 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.36

method	result
risch	$-\frac{2i\left(x^2d^2+2cdx+c^2\right)e^{i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)}-\frac{4dc\,\operatorname{arctanh}\left(e^{i(bx+a)}\right)}{b^2}-\frac{2d^2\ln\left(e^{i(bx+a)}+1\right)x}{b^2}-\frac{2d^2\ln\left(e^{i(bx+a)}+1\right)a}{b^3}+\frac{2id^2}{b}$
derivativedivides	$-\frac{a^2d^2}{b^2\sin(bx+a)}+\frac{2acd}{b\sin(bx+a)}-\frac{2a\,d^2\left(-\frac{bx+a}{\sin(bx+a)}+\ln(\csc(bx+a)-\cot(bx+a))\right)}{b^2}-\frac{c^2}{\sin(bx+a)}+\frac{2cd\left(-\frac{bx+a}{\sin(bx+a)}+\ln(\csc(bx+a)-c\right.}{b}$
default	$-\frac{a^2d^2}{b^2\sin(bx+a)}+\frac{2acd}{b\sin(bx+a)}-\frac{2a\,d^2\left(-\frac{bx+a}{\sin(bx+a)}+\ln(\csc(bx+a)-\cot(bx+a))\right)}{b^2}-\frac{c^2}{\sin(bx+a)}+\frac{2cd\left(-\frac{bx+a}{\sin(bx+a)}+\ln(\csc(bx+a)-c\right.}{b}$

input

```
int((d*x+c)^2*cot(b*x+a)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
-2*I*(d^2*x^2+2*c*d*x+c^2)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)-4*d/b^2*c
*arctanh(exp(I*(b*x+a)))-2*d^2/b^2*ln(exp(I*(b*x+a))+1)*x-2*d^2/b^3*ln(exp
(I*(b*x+a))+1)*a+2*I*d^2*polylog(2,-exp(I*(b*x+a)))/b^3+2*d^2/b^2*ln(1-exp
(I*(b*x+a)))*x+2*d^2/b^3*ln(1-exp(I*(b*x+a)))*a-2*I*d^2*polylog(2,exp(I*(b
*x+a)))/b^3+4*d^2/b^3*a*arctanh(exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(78) = 156$.

Time = 0.09 (sec) , antiderivative size = 375, normalized size of antiderivative = 4.17

$$\int (c+dx)^2 \cot(a+bx) \csc(a+bx)\, dx =$$

$$-\frac{b^2d^2x^2 + 2\,b^2cdx + b^2c^2 + i\,d^2\mathrm{Li}_2(\cos(bx+a) + i\,\sin(bx+a))\sin(bx+a) - i\,d^2\mathrm{Li}_2(\cos(bx+a) - i}{}$$

input
```
integrate((d*x+c)^2*cot(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

output
```
-(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*dilog(cos(b*x + a) + I*sin(b
*x + a))*sin(b*x + a) - I*d^2*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x
 + a) + I*d^2*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - I*d^2*d
ilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + (b*d^2*x + b*c*d)*log(
cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + (b*d^2*x + b*c*d)*log(co
s(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - (b*c*d - a*d^2)*log(-1/2*c
os(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - (b*c*d - a*d^2)*log
(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - (b*d^2*x + a
*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - (b*d^2*x + a*
d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a))/(b^3*sin(b*x +
a))
```

Sympy [F]

$$\int (c+dx)^2 \cot(a+bx) \csc(a+bx)\, dx = \int (c+dx)^2 \cot(a+bx) \csc(a+bx)\, dx$$

input
```
integrate((d*x+c)**2*cot(b*x+a)*csc(b*x+a),x)
```

output
```
Integral((c + d*x)**2*cot(a + b*x)*csc(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 553 vs. $2(78) = 156$.

Time = 0.16 (sec) , antiderivative size = 553, normalized size of antiderivative = 6.14

$$\int (c+dx)^2 \cot(a+bx) \csc(a+bx)\, dx$$

$$= \frac{2\left(bd^2x + bcd - \left(bd^2x + bcd\right)\cos\left(2\,bx + 2\,a\right) + \left(-i\,bd^2x - i\,bcd\right)\sin\left(2\,bx + 2\,a\right)\right)\arctan\left(\sin\left(bx+a\right), \text{co}\right.}{}$$

input
```
integrate((d*x+c)^2*cot(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

output
```
(2*(b*d^2*x + b*c*d - (b*d^2*x + b*c*d)*cos(2*b*x + 2*a) + (-I*b*d^2*x - I
*b*c*d)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 2*(b*c
*d*cos(2*b*x + 2*a) + I*b*c*d*sin(2*b*x + 2*a) - b*c*d)*arctan2(sin(b*x +
a), cos(b*x + a) - 1) - 2*(b*d^2*x*cos(2*b*x + 2*a) + I*b*d^2*x*sin(2*b*x
+ 2*a) - b*d^2*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a) + 2*(d^2*cos(2*b*x + 2*a) + I*d^2*
sin(2*b*x + 2*a) - d^2)*dilog(-e^(I*b*x + I*a)) - 2*(d^2*cos(2*b*x + 2*a)
+ I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(e^(I*b*x + I*a)) - (I*b*d^2*x + I*b*
c*d + (-I*b*d^2*x - I*b*c*d)*cos(2*b*x + 2*a) + (b*d^2*x + b*c*d)*sin(2*b*
x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-I*
b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(2*b*x + 2*a) - (b*d^2*x + b*
c*d)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a
) + 1) + 2*(-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x - I*b^2*c^2)*sin(b*x + a))/(-I*
b^3*cos(2*b*x + 2*a) + b^3*sin(2*b*x + 2*a) + I*b^3)
```

Giac [F]

$$\int (c+dx)^2 \cot(a+bx) \csc(a+bx)\, dx = \int (dx+c)^2 \cot\left(bx+a\right) \csc\left(bx+a\right)\, dx$$

input
```
integrate((d*x+c)^2*cot(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*cot(b*x + a)*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \cot(a+bx) \csc(a+bx)\, dx = \int \frac{\cot(a+b\,x)\ (c+d\,x)^2}{\sin(a+b\,x)}\, dx$$

input
```
int((cot(a + b*x)*(c + d*x)^2)/sin(a + b*x),x)
```

output
```
int((cot(a + b*x)*(c + d*x)^2)/sin(a + b*x), x)
```

Reduce [F]

$$\int (c+dx)^2 \cot(a+bx) \csc(a+bx)\, dx$$

$$= \frac{\cos(bx+a)\tan\left(\frac{bx}{2}+\frac{a}{2}\right) b\, d^2 x^2 + 2\left(\int \frac{x}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)} dx\right) \sin(bx+a)\tan\left(\frac{bx}{2}+\frac{a}{2}\right) b\, d^2 + 2\left(\int \tan\left(\frac{bx}{2}+\frac{a}{2}\right) x\right.}{}$$

input
```
int((d*x+c)^2*cot(b*x+a)*csc(b*x+a),x)
```

output
```
(cos(a + b*x)*tan((a + b*x)/2)*b*d**2*x**2 + 2*int(x/tan((a + b*x)/2),x)*s
in(a + b*x)*tan((a + b*x)/2)*b*d**2 + 2*int(tan((a + b*x)/2)*x,x)*sin(a +
b*x)*tan((a + b*x)/2)*b*d**2 + 4*log(tan((a + b*x)/2))*sin(a + b*x)*tan((a
 + b*x)/2)*c*d - sin(a + b*x)*b*d**2*x**2 - 2*tan((a + b*x)/2)*b*c**2 - 4*
tan((a + b*x)/2)*b*c*d*x - tan((a + b*x)/2)*b*d**2*x**2)/(2*sin(a + b*x)*t
an((a + b*x)/2)*b**2)
```

3.42 $\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx$

Optimal result . . . 481
Mathematica [B] (verified) . . . 481
Rubi [A] (verified) . . . 482
Maple [B] (verified) . . . 483
Fricas [B] (verification not implemented) . . . 484
Sympy [F] . . . 484
Maxima [B] (verification not implemented) . . . 484
Giac [B] (verification not implemented) . . . 485
Mupad [B] (verification not implemented) . . . 486
Reduce [B] (verification not implemented) . . . 487

Optimal result

Integrand size = 18, antiderivative size = 30

$$\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx = -\frac{d\operatorname{arctanh}(\cos(a+bx))}{b^2} - \frac{(c+dx)\csc(a+bx)}{b}$$

output
```
-d*arctanh(cos(b*x+a))/b^2-(d*x+c)*csc(b*x+a)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 131 vs. 2(30) = 60.

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 4.37

$$\begin{aligned}\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx = &-\frac{dx\csc(a)}{b} - \frac{c\csc(a+bx)}{b} \\ &- \frac{d\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2} + \frac{d\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2} \\ &+ \frac{dx\csc\left(\frac{a}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{bx}{2}\right)}{2b} \\ &- \frac{dx\sec\left(\frac{a}{2}\right)\sec\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{bx}{2}\right)}{2b}\end{aligned}$$

input
```
Integrate[(c + d*x)*Cot[a + b*x]*Csc[a + b*x],x]
```

output
```
-((d*x*Csc[a])/b) - (c*Csc[a + b*x])/b - (d*Log[Cos[a/2 + (b*x)/2]])/b^2 +
 (d*Log[Sin[a/2 + (b*x)/2]])/b^2 + (d*x*Csc[a/2]*Csc[a/2 + (b*x)/2]*Sin[(b
*x)/2])/(2*b) - (d*x*Sec[a/2]*Sec[a/2 + (b*x)/2]*Sin[(b*x)/2])/(2*b)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4910, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx$$

$$\downarrow 4910$$

$$\frac{d\int \csc(a+bx)dx}{b} - \frac{(c+dx)\csc(a+bx)}{b}$$

$$\downarrow 3042$$

$$\frac{d\int \csc(a+bx)dx}{b} - \frac{(c+dx)\csc(a+bx)}{b}$$

$$\downarrow 4257$$

$$-\frac{d\operatorname{arctanh}(\cos(a+bx))}{b^2} - \frac{(c+dx)\csc(a+bx)}{b}$$

input
```
Int[(c + d*x)*Cot[a + b*x]*Csc[a + b*x],x]
```

output
```
-((d*ArcTanh[Cos[a + b*x]])/b^2) - ((c + d*x)*Csc[a + b*x])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(30) = 60$.

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.27

method	result	size
derivativedivides	$\frac{\frac{da}{b\sin(bx+a)}-\frac{c}{\sin(bx+a)}+\frac{d\left(-\frac{bx+a}{\sin(bx+a)}+\ln(\csc(bx+a)-\cot(bx+a))\right)}{b}}{b}$	68
default	$\frac{\frac{da}{b\sin(bx+a)}-\frac{c}{\sin(bx+a)}+\frac{d\left(-\frac{bx+a}{\sin(bx+a)}+\ln(\csc(bx+a)-\cot(bx+a))\right)}{b}}{b}$	68
risch	$-\frac{2i(dx+c)e^{i(bx+a)}}{b\left(e^{2i(bx+a)}-1\right)}+\frac{d\ln\left(e^{i(bx+a)}-1\right)}{b^2}-\frac{d\ln\left(e^{i(bx+a)}+1\right)}{b^2}$	70

input
```
int((d*x+c)*cot(b*x+a)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/b*d*a/sin(b*x+a)-c/sin(b*x+a)+1/b*d*(-(b*x+a)/sin(b*x+a)+ln(csc(b*x
+a)-cot(b*x+a))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(30) = 60$.

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.07

$$\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx =$$
$$-\frac{2\,bdx + d\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)\sin(bx+a) - d\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)\sin(bx+a) + 2\,bc}{2\,b^2\sin(bx+a)}$$

input
```
integrate((d*x+c)*cot(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*b*d*x + d*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - d*log(-1/2*co
s(b*x + a) + 1/2)*sin(b*x + a) + 2*b*c)/(b^2*sin(b*x + a))
```

Sympy [F]

$$\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx = \int (c+dx)\cot(a+bx)\csc(a+bx)\,dx$$

input
```
integrate((d*x+c)*cot(b*x+a)*csc(b*x+a),x)
```

output
```
Integral((c + d*x)*cot(a + b*x)*csc(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(30) = 60$.

Time = 0.04 (sec) , antiderivative size = 259, normalized size of antiderivative = 8.63

$$\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx =$$
$$-\frac{\left(4\,(bx+a)\cos(bx+a)\sin(2\,bx+2\,a) - 4\,(bx+a)\cos(2\,bx+2\,a)\sin(bx+a) + \left(\cos(2\,bx+2\,a)^2 + \sin(2\,bx+2\,a)^2 - 2\,\cos(2\,bx+2\,a) + 1\right)\log\left(\cos(bx+\right.\right.}{\left(\cos(2\,bx+2\,a)^2\right.}$$

input
```
integrate((d*x+c)*cot(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*((4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x +
 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*
x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) -
(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos
(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x +
 a))*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)
*b) + 2*c/sin(b*x + a) - 2*a*d/(b*sin(b*x + a)))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(30) = 60$.

Time $= 0.35$ (sec) , antiderivative size $= 697$, normalized size of antiderivative $= 23.23$

$$\int (c + dx)\cot(a + bx)\csc(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*cot(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

output

```
1/2*(b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*c*tan(1/2*b*x)^2*tan(1/2*a)^2 +
 b*d*x*tan(1/2*b*x)^2 - d*log(4*(tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b
*x)*tan(1/2*a) + 1)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/
2*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a) + d*log(4*(tan(1/2*b*x)^2 + 2*tan(1
/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*
b*x)^2 + tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a) + b*d*x*tan(1/2*a)^2
 - d*log(4*(tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(
tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 1))*tan(1/2*
b*x)*tan(1/2*a)^2 + d*log(4*(tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) +
tan(1/2*a)^2)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2
 + 1))*tan(1/2*b*x)*tan(1/2*a)^2 + b*c*tan(1/2*b*x)^2 + b*c*tan(1/2*a)^2 +
 b*d*x + d*log(4*(tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)
+ 1)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 1))*ta
n(1/2*b*x) - d*log(4*(tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2
*a)^2)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 1))*
tan(1/2*b*x) + d*log(4*(tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1
/2*a) + 1)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 +
1))*tan(1/2*a) - d*log(4*(tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan
(1/2*a)^2)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 +
1))*tan(1/2*a) + b*c)/(b^2*tan(1/2*b*x)^2*tan(1/2*a) + b^2*tan(1/2*b*x)...
```

Mupad [B] (verification not implemented)

Time = 20.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.93

$$\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx = -\frac{d\,\ln\left(\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}{b^2} + \frac{d\,\ln\left(d\,2\mathrm{i}-d\,\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,1\mathrm{i}}\,2\mathrm{i}\right)}{b^2} - \frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,(c+d\,x)\,2\mathrm{i}}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}$$

input

```
int((cot(a + b*x)*(c + d*x))/sin(a + b*x),x)
```

output

```
(d*log(d*2i - d*exp(a*1i)*exp(b*x*1i)*2i))/b^2 - (d*log(exp(a*1i + b*x*1i)
*1i + 1i))/b^2 - (exp(a*1i + b*x*1i)*(c + d*x)*2i)/(b*(exp(a*2i + b*x*2i)
- 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx = \frac{\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin(bx+a)\,d - bc - bdx}{\sin(bx+a)\,b^2}$$

input

```
int((d*x+c)*cot(b*x+a)*csc(b*x+a),x)
```

output

```
(log(tan((a + b*x)/2))*sin(a + b*x)*d - b*c - b*d*x)/(sin(a + b*x)*b**2)
```

3.43 $\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$

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Mathematica [N/A] 488
Rubi [N/A] 489
Maple [N/A] 489
Fricas [N/A] 490
Sympy [N/A] 490
Maxima [N/A] 491
Giac [N/A] 491
Mupad [N/A] 492
Reduce [N/A] 492

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\cot(a+bx)\csc(a+bx)}{c+dx}, x\right)$$

output
```
Defer(Int)(cot(b*x+a)*csc(b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 11.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x),x]
```

output
```
Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$$
$$\downarrow 7299$$
$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}dx$$

input
```
Int[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cot(bx+a)\csc(bx+a)}{dx+c}dx$$

input
```
int(cot(b*x+a)*csc(b*x+a)/(d*x+c),x)
```

output
```
int(cot(b*x+a)*csc(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cot(bx+a)\csc(bx+a)}{dx+c}\,dx$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(cot(b*x + a)*csc(b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)/(d*x+c),x)
```

output
```
Integral(cot(a + b*x)*csc(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 508, normalized size of antiderivative = 25.40

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cot(bx+a)\csc(bx+a)}{dx+c}\,dx$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
-((b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c
*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))*integrate(s
in(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^
2)*cos(b*x + a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 + 2*(b*
d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)), x) + (b*d^2*x + b*c*d + (b*d^2
*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 - 2*
(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^2*x^2 + 2*
b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2 + (b*d^2*
x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2
)*cos(b*x + a)), x) + 2*cos(b*x + a)*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a)
*sin(b*x + a) + 2*sin(b*x + a))/(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2
+ (b*d*x + b*c)*sin(2*b*x + 2*a)^2 + b*c - 2*(b*d*x + b*c)*cos(2*b*x + 2*a
))
```

Giac [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cot(bx+a)\csc(bx+a)}{dx+c}\,dx$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(cot(b*x + a)*csc(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 19.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+b\,x)}{\sin(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(cot(a + b*x)/(sin(a + b*x)*(c + d*x)),x)
```

output
```
int(cot(a + b*x)/(sin(a + b*x)*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cot(bx+a)\csc(bx+a)}{dx+c}dx$$

input
```
int(cot(b*x+a)*csc(b*x+a)/(d*x+c),x)
```

output
```
int((cot(a + b*x)*csc(a + b*x))/(c + d*x),x)
```

3.44 $\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$

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Giac [N/A] . . . 496
Mupad [N/A] . . . 497
Reduce [N/A] . . . 497

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(cot(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 14.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$
$$\downarrow 7299$$
$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

input
```
Int[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cot(bx+a)\csc(bx+a)}{(dx+c)^2}\,dx$$

input
```
int(cot(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)
```

output
```
int(cot(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot\left(bx+a\right)\csc\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cot(b*x + a)*csc(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot\left(a+bx\right)\csc\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(cot(a + b*x)*csc(a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 745, normalized size of antiderivative = 37.25

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot\left(bx+a\right)\csc\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-2*((b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*
d)*cos(2*b*x + 2*a)^2 + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(2*b*x + 2*
a)^2 - 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a))*integrate(s
in(b*x + a)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3
 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)^2 + (b*d^3*x^3 + 3*b*c
*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(b*x + a)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*
x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)), x) + (b*d^3*x^2 + 2*b*c*d^2*x +
b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a)^2 + (b*d^3*
x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^3*x^2 + 2*b*c*d^2
*x + b*c^2*d)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^3*x^3 + 3*b*c*
d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x +
 b*c^3)*cos(b*x + a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)
*sin(b*x + a)^2 - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(
b*x + a)), x) + cos(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)*sin(b*x +
 a) + sin(b*x + a))/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*
x + b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x
+ 2*a)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a))
```

Giac [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot\left(bx+a\right)\csc\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(cot(b*x + a)*csc(b*x + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 18.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(a+b\,x)}{\sin(a+b\,x)\,(c+d\,x)^2}\,dx$$

input
```
int(cot(a + b*x)/(sin(a + b*x)*(c + d*x)^2),x)
```

output
```
int(cot(a + b*x)/(sin(a + b*x)*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(bx+a)\csc(bx+a)}{d^2x^2+2cdx+c^2}dx$$

input
```
int(cot(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)
```

output
```
int((cot(a + b*x)*csc(a + b*x))/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.45 $\int (c+dx)^m \cot(a+bx) \csc^2(a+bx)\, dx$

Optimal result 498
Mathematica [N/A] 498
Rubi [N/A] 499
Maple [N/A] 499
Fricas [N/A] 500
Sympy [N/A] 500
Maxima [N/A] 501
Giac [N/A] 501
Mupad [N/A] 501
Reduce [N/A] 502

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (c+dx)^m \cot(a+bx) \csc^2(a+bx)\, dx = \text{Int}\big((c+dx)^m \cot(a+bx) \csc^2(a+bx), x\big)$$

output
```
Defer(Int)((d*x+c)^m*cot(b*x+a)*csc(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 4.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cot(a+bx) \csc^2(a+bx)\, dx = \int (c+dx)^m \cot(a+bx) \csc^2(a+bx)\, dx$$

input
```
Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(a+bx)\csc^2(a+bx)(c+dx)^m\,dx$$

$$\downarrow 7299$$

$$\int \cot(a+bx)\csc^2(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \cot(bx+a)\csc(bx+a)^2\,dx$$

input
```
int((d*x+c)^m*cot(b*x+a)*csc(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*cot(b*x+a)*csc(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cot(a+bx) \csc^2(a+bx)\,dx = \int (dx+c)^m \cot\left(bx+a\right) \csc\left(bx+a\right)^2\,dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*cot(b*x + a)*csc(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 8.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \cot(a+bx) \csc^2(a+bx)\,dx = \int (c+dx)^m \cot\left(a+bx\right) \csc^2\left(a+bx\right)\,dx$$

input
```
integrate((d*x+c)**m*cot(b*x+a)*csc(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*cot(a + b*x)*csc(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cot(a+bx) \csc^2(a+bx)\, dx = \int (dx+c)^m \cot(bx+a) \csc(bx+a)^2\, dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cot(b*x + a)*csc(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cot(a+bx) \csc^2(a+bx)\, dx = \int (dx+c)^m \cot(bx+a) \csc(bx+a)^2\, dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cot(b*x + a)*csc(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 18.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cot(a+bx) \csc^2(a+bx)\, dx = \int \frac{\cot(a+b\,x)\ (c+d\,x)^m}{\sin(a+b\,x)^2}\, dx$$

input
```
int((cot(a + b*x)*(c + d*x)^m)/sin(a + b*x)^2,x)
```

output
```
int((cot(a + b*x)*(c + d*x)^m)/sin(a + b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cot(a+bx) \csc^2(a+bx)\, dx = \int (dx+c)^m \cot(bx+a) \csc(bx+a)^2\, dx$$

input
```
int((d*x+c)^m*cot(b*x+a)*csc(b*x+a)^2,x)
```

output
```
int((c + d*x)**m*cot(a + b*x)*csc(a + b*x)**2,x)
```

3.46 $\int (c+dx)^4 \cot(a+bx) \csc^2(a+bx)\, dx$

Optimal result . . . 503
Mathematica [B] (warning: unable to verify) . . . 504
Rubi [A] (verified) . . . 505
Maple [B] (verified) . . . 508
Fricas [B] (verification not implemented) . . . 509
Sympy [F] . . . 510
Maxima [B] (verification not implemented) . . . 511
Giac [F] . . . 512
Mupad [F(-1)] . . . 512
Reduce [F] . . . 512

Optimal result

Integrand size = 22, antiderivative size = 137

$$\begin{aligned}\int (c+dx)^4 \cot(a+bx) \csc^2(a+bx)\, dx = &-\frac{2id(c+dx)^3}{b^2} - \frac{2d(c+dx)^3 \cot(a+bx)}{b^2} \\ &- \frac{(c+dx)^4 \csc^2(a+bx)}{2b} \\ &+ \frac{6d^2(c+dx)^2 \log\left(1-e^{2i(a+bx)}\right)}{b^3} \\ &- \frac{6id^3(c+dx)\,\mathrm{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^4} \\ &+ \frac{3d^4\,\mathrm{PolyLog}\left(3, e^{2i(a+bx)}\right)}{b^5}\end{aligned}$$

output

```
-2*I*d*(d*x+c)^3/b^2-2*d*(d*x+c)^3*cot(b*x+a)/b^2-1/2*(d*x+c)^4*csc(b*x+a)
^2/b+6*d^2*(d*x+c)^2*ln(1-exp(2*I*(b*x+a)))/b^3-6*I*d^3*(d*x+c)*polylog(2,
exp(2*I*(b*x+a)))/b^4+3*d^4*polylog(3,exp(2*I*(b*x+a)))/b^5
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 512 vs. $2(137) = 274$.

Time = 6.40 (sec) , antiderivative size = 512, normalized size of antiderivative = 3.74

$$\int (c+dx)^4 \cot(a+bx)\csc^2(a+bx)\,dx = -\frac{(c+dx)^4\csc^2(a+bx)}{2b}$$
$$-\frac{d^4 e^{ia}\csc(a)\left(2b^3e^{-2ia}x^3+3ib^2(1-e^{-2ia})x^2\log\left(1-e^{-i(a+bx)}\right)+3ib^2(1-e^{-2ia})x^2\log\left(1+e^{-i(a+bx)}\right)\right.}{}$$
$$+\frac{6c^2d^2\csc(a)(-bx\cos(a)+\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a))}{b^3\left(\cos^2(a)+\sin^2(a)\right)}$$
$$+\frac{2\csc(a)\csc(a+bx)\left(c^3d\sin(bx)+3c^2d^2x\sin(bx)+3cd^3x^2\sin(bx)+d^4x^3\sin(bx)\right)}{b^2}$$
$$-\frac{6cd^3\csc(a)\sec(a)\left(b^2e^{i\arctan(\tan(a))}x^2+\frac{(ibx(-\pi+2\arctan(\tan(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx+\arctan(\tan(a)))\log\left(1-e^{2i(bx+a}\right.\right.}{b^4\sqrt{\sec^2(a)\left(\cos^2(a)\right.}}$$

input
```
Integrate[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

output
```
-1/2*((c + d*x)^4*Csc[a + b*x]^2)/b - (d^4*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^(
(2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] +
(3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 - E^
((-2*I)*a))*x*PolyLog[2, -E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*P
olyLog[2, E^((-I)*(a + b*x))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^((-
I)*(a + b*x))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]))
/b^5 + (6*c^2*d^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin
[b*x]]*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (2*Csc[a]*Csc[a + b*x]*(c^3*
d*Sin[b*x] + 3*c^2*d^2*x*Sin[b*x] + 3*c*d^3*x^2*Sin[b*x] + d^4*x^3*Sin[b*x
]))/b^2 - (6*c*d^3*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(
-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan
[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*Arc
Tan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x +
ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^4*Sqrt[Sec[a]^2*(Cos[a]
^2 + Sin[a]^2)])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.31, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {4910, 3042, 4672, 3042, 25, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \cot(a+bx) \csc^2(a+bx)\, dx$$

$$\downarrow 4910$$

$$\frac{2d \int (c+dx)^3 \csc^2(a+bx) dx}{b} - \frac{(c+dx)^4 \csc^2(a+bx)}{2b}$$

$$\downarrow 3042$$

$$\frac{2d \int (c+dx)^3 \csc(a+bx)^2 dx}{b} - \frac{(c+dx)^4 \csc^2(a+bx)}{2b}$$

$$\downarrow 4672$$

$$\frac{2d\left(\frac{3d \int (c+dx)^2 \cot(a+bx) dx}{b} - \frac{(c+dx)^3 \cot(a+bx)}{b}\right)}{b} - \frac{(c+dx)^4 \csc^2(a+bx)}{2b}$$

$$\downarrow 3042$$

$$\frac{2d\left(\frac{3d \int -(c+dx)^2 \tan\left(a+bx+\frac{\pi}{2}\right) dx}{b} - \frac{(c+dx)^3 \cot(a+bx)}{b}\right)}{b} - \frac{(c+dx)^4 \csc^2(a+bx)}{2b}$$

$$\downarrow 25$$

$$\frac{2d\left(-\frac{3d \int (c+dx)^2 \tan\left(\frac{1}{2}(2a+\pi)+bx\right) dx}{b} - \frac{(c+dx)^3 \cot(a+bx)}{b}\right)}{b} - \frac{(c+dx)^4 \csc^2(a+bx)}{2b}$$

$$\downarrow 4202$$

$$-\frac{(c+dx)^4 \csc^2(a+bx)}{2b} + \frac{2d\left(-\frac{(c+dx)^3 \cot(a+bx)}{b} - \frac{3d\left(\frac{i(c+dx)^3}{3d} - 2i \int \frac{e^{i(2a+2bx+\pi)}(c+dx)^2}{1+e^{i(2a+2bx+\pi)}} dx\right)}{b}\right)}{b}$$

$$\downarrow 2620$$

$$\frac{2d\left(-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\int(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}\right)}{b}-\frac{(c+dx)^4\csc^2(a+bx)}{2b}+$$

$$\downarrow 3011$$

$$\frac{2d\left(-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i}\right.}{2b}\right.\right.\right.}{b}\right)}{b}-\frac{(c+dx)^4\csc^2(a+bx)}{2b}+$$

$$\downarrow 2720$$

$$\frac{2d\left(-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2b}}{4b^2}\right)}{b}\right.\right.\right.}{b}\right)}{b}-\frac{(c+dx)^4\csc^2(a+bx)}{2b}+$$

$$\downarrow 7143$$

$$\frac{2d\left(-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+\right.}{2b}\right.\right.\right.}{b}\right)}{b}-\frac{(c+dx)^4\csc^2(a+bx)}{2b}+$$

input

```
Int[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

output
```
-1/2*((c + d*x)^4*Csc[a + b*x]^2)/b + (2*d*(-(((c + d*x)^3*Cot[a + b*x])/b
) - (3*d*(((I/3)*(c + d*x)^3)/d - (2*I)*(((-1/2*I)*(c + d*x)^2*Log[1 + E^(
I*(2*a + Pi + 2*b*x))])/b + (I*d*(((I/2)*(c + d*x)*PolyLog[2, -E^(I*(2*a +
 Pi + 2*b*x))])/b - (d*PolyLog[3, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)))/b)
))/b))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 715 vs. $2(127) = 254$.

Time = 0.63 (sec) , antiderivative size = 716, normalized size of antiderivative = 5.23

method	result
risch	$\frac{24d^3ca\ln\left(e^{i(bx+a)}\right)}{b^4} - \frac{12d^3ca\ln\left(e^{i(bx+a)}-1\right)}{b^4} - \frac{12id^4\,\mathrm{polylog}\left(2,e^{i(bx+a)}\right)x}{b^4} + \frac{12id^4a^2x}{b^4} - \frac{12id^3c\,\mathrm{polylog}\left(2,-e^{i(bx+a)}\right)}{b^4} -$

input
```
int((d*x+c)^4*cot(b*x+a)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
12*d^3/b^3*c*ln(1-exp(I*(b*x+a)))*x-12*I*d^4/b^4*polylog(2,exp(I*(b*x+a)))
*x-12*I*d^3/b^2*c*x^2-12*I*d^3/b^4*c*a^2-12*I*d^4/b^4*polylog(2,-exp(I*(b*
x+a)))*x+12*I*d^4/b^4*a^2*x-12*I*d^3/b^4*c*polylog(2,-exp(I*(b*x+a)))-12*I
*d^3/b^4*c*polylog(2,exp(I*(b*x+a)))+12*d^3/b^3*c*ln(exp(I*(b*x+a))+1)*x+1
2*d^3/b^4*c*ln(1-exp(I*(b*x+a)))*a+24*d^3/b^4*c*a*ln(exp(I*(b*x+a)))-12*d^
3/b^4*c*a*ln(exp(I*(b*x+a))-1)+6*d^4/b^3*ln(exp(I*(b*x+a))+1)*x^2+6*d^4/b^
3*ln(1-exp(I*(b*x+a)))*x^2-12*d^2/b^3*c^2*ln(exp(I*(b*x+a)))+6*d^2/b^3*c^2
*ln(exp(I*(b*x+a))-1)+6*d^2/b^3*c^2*ln(exp(I*(b*x+a))+1)-6*d^4/b^5*ln(1-ex
p(I*(b*x+a)))*a^2-12*d^4/b^5*a^2*ln(exp(I*(b*x+a)))+6*d^4/b^5*a^2*ln(exp(I
*(b*x+a))-1)-4*I*d^4/b^2*x^3+8*I*d^4/b^5*a^3-24*I*d^3/b^3*c*a*x+2*(b*d^4*x
^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^3*exp(2*I*(b*x+a))+6*b*c^2*d^2*x^2*exp(2*I
*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*x+a))-2*I*d^4*x^3*exp(2*I*(b*x+a))+b*c^4*
exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp(2*I*(b*x+a))-6*I*c^2*d^2*x*exp(2*I*(b*x
+a))+2*I*d^4*x^3-2*I*c^3*d*exp(2*I*(b*x+a))+6*I*c*d^3*x^2+6*I*c^2*d^2*x+2*
I*c^3*d)/b^2/(exp(2*I*(b*x+a))-1)^2+12*d^4*polylog(3,-exp(I*(b*x+a)))/b^5+
12*d^4*polylog(3,exp(I*(b*x+a)))/b^5
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1075 vs. $2(124) = 248$.

Time = 0.12 (sec) , antiderivative size = 1075, normalized size of antiderivative = 7.85

$$\int (c+dx)^4 \cot(a+bx) \csc^2(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^4*cot(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b
^4*c^4 + 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*c
os(b*x + a)*sin(b*x + a) - 12*(-I*b*d^4*x - I*b*c*d^3 + (I*b*d^4*x + I*b*c
*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) + I*sin(b*x + a)) - 12*(I*b*d^4*x
 + I*b*c*d^3 + (-I*b*d^4*x - I*b*c*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a)
 - I*sin(b*x + a)) - 12*(I*b*d^4*x + I*b*c*d^3 + (-I*b*d^4*x - I*b*c*d^3)*
cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 12*(-I*b*d^4*x - I
*b*c*d^3 + (I*b*d^4*x + I*b*c*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) - I
*sin(b*x + a)) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - (b^2*d^4*x
^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin
(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - (b^2*d^4*x
^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) - I*sin
(b*x + a) + 1) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4 - (b^2*c^2*d^2 - 2
*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*
x + a) + 1/2) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4 - (b^2*c^2*d^2 - 2*
a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x
 + a) + 1/2) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4 - (b
^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*cos(b*x + a)^2)*log(-c
os(b*x + a) + I*sin(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b
*c*d^3 - a^2*d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4...
```

Sympy [F]

$$\int (c+dx)^4 \cot(a+bx) \csc^2(a+bx)\,dx = \int (c+dx)^4 \cot(a+bx) \csc^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**4*cot(b*x+a)*csc(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**4*cot(a + b*x)*csc(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4549 vs. $2(124) = 248$.

Time = 0.29 (sec) , antiderivative size = 4549, normalized size of antiderivative = 33.20

$$\int (c+dx)^4 \cot(a+bx) \csc^2(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^4*cot(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/2*(c^4*cot(b*x + a)^2 - 4*a*c^3*d*cot(b*x + a)^2/b + 6*a^2*c^2*d^2*cot(
b*x + a)^2/b^2 - 4*a^3*c*d^3*cot(b*x + a)^2/b^3 + a^4*d^4*cot(b*x + a)^2/b
^4 - 8*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 -
(2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*
x + a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)
 + 1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*c^3*d/((2*(2*cos(2*b*x + 2*a) -
 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b
*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 +
 4*cos(2*b*x + 2*a) - 1)*b) + 24*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x
+ a)*sin(2*b*x + 2*a)^2 - (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)
)*cos(4*b*x + 4*a) - 2*(b*x + a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x
 + 2*a) - cos(2*b*x + 2*a) + 1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*a*c^2
*d^2/((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 -
4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x +
 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^2) - 24*(4*(b*x +
 a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (2*(b*x + a)*cos
(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x + a)*cos(2*b*x
 + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*sin(4*b*x
+ 4*a) + sin(2*b*x + 2*a))*a^2*c*d^3/((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*
x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)...
```

Giac [F]

$$\int (c+dx)^4 \cot(a+bx) \csc^2(a+bx)\, dx = \int (dx+c)^4 \cot(bx+a) \csc(bx+a)^2\, dx$$

input
```
integrate((d*x+c)^4*cot(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*cot(b*x + a)*csc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \cot(a+bx) \csc^2(a+bx)\, dx = \int \frac{\cot(a+b\,x)\ (c+d\,x)^4}{\sin(a+b\,x)^2}\, dx$$

input
```
int((cot(a + b*x)*(c + d*x)^4)/sin(a + b*x)^2,x)
```

output
```
int((cot(a + b*x)*(c + d*x)^4)/sin(a + b*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^4 \cot(a+bx) \csc^2(a+bx)\, dx$$

$$= \frac{-48 \log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right) \sin(bx+a)^2 d^4 + 96 \log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right) \sin(bx+a)^2 d^4 - 48 \cos(bx+a) b^2 c}{}$$

input
```
int((d*x+c)^4*cot(b*x+a)*csc(b*x+a)^2,x)
```

output

```
( - 8*cos(a + b*x)*sin(a + b*x)*b**3*c**3*d - 24*cos(a + b*x)*sin(a + b*x)
*b**3*c**2*d**2*x - 24*cos(a + b*x)*sin(a + b*x)*b**3*c*d**3*x**2 - 8*cos(
a + b*x)*sin(a + b*x)*b**3*d**4*x**3 - 48*cos(a + b*x)*sin(a + b*x)*b*c*d*
*3 - 48*cos(a + b*x)*sin(a + b*x)*b*d**4*x - 48*cos(a + b*x)*b**2*c*d**3*x
 - 24*cos(a + b*x)*b**2*d**4*x**2 - 12*int(x**2/tan((a + b*x)/2)**3,x)*sin
(a + b*x)**2*b**3*d**4 - 24*int(x/tan((a + b*x)/2)**3,x)*sin(a + b*x)**2*b
**3*c*d**3 - 12*int(tan((a + b*x)/2)*x**2,x)*sin(a + b*x)**2*b**3*d**4 - 2
4*int(tan((a + b*x)/2)*x,x)*sin(a + b*x)**2*b**3*c*d**3 - 24*log(tan((a +
b*x)/2)**2 + 1)*sin(a + b*x)**2*b**2*c**2*d**2 - 48*log(tan((a + b*x)/2)**
2 + 1)*sin(a + b*x)**2*d**4 + 24*log(tan((a + b*x)/2))*sin(a + b*x)**2*b**
2*c**2*d**2 + 96*log(tan((a + b*x)/2))*sin(a + b*x)**2*d**4 + sin(a + b*x)
**2*b**4*c**4 - 48*sin(a + b*x)*b*c*d**3 - 48*sin(a + b*x)*b*d**4*x - 2*b*
*4*c**4 - 8*b**4*c**3*d*x - 12*b**4*c**2*d**2*x**2 - 8*b**4*c*d**3*x**3 -
2*b**4*d**4*x**4 - 48*b**2*c*d**3*x - 24*b**2*d**4*x**2)/(4*sin(a + b*x)**
2*b**5)
```

3.47 $\int (c+dx)^3 \cot(a+bx) \csc^2(a+bx)\, dx$

Optimal result . . . 514
Mathematica [A] (warning: unable to verify) . . . 515
Rubi [A] (verified) . . . 515
Maple [B] (verified) . . . 518
Fricas [B] (verification not implemented) . . . 519
Sympy [F] . . . 519
Maxima [B] (verification not implemented) . . . 520
Giac [F] . . . 521
Mupad [F(-1)] . . . 521
Reduce [F] . . . 521

Optimal result

Integrand size = 22, antiderivative size = 115

$$\int (c+dx)^3 \cot(a+bx) \csc^2(a+bx)\, dx = -\frac{3id(c+dx)^2}{2b^2} - \frac{3d(c+dx)^2 \cot(a+bx)}{2b^2} - \frac{(c+dx)^3 \csc^2(a+bx)}{2b} + \frac{3d^2(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^3} - \frac{3id^3 \operatorname{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^4}$$

output

```
-3/2*I*d*(d*x+c)^2/b^2-3/2*d*(d*x+c)^2*cot(b*x+a)/b^2-1/2*(d*x+c)^3*csc(b*
x+a)^2/b+3*d^2*(d*x+c)*ln(1-exp(2*I*(b*x+a)))/b^3-3/2*I*d^3*polylog(2,exp(
2*I*(b*x+a)))/b^4
```

Mathematica [A] (warning: unable to verify)

Time = 4.97 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.76

$$\int (c+dx)^3 \cot(a+bx) \csc^2(a+bx)\, dx$$

$$= \frac{-b^3(c+dx)^3 \csc^2(a+bx) - 6bcd^2(bx\cot(a) - \log(\sin(a+bx))) + 3d^3\Big(ibx(\pi - 2\arctan(\tan(a))) + \pi \log$$

input

```
Integrate[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

output

```
(-(b^3*(c + d*x)^3*Csc[a + b*x]^2) - 6*b*c*d^2*(b*x*Cot[a] - Log[Sin[a + b
*x]]) + 3*d^3*(I*b*x*(Pi - 2*ArcTan[Tan[a]]) + Pi*Log[1 + E^((-2*I)*b*x)]
+ 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] - Pi*
Log[Cos[b*x]] - 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] - I*PolyLo
g[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))] - b^2*E^(I*ArcTan[Tan[a]])*x^2*Cot[
a]*Sqrt[Sec[a]^2]) + 3*b^2*d*(c + d*x)^2*Csc[a]*Csc[a + b*x]*Sin[b*x])/(2*
b^4)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {4910, 3042, 4672, 3042, 25, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \cot(a+bx) \csc^2(a+bx)\, dx$$

$$\downarrow 4910$$

$$\frac{3d\int (c+dx)^2 \csc^2(a+bx)dx}{2b} - \frac{(c+dx)^3 \csc^2(a+bx)}{2b}$$

$$\downarrow 3042$$

$$\frac{3d\int (c+dx)^2 \csc(a+bx)^2 dx}{2b} - \frac{(c+dx)^3 \csc^2(a+bx)}{2b}$$

$$\downarrow 4672$$

$$\frac{3d\left(\frac{2d\int(c+dx)\cot(a+bx)dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}\right)}{2b}-\frac{(c+dx)^3\csc^2(a+bx)}{2b}$$

$$\downarrow 3042$$

$$\frac{3d\left(\frac{2d\int-((c+dx)\tan\left(a+bx+\frac{\pi}{2}\right))dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}\right)}{2b}-\frac{(c+dx)^3\csc^2(a+bx)}{2b}$$

$$\downarrow 25$$

$$\frac{3d\left(-\frac{2d\int(c+dx)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}\right)}{2b}-\frac{(c+dx)^3\csc^2(a+bx)}{2b}$$

$$\downarrow 4202$$

$$-\frac{(c+dx)^3\csc^2(a+bx)}{2b}+\frac{3d\left(-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\int\frac{e^{i(2a+2bx+\pi)}(c+dx)}{1+e^{i(2a+2bx+\pi)}}dx\right)}{b}\right)}{2b}$$

$$\downarrow 2620$$

$$\frac{-\frac{(c+dx)^3\csc^2(a+bx)}{2b}+3d\left(-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{id\int\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}\right)}{2b}$$

$$\downarrow 2715$$

$$\frac{-\frac{(c+dx)^3\csc^2(a+bx)}{2b}+3d\left(-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{d\int e^{-i(2a+2bx+\pi)}\log\left(1+e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}\right)}{2b}$$

$$\downarrow 2838$$

$$\frac{-\frac{(c+dx)^3\csc^2(a+bx)}{2b}+3d\left(-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}\right)}{2b}$$

input
```
Int[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

output
```
-1/2*((c + d*x)^3*Csc[a + b*x]^2)/b + (3*d*(-(((c + d*x)^2*Cot[a + b*x])/b
) - (2*d*(((I/2)*(c + d*x)^2)/d - (2*I)*(((-1/2*I)*(c + d*x)*Log[1 + E^(I*
(2*a + Pi + 2*b*x))])/b - (d*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2
))))/b))/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4910

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(101) = 202$.

Time = 0.51 (sec) , antiderivative size = 409, normalized size of antiderivative = 3.56

method	result
risch	$\frac{2b\,d^3x^3e^{2i(bx+a)}-3id^3x^2e^{2i(bx+a)}+6bc\,d^2x^2e^{2i(bx+a)}-6ic\,d^2x\,e^{2i(bx+a)}+6b\,c^2dx\,e^{2i(bx+a)}-3ic^2d\,e^{2i(bx+a)}+3id^3x^2+2b\,c^3e^{2i(bx}}{b^2\left(e^{2i(bx+a)}-1\right)^2}$

input

```
int((d*x+c)^3*cot(b*x+a)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
(2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*e
xp(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-
3*I*c^2*d*exp(2*I*(b*x+a))+3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))+6*I*c*d^2*
x+3*I*c^2*d)/b^2/(exp(2*I*(b*x+a))-1)^2-6*d^2/b^3*c*ln(exp(I*(b*x+a)))+3*d
^2/b^3*c*ln(exp(I*(b*x+a))-1)+3*d^2/b^3*c*ln(exp(I*(b*x+a))+1)-3*I*d^3/b^2
*x^2-6*I*d^3/b^3*a*x-3*I*d^3/b^4*a^2+3*d^3/b^3*ln(exp(I*(b*x+a))+1)*x-3*I*
d^3/b^4*polylog(2,-exp(I*(b*x+a)))+3*d^3/b^3*ln(1-exp(I*(b*x+a)))*x+3*d^3/
b^4*ln(1-exp(I*(b*x+a)))*a-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4+6*d^3/b^4
*a*ln(exp(I*(b*x+a)))-3*d^3/b^4*a*ln(exp(I*(b*x+a))-1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 591 vs. $2(98) = 196$.

Time = 0.10 (sec) , antiderivative size = 591, normalized size of antiderivative = 5.14

$$\int (c+dx)^3 \cot(a+bx) \csc^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cot(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 + 3*(b^2*d^3*
x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a) - 3*(I*d^3*cos(
b*x + a)^2 - I*d^3)*dilog(cos(b*x + a) + I*sin(b*x + a)) - 3*(-I*d^3*cos(b
*x + a)^2 + I*d^3)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*(-I*d^3*cos(b*
x + a)^2 + I*d^3)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 3*(I*d^3*cos(b*x
 + a)^2 - I*d^3)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*(b*d^3*x + b*c*
d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a
) + 1) - 3*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*log(co
s(b*x + a) - I*sin(b*x + a) + 1) - 3*(b*c*d^2 - a*d^3 - (b*c*d^2 - a*d^3)*
cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 3*(b*c
*d^2 - a*d^3 - (b*c*d^2 - a*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1
/2*I*sin(b*x + a) + 1/2) - 3*(b*d^3*x + a*d^3 - (b*d^3*x + a*d^3)*cos(b*x
 + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b*d^3*x + a*d^3 - (b*
d^3*x + a*d^3)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1))/(b
^4*cos(b*x + a)^2 - b^4)
```

Sympy [F]

$$\int (c+dx)^3 \cot(a+bx) \csc^2(a+bx)\, dx = \int (c+dx)^3 \cot(a+bx) \csc^2(a+bx)\, dx$$

input
```
integrate((d*x+c)**3*cot(b*x+a)*csc(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**3*cot(a + b*x)*csc(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1044 vs. $2(98) = 196$.

Time = 0.33 (sec) , antiderivative size = 1044, normalized size of antiderivative = 9.08

$$\int (c+dx)^3 \cot(a+bx) \csc^2(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*cot(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

output

```
(6*b^2*c^2*d + 6*(b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*cos(4*b*x + 4*a)
 - 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a) - (-I*b*d^3*x - I*b*c*d^2)*sin(4
*b*x + 4*a) - 2*(I*b*d^3*x + I*b*c*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x
+ a), cos(b*x + a) + 1) + 6*(b*c*d^2*cos(4*b*x + 4*a) - 2*b*c*d^2*cos(2*b*
x + 2*a) + I*b*c*d^2*sin(4*b*x + 4*a) - 2*I*b*c*d^2*sin(2*b*x + 2*a) + b*c
*d^2)*arctan2(sin(b*x + a), cos(b*x + a) - 1) - 6*(b*d^3*x*cos(4*b*x + 4*a
) - 2*b*d^3*x*cos(2*b*x + 2*a) + I*b*d^3*x*sin(4*b*x + 4*a) - 2*I*b*d^3*x*
sin(2*b*x + 2*a) + b*d^3*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 6*(
b^2*d^3*x^2 + 2*b^2*c*d^2*x)*cos(4*b*x + 4*a) - 2*(2*I*b^3*d^3*x^3 + 2*I*b
^3*c^3 + 3*b^2*c^2*d + 3*(2*I*b^3*c*d^2 - b^2*d^3)*x^2 + 6*(I*b^3*c^2*d -
b^2*c*d^2)*x)*cos(2*b*x + 2*a) - 6*(d^3*cos(4*b*x + 4*a) - 2*d^3*cos(2*b*x
 + 2*a) + I*d^3*sin(4*b*x + 4*a) - 2*I*d^3*sin(2*b*x + 2*a) + d^3)*dilog(-
e^(I*b*x + I*a)) - 6*(d^3*cos(4*b*x + 4*a) - 2*d^3*cos(2*b*x + 2*a) + I*d^
3*sin(4*b*x + 4*a) - 2*I*d^3*sin(2*b*x + 2*a) + d^3)*dilog(e^(I*b*x + I*a)
) - 3*(I*b*d^3*x + I*b*c*d^2 + (I*b*d^3*x + I*b*c*d^2)*cos(4*b*x + 4*a) +
2*(-I*b*d^3*x - I*b*c*d^2)*cos(2*b*x + 2*a) - (b*d^3*x + b*c*d^2)*sin(4*b*
x + 4*a) + 2*(b*d^3*x + b*c*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + si
n(b*x + a)^2 + 2*cos(b*x + a) + 1) - 3*(I*b*d^3*x + I*b*c*d^2 + (I*b*d^3*x
 + I*b*c*d^2)*cos(4*b*x + 4*a) + 2*(-I*b*d^3*x - I*b*c*d^2)*cos(2*b*x + 2*
a) - (b*d^3*x + b*c*d^2)*sin(4*b*x + 4*a) + 2*(b*d^3*x + b*c*d^2)*sin(2...
```

Giac [F]

$$\int (c+dx)^3 \cot(a+bx) \csc^2(a+bx)\, dx = \int (dx+c)^3 \cot(bx+a) \csc(bx+a)^2\, dx$$

input
```
integrate((d*x+c)^3*cot(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*cot(b*x + a)*csc(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \cot(a+bx) \csc^2(a+bx)\, dx = \int \frac{\cot(a+b\,x)\ (c+d\,x)^3}{\sin(a+b\,x)^2}\, dx$$

input
```
int((cot(a + b*x)*(c + d*x)^3)/sin(a + b*x)^2,x)
```

output
```
int((cot(a + b*x)*(c + d*x)^3)/sin(a + b*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^3 \cot(a+bx) \csc^2(a+bx)\, dx$$

$$= \frac{-6\cos(bx+a)\sin(bx+a)\,b^2c^2d - 12\cos(bx+a)\sin(bx+a)\,b^2c\,d^2x - 6\cos(bx+a)\sin(bx+a)\,b^2d^3x}{}$$

input
```
int((d*x+c)^3*cot(b*x+a)*csc(b*x+a)^2,x)
```

output

```
( - 6*cos(a + b*x)*sin(a + b*x)*b**2*c**2*d - 12*cos(a + b*x)*sin(a + b*x)
*b**2*c*d**2*x - 6*cos(a + b*x)*sin(a + b*x)*b**2*d**3*x**2 - 12*cos(a + b
*x)*sin(a + b*x)*d**3 - 12*cos(a + b*x)*b*d**3*x - 6*int(x/tan((a + b*x)/2
)**3,x)*sin(a + b*x)**2*b**2*d**3 - 6*int(tan((a + b*x)/2)*x,x)*sin(a + b*
x)**2*b**2*d**3 - 12*log(tan((a + b*x)/2)**2 + 1)*sin(a + b*x)**2*b*c*d**2
 + 12*log(tan((a + b*x)/2))*sin(a + b*x)**2*b*c*d**2 + sin(a + b*x)**2*b**
3*c**3 - 12*sin(a + b*x)*d**3 - 2*b**3*c**3 - 6*b**3*c**2*d*x - 6*b**3*c*d
**2*x**2 - 2*b**3*d**3*x**3 - 12*b*d**3*x)/(4*sin(a + b*x)**2*b**4)
```

3.48 $\int (c+dx)^2 \cot(a+bx) \csc^2(a+bx)\, dx$

Optimal result . . . 523
Mathematica [C] (verified) . . . 523
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Giac [B] (verification not implemented) . . . 528
Mupad [B] (verification not implemented) . . . 529
Reduce [B] (verification not implemented) . . . 530

Optimal result

Integrand size = 22, antiderivative size = 54

$$\int (c+dx)^2 \cot(a+bx) \csc^2(a+bx)\, dx = -\frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2 \csc^2(a+bx)}{2b} + \frac{d^2 \log(\sin(a+bx))}{b^3}$$

output

```
-d*(d*x+c)*cot(b*x+a)/b^2-1/2*(d*x+c)^2*csc(b*x+a)^2/b+d^2*ln(sin(b*x+a))/
b^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.74

$$\int (c+dx)^2 \cot(a+bx) \csc^2(a+bx)\, dx = \frac{2ibd^2x - 2id^2 \arctan(\tan(a+bx)) - 2bd^2x\cot(a) - b^2(c+dx)^2 \csc^2(a+bx) + d^2 \log\left(\sin^2(a+bx)\right) + 2}{2b^3}$$

input

```
Integrate[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

output

```
((2*I)*b*d^2*x - (2*I)*d^2*ArcTan[Tan[a + b*x]] - 2*b*d^2*x*Cot[a] - b^2*(
c + d*x)^2*Csc[a + b*x]^2 + d^2*Log[Sin[a + b*x]^2] + 2*b*d*(c + d*x)*Csc[
a]*Csc[a + b*x]*Sin[b*x])/(2*b^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4910, 3042, 4672, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \cot(a+bx) \csc^2(a+bx)\, dx$$

$$\downarrow 4910$$

$$\frac{d\int (c+dx)\csc^2(a+bx)dx}{b} - \frac{(c+dx)^2\csc^2(a+bx)}{2b}$$

$$\downarrow 3042$$

$$\frac{d\int (c+dx)\csc(a+bx)^2dx}{b} - \frac{(c+dx)^2\csc^2(a+bx)}{2b}$$

$$\downarrow 4672$$

$$\frac{d\left(\frac{d\int \cot(a+bx)dx}{b} - \frac{(c+dx)\cot(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2\csc^2(a+bx)}{2b}$$

$$\downarrow 3042$$

$$\frac{d\left(\frac{d\int -\tan\left(a+bx+\frac{\pi}{2}\right)dx}{b} - \frac{(c+dx)\cot(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2\csc^2(a+bx)}{2b}$$

$$\downarrow 25$$

$$\frac{d\left(-\frac{d\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx}{b} - \frac{(c+dx)\cot(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2\csc^2(a+bx)}{2b}$$

$$\downarrow 3956$$

$$\frac{d\left(\frac{d\log(-\sin(a+bx))}{b^2} - \frac{(c+dx)\cot(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2\csc^2(a+bx)}{2b}$$

input
```
Int[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

output
```
-1/2*((c + d*x)^2*Csc[a + b*x]^2)/b + (d*(-(((c + d*x)*Cot[a + b*x])/b) +
(d*Log[-Sin[a + b*x]])/b^2))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.74

method	result
risch	$-\frac{2id^2x}{b^2}-\frac{2id^2a}{b^3}+\frac{2b\,d^2x^2e^{2i(bx+a)}+4bcdx\,e^{2i(bx+a)}+2b\,c^2e^{2i(bx+a)}-2id^2x\,e^{2i(bx+a)}-2icd\,e^{2i(bx+a)}+2id^2x+2i}{b^2\left(e^{2i(bx+a)}-1\right)^2}$
derivativedivides	$\frac{-\frac{a^2d^2}{2b^2\sin(bx+a)^2}+\frac{acd}{b\sin(bx+a)^2}-\frac{2a\,d^2\left(-\frac{bx+a}{2\sin(bx+a)^2}-\frac{\cot(bx+a)}{2}\right)}{b^2}-\frac{c^2}{2\sin(bx+a)^2}+\frac{2cd\left(-\frac{bx+a}{2\sin(bx+a)^2}-\frac{\cot(bx+a)}{2}\right)}{b}+\frac{d^2}{}}{b}$
default	$\frac{-\frac{a^2d^2}{2b^2\sin(bx+a)^2}+\frac{acd}{b\sin(bx+a)^2}-\frac{2a\,d^2\left(-\frac{bx+a}{2\sin(bx+a)^2}-\frac{\cot(bx+a)}{2}\right)}{b^2}-\frac{c^2}{2\sin(bx+a)^2}+\frac{2cd\left(-\frac{bx+a}{2\sin(bx+a)^2}-\frac{\cot(bx+a)}{2}\right)}{b}+\frac{d^2}{}}{b}$

input
```
int((d*x+c)^2*cot(b*x+a)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*I*d^2/b^2*x-2*I*d^2/b^3*a+2*(b*d^2*x^2*exp(2*I*(b*x+a))+2*b*c*d*x*exp(2
*I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^2*x*exp(2*I*(b*x+a))-I*c*d*exp(2*I*
(b*x+a))+I*d^2*x+I*d*c)/b^2/(exp(2*I*(b*x+a))-1)^2+d^2/b^3*ln(exp(2*I*(b*x
+a))-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.89

$$\int (c+dx)^2\cot(a+bx)\csc^2(a+bx)\,dx$$
$$=\frac{b^2d^2x^2+2\,b^2cdx+b^2c^2+2\left(bd^2x+bcd\right)\cos\left(bx+a\right)\sin\left(bx+a\right)+2\left(d^2\cos\left(bx+a\right)^2-d^2\right)\log\left(\frac{1}{2}\sin\left(\right.\right.}{2\left(b^3\cos\left(bx+a\right)^2-b^3\right)}$$

input
```
integrate((d*x+c)^2*cot(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a
)*sin(b*x + a) + 2*(d^2*cos(b*x + a)^2 - d^2)*log(1/2*sin(b*x + a)))/(b^3*
cos(b*x + a)^2 - b^3)
```

Sympy [F]

$$\int (c+dx)^2 \cot(a+bx) \csc^2(a+bx)\, dx = \int (c+dx)^2 \cot(a+bx) \csc^2(a+bx)\, dx$$

input
```
integrate((d*x+c)**2*cot(b*x+a)*csc(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*cot(a + b*x)*csc(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1129 vs. $2(52) = 104$.

Time $= 0.05$ (sec) , antiderivative size $= 1129$, normalized size of antiderivative $= 20.91$

$$\int (c+dx)^2 \cot(a+bx) \csc^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*cot(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/2*(c^2*cot(b*x + a)^2 - 2*a*c*d*cot(b*x + a)^2/b + a^2*d^2*cot(b*x + a)
^2/b^2 - 4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^
2 - (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2
*(b*x + a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x +
2*a) + 1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*c*d/((2*(2*cos(2*b*x + 2*a)
 - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4
*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2
 + 4*cos(2*b*x + 2*a) - 1)*b) + 4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x
 + a)*sin(2*b*x + 2*a)^2 - (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a
))*cos(4*b*x + 4*a) - 2*(b*x + a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*
x + 2*a) - cos(2*b*x + 2*a) + 1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*a*d^
2/((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*c
os(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*
a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^2) - (8*(b*x + a)^2*
cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 - 4*(b*x + a)^2*cos(
2*b*x + 2*a) - 4*((b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a
))*cos(4*b*x + 4*a) + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4
*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x +
4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log
(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (2*(2*cos(2*b*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2978 vs. $2(52) = 104$.

Time $= 1.02$ (sec) , antiderivative size $= 2978$, normalized size of antiderivative $= 55.15$

$$\int (c + dx)^2 \cot(a + bx) \csc^2(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^2*cot(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

output

```
-1/8*(b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4
*tan(1/2*a)^4 + 2*b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*d^2*x^2*
tan(1/2*b*x)^2*tan(1/2*a)^4 + b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b^2*
c*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 4*b*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^3
+ 4*b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 - 4*b*d^2*x*tan(1/2*b*x)^3*tan(1
/2*a)^4 + b^2*d^2*x^2*tan(1/2*b*x)^4 + 4*b^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/
2*a)^2 + 2*b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^2 - 4*b*c*d*tan(1/2*b*x)^4*ta
n(1/2*a)^3 + b^2*d^2*x^2*tan(1/2*a)^4 + 2*b^2*c^2*tan(1/2*b*x)^2*tan(1/2*a
)^4 - 4*b*c*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4 + 4
*b*d^2*x*tan(1/2*b*x)^4*tan(1/2*a) + 8*b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)
^2 + 24*b*d^2*x*tan(1/2*b*x)^3*tan(1/2*a)^2 - 4*d^2*log(16*(tan(1/2*b*x)^4
*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^
4 - 2*tan(1/2*b*x)^3*tan(1/2*a) - 4*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/
2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2
*a)^2)/(tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*ta
n(1/2*b*x)^2*tan(1/2*a)^4 + tan(1/2*b*x)^4 + 4*tan(1/2*b*x)^2*tan(1/2*a)^2
 + tan(1/2*a)^4 + 2*tan(1/2*b*x)^2 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*t
an(1/2*a)^2 + 24*b*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^3 - 8*d^2*log(16*(tan(1
/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*ta
n(1/2*a)^4 - 2*tan(1/2*b*x)^3*tan(1/2*a) - 4*tan(1/2*b*x)^2*tan(1/2*a)^...
```

Mupad [B] (verification not implemented)

Time = 19.87 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.72

$$\int (c+dx)^2 \cot(a+bx)\csc^2(a+bx)\,dx = \frac{\frac{(c+dx)^2}{b} + \frac{e^{a\,2i+b\,x\,2i}\,(c+dx)^2}{b}}{1+e^{a\,4i+b\,x\,4i}-2\,e^{a\,2i+b\,x\,2i}} - \frac{d^2\,x\,2i}{b^2} + \frac{b\,c^2+2\,b\,c\,d\,x-c\,d\,2i+b\,d^2\,x^2-d^2\,x\,2i}{b^2\,\left(e^{a\,2i+b\,x\,2i}-1\right)} + \frac{d^2\,\ln\left(e^{a\,2i}\,e^{b\,x\,2i}-1\right)}{b^3}$$

input

```
int((cot(a + b*x)*(c + d*x)^2)/sin(a + b*x)^2,x)
```

output

```
((c + d*x)^2/b + (exp(a*2i + b*x*2i)*(c + d*x)^2)/b)/(exp(a*4i + b*x*4i) -
 2*exp(a*2i + b*x*2i) + 1) - (d^2*x*2i)/b^2 + (b*c^2 - c*d*2i - d^2*x*2i +
 b*d^2*x^2 + 2*b*c*d*x)/(b^2*(exp(a*2i + b*x*2i) - 1)) + (d^2*log(exp(a*2i
)*exp(b*x*2i) - 1))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.63

$$\int (c+dx)^2 \cot(a+bx) \csc^2(a+bx)\,dx$$

$$= \frac{-4\cos(bx+a)\sin(bx+a)\,bcd - 4\cos(bx+a)\sin(bx+a)\,bd^2x - 4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right)\sin(bx+a$$

$$4\sin(bx+a)^2$$

input

```
int((d*x+c)^2*cot(b*x+a)*csc(b*x+a)^2,x)
```

output

```
( - 4*cos(a + b*x)*sin(a + b*x)*b*c*d - 4*cos(a + b*x)*sin(a + b*x)*b*d**2
*x - 4*log(tan((a + b*x)/2)**2 + 1)*sin(a + b*x)**2*d**2 + 4*log(tan((a +
b*x)/2))*sin(a + b*x)**2*d**2 + sin(a + b*x)**2*b**2*c**2 - 2*b**2*c**2 -
4*b**2*c*d*x - 2*b**2*d**2*x**2)/(4*sin(a + b*x)**2*b**3)
```

3.49 $\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 35

$$\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx = -\frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\csc^2(a+bx)}{2b}$$

output
```
-1/2*d*cot(b*x+a)/b^2-1/2*(d*x+c)*csc(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx = -\frac{d\cot(a+bx)}{2b^2} - \frac{c\csc^2(a+bx)}{2b} - \frac{dx\csc^2(a+bx)}{2b}$$

input
```
Integrate[(c + d*x)*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

output
```
-1/2*(d*Cot[a + b*x])/b^2 - (c*Csc[a + b*x]^2)/(2*b) - (d*x*Csc[a + b*x]^2
)/(2*b)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4910, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx$$

$$\downarrow 4910$$

$$\frac{d\int \csc^2(a+bx)dx}{2b} - \frac{(c+dx)\csc^2(a+bx)}{2b}$$

$$\downarrow 3042$$

$$\frac{d\int \csc(a+bx)^2dx}{2b} - \frac{(c+dx)\csc^2(a+bx)}{2b}$$

$$\downarrow 4254$$

$$-\frac{d\int 1d\cot(a+bx)}{2b^2} - \frac{(c+dx)\csc^2(a+bx)}{2b}$$

$$\downarrow 24$$

$$-\frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\csc^2(a+bx)}{2b}$$

input
```
Int[(c + d*x)*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

output
```
-1/2*(d*Cot[a + b*x])/b^2 - ((c + d*x)*Csc[a + b*x]^2)/(2*b)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

method	result	size
derivativedivides	$\frac{\frac{da}{2b\sin(bx+a)^2}-\frac{c}{2\sin(bx+a)^2}+\frac{d\left(-\frac{bx+a}{2\sin(bx+a)^2}-\frac{\cot(bx+a)}{2}\right)}{b}}{b}$	61
default	$\frac{\frac{da}{2b\sin(bx+a)^2}-\frac{c}{2\sin(bx+a)^2}+\frac{d\left(-\frac{bx+a}{2\sin(bx+a)^2}-\frac{\cot(bx+a)}{2}\right)}{b}}{b}$	61
risch	$\frac{2bdx\,e^{2i(bx+a)}-id\,e^{2i(bx+a)}+2bc\,e^{2i(bx+a)}+id}{b^2\left(e^{2i(bx+a)}-1\right)^2}$	63

input
```
int((d*x+c)*cot(b*x+a)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2/b*d*a/sin(b*x+a)^2-1/2*c/sin(b*x+a)^2+1/b*d*(-1/2*(b*x+a)/sin(b*x
+a)^2-1/2*cot(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx = \frac{bdx + d\cos(bx+a)\sin(bx+a) + bc}{2\left(b^2\cos(bx+a)^2 - b^2\right)}$$

input
```
integrate((d*x+c)*cot(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(b*d*x + d*cos(b*x + a)*sin(b*x + a) + b*c)/(b^2*cos(b*x + a)^2 - b^2)
```

Sympy [F]

$$\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx = \int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx$$

input
```
integrate((d*x+c)*cot(b*x+a)*csc(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*cot(a + b*x)*csc(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(31) = 62.

Time = 0.04 (sec) , antiderivative size = 287, normalized size of antiderivative = 8.20

$$\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx =$$

$$-\frac{c\cot(bx+a)^2 - \frac{ad\cot(bx+a)^2}{b} - \frac{2\left(4(bx+a)\cos(2bx+2a)^2+4(bx+a)\sin(2bx+2a)^2-(2(bx+a)\cos(2bx+2a)+\sin(2bx+2a))\cos\right.}{\left(2(2\cos(2bx+2a)-1)\cos(4bx+4a)-\cos(4bx+4a)^2-4\cos(2bx+2a)^2\right.}}{2b}$$

input
```
integrate((d*x+c)*cot(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(c*cot(b*x + a)^2 - a*d*cot(b*x + a)^2/b - 2*(4*(b*x + a)*cos(2*b*x +
 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (2*(b*x + a)*cos(2*b*x + 2*a) +
 sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x + a)*cos(2*b*x + 2*a) - (2*(b
*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*sin(4*b*x + 4*a) + sin(2*
b*x + 2*a))*d/((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x +
4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*si
n(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(31) = 62$.

Time $= 0.20$ (sec) , antiderivative size $= 526$, normalized size of antiderivative $= 15.03$

$$\int (c + dx)\cot(a + bx)\csc^2(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*cot(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/8*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4
+ 2*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^
4 + 2*b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*d*tan(1/2*b*x)^4*tan(1/2*a)^3 +
2*b*c*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + b*d*
x*tan(1/2*b*x)^4 + 4*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*d*x*tan(1/2*a)^
4 + b*c*tan(1/2*b*x)^4 + 2*d*tan(1/2*b*x)^4*tan(1/2*a) + 4*b*c*tan(1/2*b*x
)^2*tan(1/2*a)^2 + 12*d*tan(1/2*b*x)^3*tan(1/2*a)^2 + 12*d*tan(1/2*b*x)^2*
tan(1/2*a)^3 + b*c*tan(1/2*a)^4 + 2*d*tan(1/2*b*x)*tan(1/2*a)^4 + 2*b*d*x*
tan(1/2*b*x)^2 + 2*b*d*x*tan(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2 - 2*d*tan(1/2
*b*x)^3 - 12*d*tan(1/2*b*x)^2*tan(1/2*a) + 2*b*c*tan(1/2*a)^2 - 12*d*tan(1
/2*b*x)*tan(1/2*a)^2 - 2*d*tan(1/2*a)^3 + b*d*x + b*c + 2*d*tan(1/2*b*x) +
 2*d*tan(1/2*a))/(b^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*tan(1/2*b*x)^3*t
an(1/2*a)^3 + b^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*b^2*tan(1/2*b*x)^3*tan(1
/2*a) - 4*b^2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^2*tan(1/2*b*x)*tan(1/2*a)^
3 + b^2*tan(1/2*b*x)^2 + 2*b^2*tan(1/2*b*x)*tan(1/2*a) + b^2*tan(1/2*a)^2)
```

Mupad [B] (verification not implemented)

Time = 19.42 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx = \frac{d\,1\mathrm{i} - \mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\left(-b\left(2\,c+2\,d\,x\right)+d\,1\mathrm{i}\right)}{b^2\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^2}$$

input
```
int((cot(a + b*x)*(c + d*x))/sin(a + b*x)^2,x)
```

output
```
(d*1i - exp(a*2i + b*x*2i)*(d*1i - b*(2*c + 2*d*x)))/(b^2*(exp(a*2i + b*x*
2i) - 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.40

$$\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx$$
$$= \frac{-2\cos(bx+a)\sin(bx+a)\,d + \sin(bx+a)^2\,bc - 2bc - 2bdx}{4\sin(bx+a)^2\,b^2}$$

input
```
int((d*x+c)*cot(b*x+a)*csc(b*x+a)^2,x)
```

output
```
( - 2*cos(a + b*x)*sin(a + b*x)*d + sin(a + b*x)**2*b*c - 2*b*c - 2*b*d*x)
/(4*sin(a + b*x)**2*b**2)
```

3.50 $\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$

Optimal result . . . 537
Mathematica [N/A] . . . 537
Rubi [N/A] . . . 538
Maple [N/A] . . . 538
Fricas [N/A] . . . 539
Sympy [N/A] . . . 539
Maxima [N/A] . . . 540
Giac [N/A] . . . 541
Mupad [N/A] . . . 541
Reduce [N/A] . . . 541

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}, x\right)$$

output
```
Defer(Int)(cot(b*x+a)*csc(b*x+a)^2/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 8.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x),x]
```

output
```
Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$$
$$\downarrow 7299$$
$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$$

input
```
Int[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cot\left(bx+a\right)\csc\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
int(cot(b*x+a)*csc(b*x+a)^2/(d*x+c),x)
```

output
```
int(cot(b*x+a)*csc(b*x+a)^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx = \int \frac{\cot\left(bx+a\right)\csc\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(cot(b*x + a)*csc(b*x + a)^2/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx = \int \frac{\cot\left(a+bx\right)\csc^2\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(cot(a + b*x)*csc(a + b*x)**2/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 1664, normalized size of antiderivative = 75.64

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx = \int \frac{\cot\left(bx+a\right)\csc\left(bx+a\right)^2}{dx+c}\,dx$$

input

```
integrate(cot(b*x+a)*csc(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output

```
-(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2
- (2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
 - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c
^2*d^2 + (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(4*b*x + 4*a)^2 +
4*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(2*b*x + 2*a)^2 + (b^2*d^
4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^4*x^2 +
 2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d
^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^4*x^2
+ 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d
^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^4*x^2 + 2*b^2*c*d^3*x +
b^2*c^2*d^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b^2*d^3*x^3 + 3*b^2
*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3)*sin(b*x + a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
+ 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)), x) - (b^2*d^4*x^2 + 2*b^2*c*d^3*
x + b^2*c^2*d^2 + (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(4*b*x +
4*a)^2 + 4*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(2*b*x + 2*a)^2
+ (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*
d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) +
 4*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(2*b*x + 2*a)^2 + 2*(...
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx = \int \frac{\cot(bx+a)\csc(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(cot(b*x + a)*csc(b*x + a)^2/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 18.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+b\,x)}{\sin(a+b\,x)^2\,(c+d\,x)}\,dx$$

input
```
int(cot(a + b*x)/(sin(a + b*x)^2*(c + d*x)),x)
```

output
```
int(cot(a + b*x)/(sin(a + b*x)^2*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx = \int \frac{\cot(bx+a)\csc(bx+a)^2}{dx+c}\,dx$$

input
```
int(cot(b*x+a)*csc(b*x+a)^2/(d*x+c),x)
```

output
```
int((cot(a + b*x)*csc(a + b*x)**2)/(c + d*x),x)
```

3.51 $\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 543
Mathematica [N/A] . . . 543
Rubi [N/A] . . . 544
Maple [N/A] . . . 544
Fricas [N/A] . . . 545
Sympy [N/A] . . . 545
Maxima [N/A] . . . 546
Giac [N/A] . . . 547
Mupad [N/A] . . . 547
Reduce [N/A] . . . 547

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(cot(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 8.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx$$
$$\downarrow 7299$$
$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}dx$$

input
```
Int[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cot\left(bx+a\right)\csc\left(bx+a\right)^2}{\left(dx+c\right)^2}dx$$

input
```
int(cot(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(cot(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot\left(bx+a\right)\csc\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cot(b*x + a)*csc(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot\left(a+bx\right)\csc^2\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(cot(a + b*x)*csc(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 2.25 (sec) , antiderivative size = 2153, normalized size of antiderivative = 97.86

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot\left(bx+a\right)\csc\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input

```
integrate(cot(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output

```
-(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2
- 2*((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
 - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + 3*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3
*b^2*c^2*d^3*x + b^2*c^3*d^2 + (b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*
d^3*x + b^2*c^3*d^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2
 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*cos(2*b*x + 2*a)^2 + (b^2*d^5*x^3 + 3*b^
2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d
^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*sin(4*b*x + 4*a)
*sin(2*b*x + 2*a) + 4*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b
^2*c^3*d^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*
c^2*d^3*x + b^2*c^3*d^2 - 2*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3
*x + b^2*c^3*d^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^5*x^3 + 3*
b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*cos(2*b*x + 2*a))*integrate
(sin(b*x + a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c
^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*
b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6
*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(b*x + a)^2 + 2*(b^2*d^4*x^
4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x
 + a)), x) - 3*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*
d^2 + (b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*c...
```

Giac [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(bx+a)\csc(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(cot(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(cot(b*x + a)*csc(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 19.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(a+b\,x)}{\sin(a+b\,x)^2\,(c+d\,x)^2}\,dx$$

input
```
int(cot(a + b*x)/(sin(a + b*x)^2*(c + d*x)^2),x)
```

output
```
int(cot(a + b*x)/(sin(a + b*x)^2*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(bx+a)\csc(bx+a)^2}{d^2x^2+2cdx+c^2}\,dx$$

input
```
int(cot(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int((cot(a + b*x)*csc(a + b*x)**2)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.52 $\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 549
Mathematica [C] (verified) . . . 550
Rubi [A] (verified) . . . 550
Maple [A] (verified) . . . 556
Fricas [A] (verification not implemented) . . . 557
Sympy [F] . . . 557
Maxima [C] (verification not implemented) . . . 558
Giac [C] (verification not implemented) . . . 558
Mupad [F(-1)] . . . 559
Reduce [F] . . . 560

Optimal result

Integrand size = 22, antiderivative size = 196

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} - \frac{15d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}} + \frac{15d^{5/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{128b^{7/2}} + \frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{16b^2}$$

output

```
15/64*d^2*(d*x+c)^(1/2)*cos(2*b*x+2*a)/b^3-1/4*(d*x+c)^(5/2)*cos(2*b*x+2*a
)/b-15/128*d^(5/2)*Pi^(1/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1
/2)/d^(1/2)/Pi^(1/2))/b^(7/2)+15/128*d^(5/2)*Pi^(1/2)*FresnelS(2*b^(1/2)*(
d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/b^(7/2)+5/16*d*(d*x+c)^(3/
2)*sin(2*b*x+2*a)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.73

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \frac{e^{-\frac{2i(bc+ad)}{d}}(c+dx)^{5/2}\left(e^{4ia}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{2ib(c+dx)}{d}\right)+e^{\frac{4ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{2ib(c+dx)}{d}\right)\right)}{32\sqrt{2}b\left(\frac{b^2(c+dx)^2}{d^2}\right)^{3/2}}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
((c + d*x)^(5/2)*(E^((4*I)*a)*Sqrt[(I*b*(c + d*x))/d]*Gamma[7/2, ((-2*I)*b
*(c + d*x))/d] + E^(((4*I)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[7/2, (
(2*I)*b*(c + d*x))/d]))/(32*Sqrt[2]*b*E^(((2*I)*(b*c + a*d))/d)*((b^2*(c +
 d*x)^2)/d^2)^(3/2))
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.11, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {4906, 27, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin(a+bx)\cos(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \frac{1}{2}(c+dx)^{5/2}\sin(2a+2bx)dx$$

$$\downarrow 27$$

$$\frac{1}{2}\int (c+dx)^{5/2}\sin(2a+2bx)dx$$

$$\downarrow 3042$$

$$\frac{1}{2}\int (c+dx)^{5/2}\sin(2a+2bx)dx$$

$$\downarrow 3777$$

$$\frac{1}{2}\left(\frac{5d\int (c+dx)^{3/2}\cos(2a+2bx)dx}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{5d\int (c+dx)^{3/2}\sin\left(2a+2bx+\frac{\pi}{2}\right)dx}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3777$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{3d\int -\sqrt{c+dx}\sin(2a+2bx)dx}{4b}+\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}\right)}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 25$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\int \sqrt{c+dx}\sin(2a+2bx)dx}{4b}\right)}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\int \sqrt{c+dx}\sin(2a+2bx)dx}{4b}\right)}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3777$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}dx}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)}{4b}\right)}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\int\frac{\sin\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)}{4b}\right)}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3787$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)}{4b}\right)}{4b}-\frac{(c+\right.$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)}{4b}\right)}{4b}-\frac{(}{}\right.$$

↓ 3785

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)}{4b}\right)}{4b}\right)$$

↓ 3786

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\sin\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2b}{2b}\right)}{4b}\right)}{4b}\right)$$

↓ 3832

$$\frac{1}{2}\left(5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx}{2b}\right.}{4b}\right.\right.$$

$$\downarrow 3833$$

$$\frac{1}{2}\left(5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx}{2b}\right.}{4b}\right.\right.$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-1/2*((c + d*x)^(5/2)*Cos[2*a + 2*b*x])/b + (5*d*((-3*d*(-1/2*(Sqrt[c + d
*x]*Cos[2*a + 2*b*x])/b + (d*((Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*S
qrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(Sqrt[b]*Sqrt[d]) - (Sqrt[Pi]*F
resnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(Sqrt[b]*Sqrt[d])))/(4*b)))/(4*b) + ((c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(
2*b)))/(4*b))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.19

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}\right.\right.}{d}$
default	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}\right.\right.}{d}$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+5/8/b*d*(1/4/
b*d*(d*x+c)^(3/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+
c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(co
s(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(2*
(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.13

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{15\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\,(1$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/128*(15*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(
d*x + c)*sqrt(b/(pi*d))) - 15*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x
 + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(16*b^3*d^2*x^2 + 32*b^3*c
*d*x + 16*b^3*c^2 - 15*b*d^2 - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c
^2 - 15*b*d^2)*cos(b*x + a)^2 + 40*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)*sin(
b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [F]

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \int (c+dx)^{\frac{5}{2}}\sin(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**(5/2)*sin(a + b*x)*cos(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.41

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \frac{\sqrt{2}\left(160\sqrt{2}(dx+c)^{\frac{3}{2}}b^2 d\sin\left(\frac{2((dx+c)b-bc+ad)}{d}\right) - 8\left(16\sqrt{2}(dx+c)^{\frac{5}{2}}b^3 - 15\sqrt{2}\sqrt{dx+c}bd^2\right)\cos\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/1024*sqrt(2)*(160*sqrt(2)*(d*x + c)^(3/2)*b^2*d*sin(2*((d*x + c)*b - b*c
 + a*d)/d) - 8*(16*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x + c)*
b*d^2)*cos(2*((d*x + c)*b - b*c + a*d)/d) - 15*(-(I - 1)*4^(1/4)*sqrt(pi)*
d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(pi)*d^3*(
b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 1
5*((I + 1)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I
 - 1)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(-2*I*b/d)))/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 1192, normalized size of antiderivative = 6.08

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output

```
1/256*(64*(sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + s
qrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 - d^3*((
sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 15*I*d^3)*d*erf(-I*sq
rt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d
)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^
2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*
x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^
(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + (sqrt(pi)*(64*b^3*c^3 +
 48*I*b^2*c^2*d - 36*b*c*d^2 - 15*I*d^3)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)
^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 -
 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-2*(I*(d*x + c)*b -
 I*b*c + I*a*d)/d)/b^3)/d^3) - 48*(sqrt(pi)*(4*b*c - I*d)*d*erf(-I*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(pi)*(4*b*c + I*d)*d*erf(I*sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sqrt(d*x + c)*d*e^(-2*(I...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)\,(c+d\,x)^{5/2}\,dx$$

input

```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(5/2),x)
```

output

```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c + dx)^{5/2} \cos(a+bx) \sin(a+bx)\, dx = \left(\int \sqrt{dx+c} \cos(bx+a) \sin(bx+a)\, x^2 dx \right) d^2 + 2\left(\int \sqrt{dx+c} \cos(bx+a) \sin(bx+a)\, x dx \right) cd + \left(\int \sqrt{dx+c} \cos(bx+a) \sin(bx+a)\, dx \right) c^2$$

input

```
int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x)
```

output

```
int(sqrt(c + d*x)*cos(a + b*x)*sin(a + b*x)*x**2,x)*d**2 + 2*int(sqrt(c +
d*x)*cos(a + b*x)*sin(a + b*x)*x,x)*c*d + int(sqrt(c + d*x)*cos(a + b*x)*s
in(a + b*x),x)*c**2
```

3.53 $\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx$

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Mupad [F(-1)] . . . 570
Reduce [F] . . . 570

Optimal result

Integrand size = 22, antiderivative size = 168

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} - \frac{3d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}} - \frac{3d^{3/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{32b^{5/2}} + \frac{3d\sqrt{c+dx}\sin(2a+2bx)}{16b^2}$$

output

```
-1/4*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b-3/32*d^(3/2)*Pi^(1/2)*cos(2*a-2*b*c/d)
*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b^(5/2)-3/32*d^(3/2)*P
i^(1/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d
)/b^(5/2)+3/16*d*(d*x+c)^(1/2)*sin(2*b*x+2*a)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.76

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \frac{e^{-\frac{2i(bc+ad)}{d}}(c+dx)^{3/2}\left(-\frac{e^{4ia}\Gamma\left(\frac{5}{2},-\frac{2ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{3/2}}-\frac{e^{\frac{4ibc}{d}}\Gamma\left(\frac{5}{2},\frac{2ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{3/2}}\right)}{16\sqrt{2}b}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
((c + d*x)^(3/2)*(-((E^((4*I)*a)*Gamma[5/2, ((-2*I)*b*(c + d*x))/d])/(((-I
)*b*(c + d*x))/d)^(3/2)) - (E^(((4*I)*b*c)/d)*Gamma[5/2, ((2*I)*b*(c + d*x
))/d])/((I*b*(c + d*x))/d)^(3/2)))/(16*Sqrt[2]*b*E^(((2*I)*(b*c + a*d))/d)
)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {4906, 27, 3042, 3777, 3042, 3777, 25, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin(a+bx)\cos(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \frac{1}{2}(c+dx)^{3/2}\sin(2a+2bx)dx$$

$$\downarrow 27$$

$$\frac{1}{2}\int (c+dx)^{3/2}\sin(2a+2bx)dx$$

$$\downarrow 3042$$

$$\frac{1}{2}\int (c+dx)^{3/2}\sin(2a+2bx)dx$$

$$\downarrow 3777$$

$$\frac{1}{2}\left(\frac{3d\int \sqrt{c+dx}\cos(2a+2bx)dx}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{3d\int \sqrt{c+dx}\sin\left(2a+2bx+\frac{\pi}{2}\right)dx}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3777$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{d\int -\frac{\sin(2a+2bx)}{\sqrt{c+dx}}dx}{4b}+\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 25$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}dx}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}dx}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3787$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx+\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2b}{2b}\right.$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx+\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a+$$

$$\downarrow 3785$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a}{2b}$$

$$\downarrow 3786$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\sin\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos}{2b}$$

$$\downarrow 3832$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos}{2b}$$

$$\downarrow 3833$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos\right.$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-1/2*((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/b + (3*d*(-1/4*(d*((Sqrt[Pi]*Cos[
2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(
Sqrt[b]*Sqrt[d]) + (Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*S
qrt[Pi])]*Sin[2*a - (2*b*c)/d])/(Sqrt[b]*Sqrt[d])))/b + (Sqrt[c + d*x]*Sin
[2*a + 2*b*x])/(2*b)))/(4*b))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.11

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{8b\sqrt{\frac{b}{d}}}\right.}{4b}}{d}$
default	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{8b\sqrt{\frac{b}{d}}}\right.}{4b}}{d}$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+3/8/b*d*(1/4/
b*d*(d*x+c)^(1/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^
(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/
d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{3\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-4\,(2\,b^2}{32\,b^3}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/32*(3*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*
x + c)*sqrt(b/(pi*d))) + 3*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x +
c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(2*b^2*d*x + 3*b*d*cos(b*x +
a)*sin(b*x + a) + 2*b^2*c - 4*(b^2*d*x + b^2*c)*cos(b*x + a)^2)*sqrt(d*x +
 c))/b^3
```

Sympy [F]

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)*cos(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.52

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{\sqrt{2}\left(32\sqrt{2}(dx+c)^{\frac{3}{2}}b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)-24\sqrt{2}\sqrt{dx+c}bd\sin\left(\frac{2((dx+c)b-bc+ad)}{d}\right)+3\left((i+1)\cdot 4^{\frac{1}{4}}\sqrt{}\right.\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/256*sqrt(2)*(32*sqrt(2)*(d*x + c)^(3/2)*b^2*cos(2*((d*x + c)*b - b*c +
a*d)/d) - 24*sqrt(2)*sqrt(d*x + c)*b*d*sin(2*((d*x + c)*b - b*c + a*d)/d)
+ 3*((I + 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) -
(I - 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(2*I*b/d)) + 3*(-(I - 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1
/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*s
in(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 732, normalized size of antiderivative = 4.36

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
1/64*(16*(sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sq
rt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2
*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 - 8*(sqrt
(pi)*(4*b*c - I*d)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) +
sqrt(pi)*(4*b*c + I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
*b) + 2*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 2*sqr
t(d*x + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c + sqrt(pi)*(16
*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt
(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2)
 + 1)*b^2) + sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(I*sqrt(b*d)*s
qrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I
*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c +
 I*a*d)/d)/b^2 + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d -
3*sqrt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)\,(c+d\,x)^{3/2}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c + dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \left(\int \sqrt{dx+c}\,\cos(bx+a)\sin(bx+a)\,x dx\right) d + \left(\int \sqrt{dx+c}\,\cos(bx+a)\sin(bx+a)\,dx\right) c$$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)*sin(a + b*x)*x,x)*d + int(sqrt(c + d*x)*cos
(a + b*x)*sin(a + b*x),x)*c
```

3.54 $\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 571
Mathematica [C] (verified) . . . 572
Rubi [A] (verified) . . . 572
Maple [A] (verified) . . . 576
Fricas [A] (verification not implemented) . . . 576
Sympy [F] . . . 577
Maxima [C] (verification not implemented) . . . 577
Giac [C] (verification not implemented) . . . 578
Mupad [F(-1)] . . . 578
Reduce [F] . . . 579

Optimal result

Integrand size = 22, antiderivative size = 142

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}} - \frac{\sqrt{d}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{8b^{3/2}}$$

output

```
-1/4*(d*x+c)^(1/2)*cos(2*b*x+2*a)/b+1/8*d^(1/2)*Pi^(1/2)*cos(2*a-2*b*c/d)*
FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b^(3/2)-1/8*d^(1/2)*Pi^
(1/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/
b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = \frac{1}{2}\left(-\frac{e^{2i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\,\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{-2i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\,\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{\frac{ib(c+dx)}{d}}}\right)$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-1/4*(E^((2*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-2*I)*b*(c + d*x
))/d])/(Sqrt[2]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma[3/2,
((2*I)*b*(c + d*x))/d])/(4*Sqrt[2]*b*E^((2*I)*(a - (b*c)/d))*Sqrt[(I*b*(c
+ d*x))/d]))/2
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4906, 27, 3042, 3777, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin(a+bx)\cos(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \frac{1}{2}\sqrt{c+dx}\sin(2a+2bx)dx$$

$$\downarrow 27$$

$$\frac{1}{2}\int \sqrt{c+dx}\sin(2a+2bx)dx$$

$$\downarrow 3042$$

$$\frac{1}{2}\int \sqrt{c+dx}\sin(2a+2bx)dx$$

$$\downarrow 3777$$

$$\frac{1}{2}\left(\frac{d\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}dx}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{d\int \frac{\sin\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3787$$

$$\frac{1}{2}\left(\frac{d\left(\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{d\left(\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3785$$

$$\frac{1}{2}\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int \cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3786$$

$$\frac{1}{2}\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\sin\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3832$$

$$\frac{1}{2}\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3833$$

$$\frac{1}{2}\left(\frac{d\left(\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-1/2*(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/b + (d*((Sqrt[Pi]*Cos[2*a - (2*b*c)
/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(Sqrt[b]*Sqrt[
d]) - (Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin
[2*a - (2*b*c)/d])/(Sqrt[b]*Sqrt[d])))/(4*b))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}}{d}$
default	$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}}{d}$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/16/b*d*Pi^(
1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*
x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)
^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.88

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx$$
$$= \frac{\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\left(2\,b\cos(\right.}{8\,b^2}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/8*(pi*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)
*sqrt(b/(pi*d))) - pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/
(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(2*b*cos(b*x + a)^2 - b)*sqrt(d*x + c))
/b^2
```

Sympy [F]

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = \int \sqrt{c+dx}\sin(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.47

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{\sqrt{2}\left(8\sqrt{2}\sqrt{dx+c}b\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)+\left((i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)+(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/64*sqrt(2)*(8*sqrt(2)*sqrt(d*x + c)*b*cos(2*((d*x + c)*b - b*c + a*d)/d
) + ((I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I
 + 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d
*x + c)*sqrt(2*I*b/d)) + (-(I + 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(
-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c
 - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/b^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.86

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx$$

$$= \frac{4\left(\frac{\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{i\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}+\frac{\sqrt{\pi}d\,\mathrm{erf}\left(\frac{i\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(-ibc+iad)}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}\right)c-\frac{\sqrt{\pi}(4bc-id)}{}}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
1/16*(4*(sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqr
t(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*
(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - sqrt(pi)*(
4*b*c - I*d)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - sqrt(p
i)*(4*b*c + I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) -
2*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 2*sqrt(d*x
+ c)*d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)\,\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = \int \sqrt{dx+c}\cos\left(bx+a\right)\sin\left(bx+a\right)dx$$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)*sin(a + b*x),x)
```

3.55 $\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 580
Mathematica [C] (verified) . . . 581
Rubi [A] (verified) . . . 581
Maple [A] (verified) . . . 585
Fricas [A] (verification not implemented) . . . 585
Sympy [F] . . . 586
Maxima [C] (verification not implemented) . . . 586
Giac [C] (verification not implemented) . . . 587
Mupad [F(-1)] . . . 587
Reduce [F] . . . 588

Optimal result

Integrand size = 22, antiderivative size = 142

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}} - \frac{\sqrt{d}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{8b^{3/2}}$$

output

```
-1/4*(d*x+c)^(1/2)*cos(2*b*x+2*a)/b+1/8*d^(1/2)*Pi^(1/2)*cos(2*a-2*b*c/d)*
FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b^(3/2)-1/8*d^(1/2)*Pi^
(1/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/
b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = \frac{1}{2}\left(-\frac{e^{2i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\,\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{-2i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\,\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{\frac{ib(c+dx)}{d}}}\right)$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-1/4*(E^((2*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-2*I)*b*(c + d*x
))/d])/(Sqrt[2]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma[3/2,
((2*I)*b*(c + d*x))/d])/(4*Sqrt[2]*b*E^((2*I)*(a - (b*c)/d))*Sqrt[(I*b*(c
+ d*x))/d]))/2
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4906, 27, 3042, 3777, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin(a+bx)\cos(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \frac{1}{2}\sqrt{c+dx}\sin(2a+2bx)dx$$

$$\downarrow 27$$

$$\frac{1}{2}\int \sqrt{c+dx}\sin(2a+2bx)dx$$

$$\downarrow 3042$$

$$\frac{1}{2}\int \sqrt{c+dx}\sin(2a+2bx)dx$$

$$\downarrow 3777$$

$$\frac{1}{2}\left(\frac{d\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}dx}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{d\int \frac{\sin\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3787$$

$$\frac{1}{2}\left(\frac{d\left(\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{d\left(\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3785$$

$$\frac{1}{2}\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int \cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3786$$

$$\frac{1}{2}\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\sin\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3832$$

$$\frac{1}{2}\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3833$$

$$\frac{1}{2}\left(\frac{d\left(\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-1/2*(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/b + (d*((Sqrt[Pi]*Cos[2*a - (2*b*c)
/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(Sqrt[b]*Sqrt[
d]) - (Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin
[2*a - (2*b*c)/d])/(Sqrt[b]*Sqrt[d])))/(4*b))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}}{d}$
default	$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}}{d}$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/16/b*d*Pi^(
1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*
x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)
^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.88

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx$$
$$= \frac{\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\left(2\,b\cos(\right.}{8\,b^2}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/8*(pi*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)
*sqrt(b/(pi*d))) - pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/
(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(2*b*cos(b*x + a)^2 - b)*sqrt(d*x + c))
/b^2
```

Sympy [F]

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = \int \sqrt{c+dx}\sin(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.47

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{\sqrt{2}\left(8\sqrt{2}\sqrt{dx+c}b\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)+\left((i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)+(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/64*sqrt(2)*(8*sqrt(2)*sqrt(d*x + c)*b*cos(2*((d*x + c)*b - b*c + a*d)/d
) + ((I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I
 + 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d
*x + c)*sqrt(2*I*b/d)) + (-(I + 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(
-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c
 - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/b^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.86

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx$$

$$= \frac{4\left(\frac{\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{i\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}+\frac{\sqrt{\pi}d\,\mathrm{erf}\left(\frac{i\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(-ibc+iad)}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}\right)c-\frac{\sqrt{\pi}(4bc-id}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
1/16*(4*(sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqr
t(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*
(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - sqrt(pi)*(
4*b*c - I*d)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - sqrt(p
i)*(4*b*c + I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) -
2*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 2*sqrt(d*x
+ c)*d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)\,\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = \int \sqrt{dx+c}\cos(bx+a)\sin(bx+a)\,dx$$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)*sin(a + b*x),x)
```

3.56 $\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 589
Mathematica [C] (verified) . . . 590
Rubi [A] (verified) . . . 590
Maple [A] (verified) . . . 595
Fricas [A] (verification not implemented) . . . 595
Sympy [F] . . . 596
Maxima [C] (verification not implemented) . . . 596
Giac [C] (verification not implemented) . . . 597
Mupad [F(-1)] . . . 598
Reduce [F] . . . 598

Optimal result

Integrand size = 22, antiderivative size = 168

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} - \frac{3d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}} - \frac{3d^{3/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{32b^{5/2}} + \frac{3d\sqrt{c+dx}\sin(2a+2bx)}{16b^2}$$

output

```
-1/4*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b-3/32*d^(3/2)*Pi^(1/2)*cos(2*a-2*b*c/d)
*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b^(5/2)-3/32*d^(3/2)*P
i^(1/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d
)/b^(5/2)+3/16*d*(d*x+c)^(1/2)*sin(2*b*x+2*a)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.76

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \frac{e^{-\frac{2i(bc+ad)}{d}}(c+dx)^{3/2}\left(-\frac{e^{4ia}\Gamma\left(\frac{5}{2},-\frac{2ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{3/2}}-\frac{e^{\frac{4ibc}{d}}\Gamma\left(\frac{5}{2},\frac{2ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{3/2}}\right)}{16\sqrt{2}b}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
((c + d*x)^(3/2)*(-((E^((4*I)*a)*Gamma[5/2, ((-2*I)*b*(c + d*x))/d])/(((-I
)*b*(c + d*x))/d)^(3/2)) - (E^(((4*I)*b*c)/d)*Gamma[5/2, ((2*I)*b*(c + d*x
))/d])/((I*b*(c + d*x))/d)^(3/2)))/(16*Sqrt[2]*b*E^(((2*I)*(b*c + a*d))/d)
)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {4906, 27, 3042, 3777, 3042, 3777, 25, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin(a+bx)\cos(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \frac{1}{2}(c+dx)^{3/2}\sin(2a+2bx)dx$$

$$\downarrow 27$$

$$\frac{1}{2}\int (c+dx)^{3/2}\sin(2a+2bx)dx$$

$$\downarrow 3042$$

$$\frac{1}{2}\int (c+dx)^{3/2}\sin(2a+2bx)dx$$

$$\downarrow 3777$$

$$\frac{1}{2}\left(\frac{3d\int \sqrt{c+dx}\cos(2a+2bx)dx}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{3d\int \sqrt{c+dx}\sin\left(2a+2bx+\frac{\pi}{2}\right)dx}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3777$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{d\int -\frac{\sin(2a+2bx)}{\sqrt{c+dx}}dx}{4b}+\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 25$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}dx}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}dx}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3787$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx+\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2b}{2b}\right.$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx+\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a+$$

$$\downarrow 3785$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a}{2b}$$

$$\downarrow 3786$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\sin\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos}{2b}$$

$$\downarrow 3832$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos}{2b}$$

$$\downarrow 3833$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos\right.$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-1/2*((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/b + (3*d*(-1/4*(d*((Sqrt[Pi]*Cos[
2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(
Sqrt[b]*Sqrt[d]) + (Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*S
qrt[Pi])]*Sin[2*a - (2*b*c)/d])/(Sqrt[b]*Sqrt[d])))/b + (Sqrt[c + d*x]*Sin
[2*a + 2*b*x])/(2*b)))/(4*b))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.11

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{8b\sqrt{\frac{b}{d}}}\right.}{4b}}{d}$
default	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{8b\sqrt{\frac{b}{d}}}\right.}{4b}}{d}$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+3/8/b*d*(1/4/
b*d*(d*x+c)^(1/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^
(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/
d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{3\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-4\,(2\,b^2}{32\,b^3}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/32*(3*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*
x + c)*sqrt(b/(pi*d))) + 3*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x +
c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(2*b^2*d*x + 3*b*d*cos(b*x +
a)*sin(b*x + a) + 2*b^2*c - 4*(b^2*d*x + b^2*c)*cos(b*x + a)^2)*sqrt(d*x +
 c))/b^3
```

Sympy [F]

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)*cos(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.52

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{\sqrt{2}\left(32\sqrt{2}(dx+c)^{\frac{3}{2}}b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)-24\sqrt{2}\sqrt{dx+c}bd\sin\left(\frac{2((dx+c)b-bc+ad)}{d}\right)+3\left((i+1)\cdot 4^{\frac{1}{4}}\sqrt{}\right.\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/256*sqrt(2)*(32*sqrt(2)*(d*x + c)^(3/2)*b^2*cos(2*((d*x + c)*b - b*c +
a*d)/d) - 24*sqrt(2)*sqrt(d*x + c)*b*d*sin(2*((d*x + c)*b - b*c + a*d)/d)
+ 3*((I + 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) -
(I - 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(2*I*b/d)) + 3*(-(I - 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1
/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*s
in(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 732, normalized size of antiderivative = 4.36

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
1/64*(16*(sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sq
rt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2
*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 - 8*(sqrt
(pi)*(4*b*c - I*d)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) +
sqrt(pi)*(4*b*c + I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
*b) + 2*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 2*sqr
t(d*x + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c + sqrt(pi)*(16
*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt
(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2)
 + 1)*b^2) + sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(I*sqrt(b*d)*s
qrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I
*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c +
 I*a*d)/d)/b^2 + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d -
3*sqrt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)\,(c+d\,x)^{3/2}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c + dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \left(\int \sqrt{dx+c}\,\cos(bx+a)\sin(bx+a)\,x dx\right) d + \left(\int \sqrt{dx+c}\,\cos(bx+a)\sin(bx+a)\,dx\right) c$$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)*sin(a + b*x)*x,x)*d + int(sqrt(c + d*x)*cos
(a + b*x)*sin(a + b*x),x)*c
```

3.57 $\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 599
Mathematica [C] (verified) . . . 600
Rubi [A] (verified) . . . 600
Maple [A] (verified) . . . 606
Fricas [A] (verification not implemented) . . . 607
Sympy [F] . . . 607
Maxima [C] (verification not implemented) . . . 608
Giac [C] (verification not implemented) . . . 608
Mupad [F(-1)] . . . 609
Reduce [F] . . . 610

Optimal result

Integrand size = 22, antiderivative size = 196

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} - \frac{15d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}} + \frac{15d^{5/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{128b^{7/2}} + \frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{16b^2}$$

output

```
15/64*d^2*(d*x+c)^(1/2)*cos(2*b*x+2*a)/b^3-1/4*(d*x+c)^(5/2)*cos(2*b*x+2*a
)/b-15/128*d^(5/2)*Pi^(1/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1
/2)/d^(1/2)/Pi^(1/2))/b^(7/2)+15/128*d^(5/2)*Pi^(1/2)*FresnelS(2*b^(1/2)*(
d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/b^(7/2)+5/16*d*(d*x+c)^(3/
2)*sin(2*b*x+2*a)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.73

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \frac{e^{-\frac{2i(bc+ad)}{d}}(c+dx)^{5/2}\left(e^{4ia}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{2ib(c+dx)}{d}\right)+e^{\frac{4ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{2ib(c+dx)}{d}\right)\right)}{32\sqrt{2}b\left(\frac{b^2(c+dx)^2}{d^2}\right)^{3/2}}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
((c + d*x)^(5/2)*(E^((4*I)*a)*Sqrt[(I*b*(c + d*x))/d]*Gamma[7/2, ((-2*I)*b
*(c + d*x))/d] + E^(((4*I)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[7/2, (
(2*I)*b*(c + d*x))/d]))/(32*Sqrt[2]*b*E^(((2*I)*(b*c + a*d))/d)*((b^2*(c +
 d*x)^2)/d^2)^(3/2))
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.11, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {4906, 27, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin(a+bx)\cos(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \frac{1}{2}(c+dx)^{5/2}\sin(2a+2bx)dx$$

$$\downarrow 27$$

$$\frac{1}{2}\int (c+dx)^{5/2}\sin(2a+2bx)dx$$

$$\downarrow 3042$$

$$\frac{1}{2}\int (c+dx)^{5/2}\sin(2a+2bx)dx$$

$$\downarrow 3777$$

$$\frac{1}{2}\left(\frac{5d\int (c+dx)^{3/2}\cos(2a+2bx)dx}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{5d\int (c+dx)^{3/2}\sin\left(2a+2bx+\frac{\pi}{2}\right)dx}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3777$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{3d\int -\sqrt{c+dx}\sin(2a+2bx)dx}{4b}+\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}\right)}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 25$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\int \sqrt{c+dx}\sin(2a+2bx)dx}{4b}\right)}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\int \sqrt{c+dx}\sin(2a+2bx)dx}{4b}\right)}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3777$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}dx}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)}{4b}\right)}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\int\frac{\sin\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)}{4b}\right)}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\downarrow 3787$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)}{4b}\right)}{4b}-\frac{(c+$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)}{4b}\right)}{4b}-($$

$$\downarrow 3785$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)}{4b}\right)}{4b}\right)$$

$$\downarrow 3786$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\sin\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2b}{2b}\right)}{4b}\right)}{4b}\right)$$

$$\downarrow 3832$$

$$\frac{1}{2}\left(5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx}{2b}\right.}{4b}\right.\right.$$

$$\downarrow 3833$$

$$\frac{1}{2}\left(5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx}{2b}\right.}{4b}\right.\right.$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-1/2*((c + d*x)^(5/2)*Cos[2*a + 2*b*x])/b + (5*d*((-3*d*(-1/2*(Sqrt[c + d
*x]*Cos[2*a + 2*b*x])/b + (d*((Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*S
qrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(Sqrt[b]*Sqrt[d]) - (Sqrt[Pi]*F
resnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(Sqrt[b]*Sqrt[d])))/(4*b)))/(4*b) + ((c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(
2*b)))/(4*b))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.19

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}\right.\right.}{d}$
default	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}\right.\right.}{d}$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+5/8/b*d*(1/4/
b*d*(d*x+c)^(3/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+
c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(co
s(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(2*
(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.13

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{15\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\,(1$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/128*(15*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(
d*x + c)*sqrt(b/(pi*d))) - 15*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x
 + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(16*b^3*d^2*x^2 + 32*b^3*c
*d*x + 16*b^3*c^2 - 15*b*d^2 - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c
^2 - 15*b*d^2)*cos(b*x + a)^2 + 40*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)*sin(
b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [F]

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \int (c+dx)^{\frac{5}{2}}\sin(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**(5/2)*sin(a + b*x)*cos(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.41

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \frac{\sqrt{2}\left(160\sqrt{2}(dx+c)^{\frac{3}{2}}b^2 d\sin\left(\frac{2((dx+c)b-bc+ad)}{d}\right) - 8\left(16\sqrt{2}(dx+c)^{\frac{5}{2}}b^3 - 15\sqrt{2}\sqrt{dx+c}bd^2\right)\cos\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/1024*sqrt(2)*(160*sqrt(2)*(d*x + c)^(3/2)*b^2*d*sin(2*((d*x + c)*b - b*c
 + a*d)/d) - 8*(16*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x + c)*
b*d^2)*cos(2*((d*x + c)*b - b*c + a*d)/d) - 15*(-(I - 1)*4^(1/4)*sqrt(pi)*
d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(pi)*d^3*(
b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 1
5*((I + 1)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I
 - 1)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(-2*I*b/d)))/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 1192, normalized size of antiderivative = 6.08

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output

```
1/256*(64*(sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + s
qrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 - d^3*((
sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 15*I*d^3)*d*erf(-I*sq
rt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d
)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^
2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*
x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^
(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + (sqrt(pi)*(64*b^3*c^3 +
 48*I*b^2*c^2*d - 36*b*c*d^2 - 15*I*d^3)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)
^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 -
 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-2*(I*(d*x + c)*b -
 I*b*c + I*a*d)/d)/b^3)/d^3) - 48*(sqrt(pi)*(4*b*c - I*d)*d*erf(-I*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(pi)*(4*b*c + I*d)*d*erf(I*sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sqrt(d*x + c)*d*e^(-2*(I...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)\,(c+d\,x)^{5/2}\,dx$$

input

```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(5/2),x)
```

output

```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c + dx)^{5/2} \cos(a+bx) \sin(a+bx)\, dx = \left(\int \sqrt{dx+c} \cos(bx+a) \sin(bx+a)\, x^2 dx \right) d^2 + 2\left(\int \sqrt{dx+c} \cos(bx+a) \sin(bx+a)\, x dx \right) cd + \left(\int \sqrt{dx+c} \cos(bx+a) \sin(bx+a)\, dx \right) c^2$$

input

```
int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x)
```

output

```
int(sqrt(c + d*x)*cos(a + b*x)*sin(a + b*x)*x**2,x)*d**2 + 2*int(sqrt(c +
d*x)*cos(a + b*x)*sin(a + b*x)*x,x)*c*d + int(sqrt(c + d*x)*cos(a + b*x)*s
in(a + b*x),x)*c**2
```

3.58 $\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx$

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Optimal result

Integrand size = 24, antiderivative size = 406

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} - \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}} - \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{144b^{7/2}} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{16b^{7/2}} - \frac{15d^2\sqrt{c+dx}\sin(a+bx)}{16b^3} + \frac{(c+dx)^{5/2}\sin(a+bx)}{4b} + \frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{144b^3} - \frac{(c+dx)^{5/2}\sin(3a+3bx)}{12b}$$

output

```
5/8*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2-5/72*d*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b^2
+15/32*d^(5/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(
1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-5/864*d^(5/2)*6^(1/2)*Pi^(1/2)*cos(3*a
-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)
-5/864*d^(5/2)*6^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*sin(3*a-3*b*c/d)/b^(7/2)+15/32*d^(5/2)*2^(1/2)*Pi^(1/2)*Fre
snelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(7/2)
-15/16*d^2*(d*x+c)^(1/2)*sin(b*x+a)/b^3+1/4*(d*x+c)^(5/2)*sin(b*x+a)/b+5/1
44*d^2*(d*x+c)^(1/2)*sin(3*b*x+3*a)/b^3-1/12*(d*x+c)^(5/2)*sin(3*b*x+3*a)/
b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.70 (sec) , antiderivative size = 1418, normalized size of antiderivative = 3.49

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input

```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output

```
-1/288*(c*Sqrt[d]*(12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(1 +
 (2*I)*b*x + E^((6*I)*(a + b*x))*(1 - (2*I)*b*x)) + (1 + I)*(2*b*c + I*d)*
E^(((3*I)*b*(2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt
[c + d*x])/Sqrt[d]] - (1 + I)*(2*b*c - I*d)*E^((3*I)*(2*a + b*x))*Sqrt[6*P
i]*Erfi[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^((
(3*I)*(a*d + b*(c + d*x)))/d)) + (c^2*d*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x
))/d]*Gamma[3/2, ((-I)*b*(c + d*x))/d] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c +
d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d]))/(8*b^2*E^((I*(b*c + a*d))/d)*Sqrt
[c + d*x]) - (c^2*(c + d*x)^(3/2)*(-((E^((6*I)*a)*Gamma[3/2, ((-3*I)*b*(c
+ d*x))/d])/(((-I)*b*(c + d*x))/d)^(3/2)) - (E^(((6*I)*b*c)/d)*Gamma[3/2,
((3*I)*b*(c + d*x))/d])/((I*b*(c + d*x))/d)^(3/2)))/(24*Sqrt[3]*d*E^(((3*I
)*(b*c + a*d))/d)) + (c*Sqrt[d]*(E^(I*(a - (b*c)/d))*(2*Sqrt[b]*Sqrt[d]*E^
((I*b*(c + d*x))/d)*(3 - (2*I)*b*x)*Sqrt[c + d*x] + (-1)^(1/4)*(-2*b*c + (
3*I)*d)*Sqrt[Pi]*Erfi[(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x]/Sqrt[d]]) + (2*Sq
rt[b]*Sqrt[d]*(3 + (2*I)*b*x)*Sqrt[c + d*x] + (1 + I)*(2*b*c + (3*I)*d)*Sq
rt[Pi/2]*Erf[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[2]*Sqrt[d])]*(Cos[b*(c/
d + x)] + I*Sin[b*(c/d + x)]))*(Cos[a + b*x] - I*Sin[a + b*x])))/(16*b^(5/
2)) + (Sqrt[d]*((Cos[a - (b*c)/d] + I*Sin[a - (b*c)/d])*((1 + I)*(4*b^2*c^
2 - (12*I)*b*c*d - 15*d^2)*Sqrt[Pi/2]*Erfi[((1 + I)*Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[2]*Sqrt[d])] + 2*Sqrt[b]*Sqrt[d]*Sqrt[c + d*x]*((15*I)*d - (4*I)...
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2} \sin^2(a+bx) \cos(a+bx)\, dx$$

$$\downarrow \text{4906}$$

$$\int \left(\frac{1}{4}(c+dx)^{5/2} \cos(a+bx) - \frac{1}{4}(c+dx)^{5/2} \cos(3a+3bx) \right) dx$$

$$\downarrow \text{2009}$$

$$-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}-\frac{15d^2\sqrt{c+dx}\sin(a+bx)}{16b^3}+\frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{144b^3}+\frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)}{4b}-\frac{(c+dx)^{5/2}\sin(3a+3bx)}{12b}$$

input

```
Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output

```
(5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(8*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a
+ 3*b*x])/(72*b^2) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqr
t[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi
/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[
d]])/(144*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/2)*Sq
rt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c
)/d])/(16*b^(7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(16*b^3) + ((c +
d*x)^(5/2)*Sin[a + b*x])/(4*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(1
44*b^3) - ((c + d*x)^(5/2)*Sin[3*a + 3*b*x])/(12*b)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\sqrt{\pi}\left(\right.\right.}{4b}$
default	$\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\sqrt{\pi}\left(\right.\right.}{4b}$

input

```
int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/8/b*d*(d*x+c)^(5/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-5/8/b*d*(-1/2/b*d*
(d*x+c)^(3/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2)*
sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*
d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((a*
d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1/24
/b*d*(d*x+c)^(5/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+5/24/b*d*(-1/6/b*d*(d*
x+c)^(3/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2)
*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*
(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d
)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.91

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{5\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(
sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 40
5*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(
pi*d)))*sin(-(b*c - a*d)/d) + 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(
sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(10*(b^2*
d^2*x + b^2*c*d)*cos(b*x + a)^3 - 30*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) -
(12*b^3*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 35*b*d^2 - (12*b^3*d^2*x^2 +
 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d
*x + c))/b^4
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.35

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/3456*(240*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d) - 2160
*(d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d) - 5*(-(I + 1)*9^(1/4
)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9
^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(3*I*b/d)) - 405*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2
)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/
4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 405*(-(I - 1)*sqr
t(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(2)*
sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt
(-I*b/d)) - 5*((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(
-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)
*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + 24*(12*(d*x +
c)^(5/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*sin(3*((d*x + c)*b - b*c + a*d)/d)
 - 216*(4*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*sin(((d*x + c)*b
 - b*c + a*d)/d))*d/b^5
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 2453, normalized size of antiderivative = 6.04

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/1728*(72*(3*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*s
qrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b
*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/
d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(1/2*I*
sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*
c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + d^3*(27*(-I*sq
rt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3)*d*erf(
1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*
b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 2*(-4*I*(d*x
+ c)^(5/2)*b^2*d + 12*I*(d*x + c)^(3/2)*b^2*c*d - 12*I*sqrt(d*x + c)*b^2*c
^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x
 + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + (-I*sqrt(6)*s
qrt(pi)*(72*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*I*
sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c
- I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 6*(-12*I*(d*x + c)
^(5/2)*b^2*d + 36*I*(d*x + c)^(3/2)*b^2*c*d - 36*I*sqrt(d*x + c)*b^2*c^2*d
 + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x +...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)^2\,(c+d\,x)^{5/2}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \left(\int \sqrt{dx+c}\,\cos(bx+a)\sin(bx+a)^2\,x^2dx\right)d^2$$
$$+2\left(\int \sqrt{dx+c}\,\cos(bx+a)\sin(bx+a)^2\,xdx\right)cd$$
$$+\left(\int \sqrt{dx+c}\,\cos(bx+a)\sin(bx+a)^2\,dx\right)c^2$$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)*sin(a + b*x)**2*x**2,x)*d**2 + 2*int(sqrt(c
 + d*x)*cos(a + b*x)*sin(a + b*x)**2*x,x)*c*d + int(sqrt(c + d*x)*cos(a +
b*x)*sin(a + b*x)**2,x)*c**2
```

3.59 $\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal result . . . 620
Mathematica [C] (verified) . . . 621
Rubi [A] (verified) . . . 622
Maple [A] (verified) . . . 624
Fricas [A] (verification not implemented) . . . 624
Sympy [F] . . . 625
Maxima [C] (verification not implemented) . . . 625
Giac [C] (verification not implemented) . . . 626
Mupad [F(-1)] . . . 627
Reduce [F] . . . 628

Optimal result

Integrand size = 24, antiderivative size = 353

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} + \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}} - \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{24b^{5/2}} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{8b^{5/2}} + \frac{(c+dx)^{3/2}\sin(a+bx)}{4b} - \frac{(c+dx)^{3/2}\sin(3a+3bx)}{12b}$$

output

```
3/8*d*(d*x+c)^(1/2)*cos(b*x+a)/b^2-1/24*d*(d*x+c)^(1/2)*cos(3*b*x+3*a)/b^2
-3/16*d^(3/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)+1/144*d^(3/2)*6^(1/2)*Pi^(1/2)*cos(3*a-
3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)-
1/144*d^(3/2)*6^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(
1/2)/d^(1/2))*sin(3*a-3*b*c/d)/b^(5/2)+3/16*d^(3/2)*2^(1/2)*Pi^(1/2)*Fresn
elS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(5/2)+1
/4*(d*x+c)^(3/2)*sin(b*x+a)/b-1/12*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 730, normalized size of antiderivative = 2.07

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{\sqrt{d}e^{-\frac{3i(ad+b(c+dx))}{d}}\left(12\sqrt{b}\sqrt{d}e^{\frac{3ibc}{d}}\sqrt{c+dx}\left(1+2ibx+e^{6i(a+bx)}(1-2ibx)\right)+(1+i)(2bc+id)e^{\frac{3ib(2c+dx)}{d}}\sqrt{6}\right.}{576b^{5/2}}$$

$$+\frac{cde^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)\right)}{8b^2\sqrt{c+dx}}$$

$$-\frac{ce^{-\frac{3i(bc+ad)}{d}}(c+dx)^{3/2}\left(-\frac{e^{6ia}\Gamma\left(\frac{3}{2},-\frac{3ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{3/2}}-\frac{e^{\frac{6ibc}{d}}\Gamma\left(\frac{3}{2},\frac{3ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{3/2}}\right)}{24\sqrt{3}d}$$

$$+\frac{\sqrt{d}\left(e^{i\left(a-\frac{bc}{d}\right)}\left(2\sqrt{b}\sqrt{d}e^{\frac{ib(c+dx)}{d}}(3-2ibx)\sqrt{c+dx}+\sqrt[4]{-1}(-2bc+3id)\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)\right)+\left(2\sqrt{b}\sqrt{}\right.\right.}{}$$

input

```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output

```
-1/576*(Sqrt[d]*(12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(1 + (
2*I)*b*x + E^((6*I)*(a + b*x))*(1 - (2*I)*b*x)) + (1 + I)*(2*b*c + I*d)*E^
(((3*I)*b*(2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c
 + d*x])/Sqrt[d]] - (1 + I)*(2*b*c - I*d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi]
*Erfi[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^(((3
*I)*(a*d + b*(c + d*x)))/d)) + (c*d*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d
]*Gamma[3/2, ((-I)*b*(c + d*x))/d] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x)
)/d]*Gamma[3/2, (I*b*(c + d*x))/d]))/(8*b^2*E^((I*(b*c + a*d))/d)*Sqrt[c +
 d*x]) - (c*(c + d*x)^(3/2)*(-((E^((6*I)*a)*Gamma[3/2, ((-3*I)*b*(c + d*x)
)/d])/(((-I)*b*(c + d*x))/d)^(3/2)) - (E^(((6*I)*b*c)/d)*Gamma[3/2, ((3*I)
*b*(c + d*x))/d])/((I*b*(c + d*x))/d)^(3/2)))/(24*Sqrt[3]*d*E^(((3*I)*(b*c
 + a*d))/d)) + (Sqrt[d]*(E^(I*(a - (b*c)/d))*(2*Sqrt[b]*Sqrt[d]*E^((I*b*(c
 + d*x))/d)*(3 - (2*I)*b*x)*Sqrt[c + d*x] + (-1)^(1/4)*(-2*b*c + (3*I)*d)*
Sqrt[Pi]*Erfi[((-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]) + (2*Sqrt[b]*Sq
rt[d]*(3 + (2*I)*b*x)*Sqrt[c + d*x] + (1 + I)*(2*b*c + (3*I)*d)*Sqrt[Pi/2]
*Erf[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[2]*Sqrt[d])]*(Cos[b*(c/d + x)]
+ I*Sin[b*(c/d + x)]))*(Cos[a + b*x] - I*Sin[a + b*x])))/(32*b^(5/2))
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin^2(a+bx)\cos(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}(c+dx)^{3/2}\cos(a+bx) - \frac{1}{4}(c+dx)^{3/2}\cos(3a+3bx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2}-\frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)}{4b}-\frac{(c+dx)^{3/2}\sin(3a+3bx)}{12b}$$

input

```
Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output

```
(3*d*Sqrt[c + d*x]*Cos[a + b*x])/(8*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*
x])/(24*b^2) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sq
rt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*
a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b
^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/
Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*Fresne
lS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2
)) + ((c + d*x)^(3/2)*Sin[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b
*x])/(12*b)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\right.\right.}{4b\sqrt{\frac{b}{d}}}\right.}{4b}$
default	$\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\right.\right.}{4b\sqrt{\frac{b}{d}}}\right.}{4b}$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/8/b*d*(d*x+c)^(3/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-3/8/b*d*(-1/2/b*d*
(d*x+c)^(1/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^
(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)))-1/24/b*d*(d*x+c)^(3/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/8/b*d*(
-1/6/b*d*(d*x+c)^(1/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*P
i^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*
3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)
/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.84

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \frac{\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-27\,\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqr
t(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(
-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqr
t(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)
))*sin(-(b*c - a*d)/d) - sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)
*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*d*cos(b*x + a
)^3 - 3*b*d*cos(b*x + a) - 2*(b^2*d*x + b^2*c - (b^2*d*x + b^2*c)*cos(b*x
+ a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F]

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin^2(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.41

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/576*(48*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d)/d - 144*
(d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d + 24*sqrt(d*x + c)*
b^2*cos(3*((d*x + c)*b - b*c + a*d)/d) - 216*sqrt(d*x + c)*b^2*cos(((d*x +
 c)*b - b*c + a*d)/d) + ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1
/4)*cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)
^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 27*((I -
1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt
(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(I*b/d)) - 27*(-(I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c -
 a*d)/d) - (I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*
d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)
*b*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/
d)))*d/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 1520, normalized size of antiderivative = 4.31

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output

```
-1/288*(12*(3*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d
)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(1/2*I*s
qrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c
 + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + 4*(-9*I*sqrt(2)
*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*
d^2) + 1)*b) - I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*I*sqrt(6)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(
sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*(2*b*c - I*d
)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(
6)*sqrt(pi)*(6*b*c + I*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)*b) + 18*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*
d)/d)/b + 6*I*sqrt(d*x + c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

input

```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

output

```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c + dx)^{3/2} \cos(a+bx) \sin^2(a+bx)\, dx = \left(\int \sqrt{dx+c} \cos(bx+a) \sin(bx+a)^2 x dx \right) d + \left(\int \sqrt{dx+c} \cos(bx+a) \sin(bx+a)^2 dx \right) c$$

input

```
int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

output

```
int(sqrt(c + d*x)*cos(a + b*x)*sin(a + b*x)**2*x,x)*d + int(sqrt(c + d*x)*
cos(a + b*x)*sin(a + b*x)**2,x)*c
```

3.60 $\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal result . . . 629
Mathematica [C] (verified) . . . 630
Rubi [A] (verified) . . . 631
Maple [A] (verified) . . . 632
Fricas [A] (verification not implemented) . . . 633
Sympy [F] . . . 633
Maxima [C] (verification not implemented) . . . 634
Giac [C] (verification not implemented) . . . 634
Mupad [F(-1)] . . . 635
Reduce [F] . . . 636

Optimal result

Integrand size = 24, antiderivative size = 304

$$\begin{aligned}
&\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx \\
&= -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{12b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{4b^{3/2}} \\
&\quad + \frac{\sqrt{c+dx}\sin(a+bx)}{4b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}
\end{aligned}$$

output
```
-1/8*d^(1/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+1/72*d^(1/2)*6^(1/2)*Pi^(1/2)*cos(3*a-3*
b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+1/
72*d^(1/2)*6^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2
)/d^(1/2))*sin(3*a-3*b*c/d)/b^(3/2)-1/8*d^(1/2)*2^(1/2)*Pi^(1/2)*FresnelC(
b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(3/2)+1/4*(
d*x+c)^(1/2)*sin(b*x+a)/b-1/12*(d*x+c)^(1/2)*sin(3*b*x+3*a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.83

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{de^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)\right)}{8b^2\sqrt{c+dx}}$$
$$- \frac{e^{-\frac{3i(bc+ad)}{d}}(c+dx)^{3/2}\left(-\frac{e^{6ia}\Gamma\left(\frac{3}{2},-\frac{3ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{3/2}}-\frac{e^{\frac{6ibc}{d}}\Gamma\left(\frac{3}{2},\frac{3ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{3/2}}\right)}{24\sqrt{3}d}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
(d*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(c + d*x))/d
] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d
]))/(8*b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x]) - ((c + d*x)^(3/2)*(-((E^(
(6*I)*a)*Gamma[3/2, ((-3*I)*b*(c + d*x))/d])/(((-I)*b*(c + d*x))/d)^(3/2))
 - (E^(((6*I)*b*c)/d)*Gamma[3/2, ((3*I)*b*(c + d*x))/d])/((I*b*(c + d*x))/
d)^(3/2)))/(24*Sqrt[3]*d*E^(((3*I)*(b*c + a*d))/d))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^2(a+bx)\cos(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}\sqrt{c+dx}\cos(a+bx) - \frac{1}{4}\sqrt{c+dx}\cos(3a+3bx)\right)dx$$

$$\downarrow 2009$$

$$\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} + \frac{\sqrt{c+dx}\sin(a+bx)}{4b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}$$

input

```
Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output

```
-1/4*(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqr
t[c + d*x])/Sqrt[d]])/b^(3/2) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*F
resnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) + (Sqrt[
d]*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a
 - (3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2
/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2)) + (Sqrt[c + d*x
]*Sin[a + b*x])/(4*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(12*b)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{}}{}$
default	$\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{}}{}$

input

```
int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
2/d*(1/8/b*d*(d*x+c)^(1/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/16/b*d*2^(1/2)*P
i^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d))-1/24/b*d*(d*x+c)^(1/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)
/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fre
snelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b
*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.81

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx$$

$$= \frac{\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 9\,\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 9\,\sqrt{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6
)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c
- a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi
*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(
b*c - a*d)/d) + sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*cos(b*x + a)^2 - b)*sqrt
(d*x + c)*sin(b*x + a))/b^2
```

Sympy [F]

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = \int \sqrt{c+dx}\sin^2(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.39

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = -\frac{\left(\frac{24\sqrt{dx+c}b^2\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d} - \frac{72\sqrt{dx+c}b^2\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d} + \left(-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-a}{d}\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/288*(24*sqrt(d*x + c)*b^2*sin(3*((d*x + c)*b - b*c + a*d)/d)/d - 72*sqr
t(d*x + c)*b^2*sin(((d*x + c)*b - b*c + a*d)/d)/d + (-(I + 1)*9^(1/4)*sqrt
(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqr
t(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(3*I*b/d)) - 9*(-(I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c -
 a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))
*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 9*((I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^
(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin
(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((I - 1)*9^(1/4)*sqrt(
2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt
(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(-3*I*b/d)))*d/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 848, normalized size of antiderivative = 2.79

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output

```
-1/144*(-9*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(
-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*
(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*s
qrt(pi)*(2*b*c - I*d)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b) + I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(s
qrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 6*(3*I*sqrt(2)*sqrt(pi)*d*erf(1/2
*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d
*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2
)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) -
I*sqrt(6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d
^2) + 1)))*c + 18*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/
b + 6*I*sqrt(d*x + c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 18...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)^2\,\sqrt{c+d\,x}\,dx$$

input

```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

output

```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = \int \sqrt{dx+c}\,\cos\left(bx+a\right)\sin\left(bx+a\right)^2\,dx$$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)*sin(a + b*x)**2,x)
```

3.61 $\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx$

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Optimal result

Integrand size = 24, antiderivative size = 304

$$\begin{aligned}
&\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx \\
&= -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{12b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{4b^{3/2}} \\
&\quad + \frac{\sqrt{c+dx}\sin(a+bx)}{4b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}
\end{aligned}$$

output

```
-1/8*d^(1/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+1/72*d^(1/2)*6^(1/2)*Pi^(1/2)*cos(3*a-3*
b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+1/
72*d^(1/2)*6^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2
)/d^(1/2))*sin(3*a-3*b*c/d)/b^(3/2)-1/8*d^(1/2)*2^(1/2)*Pi^(1/2)*FresnelC(
b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(3/2)+1/4*(
d*x+c)^(1/2)*sin(b*x+a)/b-1/12*(d*x+c)^(1/2)*sin(3*b*x+3*a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.83

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{de^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)\right)}{8b^2\sqrt{c+dx}}$$
$$- \frac{e^{-\frac{3i(bc+ad)}{d}}(c+dx)^{3/2}\left(-\frac{e^{6ia}\Gamma\left(\frac{3}{2},-\frac{3ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{3/2}}-\frac{e^{\frac{6ibc}{d}}\Gamma\left(\frac{3}{2},\frac{3ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{3/2}}\right)}{24\sqrt{3}d}$$

input

```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output

```
(d*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(c + d*x))/d
] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d
]))/(8*b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x]) - ((c + d*x)^(3/2)*(-((E^(
(6*I)*a)*Gamma[3/2, ((-3*I)*b*(c + d*x))/d])/(((-I)*b*(c + d*x))/d)^(3/2))
 - (E^(((6*I)*b*c)/d)*Gamma[3/2, ((3*I)*b*(c + d*x))/d])/((I*b*(c + d*x))/
d)^(3/2)))/(24*Sqrt[3]*d*E^(((3*I)*(b*c + a*d))/d))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^2(a+bx)\cos(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}\sqrt{c+dx}\cos(a+bx) - \frac{1}{4}\sqrt{c+dx}\cos(3a+3bx)\right)dx$$

$$\downarrow 2009$$

$$\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} + \frac{\sqrt{c+dx}\sin(a+bx)}{4b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}$$

input

```
Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output

```
-1/4*(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqr
t[c + d*x])/Sqrt[d]])/b^(3/2) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*F
resnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) + (Sqrt[
d]*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a
 - (3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2
/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2)) + (Sqrt[c + d*x
]*Sin[a + b*x])/(4*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(12*b)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{}}{}$
default	$\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{}}{}$

input

```
int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
2/d*(1/8/b*d*(d*x+c)^(1/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/16/b*d*2^(1/2)*P
i^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d))-1/24/b*d*(d*x+c)^(1/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)
/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fre
snelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b
*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.81

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx$$

$$= \frac{\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 9\,\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 9\,\sqrt{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6
)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c
- a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi
*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(
b*c - a*d)/d) + sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*cos(b*x + a)^2 - b)*sqrt
(d*x + c)*sin(b*x + a))/b^2
```

Sympy [F]

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = \int \sqrt{c+dx}\sin^2(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.39

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = -\frac{\left(\frac{24\sqrt{dx+c}b^2\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d} - \frac{72\sqrt{dx+c}b^2\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d} + \left(-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-a}{d}\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/288*(24*sqrt(d*x + c)*b^2*sin(3*((d*x + c)*b - b*c + a*d)/d)/d - 72*sqr
t(d*x + c)*b^2*sin(((d*x + c)*b - b*c + a*d)/d)/d + (-(I + 1)*9^(1/4)*sqrt
(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqr
t(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(3*I*b/d)) - 9*(-(I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c -
 a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))
*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 9*((I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^
(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin
(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((I - 1)*9^(1/4)*sqrt(
2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt
(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(-3*I*b/d)))*d/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 848, normalized size of antiderivative = 2.79

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output

```
-1/144*(-9*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(
-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*
(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*s
qrt(pi)*(2*b*c - I*d)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b) + I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(s
qrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 6*(3*I*sqrt(2)*sqrt(pi)*d*erf(1/2
*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d
*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2
)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) -
I*sqrt(6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d
^2) + 1)))*c + 18*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/
b + 6*I*sqrt(d*x + c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 18...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)^2\,\sqrt{c+d\,x}\,dx$$

input

```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

output

```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = \int \sqrt{dx+c}\,\cos\left(bx+a\right)\sin\left(bx+a\right)^2\,dx$$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)*sin(a + b*x)**2,x)
```

3.62 $\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal result . . . 645
Mathematica [C] (verified) . . . 646
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Reduce [F] . . . 653

Optimal result

Integrand size = 24, antiderivative size = 353

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} + \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}} - \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{24b^{5/2}} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{8b^{5/2}} + \frac{(c+dx)^{3/2}\sin(a+bx)}{4b} - \frac{(c+dx)^{3/2}\sin(3a+3bx)}{12b}$$

output

```
3/8*d*(d*x+c)^(1/2)*cos(b*x+a)/b^2-1/24*d*(d*x+c)^(1/2)*cos(3*b*x+3*a)/b^2
-3/16*d^(3/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)+1/144*d^(3/2)*6^(1/2)*Pi^(1/2)*cos(3*a-
3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)-
1/144*d^(3/2)*6^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(
1/2)/d^(1/2))*sin(3*a-3*b*c/d)/b^(5/2)+3/16*d^(3/2)*2^(1/2)*Pi^(1/2)*Fresn
elS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(5/2)+1
/4*(d*x+c)^(3/2)*sin(b*x+a)/b-1/12*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 730, normalized size of antiderivative = 2.07

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{\sqrt{d}e^{-\frac{3i(ad+b(c+dx))}{d}}\left(12\sqrt{b}\sqrt{d}e^{\frac{3ibc}{d}}\sqrt{c+dx}\left(1+2ibx+e^{6i(a+bx)}(1-2ibx)\right)+(1+i)(2bc+id)e^{\frac{3ib(2c+dx)}{d}}\sqrt{6}\right.}{576b^{5/2}}$$

$$+\frac{cde^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)\right)}{8b^2\sqrt{c+dx}}$$

$$-\frac{ce^{-\frac{3i(bc+ad)}{d}}(c+dx)^{3/2}\left(-\frac{e^{6ia}\Gamma\left(\frac{3}{2},-\frac{3ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{3/2}}-\frac{e^{\frac{6ibc}{d}}\Gamma\left(\frac{3}{2},\frac{3ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{3/2}}\right)}{24\sqrt{3}d}$$

$$+\frac{\sqrt{d}\left(e^{i\left(a-\frac{bc}{d}\right)}\left(2\sqrt{b}\sqrt{d}e^{\frac{ib(c+dx)}{d}}(3-2ibx)\sqrt{c+dx}+\sqrt[4]{-1}(-2bc+3id)\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)\right)+\left(2\sqrt{b}\sqrt{}\right.\right.}{}$$

input

```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output

```
-1/576*(Sqrt[d]*(12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(1 + (
2*I)*b*x + E^((6*I)*(a + b*x))*(1 - (2*I)*b*x)) + (1 + I)*(2*b*c + I*d)*E^
(((3*I)*b*(2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c
 + d*x])/Sqrt[d]] - (1 + I)*(2*b*c - I*d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi]
*Erfi[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^(((3
*I)*(a*d + b*(c + d*x)))/d)) + (c*d*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d
]*Gamma[3/2, ((-I)*b*(c + d*x))/d] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x)
)/d]*Gamma[3/2, (I*b*(c + d*x))/d]))/(8*b^2*E^((I*(b*c + a*d))/d)*Sqrt[c +
 d*x]) - (c*(c + d*x)^(3/2)*(-((E^((6*I)*a)*Gamma[3/2, ((-3*I)*b*(c + d*x)
)/d])/(((-I)*b*(c + d*x))/d)^(3/2)) - (E^(((6*I)*b*c)/d)*Gamma[3/2, ((3*I)
*b*(c + d*x))/d])/((I*b*(c + d*x))/d)^(3/2)))/(24*Sqrt[3]*d*E^(((3*I)*(b*c
 + a*d))/d)) + (Sqrt[d]*(E^(I*(a - (b*c)/d))*(2*Sqrt[b]*Sqrt[d]*E^((I*b*(c
 + d*x))/d)*(3 - (2*I)*b*x)*Sqrt[c + d*x] + (-1)^(1/4)*(-2*b*c + (3*I)*d)*
Sqrt[Pi]*Erfi[((-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]) + (2*Sqrt[b]*Sq
rt[d]*(3 + (2*I)*b*x)*Sqrt[c + d*x] + (1 + I)*(2*b*c + (3*I)*d)*Sqrt[Pi/2]
*Erf[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[2]*Sqrt[d])]*(Cos[b*(c/d + x)]
+ I*Sin[b*(c/d + x)]))*(Cos[a + b*x] - I*Sin[a + b*x])))/(32*b^(5/2))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin^2(a+bx)\cos(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}(c+dx)^{3/2}\cos(a+bx) - \frac{1}{4}(c+dx)^{3/2}\cos(3a+3bx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+$$
$$\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-$$
$$\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}+$$
$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2}-$$
$$\frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)}{4b}-\frac{(c+dx)^{3/2}\sin(3a+3bx)}{12b}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
(3*d*Sqrt[c + d*x]*Cos[a + b*x])/(8*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*
x])/(24*b^2) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sq
rt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*
a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b
^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/
Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*Fresne
lS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2
)) + ((c + d*x)^(3/2)*Sin[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b
*x])/(12*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\right.\right.}{4b\sqrt{\frac{b}{d}}}\right.}{4b}$
default	$\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\right.\right.}{4b\sqrt{\frac{b}{d}}}\right.}{4b}$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/8/b*d*(d*x+c)^(3/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-3/8/b*d*(-1/2/b*d*
(d*x+c)^(1/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^
(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)))-1/24/b*d*(d*x+c)^(3/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/8/b*d*(
-1/6/b*d*(d*x+c)^(1/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*P
i^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*
3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)
/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.84

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \frac{\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-27\,\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\right.$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqr
t(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(
-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqr
t(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)
))*sin(-(b*c - a*d)/d) - sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)
*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*d*cos(b*x + a
)^3 - 3*b*d*cos(b*x + a) - 2*(b^2*d*x + b^2*c - (b^2*d*x + b^2*c)*cos(b*x
+ a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F]

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin^2(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.41

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/576*(48*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d)/d - 144*
(d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d + 24*sqrt(d*x + c)*
b^2*cos(3*((d*x + c)*b - b*c + a*d)/d) - 216*sqrt(d*x + c)*b^2*cos(((d*x +
 c)*b - b*c + a*d)/d) + ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1
/4)*cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)
^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 27*((I -
1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt
(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(I*b/d)) - 27*(-(I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c -
 a*d)/d) - (I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*
d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)
*b*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/
d)))*d/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 1520, normalized size of antiderivative = 4.31

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output

```
-1/288*(12*(3*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d
)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(1/2*I*s
qrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c
 + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + 4*(-9*I*sqrt(2)
*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*
d^2) + 1)*b) - I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*I*sqrt(6)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(
sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*(2*b*c - I*d
)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(
6)*sqrt(pi)*(6*b*c + I*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)*b) + 18*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*
d)/d)/b + 6*I*sqrt(d*x + c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

input

```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

output

```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c + dx)^{3/2} \cos(a+bx) \sin^2(a+bx)\, dx = \left(\int \sqrt{dx+c} \cos(bx+a) \sin(bx+a)^2 x dx \right) d + \left(\int \sqrt{dx+c} \cos(bx+a) \sin(bx+a)^2 dx \right) c$$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)*sin(a + b*x)**2*x,x)*d + int(sqrt(c + d*x)*
cos(a + b*x)*sin(a + b*x)**2,x)*c
```

3.63 $\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal result . . . 654
Mathematica [C] (verified) . . . 655
Rubi [A] (verified) . . . 656
Maple [A] (verified) . . . 658
Fricas [A] (verification not implemented) . . . 659
Sympy [F(-1)] . . . 660
Maxima [C] (verification not implemented) . . . 660
Giac [C] (verification not implemented) . . . 661
Mupad [F(-1)] . . . 662
Reduce [F] . . . 662

Optimal result

Integrand size = 24, antiderivative size = 406

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} - \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}} - \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{144b^{7/2}} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{16b^{7/2}} - \frac{15d^2\sqrt{c+dx}\sin(a+bx)}{16b^3} + \frac{(c+dx)^{5/2}\sin(a+bx)}{4b} + \frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{144b^3} - \frac{(c+dx)^{5/2}\sin(3a+3bx)}{12b}$$

output

```
5/8*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2-5/72*d*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b^2
+15/32*d^(5/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(
1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-5/864*d^(5/2)*6^(1/2)*Pi^(1/2)*cos(3*a
-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)
-5/864*d^(5/2)*6^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*sin(3*a-3*b*c/d)/b^(7/2)+15/32*d^(5/2)*2^(1/2)*Pi^(1/2)*Fre
snelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(7/2)
-15/16*d^2*(d*x+c)^(1/2)*sin(b*x+a)/b^3+1/4*(d*x+c)^(5/2)*sin(b*x+a)/b+5/1
44*d^2*(d*x+c)^(1/2)*sin(3*b*x+3*a)/b^3-1/12*(d*x+c)^(5/2)*sin(3*b*x+3*a)/
b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.83 (sec) , antiderivative size = 1418, normalized size of antiderivative = 3.49

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input

```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output

```
-1/288*(c*Sqrt[d]*(12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(1 +
 (2*I)*b*x + E^((6*I)*(a + b*x))*(1 - (2*I)*b*x)) + (1 + I)*(2*b*c + I*d)*
E^(((3*I)*b*(2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt
[c + d*x])/Sqrt[d]] - (1 + I)*(2*b*c - I*d)*E^((3*I)*(2*a + b*x))*Sqrt[6*P
i]*Erfi[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^((
(3*I)*(a*d + b*(c + d*x)))/d)) + (c^2*d*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x
))/d]*Gamma[3/2, ((-I)*b*(c + d*x))/d] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c +
d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d]))/(8*b^2*E^((I*(b*c + a*d))/d)*Sqrt
[c + d*x]) - (c^2*(c + d*x)^(3/2)*(-((E^((6*I)*a)*Gamma[3/2, ((-3*I)*b*(c
+ d*x))/d])/(((-I)*b*(c + d*x))/d)^(3/2)) - (E^(((6*I)*b*c)/d)*Gamma[3/2,
((3*I)*b*(c + d*x))/d])/((I*b*(c + d*x))/d)^(3/2)))/(24*Sqrt[3]*d*E^(((3*I
)*(b*c + a*d))/d)) + (c*Sqrt[d]*(E^(I*(a - (b*c)/d))*(2*Sqrt[b]*Sqrt[d]*E^
((I*b*(c + d*x))/d)*(3 - (2*I)*b*x)*Sqrt[c + d*x] + (-1)^(1/4)*(-2*b*c + (
3*I)*d)*Sqrt[Pi]*Erfi[((-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]) + (2*Sq
rt[b]*Sqrt[d]*(3 + (2*I)*b*x)*Sqrt[c + d*x] + (1 + I)*(2*b*c + (3*I)*d)*Sq
rt[Pi/2]*Erf[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[2]*Sqrt[d])]*(Cos[b*(c/
d + x)] + I*Sin[b*(c/d + x)]))*(Cos[a + b*x] - I*Sin[a + b*x])))/(16*b^(5/
2)) + (Sqrt[d]*((Cos[a - (b*c)/d] + I*Sin[a - (b*c)/d])*((1 + I)*(4*b^2*c^
2 - (12*I)*b*c*d - 15*d^2)*Sqrt[Pi/2]*Erfi[((1 + I)*Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[2]*Sqrt[d])] + 2*Sqrt[b]*Sqrt[d]*Sqrt[c + d*x]*((15*I)*d - (4*I)...
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2} \sin^2(a+bx) \cos(a+bx)\, dx$$

$$\downarrow \text{4906}$$

$$\int \left(\frac{1}{4}(c+dx)^{5/2} \cos(a+bx) - \frac{1}{4}(c+dx)^{5/2} \cos(3a+3bx) \right) dx$$

$$\downarrow \text{2009}$$

$$-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}-\frac{15d^2\sqrt{c+dx}\sin(a+bx)}{16b^3}+\frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{144b^3}+\frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)}{4b}-\frac{(c+dx)^{5/2}\sin(3a+3bx)}{12b}$$

input

```
Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output

```
(5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(8*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a
+ 3*b*x])/(72*b^2) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqr
t[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi
/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[
d]])/(144*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/2)*Sq
rt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c
)/d])/(16*b^(7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(16*b^3) + ((c +
d*x)^(5/2)*Sin[a + b*x])/(4*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(1
44*b^3) - ((c + d*x)^(5/2)*Sin[3*a + 3*b*x])/(12*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\sqrt{\pi}\left(\right.\right.}{4b}$
default	$\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\sqrt{\pi}\left(\right.\right.}{4b}$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/8/b*d*(d*x+c)^(5/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-5/8/b*d*(-1/2/b*d*
(d*x+c)^(3/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2)*
sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*
d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((a*
d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1/24
/b*d*(d*x+c)^(5/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+5/24/b*d*(-1/6/b*d*(d*
x+c)^(3/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2)
*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*
(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d
)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.91

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{5\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(
sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 40
5*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(
pi*d)))*sin(-(b*c - a*d)/d) + 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(
sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(10*(b^2*
d^2*x + b^2*c*d)*cos(b*x + a)^3 - 30*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) -
(12*b^3*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 35*b*d^2 - (12*b^3*d^2*x^2 +
 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d
*x + c))/b^4
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.35

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/3456*(240*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d) - 2160
*(d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d) - 5*(-(I + 1)*9^(1/4
)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9
^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(3*I*b/d)) - 405*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2
)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/
4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 405*(-(I - 1)*sqr
t(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(2)*
sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt
(-I*b/d)) - 5*((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(
-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)
*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + 24*(12*(d*x +
c)^(5/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*sin(3*((d*x + c)*b - b*c + a*d)/d)
 - 216*(4*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*sin(((d*x + c)*b
 - b*c + a*d)/d))*d/b^5
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 2453, normalized size of antiderivative = 6.04

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/1728*(72*(3*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*s
qrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b
*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/
d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(1/2*I*
sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*
c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + d^3*(27*(-I*sq
rt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3)*d*erf(
1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*
b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 2*(-4*I*(d*x
+ c)^(5/2)*b^2*d + 12*I*(d*x + c)^(3/2)*b^2*c*d - 12*I*sqrt(d*x + c)*b^2*c
^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x
 + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + (-I*sqrt(6)*s
qrt(pi)*(72*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*I*
sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c
- I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 6*(-12*I*(d*x + c)
^(5/2)*b^2*d + 36*I*(d*x + c)^(3/2)*b^2*c*d - 36*I*sqrt(d*x + c)*b^2*c^2*d
 + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x +...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)^2\,(c+d\,x)^{5/2}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \left(\int \sqrt{dx+c}\,\cos(bx+a)\sin(bx+a)^2\,x^2dx\right)d^2$$
$$+ 2\left(\int \sqrt{dx+c}\,\cos(bx+a)\sin(bx+a)^2\,xdx\right)cd$$
$$+ \left(\int \sqrt{dx+c}\,\cos(bx+a)\sin(bx+a)^2\,dx\right)c^2$$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)*sin(a + b*x)**2*x**2,x)*d**2 + 2*int(sqrt(c
 + d*x)*cos(a + b*x)*sin(a + b*x)**2*x,x)*c*d + int(sqrt(c + d*x)*cos(a +
b*x)*sin(a + b*x)**2,x)*c**2
```

3.64 $\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal result . . . 663
Mathematica [C] (verified) . . . 664
Rubi [A] (verified) . . . 665
Maple [A] (verified) . . . 667
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Sympy [F(-1)] . . . 669
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Giac [C] (verification not implemented) . . . 670
Mupad [F(-1)] . . . 671
Reduce [F] . . . 671

Optimal result

Integrand size = 24, antiderivative size = 407

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d^2\sqrt{c+dx}\cos(4a+4bx)}{2048b^3} + \frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} - \frac{15d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}} - \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{4096b^{7/2}} + \frac{15d^{5/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{256b^{7/2}} + \frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{32b^2} - \frac{5d(c+dx)^{3/2}\sin(4a+4bx)}{256b^2}$$

output

```
15/128*d^2*(d*x+c)^(1/2)*cos(2*b*x+2*a)/b^3-1/8*(d*x+c)^(5/2)*cos(2*b*x+2*
a)/b-15/2048*d^2*(d*x+c)^(1/2)*cos(4*b*x+4*a)/b^3+1/32*(d*x+c)^(5/2)*cos(4
*b*x+4*a)/b+15/8192*d^(5/2)*2^(1/2)*Pi^(1/2)*cos(4*a-4*b*c/d)*FresnelC(2*b
^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-15/256*d^(5/2)*Pi^(
1/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b
^(7/2)-15/8192*d^(5/2)*2^(1/2)*Pi^(1/2)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2
)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)/b^(7/2)+15/256*d^(5/2)*Pi^(1/2)*
FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/b^(7/2
)+5/32*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2-5/256*d*(d*x+c)^(3/2)*sin(4*b*x+
4*a)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.25 (sec) , antiderivative size = 1332, normalized size of antiderivative = 3.27

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input

```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output

```
((1/128 + I/128)*c*Sqrt[d]*((2 + 2*I)*Sqrt[b]*Sqrt[d]*E^(((2*I)*b*c)/d)*Sq
rt[c + d*x]*(3 + (4*I)*b*x + E^((4*I)*(a + b*x))*(-3 + (4*I)*b*x)) + I*(4*
b*c + (3*I)*d)*E^(((2*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[((1 + I)*Sqrt[b]*S
qrt[c + d*x])/Sqrt[d]] + ((4*I)*b*c + 3*d)*E^((2*I)*(2*a + b*x))*Sqrt[Pi]*
Erfi[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^(((2*I)*(a*d +
b*(c + d*x)))/d)) - (c*Sqrt[d]*(-4*Sqrt[b]*Sqrt[d]*E^(((4*I)*b*c)/d)*Sqrt[
c + d*x]*(-3*I + 8*b*x + E^((8*I)*(a + b*x))*(3*I + 8*b*x)) + (-1)^(3/4)*(
8*b*c + (3*I)*d)*E^(((4*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[(2*(-1)^(1/4)*Sq
rt[b]*Sqrt[c + d*x])/Sqrt[d]] + (-1)^(1/4)*((8*I)*b*c + 3*d)*E^((4*I)*(2*a
 + b*x))*Sqrt[Pi]*Erfi[(2*(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(10
24*b^(5/2)*E^(((4*I)*(a*d + b*(c + d*x)))/d)) + (c^2*(-1/4*(E^((2*I)*(a -
(b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d])/(Sqrt[2]*b*Sqr
t[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma[3/2, ((2*I)*b*(c + d*x))/d
])/(4*Sqrt[2]*b*E^((2*I)*(a - (b*c)/d))*Sqrt[(I*b*(c + d*x))/d])))/4 - (c^
2*Sqrt[c + d*x]*(-((E^((8*I)*a)*Gamma[3/2, ((-4*I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((8*I)*b*c)/d)*Gamma[3/2, ((4*I)*b*(c + d*x))/
d])/Sqrt[(I*b*(c + d*x))/d]))/(128*b*E^(((4*I)*(b*c + a*d))/d)) + (Sqrt[d]
*((1 - I)*E^((2*I)*(a - (b*c)/d))*((2 + 2*I)*Sqrt[b]*Sqrt[d]*E^(((2*I)*b*(
c + d*x))/d)*Sqrt[c + d*x]*(15*d - 16*b^2*d*x^2 + (4*I)*b*(c - 5*d*x)) + (
16*b^2*c^2 - (24*I)*b*c*d - 15*d^2)*Sqrt[Pi]*Erfi[((1 + I)*Sqrt[b]*Sqrt...
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2} \sin^3(a+bx) \cos(a+bx)\, dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}(c+dx)^{5/2} \sin(2a+2bx) - \frac{1}{8}(c+dx)^{5/2} \sin(4a+4bx) \right) dx$$

$$\downarrow 2009$$

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15\sqrt{\pi}d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{15d^2\sqrt{c+dx}\cos(4a+4bx)}{2048b^3}+\frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{32b^2}-\frac{5d(c+dx)^{3/2}\sin(4a+4bx)}{256b^2}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}$$

input

```
Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output

```
(15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2
*a + 2*b*x])/(8*b) - (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) +
((c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a
 - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(409
6*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]
*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2
]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)
/d])/(4096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*
x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*
x)^(3/2)*Sin[2*a + 2*b*x])/(32*b^2) - (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x
])/(256*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 3.06 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.15

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}\right.\right.}{\left.\left.\right.\right.}$
default	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}\right.\right.}{\left.\left.\right.\right.}$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+5/16/b*d*(1/
4/b*d*(d*x+c)^(3/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*
x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(
cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(
2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))+1/64/
b*d*(d*x+c)^(5/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-5/64/b*d*(1/8/b*d*(d*x+
c)^(3/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*
cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(co
s(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d
)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/
2)/d)))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \frac{15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\mathrm{s}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d)
)*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d
) - 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 4*(320*b^3*d^2*x^2 + 640*b^3*c
*d*x + 320*b^3*c^2 + 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*x + 64*b^3*c^2 - 15*b
*d^2)*cos(b*x + a)^4 - 255*b*d^2 - 8*(128*b^3*d^2*x^2 + 256*b^3*c*d*x + 12
8*b^3*c^2 - 75*b*d^2)*cos(b*x + a)^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*cos(b*
x + a)^3 - 5*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x +
c))/b^4
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.35

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/32768*(640*(d*x + c)^(3/2)*b^3*sin(4*((d*x + c)*b - b*c + a*d)/d) - 512
0*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 16*(64*(d*x + c
)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(4*((d*x + c)*b - b*c + a*d)/d)
 + 256*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c
)*b - b*c + a*d)/d) - 240*((I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2
)^(1/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^
2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 15*
(-(I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (
I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2
*sqrt(d*x + c)*sqrt(I*b/d)) - 15*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1
/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 240*(-(I +
1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) -
(I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/
d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^5
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 2412, normalized size of antiderivative = 5.93

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/16384*(512*(sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)) - 4*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)))*c^3 - d^3*((sqrt(2)*sqrt(pi)*(512*b^3*c^3 - 192*I*b^2*c^2*d - 72*b*c
*d^2 + 15*I*d^3)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b^3) + 4*I*(-64*I*(d*x + c)^(5/2)*b^2*d + 192*I*(d*x + c)^(3/2)*b^2*c*d -
 192*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x +
c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d
)/d)/b^3)/d^3 + (sqrt(2)*sqrt(pi)*(512*b^3*c^3 + 192*I*b^2*c^2*d - 72*b*c*
d^2 - 15*I*d^3)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)*b^3) + 4*I*(-64*I*(d*x + c)^(5/2)*b^2*d + 192*I*(d*x + c)^(3/2)*b^2*c*d
- 192*I*sqrt(d*x + c)*b^2*c^2*d - 40*(d*x + c)^(3/2)*b*d^2 + 72*sqrt(d*x +
 c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^(-4*(I*(d*x + c)*b - I*b*c + I*...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)^3\,(c+d\,x)^{5/2}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \left(\int \sqrt{dx+c}\,\cos(bx+a)\sin(bx+a)^3\,x^2dx\right)d^2$$
$$+2\left(\int \sqrt{dx+c}\,\cos(bx+a)\sin(bx+a)^3\,xdx\right)cd$$
$$+\left(\int \sqrt{dx+c}\,\cos(bx+a)\sin(bx+a)^3\,dx\right)c^2$$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)*sin(a + b*x)**3*x**2,x)*d**2 + 2*int(sqrt(c
 + d*x)*cos(a + b*x)*sin(a + b*x)**3*x,x)*c*d + int(sqrt(c + d*x)*cos(a +
b*x)*sin(a + b*x)**3,x)*c**2
```

3.65 $\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal result . . . 672
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Giac [C] (verification not implemented) . . . 678
Mupad [F(-1)] . . . 679
Reduce [F] . . . 680

Optimal result

Integrand size = 24, antiderivative size = 351

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}$$
$$+\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}$$
$$-\frac{3d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}$$
$$+\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{512b^{5/2}}$$
$$-\frac{3d^{3/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{64b^{5/2}}$$
$$+\frac{3d\sqrt{c+dx}\sin(2a+2bx)}{32b^2}-\frac{3d\sqrt{c+dx}\sin(4a+4bx)}{256b^2}$$

output

```
-1/8*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b+1/32*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b+3/
1024*d^(3/2)*2^(1/2)*Pi^(1/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/
Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)-3/64*d^(3/2)*Pi^(1/2)*cos(2*a-2*b*
c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b^(5/2)+3/1024*d^(
3/2)*2^(1/2)*Pi^(1/2)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*sin(4*a-4*b*c/d)/b^(5/2)-3/64*d^(3/2)*Pi^(1/2)*FresnelC(2*b^(1/2)*(
d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/b^(5/2)+3/32*d*(d*x+c)^(1/
2)*sin(2*b*x+2*a)/b^2-3/256*d*(d*x+c)^(1/2)*sin(4*b*x+4*a)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.16 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.97

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \frac{e^{-4ia}\left(8e^{2i\left(a-\frac{b(c+dx)}{d}\right)}\left(-4\sqrt{b}de^{\frac{2ibc}{d}}\sqrt{c+dx}\left(-3i+4bx+e^{4i(a+bx)}(3i+4bx)\right)-(1-i)(4bc+3id\right.\right.}{}$$

input

```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output

```
(8*E^((2*I)*(a - (b*(c + d*x))/d))*(-4*Sqrt[b]*d*E^(((2*I)*b*c)/d)*Sqrt[c
+ d*x]*(-3*I + 4*b*x + E^((4*I)*(a + b*x))*(3*I + 4*b*x)) - (1 - I)*(4*b*c
 + (3*I)*d)*Sqrt[d]*E^(((2*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[((1 + I)*Sqrt
[b]*Sqrt[c + d*x])/Sqrt[d]] + (1 + I)*Sqrt[d]*((4*I)*b*c + 3*d)*E^((2*I)*(
2*a + b*x))*Sqrt[Pi]*Erfi[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]) - (Sqr
t[d]*(-4*Sqrt[b]*Sqrt[d]*E^(((4*I)*b*c)/d)*Sqrt[c + d*x]*(-3*I + 8*b*x + E
^((8*I)*(a + b*x))*(3*I + 8*b*x)) + (-1)^(3/4)*(8*b*c + (3*I)*d)*E^(((4*I)
*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[(2*(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[
d]] + (-1)^(1/4)*((8*I)*b*c + 3*d)*E^((4*I)*(2*a + b*x))*Sqrt[Pi]*Erfi[(2*
(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/E^(((4*I)*b*(c + d*x))/d) - (
64*Sqrt[2]*b^(3/2)*c*E^((2*I)*(a - (b*c)/d))*Sqrt[c + d*x]*(E^((4*I)*a)*Sq
rt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d] + E^(((4*I)*b*c)/
d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((2*I)*b*(c + d*x))/d]))/Sqrt[(b^
2*(c + d*x)^2)/d^2] - (16*b^(3/2)*c*Sqrt[c + d*x]*(-((E^((8*I)*a)*Gamma[3/
2, ((-4*I)*b*(c + d*x))/d])/Sqrt[((-I)*b*(c + d*x))/d]) - (E^(((8*I)*b*c)/
d)*Gamma[3/2, ((4*I)*b*(c + d*x))/d])/Sqrt[(I*b*(c + d*x))/d]))/E^(((4*I)*
b*c)/d))/(2048*b^(5/2)*E^((4*I)*a))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin^3(a+bx)\cos(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx) - \frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right)dx$$

$$\downarrow 2009$$

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-$$
$$\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}+$$
$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-$$
$$\frac{3\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(2a+2bx)}{32b^2}-$$
$$\frac{3d\sqrt{c+dx}\sin(4a+4bx)}{256b^2}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}+\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}$$

input

```
Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output

```
-1/8*((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/b + ((c + d*x)^(3/2)*Cos[4*a + 4*b
*x])/(32*b) + (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[
b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi]
)])/(64*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqr
t[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*
b*c)/d])/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) - (3
*d*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.07

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{8b\sqrt{\frac{b}{d}}}\right.}{8b}$
default	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{8b\sqrt{\frac{b}{d}}}\right.}{8b}$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+3/16/b*d*(1/
4/b*d*(d*x+c)^(1/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d
)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2
)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)
))+1/64/b*d*(d*x+c)^(3/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-3/64/b*d*(1/8/b
*d*(d*x+c)^(1/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2
)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*
b*(d*x+c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.90

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*
fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d)
- 48*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x +
c)*sqrt(b/(pi*d))) - 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*
sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x +
 a)^4 + 10*b^2*d*x + 10*b^2*c - 32*(b^2*d*x + b^2*c)*cos(b*x + a)^2 - 3*(2
*b*d*cos(b*x + a)^3 - 5*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F]

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin^3(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**3*cos(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.43

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output

```
1/4096*(128*(d*x + c)^(3/2)*b^3*cos(4*((d*x + c)*b - b*c + a*d)/d)/d - 512
*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 48*sqrt(d*x +
c)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) + 384*sqrt(d*x + c)*b^2*sin(2*((
d*x + c)*b - b*c + a*d)/d) + 24*(-(I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^
2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*
(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) +
3*((I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I
 - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sq
rt(d*x + c)*sqrt(I*b/d)) + 3*(-(I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4
)*cos(-4*(b*c - a*d)/d) + (I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin
(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + 24*((I - 1)*4^(1/4
)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(
1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(
d*x + c)*sqrt(-2*I*b/d)))*d/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 1490, normalized size of antiderivative = 4.25

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output

```
-1/2048*(64*(sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)) - 4*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)) - 4*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)))*c^2 - 16*(sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-I*sqrt(2)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(I*s
qrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c
 + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*sqrt(pi)*(4*b*c
- I*d)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2
*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*sqrt(pi)*(
4*b*c + I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 16*s
qrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 4*sqrt(d*x + c
)*d*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 16*sqrt(d*x + c)*d*e^(-2*
(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4*(-I*(d*...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

input

```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

output

```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c + dx)^{3/2} \cos(a+bx) \sin^3(a+bx)\, dx = \left(\int \sqrt{dx+c} \cos(bx+a) \sin(bx+a)^3 x dx \right) d + \left(\int \sqrt{dx+c} \cos(bx+a) \sin(bx+a)^3 dx \right) c$$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)*sin(a + b*x)**3*x,x)*d + int(sqrt(c + d*x)*
cos(a + b*x)*sin(a + b*x)**3,x)*c
```

3.66 $\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal result . . . 681
Mathematica [C] (verified) . . . 682
Rubi [A] (verified) . . . 683
Maple [A] (verified) . . . 684
Fricas [A] (verification not implemented) . . . 685
Sympy [F] . . . 685
Maxima [C] (verification not implemented) . . . 686
Giac [C] (verification not implemented) . . . 686
Mupad [F(-1)] . . . 687
Reduce [F] . . . 688

Optimal result

Integrand size = 24, antiderivative size = 299

$$\begin{aligned}
&\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx \\
&= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b} \\
&\quad - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{64b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{16b^{3/2}}
\end{aligned}$$

output
```
-1/8*(d*x+c)^(1/2)*cos(2*b*x+2*a)/b+1/32*(d*x+c)^(1/2)*cos(4*b*x+4*a)/b-1/
128*d^(1/2)*2^(1/2)*Pi^(1/2)*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+1/16*d^(1/2)*Pi^(1/2)*cos(2*a-2*b*c
/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b^(3/2)+1/128*d^(1/
2)*2^(1/2)*Pi^(1/2)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*sin(4*a-4*b*c/d)/b^(3/2)-1/16*d^(1/2)*Pi^(1/2)*FresnelS(2*b^(1/2)*(d*
x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.84

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{e^{-\frac{4i(bc+ad)}{d}}\sqrt{c+dx}\left(-4\sqrt{2}e^{2i\left(3a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)-4\sqrt{2}e^{2i\left(a+\frac{3bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)\right.}{128b\sqrt{\frac{b^2(c+dx)^2}{d^2}}}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
(Sqrt[c + d*x]*(-4*Sqrt[2]*E^((2*I)*(3*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/
d]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d] - 4*Sqrt[2]*E^((2*I)*(a + (3*b*c)/d)
)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((2*I)*b*(c + d*x))/d] + E^((8*I)*
a)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-4*I)*b*(c + d*x))/d] + E^(((8*I)*
b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((4*I)*b*(c + d*x))/d]))/(12
8*b*E^(((4*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^3(a+bx)\cos(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}\sqrt{c+dx}\sin(2a+2bx) - \frac{1}{8}\sqrt{c+dx}\sin(4a+4bx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} + \frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} - \frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}} - \frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}$$

input

```
Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output

```
-1/8*(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/b + (Sqrt[c + d*x]*Cos[4*a + 4*b*x])
/(32*b) - (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a
- (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b
^(3/2)) + (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x]
)/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS
[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b
^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.96

method	result
derivativedivides	$-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}+$
default	$-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}+$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/32/b*d*Pi^
(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c
)^(1/2)/d))+1/64/b*d*(d*x+c)^(1/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-1/256/
b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi
^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2
)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.82

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sq
rt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_
sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 8*pi*d
*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(
pi*d))) + 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d))
)*sin(-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 16*b*cos(b*x + a)^2 + 5*
b)*sqrt(d*x + c))/b^2
```

Sympy [F]

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx = \int \sqrt{c+dx}\sin^3(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.43

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{\left(\frac{16\sqrt{dx+c}b^2\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)}{d} - \frac{64\sqrt{dx+c}b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d} - 4\left((i-1)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-a\right.\right.}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/512*(16*sqrt(d*x + c)*b^2*cos(4*((d*x + c)*b - b*c + a*d)/d)/d - 64*sqrt
(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 4*((I - 1)*4^(1/4)*sq
rt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*s
qrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)
*sqrt(2*I*b/d)) - (-(I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-4*(b*c
 - a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)
/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - ((I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d
^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/
4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 4*(-(I + 1)*
4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)
*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(-2*I*b/d)))*d/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 826, normalized size of antiderivative = 2.76

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output

```
1/256*(sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(I*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*(sqrt(2)*sqrt(pi)*d*erf
(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*
b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d
*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4
*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(pi)*d
*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c
- I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(pi)*d*erf(I*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 8*sqrt(pi)*(4*b*c - I*d)*d*
erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c -
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*sqrt(pi)*(4*b*c + I
*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-
I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 16*sqrt(d*x +
 c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4
*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 16*sqrt(d*x + c)*d*e^(-2*(-I*(d*x
+ c)*b + I*b*c - I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4*(-I*(d*x + c)*b ...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)^3\,\sqrt{c+d\,x}\,dx$$

input

```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

output

```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx = \int \sqrt{dx+c}\,\cos\left(bx+a\right)\sin\left(bx+a\right)^3\,dx$$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)*sin(a + b*x)**3,x)
```

3.67 $\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal result . . . 689
Mathematica [C] (verified) . . . 690
Rubi [A] (verified) . . . 691
Maple [A] (verified) . . . 692
Fricas [A] (verification not implemented) . . . 693
Sympy [F] . . . 693
Maxima [C] (verification not implemented) . . . 694
Giac [C] (verification not implemented) . . . 694
Mupad [F(-1)] . . . 695
Reduce [F] . . . 696

Optimal result

Integrand size = 24, antiderivative size = 299

$$\begin{aligned}
&\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx \\
&= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b} \\
&\quad - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{64b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{16b^{3/2}}
\end{aligned}$$

output
```
-1/8*(d*x+c)^(1/2)*cos(2*b*x+2*a)/b+1/32*(d*x+c)^(1/2)*cos(4*b*x+4*a)/b-1/
128*d^(1/2)*2^(1/2)*Pi^(1/2)*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+1/16*d^(1/2)*Pi^(1/2)*cos(2*a-2*b*c
/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b^(3/2)+1/128*d^(1/
2)*2^(1/2)*Pi^(1/2)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*sin(4*a-4*b*c/d)/b^(3/2)-1/16*d^(1/2)*Pi^(1/2)*FresnelS(2*b^(1/2)*(d*
x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.84

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{e^{-\frac{4i(bc+ad)}{d}}\sqrt{c+dx}\left(-4\sqrt{2}e^{2i\left(3a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)-4\sqrt{2}e^{2i\left(a+\frac{3bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)\right.}{128b\sqrt{\frac{b^2(c+dx)^2}{d^2}}}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
(Sqrt[c + d*x]*(-4*Sqrt[2]*E^((2*I)*(3*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/
d]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d] - 4*Sqrt[2]*E^((2*I)*(a + (3*b*c)/d)
)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((2*I)*b*(c + d*x))/d] + E^((8*I)*
a)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-4*I)*b*(c + d*x))/d] + E^(((8*I)*
b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((4*I)*b*(c + d*x))/d]))/(12
8*b*E^(((4*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^3(a+bx)\cos(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}\sqrt{c+dx}\sin(2a+2bx) - \frac{1}{8}\sqrt{c+dx}\sin(4a+4bx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} + \frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} - \frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}} - \frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
-1/8*(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/b + (Sqrt[c + d*x]*Cos[4*a + 4*b*x])
/(32*b) - (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a
- (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b
^(3/2)) + (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x]
)/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS
[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b
^(3/2))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.96

method	result
derivativedivides	$-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}+$
default	$-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}+$

input

```
int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output

```
2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/32/b*d*Pi^
(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c
)^(1/2)/d))+1/64/b*d*(d*x+c)^(1/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-1/256/
b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi
^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2
)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.82

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sq
rt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_
sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 8*pi*d
*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(
pi*d))) + 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d))
)*sin(-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 16*b*cos(b*x + a)^2 + 5*
b)*sqrt(d*x + c))/b^2
```

Sympy [F]

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx = \int \sqrt{c+dx}\sin^3(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.43

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{\left(\frac{16\sqrt{dx+c}b^2\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)}{d} - \frac{64\sqrt{dx+c}b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d} - 4\left((i-1)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-a\right.\right.}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/512*(16*sqrt(d*x + c)*b^2*cos(4*((d*x + c)*b - b*c + a*d)/d)/d - 64*sqrt
(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 4*((I - 1)*4^(1/4)*sq
rt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*s
qrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)
*sqrt(2*I*b/d)) - (-(I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-4*(b*c
 - a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)
/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - ((I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d
^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/
4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 4*(-(I + 1)*
4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)
*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(-2*I*b/d)))*d/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 826, normalized size of antiderivative = 2.76

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output

```
1/256*(sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(I*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*(sqrt(2)*sqrt(pi)*d*erf
(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*
b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d
*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4
*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(pi)*d
*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c
- I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(pi)*d*erf(I*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 8*sqrt(pi)*(4*b*c - I*d)*d*
erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c -
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*sqrt(pi)*(4*b*c + I
*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-
I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 16*sqrt(d*x +
 c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4
*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 16*sqrt(d*x + c)*d*e^(-2*(-I*(d*x
+ c)*b + I*b*c - I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4*(-I*(d*x + c)*b ...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)^3\,\sqrt{c+d\,x}\,dx$$

input

```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

output

```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx = \int \sqrt{dx+c}\,\cos\left(bx+a\right)\sin\left(bx+a\right)^3\,dx$$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)*sin(a + b*x)**3,x)
```

3.68 $\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal result . . . 697
Mathematica [C] (verified) . . . 698
Rubi [A] (verified) . . . 699
Maple [A] (verified) . . . 701
Fricas [A] (verification not implemented) . . . 701
Sympy [F] . . . 702
Maxima [C] (verification not implemented) . . . 702
Giac [C] (verification not implemented) . . . 703
Mupad [F(-1)] . . . 704
Reduce [F] . . . 705

Optimal result

Integrand size = 24, antiderivative size = 351

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{512b^{5/2}} - \frac{3d^{3/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{64b^{5/2}} + \frac{3d\sqrt{c+dx}\sin(2a+2bx)}{32b^2} - \frac{3d\sqrt{c+dx}\sin(4a+4bx)}{256b^2}$$

output

```
-1/8*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b+1/32*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b+3/
1024*d^(3/2)*2^(1/2)*Pi^(1/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/
Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)-3/64*d^(3/2)*Pi^(1/2)*cos(2*a-2*b*
c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b^(5/2)+3/1024*d^(
3/2)*2^(1/2)*Pi^(1/2)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*sin(4*a-4*b*c/d)/b^(5/2)-3/64*d^(3/2)*Pi^(1/2)*FresnelC(2*b^(1/2)*(
d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/b^(5/2)+3/32*d*(d*x+c)^(1/
2)*sin(2*b*x+2*a)/b^2-3/256*d*(d*x+c)^(1/2)*sin(4*b*x+4*a)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.97

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \frac{e^{-4ia}\left(8e^{2i\left(a-\frac{b(c+dx)}{d}\right)}\left(-4\sqrt{b}de^{\frac{2ibc}{d}}\sqrt{c+dx}\left(-3i+4bx+e^{4i(a+bx)}(3i+4bx)\right)-(1-i)(4bc+3id\right.\right.}{}$$

input

```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output

```
(8*E^((2*I)*(a - (b*(c + d*x))/d))*(-4*Sqrt[b]*d*E^(((2*I)*b*c)/d)*Sqrt[c
+ d*x]*(-3*I + 4*b*x + E^((4*I)*(a + b*x))*(3*I + 4*b*x)) - (1 - I)*(4*b*c
 + (3*I)*d)*Sqrt[d]*E^(((2*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[((1 + I)*Sqrt
[b]*Sqrt[c + d*x])/Sqrt[d]] + (1 + I)*Sqrt[d]*((4*I)*b*c + 3*d)*E^((2*I)*(
2*a + b*x))*Sqrt[Pi]*Erfi[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]) - (Sqr
t[d]*(-4*Sqrt[b]*Sqrt[d]*E^(((4*I)*b*c)/d)*Sqrt[c + d*x]*(-3*I + 8*b*x + E
^((8*I)*(a + b*x))*(3*I + 8*b*x)) + (-1)^(3/4)*(8*b*c + (3*I)*d)*E^(((4*I)
*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[(2*(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[
d]] + (-1)^(1/4)*((8*I)*b*c + 3*d)*E^((4*I)*(2*a + b*x))*Sqrt[Pi]*Erfi[(2*
(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/E^(((4*I)*b*(c + d*x))/d) - (
64*Sqrt[2]*b^(3/2)*c*E^((2*I)*(a - (b*c)/d))*Sqrt[c + d*x]*(E^((4*I)*a)*Sq
rt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d] + E^(((4*I)*b*c)/
d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((2*I)*b*(c + d*x))/d]))/Sqrt[(b^
2*(c + d*x)^2)/d^2] - (16*b^(3/2)*c*Sqrt[c + d*x]*(-((E^((8*I)*a)*Gamma[3/
2, ((-4*I)*b*(c + d*x))/d])/Sqrt[((-I)*b*(c + d*x))/d]) - (E^(((8*I)*b*c)/
d)*Gamma[3/2, ((4*I)*b*(c + d*x))/d])/Sqrt[(I*b*(c + d*x))/d]))/E^(((4*I)*
b*c)/d))/(2048*b^(5/2)*E^((4*I)*a))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin^3(a+bx)\cos(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx) - \frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right)dx$$

$$\downarrow 2009$$

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-$$
$$\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}+$$
$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-$$
$$\frac{3\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(2a+2bx)}{32b^2}-$$
$$\frac{3d\sqrt{c+dx}\sin(4a+4bx)}{256b^2}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}+\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
-1/8*((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/b + ((c + d*x)^(3/2)*Cos[4*a + 4*b
*x])/(32*b) + (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[
b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi]
)])/(64*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqr
t[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*
b*c)/d])/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) - (3
*d*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.07

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{8b\sqrt{\frac{b}{d}}}\right.}{8b}$
default	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{8b\sqrt{\frac{b}{d}}}\right.}{8b}$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+3/16/b*d*(1/
4/b*d*(d*x+c)^(1/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d
)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2
)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)
))+1/64/b*d*(d*x+c)^(3/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-3/64/b*d*(1/8/b
*d*(d*x+c)^(1/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2
)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*
b*(d*x+c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.90

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*
fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d)
- 48*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x +
c)*sqrt(b/(pi*d))) - 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*
sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x +
 a)^4 + 10*b^2*d*x + 10*b^2*c - 32*(b^2*d*x + b^2*c)*cos(b*x + a)^2 - 3*(2
*b*d*cos(b*x + a)^3 - 5*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F]

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin^3(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**3*cos(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.43

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output

```
1/4096*(128*(d*x + c)^(3/2)*b^3*cos(4*((d*x + c)*b - b*c + a*d)/d)/d - 512
*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 48*sqrt(d*x +
c)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) + 384*sqrt(d*x + c)*b^2*sin(2*((
d*x + c)*b - b*c + a*d)/d) + 24*(-(I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^
2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*
(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) +
3*((I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I
 - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sq
rt(d*x + c)*sqrt(I*b/d)) + 3*(-(I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4
)*cos(-4*(b*c - a*d)/d) + (I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin
(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + 24*((I - 1)*4^(1/4
)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(
1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(
d*x + c)*sqrt(-2*I*b/d)))*d/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 1490, normalized size of antiderivative = 4.25

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output

```
-1/2048*(64*(sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)) - 4*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)) - 4*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)))*c^2 - 16*(sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-I*sqrt(2)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(I*s
qrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c
 + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*sqrt(pi)*(4*b*c
- I*d)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2
*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*sqrt(pi)*(
4*b*c + I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 16*s
qrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 4*sqrt(d*x + c
)*d*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 16*sqrt(d*x + c)*d*e^(-2*
(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4*(-I*(d*...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

input

```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

output

```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c + dx)^{3/2} \cos(a+bx) \sin^3(a+bx)\, dx = \left(\int \sqrt{dx+c} \cos(bx+a) \sin(bx+a)^3 x dx \right) d + \left(\int \sqrt{dx+c} \cos(bx+a) \sin(bx+a)^3 dx \right) c$$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)*sin(a + b*x)**3*x,x)*d + int(sqrt(c + d*x)*
cos(a + b*x)*sin(a + b*x)**3,x)*c
```

3.69 $\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal result . . . 706
Mathematica [C] (verified) . . . 707
Rubi [A] (verified) . . . 708
Maple [A] (verified) . . . 710
Fricas [A] (verification not implemented) . . . 711
Sympy [F(-1)] . . . 712
Maxima [C] (verification not implemented) . . . 712
Giac [C] (verification not implemented) . . . 713
Mupad [F(-1)] . . . 714
Reduce [F] . . . 714

Optimal result

Integrand size = 24, antiderivative size = 407

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d^2\sqrt{c+dx}\cos(4a+4bx)}{2048b^3} + \frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} - \frac{15d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}} - \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{4096b^{7/2}} + \frac{15d^{5/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{256b^{7/2}} + \frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{32b^2} - \frac{5d(c+dx)^{3/2}\sin(4a+4bx)}{256b^2}$$

output

```
15/128*d^2*(d*x+c)^(1/2)*cos(2*b*x+2*a)/b^3-1/8*(d*x+c)^(5/2)*cos(2*b*x+2*
a)/b-15/2048*d^2*(d*x+c)^(1/2)*cos(4*b*x+4*a)/b^3+1/32*(d*x+c)^(5/2)*cos(4
*b*x+4*a)/b+15/8192*d^(5/2)*2^(1/2)*Pi^(1/2)*cos(4*a-4*b*c/d)*FresnelC(2*b
^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-15/256*d^(5/2)*Pi^(
1/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b
^(7/2)-15/8192*d^(5/2)*2^(1/2)*Pi^(1/2)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2
)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)/b^(7/2)+15/256*d^(5/2)*Pi^(1/2)*
FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/b^(7/2
)+5/32*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2-5/256*d*(d*x+c)^(3/2)*sin(4*b*x+
4*a)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.02 (sec) , antiderivative size = 1332, normalized size of antiderivative = 3.27

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input

```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output

```
((1/128 + I/128)*c*Sqrt[d]*((2 + 2*I)*Sqrt[b]*Sqrt[d]*E^(((2*I)*b*c)/d)*Sq
rt[c + d*x]*(3 + (4*I)*b*x + E^((4*I)*(a + b*x))*(-3 + (4*I)*b*x)) + I*(4*
b*c + (3*I)*d)*E^(((2*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[((1 + I)*Sqrt[b]*S
qrt[c + d*x])/Sqrt[d]] + ((4*I)*b*c + 3*d)*E^((2*I)*(2*a + b*x))*Sqrt[Pi]*
Erfi[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^(((2*I)*(a*d +
b*(c + d*x)))/d)) - (c*Sqrt[d]*(-4*Sqrt[b]*Sqrt[d]*E^(((4*I)*b*c)/d)*Sqrt[
c + d*x]*(-3*I + 8*b*x + E^((8*I)*(a + b*x))*(3*I + 8*b*x)) + (-1)^(3/4)*(
8*b*c + (3*I)*d)*E^(((4*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[(2*(-1)^(1/4)*Sq
rt[b]*Sqrt[c + d*x])/Sqrt[d]] + (-1)^(1/4)*((8*I)*b*c + 3*d)*E^((4*I)*(2*a
 + b*x))*Sqrt[Pi]*Erfi[(2*(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(10
24*b^(5/2)*E^(((4*I)*(a*d + b*(c + d*x)))/d)) + (c^2*(-1/4*(E^((2*I)*(a -
(b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d])/(Sqrt[2]*b*Sqr
t[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma[3/2, ((2*I)*b*(c + d*x))/d
])/(4*Sqrt[2]*b*E^((2*I)*(a - (b*c)/d))*Sqrt[(I*b*(c + d*x))/d])))/4 - (c^
2*Sqrt[c + d*x]*(-((E^((8*I)*a)*Gamma[3/2, ((-4*I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((8*I)*b*c)/d)*Gamma[3/2, ((4*I)*b*(c + d*x))/
d])/Sqrt[(I*b*(c + d*x))/d]))/(128*b*E^(((4*I)*(b*c + a*d))/d)) + (Sqrt[d]
*((1 - I)*E^((2*I)*(a - (b*c)/d))*((2 + 2*I)*Sqrt[b]*Sqrt[d]*E^(((2*I)*b*(
c + d*x))/d)*Sqrt[c + d*x]*(15*d - 16*b^2*d*x^2 + (4*I)*b*(c - 5*d*x)) + (
16*b^2*c^2 - (24*I)*b*c*d - 15*d^2)*Sqrt[Pi]*Erfi[((1 + I)*Sqrt[b]*Sqrt...
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2} \sin^3(a+bx) \cos(a+bx)\, dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}(c+dx)^{5/2} \sin(2a+2bx) - \frac{1}{8}(c+dx)^{5/2} \sin(4a+4bx) \right) dx$$

$$\downarrow 2009$$

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15\sqrt{\pi}d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{15d^2\sqrt{c+dx}\cos(4a+4bx)}{2048b^3}+\frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{32b^2}-\frac{5d(c+dx)^{3/2}\sin(4a+4bx)}{256b^2}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}$$

input

```
Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output

```
(15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2
*a + 2*b*x])/(8*b) - (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) +
((c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a
 - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(409
6*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]
*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2
]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)
/d])/(4096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*
x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*
x)^(3/2)*Sin[2*a + 2*b*x])/(32*b^2) - (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x
])/(256*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}\right.}{}\right.}{}}{}$
default	$\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}\right.}{}\right.}{}}{}$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+5/16/b*d*(1/
4/b*d*(d*x+c)^(3/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*
x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(
cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(
2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))+1/64/
b*d*(d*x+c)^(5/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-5/64/b*d*(1/8/b*d*(d*x+
c)^(3/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*
cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(co
s(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d
)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/
2)/d)))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \frac{15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\mathrm{s}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d)
)*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d
) - 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 4*(320*b^3*d^2*x^2 + 640*b^3*c
*d*x + 320*b^3*c^2 + 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*x + 64*b^3*c^2 - 15*b
*d^2)*cos(b*x + a)^4 - 255*b*d^2 - 8*(128*b^3*d^2*x^2 + 256*b^3*c*d*x + 12
8*b^3*c^2 - 75*b*d^2)*cos(b*x + a)^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*cos(b*
x + a)^3 - 5*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x +
c))/b^4
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.35

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/32768*(640*(d*x + c)^(3/2)*b^3*sin(4*((d*x + c)*b - b*c + a*d)/d) - 512
0*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 16*(64*(d*x + c
)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(4*((d*x + c)*b - b*c + a*d)/d)
 + 256*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c
)*b - b*c + a*d)/d) - 240*((I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2
)^(1/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^
2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 15*
(-(I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (
I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2
*sqrt(d*x + c)*sqrt(I*b/d)) - 15*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1
/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 240*(-(I +
1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) -
(I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/
d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^5
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 2412, normalized size of antiderivative = 5.93

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/16384*(512*(sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)) - 4*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)))*c^3 - d^3*((sqrt(2)*sqrt(pi)*(512*b^3*c^3 - 192*I*b^2*c^2*d - 72*b*c
*d^2 + 15*I*d^3)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b^3) + 4*I*(-64*I*(d*x + c)^(5/2)*b^2*d + 192*I*(d*x + c)^(3/2)*b^2*c*d -
 192*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x +
c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d
)/d)/b^3)/d^3 + (sqrt(2)*sqrt(pi)*(512*b^3*c^3 + 192*I*b^2*c^2*d - 72*b*c*
d^2 - 15*I*d^3)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)*b^3) + 4*I*(-64*I*(d*x + c)^(5/2)*b^2*d + 192*I*(d*x + c)^(3/2)*b^2*c*d
- 192*I*sqrt(d*x + c)*b^2*c^2*d - 40*(d*x + c)^(3/2)*b*d^2 + 72*sqrt(d*x +
 c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^(-4*(I*(d*x + c)*b - I*b*c + I*...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)^3\,(c+d\,x)^{5/2}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \left(\int \sqrt{dx+c}\,\cos(bx+a)\sin(bx+a)^3\,x^2dx\right)d^2$$
$$+ 2\left(\int \sqrt{dx+c}\,\cos(bx+a)\sin(bx+a)^3\,xdx\right)cd$$
$$+ \left(\int \sqrt{dx+c}\,\cos(bx+a)\sin(bx+a)^3\,dx\right)c^2$$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)*sin(a + b*x)**3*x**2,x)*d**2 + 2*int(sqrt(c
 + d*x)*cos(a + b*x)*sin(a + b*x)**3*x,x)*c*d + int(sqrt(c + d*x)*cos(a +
b*x)*sin(a + b*x)**3,x)*c**2
```

3.70 $\int (c+dx)^m \cos^2(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 715
Mathematica [A] (verified) . . . 716
Rubi [A] (verified) . . . 716
Maple [F] . . . 718
Fricas [A] (verification not implemented) . . . 718
Sympy [F] . . . 718
Maxima [F] . . . 719
Giac [F] . . . 719
Mupad [F(-1)] . . . 719
Reduce [F] . . . 720

Optimal result

Integrand size = 22, antiderivative size = 267

$$\begin{aligned}
&\int (c+dx)^m \cos^2(a+bx)\sin(a+bx)\,dx \\
&= -\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b} \\
&\quad -\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{8b} \\
&\quad -\frac{3^{-1-m}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3ib(c+dx)}{d}\right)}{8b} \\
&\quad -\frac{3^{-1-m}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3ib(c+dx)}{d}\right)}{8b}
\end{aligned}$$

output

```
-1/8*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)
/d)^m)-1/8*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*
x+c)/d)^m)-1/8*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*b*(d*x
+c)/d)/b/((-I*b*(d*x+c)/d)^m)-1/8*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I*b*(d*x+
c)/d)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.94

$$\int (c+dx)^m \cos^2(a+bx)\sin(a+bx)\,dx$$
$$= \frac{e^{-\frac{3i(bc+ad)}{d}}(c+dx)^m\left(3e^{\frac{2i(bc+ad)}{d}}\left(-e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)-e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\right.\right.\right.}{}$$

input
```
Integrate[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
((c + d*x)^m*(3*E^(((2*I)*(b*c + a*d))/d)*(-((E^((2*I)*a)*Gamma[1 + m, ((-
I)*b*(c + d*x))/d])/(((-I)*b*(c + d*x))/d)^m) - (E^(((2*I)*b*c)/d)*Gamma[1
 + m, (I*b*(c + d*x))/d])/((I*b*(c + d*x))/d)^m) - (E^((6*I)*a)*((I*b*(c +
 d*x))/d)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d] + E^(((6*I)*b*c)/d)*(((-I
)*b*(c + d*x))/d)^m*Gamma[1 + m, ((3*I)*b*(c + d*x))/d])/(3^m*((b^2*(c + d
*x)^2)/d^2)^m)))/(24*b*E^(((3*I)*(b*c + a*d))/d))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cos^2(a+bx)(c+dx)^m\,dx$$
$$\downarrow 4906$$
$$\int \left(\frac{1}{4}\sin(a+bx)(c+dx)^m+\frac{1}{4}\sin(3a+3bx)(c+dx)^m\right)dx$$
$$\downarrow 2009$$

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}-\frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{8b}-\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{8b}-\frac{3^{-m-1}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{3ib(c+dx)}{d}\right)}{8b}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/8*(E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/
(b*(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d
])/(8*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - (3^(-1 - m)*E^((3*I)*
(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(8*b*(((-
I)*b*(c + d*x))/d)^m) - (3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((3*I)*b*(c +
 d*x))/d])/(8*b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [F]

$$\int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)\,dx$$

input
```
int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x)
```

output
```
int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.70

$$\int (c+dx)^m \cos^2(a+bx)\sin(a+bx)\,dx = -\frac{3\,e^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-ibc+iad}{d}\right)}\Gamma\left(m+1,\frac{ibdx+ibc}{d}\right)+e^{\left(-\frac{dm\log\left(-\frac{3ib}{d}\right)+3ibc-3iad}{d}\right)}\Gamma\left(m+1,-\frac{3\,(ibdx+ibc)}{d}\right)+3\,e^{\left(-\frac{dm}{}\right.}}{24\,b}$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/24*(3*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x + I
*b*c)/d) + e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d)*gamma(m + 1, -3*
(I*b*d*x + I*b*c)/d) + 3*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma(m
+ 1, (-I*b*d*x - I*b*c)/d) + e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d)
*gamma(m + 1, -3*(-I*b*d*x - I*b*c)/d))/b
```

Sympy [F]

$$\int (c+dx)^m \cos^2(a+bx)\sin(a+bx)\,dx = \int (c+dx)^m \sin(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cos(b*x+a)**2*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**m*sin(a + b*x)*cos(a + b*x)**2, x)
```

Maxima [F]

$$\int (c+dx)^m \cos^2(a+bx)\sin(a+bx)\,dx = \int (dx+c)^m \cos\left(bx+a\right)^2 \sin\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a), x)
```

Giac [F]

$$\int (c+dx)^m \cos^2(a+bx)\sin(a+bx)\,dx = \int (dx+c)^m \cos\left(bx+a\right)^2 \sin\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos^2(a+bx)\sin(a+bx)\,dx = \int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)^m\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \cos^2(a+bx)\sin(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)\,dx$$

input
```
int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x)
```

output
```
int((c + d*x)**m*cos(a + b*x)**2*sin(a + b*x),x)
```

3.71 $\int (c+dx)^4 \cos^2(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 721
Mathematica [A] (verified) . . . 722
Rubi [A] (verified) . . . 722
Maple [A] (verified) . . . 728
Fricas [A] (verification not implemented) . . . 728
Sympy [B] (verification not implemented) . . . 729
Maxima [B] (verification not implemented) . . . 730
Giac [A] (verification not implemented) . . . 731
Mupad [B] (verification not implemented) . . . 732
Reduce [B] (verification not implemented) . . . 733

Optimal result

Integrand size = 22, antiderivative size = 205

$$\begin{aligned}\int (c+dx)^4 \cos^2(a+bx)\sin(a+bx)\,dx = &-\frac{160d^4\cos(a+bx)}{27b^5}\\ &+\frac{8d^2(c+dx)^2\cos(a+bx)}{3b^3}-\frac{8d^4\cos^3(a+bx)}{81b^5}\\ &+\frac{4d^2(c+dx)^2\cos^3(a+bx)}{9b^3}\\ &-\frac{(c+dx)^4\cos^3(a+bx)}{3b}\\ &-\frac{160d^3(c+dx)\sin(a+bx)}{27b^4}\\ &+\frac{8d(c+dx)^3\sin(a+bx)}{9b^2}\\ &-\frac{8d^3(c+dx)\cos^2(a+bx)\sin(a+bx)}{27b^4}\\ &+\frac{4d(c+dx)^3\cos^2(a+bx)\sin(a+bx)}{9b^2}\end{aligned}$$

output

```
-160/27*d^4*cos(b*x+a)/b^5+8/3*d^2*(d*x+c)^2*cos(b*x+a)/b^3-8/81*d^4*cos(b
*x+a)^3/b^5+4/9*d^2*(d*x+c)^2*cos(b*x+a)^3/b^3-1/3*(d*x+c)^4*cos(b*x+a)^3/
b-160/27*d^3*(d*x+c)*sin(b*x+a)/b^4+8/9*d*(d*x+c)^3*sin(b*x+a)/b^2-8/27*d^
3*(d*x+c)*cos(b*x+a)^2*sin(b*x+a)/b^4+4/9*d*(d*x+c)^3*cos(b*x+a)^2*sin(b*x
+a)/b^2
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.73

$$\int (c+dx)^4 \cos^2(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{81(24d^4-12b^2d^2(c+dx)^2+b^4(c+dx)^4)\cos(a+bx)+(8d^4-36b^2d^2(c+dx)^2+27b^4(c+dx)^4)\cos(}{32}$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/324*(81*(24*d^4 - 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cos[a + b*x
] + (8*d^4 - 36*b^2*d^2*(c + d*x)^2 + 27*b^4*(c + d*x)^4)*Cos[3*(a + b*x)]
 - 24*b*d*(c + d*x)*(-82*d^2 + 15*b^2*(c + d*x)^2 + (-2*d^2 + 3*b^2*(c + d
*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/b^5
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.20, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.864$, Rules used = {4905, 3042, 3792, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118, 3791, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sin(a+bx)\cos^2(a+bx)\,dx$$
$$\downarrow 4905$$
$$\frac{4d\int (c+dx)^3\cos^3(a+bx)dx}{3b} - \frac{(c+dx)^4\cos^3(a+bx)}{3b}$$
$$\downarrow 3042$$
$$\frac{4d\int (c+dx)^3\sin\left(a+bx+\frac{\pi}{2}\right)^3 dx}{3b} - \frac{(c+dx)^4\cos^3(a+bx)}{3b}$$
$$\downarrow 3792$$

$$\frac{4d\left(-\frac{2d^2\int(c+dx)\cos^3(a+bx)dx}{3b^2}+\frac{2}{3}\int(c+dx)^3\cos(a+bx)dx+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b}-\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$$\downarrow 3042$$

$$\frac{4d\left(-\frac{2d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\int(c+dx)^3\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b}-\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$$\downarrow 3777$$

$$\frac{4d\left(-\frac{2d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{3d\int-(c+dx)^2\sin(a+bx)dx}{b}+\frac{(c+dx)^3\sin(a+bx)}{b}\right)+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b}-\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$$\downarrow 25$$

$$\frac{4d\left(-\frac{2d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\int(c+dx)^2\sin(a+bx)dx}{b}\right)+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b}-\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$$\downarrow 3042$$

$$\frac{4d\left(-\frac{2d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\int(c+dx)^2\sin(a+bx)dx}{b}\right)+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b}-\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$$\downarrow 3777$$

$$\frac{4d\left(-\frac{2d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\int(c+dx)\cos(a+bx)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b}-\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$$\downarrow 3042$$

$$\frac{4d\left(-\frac{2d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+\frac{d(c+dx)^2\cos^3}{3b^2}\right.}{3b}$$
$$\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$$\downarrow 3777$$

$$\frac{4d\left(-\frac{2d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\int-\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+\right.}{3b}$$
$$\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$$\downarrow 25$$

$$\frac{4d\left(-\frac{2d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+\frac{d}{}\right.}{3b}$$
$$\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$$\downarrow 3042$$

$$\frac{4d\left(-\frac{2d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+\frac{d}{}\right.}{3b}$$
$$\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$$\downarrow 3118$$

$$\frac{4d\left(-\frac{2d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx\right.\right.\right.}{b}\right.\right.\right.}{3b}}{\frac{(c+dx)^4\cos^3(a+bx)}{3b}}$$

$$\downarrow 3791$$

$$\frac{4d\left(-\frac{2d^2\left(\frac{2}{3}\int(c+dx)\cos(a+bx)dx+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\right.\right.\right.}{3b}}{\frac{(c+dx)^4\cos^3(a+bx)}{3b}}$$

$$\downarrow 3042$$

$$\frac{4d\left(-\frac{2d^2\left(\frac{2}{3}\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\right.\right.}{3b}}{\frac{(c+dx)^4\cos^3(a+bx)}{3b}}$$

$$\downarrow 3777$$

$$\frac{4d\left(-\frac{2d^2\left(\frac{2}{3}\left(\frac{d\int-\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin\right.\right.}{3b}}{\frac{(c+dx)^4\cos^3(a+bx)}{3b}}$$

$$\downarrow 25$$

$$\frac{4d\left(-\frac{2d^2\left(\frac{2}{3}\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a\right.\right.}{3b}}{\frac{(c+dx)^4\cos^3(a+bx)}{3b}}$$

$$\downarrow 3042$$

$$\frac{4d\left(-\frac{2d^2\left(\frac{2}{3}\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a\right.\right.}{3b}}{\frac{(c+dx)^4\cos^3(a+bx)}{3b}}$$

$$\downarrow 3118$$

$$\frac{4d\left(-\frac{2d^2\left(\frac{2}{3}\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx\right.\right.}{3b}}{\frac{(c+dx)^4\cos^3(a+bx)}{3b}}$$

input

```
Int[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output

```
-1/3*((c + d*x)^4*Cos[a + b*x]^3)/b + (4*d*((d*(c + d*x)^2*Cos[a + b*x]^3)
/(3*b^2) + ((c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x])/(3*b) - (2*d^2*((d*Co
s[a + b*x]^3)/(9*b^2) + ((c + d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(3*b) + (2
*((d*Cos[a + b*x])/b^2 + ((c + d*x)*Sin[a + b*x])/b))/3))/(3*b^2) + (2*(((
c + d*x)^3*Sin[a + b*x])/b - (3*d*(-(((c + d*x)^2*Cos[a + b*x])/b) + (2*d*
((d*Cos[a + b*x])/b^2 + ((c + d*x)*Sin[a + b*x])/b))/b))/b))/3))/(3*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.88

method	result
parallelrisch	$\frac{\left(-27(dx+c)^4b^4+36d^2(dx+c)^2b^2-8d^4\right)\cos(3bx+3a)+36d\left((dx+c)^2b^2-\frac{2d^2}{3}\right)(dx+c)b\sin(3bx+3a)+\left(-81(dx+c)^4b^4$
risch	$-\frac{(d^4x^4b^4+4b^4cd^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+b^4c^4-12b^2d^4x^2-24b^2cd^3x-12b^2c^2d^2+24d^4)\cos(bx+a)}{4b^5}+\frac{d(b^2d^3x^3-$
derivativedivides	Expression too large to display
default	Expression too large to display
orering	$\frac{16d(135b^6d^6x^6+810b^6cd^5x^5+2025b^6c^2d^4x^4+2700b^6c^3d^3x^3+2025b^6c^4d^2x^2-891b^4d^6x^4+810b^6c^5dx-3564b^4cd^5x^3+$

input
```
int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/324*((-27*(d*x+c)^4*b^4+36*d^2*(d*x+c)^2*b^2-8*d^4)*cos(3*b*x+3*a)+36*d*
((d*x+c)^2*b^2-2/3*d^2)*(d*x+c)*b*sin(3*b*x+3*a)+(-81*(d*x+c)^4*b^4+972*d^
2*(d*x+c)^2*b^2-1944*d^4)*cos(b*x+a)+324*d*(d*x+c)*b*((d*x+c)^2*b^2-6*d^2)
*sin(b*x+a)-108*b^4*c^4+1008*b^2*c^2*d^2-1952*d^4)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.43

$$\int (c+dx)^4\cos^2(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{(27\,b^4d^4x^4+108\,b^4cd^3x^3+27\,b^4c^4-36\,b^2c^2d^2+8\,d^4+18\,(9\,b^4c^2d^2-2\,b^2d^4)x^2+36\,(3\,b^4c^3d-2\,b^2cd^3$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output

```
-1/81*((27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 - 36*b^2*c^2*d^2 +
 8*d^4 + 18*(9*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 36*(3*b^4*c^3*d - 2*b^2*c*d^
3)*x)*cos(b*x + a)^3 - 24*(9*b^2*d^4*x^2 + 18*b^2*c*d^3*x + 9*b^2*c^2*d^2
- 20*d^4)*cos(b*x + a) - 12*(6*b^3*d^4*x^3 + 18*b^3*c*d^3*x^2 + 6*b^3*c^3*
d - 40*b*c*d^3 + (3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d - 2*b*c*d^
3 + (9*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 + 2*(9*b^3*c^2*d^2 - 20*b*
d^4)*x)*sin(b*x + a))/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(207) = 414$.

Time = 0.65 (sec) , antiderivative size = 646, normalized size of antiderivative = 3.15

$$\int (c+dx)^4 \cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)**4*cos(b*x+a)**2*sin(b*x+a),x)
```

output

```
Piecewise((-c**4*cos(a + b*x)**3/(3*b) - 4*c**3*d*x*cos(a + b*x)**3/(3*b)
- 2*c**2*d**2*x**2*cos(a + b*x)**3/b - 4*c*d**3*x**3*cos(a + b*x)**3/(3*b)
 - d**4*x**4*cos(a + b*x)**3/(3*b) + 8*c**3*d*sin(a + b*x)**3/(9*b**2) + 4
*c**3*d*sin(a + b*x)*cos(a + b*x)**2/(3*b**2) + 8*c**2*d**2*x*sin(a + b*x)
**3/(3*b**2) + 4*c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**2/b**2 + 8*c*d**3*
x**2*sin(a + b*x)**3/(3*b**2) + 4*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**2
/b**2 + 8*d**4*x**3*sin(a + b*x)**3/(9*b**2) + 4*d**4*x**3*sin(a + b*x)*co
s(a + b*x)**2/(3*b**2) + 8*c**2*d**2*sin(a + b*x)**2*cos(a + b*x)/(3*b**3)
 + 28*c**2*d**2*cos(a + b*x)**3/(9*b**3) + 16*c*d**3*x*sin(a + b*x)**2*cos
(a + b*x)/(3*b**3) + 56*c*d**3*x*cos(a + b*x)**3/(9*b**3) + 8*d**4*x**2*si
n(a + b*x)**2*cos(a + b*x)/(3*b**3) + 28*d**4*x**2*cos(a + b*x)**3/(9*b**3
) - 160*c*d**3*sin(a + b*x)**3/(27*b**4) - 56*c*d**3*sin(a + b*x)*cos(a +
b*x)**2/(9*b**4) - 160*d**4*x*sin(a + b*x)**3/(27*b**4) - 56*d**4*x*sin(a
+ b*x)*cos(a + b*x)**2/(9*b**4) - 160*d**4*sin(a + b*x)**2*cos(a + b*x)/(2
7*b**5) - 488*d**4*cos(a + b*x)**3/(81*b**5), Ne(b, 0)), ((c**4*x + 2*c**3
*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*cos(a)**2,
True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(187) = 374$.

Time = 0.08 (sec) , antiderivative size = 889, normalized size of antiderivative = 4.34

$$\int (c+dx)^4 \cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output

```
-1/324*(108*c^4*cos(b*x + a)^3 - 432*a*c^3*d*cos(b*x + a)^3/b + 648*a^2*c^
2*d^2*cos(b*x + a)^3/b^2 - 432*a^3*c*d^3*cos(b*x + a)^3/b^3 + 108*a^4*d^4*
cos(b*x + a)^3/b^4 + 36*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*
x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*c^3*d/b - 108*(3*(b*x + a)*cos
(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x +
a))*a*c^2*d^2/b^2 + 108*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*
x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*a^2*c*d^3/b^3 - 36*(3*(b*x + a
)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b
*x + a))*a^3*d^4/b^4 + 18*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x
 + a)^2 - 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*si
n(b*x + a))*c^2*d^2/b^2 - 36*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((
b*x + a)^2 - 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)
*sin(b*x + a))*a*c*d^3/b^3 + 18*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27
*((b*x + a)^2 - 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x +
 a)*sin(b*x + a))*a^2*d^4/b^4 + 12*(3*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*
b*x + 3*a) + 27*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) - (9*(b*x + a)^2
- 2)*sin(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*sin(b*x + a))*c*d^3/b^3 - 12*
(3*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) + 27*((b*x + a)^3 - 6*b*
x - 6*a)*cos(b*x + a) - (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 81*((b*x +
a)^2 - 2)*sin(b*x + a))*a*d^4/b^4 + ((27*(b*x + a)^4 - 36*(b*x + a)^2 +...
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.71

$$\int (c+dx)^4 \cos^2(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{(27\,b^4d^4x^4+108\,b^4cd^3x^3+162\,b^4c^2d^2x^2+108\,b^4c^3dx+27\,b^4c^4-36\,b^2d^4x^2-72\,b^2cd^3x-36\,b^2c^2d^2+}{324\,b^5}$$
$$-\frac{(b^4d^4x^4+4\,b^4cd^3x^3+6\,b^4c^2d^2x^2+4\,b^4c^3dx+b^4c^4-12\,b^2d^4x^2-24\,b^2cd^3x-12\,b^2c^2d^2+24\,d^4)\cos(b}{4\,b^5}$$
$$+\frac{(3\,b^3d^4x^3+9\,b^3cd^3x^2+9\,b^3c^2d^2x+3\,b^3c^3d-2\,bd^4x-2\,bcd^3)\sin(3\,bx+3\,a)}{27\,b^5}$$
$$+\frac{(b^3d^4x^3+3\,b^3cd^3x^2+3\,b^3c^2d^2x+b^3c^3d-6\,bd^4x-6\,bcd^3)\sin(bx+a)}{b^5}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/324*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4
*c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 +
 8*d^4)*cos(3*b*x + 3*a)/b^5 - 1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*
c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x -
12*b^2*c^2*d^2 + 24*d^4)*cos(b*x + a)/b^5 + 1/27*(3*b^3*d^4*x^3 + 9*b^3*c*
d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*sin(3*b*x
 + 3*a)/b^5 + (b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d
 - 6*b*d^4*x - 6*b*c*d^3)*sin(b*x + a)/b^5
```

Mupad [B] (verification not implemented)

Time = 19.73 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.19

$$\int (c+dx)^4 \cos^2(a+bx)\sin(a+bx)\,dx$$

$$\begin{aligned}
&= \frac{4x\cos(a+bx)^3\left(14cd^3-3b^2c^3d\right)}{9b^3} - \frac{\cos(a+bx)^3\left(27b^4c^4-252b^2c^2d^2+488d^4\right)}{81b^5} \\
&\quad - \frac{8\cos(a+bx)\sin(a+bx)^2\left(20d^4-9b^2c^2d^2\right)}{27b^5} \\
&\quad - \frac{4\cos(a+bx)^2\sin(a+bx)\left(14cd^3-3b^2c^3d\right)}{9b^4} \\
&\quad - \frac{d^4x^4\cos(a+bx)^3}{3b} - \frac{8\sin(a+bx)^3\left(20cd^3-3b^2c^3d\right)}{27b^4} \\
&\quad + \frac{8d^4x^3\sin(a+bx)^3}{9b^2} - \frac{8x\sin(a+bx)^3\left(20d^4-9b^2c^2d^2\right)}{27b^4} \\
&\quad + \frac{2x^2\cos(a+bx)^3\left(14d^4-9b^2c^2d^2\right)}{9b^3} - \frac{4cd^3x^3\cos(a+bx)^3}{3b} \\
&\quad + \frac{4d^4x^3\cos(a+bx)^2\sin(a+bx)}{3b^2} + \frac{8d^4x^2\cos(a+bx)\sin(a+bx)^2}{3b^3} \\
&\quad + \frac{8cd^3x^2\sin(a+bx)^3}{3b^2} - \frac{4x\cos(a+bx)^2\sin(a+bx)\left(14d^4-9b^2c^2d^2\right)}{9b^4} \\
&\quad + \frac{4cd^3x^2\cos(a+bx)^2\sin(a+bx)}{b^2} + \frac{16cd^3x\cos(a+bx)\sin(a+bx)^2}{3b^3}
\end{aligned}$$

input

```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^4,x)
```

output

```
(4*x*cos(a + b*x)^3*(14*c*d^3 - 3*b^2*c^3*d))/(9*b^3) - (cos(a + b*x)^3*(4
88*d^4 + 27*b^4*c^4 - 252*b^2*c^2*d^2))/(81*b^5) - (8*cos(a + b*x)*sin(a +
 b*x)^2*(20*d^4 - 9*b^2*c^2*d^2))/(27*b^5) - (4*cos(a + b*x)^2*sin(a + b*x
)*(14*c*d^3 - 3*b^2*c^3*d))/(9*b^4) - (d^4*x^4*cos(a + b*x)^3)/(3*b) - (8*
sin(a + b*x)^3*(20*c*d^3 - 3*b^2*c^3*d))/(27*b^4) + (8*d^4*x^3*sin(a + b*x
)^3)/(9*b^2) - (8*x*sin(a + b*x)^3*(20*d^4 - 9*b^2*c^2*d^2))/(27*b^4) + (2
*x^2*cos(a + b*x)^3*(14*d^4 - 9*b^2*c^2*d^2))/(9*b^3) - (4*c*d^3*x^3*cos(a
 + b*x)^3)/(3*b) + (4*d^4*x^3*cos(a + b*x)^2*sin(a + b*x))/(3*b^2) + (8*d^
4*x^2*cos(a + b*x)*sin(a + b*x)^2)/(3*b^3) + (8*c*d^3*x^2*sin(a + b*x)^3)/
(3*b^2) - (4*x*cos(a + b*x)^2*sin(a + b*x)*(14*d^4 - 9*b^2*c^2*d^2))/(9*b^
4) + (4*c*d^3*x^2*cos(a + b*x)^2*sin(a + b*x))/b^2 + (16*c*d^3*x*cos(a + b
*x)*sin(a + b*x)^2)/(3*b^3)
```

Reduce [B] (verification not implemented)

Time = 15.32 (sec) , antiderivative size = 594, normalized size of antiderivative = 2.90

$$\int (c+dx)^4 \cos^2(a+bx)\sin(a+bx)\,dx$$
$$= \frac{108\cos(bx+a)\sin(bx+a)^2 b^4c^3dx + 27\cos(bx+a)\sin(bx+a)^2 b^4d^4x^4 - 36\cos(bx+a)\sin(bx+a)^2}$$

input

```
int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x)
```

output

```
(27*cos(a + b*x)*sin(a + b*x)**2*b**4*c**4 + 108*cos(a + b*x)*sin(a + b*x)
**2*b**4*c**3*d*x + 162*cos(a + b*x)*sin(a + b*x)**2*b**4*c**2*d**2*x**2 +
 108*cos(a + b*x)*sin(a + b*x)**2*b**4*c*d**3*x**3 + 27*cos(a + b*x)*sin(a
 + b*x)**2*b**4*d**4*x**4 - 36*cos(a + b*x)*sin(a + b*x)**2*b**2*c**2*d**2
 - 72*cos(a + b*x)*sin(a + b*x)**2*b**2*c*d**3*x - 36*cos(a + b*x)*sin(a +
 b*x)**2*b**2*d**4*x**2 + 8*cos(a + b*x)*sin(a + b*x)**2*d**4 - 27*cos(a +
 b*x)*b**4*c**4 - 108*cos(a + b*x)*b**4*c**3*d*x - 162*cos(a + b*x)*b**4*c
**2*d**2*x**2 - 108*cos(a + b*x)*b**4*c*d**3*x**3 - 27*cos(a + b*x)*b**4*d
**4*x**4 + 252*cos(a + b*x)*b**2*c**2*d**2 + 504*cos(a + b*x)*b**2*c*d**3*
x + 252*cos(a + b*x)*b**2*d**4*x**2 - 488*cos(a + b*x)*d**4 - 36*sin(a + b
*x)**3*b**3*c**3*d - 108*sin(a + b*x)**3*b**3*c**2*d**2*x - 108*sin(a + b*
x)**3*b**3*c*d**3*x**2 - 36*sin(a + b*x)**3*b**3*d**4*x**3 + 24*sin(a + b*
x)**3*b*c*d**3 + 24*sin(a + b*x)**3*b*d**4*x + 108*sin(a + b*x)*b**3*c**3*
d + 324*sin(a + b*x)*b**3*c**2*d**2*x + 324*sin(a + b*x)*b**3*c*d**3*x**2
+ 108*sin(a + b*x)*b**3*d**4*x**3 - 504*sin(a + b*x)*b*c*d**3 - 504*sin(a
+ b*x)*b*d**4*x + 27*b**4*c**4 + 36*b**2*c**2*d**2 - 152*d**4)/(81*b**5)
```

3.72 $\int (c+dx)^3 \cos^2(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 734
Mathematica [A] (verified) . . . 735
Rubi [A] (verified) . . . 735
Maple [A] (verified) . . . 739
Fricas [A] (verification not implemented) . . . 739
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Giac [A] (verification not implemented) . . . 741
Mupad [B] (verification not implemented) . . . 742
Reduce [B] (verification not implemented) . . . 743

Optimal result

Integrand size = 22, antiderivative size = 151

$$\begin{aligned}
&\int (c+dx)^3 \cos^2(a+bx)\sin(a+bx)\,dx \\
&= \frac{4d^2(c+dx)\cos(a+bx)}{3b^3} + \frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3} \\
&\quad - \frac{(c+dx)^3\cos^3(a+bx)}{3b} - \frac{14d^3\sin(a+bx)}{9b^4} + \frac{2d(c+dx)^2\sin(a+bx)}{3b^2} \\
&\quad + \frac{d(c+dx)^2\cos^2(a+bx)\sin(a+bx)}{3b^2} + \frac{2d^3\sin^3(a+bx)}{27b^4}
\end{aligned}$$

output

```
4/3*d^2*(d*x+c)*cos(b*x+a)/b^3+2/9*d^2*(d*x+c)*cos(b*x+a)^3/b^3-1/3*(d*x+c
)^3*cos(b*x+a)^3/b-14/9*d^3*sin(b*x+a)/b^4+2/3*d*(d*x+c)^2*sin(b*x+a)/b^2+
1/3*d*(d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)/b^2+2/27*d^3*sin(b*x+a)^3/b^4
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.84

$$\int (c+dx)^3 \cos^2(a+bx)\sin(a+bx)\,dx$$
$$= \frac{-27b(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\cos(a+bx) - 3b(c+dx)\left(-2d^2+3b^2(c+dx)^2\right)\cos(3(a+bx)) + 2d}{108b^4}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
(-27*b*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 3*b*(c + d*x)*(
-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 2*d*(-82*d^2 + 45*b^2*(c +
d*x)^2 + (-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/(108
*b^4)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {4905, 3042, 3792, 3042, 3113, 2009, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin(a+bx)\cos^2(a+bx)\,dx$$
$$\downarrow 4905$$
$$\frac{d\int (c+dx)^2\cos^3(a+bx)dx}{b} - \frac{(c+dx)^3\cos^3(a+bx)}{3b}$$
$$\downarrow 3042$$
$$\frac{d\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^3 dx}{b} - \frac{(c+dx)^3\cos^3(a+bx)}{3b}$$
$$\downarrow 3792$$

$$\frac{d\left(-\frac{2d^2\int\cos^3(a+bx)dx}{9b^2}+\frac{2}{3}\int(c+dx)^2\cos(a+bx)dx+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{b}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$

$$\downarrow 3042$$

$$\frac{d\left(-\frac{2d^2\int\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{9b^2}+\frac{2}{3}\int(c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{b}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$

$$\downarrow 3113$$

$$\frac{d\left(\frac{2d^2\int(1-\sin^2(a+bx))d(-\sin(a+bx))}{9b^3}+\frac{2}{3}\int(c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+\right.}{b}$$

$$\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$

$$\downarrow 2009$$

$$\frac{d\left(\frac{2}{3}\int(c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{b}$$

$$\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$

$$\downarrow 3777$$

$$\frac{d\left(\frac{2}{3}\left(\frac{2d\int-((c+dx)\sin(a+bx))dx}{b}+\frac{(c+dx)^2\sin(a+bx)}{b}\right)+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)}{3b}\right.}{b}$$

$$\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$

$$\downarrow 25$$

$$\frac{d\left(\frac{2}{3}\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int(c+dx)\sin(a+bx)dx}{b}\right)+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos}{3b}\right.}{b}$$

$$\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$

$$\downarrow 3042$$

$$\frac{d\left(\frac{2}{3}\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int(c+dx)\sin(a+bx)dx}{b}\right)+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos}{3b}\right.}{b}$$
$$\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$
$$\downarrow 3777$$
$$\frac{d\left(\frac{2}{3}\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\cos(a+bx)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx}{}\right.}{b}$$
$$\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$
$$\downarrow 3042$$
$$\frac{d\left(\frac{2}{3}\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\right.}{b}$$
$$\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$
$$\downarrow 3117$$
$$\frac{d\left(\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)+\frac{(c+dx)^2\,s}{}\right.}{b}$$
$$\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/3*((c + d*x)^3*Cos[a + b*x]^3)/b + (d*((2*d*(c + d*x)*Cos[a + b*x]^3)/(
9*b^2) + ((c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x])/(3*b) + (2*d^2*(-Sin[a
+ b*x] + Sin[a + b*x]^3/3))/(9*b^3) + (2*(((c + d*x)^2*Sin[a + b*x])/b - (
2*d*(-(((c + d*x)*Cos[a + b*x])/b) + (d*Sin[a + b*x])/b^2))/b))/3))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.48

method	result
risch	$-\frac{\left(b^2d^3x^3+3b^2cd^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6cd^2\right)\cos(bx+a)}{4b^3}+\frac{3d\left(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2\right)\sin(bx+a)}{4b^4}-\frac{(3}{}$
parallelrisch	$\frac{3dx\left(\left(\frac{1}{3}x^2d^2+cdx+c^2\right)b^2-\frac{14d^2}{9}\right)b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^6+6d\left((dx+c)^2b^2-\frac{14d^2}{9}\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^5-6\left(\left(\frac{dx}{2}+c\right)(x^2d^2+cdx+c^2)b$
derivativedivides	$\frac{a^3d^3\cos(bx+a)^3}{3b^3}-\frac{a^2cd^2\cos(bx+a)^3}{b^2}+\frac{3a^2d^3\left(-\frac{(bx+a)\cos(bx+a)^3}{3}+\frac{\left(2+\cos(bx+a)^2\right)\sin(bx+a)}{9}\right)}{b^3}+\frac{ac^2d\cos(bx+a)^3}{b}-\frac{6acd^2}{}$
default	$\frac{a^3d^3\cos(bx+a)^3}{3b^3}-\frac{a^2cd^2\cos(bx+a)^3}{b^2}+\frac{3a^2d^3\left(-\frac{(bx+a)\cos(bx+a)^3}{3}+\frac{\left(2+\cos(bx+a)^2\right)\sin(bx+a)}{9}\right)}{b^3}+\frac{ac^2d\cos(bx+a)^3}{b}-\frac{6acd^2}{}$
norman	$\frac{\left(-2b^2c^3+4cd^2\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4}{b^3}+\frac{d^3x^3\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{b}+\frac{cd^2x^2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^6}{b}+\frac{-6b^2c^3+28cd^2}{9b^3}-\frac{d^3x^3}{3b}+\frac{16cd^2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{3b^3}+\frac{2}{}$
orering	$\frac{20d\left(9d^4x^4b^4+36b^4cd^3x^3+54b^4c^2d^2x^2+36b^4c^3dx+9b^4c^4-22b^2d^4x^2-44b^2cd^3x-22b^2c^2d^2-72d^4\right)\cos(bx+a)^2\sin(bx}{27b^6(dx+c)^2}$

input
```
int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^3*(b^2*d^3*x^3+3*b^2*c*d^2*x^2+3*b^2*c^2*d*x+b^2*c^3-6*d^3*x-6*c*d^
2)*cos(b*x+a)+3/4*d*(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2-2*d^2)/b^4*sin(b*x+a)
-1/36/b^3*(3*b^2*d^3*x^3+9*b^2*c*d^2*x^2+9*b^2*c^2*d*x+3*b^2*c^3-2*d^3*x-2
*c*d^2)*cos(3*b*x+3*a)+1/108*d*(9*b^2*d^2*x^2+18*b^2*c*d*x+9*b^2*c^2-2*d^2
)/b^4*sin(3*b*x+3*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.21

$$\int (c+dx)^3\cos^2(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{3\left(3\,b^3d^3x^3+9\,b^3cd^2x^2+3\,b^3c^3-2\,bcd^2+\left(9\,b^3c^2d-2\,bd^3\right)x\right)\cos\left(bx+a\right)^3-36\left(bd^3x+bcd^2\right)\cos\left(bx\right.}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/27*(3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 2*b*c*d^2 + (9*b^3
*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^3 - 36*(b*d^3*x + b*c*d^2)*cos(b*x + a)
- (18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - 40*d^3 + (9*b^2*d^3*x^
2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a)^2)*sin(b*x + a))/b^
4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(150) = 300$.

Time $= 0.48$ (sec) , antiderivative size $= 391$, normalized size of antiderivative $= 2.59$

$$\int (c+dx)^3 \cos^2(a+bx)\sin(a+bx)\,dx$$
$$= \begin{cases} -\frac{c^3\cos^3(a+bx)}{3b} - \frac{c^2 dx\cos^3(a+bx)}{b} - \frac{cd^2x^2\cos^3(a+bx)}{b} - \frac{d^3x^3\cos^3(a+bx)}{3b} + \frac{2c^2 d\sin^3(a+bx)}{3b^2} + \frac{c^2 d\sin(a+bx)\cos^2(a+bx)}{b^2} + \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin(a)\cos^2(a) \end{cases}$$

input
```
integrate((d*x+c)**3*cos(b*x+a)**2*sin(b*x+a),x)
```

output
```
Piecewise((-c**3*cos(a + b*x)**3/(3*b) - c**2*d*x*cos(a + b*x)**3/b - c*d*
*2*x**2*cos(a + b*x)**3/b - d**3*x**3*cos(a + b*x)**3/(3*b) + 2*c**2*d*sin
(a + b*x)**3/(3*b**2) + c**2*d*sin(a + b*x)*cos(a + b*x)**2/b**2 + 4*c*d**
2*x*sin(a + b*x)**3/(3*b**2) + 2*c*d**2*x*sin(a + b*x)*cos(a + b*x)**2/b**
2 + 2*d**3*x**2*sin(a + b*x)**3/(3*b**2) + d**3*x**2*sin(a + b*x)*cos(a +
b*x)**2/b**2 + 4*c*d**2*sin(a + b*x)**2*cos(a + b*x)/(3*b**3) + 14*c*d**2*
cos(a + b*x)**3/(9*b**3) + 4*d**3*x*sin(a + b*x)**2*cos(a + b*x)/(3*b**3)
+ 14*d**3*x*cos(a + b*x)**3/(9*b**3) - 40*d**3*sin(a + b*x)**3/(27*b**4) -
 14*d**3*sin(a + b*x)*cos(a + b*x)**2/(9*b**4), Ne(b, 0)), ((c**3*x + 3*c*
*2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)*cos(a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(137) = 274$.

Time = 0.05 (sec) , antiderivative size = 505, normalized size of antiderivative = 3.34

$$\int (c+dx)^3 \cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/108*(36*c^3*cos(b*x + a)^3 - 108*a*c^2*d*cos(b*x + a)^3/b + 108*a^2*c*d
^2*cos(b*x + a)^3/b^2 - 36*a^3*d^3*cos(b*x + a)^3/b^3 + 9*(3*(b*x + a)*cos
(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x +
a))*c^2*d/b - 18*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a)
- sin(3*b*x + 3*a) - 9*sin(b*x + a))*a*c*d^2/b^2 + 9*(3*(b*x + a)*cos(3*b*
x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*a
^2*d^3/b^3 + 3*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2 - 2
)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a))
*c*d^2/b^2 - 3*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2 - 2
)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a))
*a*d^3/b^3 + (3*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) + 27*((b*x
+ a)^3 - 6*b*x - 6*a)*cos(b*x + a) - (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a)
- 81*((b*x + a)^2 - 2)*sin(b*x + a))*d^3/b^3)/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.53

$$\begin{aligned}
&\int (c+dx)^3 \cos^2(a+bx)\sin(a+bx)\,dx \\
&= -\frac{(3\,b^3d^3x^3 + 9\,b^3cd^2x^2 + 9\,b^3c^2dx + 3\,b^3c^3 - 2\,bd^3x - 2\,bcd^2)\cos(3\,bx+3\,a)}{36\,b^4} \\
&\quad - \frac{(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3 - 6\,bd^3x - 6\,bcd^2)\cos(bx+a)}{4\,b^4} \\
&\quad + \frac{(9\,b^2d^3x^2 + 18\,b^2cd^2x + 9\,b^2c^2d - 2\,d^3)\sin(3\,bx+3\,a)}{108\,b^4} \\
&\quad + \frac{3\,(b^2d^3x^2 + 2\,b^2cd^2x + b^2c^2d - 2\,d^3)\sin(bx+a)}{4\,b^4}
\end{aligned}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/36*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d
^3*x - 2*b*c*d^2)*cos(3*b*x + 3*a)/b^4 - 1/4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^
2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*cos(b*x + a)/b^4 + 1/
108*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*sin(3*b*x + 3*a
)/b^4 + 3/4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a)
/b^4
```

Mupad [B] (verification not implemented)

Time = 18.65 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.92

$$\begin{aligned}\int (c+dx)^3\cos^2(a+bx)\sin(a+bx)\,dx = {} & \frac{\cos(a+bx)^3\,(14\,c\,d^2-3\,b^2\,c^3)}{9\,b^3} \\ & -\frac{2\sin(a+bx)^3\,(20\,d^3-9\,b^2\,c^2\,d)}{27\,b^4} \\ & -\frac{\cos(a+bx)^2\,\sin(a+bx)\,(14\,d^3-9\,b^2\,c^2\,d)}{9\,b^4} \\ & +\frac{x\cos(a+bx)^3\,(14\,d^3-9\,b^2\,c^2\,d)}{9\,b^3} \\ & -\frac{d^3\,x^3\cos(a+bx)^3}{3\,b}+\frac{2\,d^3\,x^2\sin(a+bx)^3}{3\,b^2} \\ & +\frac{4\,c\,d^2\,\cos(a+bx)\,\sin(a+bx)^2}{3\,b^3} \\ & +\frac{4\,d^3\,x\,\cos(a+bx)\,\sin(a+bx)^2}{3\,b^3} \\ & +\frac{4\,c\,d^2\,x\sin(a+bx)^3}{3\,b^2}-\frac{c\,d^2\,x^2\cos(a+bx)^3}{b} \\ & +\frac{d^3\,x^2\cos(a+bx)^2\,\sin(a+bx)}{b^2} \\ & +\frac{2\,c\,d^2\,x\cos(a+bx)^2\,\sin(a+bx)}{b^2}\end{aligned}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^3,x)
```

output
```
(cos(a + b*x)^3*(14*c*d^2 - 3*b^2*c^3))/(9*b^3) - (2*sin(a + b*x)^3*(20*d^
3 - 9*b^2*c^2*d))/(27*b^4) - (cos(a + b*x)^2*sin(a + b*x)*(14*d^3 - 9*b^2*
c^2*d))/(9*b^4) + (x*cos(a + b*x)^3*(14*d^3 - 9*b^2*c^2*d))/(9*b^3) - (d^3
*x^3*cos(a + b*x)^3)/(3*b) + (2*d^3*x^2*sin(a + b*x)^3)/(3*b^2) + (4*c*d^2
*cos(a + b*x)*sin(a + b*x)^2)/(3*b^3) + (4*d^3*x*cos(a + b*x)*sin(a + b*x)
^2)/(3*b^3) + (4*c*d^2*x*sin(a + b*x)^3)/(3*b^2) - (c*d^2*x^2*cos(a + b*x)
^3)/b + (d^3*x^2*cos(a + b*x)^2*sin(a + b*x))/b^2 + (2*c*d^2*x*cos(a + b*x
)^2*sin(a + b*x))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.50

$$\int (c+dx)^3 \cos^2(a+bx)\sin(a+bx)\,dx$$
$$= \frac{9\cos(bx+a)\sin(bx+a)^2 b^3c^3 + 27\cos(bx+a)\sin(bx+a)^2 b^3c^2dx + 27\cos(bx+a)\sin(bx+a)^2 b^3c\,d}{}$$

input
```
int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x)
```

output
```
(9*cos(a + b*x)*sin(a + b*x)**2*b**3*c**3 + 27*cos(a + b*x)*sin(a + b*x)**
2*b**3*c**2*d*x + 27*cos(a + b*x)*sin(a + b*x)**2*b**3*c*d**2*x**2 + 9*cos
(a + b*x)*sin(a + b*x)**2*b**3*d**3*x**3 - 6*cos(a + b*x)*sin(a + b*x)**2*
b*c*d**2 - 6*cos(a + b*x)*sin(a + b*x)**2*b*d**3*x - 9*cos(a + b*x)*b**3*c
**3 - 27*cos(a + b*x)*b**3*c**2*d*x - 27*cos(a + b*x)*b**3*c*d**2*x**2 - 9
*cos(a + b*x)*b**3*d**3*x**3 + 42*cos(a + b*x)*b*c*d**2 + 42*cos(a + b*x)*
b*d**3*x - 9*sin(a + b*x)**3*b**2*c**2*d - 18*sin(a + b*x)**3*b**2*c*d**2*
x - 9*sin(a + b*x)**3*b**2*d**3*x**2 + 2*sin(a + b*x)**3*d**3 + 27*sin(a +
 b*x)*b**2*c**2*d + 54*sin(a + b*x)*b**2*c*d**2*x + 27*sin(a + b*x)*b**2*d
**3*x**2 - 42*sin(a + b*x)*d**3 + 9*b**3*c**3 + 6*b*c*d**2)/(27*b**4)
```

3.73 $\int (c+dx)^2 \cos^2(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 744
Mathematica [A] (verified) . . . 744
Rubi [A] (verified) . . . 745
Maple [A] (verified) . . . 747
Fricas [A] (verification not implemented) . . . 748
Sympy [B] (verification not implemented) . . . 749
Maxima [B] (verification not implemented) . . . 749
Giac [A] (verification not implemented) . . . 750
Mupad [B] (verification not implemented) . . . 750
Reduce [B] (verification not implemented) . . . 751

Optimal result

Integrand size = 22, antiderivative size = 103

$$\begin{aligned}\int (c+dx)^2\cos^2(a+bx)\sin(a+bx)\,dx = &\frac{4d^2\cos(a+bx)}{9b^3}+\frac{2d^2\cos^3(a+bx)}{27b^3}\\ &-\frac{(c+dx)^2\cos^3(a+bx)}{3b}\\ &+\frac{4d(c+dx)\sin(a+bx)}{9b^2}\\ &+\frac{2d(c+dx)\cos^2(a+bx)\sin(a+bx)}{9b^2}\end{aligned}$$

output

```
4/9*d^2*cos(b*x+a)/b^3+2/27*d^2*cos(b*x+a)^3/b^3-1/3*(d*x+c)^2*cos(b*x+a)^
3/b+4/9*d*(d*x+c)*sin(b*x+a)/b^2+2/9*d*(d*x+c)*cos(b*x+a)^2*sin(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int (c+dx)^2\cos^2(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{27(-2d^2+b^2(c+dx)^2)\cos(a+bx)+(-2d^2+9b^2(c+dx)^2)\cos(3(a+bx))-6bd(c+dx)(9\sin(a+$$
$$108b^3$$

input
```
Integrate[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/108*(27*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + (-2*d^2 + 9*b^2*(c +
d*x)^2)*Cos[3*(a + b*x)] - 6*b*d*(c + d*x)*(9*Sin[a + b*x] + Sin[3*(a + b*
x)]))/b^3
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4905, 3042, 3791, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin(a+bx)\cos^2(a+bx)\,dx$$

$$\downarrow 4905$$

$$\frac{2d\int (c+dx)\cos^3(a+bx)dx}{3b} - \frac{(c+dx)^2\cos^3(a+bx)}{3b}$$

$$\downarrow 3042$$

$$\frac{2d\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3 dx}{3b} - \frac{(c+dx)^2\cos^3(a+bx)}{3b}$$

$$\downarrow 3791$$

$$\frac{2d\left(\frac{2}{3}\int (c+dx)\cos(a+bx)dx + \frac{d\cos^3(a+bx)}{9b^2} + \frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b} - \frac{(c+dx)^2\cos^3(a+bx)}{3b}$$

$$\downarrow 3042$$

$$\frac{2d\left(\frac{2}{3}\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx + \frac{d\cos^3(a+bx)}{9b^2} + \frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b} - \frac{(c+dx)^2\cos^3(a+bx)}{3b}$$

$$\downarrow 3777$$

$$\frac{2d\left(\frac{2}{3}\left(\frac{d\int -\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}$$

$$\downarrow 25$$

$$\frac{2d\left(\frac{2}{3}\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int \sin(a+bx)dx}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}$$

$$\downarrow 3042$$

$$\frac{2d\left(\frac{2}{3}\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int \sin(a+bx)dx}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}$$

$$\downarrow 3118$$

$$\frac{2d\left(\frac{2}{3}\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/3*((c + d*x)^2*Cos[a + b*x]^3)/b + (2*d*((d*Cos[a + b*x]^3)/(9*b^2) + (
(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(3*b) + (2*((d*Cos[a + b*x])/b^2 +
((c + d*x)*Sin[a + b*x])/b))/3))/(3*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\cos(bx+a)}{4b^3}+\frac{d(dx+c)\sin(bx+a)}{2b^2}-\frac{(9x^2d^2b^2+18b^2cdx+9b^2c^2-2d^2)\cos(3bx+3a)}{108b^3}$
derivativedivides	$-\frac{a^2d^2\cos(bx+a)^3}{3b^2}+\frac{2acd\cos(bx+a)^3}{3b}-\frac{2a\,d^2\left(-\frac{(bx+a)\cos(bx+a)^3}{3}+\frac{\left(2+\cos(bx+a)^2\right)\sin(bx+a)}{9}\right)}{b^2}-\frac{c^2\cos(bx+a)^3}{3}+\frac{2cd\left(-\frac{(b}{}$
default	$-\frac{a^2d^2\cos(bx+a)^3}{3b^2}+\frac{2acd\cos(bx+a)^3}{3b}-\frac{2a\,d^2\left(-\frac{(bx+a)\cos(bx+a)^3}{3}+\frac{\left(2+\cos(bx+a)^2\right)\sin(bx+a)}{9}\right)}{b^2}-\frac{c^2\cos(bx+a)^3}{3}+\frac{2cd\left(-\frac{(b}{}$
parallelrisch	$\frac{18\left(\frac{dx}{2}+c\right)dx\,b^2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^6+36bd(dx+c)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^5+\left(\left(-27x^2d^2-54cdx-54c^2\right)b^2+36d^2\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4+24bd($
	$27b^3\left(1+$
norman	$\frac{d^2x^2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{b}+\frac{-18b^2c^2+28d^2}{27b^3}-\frac{d^2x^2}{3b}+\frac{16d^2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{9b^3}+\frac{\left(-6b^2c^2+4d^2\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4}{3b^3}+\frac{4cd\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{3b^2}+\frac{8cd\tan\left($
orering	$\frac{40d(9d^4x^4b^4+36b^4cd^3x^3+54b^4c^2d^2x^2+36b^4c^3dx+9b^4c^4-b^2d^4x^2-2b^2cd^3x-b^2c^2d^2-12d^4)\cos(bx+a)^2\sin(bx+a)}{81b^6(dx+c)^3}$

input
```
int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2-2*d^2)/b^3*cos(b*x+a)+1/2*d*(d*x+c)*
sin(b*x+a)/b^2-1/108*(9*b^2*d^2*x^2+18*b^2*c*d*x+9*b^2*c^2-2*d^2)/b^3*cos(
3*b*x+3*a)+1/18*d*(d*x+c)*sin(3*b*x+3*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.97

$$\int (c+dx)^2\cos^2(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2)\cos(bx+a)^3-12\,d^2\cos(bx+a)-6\left(2\,bd^2x+2\,bcd+(bd^2x+b\right.}{27\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/27*((9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(b*x + a)^3 -
 12*d^2*cos(b*x + a) - 6*(2*b*d^2*x + 2*b*c*d + (b*d^2*x + b*c*d)*cos(b*x
+ a)^2)*sin(b*x + a))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(102) = 204$.

Time $= 0.36$ (sec) , antiderivative size $= 216$, normalized size of antiderivative $= 2.10$

$$\int (c+dx)^2 \cos^2(a+bx)\sin(a+bx)\,dx$$
$$= \begin{cases} -\frac{c^2\cos^3(a+bx)}{3b} - \frac{2cdx\cos^3(a+bx)}{3b} - \frac{d^2x^2\cos^3(a+bx)}{3b} + \frac{4cd\sin^3(a+bx)}{9b^2} + \frac{2cd\sin(a+bx)\cos^2(a+bx)}{3b^2} + \frac{4d^2x\sin^3(a+bx)}{9b^2} + \frac{2}{} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin(a)\cos^2(a) \end{cases}$$

input
```
integrate((d*x+c)**2*cos(b*x+a)**2*sin(b*x+a),x)
```

output
```
Piecewise((-c**2*cos(a + b*x)**3/(3*b) - 2*c*d*x*cos(a + b*x)**3/(3*b) - d
**2*x**2*cos(a + b*x)**3/(3*b) + 4*c*d*sin(a + b*x)**3/(9*b**2) + 2*c*d*si
n(a + b*x)*cos(a + b*x)**2/(3*b**2) + 4*d**2*x*sin(a + b*x)**3/(9*b**2) +
2*d**2*x*sin(a + b*x)*cos(a + b*x)**2/(3*b**2) + 4*d**2*sin(a + b*x)**2*co
s(a + b*x)/(9*b**3) + 14*d**2*cos(a + b*x)**3/(27*b**3), Ne(b, 0)), ((c**2
*x + c*d*x**2 + d**2*x**3/3)*sin(a)*cos(a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(93) = 186$.

Time $= 0.04$ (sec) , antiderivative size $= 243$, normalized size of antiderivative $= 2.36$

$$\int (c+dx)^2 \cos^2(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{36\,c^2\cos(bx+a)^3 - \frac{72\,acd\cos(bx+a)^3}{b} + \frac{36\,a^2d^2\cos(bx+a)^3}{b^2} + \frac{6\,(3\,(bx+a)\cos(3\,bx+3\,a)+9\,(bx+a)\cos(bx+a)-\sin(3\,bx+3\,a)-}{b}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/108*(36*c^2*cos(b*x + a)^3 - 72*a*c*d*cos(b*x + a)^3/b + 36*a^2*d^2*cos
(b*x + a)^3/b^2 + 6*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x +
a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*c*d/b - 6*(3*(b*x + a)*cos(3*b*x +
 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*a*d^
2/b^2 + ((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2 - 2)*cos(b
*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a))*d^2/b^
2)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.33

$$\begin{aligned}
&\int (c+dx)^2 \cos^2(a+bx)\sin(a+bx)\,dx \\
&= -\frac{(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2)\cos(3\,bx+3\,a)}{108\,b^3} \\
&\quad -\frac{(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2)\cos(bx+a)}{4\,b^3} \\
&\quad +\frac{(bd^2x+bcd)\sin(3\,bx+3\,a)}{18\,b^3}+\frac{(bd^2x+bcd)\sin(bx+a)}{2\,b^3}
\end{aligned}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/108*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(3*b*x + 3*a)
/b^3 - 1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)/b^3
+ 1/18*(b*d^2*x + b*c*d)*sin(3*b*x + 3*a)/b^3 + 1/2*(b*d^2*x + b*c*d)*sin(
b*x + a)/b^3
```

Mupad [B] (verification not implemented)

Time = 18.76 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.41

$$\begin{aligned}
&\int (c+dx)^2 \cos^2(a+bx)\sin(a+bx)\,dx \\
&= \frac{12\,d^2\,\cos(a+b\,x)+2\,d^2\cos(a+b\,x)^3-9\,b^2\,c^2\cos(a+b\,x)^3+12\,b\,d^2\,x\,\sin(a+b\,x)-9\,b^2\,d^2\,x^2\cos(a+
\end{aligned}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^2,x)
```

output
```
(12*d^2*cos(a + b*x) + 2*d^2*cos(a + b*x)^3 - 9*b^2*c^2*cos(a + b*x)^3 + 1
2*b*d^2*x*sin(a + b*x) - 9*b^2*d^2*x^2*cos(a + b*x)^3 + 12*b*c*d*sin(a + b
*x) - 18*b^2*c*d*x*cos(a + b*x)^3 + 6*b*d^2*x*cos(a + b*x)^2*sin(a + b*x)
+ 6*b*c*d*cos(a + b*x)^2*sin(a + b*x))/(27*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.15

$$\int (c+dx)^2 \cos^2(a+bx)\sin(a+bx)\,dx$$
$$= \frac{9\cos(bx+a)\sin(bx+a)^2 b^2c^2 + 18\cos(bx+a)\sin(bx+a)^2 b^2cdx + 9\cos(bx+a)\sin(bx+a)^2 b^2d^2x^2}$$

input
```
int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x)
```

output
```
(9*cos(a + b*x)*sin(a + b*x)**2*b**2*c**2 + 18*cos(a + b*x)*sin(a + b*x)**
2*b**2*c*d*x + 9*cos(a + b*x)*sin(a + b*x)**2*b**2*d**2*x**2 - 2*cos(a + b
*x)*sin(a + b*x)**2*d**2 - 9*cos(a + b*x)*b**2*c**2 - 18*cos(a + b*x)*b**2
*c*d*x - 9*cos(a + b*x)*b**2*d**2*x**2 + 14*cos(a + b*x)*d**2 - 6*sin(a +
b*x)**3*b*c*d - 6*sin(a + b*x)**3*b*d**2*x + 18*sin(a + b*x)*b*c*d + 18*si
n(a + b*x)*b*d**2*x - 18*a*b*c*d + 9*b**2*c**2 + 2*d**2)/(27*b**3)
```

3.74 $\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 752
Mathematica [A] (verified) . . . 752
Rubi [A] (verified) . . . 753
Maple [A] (verified) . . . 754
Fricas [A] (verification not implemented) . . . 755
Sympy [A] (verification not implemented) . . . 755
Maxima [A] (verification not implemented) . . . 756
Giac [A] (verification not implemented) . . . 756
Mupad [B] (verification not implemented) . . . 757
Reduce [B] (verification not implemented) . . . 757

Optimal result

Integrand size = 20, antiderivative size = 51

$$\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)\cos^3(a+bx)}{3b} + \frac{d\sin(a+bx)}{3b^2} - \frac{d\sin^3(a+bx)}{9b^2}$$

output
```
-1/3*(d*x+c)*cos(b*x+a)^3/b+1/3*d*sin(b*x+a)/b^2-1/9*d*sin(b*x+a)^3/b^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

$$\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{c\cos^3(a+bx)}{3b} + \frac{d(-bx\cos(a+bx)+\sin(a+bx))}{4b^2} + \frac{d(-3bx\cos(3(a+bx))+\sin(3(a+bx)))}{36b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/3*(c*Cos[a + b*x]^3)/b + (d*(-(b*x*Cos[a + b*x]) + Sin[a + b*x]))/(4*b^
2) + (d*(-3*b*x*Cos[3*(a + b*x)] + Sin[3*(a + b*x)]))/(36*b^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4905, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin(a+bx)\cos^2(a+bx)\,dx$$

$$\downarrow 4905$$

$$\frac{d\int \cos^3(a+bx)dx}{3b} - \frac{(c+dx)\cos^3(a+bx)}{3b}$$

$$\downarrow 3042$$

$$\frac{d\int \sin\left(a+bx+\frac{\pi}{2}\right)^3 dx}{3b} - \frac{(c+dx)\cos^3(a+bx)}{3b}$$

$$\downarrow 3113$$

$$-\frac{d\int \left(1-\sin^2(a+bx)\right)d(-\sin(a+bx))}{3b^2} - \frac{(c+dx)\cos^3(a+bx)}{3b}$$

$$\downarrow 2009$$

$$-\frac{d\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{3b^2} - \frac{(c+dx)\cos^3(a+bx)}{3b}$$

input
```
Int[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/3*((c + d*x)*Cos[a + b*x]^3)/b - (d*(-Sin[a + b*x] + Sin[a + b*x]^3/3))
/(3*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.25

method	result
risch	$-\frac{(dx+c)\cos(bx+a)}{4b}+\frac{d\sin(bx+a)}{4b^2}-\frac{(dx+c)\cos(3bx+3a)}{12b}+\frac{d\sin(3bx+3a)}{36b^2}$
derivativedivides	$\frac{\frac{da\cos(bx+a)^3}{3b}-\frac{c\cos(bx+a)^3}{3}+\frac{d\left(-\frac{(bx+a)\cos(bx+a)^3}{3}+\frac{\left(2+\cos(bx+a)^2\right)\sin(bx+a)}{9}\right)}{b}}{b}$
default	$\frac{\frac{da\cos(bx+a)^3}{3b}-\frac{c\cos(bx+a)^3}{3}+\frac{d\left(-\frac{(bx+a)\cos(bx+a)^3}{3}+\frac{\left(2+\cos(bx+a)^2\right)\sin(bx+a)}{9}\right)}{b}}{b}$
parallelrisch	$\frac{3d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^6xb+6d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^5-18\left(\frac{dx}{2}+c\right)b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4+4d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3+9d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2xb+6\tan\left(\frac{a}{2}+\frac{b}{2}\right.}{9b^2\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)^3}$
norman	$\frac{-\frac{2c\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4}{b}+\frac{dx\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{b}-\frac{2c}{3b}+\frac{2d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{3b^2}+\frac{4d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3}{9b^2}+\frac{2d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^5}{3b^2}-\frac{dx}{3b}-\frac{dx\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4}{b}+\frac{dx}{}}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)^3}$
orering	$\frac{4d(5x^2d^2b^2+10b^2cdx+5b^2c^2+2d^2)\cos(bx+a)^2\sin(bx+a)}{9b^4(dx+c)^2}-\frac{2(5x^2d^2b^2+10b^2cdx+5b^2c^2+4d^2)\left(d\cos(bx+a)^2\sin(b\right.}{9b}$

input
```
int((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(d*x+c)*cos(b*x+a)/b+1/4*d*sin(b*x+a)/b^2-1/12*(d*x+c)*cos(3*b*x+3*a)
/b+1/36*d*sin(3*b*x+3*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx$$
$$= -\frac{3\,(bdx+bc)\cos\left(bx+a\right)^3 - \left(d\cos\left(bx+a\right)^2+2\,d\right)\sin\left(bx+a\right)}{9\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/9*(3*(b*d*x + b*c)*cos(b*x + a)^3 - (d*cos(b*x + a)^2 + 2*d)*sin(b*x +
a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.67

$$\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx$$
$$= \begin{cases} -\frac{c\cos^3(a+bx)}{3b} - \frac{dx\cos^3(a+bx)}{3b} + \frac{2d\sin^3(a+bx)}{9b^2} + \frac{d\sin(a+bx)\cos^2(a+bx)}{3b^2} & \text{for } b \neq 0 \\ \left(cx+\frac{dx^2}{2}\right)\sin(a)\cos^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*cos(b*x+a)**2*sin(b*x+a),x)
```

output
```
Piecewise((-c*cos(a + b*x)**3/(3*b) - d*x*cos(a + b*x)**3/(3*b) + 2*d*sin(
a + b*x)**3/(9*b**2) + d*sin(a + b*x)*cos(a + b*x)**2/(3*b**2), Ne(b, 0)),
 ((c*x + d*x**2/2)*sin(a)*cos(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.69

$$\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{12\,c\cos(bx+a)^3 - \frac{12\,ad\cos(bx+a)^3}{b} + \frac{(3\,(bx+a)\cos(3\,bx+3\,a)+9\,(bx+a)\cos(bx+a)-\sin(3\,bx+3\,a)-9\,\sin(bx+a))d}{b}}{36\,b}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/36*(12*c*cos(b*x + a)^3 - 12*a*d*cos(b*x + a)^3/b + (3*(b*x + a)*cos(3*
b*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))
*d/b)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.35

$$\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{(bdx+bc)\cos(3\,bx+3\,a)}{12\,b^2} - \frac{(bdx+bc)\cos(bx+a)}{4\,b^2} + \frac{d\sin(3\,bx+3\,a)}{36\,b^2} + \frac{d\sin(bx+a)}{4\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/12*(b*d*x + b*c)*cos(3*b*x + 3*a)/b^2 - 1/4*(b*d*x + b*c)*cos(b*x + a)/
b^2 + 1/36*d*sin(3*b*x + 3*a)/b^2 + 1/4*d*sin(b*x + a)/b^2
```

Mupad [B] (verification not implemented)

Time = 18.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx = \frac{\frac{2d\sin(a+bx)}{9} - b\left(\frac{c\cos(a+bx)^3}{3} + \frac{dx\cos(a+bx)^3}{3}\right) + \frac{d\cos(a+bx)^2\sin(a+bx)}{9}}{b^2}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x),x)
```

output
```
((2*d*sin(a + b*x))/9 - b*((c*cos(a + b*x)^3)/3 + (d*x*cos(a + b*x)^3)/3)
+ (d*cos(a + b*x)^2*sin(a + b*x))/9)/b^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.80

$$\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx = \frac{3\cos(bx+a)\sin(bx+a)^2 bc + 3\cos(bx+a)\sin(bx+a)^2 bdx - 3\cos(bx+a)bc - 3\cos(bx+a)bdx -}{9b^2}$$

input
```
int((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x)
```

output
```
(3*cos(a + b*x)*sin(a + b*x)**2*b*c + 3*cos(a + b*x)*sin(a + b*x)**2*b*d*x
 - 3*cos(a + b*x)*b*c - 3*cos(a + b*x)*b*d*x - sin(a + b*x)**3*d + 3*sin(a
 + b*x)*d - 3*a*d + 3*b*c)/(9*b**2)
```

3.75 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx$

Optimal result . . . 758
Mathematica [A] (verified) . . . 759
Rubi [A] (verified) . . . 759
Maple [A] (verified) . . . 760
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Giac [C] (verification not implemented) . . . 762
Mupad [F(-1)] . . . 763
Reduce [F] . . . 764

Optimal result

Integrand size = 22, antiderivative size = 121

$$\begin{aligned}\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx = &\frac{\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d}\\ &+\frac{\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{4d}\\ &+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}\\ &+\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}\end{aligned}$$

output

```
1/4*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d+1/4*Ci(b*c/d+b*x)*sin(a-b*c/d)/d+
1/4*cos(a-b*c/d)*Si(b*c/d+b*x)/d+1/4*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx$$
$$= \frac{\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right)+\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right)+\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)}{4d}$$

input
```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x),x]
```

output
```
(CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + CosIntegral[b*(c/d
+ x)]*Sin[a - (b*c)/d] + Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Cos[3
*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos^2(a+bx)}{c+dx}\,dx$$
$$\downarrow 4906$$
$$\int \left(\frac{\sin(a+bx)}{4(c+dx)}+\frac{\sin(3a+3bx)}{4(c+dx)}\right)dx$$
$$\downarrow 2009$$
$$\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}+$$
$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x),x]
```

output
```
(CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(4*d) + (CosIntegral
[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(4*d) + (Cos[a - (b*c)/d]*SinIntegral[(b
*c)/d + b*x])/(4*d) + (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x]
)/(4*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\frac{b\left(-\frac{3\,\mathrm{Si}\left(-3bx-3a-\frac{3(-ad+cb)}{d}\right)\cos\left(\frac{-3ad+3cb}{d}\right)}{d}-\frac{3\,\mathrm{Ci}\left(3bx+3a+\frac{-3ad+3cb}{d}\right)\sin\left(\frac{-3ad+3cb}{d}\right)}{d}\right)}{12}+\frac{b\left(-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+cb}{d}\right)}{d}\right.}{b}$
default	$\frac{b\left(-\frac{3\,\mathrm{Si}\left(-3bx-3a-\frac{3(-ad+cb)}{d}\right)\cos\left(\frac{-3ad+3cb}{d}\right)}{d}-\frac{3\,\mathrm{Ci}\left(3bx+3a+\frac{-3ad+3cb}{d}\right)\sin\left(\frac{-3ad+3cb}{d}\right)}{d}\right)}{12}+\frac{b\left(-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+cb}{d}\right)}{d}\right.}{b}$
risch	$-\frac{ie^{-\frac{3i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{8d}-\frac{ie^{-\frac{i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(ibx+ia-\frac{i(ad-cb)}{d}\right)}{8d}+\frac{ie^{\frac{i(ad-cb)}{d}}}{d}$

input
```
int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/12*b*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3
*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)+1/4*b*(-Si(-b*x-a-(-a*d+b*
c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx$$
$$= \frac{\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)\sin\left(-\frac{bc-ad}{d}\right) + \mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right) + \cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bdx+bc)}{d}\right) + \cos\left(-\frac{bc-ad}{d}\right.}{4\,d}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
1/4*(cos_integral((b*d*x + b*c)/d)*sin(-(b*c - a*d)/d) + cos_integral(3*(b
*d*x + b*c)/d)*sin(-3*(b*c - a*d)/d) + cos(-3*(b*c - a*d)/d)*sin_integral(
3*(b*d*x + b*c)/d) + cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx = \int \frac{\sin(a+bx)\cos^2(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.27

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx =$$
$$-\frac{b\left(i\,E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right) - i\,E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right) + b\left(-i\,E_1\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right) +\right.}{}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
-1/8*(b*(I*exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_in
tegral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b*(
-I*exp_integral_e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integra
l_e(1, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) + b*(
exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(1, -
(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(exp_integral_
e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1, -3*(-I*b*c
- I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/(b*d)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 6279, normalized size of antiderivative = 51.89

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

output

```
1/8*(imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral(b*x + b*c/d))*tan
(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_part(cos_i
ntegral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)
*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integ
ral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*
c/d)^2 + 2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*real_part(cos_integral(-b*x - b*c/d))*
tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*real_part(co
s_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(
1/2*b*c/d)^2 + 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(b*x
+ b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*re
al_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)
^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a
)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integra
l(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx = \int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{c+d\,x}\,dx$$

input

```
int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x),x)
```

output

```
int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx = \int \frac{\cos\left(bx+a\right)^2\sin\left(bx+a\right)}{dx+c}dx$$

input
```
int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x)
```

output
```
int((cos(a + b*x)**2*sin(a + b*x))/(c + d*x),x)
```

3.76 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 765
Mathematica [A] (verified) . . . 766
Rubi [A] (verified) . . . 766
Maple [A] (verified) . . . 767
Fricas [A] (verification not implemented) . . . 768
Sympy [F] . . . 768
Maxima [C] (verification not implemented) . . . 769
Giac [C] (verification not implemented) . . . 769
Mupad [F(-1)] . . . 770
Reduce [F] . . . 771

Optimal result

Integrand size = 22, antiderivative size = 168

$$\begin{aligned}\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = &\frac{b\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2}\\ &+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2}\\ &-\frac{\sin(a+bx)}{4d(c+dx)}-\frac{\sin(3a+3bx)}{4d(c+dx)}\\ &-\frac{b\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}\\ &-\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{4d^2}\end{aligned}$$

output

```
1/4*b*cos(a-b*c/d)*Ci(b*c/d+b*x)/d^2+3/4*b*cos(3*a-3*b*c/d)*Ci(3*b*c/d+3*b
*x)/d^2-1/4*sin(b*x+a)/d/(d*x+c)-1/4*sin(3*b*x+3*a)/d/(d*x+c)-1/4*b*sin(a-
b*c/d)*Si(b*c/d+b*x)/d^2-3/4*b*sin(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^2
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.83

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{-b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)-3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)+\frac{d\sin(a+bx)}{c+dx}+\frac{d\sin(3}{c+}}{4d^2}$$

input
```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^2,x]
```

output
```
-1/4*(-(b*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)]) - 3*b*Cos[3*a - (3*b*
c)/d]*CosIntegral[(3*b*(c + d*x))/d] + (d*Sin[a + b*x])/(c + d*x) + (d*Sin
[3*(a + b*x)])/(c + d*x) + b*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 3
*b*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/d^2
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos^2(a+bx)}{(c+dx)^2}\,dx$$
$$\downarrow 4906$$
$$\int \left(\frac{\sin(a+bx)}{4(c+dx)^2}+\frac{\sin(3a+3bx)}{4(c+dx)^2}\right)dx$$
$$\downarrow 2009$$
$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-$$
$$\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{\sin(a+bx)}{4d(c+dx)}-\frac{\sin(3a+3bx)}{4d(c+dx)}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^2,x]
```

output
```
(b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d^2) + (3*b*Cos[3*a - (
3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d^2) - Sin[a + b*x]/(4*d*(c +
 d*x)) - Sin[3*a + 3*b*x]/(4*d*(c + d*x)) - (b*Sin[a - (b*c)/d]*SinIntegra
l[(b*c)/d + b*x])/(4*d^2) - (3*b*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/
d + 3*b*x])/(4*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.46

method	result
derivativedivides	$\frac{b^2\left(-\frac{3\sin(3bx+3a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{9\,\mathrm{Si}\left(-3bx-3a-\frac{3(-ad+cb)}{d}\right)\sin\left(\frac{-3ad+3cb}{d}\right)}{d}+\frac{9\,\mathrm{Ci}\left(3bx+3a+\frac{-3ad+3cb}{d}\right)\cos\left(\frac{-3ad+3cb}{d}\right)}{d}}{d}\right)}{12}+\frac{}{b}$
default	$\frac{b^2\left(-\frac{3\sin(3bx+3a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{9\,\mathrm{Si}\left(-3bx-3a-\frac{3(-ad+cb)}{d}\right)\sin\left(\frac{-3ad+3cb}{d}\right)}{d}+\frac{9\,\mathrm{Ci}\left(3bx+3a+\frac{-3ad+3cb}{d}\right)\cos\left(\frac{-3ad+3cb}{d}\right)}{d}}{d}\right)}{12}+\frac{}{b}$
risch	$-\frac{3b\,\mathrm{e}^{-\frac{3i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{8d^2}-\frac{b\,\mathrm{e}^{-\frac{i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(ibx+ia-\frac{i(ad-cb)}{d}\right)}{8d^2}-\frac{b\,\mathrm{e}^{\frac{i(ad}{}}$

input
```
int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/12*b^2*(-3*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d+3*(-3*Si(-3*b*x-3*
a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos
(3*(-a*d+b*c)/d)/d)/d)+1/4*b^2*(-sin(b*x+a)/(-a*d+c*b+d*(b*x+a))/d+(-Si(-b
*x-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*
c)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.08

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{4\,d\cos(bx+a)^2\sin(bx+a) - 3\,(bdx+bc)\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right) - (bdx+bc)\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}}{4\,(d^3x+cd}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*d*cos(b*x + a)^2*sin(b*x + a) - 3*(b*d*x + b*c)*cos(-3*(b*c - a*d)
/d)*cos_integral(3*(b*d*x + b*c)/d) - (b*d*x + b*c)*cos(-(b*c - a*d)/d)*co
s_integral((b*d*x + b*c)/d) + 3*(b*d*x + b*c)*sin(-3*(b*c - a*d)/d)*sin_in
tegral(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*sin(-(b*c - a*d)/d)*sin_integral
((b*d*x + b*c)/d))/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(a+bx)\cos^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.80

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = -\frac{b^2\left(i\,E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^2\left(-i\,E_2\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right.}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/8*(b^2*(I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_
integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b
^2*(-I*exp_integral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_int
egral_e(2, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) +
 b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_
e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^2*(exp_i
ntegral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(2, -3*
(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b*c*d + (b*x
 + a)*d^2 - a*d^2)*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.11 (sec) , antiderivative size = 66726, normalized size of antiderivative = 397.18

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output

```
1/8*(3*b*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + b*
d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan
(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + b*d*x*real_part
(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b*d*x*real_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(b*
x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d) + 2*b*d*x*imag_part(cos_integral(-b*x - b*c/d))*
tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d) - 4*b*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b*
d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2
*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 6*b*d*x*imag_
part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2
*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 12*b*d*x*sin_integral
(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^
2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*b*d*x*imag_part(cos_integral(b*x + b
*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{\left(c+d\,x\right)^2}\,dx$$

input

```
int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^2,x)
```

output

```
int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(bx+a\right)^2\sin\left(bx+a\right)}{d^2x^2+2cdx+c^2}dx$$

input
```
int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x)
```

output
```
int((cos(a + b*x)**2*sin(a + b*x))/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.77 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$

Optimal result . . . 772
Mathematica [A] (verified) . . . 773
Rubi [A] (verified) . . . 773
Maple [A] (verified) . . . 775
Fricas [A] (verification not implemented) . . . 775
Sympy [F] . . . 776
Maxima [C] (verification not implemented) . . . 776
Giac [C] (verification not implemented) . . . 777
Mupad [F(-1)] . . . 778
Reduce [F] . . . 779

Optimal result

Integrand size = 22, antiderivative size = 221

$$\begin{aligned}\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = &-\frac{b\cos(a+bx)}{8d^2(c+dx)} - \frac{3b\cos(3a+3bx)}{8d^2(c+dx)} \\ &- \frac{9b^2\,\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{8d^3} \\ &- \frac{b^2\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{8d^3} - \frac{\sin(a+bx)}{8d(c+dx)^2} \\ &- \frac{\sin(3a+3bx)}{8d(c+dx)^2} - \frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3} \\ &- \frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^3}\end{aligned}$$

output

```
-1/8*b*cos(b*x+a)/d^2/(d*x+c)-3/8*b*cos(3*b*x+3*a)/d^2/(d*x+c)-9/8*b^2*Ci(
3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^3-1/8*b^2*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^3
-1/8*sin(b*x+a)/d/(d*x+c)^2-1/8*sin(3*b*x+3*a)/d/(d*x+c)^2-1/8*b^2*cos(a-b
*c/d)*Si(b*c/d+b*x)/d^3-9/8*b^2*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^3
```

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.82

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx =$$

$$-\frac{9b^2\,\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right)+b^2\,\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right)+\frac{d(b(c+dx)\cos(a+bx)+d}{(c+dx)^2}}$$

input

```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^3,x]
```

output

```
-1/8*(9*b^2*CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + b^2*CosI
ntegral[b*(c/d + x)]*Sin[a - (b*c)/d] + (d*(b*(c + d*x)*Cos[a + b*x] + d*S
in[a + b*x]))/(c + d*x)^2 + (d*(3*b*(c + d*x)*Cos[3*(a + b*x)] + d*Sin[3*(
a + b*x)]))/(c + d*x)^2 + b^2*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] +
9*b^2*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/d^3
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos^2(a+bx)}{(c+dx)^3}\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{\sin(a+bx)}{4(c+dx)^3}+\frac{\sin(3a+3bx)}{4(c+dx)^3}\right)dx$$

$$\downarrow 2009$$

$$-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}-\frac{b^2\sin\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^3}-\frac{b\cos(a+bx)}{8d^2(c+dx)}-\frac{3b\cos(3a+3bx)}{8d^2(c+dx)}-\frac{\sin(a+bx)}{8d(c+dx)^2}-\frac{\sin(3a+3bx)}{8d(c+dx)^2}$$

input

```
Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^3,x]
```

output

```
-1/8*(b*Cos[a + b*x])/(d^2*(c + d*x)) - (3*b*Cos[3*a + 3*b*x])/(8*d^2*(c +
 d*x)) - (9*b^2*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(8*d^
3) - (b^2*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(8*d^3) - Sin[a + b
*x]/(8*d*(c + d*x)^2) - Sin[3*a + 3*b*x]/(8*d*(c + d*x)^2) - (b^2*Cos[a -
(b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^3) - (9*b^2*Cos[3*a - (3*b*c)/d]
*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{b^3\left(-\frac{3\sin(3bx+3a)}{2(-ad+cb+d(bx+a))^2d}+\frac{-\frac{9\cos(3bx+3a)}{2(-ad+cb+d(bx+a))d}-\frac{9\left(-\frac{3\,\mathrm{Si}\left(-3bx-3a-\frac{3(-ad+cb)}{d}\right)\cos\left(\frac{-3ad+3cb}{d}\right)}{d}-\frac{3\,\mathrm{Ci}\left(3bx+3a+\right.}{2d}\right.}{d}\right.}{12}$
default	$\frac{b^3\left(-\frac{3\sin(3bx+3a)}{2(-ad+cb+d(bx+a))^2d}+\frac{-\frac{9\cos(3bx+3a)}{2(-ad+cb+d(bx+a))d}-\frac{9\left(-\frac{3\,\mathrm{Si}\left(-3bx-3a-\frac{3(-ad+cb)}{d}\right)\cos\left(\frac{-3ad+3cb}{d}\right)}{d}-\frac{3\,\mathrm{Ci}\left(3bx+3a+\right.}{2d}\right.}{d}\right.}{12}$
risch	$\frac{9ib^2\mathrm{e}^{-\frac{3i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{16d^3}+\frac{ib^2\mathrm{e}^{-\frac{i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(ibx+ia-\frac{i(ad-cb)}{d}\right)}{16d^3}-\frac{ib^2\mathrm{e}^{\frac{i(}{}}}{}$

input
```
int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/12*b^3*(-3/2*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))^2/d+3/2*(-3*cos(3*
b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d-3*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*cos(3*
(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d)/d
)+1/4*b^3*(-1/2*sin(b*x+a)/(-a*d+c*b+d*(b*x+a))^2/d+1/2*(-cos(b*x+a)/(-a*d
+c*b+d*(b*x+a))/d-(-Si(-b*x-a-(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(
-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.38

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx =$$

$$-\frac{4\,d^2\cos(bx+a)^2\sin(bx+a)+12\,(bd^2x+bcd)\cos(bx+a)^3+(b^2d^2x^2+2\,b^2cdx+b^2c^2)\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)\mathrm{s}}{}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/8*(4*d^2*cos(b*x + a)^2*sin(b*x + a) + 12*(b*d^2*x + b*c*d)*cos(b*x + a
)^3 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral((b*d*x + b*c)/d)*
sin(-(b*c - a*d)/d) + 9*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral
(3*(b*d*x + b*c)/d)*sin(-3*(b*c - a*d)/d) + 9*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*cos(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) + (b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*
c)/d) - 8*(b*d^2*x + b*c*d)*cos(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin(a+bx)\cos^2(a+bx)}{(c+dx)^3}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.52

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx =$$
$$-\frac{b^3\left(i\,E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^3\left(-i\,E_3\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right.}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output

```
-1/8*(b^3*(I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_
integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b
^3*(-I*exp_integral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_int
egral_e(3, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) +
 b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_
e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^3*(exp_i
ntegral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(3, -3*
(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^2*c^2*d -
2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b
)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.16 (sec) , antiderivative size = 118262, normalized size of antiderivative = 535.12

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input

```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output

```
-1/16*(9*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)
^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 + b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(
1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - b
^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*
x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*d
^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*
x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*
d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(
3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^2*x^2*si
n_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan
(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^2*x^2*real_part(cos_
integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*b^2*d^2*x^2*real_part(cos_integra
l(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d) + 18*b^2*d^2*x^2*real_part(cos_integral(3*b
*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan
(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*real_part(cos_integral(-3*b*
x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(
3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*real_part(cos_integral(b*x ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{\left(c+d\,x\right)^3}\,dx$$

input

```
int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^3,x)
```

output

```
int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos(bx+a)^2\sin(bx+a)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx$$

input
```
int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x)
```

output
```
int((cos(a + b*x)**2*sin(a + b*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d*
*3*x**3),x)
```

3.78 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$

Optimal result . . . 780
Mathematica [A] (verified) . . . 781
Rubi [A] (verified) . . . 781
Maple [A] (verified) . . . 783
Fricas [A] (verification not implemented) . . . 784
Sympy [F] . . . 784
Maxima [C] (verification not implemented) . . . 785
Giac [C] (verification not implemented) . . . 785
Mupad [F(-1)] . . . 786
Reduce [F] . . . 787

Optimal result

Integrand size = 22, antiderivative size = 270

$$\begin{aligned}\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = &-\frac{b\cos(a+bx)}{24d^2(c+dx)^2}-\frac{b\cos(3a+3bx)}{8d^2(c+dx)^2}\\ &-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{24d^4}\\ &-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^4}\\ &-\frac{\sin(a+bx)}{12d(c+dx)^3}+\frac{b^2\sin(a+bx)}{24d^3(c+dx)}-\frac{\sin(3a+3bx)}{12d(c+dx)^3}\\ &+\frac{3b^2\sin(3a+3bx)}{8d^3(c+dx)}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{24d^4}\\ &+\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^4}\end{aligned}$$

output

```
-1/24*b*cos(b*x+a)/d^2/(d*x+c)^2-1/8*b*cos(3*b*x+3*a)/d^2/(d*x+c)^2-1/24*b
^3*cos(a-b*c/d)*Ci(b*c/d+b*x)/d^4-9/8*b^3*cos(3*a-3*b*c/d)*Ci(3*b*c/d+3*b*
x)/d^4-1/12*sin(b*x+a)/d/(d*x+c)^3+1/24*b^2*sin(b*x+a)/d^3/(d*x+c)-1/12*si
n(3*b*x+3*a)/d/(d*x+c)^3+3/8*b^2*sin(3*b*x+3*a)/d^3/(d*x+c)+1/24*b^3*sin(a
-b*c/d)*Si(b*c/d+b*x)/d^4+9/8*b^3*sin(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^4
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.11

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx =$$

$$-\frac{d\cos(bx)\left(bd(c+dx)\cos(a)-\left(-2d^2+b^2(c+dx)^2\right)\sin(a)\right)+d\cos(3bx)\left(3bd(c+dx)\cos(3a)-(-2d$$

input

```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^4,x]
```

output

```
-1/24*(d*Cos[b*x]*(b*d*(c + d*x)*Cos[a] - (-2*d^2 + b^2*(c + d*x)^2)*Sin[a
]) + d*Cos[3*b*x]*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)
*Sin[3*a]) - d*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*d*(c + d*x)*Sin[a])*
Sin[b*x] - d*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*x)*Sin[
3*a])*Sin[3*b*x] + b^3*(c + d*x)^3*(Cos[a - (b*c)/d]*CosIntegral[b*(c/d +
x)] - Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)]) + 27*b^3*(c + d*x)^3*(Cos
[3*a - (3*b*c)/d]*CosIntegral[(3*b*(c + d*x))/d] - Sin[3*a - (3*b*c)/d]*Si
nIntegral[(3*b*(c + d*x))/d]))/(d^4*(c + d*x)^3)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos^2(a+bx)}{(c+dx)^4}\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{\sin(a+bx)}{4(c+dx)^4}+\frac{\sin(3a+3bx)}{4(c+dx)^4}\right)dx$$

$$\downarrow 2009$$

$$-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{24d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+$$
$$\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{24d^4}+\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^2\sin(a+bx)}{24d^3(c+dx)}+$$
$$\frac{3b^2\sin(3a+3bx)}{8d^3(c+dx)}-\frac{b\cos(a+bx)}{24d^2(c+dx)^2}-\frac{b\cos(3a+3bx)}{8d^2(c+dx)^2}-\frac{\sin(a+bx)}{12d(c+dx)^3}-\frac{\sin(3a+3bx)}{12d(c+dx)^3}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^4,x]
```

output
```
-1/24*(b*Cos[a + b*x])/(d^2*(c + d*x)^2) - (b*Cos[3*a + 3*b*x])/(8*d^2*(c
+ d*x)^2) - (b^3*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(24*d^4) - (
9*b^3*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(8*d^4) - Sin[a
 + b*x]/(12*d*(c + d*x)^3) + (b^2*Sin[a + b*x])/(24*d^3*(c + d*x)) - Sin[3
*a + 3*b*x]/(12*d*(c + d*x)^3) + (3*b^2*Sin[3*a + 3*b*x])/(8*d^3*(c + d*x)
) + (b^3*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(24*d^4) + (9*b^3*Si
n[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{b^4\left(-\frac{\sin(3bx+3a)}{(-ad+cb+d(bx+a))^3 d}+\frac{-\frac{3\cos(3bx+3a)}{2(-ad+cb+d(bx+a))^2 d}-\frac{3\left(-\frac{3\sin(3bx+3a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{9\,\mathrm{Si}\left(-3bx-3a-\frac{3(-ad+cb)}{d}\right)\sin\left(\frac{-3ad}{d}\right.}{d}}{2d}\right.}{d}\right.}{12}$
default	$\frac{b^4\left(-\frac{\sin(3bx+3a)}{(-ad+cb+d(bx+a))^3 d}+\frac{-\frac{3\cos(3bx+3a)}{2(-ad+cb+d(bx+a))^2 d}-\frac{3\left(-\frac{3\sin(3bx+3a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{9\,\mathrm{Si}\left(-3bx-3a-\frac{3(-ad+cb)}{d}\right)\sin\left(\frac{-3ad}{d}\right.}{d}}{2d}\right.}{d}\right.}{12}$
risch	$\frac{9b^3 e^{-\frac{3i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{16d^4}+\frac{b^3 e^{-\frac{i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(ibx+ia-\frac{i(ad-cb)}{d}\right)}{48d^4}+\frac{b^3 e^{\frac{i(ad-}{d}}}{}$

input
```
int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/12*b^4*(-sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))^3/d+(-3/2*cos(3*b*x+3*
a)/(-a*d+c*b+d*(b*x+a))^2/d-3/2*(-3*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d+
3*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3
*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d)/d)/d)+1/4*b^4*(-1/3*sin(b*x+a)/(-
a*d+c*b+d*(b*x+a))^3/d+1/3*(-1/2*cos(b*x+a)/(-a*d+c*b+d*(b*x+a))^2/d-1/2*(
-sin(b*x+a)/(-a*d+c*b+d*(b*x+a))/d+(-Si(-b*x-a-(-a*d+b*c)/d)*sin((-a*d+b*c
)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.64

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx =$$
$$-\frac{12\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^3+27\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{3\,(bdx+bc}{d}\right.}{}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/24*(12*(b*d^3*x + b*c*d^2)*cos(b*x + a)^3 + 27*(b^3*d^3*x^3 + 3*b^3*c*d
^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-3*(b*c - a*d)/d)*cos_integral(3*(b*
d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*
cos(-(b*c - a*d)/d)*cos_integral((b*d*x + b*c)/d) - 27*(b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-3*(b*c - a*d)/d)*sin_integral(
3*(b*d*x + b*c)/d) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*
c^3)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) - 8*(b*d^3*x + b*c*
d^2)*cos(b*x + a) + 4*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - (9*b^
2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a)^2)*sin(b*x
+ a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sin\left(a+bx\right)\cos^2\left(a+bx\right)}{(c+dx)^4}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c)**4,x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x)**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.43

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = -\frac{b^4\left(i\,E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^4\left(-i\,E_4\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right.}{8\left(b^3c^3d-3\,a\right.}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/8*(b^4*(I*exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_
integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b
^4*(-I*exp_integral_e(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_int
egral_e(4, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) +
 b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_
e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^4*(exp_i
ntegral_e(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(4, -3*
(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^3*c^3*d -
3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 -
 a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b
)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.25 (sec) , antiderivative size = 166374, normalized size of antiderivative = 616.20

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

output

```
-1/48*(27*b^3*d^3*x^3*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + b^3*d^3*x^3*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan
(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 +
b^3*d^3*x^3*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b
*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 27*b^3
*d^3*x^3*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b^3
*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^
2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*b^3*d^3*x^
3*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(
3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*b^3*d^3*x^3*sin_
integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 54*b^3*d^3*x^3*imag_part(cos_int
egral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2
*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 54*b^3*d^3*x^3*imag_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 108*b^3*d^3*x^3*sin_integral(3*(b*d
*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3
/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*b^3*d^3*x^3*imag_part(cos_integral(b*x +...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{\left(c+d\,x\right)^4}\,dx$$

input

```
int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^4,x)
```

output

```
int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^4, x)
```

Reduce [F]

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \text{too large to display}$$

input

```
int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x)
```

output

```
(28*cos(a + b*x)*sin(a + b*x)**2*b*c*d**2 + 28*cos(a + b*x)*sin(a + b*x)**
2*b*d**3*x + 378*cos(a + b*x)*sin(a + b*x)*b**2*c**2*d + 756*cos(a + b*x)*
sin(a + b*x)*b**2*c*d**2*x + 378*cos(a + b*x)*sin(a + b*x)*b**2*d**3*x**2
+ 732*cos(a + b*x)*sin(a + b*x)*d**3 - 952*cos(a + b*x)*b*c*d**2 - 952*cos
(a + b*x)*b*d**3*x - 2016*int(x**3/(tan((a + b*x)/2)**6*c**4 + 4*tan((a +
b*x)/2)**6*c**3*d*x + 6*tan((a + b*x)/2)**6*c**2*d**2*x**2 + 4*tan((a + b*
x)/2)**6*c*d**3*x**3 + tan((a + b*x)/2)**6*d**4*x**4 + 3*tan((a + b*x)/2)*
*4*c**4 + 12*tan((a + b*x)/2)**4*c**3*d*x + 18*tan((a + b*x)/2)**4*c**2*d*
*2*x**2 + 12*tan((a + b*x)/2)**4*c*d**3*x**3 + 3*tan((a + b*x)/2)**4*d**4*
x**4 + 3*tan((a + b*x)/2)**2*c**4 + 12*tan((a + b*x)/2)**2*c**3*d*x + 18*t
an((a + b*x)/2)**2*c**2*d**2*x**2 + 12*tan((a + b*x)/2)**2*c*d**3*x**3 + 3
*tan((a + b*x)/2)**2*d**4*x**4 + c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*
c*d**3*x**3 + d**4*x**4),x)*b**3*c**3*d**4 - 6048*int(x**3/(tan((a + b*x)/
2)**6*c**4 + 4*tan((a + b*x)/2)**6*c**3*d*x + 6*tan((a + b*x)/2)**6*c**2*d
**2*x**2 + 4*tan((a + b*x)/2)**6*c*d**3*x**3 + tan((a + b*x)/2)**6*d**4*x*
*4 + 3*tan((a + b*x)/2)**4*c**4 + 12*tan((a + b*x)/2)**4*c**3*d*x + 18*tan
((a + b*x)/2)**4*c**2*d**2*x**2 + 12*tan((a + b*x)/2)**4*c*d**3*x**3 + 3*t
an((a + b*x)/2)**4*d**4*x**4 + 3*tan((a + b*x)/2)**2*c**4 + 12*tan((a + b*
x)/2)**2*c**3*d*x + 18*tan((a + b*x)/2)**2*c**2*d**2*x**2 + 12*tan((a + b*
x)/2)**2*c*d**3*x**3 + 3*tan((a + b*x)/2)**2*d**4*x**4 + c**4 + 4*c**3*...
```

3.79 $\int (c+dx)^m \cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal result . . . 788
Mathematica [A] (verified) . . . 789
Rubi [A] (verified) . . . 789
Maple [F] . . . 790
Fricas [A] (verification not implemented) . . . 791
Sympy [F] . . . 791
Maxima [F] . . . 791
Giac [F] . . . 792
Mupad [F(-1)] . . . 792
Reduce [F] . . . 792

Optimal result

Integrand size = 24, antiderivative size = 162

$$\int (c+dx)^m \cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{(c+dx)^{1+m}}{8d(1+m)} + \frac{i2^{-2(3+m)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{4ib(c+dx)}{d}\right)}{b}$$
$$- \frac{i2^{-2(3+m)}e^{-4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{4ib(c+dx)}{d}\right)}{b}$$

output

```
1/8*(d*x+c)^(1+m)/d/(1+m)+I*exp(4*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-4*I*b*
(d*x+c)/d)/(2^(6+2*m))/b/((-I*b*(d*x+c)/d)^m)-I*(d*x+c)^m*GAMMA(1+m,4*I*b*
(d*x+c)/d)/(2^(6+2*m))/b/exp(4*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.96

$$\int (c+dx)^m \cos^2(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{(c+dx)^m\left(-8b(c+dx)-i4^{-m}de^{4i\left(a-\frac{bc}{d}\right)}(1+m)\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{4ib(c+dx)}{d}\right)+i4^{-m}de^{-4i(}\right.}{64bd(1+m)}$$

input
```
Integrate[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
-1/64*((c + d*x)^m*(-8*b*(c + d*x) - (I*d*E^((4*I)*(a - (b*c)/d))*(1 + m)*
Gamma[1 + m, ((-4*I)*b*(c + d*x))/d])/(4^m*(((-I)*b*(c + d*x))/d)^m) + (I*
d*(1 + m)*Gamma[1 + m, ((4*I)*b*(c + d*x))/d])/(4^m*E^((4*I)*(a - (b*c)/d)
)*((I*b*(c + d*x))/d)^m)))/(b*d*(1 + m))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos^2(a+bx)(c+dx)^m\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8}(c+dx)^m - \frac{1}{8}\cos(4a+4bx)(c+dx)^m\right)dx$$

$$\downarrow 2009$$

$$\frac{i2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{4ib(c+dx)}{d}\right)}{b} -$$
$$\frac{i2^{-2(m+3)}e^{-4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{4ib(c+dx)}{d}\right)}{b}+\frac{(c+dx)^{m+1}}{8d(m+1)}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^(1 + m)/(8*d*(1 + m)) + (I*E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*G
amma[1 + m, ((-4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*(((-I)*b*(c + d*x))/
d)^m) - (I*(c + d*x)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))
*b*E^((4*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [F]

$$\int (dx+c)^m\cos(bx+a)^2\sin(bx+a)^2\,dx$$

input
```
int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84

$$\int (c+dx)^m \cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{(i\,dm+i\,d)e^{\left(-\frac{dm\log\left(-\frac{4ib}{d}\right)+4i\,bc-4i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{4\,(i\,bdx+i\,bc)}{d}\right)+(-i\,dm-i\,d)e^{\left(-\frac{dm\log\left(\frac{4ib}{d}\right)-4i\,bc+4i\,ad}{d}\right)}\Gamma\left(m\right.}{64\,(bdm+bd)}$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/64*((I*d*m + I*d)*e^(-(d*m*log(-4*I*b/d) + 4*I*b*c - 4*I*a*d)/d)*gamma(m
 + 1, -4*(I*b*d*x + I*b*c)/d) + (-I*d*m - I*d)*e^(-(d*m*log(4*I*b/d) - 4*I
*b*c + 4*I*a*d)/d)*gamma(m + 1, -4*(-I*b*d*x - I*b*c)/d) + 8*(b*d*x + b*c)
*(d*x + c)^m)/(b*d*m + b*d)
```

Sympy [F]

$$\int (c+dx)^m \cos^2(a+bx)\sin^2(a+bx)\,dx = \int (c+dx)^m \sin^2(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*sin(a + b*x)**2*cos(a + b*x)**2, x)
```

Maxima [F]

$$\int (c+dx)^m \cos^2(a+bx)\sin^2(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/8*((d*m + d)*integrate((d*x + c)^m*cos(4*b*x + 4*a), x) - e^(m*log(d*x
+ c) + log(d*x + c)))/(d*m + d)
```

Giac [F]

$$\int (c+dx)^m \cos^2(a+bx) \sin^2(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos^2(a+bx) \sin^2(a+bx)\,dx = \int \cos(a+b\,x)^2 \sin(a+b\,x)^2 (c+d\,x)^m\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \cos^2(a+bx) \sin^2(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)^2\,dx$$

input
```
int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

output
```
int((c + d*x)**m*cos(a + b*x)**2*sin(a + b*x)**2,x)
```

3.80 $\int (c+dx)^4 \cos^2(a+bx) \sin^2(a+bx)\, dx$

Optimal result . . . 793
Mathematica [A] (verified) . . . 794
Rubi [A] (verified) . . . 794
Maple [A] (verified) . . . 795
Fricas [B] (verification not implemented) . . . 796
Sympy [B] (verification not implemented) . . . 796
Maxima [B] (verification not implemented) . . . 797
Giac [A] (verification not implemented) . . . 798
Mupad [B] (verification not implemented) . . . 799
Reduce [B] (verification not implemented) . . . 799

Optimal result

Integrand size = 24, antiderivative size = 131

$$\int (c+dx)^4 \cos^2(a+bx) \sin^2(a+bx)\, dx = \frac{(c+dx)^5}{40d} + \frac{3d^3(c+dx)\cos(4a+4bx)}{256b^4} - \frac{d(c+dx)^3\cos(4a+4bx)}{32b^2} - \frac{3d^4\sin(4a+4bx)}{1024b^5} + \frac{3d^2(c+dx)^2\sin(4a+4bx)}{128b^3} - \frac{(c+dx)^4\sin(4a+4bx)}{32b}$$

output

```
1/40*(d*x+c)^5/d+3/256*d^3*(d*x+c)*cos(4*b*x+4*a)/b^4-1/32*d*(d*x+c)^3*cos
(4*b*x+4*a)/b^2-3/1024*d^4*sin(4*b*x+4*a)/b^5+3/128*d^2*(d*x+c)^2*sin(4*b*
x+4*a)/b^3-1/32*(d*x+c)^4*sin(4*b*x+4*a)/b
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.01

$$\int (c+dx)^4 \cos^2(a+bx) \sin^2(a+bx)\, dx$$
$$= \frac{128b^5x(5c^4+10c^3dx+10c^2d^2x^2+5cd^3x^3+d^4x^4)+20bd(c+dx)\left(3d^2-8b^2(c+dx)^2\right)\cos(4(a+bx))-}{5120b^5}$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(128*b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4) +
 20*b*d*(c + d*x)*(3*d^2 - 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] - 5*(3*d^4
- 24*b^2*d^2*(c + d*x)^2 + 32*b^4*(c + d*x)^4)*Sin[4*(a + b*x)])/(5120*b^5
)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sin^2(a+bx) \cos^2(a+bx)\, dx$$
$$\downarrow 4906$$
$$\int \left(\frac{1}{8}(c+dx)^4 - \frac{1}{8}(c+dx)^4 \cos(4a+4bx)\right) dx$$
$$\downarrow 2009$$
$$-\frac{3d^4 \sin(4a+4bx)}{1024b^5} + \frac{3d^3(c+dx)\cos(4a+4bx)}{256b^4} + \frac{3d^2(c+dx)^2 \sin(4a+4bx)}{128b^3} - \frac{d(c+dx)^3 \cos(4a+4bx)}{32b^2} - \frac{(c+dx)^4 \sin(4a+4bx)}{32b} + \frac{(c+dx)^5}{40d}$$

input
```
Int[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^5/(40*d) + (3*d^3*(c + d*x)*Cos[4*a + 4*b*x])/(256*b^4) - (d*(c
+ d*x)^3*Cos[4*a + 4*b*x])/(32*b^2) - (3*d^4*Sin[4*a + 4*b*x])/(1024*b^5)
+ (3*d^2*(c + d*x)^2*Sin[4*a + 4*b*x])/(128*b^3) - ((c + d*x)^4*Sin[4*a +
4*b*x])/(32*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

method	result
parallelrisch	$\frac{\left(-32(dx+c)^4b^4+24d^2(dx+c)^2b^2-3d^4\right)\sin(4bx+4a)+128\left(-\frac{d\left((dx+c)^2b^2-\frac{3d^2}{8}\right)(dx+c)\cos(4bx+4a)}{4}+x\left(\frac{1}{5}d^4x^4+c\,d^3x\right.\right.}{1024b^5}$
risch	$\frac{d^4x^5}{40}+\frac{d^3cx^4}{8}+\frac{d^2c^2x^3}{4}+\frac{dc^3x^2}{4}+\frac{c^4x}{8}+\frac{c^5}{40d}-\frac{d\left(8b^2d^3x^3+24b^2cd^2x^2+24b^2c^2dx+8b^2c^3-3d^3x-3cd^2\right)c}{256b^4}$
orering	$\frac{\left(128b^6d^6x^7+896b^6cd^5x^6+2688b^6c^2d^4x^5+4480b^6c^3d^3x^4+4480b^6c^4d^2x^3+320b^4d^6x^5+2560b^6c^5dx^2+1600b^4cd^5x^4+\right.}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/1024*((-32*(d*x+c)^4*b^4+24*d^2*(d*x+c)^2*b^2-3*d^4)*sin(4*b*x+4*a)+128*
(-1/4*d*((d*x+c)^2*b^2-3/8*d^2)*(d*x+c)*cos(4*b*x+4*a)+x*(1/5*d^4*x^4+c*d^
3*x^3+2*c^2*d^2*x^2+2*c^3*d*x+c^4)*b^4+1/4*b^2*c^3*d-3/32*d^3*c)*b)/b^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(119) = 238$.

Time $= 0.08$ (sec) , antiderivative size $= 466$, normalized size of antiderivative $= 3.56$

$$\int (c+dx)^4 \cos^2(a+bx) \sin^2(a+bx)\, dx$$
$$= \frac{32\, b^5 d^4 x^5 + 160\, b^5 c d^3 x^4 - 40\left(8\, b^3 d^4 x^3 + 24\, b^3 c d^3 x^2 + 8\, b^3 c^3 d - 3\, b c d^3 + 3\left(8\, b^3 c^2 d^2 - b d^4\right) x\right) \cos\left(b x + a\right.$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/1280*(32*b^5*d^4*x^5 + 160*b^5*c*d^3*x^4 - 40*(8*b^3*d^4*x^3 + 24*b^3*c*
d^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(8*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x +
 a)^4 + 40*(8*b^5*c^2*d^2 - b^3*d^4)*x^3 + 40*(8*b^5*c^3*d - 3*b^3*c*d^3)*
x^2 + 40*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(
8*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^2 + 5*(32*b^5*c^4 - 24*b^3*c^2*d^2
+ 3*b*d^4)*x - 5*(2*(32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 32*b^4*c^4 - 24*
b^2*c^2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*x^2 + 16*(8*b^4*c^3*d -
 3*b^2*c*d^3)*x)*cos(b*x + a)^3 - (32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 32
*b^4*c^4 - 24*b^2*c^2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*x^2 + 16*
(8*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a))*sin(b*x + a))/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1231 vs. $2(126) = 252$.

Time $= 0.93$ (sec) , antiderivative size $= 1231$, normalized size of antiderivative $= 9.40$

$$\int (c+dx)^4 \cos^2(a+bx) \sin^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**4*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output

```
Piecewise((c**4*x*sin(a + b*x)**4/8 + c**4*x*sin(a + b*x)**2*cos(a + b*x)*
*2/4 + c**4*x*cos(a + b*x)**4/8 + c**3*d*x**2*sin(a + b*x)**4/4 + c**3*d*x
**2*sin(a + b*x)**2*cos(a + b*x)**2/2 + c**3*d*x**2*cos(a + b*x)**4/4 + c*
*2*d**2*x**3*sin(a + b*x)**4/4 + c**2*d**2*x**3*sin(a + b*x)**2*cos(a + b*
x)**2/2 + c**2*d**2*x**3*cos(a + b*x)**4/4 + c*d**3*x**4*sin(a + b*x)**4/8
 + c*d**3*x**4*sin(a + b*x)**2*cos(a + b*x)**2/4 + c*d**3*x**4*cos(a + b*x
)**4/8 + d**4*x**5*sin(a + b*x)**4/40 + d**4*x**5*sin(a + b*x)**2*cos(a +
b*x)**2/20 + d**4*x**5*cos(a + b*x)**4/40 + c**4*sin(a + b*x)**3*cos(a + b
*x)/(8*b) - c**4*sin(a + b*x)*cos(a + b*x)**3/(8*b) + c**3*d*x*sin(a + b*x
)**3*cos(a + b*x)/(2*b) - c**3*d*x*sin(a + b*x)*cos(a + b*x)**3/(2*b) + 3*
c**2*d**2*x**2*sin(a + b*x)**3*cos(a + b*x)/(4*b) - 3*c**2*d**2*x**2*sin(a
 + b*x)*cos(a + b*x)**3/(4*b) + c*d**3*x**3*sin(a + b*x)**3*cos(a + b*x)/(
2*b) - c*d**3*x**3*sin(a + b*x)*cos(a + b*x)**3/(2*b) + d**4*x**4*sin(a +
b*x)**3*cos(a + b*x)/(8*b) - d**4*x**4*sin(a + b*x)*cos(a + b*x)**3/(8*b)
- c**3*d*sin(a + b*x)**4/(8*b**2) - c**3*d*cos(a + b*x)**4/(8*b**2) - 3*c*
*2*d**2*x*sin(a + b*x)**4/(32*b**2) + 9*c**2*d**2*x*sin(a + b*x)**2*cos(a
+ b*x)**2/(16*b**2) - 3*c**2*d**2*x*cos(a + b*x)**4/(32*b**2) - 3*c*d**3*x
**2*sin(a + b*x)**4/(32*b**2) + 9*c*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)
**2/(16*b**2) - 3*c*d**3*x**2*cos(a + b*x)**4/(32*b**2) - d**4*x**3*sin(a
+ b*x)**4/(32*b**2) + 3*d**4*x**3*sin(a + b*x)**2*cos(a + b*x)**2/(16*b...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 735 vs. $2(119) = 238$.

Time = 0.07 (sec) , antiderivative size = 735, normalized size of antiderivative = 5.61

$$\int (c+dx)^4 \cos^2(a+bx) \sin^2(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output

```
1/5120*(160*(4*b*x + 4*a - sin(4*b*x + 4*a))*c^4 - 640*(4*b*x + 4*a - sin(
4*b*x + 4*a))*a*c^3*d/b + 960*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^2*c^2*d^2
/b^2 - 640*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^3*c*d^3/b^3 + 160*(4*b*x + 4
*a - sin(4*b*x + 4*a))*a^4*d^4/b^4 + 160*(8*(b*x + a)^2 - 4*(b*x + a)*sin(
4*b*x + 4*a) - cos(4*b*x + 4*a))*c^3*d/b - 480*(8*(b*x + a)^2 - 4*(b*x + a
)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a*c^2*d^2/b^2 + 480*(8*(b*x + a)^2
 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a^2*c*d^3/b^3 - 160*(8*
(b*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a^3*d^4/b^4
 + 40*(32*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 -
 1)*sin(4*b*x + 4*a))*c^2*d^2/b^2 - 80*(32*(b*x + a)^3 - 12*(b*x + a)*cos(
4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a))*a*c*d^3/b^3 + 40*(3
2*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*sin(
4*b*x + 4*a))*a^2*d^4/b^4 + 20*(32*(b*x + a)^4 - 3*(8*(b*x + a)^2 - 1)*cos
(4*b*x + 4*a) - 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*b*x + 4*a))*c*d^3/b^
3 - 20*(32*(b*x + a)^4 - 3*(8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 4*(8*(b*
x + a)^3 - 3*b*x - 3*a)*sin(4*b*x + 4*a))*a*d^4/b^4 + (128*(b*x + a)^5 - 2
0*(8*(b*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) - 5*(32*(b*x + a)^4 - 24*
(b*x + a)^2 + 3)*sin(4*b*x + 4*a))*d^4/b^4)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.71

$$\int (c+dx)^4 \cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{1}{40}d^4x^5 + \frac{1}{8}cd^3x^4 + \frac{1}{4}c^2d^2x^3 + \frac{1}{4}c^3dx^2 + \frac{1}{8}c^4x$$
$$- \frac{(8\,b^3d^4x^3 + 24\,b^3cd^3x^2 + 24\,b^3c^2d^2x + 8\,b^3c^3d - 3\,bd^4x - 3\,bcd^3)\cos(4\,bx + 4\,a)}{256\,b^5}$$
$$- \frac{(32\,b^4d^4x^4 + 128\,b^4cd^3x^3 + 192\,b^4c^2d^2x^2 + 128\,b^4c^3dx + 32\,b^4c^4 - 24\,b^2d^4x^2 - 48\,b^2cd^3x - 24\,b^2c^2d^2 +}{1024\,b^5}$$

input

```
integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output

```
1/40*d^4*x^5 + 1/8*c*d^3*x^4 + 1/4*c^2*d^2*x^3 + 1/4*c^3*d*x^2 + 1/8*c^4*x
 - 1/256*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 24*b^3*c^2*d^2*x + 8*b^3*c^3*
d - 3*b*d^4*x - 3*b*c*d^3)*cos(4*b*x + 4*a)/b^5 - 1/1024*(32*b^4*d^4*x^4 +
 128*b^4*c*d^3*x^3 + 192*b^4*c^2*d^2*x^2 + 128*b^4*c^3*d*x + 32*b^4*c^4 -
24*b^2*d^4*x^2 - 48*b^2*c*d^3*x - 24*b^2*c^2*d^2 + 3*d^4)*sin(4*b*x + 4*a)
/b^5
```

Mupad [B] (verification not implemented)

Time = 18.71 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.66

$$\int (c+dx)^4 \cos^2(a+bx) \sin^2(a+bx)\, dx =$$

$$-\frac{15\,d^4\,\sin(4\,a+4\,b\,x) - 640\,b^5\,c^4\,x + 160\,b^4\,c^4\,\sin(4\,a+4\,b\,x) - 128\,b^5\,d^4\,x^5 + 160\,b^3\,c^3\,d\,\cos(4\,a+4\,b$$

input

```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^4,x)
```

output

```
-(15*d^4*sin(4*a + 4*b*x) - 640*b^5*c^4*x + 160*b^4*c^4*sin(4*a + 4*b*x) -
 128*b^5*d^4*x^5 + 160*b^3*c^3*d*cos(4*a + 4*b*x) - 1280*b^5*c^3*d*x^2 - 6
40*b^5*c*d^3*x^4 - 120*b^2*c^2*d^2*sin(4*a + 4*b*x) + 160*b^3*d^4*x^3*cos(
4*a + 4*b*x) - 1280*b^5*c^2*d^2*x^3 - 120*b^2*d^4*x^2*sin(4*a + 4*b*x) + 1
60*b^4*d^4*x^4*sin(4*a + 4*b*x) - 60*b*c*d^3*cos(4*a + 4*b*x) - 60*b*d^4*x
*cos(4*a + 4*b*x) + 960*b^4*c^2*d^2*x^2*sin(4*a + 4*b*x) - 240*b^2*c*d^3*x
*sin(4*a + 4*b*x) + 640*b^4*c^3*d*x*sin(4*a + 4*b*x) + 480*b^3*c^2*d^2*x*c
os(4*a + 4*b*x) + 480*b^3*c*d^3*x^2*cos(4*a + 4*b*x) + 640*b^4*c*d^3*x^3*s
in(4*a + 4*b*x))/(5120*b^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 779, normalized size of antiderivative = 5.95

$$\int (c+dx)^4 \cos^2(a+bx) \sin^2(a+bx)\, dx = \text{Too large to display}$$

input

```
int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

output

```
(320*cos(a + b*x)*sin(a + b*x)**3*b**4*c**4 + 1280*cos(a + b*x)*sin(a + b*
x)**3*b**4*c**3*d*x + 1920*cos(a + b*x)*sin(a + b*x)**3*b**4*c**2*d**2*x**
2 + 1280*cos(a + b*x)*sin(a + b*x)**3*b**4*c*d**3*x**3 + 320*cos(a + b*x)*
sin(a + b*x)**3*b**4*d**4*x**4 - 240*cos(a + b*x)*sin(a + b*x)**3*b**2*c**
2*d**2 - 480*cos(a + b*x)*sin(a + b*x)**3*b**2*c*d**3*x - 240*cos(a + b*x)
*sin(a + b*x)**3*b**2*d**4*x**2 + 30*cos(a + b*x)*sin(a + b*x)**3*d**4 - 1
60*cos(a + b*x)*sin(a + b*x)*b**4*c**4 - 640*cos(a + b*x)*sin(a + b*x)*b**
4*c**3*d*x - 960*cos(a + b*x)*sin(a + b*x)*b**4*c**2*d**2*x**2 - 640*cos(a
 + b*x)*sin(a + b*x)*b**4*c*d**3*x**3 - 160*cos(a + b*x)*sin(a + b*x)*b**4
*d**4*x**4 + 120*cos(a + b*x)*sin(a + b*x)*b**2*c**2*d**2 + 240*cos(a + b*
x)*sin(a + b*x)*b**2*c*d**3*x + 120*cos(a + b*x)*sin(a + b*x)*b**2*d**4*x*
*2 - 15*cos(a + b*x)*sin(a + b*x)*d**4 - 320*sin(a + b*x)**4*b**3*c**3*d -
 960*sin(a + b*x)**4*b**3*c**2*d**2*x - 960*sin(a + b*x)**4*b**3*c*d**3*x*
*2 - 320*sin(a + b*x)**4*b**3*d**4*x**3 + 120*sin(a + b*x)**4*b*c*d**3 + 1
20*sin(a + b*x)**4*b*d**4*x + 320*sin(a + b*x)**2*b**3*c**3*d + 960*sin(a
+ b*x)**2*b**3*c**2*d**2*x + 960*sin(a + b*x)**2*b**3*c*d**3*x**2 + 320*si
n(a + b*x)**2*b**3*d**4*x**3 - 120*sin(a + b*x)**2*b*c*d**3 - 120*sin(a +
b*x)**2*b*d**4*x + 160*a*b**4*c**4 + 840*a*b**2*c**2*d**2 - 105*a*d**4 + 1
60*b**5*c**4*x + 320*b**5*c**3*d*x**2 + 320*b**5*c**2*d**2*x**3 + 160*b**5
*c*d**3*x**4 + 32*b**5*d**4*x**5 - 320*b**3*c**3*d - 120*b**3*c**2*d**2...
```

3.81 $\int (c+dx)^3 \cos^2(a+bx) \sin^2(a+bx)\, dx$

Optimal result 801
Mathematica [A] (verified) 802
Rubi [A] (verified) 802
Maple [A] (verified) 803
Fricas [B] (verification not implemented) 804
Sympy [B] (verification not implemented) 804
Maxima [B] (verification not implemented) 805
Giac [A] (verification not implemented) 806
Mupad [B] (verification not implemented) 807
Reduce [B] (verification not implemented) 808

Optimal result

Integrand size = 24, antiderivative size = 105

$$\int (c+dx)^3 \cos^2(a+bx) \sin^2(a+bx)\, dx = \frac{(c+dx)^4}{32d} + \frac{3d^3 \cos(4a+4bx)}{1024b^4} - \frac{3d(c+dx)^2 \cos(4a+4bx)}{128b^2} + \frac{3d^2(c+dx)\sin(4a+4bx)}{256b^3} - \frac{(c+dx)^3 \sin(4a+4bx)}{32b}$$

output

```
1/32*(d*x+c)^4/d+3/1024*d^3*cos(4*b*x+4*a)/b^4-3/128*d*(d*x+c)^2*cos(4*b*x
+4*a)/b^2+3/256*d^2*(d*x+c)*sin(4*b*x+4*a)/b^3-1/32*(d*x+c)^3*sin(4*b*x+4*
a)/b
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.01

$$\int (c+dx)^3 \cos^2(a+bx) \sin^2(a+bx)\, dx$$
$$= \frac{32b^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3) - 3d(-d^2+8b^2(c+dx)^2)\cos(4(a+bx)) - 4b(c+dx)\left(-3d^2+8b^2\right.}{1024b^4}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(32*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 3*d*(-d^2 + 8*b^2*
(c + d*x)^2)*Cos[4*(a + b*x)] - 4*b*(c + d*x)*(-3*d^2 + 8*b^2*(c + d*x)^2)
*Sin[4*(a + b*x)])/(1024*b^4)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin^2(a+bx) \cos^2(a+bx)\, dx$$
$$\downarrow 4906$$
$$\int \left(\frac{1}{8}(c+dx)^3 - \frac{1}{8}(c+dx)^3 \cos(4a+4bx)\right) dx$$
$$\downarrow 2009$$
$$\frac{3d^3\cos(4a+4bx)}{1024b^4} + \frac{3d^2(c+dx)\sin(4a+4bx)}{256b^3} - \frac{3d(c+dx)^2\cos(4a+4bx)}{128b^2} - \frac{(c+dx)^3\sin(4a+4bx)}{32b} + \frac{(c+dx)^4}{32d}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^4/(32*d) + (3*d^3*Cos[4*a + 4*b*x])/(1024*b^4) - (3*d*(c + d*x)^
2*Cos[4*a + 4*b*x])/(128*b^2) + (3*d^2*(c + d*x)*Sin[4*a + 4*b*x])/(256*b^
3) - ((c + d*x)^3*Sin[4*a + 4*b*x])/(32*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.15

method	result
parallelrisch	$\frac{-32\left((dx+c)^2b^2-\frac{3d^2}{8}\right)(dx+c)b\sin(4bx+4a)-24d\left((dx+c)^2b^2-\frac{d^2}{8}\right)\cos(4bx+4a)+32\left(d^3x^4+4d^2cx^3+6dc^2x^2+4c^3x\right)$
risch	$\frac{d^3x^4}{32}+\frac{d^2cx^3}{8}+\frac{3dc^2x^2}{16}+\frac{c^3x}{8}+\frac{c^4}{32d}-\frac{3d\left(8x^2d^2b^2+16b^2cdx+8b^2c^2-d^2\right)\cos(4bx+4a)}{1024b^4}-\frac{(8b^2d^3x^3+24b^2$
orering	$\frac{\left(16b^4d^5x^6+96b^4cd^4x^5+240b^4c^2d^3x^4+320b^4c^3d^2x^3+224b^4c^4dx^2+24b^2d^5x^4+64b^4c^5x+96b^2cd^4x^3+156b^2c^2d^3x^2+1\right.}{64b^4(dx+c)^2}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/1024*(-32*((d*x+c)^2*b^2-3/8*d^2)*(d*x+c)*b*sin(4*b*x+4*a)-24*d*((d*x+c)
^2*b^2-1/8*d^2)*cos(4*b*x+4*a)+32*(d^3*x^4+4*c*d^2*x^3+6*c^2*d*x^2+4*c^3*x
)*b^4+24*b^2*c^2*d-3*d^3)/b^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(95) = 190$.

Time $= 0.08$ (sec) , antiderivative size $= 308$, normalized size of antiderivative $= 2.93$

$$\int (c+dx)^3 \cos^2(a+bx) \sin^2(a+bx)\, dx$$
$$= \frac{4\,b^4d^3x^4 + 16\,b^4cd^2x^3 - 3\,(8\,b^2d^3x^2 + 16\,b^2cd^2x + 8\,b^2c^2d - d^3)\cos(bx+a)^4 + 3\,(8\,b^4c^2d - b^2d^3)x^2 + 3\,(}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/128*(4*b^4*d^3*x^4 + 16*b^4*c*d^2*x^3 - 3*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*
x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^4 + 3*(8*b^4*c^2*d - b^2*d^3)*x^2 + 3*
(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 2*(8
*b^4*c^3 - 3*b^2*c*d^2)*x - 2*(2*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 8*b^3
*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^3 - (8*b^3*d^3*
x^3 + 24*b^3*c*d^2*x^2 + 8*b^3*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x
)*cos(b*x + a))*sin(b*x + a))/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(100) = 200$.

Time $= 0.66$ (sec) , antiderivative size $= 835$, normalized size of antiderivative $= 7.95$

$$\int (c+dx)^3 \cos^2(a+bx) \sin^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output

```
Piecewise((c**3*x*sin(a + b*x)**4/8 + c**3*x*sin(a + b*x)**2*cos(a + b*x)*
*2/4 + c**3*x*cos(a + b*x)**4/8 + 3*c**2*d*x**2*sin(a + b*x)**4/16 + 3*c**
2*d*x**2*sin(a + b*x)**2*cos(a + b*x)**2/8 + 3*c**2*d*x**2*cos(a + b*x)**4
/16 + c*d**2*x**3*sin(a + b*x)**4/8 + c*d**2*x**3*sin(a + b*x)**2*cos(a +
b*x)**2/4 + c*d**2*x**3*cos(a + b*x)**4/8 + d**3*x**4*sin(a + b*x)**4/32 +
 d**3*x**4*sin(a + b*x)**2*cos(a + b*x)**2/16 + d**3*x**4*cos(a + b*x)**4/
32 + c**3*sin(a + b*x)**3*cos(a + b*x)/(8*b) - c**3*sin(a + b*x)*cos(a + b
*x)**3/(8*b) + 3*c**2*d*x*sin(a + b*x)**3*cos(a + b*x)/(8*b) - 3*c**2*d*x*
sin(a + b*x)*cos(a + b*x)**3/(8*b) + 3*c*d**2*x**2*sin(a + b*x)**3*cos(a +
 b*x)/(8*b) - 3*c*d**2*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b) + d**3*x**3
*sin(a + b*x)**3*cos(a + b*x)/(8*b) - d**3*x**3*sin(a + b*x)*cos(a + b*x)*
*3/(8*b) - 3*c**2*d*sin(a + b*x)**4/(32*b**2) - 3*c**2*d*cos(a + b*x)**4/(
32*b**2) - 3*c*d**2*x*sin(a + b*x)**4/(64*b**2) + 9*c*d**2*x*sin(a + b*x)*
*2*cos(a + b*x)**2/(32*b**2) - 3*c*d**2*x*cos(a + b*x)**4/(64*b**2) - 3*d*
*3*x**2*sin(a + b*x)**4/(128*b**2) + 9*d**3*x**2*sin(a + b*x)**2*cos(a + b
*x)**2/(64*b**2) - 3*d**3*x**2*cos(a + b*x)**4/(128*b**2) - 3*c*d**2*sin(a
 + b*x)**3*cos(a + b*x)/(64*b**3) + 3*c*d**2*sin(a + b*x)*cos(a + b*x)**3/
(64*b**3) - 3*d**3*x*sin(a + b*x)**3*cos(a + b*x)/(64*b**3) + 3*d**3*x*sin
(a + b*x)*cos(a + b*x)**3/(64*b**3) + 3*d**3*sin(a + b*x)**4/(256*b**4) +
3*d**3*cos(a + b*x)**4/(256*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(95) = 190$.

Time $= 0.05$ (sec) , antiderivative size $= 442$, normalized size of antiderivative $= 4.21$

$$\int (c+dx)^3 \cos^2(a+bx) \sin^2(a+bx)\, dx$$

$$= \frac{32\,(4\,bx+4\,a-\sin(4\,bx+4\,a))c^3 - \frac{96\,(4\,bx+4\,a-\sin(4\,bx+4\,a))ac^2d}{b} + \frac{96\,(4\,bx+4\,a-\sin(4\,bx+4\,a))a^2cd^2}{b^2} - \frac{32\,(4\,bx+4\,a-s}{}$$

input

```
integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/1024*(32*(4*b*x + 4*a - sin(4*b*x + 4*a))*c^3 - 96*(4*b*x + 4*a - sin(4*
b*x + 4*a))*a*c^2*d/b + 96*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^2*c*d^2/b^2
 - 32*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^3*d^3/b^3 + 24*(8*(b*x + a)^2 - 4*
(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*c^2*d/b - 48*(8*(b*x + a)^2
 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a*c*d^2/b^2 + 24*(8*(b
*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a^2*d^3/b^3 +
 4*(32*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)
*sin(4*b*x + 4*a))*c*d^2/b^2 - 4*(32*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x
 + 4*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a))*a*d^3/b^3 + (32*(b*x + a)
^4 - 3*(8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 4*(8*(b*x + a)^3 - 3*b*x - 3
*a)*sin(4*b*x + 4*a))*d^3/b^3)/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.46

$$\begin{aligned}
&\int (c+dx)^3 \cos^2(a+bx) \sin^2(a+bx)\, dx \\
&\quad = \frac{1}{32} d^3 x^4 + \frac{1}{8} c d^2 x^3 + \frac{3}{16} c^2 d x^2 + \frac{1}{8} c^3 x \\
&\qquad - \frac{3\left(8\, b^2 d^3 x^2 + 16\, b^2 c d^2 x + 8\, b^2 c^2 d - d^3\right) \cos\left(4\, bx + 4\, a\right)}{1024\, b^4} \\
&\qquad - \frac{\left(8\, b^3 d^3 x^3 + 24\, b^3 c d^2 x^2 + 24\, b^3 c^2 d x + 8\, b^3 c^3 - 3\, b d^3 x - 3\, b c d^2\right) \sin\left(4\, bx + 4\, a\right)}{256\, b^4}
\end{aligned}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/32*d^3*x^4 + 1/8*c*d^2*x^3 + 3/16*c^2*d*x^2 + 1/8*c^3*x - 3/1024*(8*b^2*
d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(4*b*x + 4*a)/b^4 - 1/256
*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 3*b*d^3*
x - 3*b*c*d^2)*sin(4*b*x + 4*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.13

$$\begin{aligned}\int (c+dx)^3\cos^2(a+bx)\sin^2(a+bx)\,dx &= x^2\left(\frac{3c^2d}{64}+\frac{9d^3}{512b^2}\right)+x^2\left(\frac{9c^2d}{64}-\frac{9d^3}{512b^2}\right)\\ &+x\left(\frac{c^3}{32}+\frac{9cd^2}{256b^2}\right)+x\left(\frac{3c^3}{32}-\frac{9cd^2}{256b^2}\right)\\ &+\frac{d^3x^4}{32}-\frac{x\cos(4a+4bx)\left(\frac{c^3}{4}+\frac{9cd^2}{32b^2}\right)}{8}\\ &+\frac{x\cos(4a+4bx)\left(\frac{c^3}{8}-\frac{3cd^2}{64b^2}\right)}{4}+\frac{cd^2x^3}{8}\\ &+\frac{\cos(4a+4bx)\left(\frac{3d^3}{128}-\frac{3b^2c^2d}{16}\right)}{8b^4}\\ &+\frac{\sin(4a+4bx)\left(3cd^2-8b^2c^3\right)}{256b^3}\\ &-\frac{x^2\cos(4a+4bx)\left(\frac{3c^2d}{8}+\frac{9d^3}{64b^2}\right)}{8}\\ &+\frac{x^2\cos(4a+4bx)\left(\frac{3c^2d}{16}-\frac{3d^3}{128b^2}\right)}{4}\\ &-\frac{d^3x^3\sin(4a+4bx)}{32b}\\ &+\frac{3x\sin(4a+4bx)\left(d^3-8b^2c^2d\right)}{256b^3}\\ &-\frac{3cd^2x^2\sin(4a+4bx)}{32b}\end{aligned}$$

input

```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^3,x)
```

output

```
x^2*((3*c^2*d)/64 + (9*d^3)/(512*b^2)) + x^2*((9*c^2*d)/64 - (9*d^3)/(512*
b^2)) + x*(c^3/32 + (9*c*d^2)/(256*b^2)) + x*((3*c^3)/32 - (9*c*d^2)/(256*
b^2)) + (d^3*x^4)/32 - (x*cos(4*a + 4*b*x)*(c^3/4 + (9*c*d^2)/(32*b^2)))/8
 + (x*cos(4*a + 4*b*x)*(c^3/8 - (3*c*d^2)/(64*b^2)))/4 + (c*d^2*x^3)/8 + (
cos(4*a + 4*b*x)*((3*d^3)/128 - (3*b^2*c^2*d)/16))/(8*b^4) + (sin(4*a + 4*
b*x)*(3*c*d^2 - 8*b^2*c^3))/(256*b^3) - (x^2*cos(4*a + 4*b*x)*((3*c^2*d)/8
 + (9*d^3)/(64*b^2)))/8 + (x^2*cos(4*a + 4*b*x)*((3*c^2*d)/16 - (3*d^3)/(1
28*b^2)))/4 - (d^3*x^3*sin(4*a + 4*b*x))/(32*b) + (3*x*sin(4*a + 4*b*x)*(d
^3 - 8*b^2*c^2*d))/(256*b^3) - (3*c*d^2*x^2*sin(4*a + 4*b*x))/(32*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 502, normalized size of antiderivative = 4.78

$$\int (c+dx)^3 \cos^2(a+bx) \sin^2(a+bx)\, dx$$

$$= \frac{16a\,b^3c^3 + 16b^4c^3x + 4b^4d^3x^4 - 3b^2d^3x^2 + 96\cos(bx+a)\sin(bx+a)^3\,b^3c^2dx + 96\cos(bx+a)\sin(bx+}$$

input

```
int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

output

```
(32*cos(a + b*x)*sin(a + b*x)**3*b**3*c**3 + 96*cos(a + b*x)*sin(a + b*x)*
*3*b**3*c**2*d*x + 96*cos(a + b*x)*sin(a + b*x)**3*b**3*c*d**2*x**2 + 32*c
os(a + b*x)*sin(a + b*x)**3*b**3*d**3*x**3 - 12*cos(a + b*x)*sin(a + b*x)*
*3*b*c*d**2 - 12*cos(a + b*x)*sin(a + b*x)**3*b*d**3*x - 16*cos(a + b*x)*s
in(a + b*x)*b**3*c**3 - 48*cos(a + b*x)*sin(a + b*x)*b**3*c**2*d*x - 48*co
s(a + b*x)*sin(a + b*x)*b**3*c*d**2*x**2 - 16*cos(a + b*x)*sin(a + b*x)*b*
*3*d**3*x**3 + 6*cos(a + b*x)*sin(a + b*x)*b*c*d**2 + 6*cos(a + b*x)*sin(a
 + b*x)*b*d**3*x - 24*sin(a + b*x)**4*b**2*c**2*d - 48*sin(a + b*x)**4*b**
2*c*d**2*x - 24*sin(a + b*x)**4*b**2*d**3*x**2 + 3*sin(a + b*x)**4*d**3 +
24*sin(a + b*x)**2*b**2*c**2*d + 48*sin(a + b*x)**2*b**2*c*d**2*x + 24*sin
(a + b*x)**2*b**2*d**3*x**2 - 3*sin(a + b*x)**2*d**3 + 16*a*b**3*c**3 + 42
*a*b*c*d**2 + 16*b**4*c**3*x + 24*b**4*c**2*d*x**2 + 16*b**4*c*d**2*x**3 +
 4*b**4*d**3*x**4 - 24*b**2*c**2*d - 6*b**2*c*d**2*x - 3*b**2*d**3*x**2 +
3*d**3)/(128*b**4)
```

3.82 $\int (c+dx)^2 \cos^2(a+bx) \sin^2(a+bx)\, dx$

Optimal result . . . 809
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Rubi [A] (verified) . . . 810
Maple [A] (verified) . . . 811
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Mupad [B] (verification not implemented) . . . 814
Reduce [B] (verification not implemented) . . . 815

Optimal result

Integrand size = 24, antiderivative size = 79

$$\int (c+dx)^2 \cos^2(a+bx) \sin^2(a+bx)\, dx = \frac{(c+dx)^3}{24d} - \frac{d(c+dx)\cos(4a+4bx)}{64b^2} + \frac{d^2 \sin(4a+4bx)}{256b^3} - \frac{(c+dx)^2 \sin(4a+4bx)}{32b}$$

output

```
1/24*(d*x+c)^3/d-1/64*d*(d*x+c)*cos(4*b*x+4*a)/b^2+1/256*d^2*sin(4*b*x+4*a
)/b^3-1/32*(d*x+c)^2*sin(4*b*x+4*a)/b
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int (c+dx)^2 \cos^2(a+bx) \sin^2(a+bx)\, dx = \frac{32b^3x(3c^2+3cdx+d^2x^2) - 12bd(c+dx)\cos(4(a+bx)) - 3(-d^2+8b^2(c+dx)^2)\sin(4(a+bx))}{768b^3}$$

input

```
Integrate[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(32*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) - 12*b*d*(c + d*x)*Cos[4*(a + b*x)]
- 3*(-d^2 + 8*b^2*(c + d*x)^2)*Sin[4*(a + b*x)])/(768*b^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin^2(a+bx)\cos^2(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8}(c+dx)^2 - \frac{1}{8}(c+dx)^2\cos(4a+4bx)\right)dx$$

$$\downarrow 2009$$

$$\frac{d^2\sin(4a+4bx)}{256b^3} - \frac{d(c+dx)\cos(4a+4bx)}{64b^2} - \frac{(c+dx)^2\sin(4a+4bx)}{32b} + \frac{(c+dx)^3}{24d}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^3/(24*d) - (d*(c + d*x)*Cos[4*a + 4*b*x])/(64*b^2) + (d^2*Sin[4*
a + 4*b*x])/(256*b^3) - ((c + d*x)^2*Sin[4*a + 4*b*x])/(32*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{\left(-8(dx+c)^2b^2+d^2\right)\sin(4bx+4a)+32\left(-\frac{d(dx+c)\cos(4bx+4a)}{8}+x\left(\frac{1}{3}x^2d^2+cdx+c^2\right)b^2+\frac{cd}{8}\right)b}{256b^3}$
risch	$\frac{d^2x^3}{24}+\frac{cdx^2}{8}+\frac{c^2x}{8}+\frac{c^3}{24d}-\frac{d(dx+c)\cos(4bx+4a)}{64b^2}-\frac{\left(8x^2d^2b^2+16b^2cdx+8b^2c^2-d^2\right)\sin(4bx+4a)}{256b^3}$
orering	$\frac{(64b^4d^4x^5+320b^4cd^3x^4+640b^4c^2d^2x^3+576b^4c^3dx^2+192b^4c^4x+48b^2d^4x^3+168b^2cd^3x^2+192b^2c^2d^2x+48b^2c^3d-12d}{192(dx+c)^2b^4}$
derivativedivides	$\frac{a^2d^2\left(-\frac{\sin(bx+a)\cos(bx+a)^3}{4}+\frac{\sin(bx+a)\cos(bx+a)}{8}+\frac{bx}{8}+\frac{a}{8}\right)}{b^2}-\frac{2acd\left(-\frac{\sin(bx+a)\cos(bx+a)^3}{4}+\frac{\sin(bx+a)\cos(bx+a)}{8}+\frac{bx}{8}+\frac{a}{8}\right)}{b}$
default	$\frac{a^2d^2\left(-\frac{\sin(bx+a)\cos(bx+a)^3}{4}+\frac{\sin(bx+a)\cos(bx+a)}{8}+\frac{bx}{8}+\frac{a}{8}\right)}{b^2}-\frac{2acd\left(-\frac{\sin(bx+a)\cos(bx+a)^3}{4}+\frac{\sin(bx+a)\cos(bx+a)}{8}+\frac{bx}{8}+\frac{a}{8}\right)}{b}$
norman	$\frac{d^2x^3}{24}+\frac{d^2x^3\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{6}+\frac{d^2x^3\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4}{4}+\frac{d^2x^3\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^6}{6}+\frac{d^2x^3\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^8}{24}+\frac{cdx^2}{8}-\frac{\left(8b^2c^2-d^2\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{32b^3}$

input

```
int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
1/256*((-8*(d*x+c)^2*b^2+d^2)*sin(4*b*x+4*a)+32*(-1/8*d*(d*x+c)*cos(4*b*x+
4*a)+x*(1/3*x^2*d^2+c*d*x+c^2)*b^2+1/8*c*d)*b)/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(71) = 142$.

Time $= 0.08$ (sec) , antiderivative size $= 180$, normalized size of antiderivative $= 2.28$

$$\int (c+dx)^2 \cos^2(a+bx) \sin^2(a+bx)\, dx$$
$$= \frac{8\,b^3d^2x^3 + 24\,b^3cdx^2 - 24\,(bd^2x + bcd)\cos(bx+a)^4 + 24\,(bd^2x + bcd)\cos(bx+a)^2 + 3\,(8\,b^3c^2 - bd^2)x -$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/192*(8*b^3*d^2*x^3 + 24*b^3*c*d*x^2 - 24*(b*d^2*x + b*c*d)*cos(b*x + a)^
4 + 24*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 3*(8*b^3*c^2 - b*d^2)*x - 3*(2*(
8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(b*x + a)^3 - (8*b^2*d^
2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(b*x + a))*sin(b*x + a))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(70) = 140$.

Time $= 0.47$ (sec) , antiderivative size $= 493$, normalized size of antiderivative $= 6.24$

$$\int (c+dx)^2 \cos^2(a+bx) \sin^2(a+bx)\, dx$$
$$= \begin{cases} \frac{c^2x\sin^4(a+bx)}{8} + \frac{c^2x\sin^2(a+bx)\cos^2(a+bx)}{4} + \frac{c^2x\cos^4(a+bx)}{8} + \frac{cdx^2\sin^4(a+bx)}{8} + \frac{cdx^2\sin^2(a+bx)\cos^2(a+bx)}{4} + \frac{cdx^2\cos^4(a+bx)}{8} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin^2(a)\cos^2(a) \end{cases}$$

input
```
integrate((d*x+c)**2*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output

```
Piecewise((c**2*x*sin(a + b*x)**4/8 + c**2*x*sin(a + b*x)**2*cos(a + b*x)*
*2/4 + c**2*x*cos(a + b*x)**4/8 + c*d*x**2*sin(a + b*x)**4/8 + c*d*x**2*si
n(a + b*x)**2*cos(a + b*x)**2/4 + c*d*x**2*cos(a + b*x)**4/8 + d**2*x**3*s
in(a + b*x)**4/24 + d**2*x**3*sin(a + b*x)**2*cos(a + b*x)**2/12 + d**2*x*
*3*cos(a + b*x)**4/24 + c**2*sin(a + b*x)**3*cos(a + b*x)/(8*b) - c**2*sin
(a + b*x)*cos(a + b*x)**3/(8*b) + c*d*x*sin(a + b*x)**3*cos(a + b*x)/(4*b)
 - c*d*x*sin(a + b*x)*cos(a + b*x)**3/(4*b) + d**2*x**2*sin(a + b*x)**3*co
s(a + b*x)/(8*b) - d**2*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b) - c*d*sin(
a + b*x)**4/(16*b**2) - c*d*cos(a + b*x)**4/(16*b**2) - d**2*x*sin(a + b*x
)**4/(64*b**2) + 3*d**2*x*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**2) - d**2
*x*cos(a + b*x)**4/(64*b**2) - d**2*sin(a + b*x)**3*cos(a + b*x)/(64*b**3)
 + d**2*sin(a + b*x)*cos(a + b*x)**3/(64*b**3), Ne(b, 0)), ((c**2*x + c*d*
x**2 + d**2*x**3/3)*sin(a)**2*cos(a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(71) = 142$.

Time $= 0.04$ (sec) , antiderivative size $= 232$, normalized size of antiderivative $= 2.94$

$$\int (c+dx)^2 \cos^2(a+bx) \sin^2(a+bx)\, dx$$

$$= \frac{24\,(4\,bx+4\,a-\sin(4\,bx+4\,a))c^2 - \frac{48\,(4\,bx+4\,a-\sin(4\,bx+4\,a))acd}{b} + \frac{24\,(4\,bx+4\,a-\sin(4\,bx+4\,a))a^2d^2}{b^2} + \frac{12\left(8\,(bx+a)^2-4\right.}{}$$

input

```
integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output

```
1/768*(24*(4*b*x + 4*a - sin(4*b*x + 4*a))*c^2 - 48*(4*b*x + 4*a - sin(4*b
*x + 4*a))*a*c*d/b + 24*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^2*d^2/b^2 + 12*
(8*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*c*d/b -
12*(8*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a*d^2
/b^2 + (32*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2
- 1)*sin(4*b*x + 4*a))*d^2/b^2)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19

$$\begin{aligned}&\int (c+dx)^2\cos^2(a+bx)\sin^2(a+bx)\,dx\\&=\frac{1}{24}d^2x^3+\frac{1}{8}cdx^2+\frac{1}{8}c^2x-\frac{(bd^2x+bcd)\cos(4bx+4a)}{64b^3}\\&\quad-\frac{(8b^2d^2x^2+16b^2cdx+8b^2c^2-d^2)\sin(4bx+4a)}{256b^3}\end{aligned}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/24*d^2*x^3 + 1/8*c*d*x^2 + 1/8*c^2*x - 1/64*(b*d^2*x + b*c*d)*cos(4*b*x
+ 4*a)/b^3 - 1/256*(8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*sin(4*
b*x + 4*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 18.69 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.27

$$\begin{aligned}\int (c+dx)^2\cos^2(a+bx)\sin^2(a+bx)\,dx &= x\left(\frac{c^2}{32}+\frac{3d^2}{256b^2}\right)+x\left(\frac{3c^2}{32}-\frac{3d^2}{256b^2}\right)\\&\quad+\frac{d^2x^3}{24}+\frac{\sin(4a+4bx)\,(d^2-8b^2c^2)}{256b^3}\\&\quad-\frac{x\cos(4a+4bx)\left(\frac{c^2}{4}+\frac{3d^2}{32b^2}\right)}{8}\\&\quad+\frac{x\cos(4a+4bx)\left(\frac{c^2}{8}-\frac{d^2}{64b^2}\right)}{4}\\&\quad+\frac{cdx^2}{8}-\frac{d^2x^2\sin(4a+4bx)}{32b}\\&\quad-\frac{cd\cos(4a+4bx)}{64b^2}-\frac{cdx\sin(4a+4bx)}{16b}\end{aligned}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^2,x)
```

output
```
x*(c^2/32 + (3*d^2)/(256*b^2)) + x*((3*c^2)/32 - (3*d^2)/(256*b^2)) + (d^2
*x^3)/24 + (sin(4*a + 4*b*x)*(d^2 - 8*b^2*c^2))/(256*b^3) - (x*cos(4*a + 4
*b*x)*(c^2/4 + (3*d^2)/(32*b^2)))/8 + (x*cos(4*a + 4*b*x)*(c^2/8 - d^2/(64
*b^2)))/4 + (c*d*x^2)/8 - (d^2*x^2*sin(4*a + 4*b*x))/(32*b) - (c*d*cos(4*a
 + 4*b*x))/(64*b^2) - (c*d*x*sin(4*a + 4*b*x))/(16*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 287, normalized size of antiderivative = 3.63

$$\int (c+dx)^2 \cos^2(a+bx) \sin^2(a+bx)\, dx$$
$$= \frac{48\cos(bx+a)\sin(bx+a)^3 b^2c^2 + 96\cos(bx+a)\sin(bx+a)^3 b^2cdx + 48\cos(bx+a)\sin(bx+a)^3 b^2d^2$$

input
```
int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

output
```
(48*cos(a + b*x)*sin(a + b*x)**3*b**2*c**2 + 96*cos(a + b*x)*sin(a + b*x)*
*3*b**2*c*d*x + 48*cos(a + b*x)*sin(a + b*x)**3*b**2*d**2*x**2 - 6*cos(a +
 b*x)*sin(a + b*x)**3*d**2 - 24*cos(a + b*x)*sin(a + b*x)*b**2*c**2 - 48*c
os(a + b*x)*sin(a + b*x)*b**2*c*d*x - 24*cos(a + b*x)*sin(a + b*x)*b**2*d*
*2*x**2 + 3*cos(a + b*x)*sin(a + b*x)*d**2 - 24*sin(a + b*x)**4*b*c*d - 24
*sin(a + b*x)**4*b*d**2*x + 24*sin(a + b*x)**2*b*c*d + 24*sin(a + b*x)**2*
b*d**2*x + 24*a*b**2*c**2 + 21*a*d**2 + 24*b**3*c**2*x + 24*b**3*c*d*x**2
+ 8*b**3*d**2*x**3 - 24*b*c*d - 3*b*d**2*x)/(192*b**3)
```

3.83 $\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal result 816
Mathematica [A] (verified) 816
Rubi [A] (verified) 817
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Optimal result

Integrand size = 22, antiderivative size = 53

$$\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{(c+dx)^2}{16d} - \frac{d\cos(4a+4bx)}{128b^2} - \frac{(c+dx)\sin(4a+4bx)}{32b}$$

output
```
1/16*(d*x+c)^2/d-1/128*d*cos(4*b*x+4*a)/b^2-1/32*(d*x+c)*sin(4*b*x+4*a)/b
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx = -\frac{8(a+bx)(-2bc+ad-bdx)+d\cos(4(a+bx))+4b(c+dx)\sin(4(a+bx))}{128b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
-1/128*(8*(a + b*x)*(-2*b*c + a*d - b*d*x) + d*Cos[4*(a + b*x)] + 4*b*(c +
 d*x)*Sin[4*(a + b*x)])/b^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin^2(a+bx)\cos^2(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8}(c+dx) - \frac{1}{8}(c+dx)\cos(4a+4bx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{d\cos(4a+4bx)}{128b^2} - \frac{(c+dx)\sin(4a+4bx)}{32b} + \frac{(c+dx)^2}{16d}$$

input
```
Int[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^2/(16*d) - (d*Cos[4*a + 4*b*x])/(128*b^2) - ((c + d*x)*Sin[4*a +
 4*b*x])/(32*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

method	result
risch	$\frac{d\,x^2}{16}+\frac{xc}{8}-\frac{d\cos(4bx+4a)}{128b^2}-\frac{(dx+c)\sin(4bx+4a)}{32b}$
derivativedivides	$-\frac{da\left(-\frac{\sin(bx+a)\cos(bx+a)^3}{4}+\frac{\sin(bx+a)\cos(bx+a)}{8}+\frac{bx}{8}+\frac{a}{8}\right)}{b}+c\left(-\frac{\sin(bx+a)\cos(bx+a)^3}{4}+\frac{\sin(bx+a)\cos(bx+a)}{8}+\frac{bx}{8}+\frac{a}{8}\right)+$
default	$-\frac{da\left(-\frac{\sin(bx+a)\cos(bx+a)^3}{4}+\frac{\sin(bx+a)\cos(bx+a)}{8}+\frac{bx}{8}+\frac{a}{8}\right)}{b}+c\left(-\frac{\sin(bx+a)\cos(bx+a)^3}{4}+\frac{\sin(bx+a)\cos(bx+a)}{8}+\frac{bx}{8}+\frac{a}{8}\right)+$
orering	$\frac{\left(8b^2d^3x^4+32b^2cd^2x^3+40b^2c^2dx^2+16b^2c^3x+3d^3x^2+6cd^2x+2c^2d\right)\cos(bx+a)^2\sin(bx+a)^2}{16b^2(dx+c)^2}-\frac{\left(2x^2d^2+4cdx+c^2\right)\left(d\right.}{}$
norman	$\frac{d\,x^2}{16}+\frac{xc}{8}-\frac{c\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{4b}+\frac{7c\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3}{4b}-\frac{7c\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^5}{4b}+\frac{c\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^7}{4b}+\frac{d\,x^2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{4}+\frac{3d\,x^2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4}{8}+\frac{d}{}$

input
```
int((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*d*x^2+1/8*x*c-1/128*d*cos(4*b*x+4*a)/b^2-1/32*(d*x+c)*sin(4*b*x+4*a)/
b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.60

$$\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$=\frac{b^2dx^2-d\cos(bx+a)^4+2\,b^2cx+d\cos(bx+a)^2-2\left(2\,(bdx+bc)\cos(bx+a)^3-(bdx+bc)\cos(bx+\right.}{16\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/16*(b^2*d*x^2 - d*cos(b*x + a)^4 + 2*b^2*c*x + d*cos(b*x + a)^2 - 2*(2*(
b*d*x + b*c)*cos(b*x + a)^3 - (b*d*x + b*c)*cos(b*x + a))*sin(b*x + a))/b^
2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(46) = 92$.

Time = 0.34 (sec) , antiderivative size = 238, normalized size of antiderivative = 4.49

$$\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \begin{cases} \frac{cx\sin^4(a+bx)}{8} + \frac{cx\sin^2(a+bx)\cos^2(a+bx)}{4} + \frac{cx\cos^4(a+bx)}{8} + \frac{dx^2\sin^4(a+bx)}{16} + \frac{dx^2\sin^2(a+bx)\cos^2(a+bx)}{8} + \frac{dx^2\cos^4(a+bx)}{16} \\ \left(cx + \frac{dx^2}{2}\right)\sin^2(a)\cos^2(a) \end{cases}$$

input
```
integrate((d*x+c)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output
```
Piecewise((c*x*sin(a + b*x)**4/8 + c*x*sin(a + b*x)**2*cos(a + b*x)**2/4 +
 c*x*cos(a + b*x)**4/8 + d*x**2*sin(a + b*x)**4/16 + d*x**2*sin(a + b*x)**
2*cos(a + b*x)**2/8 + d*x**2*cos(a + b*x)**4/16 + c*sin(a + b*x)**3*cos(a
+ b*x)/(8*b) - c*sin(a + b*x)*cos(a + b*x)**3/(8*b) + d*x*sin(a + b*x)**3*
cos(a + b*x)/(8*b) - d*x*sin(a + b*x)*cos(a + b*x)**3/(8*b) - d*sin(a + b*
x)**4/(32*b**2) - d*cos(a + b*x)**4/(32*b**2), Ne(b, 0)), ((c*x + d*x**2/2
)*sin(a)**2*cos(a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(47) = 94$.

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.81

$$\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{4\,(4\,bx+4\,a-\sin(4\,bx+4\,a))c - \frac{4\,(4\,bx+4\,a-\sin(4\,bx+4\,a))ad}{b} + \frac{\left(8\,(bx+a)^2-4\,(bx+a)\sin(4\,bx+4\,a)-\cos(4\,bx+4\,a)\right)d}{b}}{128\,b}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/128*(4*(4*b*x + 4*a - sin(4*b*x + 4*a))*c - 4*(4*b*x + 4*a - sin(4*b*x +
 4*a))*a*d/b + (8*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x +
 4*a))*d/b)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{1}{16}\,dx^2 + \frac{1}{8}\,cx - \frac{d\cos(4\,bx+4\,a)}{128\,b^2} - \frac{(bdx+bc)\sin(4\,bx+4\,a)}{32\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/16*d*x^2 + 1/8*c*x - 1/128*d*cos(4*b*x + 4*a)/b^2 - 1/32*(b*d*x + b*c)*s
in(4*b*x + 4*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 18.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{c\,x}{8} + \frac{d\,x^2}{16} - \frac{d\,\cos(4\,a+4\,b\,x)}{128\,b^2} - \frac{c\sin(4\,a+4\,b\,x)}{32\,b} - \frac{d\,x\,\sin(4\,a+4\,b\,x)}{32\,b}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x),x)
```

output
```
(c*x)/8 + (d*x^2)/16 - (d*cos(4*a + 4*b*x))/(128*b^2) - (c*sin(4*a + 4*b*x
))/(32*b) - (d*x*sin(4*a + 4*b*x))/(32*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.26

$$\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{4\cos(bx+a)\sin(bx+a)^3 bc + 4\cos(bx+a)\sin(bx+a)^3 bdx - 2\cos(bx+a)\sin(bx+a)bc - 2\cos(b}{16b^2}$$

input
```
int((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

output
```
(4*cos(a + b*x)*sin(a + b*x)**3*b*c + 4*cos(a + b*x)*sin(a + b*x)**3*b*d*x
 - 2*cos(a + b*x)*sin(a + b*x)*b*c - 2*cos(a + b*x)*sin(a + b*x)*b*d*x - s
in(a + b*x)**4*d + sin(a + b*x)**2*d + 2*a*b*c + 2*b**2*c*x + b**2*d*x**2
- d)/(16*b**2)
```

3.84 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx$

Optimal result . . . 822
Mathematica [A] (verified) . . . 822
Rubi [A] (verified) . . . 823
Maple [C] (verified) . . . 824
Fricas [A] (verification not implemented) . . . 824
Sympy [F] . . . 825
Maxima [C] (verification not implemented) . . . 825
Giac [C] (verification not implemented) . . . 826
Mupad [F(-1)] . . . 827
Reduce [F] . . . 827

Optimal result

Integrand size = 24, antiderivative size = 78

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx = -\frac{\cos\left(4a-\frac{4bc}{d}\right)\operatorname{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d} + \frac{\log(c+dx)}{8d} + \frac{\sin\left(4a-\frac{4bc}{d}\right)\operatorname{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}$$

output

```
-1/8*cos(4*a-4*b*c/d)*Ci(4*b*c/d+4*b*x)/d+1/8*ln(d*x+c)/d+1/8*sin(4*a-4*b*
c/d)*Si(4*b*c/d+4*b*x)/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \frac{-\cos\left(4a-\frac{4bc}{d}\right)\operatorname{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)+\log(c+dx)+\sin\left(4a-\frac{4bc}{d}\right)\operatorname{Si}\left(\frac{4b(c+dx)}{d}\right)}{8d}$$

input

```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x),x]
```

output
```
(-(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d]) + Log[c + d*x] + S
in[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(8*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos^2(a+bx)}{c+dx}\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8(c+dx)} - \frac{\cos(4a+4bx)}{8(c+dx)}\right)dx$$

$$\downarrow 2009$$

$$-\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d} + \frac{\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{8d} + \frac{\log(c+dx)}{8d}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x),x]
```

output
```
-1/8*(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/d + Log[c + d*x
]/(8*d) + (Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(8*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.37

method	result
risch	$\frac{\ln(dx+c)}{8d}+\frac{e^{-\frac{4i(ad-cb)}{d}}\operatorname{expIntegral}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{16d}+\frac{e^{\frac{4i(ad-cb)}{d}}\operatorname{expIntegral}_1\left(-4ibx-4ia-\frac{4(-iad+icb}{d}\right.}{16d}$
derivativedivides	$\frac{\ln(-4ad+4cb+4d(bx+a))}{8d}+\frac{\operatorname{Si}\left(-4bx-4a-\frac{-4ad+4cb}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{8d}-\frac{\operatorname{Ci}\left(4bx+4a+\frac{-4ad+4cb}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right.}{8d}$
default	$\frac{\ln(-4ad+4cb+4d(bx+a))}{8d}+\frac{\operatorname{Si}\left(-4bx-4a-\frac{-4ad+4cb}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{8d}-\frac{\operatorname{Ci}\left(4bx+4a+\frac{-4ad+4cb}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right.}{8d}$

input

```
int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output

```
1/8*ln(d*x+c)/d+1/16/d*exp(-4*I*(a*d-b*c)/d)*Ei(1,4*I*b*x+4*I*a-4*I*(a*d-b
*c)/d)+1/16/d*exp(4*I*(a*d-b*c)/d)*Ei(1,-4*I*b*x-4*I*a-4*(-I*a*d+I*c*b)/d)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx$$
$$= -\frac{\cos\left(-\frac{4\,(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)-\sin\left(-\frac{4\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{4\,(bdx+bc)}{d}\right)-\log\left(dx+c\right)}{8\,d}$$

input

```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
-1/8*(cos(-4*(b*c - a*d)/d)*cos_integral(4*(b*d*x + b*c)/d) - sin(-4*(b*c
- a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) - log(d*x + c))/d
```

Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \int \frac{\sin^2(a+bx)\cos^2(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.08

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx$$
$$= \frac{b\left(E_1\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_1\left(-\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)+b\left(i\,E_1\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right.\right.}{16\,bd}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
1/16*(b*(exp_integral_e(1, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_int
egral_e(1, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a*d)/d) +
 b*(I*exp_integral_e(1, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_inte
gral_e(1, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d)/d) +
2*b*log(b*c + (b*x + a)*d - a*d))/(b*d)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 669, normalized size of antiderivative = 8.58

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
1/16*(2*log(abs(d*x + c))*tan(2*a)^2*tan(2*b*c/d)^2 - real_part(cos_integr
al(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 - real_part(cos_integral(-4
*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(4*b*
x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) - 2*imag_part(cos_integral(-4*b*x -
4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 4*sin_integral(4*(b*d*x + b*c)/d)*tan(
2*a)^2*tan(2*b*c/d) - 2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*
tan(2*b*c/d)^2 + 2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(
2*b*c/d)^2 - 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d)^2 + 2
*log(abs(d*x + c))*tan(2*a)^2 + real_part(cos_integral(4*b*x + 4*b*c/d))*t
an(2*a)^2 + real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2 - 4*real_
part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d) - 4*real_part(co
s_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d) + 2*log(abs(d*x + c))*
tan(2*b*c/d)^2 + real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2 +
 real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d)^2 + 2*imag_part(co
s_integral(4*b*x + 4*b*c/d))*tan(2*a) - 2*imag_part(cos_integral(-4*b*x -
4*b*c/d))*tan(2*a) + 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a) - 2*imag_p
art(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d) + 2*imag_part(cos_integral
(-4*b*x - 4*b*c/d))*tan(2*b*c/d) - 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2
*b*c/d) + 2*log(abs(d*x + c)) - real_part(cos_integral(4*b*x + 4*b*c/d)) -
 real_part(cos_integral(-4*b*x - 4*b*c/d)))/(d*tan(2*a)^2*tan(2*b*c/d)^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+b\,x)^2\sin(a+b\,x)^2}{c+d\,x}\,dx$$

input
```
int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x),x)
```

output
```
int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)^2\sin(bx+a)^2}{dx+c}\,dx$$

input
```
int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x)
```

output
```
int((cos(a + b*x)**2*sin(a + b*x)**2)/(c + d*x),x)
```

3.85 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 828
Mathematica [A] (verified) . . . 828
Rubi [A] (verified) . . . 829
Maple [A] (verified) . . . 830
Fricas [A] (verification not implemented) . . . 831
Sympy [F] . . . 831
Maxima [C] (verification not implemented) . . . 831
Giac [C] (verification not implemented) . . . 832
Mupad [F(-1)] . . . 833
Reduce [F] . . . 834

Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = -\frac{1}{8d(c+dx)} + \frac{\cos(4a+4bx)}{8d(c+dx)} + \frac{b\,\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{2d^2} + \frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{2d^2}$$

output

```
-1/8/d/(d*x+c)+1/8*cos(4*b*x+4*a)/d/(d*x+c)+1/2*b*Ci(4*b*c/d+4*b*x)*sin(4*
a-4*b*c/d)/d^2+1/2*b*cos(4*a-4*b*c/d)*Si(4*b*c/d+4*b*x)/d^2
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.78

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \frac{\frac{d(-1+\cos(4(a+bx)))}{c+dx} + 4b\,\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)\sin\left(4a-\frac{4bc}{d}\right) + 4b\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4b(c+dx)}{d}\right)}{8d^2}$$

input
```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
((d*(-1 + Cos[4*(a + b*x)]))/(c + d*x) + 4*b*CosIntegral[(4*b*(c + d*x))/d
]*Sin[4*a - (4*b*c)/d] + 4*b*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*
x))/d])/(8*d^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos^2(a+bx)}{(c+dx)^2}\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8(c+dx)^2} - \frac{\cos(4a+4bx)}{8(c+dx)^2}\right)dx$$

$$\downarrow 2009$$

$$\frac{b\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2} + \frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{2d^2} + \frac{\cos(4a+4bx)}{8d(c+dx)} - \frac{1}{8d(c+dx)}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
-1/8*1/(d*(c + d*x)) + Cos[4*a + 4*b*x]/(8*d*(c + d*x)) + (b*CosIntegral[(
4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(2*d^2) + (b*Cos[4*a - (4*b*c)/d]*
SinIntegral[(4*b*c)/d + 4*b*x])/(2*d^2)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.45

method	result
derivativedivides	$-\frac{b\left(\frac{4}{(-4ad+4cb+4d(bx+a))d}-\frac{4\cos(4bx+4a)}{(-4ad+4cb+4d(bx+a))d}-\frac{4\left(-\frac{\mathrm{Si}\left(-4bx-4a-\frac{-4ad+4cb}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}-\frac{\mathrm{Ci}\left(4bx+4a+\frac{-4ad+4cb}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{d}\right.}{d}\right.}{8}$
default	$-\frac{b\left(\frac{4}{(-4ad+4cb+4d(bx+a))d}-\frac{4\cos(4bx+4a)}{(-4ad+4cb+4d(bx+a))d}-\frac{4\left(-\frac{\mathrm{Si}\left(-4bx-4a-\frac{-4ad+4cb}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}-\frac{\mathrm{Ci}\left(4bx+4a+\frac{-4ad+4cb}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{d}\right.}{d}\right.}{8}$
risch	$-\frac{1}{8d(dx+c)}-\frac{ib\,\mathrm{e}^{-\frac{4i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{4d^2}+\frac{ib\,\mathrm{e}^{\frac{4i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(-4ibx-4ia-\frac{4(-}{4d^2}\right.$

input

```
int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/8*b*(4/(-4*a*d+4*c*b+4*d*(b*x+a))/d-4*cos(4*b*x+4*a)/(-4*a*d+4*c*b+4*d*
(b*x+a))/d-4*(-Si(-4*b*x-4*a-(-4*a*d+4*b*c)/d)*cos((-4*a*d+4*b*c)/d)/d-Ci(
4*b*x+4*a+(-4*a*d+4*b*c)/d)*sin((-4*a*d+4*b*c)/d)/d)/d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{2\,d\cos(bx+a)^4 - 2\,d\cos(bx+a)^2 + (bdx+bc)\,\mathrm{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right) + (bdx+bc)\cos\left(-\frac{4\,(bc-}{d}\right.}{2\,(d^3x+cd^2)}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(2*d*cos(b*x + a)^4 - 2*d*cos(b*x + a)^2 + (b*d*x + b*c)*cos_integral(
4*(b*d*x + b*c)/d)*sin(-4*(b*c - a*d)/d) + (b*d*x + b*c)*cos(-4*(b*c - a*d
)/d)*sin_integral(4*(b*d*x + b*c)/d))/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin^2(a+bx)\cos^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.64

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{b^2\left(E_2\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_2\left(-\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)+b^2\left(i\,E_2\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,a}{d}\right.\right.}{16\,(bcd+(bx+a)d^2-ad^2)b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/16*(b^2*(exp_integral_e(2, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_i
ntegral_e(2, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a*d)/d)
 + b^2*(I*exp_integral_e(2, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_
integral_e(2, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d)/d
) - 2*b^2)/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.55 (sec) , antiderivative size = 3218, normalized size of antiderivative = 30.94

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output

```
1/4*(b*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^
2*tan(2*b*c/d)^2 - b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b
*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*b*d*x*sin_integral(4*(b*d*x + b*c)/d)*
tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*b*d*x*real_part(cos_integral(4*
b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 2*b*d*x*real_part(c
os_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 2*b*
d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b
*c/d)^2 - 2*b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*t
an(2*a)*tan(2*b*c/d)^2 + b*c*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(
2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - b*c*imag_part(cos_integral(-4*b*x - 4
*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*b*c*sin_integral(4*(b*
d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - b*d*x*imag_part(cos
_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + b*d*x*imag_part(cos_
integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 2*b*d*x*sin_integral
(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2 + 4*b*d*x*imag_part(cos_integr
al(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) - 4*b*d*x*imag_par
t(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 8*b
*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) +
2*b*c*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan
(2*b*c/d) + 2*b*c*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\,dx$$

input

```
int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^2,x)
```

output

```
int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(bx+a)^2\sin(bx+a)^2}{d^2x^2+2cdx+c^2}dx$$

input
```
int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int((cos(a + b*x)**2*sin(a + b*x)**2)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.86 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$

Optimal result . . . 835
Mathematica [A] (verified) . . . 835
Rubi [A] (verified) . . . 836
Maple [A] (verified) . . . 837
Fricas [A] (verification not implemented) . . . 838
Sympy [F] . . . 838
Maxima [C] (verification not implemented) . . . 839
Giac [C] (verification not implemented) . . . 839
Mupad [F(-1)] . . . 840
Reduce [F] . . . 841

Optimal result

Integrand size = 24, antiderivative size = 127

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = -\frac{1}{16d(c+dx)^2} + \frac{\cos(4a+4bx)}{16d(c+dx)^2} + \frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\operatorname{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3} - \frac{b\sin(4a+4bx)}{4d^2(c+dx)} - \frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\operatorname{Si}\left(\frac{4bc}{d}+4bx\right)}{d^3}$$

output

```
-1/16/d/(d*x+c)^2+1/16*cos(4*b*x+4*a)/d/(d*x+c)^2+b^2*cos(4*a-4*b*c/d)*Ci(
4*b*c/d+4*b*x)/d^3-1/4*b*sin(4*b*x+4*a)/d^2/(d*x+c)-b^2*sin(4*a-4*b*c/d)*S
i(4*b*c/d+4*b*x)/d^3
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \frac{16b^2\cos\left(4a-\frac{4bc}{d}\right)\operatorname{CosIntegral}\left(\frac{4b(c+dx)}{d}\right) + \frac{d(-d+d\cos(4(a+bx))-4b(c+dx)\sin(4(a+bx)))}{(c+dx)^2} - 16b^2\sin\left(4a-\frac{4bc}{d}\right)\operatorname{Si}}{16d^3}$$

input
```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

output
```
(16*b^2*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] + (d*(-d + d*C
os[4*(a + b*x)] - 4*b*(c + d*x)*Sin[4*(a + b*x)]))/(c + d*x)^2 - 16*b^2*Si
n[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(16*d^3)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos^2(a+bx)}{(c+dx)^3}\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8(c+dx)^3} - \frac{\cos(4a+4bx)}{8(c+dx)^3}\right)dx$$

$$\downarrow 2009$$

$$\frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3} - \frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{d^3} - \frac{b\sin(4a+4bx)}{4d^2(c+dx)} + \frac{\cos(4a+4bx)}{16d(c+dx)^2} - \frac{1}{16d(c+dx)^2}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

output
```
-1/16*1/(d*(c + d*x)^2) + Cos[4*a + 4*b*x]/(16*d*(c + d*x)^2) + (b^2*Cos[4
*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/d^3 - (b*Sin[4*a + 4*b*x])
/(4*d^2*(c + d*x)) - (b^2*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b
*x])/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.50

method	result
derivativedivides	$\frac{b^2\left(-\frac{8}{(-4ad+4cb+4d(bx+a))^2d}+\frac{8\cos(4bx+4a)}{(-4ad+4cb+4d(bx+a))^2d}+\frac{-\frac{8\sin(4bx+4a)}{(-4ad+4cb+4d(bx+a))d}+\frac{8\left(-\frac{\mathrm{Si}\left(-4bx-4a-\frac{-4ad+4cb}{d}\right)\sin\left(\right.}{d}\right.}{d}\right.}{8}$
default	$\frac{b^2\left(-\frac{8}{(-4ad+4cb+4d(bx+a))^2d}+\frac{8\cos(4bx+4a)}{(-4ad+4cb+4d(bx+a))^2d}+\frac{-\frac{8\sin(4bx+4a)}{(-4ad+4cb+4d(bx+a))d}+\frac{8\left(-\frac{\mathrm{Si}\left(-4bx-4a-\frac{-4ad+4cb}{d}\right)\sin\left(\right.}{d}\right.}{d}\right.}{8}$
risch	$-\frac{1}{16d(dx+c)^2}-\frac{b^2\mathrm{e}^{-\frac{4i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{2d^3}-\frac{b^2\mathrm{e}^{\frac{4i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(-4ibx-4ia-\right.}{2d^3}$

input
```
int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*b^2*(-8/(-4*a*d+4*c*b+4*d*(b*x+a))^2/d+8*cos(4*b*x+4*a)/(-4*a*d+4*c*b+
4*d*(b*x+a))^2/d+8*(-sin(4*b*x+4*a)/(-4*a*d+4*c*b+4*d*(b*x+a))/d+(-Si(-4*b
*x-4*a-(-4*a*d+4*b*c)/d)*sin((-4*a*d+4*b*c)/d)/d+Ci(4*b*x+4*a+(-4*a*d+4*b*
c)/d)*cos((-4*a*d+4*b*c)/d)/d)/d)/d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.68

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{d^2\cos(bx+a)^4 - d^2\cos(bx+a)^2 + 2\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{4\,(bdx+bc)}{d}\right) - 2\,(b^2d^2}{2}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*cos(-4*(b*c - a*d)/d)*cos_integral(4*(b*d*x + b*c)/d) - 2*(b^
2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b
*d*x + b*c)/d) - 2*(2*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - (b*d^2*x + b*c*d)
*cos(b*x + a))*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin^2(a+bx)\cos^2(a+bx)}{(c+dx)^3}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.62

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{b^3\left(E_3\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_3\left(-\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)+b^3\left(i\,E_3\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,a}{d}\right.\right.}{16\left(b^2c^2d-2\,abcd^2+(bx+a)^2d^3+a^2d^3+2\left(bcd^2-ad^3\right)\right.}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/16*(b^3*(exp_integral_e(3, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_i
ntegral_e(3, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a*d)/d)
 + b^3*(I*exp_integral_e(3, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_
integral_e(3, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d)/d
) - b^3)/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^
2 - a*d^3)*(b*x + a))*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 5600, normalized size of antiderivative = 44.09

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

output

```
1/8*(4*b^2*d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*t
an(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*d^2*x^2*real_part(cos_integral(-4*b*x - 4
*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(
cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 8*b^
2*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^
2*tan(2*b*c/d) - 16*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)
^2*tan(2*a)^2*tan(2*b*c/d) + 8*b^2*d^2*x^2*imag_part(cos_integral(4*b*x +
4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(c
os_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 16*b
^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c
/d)^2 + 8*b^2*c*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*
tan(2*a)^2*tan(2*b*c/d)^2 + 8*b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*
b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 4*b^2*d^2*x^2*real_part(c
os_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 4*b^2*d^2*x^2*real
_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + 16*b^2*d^2
*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*
b*c/d) + 16*b^2*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*
x)^2*tan(2*a)*tan(2*b*c/d) - 16*b^2*c*d*x*imag_part(cos_integral(4*b*x + 4
*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 16*b^2*c*d*x*imag_part(cos
_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 32*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^3}\,dx$$

input

```
int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^3,x)
```

output

```
int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos(bx+a)^2\sin(bx+a)^2}{d^3x^3+3cd^2x^2+3c^2dx+c^3}dx$$

input
```
int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x)
```

output
```
int((cos(a + b*x)**2*sin(a + b*x)**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
 d**3*x**3),x)
```

3.87 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$

Optimal result . . . 842
Mathematica [A] (verified) . . . 842
Rubi [A] (verified) . . . 843
Maple [A] (verified) . . . 844
Fricas [B] (verification not implemented) . . . 845
Sympy [F] . . . 845
Maxima [C] (verification not implemented) . . . 846
Giac [C] (verification not implemented) . . . 846
Mupad [F(-1)] . . . 847
Reduce [F] . . . 848

Optimal result

Integrand size = 24, antiderivative size = 158

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = -\frac{1}{24d(c+dx)^3} + \frac{\cos(4a+4bx)}{24d(c+dx)^3} - \frac{b^2\cos(4a+4bx)}{3d^3(c+dx)} - \frac{4b^3\,\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{3d^4} - \frac{b\sin(4a+4bx)}{12d^2(c+dx)^2} - \frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{3d^4}$$

output

```
-1/24/d/(d*x+c)^3+1/24*cos(4*b*x+4*a)/d/(d*x+c)^3-1/3*b^2*cos(4*b*x+4*a)/d
^3/(d*x+c)-4/3*b^3*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^4-1/12*b*sin(4*b*x
+4*a)/d^2/(d*x+c)^2-4/3*b^3*cos(4*a-4*b*c/d)*Si(4*b*c/d+4*b*x)/d^4
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.78

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = -\frac{32b^3\,\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)\sin\left(4a-\frac{4bc}{d}\right) + \frac{d\left(\left(-d^2+8b^2(c+dx)^2\right)\cos(4(a+bx)) + d(d+2b(c+dx)\sin(4(a+bx)))\right)}{(c+dx)^3} + 32b^3}{24d^4}$$

input
```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

output
```
-1/24*(32*b^3*CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + (d*((-
d^2 + 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] + d*(d + 2*b*(c + d*x)*Sin[4*(a
+ b*x)])))/(c + d*x)^3 + 32*b^3*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c +
 d*x))/d])/d^4
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos^2(a+bx)}{(c+dx)^4}\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8(c+dx)^4} - \frac{\cos(4a+4bx)}{8(c+dx)^4}\right)dx$$

$$\downarrow 2009$$

$$-\frac{4b^3 \sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4} - \frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{3d^4} - \frac{b^2\cos(4a+4bx)}{3d^3(c+dx)} - \frac{b\sin(4a+4bx)}{12d^2(c+dx)^2} + \frac{\cos(4a+4bx)}{24d(c+dx)^3} - \frac{1}{24d(c+dx)^3}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

output
```
-1/24*1/(d*(c + d*x)^3) + Cos[4*a + 4*b*x]/(24*d*(c + d*x)^3) - (b^2*Cos[4
*a + 4*b*x])/(3*d^3*(c + d*x)) - (4*b^3*CosIntegral[(4*b*c)/d + 4*b*x]*Sin
[4*a - (4*b*c)/d])/(3*d^4) - (b*Sin[4*a + 4*b*x])/(12*d^2*(c + d*x)^2) - (
4*b^3*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.46

method	result
derivativedivides	$-\frac{b^3\left(\frac{64}{3(-4ad+4cb+4d(bx+a))^3d}-\frac{64\cos(4bx+4a)}{3(-4ad+4cb+4d(bx+a))^3d}-\frac{64\left(-\frac{\sin(4bx+4a)}{2(-4ad+4cb+4d(bx+a))^2d}+\frac{-\frac{\cos(4bx+4a)}{(-4ad+4cb+4d(bx+a))d}}{}\right.\right.}{8}$
default	$-\frac{b^3\left(\frac{64}{3(-4ad+4cb+4d(bx+a))^3d}-\frac{64\cos(4bx+4a)}{3(-4ad+4cb+4d(bx+a))^3d}-\frac{64\left(-\frac{\sin(4bx+4a)}{2(-4ad+4cb+4d(bx+a))^2d}+\frac{-\frac{\cos(4bx+4a)}{(-4ad+4cb+4d(bx+a))d}}{}\right.\right.}{8}$
risch	$-\frac{1}{24d(dx+c)^3}+\frac{2ib^3e^{-\frac{4i(ad-cb)}{d}}\operatorname{expIntegral}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{3d^4}-\frac{2ib^3e^{\frac{4i(ad-cb)}{d}}\operatorname{expIntegral}_1\left(-4ibx-4i\right.}{3d^4}$

input
```
int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/8*b^3*(64/3/(-4*a*d+4*c*b+4*d*(b*x+a))^3/d-64/3*cos(4*b*x+4*a)/(-4*a*d+
4*c*b+4*d*(b*x+a))^3/d-64/3*(-1/2*sin(4*b*x+4*a)/(-4*a*d+4*c*b+4*d*(b*x+a)
)^2/d+1/2*(-cos(4*b*x+4*a)/(-4*a*d+4*c*b+4*d*(b*x+a))/d-(-Si(-4*b*x-4*a-(-
4*a*d+4*b*c)/d)*cos((-4*a*d+4*b*c)/d)/d-Ci(4*b*x+4*a+(-4*a*d+4*b*c)/d)*sin
((-4*a*d+4*b*c)/d)/d)/d)/d)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(146) = 292$.

Time $= 0.09$ (sec) , antiderivative size $= 349$, normalized size of antiderivative $= 2.21$

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx =$$

$$-\frac{b^2d^3x^2 + 2\,b^2cd^2x + b^2c^2d + (8\,b^2d^3x^2 + 16\,b^2cd^2x + 8\,b^2c^2d - d^3)\cos(bx+a)^4 - (8\,b^2d^3x^2 + 16\,b^2cd^2}{}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d + (8*b^2*d^3*x^2 + 16*b^2*c*
d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^4 - (8*b^2*d^3*x^2 + 16*b^2*c*d^2*
x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(4*(b*d*x + b*c)/d)*sin(-4*(b*c - a*
d)/d) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-4
*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + (2*(b*d^3*x + b*c*d^2)*c
os(b*x + a)^3 - (b*d^3*x + b*c*d^2)*cos(b*x + a))*sin(b*x + a))/(d^7*x^3 +
 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sin^2(a+bx)\cos^2(a+bx)}{(c+dx)^4}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c)**4,x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x)**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.63

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$$
$$= \frac{3\,b^4\left(E_4\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_4\left(-\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)+3\,b^4\left(i\,E_4\left(\frac{4\,(-i\,bc-i\,(bx+a)d}{d}\right.\right.}{48\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3+(bx+a)^3d^4-a^3d^4+3\,(bcd^3-ad^4)(bx+a)^2+\right.}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/48*(3*b^4*(exp_integral_e(4, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp
_integral_e(4, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a*d)/
d) + 3*b^4*(I*exp_integral_e(4, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*
exp_integral_e(4, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*
d)/d) - 2*b^4)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3
*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*
c*d^3 + a^2*d^4)*(b*x + a))*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 8508, normalized size of antiderivative = 53.85

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

output

```
-1/12*(8*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2
*tan(2*a)^2*tan(2*b*c/d)^2 - 8*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x -
 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 16*b^3*d^3*x^3*sin_int
egral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 16*b^3*d
^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*ta
n(2*b*c/d) + 16*b^3*d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(
2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 16*b^3*d^3*x^3*real_part(cos_integral(4
*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*rea
l_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^
2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2
*tan(2*a)^2*tan(2*b*c/d)^2 - 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*
x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 48*b^3*c*d^2*x^2*si
n_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 8*b
^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^
2 + 8*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*t
an(2*a)^2 - 16*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*ta
n(2*a)^2 + 32*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b
*x)^2*tan(2*a)*tan(2*b*c/d) - 32*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x
 - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 64*b^3*d^3*x^3*sin_integ
ral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 48*b^3*c*d^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^4}\,dx$$

input

```
int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^4,x)
```

output

```
int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^4, x)
```

Reduce [F]

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos(bx+a)^2\sin(bx+a)^2}{d^4x^4+4c\,d^3x^3+6c^2d^2x^2+4c^3dx+c^4}\,dx$$

input
```
int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x)
```

output
```
int((cos(a + b*x)**2*sin(a + b*x)**2)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**
2 + 4*c*d**3*x**3 + d**4*x**4),x)
```

3.88 $\int (c+dx)^m \cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal result . . . 849
Mathematica [A] (verified) . . . 850
Rubi [A] (verified) . . . 851
Maple [F] . . . 852
Fricas [A] (verification not implemented) . . . 853
Sympy [F] . . . 853
Maxima [F] . . . 854
Giac [F] . . . 854
Mupad [F(-1)] . . . 854
Reduce [F] . . . 855

Optimal result

Integrand size = 24, antiderivative size = 407

$$\begin{aligned}
&\int (c+dx)^m \cos^2(a+bx)\sin^3(a+bx)\,dx \\
&= -\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{16b} \\
&\quad -\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{16b} \\
&\quad -\frac{3^{-1-m}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3ib(c+dx)}{d}\right)}{32b} \\
&\quad -\frac{3^{-1-m}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3ib(c+dx)}{d}\right)}{32b} \\
&\quad +\frac{5^{-1-m}e^{5i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{5ib(c+dx)}{d}\right)}{32b} \\
&\quad +\frac{5^{-1-m}e^{-5i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{5ib(c+dx)}{d}\right)}{32b}
\end{aligned}$$

output

```
-1/16*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c
)/d)^m)-1/16*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(
d*x+c)/d)^m)-1/32*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*b*(
d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)-1/32*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I*b*(
d*x+c)/d)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)+1/32*5^(-1-m)*exp(5*I*(
a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-5*I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)+1/3
2*5^(-1-m)*(d*x+c)^m*GAMMA(1+m,5*I*b*(d*x+c)/d)/b/exp(5*I*(a-b*c/d))/((I*b
*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.92

$$\int (c+dx)^m \cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{e^{-\frac{5i(bc+ad)}{d}}(c+dx)^m\left(30e^{\frac{4i(bc+ad)}{d}}\left(-e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)-e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m\right.\right.\right.}{}$$

input

```
Integrate[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output

```
((c + d*x)^m*(30*E^(((4*I)*(b*c + a*d))/d)*(-((E^((2*I)*a)*Gamma[1 + m, ((
-I)*b*(c + d*x))/d])/(((-I)*b*(c + d*x))/d)^m) - (E^(((2*I)*b*c)/d)*Gamma[
1 + m, (I*b*(c + d*x))/d])/((I*b*(c + d*x))/d)^m) - (5*E^(((2*I)*(b*c + a*
d))/d)*(E^((6*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x)
)/d] + E^(((6*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((3*I)*b*(c
 + d*x))/d]))/(3^m*((b^2*(c + d*x)^2)/d^2)^m) + (3*(E^((10*I)*a)*((I*b*(c
+ d*x))/d)^m*Gamma[1 + m, ((-5*I)*b*(c + d*x))/d] + E^(((10*I)*b*c)/d)*(((
-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((5*I)*b*(c + d*x))/d]))/(5^m*((b^2*(c
+ d*x)^2)/d^2)^m)))/(480*b*E^(((5*I)*(b*c + a*d))/d))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\cos^2(a+bx)(c+dx)^m\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8}\sin(a+bx)(c+dx)^m + \frac{1}{16}\sin(3a+3bx)(c+dx)^m - \frac{1}{16}\sin(5a+5bx)(c+dx)^m\right)dx$$

$$\downarrow 2009$$

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{16b} - \frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{32b} + \frac{5^{-m-1}e^{5i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{5ib(c+dx)}{d}\right)}{32b} - \frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{16b} - \frac{3^{-m-1}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{3ib(c+dx)}{d}\right)}{32b} + \frac{5^{-m-1}e^{-5i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{5ib(c+dx)}{d}\right)}{32b}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
-1/16*(E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])
/(b*(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/
d])/(16*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - (3^(-1 - m)*E^((3*I
)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(32*b*(
((-I)*b*(c + d*x))/d)^m) - (3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((3*I)*b*(
c + d*x))/d])/(32*b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + (5^(-
1 - m)*E^((5*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-5*I)*b*(c + d*x
))/d])/(32*b*(((-I)*b*(c + d*x))/d)^m) + (5^(-1 - m)*(c + d*x)^m*Gamma[1 +
 m, ((5*I)*b*(c + d*x))/d])/(32*b*E^((5*I)*(a - (b*c)/d))*((I*b*(c + d*x))
/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [F]

$$\int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)^3 \, dx$$

input
```
int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

output
```
int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.69

$$\int (c+dx)^m \cos^2(a+bx)\sin^3(a+bx)\,dx = -\frac{30\,e^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-ibc+iad}{d}\right)}\Gamma\left(m+1,\frac{ibdx+ibc}{d}\right)+5\,e^{\left(-\frac{dm\log\left(-\frac{3ib}{d}\right)+3ibc-3iad}{d}\right)}\Gamma\left(m+1,-\frac{3\,(ibdx+ibc)}{d}\right)-3\,e^{\left(}$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/480*(30*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x +
 I*b*c)/d) + 5*e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d)*gamma(m + 1,
 -3*(I*b*d*x + I*b*c)/d) - 3*e^(-(d*m*log(-5*I*b/d) + 5*I*b*c - 5*I*a*d)/d
)*gamma(m + 1, -5*(I*b*d*x + I*b*c)/d) + 30*e^(-(d*m*log(-I*b/d) + I*b*c -
 I*a*d)/d)*gamma(m + 1, (-I*b*d*x - I*b*c)/d) + 5*e^(-(d*m*log(3*I*b/d) -
3*I*b*c + 3*I*a*d)/d)*gamma(m + 1, -3*(-I*b*d*x - I*b*c)/d) - 3*e^(-(d*m*l
og(5*I*b/d) - 5*I*b*c + 5*I*a*d)/d)*gamma(m + 1, -5*(-I*b*d*x - I*b*c)/d))
/b
```

Sympy [F]

$$\int (c+dx)^m \cos^2(a+bx)\sin^3(a+bx)\,dx = \int (c+dx)^m \sin^3(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**m*sin(a + b*x)**3*cos(a + b*x)**2, x)
```

Maxima [F]

$$\int (c+dx)^m \cos^2(a+bx) \sin^3(a+bx)\, dx = \int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)^3\, dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a)^3, x)
```

Giac [F]

$$\int (c+dx)^m \cos^2(a+bx) \sin^3(a+bx)\, dx = \int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)^3\, dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos^2(a+bx) \sin^3(a+bx)\, dx = \int \cos(a+b\,x)^2 \sin(a+b\,x)^3 (c+d\,x)^m\, dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \cos^2(a+bx) \sin^3(a+bx)\, dx = \int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)^3\, dx$$

input
```
int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

output
```
int((c + d*x)**m*cos(a + b*x)**2*sin(a + b*x)**3,x)
```

3.89 $\int (c+dx)^4 \cos^2(a+bx) \sin^3(a+bx)\, dx$

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Optimal result

Integrand size = 24, antiderivative size = 330

$$\begin{aligned}
&\int (c+dx)^4 \cos^2(a+bx) \sin^3(a+bx)\, dx \\
&= -\frac{3d^4 \cos(a+bx)}{b^5} + \frac{3d^2(c+dx)^2 \cos(a+bx)}{2b^3} - \frac{(c+dx)^4 \cos(a+bx)}{8b} \\
&\quad - \frac{d^4 \cos(3a+3bx)}{162b^5} + \frac{d^2(c+dx)^2 \cos(3a+3bx)}{36b^3} - \frac{(c+dx)^4 \cos(3a+3bx)}{48b} \\
&\quad + \frac{3d^4 \cos(5a+5bx)}{6250b^5} - \frac{3d^2(c+dx)^2 \cos(5a+5bx)}{500b^3} + \frac{(c+dx)^4 \cos(5a+5bx)}{80b} \\
&\quad - \frac{3d^3(c+dx) \sin(a+bx)}{b^4} + \frac{d(c+dx)^3 \sin(a+bx)}{2b^2} - \frac{d^3(c+dx) \sin(3a+3bx)}{54b^4} \\
&\quad + \frac{d(c+dx)^3 \sin(3a+3bx)}{36b^2} + \frac{3d^3(c+dx) \sin(5a+5bx)}{1250b^4} - \frac{d(c+dx)^3 \sin(5a+5bx)}{100b^2}
\end{aligned}$$

output

```
-3*d^4*cos(b*x+a)/b^5+3/2*d^2*(d*x+c)^2*cos(b*x+a)/b^3-1/8*(d*x+c)^4*cos(b
*x+a)/b-1/162*d^4*cos(3*b*x+3*a)/b^5+1/36*d^2*(d*x+c)^2*cos(3*b*x+3*a)/b^3
-1/48*(d*x+c)^4*cos(3*b*x+3*a)/b+3/6250*d^4*cos(5*b*x+5*a)/b^5-3/500*d^2*(
d*x+c)^2*cos(5*b*x+5*a)/b^3+1/80*(d*x+c)^4*cos(5*b*x+5*a)/b-3*d^3*(d*x+c)*
sin(b*x+a)/b^4+1/2*d*(d*x+c)^3*sin(b*x+a)/b^2-1/54*d^3*(d*x+c)*sin(3*b*x+3
*a)/b^4+1/36*d*(d*x+c)^3*sin(3*b*x+3*a)/b^2+3/1250*d^3*(d*x+c)*sin(5*b*x+5
*a)/b^4-1/100*d*(d*x+c)^3*sin(5*b*x+5*a)/b^2
```

Mathematica [A] (verified)

Time = 2.20 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.72

$$\int (c+dx)^4 \cos^2(a+bx) \sin^3(a+bx)\, dx$$
$$= \frac{-506250(24d^4 - 12b^2d^2(c+dx)^2 + b^4(c+dx)^4)\cos(a+bx) - 3125(8d^4 - 36b^2d^2(c+dx)^2 + 27b^4(c+d$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(-506250*(24*d^4 - 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cos[a + b*x]
 - 3125*(8*d^4 - 36*b^2*d^2*(c + d*x)^2 + 27*b^4*(c + d*x)^4)*Cos[3*(a + b*
x)] + 81*(24*d^4 - 300*b^2*d^2*(c + d*x)^2 + 625*b^4*(c + d*x)^4)*Cos[5*(a
 + b*x)] + 120*b*d*(c + d*x)*(17475*b^2*c^2 - 101794*d^2 + 34950*b^2*c*d*x
 + 17475*b^2*d^2*x^2 + 16*(-68*d^2 + 75*b^2*(c + d*x)^2)*Cos[2*(a + b*x)]
 - 27*(-6*d^2 + 25*b^2*(c + d*x)^2)*Cos[4*(a + b*x)])*Sin[a + b*x])/(405000
0*b^5)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sin^3(a+bx) \cos^2(a+bx)\, dx$$
$$\downarrow \text{4906}$$
$$\int \left(\frac{1}{8}(c+dx)^4 \sin(a+bx) + \frac{1}{16}(c+dx)^4 \sin(3a+3bx) - \frac{1}{16}(c+dx)^4 \sin(5a+5bx)\right) dx$$
$$\downarrow \text{2009}$$

$$-\frac{3d^4\cos(a+bx)}{b^5}-\frac{d^4\cos(3a+3bx)}{162b^5}+\frac{3d^4\cos(5a+5bx)}{6250b^5}-\frac{3d^3(c+dx)\sin(a+bx)}{b^4}-\frac{d^3(c+dx)\sin(3a+3bx)}{54b^4}+\frac{3d^3(c+dx)\sin(5a+5bx)}{1250b^4}+\frac{3d^2(c+dx)^2\cos(a+bx)}{2b^3}+\frac{d^2(c+dx)^2\cos(3a+3bx)}{36b^3}-\frac{3d^2(c+dx)^2\cos(5a+5bx)}{500b^3}+\frac{d(c+dx)^3\sin(a+bx)}{2b^2}+\frac{d(c+dx)^3\sin(3a+3bx)}{36b^2}-\frac{d(c+dx)^3\sin(5a+5bx)}{100b^2}-\frac{(c+dx)^4\cos(a+bx)}{8b}-\frac{(c+dx)^4\cos(3a+3bx)}{48b}+\frac{(c+dx)^4\cos(5a+5bx)}{80b}$$

input

```
Int[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output

```
(-3*d^4*Cos[a + b*x])/b^5 + (3*d^2*(c + d*x)^2*Cos[a + b*x])/(2*b^3) - ((c
 + d*x)^4*Cos[a + b*x])/(8*b) - (d^4*Cos[3*a + 3*b*x])/(162*b^5) + (d^2*(c
 + d*x)^2*Cos[3*a + 3*b*x])/(36*b^3) - ((c + d*x)^4*Cos[3*a + 3*b*x])/(48*
b) + (3*d^4*Cos[5*a + 5*b*x])/(6250*b^5) - (3*d^2*(c + d*x)^2*Cos[5*a + 5*
b*x])/(500*b^3) + ((c + d*x)^4*Cos[5*a + 5*b*x])/(80*b) - (3*d^3*(c + d*x)
*Sin[a + b*x])/b^4 + (d*(c + d*x)^3*Sin[a + b*x])/(2*b^2) - (d^3*(c + d*x)
*Sin[3*a + 3*b*x])/(54*b^4) + (d*(c + d*x)^3*Sin[3*a + 3*b*x])/(36*b^2) +
(3*d^3*(c + d*x)*Sin[5*a + 5*b*x])/(1250*b^4) - (d*(c + d*x)^3*Sin[5*a + 5
*b*x])/(100*b^2)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.78

method	result
parallelrisch	$\frac{\left(-84375(dx+c)^4b^4+112500d^2(dx+c)^2b^2-25000d^4\right)\cos(3bx+3a)+\left(50625(dx+c)^4b^4-24300d^2(dx+c)^2b^2+1944d^4\right)$
risch	$-\frac{(d^4x^4b^4+4b^4cd^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+b^4c^4-12b^2d^4x^2-24b^2cd^3x-12b^2c^2d^2+24d^4)\cos(bx+a)}{8b^5}+\frac{d(b^2d^3x^3-}$
derivativedivides	Expression too large to display
default	Expression too large to display
orering	Expression too large to display

input
```
int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4050000*((-84375*(d*x+c)^4*b^4+112500*d^2*(d*x+c)^2*b^2-25000*d^4)*cos(3
*b*x+3*a)+(50625*(d*x+c)^4*b^4-24300*d^2*(d*x+c)^2*b^2+1944*d^4)*cos(5*b*x
+5*a)+112500*d*((d*x+c)^2*b^2-2/3*d^2)*(d*x+c)*b*sin(3*b*x+3*a)-40500*d*((
d*x+c)^2*b^2-6/25*d^2)*(d*x+c)*b*sin(5*b*x+5*a)+(-506250*(d*x+c)^4*b^4+607
5000*d^2*(d*x+c)^2*b^2-12150000*d^4)*cos(b*x+a)+2025000*d*(d*x+c)*b*((d*x+
c)^2*b^2-6*d^2)*sin(b*x+a)-540000*b^4*c^4+6163200*b^2*c^2*d^2-12173056*d^4
)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.43

$$\int (c+dx)^4\cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{81\,(625\,b^4d^4x^4+2500\,b^4cd^3x^3+625\,b^4c^4-300\,b^2c^2d^2+24\,d^4+150\,(25\,b^4c^2d^2-2\,b^2d^4)x^2+100\,(25\,b^4c^3$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output

```
1/253125*(81*(625*b^4*d^4*x^4 + 2500*b^4*c*d^3*x^3 + 625*b^4*c^4 - 300*b^2
*c^2*d^2 + 24*d^4 + 150*(25*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 100*(25*b^4*c^3
*d - 6*b^2*c*d^3)*x)*cos(b*x + a)^5 - 5*(16875*b^4*d^4*x^4 + 67500*b^4*c*d
^3*x^3 + 16875*b^4*c^4 - 11700*b^2*c^2*d^2 + 1736*d^4 + 450*(225*b^4*c^2*d
^2 - 26*b^2*d^4)*x^2 + 900*(75*b^4*c^3*d - 26*b^2*c*d^3)*x)*cos(b*x + a)^3
 + 120*(2925*b^2*d^4*x^2 + 5850*b^2*c*d^3*x + 2925*b^2*c^2*d^2 - 6284*d^4)
*cos(b*x + a) + 60*(1950*b^3*d^4*x^3 + 5850*b^3*c*d^3*x^2 + 1950*b^3*c^3*d
 - 12568*b*c*d^3 - 27*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 25*b^3*c^3*d -
6*b*c*d^3 + 3*(25*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^4 + (975*b^3*d^4*
x^3 + 2925*b^3*c*d^3*x^2 + 975*b^3*c^3*d - 434*b*c*d^3 + (2925*b^3*c^2*d^2
 - 434*b*d^4)*x)*cos(b*x + a)^2 + 2*(2925*b^3*c^2*d^2 - 6284*b*d^4)*x)*sin
(b*x + a))/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. $2(325) = 650$.

Time $= 1.17$ (sec) , antiderivative size $= 1098$, normalized size of antiderivative $= 3.33$

$$\int (c+dx)^4 \cos^2(a+bx) \sin^3(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)**4*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output

```
Piecewise((-c**4*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c**4*cos(a + b*
x)**5/(15*b) - 4*c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 8*c**3*d
*x*cos(a + b*x)**5/(15*b) - 2*c**2*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)*
*3/b - 4*c**2*d**2*x**2*cos(a + b*x)**5/(5*b) - 4*c*d**3*x**3*sin(a + b*x)
**2*cos(a + b*x)**3/(3*b) - 8*c*d**3*x**3*cos(a + b*x)**5/(15*b) - d**4*x*
*4*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*d**4*x**4*cos(a + b*x)**5/(15
*b) + 104*c**3*d*sin(a + b*x)**5/(225*b**2) + 52*c**3*d*sin(a + b*x)**3*co
s(a + b*x)**2/(45*b**2) + 8*c**3*d*sin(a + b*x)*cos(a + b*x)**4/(15*b**2)
+ 104*c**2*d**2*x*sin(a + b*x)**5/(75*b**2) + 52*c**2*d**2*x*sin(a + b*x)*
*3*cos(a + b*x)**2/(15*b**2) + 8*c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**4/
(5*b**2) + 104*c*d**3*x**2*sin(a + b*x)**5/(75*b**2) + 52*c*d**3*x**2*sin(
a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 8*c*d**3*x**2*sin(a + b*x)*cos(a +
 b*x)**4/(5*b**2) + 104*d**4*x**3*sin(a + b*x)**5/(225*b**2) + 52*d**4*x**
3*sin(a + b*x)**3*cos(a + b*x)**2/(45*b**2) + 8*d**4*x**3*sin(a + b*x)*cos
(a + b*x)**4/(15*b**2) + 104*c**2*d**2*sin(a + b*x)**4*cos(a + b*x)/(75*b*
*3) + 676*c**2*d**2*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 1712*c**2
*d**2*cos(a + b*x)**5/(1125*b**3) + 208*c*d**3*x*sin(a + b*x)**4*cos(a + b
*x)/(75*b**3) + 1352*c*d**3*x*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) +
 3424*c*d**3*x*cos(a + b*x)**5/(1125*b**3) + 104*d**4*x**2*sin(a + b*x)**4
*cos(a + b*x)/(75*b**3) + 676*d**4*x**2*sin(a + b*x)**2*cos(a + b*x)**3...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1339 vs. $2(304) = 608$.

Time $= 0.09$ (sec) , antiderivative size $= 1339$, normalized size of antiderivative $= 4.06$

$$\int (c + dx)^4 \cos^2(a + bx) \sin^3(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output

```
1/4050000*(270000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c^4 - 1080000*(3*c
os(b*x + a)^5 - 5*cos(b*x + a)^3)*a*c^3*d/b + 1620000*(3*cos(b*x + a)^5 -
5*cos(b*x + a)^3)*a^2*c^2*d^2/b^2 - 1080000*(3*cos(b*x + a)^5 - 5*cos(b*x
+ a)^3)*a^3*c*d^3/b^3 + 270000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*a^4*d
^4/b^4 + 4500*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*
a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a)
 + 450*sin(b*x + a))*c^3*d/b - 13500*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(
b*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a
) + 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a*c^2*d^2/b^2 + 13500*(45*(b*x
 + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos
(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a
^2*c*d^3/b^3 - 4500*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*
x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x
+ 3*a) + 450*sin(b*x + a))*a^3*d^4/b^4 + 450*(27*(25*(b*x + a)^2 - 2)*cos(
5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 6750*((b*x + a)^
2 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*x + a)*sin(3
*b*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*c^2*d^2/b^2 - 900*(27*(25*(b*x
 + a)^2 - 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) -
 6750*((b*x + a)^2 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 75
0*(b*x + a)*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*a*c*d^3/b^...
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.61

$$
\begin{aligned}
&\int (c+dx)^4 \cos^2(a+bx) \sin^3(a+bx)\, dx \\
&= \frac{(625\, b^4 d^4 x^4 + 2500\, b^4 c d^3 x^3 + 3750\, b^4 c^2 d^2 x^2 + 2500\, b^4 c^3 d x + 625\, b^4 c^4 - 300\, b^2 d^4 x^2 - 600\, b^2 c d^3 x - 300\, b^2
\\
&\quad - \frac{(27\, b^4 d^4 x^4 + 108\, b^4 c d^3 x^3 + 162\, b^4 c^2 d^2 x^2 + 108\, b^4 c^3 d x + 27\, b^4 c^4 - 36\, b^2 d^4 x^2 - 72\, b^2 c d^3 x - 36\, b^2 c^2 d^2 +
\\
&\quad - \frac{(b^4 d^4 x^4 + 4\, b^4 c d^3 x^3 + 6\, b^4 c^2 d^2 x^2 + 4\, b^4 c^3 d x + b^4 c^4 - 12\, b^2 d^4 x^2 - 24\, b^2 c d^3 x - 12\, b^2 c^2 d^2 + 24\, d^4) \cos(bx
\\
&\quad - \frac{(25\, b^3 d^4 x^3 + 75\, b^3 c d^3 x^2 + 75\, b^3 c^2 d^2 x + 25\, b^3 c^3 d - 6\, b d^4 x - 6\, b c d^3) \sin(5\, bx + 5\, a)}{2500\, b^5}
\\
&\quad + \frac{(3\, b^3 d^4 x^3 + 9\, b^3 c d^3 x^2 + 9\, b^3 c^2 d^2 x + 3\, b^3 c^3 d - 2\, b d^4 x - 2\, b c d^3) \sin(3\, bx + 3\, a)}{108\, b^5}
\\
&\quad + \frac{(b^3 d^4 x^3 + 3\, b^3 c d^3 x^2 + 3\, b^3 c^2 d^2 x + b^3 c^3 d - 6\, b d^4 x - 6\, b c d^3) \sin(bx + a)}{2\, b^5}
\end{aligned}
$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/50000*(625*b^4*d^4*x^4 + 2500*b^4*c*d^3*x^3 + 3750*b^4*c^2*d^2*x^2 + 250
0*b^4*c^3*d*x + 625*b^4*c^4 - 300*b^2*d^4*x^2 - 600*b^2*c*d^3*x - 300*b^2*
c^2*d^2 + 24*d^4)*cos(5*b*x + 5*a)/b^5 - 1/1296*(27*b^4*d^4*x^4 + 108*b^4*
c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*c^3*d*x + 27*b^4*c^4 - 36*b^2*d^
4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8*d^4)*cos(3*b*x + 3*a)/b^5 - 1/
8*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4
*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^2*c^2*d^2 + 24*d^4)*cos(b*x
+ a)/b^5 - 1/2500*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 75*b^3*c^2*d^2*x +
25*b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*sin(5*b*x + 5*a)/b^5 + 1/108*(3*b^3*
d^4*x^3 + 9*b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*
b*c*d^3)*sin(3*b*x + 3*a)/b^5 + 1/2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3
*c^2*d^2*x + b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*sin(b*x + a)/b^5
```

Mupad [B] (verification not implemented)

Time = 22.93 (sec) , antiderivative size = 816, normalized size of antiderivative = 2.47

$$\int (c+dx)^4 \cos^2(a+bx) \sin^3(a+bx)\, dx = \text{Too large to display}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^4,x)
```

output

```
-(3*d^4*cos(a + b*x) + (d^4*cos(3*a + 3*b*x))/162 - (3*d^4*cos(5*a + 5*b*x
))/6250 + (b^4*c^4*cos(a + b*x))/8 + (b^4*c^4*cos(3*a + 3*b*x))/48 - (b^4*
c^4*cos(5*a + 5*b*x))/80 - (3*b^2*c^2*d^2*cos(a + b*x))/2 - (b^3*c^3*d*sin
(3*a + 3*b*x))/36 + (b^3*c^3*d*sin(5*a + 5*b*x))/100 - (3*b^2*d^4*x^2*cos(
a + b*x))/2 + (b^4*d^4*x^4*cos(a + b*x))/8 - (b^3*d^4*x^3*sin(a + b*x))/2
+ 3*b*c*d^3*sin(a + b*x) - (b^2*c^2*d^2*cos(3*a + 3*b*x))/36 + (3*b^2*c^2*
d^2*cos(5*a + 5*b*x))/500 + 3*b*d^4*x*sin(a + b*x) - (b^2*d^4*x^2*cos(3*a
+ 3*b*x))/36 + (3*b^2*d^4*x^2*cos(5*a + 5*b*x))/500 + (b^4*d^4*x^4*cos(3*a
 + 3*b*x))/48 - (b^4*d^4*x^4*cos(5*a + 5*b*x))/80 - (b^3*d^4*x^3*sin(3*a +
 3*b*x))/36 + (b^3*d^4*x^3*sin(5*a + 5*b*x))/100 + (b*c*d^3*sin(3*a + 3*b*
x))/54 - (3*b*c*d^3*sin(5*a + 5*b*x))/1250 - (b^3*c^3*d*sin(a + b*x))/2 +
(b*d^4*x*sin(3*a + 3*b*x))/54 - (3*b*d^4*x*sin(5*a + 5*b*x))/1250 - 3*b^2*
c*d^3*x*cos(a + b*x) + (b^4*c^3*d*x*cos(a + b*x))/2 + (b^4*c^2*d^2*x^2*cos
(3*a + 3*b*x))/8 - (3*b^4*c^2*d^2*x^2*cos(5*a + 5*b*x))/40 - (b^2*c*d^3*x*
cos(3*a + 3*b*x))/18 + (b^4*c^3*d*x*cos(3*a + 3*b*x))/12 + (3*b^2*c*d^3*x*
cos(5*a + 5*b*x))/250 - (b^4*c^3*d*x*cos(5*a + 5*b*x))/20 + (b^4*c*d^3*x^3
*cos(a + b*x))/2 - (3*b^3*c^2*d^2*x*sin(a + b*x))/2 - (3*b^3*c*d^3*x^2*sin
(a + b*x))/2 + (b^4*c*d^3*x^3*cos(3*a + 3*b*x))/12 - (b^4*c*d^3*x^3*cos(5*
a + 5*b*x))/20 + (3*b^4*c^2*d^2*x^2*cos(a + b*x))/4 - (b^3*c^2*d^2*x*sin(3
*a + 3*b*x))/12 - (b^3*c*d^3*x^2*sin(3*a + 3*b*x))/12 + (3*b^3*c^2*d^2*...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 936, normalized size of antiderivative = 2.84

$$\int (c+dx)^4 \cos^2(a+bx) \sin^3(a+bx)\, dx = \text{Too large to display}$$

input

```
int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

output

```
(50625*cos(a + b*x)*sin(a + b*x)**4*b**4*c**4 + 202500*cos(a + b*x)*sin(a
+ b*x)**4*b**4*c**3*d*x + 303750*cos(a + b*x)*sin(a + b*x)**4*b**4*c**2*d*
*2*x**2 + 202500*cos(a + b*x)*sin(a + b*x)**4*b**4*c*d**3*x**3 + 50625*cos
(a + b*x)*sin(a + b*x)**4*b**4*d**4*x**4 - 24300*cos(a + b*x)*sin(a + b*x)
**4*b**2*c**2*d**2 - 48600*cos(a + b*x)*sin(a + b*x)**4*b**2*c*d**3*x - 24
300*cos(a + b*x)*sin(a + b*x)**4*b**2*d**4*x**2 + 1944*cos(a + b*x)*sin(a
+ b*x)**4*d**4 - 16875*cos(a + b*x)*sin(a + b*x)**2*b**4*c**4 - 67500*cos(
a + b*x)*sin(a + b*x)**2*b**4*c**3*d*x - 101250*cos(a + b*x)*sin(a + b*x)*
*2*b**4*c**2*d**2*x**2 - 67500*cos(a + b*x)*sin(a + b*x)**2*b**4*c*d**3*x*
*3 - 16875*cos(a + b*x)*sin(a + b*x)**2*b**4*d**4*x**4 - 9900*cos(a + b*x)
*sin(a + b*x)**2*b**2*c**2*d**2 - 19800*cos(a + b*x)*sin(a + b*x)**2*b**2*
c*d**3*x - 9900*cos(a + b*x)*sin(a + b*x)**2*b**2*d**4*x**2 + 4792*cos(a +
 b*x)*sin(a + b*x)**2*d**4 - 33750*cos(a + b*x)*b**4*c**4 - 135000*cos(a +
 b*x)*b**4*c**3*d*x - 202500*cos(a + b*x)*b**4*c**2*d**2*x**2 - 135000*cos
(a + b*x)*b**4*c*d**3*x**3 - 33750*cos(a + b*x)*b**4*d**4*x**4 + 385200*co
s(a + b*x)*b**2*c**2*d**2 + 770400*cos(a + b*x)*b**2*c*d**3*x + 385200*cos
(a + b*x)*b**2*d**4*x**2 - 760816*cos(a + b*x)*d**4 - 40500*sin(a + b*x)**
5*b**3*c**3*d - 121500*sin(a + b*x)**5*b**3*c**2*d**2*x - 121500*sin(a + b
*x)**5*b**3*c*d**3*x**2 - 40500*sin(a + b*x)**5*b**3*d**4*x**3 + 9720*sin(
a + b*x)**5*b*c*d**3 + 9720*sin(a + b*x)**5*b*d**4*x + 22500*sin(a + b*...
```

3.90 $\int (c+dx)^3 \cos^2(a+bx) \sin^3(a+bx)\, dx$

Optimal result . . . 866
Mathematica [A] (verified) . . . 867
Rubi [A] (verified) . . . 867
Maple [A] (verified) . . . 868
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Maxima [B] (verification not implemented) . . . 870
Giac [A] (verification not implemented) . . . 871
Mupad [B] (verification not implemented) . . . 872
Reduce [B] (verification not implemented) . . . 873

Optimal result

Integrand size = 24, antiderivative size = 259

$$\begin{aligned}
&\int (c+dx)^3 \cos^2(a+bx) \sin^3(a+bx)\, dx \\
&= \frac{3d^2(c+dx)\cos(a+bx)}{4b^3} - \frac{(c+dx)^3\cos(a+bx)}{8b} + \frac{d^2(c+dx)\cos(3a+3bx)}{72b^3} \\
&\quad - \frac{(c+dx)^3\cos(3a+3bx)}{48b} - \frac{3d^2(c+dx)\cos(5a+5bx)}{1000b^3} + \frac{(c+dx)^3\cos(5a+5bx)}{80b} \\
&\quad - \frac{3d^3\sin(a+bx)}{4b^4} + \frac{3d(c+dx)^2\sin(a+bx)}{8b^2} - \frac{d^3\sin(3a+3bx)}{216b^4} \\
&\quad + \frac{d(c+dx)^2\sin(3a+3bx)}{48b^2} + \frac{3d^3\sin(5a+5bx)}{5000b^4} - \frac{3d(c+dx)^2\sin(5a+5bx)}{400b^2}
\end{aligned}$$

output

```
3/4*d^2*(d*x+c)*cos(b*x+a)/b^3-1/8*(d*x+c)^3*cos(b*x+a)/b+1/72*d^2*(d*x+c)
*cos(3*b*x+3*a)/b^3-1/48*(d*x+c)^3*cos(3*b*x+3*a)/b-3/1000*d^2*(d*x+c)*cos
(5*b*x+5*a)/b^3+1/80*(d*x+c)^3*cos(5*b*x+5*a)/b-3/4*d^3*sin(b*x+a)/b^4+3/8
*d*(d*x+c)^2*sin(b*x+a)/b^2-1/216*d^3*sin(3*b*x+3*a)/b^4+1/48*d*(d*x+c)^2*
sin(3*b*x+3*a)/b^2+3/5000*d^3*sin(5*b*x+5*a)/b^4-3/400*d*(d*x+c)^2*sin(5*b
*x+5*a)/b^2
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.42

$$\int (c+dx)^3 \cos^2(a+bx) \sin^3(a+bx)\, dx$$
$$= \frac{-33750b(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\cos(a+bx) - 1875b(c+dx)\left(-2d^2+3b^2(c+dx)^2\right)\cos(3(a+bx$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(-33750*b*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 1875*b*(c +
d*x)*(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 3375*b^3*c^3*Cos[5*(a
 + b*x)] - 810*b*c*d^2*Cos[5*(a + b*x)] + 10125*b^3*c^2*d*x*Cos[5*(a + b*x
)] - 810*b*d^3*x*Cos[5*(a + b*x)] + 10125*b^3*c*d^2*x^2*Cos[5*(a + b*x)] +
 3375*b^3*d^3*x^3*Cos[5*(a + b*x)] + 101250*b^2*c^2*d*Sin[a + b*x] - 20250
0*d^3*Sin[a + b*x] + 202500*b^2*c*d^2*x*Sin[a + b*x] + 101250*b^2*d^3*x^2*
Sin[a + b*x] + 5625*b^2*c^2*d*Sin[3*(a + b*x)] - 1250*d^3*Sin[3*(a + b*x)]
 + 11250*b^2*c*d^2*x*Sin[3*(a + b*x)] + 5625*b^2*d^3*x^2*Sin[3*(a + b*x)]
- 2025*b^2*c^2*d*Sin[5*(a + b*x)] + 162*d^3*Sin[5*(a + b*x)] - 4050*b^2*c*
d^2*x*Sin[5*(a + b*x)] - 2025*b^2*d^3*x^2*Sin[5*(a + b*x)])/(270000*b^4)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin^3(a+bx) \cos^2(a+bx)\, dx$$
$$\downarrow 4906$$
$$\int \left(\frac{1}{8}(c+dx)^3 \sin(a+bx) + \frac{1}{16}(c+dx)^3 \sin(3a+3bx) - \frac{1}{16}(c+dx)^3 \sin(5a+5bx)\right) dx$$
$$\downarrow 2009$$

$$-\frac{3d^3\sin(a+bx)}{4b^4}-\frac{d^3\sin(3a+3bx)}{216b^4}+\frac{3d^3\sin(5a+5bx)}{5000b^4}+\frac{3d^2(c+dx)\cos(a+bx)}{4b^3}+\frac{d^2(c+dx)\cos(3a+3bx)}{72b^3}-\frac{3d^2(c+dx)\cos(5a+5bx)}{1000b^3}+\frac{3d(c+dx)^2\sin(a+bx)}{8b^2}+\frac{d(c+dx)^2\sin(3a+3bx)}{48b^2}-\frac{3d(c+dx)^2\sin(5a+5bx)}{400b^2}-\frac{(c+dx)^3\cos(a+bx)}{8b}-\frac{(c+dx)^3\cos(3a+3bx)}{48b}+\frac{(c+dx)^3\cos(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(3*d^2*(c + d*x)*Cos[a + b*x])/(4*b^3) - ((c + d*x)^3*Cos[a + b*x])/(8*b)
+ (d^2*(c + d*x)*Cos[3*a + 3*b*x])/(72*b^3) - ((c + d*x)^3*Cos[3*a + 3*b*x
])/(48*b) - (3*d^2*(c + d*x)*Cos[5*a + 5*b*x])/(1000*b^3) + ((c + d*x)^3*C
os[5*a + 5*b*x])/(80*b) - (3*d^3*Sin[a + b*x])/(4*b^4) + (3*d*(c + d*x)^2*
Sin[a + b*x])/(8*b^2) - (d^3*Sin[3*a + 3*b*x])/(216*b^4) + (d*(c + d*x)^2*
Sin[3*a + 3*b*x])/(48*b^2) + (3*d^3*Sin[5*a + 5*b*x])/(5000*b^4) - (3*d*(c
 + d*x)^2*Sin[5*a + 5*b*x])/(400*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.79

method	result
parallelrisch	$\frac{-375(dx+c)\left((dx+c)^2b^2-\frac{2d^2}{3}\right)b\cos(3bx+3a)+225\left((dx+c)^2b^2-\frac{6d^2}{25}\right)(dx+c)b\cos(5bx+5a)+375d\left((dx+c)^2b^2-\frac{2d^2}{9}\right)$
risch	$-\frac{(b^2d^3x^3+3b^2cd^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6cd^2)\cos(bx+a)}{8b^3}+\frac{3d(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\sin(bx+a)}{8b^4}+\frac{(2$
derivativedivides	Expression too large to display
default	Expression too large to display
orering	Expression too large to display

input
```
int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/18000*(-375*(d*x+c)*((d*x+c)^2*b^2-2/3*d^2)*b*cos(3*b*x+3*a)+225*((d*x+c
)^2*b^2-6/25*d^2)*(d*x+c)*b*cos(5*b*x+5*a)+375*d*((d*x+c)^2*b^2-2/9*d^2)*s
in(3*b*x+3*a)-135*d*((d*x+c)^2*b^2-2/25*d^2)*sin(5*b*x+5*a)-2250*((d*x+c)^
2*b^2-6*d^2)*(d*x+c)*b*cos(b*x+a)+6750*d*((d*x+c)^2*b^2-2*d^2)*sin(b*x+a)-
2400*b^3*c^3+13696*c*d^2*b)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.14

$$\int (c+dx)^3\cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$=\frac{135\left(25\,b^3d^3x^3+75\,b^3cd^2x^2+25\,b^3c^3-6\,bcd^2+3\left(25\,b^3c^2d-2\,bd^3\right)x\right)\cos\left(bx+a\right)^5-75\left(75\,b^3d^3x^3+22$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/16875*(135*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 25*b^3*c^3 - 6*b*c*d^2 +
 3*(25*b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^5 - 75*(75*b^3*d^3*x^3 + 225*b
^3*c*d^2*x^2 + 75*b^3*c^3 - 26*b*c*d^2 + (225*b^3*c^2*d - 26*b*d^3)*x)*cos
(b*x + a)^3 + 11700*(b*d^3*x + b*c*d^2)*cos(b*x + a) + (5850*b^2*d^3*x^2 +
 11700*b^2*c*d^2*x + 5850*b^2*c^2*d - 81*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x
+ 25*b^2*c^2*d - 2*d^3)*cos(b*x + a)^4 - 12568*d^3 + (2925*b^2*d^3*x^2 + 5
850*b^2*c*d^2*x + 2925*b^2*c^2*d - 434*d^3)*cos(b*x + a)^2)*sin(b*x + a))/
b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(255) = 510$.

Time = 0.85 (sec) , antiderivative size = 690, normalized size of antiderivative = 2.66

$$\int (c+dx)^3 \cos^2(a+bx) \sin^3(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output
```
Piecewise((-c**3*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c**3*cos(a + b*
x)**5/(15*b) - c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**3/b - 2*c**2*d*x*cos
(a + b*x)**5/(5*b) - c*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**3/b - 2*c*d
**2*x**2*cos(a + b*x)**5/(5*b) - d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**3
/(3*b) - 2*d**3*x**3*cos(a + b*x)**5/(15*b) + 26*c**2*d*sin(a + b*x)**5/(7
5*b**2) + 13*c**2*d*sin(a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 2*c**2*d*s
in(a + b*x)*cos(a + b*x)**4/(5*b**2) + 52*c*d**2*x*sin(a + b*x)**5/(75*b**
2) + 26*c*d**2*x*sin(a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 4*c*d**2*x*si
n(a + b*x)*cos(a + b*x)**4/(5*b**2) + 26*d**3*x**2*sin(a + b*x)**5/(75*b**
2) + 13*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 2*d**3*x**2*
sin(a + b*x)*cos(a + b*x)**4/(5*b**2) + 52*c*d**2*sin(a + b*x)**4*cos(a +
b*x)/(75*b**3) + 338*c*d**2*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 8
56*c*d**2*cos(a + b*x)**5/(1125*b**3) + 52*d**3*x*sin(a + b*x)**4*cos(a +
b*x)/(75*b**3) + 338*d**3*x*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 8
56*d**3*x*cos(a + b*x)**5/(1125*b**3) - 12568*d**3*sin(a + b*x)**5/(16875*
b**4) - 5114*d**3*sin(a + b*x)**3*cos(a + b*x)**2/(3375*b**4) - 856*d**3*s
in(a + b*x)*cos(a + b*x)**4/(1125*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x*
*2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**3*cos(a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 766 vs. $2(235) = 470$.

Time = 0.07 (sec) , antiderivative size = 766, normalized size of antiderivative = 2.96

$$\int (c+dx)^3 \cos^2(a+bx) \sin^3(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output

```
1/270000*(18000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c^3 - 54000*(3*cos(b
*x + a)^5 - 5*cos(b*x + a)^3)*a*c^2*d/b + 54000*(3*cos(b*x + a)^5 - 5*cos(
b*x + a)^3)*a^2*c*d^2/b^2 - 18000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*a^
3*d^3/b^3 + 225*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x +
3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*
a) + 450*sin(b*x + a))*c^2*d/b - 450*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(
b*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a
) + 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a*c*d^2/b^2 + 225*(45*(b*x + a
)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x
 + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a^2*d
^3/b^3 + 15*(27*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2
 - 2)*cos(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*cos(b*x + a) - 270*(b*x +
a)*sin(5*b*x + 5*a) + 750*(b*x + a)*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin
(b*x + a))*c*d^2/b^2 - 15*(27*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) - 125*
(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*cos(b*x + a)
 - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*x + a)*sin(3*b*x + 3*a) + 13500
*(b*x + a)*sin(b*x + a))*a*d^3/b^3 + (135*(25*(b*x + a)^3 - 6*b*x - 6*a)*c
os(5*b*x + 5*a) - 1875*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) - 33
750*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) - 81*(25*(b*x + a)^2 - 2)*sin
(5*b*x + 5*a) + 625*(9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 101250*((b*x...
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.36

$$
\begin{aligned}
&\int (c+dx)^3 \cos^2(a+bx) \sin^3(a+bx)\, dx \\
&= \frac{(25\,b^3d^3x^3 + 75\,b^3cd^2x^2 + 75\,b^3c^2dx + 25\,b^3c^3 - 6\,bd^3x - 6\,bcd^2)\cos(5\,bx+5\,a)}{2000\,b^4} \\
&\quad - \frac{(3\,b^3d^3x^3 + 9\,b^3cd^2x^2 + 9\,b^3c^2dx + 3\,b^3c^3 - 2\,bd^3x - 2\,bcd^2)\cos(3\,bx+3\,a)}{144\,b^4} \\
&\quad - \frac{(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3 - 6\,bd^3x - 6\,bcd^2)\cos(bx+a)}{8\,b^4} \\
&\quad - \frac{3\,(25\,b^2d^3x^2 + 50\,b^2cd^2x + 25\,b^2c^2d - 2\,d^3)\sin(5\,bx+5\,a)}{10000\,b^4} \\
&\quad + \frac{(9\,b^2d^3x^2 + 18\,b^2cd^2x + 9\,b^2c^2d - 2\,d^3)\sin(3\,bx+3\,a)}{432\,b^4} \\
&\quad + \frac{3\,(b^2d^3x^2 + 2\,b^2cd^2x + b^2c^2d - 2\,d^3)\sin(bx+a)}{8\,b^4}
\end{aligned}
$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2000*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 75*b^3*c^2*d*x + 25*b^3*c^3 -
6*b*d^3*x - 6*b*c*d^2)*cos(5*b*x + 5*a)/b^4 - 1/144*(3*b^3*d^3*x^3 + 9*b^3
*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^3*x - 2*b*c*d^2)*cos(3*b*x
+ 3*a)/b^4 - 1/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3
- 6*b*d^3*x - 6*b*c*d^2)*cos(b*x + a)/b^4 - 3/10000*(25*b^2*d^3*x^2 + 50*b
^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*sin(5*b*x + 5*a)/b^4 + 1/432*(9*b^2*d^3
*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*sin(3*b*x + 3*a)/b^4 + 3/8*(b
^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a)/b^4
```

Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.99

$$\int (c+dx)^3 \cos^2(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{3d^3\sin(a+bx)}{4}+\frac{d^3\sin(3a+3bx)}{216}-\frac{3d^3\sin(5a+5bx)}{5000}+\frac{b^3c^3\cos(a+bx)}{8}+\frac{b^3c^3\cos(3a+3bx)}{48}-\frac{b^3c^3\cos(5a+5bx)}{80}-\frac{b^2c^2}{}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^3,x)
```

output
```
-((3*d^3*sin(a + b*x))/4 + (d^3*sin(3*a + 3*b*x))/216 - (3*d^3*sin(5*a + 5
*b*x))/5000 + (b^3*c^3*cos(a + b*x))/8 + (b^3*c^3*cos(3*a + 3*b*x))/48 - (
b^3*c^3*cos(5*a + 5*b*x))/80 - (b^2*c^2*d*sin(3*a + 3*b*x))/48 + (3*b^2*c^
2*d*sin(5*a + 5*b*x))/400 + (b^3*d^3*x^3*cos(a + b*x))/8 - (3*b^2*d^3*x^2*
sin(a + b*x))/8 - (3*b*c*d^2*cos(a + b*x))/4 - (3*b*d^3*x*cos(a + b*x))/4
+ (b^3*d^3*x^3*cos(3*a + 3*b*x))/48 - (b^3*d^3*x^3*cos(5*a + 5*b*x))/80 -
(b^2*d^3*x^2*sin(3*a + 3*b*x))/48 + (3*b^2*d^3*x^2*sin(5*a + 5*b*x))/400 -
 (b*c*d^2*cos(3*a + 3*b*x))/72 + (3*b*c*d^2*cos(5*a + 5*b*x))/1000 - (3*b^
2*c^2*d*sin(a + b*x))/8 - (b*d^3*x*cos(3*a + 3*b*x))/72 + (3*b*d^3*x*cos(5
*a + 5*b*x))/1000 + (3*b^3*c^2*d*x*cos(a + b*x))/8 - (3*b^2*c*d^2*x*sin(a
+ b*x))/4 + (b^3*c^2*d*x*cos(3*a + 3*b*x))/16 - (3*b^3*c^2*d*x*cos(5*a + 5
*b*x))/80 + (3*b^3*c*d^2*x^2*cos(a + b*x))/8 - (b^2*c*d^2*x*sin(3*a + 3*b*
x))/24 + (3*b^2*c*d^2*x*sin(5*a + 5*b*x))/200 + (b^3*c*d^2*x^2*cos(3*a + 3
*b*x))/16 - (3*b^3*c*d^2*x^2*cos(5*a + 5*b*x))/80)/b^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 599, normalized size of antiderivative = 2.31

$$\int (c+dx)^3 \cos^2(a+bx) \sin^3(a+bx)\, dx$$
$$= \frac{-1125 \cos(bx+a) \sin(bx+a)^2 b^3 d^3 x^3 - 330 \cos(bx+a) \sin(bx+a)^2 bc\, d^2 - 330 \cos(bx+a) \sin(bx+$$

input
```
int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

output
```
(3375*cos(a + b*x)*sin(a + b*x)**4*b**3*c**3 + 10125*cos(a + b*x)*sin(a +
b*x)**4*b**3*c**2*d*x + 10125*cos(a + b*x)*sin(a + b*x)**4*b**3*c*d**2*x**
2 + 3375*cos(a + b*x)*sin(a + b*x)**4*b**3*d**3*x**3 - 810*cos(a + b*x)*si
n(a + b*x)**4*b*c*d**2 - 810*cos(a + b*x)*sin(a + b*x)**4*b*d**3*x - 1125*
cos(a + b*x)*sin(a + b*x)**2*b**3*c**3 - 3375*cos(a + b*x)*sin(a + b*x)**2
*b**3*c**2*d*x - 3375*cos(a + b*x)*sin(a + b*x)**2*b**3*c*d**2*x**2 - 1125
*cos(a + b*x)*sin(a + b*x)**2*b**3*d**3*x**3 - 330*cos(a + b*x)*sin(a + b*
x)**2*b*c*d**2 - 330*cos(a + b*x)*sin(a + b*x)**2*b*d**3*x - 2250*cos(a +
b*x)*b**3*c**3 - 6750*cos(a + b*x)*b**3*c**2*d*x - 6750*cos(a + b*x)*b**3*
c*d**2*x**2 - 2250*cos(a + b*x)*b**3*d**3*x**3 + 12840*cos(a + b*x)*b*c*d*
*2 + 12840*cos(a + b*x)*b*d**3*x - 2025*sin(a + b*x)**5*b**2*c**2*d - 4050
*sin(a + b*x)**5*b**2*c*d**2*x - 2025*sin(a + b*x)**5*b**2*d**3*x**2 + 162
*sin(a + b*x)**5*d**3 + 1125*sin(a + b*x)**3*b**2*c**2*d + 2250*sin(a + b*
x)**3*b**2*c*d**2*x + 1125*sin(a + b*x)**3*b**2*d**3*x**2 + 110*sin(a + b*
x)**3*d**3 + 6750*sin(a + b*x)*b**2*c**2*d + 13500*sin(a + b*x)*b**2*c*d**
2*x + 6750*sin(a + b*x)*b**2*d**3*x**2 - 12840*sin(a + b*x)*d**3 + 6750*a*
b**2*c**2*d - 7440*a*d**3 - 2250*b**3*c**3 + 7440*b*c*d**2)/(16875*b**4)
```

3.91 $\int (c+dx)^2 \cos^2(a+bx) \sin^3(a+bx)\, dx$

Optimal result . . . 874
Mathematica [A] (verified) . . . 875
Rubi [A] (verified) . . . 875
Maple [A] (verified) . . . 876
Fricas [A] (verification not implemented) . . . 877
Sympy [B] (verification not implemented) . . . 877
Maxima [B] (verification not implemented) . . . 878
Giac [A] (verification not implemented) . . . 879
Mupad [B] (verification not implemented) . . . 879
Reduce [B] (verification not implemented) . . . 880

Optimal result

Integrand size = 24, antiderivative size = 184

$$\begin{aligned}\int (c+dx)^2 \cos^2(a+bx) \sin^3(a+bx)\, dx = {} & \frac{d^2 \cos(a+bx)}{4b^3} - \frac{(c+dx)^2 \cos(a+bx)}{8b} \\ & + \frac{d^2 \cos(3a+3bx)}{216b^3} - \frac{(c+dx)^2 \cos(3a+3bx)}{48b} \\ & - \frac{d^2 \cos(5a+5bx)}{1000b^3} + \frac{(c+dx)^2 \cos(5a+5bx)}{80b} \\ & + \frac{d(c+dx)\sin(a+bx)}{4b^2} \\ & + \frac{d(c+dx)\sin(3a+3bx)}{72b^2} \\ & - \frac{d(c+dx)\sin(5a+5bx)}{200b^2}\end{aligned}$$

output

```
1/4*d^2*cos(b*x+a)/b^3-1/8*(d*x+c)^2*cos(b*x+a)/b+1/216*d^2*cos(3*b*x+3*a)
/b^3-1/48*(d*x+c)^2*cos(3*b*x+3*a)/b-1/1000*d^2*cos(5*b*x+5*a)/b^3+1/80*(d
*x+c)^2*cos(5*b*x+5*a)/b+1/4*d*(d*x+c)*sin(b*x+a)/b^2+1/72*d*(d*x+c)*sin(3
*b*x+3*a)/b^2-1/200*d*(d*x+c)*sin(5*b*x+5*a)/b^2
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

$$\int (c+dx)^2 \cos^2(a+bx) \sin^3(a+bx)\, dx$$
$$= \frac{-6750(-2d^2+b^2(c+dx)^2)\cos(a+bx) - 125(-2d^2+9b^2(c+dx)^2)\cos(3(a+bx)) + 27(-2d^2+25b^2(c$$
$$54000b^3$$

input
```
Integrate[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(-6750*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 125*(-2*d^2 + 9*b^2*(c +
d*x)^2)*Cos[3*(a + b*x)] + 27*(-2*d^2 + 25*b^2*(c + d*x)^2)*Cos[5*(a + b*x
)] + 30*b*d*(c + d*x)*(450*Sin[a + b*x] + 25*Sin[3*(a + b*x)] - 9*Sin[5*(a
 + b*x)]))/(54000*b^3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin^3(a+bx) \cos^2(a+bx)\, dx$$
$$\downarrow 4906$$
$$\int \left(\frac{1}{8}(c+dx)^2 \sin(a+bx) + \frac{1}{16}(c+dx)^2 \sin(3a+3bx) - \frac{1}{16}(c+dx)^2 \sin(5a+5bx)\right) dx$$
$$\downarrow 2009$$
$$\frac{d^2\cos(a+bx)}{4b^3} + \frac{d^2\cos(3a+3bx)}{216b^3} - \frac{d^2\cos(5a+5bx)}{1000b^3} + \frac{d(c+dx)\sin(a+bx)}{4b^2} +$$
$$\frac{d(c+dx)\sin(3a+3bx)}{72b^2} - \frac{d(c+dx)\sin(5a+5bx)}{200b^2} - \frac{(c+dx)^2\cos(a+bx)}{8b} -$$
$$\frac{(c+dx)^2\cos(3a+3bx)}{48b} + \frac{(c+dx)^2\cos(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(d^2*Cos[a + b*x])/(4*b^3) - ((c + d*x)^2*Cos[a + b*x])/(8*b) + (d^2*Cos[3
*a + 3*b*x])/(216*b^3) - ((c + d*x)^2*Cos[3*a + 3*b*x])/(48*b) - (d^2*Cos[
5*a + 5*b*x])/(1000*b^3) + ((c + d*x)^2*Cos[5*a + 5*b*x])/(80*b) + (d*(c +
 d*x)*Sin[a + b*x])/(4*b^2) + (d*(c + d*x)*Sin[3*a + 3*b*x])/(72*b^2) - (d
*(c + d*x)*Sin[5*a + 5*b*x])/(200*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{\left(-1125(dx+c)^2b^2+250d^2\right)\cos(3bx+3a)+\left(675(dx+c)^2b^2-54d^2\right)\cos(5bx+5a)+750bd(dx+c)\sin(3bx+3a)-270bd(d$ $54000b^3$
risch	$-\frac{\left(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2\right)\cos(bx+a)}{8b^3}+\frac{d(dx+c)\sin(bx+a)}{4b^2}+\frac{\left(25x^2d^2b^2+50b^2cdx+25b^2c^2-2d^2\right)\cos(5bx+5$
derivativedivides	$\frac{a^2d^2\left(-\frac{\sin(bx+a)^2\cos(bx+a)^3}{5}-\frac{2\cos(bx+a)^3}{15}\right)}{b^2}-\frac{2acd\left(-\frac{\sin(bx+a)^2\cos(bx+a)^3}{5}-\frac{2\cos(bx+a)^3}{15}\right)}{b}-\frac{2a\,d^2\left(-\frac{(bx+a)\left(2+\sin(bx\right.}{}\right.$
default	$\frac{a^2d^2\left(-\frac{\sin(bx+a)^2\cos(bx+a)^3}{5}-\frac{2\cos(bx+a)^3}{15}\right)}{b^2}-\frac{2acd\left(-\frac{\sin(bx+a)^2\cos(bx+a)^3}{5}-\frac{2\cos(bx+a)^3}{15}\right)}{b}-\frac{2a\,d^2\left(-\frac{(bx+a)\left(2+\sin(bx\right.}{}\right.$
orering	Expression too large to display

input
```
int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/54000*((-1125*(d*x+c)^2*b^2+250*d^2)*cos(3*b*x+3*a)+(675*(d*x+c)^2*b^2-5
4*d^2)*cos(5*b*x+5*a)+750*b*d*(d*x+c)*sin(3*b*x+3*a)-270*b*d*(d*x+c)*sin(5
*b*x+5*a)+(-6750*(d*x+c)^2*b^2+13500*d^2)*cos(b*x+a)+13500*b*d*(d*x+c)*sin
(b*x+a)-7200*b^2*c^2+13696*d^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.90

$$\int (c+dx)^2 \cos^2(a+bx) \sin^3(a+bx)\, dx$$
$$= \frac{27\left(25\,b^2d^2x^2+50\,b^2cdx+25\,b^2c^2-2\,d^2\right)\cos\left(bx+a\right)^5-5\left(225\,b^2d^2x^2+450\,b^2cdx+225\,b^2c^2-26\,d^2\right)c}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/3375*(27*(25*b^2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*cos(b*x +
a)^5 - 5*(225*b^2*d^2*x^2 + 450*b^2*c*d*x + 225*b^2*c^2 - 26*d^2)*cos(b*x
+ a)^3 + 780*d^2*cos(b*x + a) - 30*(9*(b*d^2*x + b*c*d)*cos(b*x + a)^4 - 2
6*b*d^2*x - 26*b*c*d - 13*(b*d^2*x + b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/
b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(172) = 344$.

Time = 0.59 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.08

$$\int (c+dx)^2 \cos^2(a+bx) \sin^3(a+bx)\, dx$$
$$= \begin{cases} -\frac{c^2 \sin^2(a+bx)\cos^3(a+bx)}{3b} - \frac{2c^2\cos^5(a+bx)}{15b} - \frac{2cdx\sin^2(a+bx)\cos^3(a+bx)}{3b} - \frac{4cdx\cos^5(a+bx)}{15b} - \frac{d^2x^2\sin^2(a+bx)\cos^3(a+bx)}{3b} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin^3(a)\cos^2(a) \end{cases}$$

input
```
integrate((d*x+c)**2*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output
```
Piecewise((-c**2*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c**2*cos(a + b*
x)**5/(15*b) - 2*c*d*x*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 4*c*d*x*cos
(a + b*x)**5/(15*b) - d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*
d**2*x**2*cos(a + b*x)**5/(15*b) + 52*c*d*sin(a + b*x)**5/(225*b**2) + 26*
c*d*sin(a + b*x)**3*cos(a + b*x)**2/(45*b**2) + 4*c*d*sin(a + b*x)*cos(a +
 b*x)**4/(15*b**2) + 52*d**2*x*sin(a + b*x)**5/(225*b**2) + 26*d**2*x*sin(
a + b*x)**3*cos(a + b*x)**2/(45*b**2) + 4*d**2*x*sin(a + b*x)*cos(a + b*x)
**4/(15*b**2) + 52*d**2*sin(a + b*x)**4*cos(a + b*x)/(225*b**3) + 338*d**2
*sin(a + b*x)**2*cos(a + b*x)**3/(675*b**3) + 856*d**2*cos(a + b*x)**5/(33
75*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**3*cos(a)**
2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(166) = 332$.

Time $= 0.05$ (sec) , antiderivative size $= 375$, normalized size of antiderivative $= 2.04$

$$\int (c+dx)^2 \cos^2(a+bx) \sin^3(a+bx)\, dx$$

$$= \frac{3600\left(3\cos(bx+a)^5 - 5\cos(bx+a)^3\right)c^2 - \frac{7200\left(3\cos(bx+a)^5 - 5\cos(bx+a)^3\right)acd}{b} + \frac{3600\left(3\cos(bx+a)^5 - 5\cos(bx+a)^3\right.}{b^2}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/54000*(3600*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c^2 - 7200*(3*cos(b*x
+ a)^5 - 5*cos(b*x + a)^3)*a*c*d/b + 3600*(3*cos(b*x + a)^5 - 5*cos(b*x + 
a)^3)*a^2*d^2/b^2 + 30*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3
*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b
*x + 3*a) + 450*sin(b*x + a))*c*d/b - 30*(45*(b*x + a)*cos(5*b*x + 5*a) - 
75*(b*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 
 5*a) + 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a*d^2/b^2 + (27*(25*(b*x + 
 a)^2 - 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 6
750*((b*x + a)^2 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*
(b*x + a)*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*d^2/b^2)/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.14

$$\begin{aligned}
&\int (c+dx)^2 \cos^2(a+bx) \sin^3(a+bx)\, dx \\
&= \frac{(25\,b^2d^2x^2 + 50\,b^2cdx + 25\,b^2c^2 - 2\,d^2)\cos(5\,bx+5\,a)}{2000\,b^3} \\
&\quad - \frac{(9\,b^2d^2x^2 + 18\,b^2cdx + 9\,b^2c^2 - 2\,d^2)\cos(3\,bx+3\,a)}{432\,b^3} \\
&\quad - \frac{(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 - 2\,d^2)\cos(bx+a)}{8\,b^3} - \frac{(bd^2x + bcd)\sin(5\,bx+5\,a)}{200\,b^3} \\
&\quad + \frac{(bd^2x + bcd)\sin(3\,bx+3\,a)}{72\,b^3} + \frac{(bd^2x + bcd)\sin(bx+a)}{4\,b^3}
\end{aligned}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2000*(25*b^2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*cos(5*b*x + 5*
a)/b^3 - 1/432*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(3*b*
x + 3*a)/b^3 - 1/8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x +
 a)/b^3 - 1/200*(b*d^2*x + b*c*d)*sin(5*b*x + 5*a)/b^3 + 1/72*(b*d^2*x + b
*c*d)*sin(3*b*x + 3*a)/b^3 + 1/4*(b*d^2*x + b*c*d)*sin(b*x + a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.35

$$\begin{aligned}
&\int (c+dx)^2 \cos^2(a+bx) \sin^3(a+bx)\, dx \\
&= \frac{780\,d^2\,\cos(a+b\,x) + 130\,d^2\cos(a+b\,x)^3 - 54\,d^2\cos(a+b\,x)^5 - 1125\,b^2\,c^2\cos(a+b\,x)^3 + 675\,b^2\,c^2\cos}{}
\end{aligned}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^2,x)
```

output
```
(780*d^2*cos(a + b*x) + 130*d^2*cos(a + b*x)^3 - 54*d^2*cos(a + b*x)^5 - 1
125*b^2*c^2*cos(a + b*x)^3 + 675*b^2*c^2*cos(a + b*x)^5 + 780*b*d^2*x*sin(
a + b*x) - 1125*b^2*d^2*x^2*cos(a + b*x)^3 + 675*b^2*d^2*x^2*cos(a + b*x)^
5 + 780*b*c*d*sin(a + b*x) - 2250*b^2*c*d*x*cos(a + b*x)^3 + 1350*b^2*c*d*
x*cos(a + b*x)^5 + 390*b*d^2*x*cos(a + b*x)^2*sin(a + b*x) - 270*b*d^2*x*c
os(a + b*x)^4*sin(a + b*x) + 390*b*c*d*cos(a + b*x)^2*sin(a + b*x) - 270*b
*c*d*cos(a + b*x)^4*sin(a + b*x))/(3375*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.83

$$\int (c+dx)^2 \cos^2(a+bx) \sin^3(a+bx)\, dx$$
$$= \frac{675 \cos(bx+a) \sin(bx+a)^4 b^2c^2 + 1350 \cos(bx+a) \sin(bx+a)^4 b^2cdx + 675 \cos(bx+a) \sin(bx+a)^4$$

input
```
int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

output
```
(675*cos(a + b*x)*sin(a + b*x)**4*b**2*c**2 + 1350*cos(a + b*x)*sin(a + b*
x)**4*b**2*c*d*x + 675*cos(a + b*x)*sin(a + b*x)**4*b**2*d**2*x**2 - 54*co
s(a + b*x)*sin(a + b*x)**4*d**2 - 225*cos(a + b*x)*sin(a + b*x)**2*b**2*c*
*2 - 450*cos(a + b*x)*sin(a + b*x)**2*b**2*c*d*x - 225*cos(a + b*x)*sin(a
 + b*x)**2*b**2*d**2*x**2 - 22*cos(a + b*x)*sin(a + b*x)**2*d**2 - 450*cos(
a + b*x)*b**2*c**2 - 900*cos(a + b*x)*b**2*c*d*x - 450*cos(a + b*x)*b**2*d
**2*x**2 + 856*cos(a + b*x)*d**2 - 270*sin(a + b*x)**5*b*c*d - 270*sin(a +
 b*x)**5*b*d**2*x + 150*sin(a + b*x)**3*b*c*d + 150*sin(a + b*x)**3*b*d**2
*x + 900*sin(a + b*x)*b*c*d + 900*sin(a + b*x)*b*d**2*x + 900*a*b*c*d - 45
0*b**2*c**2 + 496*d**2)/(3375*b**3)
```

3.92 $\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal result . . . 881
Mathematica [A] (verified) . . . 882
Rubi [A] (verified) . . . 882
Maple [A] (verified) . . . 883
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Giac [A] (verification not implemented) . . . 885
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Reduce [B] (verification not implemented) . . . 886

Optimal result

Integrand size = 22, antiderivative size = 109

$$\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx = -\frac{(c+dx)\cos(a+bx)}{8b} - \frac{(c+dx)\cos(3a+3bx)}{48b} + \frac{(c+dx)\cos(5a+5bx)}{80b} + \frac{d\sin(a+bx)}{8b^2} + \frac{d\sin(3a+3bx)}{144b^2} - \frac{d\sin(5a+5bx)}{400b^2}$$

output

```
-1/8*(d*x+c)*cos(b*x+a)/b-1/48*(d*x+c)*cos(3*b*x+3*a)/b+1/80*(d*x+c)*cos(5
*b*x+5*a)/b+1/8*d*sin(b*x+a)/b^2+1/144*d*sin(3*b*x+3*a)/b^2-1/400*d*sin(5*
b*x+5*a)/b^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{-450b(c+dx)\cos(a+bx) - 75b(c+dx)\cos(3(a+bx)) + 45bc\cos(5(a+bx)) + 45bdx\cos(5(a+bx)) +}{3600b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(-450*b*(c + d*x)*Cos[a + b*x] - 75*b*(c + d*x)*Cos[3*(a + b*x)] + 45*b*c*
Cos[5*(a + b*x)] + 45*b*d*x*Cos[5*(a + b*x)] + 450*d*Sin[a + b*x] + 25*d*S
in[3*(a + b*x)] - 9*d*Sin[5*(a + b*x)])/(3600*b^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin^3(a+bx)\cos^2(a+bx)\,dx$$
$$\downarrow 4906$$
$$\int \left(\frac{1}{8}(c+dx)\sin(a+bx) + \frac{1}{16}(c+dx)\sin(3a+3bx) - \frac{1}{16}(c+dx)\sin(5a+5bx)\right)dx$$
$$\downarrow 2009$$
$$\frac{d\sin(a+bx)}{8b^2} + \frac{d\sin(3a+3bx)}{144b^2} - \frac{d\sin(5a+5bx)}{400b^2} - \frac{(c+dx)\cos(a+bx)}{8b} - \frac{(c+dx)\cos(3a+3bx)}{48b} + \frac{(c+dx)\cos(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
-1/8*((c + d*x)*Cos[a + b*x])/b - ((c + d*x)*Cos[3*a + 3*b*x])/(48*b) + ((
c + d*x)*Cos[5*a + 5*b*x])/(80*b) + (d*Sin[a + b*x])/(8*b^2) + (d*Sin[3*a
+ 3*b*x])/(144*b^2) - (d*Sin[5*a + 5*b*x])/(400*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

method	result
parallelrisch	$\frac{-75b(dx+c)\cos(3bx+3a)+45b(dx+c)\cos(5bx+5a)+25d\sin(3bx+3a)-9\sin(5bx+5a)d-450b(dx+c)\cos(bx+a)-480c}{3600b^2}$
risch	$-\frac{(dx+c)\cos(bx+a)}{8b}-\frac{(dx+c)\cos(3bx+3a)}{48b}+\frac{(dx+c)\cos(5bx+5a)}{80b}+\frac{d\sin(bx+a)}{8b^2}+\frac{d\sin(3bx+3a)}{144b^2}-\frac{d\sin(5}{40}$
derivativedivides	$\frac{-\frac{da\left(-\frac{\sin(bx+a)^2\cos(bx+a)^3}{5}-\frac{2\cos(bx+a)^3}{15}\right)}{b}+c\left(-\frac{\sin(bx+a)^2\cos(bx+a)^3}{5}-\frac{2\cos(bx+a)^3}{15}\right)+\frac{d\left(-\frac{(bx+a)\left(2+\sin(bx+a)^2\right)c}{3}\right.}{b}$
default	$\frac{-\frac{da\left(-\frac{\sin(bx+a)^2\cos(bx+a)^3}{5}-\frac{2\cos(bx+a)^3}{15}\right)}{b}+c\left(-\frac{\sin(bx+a)^2\cos(bx+a)^3}{5}-\frac{2\cos(bx+a)^3}{15}\right)+\frac{d\left(-\frac{(bx+a)\left(2+\sin(bx+a)^2\right)c}{3}\right.}{b}$
norman	$\frac{-\frac{4c}{15b}+\frac{4d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{15b^2}+\frac{56d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3}{45b^2}+\frac{152d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^5}{225b^2}+\frac{56d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^7}{45b^2}+\frac{4d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^9}{15b^2}-\frac{2dx}{15b}-\frac{4c\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{b}$
	$\left(1+\right.$
orering	$\frac{2d\left(259d^4x^4b^4+1036b^4cd^3x^3+1554b^4c^2d^2x^2+1036b^4c^3dx+259b^4c^4+140b^2d^4x^2+280b^2cd^3x+140b^2c^2d^2+72d^4\right)\cos($
	$225b^6(dx+c)^4$

input
```
int((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3600*(-75*b*(d*x+c)*cos(3*b*x+3*a)+45*b*(d*x+c)*cos(5*b*x+5*a)+25*d*sin(
3*b*x+3*a)-9*sin(5*b*x+5*a)*d-450*b*(d*x+c)*cos(b*x+a)-480*c*b+450*sin(b*x
+a)*d)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.70

$$\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{45\,(bdx+bc)\cos(bx+a)^5 - 75\,(bdx+bc)\cos(bx+a)^3 - \left(9\,d\cos(bx+a)^4 - 13\,d\cos(bx+a)^2 - 26\,d\right)}{225\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/225*(45*(b*d*x + b*c)*cos(b*x + a)^5 - 75*(b*d*x + b*c)*cos(b*x + a)^3 -
 (9*d*cos(b*x + a)^4 - 13*d*cos(b*x + a)^2 - 26*d)*sin(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.50

$$\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$= \begin{cases} -\frac{c\sin^2(a+bx)\cos^3(a+bx)}{3b} - \frac{2c\cos^5(a+bx)}{15b} - \frac{dx\sin^2(a+bx)\cos^3(a+bx)}{3b} - \frac{2dx\cos^5(a+bx)}{15b} + \frac{26d\sin^5(a+bx)}{225b^2} + \frac{13d\sin^3(a+bx)}{45b} \\ \left(cx + \frac{dx^2}{2}\right)\sin^3(a)\cos^2(a) \end{cases}$$

input
```
integrate((d*x+c)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output
```
Piecewise((-c*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c*cos(a + b*x)**5/
(15*b) - d*x*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*d*x*cos(a + b*x)**5
/(15*b) + 26*d*sin(a + b*x)**5/(225*b**2) + 13*d*sin(a + b*x)**3*cos(a + b
*x)**2/(45*b**2) + 2*d*sin(a + b*x)*cos(a + b*x)**4/(15*b**2), Ne(b, 0)),
((c*x + d*x**2/2)*sin(a)**3*cos(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.28

$$\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx$$

$$= \frac{240\left(3\cos(bx+a)^5 - 5\cos(bx+a)^3\right)c - \frac{240\left(3\cos(bx+a)^5 - 5\cos(bx+a)^3\right)ad}{b} + \frac{(45(bx+a)\cos(5bx+5a) - 75(bx+a)\cos(}{3600\,b}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/3600*(240*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c - 240*(3*cos(b*x + a)^
5 - 5*cos(b*x + a)^3)*a*d/b + (45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a
)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*
sin(3*b*x + 3*a) + 450*sin(b*x + a))*d/b)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.97

$$\begin{aligned}\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx = {} & \frac{(bdx+bc)\cos(5\,bx+5\,a)}{80\,b^2} \\ & - \frac{(bdx+bc)\cos(3\,bx+3\,a)}{48\,b^2} \\ & - \frac{(bdx+bc)\cos(bx+a)}{8\,b^2} - \frac{d\sin(5\,bx+5\,a)}{400\,b^2} \\ & + \frac{d\sin(3\,bx+3\,a)}{144\,b^2} + \frac{d\sin(bx+a)}{8\,b^2}\end{aligned}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/80*(b*d*x + b*c)*cos(5*b*x + 5*a)/b^2 - 1/48*(b*d*x + b*c)*cos(3*b*x + 3
*a)/b^2 - 1/8*(b*d*x + b*c)*cos(b*x + a)/b^2 - 1/400*d*sin(5*b*x + 5*a)/b^
2 + 1/144*d*sin(3*b*x + 3*a)/b^2 + 1/8*d*sin(b*x + a)/b^2
```

Mupad [B] (verification not implemented)

Time = 19.53 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91

$$\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{26\,d\,\sin(a+b\,x) - 75\,b\,c\cos(a+b\,x)^3 + 45\,b\,c\cos(a+b\,x)^5 + 13\,d\cos(a+b\,x)^2\,\sin(a+b\,x) - 9\,d\cos}{225\,b^2}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x),x)
```

output
```
(26*d*sin(a + b*x) - 75*b*c*cos(a + b*x)^3 + 45*b*c*cos(a + b*x)^5 + 13*d*
cos(a + b*x)^2*sin(a + b*x) - 9*d*cos(a + b*x)^4*sin(a + b*x) - 75*b*d*x*c
os(a + b*x)^3 + 45*b*d*x*cos(a + b*x)^5)/(225*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.28

$$\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{45\cos(bx+a)\sin(bx+a)^4\,bc + 45\cos(bx+a)\sin(bx+a)^4\,bdx - 15\cos(bx+a)\sin(bx+a)^2\,bc - 15}{}$$

input
```
int((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

output
```
(45*cos(a + b*x)*sin(a + b*x)**4*b*c + 45*cos(a + b*x)*sin(a + b*x)**4*b*d
*x - 15*cos(a + b*x)*sin(a + b*x)**2*b*c - 15*cos(a + b*x)*sin(a + b*x)**2
*b*d*x - 30*cos(a + b*x)*b*c - 30*cos(a + b*x)*b*d*x - 9*sin(a + b*x)**5*d
 + 5*sin(a + b*x)**3*d + 30*sin(a + b*x)*d - 30*a*d + 60*b*c)/(225*b**2)
```

3.93 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx$

Optimal result . . . 887
Mathematica [A] (verified) . . . 888
Rubi [A] (verified) . . . 888
Maple [A] (verified) . . . 890
Fricas [A] (verification not implemented) . . . 890
Sympy [F] . . . 891
Maxima [C] (verification not implemented) . . . 891
Giac [C] (verification not implemented) . . . 892
Mupad [F(-1)] . . . 893
Reduce [F] . . . 894

Optimal result

Integrand size = 24, antiderivative size = 185

$$\begin{aligned}\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx = &-\frac{\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)\sin\left(5a-\frac{5bc}{d}\right)}{16d}\\ &+\frac{\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{16d}\\ &+\frac{\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{8d}\\ &+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d}\\ &+\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{16d}\\ &-\frac{\cos\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{16d}\end{aligned}$$

output

```
-1/16*Ci(5*b*c/d+5*b*x)*sin(5*a-5*b*c/d)/d+1/16*Ci(3*b*c/d+3*b*x)*sin(3*a-
3*b*c/d)/d+1/8*Ci(b*c/d+b*x)*sin(a-b*c/d)/d+1/8*cos(a-b*c/d)*Si(b*c/d+b*x)
/d+1/16*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d-1/16*cos(5*a-5*b*c/d)*Si(5*b*
c/d+5*b*x)/d
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.83

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx$$
$$= \frac{-\operatorname{CosIntegral}\left(\frac{5b(c+dx)}{d}\right)\sin\left(5a-\frac{5bc}{d}\right)+\operatorname{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right)+2\operatorname{CosIntegral}\left(b\left(\frac{c}{d}+x\right.\right.}{16}$$

input
```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x),x]
```

output
```
(-(CosIntegral[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d]) + CosIntegral[(3*b
*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + 2*CosIntegral[b*(c/d + x)]*Sin[a - (
b*c)/d] + 2*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Cos[3*a - (3*b*c)/
d]*SinIntegral[(3*b*(c + d*x))/d] - Cos[5*a - (5*b*c)/d]*SinIntegral[(5*b*
(c + d*x))/d])/(16*d)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos^2(a+bx)}{c+dx}\,dx$$
$$\downarrow 4906$$
$$\int \left(\frac{\sin(a+bx)}{8(c+dx)}+\frac{\sin(3a+3bx)}{16(c+dx)}-\frac{\sin(5a+5bx)}{16(c+dx)}\right)dx$$
$$\downarrow 2009$$

$$-\frac{\sin\left(5a-\frac{5bc}{d}\right)\operatorname{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d}+\frac{\sin\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d}+$$
$$\frac{\sin\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d}+\frac{\cos\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{8d}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{16d}-$$
$$\frac{\cos\left(5a-\frac{5bc}{d}\right)\operatorname{Si}\left(\frac{5bc}{d}+5bx\right)}{16d}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x),x]
```

output
```
-1/16*(CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a - (5*b*c)/d])/d + (CosIntegr
al[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(16*d) + (CosIntegral[(b*c)/d
+ b*x]*Sin[a - (b*c)/d])/(8*d) + (Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b
*x])/(8*d) + (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d)
- (Cos[5*a - (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{b\left(-\frac{5\,\mathrm{Si}\left(-5bx-5a-\frac{5(-ad+cb)}{d}\right)\cos\left(\frac{-5ad+5cb}{d}\right)}{d}-\frac{5\,\mathrm{Ci}\left(5bx+5a+\frac{-5ad+5cb}{d}\right)\sin\left(\frac{-5ad+5cb}{d}\right)}{d}\right)}{80}+\frac{b\left(-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+c}{d}\right.\right.}{d}\right.$
default	$\frac{b\left(-\frac{5\,\mathrm{Si}\left(-5bx-5a-\frac{5(-ad+cb)}{d}\right)\cos\left(\frac{-5ad+5cb}{d}\right)}{d}-\frac{5\,\mathrm{Ci}\left(5bx+5a+\frac{-5ad+5cb}{d}\right)\sin\left(\frac{-5ad+5cb}{d}\right)}{d}\right)}{80}+\frac{b\left(-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+c}{d}\right.\right.}{d}\right.$
risch	$\frac{i\mathrm{e}^{-\frac{5i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(5ibx+5ia-\frac{5i(ad-cb)}{d}\right)}{32d}-\frac{i\mathrm{e}^{-\frac{3i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{32d}-\frac{i\mathrm{e}^{-\frac{i(ad}}$

input
```
int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/80*b*(-5*Si(-5*b*x-5*a-5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d-5*Ci(
5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d)+1/8*b*(-Si(-b*x-a-(-a*d+b
*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)+1/4
8*b*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a
+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.98

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx$$
$$= \frac{2\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)\sin\left(-\frac{bc-ad}{d}\right)+\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)-\mathrm{Ci}\left(\frac{5\,(bdx+bc)}{d}\right)\sin\left(-\frac{5\,(bc-ad)}{d}\right)-\cos\left(-\frac{5\,(}{}\right.}{16\,d}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
1/16*(2*cos_integral((b*d*x + b*c)/d)*sin(-(b*c - a*d)/d) + cos_integral(3
*(b*d*x + b*c)/d)*sin(-3*(b*c - a*d)/d) - cos_integral(5*(b*d*x + b*c)/d)*
sin(-5*(b*c - a*d)/d) - cos(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)
/d) + cos(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) + 2*cos(-(b*c
- a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \int \frac{\sin^3(a+bx)\cos^2(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.24

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx =$$
$$-\frac{2\,b\left(i\,E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b\left(i\,E_1\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-\right.}{}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output

```
-1/32*(2*b*(I*exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp
_integral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) -
b*(I*exp_integral_e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integ
ral_e(1, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b
*(-I*exp_integral_e(1, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integ
ral_e(1, -5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/d) + 2
*b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(
1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(exp_integ
ral_e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1, -3*(-I*
b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) - b*(exp_integral_e
(1, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1, -5*(-I*b*c -
 I*(b*x + a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/(b*d)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.57 (sec) , antiderivative size = 46675, normalized size of antiderivative = 252.30

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

input

```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output

```
-1/32*(imag_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*
tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_par
t(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integra
l(b*x + b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(-b*x - b*c/d))*
tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d)^2 + imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*a)^2*t
an(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
2 - imag_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_inte
gral(5*(b*d*x + b*c)/d)*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c
/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*sin_integral(3*(b*d*x + b*c)/d
)*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d)^2 - 4*sin_integral((b*d*x + b*c)/d)*tan(5/2*a)^2*tan(3/2*a
)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 4*re
al_part(cos_integral(b*x + b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*
tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*real_part(cos_integra
l(-b*x - b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*real_part(cos_integral(3*b*x + 3*b*c...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+b\,x)^2\sin(a+b\,x)^3}{c+d\,x}\,dx$$

input

```
int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x),x)
```

output

```
int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)^2\sin(bx+a)^3}{dx+c}dx$$

input
```
int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x)
```

output
```
int((cos(a + b*x)**2*sin(a + b*x)**3)/(c + d*x),x)
```

3.94 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 895
Mathematica [A] (verified) . . . 896
Rubi [A] (verified) . . . 896
Maple [A] (verified) . . . 898
Fricas [A] (verification not implemented) . . . 898
Sympy [F] . . . 899
Maxima [C] (verification not implemented) . . . 899
Giac [C] (verification not implemented) . . . 900
Mupad [F(-1)] . . . 901
Reduce [F] . . . 902

Optimal result

Integrand size = 24, antiderivative size = 257

$$\begin{aligned}\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = {}& \frac{b\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^2} \\ &+ \frac{3b\cos\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d^2} \\ &- \frac{5b\cos\left(5a-\frac{5bc}{d}\right)\operatorname{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d^2} \\ &- \frac{\sin(a+bx)}{8d(c+dx)} - \frac{\sin(3a+3bx)}{16d(c+dx)} \\ &+ \frac{\sin(5a+5bx)}{16d(c+dx)} - \frac{b\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{8d^2} \\ &- \frac{3b\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{16d^2} \\ &+ \frac{5b\sin\left(5a-\frac{5bc}{d}\right)\operatorname{Si}\left(\frac{5bc}{d}+5bx\right)}{16d^2}\end{aligned}$$

output

```
1/8*b*cos(a-b*c/d)*Ci(b*c/d+b*x)/d^2+3/16*b*cos(3*a-3*b*c/d)*Ci(3*b*c/d+3*
b*x)/d^2-5/16*b*cos(5*a-5*b*c/d)*Ci(5*b*c/d+5*b*x)/d^2-1/8*sin(b*x+a)/d/(d
*x+c)-1/16*sin(3*b*x+3*a)/d/(d*x+c)+1/16*sin(5*b*x+5*a)/d/(d*x+c)-1/8*b*si
n(a-b*c/d)*Si(b*c/d+b*x)/d^2-3/16*b*sin(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^2
+5/16*b*sin(5*a-5*b*c/d)*Si(5*b*c/d+5*b*x)/d^2
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.83

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{2b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)-5b\cos\left(5a-\frac{5bc}{d}\right)\text{Cos}}{}$$

input
```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
(2*b*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] + 3*b*Cos[3*a - (3*b*c)/d]*
CosIntegral[(3*b*(c + d*x))/d] - 5*b*Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b
*(c + d*x))/d] - (2*d*Sin[a + b*x])/(c + d*x) - (d*Sin[3*(a + b*x)])/(c +
d*x) + (d*Sin[5*(a + b*x)])/(c + d*x) - 2*b*Sin[a - (b*c)/d]*SinIntegral[b
*(c/d + x)] - 3*b*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] + 5*
b*Sin[5*a - (5*b*c)/d]*SinIntegral[(5*b*(c + d*x))/d])/(16*d^2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos^2(a+bx)}{(c+dx)^2}\,dx$$
$$\downarrow 4906$$
$$\int \left(\frac{\sin(a+bx)}{8(c+dx)^2}+\frac{\sin(3a+3bx)}{16(c+dx)^2}-\frac{\sin(5a+5bx)}{16(c+dx)^2}\right)dx$$
$$\downarrow 2009$$

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d^2}-\frac{5b\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^2}-\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{16d^2}+\frac{5b\sin\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{16d^2}-\frac{\sin(a+bx)}{8d(c+dx)}-\frac{\sin(3a+3bx)}{16d(c+dx)}+\frac{\sin(5a+5bx)}{16d(c+dx)}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
(b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d^2) + (3*b*Cos[3*a - (
3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) - (5*b*Cos[5*a - (5*b*c
)/d]*CosIntegral[(5*b*c)/d + 5*b*x])/(16*d^2) - Sin[a + b*x]/(8*d*(c + d*x
)) - Sin[3*a + 3*b*x]/(16*d*(c + d*x)) + Sin[5*a + 5*b*x]/(16*d*(c + d*x))
 - (b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^2) - (3*b*Sin[3*a
- (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) + (5*b*Sin[5*a - (5*
b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{b^2\left(-\frac{5\sin(5bx+5a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{25\ \mathrm{Si}\left(-5bx-5a-\frac{5(-ad+cb)}{d}\right)\sin\left(\frac{-5ad+5cb}{d}\right)}{d}+\frac{25\ \mathrm{Ci}\left(5bx+5a+\frac{-5ad+5cb}{d}\right)\cos\left(\frac{-5ad+5cb}{d}\right)}{d}}{d}\right.}{80}$
default	$\frac{b^2\left(-\frac{5\sin(5bx+5a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{25\ \mathrm{Si}\left(-5bx-5a-\frac{5(-ad+cb)}{d}\right)\sin\left(\frac{-5ad+5cb}{d}\right)}{d}+\frac{25\ \mathrm{Ci}\left(5bx+5a+\frac{-5ad+5cb}{d}\right)\cos\left(\frac{-5ad+5cb}{d}\right)}{d}}{d}\right.}{80}$
risch	$\frac{5b\,\mathrm{e}^{-\frac{5i(ad-cb)}{d}}\ \mathrm{expIntegral}_1\left(5ibx+5ia-\frac{5i(ad-cb)}{d}\right)}{32d^2}-\frac{3b\,\mathrm{e}^{-\frac{3i(ad-cb)}{d}}\ \mathrm{expIntegral}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{32d^2}-\frac{b\,\mathrm{e}^{-}}{}$

input
```
int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/80*b^2*(-5*sin(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))/d+5*(-5*Si(-5*b*x-5
*a-5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*co
s(5*(-a*d+b*c)/d)/d)/d)+1/8*b^2*(-sin(b*x+a)/(-a*d+c*b+d*(b*x+a))/d+(-Si(-
b*x-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b
*c)/d)/d)/d)+1/48*b^2*(-3*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d+3*(-3*Si(-
3*b*x-3*a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c
)/d)*cos(3*(-a*d+b*c)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.06

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{5\,(bdx+bc)\cos\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{5\,(bdx+bc)}{d}\right)-3\,(bdx+bc)\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)-2\,(bdx+b}{}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/16*(5*(b*d*x + b*c)*cos(-5*(b*c - a*d)/d)*cos_integral(5*(b*d*x + b*c)/
d) - 3*(b*d*x + b*c)*cos(-3*(b*c - a*d)/d)*cos_integral(3*(b*d*x + b*c)/d)
 - 2*(b*d*x + b*c)*cos(-(b*c - a*d)/d)*cos_integral((b*d*x + b*c)/d) - 5*(
b*d*x + b*c)*sin(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) + 3*(b*
d*x + b*c)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) + 2*(b*d*
x + b*c)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) - 16*(d*cos(b*x
 + a)^4 - d*cos(b*x + a)^2)*sin(b*x + a))/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin^3(a+bx)\cos^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.73

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{2\,b^2\left(i\,E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^2\left(i\,E_2\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right.$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output

```
-1/32*(2*b^2*(I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*e
xp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
 - b^2*(I*exp_integral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_i
ntegral_e(2, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d)
  - b^2*(-I*exp_integral_e(2, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp
_integral_e(2, -5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/
d) + 2*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_int
egral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^2*
(exp_integral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(
2, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) - b^2*(ex
p_integral_e(2, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(2,
-5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/((b*c*d + (
b*x + a)*d^2 - a*d^2)*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 41.30 (sec) , antiderivative size = 1014406, normalized size of antiderivative = 3947.11

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input

```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output

```
-1/32*(5*b*d*x*real_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*x)^2*tan
(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*
b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b*d*x*real_part(cos_integra
l(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a
)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*
c/d)^2 - 2*b*d*x*real_part(cos_integral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3
/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*
c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b*d*x*real_part(cos_integral(
-b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*t
an(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
2 - 3*b*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*b*x)^2*tan(3
/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*
c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 5*b*d*x*real_part(cos_integral(
-5*b*x - 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)
^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d)^2 + 4*b*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/
2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c
/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*b*d*x*imag_part(cos_integral(-b*
x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(
3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^2}\,dx$$

input

```
int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^2,x)
```

output

```
int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(bx+a)^2\sin(bx+a)^3}{d^2x^2+2cdx+c^2}dx$$

input
```
int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int((cos(a + b*x)**2*sin(a + b*x)**3)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.95 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$

Optimal result . . . 903
Mathematica [A] (verified) . . . 904
Rubi [A] (verified) . . . 904
Maple [A] (verified) . . . 906
Fricas [A] (verification not implemented) . . . 907
Sympy [F] . . . 907
Maxima [C] (verification not implemented) . . . 908
Giac [C] (verification not implemented) . . . 908
Mupad [F(-1)] . . . 909
Reduce [F] . . . 910

Optimal result

Integrand size = 24, antiderivative size = 338

$$\begin{aligned}\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = &-\frac{b\cos(a+bx)}{16d^2(c+dx)} - \frac{3b\cos(3a+3bx)}{32d^2(c+dx)} + \frac{5b\cos(5a+5bx)}{32d^2(c+dx)} \\ &+ \frac{25b^2\,\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)\sin\left(5a-\frac{5bc}{d}\right)}{32d^3} \\ &- \frac{9b^2\,\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{32d^3} \\ &- \frac{b^2\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{16d^3} \\ &- \frac{\sin(a+bx)}{16d(c+dx)^2} - \frac{\sin(3a+3bx)}{32d(c+dx)^2} \\ &+ \frac{\sin(5a+5bx)}{32d(c+dx)^2} - \frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{16d^3} \\ &- \frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{32d^3} \\ &+ \frac{25b^2\cos\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{32d^3}\end{aligned}$$

output
```
-1/16*b*cos(b*x+a)/d^2/(d*x+c)-3/32*b*cos(3*b*x+3*a)/d^2/(d*x+c)+5/32*b*co
s(5*b*x+5*a)/d^2/(d*x+c)+25/32*b^2*Ci(5*b*c/d+5*b*x)*sin(5*a-5*b*c/d)/d^3-
9/32*b^2*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^3-1/16*b^2*Ci(b*c/d+b*x)*sin
(a-b*c/d)/d^3-1/16*sin(b*x+a)/d/(d*x+c)^2-1/32*sin(3*b*x+3*a)/d/(d*x+c)^2+
1/32*sin(5*b*x+5*a)/d/(d*x+c)^2-1/16*b^2*cos(a-b*c/d)*Si(b*c/d+b*x)/d^3-9/
32*b^2*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^3+25/32*b^2*cos(5*a-5*b*c/d)*S
i(5*b*c/d+5*b*x)/d^3
```

Mathematica [A] (verified)

Time = 2.55 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.83

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{25b^2\,\text{CosIntegral}\left(\frac{5b(c+dx)}{d}\right)\sin\left(5a-\frac{5bc}{d}\right) - 9b^2\,\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right) - \frac{d(3b(c+dx)\cos(3(a+b}{(c+d}$$

input
```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

output
```
(25*b^2*CosIntegral[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d] - 9*b^2*CosInt
egral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] - (d*(3*b*(c + d*x)*Cos[3*(a
 + b*x)] + d*Sin[3*(a + b*x)]))/(c + d*x)^2 + (d*(5*b*(c + d*x)*Cos[5*(a +
 b*x)] + d*Sin[5*(a + b*x)]))/(c + d*x)^2 - 2*(b^2*CosIntegral[b*(c/d + x)
]*Sin[a - (b*c)/d] + (d*(b*(c + d*x)*Cos[a + b*x] + d*Sin[a + b*x]))/(c +
d*x)^2 + b^2*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)]) - 9*b^2*Cos[3*a -
(3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] + 25*b^2*Cos[5*a - (5*b*c)/d]*Si
nIntegral[(5*b*(c + d*x))/d])/(32*d^3)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos^2(a+bx)}{(c+dx)^3}\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{\sin(a+bx)}{8(c+dx)^3}+\frac{\sin(3a+3bx)}{16(c+dx)^3}-\frac{\sin(5a+5bx)}{16(c+dx)^3}\right)dx$$

$$\downarrow 2009$$

$$\frac{25b^2\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{32d^3}-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^3}-\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{16d^3}-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{16d^3}-\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{32d^3}+\frac{25b^2\cos\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{32d^3}-\frac{b\cos(a+bx)}{16d^2(c+dx)}-\frac{3b\cos(3a+3bx)}{32d^2(c+dx)}+\frac{5b\cos(5a+5bx)}{32d^2(c+dx)}-\frac{\sin(a+bx)}{16d(c+dx)^2}-\frac{\sin(3a+3bx)}{32d(c+dx)^2}+\frac{\sin(5a+5bx)}{32d(c+dx)^2}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

output
```
-1/16*(b*Cos[a + b*x])/(d^2*(c + d*x)) - (3*b*Cos[3*a + 3*b*x])/(32*d^2*(c
 + d*x)) + (5*b*Cos[5*a + 5*b*x])/(32*d^2*(c + d*x)) + (25*b^2*CosIntegral
[(5*b*c)/d + 5*b*x]*Sin[5*a - (5*b*c)/d])/(32*d^3) - (9*b^2*CosIntegral[(3
*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(32*d^3) - (b^2*CosIntegral[(b*c)/d
 + b*x]*Sin[a - (b*c)/d])/(16*d^3) - Sin[a + b*x]/(16*d*(c + d*x)^2) - Sin
[3*a + 3*b*x]/(32*d*(c + d*x)^2) + Sin[5*a + 5*b*x]/(32*d*(c + d*x)^2) - (
b^2*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(16*d^3) - (9*b^2*Cos[3*a
 - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(32*d^3) + (25*b^2*Cos[5*a -
 (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(32*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.42

method	result
derivativedivides	$b^3\left(-\frac{5\sin(5bx+5a)}{2(-ad+cb+d(bx+a))^2d}+\frac{-\frac{25\cos(5bx+5a)}{2(-ad+cb+d(bx+a))d}-\frac{25\left(-\frac{5\,\mathrm{Si}\left(-5bx-5a-\frac{5(-ad+cb)}{d}\right)\cos\left(\frac{-5ad+5cb}{d}\right)}{d}-\frac{5\,\mathrm{Ci}\left(5bx+5a\right.\right.}{2d}}{d}\right)}{80}$
default	$b^3\left(-\frac{5\sin(5bx+5a)}{2(-ad+cb+d(bx+a))^2d}+\frac{-\frac{25\cos(5bx+5a)}{2(-ad+cb+d(bx+a))d}-\frac{25\left(-\frac{5\,\mathrm{Si}\left(-5bx-5a-\frac{5(-ad+cb)}{d}\right)\cos\left(\frac{-5ad+5cb}{d}\right)}{d}-\frac{5\,\mathrm{Ci}\left(5bx+5a\right.\right.}{2d}}{d}\right)}{80}$
risch	$-\frac{25ib^2\mathrm{e}^{-\frac{5i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(5ibx+5ia-\frac{5i(ad-cb)}{d}\right)}{64d^3}+\frac{9ib^2\mathrm{e}^{-\frac{3i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{64d^3}$

input
```
int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/80*b^3*(-5/2*sin(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))^2/d+5/2*(-5*cos(5
*b*x+5*a)/(-a*d+c*b+d*(b*x+a))/d-5*(-5*Si(-5*b*x-5*a-5*(-a*d+b*c)/d)*cos(5
*(-a*d+b*c)/d)/d-5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d)/d)/
d)+1/8*b^3*(-1/2*sin(b*x+a)/(-a*d+c*b+d*(b*x+a))^2/d+1/2*(-cos(b*x+a)/(-a*
d+c*b+d*(b*x+a))/d-(-Si(-b*x-a-(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+
(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)+1/48*b^3*(-3/2*sin(3*b*x+3*a)/(-a
*d+c*b+d*(b*x+a))^2/d+3/2*(-3*cos(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d-3*(-3*
Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d
+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.35

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$$

$$= \frac{80\,(bd^2x+bcd)\cos(bx+a)^5 - 112\,(bd^2x+bcd)\cos(bx+a)^3 - 2\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)\,s$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/32*(80*(b*d^2*x + b*c*d)*cos(b*x + a)^5 - 112*(b*d^2*x + b*c*d)*cos(b*x
+ a)^3 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral((b*d*x + b*c
)/d)*sin(-(b*c - a*d)/d) - 9*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_int
egral(3*(b*d*x + b*c)/d)*sin(-3*(b*c - a*d)/d) + 25*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*cos_integral(5*(b*d*x + b*c)/d)*sin(-5*(b*c - a*d)/d) + 25
*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-5*(b*c - a*d)/d)*sin_integral(
5*(b*d*x + b*c)/d) - 9*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-3*(b*c -
 a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 32*(b*d^2*x +
 b*c*d)*cos(b*x + a) + 16*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)*sin(b*
x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin^3(a+bx)\cos^2(a+bx)}{(c+dx)^3}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.42

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx =$$
$$-\frac{2\,b^3\left(i\,E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^3\left(i\,E_3\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right.$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/32*(2*b^3*(I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*e
xp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
 - b^3*(I*exp_integral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_i
ntegral_e(3, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d)
 - b^3*(-I*exp_integral_e(3, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp
_integral_e(3, -5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/
d) + 2*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_int
egral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^3*
(exp_integral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(
3, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) - b^3*(ex
p_integral_e(3, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(3,
-5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/((b^2*c^2*d
 - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a)
)*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 80.47 (sec) , antiderivative size = 1737414, normalized size of antiderivative = 5140.28

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

output

```
1/64*(25*b^2*d^2*x^2*imag_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*x)
^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*imag_part
(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)
^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*ta
n(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^2*x^2
*imag_part(cos_integral(-b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*
b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x -
3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/
2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2
5*b^2*d^2*x^2*imag_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*b*x)^2*tan
(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*
b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 50*b^2*d^2*x^2*sin_integral(5
*(b*d*x + b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^
2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 - 18*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(5/2*b*x)^2*tan(3
/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^3}\,dx$$

input

```
int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^3,x)
```

output

```
int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos(bx+a)^2\sin(bx+a)^3}{d^3x^3+3cd^2x^2+3c^2dx+c^3}dx$$

input
```
int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x)
```

output
```
int((cos(a + b*x)**2*sin(a + b*x)**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
 d**3*x**3),x)
```

3.96 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$

Optimal result . . . 911
Mathematica [A] (verified) . . . 912
Rubi [A] (verified) . . . 913
Maple [A] (verified) . . . 914
Fricas [A] (verification not implemented) . . . 916
Sympy [F] . . . 916
Maxima [C] (verification not implemented) . . . 917
Giac [C] (verification not implemented) . . . 917
Mupad [F(-1)] . . . 918
Reduce [F] . . . 919

Optimal result

Integrand size = 24, antiderivative size = 413

$$\begin{aligned}\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = &-\frac{b\cos(a+bx)}{48d^2(c+dx)^2}-\frac{b\cos(3a+3bx)}{32d^2(c+dx)^2}+\frac{5b\cos(5a+5bx)}{96d^2(c+dx)^2}\\ &-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{48d^4}\\ &-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^4}\\ &+\frac{125b^3\cos\left(5a-\frac{5bc}{d}\right)\operatorname{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{96d^4}\\ &-\frac{\sin(a+bx)}{24d(c+dx)^3}+\frac{b^2\sin(a+bx)}{48d^3(c+dx)}-\frac{\sin(3a+3bx)}{48d(c+dx)^3}\\ &+\frac{3b^2\sin(3a+3bx)}{32d^3(c+dx)}+\frac{\sin(5a+5bx)}{48d(c+dx)^3}\\ &-\frac{25b^2\sin(5a+5bx)}{96d^3(c+dx)}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{48d^4}\\ &+\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{32d^4}\\ &-\frac{125b^3\sin\left(5a-\frac{5bc}{d}\right)\operatorname{Si}\left(\frac{5bc}{d}+5bx\right)}{96d^4}\end{aligned}$$

output
```
-1/48*b*cos(b*x+a)/d^2/(d*x+c)^2-1/32*b*cos(3*b*x+3*a)/d^2/(d*x+c)^2+5/96*
b*cos(5*b*x+5*a)/d^2/(d*x+c)^2-1/48*b^3*cos(a-b*c/d)*Ci(b*c/d+b*x)/d^4-9/3
2*b^3*cos(3*a-3*b*c/d)*Ci(3*b*c/d+3*b*x)/d^4+125/96*b^3*cos(5*a-5*b*c/d)*C
i(5*b*c/d+5*b*x)/d^4-1/24*sin(b*x+a)/d/(d*x+c)^3+1/48*b^2*sin(b*x+a)/d^3/(
d*x+c)-1/48*sin(3*b*x+3*a)/d/(d*x+c)^3+3/32*b^2*sin(3*b*x+3*a)/d^3/(d*x+c)
+1/48*sin(5*b*x+5*a)/d/(d*x+c)^3-25/96*b^2*sin(5*b*x+5*a)/d^3/(d*x+c)+1/48
*b^3*sin(a-b*c/d)*Si(b*c/d+b*x)/d^4+9/32*b^3*sin(3*a-3*b*c/d)*Si(3*b*c/d+3
*b*x)/d^4-125/96*b^3*sin(5*a-5*b*c/d)*Si(5*b*c/d+5*b*x)/d^4
```

Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.11

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$$

$$= \frac{-d\cos(3bx)\left(3bd(c+dx)\cos(3a)-\left(-2d^2+9b^2(c+dx)^2\right)\sin(3a)\right)+d\cos(5bx)\left(5bd(c+dx)\cos(5a)-\right.}{}$$

input
```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

output
```
(-(d*Cos[3*b*x]*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)*S
in[3*a])) + d*Cos[5*b*x]*(5*b*d*(c + d*x)*Cos[5*a] - (-2*d^2 + 25*b^2*(c +
 d*x)^2)*Sin[5*a]) + d*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c +
 d*x)*Sin[3*a])*Sin[3*b*x] - d*((-2*d^2 + 25*b^2*(c + d*x)^2)*Cos[5*a] + 5
*b*d*(c + d*x)*Sin[5*a])*Sin[5*b*x] - 2*(d*Cos[b*x]*(b*d*(c + d*x)*Cos[a]
- (-2*d^2 + b^2*(c + d*x)^2)*Sin[a]) - d*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a
] + b*d*(c + d*x)*Sin[a])*Sin[b*x] + b^3*(c + d*x)^3*(Cos[a - (b*c)/d]*Cos
Integral[b*(c/d + x)] - Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)])) - 27*b
^3*(c + d*x)^3*(Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*(c + d*x))/d] - Sin[
3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d]) + 125*b^3*(c + d*x)^3*(Co
s[5*a - (5*b*c)/d]*CosIntegral[(5*b*(c + d*x))/d] - Sin[5*a - (5*b*c)/d]*S
inIntegral[(5*b*(c + d*x))/d]))/(96*d^4*(c + d*x)^3)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos^2(a+bx)}{(c+dx)^4}\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{\sin(a+bx)}{8(c+dx)^4} + \frac{\sin(3a+3bx)}{16(c+dx)^4} - \frac{\sin(5a+5bx)}{16(c+dx)^4}\right)dx$$

$$\downarrow 2009$$

$$-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{48d^4} - \frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^4} + \frac{125b^3\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{96d^4} + \frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{48d^4} + \frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{32d^4} - \frac{125b^3\sin\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{96d^4} + \frac{b^2\sin(a+bx)}{48d^3(c+dx)} + \frac{3b^2\sin(3a+3bx)}{32d^3(c+dx)} - \frac{25b^2\sin(5a+5bx)}{96d^3(c+dx)} - \frac{b\cos(a+bx)}{48d^2(c+dx)^2} - \frac{b\cos(3a+3bx)}{32d^2(c+dx)^2} + \frac{5b\cos(5a+5bx)}{96d^2(c+dx)^2} - \frac{\sin(a+bx)}{24d(c+dx)^3} - \frac{\sin(3a+3bx)}{48d(c+dx)^3} + \frac{\sin(5a+5bx)}{48d(c+dx)^3}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

output
```
-1/48*(b*Cos[a + b*x])/(d^2*(c + d*x)^2) - (b*Cos[3*a + 3*b*x])/(32*d^2*(c
 + d*x)^2) + (5*b*Cos[5*a + 5*b*x])/(96*d^2*(c + d*x)^2) - (b^3*Cos[a - (b
*c)/d]*CosIntegral[(b*c)/d + b*x])/(48*d^4) - (9*b^3*Cos[3*a - (3*b*c)/d]*
CosIntegral[(3*b*c)/d + 3*b*x])/(32*d^4) + (125*b^3*Cos[5*a - (5*b*c)/d]*C
osIntegral[(5*b*c)/d + 5*b*x])/(96*d^4) - Sin[a + b*x]/(24*d*(c + d*x)^3)
+ (b^2*Sin[a + b*x])/(48*d^3*(c + d*x)) - Sin[3*a + 3*b*x]/(48*d*(c + d*x)
^3) + (3*b^2*Sin[3*a + 3*b*x])/(32*d^3*(c + d*x)) + Sin[5*a + 5*b*x]/(48*d
*(c + d*x)^3) - (25*b^2*Sin[5*a + 5*b*x])/(96*d^3*(c + d*x)) + (b^3*Sin[a
 - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(48*d^4) + (9*b^3*Sin[3*a - (3*b*c)
/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(32*d^4) - (125*b^3*Sin[5*a - (5*b*c)/
d]*SinIntegral[(5*b*c)/d + 5*b*x])/(96*d^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 4.20 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.42

method	result
derivativedivides	
default	
risch	Expression too large to display

input
```
int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/80*b^4*(-5/3*sin(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))^3/d+5/3*(-5/2*cos
(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))^2/d-5/2*(-5*sin(5*b*x+5*a)/(-a*d+c*b+d*(b
*x+a))/d+5*(-5*Si(-5*b*x-5*a-5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*
b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d)/d)/d)/d)+1/8*b^4*(-1/3*sin(
b*x+a)/(-a*d+c*b+d*(b*x+a))^3/d+1/3*(-1/2*cos(b*x+a)/(-a*d+c*b+d*(b*x+a))^
2/d-1/2*(-sin(b*x+a)/(-a*d+c*b+d*(b*x+a))/d+(-Si(-b*x-a-(-a*d+b*c)/d)*sin(
(-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)+1/48*
b^4*(-sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))^3/d+(-3/2*cos(3*b*x+3*a)/(-a*d+c
*b+d*(b*x+a))^2/d-3/2*(-3*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d+3*(-3*Si(-
3*b*x-3*a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c
)/d)*cos(3*(-a*d+b*c)/d)/d)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.58

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/96*(80*(b*d^3*x + b*c*d^2)*cos(b*x + a)^5 - 112*(b*d^3*x + b*c*d^2)*cos(
b*x + a)^3 + 125*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)
*cos(-5*(b*c - a*d)/d)*cos_integral(5*(b*d*x + b*c)/d) - 27*(b^3*d^3*x^3 +
 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-3*(b*c - a*d)/d)*cos_inte
gral(3*(b*d*x + b*c)/d) - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x
 + b^3*c^3)*cos(-(b*c - a*d)/d)*cos_integral((b*d*x + b*c)/d) - 125*(b^3*d
^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-5*(b*c - a*d)/d)*
sin_integral(5*(b*d*x + b*c)/d) + 27*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + b^3*c^3)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d)
 + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-(b*c -
 a*d)/d)*sin_integral((b*d*x + b*c)/d) + 32*(b*d^3*x + b*c*d^2)*cos(b*x +
a) - 16*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d + (25*b^2*d^3*x^2 + 5
0*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*cos(b*x + a)^4 - (21*b^2*d^3*x^2 + 4
2*b^2*c*d^2*x + 21*b^2*c^2*d - 2*d^3)*cos(b*x + a)^2)*sin(b*x + a))/(d^7*x
^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sin^3(a+bx)\cos^2(a+bx)}{(c+dx)^4}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c)**4,x)
```

output
```
Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x)**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.28

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx =$$
$$-\frac{2b^4\left(iE_4\left(\frac{ibc+i(bx+a)d-iad}{d}\right)-iE_4\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^4\left(iE_4\left(\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)\right.}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/32*(2*b^4*(I*exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*e
xp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
 - b^4*(I*exp_integral_e(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_i
ntegral_e(4, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d)
  - b^4*(-I*exp_integral_e(4, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp
_integral_e(4, -5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/
d) + 2*b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_int
egral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^4*
(exp_integral_e(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(
4, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) - b^4*(ex
p_integral_e(4, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(4,
-5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/((b^3*c^3*d
 - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^
3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)
)*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 138.55 (sec) , antiderivative size = 2449286, normalized size of antiderivative = 5930.47

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")
```

output
```
1/192*(125*b^3*d^3*x^3*real_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*
x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*
tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 27*b^3*d^3*x^3*real_p
art(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b
*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/
d)^2*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*real_part(cos_integral(b*x + b*c/d))
*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b^3*d^3*
x^3*real_part(cos_integral(-b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*ta
n(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 27*b^3*d^3*x^3*real_part(cos_integral(-3*b*
x - 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 + 125*b^3*d^3*x^3*real_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*b*x)^
2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan
(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*b^3*d^3*x^3*imag_part(
cos_integral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*ta
n(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d) - 4*b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*tan(5/2*
b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos(a+b\,x)^2\sin(a+b\,x)^3}{(c+d\,x)^4}\,dx$$

input
```
int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^4,x)
```

output
```
int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^4, x)
```

Reduce [F]

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \text{too large to display}$$

input
```
int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x)
```

output
```
(144*cos(a + b*x)*sin(a + b*x)**4*b*c*d**2 + 144*cos(a + b*x)*sin(a + b*x)
**4*b*d**3*x + 4500*cos(a + b*x)*sin(a + b*x)**3*b**2*c**2*d + 9000*cos(a
+ b*x)*sin(a + b*x)**3*b**2*c*d**2*x + 4500*cos(a + b*x)*sin(a + b*x)**3*b
**2*d**3*x**2 + 12610*cos(a + b*x)*sin(a + b*x)**3*d**3 - 4608*cos(a + b*x
)*sin(a + b*x)**2*b*c*d**2 - 4608*cos(a + b*x)*sin(a + b*x)**2*b*d**3*x -
29250*cos(a + b*x)*sin(a + b*x)*b**2*c**2*d - 58500*cos(a + b*x)*sin(a + b
*x)*b**2*c*d**2*x - 29250*cos(a + b*x)*sin(a + b*x)*b**2*d**3*x**2 - 71840
*cos(a + b*x)*sin(a + b*x)*d**3 + 48384*cos(a + b*x)*b*c*d**2 + 48384*cos(
a + b*x)*b*d**3*x - 1360800*int(tan((a + b*x)/2)**5/(tan((a + b*x)/2)**10*
c**4 + 4*tan((a + b*x)/2)**10*c**3*d*x + 6*tan((a + b*x)/2)**10*c**2*d**2*
x**2 + 4*tan((a + b*x)/2)**10*c*d**3*x**3 + tan((a + b*x)/2)**10*d**4*x**4
 + 5*tan((a + b*x)/2)**8*c**4 + 20*tan((a + b*x)/2)**8*c**3*d*x + 30*tan((
a + b*x)/2)**8*c**2*d**2*x**2 + 20*tan((a + b*x)/2)**8*c*d**3*x**3 + 5*tan
((a + b*x)/2)**8*d**4*x**4 + 10*tan((a + b*x)/2)**6*c**4 + 40*tan((a + b*x
)/2)**6*c**3*d*x + 60*tan((a + b*x)/2)**6*c**2*d**2*x**2 + 40*tan((a + b*x
)/2)**6*c*d**3*x**3 + 10*tan((a + b*x)/2)**6*d**4*x**4 + 10*tan((a + b*x)/
2)**4*c**4 + 40*tan((a + b*x)/2)**4*c**3*d*x + 60*tan((a + b*x)/2)**4*c**2
*d**2*x**2 + 40*tan((a + b*x)/2)**4*c*d**3*x**3 + 10*tan((a + b*x)/2)**4*d
**4*x**4 + 5*tan((a + b*x)/2)**2*c**4 + 20*tan((a + b*x)/2)**2*c**3*d*x +
30*tan((a + b*x)/2)**2*c**2*d**2*x**2 + 20*tan((a + b*x)/2)**2*c*d**3*x...
```

3.97 $\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx$

Optimal result . . . 920
Mathematica [N/A] . . . 920
Rubi [N/A] . . . 921
Maple [N/A] . . . 923
Fricas [N/A] . . . 923
Sympy [N/A] . . . 924
Maxima [N/A] . . . 924
Giac [N/A] . . . 924
Mupad [N/A] . . . 925
Reduce [N/A] . . . 925

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx$$
$$= \frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}$$
$$+\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{2b}+\text{Int}((c+dx)^m\csc(a+bx),x)$$

output

```
1/2*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/
d)^m)+1/2*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*x
+c)/d)^m)+Defer(Int)((d*x+c)^m*csc(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 9.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4908, 3042, 3789, 2612, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cot(a+bx)(c+dx)^m\,dx$$

$$\downarrow 4908$$

$$\int (c+dx)^m \csc(a+bx)dx - \int (c+dx)^m \sin(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)^m \csc(a+bx)dx - \int (c+dx)^m \sin(a+bx)dx$$

$$\downarrow 3789$$

$$-\frac{1}{2}i\int e^{-i(a+bx)}(c+dx)^m dx + \frac{1}{2}i\int e^{i(a+bx)}(c+dx)^m dx + \int (c+dx)^m \csc(a+bx)dx$$

$$\downarrow 2612$$

$$\int (c+dx)^m \csc(a+bx)dx + \frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b} + \frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}$$

$$\downarrow 4680$$

$$\int (c+dx)^m \csc(a+bx)dx + \frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b} + \frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4908

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \cos(bx+a) \cot(bx+a)\, dx$$

input

```
int((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x)
```

output

```
int((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \cos(a+bx) \cot(a+bx)\, dx = \int (dx+c)^m \cos(bx+a) \cot(bx+a)\ dx$$

input

```
integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

output

```
integral((d*x + c)^m*cos(b*x + a)*cot(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 7.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cos(b*x+a)*cot(b*x+a),x)
```

output
```
Integral((c + d*x)**m*cos(a + b*x)*cot(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\cot(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\cot(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 19.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx = \int \cos(a+b\,x)\,\cot(a+b\,x)\,(c+d\,x)^m\,dx$$

input
```
int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\cot(bx+a)\,dx$$

input
```
int((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x)
```

output
```
int((c + d*x)**m*cos(a + b*x)*cot(a + b*x),x)
```

3.98 $\int (c+dx)^4 \cos(a+bx)\cot(a+bx)\,dx$

Optimal result . . . 926
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Optimal result

Integrand size = 20, antiderivative size = 333

$$
\begin{aligned}
&\int (c+dx)^4 \cos(a+bx)\cot(a+bx)\,dx \\
&= -\frac{2(c+dx)^4 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} + \frac{24d^4\cos(a+bx)}{b^5} - \frac{12d^2(c+dx)^2\cos(a+bx)}{b^3} \\
&\quad + \frac{(c+dx)^4\cos(a+bx)}{b} + \frac{4id(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{4id(c+dx)^3\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} - \frac{12d^2(c+dx)^2\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{12d^2(c+dx)^2\,\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} - \frac{24id^3(c+dx)\,\mathrm{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4} \\
&\quad + \frac{24id^3(c+dx)\,\mathrm{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4} + \frac{24d^4\,\mathrm{PolyLog}\left(5,-e^{i(a+bx)}\right)}{b^5} \\
&\quad - \frac{24d^4\,\mathrm{PolyLog}\left(5,e^{i(a+bx)}\right)}{b^5} + \frac{24d^3(c+dx)\sin(a+bx)}{b^4} - \frac{4d(c+dx)^3\sin(a+bx)}{b^2}
\end{aligned}
$$

output

```
-2*(d*x+c)^4*arctanh(exp(I*(b*x+a)))/b+24*d^4*cos(b*x+a)/b^5-12*d^2*(d*x+c
)^2*cos(b*x+a)/b^3+(d*x+c)^4*cos(b*x+a)/b+4*I*d*(d*x+c)^3*polylog(2,-exp(I
*(b*x+a)))/b^2-4*I*d*(d*x+c)^3*polylog(2,exp(I*(b*x+a)))/b^2-12*d^2*(d*x+c
)^2*polylog(3,-exp(I*(b*x+a)))/b^3+12*d^2*(d*x+c)^2*polylog(3,exp(I*(b*x+a
)))/b^3-24*I*d^3*(d*x+c)*polylog(4,-exp(I*(b*x+a)))/b^4+24*I*d^3*(d*x+c)*p
olylog(4,exp(I*(b*x+a)))/b^4+24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-24*d^4*
polylog(5,exp(I*(b*x+a)))/b^5+24*d^3*(d*x+c)*sin(b*x+a)/b^4-4*d*(d*x+c)^3*
sin(b*x+a)/b^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 837 vs. $2(333) = 666$.

Time $= 0.82$ (sec) , antiderivative size $= 837$, normalized size of antiderivative $= 2.51$

$$\int (c+dx)^4 \cos(a+bx) \cot(a+bx)\, dx$$
$$= \frac{b^4c^4 \cos(a+bx) - 12b^2c^2d^2 \cos(a+bx) + 24d^4 \cos(a+bx) + 4b^4c^3dx \cos(a+bx) - 24b^2cd^3x \cos(a+bx$$

input

```
Integrate[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x],x]
```

output

```
(b^4*c^4*Cos[a + b*x] - 12*b^2*c^2*d^2*Cos[a + b*x] + 24*d^4*Cos[a + b*x]
+ 4*b^4*c^3*d*x*Cos[a + b*x] - 24*b^2*c*d^3*x*Cos[a + b*x] + 6*b^4*c^2*d^2
*x^2*Cos[a + b*x] - 12*b^2*d^4*x^2*Cos[a + b*x] + 4*b^4*c*d^3*x^3*Cos[a +
b*x] + b^4*d^4*x^4*Cos[a + b*x] + b^4*c^4*Log[1 - E^(I*(a + b*x))] + 4*b^4
*c^3*d*x*Log[1 - E^(I*(a + b*x))] + 6*b^4*c^2*d^2*x^2*Log[1 - E^(I*(a + b*
x))] + 4*b^4*c*d^3*x^3*Log[1 - E^(I*(a + b*x))] + b^4*d^4*x^4*Log[1 - E^(I
*(a + b*x))] - b^4*c^4*Log[1 + E^(I*(a + b*x))] - 4*b^4*c^3*d*x*Log[1 + E^
(I*(a + b*x))] - 6*b^4*c^2*d^2*x^2*Log[1 + E^(I*(a + b*x))] - 4*b^4*c*d^3*
x^3*Log[1 + E^(I*(a + b*x))] - b^4*d^4*x^4*Log[1 + E^(I*(a + b*x))] + (4*I
)*b^3*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))] - (4*I)*b^3*d*(c + d*x)^3
*PolyLog[2, E^(I*(a + b*x))] - 12*b^2*c^2*d^2*PolyLog[3, -E^(I*(a + b*x))]
 - 24*b^2*c*d^3*x*PolyLog[3, -E^(I*(a + b*x))] - 12*b^2*d^4*x^2*PolyLog[3,
 -E^(I*(a + b*x))] + 12*b^2*c^2*d^2*PolyLog[3, E^(I*(a + b*x))] + 24*b^2*c
*d^3*x*PolyLog[3, E^(I*(a + b*x))] + 12*b^2*d^4*x^2*PolyLog[3, E^(I*(a + b
*x))] - (24*I)*b*c*d^3*PolyLog[4, -E^(I*(a + b*x))] - (24*I)*b*d^4*x*PolyL
og[4, -E^(I*(a + b*x))] + (24*I)*b*c*d^3*PolyLog[4, E^(I*(a + b*x))] + (24
*I)*b*d^4*x*PolyLog[4, E^(I*(a + b*x))] + 24*d^4*PolyLog[5, -E^(I*(a + b*x
))] - 24*d^4*PolyLog[5, E^(I*(a + b*x))] - 4*b^3*c^3*d*Sin[a + b*x] + 24*b
*c*d^3*Sin[a + b*x] - 12*b^3*c^2*d^2*x*Sin[a + b*x] + 24*b*d^4*x*Sin[a + b
*x] - 12*b^3*c*d^3*x^2*Sin[a + b*x] - 4*b^3*d^4*x^3*Sin[a + b*x])/b^5
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.13, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.950$, Rules used = {4908, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118, 4671, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \cos(a+bx) \cot(a+bx)\, dx$$

$$\downarrow 4908$$

$$\int (c+dx)^4 \csc(a+bx) dx - \int (c+dx)^4 \sin(a+bx) dx$$

$$\downarrow 3042$$

$$\int (c+dx)^4 \csc(a+bx)dx - \int (c+dx)^4 \sin(a+bx)dx$$

$$\downarrow 3777$$

$$-\frac{4d\int (c+dx)^3 \cos(a+bx)dx}{b} + \int (c+dx)^4 \csc(a+bx)dx + \frac{(c+dx)^4 \cos(a+bx)}{b}$$

$$\downarrow 3042$$

$$-\frac{4d\int (c+dx)^3 \sin\left(a+bx+\frac{\pi}{2}\right)dx}{b} + \int (c+dx)^4 \csc(a+bx)dx + \frac{(c+dx)^4 \cos(a+bx)}{b}$$

$$\downarrow 3777$$

$$-\frac{4d\left(\frac{3d\int -(c+dx)^2 \sin(a+bx)dx}{b} + \frac{(c+dx)^3 \sin(a+bx)}{b}\right)}{b} + \int (c+dx)^4 \csc(a+bx)dx + \frac{(c+dx)^4 \cos(a+bx)}{b}$$

$$\downarrow 25$$

$$-\frac{4d\left(\frac{(c+dx)^3 \sin(a+bx)}{b} - \frac{3d\int (c+dx)^2 \sin(a+bx)dx}{b}\right)}{b} + \int (c+dx)^4 \csc(a+bx)dx + \frac{(c+dx)^4 \cos(a+bx)}{b}$$

$$\downarrow 3042$$

$$-\frac{4d\left(\frac{(c+dx)^3 \sin(a+bx)}{b} - \frac{3d\int (c+dx)^2 \sin(a+bx)dx}{b}\right)}{b} + \int (c+dx)^4 \csc(a+bx)dx + \frac{(c+dx)^4 \cos(a+bx)}{b}$$

$$\downarrow 3777$$

$$\int (c+dx)^4 \csc(a+bx)dx - \frac{4d\left(\frac{(c+dx)^3 \sin(a+bx)}{b} - \frac{3d\left(\frac{2d\int (c+dx)\cos(a+bx)dx}{b} - \frac{(c+dx)^2 \cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^4 \cos(a+bx)}{b}$$

$$\downarrow 3042$$

$$\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^4\cos(a+bx)}{b}-\int(c+dx)^4\csc(a+bx)dx$$

$$\downarrow 3777$$

$$\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\int-\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^4\cos(a+bx)}{b}-\int(c+dx)^4\csc(a+bx)dx$$

$$\downarrow 25$$

$$\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^4\cos(a+bx)}{b}-\int(c+dx)^4\csc(a+bx)dx$$

$$\downarrow 3042$$

$$\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^4\cos(a+bx)}{b}-\int(c+dx)^4\csc(a+bx)dx$$

$$\downarrow 3118$$

$$\int (c+dx)^4 \csc(a+bx)dx - \frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b} - \frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^4\cos(a+bx)}{b}$$

$$\downarrow 4671$$

$$-\frac{4d\int (c+dx)^3\log\left(1-e^{i(a+bx)}\right)dx}{b} + \frac{4d\int (c+dx)^3\log\left(1+e^{i(a+bx)}\right)dx}{b} - \frac{2(c+dx)^4\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b} - \frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^4\cos(a+bx)}{b}$$

$$\downarrow 3011$$

$$\frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b} - \frac{3id\int (c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b} - \frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b} - \frac{3id\int (c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b} - \frac{2(c+dx)^4\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b} - \frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^4\cos(a+bx)}{b}$$

$$\downarrow 7163$$

$$\frac{4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\int(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\int(c+dx)\,\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{2(c+dx)^4\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^4\cos(a+bx)}{b}$$

$$\downarrow 7163$$

$$4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{id\int\mathrm{PolyLog}\left(4,-e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right.}{b}\right.$$

$$\left.\left.4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{id\int\mathrm{PolyLog}\left(4,e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(4,e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)\right.$$

$$\frac{2(c+dx)^4\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-$$

$$4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+$$

$$\frac{(c+dx)^4\cos(a+bx)}{b}$$

$$\downarrow 2720$$

$$4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(4,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\,\mathrm{Poly}}{b}\right.}{b}\right.$$

$$4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(4,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(4,e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(3\right.}{b}\right.}{b}\right.$$

$$\frac{2(c+dx)^4\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}-$$

$$4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+$$

$$\frac{(c+dx)^4\cos(a+bx)}{b}$$

$$\downarrow 7143$$

$$-\frac{2(c+dx)^4\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{d\operatorname{PolyLog}\left(5,-e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)/b+4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{d\operatorname{PolyLog}\left(5,e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)/b-4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)/b+\frac{(c+dx)^4\cos(a+bx)}{b}$$

input
```
Int[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x],x]
```

output
```
(-2*(c + d*x)^4*ArcTanh[E^(I*(a + b*x))])/b + ((c + d*x)^4*Cos[a + b*x])/b
 + (4*d*((I*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b - ((3*I)*d*(((-I)*
(c + d*x)^2*PolyLog[3, -E^(I*(a + b*x))])/b + ((2*I)*d*((-I)*(c + d*x)*Po
lyLog[4, -E^(I*(a + b*x))])/b + (d*PolyLog[5, -E^(I*(a + b*x))])/b^2))/b))
/b))/b - (4*d*((I*(c + d*x)^3*PolyLog[2, E^(I*(a + b*x))])/b - ((3*I)*d*((
(-I)*(c + d*x)^2*PolyLog[3, E^(I*(a + b*x))])/b + ((2*I)*d*((-I)*(c + d*x
)*PolyLog[4, E^(I*(a + b*x))])/b + (d*PolyLog[5, E^(I*(a + b*x))])/b^2))/b
))/b))/b - (4*d*(((c + d*x)^3*Sin[a + b*x])/b - (3*d*(-(((c + d*x)^2*Cos[a
 + b*x])/b) + (2*d*((d*Cos[a + b*x])/b^2 + ((c + d*x)*Sin[a + b*x])/b))/b)
)/b))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))  Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1294 vs. $2(311) = 622$.

Time = 1.36 (sec) , antiderivative size = 1295, normalized size of antiderivative = 3.89

method	result	size
risch	Expression too large to display	1295

input
```
int((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
-2/b*c^4*arctanh(exp(I*(b*x+a)))+1/2*(d^4*x^4*b^4+4*b^4*c*d^3*x^3+6*b^4*c^
2*d^2*x^2+4*b^4*c^3*d*x-4*I*b^3*d^4*x^3+b^4*c^4-12*b^2*d^4*x^2-12*I*b^3*c*
d^3*x^2-24*b^2*c*d^3*x-12*I*b^3*c^2*d^2*x-12*b^2*c^2*d^2-4*I*b^3*c^3*d+24*
I*b*d^4*x+24*d^4+24*I*b*c*d^3)/b^5*exp(-I*(b*x+a))+1/b^5*d^4*ln(exp(I*(b*x
+a))+1)*a^4-1/b^5*d^4*ln(1-exp(I*(b*x+a)))*a^4+12/b^3*d^4*polylog(3,exp(I*
(b*x+a)))*x^2-12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2-2/b^5*d^4*a^4*arct
anh(exp(I*(b*x+a)))-12/b^3*d^2*c^2*polylog(3,-exp(I*(b*x+a)))+12/b^3*d^2*c
^2*polylog(3,exp(I*(b*x+a)))+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4-1/b*d^4*ln(e
xp(I*(b*x+a))+1)*x^4-4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x+4/b*c^3*d*ln(1-exp(I
*(b*x+a)))*x-6/b*d^2*c^2*ln(exp(I*(b*x+a))+1)*x^2+6/b*d^2*c^2*ln(1-exp(I*(
b*x+a)))*x^2-4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3+4/b*c*d^3*ln(1-exp(I*(b*x+
a)))*x^3+6/b^3*d^2*c^2*ln(exp(I*(b*x+a))+1)*a^2-6/b^3*d^2*c^2*ln(1-exp(I*(
b*x+a)))*a^2-4/b^2*c^3*d*ln(exp(I*(b*x+a))+1)*a+4/b^2*c^3*d*ln(1-exp(I*(b*
x+a)))*a-4/b^4*c*d^3*ln(exp(I*(b*x+a))+1)*a^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+
a)))*a^3-24/b^3*c*d^3*polylog(3,-exp(I*(b*x+a)))*x+24/b^3*c*d^3*polylog(3,
exp(I*(b*x+a)))*x+8/b^4*c*d^3*a^3*arctanh(exp(I*(b*x+a)))-12/b^3*c^2*d^2*a
^2*arctanh(exp(I*(b*x+a)))+8/b^2*c^3*d*a*arctanh(exp(I*(b*x+a)))-4*I/b^2*d
^4*polylog(2,exp(I*(b*x+a)))*x^3-12*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*
x^2+12*I/b^2*d^2*c^2*polylog(2,-exp(I*(b*x+a)))*x-12*I/b^2*d^2*c^2*polylog
(2,exp(I*(b*x+a)))*x+12*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+4*I/...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1367 vs. $2(305) = 610$.

Time $= 0.15$ (sec) , antiderivative size $= 1367$, normalized size of antiderivative $= 4.11$

$$\int (c + dx)^4 \cos(a + bx) \cot(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

output

```
-1/2*(24*d^4*polylog(5, cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*polylog(5,
 cos(b*x + a) - I*sin(b*x + a)) - 24*d^4*polylog(5, -cos(b*x + a) + I*sin(
b*x + a)) - 24*d^4*polylog(5, -cos(b*x + a) - I*sin(b*x + a)) - 2*(b^4*d^4
*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 24*d^4 + 6*(b^4*c^2*d^
2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a) + 4*(I*b^
3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(cos
(b*x + a) + I*sin(b*x + a)) + 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*
b^3*c^2*d^2*x - I*b^3*c^3*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) + 4*(I*b
^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(-c
os(b*x + a) + I*sin(b*x + a)) + 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*
I*b^3*c^2*d^2*x - I*b^3*c^3*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b^
4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)
*log(cos(b*x + a) + I*sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 +
 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*x + a) - I*sin(b*x
 + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - (b^4*c^4 -
4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-1/2*cos(
b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*
b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^
3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*d^4...
```

Sympy [F]

$$\int (c+dx)^4 \cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)^4 \cos\left(a+bx\right)\cot\left(a+bx\right)dx$$

input

```
integrate((d*x+c)**4*cos(b*x+a)*cot(b*x+a),x)
```

output

```
Integral((c + d*x)**4*cos(a + b*x)*cot(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1548 vs. $2(305) = 610$.

Time = 0.25 (sec) , antiderivative size = 1548, normalized size of antiderivative = 4.65

$$\int (c+dx)^4 \cos(a+bx) \cot(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

output

```
1/2*(c^4*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
- 4*a*c^3*d*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1
))/b + 6*a^2*c^2*d^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x
 + a) - 1))/b^2 - 4*a^3*c*d^3*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + lo
g(cos(b*x + a) - 1))/b^3 + a^4*d^4*(2*cos(b*x + a) - log(cos(b*x + a) + 1)
 + log(cos(b*x + a) - 1))/b^4 + (48*d^4*polylog(5, -e^(I*b*x + I*a)) - 48*
d^4*polylog(5, e^(I*b*x + I*a)) + 2*(-I*(b*x + a)^4*d^4 + 4*(-I*b*c*d^3 +
I*a*d^4)*(b*x + a)^3 + 6*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x
 + a)^2 + 4*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 - 3*I*a^2*b*c*d^3 + I*a^3*d^
4)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 2*(-I*(b*x + a)^4*
d^4 + 4*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 6*(-I*b^2*c^2*d^2 + 2*I*a*b*c
*d^3 - I*a^2*d^4)*(b*x + a)^2 + 4*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 - 3*I*
a^2*b*c*d^3 + I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) +
1) + 2*((b*x + a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*(a^2 - 2)*d^4
 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 -
 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^
3 - (a^3 - 6*a)*d^4)*(b*x + a))*cos(b*x + a) + 8*(I*b^3*c^3*d - 3*I*a*b^2*
c^2*d^2 + 3*I*a^2*b*c*d^3 + I*(b*x + a)^3*d^4 - I*a^3*d^4 + 3*(I*b*c*d^3 -
 I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x
 + a))*dilog(-e^(I*b*x + I*a)) + 8*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 - ...
```

Giac [F]

$$\int (c+dx)^4 \cos(a+bx) \cot(a+bx)\, dx = \int (dx+c)^4 \cos(bx+a) \cot(bx+a)\, dx$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*cos(b*x + a)*cot(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \cos(a+bx) \cot(a+bx)\, dx = \int \cos(a+b\,x)\, \cot(a+b\,x)\, (c+d\,x)^4\, dx$$

input
```
int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^4,x)
```

output
```
int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^4, x)
```

Reduce [F]

$$\int (c+dx)^4 \cos(a+bx) \cot(a+bx)\, dx$$
$$= \frac{\cos(bx+a)\, c^4 + \left(\int \cos(bx+a) \cot(bx+a)\, x^4 dx\right) b\, d^4 + 4\left(\int \cos(bx+a) \cot(bx+a)\, x^3 dx\right) bc\, d^3 + 6\left(\int}{}$$

input
```
int((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x)
```

output
```
(cos(a + b*x)*c**4 + int(cos(a + b*x)*cot(a + b*x)*x**4,x)*b*d**4 + 4*int(
cos(a + b*x)*cot(a + b*x)*x**3,x)*b*c*d**3 + 6*int(cos(a + b*x)*cot(a + b*
x)*x**2,x)*b*c**2*d**2 + 4*int(cos(a + b*x)*cot(a + b*x)*x,x)*b*c**3*d + l
og(tan((a + b*x)/2))*c**4 - c**4)/b
```

3.99 $\int (c+dx)^3 \cos(a+bx) \cot(a+bx)\, dx$

Optimal result . . . 942
Mathematica [A] (verified) . . . 943
Rubi [A] (verified) . . . 943
Maple [B] (verified) . . . 948
Fricas [B] (verification not implemented) . . . 949
Sympy [F] . . . 950
Maxima [B] (verification not implemented) . . . 951
Giac [F] . . . 952
Mupad [F(-1)] . . . 952
Reduce [F] . . . 952

Optimal result

Integrand size = 20, antiderivative size = 254

$$
\begin{aligned}
&\int (c+dx)^3 \cos(a+bx) \cot(a+bx)\, dx \\
&= -\frac{2(c+dx)^3 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} \\
&\quad + \frac{(c+dx)^3 \cos(a+bx)}{b} + \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^2} - \frac{6d^2(c+dx)\operatorname{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{6d^2(c+dx)\operatorname{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3} - \frac{6id^3 \operatorname{PolyLog}\left(4, -e^{i(a+bx)}\right)}{b^4} \\
&\quad + \frac{6id^3 \operatorname{PolyLog}\left(4, e^{i(a+bx)}\right)}{b^4} + \frac{6d^3 \sin(a+bx)}{b^4} - \frac{3d(c+dx)^2 \sin(a+bx)}{b^2}
\end{aligned}
$$

output

```
-2*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b-6*d^2*(d*x+c)*cos(b*x+a)/b^3+(d*x+c
)^3*cos(b*x+a)/b+3*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^2-3*I*d*(d*x
+c)^2*polylog(2,exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)*polylog(3,-exp(I*(b*x+a)
))/b^3+6*d^2*(d*x+c)*polylog(3,exp(I*(b*x+a)))/b^3-6*I*d^3*polylog(4,-exp(
I*(b*x+a)))/b^4+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+6*d^3*sin(b*x+a)/b^4
-3*d*(d*x+c)^2*sin(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.30

$$\int (c+dx)^3 \cos(a+bx)\cot(a+bx)\,dx$$
$$= \frac{-2b^3(c+dx)^3\operatorname{arctanh}(\cos(a+bx)+i\sin(a+bx)) + 3id(b^2(c+dx)^2\operatorname{PolyLog}(2,-\cos(a+bx)-i\sin(a+bx)) - 2b(c+dx)\operatorname{PolyLog}(3,-\cos(a+bx)-i\sin(a+bx)) + 2d^2\operatorname{PolyLog}(4,-\cos(a+bx)-i\sin(a+bx))) - 3id(b^2(c+dx)^2\operatorname{PolyLog}(2,\cos(a+bx)+i\sin(a+bx)) - 2b(c+dx)\operatorname{PolyLog}(3,\cos(a+bx)+i\sin(a+bx)) + 2d^2\operatorname{PolyLog}(4,\cos(a+bx)+i\sin(a+bx))) + \cos(bx)(b(c+dx)(-6d^2+b^2(c+dx)^2)\cos(a) - 3d(-2d^2+b^2(c+dx)^2)\sin(a)) - (3d(-2d^2+b^2(c+dx)^2)\cos(a) + b(c+dx)(-6d^2+b^2(c+dx)^2)\sin(a))\sin(bx)}{b^4}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x],x]
```

output
```
(-2*b^3*(c + d*x)^3*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]] + (3*I)*d*(b^2*
(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]] + (2*I)*b*d*(c + d*
x)*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] - 2*d^2*PolyLog[4, -Cos[a +
b*x] - I*Sin[a + b*x]]) - (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, Cos[a + b*x]
 + I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, Cos[a + b*x] + I*Sin[a
 + b*x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Sin[a + b*x]]) + Cos[b*x]*(b*
(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a] - 3*d*(-2*d^2 + b^2*(c + d*x)^
2)*Sin[a]) - (3*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*(c + d*x)*(-6*d^2
+ b^2*(c + d*x)^2)*Sin[a])*Sin[b*x])/b^4
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {4908, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \cos(a+bx)\cot(a+bx)\,dx$$
$$\downarrow 4908$$
$$\int (c+dx)^3 \csc(a+bx)dx - \int (c+dx)^3 \sin(a+bx)dx$$
$$\downarrow 3042$$

$$\int (c+dx)^3 \csc(a+bx)dx - \int (c+dx)^3 \sin(a+bx)dx$$

$$\downarrow 3777$$

$$-\frac{3d\int (c+dx)^2 \cos(a+bx)dx}{b} + \int (c+dx)^3 \csc(a+bx)dx + \frac{(c+dx)^3 \cos(a+bx)}{b}$$

$$\downarrow 3042$$

$$-\frac{3d\int (c+dx)^2 \sin\left(a+bx+\frac{\pi}{2}\right)dx}{b} + \int (c+dx)^3 \csc(a+bx)dx + \frac{(c+dx)^3 \cos(a+bx)}{b}$$

$$\downarrow 3777$$

$$-\frac{3d\left(\frac{2d\int -((c+dx)\sin(a+bx))dx}{b} + \frac{(c+dx)^2 \sin(a+bx)}{b}\right)}{b} + \int (c+dx)^3 \csc(a+bx)dx + \frac{(c+dx)^3 \cos(a+bx)}{b}$$

$$\downarrow 25$$

$$-\frac{3d\left(\frac{(c+dx)^2 \sin(a+bx)}{b} - \frac{2d\int (c+dx)\sin(a+bx)dx}{b}\right)}{b} + \int (c+dx)^3 \csc(a+bx)dx + \frac{(c+dx)^3 \cos(a+bx)}{b}$$

$$\downarrow 3042$$

$$-\frac{3d\left(\frac{(c+dx)^2 \sin(a+bx)}{b} - \frac{2d\int (c+dx)\sin(a+bx)dx}{b}\right)}{b} + \int (c+dx)^3 \csc(a+bx)dx + \frac{(c+dx)^3 \cos(a+bx)}{b}$$

$$\downarrow 3777$$

$$\int (c+dx)^3 \csc(a+bx)dx - \frac{3d\left(\frac{(c+dx)^2 \sin(a+bx)}{b} - \frac{2d\left(\frac{d\int \cos(a+bx)dx}{b} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^3 \cos(a+bx)}{b}$$

$$\downarrow 3042$$

$$\int (c+dx)^3 \csc(a+bx)dx - \frac{3d\left(\frac{(c+dx)^2 \sin(a+bx)}{b} - \frac{2d\left(\frac{d\int \sin\left(a+bx+\frac{\pi}{2}\right)dx}{b} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^3 \cos(a+bx)}{b}$$

$$\downarrow 3117$$

$$\int (c+dx)^3 \csc(a+bx)dx - \frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b} - \frac{2d\left(\frac{d\sin(a+bx)}{b^2} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\downarrow 4671$$

$$-\frac{3d\int (c+dx)^2\log\left(1-e^{i(a+bx)}\right)dx}{b} + \frac{3d\int (c+dx)^2\log\left(1+e^{i(a+bx)}\right)dx}{b} - \frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b} - \frac{2d\left(\frac{d\sin(a+bx)}{b^2} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\downarrow 3011$$

$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b} - \frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b} - \frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b} - \frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b} - \frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b} - \frac{2d\left(\frac{d\sin(a+bx)}{b^2} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\downarrow 7163$$

$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b} - \frac{2id\left(\frac{id\int \operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)dx}{b} - \frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b} - \frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b} - \frac{2id\left(\frac{id\int \operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)dx}{b} - \frac{i(c+dx)\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b} - \frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b} - \frac{2d\left(\frac{d\sin(a+bx)}{b^2} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\downarrow 2720$$

$$\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{2(c+dx)^3\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\downarrow 7143$$

$$-\frac{2(c+dx)^3\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\,\mathrm{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\,\mathrm{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^3\cos(a+bx)}{b}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x],x]
```

output
```
(-2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b + ((c + d*x)^3*Cos[a + b*x])/b
 + (3*d*((I*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*
(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b + (d*PolyLog[4, -E^(I*(a + b*x))
])/b^2))/b))/b - (3*d*((I*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b - ((2
*I)*d*(((-I)*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b + (d*PolyLog[4, E^(I
*(a + b*x))])/b^2))/b))/b - (3*d*(((c + d*x)^2*Sin[a + b*x])/b - (2*d*(-((
(c + d*x)*Cos[a + b*x])/b) + (d*Sin[a + b*x])/b^2))/b))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))  Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 846 vs. $2(236) = 472$.

Time = 0.79 (sec) , antiderivative size = 847, normalized size of antiderivative = 3.33

method	result
risch	$-\frac{6icd^2\,\mathrm{polylog}(2,e^{i(bx+a)})x}{b^2}+\frac{6icd^2\,\mathrm{polylog}(2,-e^{i(bx+a)})x}{b^2}+\frac{3id^3\,\mathrm{polylog}(2,-e^{i(bx+a)})x^2}{b^2}-\frac{3id^3\,\mathrm{polylog}(2,e^{i(bx+a)})x^2}{b^2}$

input
```
int((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
1/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b
*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))
-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-exp(I*(b*x+a))
)*a+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-6/b^3*d^3*polylog(3,-exp(I*(b*x+
a)))*x-1/b^4*d^3*ln(exp(I*(b*x+a))+1)*a^3+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a
^3-6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,exp(I*(b*x
+a)))+2/b^4*d^3*a^3*arctanh(exp(I*(b*x+a)))+1/b*d^3*ln(1-exp(I*(b*x+a)))*x
^3-1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3-3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2-6
/b^3*c*d^2*a^2*arctanh(exp(I*(b*x+a)))+1/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*
b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*
I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))+6/b^2*c^2*d*a*arctanh(exp(I*(b*x+
a)))-3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2
-3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x-2/b*c^3
*arctanh(exp(I*(b*x+a)))+6*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+a)))*x-3/b^2*
c^2*d*ln(exp(I*(b*x+a))+1)*a+3/b^3*c*d^2*ln(exp(I*(b*x+a))+1)*a^2+3*I/b^2*
c^2*d*polylog(2,-exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))+3
*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog(2,exp(I*(b*x
+a)))*x^2+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4-6*I*d^3*polylog(4,-exp(I*(
b*x+a)))/b^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(230) = 460$.

Time = 0.13 (sec) , antiderivative size = 925, normalized size of antiderivative = 3.64

$$\int (c + dx)^3 \cos(a + bx) \cot(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

output

```
1/2*(6*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4
, cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*polylog(4, -cos(b*x + a) + I*si
n(b*x + a)) - 6*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 2*(b^3*
d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*
x)*cos(b*x + a) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(
cos(b*x + a) + I*sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b
^2*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^
2*c*d^2*x + I*b^2*c^2*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 3*(-I*b^2
*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*sin(b*x
+ a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(
b*x + a) + I*sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^
2*d*x + b^3*c^3)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x
+ a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2
*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x +
 a) + I*sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x
 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x
+ a) + 1) + 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)
) + 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 6...
```

Sympy [F]

$$\int (c+dx)^3 \cos(a+bx) \cot(a+bx)\, dx = \int (c+dx)^3 \cos(a+bx) \cot(a+bx)\, dx$$

input

```
integrate((d*x+c)**3*cos(b*x+a)*cot(b*x+a),x)
```

output

```
Integral((c + d*x)**3*cos(a + b*x)*cot(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 929 vs. $2(230) = 460$.

Time = 0.18 (sec) , antiderivative size = 929, normalized size of antiderivative = 3.66

$$\int (c+dx)^3 \cos(a+bx) \cot(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

output

```
1/2*(c^3*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
- 3*a*c^2*d*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1
))/b + 3*a^2*c*d^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x +
 a) - 1))/b^2 - a^3*d^3*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(
b*x + a) - 1))/b^3 - (12*I*d^3*polylog(4, -e^(I*b*x + I*a)) - 12*I*d^3*pol
ylog(4, e^(I*b*x + I*a)) - 2*(-I*(b*x + a)^3*d^3 + 3*(-I*b*c*d^2 + I*a*d^3
)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*(b*x + a))*ar
ctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(-I*(b*x + a)^3*d^3 + 3*(-I*b*c*
d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*
(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*((b*x + a)^3*d^3 -
 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*
a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos(b*x + a) - 6*(I*b^2*c^2*d - 2*I*
a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x +
 a))*dilog(-e^(I*b*x + I*a)) - 6*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x +
a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*dilog(e^(I*b*x
+ I*a)) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2
+ 2*cos(b*x + a) + 1) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2
 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + s
in(b*x + a)^2 - 2*cos(b*x + a) + 1) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d...
```

Giac [F]

$$\int (c+dx)^3 \cos(a+bx) \cot(a+bx)\, dx = \int (dx+c)^3 \cos(bx+a) \cot(bx+a)\, dx$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*cos(b*x + a)*cot(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \cos(a+bx) \cot(a+bx)\, dx = \int \cos(a+b\,x)\, \cot(a+b\,x)\, (c+d\,x)^3\, dx$$

input
```
int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^3,x)
```

output
```
int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \cos(a+bx) \cot(a+bx)\, dx$$
$$= \frac{\cos(bx+a)\, c^3 + \left(\int \cos(bx+a) \cot(bx+a)\, x^3 dx\right) b\, d^3 + 3\left(\int \cos(bx+a) \cot(bx+a)\, x^2 dx\right) bc\, d^2 + 3\left(\int$$
$$b$$

input
```
int((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x)
```

output
```
(cos(a + b*x)*c**3 + int(cos(a + b*x)*cot(a + b*x)*x**3,x)*b*d**3 + 3*int(
cos(a + b*x)*cot(a + b*x)*x**2,x)*b*c*d**2 + 3*int(cos(a + b*x)*cot(a + b*
x)*x,x)*b*c**2*d + log(tan((a + b*x)/2))*c**3 - c**3)/b
```

3.100 $\int (c+dx)^2 \cos(a+bx) \cot(a+bx)\, dx$

Optimal result . . . 953
Mathematica [A] (verified) . . . 954
Rubi [A] (verified) . . . 954
Maple [B] (verified) . . . 958
Fricas [B] (verification not implemented) . . . 958
Sympy [F] . . . 959
Maxima [B] (verification not implemented) . . . 959
Giac [F] . . . 960
Mupad [F(-1)] . . . 960
Reduce [F] . . . 961

Optimal result

Integrand size = 20, antiderivative size = 171

$$\begin{aligned}\int (c+dx)^2 \cos(a+bx) \cot(a+bx)\, dx = &-\frac{2(c+dx)^2 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} \\ &- \frac{2d^2 \cos(a+bx)}{b^3} + \frac{(c+dx)^2 \cos(a+bx)}{b} \\ &+ \frac{2id(c+dx)\,\operatorname{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^2} \\ &- \frac{2id(c+dx)\,\operatorname{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^2} \\ &- \frac{2d^2\,\operatorname{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^3} \\ &+ \frac{2d^2\,\operatorname{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3} \\ &- \frac{2d(c+dx)\sin(a+bx)}{b^2}\end{aligned}$$

output

```
-2*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b-2*d^2*cos(b*x+a)/b^3+(d*x+c)^2*cos(
b*x+a)/b+2*I*d*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^2-2*I*d*(d*x+c)*polylo
g(2,exp(I*(b*x+a)))/b^2-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog
(3,exp(I*(b*x+a)))/b^3-2*d*(d*x+c)*sin(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.29

$$\int (c+dx)^2 \cos(a+bx)\cot(a+bx)\,dx$$
$$= \frac{b^2(c+dx)^2 \log\left(1-e^{i(a+bx)}\right) - b^2(c+dx)^2 \log\left(1+e^{i(a+bx)}\right) + 2ibd(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right) - 2ib$$

input
```
Integrate[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x],x]
```

output
```
(b^2*(c + d*x)^2*Log[1 - E^(I*(a + b*x))] - b^2*(c + d*x)^2*Log[1 + E^(I*(
a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] - (2*I)*b*d*
(c + d*x)*PolyLog[2, E^(I*(a + b*x))] - 2*d^2*PolyLog[3, -E^(I*(a + b*x))]
 + 2*d^2*PolyLog[3, E^(I*(a + b*x))] + Cos[b*x]*((-2*d^2 + b^2*(c + d*x)^2
)*Cos[a] - 2*b*d*(c + d*x)*Sin[a]) - (2*b*d*(c + d*x)*Cos[a] + (-2*d^2 + b
^2*(c + d*x)^2)*Sin[a])*Sin[b*x])/b^3
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4908, 3042, 3777, 3042, 3777, 25, 3042, 3118, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \cos(a+bx)\cot(a+bx)\,dx$$
$$\downarrow 4908$$
$$\int (c+dx)^2 \csc(a+bx)dx - \int (c+dx)^2 \sin(a+bx)dx$$
$$\downarrow 3042$$
$$\int (c+dx)^2 \csc(a+bx)dx - \int (c+dx)^2 \sin(a+bx)dx$$
$$\downarrow 3777$$

$$-\frac{2d\int(c+dx)\cos(a+bx)dx}{b}+\int(c+dx)^2\csc(a+bx)dx+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\downarrow 3042$$

$$-\frac{2d\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}+\int(c+dx)^2\csc(a+bx)dx+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\downarrow 3777$$

$$-\frac{2d\left(\frac{d\int-\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\int(c+dx)^2\csc(a+bx)dx+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\downarrow 25$$

$$-\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}+\int(c+dx)^2\csc(a+bx)dx+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\downarrow 3042$$

$$-\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}+\int(c+dx)^2\csc(a+bx)dx+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\downarrow 3118$$

$$\int(c+dx)^2\csc(a+bx)dx-\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\downarrow 4671$$

$$-\frac{2d\int(c+dx)\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{2d\int(c+dx)\log\left(1+e^{i(a+bx)}\right)dx}{b}-$$
$$\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\downarrow 3011$$

$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{id\int\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}-$$
$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{id\int\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-$$
$$\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\downarrow 2720$$

$$\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-$$
$$\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-$$
$$\frac{2(c+dx)^2\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\downarrow 7143$$

$$-\frac{2(c+dx)^2\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^2}\right)}{b}-$$
$$\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\,\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+$$
$$\frac{(c+dx)^2\cos(a+bx)}{b}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x],x]
```

output
```
(-2*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b + ((c + d*x)^2*Cos[a + b*x])/b
 + (2*d*((I*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b - (d*PolyLog[3, -E^(
I*(a + b*x))])/b^2))/b - (2*d*((I*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b
 - (d*PolyLog[3, E^(I*(a + b*x))])/b^2))/b - (2*d*((d*Cos[a + b*x])/b^2 +
((c + d*x)*Sin[a + b*x])/b))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4671

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4908

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(159) = 318.

Time = 0.52 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.80

method	result
risch	$\frac{(x^2d^2b^2+2b^2cdx+2ib\,d^2x+b^2c^2+2ibcd-2d^2)e^{i(bx+a)}}{2b^3}+\frac{(x^2d^2b^2+2b^2cdx-2ib\,d^2x+b^2c^2-2ibcd-2d^2)e^{-i(bx+a)}}{2b^3}-\frac{2idc\,\text{polylog}}{b}$

input
```
int((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(
I*(b*x+a))+1/2*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*
d)/b^3*exp(-I*(b*x+a))-2*I/b^2*d*c*polylog(2,exp(I*(b*x+a)))+4/b^2*c*d*a*a
rctanh(exp(I*(b*x+a)))+2*d^2*polylog(3,exp(I*(b*x+a)))/b^3-2*I/b^2*d^2*pol
ylog(2,exp(I*(b*x+a)))*x-2/b*d*c*ln(exp(I*(b*x+a))+1)*x+2/b*d*c*ln(1-exp(I
*(b*x+a)))*x+2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))+2/b^2*d*c*ln(1-exp(I*(
b*x+a)))*a+2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+1/b^3*d^2*ln(exp(I*(b*
x+a))+1)*a^2-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2-2/b^3*d^2*a^2*arctanh(exp(
I*(b*x+a)))-2/b^2*c*d*ln(exp(I*(b*x+a))+1)*a-2/b*c^2*arctanh(exp(I*(b*x+a)
))-1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+1
/b*d^2*ln(1-exp(I*(b*x+a)))*x^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 562 vs. 2(155) = 310.

Time = 0.11 (sec) , antiderivative size = 562, normalized size of antiderivative = 3.29

$$\int (c+dx)^2\cos(a+bx)\cot(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3, co
s(b*x + a) - I*sin(b*x + a)) - 2*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x
+ a)) - 2*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 2*(b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a) - 2*(I*b*d^2*x + I*b*c*d)*di
log(cos(b*x + a) + I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog(cos(b*
x + a) - I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(-cos(b*x + a) + I
*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog(-cos(b*x + a) - I*sin(b*x
+ a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) + I*sin(b*x
 + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) - I*si
n(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x + a) +
1/2*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b
*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c
*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b
^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) -
4*(b*d^2*x + b*c*d)*sin(b*x + a))/b^3
```

Sympy [F]

$$\int (c+dx)^2 \cos(a+bx) \cot(a+bx)\, dx = \int (c+dx)^2 \cos\left(a+bx\right) \cot\left(a+bx\right) dx$$

input
```
integrate((d*x+c)**2*cos(b*x+a)*cot(b*x+a),x)
```

output
```
Integral((c + d*x)**2*cos(a + b*x)*cot(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(155) = 310$.

Time $= 0.12$ (sec) , antiderivative size $= 513$, normalized size of antiderivative $= 3.00$

$$\int (c+dx)^2 \cos(a+bx) \cot(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(c^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
- 2*a*c*d*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b + a^2*d^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) -
1))/b^2 - (4*d^2*polylog(3, -e^(I*b*x + I*a)) - 4*d^2*polylog(3, e^(I*b*x
+ I*a)) - 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan
2(sin(b*x + a), cos(b*x + a) + 1) - 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d +
I*a*d^2)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*((b*x + a
)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(b*x + a) - 4*(I*b*c*d +
 I*(b*x + a)*d^2 - I*a*d^2)*dilog(-e^(I*b*x + I*a)) - 4*(-I*b*c*d - I*(b*x
 + a)*d^2 + I*a*d^2)*dilog(e^(I*b*x + I*a)) + ((b*x + a)^2*d^2 + 2*(b*c*d
- a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) +
 1) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 +
 sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*
sin(b*x + a))/b^2)/b
```

Giac [F]

$$\int (c+dx)^2 \cos(a+bx) \cot(a+bx)\,dx = \int (dx+c)^2 \cos(bx+a) \cot(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*cos(b*x + a)*cot(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \cos(a+bx) \cot(a+bx)\,dx = \int \cos(a+b\,x)\,\cot(a+b\,x)\,(c+d\,x)^2\,dx$$

input
```
int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^2,x)
```

output
```
int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \cos(a+bx) \cot(a+bx)\, dx$$
$$= \frac{\cos(bx+a)\, c^2 + \left(\int \cos(bx+a) \cot(bx+a)\, x^2 dx\right) b\, d^2 + 2\left(\int \cos(bx+a) \cot(bx+a)\, x dx\right) bcd + \log(\text{ta}}{b}$$

input
```
int((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x)
```

output
```
(cos(a + b*x)*c**2 + int(cos(a + b*x)*cot(a + b*x)*x**2,x)*b*d**2 + 2*int(
cos(a + b*x)*cot(a + b*x)*x,x)*b*c*d + log(tan((a + b*x)/2))*c**2 - c**2)/
b
```

3.101 $\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx$

Optimal result . . . 962
Mathematica [A] (verified) . . . 963
Rubi [A] (verified) . . . 963
Maple [B] (verified) . . . 966
Fricas [B] (verification not implemented) . . . 966
Sympy [F] . . . 967
Maxima [B] (verification not implemented) . . . 967
Giac [F] . . . 968
Mupad [F(-1)] . . . 968
Reduce [F] . . . 969

Optimal result

Integrand size = 18, antiderivative size = 94

$$\begin{aligned}\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx = &-\frac{2(c+dx)\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}\\ &+\frac{(c+dx)\cos(a+bx)}{b}\\ &+\frac{id\,\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}\\ &-\frac{id\,\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{d\sin(a+bx)}{b^2}\end{aligned}$$

output

```
-2*(d*x+c)*arctanh(exp(I*(b*x+a)))/b+(d*x+c)*cos(b*x+a)/b+I*d*polylog(2,-e
xp(I*(b*x+a)))/b^2-I*d*polylog(2,exp(I*(b*x+a)))/b^2-d*sin(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.87

$$\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx$$
$$= \frac{c\cos(a+bx)}{b} - \frac{c\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{c\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$
$$+ \frac{d\big((a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)-a\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)+i\big(\text{PolyLog}\left(2,-e^{i(a+bx)}\right)$$
$$+ \frac{d\cos(bx)(bx\cos(a)-\sin(a))}{b^2} - \frac{d(\cos(a)+bx\sin(a))\sin(bx)}{b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]*Cot[a + b*x],x]
```

output
```
(c*Cos[a + b*x])/b - (c*Log[Cos[(a + b*x)/2]])/b + (c*Log[Sin[(a + b*x)/2]
])/b + (d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))])
 - a*Log[Tan[(a + b*x)/2]] + I*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2,
E^(I*(a + b*x))])))/b^2 + (d*Cos[b*x]*(b*x*Cos[a] - Sin[a]))/b^2 - (d*(Cos
[a] + b*x*Sin[a])*Sin[b*x])/b^2
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4908, 3042, 3777, 3042, 3117, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx$$
$$\downarrow 4908$$
$$\int (c+dx)\csc(a+bx)dx - \int (c+dx)\sin(a+bx)dx$$
$$\downarrow 3042$$

$$\int (c+dx)\csc(a+bx)dx - \int (c+dx)\sin(a+bx)dx$$

$$\downarrow 3777$$

$$\int (c+dx)\csc(a+bx)dx - \frac{d\int \cos(a+bx)dx}{b} + \frac{(c+dx)\cos(a+bx)}{b}$$

$$\downarrow 3042$$

$$\int (c+dx)\csc(a+bx)dx - \frac{d\int \sin\left(a+bx+\frac{\pi}{2}\right)dx}{b} + \frac{(c+dx)\cos(a+bx)}{b}$$

$$\downarrow 3117$$

$$\int (c+dx)\csc(a+bx)dx - \frac{d\sin(a+bx)}{b^2} + \frac{(c+dx)\cos(a+bx)}{b}$$

$$\downarrow 4671$$

$$-\frac{d\int \log\left(1-e^{i(a+bx)}\right)dx}{b} + \frac{d\int \log\left(1+e^{i(a+bx)}\right)dx}{b} - \frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{d\sin(a+bx)}{b^2} + \frac{(c+dx)\cos(a+bx)}{b}$$

$$\downarrow 2715$$

$$\frac{id\int e^{-i(a+bx)}\log\left(1-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2} - \frac{id\int e^{-i(a+bx)}\log\left(1+e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2} - \frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{d\sin(a+bx)}{b^2} + \frac{(c+dx)\cos(a+bx)}{b}$$

$$\downarrow 2838$$

$$-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b} + \frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} - \frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} - \frac{d\sin(a+bx)}{b^2} + \frac{(c+dx)\cos(a+bx)}{b}$$

input

```
Int[(c + d*x)*Cos[a + b*x]*Cot[a + b*x],x]
```

output

```
(-2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + ((c + d*x)*Cos[a + b*x])/b + (
I*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^(I*(a + b*x))])/
b^2 - (d*Sin[a + b*x])/b^2
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(86) = 172$.

Time = 0.41 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.88

method	result
default	$\frac{-\frac{da\cos(bx+a)}{b}+c\cos(bx+a)-\frac{d(\sin(bx+a)-(bx+a)\cos(bx+a))}{b}}{b}+\frac{-\frac{da\ln(\csc(bx+a)-\cot(bx+a))}{b}+c\ln(\csc(bx+a)-\cot(bx+a))+\frac{d\left((bx+\right.}$
risch	$\frac{(bdx+cb+id)e^{i(bx+a)}}{2b^2}+\frac{(bdx+cb-id)e^{-i(bx+a)}}{2b^2}-\frac{2c\,\operatorname{arctanh}\left(e^{i(bx+a)}\right)}{b}-\frac{d\ln\left(e^{i(bx+a)}+1\right)x}{b}-\frac{d\ln\left(e^{i(bx+a)}+1\right)a}{b^2}+\frac{id\,p}{}$

input
```
int((d*x+c)*cos(b*x+a)*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/b*d*a*cos(b*x+a)+c*cos(b*x+a)-1/b*d*(sin(b*x+a)-(b*x+a)*cos(b*x+a)
))+1/b*(-1/b*d*a*ln(csc(b*x+a)-cot(b*x+a))+c*ln(csc(b*x+a)-cot(b*x+a))+1/b
*d*((b*x+a)*ln(1-exp(I*(b*x+a)))-(b*x+a)*ln(exp(I*(b*x+a))+1)+I*dilog(exp(
I*(b*x+a))+1)-I*dilog(1-exp(I*(b*x+a)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(82) = 164$.

Time = 0.10 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.95

$$\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx$$
$$= \frac{2\,(bdx+bc)\cos(bx+a) - i\,d\mathrm{Li}_2(\cos(bx+a)+i\,\sin(bx+a)) + i\,d\mathrm{Li}_2(\cos(bx+a)-i\,\sin(bx+a)) -}{}$$

input
```
integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*(b*d*x + b*c)*cos(b*x + a) - I*d*dilog(cos(b*x + a) + I*sin(b*x + a
)) + I*d*dilog(cos(b*x + a) - I*sin(b*x + a)) - I*d*dilog(-cos(b*x + a) +
I*sin(b*x + a)) + I*d*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b*d*x + b*c
)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b*d*x + b*c)*log(cos(b*x + a)
- I*sin(b*x + a) + 1) + (b*c - a*d)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x
+ a) + 1/2) + (b*c - a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2
) + (b*d*x + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b*d*x + a*d)*
log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 2*d*sin(b*x + a))/b^2
```

Sympy [F]

$$\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)\cos(a+bx)\cot(a+bx)\,dx$$

input
```
integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x)
```

output
```
Integral((c + d*x)*cos(a + b*x)*cot(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(82) = 164$.

Time = 0.11 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.13

$$\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx =$$

$$-\frac{2i\,bdx\arctan\left(\sin\left(bx+a\right),-\cos\left(bx+a\right)+1\right)-2i\,bc\arctan\left(\sin\left(bx+a\right),\cos\left(bx+a\right)-1\right)-2\left(-i\,b\right.}{}$$

input
```
integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(2*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*I*b*c*arctan2
(sin(b*x + a), cos(b*x + a) - 1) - 2*(-I*b*d*x - I*b*c)*arctan2(sin(b*x +
a), cos(b*x + a) + 1) - 2*(b*d*x + b*c)*cos(b*x + a) - 2*I*d*dilog(-e^(I*b
*x + I*a)) + 2*I*d*dilog(e^(I*b*x + I*a)) + (b*d*x + b*c)*log(cos(b*x + a)
^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (b*d*x + b*c)*log(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 2*d*sin(b*x + a))/b^2
```

Giac [F]

$$\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx = \int (dx+c)\cos\left(bx+a\right)\cot\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*cos(b*x + a)*cot(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx = \int \cos\left(a+b\,x\right)\,\cot(a+b\,x)\,\left(c+d\,x\right)\,dx$$

input
```
int(cos(a + b*x)*cot(a + b*x)*(c + d*x),x)
```

output
```
int(cos(a + b*x)*cot(a + b*x)*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx$$
$$= \frac{\cos(bx+a)\,c + \left(\int \cos(bx+a)\cot(bx+a)\,x dx\right) bd + \log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right) c - c}{b}$$

input
```
int((d*x+c)*cos(b*x+a)*cot(b*x+a),x)
```

output
```
(cos(a + b*x)*c + int(cos(a + b*x)*cot(a + b*x)*x,x)*b*d + log(tan((a + b*
x)/2))*c - c)/b
```

3.102 $\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$

Optimal result 970
Mathematica [N/A] 970
Rubi [N/A] 971
Maple [N/A] 973
Fricas [N/A] 973
Sympy [N/A] 973
Maxima [N/A] 974
Giac [N/A] 974
Mupad [N/A] 975
Reduce [N/A] 975

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx = -\frac{\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d} - \frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d} + \text{Int}\left(\frac{\csc(a+bx)}{c+dx}, x\right)$$

output

```
-Ci(b*c/d+b*x)*sin(a-b*c/d)/d-cos(a-b*c/d)*Si(b*c/d+b*x)/d+Defer(Int)(csc(
b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 6.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$$

input

```
Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x),x]
```

output

```
Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4908, 3042, 3784, 3042, 3780, 3783, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$$

$$\downarrow 4908$$

$$\int \frac{\csc(a+bx)}{c+dx}dx - \int \frac{\sin(a+bx)}{c+dx}dx$$

$$\downarrow 3042$$

$$\int \frac{\csc(a+bx)}{c+dx}dx - \int \frac{\sin(a+bx)}{c+dx}dx$$

$$\downarrow 3784$$

$$\int \frac{\csc(a+bx)}{c+dx}dx - \sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx - \cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx$$

$$\downarrow 3042$$

$$-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx + \int \frac{\csc(a+bx)}{c+dx}dx - \cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx$$

$$\downarrow 3780$$

$$-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx + \int \frac{\csc(a+bx)}{c+dx}dx - \frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

$$\downarrow 3783$$

$$\int \frac{\csc(a+bx)}{c+dx}dx - \frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d} - \frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

$$\downarrow 4680$$

$$\int \frac{\csc(a+bx)}{c+dx}dx - \frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d} - \frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

input
```
Int[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos(bx+a)\cot(bx+a)}{dx+c}dx$$

input
```
int(cos(b*x+a)*cot(b*x+a)/(d*x+c),x)
```

output
```
int(cos(b*x+a)*cot(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)\cot(bx+a)}{dx+c}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*cot(b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x)
```

output
```
Integral(cos(a + b*x)*cot(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 228, normalized size of antiderivative = 11.40

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)\cot(bx+a)}{dx+c}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*((I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(1, -(I*b
*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + 2*d*integrate(sin(b*x + a)/((d*x +
 c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)*cos(b*x
+ a) + c), x) + 2*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*
x + c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) + (exp_int
egral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))
*sin(-(b*c - a*d)/d))/d
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)\cot(bx+a)}{dx+c}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*cot(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 19.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+b\,x)\,\cot(a+b\,x)}{c+d\,x}\,dx$$

input
```
int((cos(a + b*x)*cot(a + b*x))/(c + d*x),x)
```

output
```
int((cos(a + b*x)*cot(a + b*x))/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)\cot(bx+a)}{dx+c}dx$$

input
```
int(cos(b*x+a)*cot(b*x+a)/(d*x+c),x)
```

output
```
int((cos(a + b*x)*cot(a + b*x))/(c + d*x),x)
```

3.103 $\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 976
Mathematica [N/A] . . . 976
Rubi [N/A] . . . 977
Maple [N/A] . . . 979
Fricas [N/A] . . . 980
Sympy [N/A] . . . 980
Maxima [N/A] . . . 981
Giac [N/A] . . . 981
Mupad [N/A] . . . 982
Reduce [N/A] . . . 982

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = -\frac{b\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2} + \frac{\sin(a+bx)}{d(c+dx)} + \frac{b\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{d^2} + \operatorname{Int}\left(\frac{\csc(a+bx)}{(c+dx)^2}, x\right)$$

output
```
-b*cos(a-b*c/d)*Ci(b*c/d+b*x)/d^2+sin(b*x+a)/d/(d*x+c)+b*sin(a-b*c/d)*Si(b
*c/d+b*x)/d^2+Defer(Int)(csc(b*x+a)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 3.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4908, 3042, 3778, 3042, 3784, 3042, 3780, 3783, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

$$\downarrow 4908$$

$$\int \frac{\csc(a+bx)}{(c+dx)^2}dx - \int \frac{\sin(a+bx)}{(c+dx)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\csc(a+bx)}{(c+dx)^2}dx - \int \frac{\sin(a+bx)}{(c+dx)^2}dx$$

$$\downarrow 3778$$

$$-\frac{b\int \frac{\cos(a+bx)}{c+dx}dx}{d} + \int \frac{\csc(a+bx)}{(c+dx)^2}dx + \frac{\sin(a+bx)}{d(c+dx)}$$

$$\downarrow 3042$$

$$-\frac{b\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx}{d} + \int \frac{\csc(a+bx)}{(c+dx)^2}dx + \frac{\sin(a+bx)}{d(c+dx)}$$

$$\downarrow 3784$$

$$\int \frac{\csc(a+bx)}{(c+dx)^2}dx - \frac{b\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx - \sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d} + \frac{\sin(a+bx)}{d(c+dx)}$$

$$\downarrow 3042$$

$$\int \frac{\csc(a+bx)}{(c+dx)^2}dx - \frac{b\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx - \sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d} + \frac{\sin(a+bx)}{d(c+dx)}$$

$$\downarrow 3780$$

$$-\frac{b\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx - \frac{\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d} + \int \frac{\csc(a+bx)}{(c+dx)^2}dx + \frac{\sin(a+bx)}{d(c+dx)}$$

$$\downarrow 3783$$

$$\int \frac{\csc(a+bx)}{(c+dx)^2}dx - \frac{b\left(\frac{\cos\left(a-\frac{bc}{d}\right)\mathrm{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d} - \frac{\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d} + \frac{\sin(a+bx)}{d(c+dx)}$$

$$\downarrow 4680$$

$$\int \frac{\csc(a+bx)}{(c+dx)^2}dx - \frac{b\left(\frac{\cos\left(a-\frac{bc}{d}\right)\mathrm{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d} - \frac{\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d} + \frac{\sin(a+bx)}{d(c+dx)}$$

input
```
Int[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos(bx+a)\cot(bx+a)}{(dx+c)^2}dx$$

input
```
int(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x)
```

output
```
int(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(bx+a\right)\cot\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*cot(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+bx\right)\cot\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(cos(a + b*x)*cot(a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 340, normalized size of antiderivative = 17.00

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(bx+a)\cot(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*((I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(2, -(I*b
*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + 2*(d^2*x + c*d)*integrate(sin(b*x
+ a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*
x^2 + 2*c*d*x + c^2)*sin(b*x + a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*co
s(b*x + a)), x) + 2*(d^2*x + c*d)*integrate(sin(b*x + a)/(d^2*x^2 + 2*c*d*
x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*s
in(b*x + a)^2 + c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)), x) + (exp
_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)
/d))*sin(-(b*c - a*d)/d))/(d^2*x + c*d)
```

Giac [N/A]

Not integrable

Time = 2.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(bx+a)\cot(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*cot(b*x + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 19.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(a+b\,x)\,\cot(a+b\,x)}{(c+d\,x)^2}\,dx$$

input
```
int((cos(a + b*x)*cot(a + b*x))/(c + d*x)^2,x)
```

output
```
int((cos(a + b*x)*cot(a + b*x))/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(bx+a)\cot(bx+a)}{d^2x^2+2cdx+c^2}dx$$

input
```
int(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x)
```

output
```
int((cos(a + b*x)*cot(a + b*x))/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.104 $\int (c+dx)^m \cot^2(a+bx)\,dx$

Optimal result . . . 983
Mathematica [N/A] . . . 983
Rubi [N/A] . . . 984
Maple [N/A] . . . 985
Fricas [N/A] . . . 985
Sympy [N/A] . . . 986
Maxima [N/A] . . . 986
Giac [N/A] . . . 987
Mupad [N/A] . . . 987
Reduce [N/A] . . . 987

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (c+dx)^m \cot^2(a+bx)\,dx = \text{Int}\big((c+dx)^m \cot^2(a+bx), x\big)$$

output
```
Defer(Int)((d*x+c)^m*cot(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \cot^2(a+bx)\,dx = \int (c+dx)^m \cot^2(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Cot[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Cot[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(a+bx)(c+dx)^m\,dx$$

$$\downarrow 3042$$

$$\int \tan\left(a+bx+\frac{\pi}{2}\right)^2 (c+dx)^m dx$$

$$\downarrow 4222$$

$$\int \cot^2(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Cot[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \cot(bx+a)^2 \, dx$$

input
```
int((d*x+c)^m*cot(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*cot(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \cot^2(a+bx) \, dx = \int (dx+c)^m \cot(bx+a)^2 \, dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*cot(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (c+dx)^m \cot^2(a+bx)\,dx = \int (c+dx)^m \cot^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cot(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*cot(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \cot^2(a+bx)\,dx = \int (dx+c)^m \cot\left(bx+a\right)^2\,dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cot(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \cot^2(a+bx)\,dx = \int (dx+c)^m \cot\left(bx+a\right)^2\,dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cot(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 18.59 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \cot^2(a+bx)\,dx = \int \cot(a+b\,x)^2\,(c+d\,x)^m\,dx$$

input
```
int(cot(a + b*x)^2*(c + d*x)^m,x)
```

output
```
int(cot(a + b*x)^2*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \cot^2(a+bx)\,dx = \int (dx+c)^m \cot\left(bx+a\right)^2\,dx$$

input
```
int((d*x+c)^m*cot(b*x+a)^2,x)
```

output

```
int((c + d*x)**m*cot(a + b*x)**2,x)
```

3.105 $\int (c+dx)^4 \cot^2(a+bx)\,dx$

Optimal result . . . 989
Mathematica [B] (warning: unable to verify) . . . 990
Rubi [A] (verified) . . . 991
Maple [B] (verified) . . . 995
Fricas [B] (verification not implemented) . . . 996
Sympy [F] . . . 997
Maxima [B] (verification not implemented) . . . 997
Giac [F] . . . 998
Mupad [F(-1)] . . . 999
Reduce [F] . . . 999

Optimal result

Integrand size = 16, antiderivative size = 155

$$\begin{aligned}\int (c+dx)^4 \cot^2(a+bx)\,dx = &-\frac{i(c+dx)^4}{b} - \frac{(c+dx)^5}{5d} - \frac{(c+dx)^4\cot(a+bx)}{b} \\ &+ \frac{4d(c+dx)^3 \log\left(1-e^{2i(a+bx)}\right)}{b^2} \\ &- \frac{6id^2(c+dx)^2\,\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3} \\ &+ \frac{6d^3(c+dx)\,\mathrm{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^4} \\ &+ \frac{3id^4\,\mathrm{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^5}\end{aligned}$$

output

```
-I*(d*x+c)^4/b-1/5*(d*x+c)^5/d-(d*x+c)^4*cot(b*x+a)/b+4*d*(d*x+c)^3*ln(1-e
xp(2*I*(b*x+a)))/b^2-6*I*d^2*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^3+6*d
^3*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^4+3*I*d^4*polylog(4,exp(2*I*(b*x+
a)))/b^5
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 598 vs. $2(155) = 310$.

Time = 5.31 (sec) , antiderivative size = 598, normalized size of antiderivative = 3.86

$$\int (c+dx)^4 \cot^2(a+bx)\,dx$$

$$= -\frac{1}{5}x\left(5c^4 + 10c^3dx + 10c^2d^2x^2 + 5cd^3x^3 + d^4x^4\right) - \frac{4c^3d(bx\cot(a) - \log(\sin(a+bx)))}{b^2}$$

$$+ \frac{6c^2d^2\left(ibx(\pi - 2\arctan(\tan(a))) + \pi\log\left(1+e^{-2ibx}\right) + 2(bx + \arctan(\tan(a)))\log\left(1-e^{2i(bx+\arctan(\tan(a)))}\right)\right.}{b^2}$$

$$+ \frac{4cd^3e^{-ia}(i+\cot(a))\left(ib^3x^3 - b^3x^3\cot(a) + 3b^2x^2\log\left(1-e^{-i(a+bx)}\right) + 3b^2x^2\log\left(1+e^{-i(a+bx)}\right) + 6ibx\right.}{b^2}$$

$$+ \frac{d^4e^{-ia}(i+\cot(a))\left(ib^4x^4 - b^4x^4\cot(a) + 4b^3x^3\log\left(1-e^{-i(a+bx)}\right) + 4b^3x^3\log\left(1+e^{-i(a+bx)}\right) + 12ib^2x\right.}{b^2}$$

$$+ \frac{(c+dx)^4\csc(a)\csc(a+bx)\sin(bx)}{b}$$

input
```
Integrate[(c + d*x)^4*Cot[a + b*x]^2,x]
```

output
```
-1/5*(x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4)) - (
4*c^3*d*(b*x*Cot[a] - Log[Sin[a + b*x]]))/b^2 + (6*c^2*d^2*(I*b*x*(Pi - 2*
ArcTan[Tan[a]]) + Pi*Log[1 + E^((-2*I)*b*x)] + 2*(b*x + ArcTan[Tan[a]])*Lo
g[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] - Pi*Log[Cos[b*x]] - 2*ArcTan[Tan[
a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] - I*PolyLog[2, E^((2*I)*(b*x + ArcTan[T
an[a]]))] - b^2*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^2]))/b^3 + (4*
c*d^3*(I + Cot[a])*(I*b^3*x^3 - b^3*x^3*Cot[a] + 3*b^2*x^2*Log[1 - E^((-I)
*(a + b*x))] + 3*b^2*x^2*Log[1 + E^((-I)*(a + b*x))] + (6*I)*b*x*PolyLog[2
, -E^((-I)*(a + b*x))] + (6*I)*b*x*PolyLog[2, E^((-I)*(a + b*x))] + 6*Poly
Log[3, -E^((-I)*(a + b*x))] + 6*PolyLog[3, E^((-I)*(a + b*x))])*Sin[a])/(b
^4*E^(I*a)) + (d^4*(I + Cot[a])*(I*b^4*x^4 - b^4*x^4*Cot[a] + 4*b^3*x^3*Lo
g[1 - E^((-I)*(a + b*x))] + 4*b^3*x^3*Log[1 + E^((-I)*(a + b*x))] + (12*I)
*b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))] + (12*I)*b^2*x^2*PolyLog[2, E^((-
I)*(a + b*x))] + 24*b*x*PolyLog[3, -E^((-I)*(a + b*x))] + 24*b*x*PolyLog[3
, E^((-I)*(a + b*x))] - (24*I)*PolyLog[4, -E^((-I)*(a + b*x))] - (24*I)*Po
lyLog[4, E^((-I)*(a + b*x))])*Sin[a])/(b^5*E^(I*a)) + ((c + d*x)^4*Csc[a]*
Csc[a + b*x]*Sin[b*x])/b
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.35, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 4203, 17, 25, 3042, 25, 4202, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \cot^2(a+bx)\,dx$$

$$\downarrow 3042$$

$$\int (c+dx)^4 \tan\left(a+bx+\frac{\pi}{2}\right)^2 dx$$

$$\downarrow 4203$$

$$-\frac{4d\int -(c+dx)^3\cot(a+bx)dx}{b} - \int (c+dx)^4 dx - \frac{(c+dx)^4\cot(a+bx)}{b}$$

$$\downarrow 17$$

$$-\frac{4d\int -(c+dx)^3\cot(a+bx)dx}{b} - \frac{(c+dx)^4\cot(a+bx)}{b} - \frac{(c+dx)^5}{5d}$$

$$\downarrow 25$$

$$\frac{4d\int (c+dx)^3\cot(a+bx)dx}{b} - \frac{(c+dx)^4\cot(a+bx)}{b} - \frac{(c+dx)^5}{5d}$$

$$\downarrow 3042$$

$$\frac{4d\int -(c+dx)^3\tan\left(a+bx+\frac{\pi}{2}\right)dx}{b} - \frac{(c+dx)^4\cot(a+bx)}{b} - \frac{(c+dx)^5}{5d}$$

$$\downarrow 25$$

$$-\frac{4d\int (c+dx)^3\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx}{b} - \frac{(c+dx)^4\cot(a+bx)}{b} - \frac{(c+dx)^5}{5d}$$

$$\downarrow 4202$$

$$-\frac{4d\left(\frac{i(c+dx)^4}{4d} - 2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)^3}{1+e^{i(2a+2bx+\pi)}}dx\right)}{b} - \frac{(c+dx)^4\cot(a+bx)}{b} - \frac{(c+dx)^5}{5d}$$

$$\downarrow 2620$$

$$-\frac{4d\left(\frac{i(c+dx)^4}{4d}-2i\left(\frac{3id\int(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-$$

$$\frac{(c+dx)^4\cot(a+bx)}{b}-\frac{(c+dx)^5}{5d}$$

$$\downarrow 3011$$

$$-\frac{4d\left(\frac{i(c+dx)^4}{4d}-2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right.}{2b}\right.\right.}{b}$$

$$\frac{(c+dx)^4\cot(a+bx)}{b}-\frac{(c+dx)^5}{5d}$$

$$\downarrow 7163$$

$$-\frac{4d\left(\frac{i(c+dx)^4}{4d}-2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\left(\frac{id\int\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}\right)}{2b}\right.\right.}{b}$$

$$\frac{(c+dx)^4\cot(a+bx)}{b}-\frac{(c+dx)^5}{5d}$$

$$\downarrow 2720$$

$$-\frac{4d\left(\frac{i(c+dx)^4}{4d}-2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\left(\frac{d\int e^{-i(2a+2bx+\pi)}\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\,\mathrm{Pol}\right.}{b}\right.}{2b}\right.\right.}{b}$$

$$\frac{(c+dx)^4\cot(a+bx)}{b}-\frac{(c+dx)^5}{5d}$$

$$\downarrow 7143$$

$$-\frac{4d\left(\frac{i(c+dx)^4}{4d}-2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\left(\frac{d\,\mathrm{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}\right)}{2b}-\right.}{b}$$

$$\left.\frac{(c+dx)^4\cot(a+bx)}{b}-\frac{(c+dx)^5}{5d}\right)$$

input
```
Int[(c + d*x)^4*Cot[a + b*x]^2,x]
```

output
```
-1/5*(c + d*x)^5/d - ((c + d*x)^4*Cot[a + b*x])/b - (4*d*(((I/4)*(c + d*x)
^4)/d - (2*I)*(((-1/2*I)*(c + d*x)^3*Log[1 + E^(I*(2*a + Pi + 2*b*x))])/b
+ (((3*I)/2)*d*(((I/2)*(c + d*x)^2*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/
b - (I*d*(((-1/2*I)*(c + d*x)*PolyLog[3, -E^(I*(2*a + Pi + 2*b*x))])/b + (
d*PolyLog[4, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)))/b))/b)))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(142) = 284$.

Time = 0.50 (sec) , antiderivative size = 921, normalized size of antiderivative = 5.94

method	result	size
risch	Expression too large to display	921

input

```
int((d*x+c)^4*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
8*d^4/b^5*a^3*ln(exp(I*(b*x+a)))-4*d^4/b^5*a^3*ln(exp(I*(b*x+a))-1)-8*d/b^
2*c^3*ln(exp(I*(b*x+a)))+4*d/b^2*c^3*ln(exp(I*(b*x+a))-1)+4*d/b^2*c^3*ln(e
xp(I*(b*x+a))+1)+4*d^4/b^5*ln(1-exp(I*(b*x+a)))*a^3+4*d^4/b^2*ln(exp(I*(b*
x+a))+1)*x^3+4*d^4/b^2*ln(1-exp(I*(b*x+a)))*x^3+24*d^4/b^4*polylog(3,-exp(
I*(b*x+a)))*x+24*d^4/b^4*polylog(3,exp(I*(b*x+a)))*x+24*d^3/b^4*c*polylog(
3,-exp(I*(b*x+a)))+24*d^3/b^4*c*polylog(3,exp(I*(b*x+a)))-2*I*d^4/b*x^4-6*
I*d^4/b^5*a^4+24*I*d^4/b^5*polylog(4,-exp(I*(b*x+a)))-12*I*d^4/b^3*polylog
(2,-exp(I*(b*x+a)))*x^2-12*I*d^4/b^3*polylog(2,exp(I*(b*x+a)))*x^2-12*I*d^
2/b*c^2*x^2-12*I*d^2/b^3*c^2*a^2-12*I*d^2/b^3*c^2*polylog(2,-exp(I*(b*x+a)
))-12*I*d^2/b^3*c^2*polylog(2,exp(I*(b*x+a)))-12*d^2/b^3*c^2*a*ln(exp(I*(b
*x+a))-1)-1/5*d^4*x^5-1/5/d*c^5-d^3*c*x^4-2*I*(d^4*x^4+4*c*d^3*x^3+6*c^2*d
^2*x^2+4*c^3*d*x+c^4)/b/(exp(2*I*(b*x+a))-1)+12*d^3/b^2*c*ln(1-exp(I*(b*x+
a)))*x^2+12*d^2/b^2*c^2*ln(exp(I*(b*x+a))+1)*x+12*d^2/b^2*c^2*ln(1-exp(I*(
b*x+a)))*x-24*d^3/b^4*c*a^2*ln(exp(I*(b*x+a)))-12*d^3/b^4*c*ln(1-exp(I*(b*
x+a)))*a^2+12*d^2/b^3*c^2*ln(1-exp(I*(b*x+a)))*a-8*I*d^3/b*c*x^3+16*I*d^3/
b^4*c*a^3-8*I*d^4/b^4*a^3*x-24*I*d^3/b^3*c*polylog(2,exp(I*(b*x+a)))*x-24*
I*d^2/b^2*c^2*a*x+24*I*d^3/b^3*c*a^2*x-24*I*d^3/b^3*c*polylog(2,-exp(I*(b*
x+a)))*x-2*d^2*c^2*x^3-2*d*c^3*x^2-c^4*x+12*d^3/b^2*c*ln(exp(I*(b*x+a))+1)
*x^2+12*d^3/b^4*c*a^2*ln(exp(I*(b*x+a))-1)+24*d^2/b^3*c^2*a*ln(exp(I*(b*x+
a)))+24*I*d^4*polylog(4,exp(I*(b*x+a)))/b^5
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 856 vs. $2(138) = 276$.

Time = 0.10 (sec) , antiderivative size = 856, normalized size of antiderivative = 5.52

$$\int (c+dx)^4 \cot^2(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^4*cot(b*x+a)^2,x, algorithm="fricas")
```

output

```
-1/10*(10*b^4*d^4*x^4 + 40*b^4*c*d^3*x^3 + 60*b^4*c^2*d^2*x^2 + 40*b^4*c^3
*d*x + 10*b^4*c^4 - 15*I*d^4*polylog(4, cos(2*b*x + 2*a) + I*sin(2*b*x + 2
*a))*sin(2*b*x + 2*a) + 15*I*d^4*polylog(4, cos(2*b*x + 2*a) - I*sin(2*b*x
 + 2*a))*sin(2*b*x + 2*a) + 30*(I*b^2*d^4*x^2 + 2*I*b^2*c*d^3*x + I*b^2*c^
2*d^2)*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) + 30*
(-I*b^2*d^4*x^2 - 2*I*b^2*c*d^3*x - I*b^2*c^2*d^2)*dilog(cos(2*b*x + 2*a)
- I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 20*(b^3*c^3*d - 3*a*b^2*c^2*d^2 +
 3*a^2*b*c*d^3 - a^3*d^4)*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*
a) + 1/2)*sin(2*b*x + 2*a) - 20*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d
^3 - a^3*d^4)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2)*si
n(2*b*x + 2*a) - 20*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a
*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*log(-cos(2*b*x + 2*a) + I*sin(2*b*
x + 2*a) + 1)*sin(2*b*x + 2*a) - 20*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3
*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*log(-cos(2*b*x + 2
*a) - I*sin(2*b*x + 2*a) + 1)*sin(2*b*x + 2*a) - 30*(b*d^4*x + b*c*d^3)*po
lylog(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 30*(b*d
^4*x + b*c*d^3)*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a))*sin(2*b*
x + 2*a) + 10*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c
^3*d*x + b^4*c^4)*cos(2*b*x + 2*a) + 2*(b^5*d^4*x^5 + 5*b^5*c*d^3*x^4 + 10
*b^5*c^2*d^2*x^3 + 10*b^5*c^3*d*x^2 + 5*b^5*c^4*x)*sin(2*b*x + 2*a))/(b...
```

Sympy [F]

$$\int (c+dx)^4 \cot^2(a+bx)\,dx = \int (c+dx)^4 \cot^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**4*cot(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**4*cot(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3242 vs. $2(138) = 276$.

Time = 0.45 (sec) , antiderivative size = 3242, normalized size of antiderivative = 20.92

$$\int (c+dx)^4 \cot^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cot(b*x+a)^2,x, algorithm="maxima")
```

output

```
-((b*x + a + 1/tan(b*x + a))*c^4 - 4*(b*x + a + 1/tan(b*x + a))*a*c^3*d/b
+ 6*(b*x + a + 1/tan(b*x + a))*a^2*c^2*d^2/b^2 - 4*(b*x + a + 1/tan(b*x +
a))*a^3*c*d^3/b^3 + (b*x + a + 1/tan(b*x + a))*a^4*d^4/b^4 + 2*((b*x + a)^
2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(
2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*
cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a)
 + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)
*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*s
in(2*b*x + 2*a))*c^3*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2
*b*x + 2*a) + 1)*b) - 6*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(
2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x
 + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^
2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 -
2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*a*c^2*d^2/((cos(2*b*x
+ 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b^2) + 6*((b*x + a
)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*co
s(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 -
2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x +
a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)...
```

Giac [F]

$$\int (c+dx)^4 \cot^2(a+bx)\,dx = \int (dx+c)^4 \cot(bx+a)^2\,dx$$

input

```
integrate((d*x+c)^4*cot(b*x+a)^2,x, algorithm="giac")
```

output

```
integrate((d*x + c)^4*cot(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \cot^2(a+bx)\,dx = \int \cot(a+b\,x)^2\,(c+d\,x)^4\,dx$$

input
```
int(cot(a + b*x)^2*(c + d*x)^4,x)
```

output
```
int(cot(a + b*x)^2*(c + d*x)^4, x)
```

Reduce [F]

$$\int (c+dx)^4 \cot^2(a+bx)\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^4*cot(b*x+a)^2,x)
```

output
```
( - 10*cos(a + b*x)*tan((a + b*x)/2)*b*c**4 - 40*cos(a + b*x)*tan((a + b*x
)/2)*b*c**3*d*x - 30*cos(a + b*x)*tan((a + b*x)/2)*b*c**2*d**2*x**2 - 20*c
os(a + b*x)*tan((a + b*x)/2)*b*c*d**3*x**3 - 5*cos(a + b*x)*tan((a + b*x)/
2)*b*d**4*x**4 + 20*int(x**3/tan((a + b*x)/2),x)*sin(a + b*x)*tan((a + b*x
)/2)*b*d**4 + 60*int(x**2/tan((a + b*x)/2),x)*sin(a + b*x)*tan((a + b*x)/2
)*b*c*d**3 + 60*int(x/tan((a + b*x)/2),x)*sin(a + b*x)*tan((a + b*x)/2)*b*
c**2*d**2 - 20*int(tan((a + b*x)/2)*x**3,x)*sin(a + b*x)*tan((a + b*x)/2)*
b*d**4 - 60*int(tan((a + b*x)/2)*x**2,x)*sin(a + b*x)*tan((a + b*x)/2)*b*c
*d**3 - 60*int(tan((a + b*x)/2)*x,x)*sin(a + b*x)*tan((a + b*x)/2)*b*c**2*
d**2 - 40*log(tan((a + b*x)/2)**2 + 1)*sin(a + b*x)*tan((a + b*x)/2)*c**3*
d + 40*log(tan((a + b*x)/2))*sin(a + b*x)*tan((a + b*x)/2)*c**3*d - 10*sin
(a + b*x)*tan((a + b*x)/2)*b**2*c**4*x - 20*sin(a + b*x)*tan((a + b*x)/2)*
b**2*c**3*d*x**2 - 20*sin(a + b*x)*tan((a + b*x)/2)*b**2*c**2*d**2*x**3 -
10*sin(a + b*x)*tan((a + b*x)/2)*b**2*c*d**3*x**4 - 2*sin(a + b*x)*tan((a
+ b*x)/2)*b**2*d**4*x**5 - 30*sin(a + b*x)*b*c**2*d**2*x**2 - 20*sin(a + b
*x)*b*c*d**3*x**3 - 5*sin(a + b*x)*b*d**4*x**4 + 30*tan((a + b*x)/2)*b*c**
2*d**2*x**2 + 20*tan((a + b*x)/2)*b*c*d**3*x**3 + 5*tan((a + b*x)/2)*b*d**
4*x**4)/(10*sin(a + b*x)*tan((a + b*x)/2)*b**2)
```

3.106 $\int (c+dx)^3 \cot^2(a+bx)\,dx$

Optimal result . . . 1000
Mathematica [B] (warning: unable to verify) . . . 1001
Rubi [A] (verified) . . . 1002
Maple [B] (verified) . . . 1005
Fricas [B] (verification not implemented) . . . 1006
Sympy [F] . . . 1006
Maxima [B] (verification not implemented) . . . 1007
Giac [F] . . . 1008
Mupad [F(-1)] . . . 1008
Reduce [F] . . . 1008

Optimal result

Integrand size = 16, antiderivative size = 127

$$\int (c+dx)^3 \cot^2(a+bx)\,dx = -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} - \frac{(c+dx)^3\cot(a+bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\,\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3} + \frac{3d^3\,\mathrm{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^4}$$

output

```
-I*(d*x+c)^3/b-1/4*(d*x+c)^4/d-(d*x+c)^3*cot(b*x+a)/b+3*d*(d*x+c)^2*ln(1-e
xp(2*I*(b*x+a)))/b^2-3*I*d^2*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^3+3/2*d
^3*polylog(3,exp(2*I*(b*x+a)))/b^4
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 374 vs. $2(127) = 254$.

Time = 4.15 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.94

$$\int (c+dx)^3 \cot^2(a+bx)\,dx$$

$$= -\frac{1}{4}x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right) - \frac{3c^2d(bx\cot(a)-\log(\sin(a+bx)))}{b^2}$$

$$+\frac{3cd^2\left(ibx(\pi-2\arctan(\tan(a)))+\pi\log\left(1+e^{-2ibx}\right)+2(bx+\arctan(\tan(a)))\log\left(1-e^{2i(bx+\arctan(\tan}\right.\right.}{}$$

$$+\frac{d^3e^{-ia}(i+\cot(a))\left(ib^3x^3-b^3x^3\cot(a)+3b^2x^2\log\left(1-e^{-i(a+bx)}\right)+3b^2x^2\log\left(1+e^{-i(a+bx)}\right)+6ibx\,\mathrm{P}\right.}{}$$

$$+\frac{(c+dx)^3\csc(a)\csc(a+bx)\sin(bx)}{b}$$

input

```
Integrate[(c + d*x)^3*Cot[a + b*x]^2,x]
```

output

```
-1/4*(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)) - (3*c^2*d*(b*x*Cot[a
] - Log[Sin[a + b*x]]))/b^2 + (3*c*d^2*(I*b*x*(Pi - 2*ArcTan[Tan[a]]) + Pi
*Log[1 + E^((-2*I)*b*x)] + 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x
+ ArcTan[Tan[a]]))] - Pi*Log[Cos[b*x]] - 2*ArcTan[Tan[a]]*Log[Sin[b*x + Ar
cTan[Tan[a]]]] - I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))] - b^2*E^(I
*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^2]))/b^3 + (d^3*(I + Cot[a])*(I*b^
3*x^3 - b^3*x^3*Cot[a] + 3*b^2*x^2*Log[1 - E^((-I)*(a + b*x))] + 3*b^2*x^2
*Log[1 + E^((-I)*(a + b*x))] + (6*I)*b*x*PolyLog[2, -E^((-I)*(a + b*x))] +
 (6*I)*b*x*PolyLog[2, E^((-I)*(a + b*x))] + 6*PolyLog[3, -E^((-I)*(a + b*x
))] + 6*PolyLog[3, E^((-I)*(a + b*x))])*Sin[a])/(b^4*E^(I*a)) + ((c + d*x)
^3*Csc[a]*Csc[a + b*x]*Sin[b*x])/b
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.29, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {3042, 4203, 17, 25, 3042, 25, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \cot^2(a+bx)\,dx$$

$$\downarrow 3042$$

$$\int (c+dx)^3 \tan\left(a+bx+\frac{\pi}{2}\right)^2 dx$$

$$\downarrow 4203$$

$$-\frac{3d\int -(c+dx)^2\cot(a+bx)dx}{b} - \int (c+dx)^3 dx - \frac{(c+dx)^3\cot(a+bx)}{b}$$

$$\downarrow 17$$

$$-\frac{3d\int -(c+dx)^2\cot(a+bx)dx}{b} - \frac{(c+dx)^3\cot(a+bx)}{b} - \frac{(c+dx)^4}{4d}$$

$$\downarrow 25$$

$$\frac{3d\int (c+dx)^2\cot(a+bx)dx}{b} - \frac{(c+dx)^3\cot(a+bx)}{b} - \frac{(c+dx)^4}{4d}$$

$$\downarrow 3042$$

$$\frac{3d\int -(c+dx)^2\tan\left(a+bx+\frac{\pi}{2}\right)dx}{b} - \frac{(c+dx)^3\cot(a+bx)}{b} - \frac{(c+dx)^4}{4d}$$

$$\downarrow 25$$

$$-\frac{3d\int (c+dx)^2\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx}{b} - \frac{(c+dx)^3\cot(a+bx)}{b} - \frac{(c+dx)^4}{4d}$$

$$\downarrow 4202$$

$$-\frac{3d\left(\frac{i(c+dx)^3}{3d} - 2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)^2}{1+e^{i(2a+2bx+\pi)}}dx\right)}{b} - \frac{(c+dx)^3\cot(a+bx)}{b} - \frac{(c+dx)^4}{4d}$$

$$\downarrow 2620$$

$$-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\int(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}}{b}$$

$$\downarrow 3011$$

$$-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}}{b}$$

$$\downarrow 2720$$

$$-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log}{}\right.}{b}-\frac{\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}}{b}$$

$$\downarrow 7143$$

$$-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}}{b}$$

input

```
Int[(c + d*x)^3*Cot[a + b*x]^2,x]
```

output

```
-1/4*(c + d*x)^4/d - ((c + d*x)^3*Cot[a + b*x])/b - (3*d*(((I/3)*(c + d*x)
^3)/d - (2*I)*(((-1/2*I)*(c + d*x)^2*Log[1 + E^(I*(2*a + Pi + 2*b*x))])/b
+ (I*d*(((I/2)*(c + d*x)*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/b - (d*Pol
yLog[3, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)))/b))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(115) = 230$.

Time = 0.47 (sec) , antiderivative size = 581, normalized size of antiderivative = 4.57

method	result
risch	$-\frac{2i(d^3x^3+3cd^2x^2+3c^2dx+c^3)}{b\left(e^{2i(bx+a)}-1\right)}+\frac{3d^3\ln\left(e^{i(bx+a)}+1\right)x^2}{b^2}+\frac{3d^3\ln\left(1-e^{i(bx+a)}\right)x^2}{b^2}-\frac{6dc^2\ln\left(e^{i(bx+a)}\right)}{b^2}+\frac{3dc^2\ln\left(e^{i(bx+a)}\right.}{b^2}$

input

```
int((d*x+c)^3*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
-12*I*d^2/b^2*c*a*x-1/4*d^3*x^4-1/4/d*c^4-d^2*c*x^3-3/2*d*c^2*x^2-c^3*x-2*
I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(exp(2*I*(b*x+a))-1)-6*d^2/b^3*c*a
*ln(exp(I*(b*x+a))-1)-6*I*d^3/b^3*polylog(2,exp(I*(b*x+a)))*x-6*I*d^2/b*c*
x^2-6*I*d^2/b^3*c*a^2-6*I*d^2/b^3*c*polylog(2,-exp(I*(b*x+a)))-6*I*d^2/b^3
*c*polylog(2,exp(I*(b*x+a)))+6*I*d^3/b^3*a^2*x-6*I*d^3/b^3*polylog(2,-exp(
I*(b*x+a)))*x+6*d^2/b^3*c*ln(1-exp(I*(b*x+a)))*a+6*d^2/b^2*c*ln(exp(I*(b*x
+a))+1)*x+6*d^2/b^2*c*ln(1-exp(I*(b*x+a)))*x+12*d^2/b^3*c*a*ln(exp(I*(b*x+
a)))+3*d^3/b^2*ln(exp(I*(b*x+a))+1)*x^2+3*d^3/b^2*ln(1-exp(I*(b*x+a)))*x^2
-6*d/b^2*c^2*ln(exp(I*(b*x+a)))+3*d/b^2*c^2*ln(exp(I*(b*x+a))-1)+3*d/b^2*c
^2*ln(exp(I*(b*x+a))+1)-6*d^3/b^4*a^2*ln(exp(I*(b*x+a)))+3*d^3/b^4*a^2*ln(
exp(I*(b*x+a))-1)-3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a^2-2*I*d^3/b*x^3+4*I*d^3
/b^4*a^3+6*d^3*polylog(3,-exp(I*(b*x+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x+a
)))/b^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 599 vs. $2(112) = 224$.

Time = 0.09 (sec) , antiderivative size = 599, normalized size of antiderivative = 4.72

$$\int (c+dx)^3 \cot^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x + 4*b^3*c^3 - 3*d^
3*polylog(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 3*d
^3*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) + 6*
(I*b*d^3*x + I*b*c*d^2)*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2
*b*x + 2*a) + 6*(-I*b*d^3*x - I*b*c*d^2)*dilog(cos(2*b*x + 2*a) - I*sin(2*
b*x + 2*a))*sin(2*b*x + 2*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-
1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) - 6*
(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(
2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*
a*b*c*d^2 - a^2*d^3)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1)*sin(2
*b*x + 2*a) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(
-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1)*sin(2*b*x + 2*a) + 4*(b^3*d^3*
x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(2*b*x + 2*a) + (b^4*d
^3*x^4 + 4*b^4*c*d^2*x^3 + 6*b^4*c^2*d*x^2 + 4*b^4*c^3*x)*sin(2*b*x + 2*a)
)/(b^4*sin(2*b*x + 2*a))
```

Sympy [F]

$$\int (c+dx)^3 \cot^2(a+bx)\,dx = \int (c+dx)^3 \cot^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*cot(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**3*cot(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1953 vs. $2(112) = 224$.

Time = 0.30 (sec) , antiderivative size = 1953, normalized size of antiderivative = 15.38

$$\int (c+dx)^3 \cot^2(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*cot(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/2*(2*(b*x + a + 1/tan(b*x + a))*c^3 - 6*(b*x + a + 1/tan(b*x + a))*a*c^
2*d/b + 6*(b*x + a + 1/tan(b*x + a))*a^2*c*d^2/b^2 - 2*(b*x + a + 1/tan(b*
x + a))*a^3*d^3/b^3 + 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(
2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x
 + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^
2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 -
2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*c^2*d/((cos(2*b*x + 2*
a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b) - 6*((b*x + a)^2*co
s(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*
x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(
2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1
) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log
(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2
*b*x + 2*a))*a*c*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b
*x + 2*a) + 1)*b^2) + 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(
2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x
 + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^
2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2...
```

Giac [F]

$$\int (c+dx)^3 \cot^2(a+bx)\,dx = \int (dx+c)^3 \cot(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^3*cot(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*cot(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \cot^2(a+bx)\,dx = \int \cot(a+b\,x)^2\,(c+d\,x)^3\,dx$$

input
```
int(cot(a + b*x)^2*(c + d*x)^3,x)
```

output
```
int(cot(a + b*x)^2*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \cot^2(a+bx)\,dx$$

$$= \frac{-4\cos(bx+a)\tan\left(\frac{bx}{2}+\frac{a}{2}\right) b\,c^3 - 12\cos(bx+a)\tan\left(\frac{bx}{2}+\frac{a}{2}\right) b\,c^2 dx - 6\cos(bx+a)\tan\left(\frac{bx}{2}+\frac{a}{2}\right) bc\,d^2}{}$$

input
```
int((d*x+c)^3*cot(b*x+a)^2,x)
```

output

```
( - 4*cos(a + b*x)*tan((a + b*x)/2)*b*c**3 - 12*cos(a + b*x)*tan((a + b*x)
/2)*b*c**2*d*x - 6*cos(a + b*x)*tan((a + b*x)/2)*b*c*d**2*x**2 - 2*cos(a +
 b*x)*tan((a + b*x)/2)*b*d**3*x**3 + 6*int(x**2/tan((a + b*x)/2),x)*sin(a
+ b*x)*tan((a + b*x)/2)*b*d**3 + 12*int(x/tan((a + b*x)/2),x)*sin(a + b*x)
*tan((a + b*x)/2)*b*c*d**2 - 6*int(tan((a + b*x)/2)*x**2,x)*sin(a + b*x)*t
an((a + b*x)/2)*b*d**3 - 12*int(tan((a + b*x)/2)*x,x)*sin(a + b*x)*tan((a
+ b*x)/2)*b*c*d**2 - 12*log(tan((a + b*x)/2)**2 + 1)*sin(a + b*x)*tan((a +
 b*x)/2)*c**2*d + 12*log(tan((a + b*x)/2))*sin(a + b*x)*tan((a + b*x)/2)*c
**2*d - 4*sin(a + b*x)*tan((a + b*x)/2)*b**2*c**3*x - 6*sin(a + b*x)*tan((
a + b*x)/2)*b**2*c**2*d*x**2 - 4*sin(a + b*x)*tan((a + b*x)/2)*b**2*c*d**2
*x**3 - sin(a + b*x)*tan((a + b*x)/2)*b**2*d**3*x**4 - 6*sin(a + b*x)*b*c*
d**2*x**2 - 2*sin(a + b*x)*b*d**3*x**3 + 6*tan((a + b*x)/2)*b*c*d**2*x**2
+ 2*tan((a + b*x)/2)*b*d**3*x**3)/(4*sin(a + b*x)*tan((a + b*x)/2)*b**2)
```

3.107 $\int (c+dx)^2 \cot^2(a+bx)\,dx$

Optimal result 1010
Mathematica [B] (warning: unable to verify) 1010
Rubi [A] (verified) 1011
Maple [B] (verified) 1014
Fricas [B] (verification not implemented) 1014
Sympy [F] 1015
Maxima [B] (verification not implemented) 1015
Giac [F] 1016
Mupad [F(-1)] 1016
Reduce [F] 1017

Optimal result

Integrand size = 16, antiderivative size = 97

$$\int (c+dx)^2 \cot^2(a+bx)\,dx = -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d} - \frac{(c+dx)^2\cot(a+bx)}{b} + \frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{id^2\,\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}$$

output

```
-I*(d*x+c)^2/b-1/3*(d*x+c)^3/d-(d*x+c)^2*cot(b*x+a)/b+2*d*(d*x+c)*ln(1-exp
(2*I*(b*x+a)))/b^2-I*d^2*polylog(2,exp(2*I*(b*x+a)))/b^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 198 vs. 2(97) = 194.

Time = 4.12 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.04

$$\int (c+dx)^2 \cot^2(a+bx)\,dx = -\frac{1}{3}x\left(3c^2+3cdx+d^2x^2\right) - \frac{2cd(bx\cot(a)-\log(\sin(a+bx)))}{b^2} + \frac{d^2\left(ibx(\pi-2\arctan(\tan(a)))+\pi\log\left(1+e^{-2ibx}\right)+2(bx+\arctan(\tan(a)))\log\left(1-e^{2i(bx+\arctan(\tan(a)))}\right)\right.}{} + \frac{(c+dx)^2\csc(a)\csc(a+bx)\sin(bx)}{b}$$

input
```
Integrate[(c + d*x)^2*Cot[a + b*x]^2,x]
```

output
```
-1/3*(x*(3*c^2 + 3*c*d*x + d^2*x^2)) - (2*c*d*(b*x*Cot[a] - Log[Sin[a + b*
x]]))/b^2 + (d^2*(I*b*x*(Pi - 2*ArcTan[Tan[a]]) + Pi*Log[1 + E^((-2*I)*b*x
)] + 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] -
Pi*Log[Cos[b*x]] - 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] - I*Pol
yLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))] - b^2*E^(I*ArcTan[Tan[a]])*x^2*C
ot[a]*Sqrt[Sec[a]^2]))/b^3 + ((c + d*x)^2*Csc[a]*Csc[a + b*x]*Sin[b*x])/b
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.24, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 4203, 17, 25, 3042, 25, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \cot^2(a+bx)\,dx$$
$$\downarrow 3042$$
$$\int (c+dx)^2 \tan\left(a+bx+\frac{\pi}{2}\right)^2 dx$$
$$\downarrow 4203$$
$$-\frac{2d\int -((c+dx)\cot(a+bx))dx}{b} - \int (c+dx)^2 dx - \frac{(c+dx)^2\cot(a+bx)}{b}$$

$$\downarrow 17$$

$$-\frac{2d\int -((c+dx)\cot(a+bx))dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

$$\downarrow 25$$

$$\frac{2d\int (c+dx)\cot(a+bx)dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

$$\downarrow 3042$$

$$\frac{2d\int -\left((c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)\right)dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

$$\downarrow 25$$

$$-\frac{2d\int (c+dx)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

$$\downarrow 4202$$

$$-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\int\frac{e^{i(2a+2bx+\pi)}(c+dx)}{1+e^{i(2a+2bx+\pi)}}dx\right)}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

$$\downarrow 2620$$

$$-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{id\int\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

$$\downarrow 2715$$

$$-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{d\int e^{-i(2a+2bx+\pi)}\log\left(1+e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

$$\downarrow 2838$$

$$-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

input

```
Int[(c + d*x)^2*Cot[a + b*x]^2,x]
```

output
```
-1/3*(c + d*x)^3/d - ((c + d*x)^2*Cot[a + b*x])/b - (2*d*(((I/2)*(c + d*x)
^2)/d - (2*I)*(((-1/2*I)*(c + d*x)*Log[1 + E^(I*(2*a + Pi + 2*b*x))])/b -
(d*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2))))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(89) = 178$.

Time = 0.44 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.14

method	result
risch	$-\frac{d^2x^3}{3} - cd\,x^2 - c^2x - \frac{c^3}{3d} - \frac{2i\left(x^2d^2+2cdx+c^2\right)}{b\left(\mathrm{e}^{2i(bx+a)}-1\right)} - \frac{4dc\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^2} + \frac{2dc\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b^2} + \frac{2dc\ln\left(\mathrm{e}^{i(bx+a)}+1\right)}{b^2} -$

input

```
int((d*x+c)^2*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/3*d^2*x^3-c*d*x^2-c^2*x-1/3/d*c^3-2*I*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*
(b*x+a))-1)-4*d/b^2*c*ln(exp(I*(b*x+a)))+2*d/b^2*c*ln(exp(I*(b*x+a))-1)+2*
d/b^2*c*ln(exp(I*(b*x+a))+1)-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^3-4*I*d^2
/b^2*a*x-2*I*d^2/b^3*a^2+2*d^2/b^2*ln(exp(I*(b*x+a))+1)*x-2*I*d^2/b*x^2+2*
d^2/b^2*ln(1-exp(I*(b*x+a)))*x+2*d^2/b^3*ln(1-exp(I*(b*x+a)))*a-2*I*d^2/b^
3*polylog(2,-exp(I*(b*x+a)))+4*d^2/b^3*a*ln(exp(I*(b*x+a)))-2*d^2/b^3*a*ln
(exp(I*(b*x+a))-1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(86) = 172$.

Time = 0.08 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.96

$$\int (c+dx)^2 \cot^2(a+bx)\,dx =$$

$$-\frac{6\,b^2d^2x^2 + 12\,b^2cdx + 6\,b^2c^2 + 3i\,d^2\mathrm{Li}_2(\cos(2\,bx+2\,a) + i\,\sin(2\,bx+2\,a))\sin(2\,bx+2\,a) - 3i\,d^2\mathrm{Li}_2($$

input
```
integrate((d*x+c)^2*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/6*(6*b^2*d^2*x^2 + 12*b^2*c*d*x + 6*b^2*c^2 + 3*I*d^2*dilog(cos(2*b*x +
 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 3*I*d^2*dilog(cos(2*b*x + 2
*a) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 6*(b*c*d - a*d^2)*log(-1/2*co
s(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) - 6*(b*c*d
 - a*d^2)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*
b*x + 2*a) - 6*(b*d^2*x + a*d^2)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a
) + 1)*sin(2*b*x + 2*a) - 6*(b*d^2*x + a*d^2)*log(-cos(2*b*x + 2*a) - I*si
n(2*b*x + 2*a) + 1)*sin(2*b*x + 2*a) + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*
c^2)*cos(2*b*x + 2*a) + 2*(b^3*d^2*x^3 + 3*b^3*c*d*x^2 + 3*b^3*c^2*x)*sin(
2*b*x + 2*a))/(b^3*sin(2*b*x + 2*a))
```

Sympy [F]

$$\int (c+dx)^2 \cot^2(a+bx)\,dx = \int (c+dx)^2 \cot^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*cot(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*cot(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 645 vs. $2(86) = 172$.

Time = 0.22 (sec) , antiderivative size = 645, normalized size of antiderivative = 6.65

$$\int (c+dx)^2 \cot^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*cot(b*x+a)^2,x, algorithm="maxima")
```

output

```
(-I*b^3*d^2*x^3 - 3*I*b^3*c*d*x^2 - 3*I*b^3*c^2*x - 6*b^2*c^2 - 6*(b*d^2*x
 + b*c*d - (b*d^2*x + b*c*d)*cos(2*b*x + 2*a) + (-I*b*d^2*x - I*b*c*d)*sin
(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 6*(b*c*d*cos(2*b*
x + 2*a) + I*b*c*d*sin(2*b*x + 2*a) - b*c*d)*arctan2(sin(b*x + a), cos(b*x
 + a) - 1) - 6*(b*d^2*x*cos(2*b*x + 2*a) + I*b*d^2*x*sin(2*b*x + 2*a) - b*
d^2*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (I*b^3*d^2*x^3 - 3*(-I*b
^3*c*d + 2*b^2*d^2)*x^2 - 3*(-I*b^3*c^2 + 4*b^2*c*d)*x)*cos(2*b*x + 2*a) -
 6*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(-e^(I*b*x +
 I*a)) - 6*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(e^(
I*b*x + I*a)) - 3*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(2*b*x
+ 2*a) - (b*d^2*x + b*c*d)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x
+ a)^2 + 2*cos(b*x + a) + 1) - 3*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*
c*d)*cos(2*b*x + 2*a) - (b*d^2*x + b*c*d)*sin(2*b*x + 2*a))*log(cos(b*x +
a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (b^3*d^2*x^3 + 3*(b^3*c*d +
2*I*b^2*d^2)*x^2 + 3*(b^3*c^2 + 4*I*b^2*c*d)*x)*sin(2*b*x + 2*a))/(-3*I*b^
3*cos(2*b*x + 2*a) + 3*b^3*sin(2*b*x + 2*a) + 3*I*b^3)
```

Giac [F]

$$\int (c+dx)^2 \cot^2(a+bx)\,dx = \int (dx+c)^2 \cot\left(bx+a\right)^2\,dx$$

input

```
integrate((d*x+c)^2*cot(b*x+a)^2,x, algorithm="giac")
```

output

```
integrate((d*x + c)^2*cot(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \cot^2(a+bx)\,dx = \int \cot(a+b\,x)^2\,(c+d\,x)^2\,dx$$

input

```
int(cot(a + b*x)^2*(c + d*x)^2,x)
```

output
```
int(cot(a + b*x)^2*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \cot^2(a+bx)\,dx$$

$$= \frac{-6\cos(bx+a)\tan\left(\frac{bx}{2}+\frac{a}{2}\right) b\,c^2 - 12\cos(bx+a)\tan\left(\frac{bx}{2}+\frac{a}{2}\right) bcdx - 3\cos(bx+a)\tan\left(\frac{bx}{2}+\frac{a}{2}\right) b\,d^2x^2}{}$$

input
```
int((d*x+c)^2*cot(b*x+a)^2,x)
```

output
```
( - 6*cos(a + b*x)*tan((a + b*x)/2)*b*c**2 - 12*cos(a + b*x)*tan((a + b*x)
/2)*b*c*d*x - 3*cos(a + b*x)*tan((a + b*x)/2)*b*d**2*x**2 + 6*int(x/tan((a
 + b*x)/2),x)*sin(a + b*x)*tan((a + b*x)/2)*b*d**2 - 6*int(tan((a + b*x)/2
)*x,x)*sin(a + b*x)*tan((a + b*x)/2)*b*d**2 - 12*log(tan((a + b*x)/2)**2 +
 1)*sin(a + b*x)*tan((a + b*x)/2)*c*d + 12*log(tan((a + b*x)/2))*sin(a + b
*x)*tan((a + b*x)/2)*c*d - 6*sin(a + b*x)*tan((a + b*x)/2)*b**2*c**2*x - 6
*sin(a + b*x)*tan((a + b*x)/2)*b**2*c*d*x**2 - 2*sin(a + b*x)*tan((a + b*x
)/2)*b**2*d**2*x**3 - 3*sin(a + b*x)*b*d**2*x**2 + 3*tan((a + b*x)/2)*b*d*
*2*x**2)/(6*sin(a + b*x)*tan((a + b*x)/2)*b**2)
```

3.108 $\int (c+dx)\cot^2(a+bx)\,dx$

Optimal result 1018
Mathematica [C] (verified) 1018
Rubi [A] (verified) 1019
Maple [A] (verified) 1021
Fricas [B] (verification not implemented) 1021
Sympy [B] (verification not implemented) 1022
Maxima [B] (verification not implemented) 1022
Giac [B] (verification not implemented) 1023
Mupad [B] (verification not implemented) 1024
Reduce [B] (verification not implemented) 1025

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int (c+dx)\cot^2(a+bx)\,dx = -\frac{(c+dx)^2}{2d} - \frac{(c+dx)\cot(a+bx)}{b} + \frac{d\log(\sin(a+bx))}{b^2}$$

output
```
-1/2*(d*x+c)^2/d-(d*x+c)*cot(b*x+a)/b+d*ln(sin(b*x+a))/b^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.91

$$\begin{aligned}&\int (c+dx)\cot^2(a+bx)\,dx \\ &= -\frac{c\cot(a+bx)\,\text{Hypergeometric2F1}\left(-\tfrac{1}{2},1,\tfrac{1}{2},-\tan^2(a+bx)\right)}{b} + \frac{d\log(\sin(a+bx))}{b^2} \\ &\quad - \frac{dx\csc(a)(2\cos(a)+bx\sin(a))}{2b} + \frac{dx\csc(a)\csc(a+bx)\sin(bx)}{b}\end{aligned}$$

input
```
Integrate[(c + d*x)*Cot[a + b*x]^2,x]
```

output

```
-((c*Cot[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[a + b*x]^2])/b) + (
d*Log[Sin[a + b*x]])/b^2 - (d*x*Csc[a]*(2*Cos[a] + b*x*Sin[a]))/(2*b) + (d
*x*Csc[a]*Csc[a + b*x]*Sin[b*x])/b
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4203, 17, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cot^2(a+bx)\,dx$$

$$\downarrow 3042$$

$$\int (c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)^2 dx$$

$$\downarrow 4203$$

$$-\frac{d\int -\cot(a+bx)dx}{b} - \int (c+dx)dx - \frac{(c+dx)\cot(a+bx)}{b}$$

$$\downarrow 17$$

$$-\frac{d\int -\cot(a+bx)dx}{b} - \frac{(c+dx)\cot(a+bx)}{b} - \frac{(c+dx)^2}{2d}$$

$$\downarrow 25$$

$$\frac{d\int \cot(a+bx)dx}{b} - \frac{(c+dx)\cot(a+bx)}{b} - \frac{(c+dx)^2}{2d}$$

$$\downarrow 3042$$

$$\frac{d\int -\tan\left(a+bx+\frac{\pi}{2}\right)dx}{b} - \frac{(c+dx)\cot(a+bx)}{b} - \frac{(c+dx)^2}{2d}$$

$$\downarrow 25$$

$$-\frac{d\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx}{b} - \frac{(c+dx)\cot(a+bx)}{b} - \frac{(c+dx)^2}{2d}$$

$$\downarrow 3956$$

$$\frac{d\log(-\sin(a+bx))}{b^2} - \frac{(c+dx)\cot(a+bx)}{b} - \frac{(c+dx)^2}{2d}$$

input
```
Int[(c + d*x)*Cot[a + b*x]^2,x]
```

output
```
-1/2*(c + d*x)^2/d - ((c + d*x)*Cot[a + b*x])/b + (d*Log[-Sin[a + b*x]])/b
^2
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

method	result	size
parallelrisch	$-\frac{d\,x^2b^2+2xc\,b^2-d\left(-\ln\left(\sec(bx+a)^2\right)+2\ln(\tan(bx+a))\right)+2\cot(bx+a)b(dx+c)}{2b^2}$	60
default	$-\frac{d\,x^2}{2}-xc+\frac{\frac{da\cot(bx+a)}{b}-c\cot(bx+a)+\frac{d(-(bx+a)\cot(bx+a)+\ln(\sin(bx+a)))}{b}}{b}$	64
risch	$-\frac{d\,x^2}{2}-xc-\frac{2idx}{b}-\frac{2ida}{b^2}-\frac{2i(dx+c)}{b\left(\mathrm{e}^{2i(bx+a)}-1\right)}+\frac{d\ln\left(\mathrm{e}^{2i(bx+a)}-1\right)}{b^2}$	69
norman	$\frac{-\frac{c}{b}-\frac{dx}{b}-\frac{d\,x^2\tan(bx+a)}{2}-xc\tan(bx+a)}{\tan(bx+a)}+\frac{d\ln(\tan(bx+a))}{b^2}-\frac{d\ln\left(\tan(bx+a)^2+1\right)}{2b^2}$	76

input
```
int((d*x+c)*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^2*(d*x^2*b^2+2*x*c*b^2-d*(-ln(sec(b*x+a)^2)+2*ln(tan(b*x+a)))+2*cot
(b*x+a)*b*(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(41) = 82$.

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.26

$$\int (c+dx)\cot^2(a+bx)\,dx =$$
$$-\frac{2\,bdx-d\log\left(-\frac{1}{2}\cos\left(2\,bx+2\,a\right)+\frac{1}{2}\right)\sin\left(2\,bx+2\,a\right)+2\,bc+2\left(bdx+bc\right)\cos\left(2\,bx+2\,a\right)+\left(b^2dx^2+\right.}{2\,b^2\sin\left(2\,bx+2\,a\right)}$$

input
```
integrate((d*x+c)*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*b*d*x - d*log(-1/2*cos(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) + 2*b*
c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d*x^2 + 2*b^2*c*x)*sin(2*b*x +
 2*a))/(b^2*sin(2*b*x + 2*a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(36) = 72$.

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.42

$$\int (c+dx)\cot^2(a+bx)\,dx$$

$$= \begin{cases} \tilde{\infty}\left(cx+\frac{dx^2}{2}\right) & \text{for } a=0 \wedge b=0 \\ \left(cx+\frac{dx^2}{2}\right)\cot^2(a) & \text{for } b=0 \\ \tilde{\infty}\left(cx+\frac{dx^2}{2}\right) & \text{for } a=-bx \\ -cx-\frac{dx^2}{2}-\frac{c}{b\tan(a+bx)}-\frac{dx}{b\tan(a+bx)}-\frac{d\log\left(\tan^2(a+bx)+1\right)}{2b^2}+\frac{d\log(\tan(a+bx))}{b^2} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*cot(b*x+a)**2,x)
```

output
```
Piecewise((zoo*(c*x + d*x**2/2), Eq(a, 0) & Eq(b, 0)), ((c*x + d*x**2/2)*c
ot(a)**2, Eq(b, 0)), (zoo*(c*x + d*x**2/2), Eq(a, -b*x)), (-c*x - d*x**2/2
 - c/(b*tan(a + b*x)) - d*x/(b*tan(a + b*x)) - d*log(tan(a + b*x)**2 + 1)/
(2*b**2) + d*log(tan(a + b*x))/b**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(41) = 82$.

Time = 0.12 (sec) , antiderivative size = 292, normalized size of antiderivative = 6.79

$$\int (c+dx)\cot^2(a+bx)\,dx =$$

$$2\left(bx+a+\frac{1}{\tan(bx+a)}\right)c-\frac{2\left(bx+a+\frac{1}{\tan(bx+a)}\right)ad}{b}+\frac{\left((bx+a)^2\cos(2\,bx+2\,a)^2+(bx+a)^2\sin(2\,bx+2\,a)^2-2\,(bx+a)^2\cos(2\,bx+2\,a\right.}{}$$

$$-$$

input
```
integrate((d*x+c)*cot(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/2*(2*(b*x + a + 1/tan(b*x + a))*c - 2*(b*x + a + 1/tan(b*x + a))*a*d/b
+ ((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*
x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2
*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*
x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) +
4*(b*x + a)*sin(2*b*x + 2*a))*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
- 2*cos(2*b*x + 2*a) + 1)*b))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1151 vs. $2(41) = 82$.

Time $= 0.35$ (sec) , antiderivative size $= 1151$, normalized size of antiderivative $= 26.77$

$$\int (c + dx) \cot^2(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)*cot(b*x+a)^2,x, algorithm="giac")
```

output

```
-1/2*(b^2*d*x^2*tan(1/2*b*x)^2*tan(1/2*a) + b^2*d*x^2*tan(1/2*b*x)*tan(1/2
*a)^2 + 2*b^2*c*x*tan(1/2*b*x)^2*tan(1/2*a) + 2*b^2*c*x*tan(1/2*b*x)*tan(1
/2*a)^2 - b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - b^2*d*x^2*tan(1/2*b*x) - b^2
*d*x^2*tan(1/2*a) - b*c*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^2*c*x*tan(1/2*b*
x) + b*d*x*tan(1/2*b*x)^2 - 2*b^2*c*x*tan(1/2*a) + 4*b*d*x*tan(1/2*b*x)*ta
n(1/2*a) - d*log(16*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3*tan(1/
2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*tan(1/2*b*x)^3*tan(1/2*a) - 4*tan
(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 +
2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*b*x)^4*tan(1/2*a)^4 + 2
*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2*tan(1/2*a)^4 + tan(1/2*b*x
)^4 + 4*tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*a)^4 + 2*tan(1/2*b*x)^2 + 2*
tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a) + b*d*x*tan(1/2*a)^2 - d*log(
16*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*
b*x)^2*tan(1/2*a)^4 - 2*tan(1/2*b*x)^3*tan(1/2*a) - 4*tan(1/2*b*x)^2*tan(1
/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*ta
n(1/2*a) + tan(1/2*a)^2)/(tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4*t
an(1/2*a)^2 + 2*tan(1/2*b*x)^2*tan(1/2*a)^4 + tan(1/2*b*x)^4 + 4*tan(1/2*b
*x)^2*tan(1/2*a)^2 + tan(1/2*a)^4 + 2*tan(1/2*b*x)^2 + 2*tan(1/2*a)^2 + 1)
)*tan(1/2*b*x)*tan(1/2*a)^2 + b*c*tan(1/2*b*x)^2 + 4*b*c*tan(1/2*b*x)*tan(
1/2*a) + b*c*tan(1/2*a)^2 - b*d*x + d*log(16*(tan(1/2*b*x)^4*tan(1/2*a)...
```

Mupad [B] (verification not implemented)

Time = 19.66 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.56

$$\int (c+dx)\cot^2(a+bx)\,dx = -\frac{d\,x^2}{2} + \frac{d\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}-1\right)}{b^2} - \frac{(c+d\,x)\,2\mathrm{i}}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)} - \frac{x\,(b\,c+d\,2\mathrm{i})}{b}$$

input

```
int(cot(a + b*x)^2*(c + d*x),x)
```

output

```
(d*log(exp(a*2i)*exp(b*x*2i) - 1))/b^2 - (d*x^2)/2 - ((c + d*x)*2i)/(b*(ex
p(a*2i + b*x*2i) - 1)) - (x*(d*2i + b*c))/b
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.44

$$\int (c+dx)\cot^2(a+bx)\,dx$$

$$= \frac{-2\cos(bx+a)\,bc - 2\cos(bx+a)\,bdx - 2\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right)\sin(bx+a)\,d + 2\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)s}{2\sin(bx+a)\,b^2}$$

input

```
int((d*x+c)*cot(b*x+a)^2,x)
```

output

```
( - 2*cos(a + b*x)*b*c - 2*cos(a + b*x)*b*d*x - 2*log(tan((a + b*x)/2)**2
+ 1)*sin(a + b*x)*d + 2*log(tan((a + b*x)/2))*sin(a + b*x)*d - 2*sin(a + b
*x)*b**2*c*x - sin(a + b*x)*b**2*d*x**2)/(2*sin(a + b*x)*b**2)
```

3.109 $\int \frac{\cot^2(a+bx)}{c+dx}\,dx$

Optimal result 1026
Mathematica [N/A] 1026
Rubi [N/A] 1027
Maple [N/A] 1028
Fricas [N/A] 1028
Sympy [N/A] 1029
Maxima [N/A] 1029
Giac [N/A] 1030
Mupad [N/A] 1030
Reduce [N/A] 1031

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\cot^2(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\cot^2(a+bx)}{c+dx}, x\right)$$

output
```
Defer(Int)(cot(b*x+a)^2/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 4.82 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^2(a+bx)}{c+dx}\,dx = \int \frac{\cot^2(a+bx)}{c+dx}\,dx$$

input
```
Integrate[Cot[a + b*x]^2/(c + d*x),x]
```

output
```
Integrate[Cot[a + b*x]^2/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(a+bx)}{c+dx}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\tan\left(a+bx+\frac{\pi}{2}\right)^2}{c+dx}\,dx$$
$$\downarrow 4222$$
$$\int \frac{\cot^2(a+bx)}{c+dx}\,dx$$

input
```
Int[Cot[a + b*x]^2/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cot\left(bx+a\right)^2}{dx+c}dx$$

input

```
int(cot(b*x+a)^2/(d*x+c),x)
```

output

```
int(cot(b*x+a)^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^2(a+bx)}{c+dx}\,dx = \int \frac{\cot\left(bx+a\right)^2}{dx+c}\,dx$$

input

```
integrate(cot(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(cot(b*x + a)^2/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\cot^2(a+bx)}{c+dx}\,dx = \int \frac{\cot^2(a+bx)}{c+dx}\,dx$$

input
```
integrate(cot(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(cot(a + b*x)**2/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 581, normalized size of antiderivative = 36.31

$$\int \frac{\cot^2(a+bx)}{c+dx}\,dx = \int \frac{\cot\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(cot(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
((b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a)^2 + (b*d^3*x +
b*c*d^2)*sin(2*b*x + 2*a)^2 - 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*inte
grate(sin(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x
 + b*c^2)*cos(b*x + a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2
+ 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)), x) - (b*d^3*x + b*c*d^2
 + (b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a)^2 + (b*d^3*x + b*c*d^2)*sin(2*b*x
+ 2*a)^2 - 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/
(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x +
 a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 - 2*(b*d^2*x^2 + 2*
b*c*d*x + b*c^2)*cos(b*x + a)), x) - (b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*
a)^2 + (b*d*x + b*c)*sin(2*b*x + 2*a)^2 + b*c - 2*(b*d*x + b*c)*cos(2*b*x
+ 2*a))*log(d*x + c) - 2*d*sin(2*b*x + 2*a))/(b*d^2*x + b*c*d + (b*d^2*x +
 b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 - 2*(b*d
^2*x + b*c*d)*cos(2*b*x + 2*a))
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^2(a+bx)}{c+dx}\,dx = \int \frac{\cot\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(cot(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(cot(b*x + a)^2/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 19.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^2(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+b\,x)^2}{c+d\,x}\,dx$$

input
```
int(cot(a + b*x)^2/(c + d*x),x)
```

output
```
int(cot(a + b*x)^2/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^2(a+bx)}{c+dx}\,dx = \int \frac{\cot\left(bx+a\right)^2}{dx+c}dx$$

input
```
int(cot(b*x+a)^2/(d*x+c),x)
```

output
```
int(cot(a + b*x)**2/(c + d*x),x)
```

3.110 $\int \frac{\cot^2(a+bx)}{(c+dx)^2}\,dx$

Optimal result 1032
Mathematica [N/A] 1032
Rubi [N/A] 1033
Maple [N/A] 1034
Fricas [N/A] 1034
Sympy [N/A] 1035
Maxima [N/A] 1035
Giac [N/A] 1036
Mupad [N/A] 1037
Reduce [N/A] 1037

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\cot^2(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(cot(b*x+a)^2/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[Cot[a + b*x]^2/(c + d*x)^2,x]
```

output
```
Integrate[Cot[a + b*x]^2/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\tan\left(a+bx+\frac{\pi}{2}\right)^2}{(c+dx)^2}\,dx$$

$$\downarrow 4222$$

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
Int[Cot[a + b*x]^2/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cot\left(bx+a\right)^2}{\left(dx+c\right)^2}dx$$

input

```
int(cot(b*x+a)^2/(d*x+c)^2,x)
```

output

```
int(cot(b*x+a)^2/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input

```
integrate(cot(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cot(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(cot(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(cot(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 813, normalized size of antiderivative = 50.81

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(cot(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output

```
(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x + 2*a)
^2 + b*c - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + 2*(b*d^4*x^2 + 2*b*c*d^3*x +
 b*c^2*d^2 + (b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2)*cos(2*b*x + 2*a)^2 + (b
*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2)*sin(2*b*x + 2*a)^2 - 2*(b*d^4*x^2 + 2*
b*c*d^3*x + b*c^2*d^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^3*x^3
 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*
c^2*d*x + b*c^3)*cos(b*x + a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x
 + b*c^3)*sin(b*x + a)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*
c^3)*cos(b*x + a)), x) - 2*(b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2 + (b*d^4*x
^2 + 2*b*c*d^3*x + b*c^2*d^2)*cos(2*b*x + 2*a)^2 + (b*d^4*x^2 + 2*b*c*d^3*
x + b*c^2*d^2)*sin(2*b*x + 2*a)^2 - 2*(b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2
)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*
b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(
b*x + a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(b*x + a
)^2 - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)), x
) - 2*d*sin(2*b*x + 2*a))/(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d + (b*d^3*x^2
+ 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a)^2 + (b*d^3*x^2 + 2*b*c*d^2*x + b
*c^2*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b
*x + 2*a))
```

Giac [N/A]

Not integrable

Time = 2.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input

```
integrate(cot(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output

```
integrate(cot(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 18.96 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(a+b\,x)^2}{(c+d\,x)^2}\,dx$$

input
```
int(cot(a + b*x)^2/(c + d*x)^2,x)
```

output
```
int(cot(a + b*x)^2/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot\left(bx+a\right)^2}{d^2x^2+2cdx+c^2}dx$$

input
```
int(cot(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(cot(a + b*x)**2/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.111 $\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx$

Optimal result 1038
Mathematica [N/A] 1038
Rubi [N/A] 1039
Maple [N/A] 1040
Fricas [N/A] 1040
Sympy [N/A] 1041
Maxima [N/A] 1041
Giac [N/A] 1041
Mupad [N/A] 1042
Reduce [N/A] 1042

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx = -\text{Int}((c+dx)^m \csc(a+bx), x) + \text{Int}\left((c+dx)^m \csc^3(a+bx), x\right)$$

output

```
-Defer(Int)((d*x+c)^m*csc(b*x+a),x)+Defer(Int)((d*x+c)^m*csc(b*x+a)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 24.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx = \int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx$$

input

```
Integrate[(c + d*x)^m*Cot[a + b*x]^2*Csc[a + b*x],x]
```

output

```
Integrate[(c + d*x)^m*Cot[a + b*x]^2*Csc[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4915, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(a+bx)\csc(a+bx)(c+dx)^m\,dx$$

$$\downarrow 4915$$

$$\int (c+dx)^m \csc^3(a+bx)dx - \int (c+dx)^m \csc(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)^m \csc(a+bx)^3dx - \int (c+dx)^m \csc(a+bx)dx$$

$$\downarrow 4680$$

$$\int (c+dx)^m \csc^3(a+bx)dx - \int (c+dx)^m \csc(a+bx)dx$$

input
```
Int[(c + d*x)^m*Cot[a + b*x]^2*Csc[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4915

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \cot(bx+a)^2 \csc(bx+a)\,dx$$

input

```
int((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x)
```

output

```
int((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx = \int (dx+c)^m \cot(bx+a)^2 \csc(bx+a)\,dx$$

input

```
integrate((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

output

```
integral((d*x + c)^m*cot(b*x + a)^2*csc(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 5.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx = \int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cot(b*x+a)**2*csc(b*x+a),x)
```

output
```
Integral((c + d*x)**m*cot(a + b*x)**2*csc(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx = \int (dx+c)^m \cot(bx+a)^2 \csc(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cot(b*x + a)^2*csc(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx = \int (dx+c)^m \cot(bx+a)^2 \csc(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cot(b*x + a)^2*csc(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 18.90 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (c+dx)^m \cot^2(a+bx) \csc(a+bx)\,dx = \int \frac{\cot(a+b\,x)^2\,(c+d\,x)^m}{\sin(a+b\,x)}\,dx$$

input
```
int((cot(a + b*x)^2*(c + d*x)^m)/sin(a + b*x),x)
```

output
```
int((cot(a + b*x)^2*(c + d*x)^m)/sin(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cot^2(a+bx) \csc(a+bx)\,dx = \int (dx+c)^m \cot(bx+a)^2 \csc(bx+a)\,dx$$

input
```
int((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x)
```

output
```
int((c + d*x)**m*cot(a + b*x)**2*csc(a + b*x),x)
```

3.112 $\int (c+dx)^4 \cot^2(a+bx) \csc(a+bx)\, dx$

Optimal result . . . 1044
Mathematica [B] (verified) . . . 1045
Rubi [A] (verified) . . . 1046
Maple [B] (verified) . . . 1056
Fricas [B] (verification not implemented) . . . 1057
Sympy [F] . . . 1058
Maxima [B] (verification not implemented) . . . 1058
Giac [F] . . . 1059
Mupad [F(-1)] . . . 1060
Reduce [F] . . . 1060

Optimal result

Integrand size = 22, antiderivative size = 416

$$\begin{aligned}\int (c+dx)^4 \cot^2(a+bx)\csc(a+bx)\,dx = &-\frac{12d^2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b^3}\\ &+\frac{(c+dx)^4\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}\\ &-\frac{2d(c+dx)^3\csc(a+bx)}{b^2}\\ &-\frac{(c+dx)^4\cot(a+bx)\csc(a+bx)}{2b}\\ &+\frac{12id^3(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^4}\\ &-\frac{2id(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}\\ &-\frac{12id^3(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^4}\\ &+\frac{2id(c+dx)^3\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}\\ &-\frac{12d^4\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^5}\\ &+\frac{6d^2(c+dx)^2\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}\\ &+\frac{12d^4\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^5}\\ &-\frac{6d^2(c+dx)^2\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}\\ &+\frac{12id^3(c+dx)\operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4}\\ &-\frac{12id^3(c+dx)\operatorname{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4}\\ &-\frac{12d^4\operatorname{PolyLog}\left(5,-e^{i(a+bx)}\right)}{b^5}\\ &+\frac{12d^4\operatorname{PolyLog}\left(5,e^{i(a+bx)}\right)}{b^5}\end{aligned}$$

output

```
-12*d^2*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b^3+(d*x+c)^4*arctanh(exp(I*(b*x
+a)))/b-2*d*(d*x+c)^3*csc(b*x+a)/b^2-1/2*(d*x+c)^4*cot(b*x+a)*csc(b*x+a)/b
-12*I*d^3*(d*x+c)*polylog(4,exp(I*(b*x+a)))/b^4+12*I*d^3*(d*x+c)*polylog(4
,-exp(I*(b*x+a)))/b^4+2*I*d*(d*x+c)^3*polylog(2,exp(I*(b*x+a)))/b^2-2*I*d*
(d*x+c)^3*polylog(2,-exp(I*(b*x+a)))/b^2-12*d^4*polylog(3,-exp(I*(b*x+a)))
/b^5+6*d^2*(d*x+c)^2*polylog(3,-exp(I*(b*x+a)))/b^3+12*d^4*polylog(3,exp(I
*(b*x+a)))/b^5-6*d^2*(d*x+c)^2*polylog(3,exp(I*(b*x+a)))/b^3-12*I*d^3*(d*x
+c)*polylog(2,exp(I*(b*x+a)))/b^4+12*I*d^3*(d*x+c)*polylog(2,-exp(I*(b*x+a
)))/b^4-12*d^4*polylog(5,-exp(I*(b*x+a)))/b^5+12*d^4*polylog(5,exp(I*(b*x+
a)))/b^5
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 854 vs. $2(416) = 832$.

Time $= 5.66$ (sec) , antiderivative size $= 854$, normalized size of antiderivative $= 2.05$

$$\int (c+dx)^4 \cot^2(a+bx)\csc(a+bx)\,dx =$$

$$-\frac{b^3(c+dx)^3(4d+b(c+dx)\cot(a+bx))\csc(a+bx)+b^4c^4\log\left(1-e^{i(a+bx)}\right)-12b^2c^2d^2\log\left(1-e^{i(a+b}\right.}{}$$

input

```
Integrate[(c + d*x)^4*Cot[a + b*x]^2*Csc[a + b*x],x]
```

output

```
-1/2*(b^3*(c + d*x)^3*(4*d + b*(c + d*x)*Cot[a + b*x])*Csc[a + b*x] + b^4*
c^4*Log[1 - E^(I*(a + b*x))] - 12*b^2*c^2*d^2*Log[1 - E^(I*(a + b*x))] + 4
*b^4*c^3*d*x*Log[1 - E^(I*(a + b*x))] - 24*b^2*c*d^3*x*Log[1 - E^(I*(a + b
*x))] + 6*b^4*c^2*d^2*x^2*Log[1 - E^(I*(a + b*x))] - 12*b^2*d^4*x^2*Log[1
 - E^(I*(a + b*x))] + 4*b^4*c*d^3*x^3*Log[1 - E^(I*(a + b*x))] + b^4*d^4*x^
4*Log[1 - E^(I*(a + b*x))] - b^4*c^4*Log[1 + E^(I*(a + b*x))] + 12*b^2*c^2
*d^2*Log[1 + E^(I*(a + b*x))] - 4*b^4*c^3*d*x*Log[1 + E^(I*(a + b*x))] + 2
4*b^2*c*d^3*x*Log[1 + E^(I*(a + b*x))] - 6*b^4*c^2*d^2*x^2*Log[1 + E^(I*(a
 + b*x))] + 12*b^2*d^4*x^2*Log[1 + E^(I*(a + b*x))] - 4*b^4*c*d^3*x^3*Log[
1 + E^(I*(a + b*x))] - b^4*d^4*x^4*Log[1 + E^(I*(a + b*x))] + (4*I)*b*d*(c
 + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*PolyLog[2, -E^(I*(a + b*x))] - (4*I)*b*
d*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*PolyLog[2, E^(I*(a + b*x))] - 12*b^
2*c^2*d^2*PolyLog[3, -E^(I*(a + b*x))] + 24*d^4*PolyLog[3, -E^(I*(a + b*x)
)] - 24*b^2*c*d^3*x*PolyLog[3, -E^(I*(a + b*x))] - 12*b^2*d^4*x^2*PolyLog[
3, -E^(I*(a + b*x))] + 12*b^2*c^2*d^2*PolyLog[3, E^(I*(a + b*x))] - 24*d^4
*PolyLog[3, E^(I*(a + b*x))] + 24*b^2*c*d^3*x*PolyLog[3, E^(I*(a + b*x))]
+ 12*b^2*d^4*x^2*PolyLog[3, E^(I*(a + b*x))] - (24*I)*b*c*d^3*PolyLog[4, -
E^(I*(a + b*x))] - (24*I)*b*d^4*x*PolyLog[4, -E^(I*(a + b*x))] + (24*I)*b*
c*d^3*PolyLog[4, E^(I*(a + b*x))] + (24*I)*b*d^4*x*PolyLog[4, E^(I*(a + b*
x))] + 24*d^4*PolyLog[5, -E^(I*(a + b*x))] - 24*d^4*PolyLog[5, E^(I*(a ...
```

Rubi [A] (verified)

Time = 2.90 (sec) , antiderivative size = 742, normalized size of antiderivative = 1.78, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {4915, 3042, 4671, 3011, 4674, 3042, 4671, 3011, 2720, 7143, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \cot^2(a+bx) \csc(a+bx)\, dx$$

$$\downarrow 4915$$

$$\int (c+dx)^4 \csc^3(a+bx) dx - \int (c+dx)^4 \csc(a+bx) dx$$

$$\downarrow 3042$$

$$\int (c+dx)^4 \csc(a+bx)^3 dx - \int (c+dx)^4 \csc(a+bx) dx$$

$$\downarrow 4671$$

$$\frac{4d\int (c+dx)^3 \log\left(1-e^{i(a+bx)}\right) dx}{b} - \frac{4d\int (c+dx)^3 \log\left(1+e^{i(a+bx)}\right) dx}{b} + \int (c+dx)^4 \csc(a+bx)^3 dx + \frac{2(c+dx)^4 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\downarrow 3011$$

$$-\frac{4d\left(\frac{i(c+dx)^3 \operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b} - \frac{3id\int (c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right) dx}{b}\right)}{b} + \frac{4d\left(\frac{i(c+dx)^3 \operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b} - \frac{3id\int (c+dx)^2 \operatorname{PolyLog}\left(2,e^{i(a+bx)}\right) dx}{b}\right)}{b} + \int (c+dx)^4 \csc(a+bx)^3 dx + \frac{2(c+dx)^4 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\downarrow 4674$$

$$\frac{6d^2\int (c+dx)^2 \csc(a+bx) dx}{b^2} - \frac{4d\left(\frac{i(c+dx)^3 \operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b} - \frac{3id\int (c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right) dx}{b}\right)}{b} + \frac{4d\left(\frac{i(c+dx)^3 \operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b} - \frac{3id\int (c+dx)^2 \operatorname{PolyLog}\left(2,e^{i(a+bx)}\right) dx}{b}\right)}{b} + \frac{1}{2}\int (c+dx)^4 \csc(a+bx) dx + \frac{2(c+dx)^4 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)^3 \csc(a+bx)}{b^2} - \frac{(c+dx)^4 \cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 3042$$

$$\frac{6d^2\int (c+dx)^2 \csc(a+bx) dx}{b^2} - \frac{4d\left(\frac{i(c+dx)^3 \operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b} - \frac{3id\int (c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right) dx}{b}\right)}{b} + \frac{4d\left(\frac{i(c+dx)^3 \operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b} - \frac{3id\int (c+dx)^2 \operatorname{PolyLog}\left(2,e^{i(a+bx)}\right) dx}{b}\right)}{b} + \frac{1}{2}\int (c+dx)^4 \csc(a+bx) dx + \frac{2(c+dx)^4 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)^3 \csc(a+bx)}{b^2} - \frac{(c+dx)^4 \cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 4671$$

$$\frac{6d^2\left(-\frac{2d\int(c+dx)\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{2d\int(c+dx)\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)}{b^2}+$$
$$\frac{1}{2}\left(-\frac{4d\int(c+dx)^3\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{4d\int(c+dx)^3\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^4\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)-$$
$$\frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\int(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}+$$
$$\frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\int(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}+$$
$$\frac{2(c+dx)^4\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{2d(c+dx)^3\csc(a+bx)}{b^2}-\frac{(c+dx)^4\cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 3011$$

$$\frac{6d^2\left(\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{id\int\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{id\int\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2(}{}\right.}{b^2}$$
$$\frac{1}{2}\left(\frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\int(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\int(c+dx)}{}\right.}{b}\right.$$
$$\frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\int(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}+$$
$$\frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\int(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}+$$
$$\frac{2(c+dx)^4\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{2d(c+dx)^3\csc(a+bx)}{b^2}-\frac{(c+dx)^4\cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 2720$$

$$6d^2\left(\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{Poly}}{b}\right.\right.$$

$$\frac{1}{2}\left(\frac{4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\int(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\int(c+dx)}{b}\right.\right.$$

$$\frac{4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\int(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}+$$

$$\frac{4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\int(c+dx)^2\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}+$$

$$\frac{2(c+dx)^4\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{2d(c+dx)^3\csc(a+bx)}{b^2}-\frac{(c+dx)^4\cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 7143$$

$$\frac{1}{2}\left(\frac{4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\int(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\int(c+dx)}{b}\right.\right.$$

$$\frac{4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\int(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}+$$

$$\frac{4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\int(c+dx)^2\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}+$$

$$6d^2\left(-\frac{2(c+dx)^2\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d}{b}\right.\right.$$

$$\frac{2(c+dx)^4\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{2d(c+dx)^3\csc(a+bx)}{b^2}-\frac{(c+dx)^4\cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 7163$$

$$\frac{1}{2}\left(\frac{4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\int(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{4d\left(\frac{i(c+dx)^3}{b}\right.}{b}\right.$$

$$\frac{4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\int(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+$$

$$\frac{4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\int(c+dx)\,\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+$$

$$\frac{6d^2\left(-\frac{2(c+dx)^2\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d}{b}\right.}{b}\right.}{b^2}$$

$$\frac{2(c+dx)^4\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{2d(c+dx)^3\csc(a+bx)}{b^2}-\frac{(c+dx)^4\cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 7163$$

$$\frac{1}{2}\left(\frac{4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{id\int \mathrm{PolyLog}\left(4,-e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}\right.$$

$$\left.\frac{4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{id\int \mathrm{PolyLog}\left(4,-e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}\right.$$

$$\left.\frac{4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{id\int \mathrm{PolyLog}\left(4,e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(4,e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}\right.$$

$$\left.6d^2\left(-\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d}{b}\right.\right.\right.$$

$$\left.\left.\left.\frac{2(c+dx)^4\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{2d(c+dx)^3\csc(a+bx)}{b^2}-\frac{(c+dx)^4\cot(a+bx)\csc(a+bx)}{2b}\right)\right)\right)$$

$$\downarrow 2720$$

$$\frac{2\operatorname{arctanh}\left(e^{i(a+bx)}\right)(c+dx)^4}{b}-\frac{\cot(a+bx)\csc(a+bx)(c+dx)^4}{2b}-\frac{2d\csc(a+bx)(c+dx)^3}{b^2}+$$

$$6d^2\left(-\frac{2\operatorname{arctanh}\left(e^{i(a+bx)}\right)(c+dx)^2}{b}+\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d}{b^2}\right.\right.$$

$$4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\operatorname{Poly}\right.}{b}\right.$$

$$4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(4,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\operatorname{PolyLog}\left(3\right.}{b}\right.$$

$$\frac{1}{2}\left(-\frac{2\operatorname{arctanh}\left(e^{i(a+bx)}\right)(c+dx)^4}{b}+\frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right.\right.\right.\right.}{b}\right.$$

$$\downarrow 7143$$

$$\frac{6d^2\left(-\frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\,\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d}{}\right.}{b}\right.}{b^2}$$

$$\frac{1}{2}\left(-\frac{2(c+dx)^4\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{4d\left(\frac{i(c+dx)^3\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{d\,\text{PolyLog}\left(5,-e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\,\text{PolyLog}\left(4,-\right.}{b}\right.}{b}\right.}{b}\right.}{b}\right.$$

$$\frac{2(c+dx)^4\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-$$

$$\frac{4d\left(\frac{i(c+dx)^3\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{d\,\text{PolyLog}\left(5,-e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\,\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\,\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}$$

$$\frac{4d\left(\frac{i(c+dx)^3\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{d\,\text{PolyLog}\left(5,e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\,\text{PolyLog}\left(4,e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\,\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-$$

$$\frac{2d(c+dx)^3\csc(a+bx)}{b^2}-\frac{(c+dx)^4\cot(a+bx)\csc(a+bx)}{2b}$$

input
```
Int[(c + d*x)^4*Cot[a + b*x]^2*Csc[a + b*x],x]
```

output

```
(2*(c + d*x)^4*ArcTanh[E^(I*(a + b*x))])/b - (2*d*(c + d*x)^3*Csc[a + b*x]
)/b^2 - ((c + d*x)^4*Cot[a + b*x]*Csc[a + b*x])/(2*b) + (6*d^2*((-2*(c + d
*x)^2*ArcTanh[E^(I*(a + b*x))])/b + (2*d*((I*(c + d*x)*PolyLog[2, -E^(I*(a
 + b*x))])/b - (d*PolyLog[3, -E^(I*(a + b*x))])/b^2))/b - (2*d*((I*(c + d*
x)*PolyLog[2, E^(I*(a + b*x))])/b - (d*PolyLog[3, E^(I*(a + b*x))])/b^2))/
b))/b^2 - (4*d*((I*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b - ((3*I)*d*
(((-I)*(c + d*x)^2*PolyLog[3, -E^(I*(a + b*x))])/b + ((2*I)*d*(((-I)*(c +
d*x)*PolyLog[4, -E^(I*(a + b*x))])/b + (d*PolyLog[5, -E^(I*(a + b*x))])/b^
2))/b))/b))/b + (4*d*((I*(c + d*x)^3*PolyLog[2, E^(I*(a + b*x))])/b - ((3*
I)*d*(((-I)*(c + d*x)^2*PolyLog[3, E^(I*(a + b*x))])/b + ((2*I)*d*(((-I)*(
c + d*x)*PolyLog[4, E^(I*(a + b*x))])/b + (d*PolyLog[5, E^(I*(a + b*x))])/
b^2))/b))/b))/b + ((-2*(c + d*x)^4*ArcTanh[E^(I*(a + b*x))])/b + (4*d*((I*
(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b - ((3*I)*d*(((-I)*(c + d*x)^2*
PolyLog[3, -E^(I*(a + b*x))])/b + ((2*I)*d*(((-I)*(c + d*x)*PolyLog[4, -E^
(I*(a + b*x))])/b + (d*PolyLog[5, -E^(I*(a + b*x))])/b^2))/b))/b))/b - (4*
d*((I*(c + d*x)^3*PolyLog[2, E^(I*(a + b*x))])/b - ((3*I)*d*(((-I)*(c + d*
x)^2*PolyLog[3, E^(I*(a + b*x))])/b + ((2*I)*d*(((-I)*(c + d*x)*PolyLog[4,
 E^(I*(a + b*x))])/b + (d*PolyLog[5, E^(I*(a + b*x))])/b^2))/b))/b))/b)/2
```

Defintions of rubi rules used

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4674

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4915

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1672 vs. 2(380) = 760.

Time = 0.56 (sec) , antiderivative size = 1673, normalized size of antiderivative = 4.02

method	result	size
risch	Expression too large to display	1673

input
```
int((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
3/b^3*c^2*d^2*ln(1-exp(I*(b*x+a)))*a^2-3/b^3*c^2*d^2*ln(exp(I*(b*x+a))+1)*
a^2+12/b^3*c*d^3*polylog(3,-exp(I*(b*x+a)))*x-12/b^3*c*d^3*polylog(3,exp(I
*(b*x+a)))*x-12/b^3*c*d^3*ln(exp(I*(b*x+a))+1)*x+12/b^3*c*d^3*ln(1-exp(I*(
b*x+a)))*x+3/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2-6*I/b^2*c*d^3*polylog(2,-e
xp(I*(b*x+a)))*x^2+6*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2-6*I/b^2*c^2
*d^2*polylog(2,-exp(I*(b*x+a)))*x+6*I/b^2*c^2*d^2*polylog(2,exp(I*(b*x+a))
)*x+1/b^2/(exp(2*I*(b*x+a))-1)^2*(d^4*x^4*b*exp(3*I*(b*x+a))+4*c*d^3*x^3*b
*exp(3*I*(b*x+a))+6*c^2*d^2*x^2*b*exp(3*I*(b*x+a))+d^4*x^4*b*exp(I*(b*x+a)
)+4*c^3*d*x*b*exp(3*I*(b*x+a))+4*c*d^3*x^3*b*exp(I*(b*x+a))+4*I*d^4*x^3*ex
p(I*(b*x+a))+b*c^4*exp(3*I*(b*x+a))+6*c^2*d^2*x^2*b*exp(I*(b*x+a))-12*I*c*
d^3*x^2*exp(3*I*(b*x+a))+4*c^3*d*x*b*exp(I*(b*x+a))-4*I*d^4*x^3*exp(3*I*(b
*x+a))+12*I*c*d^3*x^2*exp(I*(b*x+a))+b*c^4*exp(I*(b*x+a))+12*I*c^2*d^2*x*e
xp(I*(b*x+a))+4*I*c^3*d*exp(I*(b*x+a))-12*I*c^2*d^2*x*exp(3*I*(b*x+a))-4*I
*c^3*d*exp(3*I*(b*x+a)))-12/b^4*c*d^3*ln(exp(I*(b*x+a))+1)*a+12/b^4*c*d^3*
ln(1-exp(I*(b*x+a)))*a+2/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3-2/b*c*d^3*ln(1-e
xp(I*(b*x+a)))*x^3-2/b*c^3*d*ln(1-exp(I*(b*x+a)))*x+6/b^3*c^2*d^2*a^2*arct
anh(exp(I*(b*x+a)))-2/b^4*c*d^3*a^3*ln(1-exp(I*(b*x+a)))-3/b*c^2*d^2*ln(1-
exp(I*(b*x+a)))*x^2+2/b^4*c*d^3*a^3*ln(exp(I*(b*x+a))+1)-4/b^4*c*d^3*a^3*a
rctanh(exp(I*(b*x+a)))-4/b^2*c^3*d*a*arctanh(exp(I*(b*x+a)))+24/b^4*c*d^3*
a*arctanh(exp(I*(b*x+a)))+2/b*c^3*d*ln(exp(I*(b*x+a))+1)*x-2/b^2*c^3*d*...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2770 vs. $2(370) = 740$.

Time = 0.17 (sec) , antiderivative size = 2770, normalized size of antiderivative = 6.66

$$\int (c+dx)^4 \cot^2(a+bx) \csc(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x
+ b^4*c^4)*cos(b*x + a) - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + I*b^3*c^3
*d - 6*I*b*c*d^3 + (-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - I*b^3*c^3*d + 6*I
*b*c*d^3 - 3*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 + 3*I*(b^3*c^2*d^
2 - 2*b*d^4)*x)*dilog(cos(b*x + a) + I*sin(b*x + a)) - 4*(-I*b^3*d^4*x^3 -
 3*I*b^3*c*d^3*x^2 - I*b^3*c^3*d + 6*I*b*c*d^3 + (I*b^3*d^4*x^3 + 3*I*b^3*
c*d^3*x^2 + I*b^3*c^3*d - 6*I*b*c*d^3 + 3*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*cos
(b*x + a)^2 - 3*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*dilog(cos(b*x + a) - I*sin(b*
x + a)) - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + I*b^3*c^3*d - 6*I*b*c*d^3
 + (-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - I*b^3*c^3*d + 6*I*b*c*d^3 - 3*I*(
b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 + 3*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*
dilog(-cos(b*x + a) + I*sin(b*x + a)) - 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*
x^2 - I*b^3*c^3*d + 6*I*b*c*d^3 + (I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + I*b
^3*c^3*d - 6*I*b*c*d^3 + 3*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 - 3
*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^4
*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 6*(b^4*c^2*d^2 - 2
*b^2*d^4)*x^2 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2
+ 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x
 + a)^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*log(cos(b*x + a) + I*sin(b*x + a)
 + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 6*(...
```

Sympy [F]

$$\int (c+dx)^4 \cot^2(a+bx) \csc(a+bx)\, dx = \int (c+dx)^4 \cot^2(a+bx) \csc(a+bx)\, dx$$

input
```
integrate((d*x+c)**4*cot(b*x+a)**2*csc(b*x+a),x)
```

output
```
Integral((c + d*x)**4*cot(a + b*x)**2*csc(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7004 vs. $2(370) = 740$.

Time = 3.13 (sec) , antiderivative size = 7004, normalized size of antiderivative = 16.84

$$\int (c+dx)^4 \cot^2(a+bx) \csc(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

output

```
1/4*(c^4*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - lo
g(cos(b*x + a) - 1)) - 4*a*c^3*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + lo
g(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b + 6*a^2*c^2*d^2*(2*cos(b*x
+ a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))
/b^2 - 4*a^3*c*d^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a)
 + 1) - log(cos(b*x + a) - 1))/b^3 + a^4*d^4*(2*cos(b*x + a)/(cos(b*x + a)
^2 - 1) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b^4 + 4*(2*((b*x
+ a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d^4 + 4*(b*c*d^3 - a*d
^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^
2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4
)*(b*x + a) + ((b*x + a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d^
4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2
- 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d
^3 - (a^3 - 6*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 2*((b*x + a)^4*d^4 - 1
2*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^
3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3
*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*c
os(2*b*x + 2*a) - (-I*(b*x + a)^4*d^4 + 12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3
+ 12*I*a^2*d^4 + 4*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 6*(-I*b^2*c^2*d^2
+ 2*I*a*b*c*d^3 + (-I*a^2 + 2*I)*d^4)*(b*x + a)^2 + 4*(-I*b^3*c^3*d + 3...
```

Giac [F]

$$\int (c+dx)^4 \cot^2(a+bx)\csc(a+bx)\,dx = \int (dx+c)^4 \cot\left(bx+a\right)^2 \csc\left(bx+a\right)\,dx$$

input

```
integrate((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

output

```
integrate((d*x + c)^4*cot(b*x + a)^2*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \cot^2(a+bx)\csc(a+bx)\,dx = \text{Hanged}$$

input
```
int((cot(a + b*x)^2*(c + d*x)^4)/sin(a + b*x),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^4 \cot^2(a+bx)\csc(a+bx)\,dx$$
$$= \frac{-\cos(bx+a)\,c^4 + 2\left(\int \cot(bx+a)^2 \csc(bx+a)\,x^4 dx\right)\sin(bx+a)^2\,b\,d^4 + 8\left(\int \cot(bx+a)^2 \csc(bx+a\right.}{}$$

input
```
int((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x)
```

output
```
( - cos(a + b*x)*c**4 + 2*int(cot(a + b*x)**2*csc(a + b*x)*x**4,x)*sin(a +
 b*x)**2*b*d**4 + 8*int(cot(a + b*x)**2*csc(a + b*x)*x**3,x)*sin(a + b*x)*
*2*b*c*d**3 + 12*int(cot(a + b*x)**2*csc(a + b*x)*x**2,x)*sin(a + b*x)**2*
b*c**2*d**2 + 8*int(cot(a + b*x)**2*csc(a + b*x)*x,x)*sin(a + b*x)**2*b*c*
*3*d - log(tan((a + b*x)/2))*sin(a + b*x)**2*c**4)/(2*sin(a + b*x)**2*b)
```

3.113 $\int (c+dx)^3 \cot^2(a+bx)\csc(a+bx)\,dx$

Optimal result . . . 1061
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Rubi [A] (verified) . . . 1062
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Optimal result

Integrand size = 22, antiderivative size = 308

$$\begin{aligned}
&\int (c+dx)^3 \cot^2(a+bx)\csc(a+bx)\,dx \\
&= -\frac{6d^2(c+dx)\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} \\
&\quad - \frac{3d(c+dx)^2\csc(a+bx)}{2b^2} - \frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b} \\
&\quad + \frac{3id^3\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^4} - \frac{3id(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2} \\
&\quad - \frac{3id^3\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^4} + \frac{3id(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2} \\
&\quad + \frac{3d^2(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} - \frac{3d^2(c+dx)\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{3id^3\operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4} - \frac{3id^3\operatorname{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4}
\end{aligned}$$

output

```
-6*d^2*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^3+(d*x+c)^3*arctanh(exp(I*(b*x+a)
))/b-3/2*d*(d*x+c)^2*csc(b*x+a)/b^2-1/2*(d*x+c)^3*cot(b*x+a)*csc(b*x+a)/b+
3*I*d^3*polylog(2,-exp(I*(b*x+a)))/b^4-3/2*I*d*(d*x+c)^2*polylog(2,-exp(I*
(b*x+a)))/b^2-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4+3/2*I*d*(d*x+c)^2*poly
log(2,exp(I*(b*x+a)))/b^2+3*d^2*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/b^3-3*d
^2*(d*x+c)*polylog(3,exp(I*(b*x+a)))/b^3+3*I*d^3*polylog(4,-exp(I*(b*x+a))
)/b^4-3*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4
```

Mathematica [A] (verified)

Time = 3.36 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.71

$$\int (c+dx)^3 \cot^2(a+bx)\csc(a+bx)\,dx =$$
$$-\frac{b^2(c+dx)^2(3d+b(c+dx)\cot(a+bx))\csc(a+bx)+b^3c^3\log\left(1-e^{i(a+bx)}\right)-6bcd^2\log\left(1-e^{i(a+bx)}\right)}{b^4}$$

input
```
Integrate[(c + d*x)^3*Cot[a + b*x]^2*Csc[a + b*x],x]
```

output
```
-1/2*(b^2*(c + d*x)^2*(3*d + b*(c + d*x)*Cot[a + b*x])*Csc[a + b*x] + b^3*
c^3*Log[1 - E^(I*(a + b*x))] - 6*b*c*d^2*Log[1 - E^(I*(a + b*x))] + 3*b^3*
c^2*d*x*Log[1 - E^(I*(a + b*x))] - 6*b*d^3*x*Log[1 - E^(I*(a + b*x))] + 3*
b^3*c*d^2*x^2*Log[1 - E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1 - E^(I*(a + b*x
))] - b^3*c^3*Log[1 + E^(I*(a + b*x))] + 6*b*c*d^2*Log[1 + E^(I*(a + b*x))
] - 3*b^3*c^2*d*x*Log[1 + E^(I*(a + b*x))] + 6*b*d^3*x*Log[1 + E^(I*(a + b
*x))] - 3*b^3*c*d^2*x^2*Log[1 + E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + E^(
I*(a + b*x))] + (3*I)*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, -E^(I*(a + b
*x))] - (3*I)*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, E^(I*(a + b*x))] - 6
*b*c*d^2*PolyLog[3, -E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, -E^(I*(a + b*
x))] + 6*b*c*d^2*PolyLog[3, E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, E^(I*(
a + b*x))] - (6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4,
 E^(I*(a + b*x))])/b^4
```

Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.73, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {4915, 3042, 4671, 3011, 4674, 3042, 4671, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \cot^2(a+bx)\csc(a+bx)\,dx$$
$$\downarrow \text{4915}$$

$$\int (c+dx)^3 \csc^3(a+bx)dx - \int (c+dx)^3 \csc(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)^3 \csc(a+bx)^3 dx - \int (c+dx)^3 \csc(a+bx)dx$$

$$\downarrow 4671$$

$$\frac{3d\int (c+dx)^2 \log\left(1-e^{i(a+bx)}\right)dx}{b} - \frac{3d\int (c+dx)^2 \log\left(1+e^{i(a+bx)}\right)dx}{b} + \int (c+dx)^3 \csc(a+bx)^3 dx + \frac{2(c+dx)^3 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\downarrow 3011$$

$$-\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b} - \frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b} + \frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b} - \frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b} + \int (c+dx)^3 \csc(a+bx)^3 dx + \frac{2(c+dx)^3 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\downarrow 4674$$

$$\frac{3d^2\int (c+dx)\csc(a+bx)dx}{b^2} - \frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b} - \frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b} + \frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b} - \frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b} + \frac{1}{2}\int (c+dx)^3 \csc(a+bx)dx + \frac{2(c+dx)^3 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2 \csc(a+bx)}{2b^2} - \frac{(c+dx)^3 \cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 3042$$

$$\frac{3d^2\int (c+dx)\csc(a+bx)dx}{b^2} - \frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b} - \frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b} + \frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b} - \frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b} + \frac{1}{2}\int (c+dx)^3 \csc(a+bx)dx + \frac{2(c+dx)^3 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2 \csc(a+bx)}{2b^2} - \frac{(c+dx)^3 \cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 4671$$

$$\frac{3d^2\left(-\frac{d\int\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{d\int\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)}{b^2}+$$
$$\frac{1}{2}\left(-\frac{3d\int(c+dx)^2\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{3d\int(c+dx)^2\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)-$$
$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}+$$
$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-$$
$$\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 2715$$

$$\frac{3d^2\left(\frac{id\int e^{-i(a+bx)}\log\left(1-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log\left(1+e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{2(c+dx)\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)}{b^2}+$$
$$\frac{1}{2}\left(-\frac{3d\int(c+dx)^2\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{3d\int(c+dx)^2\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)-$$
$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}+$$
$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-$$
$$\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 2838$$

$$\frac{1}{2}\left(-\frac{3d\int(c+dx)^2\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{3d\int(c+dx)^2\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)-$$
$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}+$$
$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}+$$
$$\frac{3d^2\left(-\frac{2(c+dx)\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{id\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}\right)}{b^2}+$$
$$\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 3011$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{3d^2\left(-\frac{2(c+dx)\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}\right)}{b^2}+\frac{2(c+dx)^3\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 7163$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}}{b}\right.}{b}+\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+\frac{3d^2\left(-\frac{2(c+dx)\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}\right)}{b^2}+\frac{2(c+dx)^3\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 2720$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(}{}\right.$$

$$\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+$$

$$\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+$$

$$\frac{3d^2\left(-\frac{2(c+dx)\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}\right)}{b^2}+$$

$$\frac{2(c+dx)^3\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 7143$$

$$\frac{3d^2\left(-\frac{2(c+dx)\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{id\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}\right)}{b^2}+$$

$$\frac{1}{2}\left(-\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+\right.$$

$$\left.\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\operatorname{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-\right.$$

$$\left.\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+\right.$$

$$\left.\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\operatorname{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\right.$$

$$\left.\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}\right)$$

input
```
Int[(c + d*x)^3*Cot[a + b*x]^2*Csc[a + b*x],x]
```

output
```
(2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b - (3*d*(c + d*x)^2*Csc[a + b*x]
)/(2*b^2) - ((c + d*x)^3*Cot[a + b*x]*Csc[a + b*x])/(2*b) + (3*d^2*((-2*(c
 + d*x)*ArcTanh[E^(I*(a + b*x))])/b + (I*d*PolyLog[2, -E^(I*(a + b*x))])/b
^2 - (I*d*PolyLog[2, E^(I*(a + b*x))])/b^2))/b^2 - (3*d*((I*(c + d*x)^2*Po
lyLog[2, -E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c + d*x)*PolyLog[3, -E^(I
*(a + b*x))])/b + (d*PolyLog[4, -E^(I*(a + b*x))])/b^2))/b))/b + (3*d*((I*
(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c + d*x)*Pol
yLog[3, E^(I*(a + b*x))])/b + (d*PolyLog[4, E^(I*(a + b*x))])/b^2))/b))/b
 + ((-2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b + (3*d*((I*(c + d*x)^2*Poly
Log[2, -E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c + d*x)*PolyLog[3, -E^(I*(
a + b*x))])/b + (d*PolyLog[4, -E^(I*(a + b*x))])/b^2))/b))/b - (3*d*((I*(c
 + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c + d*x)*PolyL
og[3, E^(I*(a + b*x))])/b + (d*PolyLog[4, E^(I*(a + b*x))])/b^2))/b))/b)/2
```

Defintions of rubi rules used

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4915
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1055 vs. $2(274) = 548$.

Time = 0.44 (sec) , antiderivative size = 1056, normalized size of antiderivative = 3.43

method	result	size
risch	Expression too large to display	1056

input
```
int((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
-3/2/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a+3/2/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+3
/2/b^2*c^2*d*ln(exp(I*(b*x+a))+1)*a+3/2/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2-3
/2/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2+1/b*c^3*arctanh(exp(I*(b*x+a)))+3/b^3*
c*d^2*polylog(3,-exp(I*(b*x+a)))-3/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+3/b
^3*d^3*polylog(3,-exp(I*(b*x+a)))*x-3/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-
6/b^3*c*d^2*arctanh(exp(I*(b*x+a)))+6/b^4*d^3*a*arctanh(exp(I*(b*x+a)))-1/
2/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3-3/b^4*d^3*ln(exp(I*(b*x+a))+1)*a+1/2/b*
d^3*ln(exp(I*(b*x+a))+1)*x^3+1/2/b^4*d^3*ln(exp(I*(b*x+a))+1)*a^3-1/2/b*d^
3*ln(1-exp(I*(b*x+a)))*x^3-1/b^4*d^3*a^3*arctanh(exp(I*(b*x+a)))-3*I/b^2*c
*d^2*polylog(2,-exp(I*(b*x+a)))*x+3*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*
x+1/b^2/(exp(2*I*(b*x+a))-1)^2*(d^3*x^3*b*exp(3*I*(b*x+a))+3*c*d^2*x^2*b*e
xp(3*I*(b*x+a))+3*c^2*d*x*b*exp(3*I*(b*x+a))+d^3*x^3*b*exp(I*(b*x+a))+b*c^
3*exp(3*I*(b*x+a))+3*c*d^2*x^2*b*exp(I*(b*x+a))-3*I*d^3*x^2*exp(3*I*(b*x+a
))+3*c^2*d*x*b*exp(I*(b*x+a))-6*I*c*d^2*x*exp(3*I*(b*x+a))+b*c^3*exp(I*(b*
x+a))-3*I*c^2*d*exp(3*I*(b*x+a))+3*I*d^3*x^2*exp(I*(b*x+a))+6*I*c*d^2*x*ex
p(I*(b*x+a))+3*I*c^2*d*exp(I*(b*x+a)))-3*d^3/b^3*ln(exp(I*(b*x+a))+1)*x+3*
d^3/b^3*ln(1-exp(I*(b*x+a)))*x+3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a+3/b^3*c*d^
2*a^2*arctanh(exp(I*(b*x+a)))-3/2/b*c^2*d*ln(1-exp(I*(b*x+a)))*x-3/b^2*c^2
*d*a*arctanh(exp(I*(b*x+a)))-3/2/b^3*c*d^2*ln(exp(I*(b*x+a))+1)*a^2+3/2/b^
3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2-3/2*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1742 vs. $2(264) = 528$.

Time = 0.14 (sec) , antiderivative size = 1742, normalized size of antiderivative = 5.66

$$\int (c + dx)^3 \cot^2(a + bx) \csc(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

output

```
1/4*(2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x +
 a) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - 2*I*d^3 + (-I*b^2
*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a)^2)*dilog(
cos(b*x + a) + I*sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b
^2*c^2*d + 2*I*d^3 + (I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - 2*I*
d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*(I*b^2*d^3*x
^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - 2*I*d^3 + (-I*b^2*d^3*x^2 - 2*I*b^2*c
*d^2*x - I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*si
n(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d + 2*I*d^3
 + (I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - 2*I*d^3)*cos(b*x + a)^2
)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
 b^3*c^3 - 6*b*c*d^2 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^
2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)
*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
 b^3*c^3 - 6*b*c*d^2 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^
2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)
*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^
2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*
b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*s
in(b*x + a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (...
```

Sympy [F]

$$\int (c+dx)^3 \cot^2(a+bx) \csc(a+bx)\, dx = \int (c+dx)^3 \cot^2(a+bx) \csc(a+bx)\, dx$$

input

```
integrate((d*x+c)**3*cot(b*x+a)**2*csc(b*x+a),x)
```

output

```
Integral((c + d*x)**3*cot(a + b*x)**2*csc(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3887 vs. $2(264) = 528$.

Time = 0.99 (sec) , antiderivative size = 3887, normalized size of antiderivative = 12.62

$$\int (c+dx)^3 \cot^2(a+bx) \csc(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

output
```
1/4*(c^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - lo
g(cos(b*x + a) - 1)) - 3*a*c^2*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + lo
g(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*cos(b*x +
a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b
^2 - a^3*d^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1)
- log(cos(b*x + a) - 1))/b^3 + 4*(2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3
 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2
)*d^3)*(b*x + a) + ((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a
*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))
*cos(4*b*x + 4*a) - 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2
- a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x +
a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + 3*(
-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*
a^2 + 2*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(I*(b*x + a)^3*d^3 - 6*I*b
*c*d^2 + 6*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d
- 2*I*a*b*c*d^2 + (I*a^2 - 2*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(
sin(b*x + a), cos(b*x + a) + 1) + 12*(b*c*d^2 - a*d^3 + (b*c*d^2 - a*d^3)*
cos(4*b*x + 4*a) - 2*(b*c*d^2 - a*d^3)*cos(2*b*x + 2*a) - (-I*b*c*d^2 + I*
a*d^3)*sin(4*b*x + 4*a) - 2*(I*b*c*d^2 - I*a*d^3)*sin(2*b*x + 2*a))*arctan
2(sin(b*x + a), cos(b*x + a) - 1) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a...
```

Giac [F]

$$\int (c+dx)^3 \cot^2(a+bx) \csc(a+bx)\,dx = \int (dx+c)^3 \cot(bx+a)^2 \csc(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*cot(b*x + a)^2*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \cot^2(a+bx) \csc(a+bx)\,dx = \text{Hanged}$$

input
```
int((cot(a + b*x)^2*(c + d*x)^3)/sin(a + b*x),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^3 \cot^2(a+bx) \csc(a+bx)\,dx$$
$$= \frac{-\cos(bx+a)\,c^3 + 2\left(\int \cot(bx+a)^2 \csc(bx+a)\,x^3 dx\right) \sin(bx+a)^2\, b\, d^3 + 6\left(\int \cot(bx+a)^2 \csc(bx+a\right.}{2}$$

input
```
int((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x)
```

output
```
( - cos(a + b*x)*c**3 + 2*int(cot(a + b*x)**2*csc(a + b*x)*x**3,x)*sin(a +
 b*x)**2*b*d**3 + 6*int(cot(a + b*x)**2*csc(a + b*x)*x**2,x)*sin(a + b*x)*
*2*b*c*d**2 + 6*int(cot(a + b*x)**2*csc(a + b*x)*x,x)*sin(a + b*x)**2*b*c*
*2*d - log(tan((a + b*x)/2))*sin(a + b*x)**2*c**3)/(2*sin(a + b*x)**2*b)
```

3.114 $\int (c+dx)^2 \cot^2(a+bx) \csc(a+bx)\, dx$

Optimal result 1074
Mathematica [B] (verified) 1075
Rubi [A] (verified) 1076
Maple [B] (verified) 1080
Fricas [B] (verification not implemented) 1081
Sympy [F] 1082
Maxima [B] (verification not implemented) 1083
Giac [F] 1084
Mupad [F(-1)] 1084
Reduce [F] 1084

Optimal result

Integrand size = 22, antiderivative size = 179

$$\begin{aligned}
&\int (c+dx)^2 \cot^2(a+bx) \csc(a+bx)\, dx \\
&= \frac{(c+dx)^2 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2 \operatorname{arctanh}(\cos(a+bx))}{b^3} \\
&\quad - \frac{d(c+dx)\csc(a+bx)}{b^2} - \frac{(c+dx)^2 \cot(a+bx)\csc(a+bx)}{2b} \\
&\quad - \frac{id(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} + \frac{id(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} \\
&\quad + \frac{d^2 \operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} - \frac{d^2 \operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}
\end{aligned}$$

output

```
(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b-d^2*arctanh(cos(b*x+a))/b^3-d*(d*x+c)*
csc(b*x+a)/b^2-1/2*(d*x+c)^2*cot(b*x+a)*csc(b*x+a)/b-I*d*(d*x+c)*polylog(2
,-exp(I*(b*x+a)))/b^2+I*d*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^2+d^2*polylo
g(3,-exp(I*(b*x+a)))/b^3-d^2*polylog(3,exp(I*(b*x+a)))/b^3
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 471 vs. $2(179) = 358$.

Time = 6.96 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.63

$$\int (c+dx)^2 \cot^2(a+bx)\csc(a+bx)\,dx$$
$$= -\frac{d(c+dx)\csc(a)}{b^2} + \frac{(-c^2-2cdx-d^2x^2)\csc^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}$$
$$+ \frac{-b^2c^2\log\left(1-e^{i(a+bx)}\right)+2d^2\log\left(1-e^{i(a+bx)}\right)-2b^2cdx\log\left(1-e^{i(a+bx)}\right)-b^2d^2x^2\log\left(1-e^{i(a+bx)}\right)-}{}$$
$$+ \frac{(c^2+2cdx+d^2x^2)\sec^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}$$
$$+ \frac{\sec\left(\frac{a}{2}\right)\sec\left(\frac{a}{2}+\frac{bx}{2}\right)\left(-cd\sin\left(\frac{bx}{2}\right)-d^2x\sin\left(\frac{bx}{2}\right)\right)}{2b^2}$$
$$+ \frac{\csc\left(\frac{a}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)\left(cd\sin\left(\frac{bx}{2}\right)+d^2x\sin\left(\frac{bx}{2}\right)\right)}{2b^2}$$

input
```
Integrate[(c + d*x)^2*Cot[a + b*x]^2*Csc[a + b*x],x]
```

output
```
-((d*(c + d*x)*Csc[a])/b^2) + ((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a/2 + (b*x)/
2]^2)/(8*b) + (-(b^2*c^2*Log[1 - E^(I*(a + b*x))]) + 2*d^2*Log[1 - E^(I*(a
 + b*x))] - 2*b^2*c*d*x*Log[1 - E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 - E^(
I*(a + b*x))] + b^2*c^2*Log[1 + E^(I*(a + b*x))] - 2*d^2*Log[1 + E^(I*(a +
 b*x))] + 2*b^2*c*d*x*Log[1 + E^(I*(a + b*x))] + b^2*d^2*x^2*Log[1 + E^(I*
(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] + (2*I)*b*d
*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] + 2*d^2*PolyLog[3, -E^(I*(a + b*x))
] - 2*d^2*PolyLog[3, E^(I*(a + b*x))])/(2*b^3) + ((c^2 + 2*c*d*x + d^2*x^2
)*Sec[a/2 + (b*x)/2]^2)/(8*b) + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c*d*Sin[(b
*x)/2]) - d^2*x*Sin[(b*x)/2]))/(2*b^2) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*(c*d
*Sin[(b*x)/2] + d^2*x*Sin[(b*x)/2]))/(2*b^2)
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.81, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {4915, 3042, 4671, 3011, 2720, 4674, 3042, 4257, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \cot^2(a+bx)\csc(a+bx)\,dx$$

$$\downarrow 4915$$

$$\int (c+dx)^2 \csc^3(a+bx)dx - \int (c+dx)^2 \csc(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)^2 \csc(a+bx)^3 dx - \int (c+dx)^2 \csc(a+bx)dx$$

$$\downarrow 4671$$

$$\frac{2d\int (c+dx)\log\left(1-e^{i(a+bx)}\right)dx}{b} - \frac{2d\int (c+dx)\log\left(1+e^{i(a+bx)}\right)dx}{b} + \int (c+dx)^2\csc(a+bx)^3dx + \frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\downarrow 3011$$

$$-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b} - \frac{id\int \text{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b} + \frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b} - \frac{id\int \text{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b} + \int (c+dx)^2\csc(a+bx)^3dx + \frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\downarrow 2720$$

$$-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b} - \frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b} + \frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b} - \frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b} + \int (c+dx)^2\csc(a+bx)^3dx + \frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\downarrow 4674$$

$$-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+$$
$$\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+\frac{d^2\int\csc(a+bx)dx}{b^2}+$$
$$\frac{1}{2}\int(c+dx)^2\csc(a+bx)dx+\frac{2(c+dx)^2\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{d(c+dx)\csc(a+bx)}{b^2}-$$
$$\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 3042$$

$$-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+$$
$$\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+\frac{d^2\int\csc(a+bx)dx}{b^2}+$$
$$\frac{1}{2}\int(c+dx)^2\csc(a+bx)dx+\frac{2(c+dx)^2\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{d(c+dx)\csc(a+bx)}{b^2}-$$
$$\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 4257$$

$$-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+$$
$$\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+\frac{1}{2}\int(c+dx)^2\csc(a+$$
$$bx)dx-\frac{d^2\mathrm{arctanh}(\cos(a+bx))}{b^3}+\frac{2(c+dx)^2\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{d(c+dx)\csc(a+bx)}{b^2}-$$
$$\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 4671$$

$$\frac{1}{2}\left(-\frac{2d\int(c+dx)\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{2d\int(c+dx)\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^2\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)-$$
$$\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+$$
$$\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{d^2\mathrm{arctanh}(\cos(a+bx))}{b^3}+$$
$$\frac{2(c+dx)^2\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{d(c+dx)\csc(a+bx)}{b^2}-\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 3011$$

$$\frac{1}{2}\left(\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{id\int \mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{id\int \mathrm{PolyLog}\left(2,e^{i(a+bx)}\right.\right.}{b}\right.$$

$$\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+$$

$$\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{d^2\mathrm{arctanh}(\cos(a+bx))}{b^3}+$$

$$\frac{2(c+dx)^2\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{d(c+dx)\csc(a+bx)}{b^2}-\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 2720$$

$$\frac{1}{2}\left(\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\right.}{b}\right.$$

$$\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+$$

$$\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{d^2\mathrm{arctanh}(\cos(a+bx))}{b^3}+$$

$$\frac{2(c+dx)^2\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{d(c+dx)\csc(a+bx)}{b^2}-\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 7143$$

$$-\frac{d^2\mathrm{arctanh}(\cos(a+bx))}{b^3}+$$

$$\frac{1}{2}\left(-\frac{2(c+dx)^2\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,\right.\right.}{b}\right.$$

$$\frac{2(c+dx)^2\mathrm{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^2}\right)}{b}+$$

$$\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\,\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{d(c+dx)\csc(a+bx)}{b^2}-$$

$$\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

input
```
Int[(c + d*x)^2*Cot[a + b*x]^2*Csc[a + b*x],x]
```

output
```
(2*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (d^2*ArcTanh[Cos[a + b*x]])/b
^3 - (d*(c + d*x)*Csc[a + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]*Csc[a + b*
x])/(2*b) - (2*d*((I*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b - (d*PolyLo
g[3, -E^(I*(a + b*x))])/b^2))/b + (2*d*((I*(c + d*x)*PolyLog[2, E^(I*(a +
b*x))])/b - (d*PolyLog[3, E^(I*(a + b*x))])/b^2))/b + ((-2*(c + d*x)^2*Arc
Tanh[E^(I*(a + b*x))])/b + (2*d*((I*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))]
)/b - (d*PolyLog[3, -E^(I*(a + b*x))])/b^2))/b - (2*d*((I*(c + d*x)*PolyLo
g[2, E^(I*(a + b*x))])/b - (d*PolyLog[3, E^(I*(a + b*x))])/b^2))/b)/2
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))  Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)  Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)  Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4915
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 545 vs. $2(165) = 330$.

Time = 0.36 (sec) , antiderivative size = 546, normalized size of antiderivative = 3.05

method	result
risch	$\frac{d^2x^2b\,e^{3i(bx+a)}+2cdxb\,e^{3i(bx+a)}+c^2b\,e^{3i(bx+a)}+d^2x^2b\,e^{i(bx+a)}+2cdxb\,e^{i(bx+a)}-2id^2x\,e^{3i(bx+a)}+c^2b\,e^{i(bx+a)}-2icd\,e^{3i(bx+a)}+2}{b^2\left(e^{2i(bx+a)}-1\right)^2}$

input
```
int((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
1/b^2/(exp(2*I*(b*x+a))-1)^2*(d^2*x^2*b*exp(3*I*(b*x+a))+2*c*d*x*b*exp(3*I
*(b*x+a))+c^2*b*exp(3*I*(b*x+a))+d^2*x^2*b*exp(I*(b*x+a))+2*c*d*x*b*exp(I*
(b*x+a))-2*I*d^2*x*exp(3*I*(b*x+a))+c^2*b*exp(I*(b*x+a))-2*I*c*d*exp(3*I*(
b*x+a))+2*I*d^2*x*exp(I*(b*x+a))+2*I*c*d*exp(I*(b*x+a)))+I/b^2*c*d*polylog
(2,exp(I*(b*x+a)))-I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-2/b^2*c*d*a*arct
anh(exp(I*(b*x+a)))+1/2/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+I/b^2*d^2*polylog(2
,exp(I*(b*x+a)))*x-1/2/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-I/b^2*c*d*polylog(2,
-exp(I*(b*x+a)))+1/b*d*c*ln(exp(I*(b*x+a))+1)*x+1/b^2*c*d*ln(exp(I*(b*x+a)
)+1)*a-1/b*d*c*ln(1-exp(I*(b*x+a)))*x-1/b^2*d*c*ln(1-exp(I*(b*x+a)))*a-1/2
/b^3*d^2*ln(exp(I*(b*x+a))+1)*a^2+1/2/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+d^2
*polylog(3,-exp(I*(b*x+a)))/b^3-d^2*polylog(3,exp(I*(b*x+a)))/b^3-2/b^3*d^
2*arctanh(exp(I*(b*x+a)))+1/b*c^2*arctanh(exp(I*(b*x+a)))+1/b^3*d^2*a^2*ar
ctanh(exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 970 vs. $2(161) = 322$.

Time $= 0.11$ (sec) , antiderivative size $= 970$, normalized size of antiderivative $= 5.42$

$$\int (c+dx)^2 \cot^2(a+bx) \csc(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

output

```
1/4*(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a) - 2*(I*b*d^2*x +
 I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(b*x + a)^2)*dilog(cos(b*x + a) + I*s
in(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(b*x + a
)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d + (-I*b
*d^2*x - I*b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a)) -
2*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilog(-cos
(b*x + a) - I*sin(b*x + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - 2*d^2)*log(cos(b
*x + a) + I*sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^
2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - 2*d^2)*log(cos
(b*x + a) - I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2 - (
b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a)
 + 1/2*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2 - (b^2
*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) -
1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d
^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log
(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*
c*d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x
+ a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 2*(d^2*cos(b*x + a)^2 -
d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 2*(d^2*cos(b*x + a)^2 ...
```

Sympy [F]

$$\int (c+dx)^2 \cot^2(a+bx) \csc(a+bx)\, dx = \int (c+dx)^2 \cot^2(a+bx) \csc(a+bx)\, dx$$

input

```
integrate((d*x+c)**2*cot(b*x+a)**2*csc(b*x+a),x)
```

output

```
Integral((c + d*x)**2*cot(a + b*x)**2*csc(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1938 vs. $2(161) = 322$.

Time = 0.31 (sec) , antiderivative size = 1938, normalized size of antiderivative = 10.83

$$\int (c+dx)^2 \cot^2(a+bx) \csc(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

output
```
1/4*(c^2*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - lo
g(cos(b*x + a) - 1)) - 2*a*c*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(
cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b + a^2*d^2*(2*cos(b*x + a)/(co
s(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b^2 + 4
*(2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2 + ((b*x + a)^2*
d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(4*b*x + 4*a) - 2*((b*x + a)
^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(2*b*x + 2*a) - (-I*(b*x
+ a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(4*b*x + 4*a)
- 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(2*
b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 4*(d^2*cos(4*b*x + 4
*a) - 2*d^2*cos(2*b*x + 2*a) + I*d^2*sin(4*b*x + 4*a) - 2*I*d^2*sin(2*b*x
+ 2*a) + d^2)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 2*((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
 + a))*cos(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a)
)*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a
))*sin(4*b*x + 4*a) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x +
a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*(I*(b*x
 + a)^2*d^2 + 2*b*c*d - 2*a*d^2 + 2*(I*b*c*d + (-I*a + 1)*d^2)*(b*x + a))*
cos(3*b*x + 3*a) - 4*(I*(b*x + a)^2*d^2 - 2*b*c*d + 2*a*d^2 + 2*(I*b*c*d +
 (-I*a - 1)*d^2)*(b*x + a))*cos(b*x + a) - 4*(b*c*d + (b*x + a)*d^2 - a...
```

Giac [F]

$$\int (c+dx)^2 \cot^2(a+bx) \csc(a+bx)\, dx = \int (dx+c)^2 \cot(bx+a)^2 \csc(bx+a)\, dx$$

input
```
integrate((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*cot(b*x + a)^2*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \cot^2(a+bx) \csc(a+bx)\, dx = \text{Hanged}$$

input
```
int((cot(a + b*x)^2*(c + d*x)^2)/sin(a + b*x),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^2 \cot^2(a+bx) \csc(a+bx)\, dx$$
$$= \frac{-\cos(bx+a)\, c^2 + 2\left(\int \cot(bx+a)^2 \csc(bx+a)\, x^2 dx\right) \sin(bx+a)^2\, b\, d^2 + 4\left(\int \cot(bx+a)^2 \csc(bx+a\right.}{2\sin(bx+a)^2\, b}$$

input
```
int((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x)
```

output
```
( - cos(a + b*x)*c**2 + 2*int(cot(a + b*x)**2*csc(a + b*x)*x**2,x)*sin(a +
 b*x)**2*b*d**2 + 4*int(cot(a + b*x)**2*csc(a + b*x)*x,x)*sin(a + b*x)**2*
b*c*d - log(tan((a + b*x)/2))*sin(a + b*x)**2*c**2)/(2*sin(a + b*x)**2*b)
```

3.115 $\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx$

Optimal result . . . 1085
Mathematica [B] (verified) . . . 1086
Rubi [A] (verified) . . . 1086
Maple [B] (verified) . . . 1090
Fricas [B] (verification not implemented) . . . 1090
Sympy [F] . . . 1091
Maxima [B] (verification not implemented) . . . 1091
Giac [F] . . . 1092
Mupad [F(-1)] . . . 1093
Reduce [F] . . . 1093

Optimal result

Integrand size = 20, antiderivative size = 108

$$\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx = \frac{(c+dx)\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b} - \frac{id\,\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2} + \frac{id\,\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}$$

output

```
(d*x+c)*arctanh(exp(I*(b*x+a)))/b-1/2*d*csc(b*x+a)/b^2-1/2*(d*x+c)*cot(b*x
+a)*csc(b*x+a)/b-1/2*I*d*polylog(2,-exp(I*(b*x+a)))/b^2+1/2*I*d*polylog(2,
exp(I*(b*x+a)))/b^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 260 vs. $2(108) = 216$.

Time $= 1.18$ (sec) , antiderivative size $= 260$, normalized size of antiderivative $= 2.41$

$$\begin{aligned}
&\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx \\
&= -\frac{d\cot\left(\frac{1}{2}(a+bx)\right)}{4b^2} - \frac{c\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{dx\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b} \\
&\quad + \frac{c\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{c\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b} + \frac{ad\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)}{2b^2} \\
&\quad - \frac{d\left((a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)+i\left(\text{PolyLog}\left(2,-e^{i(a+bx)}\right)-\text{PolyLog}\left(2,e^{i(a+bx)}\right)\right)\right)}{2b^2} \\
&\quad + \frac{c\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{dx\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{d\tan\left(\frac{1}{2}(a+bx)\right)}{4b^2}
\end{aligned}$$

input
```
Integrate[(c + d*x)*Cot[a + b*x]^2*Csc[a + b*x],x]
```

output
```
-1/4*(d*Cot[(a + b*x)/2])/b^2 - (c*Csc[(a + b*x)/2]^2)/(8*b) - (d*x*Csc[(a
 + b*x)/2]^2)/(8*b) + (c*Log[Cos[(a + b*x)/2]])/(2*b) - (c*Log[Sin[(a + b*
x)/2]])/(2*b) + (a*d*Log[Tan[(a + b*x)/2]])/(2*b^2) - (d*((a + b*x)*(Log[1
 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*(PolyLog[2, -E^(I*(a +
 b*x))] - PolyLog[2, E^(I*(a + b*x))])))/(2*b^2) + (c*Sec[(a + b*x)/2]^2)/
(8*b) + (d*x*Sec[(a + b*x)/2]^2)/(8*b) - (d*Tan[(a + b*x)/2])/(4*b^2)
```

Rubi [A] (verified)

Time $= 0.70$ (sec) , antiderivative size $= 176$, normalized size of antiderivative $= 1.63$, number of steps used $= 11$, number of rules used $= 10$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used $= \{4915, 3042, 4671, 2715, 2838, 4673, 3042, 4671, 2715, 2838\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx$$

$$\downarrow 4915$$

$$\int (c+dx)\csc^3(a+bx)dx - \int (c+dx)\csc(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)\csc(a+bx)^3dx - \int (c+dx)\csc(a+bx)dx$$

$$\downarrow 4671$$

$$\int (c+dx)\csc(a+bx)^3dx + \frac{d\int \log\left(1-e^{i(a+bx)}\right)dx}{b} - \frac{d\int \log\left(1+e^{i(a+bx)}\right)dx}{b} + \frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\downarrow 2715$$

$$-\frac{id\int e^{-i(a+bx)}\log\left(1-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2} + \frac{id\int e^{-i(a+bx)}\log\left(1+e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2} + \int (c+dx)\csc(a+bx)^3dx + \frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\downarrow 2838$$

$$\int (c+dx)\csc(a+bx)^3dx + \frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} + \frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}$$

$$\downarrow 4673$$

$$\frac{1}{2}\int (c+dx)\csc(a+bx)dx + \frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} + \frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 3042$$

$$\frac{1}{2}\int (c+dx)\csc(a+bx)dx + \frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} + \frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 4671$$

$$\frac{1}{2}\left(-\frac{d\int \log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{d\int \log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)+$$
$$\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}+\frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-$$
$$\frac{d\csc(a+bx)}{2b^2}-\frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 2715$$

$$\frac{1}{2}\left(\frac{id\int e^{-i(a+bx)}\log\left(1-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log\left(1+e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{2(c+dx)\text{arctanh}\left(e^{i(a}}{b}\right.$$
$$\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}+\frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-$$
$$\frac{d\csc(a+bx)}{2b^2}-\frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b}$$

$$\downarrow 2838$$

$$\frac{1}{2}\left(-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}\right)+$$
$$\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}+\frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-$$
$$\frac{d\csc(a+bx)}{2b^2}-\frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b}$$

input
```
Int[(c + d*x)*Cot[a + b*x]^2*Csc[a + b*x],x]
```

output
```
(2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b - (d*Csc[a + b*x])/(2*b^2) - ((c
+ d*x)*Cot[a + b*x]*Csc[a + b*x])/(2*b) - (I*d*PolyLog[2, -E^(I*(a + b*x))
])/b^2 + (I*d*PolyLog[2, E^(I*(a + b*x))])/b^2 + ((-2*(c + d*x)*ArcTanh[E^
(I*(a + b*x))])/b + (I*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[
2, E^(I*(a + b*x))])/b^2)/2
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4915
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(92) = 184$.

Time = 0.27 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.28

method	result
risch	$\frac{bdx\,e^{3i(bx+a)}+cb\,e^{3i(bx+a)}+bdx\,e^{i(bx+a)}+bc\,e^{i(bx+a)}-id\,e^{3i(bx+a)}+id\,e^{i(bx+a)}}{b^2\left(e^{2i(bx+a)}-1\right)^2}+\frac{c\,\operatorname{arctanh}\left(e^{i(bx+a)}\right)}{b}+\frac{d\ln\left(e^{i(bx+a)}+1\right)x}{2b}+$

input
```
int((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2/(exp(2*I*(b*x+a))-1)^2*(b*d*x*exp(3*I*(b*x+a))+c*b*exp(3*I*(b*x+a))+
b*d*x*exp(I*(b*x+a))+b*c*exp(I*(b*x+a))-I*d*exp(3*I*(b*x+a))+I*d*exp(I*(b*
x+a)))+1/b*c*arctanh(exp(I*(b*x+a)))+1/2/b*d*ln(exp(I*(b*x+a))+1)*x+1/2/b^
2*d*ln(exp(I*(b*x+a))+1)*a-1/2*I*d*polylog(2,-exp(I*(b*x+a)))/b^2-1/2/b*d*
ln(1-exp(I*(b*x+a)))*x-1/2/b^2*d*ln(1-exp(I*(b*x+a)))*a+1/2*I*d*polylog(2,
exp(I*(b*x+a)))/b^2-1/b^2*d*a*arctanh(exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(88) = 176$.

Time = 0.10 (sec) , antiderivative size = 454, normalized size of antiderivative = 4.20

$$\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx$$
$$= \frac{2\,(bdx+bc)\cos(bx+a)+\left(i\,d\cos(bx+a)^2-i\,d\right)\mathrm{Li}_2(\cos(bx+a)+i\,\sin(bx+a))+\left(-i\,d\cos(bx+a)\right.}{}$$

input
```
integrate((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(2*(b*d*x + b*c)*cos(b*x + a) + (I*d*cos(b*x + a)^2 - I*d)*dilog(cos(b
*x + a) + I*sin(b*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(cos(b*x + a)
 - I*sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(-cos(b*x + a) + I*si
n(b*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(-cos(b*x + a) - I*sin(b*x
+ a)) - (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a) + I*
sin(b*x + a) + 1) - (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b
*x + a) - I*sin(b*x + a) + 1) - ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*l
og(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - ((b*c - a*d)*cos(b*x +
a)^2 - b*c + a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b*d
*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d)*log(-cos(b*x + a) + I*sin(b*x + a
) + 1) + (b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d)*log(-cos(b*x + a) -
I*sin(b*x + a) + 1) + 2*d*sin(b*x + a))/(b^2*cos(b*x + a)^2 - b^2)
```

Sympy [F]

$$\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx = \int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx$$

input
```
integrate((d*x+c)*cot(b*x+a)**2*csc(b*x+a),x)
```

output
```
Integral((c + d*x)*cot(a + b*x)**2*csc(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(88) = 176$.

Time = 0.17 (sec) , antiderivative size = 762, normalized size of antiderivative = 7.06

$$\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

output

```
(2*(b*d*x + b*c + (b*d*x + b*c)*cos(4*b*x + 4*a) - 2*(b*d*x + b*c)*cos(2*b
*x + 2*a) - (-I*b*d*x - I*b*c)*sin(4*b*x + 4*a) - 2*(I*b*d*x + I*b*c)*sin(
2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(b*c*cos(4*b*x +
 4*a) - 2*b*c*cos(2*b*x + 2*a) + I*b*c*sin(4*b*x + 4*a) - 2*I*b*c*sin(2*b*
x + 2*a) + b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 2*(b*d*x*cos(4*b
*x + 4*a) - 2*b*d*x*cos(2*b*x + 2*a) + I*b*d*x*sin(4*b*x + 4*a) - 2*I*b*d*
x*sin(2*b*x + 2*a) + b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*(
I*b*d*x + I*b*c + d)*cos(3*b*x + 3*a) - 4*(I*b*d*x + I*b*c - d)*cos(b*x +
a) - 2*(d*cos(4*b*x + 4*a) - 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) -
 2*I*d*sin(2*b*x + 2*a) + d)*dilog(-e^(I*b*x + I*a)) + 2*(d*cos(4*b*x + 4*
a) - 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) - 2*I*d*sin(2*b*x + 2*a)
+ d)*dilog(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(4
*b*x + 4*a) - 2*(-I*b*d*x - I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(4*
b*x + 4*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*
x + a)^2 + 2*cos(b*x + a) + 1) + (I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(
4*b*x + 4*a) - 2*(I*b*d*x + I*b*c)*cos(2*b*x + 2*a) - (b*d*x + b*c)*sin(4*
b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*
x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*d*x + b*c - I*d)*sin(3*b*x + 3*a) +
4*(b*d*x + b*c + I*d)*sin(b*x + a))/(-4*I*b^2*cos(4*b*x + 4*a) + 8*I*b^2*c
os(2*b*x + 2*a) + 4*b^2*sin(4*b*x + 4*a) - 8*b^2*sin(2*b*x + 2*a) - 4*I...
```

Giac [F]

$$\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx = \int (dx+c)\cot\left(bx+a\right)^2\csc\left(bx+a\right)\,dx$$

input

```
integrate((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

output

```
integrate((d*x + c)*cot(b*x + a)^2*csc(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx = \text{Hanged}$$

input
```
int((cot(a + b*x)^2*(c + d*x))/sin(a + b*x),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx$$
$$= \frac{-\cos(bx+a)\,c + 2\left(\int \cot(bx+a)^2\csc(bx+a)\,x dx\right)\sin(bx+a)^2\,bd - \log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin(bx+a)^2\,c}{2\sin(bx+a)^2\,b}$$

input
```
int((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x)
```

output
```
( - cos(a + b*x)*c + 2*int(cot(a + b*x)**2*csc(a + b*x)*x,x)*sin(a + b*x)*
*2*b*d - log(tan((a + b*x)/2))*sin(a + b*x)**2*c)/(2*sin(a + b*x)**2*b)
```

3.116 $\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$

Optimal result 1094
Mathematica [N/A] 1094
Rubi [N/A] 1095
Maple [N/A] 1096
Fricas [N/A] 1096
Sympy [N/A] 1097
Maxima [N/A] 1097
Giac [N/A] 1098
Mupad [N/A] 1099
Reduce [N/A] 1099

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx = -\text{Int}\left(\frac{\csc(a+bx)}{c+dx},x\right) + \text{Int}\left(\frac{\csc^3(a+bx)}{c+dx},x\right)$$

output
```
-Defer(Int)(csc(b*x+a)/(d*x+c),x)+Defer(Int)(csc(b*x+a)^3/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 29.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x),x]
```

output
```
Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4915, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$$

$$\downarrow 4915$$

$$\int \frac{\csc^3(a+bx)}{c+dx}dx - \int \frac{\csc(a+bx)}{c+dx}dx$$

$$\downarrow 3042$$

$$\int \frac{\csc(a+bx)^3}{c+dx}dx - \int \frac{\csc(a+bx)}{c+dx}dx$$

$$\downarrow 4680$$

$$\int \frac{\csc^3(a+bx)}{c+dx}dx - \int \frac{\csc(a+bx)}{c+dx}dx$$

input
```
Int[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4915

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cot(bx+a)^2 \csc(bx+a)}{dx+c} dx$$

input

```
int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x)
```

output

```
int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx} dx = \int \frac{\cot(bx+a)^2 \csc(bx+a)}{dx+c} dx$$

input

```
integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(cot(b*x + a)^2*csc(b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$$

input
```
integrate(cot(b*x+a)**2*csc(b*x+a)/(d*x+c),x)
```

output
```
Integral(cot(a + b*x)**2*csc(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 1793, normalized size of antiderivative = 81.50

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cot(bx+a)^2\csc(bx+a)}{dx+c}\,dx$$

input
```
integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output

```
(((b*d*x + b*c)*cos(3*b*x + 3*a) + (b*d*x + b*c)*cos(b*x + a) - d*sin(3*b*
x + 3*a) + d*sin(b*x + a))*cos(4*b*x + 4*a) + (b*d*x + b*c - 2*(b*d*x + b*
c)*cos(2*b*x + 2*a) - 2*d*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) - 2*((b*d*x +
 b*c)*cos(b*x + a) + d*sin(b*x + a))*cos(2*b*x + 2*a) + (b*d*x + b*c)*cos(
b*x + a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*
a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x
 + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2)*cos(2*b*x + 2*a))*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
 - 2*d^2)*sin(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^
2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x
 + a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(b*x
 + a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(
b*x + a)), x) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^
2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x
 + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*si...
```

Giac [N/A]

Not integrable

Time = 4.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cot\left(bx+a\right)^2\csc\left(bx+a\right)}{dx+c}\,dx$$

input

```
integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x, algorithm="giac")
```

output

```
integrate(cot(b*x + a)^2*csc(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 18.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+b\,x)^2}{\sin(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(cot(a + b*x)^2/(sin(a + b*x)*(c + d*x)),x)
```

output
```
int(cot(a + b*x)^2/(sin(a + b*x)*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cot(bx+a)^2\csc(bx+a)}{dx+c}dx$$

input
```
int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x)
```

output
```
int((cot(a + b*x)**2*csc(a + b*x))/(c + d*x),x)
```

3.117 $\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$

Optimal result 1100
Mathematica [N/A] 1100
Rubi [N/A] 1101
Maple [N/A] 1102
Fricas [N/A] 1102
Sympy [N/A] 1103
Maxima [N/A] 1103
Giac [N/A] 1104
Mupad [N/A] 1105
Reduce [N/A] 1105

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = -\text{Int}\left(\frac{\csc(a+bx)}{(c+dx)^2},x\right) + \text{Int}\left(\frac{\csc^3(a+bx)}{(c+dx)^2},x\right)$$

output
```
-Defer(Int)(csc(b*x+a)/(d*x+c)^2,x)+Defer(Int)(csc(b*x+a)^3/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 34.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4915, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

$$\downarrow 4915$$

$$\int \frac{\csc^3(a+bx)}{(c+dx)^2}dx - \int \frac{\csc(a+bx)}{(c+dx)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\csc(a+bx)^3}{(c+dx)^2}dx - \int \frac{\csc(a+bx)}{(c+dx)^2}dx$$

$$\downarrow 4680$$

$$\int \frac{\csc^3(a+bx)}{(c+dx)^2}dx - \int \frac{\csc(a+bx)}{(c+dx)^2}dx$$

input
```
Int[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4915

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cot(bx+a)^2 \csc(bx+a)}{(dx+c)^2} dx$$

input

```
int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x)
```

output

```
int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\, dx = \int \frac{\cot(bx+a)^2 \csc(bx+a)}{(dx+c)^2}\, dx$$

input

```
integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cot(b*x + a)^2*csc(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 4.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(cot(b*x+a)**2*csc(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(cot(a + b*x)**2*csc(a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 8.30 (sec) , antiderivative size = 2289, normalized size of antiderivative = 104.05

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(bx+a)^2\csc(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
(((b*d*x + b*c)*cos(3*b*x + 3*a) + (b*d*x + b*c)*cos(b*x + a) - 2*d*sin(3*
b*x + 3*a) + 2*d*sin(b*x + a))*cos(4*b*x + 4*a) + (b*d*x + b*c - 2*(b*d*x
+ b*c)*cos(2*b*x + 2*a) - 4*d*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) - 2*((b*d
*x + b*c)*cos(b*x + a) + 2*d*sin(b*x + a))*cos(2*b*x + 2*a) + (b*d*x + b*c
)*cos(b*x + a) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
+ (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*
a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b
*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*si
n(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*
c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
+ 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d
^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^3*x^3 +
 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*integrate(1/
2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 6*d^2)*sin(b*x + a)/(b^2*d^4*x^4
+ 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4
*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(
b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^
3*d*x + b^2*c^4)*sin(b*x + a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2
*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x) - (b^2*d^3*x^...
```

Giac [N/A]

Not integrable

Time = 16.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot\left(bx+a\right)^2\csc\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(cot(b*x + a)^2*csc(b*x + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 18.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(a+b\,x)^2}{\sin(a+b\,x)\,(c+d\,x)^2}\,dx$$

input
```
int(cot(a + b*x)^2/(sin(a + b*x)*(c + d*x)^2),x)
```

output
```
int(cot(a + b*x)^2/(sin(a + b*x)*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(bx+a)^2\csc(bx+a)}{d^2x^2+2cdx+c^2}dx$$

input
```
int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x)
```

output
```
int((cot(a + b*x)**2*csc(a + b*x))/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.118 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 1106
Mathematica [C] (verified) . . . 1107
Rubi [A] (verified) . . . 1108
Maple [A] (verified) . . . 1110
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Maxima [C] (verification not implemented) . . . 1112
Giac [C] (verification not implemented) . . . 1113
Mupad [F(-1)] . . . 1114
Reduce [F] . . . 1114

Optimal result

Integrand size = 24, antiderivative size = 406

$$\begin{aligned}\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx &= \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{16b^3} \\ &- \frac{(c+dx)^{5/2}\cos(a+bx)}{4b} + \frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{144b^3} \\ &- \frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b} - \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} \\ &- \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}} \\ &+ \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{144b^{7/2}} \\ &+ \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{16b^{7/2}} \\ &+ \frac{5d(c+dx)^{3/2}\sin(a+bx)}{8b^2} + \frac{5d(c+dx)^{3/2}\sin(3a+3bx)}{72b^2}\end{aligned}$$

output

```
15/16*d^2*(d*x+c)^(1/2)*cos(b*x+a)/b^3-1/4*(d*x+c)^(5/2)*cos(b*x+a)/b+5/14
4*d^2*(d*x+c)^(1/2)*cos(3*b*x+3*a)/b^3-1/12*(d*x+c)^(5/2)*cos(3*b*x+3*a)/b
-15/32*d^(5/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(
1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-5/864*d^(5/2)*6^(1/2)*Pi^(1/2)*cos(3*a
-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)
+5/864*d^(5/2)*6^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*sin(3*a-3*b*c/d)/b^(7/2)+15/32*d^(5/2)*2^(1/2)*Pi^(1/2)*Fre
snelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(7/2)
+5/8*d*(d*x+c)^(3/2)*sin(b*x+a)/b^2+5/72*d*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b^
2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.29 (sec) , antiderivative size = 1445, normalized size of antiderivative = 3.56

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input

```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output

```
(c*Sqrt[d]*(-12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(-I + 2*b*
x + E^((6*I)*(a + b*x))*(I + 2*b*x)) - (1 - I)*(2*b*c + I*d)*E^(((3*I)*b*(
2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sq
rt[d]] + (1 + I)*((2*I)*b*c + d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi]*Erfi[((1
 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(288*b^(5/2)*E^(((3*I)*(
a*d + b*(c + d*x)))/d)) + ((I/8)*c^2*d*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*
x))/d]*Gamma[3/2, ((-I)*b*(c + d*x))/d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c
+ d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d]))/(b^2*E^((I*(b*c + a*d))/d)*Sqrt
[c + d*x]) + (c^2*(-1/6*(E^((3*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2,
((-3*I)*b*(c + d*x))/d])/(Sqrt[3]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c
+ d*x]*Gamma[3/2, ((3*I)*b*(c + d*x))/d])/(6*Sqrt[3]*b*E^((3*I)*(a - (b*c)
/d))*Sqrt[(I*b*(c + d*x))/d])))/4 + (c*Sqrt[d]*(E^(I*(a - (b*c)/d))*(-2*Sq
rt[b]*Sqrt[d]*E^((I*b*(c + d*x))/d)*(3*I + 2*b*x)*Sqrt[c + d*x] + (-1)^(1/
4)*((2*I)*b*c + 3*d)*Sqrt[Pi]*Erfi[((-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]]) + I*(2*Sqrt[b]*Sqrt[d]*(3 + (2*I)*b*x)*Sqrt[c + d*x] + (1 + I)*(2*b*
c + (3*I)*d)*Sqrt[Pi/2]*Erf[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[2]*Sqrt[
d])]*(Cos[b*(c/d + x)] + I*Sin[b*(c/d + x)]))*(Cos[a + b*x] - I*Sin[a + b*
x])))/(16*b^(5/2)) - ((I/64)*Sqrt[d]*((Cos[a - (b*c)/d] + I*Sin[a - (b*c)/
d])*((1 + I)*(4*b^2*c^2 - (12*I)*b*c*d - 15*d^2)*Sqrt[Pi/2]*Erfi[((1 + I)*
Sqrt[b]*Sqrt[c + d*x])/(Sqrt[2]*Sqrt[d])] + 2*Sqrt[b]*Sqrt[d]*Sqrt[c + ...
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2} \sin(a+bx) \cos^2(a+bx)\, dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}(c+dx)^{5/2} \sin(a+bx) + \frac{1}{4}(c+dx)^{5/2} \sin(3a+3bx) \right) dx$$

$$\downarrow 2009$$

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{16b^3}+\frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{144b^3}+\frac{5d(c+dx)^{3/2}\sin(a+bx)}{8b^2}+\frac{5d(c+dx)^{3/2}\sin(3a+3bx)}{72b^2}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}$$

input

```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output

```
(15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^3) - ((c + d*x)^(5/2)*Cos[a + b*
x])/(4*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(144*b^3) - ((c + d*x)^
(5/2)*Cos[3*a + 3*b*x])/(12*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*F
resnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(
5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(144*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (1
5*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*
Sin[a - (b*c)/d])/(16*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[a + b*x])/(8*b^2
) + (5*d*(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(72*b^2)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.17

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\sqrt{\pi}}{}\right.}\right.}{4b}$
default	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\sqrt{\pi}}{}\right.}\right.}{4b}$

input

```
int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+5/8/b*d*(1/2/b*d*
(d*x+c)^(3/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(1/2)
*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a
*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin((a
*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1/2
4/b*d*(d*x+c)^(5/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+5/24/b*d*(1/6/b*d*(d*
x+c)^(3/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/2
)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)
^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b
*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/
d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.84

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{5\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right.$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(
sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 40
5*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(
pi*d)))*sin(-(b*c - a*d)/d) - 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(
sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(30*b*d^2
*cos(b*x + a) - (12*b^3*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos
(b*x + a)^3 + 10*(2*b^2*d^2*x + 2*b^2*c*d + (b^2*d^2*x + b^2*c*d)*cos(b*x
+ a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.35

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/3456*(240*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d) + 2160*
(d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d) - 24*(12*(d*x + c)^(5
/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*cos(3*((d*x + c)*b - b*c + a*d)/d) - 21
6*(4*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(((d*x + c)*b - b*
c + a*d)/d) - 5*(-(I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*c
os(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1
/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 405*(-(I - 1
)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqr
t(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)
*sqrt(I*b/d)) - 405*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(
b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c -
 a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - 5*((I + 1)*9^(1/4)*sqrt(2)*sqr
t(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2
)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*
sqrt(-3*I*b/d)))*d/b^5
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 2445, normalized size of antiderivative = 6.02

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
1/1728*(72*(3*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d
*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(
I*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d
)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 - d^3*(27*(sqrt(2)*sqrt(p
i)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3)*d*erf(1/2*I*sqrt(2
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)*
b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d + 10*(
d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*
e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + (sqrt(6)*sqrt(pi)*(72*b^
3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*I*sqrt(6)*sqrt(b
*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(s
qrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^(5/2)*b^2*d
- 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x +
c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^(-...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int \cos(a+b\,x)^2\,\sin(a+b\,x)\,(c+d\,x)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \left(\int \sqrt{dx+c}\,\cos(bx+a)^2\sin(bx+a)\,x^2dx\right)d^2$$
$$+2\left(\int \sqrt{dx+c}\,\cos(bx+a)^2\sin(bx+a)\,xdx\right)cd$$
$$+\left(\int \sqrt{dx+c}\,\cos(bx+a)^2\sin(bx+a)\,dx\right)c^2$$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**2*sin(a + b*x)*x**2,x)*d**2 + 2*int(sqrt(c
 + d*x)*cos(a + b*x)**2*sin(a + b*x)*x,x)*c*d + int(sqrt(c + d*x)*cos(a +
b*x)**2*sin(a + b*x),x)*c**2
```

3.119 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx$

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Optimal result

Integrand size = 24, antiderivative size = 353

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} - \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}} - \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{24b^{5/2}} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{8b^{5/2}} + \frac{3d\sqrt{c+dx}\sin(a+bx)}{8b^2} + \frac{d\sqrt{c+dx}\sin(3a+3bx)}{24b^2}$$

output

```
-1/4*(d*x+c)^(3/2)*cos(b*x+a)/b-1/12*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b-3/16*d
^(3/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*
x+c)^(1/2)/d^(1/2))/b^(5/2)-1/144*d^(3/2)*6^(1/2)*Pi^(1/2)*cos(3*a-3*b*c/d
)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)-1/144*d
^(3/2)*6^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*sin(3*a-3*b*c/d)/b^(5/2)-3/16*d^(3/2)*2^(1/2)*Pi^(1/2)*FresnelC(b^(
1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(5/2)+3/8*d*(d
*x+c)^(1/2)*sin(b*x+a)/b^2+1/24*d*(d*x+c)^(1/2)*sin(3*b*x+3*a)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 751, normalized size of antiderivative = 2.13

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \frac{\sqrt{d}e^{-\frac{3i(ad+b(c+dx))}{d}}\left(-12\sqrt{b}\sqrt{d}e^{\frac{3ibc}{d}}\sqrt{c+dx}\left(-i+2bx+e^{6i(a+bx)}(i+2bx)\right)-(1-i)(2bc+id)e^{\frac{3i}{}}\right.}{576b^{5/2}}$$

$$+\frac{icde^{-\frac{i(bc+ad)}{d}}\left(-e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)\right)}{8b^2\sqrt{c+dx}}$$

$$+\frac{1}{4}c\left(-\frac{e^{3i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\Gamma\left(\frac{3}{2},-\frac{3ib(c+dx)}{d}\right)}{6\sqrt{3}b\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{-3i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\Gamma\left(\frac{3}{2},\frac{3ib(c+dx)}{d}\right)}{6\sqrt{3}b\sqrt{\frac{ib(c+dx)}{d}}}\right)$$

$$+\frac{\sqrt{d}\left(e^{i\left(a-\frac{bc}{d}\right)}\left(-2\sqrt{b}\sqrt{d}e^{\frac{ib(c+dx)}{d}}(3i+2bx)\sqrt{c+dx}+\sqrt[4]{-1}(2ibc+3d)\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)\right)+i\left(2\sqrt{b}\right.\right.}{}$$

input

```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output

```
(Sqrt[d]*(-12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(-I + 2*b*x
+ E^((6*I)*(a + b*x))*(I + 2*b*x)) - (1 - I)*(2*b*c + I*d)*E^(((3*I)*b*(2*
c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]] + (1 + I)*((2*I)*b*c + d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi]*Erfi[((1 +
 I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(576*b^(5/2)*E^(((3*I)*(a*
d + b*(c + d*x)))/d)) + ((I/8)*c*d*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/
d]*Gamma[3/2, ((-I)*b*(c + d*x))/d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*
x))/d]*Gamma[3/2, (I*b*(c + d*x))/d]))/(b^2*E^((I*(b*c + a*d))/d)*Sqrt[c +
 d*x]) + (c*(-1/6*(E^((3*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-3*I
)*b*(c + d*x))/d])/(Sqrt[3]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]
*Gamma[3/2, ((3*I)*b*(c + d*x))/d])/(6*Sqrt[3]*b*E^((3*I)*(a - (b*c)/d))*S
qrt[(I*b*(c + d*x))/d])))/4 + (Sqrt[d]*(E^(I*(a - (b*c)/d))*(-2*Sqrt[b]*Sq
rt[d]*E^((I*b*(c + d*x))/d)*(3*I + 2*b*x)*Sqrt[c + d*x] + (-1)^(1/4)*((2*I
)*b*c + 3*d)*Sqrt[Pi]*Erfi[((-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]) +
I*(2*Sqrt[b]*Sqrt[d]*(3 + (2*I)*b*x)*Sqrt[c + d*x] + (1 + I)*(2*b*c + (3*I
)*d)*Sqrt[Pi/2]*Erf[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[2]*Sqrt[d])]*(Co
s[b*(c/d + x)] + I*Sin[b*(c/d + x)]))*(Cos[a + b*x] - I*Sin[a + b*x])))/(3
2*b^(5/2))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin(a+bx)\cos^2(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}(c+dx)^{3/2}\sin(a+bx)+\frac{1}{4}(c+dx)^{3/2}\sin(3a+3bx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(a+bx)}{8b^2}+\frac{d\sqrt{c+dx}\sin(3a+3bx)}{24b^2}-\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}$$

input

```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output

```
-1/4*((c + d*x)^(3/2)*Cos[a + b*x])/b - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])
/(12*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (
3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/2
)) - (d^(3/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[
d]]*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(S
qrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) +
(3*d*Sqrt[c + d*x]*Sin[a + b*x])/(8*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*
x])/(24*b^2)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.09

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}d}\right)+\sin\right.\right.}{4b\sqrt{\frac{b}{d}}}\right.}{4b}$
default	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}d}\right)+\sin\right.\right.}{4b\sqrt{\frac{b}{d}}}\right.}{4b}$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+3/8/b*d*(1/2/b*d*
(d*x+c)^(1/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^
(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)))-1/24/b*d*(d*x+c)^(3/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/8/b*d*(
1/6/b*d*(d*x+c)^(1/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi
^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3
^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.79

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+27\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sq
rt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos
(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sq
rt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-(b*c - a*d)/d) + sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6
)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(2*(b^2*d*x + b
^2*c)*cos(b*x + a)^3 - (b*d*cos(b*x + a)^2 + 2*b*d)*sin(b*x + a))*sqrt(d*x
 + c))/b^3
```

Sympy [F]

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)*cos(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.41

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output

```
-1/576*(48*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d)/d + 144*
(d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d - 24*sqrt(d*x + c)*
b^2*sin(3*((d*x + c)*b - b*c + a*d)/d) - 216*sqrt(d*x + c)*b^2*sin(((d*x +
 c)*b - b*c + a*d)/d) - (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(
1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2
)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + 27*((I +
 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqr
t(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(I*b/d)) + 27*(-(I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c
 - a*d)/d) + (I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/
d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*
d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)
*b*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/
d)))*d/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 1515, normalized size of antiderivative = 4.29

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output

```
1/288*(12*(3*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt
(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*sq
rt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 - 4*(9*sqrt(2)*sqrt(pi)*(2
*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b)
 + sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*I*s
qrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*(6*b*c
 + I*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b)
 + 18*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*sqrt(d*x
 + c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int \cos(a+b\,x)^2\,\sin(a+b\,x)\,(c+d\,x)^{3/2}\,dx$$

input

```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(3/2),x)
```

output

```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c + dx)^{3/2} \cos^2(a+bx)\sin(a+bx)\,dx = \left(\int \sqrt{dx+c}\,\cos(bx+a)^2\sin(bx+a)\,x dx\right) d + \left(\int \sqrt{dx+c}\,\cos(bx+a)^2\sin(bx+a)\,dx\right) c$$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**2*sin(a + b*x)*x,x)*d + int(sqrt(c + d*x)*
cos(a + b*x)**2*sin(a + b*x),x)*c
```

3.120 $\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 1124
Mathematica [C] (verified) . . . 1125
Rubi [A] (verified) . . . 1126
Maple [A] (verified) . . . 1127
Fricas [A] (verification not implemented) . . . 1128
Sympy [F] . . . 1128
Maxima [C] (verification not implemented) . . . 1129
Giac [C] (verification not implemented) . . . 1129
Mupad [F(-1)] . . . 1130
Reduce [F] . . . 1131

Optimal result

Integrand size = 24, antiderivative size = 304

$$\begin{aligned}
&\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx \\
&= -\frac{\sqrt{c+dx}\cos(a+bx)}{4b} - \frac{\sqrt{c+dx}\cos(3a+3bx)}{12b} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{12b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{4b^{3/2}}
\end{aligned}$$

output

```
-1/4*(d*x+c)^(1/2)*cos(b*x+a)/b-1/12*(d*x+c)^(1/2)*cos(3*b*x+3*a)/b+1/8*d^
(1/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x
+c)^(1/2)/d^(1/2))/b^(3/2)+1/72*d^(1/2)*6^(1/2)*Pi^(1/2)*cos(3*a-3*b*c/d)*
FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)-1/72*d^(1
/2)*6^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/
2))*sin(3*a-3*b*c/d)/b^(3/2)-1/8*d^(1/2)*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)
*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.83

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{e^{-\frac{3i(bc+ad)}{d}}\sqrt{c+dx}\left(9e^{2i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+9e^{2ia+\frac{4ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)+\sqrt{3}\left(e\right.\right.}{72b\sqrt{\frac{b^2(c+dx)^2}{d^2}}}$$

input

```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output

```
-1/72*(Sqrt[c + d*x]*(9*E^((2*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*
Gamma[3/2, ((-I)*b*(c + d*x))/d] + 9*E^((2*I)*a + ((4*I)*b*c)/d)*Sqrt[((-I
)*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d] + Sqrt[3]*(E^((6*I)*a)*Sqr
t[(I*b*(c + d*x))/d]*Gamma[3/2, ((-3*I)*b*(c + d*x))/d] + E^(((6*I)*b*c)/d
)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((3*I)*b*(c + d*x))/d])))/(b*E^(((
3*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin(a+bx)\cos^2(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}\sqrt{c+dx}\sin(a+bx)+\frac{1}{4}\sqrt{c+dx}\sin(3a+3bx)\right)dx$$

$$\downarrow 2009$$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output

```
-1/4*(Sqrt[c + d*x]*Cos[a + b*x])/b - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(12
*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)
/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) - (
Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Si
n[3*a - (3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*S
qrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.97

method	result
derivativedivides	$-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d}{}$
default	$-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d}{}$

input

```
int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/16/b*d*2^(1/2)*
Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d))-1/24/b*d*(d*x+c)^(1/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c
)/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fr
esnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-
b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.77

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx$$
$$= \frac{\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+9\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6
)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c
- a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi
*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(
b*c - a*d)/d) - sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*sqrt(d*x + c)*b*cos(b*x + a
)^3)/b^2
```

Sympy [F]

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = \int \sqrt{c+dx}\sin(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.39

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{\left(\frac{24\sqrt{dx+c}b^2\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{72\sqrt{dx+c}b^2\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+\left((i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/288*(24*sqrt(d*x + c)*b^2*cos(3*((d*x + c)*b - b*c + a*d)/d)/d + 72*sqr
t(d*x + c)*b^2*cos(((d*x + c)*b - b*c + a*d)/d)/d + ((I - 1)*9^(1/4)*sqrt(
2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt
(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(3*I*b/d)) - 9*(-(I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c -
a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*
erf(sqrt(d*x + c)*sqrt(I*b/d)) - 9*((I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(
1/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(
-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(
2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt
(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(-3*I*b/d)))*d/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 844, normalized size of antiderivative = 2.78

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/144*(9*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*
(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*
I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*
c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)
*(2*b*c - I*d)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b) + sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*
(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*(3*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*I*s
qrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c -
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(2)*sqrt(pi)*d*er
f(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi
)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c + 18
*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*sqrt(d*x + c)
*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = \int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = \int \sqrt{dx+c}\cos\left(bx+a\right)^2\sin\left(bx+a\right)dx$$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**2*sin(a + b*x),x)
```

3.121 $\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 1132
Mathematica [C] (verified) . . . 1133
Rubi [A] (verified) . . . 1134
Maple [A] (verified) . . . 1135
Fricas [A] (verification not implemented) . . . 1136
Sympy [F] . . . 1136
Maxima [C] (verification not implemented) . . . 1137
Giac [C] (verification not implemented) . . . 1137
Mupad [F(-1)] . . . 1138
Reduce [F] . . . 1139

Optimal result

Integrand size = 24, antiderivative size = 304

$$\begin{aligned}
&\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx \\
&= -\frac{\sqrt{c+dx}\cos(a+bx)}{4b} - \frac{\sqrt{c+dx}\cos(3a+3bx)}{12b} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{12b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{4b^{3/2}}
\end{aligned}$$

output
```
-1/4*(d*x+c)^(1/2)*cos(b*x+a)/b-1/12*(d*x+c)^(1/2)*cos(3*b*x+3*a)/b+1/8*d^
(1/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x
+c)^(1/2)/d^(1/2))/b^(3/2)+1/72*d^(1/2)*6^(1/2)*Pi^(1/2)*cos(3*a-3*b*c/d)*
FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)-1/72*d^(1
/2)*6^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/
2))*sin(3*a-3*b*c/d)/b^(3/2)-1/8*d^(1/2)*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)
*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.83

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{e^{-\frac{3i(bc+ad)}{d}}\sqrt{c+dx}\left(9e^{2i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+9e^{2ia+\frac{4ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)+\sqrt{3}\left(e\right.\right.}{72b\sqrt{\frac{b^2(c+dx)^2}{d^2}}}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/72*(Sqrt[c + d*x]*(9*E^((2*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*
Gamma[3/2, ((-I)*b*(c + d*x))/d] + 9*E^((2*I)*a + ((4*I)*b*c)/d)*Sqrt[((-I
)*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d] + Sqrt[3]*(E^((6*I)*a)*Sqr
t[(I*b*(c + d*x))/d]*Gamma[3/2, ((-3*I)*b*(c + d*x))/d] + E^(((6*I)*b*c)/d
)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((3*I)*b*(c + d*x))/d])))/(b*E^(((
3*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin(a+bx)\cos^2(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}\sqrt{c+dx}\sin(a+bx)+\frac{1}{4}\sqrt{c+dx}\sin(3a+3bx)\right)dx$$

$$\downarrow 2009$$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output

```
-1/4*(Sqrt[c + d*x]*Cos[a + b*x])/b - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(12
*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)
/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) - (
Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Si
n[3*a - (3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*S
qrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.97

method	result
derivativedivides	$-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d}{}$
default	$-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d}{}$

input

```
int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/16/b*d*2^(1/2)*
Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d))-1/24/b*d*(d*x+c)^(1/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c
)/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fr
esnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-
b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.77

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx$$
$$= \frac{\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+9\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6
)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c
- a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi
*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(
b*c - a*d)/d) - sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*sqrt(d*x + c)*b*cos(b*x + a
)^3)/b^2
```

Sympy [F]

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = \int \sqrt{c+dx}\sin(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.39

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{\left(\frac{24\sqrt{dx+c}b^2\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{72\sqrt{dx+c}b^2\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+\left((i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/288*(24*sqrt(d*x + c)*b^2*cos(3*((d*x + c)*b - b*c + a*d)/d)/d + 72*sqr
t(d*x + c)*b^2*cos(((d*x + c)*b - b*c + a*d)/d)/d + ((I - 1)*9^(1/4)*sqrt(
2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt
(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(3*I*b/d)) - 9*(-(I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c -
a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*
erf(sqrt(d*x + c)*sqrt(I*b/d)) - 9*((I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(
1/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(
-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(
2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt
(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(-3*I*b/d)))*d/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 844, normalized size of antiderivative = 2.78

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/144*(9*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*
(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*
I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*
c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)
*(2*b*c - I*d)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b) + sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*
(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*(3*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*I*s
qrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c -
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(2)*sqrt(pi)*d*er
f(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi
)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c + 18
*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*sqrt(d*x + c)
*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = \int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = \int \sqrt{dx+c}\cos\left(bx+a\right)^2\sin\left(bx+a\right)dx$$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**2*sin(a + b*x),x)
```

3.122 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 1140
Mathematica [C] (verified) . . . 1141
Rubi [A] (verified) . . . 1142
Maple [A] (verified) . . . 1144
Fricas [A] (verification not implemented) . . . 1144
Sympy [F] . . . 1145
Maxima [C] (verification not implemented) . . . 1145
Giac [C] (verification not implemented) . . . 1146
Mupad [F(-1)] . . . 1147
Reduce [F] . . . 1148

Optimal result

Integrand size = 24, antiderivative size = 353

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} - \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}} - \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{24b^{5/2}} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{8b^{5/2}} + \frac{3d\sqrt{c+dx}\sin(a+bx)}{8b^2} + \frac{d\sqrt{c+dx}\sin(3a+3bx)}{24b^2}$$

output

```
-1/4*(d*x+c)^(3/2)*cos(b*x+a)/b-1/12*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b-3/16*d
^(3/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*
x+c)^(1/2)/d^(1/2))/b^(5/2)-1/144*d^(3/2)*6^(1/2)*Pi^(1/2)*cos(3*a-3*b*c/d
)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)-1/144*d
^(3/2)*6^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*sin(3*a-3*b*c/d)/b^(5/2)-3/16*d^(3/2)*2^(1/2)*Pi^(1/2)*FresnelC(b^(
1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(5/2)+3/8*d*(d
*x+c)^(1/2)*sin(b*x+a)/b^2+1/24*d*(d*x+c)^(1/2)*sin(3*b*x+3*a)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 751, normalized size of antiderivative = 2.13

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \frac{\sqrt{d}e^{-\frac{3i(ad+b(c+dx))}{d}}\left(-12\sqrt{b}\sqrt{d}e^{\frac{3ibc}{d}}\sqrt{c+dx}\left(-i+2bx+e^{6i(a+bx)}(i+2bx)\right)-(1-i)(2bc+id)e^{\frac{3i}{}}\right.}{576b^{5/2}}$$

$$+\frac{icde^{-\frac{i(bc+ad)}{d}}\left(-e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)\right)}{8b^2\sqrt{c+dx}}$$

$$+\frac{1}{4}c\left(-\frac{e^{3i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\Gamma\left(\frac{3}{2},-\frac{3ib(c+dx)}{d}\right)}{6\sqrt{3}b\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{-3i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\Gamma\left(\frac{3}{2},\frac{3ib(c+dx)}{d}\right)}{6\sqrt{3}b\sqrt{\frac{ib(c+dx)}{d}}}\right)$$

$$+\frac{\sqrt{d}\left(e^{i\left(a-\frac{bc}{d}\right)}\left(-2\sqrt{b}\sqrt{d}e^{\frac{ib(c+dx)}{d}}(3i+2bx)\sqrt{c+dx}+\sqrt[4]{-1}(2ibc+3d)\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)\right)+i\left(2\sqrt{b}\right.\right.}{}$$

input

```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output

```
(Sqrt[d]*(-12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(-I + 2*b*x
+ E^((6*I)*(a + b*x))*(I + 2*b*x)) - (1 - I)*(2*b*c + I*d)*E^(((3*I)*b*(2*
c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]] + (1 + I)*((2*I)*b*c + d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi]*Erfi[((1 +
 I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(576*b^(5/2)*E^(((3*I)*(a*
d + b*(c + d*x)))/d)) + ((I/8)*c*d*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/
d]*Gamma[3/2, ((-I)*b*(c + d*x))/d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*
x))/d]*Gamma[3/2, (I*b*(c + d*x))/d]))/(b^2*E^((I*(b*c + a*d))/d)*Sqrt[c +
 d*x]) + (c*(-1/6*(E^((3*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-3*I
)*b*(c + d*x))/d])/(Sqrt[3]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]
*Gamma[3/2, ((3*I)*b*(c + d*x))/d])/(6*Sqrt[3]*b*E^((3*I)*(a - (b*c)/d))*S
qrt[(I*b*(c + d*x))/d])))/4 + (Sqrt[d]*(E^(I*(a - (b*c)/d))*(-2*Sqrt[b]*Sq
rt[d]*E^((I*b*(c + d*x))/d)*(3*I + 2*b*x)*Sqrt[c + d*x] + (-1)^(1/4)*((2*I
)*b*c + 3*d)*Sqrt[Pi]*Erfi[((-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]) +
I*(2*Sqrt[b]*Sqrt[d]*(3 + (2*I)*b*x)*Sqrt[c + d*x] + (1 + I)*(2*b*c + (3*I
)*d)*Sqrt[Pi/2]*Erf[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[2]*Sqrt[d])]*(Co
s[b*(c/d + x)] + I*Sin[b*(c/d + x)]))*(Cos[a + b*x] - I*Sin[a + b*x])))/(3
2*b^(5/2))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin(a+bx)\cos^2(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}(c+dx)^{3/2}\sin(a+bx)+\frac{1}{4}(c+dx)^{3/2}\sin(3a+3bx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(a+bx)}{8b^2}+\frac{d\sqrt{c+dx}\sin(3a+3bx)}{24b^2}-\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/4*((c + d*x)^(3/2)*Cos[a + b*x])/b - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])
/(12*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (
3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/2
)) - (d^(3/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[
d]]*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(S
qrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) +
(3*d*Sqrt[c + d*x]*Sin[a + b*x])/(8*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*
x])/(24*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.09

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}d}\right)+\sin\right.\right.}{4b\sqrt{\frac{b}{d}}}\right.}{4b}$
default	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}d}\right)+\sin\right.\right.}{4b\sqrt{\frac{b}{d}}}\right.}{4b}$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+3/8/b*d*(1/2/b*d*
(d*x+c)^(1/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^
(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)))-1/24/b*d*(d*x+c)^(3/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/8/b*d*(
1/6/b*d*(d*x+c)^(1/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi
^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3
^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.79

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+27\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sq
rt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos
(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sq
rt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-(b*c - a*d)/d) + sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6
)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(2*(b^2*d*x + b
^2*c)*cos(b*x + a)^3 - (b*d*cos(b*x + a)^2 + 2*b*d)*sin(b*x + a))*sqrt(d*x
 + c))/b^3
```

Sympy [F]

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)*cos(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.41

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output

```
-1/576*(48*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d)/d + 144*
(d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d - 24*sqrt(d*x + c)*
b^2*sin(3*((d*x + c)*b - b*c + a*d)/d) - 216*sqrt(d*x + c)*b^2*sin(((d*x +
 c)*b - b*c + a*d)/d) - (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(
1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2
)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + 27*((I +
 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqr
t(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(I*b/d)) + 27*(-(I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c
 - a*d)/d) + (I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/
d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*
d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)
*b*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/
d)))*d/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 1515, normalized size of antiderivative = 4.29

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output

```
1/288*(12*(3*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt
(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*sq
rt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 - 4*(9*sqrt(2)*sqrt(pi)*(2
*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b)
 + sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*I*s
qrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*(6*b*c
 + I*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b)
 + 18*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*sqrt(d*x
 + c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int \cos(a+b\,x)^2\,\sin(a+b\,x)\,(c+d\,x)^{3/2}\,dx$$

input

```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(3/2),x)
```

output

```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c + dx)^{3/2} \cos^2(a+bx) \sin(a+bx)\, dx = \left(\int \sqrt{dx+c} \cos(bx+a)^2 \sin(bx+a)\, x dx \right) d + \left(\int \sqrt{dx+c} \cos(bx+a)^2 \sin(bx+a)\, dx \right) c$$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**2*sin(a + b*x)*x,x)*d + int(sqrt(c + d*x)*
cos(a + b*x)**2*sin(a + b*x),x)*c
```

3.123 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 1149
Mathematica [C] (verified) . . . 1150
Rubi [A] (verified) . . . 1151
Maple [A] (verified) . . . 1153
Fricas [A] (verification not implemented) . . . 1154
Sympy [F(-1)] . . . 1155
Maxima [C] (verification not implemented) . . . 1155
Giac [C] (verification not implemented) . . . 1156
Mupad [F(-1)] . . . 1157
Reduce [F] . . . 1157

Optimal result

Integrand size = 24, antiderivative size = 406

$$\begin{aligned}\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx &= \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{16b^3}\\ &- \frac{(c+dx)^{5/2}\cos(a+bx)}{4b} + \frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{144b^3}\\ &- \frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b} - \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}\\ &- \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}\\ &+ \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{144b^{7/2}}\\ &+ \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{16b^{7/2}}\\ &+ \frac{5d(c+dx)^{3/2}\sin(a+bx)}{8b^2} + \frac{5d(c+dx)^{3/2}\sin(3a+3bx)}{72b^2}\end{aligned}$$

output

```
15/16*d^2*(d*x+c)^(1/2)*cos(b*x+a)/b^3-1/4*(d*x+c)^(5/2)*cos(b*x+a)/b+5/14
4*d^2*(d*x+c)^(1/2)*cos(3*b*x+3*a)/b^3-1/12*(d*x+c)^(5/2)*cos(3*b*x+3*a)/b
-15/32*d^(5/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(
1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-5/864*d^(5/2)*6^(1/2)*Pi^(1/2)*cos(3*a
-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)
+5/864*d^(5/2)*6^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*sin(3*a-3*b*c/d)/b^(7/2)+15/32*d^(5/2)*2^(1/2)*Pi^(1/2)*Fre
snelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(7/2)
+5/8*d*(d*x+c)^(3/2)*sin(b*x+a)/b^2+5/72*d*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b^
2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.01 (sec) , antiderivative size = 1445, normalized size of antiderivative = 3.56

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input

```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output

```
(c*Sqrt[d]*(-12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(-I + 2*b*
x + E^((6*I)*(a + b*x))*(I + 2*b*x)) - (1 - I)*(2*b*c + I*d)*E^(((3*I)*b*(
2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sq
rt[d]] + (1 + I)*((2*I)*b*c + d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi]*Erfi[((1
 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(288*b^(5/2)*E^(((3*I)*(
a*d + b*(c + d*x)))/d)) + ((I/8)*c^2*d*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*
x))/d]*Gamma[3/2, ((-I)*b*(c + d*x))/d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c
+ d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d]))/(b^2*E^((I*(b*c + a*d))/d)*Sqrt
[c + d*x]) + (c^2*(-1/6*(E^((3*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2,
((-3*I)*b*(c + d*x))/d])/(Sqrt[3]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c
+ d*x]*Gamma[3/2, ((3*I)*b*(c + d*x))/d])/(6*Sqrt[3]*b*E^((3*I)*(a - (b*c)
/d))*Sqrt[(I*b*(c + d*x))/d])))/4 + (c*Sqrt[d]*(E^(I*(a - (b*c)/d))*(-2*Sq
rt[b]*Sqrt[d]*E^((I*b*(c + d*x))/d)*(3*I + 2*b*x)*Sqrt[c + d*x] + (-1)^(1/
4)*((2*I)*b*c + 3*d)*Sqrt[Pi]*Erfi[((-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]]) + I*(2*Sqrt[b]*Sqrt[d]*(3 + (2*I)*b*x)*Sqrt[c + d*x] + (1 + I)*(2*b*
c + (3*I)*d)*Sqrt[Pi/2]*Erf[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[2]*Sqrt[
d])]*(Cos[b*(c/d + x)] + I*Sin[b*(c/d + x)]))*(Cos[a + b*x] - I*Sin[a + b*
x])))/(16*b^(5/2)) - ((I/64)*Sqrt[d]*((Cos[a - (b*c)/d] + I*Sin[a - (b*c)/
d])*((1 + I)*(4*b^2*c^2 - (12*I)*b*c*d - 15*d^2)*Sqrt[Pi/2]*Erfi[((1 + I)*
Sqrt[b]*Sqrt[c + d*x])/(Sqrt[2]*Sqrt[d])] + 2*Sqrt[b]*Sqrt[d]*Sqrt[c + ...
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2} \sin(a+bx) \cos^2(a+bx)\, dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}(c+dx)^{5/2} \sin(a+bx) + \frac{1}{4}(c+dx)^{5/2} \sin(3a+3bx) \right) dx$$

$$\downarrow 2009$$

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{16b^3}+\frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{144b^3}+\frac{5d(c+dx)^{3/2}\sin(a+bx)}{8b^2}+\frac{5d(c+dx)^{3/2}\sin(3a+3bx)}{72b^2}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}$$

input

```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output

```
(15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^3) - ((c + d*x)^(5/2)*Cos[a + b*
x])/(4*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(144*b^3) - ((c + d*x)^
(5/2)*Cos[3*a + 3*b*x])/(12*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*F
resnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(
5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(144*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (1
5*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*
Sin[a - (b*c)/d])/(16*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[a + b*x])/(8*b^2
) + (5*d*(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(72*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.17

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\sqrt{\pi}}{}\right.}\right.}{4b}$
default	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\sqrt{\pi}}{}\right.}\right.}{4b}$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+5/8/b*d*(1/2/b*d*
(d*x+c)^(3/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(1/2)
*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a
*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin((a
*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1/2
4/b*d*(d*x+c)^(5/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+5/24/b*d*(1/6/b*d*(d*
x+c)^(3/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/2
)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)
^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b
*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/
d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.84

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{5\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right.$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(
sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 40
5*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(
pi*d)))*sin(-(b*c - a*d)/d) - 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(
sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(30*b*d^2
*cos(b*x + a) - (12*b^3*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos
(b*x + a)^3 + 10*(2*b^2*d^2*x + 2*b^2*c*d + (b^2*d^2*x + b^2*c*d)*cos(b*x
+ a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.35

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/3456*(240*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d) + 2160*
(d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d) - 24*(12*(d*x + c)^(5
/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*cos(3*((d*x + c)*b - b*c + a*d)/d) - 21
6*(4*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(((d*x + c)*b - b*
c + a*d)/d) - 5*(-(I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*c
os(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1
/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 405*(-(I - 1
)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqr
t(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)
*sqrt(I*b/d)) - 405*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(
b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c -
 a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - 5*((I + 1)*9^(1/4)*sqrt(2)*sqr
t(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2
)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*
sqrt(-3*I*b/d)))*d/b^5
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 2445, normalized size of antiderivative = 6.02

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
1/1728*(72*(3*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d
*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(
I*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d
)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 - d^3*(27*(sqrt(2)*sqrt(p
i)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3)*d*erf(1/2*I*sqrt(2
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)*
b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d + 10*(
d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*
e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + (sqrt(6)*sqrt(pi)*(72*b^
3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*I*sqrt(6)*sqrt(b
*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(s
qrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^(5/2)*b^2*d
- 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x +
c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^(-...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int \cos(a+b\,x)^2\,\sin(a+b\,x)\,(c+d\,x)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \left(\int \sqrt{dx+c}\,\cos(bx+a)^2\sin(bx+a)\,x^2dx\right)d^2$$
$$+2\left(\int \sqrt{dx+c}\,\cos(bx+a)^2\sin(bx+a)\,xdx\right)cd$$
$$+\left(\int \sqrt{dx+c}\,\cos(bx+a)^2\sin(bx+a)\,dx\right)c^2$$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**2*sin(a + b*x)*x**2,x)*d**2 + 2*int(sqrt(c
 + d*x)*cos(a + b*x)**2*sin(a + b*x)*x,x)*c*d + int(sqrt(c + d*x)*cos(a +
b*x)**2*sin(a + b*x),x)*c**2
```

3.124 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal result . . . 1158
Mathematica [C] (verified) . . . 1159
Rubi [A] (verified) . . . 1159
Maple [A] (verified) . . . 1160
Fricas [A] (verification not implemented) . . . 1161
Sympy [F(-1)] . . . 1162
Maxima [C] (verification not implemented) . . . 1162
Giac [C] (verification not implemented) . . . 1163
Mupad [F(-1)] . . . 1164
Reduce [F] . . . 1165

Optimal result

Integrand size = 26, antiderivative size = 228

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} - \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} - \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{4096b^{7/2}} + \frac{15d^2\sqrt{c+dx}\sin(4a+4bx)}{2048b^3} - \frac{(c+dx)^{5/2}\sin(4a+4bx)}{32b}$$

output

```
1/28*(d*x+c)^(7/2)/d-5/256*d*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b^2-15/8192*d^(5
/2)*2^(1/2)*Pi^(1/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*
(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-15/8192*d^(5/2)*2^(1/2)*Pi^(1/2)*FresnelC(2
*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)/b^(7/2)+
15/2048*d^2*(d*x+c)^(1/2)*sin(4*b*x+4*a)/b^3-1/32*(d*x+c)^(5/2)*sin(4*b*x+
4*a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.61

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{512(c+dx)^4 + \frac{7d^4 e^{4i\left(a-\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{4ib(c+dx)}{d}\right)}{b^4} + \frac{7d^4 e^{-4i\left(a-\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{4ib(c+dx)}{d}\right)}{b^4}}{14336d\sqrt{c+dx}}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(512*(c + d*x)^4 + (7*d^4*E^((4*I)*(a - (b*c)/d))*Sqrt[((-I)*b*(c + d*x))/
d]*Gamma[7/2, ((-4*I)*b*(c + d*x))/d])/b^4 + (7*d^4*Sqrt[(I*b*(c + d*x))/d
]*Gamma[7/2, ((4*I)*b*(c + d*x))/d])/(b^4*E^((4*I)*(a - (b*c)/d))))/(14336
*d*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin^2(a+bx)\cos^2(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8}(c+dx)^{5/2} - \frac{1}{8}(c+dx)^{5/2}\cos(4a+4bx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-$$
$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15d^2\sqrt{c+dx}\sin(4a+4bx)}{2048b^3}-$$
$$\frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2}-\frac{(c+dx)^{5/2}\sin(4a+4bx)}{32b}+\frac{(c+dx)^{7/2}}{28d}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^(7/2)/(28*d) - (5*d*(c + d*x)^(3/2)*Cos[4*a + 4*b*x])/(256*b^2)
- (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/P
i]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*Fresne
lC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(40
96*b^(7/2)) + (15*d^2*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(2048*b^3) - ((c + d
*x)^(5/2)*Sin[4*a + 4*b*x])/(32*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{(dx+c)^{\frac{7}{2}}}{28}-\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{4b(dx+c)}{d}+\frac{4}{8b}\right.\right.\right.\right.}{d}$
default	$\frac{(dx+c)^{\frac{7}{2}}}{28}-\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{4b(dx+c)}{d}+\frac{4}{8b}\right.\right.\right.\right.}{d}$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/56*(d*x+c)^(7/2)-1/64/b*d*(d*x+c)^(5/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*
c)/d)+5/64/b*d*(-1/8/b*d*(d*x+c)^(3/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+3/
8/b*d*(1/8/b*d*(d*x+c)^(1/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-1/32/b*d*2^(
1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/
(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1
/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.52

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{105\sqrt{2}\pi d^4\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+105\sqrt{2}\pi d^4\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/57344*(105*sqrt(2)*pi*d^4*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_
sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 105*sqrt(2)*pi*d^4*sqrt(b/(p
i*d))*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*
d)/d) - 16*(128*b^4*d^3*x^3 + 384*b^4*c*d^2*x^2 + 128*b^4*c^3 - 70*b^2*c*d
^2 - 560*(b^2*d^3*x + b^2*c*d^2)*cos(b*x + a)^4 + 560*(b^2*d^3*x + b^2*c*d
^2)*cos(b*x + a)^2 + 2*(192*b^4*c^2*d - 35*b^2*d^3)*x - 7*(2*(64*b^3*d^3*x
^2 + 128*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a)^3 - (64*b^3*d
^3*x^2 + 128*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a))*sin(b*x
+ a))*sqrt(d*x + c))/(b^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.25

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{\sqrt{2}\left(\frac{4096\sqrt{2}(dx+c)^{\frac{7}{2}}b^4}{d} - 2240\sqrt{2}(dx+c)^{\frac{3}{2}}b^2 d\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right) + 105\left(-(i+1)\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\right.\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output

```
1/229376*sqrt(2)*(4096*sqrt(2)*(d*x + c)^(7/2)*b^4/d - 2240*sqrt(2)*(d*x +
 c)^(3/2)*b^2*d*cos(4*((d*x + c)*b - b*c + a*d)/d) + 105*(-(I + 1)*sqrt(pi
)*d^3*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(pi)*d^3*(b^2/d^
2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + 105*((I
 - 1)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I + 1)*sqrt(pi
)*d^3*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b
/d)) - 56*(64*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x + c)*b*d^2
)*sin(4*((d*x + c)*b - b*c + a*d)/d))/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 1358, normalized size of antiderivative = 5.96

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output

```
-1/573440*(17920*(I*sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 8*sqrt(d*x + c))*c^3 - d^3*(4096*(5*(d*x +
 c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x +
c)*c^3)/d^3 - 35*(-I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 - 192*I*b^2*c^2*d - 72*
b*c*d^2 + 15*I*d^3)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
 1)*b^3) + 4*(-64*I*(d*x + c)^(5/2)*b^2*d + 192*I*(d*x + c)^(3/2)*b^2*c*d
- 192*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x +
 c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*
d)/d)/b^3)/d^3 - 35*(I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 + 192*I*b^2*c^2*d - 7
2*b*c*d^2 - 15*I*d^3)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt
(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^
2) + 1)*b^3) + 4*(64*I*(d*x + c)^(5/2)*b^2*d - 192*I*(d*x + c)^(3/2)*b^2*c
*d + 192*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*
x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-4*(I*(d*x + c)*b - I*b*c + I*
a*d)/d)/b^3)/d^3) + 2240*(-3*I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-I*sqr
t(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c ...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)^2\sin(a+b\,x)^2\,(c+d\,x)^{5/2}\,dx$$

input

```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

output

```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \left(\int \sqrt{dx+c}\cos(bx+a)^2\sin(bx+a)^2 x^2 dx\right)d^2 + 2\left(\int \sqrt{dx+c}\cos(bx+a)^2\sin(bx+a)^2 x\,dx\right)cd + \left(\int \sqrt{dx+c}\cos(bx+a)^2\sin(bx+a)^2\,dx\right)c^2$$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**2*sin(a + b*x)**2*x**2,x)*d**2 + 2*int(sqr
t(c + d*x)*cos(a + b*x)**2*sin(a + b*x)**2*x,x)*c*d + int(sqrt(c + d*x)*co
s(a + b*x)**2*sin(a + b*x)**2,x)*c**2
```

3.125 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal result . . . 1166
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Optimal result

Integrand size = 26, antiderivative size = 200

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{512b^{5/2}} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b}$$

output

```
1/20*(d*x+c)^(5/2)/d-3/256*d*(d*x+c)^(1/2)*cos(4*b*x+4*a)/b^2+3/1024*d^(3/
2)*2^(1/2)*Pi^(1/2)*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))/b^(5/2)-3/1024*d^(3/2)*2^(1/2)*Pi^(1/2)*FresnelS(2*b
^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)/b^(5/2)-1/
32*(d*x+c)^(3/2)*sin(4*b*x+4*a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{\sqrt{c+dx}\left(128(c+dx)^2 - \frac{5d^2 e^{4i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{5}{2},-\frac{4ib(c+dx)}{d}\right)}{b^2\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{5d^2 e^{-4i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{5}{2},\frac{4ib(c+dx)}{d}\right)}{b^2\sqrt{\frac{ib(c+dx)}{d}}}\right)}{2560d}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(Sqrt[c + d*x]*(128*(c + d*x)^2 - (5*d^2*E^((4*I)*(a - (b*c)/d))*Gamma[5/2
, ((-4*I)*b*(c + d*x))/d])/(b^2*Sqrt[((-I)*b*(c + d*x))/d]) - (5*d^2*Gamma
[5/2, ((4*I)*b*(c + d*x))/d])/(b^2*E^((4*I)*(a - (b*c)/d))*Sqrt[(I*b*(c +
d*x))/d])))/(2560*d)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin^2(a+bx)\cos^2(a+bx)\,dx$$

$$\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)^{3/2} - \frac{1}{8}(c+dx)^{3/2}\cos(4a+4bx)\right)dx$$

$$\downarrow \text{2009}$$

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b} + \frac{(c+dx)^{5/2}}{20d}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^(5/2)/(20*d) - (3*d*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(256*b^2) +
(3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(2
*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(
5/2)) - ((c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(32*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{(dx+c)^{\frac{5}{2}}}{20}-\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{4ad-4cb}{d}\right)\text{Fresnel}\right.}{32b}\right.}{d}$
default	$\frac{(dx+c)^{\frac{5}{2}}}{20}-\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{4ad-4cb}{d}\right)\text{Fresnel}\right.}{32b}\right.}{d}$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/40*(d*x+c)^(5/2)-1/64/b*d*(d*x+c)^(3/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*
c)/d)+3/64/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+1/
32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)
/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(
1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.24

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)s$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/5120*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d)
)*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d
) + 4*(64*b^3*d^2*x^2 - 120*b*d^2*cos(b*x + a)^4 + 128*b^3*c*d*x + 64*b^3*
c^2 + 120*b*d^2*cos(b*x + a)^2 - 15*b*d^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*c
os(b*x + a)^3 - (b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x
 + c))/(b^3*d)
```

Sympy [F]

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin^2(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.32

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{\sqrt{2}\left(\frac{512\sqrt{2}(dx+c)^{\frac{5}{2}}b^3}{d} - 320\sqrt{2}(dx+c)^{\frac{3}{2}}b^2\sin\left(\frac{4((dx+c)b-bc+ad)}{d}\right) - 120\sqrt{2}\sqrt{dx+c}bd\cos\left(\frac{4((dx+c)}{d}\right.\right.$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output

```
1/20480*sqrt(2)*(512*sqrt(2)*(d*x + c)^(5/2)*b^3/d - 320*sqrt(2)*(d*x + c)
^(3/2)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) - 120*sqrt(2)*sqrt(d*x + c)*
b*d*cos(4*((d*x + c)*b - b*c + a*d)/d) + 15*(-(I - 1)*sqrt(pi)*d^2*(b^2/d^
2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I + 1)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(
-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + 15*((I + 1)*sqrt(pi)
*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(pi)*d^2*(b^2/d^2
)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 833, normalized size of antiderivative = 4.16

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output

```
-1/30720*(960*(I*sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)) - 8*sqrt(d*x + c))*c^2 - 1536*(d*x + c)^(5/2) +
 5120*(d*x + c)^(3/2)*c - 7680*sqrt(d*x + c)*c^2 + 80*(-3*I*sqrt(2)*sqrt(p
i)*(8*b*c - I*d)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b) + 3*I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*
x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*
(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 64*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c +
 12*I*sqrt(d*x + c)*d*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 12*I*sq
rt(d*x + c)*d*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c + 15*I*sqrt(2
)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(2)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 15*I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16
*I*b*c*d - 3*d^2)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)*b^2) + 60*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt
(d*x + c)*d^2)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2 + 60*(-8*I*...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^2\left(c+d\,x\right)^{3/2}dx$$

input

```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

output

```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c + dx)^{3/2} \cos^2(a+bx) \sin^2(a+bx)\, dx = \left(\int \sqrt{dx+c} \cos(bx+a)^2 \sin(bx+a)^2 x dx \right) d + \left(\int \sqrt{dx+c} \cos(bx+a)^2 \sin(bx+a)^2 dx \right) c$$

input

```
int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

output

```
int(sqrt(c + d*x)*cos(a + b*x)**2*sin(a + b*x)**2*x,x)*d + int(sqrt(c + d*
x)*cos(a + b*x)**2*sin(a + b*x)**2,x)*c
```

3.126 $\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal result . . . 1174
Mathematica [C] (verified) . . . 1175
Rubi [A] (verified) . . . 1175
Maple [A] (verified) . . . 1176
Fricas [A] (verification not implemented) . . . 1177
Sympy [F] . . . 1177
Maxima [C] (verification not implemented) . . . 1178
Giac [C] (verification not implemented) . . . 1178
Mupad [F(-1)] . . . 1179
Reduce [F] . . . 1179

Optimal result

Integrand size = 26, antiderivative size = 174

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{(c+dx)^{3/2}}{12d} + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}$$
$$+ \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{64b^{3/2}} - \frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}$$

output

```
1/12*(d*x+c)^(3/2)/d+1/128*d^(1/2)*2^(1/2)*Pi^(1/2)*cos(4*a-4*b*c/d)*Fresn
elS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+1/128*d^(1/2
)*2^(1/2)*Pi^(1/2)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/
2))*sin(4*a-4*b*c/d)/b^(3/2)-1/32*(d*x+c)^(1/2)*sin(4*b*x+4*a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.69

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{(c+dx)^{3/2}\left(32+\frac{3e^{4i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{3}{2},-\frac{4ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{3/2}}+\frac{3e^{-4i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{3}{2},\frac{4ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{3/2}}\right)}{384d}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
((c + d*x)^(3/2)*(32 + (3*E^((4*I)*(a - (b*c)/d))*Gamma[3/2, ((-4*I)*b*(c
+ d*x))/d])/(((-I)*b*(c + d*x))/d)^(3/2) + (3*Gamma[3/2, ((4*I)*b*(c + d*x
))/d])/(E^((4*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^(3/2))))/(384*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^2(a+bx)\cos^2(a+bx)\,dx$$
$$\downarrow \text{4906}$$
$$\int \left(\frac{1}{8}\sqrt{c+dx}-\frac{1}{8}\sqrt{c+dx}\cos(4a+4bx)\right)dx$$
$$\downarrow \text{2009}$$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+$$
$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{(c+dx)^{3/2}}{12d}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^(3/2)/(12*d) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS
[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sq
rt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a -
(4*b*c)/d])/(64*b^(3/2)) - (Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(32*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{(dx+c)^{\frac{3}{2}}}{12}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4ad-4cb}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{4ad-4cb}{d}\right)\text{FresnelC}\left(\frac{2}{}\right.\right.}{128b\sqrt{\frac{b}{d}}}}{d}$
default	$\frac{(dx+c)^{\frac{3}{2}}}{12}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4ad-4cb}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{4ad-4cb}{d}\right)\text{FresnelC}\left(\frac{2}{}\right.\right.}{128b\sqrt{\frac{b}{d}}}}{d}$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/24*(d*x+c)^(3/2)-1/64/b*d*(d*x+c)^(1/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*
c)/d)+1/256/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(
2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*Fresn
elC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.01

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-}{}\right.}{384\,b^2d}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/384*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2
*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*f
resnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) +
 16*(2*b^2*d*x + 2*b^2*c - 3*(2*b*d*cos(b*x + a)^3 - b*d*cos(b*x + a))*sin
(b*x + a))*sqrt(d*x + c))/(b^2*d)
```

Sympy [F]

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \sqrt{c+dx}\sin^2(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.26

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{\sqrt{2}\left(\frac{64\sqrt{2}(dx+c)^{\frac{3}{2}}b^2}{d} - 24\sqrt{2}\sqrt{dx+c}b\sin\left(\frac{4((dx+c)b-bc+ad)}{d}\right) + 3\left((i+1)\sqrt{\pi}d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right) - (i\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/1536*sqrt(2)*(64*sqrt(2)*(d*x + c)^(3/2)*b^2/d - 24*sqrt(2)*sqrt(d*x + c
)*b*sin(4*((d*x + c)*b - b*c + a*d)/d) + 3*((I + 1)*sqrt(pi)*d*(b^2/d^2)^(
1/4)*cos(-4*(b*c - a*d)/d) - (I - 1)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-4*(b*
c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + 3*(-(I - 1)*sqrt(pi)*d*(b^
2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I + 1)*sqrt(pi)*d*(b^2/d^2)^(1/4)*si
n(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/b^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.64

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{3i\sqrt{2}\sqrt{\pi}(8bc-id)d\operatorname{erf}\left(-\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{4(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b} + \frac{3i\sqrt{2}\sqrt{\pi}(8bc+id)d\operatorname{erf}\left(\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\right.}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}$$
$$-$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output

```
-1/768*(-3*I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqr
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 3*I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(
I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*
b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 24*(I*sqrt(2)*s
qrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*sqr
t(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1))
 - 8*sqrt(d*x + c))*c - 64*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c + 12*I*sq
rt(d*x + c)*d*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 12*I*sqrt(d*x +
 c)*d*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)^2\sin(a+b\,x)^2\sqrt{c+d\,x}\,dx$$

input

```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

output

```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \sqrt{dx+c}\cos(bx+a)^2\sin(bx+a)^2\,dx$$

input

```
int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

output

```
int(sqrt(c + d*x)*cos(a + b*x)**2*sin(a + b*x)**2,x)
```

3.127 $\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal result . . . 1180
Mathematica [C] (verified) . . . 1181
Rubi [A] (verified) . . . 1181
Maple [A] (verified) . . . 1182
Fricas [A] (verification not implemented) . . . 1183
Sympy [F] . . . 1183
Maxima [C] (verification not implemented) . . . 1184
Giac [C] (verification not implemented) . . . 1184
Mupad [F(-1)] . . . 1185
Reduce [F] . . . 1185

Optimal result

Integrand size = 26, antiderivative size = 174

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{(c+dx)^{3/2}}{12d} + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}$$
$$+ \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{64b^{3/2}} - \frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}$$

output

```
1/12*(d*x+c)^(3/2)/d+1/128*d^(1/2)*2^(1/2)*Pi^(1/2)*cos(4*a-4*b*c/d)*Fresn
elS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+1/128*d^(1/2
)*2^(1/2)*Pi^(1/2)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/
2))*sin(4*a-4*b*c/d)/b^(3/2)-1/32*(d*x+c)^(1/2)*sin(4*b*x+4*a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.69

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{(c+dx)^{3/2}\left(32+\frac{3e^{4i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{3}{2},-\frac{4ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{3/2}}+\frac{3e^{-4i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{3}{2},\frac{4ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{3/2}}\right)}{384d}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
((c + d*x)^(3/2)*(32 + (3*E^((4*I)*(a - (b*c)/d))*Gamma[3/2, ((-4*I)*b*(c
+ d*x))/d])/(((-I)*b*(c + d*x))/d)^(3/2) + (3*Gamma[3/2, ((4*I)*b*(c + d*x
))/d])/(E^((4*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^(3/2))))/(384*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^2(a+bx)\cos^2(a+bx)\,dx$$
$$\downarrow 4906$$
$$\int \left(\frac{1}{8}\sqrt{c+dx}-\frac{1}{8}\sqrt{c+dx}\cos(4a+4bx)\right)dx$$
$$\downarrow 2009$$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+$$
$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{(c+dx)^{3/2}}{12d}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^(3/2)/(12*d) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS
[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sq
rt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a -
(4*b*c)/d])/(64*b^(3/2)) - (Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(32*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{(dx+c)^{\frac{3}{2}}}{12}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4ad-4cb}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{4ad-4cb}{d}\right)\text{FresnelC}\left(\frac{2}{}\right.\right.}{128b\sqrt{\frac{b}{d}}}}{d}$
default	$\frac{(dx+c)^{\frac{3}{2}}}{12}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4ad-4cb}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{4ad-4cb}{d}\right)\text{FresnelC}\left(\frac{2}{}\right.\right.}{128b\sqrt{\frac{b}{d}}}}{d}$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/24*(d*x+c)^(3/2)-1/64/b*d*(d*x+c)^(1/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*
c)/d)+1/256/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(
2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*Fresn
elC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.01

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-}{384\,b^2d}\right.$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/384*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2
*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*f
resnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) +
 16*(2*b^2*d*x + 2*b^2*c - 3*(2*b*d*cos(b*x + a)^3 - b*d*cos(b*x + a))*sin
(b*x + a))*sqrt(d*x + c))/(b^2*d)
```

Sympy [F]

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \sqrt{c+dx}\sin^2(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.26

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{\sqrt{2}\left(\frac{64\sqrt{2}(dx+c)^{\frac{3}{2}}b^2}{d} - 24\sqrt{2}\sqrt{dx+c}b\sin\left(\frac{4((dx+c)b-bc+ad)}{d}\right) + 3\left((i+1)\sqrt{\pi}d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right) - (i\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/1536*sqrt(2)*(64*sqrt(2)*(d*x + c)^(3/2)*b^2/d - 24*sqrt(2)*sqrt(d*x + c
)*b*sin(4*((d*x + c)*b - b*c + a*d)/d) + 3*((I + 1)*sqrt(pi)*d*(b^2/d^2)^(
1/4)*cos(-4*(b*c - a*d)/d) - (I - 1)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-4*(b*
c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + 3*(-(I - 1)*sqrt(pi)*d*(b^
2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I + 1)*sqrt(pi)*d*(b^2/d^2)^(1/4)*si
n(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/b^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.64

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{3i\sqrt{2}\sqrt{\pi}(8bc-id)d\,\mathrm{erf}\left(-\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{4(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b} + \frac{3i\sqrt{2}\sqrt{\pi}(8bc+id)d\,\mathrm{erf}\left(\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\right.}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}$$
$$-$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output

```
-1/768*(-3*I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqr
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 3*I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(
I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*
b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 24*(I*sqrt(2)*s
qrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*sqr
t(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1))
 - 8*sqrt(d*x + c))*c - 64*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c + 12*I*sq
rt(d*x + c)*d*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 12*I*sqrt(d*x +
 c)*d*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)^2\sin(a+b\,x)^2\sqrt{c+d\,x}\,dx$$

input

```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

output

```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \sqrt{dx+c}\cos(bx+a)^2\sin(bx+a)^2\,dx$$

input

```
int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

output

```
int(sqrt(c + d*x)*cos(a + b*x)**2*sin(a + b*x)**2,x)
```

3.128 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal result . . . 1186
Mathematica [C] (verified) . . . 1187
Rubi [A] (verified) . . . 1187
Maple [A] (verified) . . . 1189
Fricas [A] (verification not implemented) . . . 1189
Sympy [F] . . . 1190
Maxima [C] (verification not implemented) . . . 1190
Giac [C] (verification not implemented) . . . 1191
Mupad [F(-1)] . . . 1192
Reduce [F] . . . 1193

Optimal result

Integrand size = 26, antiderivative size = 200

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{512b^{5/2}} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b}$$

output

```
1/20*(d*x+c)^(5/2)/d-3/256*d*(d*x+c)^(1/2)*cos(4*b*x+4*a)/b^2+3/1024*d^(3/
2)*2^(1/2)*Pi^(1/2)*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))/b^(5/2)-3/1024*d^(3/2)*2^(1/2)*Pi^(1/2)*FresnelS(2*b
^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)/b^(5/2)-1/
32*(d*x+c)^(3/2)*sin(4*b*x+4*a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{\sqrt{c+dx}\left(128(c+dx)^2 - \frac{5d^2 e^{4i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{5}{2},-\frac{4ib(c+dx)}{d}\right)}{b^2\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{5d^2 e^{-4i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{5}{2},\frac{4ib(c+dx)}{d}\right)}{b^2\sqrt{\frac{ib(c+dx)}{d}}}\right)}{2560d}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(Sqrt[c + d*x]*(128*(c + d*x)^2 - (5*d^2*E^((4*I)*(a - (b*c)/d))*Gamma[5/2
, ((-4*I)*b*(c + d*x))/d])/(b^2*Sqrt[((-I)*b*(c + d*x))/d]) - (5*d^2*Gamma
[5/2, ((4*I)*b*(c + d*x))/d])/(b^2*E^((4*I)*(a - (b*c)/d))*Sqrt[(I*b*(c +
d*x))/d])))/(2560*d)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin^2(a+bx)\cos^2(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8}(c+dx)^{3/2} - \frac{1}{8}(c+dx)^{3/2}\cos(4a+4bx)\right)dx$$

$$\downarrow 2009$$

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b} + \frac{(c+dx)^{5/2}}{20d}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^(5/2)/(20*d) - (3*d*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(256*b^2) +
(3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(2
*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(
5/2)) - ((c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(32*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{(dx+c)^{\frac{5}{2}}}{20}-\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{4ad-4cb}{d}\right)\text{Fresnel}\right.}{32b}\right.}{d}$
default	$\frac{(dx+c)^{\frac{5}{2}}}{20}-\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{4ad-4cb}{d}\right)\text{Fresnel}\right.}{32b}\right.}{d}$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/40*(d*x+c)^(5/2)-1/64/b*d*(d*x+c)^(3/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*
c)/d)+3/64/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+1/
32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)
/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(
1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.24

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)s$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/5120*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d)
)*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d
) + 4*(64*b^3*d^2*x^2 - 120*b*d^2*cos(b*x + a)^4 + 128*b^3*c*d*x + 64*b^3*
c^2 + 120*b*d^2*cos(b*x + a)^2 - 15*b*d^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*c
os(b*x + a)^3 - (b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x
 + c))/(b^3*d)
```

Sympy [F]

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin^2(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.32

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{\sqrt{2}\left(\frac{512\sqrt{2}(dx+c)^{\frac{5}{2}}b^3}{d} - 320\sqrt{2}(dx+c)^{\frac{3}{2}}b^2\sin\left(\frac{4((dx+c)b-bc+ad)}{d}\right) - 120\sqrt{2}\sqrt{dx+c}bd\cos\left(\frac{4((dx+c)}{d}\right.\right.$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/20480*sqrt(2)*(512*sqrt(2)*(d*x + c)^(5/2)*b^3/d - 320*sqrt(2)*(d*x + c)
^(3/2)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) - 120*sqrt(2)*sqrt(d*x + c)*
b*d*cos(4*((d*x + c)*b - b*c + a*d)/d) + 15*(-(I - 1)*sqrt(pi)*d^2*(b^2/d^
2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I + 1)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(
-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + 15*((I + 1)*sqrt(pi)
*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(pi)*d^2*(b^2/d^2
)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 833, normalized size of antiderivative = 4.16

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output

```
-1/30720*(960*(I*sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)) - 8*sqrt(d*x + c))*c^2 - 1536*(d*x + c)^(5/2) +
 5120*(d*x + c)^(3/2)*c - 7680*sqrt(d*x + c)*c^2 + 80*(-3*I*sqrt(2)*sqrt(p
i)*(8*b*c - I*d)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b) + 3*I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*
x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*
(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 64*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c +
 12*I*sqrt(d*x + c)*d*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 12*I*sq
rt(d*x + c)*d*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c + 15*I*sqrt(2
)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(2)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 15*I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16
*I*b*c*d - 3*d^2)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)*b^2) + 60*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt
(d*x + c)*d^2)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2 + 60*(-8*I*...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^2\left(c+d\,x\right)^{3/2}dx$$

input

```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

output

```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c + dx)^{3/2} \cos^2(a+bx) \sin^2(a+bx)\, dx = \left(\int \sqrt{dx+c} \cos(bx+a)^2 \sin(bx+a)^2 x dx \right) d + \left(\int \sqrt{dx+c} \cos(bx+a)^2 \sin(bx+a)^2 dx \right) c$$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**2*sin(a + b*x)**2*x,x)*d + int(sqrt(c + d*
x)*cos(a + b*x)**2*sin(a + b*x)**2,x)*c
```

3.129 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal result . . . 1194
Mathematica [C] (verified) . . . 1195
Rubi [A] (verified) . . . 1195
Maple [A] (verified) . . . 1196
Fricas [A] (verification not implemented) . . . 1197
Sympy [F(-1)] . . . 1198
Maxima [C] (verification not implemented) . . . 1198
Giac [C] (verification not implemented) . . . 1199
Mupad [F(-1)] . . . 1200
Reduce [F] . . . 1201

Optimal result

Integrand size = 26, antiderivative size = 228

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} - \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} - \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{4096b^{7/2}} + \frac{15d^2\sqrt{c+dx}\sin(4a+4bx)}{2048b^3} - \frac{(c+dx)^{5/2}\sin(4a+4bx)}{32b}$$

output

```
1/28*(d*x+c)^(7/2)/d-5/256*d*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b^2-15/8192*d^(5
/2)*2^(1/2)*Pi^(1/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*
(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-15/8192*d^(5/2)*2^(1/2)*Pi^(1/2)*FresnelC(2
*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)/b^(7/2)+
15/2048*d^2*(d*x+c)^(1/2)*sin(4*b*x+4*a)/b^3-1/32*(d*x+c)^(5/2)*sin(4*b*x+
4*a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.61

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{512(c+dx)^4 + \frac{7d^4 e^{4i\left(a-\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{4ib(c+dx)}{d}\right)}{b^4} + \frac{7d^4 e^{-4i\left(a-\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{4ib(c+dx)}{d}\right)}{b^4}}{14336d\sqrt{c+dx}}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(512*(c + d*x)^4 + (7*d^4*E^((4*I)*(a - (b*c)/d))*Sqrt[((-I)*b*(c + d*x))/
d]*Gamma[7/2, ((-4*I)*b*(c + d*x))/d])/b^4 + (7*d^4*Sqrt[(I*b*(c + d*x))/d
]*Gamma[7/2, ((4*I)*b*(c + d*x))/d])/(b^4*E^((4*I)*(a - (b*c)/d))))/(14336
*d*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin^2(a+bx)\cos^2(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8}(c+dx)^{5/2} - \frac{1}{8}(c+dx)^{5/2}\cos(4a+4bx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-$$
$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15d^2\sqrt{c+dx}\sin(4a+4bx)}{2048b^3}-$$
$$\frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2}-\frac{(c+dx)^{5/2}\sin(4a+4bx)}{32b}+\frac{(c+dx)^{7/2}}{28d}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^(7/2)/(28*d) - (5*d*(c + d*x)^(3/2)*Cos[4*a + 4*b*x])/(256*b^2)
- (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/P
i]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*Fresne
lC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(40
96*b^(7/2)) + (15*d^2*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(2048*b^3) - ((c + d
*x)^(5/2)*Sin[4*a + 4*b*x])/(32*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.61 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{(dx+c)^{\frac{7}{2}}}{28}-\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{4b(dx+c)}{d}+\frac{4}{8b}\right.\right.\right.\right.}{d}$
default	$\frac{(dx+c)^{\frac{7}{2}}}{28}-\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{4b(dx+c)}{d}+\frac{4}{8b}\right.\right.\right.\right.}{d}$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/56*(d*x+c)^(7/2)-1/64/b*d*(d*x+c)^(5/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*
c)/d)+5/64/b*d*(-1/8/b*d*(d*x+c)^(3/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+3/
8/b*d*(1/8/b*d*(d*x+c)^(1/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-1/32/b*d*2^(
1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/
(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1
/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.52

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{105\sqrt{2}\pi d^4\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+105\sqrt{2}\pi d^4\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/57344*(105*sqrt(2)*pi*d^4*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_
sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 105*sqrt(2)*pi*d^4*sqrt(b/(p
i*d))*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*
d)/d) - 16*(128*b^4*d^3*x^3 + 384*b^4*c*d^2*x^2 + 128*b^4*c^3 - 70*b^2*c*d
^2 - 560*(b^2*d^3*x + b^2*c*d^2)*cos(b*x + a)^4 + 560*(b^2*d^3*x + b^2*c*d
^2)*cos(b*x + a)^2 + 2*(192*b^4*c^2*d - 35*b^2*d^3)*x - 7*(2*(64*b^3*d^3*x
^2 + 128*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a)^3 - (64*b^3*d
^3*x^2 + 128*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a))*sin(b*x
+ a))*sqrt(d*x + c))/(b^4*d)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.25

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{\sqrt{2}\left(\frac{4096\sqrt{2}(dx+c)^{\frac{7}{2}}b^4}{d} - 2240\sqrt{2}(dx+c)^{\frac{3}{2}}b^2 d\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right) + 105\left(-(i+1)\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\right.\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/229376*sqrt(2)*(4096*sqrt(2)*(d*x + c)^(7/2)*b^4/d - 2240*sqrt(2)*(d*x +
 c)^(3/2)*b^2*d*cos(4*((d*x + c)*b - b*c + a*d)/d) + 105*(-(I + 1)*sqrt(pi
)*d^3*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(pi)*d^3*(b^2/d^
2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + 105*((I
 - 1)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I + 1)*sqrt(pi
)*d^3*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b
/d)) - 56*(64*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x + c)*b*d^2
)*sin(4*((d*x + c)*b - b*c + a*d)/d))/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 1358, normalized size of antiderivative = 5.96

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output

```
-1/573440*(17920*(I*sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 8*sqrt(d*x + c))*c^3 - d^3*(4096*(5*(d*x +
 c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x +
c)*c^3)/d^3 - 35*(-I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 - 192*I*b^2*c^2*d - 72*
b*c*d^2 + 15*I*d^3)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
 1)*b^3) + 4*(-64*I*(d*x + c)^(5/2)*b^2*d + 192*I*(d*x + c)^(3/2)*b^2*c*d
- 192*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x +
 c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*
d)/d)/b^3)/d^3 - 35*(I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 + 192*I*b^2*c^2*d - 7
2*b*c*d^2 - 15*I*d^3)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt
(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^
2) + 1)*b^3) + 4*(64*I*(d*x + c)^(5/2)*b^2*d - 192*I*(d*x + c)^(3/2)*b^2*c
*d + 192*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*
x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-4*(I*(d*x + c)*b - I*b*c + I*
a*d)/d)/b^3)/d^3) + 2240*(-3*I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-I*sqr
t(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c ...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)^2\sin(a+b\,x)^2\,(c+d\,x)^{5/2}\,dx$$

input

```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

output

```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \left(\int \sqrt{dx+c}\cos(bx+a)^2\sin(bx+a)^2 x^2 dx\right) d^2 + 2\left(\int \sqrt{dx+c}\cos(bx+a)^2\sin(bx+a)^2 x dx\right) cd + \left(\int \sqrt{dx+c}\cos(bx+a)^2\sin(bx+a)^2 dx\right) c^2$$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**2*sin(a + b*x)**2*x**2,x)*d**2 + 2*int(sqr
t(c + d*x)*cos(a + b*x)**2*sin(a + b*x)**2*x,x)*c*d + int(sqrt(c + d*x)*co
s(a + b*x)**2*sin(a + b*x)**2,x)*c**2
```

3.130 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal result 1203
Mathematica [C] (verified) 1204
Rubi [A] (verified) 1205
Maple [A] (verified) 1207
Fricas [A] (verification not implemented) 1209
Sympy [F(-1)] 1209
Maxima [C] (verification not implemented) 1210
Giac [C] (verification not implemented) 1211
Mupad [F(-1)] 1212
Reduce [F] 1212

Optimal result

Integrand size = 26, antiderivative size = 615

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{32b^3} - \frac{(c+dx)^{5/2}\cos(a+bx)}{8b} + \frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{576b^3} - \frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b} - \frac{3d^2\sqrt{c+dx}\cos(5a+5bx)}{1600b^3} + \frac{(c+dx)^{5/2}\cos(5a+5bx)}{80b} - \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}} - \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}} + \frac{3d^{5/2}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}} - \frac{3d^{5/2}\sqrt{\frac{\pi}{10}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{1600b^{7/2}} + \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{576b^{7/2}} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{32b^{7/2}} + \frac{5d(c+dx)^{3/2}\sin(a+bx)}{16b^2} + \frac{5d(c+dx)^{3/2}\sin(3a+3bx)}{288b^2} - \frac{d(c+dx)^{3/2}\sin(5a+5bx)}{160b^2}$$

output

```
15/32*d^2*(d*x+c)^(1/2)*cos(b*x+a)/b^3-1/8*(d*x+c)^(5/2)*cos(b*x+a)/b+5/57
6*d^2*(d*x+c)^(1/2)*cos(3*b*x+3*a)/b^3-1/48*(d*x+c)^(5/2)*cos(3*b*x+3*a)/b
-3/1600*d^2*(d*x+c)^(1/2)*cos(5*b*x+5*a)/b^3+1/80*(d*x+c)^(5/2)*cos(5*b*x+
5*a)/b-15/64*d^(5/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2
)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-5/3456*d^(5/2)*6^(1/2)*Pi^(1/2)*
cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/
b^(7/2)+3/16000*d^(5/2)*10^(1/2)*Pi^(1/2)*cos(5*a-5*b*c/d)*FresnelC(b^(1/2
)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-3/16000*d^(5/2)*10^(1/2
)*Pi^(1/2)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5
*a-5*b*c/d)/b^(7/2)+5/3456*d^(5/2)*6^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*6^(1/
2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)/b^(7/2)+15/64*d^(5/2)*
2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*
sin(a-b*c/d)/b^(7/2)+5/16*d*(d*x+c)^(3/2)*sin(b*x+a)/b^2+5/288*d*(d*x+c)^(
3/2)*sin(3*b*x+3*a)/b^2-1/160*d*(d*x+c)^(3/2)*sin(5*b*x+5*a)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.10 (sec) , antiderivative size = 2177, normalized size of antiderivative = 3.54

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Result too large to show}$$

input

```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output

```
(c*Sqrt[d]*(-12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(-I + 2*b*
x + E^((6*I)*(a + b*x))*(I + 2*b*x)) - (1 - I)*(2*b*c + I*d)*E^(((3*I)*b*(
2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sq
rt[d]] + (1 + I)*((2*I)*b*c + d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi]*Erfi[((1
 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(1152*b^(5/2)*E^(((3*I)*
(a*d + b*(c + d*x)))/d)) - (c*Sqrt[d]*(-20*Sqrt[b]*Sqrt[d]*E^(((5*I)*b*c)/
d)*Sqrt[c + d*x]*(-3*I + 10*b*x + E^((10*I)*(a + b*x))*(3*I + 10*b*x)) - (
1 - I)*(10*b*c + (3*I)*d)*E^(((5*I)*b*(2*c + d*x))/d)*Sqrt[10*Pi]*Erf[((1
+ I)*Sqrt[5/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + (1 + I)*((10*I)*b*c + 3*d
)*E^((5*I)*(2*a + b*x))*Sqrt[10*Pi]*Erfi[((1 + I)*Sqrt[5/2]*Sqrt[b]*Sqrt[c
 + d*x])/Sqrt[d]]))/(16000*b^(5/2)*E^(((5*I)*(a*d + b*(c + d*x)))/d)) + ((
I/16)*c^2*d*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(
c + d*x))/d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*
(c + d*x))/d]))/(b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x]) + (c^2*(-1/6*(E^
((3*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-3*I)*b*(c + d*x))/d])/(S
qrt[3]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma[3/2, ((3*I)*b*
(c + d*x))/d])/(6*Sqrt[3]*b*E^((3*I)*(a - (b*c)/d))*Sqrt[(I*b*(c + d*x))/d
]))))/16 - (c^2*(-1/10*(E^((5*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((
-5*I)*b*(c + d*x))/d])/(Sqrt[5]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c +
d*x]*Gamma[3/2, ((5*I)*b*(c + d*x))/d])/(10*Sqrt[5]*b*E^((5*I)*(a - (b*...
```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2} \sin^3(a+bx) \cos^2(a+bx)\, dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8}(c+dx)^{5/2} \sin(a+bx) + \frac{1}{16}(c+dx)^{5/2} \sin(3a+3bx) - \frac{1}{16}(c+dx)^{5/2} \sin(5a+5bx) \right) dx$$

$$\downarrow 2009$$

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-$$
$$\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+$$
$$\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-$$
$$\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}+$$
$$\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+$$
$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{32b^3}+$$
$$\frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{576b^3}-\frac{3d^2\sqrt{c+dx}\cos(5a+5bx)}{1600b^3}+\frac{5d(c+dx)^{3/2}\sin(a+bx)}{16b^2}+$$
$$\frac{5d(c+dx)^{3/2}\sin(3a+3bx)}{288b^2}-\frac{d(c+dx)^{3/2}\sin(5a+5bx)}{160b^2}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}-$$
$$\frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{5/2}\cos(5a+5bx)}{80b}$$

input

```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(32*b^3) - ((c + d*x)^(5/2)*Cos[a + b*
x])/(8*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^
(5/2)*Cos[3*a + 3*b*x])/(48*b) - (3*d^2*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(1
600*b^3) + ((c + d*x)^(5/2)*Cos[5*a + 5*b*x])/(80*b) - (15*d^(5/2)*Sqrt[Pi
/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])
/(32*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[
b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) + (3*d^(5/2)*Sqrt[Pi/
10]*Cos[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt
[d]])/(1600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi
]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) + (5*d^(5/2
)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a
- (3*b*c)/d])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) + (5*d*(c +
 d*x)^(3/2)*Sin[a + b*x])/(16*b^2) + (5*d*(c + d*x)^(3/2)*Sin[3*a + 3*b*x]
)/(288*b^2) - (d*(c + d*x)^(3/2)*Sin[5*a + 5*b*x])/(160*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 9.49 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.17

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\sqrt{\pi}$
default	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\sqrt{\pi}$

input

```
int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output

```
2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+5/16/b*d*(1/2/b*
d*(d*x+c)^(3/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(1/
2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(
(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1
/96/b*d*(d*x+c)^(5/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+5/96/b*d*(1/6/b*d*(
d*x+c)^(3/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1
/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/
d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)
*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(
b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))+1/160/b*d*(d*x+c)^(5/2)*cos(5*b/d*(d*x+c)
+5*(a*d-b*c)/d)-1/32/b*d*(1/10/b*d*(d*x+c)^(3/2)*sin(5*b/d*(d*x+c)+5*(a*d-
b*c)/d)-3/10/b*d*(-1/10/b*d*(d*x+c)^(1/2)*cos(5*b/d*(d*x+c)+5*(a*d-b*c)/d)
+1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*Fresne
lC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(5*(a*d-b*c)
/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 521, normalized size of antiderivative = 0.85

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{81\sqrt{10}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 625\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{}\right.$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_
cos(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 625*sqrt(6)*pi*d^3*sqrt(b/(pi
*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)
)) - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(
sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d
))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) +
 625*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(
b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresn
el_sin(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*
(9*(20*b^3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*cos(b*x + a)^5 +
 390*b*d^2*cos(b*x + a) - 5*(60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 -
 13*b*d^2)*cos(b*x + a)^3 + 10*(26*b^2*d^2*x - 9*(b^2*d^2*x + b^2*c*d)*cos
(b*x + a)^4 + 26*b^2*c*d + 13*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*sin(b*
x + a))*sqrt(d*x + c))/b^4
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.34

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output

```
-1/1728000*sqrt(2)*(5400*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(5*((d*x + c)*b -
b*c + a*d)/d)/d - 15000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(3*((d*x + c)*b - b
*c + a*d)/d)/d - 270000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(((d*x + c)*b - b*c
 + a*d)/d)/d - 540*(20*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 3*sqrt(2)*sqrt(d*
x + c)*b^3)*cos(5*((d*x + c)*b - b*c + a*d)/d) + 1500*(12*sqrt(2)*(d*x + c
)^(5/2)*b^5/d^2 - 5*sqrt(2)*sqrt(d*x + c)*b^3)*cos(3*((d*x + c)*b - b*c +
a*d)/d) + 27000*(4*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 15*sqrt(2)*sqrt(d*x +
 c)*b^3)*cos(((d*x + c)*b - b*c + a*d)/d) - 81*(-(I - 1)*25^(1/4)*sqrt(pi)
*b^2*d*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) - (I + 1)*25^(1/4)*sqrt(pi)*b
^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d
)) - 625*((I - 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d
)/d) + (I + 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 101250*((I - 1)*sqrt(pi)*b^2*d*(b^2/
d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*si
n(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 101250*(-(I + 1)*sqrt(
pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqrt(pi)*b^2*d*(b^
2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - 625*(-
(I + 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I
- 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(-3*I*b/d)) - 81*((I + 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d...
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 3673, normalized size of antiderivative = 5.97

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/864000*(1800*(30*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)
*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt
(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d*erf(-1/2*I*sqrt(1
0)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a
*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 30*sqrt(2)*sqrt(pi)*d*erf(-
1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*
b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)
*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt
(10)*sqrt(pi)*d*erf(1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)))*c^3 - d^3*(6750*(sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*
b*c*d^2 - 15*I*d^3)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
 + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d
 + 12*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x +
 c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/
d)/b^3)/d^3 + 125*(sqrt(6)*sqrt(pi)*(72*b^3*c^3 - 36*I*b^2*c^2*d - 18*b...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)^2\sin(a+b\,x)^3\,(c+d\,x)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\begin{aligned}&\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a\\&+bx)\,dx = \left(\int \sqrt{dx+c}\,\cos(bx+a)^2\sin(bx+a)^3\,x^2dx\right)d^2\\&+2\left(\int \sqrt{dx+c}\,\cos(bx+a)^2\sin(bx+a)^3\,xdx\right)cd\\&+\left(\int \sqrt{dx+c}\,\cos(bx+a)^2\sin(bx+a)^3\,dx\right)c^2\end{aligned}$$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**2*sin(a + b*x)**3*x**2,x)*d**2 + 2*int(sqr
t(c + d*x)*cos(a + b*x)**2*sin(a + b*x)**3*x,x)*c*d + int(sqrt(c + d*x)*co
s(a + b*x)**2*sin(a + b*x)**3,x)*c**2
```

3.131 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal result 1214
Mathematica [C] (verified) 1215
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Reduce [F] 1223

Optimal result

Integrand size = 26, antiderivative size = 534

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b} + \frac{(c+dx)^{3/2}\cos(5a+5bx)}{80b} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}} - \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} + \frac{3d^{3/2}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}} + \frac{3d^{3/2}\sqrt{\frac{\pi}{10}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{800b^{5/2}} - \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{96b^{5/2}} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{16b^{5/2}} + \frac{3d\sqrt{c+dx}\sin(a+bx)}{16b^2} + \frac{d\sqrt{c+dx}\sin(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\sin(5a+5bx)}{800b^2}$$

output

```
-1/8*(d*x+c)^(3/2)*cos(b*x+a)/b-1/48*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b+1/80*(
d*x+c)^(3/2)*cos(5*b*x+5*a)/b-3/32*d^(3/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*F
resnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)-1/576*d^(3
/2)*6^(1/2)*Pi^(1/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d
*x+c)^(1/2)/d^(1/2))/b^(5/2)+3/8000*d^(3/2)*10^(1/2)*Pi^(1/2)*cos(5*a-5*b*
c/d)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)+3/8
000*d^(3/2)*10^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(
1/2)/d^(1/2))*sin(5*a-5*b*c/d)/b^(5/2)-1/576*d^(3/2)*6^(1/2)*Pi^(1/2)*Fres
nelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)/b^(5
/2)-3/32*d^(3/2)*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(5/2)+3/16*d*(d*x+c)^(1/2)*sin(b*x+a)/b^2+
1/96*d*(d*x+c)^(1/2)*sin(3*b*x+3*a)/b^2-3/800*d*(d*x+c)^(1/2)*sin(5*b*x+5*
a)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.22 (sec) , antiderivative size = 1126, normalized size of antiderivative = 2.11

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input

```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output

```
(Sqrt[d]*(-12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(-I + 2*b*x
+ E^((6*I)*(a + b*x))*(I + 2*b*x)) - (1 - I)*(2*b*c + I*d)*E^(((3*I)*b*(2*
c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]] + (1 + I)*((2*I)*b*c + d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi]*Erfi[((1 +
 I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(2304*b^(5/2)*E^(((3*I)*(a
*d + b*(c + d*x)))/d)) - (Sqrt[d]*(-20*Sqrt[b]*Sqrt[d]*E^(((5*I)*b*c)/d)*S
qrt[c + d*x]*(-3*I + 10*b*x + E^((10*I)*(a + b*x))*(3*I + 10*b*x)) - (1 -
I)*(10*b*c + (3*I)*d)*E^(((5*I)*b*(2*c + d*x))/d)*Sqrt[10*Pi]*Erf[((1 + I)
*Sqrt[5/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + (1 + I)*((10*I)*b*c + 3*d)*E^
((5*I)*(2*a + b*x))*Sqrt[10*Pi]*Erfi[((1 + I)*Sqrt[5/2]*Sqrt[b]*Sqrt[c + d
*x])/Sqrt[d]]))/(32000*b^(5/2)*E^(((5*I)*(a*d + b*(c + d*x)))/d)) + ((I/16
)*c*d*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(c + d*
x))/d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d
*x))/d]))/(b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x]) + (c*(-1/6*(E^((3*I)*(
a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-3*I)*b*(c + d*x))/d])/(Sqrt[3]*b
*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma[3/2, ((3*I)*b*(c + d*x
))/d])/(6*Sqrt[3]*b*E^((3*I)*(a - (b*c)/d))*Sqrt[(I*b*(c + d*x))/d])))/16
- (c*(-1/10*(E^((5*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-5*I)*b*(c
 + d*x))/d])/(Sqrt[5]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma
[3/2, ((5*I)*b*(c + d*x))/d])/(10*Sqrt[5]*b*E^((5*I)*(a - (b*c)/d))*Sqr...
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2} \sin^3(a+bx) \cos^2(a+bx)\, dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8}(c+dx)^{3/2} \sin(a+bx) + \frac{1}{16}(c+dx)^{3/2} \sin(3a+3bx) - \frac{1}{16}(c+dx)^{3/2} \sin(5a+5bx) \right) dx$$

$$\downarrow 2009$$

$$\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(a+bx)}{16b^2}+\frac{d\sqrt{c+dx}\sin(3a+3bx)}{96b^2}-\frac{3d\sqrt{c+dx}\sin(5a+5bx)}{800b^2}-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\cos(5a+5bx)}{80b}$$

input

```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output

```
-1/8*((c + d*x)^(3/2)*Cos[a + b*x])/b - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])
/(48*b) + ((c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(80*b) - (3*d^(3/2)*Sqrt[Pi/2
]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(
16*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*C
os[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])
/(800*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt
[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt[Pi
/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)
/d])/(96*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqr
t[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + (3*d*Sqrt[c + d*x]*S
in[a + b*x])/(16*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(96*b^2) - (3*d
*Sqrt[c + d*x]*Sin[5*a + 5*b*x])/(800*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.09

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\right.}{4b\sqrt{\frac{b}{d}}}\right.}{8b}$
default	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\right.}{4b\sqrt{\frac{b}{d}}}\right.}{8b}$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output

```
2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+3/16/b*d*(1/2/b*
d*(d*x+c)^(1/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^
(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^
(1/2)/d)))-1/96/b*d*(d*x+c)^(3/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/32/b*
d*(1/6/b*d*(d*x+c)^(1/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)
*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2
)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/
2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))+1/160/b*d*(d*x+c)^(3/
2)*cos(5*b/d*(d*x+c)+5*(a*d-b*c)/d)-3/160/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(
5*b/d*(d*x+c)+5*(a*d-b*c)/d)-1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2
)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x
+c)^(1/2)/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 427, normalized size of antiderivative = 0.80

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{27\sqrt{10}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 125\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\right.$$

input

```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_s
in(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 125*sqrt(6)*pi*d^2*sqrt(b/(pi*
d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))
) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fr
esnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*
sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi
*d)))*sin(-3*(b*c - a*d)/d) + 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_co
s(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(30*(
b^2*d*x + b^2*c)*cos(b*x + a)^5 - 50*(b^2*d*x + b^2*c)*cos(b*x + a)^3 - (9
*b*d*cos(b*x + a)^4 - 13*b*d*cos(b*x + a)^2 - 26*b*d)*sin(b*x + a))*sqrt(d
*x + c))/b^3
```

Sympy [F]

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin^3(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**3*cos(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 760, normalized size of antiderivative = 1.42

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output

```
1/288000*sqrt(2)*(1800*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(5*((d*x + c)*b - b*
c + a*d)/d)/d^2 - 3000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(3*((d*x + c)*b - b*
c + a*d)/d)/d^2 - 18000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(((d*x + c)*b - b*c
 + a*d)/d)/d^2 - 540*sqrt(2)*sqrt(d*x + c)*b^3*sin(5*((d*x + c)*b - b*c +
a*d)/d)/d + 1500*sqrt(2)*sqrt(d*x + c)*b^3*sin(3*((d*x + c)*b - b*c + a*d)
/d)/d + 27000*sqrt(2)*sqrt(d*x + c)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d
 + 27*((I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)
 - (I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*er
f(sqrt(d*x + c)*sqrt(5*I*b/d)) + 125*(-(I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d
^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(
1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + 6750*(-(I +
 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqrt(pi)*b^
2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + 67
50*((I - 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqr
t(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b
/d)) + 125*((I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d
)/d) - (I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))
*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + 27*(-(I - 1)*25^(1/4)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d
^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2...
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 2279, normalized size of antiderivative = 4.27

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output

```
1/144000*(300*(30*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*
sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(
b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d*erf(-1/2*I*sqrt(10
)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 30*sqrt(2)*sqrt(pi)*d*erf(-1
/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b
*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*
d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(
10)*sqrt(pi)*d*erf(1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)))*c^2 - 20*(450*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(
sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c - I*d
)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqr
t(10)*sqrt(pi)*(10*b*c - I*d)*d*erf(-1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c
)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)^2\sin(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

input

```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

output

```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c + dx)^{3/2} \cos^2(a+bx) \sin^3(a+bx)\, dx = \left(\int \sqrt{dx+c} \cos(bx+a)^2 \sin(bx+a)^3 x dx \right) d + \left(\int \sqrt{dx+c} \cos(bx+a)^2 \sin(bx+a)^3 dx \right) c$$

input

```
int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

output

```
int(sqrt(c + d*x)*cos(a + b*x)**2*sin(a + b*x)**3*x,x)*d + int(sqrt(c + d*
x)*cos(a + b*x)**2*sin(a + b*x)**3,x)*c
```

3.132 $\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal result . . . 1224
Mathematica [C] (verified) . . . 1225
Rubi [A] (verified) . . . 1226
Maple [A] (verified) . . . 1227
Fricas [A] (verification not implemented) . . . 1228
Sympy [F] . . . 1229
Maxima [C] (verification not implemented) . . . 1229
Giac [C] (verification not implemented) . . . 1230
Mupad [F(-1)] . . . 1231
Reduce [F] . . . 1232

Optimal result

Integrand size = 26, antiderivative size = 459

$$
\begin{aligned}
&\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx \\
&= -\frac{\sqrt{c+dx}\cos(a+bx)}{8b} - \frac{\sqrt{c+dx}\cos(3a+3bx)}{48b} \\
&\quad + \frac{\sqrt{c+dx}\cos(5a+5bx)}{80b} + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{10}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{80b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{48b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{8b^{3/2}}
\end{aligned}
$$

output

```
-1/8*(d*x+c)^(1/2)*cos(b*x+a)/b-1/48*(d*x+c)^(1/2)*cos(3*b*x+3*a)/b+1/80*(
d*x+c)^(1/2)*cos(5*b*x+5*a)/b+1/16*d^(1/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*F
resnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+1/288*d^(1
/2)*6^(1/2)*Pi^(1/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d
*x+c)^(1/2)/d^(1/2))/b^(3/2)-1/800*d^(1/2)*10^(1/2)*Pi^(1/2)*cos(5*a-5*b*c
/d)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+1/80
0*d^(1/2)*10^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/
2)/d^(1/2))*sin(5*a-5*b*c/d)/b^(3/2)-1/288*d^(1/2)*6^(1/2)*Pi^(1/2)*Fresne
lS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)/b^(3/2
)-1/16*d^(1/2)*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*sin(a-b*c/d)/b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.81

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{e^{-\frac{5i(bc+ad)}{d}}\sqrt{c+dx}\left(450e^{6ia+\frac{4ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+450e^{4ia+\frac{6ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)+25\right.}{}$$

input

```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output

```
-1/7200*(Sqrt[c + d*x]*(450*E^((6*I)*a + ((4*I)*b*c)/d)*Sqrt[(I*b*(c + d*x
))/d]*Gamma[3/2, ((-I)*b*(c + d*x))/d] + 450*E^((4*I)*a + ((6*I)*b*c)/d)*S
qrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d] + 25*Sqrt[3]*E^((2
*I)*(4*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-3*I)*b*(c + d*x
))/d] + 25*Sqrt[3]*E^((2*I)*a + ((8*I)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*
Gamma[3/2, ((3*I)*b*(c + d*x))/d] - 9*Sqrt[5]*E^((10*I)*a)*Sqrt[(I*b*(c +
d*x))/d]*Gamma[3/2, ((-5*I)*b*(c + d*x))/d] - 9*Sqrt[5]*E^(((10*I)*b*c)/d)
*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((5*I)*b*(c + d*x))/d]))/(b*E^(((5*
I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^3(a+bx)\cos^2(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8}\sqrt{c+dx}\sin(a+bx)+\frac{1}{16}\sqrt{c+dx}\sin(3a+3bx)-\frac{1}{16}\sqrt{c+dx}\sin(5a+5bx)\right)dx$$

$$\downarrow 2009$$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+$$
$$\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-$$
$$\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+$$
$$\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}-$$
$$\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-$$
$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\cos(a+bx)}{8b}-$$
$$\frac{\sqrt{c+dx}\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\cos(5a+5bx)}{80b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output

```
-1/8*(Sqrt[c + d*x]*Cos[a + b*x])/b - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(48
*b) + (Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(80*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a
- (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(3/2
)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]
*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/10]*Cos[5*a - (5
*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(80*b^(3/2
)) + (Sqrt[d]*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[5*a - (5*b*c)/d])/(80*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*FresnelS[(S
qrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(48*b^(3/2
)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[
d]]*Sin[a - (b*c)/d])/(8*b^(3/2))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.97

method	result
derivativedivides	$-\frac{d\sqrt{dx+c}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d$
default	$-\frac{d\sqrt{dx+c}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d$

input

```
int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output

```
2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/32/b*d*2^(1/2)
*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d))-1/96/b*d*(d*x+c)^(1/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*
c)/d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*F
resnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d
-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))
+1/160/b*d*(d*x+c)^(1/2)*cos(5*b/d*(d*x+c)+5*(a*d-b*c)/d)-1/1600/b*d*2^(1/
2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1
/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(
1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.78

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{9\sqrt{10}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-25\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\right.}$$

input

```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output

```
-1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_cos(
sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*co
s(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 45
0*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt
(d*x + c)*sqrt(b/(pi*d))) + 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sq
rt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*
sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b
*c - a*d)/d) - 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(10)*sqrt(d*
x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^5 - 5
*b*cos(b*x + a)^3)*sqrt(d*x + c))/b^2
```

Sympy [F]

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \sqrt{c+dx}\sin^3(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.48

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output

```
1/28800*sqrt(2)*(180*sqrt(2)*sqrt(d*x + c)*b^3*cos(5*((d*x + c)*b - b*c +
a*d)/d)/d^2 - 300*sqrt(2)*sqrt(d*x + c)*b^3*cos(3*((d*x + c)*b - b*c + a*d
)/d)/d^2 - 1800*sqrt(2)*sqrt(d*x + c)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)
/d^2 - 9*(-(I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d
)/d)/d - (I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/
d)/d)*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 25*((I - 1)*9^(1/4)*sqrt(pi)*b^2*
(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d + (I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^
2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 4
50*((I - 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d + (I + 1)*s
qrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(
I*b/d)) - 450*(-(I + 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d
 - (I - 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*
x + c)*sqrt(-I*b/d)) - 25*(-(I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*c
os(-3*(b*c - a*d)/d)/d - (I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(
-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - 9*((I + 1)*25^(1/
4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)/d + (I - 1)*25^(1/4)
*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*s
qrt(-5*I*b/d)))*d^2/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 1270, normalized size of antiderivative = 2.77

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output

```
-1/14400*(450*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b
*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*er
f(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(10)*
sqrt(pi)*(10*b*c - I*d)*d*erf(-1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b) + 450*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*I*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d
)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c + I
*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9
*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b) - 30*(30*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*I
*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d
*erf(-1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)^2\sin(a+b\,x)^3\sqrt{c+d\,x}\,dx$$

input

```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

output

```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \sqrt{dx+c}\cos(bx+a)^2\sin(bx+a)^3\,dx$$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**2*sin(a + b*x)**3,x)
```

3.133 $\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal result . . . 1233
Mathematica [C] (verified) . . . 1234
Rubi [A] (verified) . . . 1235
Maple [A] (verified) . . . 1236
Fricas [A] (verification not implemented) . . . 1237
Sympy [F] . . . 1238
Maxima [C] (verification not implemented) . . . 1238
Giac [C] (verification not implemented) . . . 1239
Mupad [F(-1)] . . . 1240
Reduce [F] . . . 1241

Optimal result

Integrand size = 26, antiderivative size = 459

$$
\begin{aligned}
&\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx \\
&= -\frac{\sqrt{c+dx}\cos(a+bx)}{8b} - \frac{\sqrt{c+dx}\cos(3a+3bx)}{48b} \\
&\quad + \frac{\sqrt{c+dx}\cos(5a+5bx)}{80b} + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{10}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{80b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{48b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{8b^{3/2}}
\end{aligned}
$$

output

```
-1/8*(d*x+c)^(1/2)*cos(b*x+a)/b-1/48*(d*x+c)^(1/2)*cos(3*b*x+3*a)/b+1/80*(
d*x+c)^(1/2)*cos(5*b*x+5*a)/b+1/16*d^(1/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*F
resnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+1/288*d^(1
/2)*6^(1/2)*Pi^(1/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d
*x+c)^(1/2)/d^(1/2))/b^(3/2)-1/800*d^(1/2)*10^(1/2)*Pi^(1/2)*cos(5*a-5*b*c
/d)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+1/80
0*d^(1/2)*10^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/
2)/d^(1/2))*sin(5*a-5*b*c/d)/b^(3/2)-1/288*d^(1/2)*6^(1/2)*Pi^(1/2)*Fresne
lS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)/b^(3/2
)-1/16*d^(1/2)*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*sin(a-b*c/d)/b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.81

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{e^{-\frac{5i(bc+ad)}{d}}\sqrt{c+dx}\left(450e^{6ia+\frac{4ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+450e^{4ia+\frac{6ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)+25\right.}{}$$

input

```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output

```
-1/7200*(Sqrt[c + d*x]*(450*E^((6*I)*a + ((4*I)*b*c)/d)*Sqrt[(I*b*(c + d*x
))/d]*Gamma[3/2, ((-I)*b*(c + d*x))/d] + 450*E^((4*I)*a + ((6*I)*b*c)/d)*S
qrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d] + 25*Sqrt[3]*E^((2
*I)*(4*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-3*I)*b*(c + d*x
))/d] + 25*Sqrt[3]*E^((2*I)*a + ((8*I)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*
Gamma[3/2, ((3*I)*b*(c + d*x))/d] - 9*Sqrt[5]*E^((10*I)*a)*Sqrt[(I*b*(c +
d*x))/d]*Gamma[3/2, ((-5*I)*b*(c + d*x))/d] - 9*Sqrt[5]*E^(((10*I)*b*c)/d)
*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((5*I)*b*(c + d*x))/d]))/(b*E^(((5*
I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^3(a+bx)\cos^2(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8}\sqrt{c+dx}\sin(a+bx)+\frac{1}{16}\sqrt{c+dx}\sin(3a+3bx)-\frac{1}{16}\sqrt{c+dx}\sin(5a+5bx)\right)dx$$

$$\downarrow 2009$$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+$$
$$\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-$$
$$\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+$$
$$\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}-$$
$$\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-$$
$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\cos(a+bx)}{8b}-$$
$$\frac{\sqrt{c+dx}\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\cos(5a+5bx)}{80b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output

```
-1/8*(Sqrt[c + d*x]*Cos[a + b*x])/b - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(48
*b) + (Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(80*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a
- (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(3/2
)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]
*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/10]*Cos[5*a - (5
*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(80*b^(3/2
)) + (Sqrt[d]*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[5*a - (5*b*c)/d])/(80*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*FresnelS[(S
qrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(48*b^(3/2
)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[
d]]*Sin[a - (b*c)/d])/(8*b^(3/2))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.97

method	result
derivativedivides	$-\frac{d\sqrt{dx+c}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d$
default	$-\frac{d\sqrt{dx+c}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d$

input

```
int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/32/b*d*2^(1/2)
*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d))-1/96/b*d*(d*x+c)^(1/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*
c)/d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*F
resnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d
-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))
+1/160/b*d*(d*x+c)^(1/2)*cos(5*b/d*(d*x+c)+5*(a*d-b*c)/d)-1/1600/b*d*2^(1/
2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1
/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(
1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.78

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx =$$
$$-\frac{9\sqrt{10}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-25\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\right.}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_cos(
sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*co
s(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 45
0*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt
(d*x + c)*sqrt(b/(pi*d))) + 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sq
rt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*
sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b
*c - a*d)/d) - 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(10)*sqrt(d*
x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^5 - 5
*b*cos(b*x + a)^3)*sqrt(d*x + c))/b^2
```

Sympy [F]

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \sqrt{c+dx}\sin^3(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.48

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output

```
1/28800*sqrt(2)*(180*sqrt(2)*sqrt(d*x + c)*b^3*cos(5*((d*x + c)*b - b*c +
a*d)/d)/d^2 - 300*sqrt(2)*sqrt(d*x + c)*b^3*cos(3*((d*x + c)*b - b*c + a*d
)/d)/d^2 - 1800*sqrt(2)*sqrt(d*x + c)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)
/d^2 - 9*(-(I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d
)/d)/d - (I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/
d)/d)*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 25*((I - 1)*9^(1/4)*sqrt(pi)*b^2*
(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d + (I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^
2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 4
50*((I - 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d + (I + 1)*s
qrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(
I*b/d)) - 450*(-(I + 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d
 - (I - 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*
x + c)*sqrt(-I*b/d)) - 25*(-(I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*c
os(-3*(b*c - a*d)/d)/d - (I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(
-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - 9*((I + 1)*25^(1/
4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)/d + (I - 1)*25^(1/4)
*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*s
qrt(-5*I*b/d)))*d^2/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 1270, normalized size of antiderivative = 2.77

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output

```
-1/14400*(450*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b
*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*er
f(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(10)*
sqrt(pi)*(10*b*c - I*d)*d*erf(-1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b) + 450*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*I*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d
)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c + I
*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9
*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b) - 30*(30*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*I
*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d
*erf(-1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)^2\sin(a+b\,x)^3\sqrt{c+d\,x}\,dx$$

input

```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

output

```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \sqrt{dx+c}\cos(bx+a)^2\sin(bx+a)^3\,dx$$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**2*sin(a + b*x)**3,x)
```

3.134 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal result . . . 1243
Mathematica [C] (verified) . . . 1244
Rubi [A] (verified) . . . 1245
Maple [A] (verified) . . . 1247
Fricas [A] (verification not implemented) . . . 1248
Sympy [F] . . . 1249
Maxima [C] (verification not implemented) . . . 1249
Giac [C] (verification not implemented) . . . 1250
Mupad [F(-1)] . . . 1251
Reduce [F] . . . 1252

Optimal result

Integrand size = 26, antiderivative size = 534

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b} + \frac{(c+dx)^{3/2}\cos(5a+5bx)}{80b} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}} - \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} + \frac{3d^{3/2}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}} + \frac{3d^{3/2}\sqrt{\frac{\pi}{10}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{800b^{5/2}} - \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{96b^{5/2}} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{16b^{5/2}} + \frac{3d\sqrt{c+dx}\sin(a+bx)}{16b^2} + \frac{d\sqrt{c+dx}\sin(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\sin(5a+5bx)}{800b^2}$$

output

```
-1/8*(d*x+c)^(3/2)*cos(b*x+a)/b-1/48*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b+1/80*(
d*x+c)^(3/2)*cos(5*b*x+5*a)/b-3/32*d^(3/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*F
resnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)-1/576*d^(3
/2)*6^(1/2)*Pi^(1/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d
*x+c)^(1/2)/d^(1/2))/b^(5/2)+3/8000*d^(3/2)*10^(1/2)*Pi^(1/2)*cos(5*a-5*b*
c/d)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)+3/8
000*d^(3/2)*10^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(
1/2)/d^(1/2))*sin(5*a-5*b*c/d)/b^(5/2)-1/576*d^(3/2)*6^(1/2)*Pi^(1/2)*Fres
nelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)/b^(5
/2)-3/32*d^(3/2)*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(5/2)+3/16*d*(d*x+c)^(1/2)*sin(b*x+a)/b^2+
1/96*d*(d*x+c)^(1/2)*sin(3*b*x+3*a)/b^2-3/800*d*(d*x+c)^(1/2)*sin(5*b*x+5*
a)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 1126, normalized size of antiderivative = 2.11

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input

```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output

```
(Sqrt[d]*(-12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(-I + 2*b*x
+ E^((6*I)*(a + b*x))*(I + 2*b*x)) - (1 - I)*(2*b*c + I*d)*E^(((3*I)*b*(2*
c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]] + (1 + I)*((2*I)*b*c + d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi]*Erfi[((1 +
 I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(2304*b^(5/2)*E^(((3*I)*(a
*d + b*(c + d*x)))/d)) - (Sqrt[d]*(-20*Sqrt[b]*Sqrt[d]*E^(((5*I)*b*c)/d)*S
qrt[c + d*x]*(-3*I + 10*b*x + E^((10*I)*(a + b*x))*(3*I + 10*b*x)) - (1 -
I)*(10*b*c + (3*I)*d)*E^(((5*I)*b*(2*c + d*x))/d)*Sqrt[10*Pi]*Erf[((1 + I)
*Sqrt[5/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + (1 + I)*((10*I)*b*c + 3*d)*E^
((5*I)*(2*a + b*x))*Sqrt[10*Pi]*Erfi[((1 + I)*Sqrt[5/2]*Sqrt[b]*Sqrt[c + d
*x])/Sqrt[d]]))/(32000*b^(5/2)*E^(((5*I)*(a*d + b*(c + d*x)))/d)) + ((I/16
)*c*d*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(c + d*
x))/d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d
*x))/d]))/(b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x]) + (c*(-1/6*(E^((3*I)*(
a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-3*I)*b*(c + d*x))/d])/(Sqrt[3]*b
*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma[3/2, ((3*I)*b*(c + d*x
))/d])/(6*Sqrt[3]*b*E^((3*I)*(a - (b*c)/d))*Sqrt[(I*b*(c + d*x))/d])))/16
- (c*(-1/10*(E^((5*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-5*I)*b*(c
 + d*x))/d])/(Sqrt[5]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma
[3/2, ((5*I)*b*(c + d*x))/d])/(10*Sqrt[5]*b*E^((5*I)*(a - (b*c)/d))*Sqr...
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2} \sin^3(a+bx) \cos^2(a+bx)\, dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8}(c+dx)^{3/2} \sin(a+bx) + \frac{1}{16}(c+dx)^{3/2} \sin(3a+3bx) - \frac{1}{16}(c+dx)^{3/2} \sin(5a+5bx) \right) dx$$

$$\downarrow 2009$$

$$\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(a+bx)}{16b^2}+\frac{d\sqrt{c+dx}\sin(3a+3bx)}{96b^2}-\frac{3d\sqrt{c+dx}\sin(5a+5bx)}{800b^2}-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\cos(5a+5bx)}{80b}$$

input

```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output

```
-1/8*((c + d*x)^(3/2)*Cos[a + b*x])/b - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])
/(48*b) + ((c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(80*b) - (3*d^(3/2)*Sqrt[Pi/2
]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(
16*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*C
os[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])
/(800*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt
[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt[Pi
/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)
/d])/(96*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqr
t[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + (3*d*Sqrt[c + d*x]*S
in[a + b*x])/(16*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(96*b^2) - (3*d
*Sqrt[c + d*x]*Sin[5*a + 5*b*x])/(800*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.09

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\right.}{4b\sqrt{\frac{b}{d}}}\right.}{8b}$
default	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\right.}{4b\sqrt{\frac{b}{d}}}\right.}{8b}$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output

```
2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+3/16/b*d*(1/2/b*
d*(d*x+c)^(1/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^
(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^
(1/2)/d)))-1/96/b*d*(d*x+c)^(3/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/32/b*
d*(1/6/b*d*(d*x+c)^(1/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)
*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2
)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/
2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))+1/160/b*d*(d*x+c)^(3/
2)*cos(5*b/d*(d*x+c)+5*(a*d-b*c)/d)-3/160/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(
5*b/d*(d*x+c)+5*(a*d-b*c)/d)-1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2
)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x
+c)^(1/2)/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 427, normalized size of antiderivative = 0.80

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{27\sqrt{10}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 125\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\right.$$

input

```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_s
in(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 125*sqrt(6)*pi*d^2*sqrt(b/(pi*
d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))
) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fr
esnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*
sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi
*d)))*sin(-3*(b*c - a*d)/d) + 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_co
s(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(30*(
b^2*d*x + b^2*c)*cos(b*x + a)^5 - 50*(b^2*d*x + b^2*c)*cos(b*x + a)^3 - (9
*b*d*cos(b*x + a)^4 - 13*b*d*cos(b*x + a)^2 - 26*b*d)*sin(b*x + a))*sqrt(d
*x + c))/b^3
```

Sympy [F]

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin^3(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**3*cos(a + b*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 760, normalized size of antiderivative = 1.42

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output

```
1/288000*sqrt(2)*(1800*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(5*((d*x + c)*b - b*
c + a*d)/d)/d^2 - 3000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(3*((d*x + c)*b - b*
c + a*d)/d)/d^2 - 18000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(((d*x + c)*b - b*c
 + a*d)/d)/d^2 - 540*sqrt(2)*sqrt(d*x + c)*b^3*sin(5*((d*x + c)*b - b*c +
a*d)/d)/d + 1500*sqrt(2)*sqrt(d*x + c)*b^3*sin(3*((d*x + c)*b - b*c + a*d)
/d)/d + 27000*sqrt(2)*sqrt(d*x + c)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d
 + 27*((I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)
 - (I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*er
f(sqrt(d*x + c)*sqrt(5*I*b/d)) + 125*(-(I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d
^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(
1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + 6750*(-(I +
 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqrt(pi)*b^
2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + 67
50*((I - 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqr
t(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b
/d)) + 125*((I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d
)/d) - (I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))
*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + 27*(-(I - 1)*25^(1/4)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d
^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2...
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 2279, normalized size of antiderivative = 4.27

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output

```
1/144000*(300*(30*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*
sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(
b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d*erf(-1/2*I*sqrt(10
)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 30*sqrt(2)*sqrt(pi)*d*erf(-1
/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b
*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*
d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(
10)*sqrt(pi)*d*erf(1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)))*c^2 - 20*(450*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(
sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c - I*d
)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqr
t(10)*sqrt(pi)*(10*b*c - I*d)*d*erf(-1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c
)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)^2\sin(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

input

```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

output

```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c + dx)^{3/2} \cos^2(a+bx) \sin^3(a+bx)\, dx = \left(\int \sqrt{dx+c} \cos(bx+a)^2 \sin(bx+a)^3 x dx \right) d + \left(\int \sqrt{dx+c} \cos(bx+a)^2 \sin(bx+a)^3 dx \right) c$$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**2*sin(a + b*x)**3*x,x)*d + int(sqrt(c + d*
x)*cos(a + b*x)**2*sin(a + b*x)**3,x)*c
```

3.135 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal result 1254
Mathematica [C] (verified) 1255
Rubi [A] (verified) 1256
Maple [A] (verified) 1258
Fricas [A] (verification not implemented) 1260
Sympy [F(-1)] 1260
Maxima [C] (verification not implemented) 1261
Giac [C] (verification not implemented) 1262
Mupad [F(-1)] 1263
Reduce [F] 1263

Optimal result

Integrand size = 26, antiderivative size = 615

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{32b^3} - \frac{(c+dx)^{5/2}\cos(a+bx)}{8b} + \frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{576b^3} - \frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b} - \frac{3d^2\sqrt{c+dx}\cos(5a+5bx)}{1600b^3} + \frac{(c+dx)^{5/2}\cos(5a+5bx)}{80b} - \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}} - \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}} + \frac{3d^{5/2}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}} - \frac{3d^{5/2}\sqrt{\frac{\pi}{10}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{1600b^{7/2}} + \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{576b^{7/2}} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{32b^{7/2}} + \frac{5d(c+dx)^{3/2}\sin(a+bx)}{16b^2} + \frac{5d(c+dx)^{3/2}\sin(3a+3bx)}{288b^2} - \frac{d(c+dx)^{3/2}\sin(5a+5bx)}{160b^2}$$

output

```
15/32*d^2*(d*x+c)^(1/2)*cos(b*x+a)/b^3-1/8*(d*x+c)^(5/2)*cos(b*x+a)/b+5/57
6*d^2*(d*x+c)^(1/2)*cos(3*b*x+3*a)/b^3-1/48*(d*x+c)^(5/2)*cos(3*b*x+3*a)/b
-3/1600*d^2*(d*x+c)^(1/2)*cos(5*b*x+5*a)/b^3+1/80*(d*x+c)^(5/2)*cos(5*b*x+
5*a)/b-15/64*d^(5/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2
)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-5/3456*d^(5/2)*6^(1/2)*Pi^(1/2)*
cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/
b^(7/2)+3/16000*d^(5/2)*10^(1/2)*Pi^(1/2)*cos(5*a-5*b*c/d)*FresnelC(b^(1/2
)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-3/16000*d^(5/2)*10^(1/2
)*Pi^(1/2)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5
*a-5*b*c/d)/b^(7/2)+5/3456*d^(5/2)*6^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*6^(1/
2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)/b^(7/2)+15/64*d^(5/2)*
2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*
sin(a-b*c/d)/b^(7/2)+5/16*d*(d*x+c)^(3/2)*sin(b*x+a)/b^2+5/288*d*(d*x+c)^(
3/2)*sin(3*b*x+3*a)/b^2-1/160*d*(d*x+c)^(3/2)*sin(5*b*x+5*a)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.01 (sec) , antiderivative size = 2177, normalized size of antiderivative = 3.54

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Result too large to show}$$

input

```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output

```
(c*Sqrt[d]*(-12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(-I + 2*b*
x + E^((6*I)*(a + b*x))*(I + 2*b*x)) - (1 - I)*(2*b*c + I*d)*E^(((3*I)*b*(
2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sq
rt[d]] + (1 + I)*((2*I)*b*c + d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi]*Erfi[((1
 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(1152*b^(5/2)*E^(((3*I)*
(a*d + b*(c + d*x)))/d)) - (c*Sqrt[d]*(-20*Sqrt[b]*Sqrt[d]*E^(((5*I)*b*c)/
d)*Sqrt[c + d*x]*(-3*I + 10*b*x + E^((10*I)*(a + b*x))*(3*I + 10*b*x)) - (
1 - I)*(10*b*c + (3*I)*d)*E^(((5*I)*b*(2*c + d*x))/d)*Sqrt[10*Pi]*Erf[((1
+ I)*Sqrt[5/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + (1 + I)*((10*I)*b*c + 3*d
)*E^((5*I)*(2*a + b*x))*Sqrt[10*Pi]*Erfi[((1 + I)*Sqrt[5/2]*Sqrt[b]*Sqrt[c
 + d*x])/Sqrt[d]]))/(16000*b^(5/2)*E^(((5*I)*(a*d + b*(c + d*x)))/d)) + ((
I/16)*c^2*d*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(
c + d*x))/d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*
(c + d*x))/d]))/(b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x]) + (c^2*(-1/6*(E^
((3*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-3*I)*b*(c + d*x))/d])/(S
qrt[3]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma[3/2, ((3*I)*b*
(c + d*x))/d])/(6*Sqrt[3]*b*E^((3*I)*(a - (b*c)/d))*Sqrt[(I*b*(c + d*x))/d
]))))/16 - (c^2*(-1/10*(E^((5*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((
-5*I)*b*(c + d*x))/d])/(Sqrt[5]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c +
d*x]*Gamma[3/2, ((5*I)*b*(c + d*x))/d])/(10*Sqrt[5]*b*E^((5*I)*(a - (b*...
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2} \sin^3(a+bx) \cos^2(a+bx)\, dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8}(c+dx)^{5/2} \sin(a+bx) + \frac{1}{16}(c+dx)^{5/2} \sin(3a+3bx) - \frac{1}{16}(c+dx)^{5/2} \sin(5a+5bx) \right) dx$$

$$\downarrow 2009$$

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-$$
$$\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+$$
$$\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-$$
$$\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}+$$
$$\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+$$
$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{32b^3}+$$
$$\frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{576b^3}-\frac{3d^2\sqrt{c+dx}\cos(5a+5bx)}{1600b^3}+\frac{5d(c+dx)^{3/2}\sin(a+bx)}{16b^2}+$$
$$\frac{5d(c+dx)^{3/2}\sin(3a+3bx)}{288b^2}-\frac{d(c+dx)^{3/2}\sin(5a+5bx)}{160b^2}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}-$$
$$\frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{5/2}\cos(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(32*b^3) - ((c + d*x)^(5/2)*Cos[a + b*
x])/(8*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^
(5/2)*Cos[3*a + 3*b*x])/(48*b) - (3*d^2*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(1
600*b^3) + ((c + d*x)^(5/2)*Cos[5*a + 5*b*x])/(80*b) - (15*d^(5/2)*Sqrt[Pi
/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])
/(32*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[
b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) + (3*d^(5/2)*Sqrt[Pi/
10]*Cos[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt
[d]])/(1600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi
]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) + (5*d^(5/2
)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a
- (3*b*c)/d])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) + (5*d*(c +
 d*x)^(3/2)*Sin[a + b*x])/(16*b^2) + (5*d*(c + d*x)^(3/2)*Sin[3*a + 3*b*x]
)/(288*b^2) - (d*(c + d*x)^(3/2)*Sin[5*a + 5*b*x])/(160*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 9.11 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.17

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\sqrt{\pi}$
default	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\sqrt{\pi}$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+5/16/b*d*(1/2/b*
d*(d*x+c)^(3/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(1/
2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(
(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1
/96/b*d*(d*x+c)^(5/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+5/96/b*d*(1/6/b*d*(
d*x+c)^(3/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1
/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/
d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)
*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(
b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))+1/160/b*d*(d*x+c)^(5/2)*cos(5*b/d*(d*x+c)
+5*(a*d-b*c)/d)-1/32/b*d*(1/10/b*d*(d*x+c)^(3/2)*sin(5*b/d*(d*x+c)+5*(a*d-
b*c)/d)-3/10/b*d*(-1/10/b*d*(d*x+c)^(1/2)*cos(5*b/d*(d*x+c)+5*(a*d-b*c)/d)
+1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*Fresne
lC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(5*(a*d-b*c)
/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 521, normalized size of antiderivative = 0.85

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{81\sqrt{10}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 625\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{}\right.$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_
cos(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 625*sqrt(6)*pi*d^3*sqrt(b/(pi
*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)
)) - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(
sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d
))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) +
 625*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(
b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresn
el_sin(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*
(9*(20*b^3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*cos(b*x + a)^5 +
 390*b*d^2*cos(b*x + a) - 5*(60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 -
 13*b*d^2)*cos(b*x + a)^3 + 10*(26*b^2*d^2*x - 9*(b^2*d^2*x + b^2*c*d)*cos
(b*x + a)^4 + 26*b^2*c*d + 13*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*sin(b*
x + a))*sqrt(d*x + c))/b^4
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.34

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output

```
-1/1728000*sqrt(2)*(5400*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(5*((d*x + c)*b -
b*c + a*d)/d)/d - 15000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(3*((d*x + c)*b - b
*c + a*d)/d)/d - 270000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(((d*x + c)*b - b*c
 + a*d)/d)/d - 540*(20*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 3*sqrt(2)*sqrt(d*
x + c)*b^3)*cos(5*((d*x + c)*b - b*c + a*d)/d) + 1500*(12*sqrt(2)*(d*x + c
)^(5/2)*b^5/d^2 - 5*sqrt(2)*sqrt(d*x + c)*b^3)*cos(3*((d*x + c)*b - b*c +
a*d)/d) + 27000*(4*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 15*sqrt(2)*sqrt(d*x +
 c)*b^3)*cos(((d*x + c)*b - b*c + a*d)/d) - 81*(-(I - 1)*25^(1/4)*sqrt(pi)
*b^2*d*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) - (I + 1)*25^(1/4)*sqrt(pi)*b
^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d
)) - 625*((I - 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d
)/d) + (I + 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 101250*((I - 1)*sqrt(pi)*b^2*d*(b^2/
d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*si
n(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 101250*(-(I + 1)*sqrt(
pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqrt(pi)*b^2*d*(b^
2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - 625*(-
(I + 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I
- 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(-3*I*b/d)) - 81*((I + 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d...
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.55 (sec) , antiderivative size = 3673, normalized size of antiderivative = 5.97

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/864000*(1800*(30*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)
*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt
(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d*erf(-1/2*I*sqrt(1
0)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a
*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 30*sqrt(2)*sqrt(pi)*d*erf(-
1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*
b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)
*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt
(10)*sqrt(pi)*d*erf(1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)))*c^3 - d^3*(6750*(sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*
b*c*d^2 - 15*I*d^3)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
 + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d
 + 12*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x +
 c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/
d)/b^3)/d^3 + 125*(sqrt(6)*sqrt(pi)*(72*b^3*c^3 - 36*I*b^2*c^2*d - 18*b...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)^2\sin(a+b\,x)^3\,(c+d\,x)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\begin{aligned}&\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a\\&+bx)\,dx = \left(\int \sqrt{dx+c}\,\cos(bx+a)^2\sin(bx+a)^3\,x^2dx\right)d^2\\&+2\left(\int \sqrt{dx+c}\,\cos(bx+a)^2\sin(bx+a)^3\,xdx\right)cd\\&+\left(\int \sqrt{dx+c}\,\cos(bx+a)^2\sin(bx+a)^3\,dx\right)c^2\end{aligned}$$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**2*sin(a + b*x)**3*x**2,x)*d**2 + 2*int(sqr
t(c + d*x)*cos(a + b*x)**2*sin(a + b*x)**3*x,x)*c*d + int(sqrt(c + d*x)*co
s(a + b*x)**2*sin(a + b*x)**3,x)*c**2
```

3.136 $\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 1264
Mathematica [A] (verified) . . . 1265
Rubi [A] (verified) . . . 1265
Maple [F] . . . 1267
Fricas [A] (verification not implemented) . . . 1267
Sympy [F] . . . 1267
Maxima [F] . . . 1268
Giac [F] . . . 1268
Mupad [F(-1)] . . . 1268
Reduce [F] . . . 1269

Optimal result

Integrand size = 22, antiderivative size = 273

$$\begin{aligned}
&\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx \\
&= -\frac{2^{-4-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} \\
&\quad -\frac{2^{-4-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2ib(c+dx)}{d}\right)}{b} \\
&\quad -\frac{2^{-2(3+m)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{4ib(c+dx)}{d}\right)}{b} \\
&\quad -\frac{2^{-2(3+m)}e^{-4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{4ib(c+dx)}{d}\right)}{b}
\end{aligned}$$

output

```
-2^(-4-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*
b*(d*x+c)/d)^m)-2^(-4-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a
-b*c/d))/((I*b*(d*x+c)/d)^m)-exp(4*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-4*I*b
*(d*x+c)/d)/(2^(6+2*m))/b/((-I*b*(d*x+c)/d)^m)-(d*x+c)^m*GAMMA(1+m,4*I*b*(
d*x+c)/d)/(2^(6+2*m))/b/exp(4*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.90

$$\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{4^{-3-m}e^{-\frac{4i(bc+ad)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(2^{2+m}e^{2i\left(3a+\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^m\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)+2^{2+m}e^{2i(}\right.}{}$$

input

```
Integrate[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output

```
-((4^(-3 - m)*(c + d*x)^m*(2^(2 + m)*E^((2*I)*(3*a + (b*c)/d))*((I*b*(c +
d*x))/d)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d] + 2^(2 + m)*E^((2*I)*(a +
(3*b*c)/d))*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d] +
 E^((8*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-4*I)*b*(c + d*x))/d] +
E^(((8*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((4*I)*b*(c + d*x)
)/d]))/(b*E^(((4*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cos^3(a+bx)(c+dx)^m\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}\sin(2a+2bx)(c+dx)^m+\frac{1}{8}\sin(4a+4bx)(c+dx)^m\right)dx$$

$$\downarrow 2009$$

$$-\frac{2^{-m-4}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{4ib(c+dx)}{d}\right)}{b}-\frac{2^{-m-4}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-2(m+3)}e^{-4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{4ib(c+dx)}{d}\right)}{b}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
-((2^(-4 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(
c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)) - (2^(-4 - m)*(c + d*x)^m*Gamm
a[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x
))/d)^m) - (E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-4*I)*b*(c
+ d*x))/d])/(2^(2*(3 + m))*b*(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamm
a[1 + m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*E^((4*I)*(a - (b*c)/d))*
((I*b*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [F]

$$\int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)\,dx$$

input
```
int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x)
```

output
```
int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.69

$$\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{4\,e^{\left(-\frac{dm\log\left(-\frac{2ib}{d}\right)+2ibc-2iad}{d}\right)}\Gamma\left(m+1,-\frac{2\,(ibdx+ibc)}{d}\right)+e^{\left(-\frac{dm\log\left(-\frac{4ib}{d}\right)+4ibc-4iad}{d}\right)}\Gamma\left(m+1,-\frac{4\,(ibdx+ibc)}{d}\right)+}{64}$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/64*(4*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d)*gamma(m + 1, -2*(I
*b*d*x + I*b*c)/d) + e^(-(d*m*log(-4*I*b/d) + 4*I*b*c - 4*I*a*d)/d)*gamma(
m + 1, -4*(I*b*d*x + I*b*c)/d) + 4*e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a
*d)/d)*gamma(m + 1, -2*(-I*b*d*x - I*b*c)/d) + e^(-(d*m*log(4*I*b/d) - 4*I
*b*c + 4*I*a*d)/d)*gamma(m + 1, -4*(-I*b*d*x - I*b*c)/d))/b
```

Sympy [F]

$$\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx = \int (c+dx)^m \sin(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cos(b*x+a)**3*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**m*sin(a + b*x)*cos(a + b*x)**3, x)
```

Maxima [F]

$$\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx = \int (dx+c)^m \cos\left(bx+a\right)^3 \sin\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a), x)
```

Giac [F]

$$\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx = \int (dx+c)^m \cos\left(bx+a\right)^3 \sin\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx = \int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)^m\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)\,dx$$

input
```
int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x)
```

output
```
int((c + d*x)**m*cos(a + b*x)**3*sin(a + b*x),x)
```

3.137 $\int (c+dx)^4 \cos^3(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 1270
Mathematica [A] (verified) . . . 1271
Rubi [A] (verified) . . . 1271
Maple [A] (verified) . . . 1275
Fricas [A] (verification not implemented) . . . 1276
Sympy [B] (verification not implemented) . . . 1276
Maxima [B] (verification not implemented) . . . 1277
Giac [A] (verification not implemented) . . . 1278
Mupad [B] (verification not implemented) . . . 1279
Reduce [B] (verification not implemented) . . . 1280

Optimal result

Integrand size = 22, antiderivative size = 252

$$\begin{aligned}
&\int (c+dx)^4 \cos^3(a+bx)\sin(a+bx)\,dx \\
&= -\frac{45d^2(c+dx)^2}{128b^3} + \frac{3(c+dx)^4}{32b} - \frac{45d^4\cos^2(a+bx)}{128b^5} + \frac{9d^2(c+dx)^2\cos^2(a+bx)}{16b^3} \\
&\quad - \frac{3d^4\cos^4(a+bx)}{128b^5} + \frac{3d^2(c+dx)^2\cos^4(a+bx)}{16b^3} - \frac{(c+dx)^4\cos^4(a+bx)}{4b} \\
&\quad - \frac{45d^3(c+dx)\cos(a+bx)\sin(a+bx)}{64b^4} + \frac{3d(c+dx)^3\cos(a+bx)\sin(a+bx)}{8b^2} \\
&\quad - \frac{3d^3(c+dx)\cos^3(a+bx)\sin(a+bx)}{32b^4} + \frac{d(c+dx)^3\cos^3(a+bx)\sin(a+bx)}{4b^2}
\end{aligned}$$

output

```
-45/128*d^2*(d*x+c)^2/b^3+3/32*(d*x+c)^4/b-45/128*d^4*cos(b*x+a)^2/b^5+9/1
6*d^2*(d*x+c)^2*cos(b*x+a)^2/b^3-3/128*d^4*cos(b*x+a)^4/b^5+3/16*d^2*(d*x+
c)^2*cos(b*x+a)^4/b^3-1/4*(d*x+c)^4*cos(b*x+a)^4/b-45/64*d^3*(d*x+c)*cos(b
*x+a)*sin(b*x+a)/b^4+3/8*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2-3/32*d^3*(d
*x+c)*cos(b*x+a)^3*sin(b*x+a)/b^4+1/4*d*(d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)/
b^2
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.63

$$\int (c+dx)^4 \cos^3(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{64(3d^4-6b^2d^2(c+dx)^2+2b^4(c+dx)^4)\cos(2(a+bx))+(3d^4-24b^2d^2(c+dx)^2+32b^4(c+dx)^4)\cos$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
-1/1024*(64*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a +
 b*x)] + (3*d^4 - 24*b^2*d^2*(c + d*x)^2 + 32*b^4*(c + d*x)^4)*Cos[4*(a +
b*x)] - 8*b*d*(c + d*x)*(16*(-3*d^2 + 2*b^2*(c + d*x)^2) + (-3*d^2 + 8*b^2
*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)])/b^5
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.29, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {4905, 3042, 3792, 3042, 3791, 3042, 3791, 17, 3792, 17, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sin(a+bx)\cos^3(a+bx)\,dx$$
$$\downarrow 4905$$
$$\frac{d\int (c+dx)^3\cos^4(a+bx)dx}{b}-\frac{(c+dx)^4\cos^4(a+bx)}{4b}$$
$$\downarrow 3042$$
$$\frac{d\int (c+dx)^3\sin\left(a+bx+\frac{\pi}{2}\right)^4 dx}{b}-\frac{(c+dx)^4\cos^4(a+bx)}{4b}$$
$$\downarrow 3792$$

$$\frac{d\left(-\frac{3d^2\int(c+dx)\cos^4(a+bx)dx}{8b^2}+\frac{3}{4}\int(c+dx)^3\cos^2(a+bx)dx+\frac{3d(c+dx)^2\cos^4(a+bx)}{16b^2}+\frac{(c+dx)^3\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{b}-\frac{(c+dx)^4\cos^4(a+bx)}{4b}$$

$$\downarrow 3042$$

$$\frac{d\left(-\frac{3d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^4dx}{8b^2}+\frac{3}{4}\int(c+dx)^3\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{3d(c+dx)^2\cos^4(a+bx)}{16b^2}+\frac{(c+dx)^3\sin(a+bx)\cos^3(a+b}{4b}\right.}{b}$$
$$\left.\frac{(c+dx)^4\cos^4(a+bx)}{4b}\right)$$

$$\downarrow 3791$$

$$\frac{d\left(-\frac{3d^2\left(\frac{3}{4}\int(c+dx)\cos^2(a+bx)dx+\frac{d\cos^4(a+bx)}{16b^2}+\frac{(c+dx)\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int(c+dx)^3\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{3d(c+dx)^2}{16}\right.}{b}$$
$$\left.\frac{(c+dx)^4\cos^4(a+bx)}{4b}\right)$$

$$\downarrow 3042$$

$$\frac{d\left(-\frac{3d^2\left(\frac{3}{4}\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{d\cos^4(a+bx)}{16b^2}+\frac{(c+dx)\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int(c+dx)^3\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{3d(c+d}{b}\right.$$
$$\left.\frac{(c+dx)^4\cos^4(a+bx)}{4b}\right)$$

$$\downarrow 3791$$

$$\frac{d\left(-\frac{3d^2\left(\frac{3}{4}\left(\frac{1}{2}\int(c+dx)dx+\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{d\cos^4(a+bx)}{16b^2}+\frac{(c+dx)\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int(c+dx)^3\sin(c\right.}{b}$$
$$\left.\frac{(c+dx)^4\cos^4(a+bx)}{4b}\right)$$

$$\downarrow 17$$

$$d\left(\frac{\frac{3}{4}\int (c+dx)^3 \sin\left(a+bx+\frac{\pi}{2}\right)^2 dx - \frac{3d^2\left(\frac{3}{4}\left(\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)+\frac{d\cos^4(a+bx)}{16b^2}+\frac{(c+dx)\sin(a+bx)}{4b}\right.}{8b^2}}{b}\right.$$

$$\frac{(c+dx)^4\cos^4(a+bx)}{4b}$$

$$\downarrow 3792$$

$$d\left(\frac{\frac{3}{4}\left(-\frac{3d^2\int (c+dx)\cos^2(a+bx)dx}{2b^2}+\frac{1}{2}\int (c+dx)^3 dx+\frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}\right)-\frac{3d^2\left(\frac{3}{4}\left(\frac{d\cos^2(}{4b^2}\right.\right.}{}\right.$$

$$\frac{(c+dx)^4\cos^4(a+bx)}{4b}$$

$$\downarrow 17$$

$$d\left(\frac{\frac{3}{4}\left(-\frac{3d^2\int (c+dx)\cos^2(a+bx)dx}{2b^2}+\frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)-\frac{3d^2\left(\frac{3}{4}\left(\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+}{}\right.\right.}{b}\right.$$

$$\frac{(c+dx)^4\cos^4(a+bx)}{4b}$$

$$\downarrow 3042$$

$$d\left(\frac{\frac{3}{4}\left(-\frac{3d^2\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^2 dx}{2b^2}+\frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)-\frac{3d^2\left(\frac{3}{4}\left(\frac{d\cos^2(a+bx)}{4b^2}-\right.\right.}{}\right.$$

$$\frac{(c+dx)^4\cos^4(a+bx)}{4b}$$

$$\downarrow 3791$$

$$d\left(\frac{\frac{3}{4}\left(-\frac{3d^2\left(\frac{1}{2}\int (c+dx)dx+\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+d}{8d}\right.\right.$$

$$\frac{(c+dx)^4\cos^4(a+bx)}{4b}$$

$$\downarrow 17$$

$$d\left(\frac{\frac{3}{4}\left(-\frac{3d^2\left(\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{2b^2}+\frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)}{(c+dx)^4\cos^4(a+bx)}\right)$$
$$4b$$

input
```
Int[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
-1/4*((c + d*x)^4*Cos[a + b*x]^4)/b + (d*((3*d*(c + d*x)^2*Cos[a + b*x]^4)
/(16*b^2) + ((c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x])/(4*b) - (3*d^2*((d*C
os[a + b*x]^4)/(16*b^2) + ((c + d*x)*Cos[a + b*x]^3*Sin[a + b*x])/(4*b) +
(3*((c + d*x)^2/(4*d) + (d*Cos[a + b*x]^2)/(4*b^2) + ((c + d*x)*Cos[a + b*
x]*Sin[a + b*x])/(2*b)))/4))/(8*b^2) + (3*((c + d*x)^4/(8*d) + (3*d*(c + d
*x)^2*Cos[a + b*x]^2)/(4*b^2) + ((c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/(2
*b) - (3*d^2*((c + d*x)^2/(4*d) + (d*Cos[a + b*x]^2)/(4*b^2) + ((c + d*x)*
Cos[a + b*x]*Sin[a + b*x])/(2*b)))/(2*b^2)))/4))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)  Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)  Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))  Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4905

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))  Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{\left(-128(dx+c)^4b^4+384d^2(dx+c)^2b^2-192d^4\right)\cos(2bx+2a)+\left(-32(dx+c)^4b^4+24d^2(dx+c)^2b^2-3d^4\right)\cos(4bx+4a)+25$
risch	$-\frac{\left(32d^4x^4b^4+128b^4cd^3x^3+192b^4c^2d^2x^2+128b^4c^3dx+32b^4c^4-24b^2d^4x^2-48b^2cd^3x-24b^2c^2d^2+3d^4\right)\cos(4bx+4a)}{1024b^5}$
orering	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input

```
int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
1/1024*((-128*(d*x+c)^4*b^4+384*d^2*(d*x+c)^2*b^2-192*d^4)*cos(2*b*x+2*a)+
(-32*(d*x+c)^4*b^4+24*d^2*(d*x+c)^2*b^2-3*d^4)*cos(4*b*x+4*a)+256*d*((d*x+
c)^2*b^2-3/2*d^2)*(d*x+c)*b*sin(2*b*x+2*a)+32*d*(d*x+c)*((d*x+c)^2*b^2-3/8
*d^2)*b*sin(4*b*x+4*a)+160*b^4*c^4-408*b^2*c^2*d^2+195*d^4)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.50

$$\int (c+dx)^4 \cos^3(a+bx)\sin(a+bx)\,dx$$
$$= \frac{12\,b^4d^4x^4 + 48\,b^4cd^3x^3 - (32\,b^4d^4x^4 + 128\,b^4cd^3x^3 + 32\,b^4c^4 - 24\,b^2c^2d^2 + 3\,d^4 + 24\,(8\,b^4c^2d^2 - b^2d^4)x^2 -$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/128*(12*b^4*d^4*x^4 + 48*b^4*c*d^3*x^3 - (32*b^4*d^4*x^4 + 128*b^4*c*d^3
*x^3 + 32*b^4*c^4 - 24*b^2*c^2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*
x^2 + 16*(8*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^4 + 9*(8*b^4*c^2*d^2
- 5*b^2*d^4)*x^2 + 9*(8*b^2*d^4*x^2 + 16*b^2*c*d^3*x + 8*b^2*c^2*d^2 - 5*d
^4)*cos(b*x + a)^2 + 6*(8*b^4*c^3*d - 15*b^2*c*d^3)*x + 2*(2*(8*b^3*d^4*x^
3 + 24*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(8*b^3*c^2*d^2 - b*d^4)
*x)*cos(b*x + a)^3 + 3*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 1
5*b*c*d^3 + 3*(8*b^3*c^2*d^2 - 5*b*d^4)*x)*cos(b*x + a))*sin(b*x + a))/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 935 vs. 2(252) = 504.

Time = 0.93 (sec) , antiderivative size = 935, normalized size of antiderivative = 3.71

$$\int (c+dx)^4 \cos^3(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**4*cos(b*x+a)**3*sin(b*x+a),x)
```

output

```
Piecewise((-c**4*cos(a + b*x)**4/(4*b) + 3*c**3*d*x*sin(a + b*x)**4/(8*b)
+ 3*c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(4*b) - 5*c**3*d*x*cos(a + b*
x)**4/(8*b) + 9*c**2*d**2*x**2*sin(a + b*x)**4/(16*b) + 9*c**2*d**2*x**2*s
in(a + b*x)**2*cos(a + b*x)**2/(8*b) - 15*c**2*d**2*x**2*cos(a + b*x)**4/(
16*b) + 3*c*d**3*x**3*sin(a + b*x)**4/(8*b) + 3*c*d**3*x**3*sin(a + b*x)**
2*cos(a + b*x)**2/(4*b) - 5*c*d**3*x**3*cos(a + b*x)**4/(8*b) + 3*d**4*x**
4*sin(a + b*x)**4/(32*b) + 3*d**4*x**4*sin(a + b*x)**2*cos(a + b*x)**2/(16
*b) - 5*d**4*x**4*cos(a + b*x)**4/(32*b) + 3*c**3*d*sin(a + b*x)**3*cos(a
+ b*x)/(8*b**2) + 5*c**3*d*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 9*c**2*
d**2*x*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 15*c**2*d**2*x*sin(a + b*x)
*cos(a + b*x)**3/(8*b**2) + 9*c*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)/(8*
b**2) + 15*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 3*d**4*x**3
*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 5*d**4*x**3*sin(a + b*x)*cos(a +
b*x)**3/(8*b**2) - 9*c**2*d**2*sin(a + b*x)**4/(32*b**3) + 15*c**2*d**2*co
s(a + b*x)**4/(32*b**3) - 45*c*d**3*x*sin(a + b*x)**4/(64*b**3) - 9*c*d**3
*x*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**3) + 51*c*d**3*x*cos(a + b*x)**4
/(64*b**3) - 45*d**4*x**2*sin(a + b*x)**4/(128*b**3) - 9*d**4*x**2*sin(a +
 b*x)**2*cos(a + b*x)**2/(64*b**3) + 51*d**4*x**2*cos(a + b*x)**4/(128*b**
3) - 45*c*d**3*sin(a + b*x)**3*cos(a + b*x)/(64*b**4) - 51*c*d**3*sin(a +
b*x)*cos(a + b*x)**3/(64*b**4) - 45*d**4*x*sin(a + b*x)**3*cos(a + b*x)...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 967 vs. $2(230) = 460$.

Time $= 0.08$ (sec) , antiderivative size $= 967$, normalized size of antiderivative $= 3.84$

$$\int (c + dx)^4 \cos^3(a + bx) \sin(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output

```
-1/1024*(256*c^4*cos(b*x + a)^4 - 1024*a*c^3*d*cos(b*x + a)^4/b + 1536*a^2
*c^2*d^2*cos(b*x + a)^4/b^2 - 1024*a^3*c*d^3*cos(b*x + a)^4/b^3 + 256*a^4*
d^4*cos(b*x + a)^4/b^4 + 32*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*c
os(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*c^3*d/b - 96*(4*(
b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*
a) - 8*sin(2*b*x + 2*a))*a*c^2*d^2/b^2 + 96*(4*(b*x + a)*cos(4*b*x + 4*a)
+ 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*a
^2*c*d^3/b^3 - 32*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x +
 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*a^3*d^4/b^4 + 24*((8*(b*x +
 a)^2 - 1)*cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*
(b*x + a)*sin(4*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 -
48*((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*
x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*a
*c*d^3/b^3 + 24*((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2
 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) - 32*(b*x + a)*sin(2*
b*x + 2*a))*a^2*d^4/b^4 + 4*(4*(8*(b*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4
*a) + 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2
 - 1)*sin(4*b*x + 4*a) - 96*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*c*d^3/b^
3 - 4*(4*(8*(b*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) + 64*(2*(b*x + a)^
3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4...
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.43

$$\int (c+dx)^4 \cos^3(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{(32\,b^4d^4x^4+128\,b^4cd^3x^3+192\,b^4c^2d^2x^2+128\,b^4c^3dx+32\,b^4c^4-24\,b^2d^4x^2-48\,b^2cd^3x-24\,b^2c^2d^2+}{1024\,b^5}$$
$$-\frac{(2\,b^4d^4x^4+8\,b^4cd^3x^3+12\,b^4c^2d^2x^2+8\,b^4c^3dx+2\,b^4c^4-6\,b^2d^4x^2-12\,b^2cd^3x-6\,b^2c^2d^2+3\,d^4)\cos(2}{16\,b^5}$$
$$+\frac{(8\,b^3d^4x^3+24\,b^3cd^3x^2+24\,b^3c^2d^2x+8\,b^3c^3d-3\,bd^4x-3\,bcd^3)\sin(4\,bx+4\,a)}{256\,b^5}$$
$$+\frac{(2\,b^3d^4x^3+6\,b^3cd^3x^2+6\,b^3c^2d^2x+2\,b^3c^3d-3\,bd^4x-3\,bcd^3)\sin(2\,bx+2\,a)}{8\,b^5}$$

input

```
integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/1024*(32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 192*b^4*c^2*d^2*x^2 + 128*b^
4*c^3*d*x + 32*b^4*c^4 - 24*b^2*d^4*x^2 - 48*b^2*c*d^3*x - 24*b^2*c^2*d^2
+ 3*d^4)*cos(4*b*x + 4*a)/b^5 - 1/16*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12
*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^
3*x - 6*b^2*c^2*d^2 + 3*d^4)*cos(2*b*x + 2*a)/b^5 + 1/256*(8*b^3*d^4*x^3 +
 24*b^3*c*d^3*x^2 + 24*b^3*c^2*d^2*x + 8*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3
)*sin(4*b*x + 4*a)/b^5 + 1/8*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*
d^2*x + 2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

Mupad [B] (verification not implemented)

Time = 19.17 (sec) , antiderivative size = 576, normalized size of antiderivative = 2.29

$$\int (c+dx)^4 \cos^3(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{192\,d^4\cos(2\,a+2\,b\,x)+3\,d^4\cos(4\,a+4\,b\,x)+128\,b^4\,c^4\cos(2\,a+2\,b\,x)+32\,b^4\,c^4\cos(4\,a+4\,b\,x)-}{}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^4,x)
```

output
```
-(192*d^4*cos(2*a + 2*b*x) + 3*d^4*cos(4*a + 4*b*x) + 128*b^4*c^4*cos(2*a
+ 2*b*x) + 32*b^4*c^4*cos(4*a + 4*b*x) - 256*b^3*c^3*d*sin(2*a + 2*b*x) -
32*b^3*c^3*d*sin(4*a + 4*b*x) - 384*b^2*c^2*d^2*cos(2*a + 2*b*x) - 24*b^2*
c^2*d^2*cos(4*a + 4*b*x) - 384*b^2*d^4*x^2*cos(2*a + 2*b*x) - 24*b^2*d^4*x
^2*cos(4*a + 4*b*x) + 128*b^4*d^4*x^4*cos(2*a + 2*b*x) + 32*b^4*d^4*x^4*co
s(4*a + 4*b*x) - 256*b^3*d^4*x^3*sin(2*a + 2*b*x) - 32*b^3*d^4*x^3*sin(4*a
 + 4*b*x) + 384*b*c*d^3*sin(2*a + 2*b*x) + 12*b*c*d^3*sin(4*a + 4*b*x) + 3
84*b*d^4*x*sin(2*a + 2*b*x) + 12*b*d^4*x*sin(4*a + 4*b*x) + 768*b^4*c^2*d^
2*x^2*cos(2*a + 2*b*x) + 192*b^4*c^2*d^2*x^2*cos(4*a + 4*b*x) - 768*b^2*c*
d^3*x*cos(2*a + 2*b*x) + 512*b^4*c^3*d*x*cos(2*a + 2*b*x) - 48*b^2*c*d^3*x
*cos(4*a + 4*b*x) + 128*b^4*c^3*d*x*cos(4*a + 4*b*x) + 512*b^4*c*d^3*x^3*c
os(2*a + 2*b*x) + 128*b^4*c*d^3*x^3*cos(4*a + 4*b*x) - 768*b^3*c^2*d^2*x*s
in(2*a + 2*b*x) - 768*b^3*c*d^3*x^2*sin(2*a + 2*b*x) - 96*b^3*c^2*d^2*x*si
n(4*a + 4*b*x) - 96*b^3*c*d^3*x^2*sin(4*a + 4*b*x))/(1024*b^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 698, normalized size of antiderivative = 2.77

$$\int (c+dx)^4 \cos^3(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input

```
int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x)
```

output

```
( - 32*cos(a + b*x)*sin(a + b*x)**3*b**3*c**3*d - 96*cos(a + b*x)*sin(a +
b*x)**3*b**3*c**2*d**2*x - 96*cos(a + b*x)*sin(a + b*x)**3*b**3*c*d**3*x**
2 - 32*cos(a + b*x)*sin(a + b*x)**3*b**3*d**4*x**3 + 12*cos(a + b*x)*sin(a
 + b*x)**3*b*c*d**3 + 12*cos(a + b*x)*sin(a + b*x)**3*b*d**4*x + 80*cos(a
+ b*x)*sin(a + b*x)*b**3*c**3*d + 240*cos(a + b*x)*sin(a + b*x)*b**3*c**2*
d**2*x + 240*cos(a + b*x)*sin(a + b*x)*b**3*c*d**3*x**2 + 80*cos(a + b*x)*
sin(a + b*x)*b**3*d**4*x**3 - 102*cos(a + b*x)*sin(a + b*x)*b*c*d**3 - 102
*cos(a + b*x)*sin(a + b*x)*b*d**4*x - 32*sin(a + b*x)**4*b**4*c**4 - 128*s
in(a + b*x)**4*b**4*c**3*d*x - 192*sin(a + b*x)**4*b**4*c**2*d**2*x**2 - 1
28*sin(a + b*x)**4*b**4*c*d**3*x**3 - 32*sin(a + b*x)**4*b**4*d**4*x**4 +
24*sin(a + b*x)**4*b**2*c**2*d**2 + 48*sin(a + b*x)**4*b**2*c*d**3*x + 24*
sin(a + b*x)**4*b**2*d**4*x**2 - 3*sin(a + b*x)**4*d**4 + 64*sin(a + b*x)*
*2*b**4*c**4 + 256*sin(a + b*x)**2*b**4*c**3*d*x + 384*sin(a + b*x)**2*b**
4*c**2*d**2*x**2 + 256*sin(a + b*x)**2*b**4*c*d**3*x**3 + 64*sin(a + b*x)*
*2*b**4*d**4*x**4 - 120*sin(a + b*x)**2*b**2*c**2*d**2 - 240*sin(a + b*x)*
*2*b**2*c*d**3*x - 120*sin(a + b*x)**2*b**2*d**4*x**2 + 51*sin(a + b*x)**2
*d**4 - 64*b**4*c**4 - 80*b**4*c**3*d*x - 120*b**4*c**2*d**2*x**2 - 80*b**
4*c*d**3*x**3 - 20*b**4*d**4*x**4 + 120*b**2*c**2*d**2 + 102*b**2*c*d**3*x
 + 51*b**2*d**4*x**2 - 51*d**4)/(128*b**5)
```

3.138 $\int (c+dx)^3 \cos^3(a+bx)\sin(a+bx)\,dx$

Optimal result 1281
Mathematica [A] (verified) 1282
Rubi [A] (verified) 1282
Maple [A] (verified) 1286
Fricas [A] (verification not implemented) 1287
Sympy [B] (verification not implemented) 1287
Maxima [B] (verification not implemented) 1288
Giac [A] (verification not implemented) 1289
Mupad [B] (verification not implemented) 1290
Reduce [B] (verification not implemented) 1290

Optimal result

Integrand size = 22, antiderivative size = 196

$$\begin{aligned}
&\int (c+dx)^3 \cos^3(a+bx)\sin(a+bx)\,dx \\
&= -\frac{45d^3x}{256b^3} + \frac{3(c+dx)^3}{32b} + \frac{9d^2(c+dx)\cos^2(a+bx)}{32b^3} \\
&\quad + \frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3} - \frac{(c+dx)^3\cos^4(a+bx)}{4b} \\
&\quad - \frac{45d^3\cos(a+bx)\sin(a+bx)}{256b^4} + \frac{9d(c+dx)^2\cos(a+bx)\sin(a+bx)}{32b^2} \\
&\quad - \frac{3d^3\cos^3(a+bx)\sin(a+bx)}{128b^4} + \frac{3d(c+dx)^2\cos^3(a+bx)\sin(a+bx)}{16b^2}
\end{aligned}$$

output

```
-45/256*d^3*x/b^3+3/32*(d*x+c)^3/b+9/32*d^2*(d*x+c)*cos(b*x+a)^2/b^3+3/32*
d^2*(d*x+c)*cos(b*x+a)^4/b^3-1/4*(d*x+c)^3*cos(b*x+a)^4/b-45/256*d^3*cos(b
*x+a)*sin(b*x+a)/b^4+9/32*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^2-3/128*d^3*
cos(b*x+a)^3*sin(b*x+a)/b^4+3/16*d*(d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.69

$$\int (c+dx)^3 \cos^3(a+bx)\sin(a+bx)\,dx$$
$$= \frac{-64b(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\cos(2(a+bx)) - 4b(c+dx)\left(-3d^2+8b^2(c+dx)^2\right)\cos(4(a+bx)) -}{1024b^4}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
(-64*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] - 4*b*(c +
d*x)*(-3*d^2 + 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] + 6*d*(16*(-d^2 + 2*b^2
*(c + d*x)^2) + (-d^2 + 8*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*
x)])/(1024*b^4)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.24, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {4905, 3042, 3792, 3042, 3115, 3042, 3115, 24, 3792, 17, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin(a+bx)\cos^3(a+bx)\,dx$$
$$\downarrow 4905$$
$$\frac{3d\int (c+dx)^2\cos^4(a+bx)dx}{4b} - \frac{(c+dx)^3\cos^4(a+bx)}{4b}$$
$$\downarrow 3042$$
$$\frac{3d\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^4 dx}{4b} - \frac{(c+dx)^3\cos^4(a+bx)}{4b}$$
$$\downarrow 3792$$

$$\frac{3d\left(-\frac{d^2\int\cos^4(a+bx)dx}{8b^2}+\frac{3}{4}\int(c+dx)^2\cos^2(a+bx)dx+\frac{d(c+dx)\cos^4(a+bx)}{8b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{4b}-\frac{(c+dx)^3\cos^4(a+bx)}{4b}$$

$$\downarrow 3042$$

$$\frac{3d\left(-\frac{d^2\int\sin\left(a+bx+\frac{\pi}{2}\right)^4dx}{8b^2}+\frac{3}{4}\int(c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{d(c+dx)\cos^4(a+bx)}{8b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{4b}-\frac{(c+dx)^3\cos^4(a+bx)}{4b}$$

$$\downarrow 3115$$

$$\frac{3d\left(-\frac{d^2\left(\frac{3}{4}\int\cos^2(a+bx)dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int(c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{d(c+dx)\cos^4(a+bx)}{8b^2}+\frac{(c+dx)^2\sin(a}{4b}\right.}{4b}-\frac{(c+dx)^3\cos^4(a+bx)}{4b}$$

$$\downarrow 3042$$

$$\frac{3d\left(-\frac{d^2\left(\frac{3}{4}\int\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int(c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{d(c+dx)\cos^4(a+bx)}{8b^2}+\frac{(c+dx)^2\si}{4b}\right.}{4b}-\frac{(c+dx)^3\cos^4(a+bx)}{4b}$$

$$\downarrow 3115$$

$$\frac{3d\left(-\frac{d^2\left(\frac{3}{4}\left(\frac{\int1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int(c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{d(c+dx)\cos^4(a+bx)}{8b^2}+\frac{(}{4b}\right.}{4b}-\frac{(c+dx)^3\cos^4(a+bx)}{4b}$$

$$\downarrow 24$$

$$\frac{3d\left(\frac{3}{4}\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{d(c+dx)\cos^4(a+bx)}{8b^2}-\frac{d^2\left(\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)\right)}{8b^2}+\frac{(c+dx)}{4b}\right.}{\frac{(c+dx)^3\cos^4(a+bx)}{4b}}$$

$$\downarrow 3792$$

$$\frac{3d\left(\frac{3}{4}\left(-\frac{d^2\int\cos^2(a+bx)dx}{2b^2}+\frac{1}{2}\int(c+dx)^2dx+\frac{d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{d(c+dx)\cos^4(a+bx)}{8b^2}-\right.}{\frac{(c+dx)^3\cos^4(a+bx)}{4b}}$$

$$\downarrow 17$$

$$\frac{3d\left(\frac{3}{4}\left(-\frac{d^2\int\cos^2(a+bx)dx}{2b^2}+\frac{d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)+\frac{d(c+dx)\cos^4(a+bx)}{8b^2}-\frac{d^2\left(\frac{\sin(a+\right.}{4b}\right.}{\frac{(c+dx)^3\cos^4(a+bx)}{4b}}$$

$$\downarrow 3042$$

$$\frac{3d\left(\frac{3}{4}\left(-\frac{d^2\int\sin\left(a+bx+\frac{\pi}{2}\right)^2dx}{2b^2}+\frac{d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)+\frac{d(c+dx)\cos^4(a+bx)}{8b^2}-\frac{d^2\left(\frac{si}{4b}\right.}{\frac{(c+dx)^3\cos^4(a+bx)}{4b}}$$

$$\downarrow 3115$$

$$\frac{3d\left(\frac{3}{4}\left(-\frac{d^2\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)+\frac{d(c+dx)\cos^4(a+bx)}{8b^2}\right.}{\frac{(c+dx)^3\cos^4(a+bx)}{4b}}$$

$$\downarrow 24$$

$$\frac{3d\left(\frac{3}{4}\left(\frac{d(c+dx)\cos^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)}{2b^2}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)+\frac{d(c+dx)\cos^4(a+bx)}{8b^2}-\frac{d^2(}{}\right.}{4b}$$

$$\frac{(c+dx)^3\cos^4(a+bx)}{4b}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
-1/4*((c + d*x)^3*Cos[a + b*x]^4)/b + (3*d*((d*(c + d*x)*Cos[a + b*x]^4)/(
8*b^2) + ((c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x])/(4*b) - (d^2*((Cos[a +
b*x]^3*Sin[a + b*x])/(4*b) + (3*(x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)))
/4))/(8*b^2) + (3*((c + d*x)^3/(6*d) + (d*(c + d*x)*Cos[a + b*x]^2)/(2*b^2
) + ((c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(2*b) - (d^2*(x/2 + (Cos[a + b
*x]*Sin[a + b*x])/(2*b)))/(2*b^2)))/4))/(4*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)  Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)  Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))  Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4905

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))  Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.76

method	result
parallelrisch	$\frac{-32\left((dx+c)^2b^2-\frac{3d^2}{2}\right)(dx+c)b\cos(2bx+2a)-8\left((dx+c)^2b^2-\frac{3d^2}{8}\right)(dx+c)b\cos(4bx+4a)+48d\left((dx+c)^2b^2-\frac{d^2}{2}\right)\sin(2$
risch	$-\frac{\left(8b^2d^3x^3+24b^2cd^2x^2+24b^2c^2dx+8b^2c^3-3d^3x-3cd^2\right)\cos(4bx+4a)}{256b^3}+\frac{3d\left(8x^2d^2b^2+16b^2cdx+8b^2c^2-d^2\right)\sin(4bx$
derivativedivides	$\frac{a^3d^3\cos(bx+a)^4}{4b^3}-\frac{3a^2cd^2\cos(bx+a)^4}{4b^2}+\frac{3a^2d^3\left(-\frac{(bx+a)\cos(bx+a)^4}{4}+\frac{\left(\cos(bx+a)^3+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{16}+\frac{3bx}{32}+\frac{3a}{32}\right)}{b^3}+\frac{3a$
default	$\frac{a^3d^3\cos(bx+a)^4}{4b^3}-\frac{3a^2cd^2\cos(bx+a)^4}{4b^2}+\frac{3a^2d^3\left(-\frac{(bx+a)\cos(bx+a)^4}{4}+\frac{\left(\cos(bx+a)^3+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{16}+\frac{3bx}{32}+\frac{3a}{32}\right)}{b^3}+\frac{3a$
orering	$\frac{15d\left(8d^4x^4b^4+32b^4cd^3x^3+48b^4c^2d^2x^2+32b^4c^3dx+8b^4c^4-2b^2d^4x^2-4b^2cd^3x-2b^2c^2d^2-9d^4\right)\cos(bx+a)^3\sin(bx+a)}{64b^6(dx+c)^2}$

input

```
int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/256*(-32*((d*x+c)^2*b^2-3/2*d^2)*(d*x+c)*b*cos(2*b*x+2*a)-8*((d*x+c)^2*b
^2-3/8*d^2)*(d*x+c)*b*cos(4*b*x+4*a)+48*d*((d*x+c)^2*b^2-1/2*d^2)*sin(2*b*
x+2*a)+6*d*((d*x+c)^2*b^2-1/8*d^2)*sin(4*b*x+4*a)+40*b^3*c^3-51*c*d^2*b)/b
^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.21

$$\int (c+dx)^3 \cos^3(a+bx)\sin(a+bx)\,dx$$
$$= \frac{24\,b^3d^3x^3 + 72\,b^3cd^2x^2 - 8\,(8\,b^3d^3x^3 + 24\,b^3cd^2x^2 + 8\,b^3c^3 - 3\,bcd^2 + 3\,(8\,b^3c^2d - bd^3)x)\cos(bx+a)^4 +$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/256*(24*b^3*d^3*x^3 + 72*b^3*c*d^2*x^2 - 8*(8*b^3*d^3*x^3 + 24*b^3*c*d^2
*x^2 + 8*b^3*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^4 +
 72*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 + 9*(8*b^3*c^2*d - 5*b*d^3)*x + 3*(
2*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^3 + 3*
(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - 5*d^3)*cos(b*x + a))*sin(b
*x + a))/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. 2(197) = 394.

Time = 0.64 (sec) , antiderivative size = 602, normalized size of antiderivative = 3.07

$$\int (c+dx)^3 \cos^3(a+bx)\sin(a+bx)\,dx$$
$$= \begin{cases} -\frac{c^3\cos^4(a+bx)}{4b} + \frac{9c^2dx\sin^4(a+bx)}{32b} + \frac{9c^2dx\sin^2(a+bx)\cos^2(a+bx)}{16b} - \frac{15c^2dx\cos^4(a+bx)}{32b} + \frac{9cd^2x^2\sin^4(a+bx)}{32b} + \frac{9cd^2x^2\sin^2(} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin(a)\cos^3(a) \end{cases}$$

input
```
integrate((d*x+c)**3*cos(b*x+a)**3*sin(b*x+a),x)
```

output

```
Piecewise((-c**3*cos(a + b*x)**4/(4*b) + 9*c**2*d*x*sin(a + b*x)**4/(32*b)
 + 9*c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 15*c**2*d*x*cos(a +
 b*x)**4/(32*b) + 9*c*d**2*x**2*sin(a + b*x)**4/(32*b) + 9*c*d**2*x**2*sin
(a + b*x)**2*cos(a + b*x)**2/(16*b) - 15*c*d**2*x**2*cos(a + b*x)**4/(32*b
) + 3*d**3*x**3*sin(a + b*x)**4/(32*b) + 3*d**3*x**3*sin(a + b*x)**2*cos(a
 + b*x)**2/(16*b) - 5*d**3*x**3*cos(a + b*x)**4/(32*b) + 9*c**2*d*sin(a +
b*x)**3*cos(a + b*x)/(32*b**2) + 15*c**2*d*sin(a + b*x)*cos(a + b*x)**3/(3
2*b**2) + 9*c*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 15*c*d**2*x*
sin(a + b*x)*cos(a + b*x)**3/(16*b**2) + 9*d**3*x**2*sin(a + b*x)**3*cos(a
 + b*x)/(32*b**2) + 15*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(32*b**2) -
9*c*d**2*sin(a + b*x)**4/(64*b**3) + 15*c*d**2*cos(a + b*x)**4/(64*b**3) -
 45*d**3*x*sin(a + b*x)**4/(256*b**3) - 9*d**3*x*sin(a + b*x)**2*cos(a + b
*x)**2/(128*b**3) + 51*d**3*x*cos(a + b*x)**4/(256*b**3) - 45*d**3*sin(a +
 b*x)**3*cos(a + b*x)/(256*b**4) - 51*d**3*sin(a + b*x)*cos(a + b*x)**3/(2
56*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/
4)*sin(a)*cos(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(178) = 356$.

Time $= 0.05$ (sec) , antiderivative size $= 549$, normalized size of antiderivative $= 2.80$

$$\int (c+dx)^3 \cos^3(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/1024*(256*c^3*cos(b*x + a)^4 - 768*a*c^2*d*cos(b*x + a)^4/b + 768*a^2*c
*d^2*cos(b*x + a)^4/b^2 - 256*a^3*d^3*cos(b*x + a)^4/b^3 + 24*(4*(b*x + a)
*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*s
in(2*b*x + 2*a))*c^2*d/b - 48*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)
*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*a*c*d^2/b^2 + 2
4*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*
x + 4*a) - 8*sin(2*b*x + 2*a))*a^2*d^3/b^3 + 12*((8*(b*x + a)^2 - 1)*cos(4
*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*
b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*c*d^2/b^2 - 12*((8*(b*x + a)^2
 - 1)*cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x
+ a)*sin(4*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*a*d^3/b^3 + (4*(8*(
b*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) + 64*(2*(b*x + a)^3 - 3*b*x - 3
*a)*cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) - 96*(2*(b*x
 + a)^2 - 1)*sin(2*b*x + 2*a))*d^3/b^3)/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.23

$$\begin{aligned}
&\int (c+dx)^3 \cos^3(a+bx)\sin(a+bx)\,dx \\
&\quad = -\frac{\left(8\,b^3d^3x^3+24\,b^3cd^2x^2+24\,b^3c^2dx+8\,b^3c^3-3\,bd^3x-3\,bcd^2\right)\cos\left(4\,bx+4\,a\right)}{256\,b^4} \\
&\qquad -\frac{\left(2\,b^3d^3x^3+6\,b^3cd^2x^2+6\,b^3c^2dx+2\,b^3c^3-3\,bd^3x-3\,bcd^2\right)\cos\left(2\,bx+2\,a\right)}{16\,b^4} \\
&\qquad +\frac{3\left(8\,b^2d^3x^2+16\,b^2cd^2x+8\,b^2c^2d-d^3\right)\sin\left(4\,bx+4\,a\right)}{1024\,b^4} \\
&\qquad +\frac{3\left(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3\right)\sin\left(2\,bx+2\,a\right)}{32\,b^4}
\end{aligned}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/256*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 3*
b*d^3*x - 3*b*c*d^2)*cos(4*b*x + 4*a)/b^4 - 1/16*(2*b^3*d^3*x^3 + 6*b^3*c*
d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(2*b*x + 2
*a)/b^4 + 3/1024*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*sin(
4*b*x + 4*a)/b^4 + 3/32*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3
)*sin(2*b*x + 2*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 19.13 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.87

$$\int (c+dx)^3 \cos^3(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{24\,d^3\,\sin(2\,a+2\,b\,x)+\frac{3\,d^3\,\sin(4\,a+4\,b\,x)}{4}+32\,b^3\,c^3\,\cos(2\,a+2\,b\,x)+8\,b^3\,c^3\,\cos(4\,a+4\,b\,x)-48\,b^2\,c^2\,d}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^3,x)
```

output
```
-(24*d^3*sin(2*a + 2*b*x) + (3*d^3*sin(4*a + 4*b*x))/4 + 32*b^3*c^3*cos(2*
a + 2*b*x) + 8*b^3*c^3*cos(4*a + 4*b*x) - 48*b^2*c^2*d*sin(2*a + 2*b*x) -
6*b^2*c^2*d*sin(4*a + 4*b*x) + 32*b^3*d^3*x^3*cos(2*a + 2*b*x) + 8*b^3*d^3
*x^3*cos(4*a + 4*b*x) - 48*b^2*d^3*x^2*sin(2*a + 2*b*x) - 6*b^2*d^3*x^2*si
n(4*a + 4*b*x) - 48*b*c*d^2*cos(2*a + 2*b*x) - 3*b*c*d^2*cos(4*a + 4*b*x)
- 48*b*d^3*x*cos(2*a + 2*b*x) - 3*b*d^3*x*cos(4*a + 4*b*x) + 96*b^3*c^2*d*
x*cos(2*a + 2*b*x) + 24*b^3*c^2*d*x*cos(4*a + 4*b*x) - 96*b^2*c*d^2*x*sin(
2*a + 2*b*x) - 12*b^2*c*d^2*x*sin(4*a + 4*b*x) + 96*b^3*c*d^2*x^2*cos(2*a
+ 2*b*x) + 24*b^3*c*d^2*x^2*cos(4*a + 4*b*x))/(256*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.25

$$\int (c+dx)^3 \cos^3(a+bx)\sin(a+bx)\,dx$$
$$=\frac{120\cos(bx+a)\sin(bx+a)\,b^2c^2d+120\cos(bx+a)\sin(bx+a)\,b^2d^3x^2+384\sin(bx+a)^2\,b^3c^2dx+384}$$

input
```
int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x)
```

output

```
( - 48*cos(a + b*x)*sin(a + b*x)**3*b**2*c**2*d - 96*cos(a + b*x)*sin(a +
b*x)**3*b**2*c*d**2*x - 48*cos(a + b*x)*sin(a + b*x)**3*b**2*d**3*x**2 + 6
*cos(a + b*x)*sin(a + b*x)**3*d**3 + 120*cos(a + b*x)*sin(a + b*x)*b**2*c*
*2*d + 240*cos(a + b*x)*sin(a + b*x)*b**2*c*d**2*x + 120*cos(a + b*x)*sin(
a + b*x)*b**2*d**3*x**2 - 51*cos(a + b*x)*sin(a + b*x)*d**3 - 64*sin(a + b
*x)**4*b**3*c**3 - 192*sin(a + b*x)**4*b**3*c**2*d*x - 192*sin(a + b*x)**4
*b**3*c*d**2*x**2 - 64*sin(a + b*x)**4*b**3*d**3*x**3 + 24*sin(a + b*x)**4
*b*c*d**2 + 24*sin(a + b*x)**4*b*d**3*x + 128*sin(a + b*x)**2*b**3*c**3 +
384*sin(a + b*x)**2*b**3*c**2*d*x + 384*sin(a + b*x)**2*b**3*c*d**2*x**2 +
 128*sin(a + b*x)**2*b**3*d**3*x**3 - 120*sin(a + b*x)**2*b*c*d**2 - 120*s
in(a + b*x)**2*b*d**3*x - 128*b**3*c**3 - 120*b**3*c**2*d*x - 120*b**3*c*d
**2*x**2 - 40*b**3*d**3*x**3 + 120*b*c*d**2 + 51*b*d**3*x)/(256*b**4)
```

3.139 $\int (c+dx)^2 \cos^3(a+bx)\sin(a+bx)\,dx$

Optimal result 1292
Mathematica [A] (verified) 1293
Rubi [A] (verified) 1293
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Optimal result

Integrand size = 22, antiderivative size = 125

$$\begin{aligned}\int (c+dx)^2 \cos^3(a+bx)\sin(a+bx)\,dx = {} & \frac{3(c+dx)^2}{32b} + \frac{3d^2\cos^2(a+bx)}{32b^3} \\ & + \frac{d^2\cos^4(a+bx)}{32b^3} - \frac{(c+dx)^2\cos^4(a+bx)}{4b} \\ & + \frac{3d(c+dx)\cos(a+bx)\sin(a+bx)}{16b^2} \\ & + \frac{d(c+dx)\cos^3(a+bx)\sin(a+bx)}{8b^2}\end{aligned}$$

output

```
3/32*(d*x+c)^2/b+3/32*d^2*cos(b*x+a)^2/b^3+1/32*d^2*cos(b*x+a)^4/b^3-1/4*(
d*x+c)^2*cos(b*x+a)^4/b+3/16*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2+1/8*d*(d*
x+c)*cos(b*x+a)^3*sin(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.71

$$\int (c+dx)^2 \cos^3(a+bx)\sin(a+bx)\,dx$$
$$= \frac{-16(-d^2+2b^2(c+dx)^2)\cos(2(a+bx)) + (d^2-8b^2(c+dx)^2)\cos(4(a+bx)) + 4bd(c+dx)(8\sin(2(a+$$
$$256b^3$$

input
```
Integrate[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
(-16*(-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + (d^2 - 8*b^2*(c + d*x)^
2)*Cos[4*(a + b*x)] + 4*b*d*(c + d*x)*(8*Sin[2*(a + b*x)] + Sin[4*(a + b*x
)]))/(256*b^3)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4905, 3042, 3791, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin(a+bx)\cos^3(a+bx)\,dx$$
$$\downarrow 4905$$
$$\frac{d\int (c+dx)\cos^4(a+bx)dx}{2b} - \frac{(c+dx)^2\cos^4(a+bx)}{4b}$$
$$\downarrow 3042$$
$$\frac{d\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^4 dx}{2b} - \frac{(c+dx)^2\cos^4(a+bx)}{4b}$$
$$\downarrow 3791$$

$$\frac{d\left(\frac{3}{4}\int (c+dx)\cos^2(a+bx)dx+\frac{d\cos^4(a+bx)}{16b^2}+\frac{(c+dx)\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{2b}-\frac{(c+dx)^2\cos^4(a+bx)}{4b}$$

$$\downarrow 3042$$

$$\frac{d\left(\frac{3}{4}\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^2 dx+\frac{d\cos^4(a+bx)}{16b^2}+\frac{(c+dx)\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{2b}-\frac{(c+dx)^2\cos^4(a+bx)}{4b}$$

$$\downarrow 3791$$

$$\frac{d\left(\frac{3}{4}\left(\frac{1}{2}\int (c+dx)dx+\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{d\cos^4(a+bx)}{16b^2}+\frac{(c+dx)\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{2b}-\frac{(c+dx)^2\cos^4(a+bx)}{4b}$$

$$\downarrow 17$$

$$\frac{d\left(\frac{3}{4}\left(\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)+\frac{d\cos^4(a+bx)}{16b^2}+\frac{(c+dx)\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{2b}-\frac{(c+dx)^2\cos^4(a+bx)}{4b}$$

input

```
Int[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output

```
-1/4*((c + d*x)^2*Cos[a + b*x]^4)/b + (d*((d*Cos[a + b*x]^4)/(16*b^2) + ((
c + d*x)*Cos[a + b*x]^3*Sin[a + b*x])/(4*b) + (3*((c + d*x)^2/(4*d) + (d*C
os[a + b*x]^2)/(4*b^2) + ((c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b)))/4))
/(2*b)
```

Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4905

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.87

method	result
parallelrisch	$\frac{16\left(-2(dx+c)^2b^2+d^2\right)\cos(2bx+2a)+\left(-8(dx+c)^2b^2+d^2\right)\cos(4bx+4a)+32bd(dx+c)\sin(2bx+2a)+4bd(dx+c)\sin(4b}{256b^3}$
risch	$-\frac{\left(8x^2d^2b^2+16b^2cdx+8b^2c^2-d^2\right)\cos(4bx+4a)}{256b^3}+\frac{d(dx+c)\sin(4bx+4a)}{64b^2}-\frac{\left(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2\right)\cos(2bx}{16b^3}$
derivativedivides	$-\frac{a^2d^2\cos(bx+a)^4}{4b^2}+\frac{acd\cos(bx+a)^4}{2b}-\frac{2a\,d^2\left(-\frac{(bx+a)\cos(bx+a)^4}{4}+\frac{\left(\cos(bx+a)^3+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{16}+\frac{3bx}{32}+\frac{3a}{32}\right)}{b^2}-\frac{c^2\,co}{}$
default	$-\frac{a^2d^2\cos(bx+a)^4}{4b^2}+\frac{acd\cos(bx+a)^4}{2b}-\frac{2a\,d^2\left(-\frac{(bx+a)\cos(bx+a)^4}{4}+\frac{\left(\cos(bx+a)^3+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{16}+\frac{3bx}{32}+\frac{3a}{32}\right)}{b^2}-\frac{c^2\,co}{}$
norman	$-\frac{5d^2x^2}{32b}-\frac{3d^2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4}{4b^3}+\frac{\left(16b^2c^2-5d^2\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{8b^3}+\frac{\left(16b^2c^2-5d^2\right)\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^6}{8b^3}+\frac{5cd\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b^2}-\frac{3cd\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b^2}$
orering	$\frac{5d\left(16d^4x^4b^4+64b^4cd^3x^3+96b^4c^2d^2x^2+64b^4c^3dx+16b^4c^4+3b^2d^4x^2+6b^2cd^3x+3b^2c^2d^2-3d^4\right)\cos(bx+a)^3\sin(bx+a)}{64b^6(dx+c)^3}$

input
```
int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/256*(16*(-2*(d*x+c)^2*b^2+d^2)*cos(2*b*x+2*a)+(-8*(d*x+c)^2*b^2+d^2)*cos
(4*b*x+4*a)+32*b*d*(d*x+c)*sin(2*b*x+2*a)+4*b*d*(d*x+c)*sin(4*b*x+4*a)+40*
b^2*c^2-17*d^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.04

$$\int (c+dx)^2\cos^3(a+bx)\sin(a+bx)\,dx$$
$$=\frac{3\,b^2d^2x^2+6\,b^2cdx-\left(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2\right)\cos\left(bx+a\right)^4+3\,d^2\cos\left(bx+a\right)^2+2\left(2\left(bd^2x+\right.\right.}{32\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/32*(3*b^2*d^2*x^2 + 6*b^2*c*d*x - (8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*
c^2 - d^2)*cos(b*x + a)^4 + 3*d^2*cos(b*x + a)^2 + 2*(2*(b*d^2*x + b*c*d)*
cos(b*x + a)^3 + 3*(b*d^2*x + b*c*d)*cos(b*x + a))*sin(b*x + a))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(119) = 238$.

Time $= 0.46$ (sec) , antiderivative size $= 320$, normalized size of antiderivative $= 2.56$

$$\int (c+dx)^2 \cos^3(a+bx)\sin(a+bx)\,dx$$
$$= \begin{cases} -\frac{c^2\cos^4(a+bx)}{4b} + \frac{3cdx\sin^4(a+bx)}{16b} + \frac{3cdx\sin^2(a+bx)\cos^2(a+bx)}{8b} - \frac{5cdx\cos^4(a+bx)}{16b} + \frac{3d^2x^2\sin^4(a+bx)}{32b} + \frac{3d^2x^2\sin^2(a+bx)}{16b} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin(a)\cos^3(a) \end{cases}$$

input
```
integrate((d*x+c)**2*cos(b*x+a)**3*sin(b*x+a),x)
```

output
```
Piecewise((-c**2*cos(a + b*x)**4/(4*b) + 3*c*d*x*sin(a + b*x)**4/(16*b) +
3*c*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(8*b) - 5*c*d*x*cos(a + b*x)**4/(1
6*b) + 3*d**2*x**2*sin(a + b*x)**4/(32*b) + 3*d**2*x**2*sin(a + b*x)**2*co
s(a + b*x)**2/(16*b) - 5*d**2*x**2*cos(a + b*x)**4/(32*b) + 3*c*d*sin(a +
b*x)**3*cos(a + b*x)/(16*b**2) + 5*c*d*sin(a + b*x)*cos(a + b*x)**3/(16*b*
*2) + 3*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 5*d**2*x*sin(a + b
*x)*cos(a + b*x)**3/(16*b**2) - 3*d**2*sin(a + b*x)**4/(64*b**3) + 5*d**2*
cos(a + b*x)**4/(64*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*s
in(a)*cos(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(113) = 226$.

Time $= 0.04$ (sec) , antiderivative size $= 263$, normalized size of antiderivative $= 2.10$

$$\int (c+dx)^2 \cos^3(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{64\,c^2\cos(bx+a)^4 - \frac{128\,acd\cos(bx+a)^4}{b} + \frac{64\,a^2d^2\cos(bx+a)^4}{b^2} + \frac{4\,(4\,(bx+a)\cos(4\,bx+4\,a)+16\,(bx+a)\cos(2\,bx+2\,a)-\sin(4\,bx+}{b}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/256*(64*c^2*cos(b*x + a)^4 - 128*a*c*d*cos(b*x + a)^4/b + 64*a^2*d^2*co
s(b*x + a)^4/b^2 + 4*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*
x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*c*d/b - 4*(4*(b*x + a)*c
os(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin
(2*b*x + 2*a))*a*d^2/b^2 + ((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) + 16*(2*(
b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) - 32*(b*x
+ a)*sin(2*b*x + 2*a))*d^2/b^2)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.16

$$\begin{aligned}
&\int (c+dx)^2 \cos^3(a+bx)\sin(a+bx)\,dx \\
&= -\frac{(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2)\cos(4\,bx+4\,a)}{256\,b^3} \\
&\quad -\frac{(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2)\cos(2\,bx+2\,a)}{16\,b^3} \\
&\quad +\frac{(bd^2x+bcd)\sin(4\,bx+4\,a)}{64\,b^3}+\frac{(bd^2x+bcd)\sin(2\,bx+2\,a)}{8\,b^3}
\end{aligned}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/256*(8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(4*b*x + 4*a)/b
^3 - 1/16*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a)
/b^3 + 1/64*(b*d^2*x + b*c*d)*sin(4*b*x + 4*a)/b^3 + 1/8*(b*d^2*x + b*c*d)
*sin(2*b*x + 2*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 18.66 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.62

$$\int (c+dx)^2 \cos^3(a+bx)\sin(a+bx)\,dx$$
$$= \frac{8\,d^2\,\cos(2\,a+2\,b\,x) + \frac{d^2\,\cos(4\,a+4\,b\,x)}{2} - 16\,b^2\,c^2\,\cos(2\,a+2\,b\,x) - 4\,b^2\,c^2\,\cos(4\,a+4\,b\,x) + 16\,b\,c\,d\,\sin(2$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^2,x)
```

output
```
(8*d^2*cos(2*a + 2*b*x) + (d^2*cos(4*a + 4*b*x))/2 - 16*b^2*c^2*cos(2*a +
2*b*x) - 4*b^2*c^2*cos(4*a + 4*b*x) + 16*b*c*d*sin(2*a + 2*b*x) + 2*b*c*d*
sin(4*a + 4*b*x) - 16*b^2*d^2*x^2*cos(2*a + 2*b*x) - 4*b^2*d^2*x^2*cos(4*a
 + 4*b*x) + 16*b*d^2*x*sin(2*a + 2*b*x) + 2*b*d^2*x*sin(4*a + 4*b*x) - 32*
b^2*c*d*x*cos(2*a + 2*b*x) - 8*b^2*c*d*x*cos(4*a + 4*b*x))/(128*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.98

$$\int (c+dx)^2 \cos^3(a+bx)\sin(a+bx)\,dx$$
$$= \frac{-4\cos(bx+a)\sin(bx+a)^3\,bcd - 4\cos(bx+a)\sin(bx+a)^3\,b\,d^2x + 10\cos(bx+a)\sin(bx+a)\,bcd + 1$$

input
```
int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x)
```

output
```
( - 4*cos(a + b*x)*sin(a + b*x)**3*b*c*d - 4*cos(a + b*x)*sin(a + b*x)**3*
b*d**2*x + 10*cos(a + b*x)*sin(a + b*x)*b*c*d + 10*cos(a + b*x)*sin(a + b*
x)*b*d**2*x - 8*sin(a + b*x)**4*b**2*c**2 - 16*sin(a + b*x)**4*b**2*c*d*x
 - 8*sin(a + b*x)**4*b**2*d**2*x**2 + sin(a + b*x)**4*d**2 + 16*sin(a + b*x
)**2*b**2*c**2 + 32*sin(a + b*x)**2*b**2*c*d*x + 16*sin(a + b*x)**2*b**2*d
**2*x**2 - 5*sin(a + b*x)**2*d**2 + 22*a*b*c*d - 16*b**2*c**2 - 10*b**2*c*
d*x - 5*b**2*d**2*x**2 + 5*d**2)/(32*b**3)
```

3.140 $\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal result 1300
Mathematica [A] (verified) 1300
Rubi [A] (verified) 1301
Maple [A] (verified) 1303
Fricas [A] (verification not implemented) 1303
Sympy [A] (verification not implemented) 1304
Maxima [A] (verification not implemented) 1304
Giac [A] (verification not implemented) 1305
Mupad [B] (verification not implemented) 1305
Reduce [B] (verification not implemented) 1306

Optimal result

Integrand size = 20, antiderivative size = 72

$$\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx = \frac{3dx}{32b} - \frac{(c+dx)\cos^4(a+bx)}{4b} + \frac{3d\cos(a+bx)\sin(a+bx)}{32b^2} + \frac{d\cos^3(a+bx)\sin(a+bx)}{16b^2}$$

output

```
3/32*d*x/b-1/4*(d*x+c)*cos(b*x+a)^4/b+3/32*d*cos(b*x+a)*sin(b*x+a)/b^2+1/1
6*d*cos(b*x+a)^3*sin(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{c\cos^4(a+bx)}{4b} + \frac{d(-2bx\cos(2(a+bx)) + \sin(2(a+bx)))}{16b^2} + \frac{d(-4bx\cos(4(a+bx)) + \sin(4(a+bx)))}{128b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
-1/4*(c*Cos[a + b*x]^4)/b + (d*(-2*b*x*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]
))/(16*b^2) + (d*(-4*b*x*Cos[4*(a + b*x)] + Sin[4*(a + b*x)]))/(128*b^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4905, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin(a+bx)\cos^3(a+bx)\,dx$$

$$\downarrow 4905$$

$$\frac{d\int \cos^4(a+bx)dx}{4b} - \frac{(c+dx)\cos^4(a+bx)}{4b}$$

$$\downarrow 3042$$

$$\frac{d\int \sin\left(a+bx+\frac{\pi}{2}\right)^4 dx}{4b} - \frac{(c+dx)\cos^4(a+bx)}{4b}$$

$$\downarrow 3115$$

$$\frac{d\left(\frac{3}{4}\int \cos^2(a+bx)dx + \frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{4b} - \frac{(c+dx)\cos^4(a+bx)}{4b}$$

$$\downarrow 3042$$

$$\frac{d\left(\frac{3}{4}\int \sin\left(a+bx+\frac{\pi}{2}\right)^2 dx + \frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{4b} - \frac{(c+dx)\cos^4(a+bx)}{4b}$$

$$\downarrow 3115$$

$$\frac{d\left(\frac{3}{4}\left(\frac{\int 1dx}{2} + \frac{\sin(a+bx)\cos(a+bx)}{2b}\right) + \frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{4b} - \frac{(c+dx)\cos^4(a+bx)}{4b}$$

$$\downarrow 24$$

$$\frac{d\left(\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)\right)}{4b}-\frac{(c+dx)\cos^4(a+bx)}{4b}$$

input
```
Int[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
-1/4*((c + d*x)*Cos[a + b*x]^4)/b + (d*((Cos[a + b*x]^3*Sin[a + b*x])/(4*b
) + (3*(x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)))/4))/(4*b)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

method	result
parallelrisch	$\frac{-16(dx+c)b\cos(2bx+2a)-4(dx+c)b\cos(4bx+4a)+20cb+8d\sin(2bx+2a)+d\sin(4bx+4a)}{128b^2}$
risch	$-\frac{(dx+c)\cos(4bx+4a)}{32b}+\frac{d\sin(4bx+4a)}{128b^2}-\frac{(dx+c)\cos(2bx+2a)}{8b}+\frac{d\sin(2bx+2a)}{16b^2}$
derivativedivides	$\frac{\frac{da\cos(bx+a)^4}{4b}-\frac{c\cos(bx+a)^4}{4}+d\left(-\frac{(bx+a)\cos(bx+a)^4}{4}+\frac{\left(\cos(bx+a)^3+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{16}+\frac{3bx}{32}+\frac{3a}{32}\right)}{b}$
default	$\frac{\frac{da\cos(bx+a)^4}{4b}-\frac{c\cos(bx+a)^4}{4}+d\left(-\frac{(bx+a)\cos(bx+a)^4}{4}+\frac{\left(\cos(bx+a)^3+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{16}+\frac{3bx}{32}+\frac{3a}{32}\right)}{b}$
norman	$\frac{\frac{5d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{16b^2}-\frac{3d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3}{16b^2}+\frac{3d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^5}{16b^2}-\frac{5d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^7}{16b^2}-\frac{5dx}{32b}+\frac{2c\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{b}+\frac{2c\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^6}{b}+\frac{11dx\tan}{}}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)^4}$
orering	$\frac{d\left(5x^2d^2b^2+10b^2cdx+5b^2c^2+d^2\right)\cos(bx+a)^3\sin(bx+a)}{8b^4(dx+c)^2}-\frac{\left(5x^2d^2b^2+10b^2cdx+5b^2c^2+2d^2\right)\left(d\cos(bx+a)^3\sin(bx+a\right.}{16b^4(}$

input
```
int((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/128*(-16*(d*x+c)*b*cos(2*b*x+2*a)-4*(d*x+c)*b*cos(4*b*x+4*a)+20*c*b+8*d*
sin(2*b*x+2*a)+d*sin(4*b*x+4*a))/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{8\,(bdx+bc)\cos(bx+a)^4-3\,bdx-\left(2\,d\cos(bx+a)^3+3\,d\cos(bx+a)\right)\sin(bx+a)}{32\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/32*(8*(b*d*x + b*c)*cos(b*x + a)^4 - 3*b*d*x - (2*d*cos(b*x + a)^3 + 3*
d*cos(b*x + a))*sin(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.92

$$\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx$$
$$= \begin{cases} -\frac{c\cos^4(a+bx)}{4b} + \frac{3dx\sin^4(a+bx)}{32b} + \frac{3dx\sin^2(a+bx)\cos^2(a+bx)}{16b} - \frac{5dx\cos^4(a+bx)}{32b} + \frac{3d\sin^3(a+bx)\cos(a+bx)}{32b^2} + \frac{5d\sin(a+bx)\,c}{32b^2} \\ \left(cx + \frac{dx^2}{2}\right)\sin(a)\cos^3(a) \end{cases}$$

input
```
integrate((d*x+c)*cos(b*x+a)**3*sin(b*x+a),x)
```

output
```
Piecewise((-c*cos(a + b*x)**4/(4*b) + 3*d*x*sin(a + b*x)**4/(32*b) + 3*d*x
*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 5*d*x*cos(a + b*x)**4/(32*b) + 3
*d*sin(a + b*x)**3*cos(a + b*x)/(32*b**2) + 5*d*sin(a + b*x)*cos(a + b*x)*
*3/(32*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*sin(a)*cos(a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

$$\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{32\,c\cos(bx+a)^4 - \frac{32\,ad\cos(bx+a)^4}{b} + \frac{(4\,(bx+a)\cos(4\,bx+4\,a)+16\,(bx+a)\cos(2\,bx+2\,a)-\sin(4\,bx+4\,a)-8\,\sin(2\,bx+2\,a))d}{b}}{128\,b}$$

input
```
integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/128*(32*c*cos(b*x + a)^4 - 32*a*d*cos(b*x + a)^4/b + (4*(b*x + a)*cos(4
*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b
*x + 2*a))*d/b)/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{(bdx+bc)\cos(4\,bx+4\,a)}{32\,b^2} - \frac{(bdx+bc)\cos(2\,bx+2\,a)}{8\,b^2} + \frac{d\sin(4\,bx+4\,a)}{128\,b^2} + \frac{d\sin(2\,bx+2\,a)}{16\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/32*(b*d*x + b*c)*cos(4*b*x + 4*a)/b^2 - 1/8*(b*d*x + b*c)*cos(2*b*x + 2
*a)/b^2 + 1/128*d*sin(4*b*x + 4*a)/b^2 + 1/16*d*sin(2*b*x + 2*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.31

$$\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx = \frac{4\,d\,\sin(2\,a+2\,bx) + \frac{d\sin(4\,a+4\,bx)}{2} + 4\,bc\sin(2\,a+2\,bx)^2 + 16\,bc\sin(a+bx)^2 + 8\,bdx\left(2\sin(a+bx)^2\right.}{64\,b^2}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x),x)
```

output
```
(4*d*sin(2*a + 2*b*x) + (d*sin(4*a + 4*b*x))/2 + 4*b*c*sin(2*a + 2*b*x)^2
+ 16*b*c*sin(a + b*x)^2 + 8*b*d*x*(2*sin(a + b*x)^2 - 1) + 2*b*d*x*(2*sin(
2*a + 2*b*x)^2 - 1))/(64*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.40

$$\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx$$
$$= \frac{-2\cos(bx+a)\sin(bx+a)^3 d + 5\cos(bx+a)\sin(bx+a) d - 8\sin(bx+a)^4 bc - 8\sin(bx+a)^4 bdx +}{32b^2}$$

input
```
int((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x)
```

output
```
( - 2*cos(a + b*x)*sin(a + b*x)**3*d + 5*cos(a + b*x)*sin(a + b*x)*d - 8*s
in(a + b*x)**4*b*c - 8*sin(a + b*x)**4*b*d*x + 16*sin(a + b*x)**2*b*c + 16
*sin(a + b*x)**2*b*d*x - 5*a*d - 16*b*c - 5*b*d*x)/(32*b**2)
```

3.141 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx$

Optimal result . . . 1307
Mathematica [A] (verified) . . . 1308
Rubi [A] (verified) . . . 1308
Maple [A] (verified) . . . 1309
Fricas [A] (verification not implemented) . . . 1310
Sympy [F] . . . 1310
Maxima [C] (verification not implemented) . . . 1311
Giac [C] (verification not implemented) . . . 1311
Mupad [F(-1)] . . . 1312
Reduce [F] . . . 1313

Optimal result

Integrand size = 22, antiderivative size = 129

$$\begin{aligned}\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx = &\frac{\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{8d}\\ &+\frac{\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{4d}\\ &+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{4d}\\ &+\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}\end{aligned}$$

output

```
1/8*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d+1/4*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b
*c/d)/d+1/4*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d+1/8*cos(4*a-4*b*c/d)*Si(4
*b*c/d+4*b*x)/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.85

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx$$
$$= \frac{\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)\sin\left(4a-\frac{4bc}{d}\right)+2\,\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right)+2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c-}{8d}$$

input
```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x),x]
```

output
```
(CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + 2*CosIntegral[(2*b*
(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + 2*Cos[2*a - (2*b*c)/d]*SinIntegral[(2
*b*(c + d*x))/d] + Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(8
*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos^3(a+bx)}{c+dx}\,dx$$
$$\downarrow 4906$$
$$\int \left(\frac{\sin(2a+2bx)}{4(c+dx)}+\frac{\sin(4a+4bx)}{8(c+dx)}\right)dx$$
$$\downarrow 2009$$
$$\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{4d}+$$
$$\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{4d}+\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}$$

input
```
Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x),x]
```

output
```
(CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(8*d) + (CosIntegral
[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(4*d) + (Cos[2*a - (2*b*c)/d]*Si
nIntegral[(2*b*c)/d + 2*b*x])/(4*d) + (Cos[4*a - (4*b*c)/d]*SinIntegral[(4
*b*c)/d + 4*b*x])/(8*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{b\left(-\frac{4\,\mathrm{Si}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}-\frac{4\,\mathrm{Ci}\left(4bx+4a+\frac{-4ad+4cb}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{d}\right)}{32}+\frac{b\left(-\frac{2\,\mathrm{Si}\left(-2bx-2a-\frac{2(-a}{}\right.\right.}{}$
default	$\frac{b\left(-\frac{4\,\mathrm{Si}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}-\frac{4\,\mathrm{Ci}\left(4bx+4a+\frac{-4ad+4cb}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{d}\right)}{32}+\frac{b\left(-\frac{2\,\mathrm{Si}\left(-2bx-2a-\frac{2(-a}{}\right.\right.}{}$
risch	$-\frac{ie^{-\frac{4i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{16d}-\frac{ie^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{8d}+\frac{ie^{\frac{2i(}{}}}{}$

input
```
int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/32*b*(-4*Si(-4*b*x-4*a-4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4
*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)+1/8*b*(-2*Si(-2*b*x-2*a-2*
(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(
-a*d+b*c)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.96

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx$$
$$= \frac{2\ \mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+\mathrm{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)+\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{4\,(bdx+bc)}{d}\right)+2\ \mathrm{c}}{8\,d}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
1/8*(2*cos_integral(2*(b*d*x + b*c)/d)*sin(-2*(b*c - a*d)/d) + cos_integra
l(4*(b*d*x + b*c)/d)*sin(-4*(b*c - a*d)/d) + cos(-4*(b*c - a*d)/d)*sin_int
egral(4*(b*d*x + b*c)/d) + 2*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x +
 b*c)/d))/d
```

Sympy [F]

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx = \int \frac{\sin(a+bx)\cos^3(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.18

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx$$
$$= \frac{2\,b\left(i\,E_1\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_1\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b\left(-i\,E_1\left(\frac{4\,(-i\,bc-i\,(bx+a)}{d}\right.\right.}{}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
1/16*(2*b*(I*exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*e
xp_integral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d
)/d) - b*(-I*exp_integral_e(1, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*e
xp_integral_e(1, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a*d
)/d) - 2*b*(exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_
integral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d
) - b*(exp_integral_e(1, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integ
ral_e(1, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d)/d))/(b
*d)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.35 (sec) , antiderivative size = 6046, normalized size of antiderivative = 46.87

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

output

```
1/16*(imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b
*c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)
^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(co
s_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d
)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2
*tan(b*c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(
2*b*c/d)^2*tan(b*c/d)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2
*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 4*real_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 2*real_part(c
os_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^
2 + 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*
b*c/d)*tan(b*c/d)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^
2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*real_part(cos_integral(-2*b*x - 2
*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_i
ntegral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 -
2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)
^2*tan(b*c/d)^2 + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(
a)^2*tan(2*b*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^
2*tan(a)^2*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx = \int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)}{c+d\,x}\,dx$$

input

```
int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x),x)
```

output

```
int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx = \int \frac{\cos\left(bx+a\right)^3\sin\left(bx+a\right)}{dx+c}dx$$

input
```
int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x)
```

output
```
int((cos(a + b*x)**3*sin(a + b*x))/(c + d*x),x)
```

3.142 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 1314
Mathematica [A] (verified) . . . 1315
Rubi [A] (verified) . . . 1315
Maple [A] (verified) . . . 1316
Fricas [A] (verification not implemented) . . . 1317
Sympy [F] . . . 1317
Maxima [C] (verification not implemented) . . . 1318
Giac [C] (verification not implemented) . . . 1318
Mupad [F(-1)] . . . 1319
Reduce [F] . . . 1320

Optimal result

Integrand size = 22, antiderivative size = 179

$$\begin{aligned}\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = &\frac{b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^2}\\ &+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\operatorname{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}\\ &-\frac{\sin(2a+2bx)}{4d(c+dx)}-\frac{\sin(4a+4bx)}{8d(c+dx)}\\ &-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^2}\\ &-\frac{b\sin\left(4a-\frac{4bc}{d}\right)\operatorname{Si}\left(\frac{4bc}{d}+4bx\right)}{2d^2}\end{aligned}$$

output

```
1/2*b*cos(2*a-2*b*c/d)*Ci(2*b*c/d+2*b*x)/d^2+1/2*b*cos(4*a-4*b*c/d)*Ci(4*b
*c/d+4*b*x)/d^2-1/4*sin(2*b*x+2*a)/d/(d*x+c)-1/8*sin(4*b*x+4*a)/d/(d*x+c)-
1/2*b*sin(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^2-1/2*b*sin(4*a-4*b*c/d)*Si(4*b
*c/d+4*b*x)/d^2
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.84

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = -\frac{-4b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)-4b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)+\frac{2d\sin(2(a+bx))}{c+dx}+}{8d^2}$$

input
```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^2,x]
```

output
```
-1/8*(-4*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - 4*b*Cos[4
*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] + (2*d*Sin[2*(a + b*x)])/(c
 + d*x) + (d*Sin[4*(a + b*x)])/(c + d*x) + 4*b*Sin[2*a - (2*b*c)/d]*SinInt
egral[(2*b*(c + d*x))/d] + 4*b*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c +
d*x))/d])/d^2
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos^3(a+bx)}{(c+dx)^2}\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{\sin(2a+2bx)}{4(c+dx)^2}+\frac{\sin(4a+4bx)}{8(c+dx)^2}\right)dx$$

$$\downarrow 2009$$

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^2}-\frac{b\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{\sin(2a+2bx)}{4d(c+dx)}-\frac{\sin(4a+4bx)}{8d(c+dx)}$$

input
```
Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^2,x]
```

output
```
(b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) + (b*Cos[4
*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(2*d^2) - Sin[2*a + 2*b*x]
/(4*d*(c + d*x)) - Sin[4*a + 4*b*x]/(8*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)
/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) - (b*Sin[4*a - (4*b*c)/d]*SinI
ntegral[(4*b*c)/d + 4*b*x])/(2*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{b^2\left(-\frac{4\sin(4bx+4a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{16\,\mathrm{Si}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{d}+\frac{16\,\mathrm{Ci}\left(4bx+4a+\frac{-4ad+4cb}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}}{d}\right)}{32}$
default	$\frac{b^2\left(-\frac{4\sin(4bx+4a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{16\,\mathrm{Si}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{d}+\frac{16\,\mathrm{Ci}\left(4bx+4a+\frac{-4ad+4cb}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}}{d}\right)}{32}$
risch	$-\frac{b\,\mathrm{e}^{-\frac{4i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{4d^2}-\frac{b\,\mathrm{e}^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{4d^2}-\frac{b\,\mathrm{e}^{2}}{}$

input
```
int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/32*b^2*(-4*sin(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))/d+4*(-4*Si(-4*b*x-4*
a-4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*cos
(4*(-a*d+b*c)/d)/d)/d)+1/8*b^2*(-2*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2
*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*
(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.02

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = -\frac{2\,d\cos(bx+a)^3\sin(bx+a) - (bdx+bc)\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{4\,(bdx+bc)}{d}\right) - (bdx+bc)\cos\left(-\frac{2\,(bc-ad)}{d}\right)}{2\,(d^3x}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*d*cos(b*x + a)^3*sin(b*x + a) - (b*d*x + b*c)*cos(-4*(b*c - a*d)/d
)*cos_integral(4*(b*d*x + b*c)/d) - (b*d*x + b*c)*cos(-2*(b*c - a*d)/d)*co
s_integral(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*sin(-4*(b*c - a*d)/d)*sin_in
tegral(4*(b*d*x + b*c)/d) + (b*d*x + b*c)*sin(-2*(b*c - a*d)/d)*sin_integr
al(2*(b*d*x + b*c)/d))/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(a+bx)\cos^3(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.72

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{2\,b^2\left(i\,E_2\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_2\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^2\left(-i\,E_2\left(\frac{4\,(-i\,bc-i\,(bx+}{d}\right.\right.$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/16*(2*b^2*(I*exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I
*exp_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a
*d)/d) - b^2*(-I*exp_integral_e(2, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 I*exp_integral_e(2, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c -
 a*d)/d) - 2*b^2*(exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d)
+ exp_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c -
a*d)/d) - b^2*(exp_integral_e(2, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + e
xp_integral_e(2, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d
)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.56 (sec) , antiderivative size = 63510, normalized size of antiderivative = 354.80

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output

```
1/4*(b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^
2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + b*d*x*real_part(cos_in
tegral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2
*b*c/d)^2*tan(b*c/d)^2 + b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + b
*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan
(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*b*d*x*imag_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*
c/d)^2*tan(b*c/d) + 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 4*b*d*
x*sin_integral(2*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a
)^2*tan(2*b*c/d)^2*tan(b*c/d) - 2*b*d*x*imag_part(cos_integral(4*b*x + 4*b
*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)
^2 + 2*b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*
x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 4*b*d*x*sin_integral(
4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d
)*tan(b*c/d)^2 + 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*
x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*b*d*x*im
ag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2
*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*b*d*x*sin_integral(2*(b*d*x + b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)}{\left(c+d\,x\right)^2}\,dx$$

input

```
int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^2,x)
```

output

```
int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(bx+a\right)^3\sin\left(bx+a\right)}{d^2x^2+2cdx+c^2}dx$$

input
```
int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x)
```

output
```
int((cos(a + b*x)**3*sin(a + b*x))/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.143 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$

Optimal result . . . 1321
Mathematica [A] (verified) . . . 1322
Rubi [A] (verified) . . . 1322
Maple [A] (verified) . . . 1324
Fricas [A] (verification not implemented) . . . 1324
Sympy [F] . . . 1325
Maxima [C] (verification not implemented) . . . 1325
Giac [C] (verification not implemented) . . . 1326
Mupad [F(-1)] . . . 1327
Reduce [F] . . . 1328

Optimal result

Integrand size = 22, antiderivative size = 231

$$\begin{aligned}\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = &-\frac{b\cos(2a+2bx)}{4d^2(c+dx)} - \frac{b\cos(4a+4bx)}{4d^2(c+dx)} \\ &- \frac{b^2\,\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{d^3} \\ &- \frac{b^2\,\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d^3} \\ &- \frac{\sin(2a+2bx)}{8d(c+dx)^2} - \frac{\sin(4a+4bx)}{16d(c+dx)^2} \\ &- \frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^3} \\ &- \frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{d^3}\end{aligned}$$

output

```
-1/4*b*cos(2*b*x+2*a)/d^2/(d*x+c)-1/4*b*cos(4*b*x+4*a)/d^2/(d*x+c)-b^2*Ci(
4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^3-1/2*b^2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*
c/d)/d^3-1/8*sin(2*b*x+2*a)/d/(d*x+c)^2-1/16*sin(4*b*x+4*a)/d/(d*x+c)^2-1/
2*b^2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^3-b^2*cos(4*a-4*b*c/d)*Si(4*b*c
/d+4*b*x)/d^3
```

Mathematica [A] (verified)

Time = 2.64 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.85

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx =$$

$$-\frac{16b^2\,\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)\sin\left(4a-\frac{4bc}{d}\right)+8b^2\,\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right)+\frac{2d(2b(c+dx)\cos(2($$

input

```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^3,x]
```

output

```
-1/16*(16*b^2*CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + 8*b^2*
CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (2*d*(2*b*(c + d*x)*
Cos[2*(a + b*x)] + d*Sin[2*(a + b*x)]))/(c + d*x)^2 + (d*(4*b*(c + d*x)*Co
s[4*(a + b*x)] + d*Sin[4*(a + b*x)]))/(c + d*x)^2 + 8*b^2*Cos[2*a - (2*b*c
)/d]*SinIntegral[(2*b*(c + d*x))/d] + 16*b^2*Cos[4*a - (4*b*c)/d]*SinInteg
ral[(4*b*(c + d*x))/d])/d^3
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos^3(a+bx)}{(c+dx)^3}\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{\sin(2a+2bx)}{4(c+dx)^3}+\frac{\sin(4a+4bx)}{8(c+dx)^3}\right)dx$$

$$\downarrow 2009$$

$$-\frac{b^2 \sin\left(4a-\frac{4bc}{d}\right) \text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2 \sin\left(2a-\frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^3}-\frac{b^2 \cos\left(2a-\frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^3}-\frac{b^2 \cos\left(4a-\frac{4bc}{d}\right) \text{Si}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b\cos(2a+2bx)}{4d^2(c+dx)}-\frac{b\cos(4a+4bx)}{4d^2(c+dx)}-\frac{\sin(2a+2bx)}{8d(c+dx)^2}-\frac{\sin(4a+4bx)}{16d(c+dx)^2}$$

input

```
Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^3,x]
```

output

```
-1/4*(b*Cos[2*a + 2*b*x])/(d^2*(c + d*x)) - (b*Cos[4*a + 4*b*x])/(4*d^2*(c
 + d*x)) - (b^2*CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/d^3 -
 (b^2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d^3) - Sin[2
*a + 2*b*x]/(8*d*(c + d*x)^2) - Sin[4*a + 4*b*x]/(16*d*(c + d*x)^2) - (b^2
*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^3) - (b^2*Cos[4
*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/d^3
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\frac{b^3\left(-\frac{2\sin(4bx+4a)}{(-ad+cb+d(bx+a))^2 d}+\frac{-\frac{8\cos(4bx+4a)}{(-ad+cb+d(bx+a))d}-\frac{8\left(-\frac{4\ \mathrm{Si}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}-\frac{4\ \mathrm{Ci}\left(4bx+4a+\frac{-4}{d}\right.\right.}{d}}{d}}{32}$
default	$\frac{b^3\left(-\frac{2\sin(4bx+4a)}{(-ad+cb+d(bx+a))^2 d}+\frac{-\frac{8\cos(4bx+4a)}{(-ad+cb+d(bx+a))d}-\frac{8\left(-\frac{4\ \mathrm{Si}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}-\frac{4\ \mathrm{Ci}\left(4bx+4a+\frac{-4}{d}\right.\right.}{d}}{d}}{32}$
risch	$\frac{ib^2 e^{-\frac{4i(ad-cb)}{d}}\ \mathrm{expIntegral}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{2d^3}+\frac{ib^2 e^{-\frac{2i(ad-cb)}{d}}\ \mathrm{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{4d^3}-\frac{ib^2}{}$

input
```
int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/32*b^3*(-2*sin(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))^2/d+2*(-4*cos(4*b*x+
4*a)/(-a*d+c*b+d*(b*x+a))/d-4*(-4*Si(-4*b*x-4*a-4*(-a*d+b*c)/d)*cos(4*(-a*
d+b*c)/d)/d-4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)/d)+1/
8*b^3*(-sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2/d+(-2*cos(2*b*x+2*a)/(-a*d+c
*b+d*(b*x+a))/d-2*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-
2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.34

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx =$$

$$-\frac{d^2\cos(bx+a)^3\sin(bx+a)+4\,(bd^2x+bcd)\cos(bx+a)^4-3\,(bd^2x+bcd)\cos(bx+a)^2+(b^2d^2x^2+}{}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/2*(d^2*cos(b*x + a)^3*sin(b*x + a) + 4*(b*d^2*x + b*c*d)*cos(b*x + a)^4
 - 3*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*cos_integral(2*(b*d*x + b*c)/d)*sin(-2*(b*c - a*d)/d) + 2*(b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(4*(b*d*x + b*c)/d)*sin(-4*(b*c - a*
d)/d) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-4*(b*c - a*d)/d)*sin_
integral(4*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-2
*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d))/(d^5*x^2 + 2*c*d^4*x + c^
2*d^3)
```

Sympy [F]

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin(a+bx)\cos^3(a+bx)}{(c+dx)^3}\,dx$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.48

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{2\,b^3\left(i\,E_3\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_3\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^3\left(-i\,E_3\left(\frac{4\,(-i\,bc-i\,(bx+}{d}\right.\right.$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output

```
1/16*(2*b^3*(I*exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I
*exp_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a
*d)/d) - b^3*(-I*exp_integral_e(3, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 I*exp_integral_e(3, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c -
 a*d)/d) - 2*b^3*(exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d)
+ exp_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c -
a*d)/d) - b^3*(exp_integral_e(3, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + e
xp_integral_e(3, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d
)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 -
 a*d^3)*(b*x + a))*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.04 (sec) , antiderivative size = 111694, normalized size of antiderivative = 483.52

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input

```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output

```
-1/8*(4*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*
tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2
*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)
^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*
b*c/d)^2*tan(b*c/d)^2 - 4*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*
c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d
)^2 + 8*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^
2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*b^2*d^2*x^2*sin_inte
gral(2*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*
b*c/d)^2*tan(b*c/d)^2 + 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)
 + 4*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*ta
n(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 8*b^2*d^2*x^2*rea
l_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*t
an(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 8*b^2*d^2*x^2*real_part(cos_integral(-
4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)
*tan(b*c/d)^2 - 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan
(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*b^2
*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)}{\left(c+d\,x\right)^3}\,dx$$

input

```
int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^3,x)
```

output

```
int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos(bx+a)^3\sin(bx+a)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx$$

input
```
int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x)
```

output
```
int((cos(a + b*x)**3*sin(a + b*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d*
**3*x**3),x)
```

3.144 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$

Optimal result . . . 1329
Mathematica [A] (verified) . . . 1330
Rubi [A] (verified) . . . 1330
Maple [A] (verified) . . . 1332
Fricas [A] (verification not implemented) . . . 1333
Sympy [F] . . . 1333
Maxima [C] (verification not implemented) . . . 1334
Giac [C] (verification not implemented) . . . 1334
Mupad [F(-1)] . . . 1335
Reduce [F] . . . 1336

Optimal result

Integrand size = 22, antiderivative size = 287

$$\begin{aligned}\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = &-\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}-\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}\\ &-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}\\ &-\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}\\ &-\frac{\sin(2a+2bx)}{12d(c+dx)^3}+\frac{b^2\sin(2a+2bx)}{6d^3(c+dx)}-\frac{\sin(4a+4bx)}{24d(c+dx)^3}\\ &+\frac{b^2\sin(4a+4bx)}{3d^3(c+dx)}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}\\ &+\frac{4b^3\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{3d^4}\end{aligned}$$

output

```
-1/12*b*cos(2*b*x+2*a)/d^2/(d*x+c)^2-1/12*b*cos(4*b*x+4*a)/d^2/(d*x+c)^2-1
/3*b^3*cos(2*a-2*b*c/d)*Ci(2*b*c/d+2*b*x)/d^4-4/3*b^3*cos(4*a-4*b*c/d)*Ci(
4*b*c/d+4*b*x)/d^4-1/12*sin(2*b*x+2*a)/d/(d*x+c)^3+1/6*b^2*sin(2*b*x+2*a)/
d^3/(d*x+c)-1/24*sin(4*b*x+4*a)/d/(d*x+c)^3+1/3*b^2*sin(4*b*x+4*a)/d^3/(d*
x+c)+1/3*b^3*sin(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^4+4/3*b^3*sin(4*a-4*b*c/
d)*Si(4*b*c/d+4*b*x)/d^4
```

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.10

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx =$$

$$-\frac{2d\cos(2bx)\left(bd(c+dx)\cos(2a)+(d^2-2b^2(c+dx)^2)\sin(2a)\right)+d\cos(4bx)\left(2bd(c+dx)\cos(4a)+(d^{}$$

input

```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^4,x]
```

output

```
-1/24*(2*d*Cos[2*b*x]*(b*d*(c + d*x)*Cos[2*a] + (d^2 - 2*b^2*(c + d*x)^2)*
Sin[2*a]) + d*Cos[4*b*x]*(2*b*d*(c + d*x)*Cos[4*a] + (d^2 - 8*b^2*(c + d*x
)^2)*Sin[4*a]) - 2*d*((-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*a] + b*d*(c + d*x)*
Sin[2*a])*Sin[2*b*x] - d*((-d^2 + 8*b^2*(c + d*x)^2)*Cos[4*a] + 2*b*d*(c +
 d*x)*Sin[4*a])*Sin[4*b*x] + 8*b^3*(c + d*x)^3*(Cos[2*a - (2*b*c)/d]*CosIn
tegral[(2*b*(c + d*x))/d] - Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x
))/d]) + 32*b^3*(c + d*x)^3*(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*
x))/d] - Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d]))/(d^4*(c + d
*x)^3)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos^3(a+bx)}{(c+dx)^4}\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{\sin(2a+2bx)}{4(c+dx)^4}+\frac{\sin(4a+4bx)}{8(c+dx)^4}\right)dx$$

$$\downarrow 2009$$

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+$$
$$\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{4b^3\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+\frac{b^2\sin(2a+2bx)}{6d^3(c+dx)}+$$
$$\frac{b^2\sin(4a+4bx)}{3d^3(c+dx)}-\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}-\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}-\frac{\sin(2a+2bx)}{12d(c+dx)^3}-\frac{\sin(4a+4bx)}{24d(c+dx)^3}$$

input
```
Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^4,x]
```

output
```
-1/12*(b*Cos[2*a + 2*b*x])/(d^2*(c + d*x)^2) - (b*Cos[4*a + 4*b*x])/(12*d^
2*(c + d*x)^2) - (b^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])
/(3*d^4) - (4*b^3*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(3*
d^4) - Sin[2*a + 2*b*x]/(12*d*(c + d*x)^3) + (b^2*Sin[2*a + 2*b*x])/(6*d^3
*(c + d*x)) - Sin[4*a + 4*b*x]/(24*d*(c + d*x)^3) + (b^2*Sin[4*a + 4*b*x])
/(3*d^3*(c + d*x)) + (b^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b
*x])/(3*d^4) + (4*b^3*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])
/(3*d^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{b^4\left(-\frac{4\sin(4bx+4a)}{3(-ad+cb+d(bx+a))^3d}+\frac{-\frac{8\cos(4bx+4a)}{3(-ad+cb+d(bx+a))^2d}-\frac{8\left(-\frac{4\sin(4bx+4a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{16\,\mathrm{Si}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\sin\left(\frac{-4}{}\right.}{d}}{3d}\right.}{d}\right.}{32}$
default	$\frac{b^4\left(-\frac{4\sin(4bx+4a)}{3(-ad+cb+d(bx+a))^3d}+\frac{-\frac{8\cos(4bx+4a)}{3(-ad+cb+d(bx+a))^2d}-\frac{8\left(-\frac{4\sin(4bx+4a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{16\,\mathrm{Si}\left(-4bx-4a-\frac{4(-ad+cb)}{d}\right)\sin\left(\frac{-4}{}\right.}{d}}{3d}\right.}{d}\right.}{32}$
risch	$\frac{2b^3\mathrm{e}^{-\frac{4i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{3d^4}+\frac{b^3\mathrm{e}^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{6d^4}+\frac{b^3\mathrm{e}}{}$

input
```
int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/32*b^4*(-4/3*sin(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))^3/d+4/3*(-2*cos(4*
b*x+4*a)/(-a*d+c*b+d*(b*x+a))^2/d-2*(-4*sin(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a)
)/d+4*(-4*Si(-4*b*x-4*a-4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*b*x+4
*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d)/d)/d)+1/8*b^4*(-2/3*sin(2*b*x
+2*a)/(-a*d+c*b+d*(b*x+a))^3/d+2/3*(-cos(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2
/d-(-2*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2*(-2*Si(-2*b*x-2*a-2*(-a*d+b
*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*
c)/d)/d)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.59

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx =$$

$$-\frac{4\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^4-3\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^2+8\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right.}{}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/6*(4*(b*d^3*x + b*c*d^2)*cos(b*x + a)^4 - 3*(b*d^3*x + b*c*d^2)*cos(b*x
 + a)^2 + 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(
-4*(b*c - a*d)/d)*cos_integral(4*(b*d*x + b*c)/d) + 2*(b^3*d^3*x^3 + 3*b^3
*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-2*(b*c - a*d)/d)*cos_integral(2
*(b*d*x + b*c)/d) - 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3
*c^3)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) - 2*(b^3*d^3*x
^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-2*(b*c - a*d)/d)*sin_
integral(2*(b*d*x + b*c)/d) - 2*((8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c
^2*d - d^3)*cos(b*x + a)^3 - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*c
os(b*x + a))*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F]

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sin\left(a+bx\right)\cos^3\left(a+bx\right)}{(c+dx)^4}\,dx$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c)**4,x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x)**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.37

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$$
$$= \frac{2\,b^4\left(i\,E_4\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_4\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^4\left(-i\,E_4\left(\frac{4\,(-i\,bc-i\,(bx+}{16\,(b}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/16*(2*b^4*(I*exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I
*exp_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a
*d)/d) - b^4*(-I*exp_integral_e(4, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 I*exp_integral_e(4, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c -
 a*d)/d) - 2*b^4*(exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d)
+ exp_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c -
a*d)/d) - b^4*(exp_integral_e(4, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + e
xp_integral_e(4, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d
)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^
3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a
^2*d^4)*(b*x + a))*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.18 (sec) , antiderivative size = 157526, normalized size of antiderivative = 548.87

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

output

```
-1/12*(8*b^3*d^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2
*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^
3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a
)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2
*b*c/d)^2*tan(b*c/d)^2 + 8*b^3*d^3*x^3*real_part(cos_integral(-4*b*x - 4*b
*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/
d)^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2
*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 4*b^3*d^3*x^3*
imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)
^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 8*b^3*d^3*x^3*sin_integral(2*(b*d*
x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan
(b*c/d) - 16*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*
x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 16*b^3*d^3
*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan
(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 32*b^3*d^3*x^3*sin_integral(4
*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)
*tan(b*c/d)^2 + 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan
(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*b^3
*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)}{\left(c+d\,x\right)^4}\,dx$$

input

```
int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^4,x)
```

output

```
int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^4, x)
```

Reduce [F]

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \text{too large to display}$$

input
```
int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x)
```

output
```
(180*cos(a + b*x)*sin(a + b*x)**3*b**2*c**2*d + 360*cos(a + b*x)*sin(a + b
*x)**3*b**2*c*d**2*x + 180*cos(a + b*x)*sin(a + b*x)**3*b**2*d**3*x**2 + 4
54*cos(a + b*x)*sin(a + b*x)**3*d**3 - 320*cos(a + b*x)*sin(a + b*x)**2*b*
c*d**2 - 320*cos(a + b*x)*sin(a + b*x)**2*b*d**3*x - 2610*cos(a + b*x)*sin
(a + b*x)*b**2*c**2*d - 5220*cos(a + b*x)*sin(a + b*x)*b**2*c*d**2*x - 261
0*cos(a + b*x)*sin(a + b*x)*b**2*d**3*x**2 - 5638*cos(a + b*x)*sin(a + b*x
)*d**3 + 5120*cos(a + b*x)*b*c*d**2 + 5120*cos(a + b*x)*b*d**3*x - 68544*i
nt(tan((a + b*x)/2)**3/(tan((a + b*x)/2)**8*c**4 + 4*tan((a + b*x)/2)**8*c
**3*d*x + 6*tan((a + b*x)/2)**8*c**2*d**2*x**2 + 4*tan((a + b*x)/2)**8*c*d
**3*x**3 + tan((a + b*x)/2)**8*d**4*x**4 + 4*tan((a + b*x)/2)**6*c**4 + 16
*tan((a + b*x)/2)**6*c**3*d*x + 24*tan((a + b*x)/2)**6*c**2*d**2*x**2 + 16
*tan((a + b*x)/2)**6*c*d**3*x**3 + 4*tan((a + b*x)/2)**6*d**4*x**4 + 6*tan
((a + b*x)/2)**4*c**4 + 24*tan((a + b*x)/2)**4*c**3*d*x + 36*tan((a + b*x)
/2)**4*c**2*d**2*x**2 + 24*tan((a + b*x)/2)**4*c*d**3*x**3 + 6*tan((a + b*
x)/2)**4*d**4*x**4 + 4*tan((a + b*x)/2)**2*c**4 + 16*tan((a + b*x)/2)**2*c
**3*d*x + 24*tan((a + b*x)/2)**2*c**2*d**2*x**2 + 16*tan((a + b*x)/2)**2*c
*d**3*x**3 + 4*tan((a + b*x)/2)**2*d**4*x**4 + c**4 + 4*c**3*d*x + 6*c**2*
d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*c**3*d**4 - 205632*int(tan((a +
b*x)/2)**3/(tan((a + b*x)/2)**8*c**4 + 4*tan((a + b*x)/2)**8*c**3*d*x + 6*
tan((a + b*x)/2)**8*c**2*d**2*x**2 + 4*tan((a + b*x)/2)**8*c*d**3*x**3 ...
```

3.145 $\int (c+dx)^m \cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal result . . . 1337
Mathematica [A] (verified) . . . 1338
Rubi [A] (verified) . . . 1339
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Giac [F] . . . 1342
Mupad [F(-1)] . . . 1342
Reduce [F] . . . 1343

Optimal result

Integrand size = 24, antiderivative size = 419

$$\begin{aligned}
&\int (c+dx)^m \cos^3(a+bx)\sin^2(a+bx)\,dx \\
&= -\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{16b} \\
&\quad + \frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{16b} \\
&\quad + \frac{i3^{-1-m}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3ib(c+dx)}{d}\right)}{32b} \\
&\quad - \frac{i3^{-1-m}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3ib(c+dx)}{d}\right)}{32b} \\
&\quad + \frac{i5^{-1-m}e^{5i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{5ib(c+dx)}{d}\right)}{32b} \\
&\quad - \frac{i5^{-1-m}e^{-5i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{5ib(c+dx)}{d}\right)}{32b}
\end{aligned}$$

output

```
-1/16*I*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x
+c)/d)^m)+1/16*I*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I
*b*(d*x+c)/d)^m)+1/32*I*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3
*I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)-1/32*I*3^(-1-m)*(d*x+c)^m*GAMMA(1+m
,3*I*b*(d*x+c)/d)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)+1/32*I*5^(-1-m)
*exp(5*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-5*I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)
/d)^m)-1/32*I*5^(-1-m)*(d*x+c)^m*GAMMA(1+m,5*I*b*(d*x+c)/d)/b/exp(5*I*(a-b
*c/d))/((I*b*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.98

$$\int (c+dx)^m \cos^3(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{ie^{-\frac{i(bc+ad)}{d}}(c+dx)^m\left(e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)-e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)\right.}{16b}$$

$$-\frac{i3^{-1-m}e^{-\frac{3i(bc+ad)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-e^{6ia}\left(\frac{ib(c+dx)}{d}\right)^{m}\Gamma\left(1+m,-\frac{3ib(c+dx)}{d}\right)+e^{\frac{6ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^{m}\right.}{32b}$$

$$-\frac{i5^{-1-m}e^{-\frac{5i(bc+ad)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-e^{10ia}\left(\frac{ib(c+dx)}{d}\right)^{m}\Gamma\left(1+m,-\frac{5ib(c+dx)}{d}\right)+e^{\frac{10ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^{m}\right.}{32b}$$

input

```
Integrate[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output

```
((-1/16*I)*(c + d*x)^m*((E^((2*I)*a)*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(
((-I)*b*(c + d*x))/d)^m - (E^(((2*I)*b*c)/d)*Gamma[1 + m, (I*b*(c + d*x))/
d])/((I*b*(c + d*x))/d)^m))/(b*E^((I*(b*c + a*d))/d)) - ((I/32)*3^(-1 - m)
*(c + d*x)^m*(-(E^((6*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-3*I)*b*(
c + d*x))/d]) + E^(((6*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((
3*I)*b*(c + d*x))/d]))/(b*E^(((3*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2
)^m) - ((I/32)*5^(-1 - m)*(c + d*x)^m*(-(E^((10*I)*a)*((I*b*(c + d*x))/d)^
m*Gamma[1 + m, ((-5*I)*b*(c + d*x))/d]) + E^(((10*I)*b*c)/d)*(((-I)*b*(c +
 d*x))/d)^m*Gamma[1 + m, ((5*I)*b*(c + d*x))/d]))/(b*E^(((5*I)*(b*c + a*d)
)/d)*((b^2*(c + d*x)^2)/d^2)^m)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos^3(a+bx)(c+dx)^m\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8}\cos(a+bx)(c+dx)^m - \frac{1}{16}\cos(3a+3bx)(c+dx)^m - \frac{1}{16}\cos(5a+5bx)(c+dx)^m\right)dx$$

$$\downarrow 2009$$

$$-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{16b} + \frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{32b} + \frac{i5^{-m-1}e^{5i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{5ib(c+dx)}{d}\right)}{32b} + \frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{16b} - \frac{i3^{-m-1}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{3ib(c+dx)}{d}\right)}{32b} - \frac{i5^{-m-1}e^{-5i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{5ib(c+dx)}{d}\right)}{32b}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
((-1/16*I)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))
/d])/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/16)*(c + d*x)^m*Gamma[1 + m, (I*b*
(c + d*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + ((I/32)*3^(
-1 - m)*E^((3*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*
x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) - ((I/32)*3^(-1 - m)*(c + d*x)^m*Gamm
a[1 + m, ((3*I)*b*(c + d*x))/d])/(b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x
))/d)^m) + ((I/32)*5^(-1 - m)*E^((5*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1
+ m, ((-5*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) - ((I/32)*5^(-1
 - m)*(c + d*x)^m*Gamma[1 + m, ((5*I)*b*(c + d*x))/d])/(b*E^((5*I)*(a - (b
*c)/d))*((I*b*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [F]

$$\int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)^2 \, dx$$

input
```
int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.67

$$\int (c+dx)^m \cos^3(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{30i\,e^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-ibc+iad}{d}\right)}\Gamma\left(m+1,\frac{ibdx+ibc}{d}\right)+5i\,e^{\left(-\frac{dm\log\left(-\frac{3ib}{d}\right)+3ibc-3iad}{d}\right)}\Gamma\left(m+1,-\frac{3\,(ibdx+ibc)}{d}\right)+3i\,e^{\left(-}$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/480*(30*I*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x
+ I*b*c)/d) + 5*I*e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d)*gamma(m +
 1, -3*(I*b*d*x + I*b*c)/d) + 3*I*e^(-(d*m*log(-5*I*b/d) + 5*I*b*c - 5*I*a
*d)/d)*gamma(m + 1, -5*(I*b*d*x + I*b*c)/d) - 30*I*e^(-(d*m*log(-I*b/d) +
I*b*c - I*a*d)/d)*gamma(m + 1, (-I*b*d*x - I*b*c)/d) - 5*I*e^(-(d*m*log(3*
I*b/d) - 3*I*b*c + 3*I*a*d)/d)*gamma(m + 1, -3*(-I*b*d*x - I*b*c)/d) - 3*I
*e^(-(d*m*log(5*I*b/d) - 5*I*b*c + 5*I*a*d)/d)*gamma(m + 1, -5*(-I*b*d*x -
 I*b*c)/d))/b
```

Sympy [F]

$$\int (c+dx)^m \cos^3(a+bx)\sin^2(a+bx)\,dx = \int (c+dx)^m \sin^2(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*sin(a + b*x)**2*cos(a + b*x)**3, x)
```

Maxima [F]

$$\int (c+dx)^m \cos^3(a+bx) \sin^2(a+bx)\, dx = \int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)^2\, dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^2, x)
```

Giac [F]

$$\int (c+dx)^m \cos^3(a+bx) \sin^2(a+bx)\, dx = \int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)^2\, dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos^3(a+bx) \sin^2(a+bx)\, dx = \int \cos(a+b\,x)^3 \sin(a+b\,x)^2 (c+d\,x)^m\, dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \cos^3(a+bx) \sin^2(a+bx)\, dx = \int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)^2\, dx$$

input
```
int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

output
```
int((c + d*x)**m*cos(a + b*x)**3*sin(a + b*x)**2,x)
```

3.146 $\int (c+dx)^4 \cos^3(a+bx) \sin^2(a+bx)\, dx$

Optimal result . . . 1344
Mathematica [A] (verified) . . . 1345
Rubi [A] (verified) . . . 1345
Maple [A] (verified) . . . 1347
Fricas [A] (verification not implemented) . . . 1347
Sympy [B] (verification not implemented) . . . 1348
Maxima [B] (verification not implemented) . . . 1349
Giac [A] (verification not implemented) . . . 1350
Mupad [B] (verification not implemented) . . . 1351
Reduce [B] (verification not implemented) . . . 1352

Optimal result

Integrand size = 24, antiderivative size = 330

$$\begin{aligned}
&\int (c+dx)^4 \cos^3(a+bx) \sin^2(a+bx)\, dx \\
&= -\frac{3d^3(c+dx)\cos(a+bx)}{b^4} + \frac{d(c+dx)^3\cos(a+bx)}{2b^2} + \frac{d^3(c+dx)\cos(3a+3bx)}{54b^4} \\
&\quad - \frac{d(c+dx)^3\cos(3a+3bx)}{36b^2} + \frac{3d^3(c+dx)\cos(5a+5bx)}{1250b^4} - \frac{d(c+dx)^3\cos(5a+5bx)}{100b^2} \\
&\quad + \frac{3d^4\sin(a+bx)}{b^5} - \frac{3d^2(c+dx)^2\sin(a+bx)}{2b^3} + \frac{(c+dx)^4\sin(a+bx)}{8b} \\
&\quad - \frac{d^4\sin(3a+3bx)}{162b^5} + \frac{d^2(c+dx)^2\sin(3a+3bx)}{36b^3} - \frac{(c+dx)^4\sin(3a+3bx)}{48b} \\
&\quad - \frac{3d^4\sin(5a+5bx)}{6250b^5} + \frac{3d^2(c+dx)^2\sin(5a+5bx)}{500b^3} - \frac{(c+dx)^4\sin(5a+5bx)}{80b}
\end{aligned}$$

output

```
-3*d^3*(d*x+c)*cos(b*x+a)/b^4+1/2*d*(d*x+c)^3*cos(b*x+a)/b^2+1/54*d^3*(d*x
+c)*cos(3*b*x+3*a)/b^4-1/36*d*(d*x+c)^3*cos(3*b*x+3*a)/b^2+3/1250*d^3*(d*x
+c)*cos(5*b*x+5*a)/b^4-1/100*d*(d*x+c)^3*cos(5*b*x+5*a)/b^2+3*d^4*sin(b*x+
a)/b^5-3/2*d^2*(d*x+c)^2*sin(b*x+a)/b^3+1/8*(d*x+c)^4*sin(b*x+a)/b-1/162*d
^4*sin(3*b*x+3*a)/b^5+1/36*d^2*(d*x+c)^2*sin(3*b*x+3*a)/b^3-1/48*(d*x+c)^4
*sin(3*b*x+3*a)/b-3/6250*d^4*sin(5*b*x+5*a)/b^5+3/500*d^2*(d*x+c)^2*sin(5*
b*x+5*a)/b^3-1/80*(d*x+c)^4*sin(5*b*x+5*a)/b
```

Mathematica [A] (verified)

Time = 2.42 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.71

$$\int (c+dx)^4 \cos^3(a+bx) \sin^2(a+bx)\, dx =$$

$$-\frac{-506250b^4c^4\sin(a+bx) - 2025000b^3c^3d(\cos(a+bx) + bx\sin(a+bx)) - 2025000bcd^3(3(-2+b^2x^2)$$

input

```
Integrate[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output

```
-1/4050000*(-506250*b^4*c^4*Sin[a + b*x] - 2025000*b^3*c^3*d*(Cos[a + b*x]
 + b*x*Sin[a + b*x]) - 2025000*b*c*d^3*(3*(-2 + b^2*x^2)*Cos[a + b*x] + b*
x*(-6 + b^2*x^2)*Sin[a + b*x]) - 3037500*b^2*c^2*d^2*(2*b*x*Cos[a + b*x] +
 (-2 + b^2*x^2)*Sin[a + b*x]) - 506250*d^4*(4*b*x*(-6 + b^2*x^2)*Cos[a + b
*x] + (24 - 12*b^2*x^2 + b^4*x^4)*Sin[a + b*x]) + 84375*b^4*c^4*Sin[3*(a +
 b*x)] + 112500*b^3*c^3*d*(Cos[3*(a + b*x)] + 3*b*x*Sin[3*(a + b*x)]) + 37
500*b*c*d^3*((-2 + 9*b^2*x^2)*Cos[3*(a + b*x)] + 3*b*x*(-2 + 3*b^2*x^2)*Si
n[3*(a + b*x)]) + 56250*b^2*c^2*d^2*(6*b*x*Cos[3*(a + b*x)] + (-2 + 9*b^2*
x^2)*Sin[3*(a + b*x)]) + 3125*d^4*(12*b*x*(-2 + 3*b^2*x^2)*Cos[3*(a + b*x)
] + (8 - 36*b^2*x^2 + 27*b^4*x^4)*Sin[3*(a + b*x)]) + 50625*b^4*c^4*Sin[5*
(a + b*x)] + 40500*b^3*c^3*d*(Cos[5*(a + b*x)] + 5*b*x*Sin[5*(a + b*x)]) +
 1620*b*c*d^3*((-6 + 75*b^2*x^2)*Cos[5*(a + b*x)] + 5*b*x*(-6 + 25*b^2*x^2
)*Sin[5*(a + b*x)]) + 12150*b^2*c^2*d^2*(10*b*x*Cos[5*(a + b*x)] + (-2 + 2
5*b^2*x^2)*Sin[5*(a + b*x)]) + 81*d^4*(20*b*x*(-6 + 25*b^2*x^2)*Cos[5*(a +
 b*x)] + (24 - 300*b^2*x^2 + 625*b^4*x^4)*Sin[5*(a + b*x)]))/b^5
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sin^2(a+bx) \cos^3(a+bx)\, dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8}(c+dx)^4\cos(a+bx) - \frac{1}{16}(c+dx)^4\cos(3a+3bx) - \frac{1}{16}(c+dx)^4\cos(5a+5bx)\right)dx$$

$$\downarrow 2009$$

$$\frac{3d^4\sin(a+bx)}{b^5} - \frac{d^4\sin(3a+3bx)}{162b^5} - \frac{3d^4\sin(5a+5bx)}{6250b^5} - \frac{3d^3(c+dx)\cos(a+bx)}{b^4} + \frac{d^3(c+dx)\cos(3a+3bx)}{54b^4} + \frac{3d^3(c+dx)\cos(5a+5bx)}{1250b^4} - \frac{3d^2(c+dx)^2\sin(a+bx)}{2b^3} + \frac{d^2(c+dx)^2\sin(3a+3bx)}{36b^3} + \frac{3d^2(c+dx)^2\sin(5a+5bx)}{500b^3} + \frac{d(c+dx)^3\cos(a+bx)}{2b^2} - \frac{d(c+dx)^3\cos(3a+3bx)}{36b^2} - \frac{d(c+dx)^3\cos(5a+5bx)}{100b^2} + \frac{(c+dx)^4\sin(a+bx)}{8b} - \frac{(c+dx)^4\sin(3a+3bx)}{48b} - \frac{(c+dx)^4\sin(5a+5bx)}{80b}$$

input

```
Int[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output

```
(-3*d^3*(c + d*x)*Cos[a + b*x])/b^4 + (d*(c + d*x)^3*Cos[a + b*x])/(2*b^2)
 + (d^3*(c + d*x)*Cos[3*a + 3*b*x])/(54*b^4) - (d*(c + d*x)^3*Cos[3*a + 3*
b*x])/(36*b^2) + (3*d^3*(c + d*x)*Cos[5*a + 5*b*x])/(1250*b^4) - (d*(c + d
*x)^3*Cos[5*a + 5*b*x])/(100*b^2) + (3*d^4*Sin[a + b*x])/b^5 - (3*d^2*(c +
 d*x)^2*Sin[a + b*x])/(2*b^3) + ((c + d*x)^4*Sin[a + b*x])/(8*b) - (d^4*Si
n[3*a + 3*b*x])/(162*b^5) + (d^2*(c + d*x)^2*Sin[3*a + 3*b*x])/(36*b^3) -
((c + d*x)^4*Sin[3*a + 3*b*x])/(48*b) - (3*d^4*Sin[5*a + 5*b*x])/(6250*b^5
) + (3*d^2*(c + d*x)^2*Sin[5*a + 5*b*x])/(500*b^3) - ((c + d*x)^4*Sin[5*a
+ 5*b*x])/(80*b)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.77

method	result
parallelrisch	$\frac{\left(-84375(dx+c)^4b^4+112500d^2(dx+c)^2b^2-25000d^4\right)\sin(3bx+3a)+\left(-50625(dx+c)^4b^4+24300d^2(dx+c)^2b^2-1944d^4\right.}{}$
risch	$\frac{d\left(b^2d^3x^3+3b^2cd^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6cd^2\right)\cos(bx+a)}{2b^4}+\frac{(d^4x^4b^4+4b^4cd^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+b^4c^4-}{8b}$
derivativedivides	Expression too large to display
default	Expression too large to display
orering	Expression too large to display

input
```
int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4050000*((-84375*(d*x+c)^4*b^4+112500*d^2*(d*x+c)^2*b^2-25000*d^4)*sin(3
*b*x+3*a)+(-50625*(d*x+c)^4*b^4+24300*d^2*(d*x+c)^2*b^2-1944*d^4)*sin(5*b*
x+5*a)-112500*d*((d*x+c)^2*b^2-2/3*d^2)*(d*x+c)*b*cos(3*b*x+3*a)-40500*((d
*x+c)^2*b^2-6/25*d^2)*d*(d*x+c)*b*cos(5*b*x+5*a)+(506250*(d*x+c)^4*b^4-607
5000*d^2*(d*x+c)^2*b^2+12150000*d^4)*sin(b*x+a)+2025000*d*(((d*x+c)^2*b^2-
6*d^2)*(d*x+c)*cos(b*x+a)-208/225*c*(b^2*c^2-6284/975*d^2))*b)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.60

$$\int (c+dx)^4\cos^3(a+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{1620\left(25\,b^3d^4x^3+75\,b^3cd^3x^2+25\,b^3c^3d-6\,bcd^3+3\left(25\,b^3c^2d^2-2\,bd^4\right)x\right)\cos\left(bx+a\right)^5-300\left(75\,b^3d^4\right.}{}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output

```
-1/253125*(1620*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 25*b^3*c^3*d - 6*b*c*
d^3 + 3*(25*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^5 - 300*(75*b^3*d^4*x^3
 + 225*b^3*c*d^3*x^2 + 75*b^3*c^3*d + 22*b*c*d^3 + (225*b^3*c^2*d^2 + 22*b
*d^4)*x)*cos(b*x + a)^3 - 1800*(75*b^3*d^4*x^3 + 225*b^3*c*d^3*x^2 + 75*b^
3*c^3*d - 428*b*c*d^3 + (225*b^3*c^2*d^2 - 428*b*d^4)*x)*cos(b*x + a) - (3
3750*b^4*d^4*x^4 + 135000*b^4*c*d^3*x^3 + 33750*b^4*c^4 - 385200*b^2*c^2*d
^2 - 81*(625*b^4*d^4*x^4 + 2500*b^4*c*d^3*x^3 + 625*b^4*c^4 - 300*b^2*c^2*
d^2 + 24*d^4 + 150*(25*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 100*(25*b^4*c^3*d -
6*b^2*c*d^3)*x)*cos(b*x + a)^4 + 760816*d^4 + 900*(225*b^4*c^2*d^2 - 428*b
^2*d^4)*x^2 + (16875*b^4*d^4*x^4 + 67500*b^4*c*d^3*x^3 + 16875*b^4*c^4 + 9
900*b^2*c^2*d^2 - 4792*d^4 + 450*(225*b^4*c^2*d^2 + 22*b^2*d^4)*x^2 + 900*
(75*b^4*c^3*d + 22*b^2*c*d^3)*x)*cos(b*x + a)^2 + 1800*(75*b^4*c^3*d - 428
*b^2*c*d^3)*x)*sin(b*x + a))/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. $2(325) = 650$.

Time $= 1.15$ (sec) , antiderivative size $= 1098$, normalized size of antiderivative $= 3.33$

$$\int (c + dx)^4 \cos^3(a + bx) \sin^2(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)**4*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output

```
Piecewise((2*c**4*sin(a + b*x)**5/(15*b) + c**4*sin(a + b*x)**3*cos(a + b*
x)**2/(3*b) + 8*c**3*d*x*sin(a + b*x)**5/(15*b) + 4*c**3*d*x*sin(a + b*x)*
*3*cos(a + b*x)**2/(3*b) + 4*c**2*d**2*x**2*sin(a + b*x)**5/(5*b) + 2*c**2
*d**2*x**2*sin(a + b*x)**3*cos(a + b*x)**2/b + 8*c*d**3*x**3*sin(a + b*x)*
*5/(15*b) + 4*c*d**3*x**3*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 2*d**4*x
**4*sin(a + b*x)**5/(15*b) + d**4*x**4*sin(a + b*x)**3*cos(a + b*x)**2/(3*
b) + 8*c**3*d*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 52*c**3*d*sin(a + b
*x)**2*cos(a + b*x)**3/(45*b**2) + 104*c**3*d*cos(a + b*x)**5/(225*b**2) +
 8*c**2*d**2*x*sin(a + b*x)**4*cos(a + b*x)/(5*b**2) + 52*c**2*d**2*x*sin(
a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 104*c**2*d**2*x*cos(a + b*x)**5/(7
5*b**2) + 8*c*d**3*x**2*sin(a + b*x)**4*cos(a + b*x)/(5*b**2) + 52*c*d**3*
x**2*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 104*c*d**3*x**2*cos(a + b
*x)**5/(75*b**2) + 8*d**4*x**3*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 52
*d**4*x**3*sin(a + b*x)**2*cos(a + b*x)**3/(45*b**2) + 104*d**4*x**3*cos(a
 + b*x)**5/(225*b**2) - 1712*c**2*d**2*sin(a + b*x)**5/(1125*b**3) - 676*c
**2*d**2*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 104*c**2*d**2*sin(a
+ b*x)*cos(a + b*x)**4/(75*b**3) - 3424*c*d**3*x*sin(a + b*x)**5/(1125*b**
3) - 1352*c*d**3*x*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 208*c*d**3
*x*sin(a + b*x)*cos(a + b*x)**4/(75*b**3) - 1712*d**4*x**2*sin(a + b*x)**5
/(1125*b**3) - 676*d**4*x**2*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3)...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1339 vs. $2(304) = 608$.

Time $= 0.11$ (sec) , antiderivative size $= 1339$, normalized size of antiderivative $= 4.06$

$$\int (c + dx)^4 \cos^3(a + bx) \sin^2(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/4050000*(270000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c^4 - 1080000*(3*
sin(b*x + a)^5 - 5*sin(b*x + a)^3)*a*c^3*d/b + 1620000*(3*sin(b*x + a)^5 -
 5*sin(b*x + a)^3)*a^2*c^2*d^2/b^2 - 1080000*(3*sin(b*x + a)^5 - 5*sin(b*x
 + a)^3)*a^3*c*d^3/b^3 + 270000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*a^4*
d^4/b^4 + 4500*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3
*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a
) - 450*cos(b*x + a))*c^3*d/b - 13500*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*
(b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*
a) + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a*c^2*d^2/b^2 + 13500*(45*(b*
x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*si
n(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*
a^2*c*d^3/b^3 - 4500*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b
*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x
 + 3*a) - 450*cos(b*x + a))*a^3*d^4/b^4 + 450*(270*(b*x + a)*cos(5*b*x + 5
*a) + 750*(b*x + a)*cos(3*b*x + 3*a) - 13500*(b*x + a)*cos(b*x + a) + 27*(
25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 - 2)*sin(3*b*x +
 3*a) - 6750*((b*x + a)^2 - 2)*sin(b*x + a))*c^2*d^2/b^2 - 900*(270*(b*x +
 a)*cos(5*b*x + 5*a) + 750*(b*x + a)*cos(3*b*x + 3*a) - 13500*(b*x + a)*co
s(b*x + a) + 27*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2
 - 2)*sin(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*sin(b*x + a))*a*c*d^3/b...
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.61

$$
\begin{aligned}
&\int (c+dx)^4 \cos^3(a+bx) \sin^2(a+bx)\, dx \\
&= -\frac{(25\,b^3d^4x^3 + 75\,b^3cd^3x^2 + 75\,b^3c^2d^2x + 25\,b^3c^3d - 6\,bd^4x - 6\,bcd^3)\cos(5\,bx + 5\,a)}{2500\,b^5} \\
&\quad - \frac{(3\,b^3d^4x^3 + 9\,b^3cd^3x^2 + 9\,b^3c^2d^2x + 3\,b^3c^3d - 2\,bd^4x - 2\,bcd^3)\cos(3\,bx + 3\,a)}{108\,b^5} \\
&\quad + \frac{(b^3d^4x^3 + 3\,b^3cd^3x^2 + 3\,b^3c^2d^2x + b^3c^3d - 6\,bd^4x - 6\,bcd^3)\cos(bx + a)}{2\,b^5} \\
&\quad - \frac{(625\,b^4d^4x^4 + 2500\,b^4cd^3x^3 + 3750\,b^4c^2d^2x^2 + 2500\,b^4c^3dx + 625\,b^4c^4 - 300\,b^2d^4x^2 - 600\,b^2cd^3x - 300}{50000\,b^5} \\
&\quad - \frac{(27\,b^4d^4x^4 + 108\,b^4cd^3x^3 + 162\,b^4c^2d^2x^2 + 108\,b^4c^3dx + 27\,b^4c^4 - 36\,b^2d^4x^2 - 72\,b^2cd^3x - 36\,b^2c^2d^2 +}{1296\,b^5} \\
&\quad + \frac{(b^4d^4x^4 + 4\,b^4cd^3x^3 + 6\,b^4c^2d^2x^2 + 4\,b^4c^3dx + b^4c^4 - 12\,b^2d^4x^2 - 24\,b^2cd^3x - 12\,b^2c^2d^2 + 24\,d^4)\sin(bx}{8\,b^5}
\end{aligned}
$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2500*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 75*b^3*c^2*d^2*x + 25*b^3*c^3
*d - 6*b*d^4*x - 6*b*c*d^3)*cos(5*b*x + 5*a)/b^5 - 1/108*(3*b^3*d^4*x^3 +
9*b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*c
os(3*b*x + 3*a)/b^5 + 1/2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x
 + b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*cos(b*x + a)/b^5 - 1/50000*(625*b^4*
d^4*x^4 + 2500*b^4*c*d^3*x^3 + 3750*b^4*c^2*d^2*x^2 + 2500*b^4*c^3*d*x + 6
25*b^4*c^4 - 300*b^2*d^4*x^2 - 600*b^2*c*d^3*x - 300*b^2*c^2*d^2 + 24*d^4)
*sin(5*b*x + 5*a)/b^5 - 1/1296*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b
^4*c^2*d^2*x^2 + 108*b^4*c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*
d^3*x - 36*b^2*c^2*d^2 + 8*d^4)*sin(3*b*x + 3*a)/b^5 + 1/8*(b^4*d^4*x^4 +
4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4
*x^2 - 24*b^2*c*d^3*x - 12*b^2*c^2*d^2 + 24*d^4)*sin(b*x + a)/b^5
```

Mupad [B] (verification not implemented)

Time = 21.87 (sec) , antiderivative size = 816, normalized size of antiderivative = 2.47

$$\int (c+dx)^4 \cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^4,x)
```

output

```
-((d^4*sin(3*a + 3*b*x))/162 - 3*d^4*sin(a + b*x) + (3*d^4*sin(5*a + 5*b*x
))/6250 - (b^4*c^4*sin(a + b*x))/8 + (b^4*c^4*sin(3*a + 3*b*x))/48 + (b^4*
c^4*sin(5*a + 5*b*x))/80 + (b^3*c^3*d*cos(3*a + 3*b*x))/36 + (b^3*c^3*d*co
s(5*a + 5*b*x))/100 + (3*b^2*c^2*d^2*sin(a + b*x))/2 - (b^3*d^4*x^3*cos(a
+ b*x))/2 + (3*b^2*d^4*x^2*sin(a + b*x))/2 - (b^4*d^4*x^4*sin(a + b*x))/8
+ 3*b*c*d^3*cos(a + b*x) + 3*b*d^4*x*cos(a + b*x) - (b^2*c^2*d^2*sin(3*a +
 3*b*x))/36 - (3*b^2*c^2*d^2*sin(5*a + 5*b*x))/500 + (b^3*d^4*x^3*cos(3*a
+ 3*b*x))/36 + (b^3*d^4*x^3*cos(5*a + 5*b*x))/100 - (b^2*d^4*x^2*sin(3*a +
 3*b*x))/36 - (3*b^2*d^4*x^2*sin(5*a + 5*b*x))/500 + (b^4*d^4*x^4*sin(3*a
+ 3*b*x))/48 + (b^4*d^4*x^4*sin(5*a + 5*b*x))/80 - (b*c*d^3*cos(3*a + 3*b*
x))/54 - (3*b*c*d^3*cos(5*a + 5*b*x))/1250 - (b^3*c^3*d*cos(a + b*x))/2 -
(b*d^4*x*cos(3*a + 3*b*x))/54 - (3*b*d^4*x*cos(5*a + 5*b*x))/1250 + 3*b^2*
c*d^3*x*sin(a + b*x) - (b^4*c^3*d*x*sin(a + b*x))/2 + (b^4*c^2*d^2*x^2*sin
(3*a + 3*b*x))/8 + (3*b^4*c^2*d^2*x^2*sin(5*a + 5*b*x))/40 - (3*b^3*c^2*d^
2*x*cos(a + b*x))/2 - (3*b^3*c*d^3*x^2*cos(a + b*x))/2 - (b^2*c*d^3*x*sin(
3*a + 3*b*x))/18 + (b^4*c^3*d*x*sin(3*a + 3*b*x))/12 - (3*b^2*c*d^3*x*sin(
5*a + 5*b*x))/250 + (b^4*c^3*d*x*sin(5*a + 5*b*x))/20 - (b^4*c*d^3*x^3*sin
(a + b*x))/2 + (b^3*c^2*d^2*x*cos(3*a + 3*b*x))/12 + (b^3*c*d^3*x^2*cos(3*
a + 3*b*x))/12 + (3*b^3*c^2*d^2*x*cos(5*a + 5*b*x))/100 + (3*b^3*c*d^3*x^2
*cos(5*a + 5*b*x))/100 + (b^4*c*d^3*x^3*sin(3*a + 3*b*x))/12 + (b^4*c*d...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 807, normalized size of antiderivative = 2.45

$$\int (c+dx)^4 \cos^3(a+bx) \sin^2(a+bx)\, dx = \text{Too large to display}$$

input

```
int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

output

```
( - 40500*cos(a + b*x)*sin(a + b*x)**4*b**3*c**3*d - 121500*cos(a + b*x)*s
in(a + b*x)**4*b**3*c**2*d**2*x - 121500*cos(a + b*x)*sin(a + b*x)**4*b**3
*c*d**3*x**2 - 40500*cos(a + b*x)*sin(a + b*x)**4*b**3*d**4*x**3 + 9720*co
s(a + b*x)*sin(a + b*x)**4*b*c*d**3 + 9720*cos(a + b*x)*sin(a + b*x)**4*b*
d**4*x + 58500*cos(a + b*x)*sin(a + b*x)**2*b**3*c**3*d + 175500*cos(a + b
*x)*sin(a + b*x)**2*b**3*c**2*d**2*x + 175500*cos(a + b*x)*sin(a + b*x)**2
*b**3*c*d**3*x**2 + 58500*cos(a + b*x)*sin(a + b*x)**2*b**3*d**4*x**3 - 26
040*cos(a + b*x)*sin(a + b*x)**2*b*c*d**3 - 26040*cos(a + b*x)*sin(a + b*x
)**2*b*d**4*x + 117000*cos(a + b*x)*b**3*c**3*d + 351000*cos(a + b*x)*b**3
*c**2*d**2*x + 351000*cos(a + b*x)*b**3*c*d**3*x**2 + 117000*cos(a + b*x)*
b**3*d**4*x**3 - 754080*cos(a + b*x)*b*c*d**3 - 754080*cos(a + b*x)*b*d**4
*x - 50625*sin(a + b*x)**5*b**4*c**4 - 202500*sin(a + b*x)**5*b**4*c**3*d*
x - 303750*sin(a + b*x)**5*b**4*c**2*d**2*x**2 - 202500*sin(a + b*x)**5*b*
*4*c*d**3*x**3 - 50625*sin(a + b*x)**5*b**4*d**4*x**4 + 24300*sin(a + b*x)
**5*b**2*c**2*d**2 + 48600*sin(a + b*x)**5*b**2*c*d**3*x + 24300*sin(a + b
*x)**5*b**2*d**4*x**2 - 1944*sin(a + b*x)**5*d**4 + 84375*sin(a + b*x)**3*
b**4*c**4 + 337500*sin(a + b*x)**3*b**4*c**3*d*x + 506250*sin(a + b*x)**3*
b**4*c**2*d**2*x**2 + 337500*sin(a + b*x)**3*b**4*c*d**3*x**3 + 84375*sin(
a + b*x)**3*b**4*d**4*x**4 - 58500*sin(a + b*x)**3*b**2*c**2*d**2 - 117000
*sin(a + b*x)**3*b**2*c*d**3*x - 58500*sin(a + b*x)**3*b**2*d**4*x**2 +...
```

3.147 $\int (c+dx)^3 \cos^3(a+bx) \sin^2(a+bx)\, dx$

Optimal result 1354
Mathematica [A] (verified) 1355
Rubi [A] (verified) 1355
Maple [A] (verified) 1356
Fricas [A] (verification not implemented) 1357
Sympy [B] (verification not implemented) 1358
Maxima [B] (verification not implemented) 1358
Giac [A] (verification not implemented) 1359
Mupad [B] (verification not implemented) 1360
Reduce [B] (verification not implemented) 1361

Optimal result

Integrand size = 24, antiderivative size = 259

$$\begin{aligned}
&\int (c+dx)^3 \cos^3(a+bx) \sin^2(a+bx)\, dx \\
&= -\frac{3d^3 \cos(a+bx)}{4b^4} + \frac{3d(c+dx)^2 \cos(a+bx)}{8b^2} + \frac{d^3 \cos(3a+3bx)}{216b^4} \\
&\quad - \frac{d(c+dx)^2 \cos(3a+3bx)}{48b^2} + \frac{3d^3 \cos(5a+5bx)}{5000b^4} - \frac{3d(c+dx)^2 \cos(5a+5bx)}{400b^2} \\
&\quad - \frac{3d^2(c+dx) \sin(a+bx)}{4b^3} + \frac{(c+dx)^3 \sin(a+bx)}{8b} + \frac{d^2(c+dx) \sin(3a+3bx)}{72b^3} \\
&\quad - \frac{(c+dx)^3 \sin(3a+3bx)}{48b} + \frac{3d^2(c+dx) \sin(5a+5bx)}{1000b^3} - \frac{(c+dx)^3 \sin(5a+5bx)}{80b}
\end{aligned}$$

output

```
-3/4*d^3*cos(b*x+a)/b^4+3/8*d*(d*x+c)^2*cos(b*x+a)/b^2+1/216*d^3*cos(3*b*x
+3*a)/b^4-1/48*d*(d*x+c)^2*cos(3*b*x+3*a)/b^2+3/5000*d^3*cos(5*b*x+5*a)/b^
4-3/400*d*(d*x+c)^2*cos(5*b*x+5*a)/b^2-3/4*d^2*(d*x+c)*sin(b*x+a)/b^3+1/8*
(d*x+c)^3*sin(b*x+a)/b+1/72*d^2*(d*x+c)*sin(3*b*x+3*a)/b^3-1/48*(d*x+c)^3*
sin(3*b*x+3*a)/b+3/1000*d^2*(d*x+c)*sin(5*b*x+5*a)/b^3-1/80*(d*x+c)^3*sin(
5*b*x+5*a)/b
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.75

$$\int (c+dx)^3 \cos^3(a+bx) \sin^2(a+bx)\, dx =$$
$$-\frac{-101250d(-2d^2+b^2(c+dx)^2)\cos(a+bx)+625d(-2d^2+9b^2(c+dx)^2)\cos(3(a+bx))+81d(-2d^2$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
-1/270000*(-101250*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + 625*d*(-2*d
^2 + 9*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 81*d*(-2*d^2 + 25*b^2*(c + d*x)
^2)*Cos[5*(a + b*x)] + 30*b*(c + d*x)*(-825*b^2*c^2 + 6598*d^2 - 1650*b^2*
c*d*x - 825*b^2*d^2*x^2 + 8*(-38*d^2 + 75*b^2*(c + d*x)^2)*Cos[2*(a + b*x)
] + 9*(-6*d^2 + 25*b^2*(c + d*x)^2)*Cos[4*(a + b*x)])*Sin[a + b*x])/b^4
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin^2(a+bx) \cos^3(a+bx)\, dx$$
$$\downarrow \text{4906}$$
$$\int \left(\frac{1}{8}(c+dx)^3\cos(a+bx) - \frac{1}{16}(c+dx)^3\cos(3a+3bx) - \frac{1}{16}(c+dx)^3\cos(5a+5bx)\right) dx$$
$$\downarrow \text{2009}$$

$$-\frac{3d^3\cos(a+bx)}{4b^4}+\frac{d^3\cos(3a+3bx)}{216b^4}+\frac{3d^3\cos(5a+5bx)}{5000b^4}-\frac{3d^2(c+dx)\sin(a+bx)}{4b^3}+\frac{d^2(c+dx)\sin(3a+3bx)}{72b^3}+\frac{3d^2(c+dx)\sin(5a+5bx)}{1000b^3}+\frac{3d(c+dx)^2\cos(a+bx)}{8b^2}-\frac{d(c+dx)^2\cos(3a+3bx)}{48b^2}-\frac{3d(c+dx)^2\cos(5a+5bx)}{400b^2}+\frac{(c+dx)^3\sin(a+bx)}{8b}-\frac{(c+dx)^3\sin(3a+3bx)}{48b}-\frac{(c+dx)^3\sin(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
(-3*d^3*Cos[a + b*x])/(4*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x])/(8*b^2) + (
d^3*Cos[3*a + 3*b*x])/(216*b^4) - (d*(c + d*x)^2*Cos[3*a + 3*b*x])/(48*b^2
) + (3*d^3*Cos[5*a + 5*b*x])/(5000*b^4) - (3*d*(c + d*x)^2*Cos[5*a + 5*b*x
])/(400*b^2) - (3*d^2*(c + d*x)*Sin[a + b*x])/(4*b^3) + ((c + d*x)^3*Sin[a
 + b*x])/(8*b) + (d^2*(c + d*x)*Sin[3*a + 3*b*x])/(72*b^3) - ((c + d*x)^3*
Sin[3*a + 3*b*x])/(48*b) + (3*d^2*(c + d*x)*Sin[5*a + 5*b*x])/(1000*b^3) -
 ((c + d*x)^3*Sin[5*a + 5*b*x])/(80*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.79

method	result
parallelrisch	$\frac{-25\left((dx+c)^2b^2-\frac{2d^2}{3}\right)(dx+c)b\sin(3bx+3a)-15(dx+c)\left((dx+c)^2b^2-\frac{6d^2}{25}\right)b\sin(5bx+5a)-25\left((dx+c)^2b^2-\frac{2d^2}{9}\right)d\cos$
risch	$\frac{3d(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\cos(bx+a)}{8b^4}+\frac{(b^2d^3x^3+3b^2cd^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6cd^2)\sin(bx+a)}{8b^3}-\frac{3d(2}{}$
derivativedivides	Expression too large to display
default	Expression too large to display
orering	Expression too large to display

input
```
int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/1200*(-25*((d*x+c)^2*b^2-2/3*d^2)*(d*x+c)*b*sin(3*b*x+3*a)-15*(d*x+c)*((
d*x+c)^2*b^2-6/25*d^2)*b*sin(5*b*x+5*a)-25*((d*x+c)^2*b^2-2/9*d^2)*d*cos(3
*b*x+3*a)-9*d*((d*x+c)^2*b^2-2/25*d^2)*cos(5*b*x+5*a)+150*((d*x+c)^2*b^2-6
*d^2)*(d*x+c)*b*sin(b*x+a)+450*d*(((d*x+c)^2*b^2-2*d^2)*cos(b*x+a)+208/225
*b^2*c^2-100544/50625*d^2))/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.32

$$\int (c+dx)^3\cos^3(a+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{81\left(25\,b^2d^3x^2+50\,b^2cd^2x+25\,b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)^5-5\left(225\,b^2d^3x^2+450\,b^2cd^2x+225\,b^2c^2d+2\right.}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/16875*(81*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*cos(
b*x + a)^5 - 5*(225*b^2*d^3*x^2 + 450*b^2*c*d^2*x + 225*b^2*c^2*d + 22*d^3
)*cos(b*x + a)^3 - 30*(225*b^2*d^3*x^2 + 450*b^2*c*d^2*x + 225*b^2*c^2*d -
 428*d^3)*cos(b*x + a) - 15*(150*b^3*d^3*x^3 + 450*b^3*c*d^2*x^2 + 150*b^3
*c^3 - 9*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 25*b^3*c^3 - 6*b*c*d^2 + 3*(
25*b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^4 - 856*b*c*d^2 + (75*b^3*d^3*x^3
+ 225*b^3*c*d^2*x^2 + 75*b^3*c^3 + 22*b*c*d^2 + (225*b^3*c^2*d + 22*b*d^3)
*x)*cos(b*x + a)^2 + 2*(225*b^3*c^2*d - 428*b*d^3)*x)*sin(b*x + a))/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(255) = 510$.

Time = 0.86 (sec) , antiderivative size = 690, normalized size of antiderivative = 2.66

$$\int (c+dx)^3 \cos^3(a+bx) \sin^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output
```
Piecewise((2*c**3*sin(a + b*x)**5/(15*b) + c**3*sin(a + b*x)**3*cos(a + b*
x)**2/(3*b) + 2*c**2*d*x*sin(a + b*x)**5/(5*b) + c**2*d*x*sin(a + b*x)**3*
cos(a + b*x)**2/b + 2*c*d**2*x**2*sin(a + b*x)**5/(5*b) + c*d**2*x**2*sin(
a + b*x)**3*cos(a + b*x)**2/b + 2*d**3*x**3*sin(a + b*x)**5/(15*b) + d**3*
x**3*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 2*c**2*d*sin(a + b*x)**4*cos(
a + b*x)/(5*b**2) + 13*c**2*d*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) +
26*c**2*d*cos(a + b*x)**5/(75*b**2) + 4*c*d**2*x*sin(a + b*x)**4*cos(a + b
*x)/(5*b**2) + 26*c*d**2*x*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 52*
c*d**2*x*cos(a + b*x)**5/(75*b**2) + 2*d**3*x**2*sin(a + b*x)**4*cos(a + b
*x)/(5*b**2) + 13*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 26
*d**3*x**2*cos(a + b*x)**5/(75*b**2) - 856*c*d**2*sin(a + b*x)**5/(1125*b*
*3) - 338*c*d**2*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 52*c*d**2*si
n(a + b*x)*cos(a + b*x)**4/(75*b**3) - 856*d**3*x*sin(a + b*x)**5/(1125*b*
*3) - 338*d**3*x*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 52*d**3*x*si
n(a + b*x)*cos(a + b*x)**4/(75*b**3) - 856*d**3*sin(a + b*x)**4*cos(a + b*
x)/(1125*b**4) - 5114*d**3*sin(a + b*x)**2*cos(a + b*x)**3/(3375*b**4) - 1
2568*d**3*cos(a + b*x)**5/(16875*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**
2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**2*cos(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 766 vs. $2(235) = 470$.

Time = 0.08 (sec) , antiderivative size = 766, normalized size of antiderivative = 2.96

$$\int (c+dx)^3 \cos^3(a+bx) \sin^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/270000*(18000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c^3 - 54000*(3*sin(
b*x + a)^5 - 5*sin(b*x + a)^3)*a*c^2*d/b + 54000*(3*sin(b*x + a)^5 - 5*sin
(b*x + a)^3)*a^2*c*d^2/b^2 - 18000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*a
^3*d^3/b^3 + 225*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x +
 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3
*a) - 450*cos(b*x + a))*c^2*d/b - 450*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*
(b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*
a) + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a*c*d^2/b^2 + 225*(45*(b*x +
a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*
x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a^2*
d^3/b^3 + 15*(270*(b*x + a)*cos(5*b*x + 5*a) + 750*(b*x + a)*cos(3*b*x + 3
*a) - 13500*(b*x + a)*cos(b*x + a) + 27*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5
*a) + 125*(9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*si
n(b*x + a))*c*d^2/b^2 - 15*(270*(b*x + a)*cos(5*b*x + 5*a) + 750*(b*x + a)
*cos(3*b*x + 3*a) - 13500*(b*x + a)*cos(b*x + a) + 27*(25*(b*x + a)^2 - 2)
*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 6750*((b*x
+ a)^2 - 2)*sin(b*x + a))*a*d^3/b^3 + (81*(25*(b*x + a)^2 - 2)*cos(5*b*x +
 5*a) + 625*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 101250*((b*x + a)^2 - 2
)*cos(b*x + a) + 135*(25*(b*x + a)^3 - 6*b*x - 6*a)*sin(5*b*x + 5*a) + 187
5*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) - 33750*((b*x + a)^3 -...
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.36

$$
\begin{aligned}
&\int (c+dx)^3 \cos^3(a+bx) \sin^2(a+bx)\, dx \\
&= -\frac{3\left(25\,b^2d^3x^2 + 50\,b^2cd^2x + 25\,b^2c^2d - 2\,d^3\right)\cos\left(5\,bx + 5\,a\right)}{10000\,b^4} \\
&\quad - \frac{\left(9\,b^2d^3x^2 + 18\,b^2cd^2x + 9\,b^2c^2d - 2\,d^3\right)\cos\left(3\,bx + 3\,a\right)}{432\,b^4} \\
&\quad + \frac{3\left(b^2d^3x^2 + 2\,b^2cd^2x + b^2c^2d - 2\,d^3\right)\cos\left(bx + a\right)}{8\,b^4} \\
&\quad - \frac{\left(25\,b^3d^3x^3 + 75\,b^3cd^2x^2 + 75\,b^3c^2dx + 25\,b^3c^3 - 6\,bd^3x - 6\,bcd^2\right)\sin\left(5\,bx + 5\,a\right)}{2000\,b^4} \\
&\quad - \frac{\left(3\,b^3d^3x^3 + 9\,b^3cd^2x^2 + 9\,b^3c^2dx + 3\,b^3c^3 - 2\,bd^3x - 2\,bcd^2\right)\sin\left(3\,bx + 3\,a\right)}{144\,b^4} \\
&\quad + \frac{\left(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3 - 6\,bd^3x - 6\,bcd^2\right)\sin\left(bx + a\right)}{8\,b^4}
\end{aligned}
$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-3/10000*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*cos(5*b*
x + 5*a)/b^4 - 1/432*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3
)*cos(3*b*x + 3*a)/b^4 + 3/8*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*
d^3)*cos(b*x + a)/b^4 - 1/2000*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 75*b^3
*c^2*d*x + 25*b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(5*b*x + 5*a)/b^4 - 1/14
4*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^3*x
 - 2*b*c*d^2)*sin(3*b*x + 3*a)/b^4 + 1/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(b*x + a)/b^4
```

Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.99

$$\int (c+dx)^3 \cos^3(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{3\,d^3\cos(a+b\,x)}{4}-\frac{d^3\cos(3\,a+3\,b\,x)}{216}-\frac{3\,d^3\cos(5\,a+5\,b\,x)}{5000}-\frac{b^3\,c^3\sin(a+b\,x)}{8}+\frac{b^3\,c^3\sin(3\,a+3\,b\,x)}{48}+\frac{b^3\,c^3\sin(5\,a+5\,b\,x)}{80}+\frac{b^2\,c^2}{}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^3,x)
```

output
```
-((3*d^3*cos(a + b*x))/4 - (d^3*cos(3*a + 3*b*x))/216 - (3*d^3*cos(5*a + 5
*b*x))/5000 - (b^3*c^3*sin(a + b*x))/8 + (b^3*c^3*sin(3*a + 3*b*x))/48 + (
b^3*c^3*sin(5*a + 5*b*x))/80 + (b^2*c^2*d*cos(3*a + 3*b*x))/48 + (3*b^2*c^
2*d*cos(5*a + 5*b*x))/400 - (3*b^2*d^3*x^2*cos(a + b*x))/8 - (b^3*d^3*x^3*
sin(a + b*x))/8 + (3*b*c*d^2*sin(a + b*x))/4 + (3*b*d^3*x*sin(a + b*x))/4
+ (b^2*d^3*x^2*cos(3*a + 3*b*x))/48 + (3*b^2*d^3*x^2*cos(5*a + 5*b*x))/400
 + (b^3*d^3*x^3*sin(3*a + 3*b*x))/48 + (b^3*d^3*x^3*sin(5*a + 5*b*x))/80 -
 (3*b^2*c^2*d*cos(a + b*x))/8 - (b*c*d^2*sin(3*a + 3*b*x))/72 - (3*b*c*d^2
*sin(5*a + 5*b*x))/1000 - (b*d^3*x*sin(3*a + 3*b*x))/72 - (3*b*d^3*x*sin(5
*a + 5*b*x))/1000 - (3*b^2*c*d^2*x*cos(a + b*x))/4 - (3*b^3*c^2*d*x*sin(a
+ b*x))/8 + (b^2*c*d^2*x*cos(3*a + 3*b*x))/24 + (3*b^2*c*d^2*x*cos(5*a + 5
*b*x))/200 + (b^3*c^2*d*x*sin(3*a + 3*b*x))/16 + (3*b^3*c^2*d*x*sin(5*a +
5*b*x))/80 - (3*b^3*c*d^2*x^2*sin(a + b*x))/8 + (b^3*c*d^2*x^2*sin(3*a + 3
*b*x))/16 + (3*b^3*c*d^2*x^2*sin(5*a + 5*b*x))/80)/b^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.93

$$\int (c+dx)^3 \cos^3(a+bx) \sin^2(a+bx)\, dx$$

$$= \frac{-4050 \cos(bx+a) \sin(bx+a)^4 b^2 c\, d^2 x + 5850 \cos(bx+a) b^2 c^2 d + 5850 \cos(bx+a) b^2 d^3 x^2 + 5625 \sin(}{}$$

input

```
int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

output

```
( - 2025*cos(a + b*x)*sin(a + b*x)**4*b**2*c**2*d - 4050*cos(a + b*x)*sin(
a + b*x)**4*b**2*c*d**2*x - 2025*cos(a + b*x)*sin(a + b*x)**4*b**2*d**3*x*
*2 + 162*cos(a + b*x)*sin(a + b*x)**4*d**3 + 2925*cos(a + b*x)*sin(a + b*x
)**2*b**2*c**2*d + 5850*cos(a + b*x)*sin(a + b*x)**2*b**2*c*d**2*x + 2925*
cos(a + b*x)*sin(a + b*x)**2*b**2*d**3*x**2 - 434*cos(a + b*x)*sin(a + b*x
)**2*d**3 + 5850*cos(a + b*x)*b**2*c**2*d + 11700*cos(a + b*x)*b**2*c*d**2
*x + 5850*cos(a + b*x)*b**2*d**3*x**2 - 12568*cos(a + b*x)*d**3 - 3375*sin
(a + b*x)**5*b**3*c**3 - 10125*sin(a + b*x)**5*b**3*c**2*d*x - 10125*sin(a
 + b*x)**5*b**3*c*d**2*x**2 - 3375*sin(a + b*x)**5*b**3*d**3*x**3 + 810*si
n(a + b*x)**5*b*c*d**2 + 810*sin(a + b*x)**5*b*d**3*x + 5625*sin(a + b*x)*
*3*b**3*c**3 + 16875*sin(a + b*x)**3*b**3*c**2*d*x + 16875*sin(a + b*x)**3
*b**3*c*d**2*x**2 + 5625*sin(a + b*x)**3*b**3*d**3*x**3 - 1950*sin(a + b*x
)**3*b*c*d**2 - 1950*sin(a + b*x)**3*b*d**3*x - 11700*sin(a + b*x)*b*c*d**
2 - 11700*sin(a + b*x)*b*d**3*x - 11700*a*b*c*d**2 + 5850*b**2*c**2*d - 78
88*d**3)/(16875*b**4)
```

3.148 $\int (c+dx)^2 \cos^3(a+bx) \sin^2(a+bx)\, dx$

Optimal result . . . 1362
Mathematica [A] (verified) . . . 1363
Rubi [A] (verified) . . . 1363
Maple [A] (verified) . . . 1365
Fricas [A] (verification not implemented) . . . 1365
Sympy [B] (verification not implemented) . . . 1366
Maxima [B] (verification not implemented) . . . 1367
Giac [A] (verification not implemented) . . . 1367
Mupad [B] (verification not implemented) . . . 1368
Reduce [B] (verification not implemented) . . . 1369

Optimal result

Integrand size = 24, antiderivative size = 184

$$\begin{aligned}\int (c+dx)^2 \cos^3(a+bx) \sin^2(a+bx)\, dx = {} & \frac{d(c+dx)\cos(a+bx)}{4b^2} \\ & - \frac{d(c+dx)\cos(3a+3bx)}{72b^2} \\ & - \frac{d(c+dx)\cos(5a+5bx)}{200b^2} - \frac{d^2\sin(a+bx)}{4b^3} \\ & + \frac{(c+dx)^2\sin(a+bx)}{8b} + \frac{d^2\sin(3a+3bx)}{216b^3} \\ & - \frac{(c+dx)^2\sin(3a+3bx)}{48b} + \frac{d^2\sin(5a+5bx)}{1000b^3} \\ & - \frac{(c+dx)^2\sin(5a+5bx)}{80b}\end{aligned}$$

output

```
1/4*d*(d*x+c)*cos(b*x+a)/b^2-1/72*d*(d*x+c)*cos(3*b*x+3*a)/b^2-1/200*d*(d*
x+c)*cos(5*b*x+5*a)/b^2-1/4*d^2*sin(b*x+a)/b^3+1/8*(d*x+c)^2*sin(b*x+a)/b+
1/216*d^2*sin(3*b*x+3*a)/b^3-1/48*(d*x+c)^2*sin(3*b*x+3*a)/b+1/1000*d^2*si
n(5*b*x+5*a)/b^3-1/80*(d*x+c)^2*sin(5*b*x+5*a)/b
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.37

$$\int (c+dx)^2 \cos^3(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{-13500bd(c+dx)\cos(a+bx)+750bd(c+dx)\cos(3(a+bx))+270bcd\cos(5(a+bx))+270bd^2x\cos}{}$$

input

```
Integrate[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output

```
-1/54000*(-13500*b*d*(c + d*x)*Cos[a + b*x] + 750*b*d*(c + d*x)*Cos[3*(a +
 b*x)] + 270*b*c*d*Cos[5*(a + b*x)] + 270*b*d^2*x*Cos[5*(a + b*x)] - 6750*
b^2*c^2*Sin[a + b*x] + 13500*d^2*Sin[a + b*x] - 13500*b^2*c*d*x*Sin[a + b*
x] - 6750*b^2*d^2*x^2*Sin[a + b*x] + 1125*b^2*c^2*Sin[3*(a + b*x)] - 250*d
^2*Sin[3*(a + b*x)] + 2250*b^2*c*d*x*Sin[3*(a + b*x)] + 1125*b^2*d^2*x^2*S
in[3*(a + b*x)] + 675*b^2*c^2*Sin[5*(a + b*x)] - 54*d^2*Sin[5*(a + b*x)] +
 1350*b^2*c*d*x*Sin[5*(a + b*x)] + 675*b^2*d^2*x^2*Sin[5*(a + b*x)])/b^3
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin^2(a+bx)\cos^3(a+bx)\,dx$$

$$\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)^2\cos(a+bx)-\frac{1}{16}(c+dx)^2\cos(3a+3bx)-\frac{1}{16}(c+dx)^2\cos(5a+5bx)\right)dx$$

$$\downarrow \text{2009}$$

$$-\frac{d^2\sin(a+bx)}{4b^3}+\frac{d^2\sin(3a+3bx)}{216b^3}+\frac{d^2\sin(5a+5bx)}{1000b^3}+\frac{d(c+dx)\cos(a+bx)}{4b^2}-\frac{d(c+dx)\cos(3a+3bx)}{72b^2}-\frac{d(c+dx)\cos(5a+5bx)}{200b^2}+\frac{(c+dx)^2\sin(a+bx)}{8b}-\frac{(c+dx)^2\sin(3a+3bx)}{48b}-\frac{(c+dx)^2\sin(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
(d*(c + d*x)*Cos[a + b*x])/(4*b^2) - (d*(c + d*x)*Cos[3*a + 3*b*x])/(72*b^
2) - (d*(c + d*x)*Cos[5*a + 5*b*x])/(200*b^2) - (d^2*Sin[a + b*x])/(4*b^3)
 + ((c + d*x)^2*Sin[a + b*x])/(8*b) + (d^2*Sin[3*a + 3*b*x])/(216*b^3) - (
(c + d*x)^2*Sin[3*a + 3*b*x])/(48*b) + (d^2*Sin[5*a + 5*b*x])/(1000*b^3) -
 ((c + d*x)^2*Sin[5*a + 5*b*x])/(80*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 3.23 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.78

method	result
parallelrisch	$\frac{\left(-1125(dx+c)^2b^2+250d^2\right)\sin(3bx+3a)+\left(-675(dx+c)^2b^2+54d^2\right)\sin(5bx+5a)-750bd(dx+c)\cos(3bx+3a)-270bd(}{54000b^3}$
risch	$\frac{d(dx+c)\cos(bx+a)}{4b^2}+\frac{\left(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2\right)\sin(bx+a)}{8b^3}-\frac{d(dx+c)\cos(5bx+5a)}{200b^2}-\frac{\left(25x^2d^2b^2+50b^2cdx+}{2}$
derivativedivides	$\frac{a^2d^2\left(-\frac{\sin(bx+a)\cos(bx+a)^4}{5}+\frac{\left(2+\cos(bx+a)^2\right)\sin(bx+a)}{15}\right)}{b^2}-\frac{2acd\left(-\frac{\sin(bx+a)\cos(bx+a)^4}{5}+\frac{\left(2+\cos(bx+a)^2\right)\sin(bx+a)}{15}\right)}{b}-$
default	$\frac{a^2d^2\left(-\frac{\sin(bx+a)\cos(bx+a)^4}{5}+\frac{\left(2+\cos(bx+a)^2\right)\sin(bx+a)}{15}\right)}{b^2}-\frac{2acd\left(-\frac{\sin(bx+a)\cos(bx+a)^4}{5}+\frac{\left(2+\cos(bx+a)^2\right)\sin(bx+a)}{15}\right)}{b}-$
orering	Expression too large to display

input
```
int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/54000*((-1125*(d*x+c)^2*b^2+250*d^2)*sin(3*b*x+3*a)+(-675*(d*x+c)^2*b^2+
54*d^2)*sin(5*b*x+5*a)-750*b*d*(d*x+c)*cos(3*b*x+3*a)-270*b*d*(d*x+c)*cos(
5*b*x+5*a)+(6750*(d*x+c)^2*b^2-13500*d^2)*sin(b*x+a)+13500*d*((d*x+c)*cos(
b*x+a)+208/225*c)*b)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.05

$$\int (c+dx)^2\cos^3(a+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{270\left(bd^2x+bcd\right)\cos\left(bx+a\right)^5-150\left(bd^2x+bcd\right)\cos\left(bx+a\right)^3-900\left(bd^2x+bcd\right)\cos\left(bx+a\right)-(450}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/3375*(270*(b*d^2*x + b*c*d)*cos(b*x + a)^5 - 150*(b*d^2*x + b*c*d)*cos(
b*x + a)^3 - 900*(b*d^2*x + b*c*d)*cos(b*x + a) - (450*b^2*d^2*x^2 + 900*b
^2*c*d*x - 27*(25*b^2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*cos(b*x
 + a)^4 + 450*b^2*c^2 + (225*b^2*d^2*x^2 + 450*b^2*c*d*x + 225*b^2*c^2 + 2
2*d^2)*cos(b*x + a)^2 - 856*d^2)*sin(b*x + a))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(172) = 344$.

Time $= 0.62$ (sec) , antiderivative size $= 382$, normalized size of antiderivative $= 2.08$

$$\int (c+dx)^2 \cos^3(a+bx) \sin^2(a+bx)\, dx$$
$$= \begin{cases} \frac{2c^2 \sin^5(a+bx)}{15b} + \frac{c^2 \sin^3(a+bx)\cos^2(a+bx)}{3b} + \frac{4cdx \sin^5(a+bx)}{15b} + \frac{2cdx \sin^3(a+bx)\cos^2(a+bx)}{3b} + \frac{2d^2x^2 \sin^5(a+bx)}{15b} + \frac{d^2x^2 \sin^3(} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \sin^2(a) \cos^3(a) \end{cases}$$

input
```
integrate((d*x+c)**2*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output
```
Piecewise((2*c**2*sin(a + b*x)**5/(15*b) + c**2*sin(a + b*x)**3*cos(a + b*
x)**2/(3*b) + 4*c*d*x*sin(a + b*x)**5/(15*b) + 2*c*d*x*sin(a + b*x)**3*cos
(a + b*x)**2/(3*b) + 2*d**2*x**2*sin(a + b*x)**5/(15*b) + d**2*x**2*sin(a
+ b*x)**3*cos(a + b*x)**2/(3*b) + 4*c*d*sin(a + b*x)**4*cos(a + b*x)/(15*b
**2) + 26*c*d*sin(a + b*x)**2*cos(a + b*x)**3/(45*b**2) + 52*c*d*cos(a + b
*x)**5/(225*b**2) + 4*d**2*x*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 26*d
**2*x*sin(a + b*x)**2*cos(a + b*x)**3/(45*b**2) + 52*d**2*x*cos(a + b*x)**
5/(225*b**2) - 856*d**2*sin(a + b*x)**5/(3375*b**3) - 338*d**2*sin(a + b*x
)**3*cos(a + b*x)**2/(675*b**3) - 52*d**2*sin(a + b*x)*cos(a + b*x)**4/(22
5*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**2*cos(a)**3
, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(166) = 332.

Time = 0.05 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.04

$$\int (c+dx)^2 \cos^3(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{3600\left(3\sin(bx+a)^5-5\sin(bx+a)^3\right)c^2-\frac{7200\left(3\sin(bx+a)^5-5\sin(bx+a)^3\right)acd}{b}+\frac{3600\left(3\sin(bx+a)^5-5\sin(bx+a)\right.}{b^2}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/54000*(3600*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c^2 - 7200*(3*sin(b*x
 + a)^5 - 5*sin(b*x + a)^3)*a*c*d/b + 3600*(3*sin(b*x + a)^5 - 5*sin(b*x +
 a)^3)*a^2*d^2/b^2 + 30*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(
3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*
b*x + 3*a) - 450*cos(b*x + a))*c*d/b - 30*(45*(b*x + a)*sin(5*b*x + 5*a) +
 75*(b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x
+ 5*a) + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a*d^2/b^2 + (270*(b*x + a
)*cos(5*b*x + 5*a) + 750*(b*x + a)*cos(3*b*x + 3*a) - 13500*(b*x + a)*cos(
b*x + a) + 27*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 -
 2)*sin(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*sin(b*x + a))*d^2/b^2)/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.14

$$\begin{aligned}
&\int (c+dx)^2 \cos^3(a+bx)\sin^2(a+bx)\,dx \\
&= -\frac{(bd^2x+bcd)\cos(5\,bx+5\,a)}{200\,b^3} - \frac{(bd^2x+bcd)\cos(3\,bx+3\,a)}{72\,b^3} \\
&\quad + \frac{(bd^2x+bcd)\cos(bx+a)}{4\,b^3} - \frac{(25\,b^2d^2x^2+50\,b^2cdx+25\,b^2c^2-2\,d^2)\sin(5\,bx+5\,a)}{2000\,b^3} \\
&\quad - \frac{(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2)\sin(3\,bx+3\,a)}{432\,b^3} \\
&\quad + \frac{(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2)\sin(bx+a)}{8\,b^3}
\end{aligned}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/200*(b*d^2*x + b*c*d)*cos(5*b*x + 5*a)/b^3 - 1/72*(b*d^2*x + b*c*d)*cos
(3*b*x + 3*a)/b^3 + 1/4*(b*d^2*x + b*c*d)*cos(b*x + a)/b^3 - 1/2000*(25*b^
2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*sin(5*b*x + 5*a)/b^3 - 1/43
2*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*sin(3*b*x + 3*a)/b^3
+ 1/8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b*x + a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.60

$$
\begin{aligned}
&\int (c+dx)^2 \cos^3(a+bx)\sin^2(a+bx)\,dx \\
&= \frac{52\,d^2\,x\cos\left(a+b\,x\right)^5}{225\,b^2} - \frac{52\,d^2\cos\left(a+b\,x\right)^4\sin\left(a+b\,x\right)}{225\,b^3} \\
&\quad - \frac{\cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^3\left(338\,d^2-225\,b^2\,c^2\right)}{675\,b^3} \\
&\quad - \frac{2\sin\left(a+b\,x\right)^5\left(428\,d^2-225\,b^2\,c^2\right)}{3375\,b^3} + \frac{2\,d^2\,x^2\sin\left(a+b\,x\right)^5}{15\,b} \\
&\quad + \frac{52\,c\,d\cos\left(a+b\,x\right)^5}{225\,b^2} + \frac{4\,c\,d\,\cos\left(a+b\,x\right)\sin\left(a+b\,x\right)^4}{15\,b^2} \\
&\quad + \frac{4\,c\,d\,x\sin\left(a+b\,x\right)^5}{15\,b} + \frac{d^2\,x^2\cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^3}{3\,b} \\
&\quad + \frac{26\,c\,d\cos\left(a+b\,x\right)^3\sin\left(a+b\,x\right)^2}{45\,b^2} + \frac{4\,d^2\,x\,\cos\left(a+b\,x\right)\sin\left(a+b\,x\right)^4}{15\,b^2} \\
&\quad + \frac{26\,d^2\,x\cos\left(a+b\,x\right)^3\sin\left(a+b\,x\right)^2}{45\,b^2} + \frac{2\,c\,d\,x\cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^3}{3\,b}
\end{aligned}
$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^2,x)
```

output
```
(52*d^2*x*cos(a + b*x)^5)/(225*b^2) - (52*d^2*cos(a + b*x)^4*sin(a + b*x))
/(225*b^3) - (cos(a + b*x)^2*sin(a + b*x)^3*(338*d^2 - 225*b^2*c^2))/(675*
b^3) - (2*sin(a + b*x)^5*(428*d^2 - 225*b^2*c^2))/(3375*b^3) + (2*d^2*x^2*
sin(a + b*x)^5)/(15*b) + (52*c*d*cos(a + b*x)^5)/(225*b^2) + (4*c*d*cos(a
 + b*x)*sin(a + b*x)^4)/(15*b^2) + (4*c*d*x*sin(a + b*x)^5)/(15*b) + (d^2*x
^2*cos(a + b*x)^2*sin(a + b*x)^3)/(3*b) + (26*c*d*cos(a + b*x)^3*sin(a + b
*x)^2)/(45*b^2) + (4*d^2*x*cos(a + b*x)*sin(a + b*x)^4)/(15*b^2) + (26*d^2
*x*cos(a + b*x)^3*sin(a + b*x)^2)/(45*b^2) + (2*c*d*x*cos(a + b*x)^2*sin(a
 + b*x)^3)/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.41

$$\int (c+dx)^2 \cos^3(a+bx) \sin^2(a+bx)\, dx$$
$$= \frac{-270 \cos(bx+a) \sin(bx+a)^4 bcd - 270 \cos(bx+a) \sin(bx+a)^4 b\, d^2 x + 390 \cos(bx+a) \sin(bx+a)^2}$$

input
```
int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

output
```
( - 270*cos(a + b*x)*sin(a + b*x)**4*b*c*d - 270*cos(a + b*x)*sin(a + b*x)
**4*b*d**2*x + 390*cos(a + b*x)*sin(a + b*x)**2*b*c*d + 390*cos(a + b*x)*s
in(a + b*x)**2*b*d**2*x + 780*cos(a + b*x)*b*c*d + 780*cos(a + b*x)*b*d**2
*x - 675*sin(a + b*x)**5*b**2*c**2 - 1350*sin(a + b*x)**5*b**2*c*d*x - 675
*sin(a + b*x)**5*b**2*d**2*x**2 + 54*sin(a + b*x)**5*d**2 + 1125*sin(a + b
*x)**3*b**2*c**2 + 2250*sin(a + b*x)**3*b**2*c*d*x + 1125*sin(a + b*x)**3*
b**2*d**2*x**2 - 130*sin(a + b*x)**3*d**2 - 780*sin(a + b*x)*d**2 - 780*a*
d**2 + 780*b*c*d)/(3375*b**3)
```

3.149 $\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal result 1370
Mathematica [A] (verified) 1371
Rubi [A] (verified) 1371
Maple [A] (verified) 1372
Fricas [A] (verification not implemented) 1373
Sympy [A] (verification not implemented) 1373
Maxima [A] (verification not implemented) 1374
Giac [A] (verification not implemented) 1374
Mupad [B] (verification not implemented) 1375
Reduce [B] (verification not implemented) 1375

Optimal result

Integrand size = 22, antiderivative size = 109

$$\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx = \frac{d\cos(a+bx)}{8b^2} - \frac{d\cos(3a+3bx)}{144b^2} - \frac{d\cos(5a+5bx)}{400b^2} + \frac{(c+dx)\sin(a+bx)}{8b} - \frac{(c+dx)\sin(3a+3bx)}{48b} - \frac{(c+dx)\sin(5a+5bx)}{80b}$$

output

```
1/8*d*cos(b*x+a)/b^2-1/144*d*cos(3*b*x+3*a)/b^2-1/400*d*cos(5*b*x+5*a)/b^2
+1/8*(d*x+c)*sin(b*x+a)/b-1/48*(d*x+c)*sin(3*b*x+3*a)/b-1/80*(d*x+c)*sin(5
*b*x+5*a)/b
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01

$$\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{-450d\cos(a+bx)+25d\cos(3(a+bx))+9d\cos(5(a+bx))-450bc\sin(a+bx)-450bdx\sin(a+bx}{3600b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
-1/3600*(-450*d*Cos[a + b*x] + 25*d*Cos[3*(a + b*x)] + 9*d*Cos[5*(a + b*x)
] - 450*b*c*Sin[a + b*x] - 450*b*d*x*Sin[a + b*x] + 75*b*c*Sin[3*(a + b*x)
] + 75*b*d*x*Sin[3*(a + b*x)] + 45*b*c*Sin[5*(a + b*x)] + 45*b*d*x*Sin[5*(
a + b*x)])/b^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin^2(a+bx)\cos^3(a+bx)\,dx$$
$$\downarrow 4906$$
$$\int \left(\frac{1}{8}(c+dx)\cos(a+bx)-\frac{1}{16}(c+dx)\cos(3a+3bx)-\frac{1}{16}(c+dx)\cos(5a+5bx)\right)dx$$
$$\downarrow 2009$$
$$\frac{d\cos(a+bx)}{8b^2}-\frac{d\cos(3a+3bx)}{144b^2}-\frac{d\cos(5a+5bx)}{400b^2}+\frac{(c+dx)\sin(a+bx)}{8b}-$$
$$\frac{(c+dx)\sin(3a+3bx)}{48b}-\frac{(c+dx)\sin(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
(d*Cos[a + b*x])/(8*b^2) - (d*Cos[3*a + 3*b*x])/(144*b^2) - (d*Cos[5*a + 5
*b*x])/(400*b^2) + ((c + d*x)*Sin[a + b*x])/(8*b) - ((c + d*x)*Sin[3*a + 3
*b*x])/(48*b) - ((c + d*x)*Sin[5*a + 5*b*x])/(80*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{-75b(dx+c)\sin(3bx+3a)-45b(dx+c)\sin(5bx+5a)-25d\cos(3bx+3a)-9d\cos(5bx+5a)+450(dx+c)b\sin(bx+a)+450d}{3600b^2}$
risch	$\frac{d\cos(bx+a)}{8b^2}-\frac{d\cos(3bx+3a)}{144b^2}-\frac{d\cos(5bx+5a)}{400b^2}+\frac{(dx+c)\sin(bx+a)}{8b}-\frac{(dx+c)\sin(3bx+3a)}{48b}-\frac{(dx+c)\sin(5bx}{80b}$
derivativedivides	$-\frac{da\left(-\frac{\sin(bx+a)\cos(bx+a)^4}{5}+\frac{\left(2+\cos(bx+a)^2\right)\sin(bx+a)}{15}\right)}{b}+c\left(-\frac{\sin(bx+a)\cos(bx+a)^4}{5}+\frac{\left(2+\cos(bx+a)^2\right)\sin(bx+a)}{15}\right)+\frac{d\left(}{}$
default	$-\frac{da\left(-\frac{\sin(bx+a)\cos(bx+a)^4}{5}+\frac{\left(2+\cos(bx+a)^2\right)\sin(bx+a)}{15}\right)}{b}+c\left(-\frac{\sin(bx+a)\cos(bx+a)^4}{5}+\frac{\left(2+\cos(bx+a)^2\right)\sin(bx+a)}{15}\right)+\frac{d\left(}{}$
norman	$\frac{\frac{52d}{225b^2}+\frac{8c\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3}{3b}-\frac{16c\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^5}{15b}+\frac{8c\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^7}{3b}+\frac{4d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^6}{3b^2}+\frac{44d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4}{45b^2}+\frac{52d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{45b^2}+\frac{8d}{}}{\left(1+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)^5}$
orering	$\frac{2d\left(259d^4x^4b^4+1036b^4cd^3x^3+1554b^4c^2d^2x^2+1036b^4c^3dx+259b^4c^4+140b^2d^4x^2+280b^2cd^3x+140b^2c^2d^2+72d^4\right)\cos(}{225b^6(dx+c)^4}$

input
```
int((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3600*(-75*b*(d*x+c)*sin(3*b*x+3*a)-45*b*(d*x+c)*sin(5*b*x+5*a)-25*d*cos(
3*b*x+3*a)-9*d*cos(5*b*x+5*a)+450*(d*x+c)*b*sin(b*x+a)+450*d*cos(b*x+a)+41
6*d)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{9\,d\cos(bx+a)^5 - 5\,d\cos(bx+a)^3 - 30\,d\cos(bx+a) + 15\left(3\,(bdx+bc)\cos(bx+a)^4 - 2\,bdx - (bdx\right.}{225\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/225*(9*d*cos(b*x + a)^5 - 5*d*cos(b*x + a)^3 - 30*d*cos(b*x + a) + 15*(
3*(b*d*x + b*c)*cos(b*x + a)^4 - 2*b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 -
2*b*c)*sin(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.50

$$\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx$$
$$= \begin{cases} \frac{2c\sin^5(a+bx)}{15b} + \frac{c\sin^3(a+bx)\cos^2(a+bx)}{3b} + \frac{2dx\sin^5(a+bx)}{15b} + \frac{dx\sin^3(a+bx)\cos^2(a+bx)}{3b} + \frac{2d\sin^4(a+bx)\cos(a+bx)}{15b^2} + \frac{13d\sin^2}{} \\ \left(cx + \frac{dx^2}{2}\right)\sin^2(a)\cos^3(a) \end{cases}$$

input
```
integrate((d*x+c)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output

```
Piecewise((2*c*sin(a + b*x)**5/(15*b) + c*sin(a + b*x)**3*cos(a + b*x)**2/
(3*b) + 2*d*x*sin(a + b*x)**5/(15*b) + d*x*sin(a + b*x)**3*cos(a + b*x)**2
/(3*b) + 2*d*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 13*d*sin(a + b*x)**2
*cos(a + b*x)**3/(45*b**2) + 26*d*cos(a + b*x)**5/(225*b**2), Ne(b, 0)), (
(c*x + d*x**2/2)*sin(a)**2*cos(a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.28

$$\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{240\left(3\sin(bx+a)^5-5\sin(bx+a)^3\right)c-\frac{240\left(3\sin(bx+a)^5-5\sin(bx+a)^3\right)ad}{b}+\frac{(45(bx+a)\sin(5bx+5a)+75(bx+a)\si}{3600\,b}$$

input

```
integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/3600*(240*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c - 240*(3*sin(b*x + a)
^5 - 5*sin(b*x + a)^3)*a*d/b + (45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x +
a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25
*cos(3*b*x + 3*a) - 450*cos(b*x + a))*d/b)/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.97

$$\begin{aligned}\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx = &-\frac{d\cos(5bx+5a)}{400\,b^2}-\frac{d\cos(3bx+3a)}{144\,b^2}\\&+\frac{d\cos(bx+a)}{8\,b^2}-\frac{(bdx+bc)\sin(5bx+5a)}{80\,b^2}\\&-\frac{(bdx+bc)\sin(3bx+3a)}{48\,b^2}\\&+\frac{(bdx+bc)\sin(bx+a)}{8\,b^2}\end{aligned}$$

input

```
integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/400*d*cos(5*b*x + 5*a)/b^2 - 1/144*d*cos(3*b*x + 3*a)/b^2 + 1/8*d*cos(b
*x + a)/b^2 - 1/80*(b*d*x + b*c)*sin(5*b*x + 5*a)/b^2 - 1/48*(b*d*x + b*c)
*sin(3*b*x + 3*a)/b^2 + 1/8*(b*d*x + b*c)*sin(b*x + a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.09

$$\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{26\,d\cos(a+b\,x)^5 + 65\,d\cos(a+b\,x)^3\sin(a+b\,x)^2 + 30\,d\,\cos(a+b\,x)\,\sin(a+b\,x)^4 + 30\,b\,c\sin(a+b}{225\,b^2}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x),x)
```

output
```
(26*d*cos(a + b*x)^5 + 65*d*cos(a + b*x)^3*sin(a + b*x)^2 + 30*d*cos(a + b
*x)*sin(a + b*x)^4 + 30*b*c*sin(a + b*x)^5 + 30*b*d*x*sin(a + b*x)^5 + 75*
b*c*cos(a + b*x)^2*sin(a + b*x)^3 + 75*b*d*x*cos(a + b*x)^2*sin(a + b*x)^3
)/(225*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{-9\cos(bx+a)\sin(bx+a)^4\,d + 13\cos(bx+a)\sin(bx+a)^2\,d + 26\cos(bx+a)\,d - 45\sin(bx+a)^5\,bc -}{225b^2}$$

input
```
int((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

output
```
( - 9*cos(a + b*x)*sin(a + b*x)**4*d + 13*cos(a + b*x)*sin(a + b*x)**2*d +
 26*cos(a + b*x)*d - 45*sin(a + b*x)**5*b*c - 45*sin(a + b*x)**5*b*d*x + 7
5*sin(a + b*x)**3*b*c + 75*sin(a + b*x)**3*b*d*x + 26*d)/(225*b**2)
```

3.150 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx$

Optimal result . . . 1376
Mathematica [A] (verified) . . . 1377
Rubi [A] (verified) . . . 1377
Maple [A] (verified) . . . 1379
Fricas [A] (verification not implemented) . . . 1379
Sympy [F] . . . 1380
Maxima [C] (verification not implemented) . . . 1380
Giac [C] (verification not implemented) . . . 1381
Mupad [F(-1)] . . . 1382
Reduce [F] . . . 1383

Optimal result

Integrand size = 24, antiderivative size = 185

$$\begin{aligned}\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx = &\frac{\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d}\\ &-\frac{\cos\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d}\\ &-\frac{\cos\left(5a-\frac{5bc}{d}\right)\operatorname{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d}\\ &-\frac{\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{8d}\\ &+\frac{\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{16d}\\ &+\frac{\sin\left(5a-\frac{5bc}{d}\right)\operatorname{Si}\left(\frac{5bc}{d}+5bx\right)}{16d}\end{aligned}$$

output

```
1/8*cos(a-b*c/d)*Ci(b*c/d+b*x)/d-1/16*cos(3*a-3*b*c/d)*Ci(3*b*c/d+3*b*x)/d
-1/16*cos(5*a-5*b*c/d)*Ci(5*b*c/d+5*b*x)/d-1/8*sin(a-b*c/d)*Si(b*c/d+b*x)/
d+1/16*sin(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d+1/16*sin(5*a-5*b*c/d)*Si(5*b*c
/d+5*b*x)/d
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.83

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx$$
$$= \frac{2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right) - \cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right) - \cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegr}}{16d}$$

input
```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x),x]
```

output
```
(2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] - Cos[3*a - (3*b*c)/d]*CosInt
egral[(3*b*(c + d*x))/d] - Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b*(c + d*x)
)/d] - 2*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Sin[3*a - (3*b*c)/d]*
SinIntegral[(3*b*(c + d*x))/d] + Sin[5*a - (5*b*c)/d]*SinIntegral[(5*b*(c
+ d*x))/d])/(16*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos^3(a+bx)}{c+dx}\,dx$$
$$\downarrow 4906$$
$$\int \left(\frac{\cos(a+bx)}{8(c+dx)} - \frac{\cos(3a+3bx)}{16(c+dx)} - \frac{\cos(5a+5bx)}{16(c+dx)}\right)dx$$
$$\downarrow 2009$$

$$\frac{\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d}-\frac{\cos\left(5a-\frac{5bc}{d}\right)\operatorname{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d}-\frac{\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{8d}+\frac{\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{16d}+\frac{\sin\left(5a-\frac{5bc}{d}\right)\operatorname{Si}\left(\frac{5bc}{d}+5bx\right)}{16d}$$

input
```
Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x),x]
```

output
```
(Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d) - (Cos[3*a - (3*b*c)/d
]*CosIntegral[(3*b*c)/d + 3*b*x])/(16*d) - (Cos[5*a - (5*b*c)/d]*CosIntegr
al[(5*b*c)/d + 5*b*x])/(16*d) - (Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*
x])/(8*d) + (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d) +
 (Sin[5*a - (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{b\left(-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(bx+a+\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}\right)}{8}-\frac{b\left(-\frac{5\,\mathrm{Si}\left(-5bx-5a-\frac{5(-ad+cb)}{d}\right)\sin\left(\frac{-5ad}{d}\right.\right.}{d}$
default	$\frac{b\left(-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(bx+a+\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}\right)}{8}-\frac{b\left(-\frac{5\,\mathrm{Si}\left(-5bx-5a-\frac{5(-ad+cb)}{d}\right)\sin\left(\frac{-5ad}{d}\right.\right.}{d}$
risch	$\frac{e^{-\frac{5i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(5ibx+5ia-\frac{5i(ad-cb)}{d}\right)}{32d}+\frac{e^{-\frac{3i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{32d}-\frac{e^{-\frac{i(ad-c}{d}}}{d}$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*b*(-Si(-b*x-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*
c)/d)*cos((-a*d+b*c)/d)/d)-1/80*b*(-5*Si(-5*b*x-5*a-5*(-a*d+b*c)/d)*sin(5*
(-a*d+b*c)/d)/d+5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d)-1/48
*b*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+
3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.98

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx =$$
$$-\frac{\cos\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{5\,(bdx+bc)}{d}\right)+\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)-2\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)-\sin\left(-\right.}{16\,d}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
-1/16*(cos(-5*(b*c - a*d)/d)*cos_integral(5*(b*d*x + b*c)/d) + cos(-3*(b*c
 - a*d)/d)*cos_integral(3*(b*d*x + b*c)/d) - 2*cos(-(b*c - a*d)/d)*cos_int
egral((b*d*x + b*c)/d) - sin(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c
)/d) - sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) + 2*sin(-(b*c
 - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

Sympy [F]

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \int \frac{\sin^2(a+bx)\cos^3(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.23

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx =$$
$$-\frac{2\,b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b\left(E_1\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_1\right.}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output

```
-1/32*(2*b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_int
egral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b*(e
xp_integral_e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1,
 -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b*(exp_in
tegral_e(1, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1, -5*(
-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/d) - 2*b*(I*exp_int
egral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(1, -(I*b*
c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(-I*exp_integral_e(
1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integral_e(1, -3*(-I*b*c
- I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) + b*(-I*exp_integral_e(
1, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integral_e(1, -5*(-I*b*c
- I*(b*x + a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/(b*d)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.50 (sec) , antiderivative size = 44961, normalized size of antiderivative = 243.03

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

input

```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output

```
-1/32*(real_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*
tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + real_par
t(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integra
l(b*x + b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-b*x - b*c/d))*
tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*a)^2*t
an(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
2 + real_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*imag_par
t(cos_integral(b*x + b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/
2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*imag_part(cos_integral(-b*x
 - b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2
*b*c/d)^2*tan(1/2*b*c/d) + 8*sin_integral((b*d*x + b*c)/d)*tan(5/2*a)^2*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) -
 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(
1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_part(co
s_integral(-3*b*x - 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5
/2*b*c/d)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 4*sin_integral(3*(b*d*x +...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+b\,x)^3\sin(a+b\,x)^2}{c+d\,x}\,dx$$

input

```
int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x),x)
```

output

```
int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)^3\sin(bx+a)^2}{dx+c}dx$$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x)
```

output
```
int((cos(a + b*x)**3*sin(a + b*x)**2)/(c + d*x),x)
```

3.151 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 1384
Mathematica [A] (verified) . . . 1385
Rubi [A] (verified) . . . 1385
Maple [A] (verified) . . . 1387
Fricas [A] (verification not implemented) . . . 1387
Sympy [F] . . . 1388
Maxima [C] (verification not implemented) . . . 1388
Giac [C] (verification not implemented) . . . 1389
Mupad [F(-1)] . . . 1390
Reduce [F] . . . 1391

Optimal result

Integrand size = 24, antiderivative size = 257

$$\begin{aligned}\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = &-\frac{\cos(a+bx)}{8d(c+dx)}+\frac{\cos(3a+3bx)}{16d(c+dx)}+\frac{\cos(5a+5bx)}{16d(c+dx)}\\&+\frac{5b\,\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)\sin\left(5a-\frac{5bc}{d}\right)}{16d^2}\\&+\frac{3b\,\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{16d^2}\\&-\frac{b\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{8d^2}\\&-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^2}\\&+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{16d^2}\\&+\frac{5b\cos\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{16d^2}\end{aligned}$$

output

```
-1/8*cos(b*x+a)/d/(d*x+c)+1/16*cos(3*b*x+3*a)/d/(d*x+c)+1/16*cos(5*b*x+5*a
)/d/(d*x+c)+5/16*b*Ci(5*b*c/d+5*b*x)*sin(5*a-5*b*c/d)/d^2+3/16*b*Ci(3*b*c/
d+3*b*x)*sin(3*a-3*b*c/d)/d^2-1/8*b*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^2-1/8*b*c
os(a-b*c/d)*Si(b*c/d+b*x)/d^2+3/16*b*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^
2+5/16*b*cos(5*a-5*b*c/d)*Si(5*b*c/d+5*b*x)/d^2
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.82

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{\frac{d\cos(3(a+bx))}{c+dx} + \frac{d\cos(5(a+bx))}{c+dx} + 5b\,\text{CosIntegral}\left(\frac{5b(c+dx)}{d}\right)\sin\left(5a-\frac{5bc}{d}\right) + 3b\,\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a\right.}{}$$

input
```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
((d*Cos[3*(a + b*x)])/(c + d*x) + (d*Cos[5*(a + b*x)])/(c + d*x) + 5*b*Cos
Integral[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d] + 3*b*CosIntegral[(3*b*(c
 + d*x))/d]*Sin[3*a - (3*b*c)/d] - 2*((d*Cos[a + b*x])/(c + d*x) + b*CosIn
tegral[b*(c/d + x)]*Sin[a - (b*c)/d] + b*Cos[a - (b*c)/d]*SinIntegral[b*(c
/d + x)]) + 3*b*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] + 5*b*
Cos[5*a - (5*b*c)/d]*SinIntegral[(5*b*(c + d*x))/d])/(16*d^2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos^3(a+bx)}{(c+dx)^2}\,dx$$
$$\downarrow 4906$$
$$\int \left(\frac{\cos(a+bx)}{8(c+dx)^2} - \frac{\cos(3a+3bx)}{16(c+dx)^2} - \frac{\cos(5a+5bx)}{16(c+dx)^2}\right)dx$$
$$\downarrow 2009$$

$$\frac{5b\sin\left(5a-\frac{5bc}{d}\right)\operatorname{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d^2}+\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^2}-\frac{b\cos\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{8d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{16d^2}+\frac{5b\cos\left(5a-\frac{5bc}{d}\right)\operatorname{Si}\left(\frac{5bc}{d}+5bx\right)}{16d^2}-\frac{\cos(a+bx)}{8d(c+dx)}+\frac{\cos(3a+3bx)}{16d(c+dx)}+\frac{\cos(5a+5bx)}{16d(c+dx)}$$

input

```
Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

output

```
-1/8*Cos[a + b*x]/(d*(c + d*x)) + Cos[3*a + 3*b*x]/(16*d*(c + d*x)) + Cos[
5*a + 5*b*x]/(16*d*(c + d*x)) + (5*b*CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*
a - (5*b*c)/d])/(16*d^2) + (3*b*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (
3*b*c)/d])/(16*d^2) - (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(8*d
^2) - (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^2) + (3*b*Cos[3
*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) + (5*b*Cos[5*a -
(5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d^2)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.45

method	result
derivativedivides	$\frac{b^2\left(-\frac{\cos(bx+a)}{(-ad+cb+d(bx+a))d}-\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}}{d}\right)}{8}-\frac{b^2\left(-\frac{5\cos(5b}{(-ad+cb+d}\right.}{}$
default	$\frac{b^2\left(-\frac{\cos(bx+a)}{(-ad+cb+d(bx+a))d}-\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}}{d}\right)}{8}-\frac{b^2\left(-\frac{5\cos(5b}{(-ad+cb+d}\right.}{}$
risch	$-\frac{5ib\,\mathrm{e}^{-\frac{5i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(5ibx+5ia-\frac{5i(ad-cb)}{d}\right)}{32d^2}-\frac{3ib\,\mathrm{e}^{-\frac{3i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{32d^2}+$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*b^2*(-cos(b*x+a)/(-a*d+c*b+d*(b*x+a))/d-(-Si(-b*x-a-(-a*d+b*c)/d)
*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)-1/80*b
^2*(-5*cos(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))/d-5*(-5*Si(-5*b*x-5*a-5*(-a*d+b
*c)/d)*cos(5*(-a*d+b*c)/d)/d-5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*
c)/d)/d)/d)-1/48*b^2*(-3*cos(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d-3*(-3*Si(-3
*b*x-3*a-3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)
/d)*sin(3*(-a*d+b*c)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.03

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$$

$$= \frac{16\,d\cos(bx+a)^5 - 16\,d\cos(bx+a)^3 - 2\,(bdx+bc)\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)\sin\left(-\frac{bc-ad}{d}\right) + 3\,(bdx+bc)\,\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right.}{}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/16*(16*d*cos(b*x + a)^5 - 16*d*cos(b*x + a)^3 - 2*(b*d*x + b*c)*cos_inte
gral((b*d*x + b*c)/d)*sin(-(b*c - a*d)/d) + 3*(b*d*x + b*c)*cos_integral(3
*(b*d*x + b*c)/d)*sin(-3*(b*c - a*d)/d) + 5*(b*d*x + b*c)*cos_integral(5*(
b*d*x + b*c)/d)*sin(-5*(b*c - a*d)/d) + 5*(b*d*x + b*c)*cos(-5*(b*c - a*d)
/d)*sin_integral(5*(b*d*x + b*c)/d) + 3*(b*d*x + b*c)*cos(-3*(b*c - a*d)/d
)*sin_integral(3*(b*d*x + b*c)/d) - 2*(b*d*x + b*c)*cos(-(b*c - a*d)/d)*si
n_integral((b*d*x + b*c)/d))/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin^2(a+bx)\cos^3(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.74

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{2\,b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^2\left(E_2\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E\right.}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output

```
-1/32*(2*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_i
ntegral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b^
2*(exp_integral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_
e(2, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b^2*(
exp_integral_e(2, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(2
, -5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/d) - 2*b^2*(I
*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(2
, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^2*(I*exp_in
tegral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(2, -3
*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) - b^2*(I*exp_i
ntegral_e(2, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(2, -
5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/((b*c*d + (b
*x + a)*d^2 - a*d^2)*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 40.17 (sec) , antiderivative size = 1022022, normalized size of antiderivative = 3976.74

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input

```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output

```
1/32*(5*b*d*x*imag_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*x)^2*tan(
3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b
*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b*d*x*imag_part(cos_integral
(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)
^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d)^2 - 2*b*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/
2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c
/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*b*d*x*imag_part(cos_integral(-
b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 - 3*b*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*b*x)^2*tan(3/
2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c
/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 5*b*d*x*imag_part(cos_integral(-
5*b*x - 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^
2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + 10*b*d*x*sin_integral(5*(b*d*x + b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b*d*x*sin_integral(3*(b*d*x + b*c)/
d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*
tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 4*b*d...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+b\,x\right)^3\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\,dx$$

input

```
int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^2,x)
```

output

```
int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(bx+a)^3\sin(bx+a)^2}{d^2x^2+2cdx+c^2}dx$$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int((cos(a + b*x)**3*sin(a + b*x)**2)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.152 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$

Optimal result . . . 1392
Mathematica [A] (verified) . . . 1393
Rubi [A] (verified) . . . 1393
Maple [A] (verified) . . . 1395
Fricas [A] (verification not implemented) . . . 1396
Sympy [F] . . . 1396
Maxima [C] (verification not implemented) . . . 1397
Giac [C] (verification not implemented) . . . 1397
Mupad [F(-1)] . . . 1398
Reduce [F] . . . 1399

Optimal result

Integrand size = 24, antiderivative size = 338

$$\begin{aligned}\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx &= -\frac{\cos(a+bx)}{16d(c+dx)^2}+\frac{\cos(3a+3bx)}{32d(c+dx)^2}+\frac{\cos(5a+5bx)}{32d(c+dx)^2}\\ &-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{16d^3}\\ &+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^3}\\ &+\frac{25b^2\cos\left(5a-\frac{5bc}{d}\right)\operatorname{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{32d^3}\\ &+\frac{b\sin(a+bx)}{16d^2(c+dx)}-\frac{3b\sin(3a+3bx)}{32d^2(c+dx)}\\ &-\frac{5b\sin(5a+5bx)}{32d^2(c+dx)}+\frac{b^2\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{16d^3}\\ &-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{32d^3}\\ &-\frac{25b^2\sin\left(5a-\frac{5bc}{d}\right)\operatorname{Si}\left(\frac{5bc}{d}+5bx\right)}{32d^3}\end{aligned}$$

output
```
-1/16*cos(b*x+a)/d/(d*x+c)^2+1/32*cos(3*b*x+3*a)/d/(d*x+c)^2+1/32*cos(5*b*
x+5*a)/d/(d*x+c)^2-1/16*b^2*cos(a-b*c/d)*Ci(b*c/d+b*x)/d^3+9/32*b^2*cos(3*
a-3*b*c/d)*Ci(3*b*c/d+3*b*x)/d^3+25/32*b^2*cos(5*a-5*b*c/d)*Ci(5*b*c/d+5*b
*x)/d^3+1/16*b*sin(b*x+a)/d^2/(d*x+c)-3/32*b*sin(3*b*x+3*a)/d^2/(d*x+c)-5/
32*b*sin(5*b*x+5*a)/d^2/(d*x+c)+1/16*b^2*sin(a-b*c/d)*Si(b*c/d+b*x)/d^3-9/
32*b^2*sin(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^3-25/32*b^2*sin(5*a-5*b*c/d)*S
i(5*b*c/d+5*b*x)/d^3
```

Mathematica [A] (verified)

Time = 1.92 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.84

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{\frac{d^2\cos(3(a+bx))}{(c+dx)^2} + \frac{d^2\cos(5(a+bx))}{(c+dx)^2} - 2b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right) + 9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left($$

input
```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

output
```
((d^2*Cos[3*(a + b*x)])/(c + d*x)^2 + (d^2*Cos[5*(a + b*x)])/(c + d*x)^2 -
 2*b^2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] + 9*b^2*Cos[3*a - (3*b*c)
/d]*CosIntegral[(3*b*(c + d*x))/d] + 25*b^2*Cos[5*a - (5*b*c)/d]*CosIntegr
al[(5*b*(c + d*x))/d] + (2*d*(-(d*Cos[a + b*x]) + b*(c + d*x)*Sin[a + b*x]
))/(c + d*x)^2 - (3*b*d*Sin[3*(a + b*x)])/(c + d*x) - (5*b*d*Sin[5*(a + b*
x)])/(c + d*x) + 2*b^2*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 9*b^2*S
in[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] - 25*b^2*Sin[5*a - (5*b
*c)/d]*SinIntegral[(5*b*(c + d*x))/d])/(32*d^3)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos^3(a+bx)}{(c+dx)^3}\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{\cos(a+bx)}{8(c+dx)^3} - \frac{\cos(3a+3bx)}{16(c+dx)^3} - \frac{\cos(5a+5bx)}{16(c+dx)^3}\right) dx$$

$$\downarrow 2009$$

$$-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{16d^3} + \frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^3} + \frac{25b^2\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{32d^3} + \frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{16d^3} - \frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{32d^3} - \frac{25b^2\sin\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{32d^3} + \frac{b\sin(a+bx)}{16d^2(c+dx)} - \frac{3b\sin(3a+3bx)}{32d^2(c+dx)} - \frac{5b\sin(5a+5bx)}{32d^2(c+dx)} - \frac{\cos(a+bx)}{16d(c+dx)^2} + \frac{\cos(3a+3bx)}{32d(c+dx)^2} + \frac{\cos(5a+5bx)}{32d(c+dx)^2}$$

input

```
Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

output

```
-1/16*Cos[a + b*x]/(d*(c + d*x)^2) + Cos[3*a + 3*b*x]/(32*d*(c + d*x)^2) +
 Cos[5*a + 5*b*x]/(32*d*(c + d*x)^2) - (b^2*Cos[a - (b*c)/d]*CosIntegral[(
b*c)/d + b*x])/(16*d^3) + (9*b^2*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/
d + 3*b*x])/(32*d^3) + (25*b^2*Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b*c)/d
+ 5*b*x])/(32*d^3) + (b*Sin[a + b*x])/(16*d^2*(c + d*x)) - (3*b*Sin[3*a +
3*b*x])/(32*d^2*(c + d*x)) - (5*b*Sin[5*a + 5*b*x])/(32*d^2*(c + d*x)) + (
b^2*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(16*d^3) - (9*b^2*Sin[3*a
 - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(32*d^3) - (25*b^2*Sin[5*a -
 (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(32*d^3)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{b^3\left(-\frac{\cos(bx+a)}{2(-ad+cb+d(bx+a))^2d}-\frac{-\frac{\sin(bx+a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(bx+a+\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+}{d}\right.}{d}}{d}}{2d}\right.}{8}$
default	$\frac{b^3\left(-\frac{\cos(bx+a)}{2(-ad+cb+d(bx+a))^2d}-\frac{-\frac{\sin(bx+a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(bx+a+\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+}{d}\right.}{d}}{d}}{2d}\right.}{8}$
risch	$-\frac{25b^2\mathrm{e}^{-\frac{5i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(5ibx+5ia-\frac{5i(ad-cb)}{d}\right)}{64d^3}-\frac{9b^2\mathrm{e}^{-\frac{3i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{64d^3}+$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*b^3*(-1/2*cos(b*x+a)/(-a*d+c*b+d*(b*x+a))^2/d-1/2*(-sin(b*x+a)/(-
a*d+c*b+d*(b*x+a))/d+(-Si(-b*x-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+
a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)-1/80*b^3*(-5/2*cos(5*b*x+5*a)/(
-a*d+c*b+d*(b*x+a))^2/d-5/2*(-5*sin(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))/d+5*(-
5*Si(-5*b*x-5*a-5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*b*x+5*a+5*(-a
*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d)/d)/d)-1/48*b^3*(-3/2*cos(3*b*x+3*a)/(-a*
d+c*b+d*(b*x+a))^2/d-3/2*(-3*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d+3*(-3*S
i(-3*b*x-3*a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+
b*c)/d)*cos(3*(-a*d+b*c)/d)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.30

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{16\,d^2\cos(bx+a)^5 - 16\,d^2\cos(bx+a)^3 + 25\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\cos\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{5\,(bdx+bc)}{d}\right) +$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/32*(16*d^2*cos(b*x + a)^5 - 16*d^2*cos(b*x + a)^3 + 25*(b^2*d^2*x^2 + 2*
b^2*c*d*x + b^2*c^2)*cos(-5*(b*c - a*d)/d)*cos_integral(5*(b*d*x + b*c)/d)
 + 9*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-3*(b*c - a*d)/d)*cos_integ
ral(3*(b*d*x + b*c)/d) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-(b*c
 - a*d)/d)*cos_integral((b*d*x + b*c)/d) - 25*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*sin(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) - 9*(b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*
x + b*c)/d) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-(b*c - a*d)/d)*
sin_integral((b*d*x + b*c)/d) - 16*(5*(b*d^2*x + b*c*d)*cos(b*x + a)^4 - 3
*(b*d^2*x + b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^
2*d^3)
```

Sympy [F]

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin^2(a+bx)\cos^3(a+bx)}{(c+dx)^3}\,dx$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.42

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx =$$
$$-\frac{2\,b^3\left(E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^3\left(E_3\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E\right.}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/32*(2*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_i
ntegral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b^
3*(exp_integral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_
e(3, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b^3*(
exp_integral_e(3, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(3
, -5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/d) - 2*b^3*(I
*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(3
, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^3*(I*exp_in
tegral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(3, -3
*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) - b^3*(I*exp_i
ntegral_e(3, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(3, -
5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/((b^2*c^2*d
- 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))
*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 79.75 (sec) , antiderivative size = 1708998, normalized size of antiderivative = 5056.21

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

output

```
1/64*(25*b^2*d^2*x^2*real_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*x)
^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part
(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)
^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*ta
n(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2
*real_part(cos_integral(-b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*
b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x -
3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/
2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2
5*b^2*d^2*x^2*real_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*b*x)^2*tan
(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*
b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*b^2*d^2*x^2*imag_part(cos_i
ntegral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2
*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d) - 4*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(5/2*b*x)^
2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos\left(a+b\,x\right)^3\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^3}\,dx$$

input

```
int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^3,x)
```

output

```
int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos(bx+a)^3\sin(bx+a)^2}{d^3x^3+3cd^2x^2+3c^2dx+c^3}dx$$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x)
```

output
```
int((cos(a + b*x)**3*sin(a + b*x)**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
 d**3*x**3),x)
```

3.153 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$

Optimal result . . . 1400
Mathematica [A] (verified) . . . 1401
Rubi [A] (verified) . . . 1402
Maple [A] (verified) . . . 1403
Fricas [A] (verification not implemented) . . . 1405
Sympy [F] . . . 1405
Maxima [C] (verification not implemented) . . . 1406
Giac [F(-1)] . . . 1406
Mupad [F(-1)] . . . 1407
Reduce [F] . . . 1407

Optimal result

Integrand size = 24, antiderivative size = 413

$$\begin{aligned}\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = &-\frac{\cos(a+bx)}{24d(c+dx)^3}+\frac{b^2\cos(a+bx)}{48d^3(c+dx)}\\&+\frac{\cos(3a+3bx)}{48d(c+dx)^3}-\frac{3b^2\cos(3a+3bx)}{32d^3(c+dx)}\\&+\frac{\cos(5a+5bx)}{48d(c+dx)^3}-\frac{25b^2\cos(5a+5bx)}{96d^3(c+dx)}\\&-\frac{125b^3\,\mathrm{CosIntegral}\left(\frac{5bc}{d}+5bx\right)\sin\left(5a-\frac{5bc}{d}\right)}{96d^4}\\&-\frac{9b^3\,\mathrm{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{32d^4}\\&+\frac{b^3\,\mathrm{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{48d^4}\\&+\frac{b\sin(a+bx)}{48d^2(c+dx)^2}-\frac{b\sin(3a+3bx)}{32d^2(c+dx)^2}\\&-\frac{5b\sin(5a+5bx)}{96d^2(c+dx)^2}+\frac{b^3\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{48d^4}\\&-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Si}\left(\frac{3bc}{d}+3bx\right)}{32d^4}\\&-\frac{125b^3\cos\left(5a-\frac{5bc}{d}\right)\mathrm{Si}\left(\frac{5bc}{d}+5bx\right)}{96d^4}\end{aligned}$$

output
```
-1/24*cos(b*x+a)/d/(d*x+c)^3+1/48*b^2*cos(b*x+a)/d^3/(d*x+c)+1/48*cos(3*b*
x+3*a)/d/(d*x+c)^3-3/32*b^2*cos(3*b*x+3*a)/d^3/(d*x+c)+1/48*cos(5*b*x+5*a)
/d/(d*x+c)^3-25/96*b^2*cos(5*b*x+5*a)/d^3/(d*x+c)-125/96*b^3*Ci(5*b*c/d+5*
b*x)*sin(5*a-5*b*c/d)/d^4-9/32*b^3*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^4+
1/48*b^3*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^4+1/48*b*sin(b*x+a)/d^2/(d*x+c)^2-1/
32*b*sin(3*b*x+3*a)/d^2/(d*x+c)^2-5/96*b*sin(5*b*x+5*a)/d^2/(d*x+c)^2+1/48
*b^3*cos(a-b*c/d)*Si(b*c/d+b*x)/d^4-9/32*b^3*cos(3*a-3*b*c/d)*Si(3*b*c/d+3
*b*x)/d^4-125/96*b^3*cos(5*a-5*b*c/d)*Si(5*b*c/d+5*b*x)/d^4
```

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.09

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx =$$

$$-\frac{d\cos(3bx)\left(\left(-2d^2+9b^2(c+dx)^2\right)\cos(3a)+3bd(c+dx)\sin(3a)\right)+d\cos(5bx)\left(\left(-2d^2+25b^2(c+dx)^2\right)\cos(5a)+5bd(c+dx)\sin(5a)\right)+d\left(3bd(c+dx)\cos(3a)-\left(-2d^2+9b^2(c+dx)^2\right)\sin(3a)\right)\sin(3bx)+d\left(5bd(c+dx)\cos(5a)-\left(-2d^2+25b^2(c+dx)^2\right)\sin(5a)\right)\sin(5bx)-2\left(d\cos(bx)\left(\left(-2d^2+b^2(c+dx)^2\right)\cos(a)+bd(c+dx)\sin(a)\right)+d\left(bd(c+dx)\cos(a)-\left(-2d^2+b^2(c+dx)^2\right)\sin(a)\right)\sin(bx)+b^3(c+dx)^3\left(\operatorname{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right)+\cos\left(a-\frac{bc}{d}\right)\operatorname{SinIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\right)\right)+27b^3(c+dx)^3\left(\operatorname{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right)+\cos\left(3a-\frac{3bc}{d}\right)\operatorname{SinIntegral}\left(\frac{3b(c+dx)}{d}\right)\right)+125b^3(c+dx)^3\left(\operatorname{CosIntegral}\left(\frac{5b(c+dx)}{d}\right)\sin\left(5a-\frac{5bc}{d}\right)+\cos\left(5a-\frac{5bc}{d}\right)\operatorname{SinIntegral}\left(\frac{5b(c+dx)}{d}\right)\right)}{96d^4(c+dx)^3}$$

input
```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

output
```
-1/96*(d*Cos[3*b*x]*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*
x)*Sin[3*a]) + d*Cos[5*b*x]*((-2*d^2 + 25*b^2*(c + d*x)^2)*Cos[5*a] + 5*b*
d*(c + d*x)*Sin[5*a]) + d*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c +
 d*x)^2)*Sin[3*a])*Sin[3*b*x] + d*(5*b*d*(c + d*x)*Cos[5*a] - (-2*d^2 + 25
*b^2*(c + d*x)^2)*Sin[5*a])*Sin[5*b*x] - 2*(d*Cos[b*x]*((-2*d^2 + b^2*(c +
 d*x)^2)*Cos[a] + b*d*(c + d*x)*Sin[a]) + d*(b*d*(c + d*x)*Cos[a] - (-2*d^
2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x] + b^3*(c + d*x)^3*(CosIntegral[b*(c/
d + x)]*Sin[a - (b*c)/d] + Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)])) + 2
7*b^3*(c + d*x)^3*(CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + C
os[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d]) + 125*b^3*(c + d*x)^3*
(CosIntegral[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d] + Cos[5*a - (5*b*c)/d
]*SinIntegral[(5*b*(c + d*x))/d]))/(d^4*(c + d*x)^3)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos^3(a+bx)}{(c+dx)^4}\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{\cos(a+bx)}{8(c+dx)^4} - \frac{\cos(3a+3bx)}{16(c+dx)^4} - \frac{\cos(5a+5bx)}{16(c+dx)^4}\right)dx$$

$$\downarrow 2009$$

$$-\frac{125b^3 \sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{96d^4} - \frac{9b^3 \sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^4} + \frac{b^3 \sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{48d^4} + \frac{b^3 \cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{48d^4} - \frac{9b^3 \cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{32d^4} - \frac{125b^3 \cos\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{96d^4} + \frac{b^2\cos(a+bx)}{48d^3(c+dx)} - \frac{3b^2\cos(3a+3bx)}{32d^3(c+dx)} - \frac{25b^2\cos(5a+5bx)}{96d^3(c+dx)} + \frac{b\sin(a+bx)}{48d^2(c+dx)^2} - \frac{b\sin(3a+3bx)}{32d^2(c+dx)^2} - \frac{5b\sin(5a+5bx)}{96d^2(c+dx)^2} - \frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} + \frac{\cos(5a+5bx)}{48d(c+dx)^3}$$

input
```
Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

output
```
-1/24*Cos[a + b*x]/(d*(c + d*x)^3) + (b^2*Cos[a + b*x])/(48*d^3*(c + d*x))
 + Cos[3*a + 3*b*x]/(48*d*(c + d*x)^3) - (3*b^2*Cos[3*a + 3*b*x])/(32*d^3*
(c + d*x)) + Cos[5*a + 5*b*x]/(48*d*(c + d*x)^3) - (25*b^2*Cos[5*a + 5*b*x
])/(96*d^3*(c + d*x)) - (125*b^3*CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a -
(5*b*c)/d])/(96*d^4) - (9*b^3*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*
b*c)/d])/(32*d^4) + (b^3*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(48*
d^4) + (b*Sin[a + b*x])/(48*d^2*(c + d*x)^2) - (b*Sin[3*a + 3*b*x])/(32*d^
2*(c + d*x)^2) - (5*b*Sin[5*a + 5*b*x])/(96*d^2*(c + d*x)^2) + (b^3*Cos[a
- (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(48*d^4) - (9*b^3*Cos[3*a - (3*b*c)
/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(32*d^4) - (125*b^3*Cos[5*a - (5*b*c)/
d]*SinIntegral[(5*b*c)/d + 5*b*x])/(96*d^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.42

method	result
derivativedivides	$b^4\left(-\frac{\cos(bx+a)}{3(-ad+cb+d(bx+a))^3d}-\frac{-\frac{\sin(bx+a)}{2(-ad+cb+d(bx+a))^2d}+\frac{-\frac{\cos(bx+a)}{(-ad+cb+d(bx+a))d}-\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}-\frac{\mathrm{Ci}}{d}}{2d}}{3d}\right)$ … $/8$
default	$b^4\left(-\frac{\cos(bx+a)}{3(-ad+cb+d(bx+a))^3d}-\frac{-\frac{\sin(bx+a)}{2(-ad+cb+d(bx+a))^2d}+\frac{-\frac{\cos(bx+a)}{(-ad+cb+d(bx+a))d}-\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}-\frac{\mathrm{Ci}}{d}}{2d}}{3d}\right)$ … $/8$
risch	Expression too large to display

input
```
int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*b^4*(-1/3*cos(b*x+a)/(-a*d+c*b+d*(b*x+a))^3/d-1/3*(-1/2*sin(b*x+a
)/(-a*d+c*b+d*(b*x+a))^2/d+1/2*(-cos(b*x+a)/(-a*d+c*b+d*(b*x+a))/d-(-Si(-b
*x-a-(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*
c)/d)/d)/d)/d)/d)-1/80*b^4*(-5/3*cos(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))^3/d-5
/3*(-5/2*sin(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))^2/d+5/2*(-5*cos(5*b*x+5*a)/(-
a*d+c*b+d*(b*x+a))/d-5*(-5*Si(-5*b*x-5*a-5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/
d)/d-5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d)/d)/d)/d)-1/48*b
^4*(-cos(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))^3/d-(-3/2*sin(3*b*x+3*a)/(-a*d+c*
b+d*(b*x+a))^2/d+3/2*(-3*cos(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d-3*(-3*Si(-3
*b*x-3*a-3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)
/d)*sin(3*(-a*d+b*c)/d)/d)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.55

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx =$$

$$-\frac{16\left(25\,b^2d^3x^2+50\,b^2cd^2x+25\,b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)^5-16\left(29\,b^2d^3x^2+58\,b^2cd^2x+29\,b^2c^2d-2\,d\right.}{}$$

input

```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

output

```
-1/96*(16*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*cos(b*x
 + a)^5 - 16*(29*b^2*d^3*x^2 + 58*b^2*c*d^2*x + 29*b^2*c^2*d - 2*d^3)*cos(
b*x + a)^3 - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*c
os_integral((b*d*x + b*c)/d)*sin(-(b*c - a*d)/d) + 27*(b^3*d^3*x^3 + 3*b^3
*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(3*(b*d*x + b*c)/d)*sin(
-3*(b*c - a*d)/d) + 125*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b
^3*c^3)*cos_integral(5*(b*d*x + b*c)/d)*sin(-5*(b*c - a*d)/d) + 125*(b^3*d
^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-5*(b*c - a*d)/d)*
sin_integral(5*(b*d*x + b*c)/d) + 27*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + b^3*c^3)*cos(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d)
 - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-(b*c -
 a*d)/d)*sin_integral((b*d*x + b*c)/d) + 96*(b^2*d^3*x^2 + 2*b^2*c*d^2*x +
 b^2*c^2*d)*cos(b*x + a) + 16*(5*(b*d^3*x + b*c*d^2)*cos(b*x + a)^4 - 3*(b
*d^3*x + b*c*d^2)*cos(b*x + a)^2)*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3
*c^2*d^5*x + c^3*d^4)
```

Sympy [F]

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sin^2\left(a+bx\right)\cos^3\left(a+bx\right)}{\left(c+dx\right)^4}\,dx$$

input

```
integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c)**4,x)
```

output

```
Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x)**4, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.29

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx =$$
$$-\frac{2\,b^4\left(E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^4\left(E_4\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E\right.}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/32*(2*b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_i
ntegral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b^
4*(exp_integral_e(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_
e(4, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b^4*(
exp_integral_e(4, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(4
, -5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/d) - 2*b^4*(I
*exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(4
, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^4*(I*exp_in
tegral_e(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(4, -3
*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) - b^4*(I*exp_i
ntegral_e(4, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(4, -
5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/((b^3*c^3*d
- 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3
 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))
*b)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos(a+b\,x)^3\sin(a+b\,x)^2}{(c+d\,x)^4}\,dx$$

input
```
int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^4,x)
```

output
```
int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^4, x)
```

Reduce [F]

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos(bx+a)^3\sin(bx+a)^2}{d^4x^4+4c\,d^3x^3+6c^2d^2x^2+4c^3dx+c^4}dx$$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x)
```

output
```
int((cos(a + b*x)**3*sin(a + b*x)**2)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**
2 + 4*c*d**3*x**3 + d**4*x**4),x)
```

3.154 $\int (c+dx)^m \cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal result . . . 1408
Mathematica [A] (verified) . . . 1409
Rubi [A] (verified) . . . 1409
Maple [F] . . . 1411
Fricas [A] (verification not implemented) . . . 1411
Sympy [F] . . . 1411
Maxima [F] . . . 1412
Giac [F] . . . 1412
Mupad [F(-1)] . . . 1412
Reduce [F] . . . 1413

Optimal result

Integrand size = 24, antiderivative size = 285

$$\begin{aligned}
&\int (c+dx)^m \cos^3(a+bx)\sin^3(a+bx)\,dx \\
&= -\frac{3\,2^{-7-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} \\
&\quad -\frac{3\,2^{-7-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2ib(c+dx)}{d}\right)}{b} \\
&\quad +\frac{2^{-7-m}3^{-1-m}e^{6i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{6ib(c+dx)}{d}\right)}{b} \\
&\quad +\frac{2^{-7-m}3^{-1-m}e^{-6i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{6ib(c+dx)}{d}\right)}{b}
\end{aligned}$$

output

```
-3*2^(-7-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-
I*b*(d*x+c)/d)^m)-3*2^(-7-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*
I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)+2^(-7-m)*3^(-1-m)*exp(6*I*(a-b*c/d))*(d*x
+c)^m*GAMMA(1+m,-6*I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)+2^(-7-m)*3^(-1-m)
*(d*x+c)^m*GAMMA(1+m,6*I*b*(d*x+c)/d)/b/exp(6*I*(a-b*c/d))/((I*b*(d*x+c)/d
)^m)
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.89

$$\int (c+dx)^m \cos^3(a+bx)\sin^3(a+bx)\,dx$$

$$= \frac{2^{-7-m}3^{-1-m}e^{-\frac{6i(bc+ad)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-3^{2+m}e^{4i\left(2a+\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^m\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)-3^{2}\right.}{}$$

input

```
Integrate[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output

```
(2^(-7 - m)*3^(-1 - m)*(c + d*x)^m*(-(3^(2 + m)*E^((4*I)*(2*a + (b*c)/d))*
((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d]) - 3^(2 + m)*E^
((4*I)*a + ((8*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((2*I)*b*(
c + d*x))/d] + E^((12*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-6*I)*b*(
c + d*x))/d] + E^(((12*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((
6*I)*b*(c + d*x))/d]))/(b*E^(((6*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2
)^m)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\cos^3(a+bx)(c+dx)^m\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{3}{32}\sin(2a+2bx)(c+dx)^m - \frac{1}{32}\sin(6a+6bx)(c+dx)^m\right)dx$$

$$\downarrow 2009$$

$$-\frac{3\,2^{-m-7}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-m-7}3^{-m-1}e^{6i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{6ib(c+dx)}{d}\right)}{b}-\frac{3\,2^{-m-7}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-m-7}3^{-m-1}e^{-6i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{6ib(c+dx)}{d}\right)}{b}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(-3*2^(-7 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*
(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) - (3*2^(-7 - m)*(c + d*x)^m*Ga
mma[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d
*x))/d)^m) + (2^(-7 - m)*3^(-1 - m)*E^((6*I)*(a - (b*c)/d))*(c + d*x)^m*Ga
mma[1 + m, ((-6*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + (2^(-7
 - m)*3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((6*I)*b*(c + d*x))/d])/(b*E^((6*
I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [F]

$$\int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)^3 \, dx$$

input
```
int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

output
```
int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.67

$$\int (c+dx)^m \cos^3(a+bx) \sin^3(a+bx) \, dx =$$

$$-\frac{9\, e^{\left(-\frac{dm \log\left(-\frac{2ib}{d}\right)+2ibc-2iad}{d}\right)} \Gamma\left(m+1, -\frac{2(ibdx+ibc)}{d}\right) - e^{\left(-\frac{dm \log\left(-\frac{6ib}{d}\right)+6ibc-6iad}{d}\right)} \Gamma\left(m+1, -\frac{6(ibdx+ibc)}{d}\right) +}{384}$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/384*(9*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d)*gamma(m + 1, -2*(
I*b*d*x + I*b*c)/d) - e^(-(d*m*log(-6*I*b/d) + 6*I*b*c - 6*I*a*d)/d)*gamma
(m + 1, -6*(I*b*d*x + I*b*c)/d) + 9*e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*
a*d)/d)*gamma(m + 1, -2*(-I*b*d*x - I*b*c)/d) - e^(-(d*m*log(6*I*b/d) - 6*
I*b*c + 6*I*a*d)/d)*gamma(m + 1, -6*(-I*b*d*x - I*b*c)/d))/b
```

Sympy [F]

$$\int (c+dx)^m \cos^3(a+bx) \sin^3(a+bx) \, dx = \int (c+dx)^m \sin^3(a+bx) \cos^3(a+bx) \, dx$$

input
```
integrate((d*x+c)**m*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**m*sin(a + b*x)**3*cos(a + b*x)**3, x)
```

Maxima [F]

$$\int (c+dx)^m \cos^3(a+bx) \sin^3(a+bx)\, dx = \int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)^3\, dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^3, x)
```

Giac [F]

$$\int (c+dx)^m \cos^3(a+bx) \sin^3(a+bx)\, dx = \int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)^3\, dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos^3(a+bx) \sin^3(a+bx)\, dx = \int \cos(a+b\,x)^3 \sin(a+b\,x)^3 (c+d\,x)^m\, dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \cos^3(a+bx) \sin^3(a+bx)\, dx = \int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)^3\, dx$$

input
```
int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

output
```
int((c + d*x)**m*cos(a + b*x)**3*sin(a + b*x)**3,x)
```

3.155 $\int (c+dx)^4 \cos^3(a+bx) \sin^3(a+bx)\, dx$

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Optimal result

Integrand size = 24, antiderivative size = 233

$$\begin{aligned}
&\int (c+dx)^4 \cos^3(a+bx) \sin^3(a+bx)\, dx \\
&= -\frac{9d^4 \cos(2a+2bx)}{128b^5} + \frac{9d^2(c+dx)^2 \cos(2a+2bx)}{64b^3} - \frac{3(c+dx)^4 \cos(2a+2bx)}{64b} \\
&\quad + \frac{d^4 \cos(6a+6bx)}{10368b^5} - \frac{d^2(c+dx)^2 \cos(6a+6bx)}{576b^3} + \frac{(c+dx)^4 \cos(6a+6bx)}{192b} \\
&\quad - \frac{9d^3(c+dx) \sin(2a+2bx)}{64b^4} + \frac{3d(c+dx)^3 \sin(2a+2bx)}{32b^2} \\
&\quad + \frac{d^3(c+dx) \sin(6a+6bx)}{1728b^4} - \frac{d(c+dx)^3 \sin(6a+6bx)}{288b^2}
\end{aligned}$$

output

```
-9/128*d^4*cos(2*b*x+2*a)/b^5+9/64*d^2*(d*x+c)^2*cos(2*b*x+2*a)/b^3-3/64*(
d*x+c)^4*cos(2*b*x+2*a)/b+1/10368*d^4*cos(6*b*x+6*a)/b^5-1/576*d^2*(d*x+c)
^2*cos(6*b*x+6*a)/b^3+1/192*(d*x+c)^4*cos(6*b*x+6*a)/b-9/64*d^3*(d*x+c)*si
n(2*b*x+2*a)/b^4+3/32*d*(d*x+c)^3*sin(2*b*x+2*a)/b^2+1/1728*d^3*(d*x+c)*si
n(6*b*x+6*a)/b^4-1/288*d*(d*x+c)^3*sin(6*b*x+6*a)/b^2
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.66

$$\int (c+dx)^4 \cos^3(a+bx) \sin^3(a+bx)\, dx$$
$$= \frac{-243(3d^4 - 6b^2d^2(c+dx)^2 + 2b^4(c+dx)^4)\cos(2(a+bx)) + (d^4 - 18b^2d^2(c+dx)^2 + 54b^4(c+dx)^4)\cos$$
$$10$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(-243*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)]
 + (d^4 - 18*b^2*d^2*(c + d*x)^2 + 54*b^4*(c + d*x)^4)*Cos[6*(a + b*x)] -
12*b*d*(c + d*x)*(121*d^2 - 78*b^2*(c + d*x)^2 + (-d^2 + 6*b^2*(c + d*x)^2
)*Cos[4*(a + b*x)])*Sin[2*(a + b*x)])/(10368*b^5)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sin^3(a+bx) \cos^3(a+bx)\, dx$$
$$\downarrow 4906$$
$$\int \left(\frac{3}{32}(c+dx)^4 \sin(2a+2bx) - \frac{1}{32}(c+dx)^4 \sin(6a+6bx)\right) dx$$
$$\downarrow 2009$$

$$-\frac{9d^4\cos(2a+2bx)}{128b^5}+\frac{d^4\cos(6a+6bx)}{10368b^5}-\frac{9d^3(c+dx)\sin(2a+2bx)}{64b^4}+\frac{d^3(c+dx)\sin(6a+6bx)}{1728b^4}+\frac{9d^2(c+dx)^2\cos(2a+2bx)}{64b^3}-\frac{d^2(c+dx)^2\cos(6a+6bx)}{576b^3}+\frac{3d(c+dx)^3\sin(2a+2bx)}{32b^2}-\frac{d(c+dx)^3\sin(6a+6bx)}{288b^2}-\frac{3(c+dx)^4\cos(2a+2bx)}{64b}+\frac{(c+dx)^4\cos(6a+6bx)}{192b}$$

input

```
Int[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output

```
(-9*d^4*Cos[2*a + 2*b*x])/(128*b^5) + (9*d^2*(c + d*x)^2*Cos[2*a + 2*b*x])
/(64*b^3) - (3*(c + d*x)^4*Cos[2*a + 2*b*x])/(64*b) + (d^4*Cos[6*a + 6*b*x
])/(10368*b^5) - (d^2*(c + d*x)^2*Cos[6*a + 6*b*x])/(576*b^3) + ((c + d*x)
^4*Cos[6*a + 6*b*x])/(192*b) - (9*d^3*(c + d*x)*Sin[2*a + 2*b*x])/(64*b^4)
 + (3*d*(c + d*x)^3*Sin[2*a + 2*b*x])/(32*b^2) + (d^3*(c + d*x)*Sin[6*a +
6*b*x])/(1728*b^4) - (d*(c + d*x)^3*Sin[6*a + 6*b*x])/(288*b^2)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.79

method	result
parallelrisch	$\frac{\left(-486(dx+c)^4b^4+1458d^2(dx+c)^2b^2-729d^4\right)\cos(2bx+2a)+\left(54(dx+c)^4b^4-18d^2(dx+c)^2b^2+d^4\right)\cos(6bx+6a)+972$
risch	$\frac{(54d^4x^4b^4+216b^4cd^3x^3+324b^4c^2d^2x^2+216b^4c^3dx+54b^4c^4-18b^2d^4x^2-36b^2cd^3x-18b^2c^2d^2+d^4)\cos(6bx+6a)}{10368b^5}$
orering	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10368*((-486*(d*x+c)^4*b^4+1458*d^2*(d*x+c)^2*b^2-729*d^4)*cos(2*b*x+2*a
)+(54*(d*x+c)^4*b^4-18*d^2*(d*x+c)^2*b^2+d^4)*cos(6*b*x+6*a)+972*d*((d*x+c
)^2*b^2-3/2*d^2)*(d*x+c)*b*sin(2*b*x+2*a)-36*d*(d*x+c)*((d*x+c)^2*b^2-1/6*
d^2)*b*sin(6*b*x+6*a)+432*b^4*c^4-1440*b^2*c^2*d^2+728*d^4)/b^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 546 vs. 2(213) = 426.

Time = 0.09 (sec) , antiderivative size = 546, normalized size of antiderivative = 2.34

$$\int (c+dx)^4\cos^3(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{27\,b^4d^4x^4+108\,b^4cd^3x^3+2\,(54\,b^4d^4x^4+216\,b^4cd^3x^3+54\,b^4c^4-18\,b^2c^2d^2+d^4+18\,(18\,b^4c^2d^2-b^2d^4)x}{}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output

```
1/648*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 2*(54*b^4*d^4*x^4 + 216*b^4*c*
d^3*x^3 + 54*b^4*c^4 - 18*b^2*c^2*d^2 + d^4 + 18*(18*b^4*c^2*d^2 - b^2*d^4
)*x^2 + 36*(6*b^4*c^3*d - b^2*c*d^3)*x)*cos(b*x + a)^6 - 3*(54*b^4*d^4*x^4
 + 216*b^4*c*d^3*x^3 + 54*b^4*c^4 - 18*b^2*c^2*d^2 + d^4 + 18*(18*b^4*c^2*
d^2 - b^2*d^4)*x^2 + 36*(6*b^4*c^3*d - b^2*c*d^3)*x)*cos(b*x + a)^4 + 18*(
9*b^4*c^2*d^2 - 5*b^2*d^4)*x^2 + 18*(9*b^2*d^4*x^2 + 18*b^2*c*d^3*x + 9*b^
2*c^2*d^2 - 5*d^4)*cos(b*x + a)^2 + 36*(3*b^4*c^3*d - 5*b^2*c*d^3)*x - 12*
((6*b^3*d^4*x^3 + 18*b^3*c*d^3*x^2 + 6*b^3*c^3*d - b*c*d^3 + (18*b^3*c^2*d
^2 - b*d^4)*x)*cos(b*x + a)^5 - (6*b^3*d^4*x^3 + 18*b^3*c*d^3*x^2 + 6*b^3*
c^3*d - b*c*d^3 + (18*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^3 - 3*(3*b^3*d^
4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d - 5*b*c*d^3 + (9*b^3*c^2*d^2 - 5*b*d
^4)*x)*cos(b*x + a))*sin(b*x + a))/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1334 vs. $2(231) = 462$.

Time = 1.60 (sec) , antiderivative size = 1334, normalized size of antiderivative = 5.73

$$\int (c+dx)^4 \cos^3(a+bx) \sin^3(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)**4*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output

```
Piecewise((-c**4*sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - c**4*cos(a + b*x)
**6/(12*b) + c**3*d*x*sin(a + b*x)**6/(6*b) + c**3*d*x*sin(a + b*x)**4*cos
(a + b*x)**2/(2*b) - c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**4/(2*b) - c**3
*d*x*cos(a + b*x)**6/(6*b) + c**2*d**2*x**2*sin(a + b*x)**6/(4*b) + 3*c**2
*d**2*x**2*sin(a + b*x)**4*cos(a + b*x)**2/(4*b) - 3*c**2*d**2*x**2*sin(a
+ b*x)**2*cos(a + b*x)**4/(4*b) - c**2*d**2*x**2*cos(a + b*x)**6/(4*b) + c
*d**3*x**3*sin(a + b*x)**6/(6*b) + c*d**3*x**3*sin(a + b*x)**4*cos(a + b*x
)**2/(2*b) - c*d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**4/(2*b) - c*d**3*x*
*3*cos(a + b*x)**6/(6*b) + d**4*x**4*sin(a + b*x)**6/(24*b) + d**4*x**4*si
n(a + b*x)**4*cos(a + b*x)**2/(8*b) - d**4*x**4*sin(a + b*x)**2*cos(a + b*
x)**4/(8*b) - d**4*x**4*cos(a + b*x)**6/(24*b) + c**3*d*sin(a + b*x)**5*co
s(a + b*x)/(6*b**2) + 4*c**3*d*sin(a + b*x)**3*cos(a + b*x)**3/(9*b**2) +
c**3*d*sin(a + b*x)*cos(a + b*x)**5/(6*b**2) + c**2*d**2*x*sin(a + b*x)**5
*cos(a + b*x)/(2*b**2) + 4*c**2*d**2*x*sin(a + b*x)**3*cos(a + b*x)**3/(3*
b**2) + c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**5/(2*b**2) + c*d**3*x**2*si
n(a + b*x)**5*cos(a + b*x)/(2*b**2) + 4*c*d**3*x**2*sin(a + b*x)**3*cos(a
+ b*x)**3/(3*b**2) + c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**5/(2*b**2) + d
**4*x**3*sin(a + b*x)**5*cos(a + b*x)/(6*b**2) + 4*d**4*x**3*sin(a + b*x)*
*3*cos(a + b*x)**3/(9*b**2) + d**4*x**3*sin(a + b*x)*cos(a + b*x)**5/(6*b*
*2) - c**2*d**2*sin(a + b*x)**6/(12*b**3) + c**2*d**2*sin(a + b*x)**2*c...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1033 vs. $2(213) = 426$.

Time $= 0.08$ (sec) , antiderivative size $= 1033$, normalized size of antiderivative $= 4.43$

$$\int (c + dx)^4 \cos^3(a + bx) \sin^3(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output

```
-1/10368*(864*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c^4 - 3456*(2*sin(b*x
+ a)^6 - 3*sin(b*x + a)^4)*a*c^3*d/b + 5184*(2*sin(b*x + a)^6 - 3*sin(b*x
+ a)^4)*a^2*c^2*d^2/b^2 - 3456*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*a^3*c
*d^3/b^3 + 864*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*a^4*d^4/b^4 - 36*(6*(
b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*
a) + 27*sin(2*b*x + 2*a))*c^3*d/b + 108*(6*(b*x + a)*cos(6*b*x + 6*a) - 54
*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*a*c^
2*d^2/b^2 - 108*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2
*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*a^2*c*d^3/b^3 + 36*(6*(b*x +
 a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) +
27*sin(2*b*x + 2*a))*a^3*d^4/b^4 - 18*((18*(b*x + a)^2 - 1)*cos(6*b*x + 6*
a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*sin(6*b*x + 6*a
) + 162*(b*x + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 + 36*((18*(b*x + a)^2 - 1)
*cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*
sin(6*b*x + 6*a) + 162*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/b^3 - 18*((18*(
b*x + a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a)
 - 6*(b*x + a)*sin(6*b*x + 6*a) + 162*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^4/
b^4 - 6*(6*(6*(b*x + a)^3 - b*x - a)*cos(6*b*x + 6*a) - 162*(2*(b*x + a)^3
 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - (18*(b*x + a)^2 - 1)*sin(6*b*x + 6*a) +
 243*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3 + 6*(6*(6*(b*x + a...
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.54

$$\int (c+dx)^4 \cos^3(a+bx) \sin^3(a+bx)\, dx$$
$$= \frac{(54\,b^4d^4x^4 + 216\,b^4cd^3x^3 + 324\,b^4c^2d^2x^2 + 216\,b^4c^3dx + 54\,b^4c^4 - 18\,b^2d^4x^2 - 36\,b^2cd^3x - 18\,b^2c^2d^2 + d^4}{10368\,b^5}$$
$$- \frac{3\,(2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 12\,b^4c^2d^2x^2 + 8\,b^4c^3dx + 2\,b^4c^4 - 6\,b^2d^4x^2 - 12\,b^2cd^3x - 6\,b^2c^2d^2 + 3\,d^4)\cos}{128\,b^5}$$
$$- \frac{(6\,b^3d^4x^3 + 18\,b^3cd^3x^2 + 18\,b^3c^2d^2x + 6\,b^3c^3d - bd^4x - bcd^3)\sin(6\,bx + 6\,a)}{1728\,b^5}$$
$$+ \frac{3\,(2\,b^3d^4x^3 + 6\,b^3cd^3x^2 + 6\,b^3c^2d^2x + 2\,b^3c^3d - 3\,bd^4x - 3\,bcd^3)\sin(2\,bx + 2\,a)}{64\,b^5}$$

input

```
integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/10368*(54*b^4*d^4*x^4 + 216*b^4*c*d^3*x^3 + 324*b^4*c^2*d^2*x^2 + 216*b^
4*c^3*d*x + 54*b^4*c^4 - 18*b^2*d^4*x^2 - 36*b^2*c*d^3*x - 18*b^2*c^2*d^2
+ d^4)*cos(6*b*x + 6*a)/b^5 - 3/128*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*
b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3
*x - 6*b^2*c^2*d^2 + 3*d^4)*cos(2*b*x + 2*a)/b^5 - 1/1728*(6*b^3*d^4*x^3 +
 18*b^3*c*d^3*x^2 + 18*b^3*c^2*d^2*x + 6*b^3*c^3*d - b*d^4*x - b*c*d^3)*si
n(6*b*x + 6*a)/b^5 + 3/64*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2
*x + 2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

Mupad [B] (verification not implemented)

Time = 21.15 (sec) , antiderivative size = 576, normalized size of antiderivative = 2.47

$$\int (c+dx)^4 \cos^3(a+bx)\sin^3(a+bx)\,dx =$$
$$-\frac{729\,d^4\cos(2\,a+2\,b\,x) - d^4\cos(6\,a+6\,b\,x) + 486\,b^4\,c^4\cos(2\,a+2\,b\,x) - 54\,b^4\,c^4\cos(6\,a+6\,b\,x) - 9}{}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^4,x)
```

output
```
-(729*d^4*cos(2*a + 2*b*x) - d^4*cos(6*a + 6*b*x) + 486*b^4*c^4*cos(2*a +
2*b*x) - 54*b^4*c^4*cos(6*a + 6*b*x) - 972*b^3*c^3*d*sin(2*a + 2*b*x) + 36
*b^3*c^3*d*sin(6*a + 6*b*x) - 1458*b^2*c^2*d^2*cos(2*a + 2*b*x) + 18*b^2*c
^2*d^2*cos(6*a + 6*b*x) - 1458*b^2*d^4*x^2*cos(2*a + 2*b*x) + 486*b^4*d^4*
x^4*cos(2*a + 2*b*x) + 18*b^2*d^4*x^2*cos(6*a + 6*b*x) - 54*b^4*d^4*x^4*co
s(6*a + 6*b*x) - 972*b^3*d^4*x^3*sin(2*a + 2*b*x) + 36*b^3*d^4*x^3*sin(6*a
 + 6*b*x) + 1458*b*c*d^3*sin(2*a + 2*b*x) - 6*b*c*d^3*sin(6*a + 6*b*x) + 1
458*b*d^4*x*sin(2*a + 2*b*x) - 6*b*d^4*x*sin(6*a + 6*b*x) + 2916*b^4*c^2*d
^2*x^2*cos(2*a + 2*b*x) - 324*b^4*c^2*d^2*x^2*cos(6*a + 6*b*x) - 2916*b^2*
c*d^3*x*cos(2*a + 2*b*x) + 1944*b^4*c^3*d*x*cos(2*a + 2*b*x) + 36*b^2*c*d^
3*x*cos(6*a + 6*b*x) - 216*b^4*c^3*d*x*cos(6*a + 6*b*x) + 1944*b^4*c*d^3*x
^3*cos(2*a + 2*b*x) - 216*b^4*c*d^3*x^3*cos(6*a + 6*b*x) - 2916*b^3*c^2*d^
2*x*sin(2*a + 2*b*x) - 2916*b^3*c*d^3*x^2*sin(2*a + 2*b*x) + 108*b^3*c^2*d
^2*x*sin(6*a + 6*b*x) + 108*b^3*c*d^3*x^2*sin(6*a + 6*b*x))/(10368*b^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 919, normalized size of antiderivative = 3.94

$$\int (c+dx)^4 \cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input

```
int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

output

```
( - 72*cos(a + b*x)*sin(a + b*x)**5*b**3*c**3*d - 216*cos(a + b*x)*sin(a +
 b*x)**5*b**3*c**2*d**2*x - 216*cos(a + b*x)*sin(a + b*x)**5*b**3*c*d**3*x
**2 - 72*cos(a + b*x)*sin(a + b*x)**5*b**3*d**4*x**3 + 12*cos(a + b*x)*sin
(a + b*x)**5*b*c*d**3 + 12*cos(a + b*x)*sin(a + b*x)**5*b*d**4*x + 72*cos(
a + b*x)*sin(a + b*x)**3*b**3*c**3*d + 216*cos(a + b*x)*sin(a + b*x)**3*b*
*3*c**2*d**2*x + 216*cos(a + b*x)*sin(a + b*x)**3*b**3*c*d**3*x**2 + 72*co
s(a + b*x)*sin(a + b*x)**3*b**3*d**4*x**3 - 12*cos(a + b*x)*sin(a + b*x)**
3*b*c*d**3 - 12*cos(a + b*x)*sin(a + b*x)**3*b*d**4*x + 108*cos(a + b*x)*s
in(a + b*x)*b**3*c**3*d + 324*cos(a + b*x)*sin(a + b*x)*b**3*c**2*d**2*x +
 324*cos(a + b*x)*sin(a + b*x)*b**3*c*d**3*x**2 + 108*cos(a + b*x)*sin(a +
 b*x)*b**3*d**4*x**3 - 180*cos(a + b*x)*sin(a + b*x)*b*c*d**3 - 180*cos(a
+ b*x)*sin(a + b*x)*b*d**4*x - 108*sin(a + b*x)**6*b**4*c**4 - 432*sin(a +
 b*x)**6*b**4*c**3*d*x - 648*sin(a + b*x)**6*b**4*c**2*d**2*x**2 - 432*sin
(a + b*x)**6*b**4*c*d**3*x**3 - 108*sin(a + b*x)**6*b**4*d**4*x**4 + 36*si
n(a + b*x)**6*b**2*c**2*d**2 + 72*sin(a + b*x)**6*b**2*c*d**3*x + 36*sin(a
 + b*x)**6*b**2*d**4*x**2 - 2*sin(a + b*x)**6*d**4 + 162*sin(a + b*x)**4*b
**4*c**4 + 648*sin(a + b*x)**4*b**4*c**3*d*x + 972*sin(a + b*x)**4*b**4*c*
*2*d**2*x**2 + 648*sin(a + b*x)**4*b**4*c*d**3*x**3 + 162*sin(a + b*x)**4*
b**4*d**4*x**4 - 54*sin(a + b*x)**4*b**2*c**2*d**2 - 108*sin(a + b*x)**4*b
**2*c*d**3*x - 54*sin(a + b*x)**4*b**2*d**4*x**2 + 3*sin(a + b*x)**4*d*...
```

3.156 $\int (c+dx)^3 \cos^3(a+bx) \sin^3(a+bx)\, dx$

Optimal result 1423
Mathematica [A] (verified) 1424
Rubi [A] (verified) 1424
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Optimal result

Integrand size = 24, antiderivative size = 181

$$\begin{aligned}\int (c+dx)^3 \cos^3(a+bx) \sin^3(a+bx)\, dx = {} & \frac{9d^2(c+dx)\cos(2a+2bx)}{128b^3} \\ & - \frac{3(c+dx)^3\cos(2a+2bx)}{64b} \\ & - \frac{d^2(c+dx)\cos(6a+6bx)}{1152b^3} \\ & + \frac{(c+dx)^3\cos(6a+6bx)}{192b} \\ & - \frac{9d^3\sin(2a+2bx)}{256b^4} \\ & + \frac{9d(c+dx)^2\sin(2a+2bx)}{128b^2} \\ & + \frac{d^3\sin(6a+6bx)}{6912b^4} \\ & - \frac{d(c+dx)^2\sin(6a+6bx)}{384b^2}\end{aligned}$$

output

```
9/128*d^2*(d*x+c)*cos(2*b*x+2*a)/b^3-3/64*(d*x+c)^3*cos(2*b*x+2*a)/b-1/115
2*d^2*(d*x+c)*cos(6*b*x+6*a)/b^3+1/192*(d*x+c)^3*cos(6*b*x+6*a)/b-9/256*d^
3*sin(2*b*x+2*a)/b^4+9/128*d*(d*x+c)^2*sin(2*b*x+2*a)/b^2+1/6912*d^3*sin(6
*b*x+6*a)/b^4-1/384*d*(d*x+c)^2*sin(6*b*x+6*a)/b^2
```

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.73

$$\int (c+dx)^3 \cos^3(a+bx) \sin^3(a+bx)\, dx$$
$$= \frac{-324b(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\cos(2(a+bx)) + 12b(c+dx)\left(-d^2+6b^2(c+dx)^2\right)\cos(6(a+bx))}{13824b^4}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(-324*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 12*b*(c
+ d*x)*(-d^2 + 6*b^2*(c + d*x)^2)*Cos[6*(a + b*x)] - 4*d*(121*d^2 - 234*b^
2*(c + d*x)^2 + (-d^2 + 18*b^2*(c + d*x)^2)*Cos[4*(a + b*x)])*Sin[2*(a + b
*x)])/(13824*b^4)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin^3(a+bx) \cos^3(a+bx)\, dx$$
$$\downarrow 4906$$
$$\int \left(\frac{3}{32}(c+dx)^3 \sin(2a+2bx) - \frac{1}{32}(c+dx)^3 \sin(6a+6bx)\right) dx$$
$$\downarrow 2009$$
$$-\frac{9d^3 \sin(2a+2bx)}{256b^4} + \frac{d^3 \sin(6a+6bx)}{6912b^4} + \frac{9d^2(c+dx)\cos(2a+2bx)}{128b^3} - \frac{d^2(c+dx)\cos(6a+6bx)}{1152b^3} + \frac{9d(c+dx)^2 \sin(2a+2bx)}{128b^2} - \frac{d(c+dx)^2 \sin(6a+6bx)}{384b^2} - \frac{3(c+dx)^3 \cos(2a+2bx)}{64b} + \frac{(c+dx)^3 \cos(6a+6bx)}{192b}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(9*d^2*(c + d*x)*Cos[2*a + 2*b*x])/(128*b^3) - (3*(c + d*x)^3*Cos[2*a + 2*
b*x])/(64*b) - (d^2*(c + d*x)*Cos[6*a + 6*b*x])/(1152*b^3) + ((c + d*x)^3*
Cos[6*a + 6*b*x])/(192*b) - (9*d^3*Sin[2*a + 2*b*x])/(256*b^4) + (9*d*(c +
 d*x)^2*Sin[2*a + 2*b*x])/(128*b^2) + (d^3*Sin[6*a + 6*b*x])/(6912*b^4) -
(d*(c + d*x)^2*Sin[6*a + 6*b*x])/(384*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.82

method	result
parallelrisch	$\frac{-54\left((dx+c)^2b^2-\frac{3d^2}{2}\right)(dx+c)b\cos(2bx+2a)+6\left((dx+c)^2b^2-\frac{d^2}{6}\right)(dx+c)b\cos(6bx+6a)+81d\left((dx+c)^2b^2-\frac{d^2}{2}\right)\sin(2b}{1152b^4}$
risch	$\frac{\left(6b^2d^3x^3+18b^2cd^2x^2+18b^2c^2dx+6b^2c^3-d^3x-cd^2\right)\cos(6bx+6a)}{1152b^3}-\frac{d\left(18x^2d^2b^2+36b^2cdx+18b^2c^2-d^2\right)\sin(6bx+6}{6912b^4}$
orering	$\frac{5d\left(18d^4x^4b^4+72b^4cd^3x^3+108b^4c^2d^2x^2+72b^4c^3dx+18b^4c^4-11b^2d^4x^2-22b^2cd^3x-11b^2c^2d^2-9d^4\right)\cos(bx+a)^3\sin(b}{54b^6(dx+c)^2}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/1152*(-54*((d*x+c)^2*b^2-3/2*d^2)*(d*x+c)*b*cos(2*b*x+2*a)+6*((d*x+c)^2*
b^2-1/6*d^2)*(d*x+c)*b*cos(6*b*x+6*a)+81*d*((d*x+c)^2*b^2-1/2*d^2)*sin(2*b
*x+2*a)-3*d*((d*x+c)^2*b^2-1/18*d^2)*sin(6*b*x+6*a)+48*b^3*c^3-80*c*d^2*b)
/b^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(165) = 330$.

Time $= 0.08$ (sec) , antiderivative size $= 349$, normalized size of antiderivative $= 1.93$

$$\int (c+dx)^3 \cos^3(a+bx) \sin^3(a+bx)\, dx$$
$$= \frac{9\,b^3d^3x^3 + 27\,b^3cd^2x^2 + 6\,(6\,b^3d^3x^3 + 18\,b^3cd^2x^2 + 6\,b^3c^3 - bcd^2 + (18\,b^3c^2d - bd^3)x)\cos(bx+a)^6 - 9\,(6$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/216*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x^2 + 6*(6*b^3*d^3*x^3 + 18*b^3*c*d^2*
x^2 + 6*b^3*c^3 - b*c*d^2 + (18*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^6 - 9*(
6*b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 6*b^3*c^3 - b*c*d^2 + (18*b^3*c^2*d - b
*d^3)*x)*cos(b*x + a)^4 + 27*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 + 3*(9*b^3
*c^2*d - 5*b*d^3)*x - ((18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d
^3)*cos(b*x + a)^5 - (18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3
)*cos(b*x + a)^3 - 3*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 5*d^3
)*cos(b*x + a))*sin(b*x + a))/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(178) = 356$.

Time $= 1.12$ (sec) , antiderivative size $= 857$, normalized size of antiderivative $= 4.73$

$$\int (c+dx)^3 \cos^3(a+bx) \sin^3(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output

```
Piecewise((-c**3*sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - c**3*cos(a + b*x)
**6/(12*b) + c**2*d*x*sin(a + b*x)**6/(8*b) + 3*c**2*d*x*sin(a + b*x)**4*c
os(a + b*x)**2/(8*b) - 3*c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) -
c**2*d*x*cos(a + b*x)**6/(8*b) + c*d**2*x**2*sin(a + b*x)**6/(8*b) + 3*c*d
**2*x**2*sin(a + b*x)**4*cos(a + b*x)**2/(8*b) - 3*c*d**2*x**2*sin(a + b*x
)**2*cos(a + b*x)**4/(8*b) - c*d**2*x**2*cos(a + b*x)**6/(8*b) + d**3*x**3
*sin(a + b*x)**6/(24*b) + d**3*x**3*sin(a + b*x)**4*cos(a + b*x)**2/(8*b)
- d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) - d**3*x**3*cos(a + b*x)
**6/(24*b) + c**2*d*sin(a + b*x)**5*cos(a + b*x)/(8*b**2) + c**2*d*sin(a +
 b*x)**3*cos(a + b*x)**3/(3*b**2) + c**2*d*sin(a + b*x)*cos(a + b*x)**5/(8
*b**2) + c*d**2*x*sin(a + b*x)**5*cos(a + b*x)/(4*b**2) + 2*c*d**2*x*sin(a
 + b*x)**3*cos(a + b*x)**3/(3*b**2) + c*d**2*x*sin(a + b*x)*cos(a + b*x)**
5/(4*b**2) + d**3*x**2*sin(a + b*x)**5*cos(a + b*x)/(8*b**2) + d**3*x**2*s
in(a + b*x)**3*cos(a + b*x)**3/(3*b**2) + d**3*x**2*sin(a + b*x)*cos(a + b
*x)**5/(8*b**2) - c*d**2*sin(a + b*x)**6/(24*b**3) + c*d**2*sin(a + b*x)**
2*cos(a + b*x)**4/(6*b**3) + 7*c*d**2*cos(a + b*x)**6/(72*b**3) - 5*d**3*x
*sin(a + b*x)**6/(72*b**3) - d**3*x*sin(a + b*x)**4*cos(a + b*x)**2/(12*b*
*3) + d**3*x*sin(a + b*x)**2*cos(a + b*x)**4/(12*b**3) + 5*d**3*x*cos(a +
b*x)**6/(72*b**3) - 5*d**3*sin(a + b*x)**5*cos(a + b*x)/(72*b**4) - 31*d**
3*sin(a + b*x)**3*cos(a + b*x)**3/(216*b**4) - 5*d**3*sin(a + b*x)*cos(...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(165) = 330$.

Time $= 0.06$ (sec) , antiderivative size $= 602$, normalized size of antiderivative $= 3.33$

$$\int (c + dx)^3 \cos^3(a + bx) \sin^3(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output

```
-1/6912*(576*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c^3 - 1728*(2*sin(b*x +
 a)^6 - 3*sin(b*x + a)^4)*a*c^2*d/b + 1728*(2*sin(b*x + a)^6 - 3*sin(b*x +
 a)^4)*a^2*c*d^2/b^2 - 576*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*a^3*d^3/b
^3 - 18*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - si
n(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*c^2*d/b + 36*(6*(b*x + a)*cos(6*b*x
 + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x +
 2*a))*a*c*d^2/b^2 - 18*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2
*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*a^2*d^3/b^3 - 6*((18
*(b*x + a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*
a) - 6*(b*x + a)*sin(6*b*x + 6*a) + 162*(b*x + a)*sin(2*b*x + 2*a))*c*d^2/
b^2 + 6*((18*(b*x + a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*co
s(2*b*x + 2*a) - 6*(b*x + a)*sin(6*b*x + 6*a) + 162*(b*x + a)*sin(2*b*x +
2*a))*a*d^3/b^3 - (6*(6*(b*x + a)^3 - b*x - a)*cos(6*b*x + 6*a) - 162*(2*(
b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - (18*(b*x + a)^2 - 1)*sin(6*b*
x + 6*a) + 243*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*d^3/b^3)/b
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.33

$$
\begin{aligned}
&\int (c+dx)^3 \cos^3(a+bx) \sin^3(a+bx)\, dx \\
&= \frac{\left(6\,b^3d^3x^3 + 18\,b^3cd^2x^2 + 18\,b^3c^2dx + 6\,b^3c^3 - bd^3x - bcd^2\right)\cos\left(6\,bx + 6\,a\right)}{1152\,b^4} \\
&\quad - \frac{3\left(2\,b^3d^3x^3 + 6\,b^3cd^2x^2 + 6\,b^3c^2dx + 2\,b^3c^3 - 3\,bd^3x - 3\,bcd^2\right)\cos\left(2\,bx + 2\,a\right)}{128\,b^4} \\
&\quad - \frac{\left(18\,b^2d^3x^2 + 36\,b^2cd^2x + 18\,b^2c^2d - d^3\right)\sin\left(6\,bx + 6\,a\right)}{6912\,b^4} \\
&\quad + \frac{9\left(2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d - d^3\right)\sin\left(2\,bx + 2\,a\right)}{256\,b^4}
\end{aligned}
$$

input

```
integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/1152*(6*b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 18*b^3*c^2*d*x + 6*b^3*c^3 - b*
d^3*x - b*c*d^2)*cos(6*b*x + 6*a)/b^4 - 3/128*(2*b^3*d^3*x^3 + 6*b^3*c*d^2
*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(2*b*x + 2*a)
/b^4 - 1/6912*(18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*sin(6
*b*x + 6*a)/b^4 + 9/256*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3
)*sin(2*b*x + 2*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.02

$$\int (c+dx)^3 \cos^3(a+bx) \sin^3(a+bx)\, dx =$$
$$-\frac{243\, d^3 \sin(2\,a+2\,b\,x) - d^3 \sin(6\,a+6\,b\,x) + 324\, b^3\, c^3 \cos(2\,a+2\,b\,x) - 36\, b^3\, c^3 \cos(6\,a+6\,b\,x) - 4$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^3,x)
```

output
```
-(243*d^3*sin(2*a + 2*b*x) - d^3*sin(6*a + 6*b*x) + 324*b^3*c^3*cos(2*a +
2*b*x) - 36*b^3*c^3*cos(6*a + 6*b*x) - 486*b^2*c^2*d*sin(2*a + 2*b*x) + 18
*b^2*c^2*d*sin(6*a + 6*b*x) + 324*b^3*d^3*x^3*cos(2*a + 2*b*x) - 36*b^3*d^
3*x^3*cos(6*a + 6*b*x) - 486*b^2*d^3*x^2*sin(2*a + 2*b*x) + 18*b^2*d^3*x^2
*sin(6*a + 6*b*x) - 486*b*c*d^2*cos(2*a + 2*b*x) + 6*b*c*d^2*cos(6*a + 6*b
*x) - 486*b*d^3*x*cos(2*a + 2*b*x) + 6*b*d^3*x*cos(6*a + 6*b*x) + 972*b^3*
c^2*d*x*cos(2*a + 2*b*x) - 108*b^3*c^2*d*x*cos(6*a + 6*b*x) - 972*b^2*c*d^
2*x*sin(2*a + 2*b*x) + 36*b^2*c*d^2*x*sin(6*a + 6*b*x) + 972*b^3*c*d^2*x^2
*cos(2*a + 2*b*x) - 108*b^3*c*d^2*x^2*cos(6*a + 6*b*x))/(6912*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 569, normalized size of antiderivative = 3.14

$$\int (c+dx)^3 \cos^3(a+bx) \sin^3(a+bx)\, dx$$
$$=\frac{27\cos(bx+a)\sin(bx+a)\, b^2c^2d + 27\cos(bx+a)\sin(bx+a)\, b^2d^3x^2 - 36\sin(bx+a)^6\, b^3c^3 - 27\sin(bx$$

input
```
int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

output
```
( - 18*cos(a + b*x)*sin(a + b*x)**5*b**2*c**2*d - 36*cos(a + b*x)*sin(a +
b*x)**5*b**2*c*d**2*x - 18*cos(a + b*x)*sin(a + b*x)**5*b**2*d**3*x**2 + c
os(a + b*x)*sin(a + b*x)**5*d**3 + 18*cos(a + b*x)*sin(a + b*x)**3*b**2*c*
*2*d + 36*cos(a + b*x)*sin(a + b*x)**3*b**2*c*d**2*x + 18*cos(a + b*x)*sin
(a + b*x)**3*b**2*d**3*x**2 - cos(a + b*x)*sin(a + b*x)**3*d**3 + 27*cos(a
 + b*x)*sin(a + b*x)*b**2*c**2*d + 54*cos(a + b*x)*sin(a + b*x)*b**2*c*d**
2*x + 27*cos(a + b*x)*sin(a + b*x)*b**2*d**3*x**2 - 15*cos(a + b*x)*sin(a
+ b*x)*d**3 - 36*sin(a + b*x)**6*b**3*c**3 - 108*sin(a + b*x)**6*b**3*c**2
*d*x - 108*sin(a + b*x)**6*b**3*c*d**2*x**2 - 36*sin(a + b*x)**6*b**3*d**3
*x**3 + 6*sin(a + b*x)**6*b*c*d**2 + 6*sin(a + b*x)**6*b*d**3*x + 54*sin(a
 + b*x)**4*b**3*c**3 + 162*sin(a + b*x)**4*b**3*c**2*d*x + 162*sin(a + b*x
)**4*b**3*c*d**2*x**2 + 54*sin(a + b*x)**4*b**3*d**3*x**3 - 9*sin(a + b*x)
**4*b*c*d**2 - 9*sin(a + b*x)**4*b*d**3*x - 27*sin(a + b*x)**2*b*c*d**2 -
27*sin(a + b*x)**2*b*d**3*x - 27*a*b**2*c**2*d - 3*a*d**3 - 27*b**3*c**2*d
*x - 27*b**3*c*d**2*x**2 - 9*b**3*d**3*x**3 + 18*b*c*d**2 + 15*b*d**3*x)/(
216*b**4)
```

3.157 $\int (c+dx)^2 \cos^3(a+bx) \sin^3(a+bx)\, dx$

Optimal result 1431
Mathematica [A] (verified) 1432
Rubi [A] (verified) 1432
Maple [A] (verified) 1433
Fricas [A] (verification not implemented) 1434
Sympy [B] (verification not implemented) 1434
Maxima [B] (verification not implemented) 1435
Giac [A] (verification not implemented) 1436
Mupad [B] (verification not implemented) 1436
Reduce [B] (verification not implemented) 1437

Optimal result

Integrand size = 24, antiderivative size = 129

$$\int (c+dx)^2 \cos^3(a+bx) \sin^3(a+bx)\, dx = \frac{3d^2 \cos(2a+2bx)}{128b^3} - \frac{3(c+dx)^2 \cos(2a+2bx)}{64b} - \frac{d^2 \cos(6a+6bx)}{3456b^3} + \frac{(c+dx)^2 \cos(6a+6bx)}{192b} + \frac{3d(c+dx)\sin(2a+2bx)}{64b^2} - \frac{d(c+dx)\sin(6a+6bx)}{576b^2}$$

output

```
3/128*d^2*cos(2*b*x+2*a)/b^3-3/64*(d*x+c)^2*cos(2*b*x+2*a)/b-1/3456*d^2*co
s(6*b*x+6*a)/b^3+1/192*(d*x+c)^2*cos(6*b*x+6*a)/b+3/64*d*(d*x+c)*sin(2*b*x
+2*a)/b^2-1/576*d*(d*x+c)*sin(6*b*x+6*a)/b^2
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.71

$$\int (c+dx)^2 \cos^3(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{-81(-d^2+2b^2(c+dx)^2)\cos(2(a+bx)) + (-d^2+18b^2(c+dx)^2)\cos(6(a+bx)) - 6bd(c+dx)(-27\sin}{3456b^3}$$

input
```
Integrate[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(-81*(-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + (-d^2 + 18*b^2*(c + d*x
)^2)*Cos[6*(a + b*x)] - 6*b*d*(c + d*x)*(-27*Sin[2*(a + b*x)] + Sin[6*(a +
 b*x)]))/(3456*b^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin^3(a+bx)\cos^3(a+bx)\,dx$$
$$\downarrow 4906$$
$$\int \left(\frac{3}{32}(c+dx)^2\sin(2a+2bx) - \frac{1}{32}(c+dx)^2\sin(6a+6bx)\right)dx$$
$$\downarrow 2009$$
$$\frac{3d^2\cos(2a+2bx)}{128b^3} - \frac{d^2\cos(6a+6bx)}{3456b^3} + \frac{3d(c+dx)\sin(2a+2bx)}{64b^2} - \frac{d(c+dx)\sin(6a+6bx)}{576b^2} - \frac{3(c+dx)^2\cos(2a+2bx)}{64b} + \frac{(c+dx)^2\cos(6a+6bx)}{192b}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(3*d^2*Cos[2*a + 2*b*x])/(128*b^3) - (3*(c + d*x)^2*Cos[2*a + 2*b*x])/(64*
b) - (d^2*Cos[6*a + 6*b*x])/(3456*b^3) + ((c + d*x)^2*Cos[6*a + 6*b*x])/(1
92*b) + (3*d*(c + d*x)*Sin[2*a + 2*b*x])/(64*b^2) - (d*(c + d*x)*Sin[6*a +
 6*b*x])/(576*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.86

method	result
parallelrisch	$\frac{81\left(-2(dx+c)^2b^2+d^2\right)\cos(2bx+2a)+\left(18(dx+c)^2b^2-d^2\right)\cos(6bx+6a)+162bd(dx+c)\sin(2bx+2a)-6bd(dx+c)\sin(6b}{3456b^3}$
risch	$\frac{\left(18x^2d^2b^2+36b^2cdx+18b^2c^2-d^2\right)\cos(6bx+6a)}{3456b^3}-\frac{d(dx+c)\sin(6bx+6a)}{576b^2}-\frac{3\left(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2\right)\cos(2b}{128b^3}$
derivativedivides	$\frac{a^2d^2\left(-\frac{\sin(bx+a)^2\cos(bx+a)^4}{6}-\frac{\cos(bx+a)^4}{12}\right)}{b^2}-\frac{2acd\left(-\frac{\sin(bx+a)^2\cos(bx+a)^4}{6}-\frac{\cos(bx+a)^4}{12}\right)}{b}-\frac{2a\,d^2\left(\frac{(bx+a)\sin(bx+a)^4}{4}+\right.}{}$
default	$\frac{a^2d^2\left(-\frac{\sin(bx+a)^2\cos(bx+a)^4}{6}-\frac{\cos(bx+a)^4}{12}\right)}{b^2}-\frac{2acd\left(-\frac{\sin(bx+a)^2\cos(bx+a)^4}{6}-\frac{\cos(bx+a)^4}{12}\right)}{b}-\frac{2a\,d^2\left(\frac{(bx+a)\sin(bx+a)^4}{4}+\right.}{}$
orering	$\frac{5d\left(36d^4x^4b^4+144b^4cd^3x^3+216b^4c^2d^2x^2+144b^4c^3dx+36b^4c^4-b^2d^4x^2-2b^2cd^3x-b^2c^2d^2-3d^4\right)\cos(bx+a)^3\sin(bx+a}{162b^6(dx+c)^3}$

input
```
int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3456*(81*(-2*(d*x+c)^2*b^2+d^2)*cos(2*b*x+2*a)+(18*(d*x+c)^2*b^2-d^2)*co
s(6*b*x+6*a)+162*b*d*(d*x+c)*sin(2*b*x+2*a)-6*b*d*(d*x+c)*sin(6*b*x+6*a)+1
44*b^2*c^2-80*d^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.50

$$\int (c+dx)^2 \cos^3(a+bx) \sin^3(a+bx)\, dx$$
$$= \frac{2\left(18\,b^2d^2x^2+36\,b^2cdx+18\,b^2c^2-d^2\right)\cos\left(bx+a\right)^6+9\,b^2d^2x^2+18\,b^2cdx-3\left(18\,b^2d^2x^2+36\,b^2cdx+1\right.}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/216*(2*(18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*c^2 - d^2)*cos(b*x + a)^6
 + 9*b^2*d^2*x^2 + 18*b^2*c*d*x - 3*(18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^
2*c^2 - d^2)*cos(b*x + a)^4 + 9*d^2*cos(b*x + a)^2 - 6*(2*(b*d^2*x + b*c*d
)*cos(b*x + a)^5 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 3*(b*d^2*x + b*c*d
)*cos(b*x + a))*sin(b*x + a))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(126) = 252.

Time = 0.81 (sec) , antiderivative size = 461, normalized size of antiderivative = 3.57

$$\int (c+dx)^2 \cos^3(a+bx) \sin^3(a+bx)\, dx$$
$$= \begin{cases} -\frac{c^2 \sin^2(a+bx)\cos^4(a+bx)}{4b} - \frac{c^2\cos^6(a+bx)}{12b} + \frac{cdx\sin^6(a+bx)}{12b} + \frac{cdx\sin^4(a+bx)\cos^2(a+bx)}{4b} - \frac{cdx\sin^2(a+bx)\cos^4(a+bx)}{4b} - \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin^3(a)\cos^3(a) \end{cases}$$

input
```
integrate((d*x+c)**2*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output

```
Piecewise((-c**2*sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - c**2*cos(a + b*x)
**6/(12*b) + c*d*x*sin(a + b*x)**6/(12*b) + c*d*x*sin(a + b*x)**4*cos(a +
b*x)**2/(4*b) - c*d*x*sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - c*d*x*cos(a
+ b*x)**6/(12*b) + d**2*x**2*sin(a + b*x)**6/(24*b) + d**2*x**2*sin(a + b*
x)**4*cos(a + b*x)**2/(8*b) - d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**4/(8
*b) - d**2*x**2*cos(a + b*x)**6/(24*b) + c*d*sin(a + b*x)**5*cos(a + b*x)/
(12*b**2) + 2*c*d*sin(a + b*x)**3*cos(a + b*x)**3/(9*b**2) + c*d*sin(a + b
*x)*cos(a + b*x)**5/(12*b**2) + d**2*x*sin(a + b*x)**5*cos(a + b*x)/(12*b*
*2) + 2*d**2*x*sin(a + b*x)**3*cos(a + b*x)**3/(9*b**2) + d**2*x*sin(a + b
*x)*cos(a + b*x)**5/(12*b**2) - d**2*sin(a + b*x)**6/(72*b**3) + d**2*sin(
a + b*x)**2*cos(a + b*x)**4/(18*b**3) + 7*d**2*cos(a + b*x)**6/(216*b**3),
 Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**3*cos(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(117) = 234$.

Time = 0.04 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.35

$$\int (c+dx)^2 \cos^3(a+bx) \sin^3(a+bx)\, dx =$$

$$-\frac{288\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)c^2}{} - \frac{576\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)acd}{b} + \frac{288\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)a}{b^2}$$

input

```
integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output

```
-1/3456*(288*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c^2 - 576*(2*sin(b*x +
a)^6 - 3*sin(b*x + a)^4)*a*c*d/b + 288*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^
4)*a^2*d^2/b^2 - 6*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x
+ 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*c*d/b + 6*(6*(b*x + a)*co
s(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin
(2*b*x + 2*a))*a*d^2/b^2 - ((18*(b*x + a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*
(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*sin(6*b*x + 6*a) + 162*(b*
x + a)*sin(2*b*x + 2*a))*d^2/b^2)/b
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12

$$\begin{aligned}&\int (c+dx)^2\cos^3(a+bx)\sin^3(a+bx)\,dx\\&=\frac{(18\,b^2d^2x^2+36\,b^2cdx+18\,b^2c^2-d^2)\cos(6\,bx+6\,a)}{3456\,b^3}\\&\quad-\frac{3\,(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2)\cos(2\,bx+2\,a)}{128\,b^3}\\&\quad-\frac{(bd^2x+bcd)\sin(6\,bx+6\,a)}{576\,b^3}+\frac{3\,(bd^2x+bcd)\sin(2\,bx+2\,a)}{64\,b^3}\end{aligned}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/3456*(18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*c^2 - d^2)*cos(6*b*x + 6*a)
/b^3 - 3/128*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2
*a)/b^3 - 1/576*(b*d^2*x + b*c*d)*sin(6*b*x + 6*a)/b^3 + 3/64*(b*d^2*x + b
*c*d)*sin(2*b*x + 2*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.57

$$\begin{aligned}&\int (c+dx)^2\cos^3(a+bx)\sin^3(a+bx)\,dx\\&=\frac{81\,d^2\,\cos(2\,a+2\,b\,x)-d^2\,\cos(6\,a+6\,b\,x)-162\,b^2\,c^2\,\cos(2\,a+2\,b\,x)+18\,b^2\,c^2\,\cos(6\,a+6\,b\,x)+162\,b}{}\end{aligned}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^2,x)
```

output
```
(81*d^2*cos(2*a + 2*b*x) - d^2*cos(6*a + 6*b*x) - 162*b^2*c^2*cos(2*a + 2*
b*x) + 18*b^2*c^2*cos(6*a + 6*b*x) + 162*b*c*d*sin(2*a + 2*b*x) - 6*b*c*d*
sin(6*a + 6*b*x) - 162*b^2*d^2*x^2*cos(2*a + 2*b*x) + 18*b^2*d^2*x^2*cos(6
*a + 6*b*x) + 162*b*d^2*x*sin(2*a + 2*b*x) - 6*b*d^2*x*sin(6*a + 6*b*x) -
324*b^2*c*d*x*cos(2*a + 2*b*x) + 36*b^2*c*d*x*cos(6*a + 6*b*x))/(3456*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.27

$$\int (c+dx)^2 \cos^3(a+bx) \sin^3(a+bx)\, dx$$
$$= \frac{-12\cos(bx+a)\sin(bx+a)^5\, bcd - 12\cos(bx+a)\sin(bx+a)^5\, b\, d^2x + 12\cos(bx+a)\sin(bx+a)^3\, bcd}$$

input

```
int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

output

```
( - 12*cos(a + b*x)*sin(a + b*x)**5*b*c*d - 12*cos(a + b*x)*sin(a + b*x)**
5*b*d**2*x + 12*cos(a + b*x)*sin(a + b*x)**3*b*c*d + 12*cos(a + b*x)*sin(a
 + b*x)**3*b*d**2*x + 18*cos(a + b*x)*sin(a + b*x)*b*c*d + 18*cos(a + b*x)
*sin(a + b*x)*b*d**2*x - 36*sin(a + b*x)**6*b**2*c**2 - 72*sin(a + b*x)**6
*b**2*c*d*x - 36*sin(a + b*x)**6*b**2*d**2*x**2 + 2*sin(a + b*x)**6*d**2 +
 54*sin(a + b*x)**4*b**2*c**2 + 108*sin(a + b*x)**4*b**2*c*d*x + 54*sin(a
+ b*x)**4*b**2*d**2*x**2 - 3*sin(a + b*x)**4*d**2 - 9*sin(a + b*x)**2*d**2
 - 18*a*b*c*d - 18*b**2*c*d*x - 9*b**2*d**2*x**2 + 6*d**2)/(216*b**3)
```

3.158 $\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal result 1438
Mathematica [A] (verified) 1438
Rubi [A] (verified) 1439
Maple [A] (verified) 1440
Fricas [A] (verification not implemented) 1441
Sympy [B] (verification not implemented) 1441
Maxima [A] (verification not implemented) 1442
Giac [A] (verification not implemented) 1442
Mupad [B] (verification not implemented) 1443
Reduce [B] (verification not implemented) 1443

Optimal result

Integrand size = 22, antiderivative size = 77

$$\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx = -\frac{3(c+dx)\cos(2a+2bx)}{64b} + \frac{(c+dx)\cos(6a+6bx)}{192b} + \frac{3d\sin(2a+2bx)}{128b^2} - \frac{d\sin(6a+6bx)}{1152b^2}$$

output

```
-3/64*(d*x+c)*cos(2*b*x+2*a)/b+1/192*(d*x+c)*cos(6*b*x+6*a)/b+3/128*d*sin(
2*b*x+2*a)/b^2-1/1152*d*sin(6*b*x+6*a)/b^2
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

$$\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{-54b(c+dx)\cos(2(a+bx)) + 6b(c+dx)\cos(6(a+bx)) + d(27\sin(2(a+bx)) - \sin(6(a+bx)))}{1152b^2}$$

input

```
Integrate[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(-54*b*(c + d*x)*Cos[2*(a + b*x)] + 6*b*(c + d*x)*Cos[6*(a + b*x)] + d*(27
*Sin[2*(a + b*x)] - Sin[6*(a + b*x)]))/(1152*b^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin^3(a+bx)\cos^3(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{3}{32}(c+dx)\sin(2a+2bx) - \frac{1}{32}(c+dx)\sin(6a+6bx)\right)dx$$

$$\downarrow 2009$$

$$\frac{3d\sin(2a+2bx)}{128b^2} - \frac{d\sin(6a+6bx)}{1152b^2} - \frac{3(c+dx)\cos(2a+2bx)}{64b} + \frac{(c+dx)\cos(6a+6bx)}{192b}$$

input
```
Int[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(-3*(c + d*x)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)*Cos[6*a + 6*b*x])/(192
*b) + (3*d*Sin[2*a + 2*b*x])/(128*b^2) - (d*Sin[6*a + 6*b*x])/(1152*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{-54b(dx+c)\cos(2bx+2a)+6b\cos(6bx+6a)(dx+c)+48cb+27d\sin(2bx+2a)-d\sin(6bx+6a)}{1152b^2}$
risch	$-\frac{3(dx+c)\cos(2bx+2a)}{64b}+\frac{(dx+c)\cos(6bx+6a)}{192b}+\frac{3d\sin(2bx+2a)}{128b^2}-\frac{d\sin(6bx+6a)}{1152b^2}$
derivativedivides	$-\frac{da\left(-\frac{\sin(bx+a)^2\cos(bx+a)^4}{6}-\frac{\cos(bx+a)^4}{12}\right)}{b}+c\left(-\frac{\sin(bx+a)^2\cos(bx+a)^4}{6}-\frac{\cos(bx+a)^4}{12}\right)+\frac{d\left(\frac{(bx+a)\sin(bx+a)^4}{4}+\frac{\left(\sin(bx+\right.\right.}{b}$
default	$-\frac{da\left(-\frac{\sin(bx+a)^2\cos(bx+a)^4}{6}-\frac{\cos(bx+a)^4}{12}\right)}{b}+c\left(-\frac{\sin(bx+a)^2\cos(bx+a)^4}{6}-\frac{\cos(bx+a)^4}{12}\right)+\frac{d\left(\frac{(bx+a)\sin(bx+a)^4}{4}+\frac{\left(\sin(bx+\right.\right.}{b}$
norman	$\frac{4c\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4}{b}+\frac{4c\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^8}{b}+\frac{d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{12b^2}+\frac{17d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3}{36b^2}-\frac{d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^5}{2b^2}+\frac{d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^7}{2b^2}-\frac{17d\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^9}{36b^2}$
orering	$\frac{d\left(10x^2d^2b^2+20b^2cdx+10b^2c^2+d^2\right)\cos(bx+a)^3\sin(bx+a)^3}{18b^4(dx+c)^2}-\frac{\left(5x^2d^2b^2+10b^2cdx+5b^2c^2+d^2\right)\left(d\cos(bx+a)^3\sin(bx+\right.}{}$

input

```
int((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output

```
1/1152*(-54*b*(d*x+c)*cos(2*b*x+2*a)+6*b*cos(6*b*x+6*a)*(d*x+c)+48*c*b+27*
d*sin(2*b*x+2*a)-d*sin(6*b*x+6*a))/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{12\,(bdx+bc)\cos(bx+a)^6 - 18\,(bdx+bc)\cos(bx+a)^4 + 3\,bdx - \left(2\,d\cos(bx+a)^5 - 2\,d\cos(bx+a)^3\right.}{72\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/72*(12*(b*d*x + b*c)*cos(b*x + a)^6 - 18*(b*d*x + b*c)*cos(b*x + a)^4 +
3*b*d*x - (2*d*cos(b*x + a)^5 - 2*d*cos(b*x + a)^3 - 3*d*cos(b*x + a))*sin
(b*x + a))/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(80) = 160.

Time = 0.54 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.61

$$\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx$$
$$= \begin{cases} -\frac{c\sin^2(a+bx)\cos^4(a+bx)}{4b} - \frac{c\cos^6(a+bx)}{12b} + \frac{dx\sin^6(a+bx)}{24b} + \frac{dx\sin^4(a+bx)\cos^2(a+bx)}{8b} - \frac{dx\sin^2(a+bx)\cos^4(a+bx)}{8b} - \frac{dx\cos^{}}{2} \\ \left(cx + \frac{dx^2}{2}\right)\sin^3(a)\cos^3(a) \end{cases}$$

input
```
integrate((d*x+c)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output
```
Piecewise((-c*sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - c*cos(a + b*x)**6/(1
2*b) + d*x*sin(a + b*x)**6/(24*b) + d*x*sin(a + b*x)**4*cos(a + b*x)**2/(8
*b) - d*x*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) - d*x*cos(a + b*x)**6/(24*
b) + d*sin(a + b*x)**5*cos(a + b*x)/(24*b**2) + d*sin(a + b*x)**3*cos(a +
b*x)**3/(9*b**2) + d*sin(a + b*x)*cos(a + b*x)**5/(24*b**2), Ne(b, 0)), ((
c*x + d*x**2/2)*sin(a)**3*cos(a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.55

$$\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{96\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)c-\frac{96\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)ad}{b}-\frac{(6(bx+a)\cos(6bx+6a)-54(bx+a)\cos(2}{1152\,b}$$

input
```
integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/1152*(96*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c - 96*(2*sin(b*x + a)^6
 - 3*sin(b*x + a)^4)*a*d/b - (6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*
cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*d/b)/b
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{(bdx+bc)\cos(6bx+6a)}{192\,b^2} - \frac{3(bdx+bc)\cos(2bx+2a)}{64\,b^2} - \frac{d\sin(6bx+6a)}{1152\,b^2} + \frac{3d\sin(2bx+2a)}{128\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/192*(b*d*x + b*c)*cos(6*b*x + 6*a)/b^2 - 3/64*(b*d*x + b*c)*cos(2*b*x +
2*a)/b^2 - 1/1152*d*sin(6*b*x + 6*a)/b^2 + 3/128*d*sin(2*b*x + 2*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

$$\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{\frac{27\,d\sin(2\,a+2\,b\,x)}{4} - \frac{d\sin(6\,a+6\,b\,x)}{4} - \frac{27\,b\,c\cos(2\,a+2\,b\,x)}{2} + \frac{3\,b\,c\cos(6\,a+6\,b\,x)}{2} - \frac{27\,b\,d\,x\cos(2\,a+2\,b\,x)}{2} + \frac{3\,b\,d\,x\cos(6\,a+6\,b\,x)}{2}}{288\,b^2}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x),x)
```

output
```
((27*d*sin(2*a + 2*b*x))/4 - (d*sin(6*a + 6*b*x))/4 - (27*b*c*cos(2*a + 2*
b*x))/2 + (3*b*c*cos(6*a + 6*b*x))/2 - (27*b*d*x*cos(2*a + 2*b*x))/2 + (3*
b*d*x*cos(6*a + 6*b*x))/2)/(288*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.53

$$\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{-10\cos(bx+a)\sin(bx+a)^5\,d + 10\cos(bx+a)\sin(bx+a)^3\,d + 15\cos(bx+a)\sin(bx+a)\,d - 60\sin$$
$$360b$$

input
```
int((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

output
```
( - 10*cos(a + b*x)*sin(a + b*x)**5*d + 10*cos(a + b*x)*sin(a + b*x)**3*d
+ 15*cos(a + b*x)*sin(a + b*x)*d - 60*sin(a + b*x)**6*b*c - 60*sin(a + b*x
)**6*b*d*x + 90*sin(a + b*x)**4*b*c + 90*sin(a + b*x)**4*b*d*x - 15*a*d -
96*b*c - 15*b*d*x)/(360*b**2)
```

3.159 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx$

Optimal result . . . 1444
Mathematica [A] (verified) . . . 1445
Rubi [A] (verified) . . . 1445
Maple [A] (verified) . . . 1446
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Mupad [F(-1)] . . . 1449
Reduce [F] . . . 1450

Optimal result

Integrand size = 24, antiderivative size = 129

$$\begin{aligned}\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx = &-\frac{\operatorname{CosIntegral}\left(\frac{6bc}{d}+6bx\right)\sin\left(6a-\frac{6bc}{d}\right)}{32d}\\ &+\frac{3\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{32d}\\ &+\frac{3\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{32d}\\ &-\frac{\cos\left(6a-\frac{6bc}{d}\right)\operatorname{Si}\left(\frac{6bc}{d}+6bx\right)}{32d}\end{aligned}$$

output

```
-1/32*Ci(6*b*c/d+6*b*x)*sin(6*a-6*b*c/d)/d+3/32*Ci(2*b*c/d+2*b*x)*sin(2*a-
2*b*c/d)/d+3/32*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d-1/32*cos(6*a-6*b*c/d)
*Si(6*b*c/d+6*b*x)/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.85

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx = -\frac{\text{CosIntegral}\left(\frac{6b(c+dx)}{d}\right)\sin\left(6a-\frac{6bc}{d}\right)-3\,\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right)-3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2}{}\right.}{32d}$$

input
```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x),x]
```

output
```
-1/32*(CosIntegral[(6*b*(c + d*x))/d]*Sin[6*a - (6*b*c)/d] - 3*CosIntegral
[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - 3*Cos[2*a - (2*b*c)/d]*SinInteg
ral[(2*b*(c + d*x))/d] + Cos[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c + d*x))/
d])/d
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos^3(a+bx)}{c+dx}\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{3\sin(2a+2bx)}{32(c+dx)}-\frac{\sin(6a+6bx)}{32(c+dx)}\right)dx$$

$$\downarrow 2009$$

$$-\frac{\sin\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{32d}+\frac{3\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{32d}+\frac{3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{32d}-\frac{\cos\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6bc}{d}+6bx\right)}{32d}$$

input
```
Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x),x]
```

output
```
-1/32*(CosIntegral[(6*b*c)/d + 6*b*x]*Sin[6*a - (6*b*c)/d])/d + (3*CosInte
gral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(32*d) + (3*Cos[2*a - (2*b*c
)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(32*d) - (Cos[6*a - (6*b*c)/d]*SinInt
egral[(6*b*c)/d + 6*b*x])/(32*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.38

method	result
derivativedivides	$\frac{b\left(-\frac{6\,\mathrm{Si}\left(-6bx-6a-\frac{6(-ad+cb)}{d}\right)\cos\left(\frac{-6ad+6cb}{d}\right)}{d}-\frac{6\,\mathrm{Ci}\left(6bx+6a+\frac{-6ad+6cb}{d}\right)\sin\left(\frac{-6ad+6cb}{d}\right)}{d}\right)}{192}+\frac{3b\left(-\frac{2\,\mathrm{Si}\left(-2bx-2a-\frac{2(}{}\right.\right.}{b}$
default	$\frac{b\left(-\frac{6\,\mathrm{Si}\left(-6bx-6a-\frac{6(-ad+cb)}{d}\right)\cos\left(\frac{-6ad+6cb}{d}\right)}{d}-\frac{6\,\mathrm{Ci}\left(6bx+6a+\frac{-6ad+6cb}{d}\right)\sin\left(\frac{-6ad+6cb}{d}\right)}{d}\right)}{192}+\frac{3b\left(-\frac{2\,\mathrm{Si}\left(-2bx-2a-\frac{2(}{}\right.\right.}{b}$
risch	$\frac{ie^{-\frac{6i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(6ibx+6ia-\frac{6i(ad-cb)}{d}\right)}{64d}-\frac{3ie^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{64d}+\frac{3ie^{\frac{2i(}{}}}{}$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/192*b*(-6*Si(-6*b*x-6*a-6*(-a*d+b*c)/d)*cos(6*(-a*d+b*c)/d)/d-6*Ci
(6*b*x+6*a+6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d)+3/64*b*(-2*Si(-2*b*x-2*a
-2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(
2*(-a*d+b*c)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx$$
$$= \frac{3\ \mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right) - \mathrm{Ci}\left(\frac{6\,(bdx+bc)}{d}\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right) - \cos\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{6\,(bdx+bc)}{d}\right) + 3\ \mathrm{c}}{32\,d}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
1/32*(3*cos_integral(2*(b*d*x + b*c)/d)*sin(-2*(b*c - a*d)/d) - cos_integr
al(6*(b*d*x + b*c)/d)*sin(-6*(b*c - a*d)/d) - cos(-6*(b*c - a*d)/d)*sin_in
tegral(6*(b*d*x + b*c)/d) + 3*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x
+ b*c)/d))/d
```

Sympy [F]

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \int \frac{\sin^3(a+bx)\cos^3(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)**3*cos(a + b*x)**3/(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.18

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx =$$
$$-\frac{3\,b\left(-i\,E_1\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_1\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b\left(-i\,E_1\left(\frac{6\,(-i\,bc-i\,(}{}\right.\right.$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
-1/64*(3*b*(-I*exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I
*exp_integral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a
*d)/d) - b*(-I*exp_integral_e(1, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I
*exp_integral_e(1, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-6*(b*c - a
*d)/d) + 3*b*(exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + ex
p_integral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)
/d) - b*(exp_integral_e(1, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_int
egral_e(1, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-6*(b*c - a*d)/d))/
(b*d)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 6046, normalized size of antiderivative = 46.87

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output

```
-1/64*(imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*
b*c/d)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a
)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 3*imag_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - imag_part(c
os_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/
d)^2 + 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^
2*tan(b*c/d)^2 - 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan
(3*b*c/d)^2*tan(b*c/d)^2 - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(
3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) - 6*real_part(cos_integral(-2*b*
x - 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) + 2*real_part(
cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)
^2 + 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3
*b*c/d)*tan(b*c/d)^2 + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)
^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2 + 6*real_part(cos_integral(-2*b*x -
2*b*c/d))*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_
integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 -
 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d
)^2*tan(b*c/d)^2 + imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan
(a)^2*tan(3*b*c/d)^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)
^2*tan(a)^2*tan(3*b*c/d)^2 - 3*imag_part(cos_integral(-2*b*x - 2*b*c/d)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+b\,x)^3\sin(a+b\,x)^3}{c+d\,x}\,dx$$

input

```
int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x),x)
```

output

```
int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)^3\sin(bx+a)^3}{dx+c}dx$$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x)
```

output
```
int((cos(a + b*x)**3*sin(a + b*x)**3)/(c + d*x),x)
```

3.160 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 1451
Mathematica [A] (verified) . . . 1452
Rubi [A] (verified) . . . 1452
Maple [A] (verified) . . . 1454
Fricas [A] (verification not implemented) . . . 1454
Sympy [F] . . . 1455
Maxima [C] (verification not implemented) . . . 1455
Giac [C] (verification not implemented) . . . 1456
Mupad [F(-1)] . . . 1457
Reduce [F] . . . 1458

Optimal result

Integrand size = 24, antiderivative size = 179

$$\begin{aligned}\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = &\frac{3b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{16d^2}\\ &-\frac{3b\cos\left(6a-\frac{6bc}{d}\right)\operatorname{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{16d^2}\\ &-\frac{3\sin(2a+2bx)}{32d(c+dx)}+\frac{\sin(6a+6bx)}{32d(c+dx)}\\ &-\frac{3b\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{16d^2}\\ &+\frac{3b\sin\left(6a-\frac{6bc}{d}\right)\operatorname{Si}\left(\frac{6bc}{d}+6bx\right)}{16d^2}\end{aligned}$$

output

```
3/16*b*cos(2*a-2*b*c/d)*Ci(2*b*c/d+2*b*x)/d^2-3/16*b*cos(6*a-6*b*c/d)*Ci(6
*b*c/d+6*b*x)/d^2-3/32*sin(2*b*x+2*a)/d/(d*x+c)+1/32*sin(6*b*x+6*a)/d/(d*x
+c)-3/16*b*sin(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^2+3/16*b*sin(6*a-6*b*c/d)*
Si(6*b*c/d+6*b*x)/d^2
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.06

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{6b(c+dx)\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) - 6b(c+dx)\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6b(c+dx)}{d}\right) - 3d$$

input
```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
(6*b*(c + d*x)*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - 6*b*(
c + d*x)*Cos[6*a - (6*b*c)/d]*CosIntegral[(6*b*(c + d*x))/d] - 3*d*Cos[2*b
*x]*Sin[2*a] + d*Cos[6*b*x]*Sin[6*a] - 3*d*Cos[2*a]*Sin[2*b*x] + d*Cos[6*a
]*Sin[6*b*x] - 6*b*(c + d*x)*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*
x))/d] + 6*b*(c + d*x)*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c + d*x))/d]
)/(32*d^2*(c + d*x))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos^3(a+bx)}{(c+dx)^2}\,dx$$
$$\downarrow 4906$$
$$\int \left(\frac{3\sin(2a+2bx)}{32(c+dx)^2} - \frac{\sin(6a+6bx)}{32(c+dx)^2}\right)dx$$
$$\downarrow 2009$$

$$\frac{3b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{16d^2}-\frac{3b\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{16d^2}-\frac{3b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{16d^2}+\frac{3b\sin\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6bc}{d}+6bx\right)}{16d^2}-\frac{3\sin(2a+2bx)}{32d(c+dx)}+\frac{\sin(6a+6bx)}{32d(c+dx)}$$

input
```
Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
(3*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(16*d^2) - (3*b*
Cos[6*a - (6*b*c)/d]*CosIntegral[(6*b*c)/d + 6*b*x])/(16*d^2) - (3*Sin[2*a
 + 2*b*x])/(32*d*(c + d*x)) + Sin[6*a + 6*b*x]/(32*d*(c + d*x)) - (3*b*Sin
[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(16*d^2) + (3*b*Sin[6*a
- (6*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(16*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.35 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.43

method	result
derivativedivides	$-\frac{b^2\left(-\frac{6\sin(6bx+6a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{36\ \mathrm{Si}\left(-6bx-6a-\frac{6(-ad+cb)}{d}\right)\sin\left(\frac{-6ad+6cb}{d}\right)}{d}+\frac{36\ \mathrm{Ci}\left(6bx+6a+\frac{-6ad+6cb}{d}\right)\cos\left(\frac{-6ad+6cb}{d}\right)}{d}}{d}\right.}{192}$
default	$-\frac{b^2\left(-\frac{6\sin(6bx+6a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{36\ \mathrm{Si}\left(-6bx-6a-\frac{6(-ad+cb)}{d}\right)\sin\left(\frac{-6ad+6cb}{d}\right)}{d}+\frac{36\ \mathrm{Ci}\left(6bx+6a+\frac{-6ad+6cb}{d}\right)\cos\left(\frac{-6ad+6cb}{d}\right)}{d}}{d}\right.}{192}$
risch	$\frac{3b\,\mathrm{e}^{-\frac{6i(ad-cb)}{d}}\ \mathrm{expIntegral}_1\left(6ibx+6ia-\frac{6i(ad-cb)}{d}\right)}{32d^2}-\frac{3b\,\mathrm{e}^{-\frac{2i(ad-cb)}{d}}\ \mathrm{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{32d^2}-\frac{3b\,\mathrm{e}$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/192*b^2*(-6*sin(6*b*x+6*a)/(-a*d+c*b+d*(b*x+a))/d+6*(-6*Si(-6*b*x-
6*a-6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d+6*Ci(6*b*x+6*a+6*(-a*d+b*c)/d)*c
os(6*(-a*d+b*c)/d)/d)/d)+3/64*b^2*(-2*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/
d+2*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a
+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.11

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{3\,(bdx+bc)\cos\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{6\,(bdx+bc)}{d}\right)-3\,(bdx+bc)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)-3\,(bdx+b$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/16*(3*(b*d*x + b*c)*cos(-6*(b*c - a*d)/d)*cos_integral(6*(b*d*x + b*c)/
d) - 3*(b*d*x + b*c)*cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*d*x + b*c)/d)
 - 3*(b*d*x + b*c)*sin(-6*(b*c - a*d)/d)*sin_integral(6*(b*d*x + b*c)/d) +
 3*(b*d*x + b*c)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - 1
6*(d*cos(b*x + a)^5 - d*cos(b*x + a)^3)*sin(b*x + a))/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin^3(a+bx)\cos^3(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)**3*cos(a + b*x)**3/(c + d*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.72

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{3\,b^2\left(-i\,E_2\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_2\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^2\left(-i\,E_2\left(\frac{6\,(-i\,bc-}{}\right.\right.$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/64*(3*b^2*(-I*exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 I*exp_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c -
 a*d)/d) - b^2*(-I*exp_integral_e(2, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d)
 + I*exp_integral_e(2, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-6*(b*c
 - a*d)/d) + 3*b^2*(exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d
) + exp_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c
- a*d)/d) - b^2*(exp_integral_e(2, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 exp_integral_e(2, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-6*(b*c - a
*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.99 (sec) , antiderivative size = 63798, normalized size of antiderivative = 356.41

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output

```
-1/32*(3*b*d*x*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b
*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 3*b*d*x*real_part(
cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2
*tan(3*b*c/d)^2*tan(b*c/d)^2 - 3*b*d*x*real_part(cos_integral(-2*b*x - 2*b
*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/
d)^2 + 3*b*d*x*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*x)^2*tan(
b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 6*b*d*x*imag_part
(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^
2*tan(3*b*c/d)^2*tan(b*c/d) - 6*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d
) + 12*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3
*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) - 6*b*d*x*imag_part(cos_integral(
6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)
*tan(b*c/d)^2 + 6*b*d*x*imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*
x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 12*b*d*x*s
in_integral(6*(b*d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2
*tan(3*b*c/d)*tan(b*c/d)^2 - 6*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/
d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2
+ 6*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^
2*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2 - 12*b*d*x*sin_integral...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+b\,x\right)^3\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^2}\,dx$$

input

```
int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^2,x)
```

output

```
int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(bx+a)^3\sin(bx+a)^3}{d^2x^2+2cdx+c^2}dx$$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int((cos(a + b*x)**3*sin(a + b*x)**3)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.161 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$

Optimal result . . . 1459
Mathematica [A] (verified) . . . 1460
Rubi [A] (verified) . . . 1460
Maple [A] (verified) . . . 1462
Fricas [A] (verification not implemented) . . . 1462
Sympy [F] . . . 1463
Maxima [C] (verification not implemented) . . . 1463
Giac [C] (verification not implemented) . . . 1464
Mupad [F(-1)] . . . 1465
Reduce [F] . . . 1466

Optimal result

Integrand size = 24, antiderivative size = 235

$$\begin{aligned}\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = &-\frac{3b\cos(2a+2bx)}{32d^2(c+dx)}+\frac{3b\cos(6a+6bx)}{32d^2(c+dx)}\\&+\frac{9b^2\,\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)\sin\left(6a-\frac{6bc}{d}\right)}{16d^3}\\&-\frac{3b^2\,\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{16d^3}\\&-\frac{3\sin(2a+2bx)}{64d(c+dx)^2}+\frac{\sin(6a+6bx)}{64d(c+dx)^2}\\&-\frac{3b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{16d^3}\\&+\frac{9b^2\cos\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6bc}{d}+6bx\right)}{16d^3}\end{aligned}$$

output

```
-3/32*b*cos(2*b*x+2*a)/d^2/(d*x+c)+3/32*b*cos(6*b*x+6*a)/d^2/(d*x+c)+9/16*
b^2*Ci(6*b*c/d+6*b*x)*sin(6*a-6*b*c/d)/d^3-3/16*b^2*Ci(2*b*c/d+2*b*x)*sin(
2*a-2*b*c/d)/d^3-3/64*sin(2*b*x+2*a)/d/(d*x+c)^2+1/64*sin(6*b*x+6*a)/d/(d*
x+c)^2-3/16*b^2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^3+9/16*b^2*cos(6*a-6*
b*c/d)*Si(6*b*c/d+6*b*x)/d^3
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.02

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$$

$$= \frac{-3d\cos(2bx)(2b(c+dx)\cos(2a)+d\sin(2a))+d\cos(6bx)(6b(c+dx)\cos(6a)+d\sin(6a))+3d(-d\cos($$

input

```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

output

```
(-3*d*Cos[2*b*x]*(2*b*(c + d*x)*Cos[2*a] + d*Sin[2*a]) + d*Cos[6*b*x]*(6*b
*(c + d*x)*Cos[6*a] + d*Sin[6*a]) + 3*d*(-(d*Cos[2*a]) + 2*b*(c + d*x)*Sin
[2*a])*Sin[2*b*x] + d*(d*Cos[6*a] - 6*b*(c + d*x)*Sin[6*a])*Sin[6*b*x] + 6
*b^2*(c + d*x)^2*(6*CosIntegral[(6*b*(c + d*x))/d]*Sin[6*a - (6*b*c)/d] -
2*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - 2*Cos[2*a - (2*b*c
)/d]*SinIntegral[(2*b*(c + d*x))/d] + 6*Cos[6*a - (6*b*c)/d]*SinIntegral[(
6*b*(c + d*x))/d]))/(64*d^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos^3(a+bx)}{(c+dx)^3}\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{3\sin(2a+2bx)}{32(c+dx)^3} - \frac{\sin(6a+6bx)}{32(c+dx)^3}\right)dx$$

$$\downarrow 2009$$

$$\frac{9b^2 \sin\left(6a - \frac{6bc}{d}\right) \text{CosIntegral}\left(\frac{6bc}{d} + 6bx\right)}{16d^3} - \frac{3b^2 \sin\left(2a - \frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d} + 2bx\right)}{16d^3} - \frac{3b^2 \cos\left(2a - \frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d} + 2bx\right)}{16d^3} + \frac{9b^2 \cos\left(6a - \frac{6bc}{d}\right) \text{Si}\left(\frac{6bc}{d} + 6bx\right)}{16d^3} - \frac{3b\cos(2a + 2bx)}{32d^2(c + dx)} + \frac{3b\cos(6a + 6bx)}{32d^2(c + dx)} - \frac{3\sin(2a + 2bx)}{64d(c + dx)^2} + \frac{\sin(6a + 6bx)}{64d(c + dx)^2}$$

input

```
Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

output

```
(-3*b*Cos[2*a + 2*b*x])/(32*d^2*(c + d*x)) + (3*b*Cos[6*a + 6*b*x])/(32*d^
2*(c + d*x)) + (9*b^2*CosIntegral[(6*b*c)/d + 6*b*x]*Sin[6*a - (6*b*c)/d])
/(16*d^3) - (3*b^2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(1
6*d^3) - (3*Sin[2*a + 2*b*x])/(64*d*(c + d*x)^2) + Sin[6*a + 6*b*x]/(64*d*
(c + d*x)^2) - (3*b^2*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])
/(16*d^3) + (9*b^2*Cos[6*a - (6*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(1
6*d^3)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.40

method	result
derivativedivides	$-\frac{b^3\left(-\frac{3\sin(6bx+6a)}{(-ad+cb+d(bx+a))^2d}+\frac{-\frac{18\cos(6bx+6a)}{(-ad+cb+d(bx+a))d}-\frac{18\left(-\frac{6\,\mathrm{Si}\left(-6bx-6a-\frac{6(-ad+cb)}{d}\right)\cos\left(\frac{-6ad+6cb}{d}\right)}{d}-\frac{6\,\mathrm{Ci}\left(6bx+6a+\right.}{d}\right.}{d}\right.}{192}$
default	$-\frac{b^3\left(-\frac{3\sin(6bx+6a)}{(-ad+cb+d(bx+a))^2d}+\frac{-\frac{18\cos(6bx+6a)}{(-ad+cb+d(bx+a))d}-\frac{18\left(-\frac{6\,\mathrm{Si}\left(-6bx-6a-\frac{6(-ad+cb)}{d}\right)\cos\left(\frac{-6ad+6cb}{d}\right)}{d}-\frac{6\,\mathrm{Ci}\left(6bx+6a+\right.}{d}\right.}{d}\right.}{192}$
risch	$-\frac{9ib^2\mathrm{e}^{-\frac{6i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(6ibx+6ia-\frac{6i(ad-cb)}{d}\right)}{32d^3}+\frac{3ib^2\mathrm{e}^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{32d^3}-$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/192*b^3*(-3*sin(6*b*x+6*a)/(-a*d+c*b+d*(b*x+a))^2/d+3*(-6*cos(6*b*
x+6*a)/(-a*d+c*b+d*(b*x+a))/d-6*(-6*Si(-6*b*x-6*a-6*(-a*d+b*c)/d)*cos(6*(-
a*d+b*c)/d)/d-6*Ci(6*b*x+6*a+6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d)/d)+
3/64*b^3*(-sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2/d+(-2*cos(2*b*x+2*a)/(-a*
d+c*b+d*(b*x+a))/d-2*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)
/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.48

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$$

$$= \frac{48\left(bd^2x+bcd\right)\cos\left(bx+a\right)^6-72\left(bd^2x+bcd\right)\cos\left(bx+a\right)^4+24\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2-3\left(b^2d^2x^2\right.}{}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/16*(48*(b*d^2*x + b*c*d)*cos(b*x + a)^6 - 72*(b*d^2*x + b*c*d)*cos(b*x +
 a)^4 + 24*(b*d^2*x + b*c*d)*cos(b*x + a)^2 - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*cos_integral(2*(b*d*x + b*c)/d)*sin(-2*(b*c - a*d)/d) + 9*(b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(6*(b*d*x + b*c)/d)*sin(-6*(
b*c - a*d)/d) + 9*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-6*(b*c - a*d)
/d)*sin_integral(6*(b*d*x + b*c)/d) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) + 8*(d^2*cos(b*x
 + a)^5 - d^2*cos(b*x + a)^3)*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3
)
```

Sympy [F]

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin^3(a+bx)\cos^3(a+bx)}{(c+dx)^3}\,dx$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)**3*cos(a + b*x)**3/(c + d*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.46

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx =$$
$$-\frac{3\,b^3\left(-i\,E_3\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_3\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^3\left(-i\,E_3\left(\frac{6\,(-i\,bc-}\right.\right.$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

output

```
-1/64*(3*b^3*(-I*exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 I*exp_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c -
 a*d)/d) - b^3*(-I*exp_integral_e(3, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d)
 + I*exp_integral_e(3, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-6*(b*c
 - a*d)/d) + 3*b^3*(exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d
) + exp_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c
- a*d)/d) - b^3*(exp_integral_e(3, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 exp_integral_e(3, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-6*(b*c - a
*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2
 - a*d^3)*(b*x + a))*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.02 (sec) , antiderivative size = 111694, normalized size of antiderivative = 475.29

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input

```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

output

```
1/32*(9*b^2*d^2*x^2*imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*
tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 3*b^2*d^2*x^2
*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)
^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 3*b^2*d^2*x^2*imag_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*
b*c/d)^2*tan(b*c/d)^2 - 9*b^2*d^2*x^2*imag_part(cos_integral(-6*b*x - 6*b*
c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d
)^2 + 18*b^2*d^2*x^2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)
^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 6*b^2*d^2*x^2*sin_int
egral(2*(b*d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3
*b*c/d)^2*tan(b*c/d)^2 - 6*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*
c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d
) - 6*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*t
an(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) + 18*b^2*d^2*x^2*r
eal_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2
*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 + 18*b^2*d^2*x^2*real_part(cos_integra
l(-6*b*x - 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c
/d)*tan(b*c/d)^2 + 6*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*
tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2 + 6*
b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos\left(a+b\,x\right)^3\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^3}\,dx$$

input

```
int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^3,x)
```

output

```
int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos(bx+a)^3\sin(bx+a)^3}{d^3x^3+3cd^2x^2+3c^2dx+c^3}dx$$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x)
```

output
```
int((cos(a + b*x)**3*sin(a + b*x)**3)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
 d**3*x**3),x)
```

3.162 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$

Optimal result . . . 1467
Mathematica [A] (verified) . . . 1468
Rubi [A] (verified) . . . 1468
Maple [A] (verified) . . . 1470
Fricas [A] (verification not implemented) . . . 1471
Sympy [F(-1)] . . . 1471
Maxima [C] (verification not implemented) . . . 1472
Giac [C] (verification not implemented) . . . 1472
Mupad [F(-1)] . . . 1473
Reduce [F] . . . 1474

Optimal result

Integrand size = 24, antiderivative size = 287

$$\begin{aligned}\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = &-\frac{b\cos(2a+2bx)}{32d^2(c+dx)^2}+\frac{b\cos(6a+6bx)}{32d^2(c+dx)^2}\\ &-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{8d^4}\\ &+\frac{9b^3\cos\left(6a-\frac{6bc}{d}\right)\operatorname{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{8d^4}\\ &-\frac{\sin(2a+2bx)}{32d(c+dx)^3}+\frac{b^2\sin(2a+2bx)}{16d^3(c+dx)}+\frac{\sin(6a+6bx)}{96d(c+dx)^3}\\ &-\frac{3b^2\sin(6a+6bx)}{16d^3(c+dx)}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{8d^4}\\ &-\frac{9b^3\sin\left(6a-\frac{6bc}{d}\right)\operatorname{Si}\left(\frac{6bc}{d}+6bx\right)}{8d^4}\end{aligned}$$

output

```
-1/32*b*cos(2*b*x+2*a)/d^2/(d*x+c)^2+1/32*b*cos(6*b*x+6*a)/d^2/(d*x+c)^2-1
/8*b^3*cos(2*a-2*b*c/d)*Ci(2*b*c/d+2*b*x)/d^4+9/8*b^3*cos(6*a-6*b*c/d)*Ci(
6*b*c/d+6*b*x)/d^4-1/32*sin(2*b*x+2*a)/d/(d*x+c)^3+1/16*b^2*sin(2*b*x+2*a)
/d^3/(d*x+c)+1/96*sin(6*b*x+6*a)/d/(d*x+c)^3-3/16*b^2*sin(6*b*x+6*a)/d^3/(
d*x+c)+1/8*b^3*sin(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^4-9/8*b^3*sin(6*a-6*b*
c/d)*Si(6*b*c/d+6*b*x)/d^4
```

Mathematica [A] (verified)

Time = 4.15 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.93

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$$

$$= \frac{-3bcd^2\cos(2(a+bx)) - 3bd^3x\cos(2(a+bx)) + 3bcd^2\cos(6(a+bx)) + 3bd^3x\cos(6(a+bx)) - 12b^3(c+$$

input

```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

output

```
(-3*b*c*d^2*Cos[2*(a + b*x)] - 3*b*d^3*x*Cos[2*(a + b*x)] + 3*b*c*d^2*Cos[
6*(a + b*x)] + 3*b*d^3*x*Cos[6*(a + b*x)] - 12*b^3*(c + d*x)^3*Cos[2*a - (
2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + 108*b^3*(c + d*x)^3*Cos[6*a - (
6*b*c)/d]*CosIntegral[(6*b*(c + d*x))/d] + 6*b^2*c^2*d*Sin[2*(a + b*x)] -
3*d^3*Sin[2*(a + b*x)] + 12*b^2*c*d^2*x*Sin[2*(a + b*x)] + 6*b^2*d^3*x^2*S
in[2*(a + b*x)] - 18*b^2*c^2*d*Sin[6*(a + b*x)] + d^3*Sin[6*(a + b*x)] - 3
6*b^2*c*d^2*x*Sin[6*(a + b*x)] - 18*b^2*d^3*x^2*Sin[6*(a + b*x)] + 12*b^3*
c^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 36*b^3*c^2*d*x*S
in[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 36*b^3*c*d^2*x^2*Sin[
2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 12*b^3*d^3*x^3*Sin[2*a -
 (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] - 108*b^3*c^3*Sin[6*a - (6*b*c)
/d]*SinIntegral[(6*b*(c + d*x))/d] - 324*b^3*c^2*d*x*Sin[6*a - (6*b*c)/d]*
SinIntegral[(6*b*(c + d*x))/d] - 324*b^3*c*d^2*x^2*Sin[6*a - (6*b*c)/d]*Si
nIntegral[(6*b*(c + d*x))/d] - 108*b^3*d^3*x^3*Sin[6*a - (6*b*c)/d]*SinInt
egral[(6*b*(c + d*x))/d])/(96*d^4*(c + d*x)^3)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos^3(a+bx)}{(c+dx)^4}\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{3\sin(2a+2bx)}{32(c+dx)^4} - \frac{\sin(6a+6bx)}{32(c+dx)^4}\right)dx$$

$$\downarrow 2009$$

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{8d^4} + \frac{9b^3\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{8d^4} + \frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{8d^4} - \frac{9b^3\sin\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6bc}{d}+6bx\right)}{8d^4} + \frac{b^2\sin(2a+2bx)}{16d^3(c+dx)} - \frac{3b^2\sin(6a+6bx)}{16d^3(c+dx)} - \frac{b\cos(2a+2bx)}{32d^2(c+dx)^2} + \frac{b\cos(6a+6bx)}{32d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{32d(c+dx)^3} + \frac{\sin(6a+6bx)}{96d(c+dx)^3}$$

input
```
Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

output
```
-1/32*(b*Cos[2*a + 2*b*x])/(d^2*(c + d*x)^2) + (b*Cos[6*a + 6*b*x])/(32*d^
2*(c + d*x)^2) - (b^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])
/(8*d^4) + (9*b^3*Cos[6*a - (6*b*c)/d]*CosIntegral[(6*b*c)/d + 6*b*x])/(8*
d^4) - Sin[2*a + 2*b*x]/(32*d*(c + d*x)^3) + (b^2*Sin[2*a + 2*b*x])/(16*d^
3*(c + d*x)) + Sin[6*a + 6*b*x]/(96*d*(c + d*x)^3) - (3*b^2*Sin[6*a + 6*b*
x])/(16*d^3*(c + d*x)) + (b^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d +
 2*b*x])/(8*d^4) - (9*b^3*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b
*x])/(8*d^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 3.57 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{b^4\left(-\frac{2\sin(6bx+6a)}{(-ad+cb+d(bx+a))^3 d}+\frac{-\frac{6\cos(6bx+6a)}{(-ad+cb+d(bx+a))^2 d}-\frac{6\left(-\frac{6\sin(6bx+6a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{36\ \mathrm{Si}\left(-6bx-6a-\frac{6(-ad+cb)}{d}\right)\sin\left(\frac{-6}{d}\right.}{d}}{d}\right)}{d}\right)}{192}$
default	$\frac{b^4\left(-\frac{2\sin(6bx+6a)}{(-ad+cb+d(bx+a))^3 d}+\frac{-\frac{6\cos(6bx+6a)}{(-ad+cb+d(bx+a))^2 d}-\frac{6\left(-\frac{6\sin(6bx+6a)}{(-ad+cb+d(bx+a))d}+\frac{-\frac{36\ \mathrm{Si}\left(-6bx-6a-\frac{6(-ad+cb)}{d}\right)\sin\left(\frac{-6}{d}\right.}{d}}{d}\right)}{d}\right)}{192}$
risch	$-\frac{9b^3 e^{-\frac{6i(ad-cb)}{d}}\ \mathrm{expIntegral}_1\left(6ibx+6ia-\frac{6i(ad-cb)}{d}\right)}{16d^4}+\frac{b^3 e^{-\frac{2i(ad-cb)}{d}}\ \mathrm{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{16d^4}+\frac{b}{}$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/192*b^4*(-2*sin(6*b*x+6*a)/(-a*d+c*b+d*(b*x+a))^3/d+2*(-3*cos(6*b*
x+6*a)/(-a*d+c*b+d*(b*x+a))^2/d-3*(-6*sin(6*b*x+6*a)/(-a*d+c*b+d*(b*x+a))/
d+6*(-6*Si(-6*b*x-6*a-6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d+6*Ci(6*b*x+6*a
+6*(-a*d+b*c)/d)*cos(6*(-a*d+b*c)/d)/d)/d)/d)+3/64*b^4*(-2/3*sin(2*b*x+
2*a)/(-a*d+c*b+d*(b*x+a))^3/d+2/3*(-cos(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2/
d-(-2*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2*(-2*Si(-2*b*x-2*a-2*(-a*d+b*
c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c
)/d)/d)/d)/d)/d))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.83

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$$

$$= \frac{24\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^6 - 36\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^4 + 12\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^2 + 27\left(b^3d}{}$$

input

```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="fricas")
```

output

```
1/24*(24*(b*d^3*x + b*c*d^2)*cos(b*x + a)^6 - 36*(b*d^3*x + b*c*d^2)*cos(b
*x + a)^4 + 12*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 + 27*(b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-6*(b*c - a*d)/d)*cos_integral(
6*(b*d*x + b*c)/d) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^
3*c^3)*cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*d*x + b*c)/d) - 27*(b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-6*(b*c - a*d)/d)*si
n_integral(6*(b*d*x + b*c)/d) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c
^2*d*x + b^3*c^3)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) -
8*((18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*cos(b*x + a)^5 -
 (18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*cos(b*x + a)^3 + 3
*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a))*sin(b*x + a))/(d^
7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \text{Timed out}$$

input

```
integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c)**4,x)
```

output

```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.37

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = -\frac{3\,b^4\left(-i\,E_4\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_4\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^4\left(-i\,E_4\left(\frac{6\,(-i\,bc-}{6}\right.\right.$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/64*(3*b^4*(-I*exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 I*exp_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c -
 a*d)/d) - b^4*(-I*exp_integral_e(4, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d)
 + I*exp_integral_e(4, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-6*(b*c
 - a*d)/d) + 3*b^4*(exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d
) + exp_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c
- a*d)/d) - b^4*(exp_integral_e(4, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 exp_integral_e(4, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-6*(b*c - a
*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 -
a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 +
 a^2*d^4)*(b*x + a))*b)
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.29 (sec) , antiderivative size = 157526, normalized size of antiderivative = 548.87

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")
```

output

```
1/48*(27*b^3*d^3*x^3*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2
*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 3*b^3*d^3*x^
3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a
)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 3*b^3*d^3*x^3*real_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3
*b*c/d)^2*tan(b*c/d)^2 + 27*b^3*d^3*x^3*real_part(cos_integral(-6*b*x - 6*
b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c
/d)^2 + 6*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^
2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) - 6*b^3*d^3*x^3
*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a
)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) + 12*b^3*d^3*x^3*sin_integral(2*(b*
d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*t
an(b*c/d) - 54*b^3*d^3*x^3*imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*
b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 + 54*b^3*d
^3*x^3*imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*t
an(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 108*b^3*d^3*x^3*sin_integra
l(6*(b*d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c
/d)*tan(b*c/d)^2 - 6*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*
tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2 + 6*
b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos\left(a+b\,x\right)^3\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^4}\,dx$$

input

```
int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^4,x)
```

output

```
int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^4, x)
```

Reduce [F]

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \text{too large to display}$$

input
```
int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x)
```

output
```
(3600*cos(a + b*x)*sin(a + b*x)**5*b**2*c**2*d + 7200*cos(a + b*x)*sin(a +
 b*x)**5*b**2*c*d**2*x + 3600*cos(a + b*x)*sin(a + b*x)**5*b**2*d**3*x**2
+ 10538*cos(a + b*x)*sin(a + b*x)**5*d**3 - 5184*cos(a + b*x)*sin(a + b*x)
**4*b*c*d**2 - 5184*cos(a + b*x)*sin(a + b*x)**4*b*d**3*x - 92700*cos(a +
b*x)*sin(a + b*x)**3*b**2*c**2*d - 185400*cos(a + b*x)*sin(a + b*x)**3*b**
2*c*d**2*x - 92700*cos(a + b*x)*sin(a + b*x)**3*b**2*d**3*x**2 - 265166*co
s(a + b*x)*sin(a + b*x)**3*d**3 + 69888*cos(a + b*x)*sin(a + b*x)**2*b*c*d
**2 + 69888*cos(a + b*x)*sin(a + b*x)**2*b*d**3*x + 379350*cos(a + b*x)*si
n(a + b*x)*b**2*c**2*d + 758700*cos(a + b*x)*sin(a + b*x)*b**2*c*d**2*x +
379350*cos(a + b*x)*sin(a + b*x)*b**2*d**3*x**2 + 958848*cos(a + b*x)*sin(
a + b*x)*d**3 - 551424*cos(a + b*x)*b*c*d**2 - 551424*cos(a + b*x)*b*d**3*
x + 19958400*int(tan((a + b*x)/2)**7/(tan((a + b*x)/2)**12*c**4 + 4*tan((a
 + b*x)/2)**12*c**3*d*x + 6*tan((a + b*x)/2)**12*c**2*d**2*x**2 + 4*tan((a
 + b*x)/2)**12*c*d**3*x**3 + tan((a + b*x)/2)**12*d**4*x**4 + 6*tan((a + b
*x)/2)**10*c**4 + 24*tan((a + b*x)/2)**10*c**3*d*x + 36*tan((a + b*x)/2)**
10*c**2*d**2*x**2 + 24*tan((a + b*x)/2)**10*c*d**3*x**3 + 6*tan((a + b*x)/
2)**10*d**4*x**4 + 15*tan((a + b*x)/2)**8*c**4 + 60*tan((a + b*x)/2)**8*c*
*3*d*x + 90*tan((a + b*x)/2)**8*c**2*d**2*x**2 + 60*tan((a + b*x)/2)**8*c*
d**3*x**3 + 15*tan((a + b*x)/2)**8*d**4*x**4 + 20*tan((a + b*x)/2)**6*c**4
 + 80*tan((a + b*x)/2)**6*c**3*d*x + 120*tan((a + b*x)/2)**6*c**2*d**2*...
```

3.163 $\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx$

Optimal result . . . 1475
Mathematica [N/A] . . . 1476
Rubi [N/A] . . . 1476
Maple [N/A] . . . 1479
Fricas [N/A] . . . 1479
Sympy [N/A] . . . 1480
Maxima [N/A] . . . 1480
Giac [N/A] . . . 1480
Mupad [N/A] . . . 1481
Reduce [N/A] . . . 1481

Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}
&\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx \\
&= \frac{2^{-3-m} e^{2i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} \\
&\quad + \frac{2^{-3-m} e^{-2i\left(a-\frac{bc}{d}\right)} (c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,\frac{2ib(c+dx)}{d}\right)}{b} \\
&\quad + \text{Int}((c+dx)^m \cot(a+bx), x)
\end{aligned}$$

output

```
2^(-3-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*b
*(d*x+c)/d)^m)+2^(-3-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a-
b*c/d))/((I*b*(d*x+c)/d)^m)+Defer(Int)((d*x+c)^m*cot(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 7.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Cos[a + b*x]^2*Cot[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Cos[a + b*x]^2*Cot[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4908, 3042, 25, 4222, 4906, 27, 3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\cot(a+bx)(c+dx)^m\,dx$$

$$\downarrow 4908$$

$$\int (c+dx)^m \cot(a+bx)dx - \int (c+dx)^m \cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 3042$$

$$\int -(c+dx)^m \tan\left(a+bx+\frac{\pi}{2}\right)dx - \int (c+dx)^m \cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 25$$

$$-\int (c+dx)^m \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx - \int (c+dx)^m \cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 4222$$

$$-\int (c+dx)^m \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx - \int (c+dx)^m \cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 4906$$

$$-\int \frac{1}{2}(c+dx)^m \sin(2a+2bx)dx - \int (c+dx)^m \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\downarrow 27$$

$$-\frac{1}{2}\int (c+dx)^m \sin(2a+2bx)dx - \int (c+dx)^m \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\downarrow 3042$$

$$-\frac{1}{2}\int (c+dx)^m \sin(2a+2bx)dx - \int (c+dx)^m \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\downarrow 3789$$

$$-\int (c+dx)^m \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx + \frac{1}{2}\left(\frac{1}{2}i\int e^{2i(a+bx)}(c+dx)^m dx - \frac{1}{2}i\int e^{-2i(a+bx)}(c+dx)^m dx\right)$$

$$\downarrow 2612$$

$$-\int (c+dx)^m \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx + \frac{1}{2}\left(\frac{2^{-m-2}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b} + \frac{2^{-m-2}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}}{b}\right.$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]^2*Cot[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 4908

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \cos(bx+a)^2 \cot(bx+a)\,dx$$

input

```
int((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x)
```

output

```
int((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)^2 \cot(bx+a)\,dx$$

input

```
integrate((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

output

```
integral((d*x + c)^m*cos(b*x + a)^2*cot(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 25.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cos(b*x+a)**2*cot(b*x+a),x)
```

output
```
Integral((c + d*x)**m*cos(a + b*x)**2*cot(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)^2 \cot(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^2*cot(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)^2 \cot(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^2*cot(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 18.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx = \int \cos\left(a+b\,x\right)^2 \cot(a+b\,x)\,\left(c+d\,x\right)^m\,dx$$

input
```
int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx = \int (dx+c)^m \cos\left(bx+a\right)^2 \cot\left(bx+a\right)\,dx$$

input
```
int((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x)
```

output
```
int((c + d*x)**m*cos(a + b*x)**2*cot(a + b*x),x)
```

3.164 $\int (c+dx)^4 \cos^2(a+bx)\cot(a+bx)\,dx$

Optimal result . . . 1482
Mathematica [B] (warning: unable to verify) . . . 1483
Rubi [A] (verified) . . . 1484
Maple [B] (verified) . . . 1492
Fricas [B] (verification not implemented) . . . 1493
Sympy [F] . . . 1494
Maxima [B] (verification not implemented) . . . 1494
Giac [F] . . . 1495
Mupad [F(-1)] . . . 1496
Reduce [F] . . . 1496

Optimal result

Integrand size = 22, antiderivative size = 299

$$\begin{aligned}\int (c+dx)^4 \cos^2(a+bx)\cot(a+bx)\,dx = &-\frac{3d^2(c+dx)^2}{4b^3}+\frac{(c+dx)^4}{4b}-\frac{i(c+dx)^5}{5d}\\&+\frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b}\\&-\frac{2id(c+dx)^3\,\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}\\&+\frac{3d^2(c+dx)^2\,\mathrm{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}\\&+\frac{3id^3(c+dx)\,\mathrm{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}\\&-\frac{3d^4\,\mathrm{PolyLog}\left(5,e^{2i(a+bx)}\right)}{2b^5}\\&+\frac{3d^3(c+dx)\cos(a+bx)\sin(a+bx)}{2b^4}\\&-\frac{d(c+dx)^3\cos(a+bx)\sin(a+bx)}{b^2}\\&-\frac{3d^4\sin^2(a+bx)}{4b^5}\\&+\frac{3d^2(c+dx)^2\sin^2(a+bx)}{2b^3}\\&-\frac{(c+dx)^4\sin^2(a+bx)}{2b}\end{aligned}$$

output
```
-3/4*d^2*(d*x+c)^2/b^3+1/4*(d*x+c)^4/b-1/5*I*(d*x+c)^5/d+(d*x+c)^4*ln(1-ex
p(2*I*(b*x+a)))/b-2*I*d*(d*x+c)^3*polylog(2,exp(2*I*(b*x+a)))/b^2+3*d^2*(d
*x+c)^2*polylog(3,exp(2*I*(b*x+a)))/b^3+3*I*d^3*(d*x+c)*polylog(4,exp(2*I*
(b*x+a)))/b^4-3/2*d^4*polylog(5,exp(2*I*(b*x+a)))/b^5+3/2*d^3*(d*x+c)*cos(
b*x+a)*sin(b*x+a)/b^4-d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2-3/4*d^4*sin(b*
x+a)^2/b^5+3/2*d^2*(d*x+c)^2*sin(b*x+a)^2/b^3-1/2*(d*x+c)^4*sin(b*x+a)^2/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2918 vs. $2(299) = 598$.

Time $= 6.29$ (sec) , antiderivative size $= 2918$, normalized size of antiderivative $= 9.76$

$$\int (c+dx)^4 \cos^2(a+bx) \cot(a+bx)\, dx = \text{Result too large to show}$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]^2*Cot[a + b*x],x]
```

output

```
-((c^2*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2
*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2
*Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, -E^((-I
)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, E^((-I)*(a + b*x))] +
(6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^((-I)*(a + b*x))] + (6*I)*(1 - E^((
-2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]))/b^3) - (c*d^3*E^(I*a)*Csc[a]*((
b^4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a
 + b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] -
 6*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - 6*b^2*(1 -
 E^((-2*I)*a))*x^2*PolyLog[2, E^((-I)*(a + b*x))] + (12*I)*b*(1 - E^((-2*I
)*a))*x*PolyLog[3, -E^((-I)*(a + b*x))] + (12*I)*b*(1 - E^((-2*I)*a))*x*Po
lyLog[3, E^((-I)*(a + b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, -E^((-I)*(
a + b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, E^((-I)*(a + b*x))]))/b^4 -
(d^4*E^(I*a)*Csc[a]*((2*b^5*x^5)/E^((2*I)*a) + (5*I)*b^4*(1 - E^((-2*I)*a)
)*x^4*Log[1 - E^((-I)*(a + b*x))] + (5*I)*b^4*(1 - E^((-2*I)*a))*x^4*Log[1
 + E^((-I)*(a + b*x))] - 20*b^3*(1 - E^((-2*I)*a))*x^3*PolyLog[2, -E^((-I)
*(a + b*x))] - 20*b^3*(1 - E^((-2*I)*a))*x^3*PolyLog[2, E^((-I)*(a + b*x))
] + (60*I)*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[3, -E^((-I)*(a + b*x))] + (6
0*I)*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[3, E^((-I)*(a + b*x))] + 120*b*(1
- E^((-2*I)*a))*x*PolyLog[4, -E^((-I)*(a + b*x))] + 120*b*(1 - E^((-2*I...
```

Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.24, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {4908, 3042, 25, 4202, 2620, 3011, 4904, 3042, 3792, 17, 3042, 3791, 17, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \cos^2(a+bx)\cot(a+bx)\,dx$$

$$\downarrow 4908$$

$$\int (c+dx)^4 \cot(a+bx)dx - \int (c+dx)^4 \cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 3042$$

$$\int -(c+dx)^4 \tan\left(a+bx+\frac{\pi}{2}\right) dx - \int (c+dx)^4 \cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 25$$

$$-\int (c+dx)^4 \tan\left(\frac{1}{2}(2a+\pi)+bx\right) dx - \int (c+dx)^4 \cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 4202$$

$$2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)^4}{1+e^{i(2a+2bx+\pi)}}dx - \int (c+dx)^4 \cos(a+bx)\sin(a+bx)dx - \frac{i(c+dx)^5}{5d}$$

$$\downarrow 2620$$

$$2i\left(\frac{2id\int (c+dx)^3 \log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{b} - \frac{i(c+dx)^4 \log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right) - \int (c+dx)^4 \cos(a+bx)\sin(a+bx)dx - \frac{i(c+dx)^5}{5d}$$

$$\downarrow 3011$$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3 \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b} - \frac{3id\int (c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b} - \frac{i(c+dx)^4 \log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$
$$\int (c+dx)^4 \cos(a+bx)\sin(a+bx)dx - \frac{i(c+dx)^5}{5d}$$

$$\downarrow 4904$$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3 \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b} - \frac{3id\int (c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b} - \frac{i(c+dx)^4 \log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$
$$\frac{2d\int (c+dx)^3 \sin^2(a+bx)dx}{b} - \frac{(c+dx)^4 \sin^2(a+bx)}{2b} - \frac{i(c+dx)^5}{5d}$$

$$\downarrow 3042$$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3 \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b} - \frac{3id\int (c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b} - \frac{i(c+dx)^4 \log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$
$$\frac{2d\int (c+dx)^3 \sin(a+bx)^2 dx}{b} - \frac{(c+dx)^4 \sin^2(a+bx)}{2b} - \frac{i(c+dx)^5}{5d}$$

$$\downarrow 3792$$

$$\frac{2d\left(-\frac{3d^2\int(c+dx)\sin^2(a+bx)dx}{2b^2}+\frac{1}{2}\int(c+dx)^3dx+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}\right)}{b}+$$
$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$
$$\left.\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{i(c+dx)^5}{5d}\right)$$

$$\downarrow 17$$

$$\frac{2d\left(-\frac{3d^2\int(c+dx)\sin^2(a+bx)dx}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)}{b}+$$
$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$
$$\left.\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{i(c+dx)^5}{5d}\right)$$

$$\downarrow 3042$$

$$\frac{2d\left(-\frac{3d^2\int(c+dx)\sin(a+bx)^2dx}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)}{b}+$$
$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$
$$\left.\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{i(c+dx)^5}{5d}\right)$$

$$\downarrow 3791$$

$$\frac{2d\left(-\frac{3d^2\left(\frac{1}{2}\int(c+dx)dx+\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right.}{b}$$
$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$
$$\left.\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{i(c+dx)^5}{5d}\right)$$

$$\downarrow 17$$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$

$$\left.2d\left(-\frac{3d^2\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)\right.$$

$$\left.\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{i(c+dx)^5}{5d}\right)$$

$$\downarrow 7163$$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\left(\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)dx}{b}-\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{2b}\right)}{b}-\right.$$

$$\left.2d\left(-\frac{3d^2\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)\right.$$

$$\left.\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{i(c+dx)^5}{5d}\right)$$

$$\downarrow 7163$$

$$\frac{2i\left(2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\left(\frac{id\left(\frac{\mathrm{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}-\frac{i(c+dx)^2\,\mathrm{P}}{2b}\right)}{b}\right)}{2d\left(-\frac{3d^2\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)-}$$

$$\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{i(c+dx)^5}{5d}$$

$$\downarrow 2720$$

$$\frac{2i\left(2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\left(\frac{id\left(\frac{d\int e^{-i(2a+2bx+\pi)}\,\mathrm{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(4,-e^{i}\right.\right.}{2b}\right)}{b}-\frac{}{2b}\right)}{b}\right)}{2d\left(-\frac{3d^2\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)-}$$

$$\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{i(c+dx)^5}{5d}$$

$$\downarrow 7143$$

$$\frac{2d\left(-\frac{3d^2\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)}{b}+$$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\left(\frac{id\left(\frac{d\,\mathrm{PolyLog}\left(5,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}-\frac{i(c+dx)^2\,\mathrm{PolyL}}{2b}\right.}{b}\right.\right.$$

$$\left.\left.\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{i(c+dx)^5}{5d}\right.\right.$$

input
```
Int[(c + d*x)^4*Cos[a + b*x]^2*Cot[a + b*x],x]
```

output
```
((-1/5*I)*(c + d*x)^5)/d + (2*I)*(((-1/2*I)*(c + d*x)^4*Log[1 + E^(I*(2*a
+ Pi + 2*b*x))])/b + ((2*I)*d*(((I/2)*(c + d*x)^3*PolyLog[2, -E^(I*(2*a +
Pi + 2*b*x))])/b - (((3*I)/2)*d*(((-1/2*I)*(c + d*x)^2*PolyLog[3, -E^(I*(2
*a + Pi + 2*b*x))])/b + (I*d*(((-1/2*I)*(c + d*x)*PolyLog[4, -E^(I*(2*a +
Pi + 2*b*x))])/b + (d*PolyLog[5, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)))/b))
/b))/b) - ((c + d*x)^4*Sin[a + b*x]^2)/(2*b) + (2*d*((c + d*x)^4/(8*d) - (
(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (3*d*(c + d*x)^2*Sin[a + b*
x]^2)/(4*b^2) - (3*d^2*((c + d*x)^2/(4*d) - ((c + d*x)*Cos[a + b*x]*Sin[a
+ b*x])/(2*b) + (d*Sin[a + b*x]^2)/(4*b^2)))/(2*b^2)))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4202

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4904

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4908

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1334 vs. 2(270) = 540.

Time = 2.75 (sec) , antiderivative size = 1335, normalized size of antiderivative = 4.46

method	result	size
risch	Expression too large to display	1335

input
```
int((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^4*d*(2*b^2*d^3*x^3+6*b^2*c*d^2*x^2+6*b^2*c^2*d*x+2*b^2*c^3-3*d^3*x-
3*c*d^2)*sin(2*b*x+2*a)-2*I*d*c^3*x^2-I*d^3*c*x^4-2*I*d^2*c^2*x^3-1/b^5*d^
4*ln(1-exp(I*(b*x+a)))*a^4-2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))+1/b^5*d^4*a^4*
ln(exp(I*(b*x+a))-1)+12/b^3*d^2*c^2*polylog(3,-exp(I*(b*x+a)))+12/b^3*d^2*
c^2*polylog(3,exp(I*(b*x+a)))+1/b*d^4*ln(exp(I*(b*x+a))+1)*x^4+1/b*d^4*ln(
1-exp(I*(b*x+a)))*x^4+12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2+12/b^3*d^4
*polylog(3,exp(I*(b*x+a)))*x^2+8/5*I/b^5*d^4*a^5+1/8*(2*b^4*d^4*x^4+8*b^4*
c*d^3*x^3+12*b^4*c^2*d^2*x^2+8*b^4*c^3*d*x+2*b^4*c^4-6*b^2*d^4*x^2-12*b^2*
c*d^3*x-6*b^2*c^2*d^2+3*d^4)/b^5*cos(2*b*x+2*a)-12*I/b^2*d^3*c*polylog(2,-
exp(I*(b*x+a)))*x^2-12*I/b^2*d^3*c*polylog(2,exp(I*(b*x+a)))*x^2-8*I/b^3*d
^3*c*a^3*x-12*I/b^2*d^2*c^2*polylog(2,-exp(I*(b*x+a)))*x-12*I/b^2*d^2*c^2*
polylog(2,exp(I*(b*x+a)))*x+12*I/b^2*d^2*c^2*a^2*x-8*I/b*d*c^3*a*x+4/b*d^3
*c*ln(1-exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-4*I/b
^2*d^4*polylog(2,exp(I*(b*x+a)))*x^3-4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a))
)*x^3-4*I/b^2*d*c^3*a^2-4*I/b^2*d*c^3*polylog(2,-exp(I*(b*x+a)))-4*I/b^2*d
*c^3*polylog(2,exp(I*(b*x+a)))+8*I/b^3*d^2*c^2*a^3+24*I/b^4*d^3*c*polylog(
4,-exp(I*(b*x+a)))+24*I/b^4*d^3*c*polylog(4,exp(I*(b*x+a)))-6*I/b^4*d^3*c*
a^4+2*I/b^4*d^4*a^4*x+24*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)))*x+4/b^4*d^3*
c*ln(1-exp(I*(b*x+a)))*a^3+4/b^2*d*c^3*ln(1-exp(I*(b*x+a)))*a-6/b^3*d^2*c^
2*ln(1-exp(I*(b*x+a)))*a^2+24/b^3*d^3*c*polylog(3,-exp(I*(b*x+a)))*x+24...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1453 vs. $2(266) = 532$.

Time = 0.17 (sec) , antiderivative size = 1453, normalized size of antiderivative = 4.86

$$\int (c+dx)^4 \cos^2(a+bx)\cot(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 48*d^4*polylog(5, cos(b*x + a) + I*s
in(b*x + a)) + 48*d^4*polylog(5, cos(b*x + a) - I*sin(b*x + a)) + 48*d^4*p
olylog(5, -cos(b*x + a) + I*sin(b*x + a)) + 48*d^4*polylog(5, -cos(b*x + a
) - I*sin(b*x + a)) + 3*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 - (2*b^4*d^4*x^4 + 8
*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 - b^
2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^2 + 2*(2*b^3*d^
4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d
^4)*x)*cos(b*x + a)*sin(b*x + a) + 2*(2*b^4*c^3*d - 3*b^2*c*d^3)*x + 8*(I*
b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(c
os(b*x + a) + I*sin(b*x + a)) + 8*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*
I*b^3*c^2*d^2*x - I*b^3*c^3*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) + 8*(-
I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*dilog
(-cos(b*x + a) + I*sin(b*x + a)) + 8*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 +
3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 2
*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*
c^4)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b^4*d^4*x^4 + 4*b^4*c*d^3
*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*x + a) - I*s
in(b*x + a) + 1) - 2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*(
b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*...
```

Sympy [F]

$$\int (c+dx)^4 \cos^2(a+bx) \cot(a+bx)\, dx = \int (c+dx)^4 \cos^2(a+bx) \cot(a+bx)\, dx$$

input
```
integrate((d*x+c)**4*cos(b*x+a)**2*cot(b*x+a),x)
```

output
```
Integral((c + d*x)**4*cos(a + b*x)**2*cot(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1654 vs. $2(266) = 532$.

Time = 0.28 (sec) , antiderivative size = 1654, normalized size of antiderivative = 5.53

$$\int (c+dx)^4 \cos^2(a+bx) \cot(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

output

```
-1/40*(20*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*c^4 - 80*(sin(b*x + a)^2
- log(sin(b*x + a)^2))*a*c^3*d/b + 120*(sin(b*x + a)^2 - log(sin(b*x + a)^
2))*a^2*c^2*d^2/b^2 - 80*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a^3*c*d^3/
b^3 + 20*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a^4*d^4/b^4 - (-8*I*(b*x +
 a)^5*d^4 - 40*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^4 - 960*d^4*polylog(5, -e^(
I*b*x + I*a)) - 960*d^4*polylog(5, e^(I*b*x + I*a)) - 80*(I*b^2*c^2*d^2 -
2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a)^3 - 80*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d
^2 + 3*I*a^2*b*c*d^3 - I*a^3*d^4)*(b*x + a)^2 - 40*(-I*(b*x + a)^4*d^4 + 4
*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 6*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 -
I*a^2*d^4)*(b*x + a)^2 + 4*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 - 3*I*a^2*b*c
*d^3 + I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 40*
(I*(b*x + a)^4*d^4 + 4*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^3 + 6*(I*b^2*c^2*d^
2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a)^2 + 4*(I*b^3*c^3*d - 3*I*a*b^2*c^
2*d^2 + 3*I*a^2*b*c*d^3 - I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a), -cos
(b*x + a) + 1) + 5*(2*(b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 3*(
2*a^2 - 1)*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(2*b^2*c^2*d^2 - 4*a*
b*c*d^3 + (2*a^2 - 1)*d^4)*(b*x + a)^2 + 4*(2*b^3*c^3*d - 6*a*b^2*c^2*d^2
+ 3*(2*a^2 - 1)*b*c*d^3 - (2*a^3 - 3*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) -
 160*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 + I*(b*x + a)^3*d^
4 - I*a^3*d^4 + 3*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 ...
```

Giac [F]

$$\int (c+dx)^4 \cos^2(a+bx)\cot(a+bx)\,dx = \int (dx+c)^4 \cos\left(bx+a\right)^2 \cot\left(bx+a\right)\,dx$$

input

```
integrate((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

output

```
integrate((d*x + c)^4*cos(b*x + a)^2*cot(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \cos^2(a+bx)\cot(a+bx)\,dx = \int \cos(a+b\,x)^2 \cot(a+b\,x)\,(c+d\,x)^4\,dx$$

input
```
int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^4,x)
```

output
```
int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^4, x)
```

Reduce [F]

$$\int (c+dx)^4 \cos^2(a+bx)\cot(a+bx)\,dx$$
$$= \frac{2\left(\int \cos(bx+a)^2 \cot(bx+a)\,x^4 dx\right) b\,d^4 + 8\left(\int \cos(bx+a)^2 \cot(bx+a)\,x^3 dx\right) bc\,d^3 + 12\left(\int \cos(bx+a)\right.}{}$$

input
```
int((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x)
```

output
```
(2*int(cos(a + b*x)**2*cot(a + b*x)*x**4,x)*b*d**4 + 8*int(cos(a + b*x)**2
*cot(a + b*x)*x**3,x)*b*c*d**3 + 12*int(cos(a + b*x)**2*cot(a + b*x)*x**2,
x)*b*c**2*d**2 + 8*int(cos(a + b*x)**2*cot(a + b*x)*x,x)*b*c**3*d - 2*log(
tan((a + b*x)/2)**2 + 1)*c**4 + 2*log(tan((a + b*x)/2))*c**4 - sin(a + b*x
)**2*c**4)/(2*b)
```

3.165 $\int (c+dx)^3 \cos^2(a+bx) \cot(a+bx)\, dx$

Optimal result . . . 1497
Mathematica [B] (warning: unable to verify) . . . 1498
Rubi [A] (verified) . . . 1499
Maple [B] (verified) . . . 1505
Fricas [B] (verification not implemented) . . . 1506
Sympy [F] . . . 1507
Maxima [B] (verification not implemented) . . . 1508
Giac [F] . . . 1509
Mupad [F(-1)] . . . 1509
Reduce [F] . . . 1509

Optimal result

Integrand size = 22, antiderivative size = 246

$$\begin{aligned}\int (c+dx)^3 \cos^2(a+bx) \cot(a+bx)\, dx = &-\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} - \frac{i(c+dx)^4}{4d} \\ &+ \frac{(c+dx)^3 \log\left(1-e^{2i(a+bx)}\right)}{b} \\ &- \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^2} \\ &+ \frac{3d^2(c+dx) \operatorname{PolyLog}\left(3, e^{2i(a+bx)}\right)}{2b^3} \\ &+ \frac{3id^3 \operatorname{PolyLog}\left(4, e^{2i(a+bx)}\right)}{4b^4} \\ &+ \frac{3d^3 \cos(a+bx)\sin(a+bx)}{8b^4} \\ &- \frac{3d(c+dx)^2 \cos(a+bx)\sin(a+bx)}{4b^2} \\ &+ \frac{3d^2(c+dx)\sin^2(a+bx)}{4b^3} \\ &- \frac{(c+dx)^3 \sin^2(a+bx)}{2b}\end{aligned}$$

output
```
-3/8*d^3*x/b^3+1/4*(d*x+c)^3/b-1/4*I*(d*x+c)^4/d+(d*x+c)^3*ln(1-exp(2*I*(b
*x+a)))/b-3/2*I*d*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2+3/2*d^2*(d*x+c
)*polylog(3,exp(2*I*(b*x+a)))/b^3+3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^
4+3/8*d^3*cos(b*x+a)*sin(b*x+a)/b^4-3/4*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/
b^2+3/4*d^2*(d*x+c)*sin(b*x+a)^2/b^3-1/2*(d*x+c)^3*sin(b*x+a)^2/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1956 vs. $2(246) = 492$.

Time = 6.24 (sec) , antiderivative size = 1956, normalized size of antiderivative = 7.95

$$\int (c+dx)^3 \cos^2(a+bx)\cot(a+bx)\,dx = \text{Too large to display}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]^2*Cot[a + b*x],x]
```

output
```
-1/2*(c*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-
2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^
2*Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, -E^((-
I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, E^((-I)*(a + b*x))] +
 (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^((-I)*(a + b*x))] + (6*I)*(1 - E^(
(-2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]))/b^3 - (d^3*E^(I*a)*Csc[a]*((b^
4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a +
 b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - 6
*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - 6*b^2*(1 - E
^((-2*I)*a))*x^2*PolyLog[2, E^((-I)*(a + b*x))] + (12*I)*b*(1 - E^((-2*I)*
a))*x*PolyLog[3, -E^((-I)*(a + b*x))] + (12*I)*b*(1 - E^((-2*I)*a))*x*Poly
Log[3, E^((-I)*(a + b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, -E^((-I)*(a
+ b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, E^((-I)*(a + b*x))]))/(4*b^4)
+ (c^3*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[
a]))/(b*(Cos[a]^2 + Sin[a]^2)) + Csc[a]*(Cos[2*a + 2*b*x]/(64*b^4) - ((I/6
4)*Sin[2*a + 2*b*x])/b^4)*(32*b^4*c^3*x*Cos[a + 2*b*x] + 48*b^4*c^2*d*x^2*
Cos[a + 2*b*x] + 32*b^4*c*d^2*x^3*Cos[a + 2*b*x] + 8*b^4*d^3*x^4*Cos[a + 2
*b*x] + 32*b^4*c^3*x*Cos[3*a + 2*b*x] + 48*b^4*c^2*d*x^2*Cos[3*a + 2*b*x]
+ 32*b^4*c*d^2*x^3*Cos[3*a + 2*b*x] + 8*b^4*d^3*x^4*Cos[3*a + 2*b*x] + (4*
I)*b^3*c^3*Cos[3*a + 4*b*x] - 6*b^2*c^2*d*Cos[3*a + 4*b*x] - (6*I)*b*c*...
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.21, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {4908, 3042, 25, 4202, 2620, 3011, 4904, 3042, 3792, 17, 3042, 3115, 24, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \cos^2(a+bx)\cot(a+bx)\,dx$$

$$\downarrow 4908$$

$$\int (c+dx)^3 \cot(a+bx)dx - \int (c+dx)^3 \cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 3042$$

$$\int -(c+dx)^3 \tan\left(a+bx+\frac{\pi}{2}\right)dx - \int (c+dx)^3 \cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 25$$

$$-\int (c+dx)^3 \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx - \int (c+dx)^3 \cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 4202$$

$$2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)^3}{1+e^{i(2a+2bx+\pi)}}dx - \int (c+dx)^3 \cos(a+bx)\sin(a+bx)dx - \frac{i(c+dx)^4}{4d}$$

$$\downarrow 2620$$

$$2i\left(\frac{3id\int (c+dx)^2 \log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{2b} - \frac{i(c+dx)^3 \log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right) - \int (c+dx)^3 \cos(a+bx)\sin(a+bx)dx - \frac{i(c+dx)^4}{4d}$$

$$\downarrow 3011$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b} - \frac{id\int (c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b} - \frac{i(c+dx)^3 \log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$

$$\int (c+dx)^3 \cos(a+bx)\sin(a+bx)dx - \frac{i(c+dx)^4}{4d}$$

$$\downarrow 4904$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$
$$\frac{3d\int(c+dx)^2\sin^2(a+bx)dx}{2b}-\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\downarrow 3042$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$
$$\frac{3d\int(c+dx)^2\sin(a+bx)^2dx}{2b}-\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\downarrow 3792$$

$$\frac{3d\left(-\frac{d^2\int\sin^2(a+bx)dx}{2b^2}+\frac{1}{2}\int(c+dx)^2dx+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b}+$$
$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$
$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\downarrow 17$$

$$\frac{3d\left(-\frac{d^2\int\sin^2(a+bx)dx}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}+$$
$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$
$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\downarrow 3042$$

$$\frac{3d\left(-\frac{d^2\int \sin(a+bx)^2dx}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}+$$
$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$
$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\downarrow 3115$$

$$\frac{3d\left(-\frac{d^2\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}+$$
$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$
$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\downarrow 24$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$
$$\frac{3d\left(\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}-$$
$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\downarrow 7163$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\left(\frac{id\int \mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(c+d}{}\right.$$

$$\left.\frac{3d\left(\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}-\right.$$

$$\left.\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}\right)$$

$$\downarrow 2720$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\left(\frac{d\int e^{-i(2a+2bx+\pi)}\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(2a+2}\right.\right.}{b}\right.}{2b}$$

$$\left.\frac{3d\left(\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}-\right.$$

$$\left.\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}\right)$$

$$\downarrow 7143$$

$$\frac{3d\left(\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}+$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\left(\frac{d\,\mathrm{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(c+dx)^3}{}\right.$$

$$\left.\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}\right)$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]^2*Cot[a + b*x],x]
```

output
```
((-1/4*I)*(c + d*x)^4)/d + (2*I)*(((-1/2*I)*(c + d*x)^3*Log[1 + E^(I*(2*a
+ Pi + 2*b*x))])/b + (((3*I)/2)*d*(((I/2)*(c + d*x)^2*PolyLog[2, -E^(I*(2*
a + Pi + 2*b*x))])/b - (I*d*(((-1/2*I)*(c + d*x)*PolyLog[3, -E^(I*(2*a + P
i + 2*b*x))])/b + (d*PolyLog[4, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)))/b))/
b) - ((c + d*x)^3*Sin[a + b*x]^2)/(2*b) + (3*d*((c + d*x)^3/(6*d) - ((c +
d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (d*(c + d*x)*Sin[a + b*x]^2)/(2*
b^2) - (d^2*(x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)))/(2*b^2)))/(2*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))  Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)  Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)  Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))  Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4202

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I  Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4904

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))  Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))  Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 907 vs. $2(215) = 430$.

Time = 1.76 (sec) , antiderivative size = 908, normalized size of antiderivative = 3.69

method	result	size
risch	Expression too large to display	908

input
```
int((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
-3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2-6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a)))-6
*I/b*d*c^2*a*x+6*I/b^2*c*d^2*a^2*x-6*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+a))
)*x-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x-3/16*d*(2*b^2*d^2*x^2+4*b^2*
c*d*x+2*b^2*c^2-d^2)/b^4*sin(2*b*x+2*a)+I*c^3*x+1/4*I/d*c^4-3/2*I/b^4*d^3*
a^4+6*I/b^4*d^3*polylog(4,-exp(I*(b*x+a)))-I*d^2*c*x^3-3/2*I*d*c^2*x^2+1/8
/b^3*(2*b^2*d^3*x^3+6*b^2*c*d^2*x^2+6*b^2*c^2*d*x+2*b^2*c^3-3*d^3*x-3*c*d^
2)*cos(2*b*x+2*a)+2/b^4*d^3*a^3*ln(exp(I*(b*x+a)))-1/b^4*d^3*a^3*ln(exp(I*
(b*x+a))-1)+6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,e
xp(I*(b*x+a)))+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b*d^3*ln(exp(I*(b*x+a))+
1)*x^3+6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+6/b^3*d^3*polylog(3,exp(I*(b
*x+a)))*x+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+3/b^3*c*d^2*a^2*ln(exp(I*(b*x
+a))-1)+6/b^2*c^2*d*a*ln(exp(I*(b*x+a)))-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1
)+3/b^2*d*c^2*ln(1-exp(I*(b*x+a)))*a+3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/
b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2+3/b*d*c^2*ln(exp(I*(b*x+a))+1)*x+3/b*d*c^
2*ln(1-exp(I*(b*x+a)))*x-3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-3*I/b^
2*d^3*polylog(2,exp(I*(b*x+a)))*x^2-3*I/b^2*d*c^2*polylog(2,exp(I*(b*x+a))
)-3*I/b^2*d*c^2*a^2-3*I/b^2*d*c^2*polylog(2,-exp(I*(b*x+a)))-2*I/b^3*d^3*a
^3*x+4*I/b^3*c*d^2*a^3-1/4*I*d^3*x^4-2/b*c^3*ln(exp(I*(b*x+a)))+1/b*c^3*ln
(exp(I*(b*x+a))-1)+1/b*c^3*ln(exp(I*(b*x+a))+1)+6*I*d^3*polylog(4,exp(I*(b
*x+a)))/b^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 984 vs. $2(211) = 422$.

Time $= 0.14$ (sec) , antiderivative size $= 984$, normalized size of antiderivative $= 4.00$

$$\int (c+dx)^3 \cos^2(a+bx) \cot(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

output

```
-1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 24*I*d^3*polylog(4, cos(b*x + a) +
 I*sin(b*x + a)) + 24*I*d^3*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 24
*I*d^3*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) - 24*I*d^3*polylog(4, -c
os(b*x + a) - I*sin(b*x + a)) - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3
*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b^2*d^
3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 3*(
2*b^3*c^2*d - b*d^3)*x + 12*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d
)*dilog(cos(b*x + a) + I*sin(b*x + a)) + 12*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^
2*x - I*b^2*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) + 12*(-I*b^2*d^3*x
^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-cos(b*x + a) + I*sin(b*x + a))
+ 12*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(-cos(b*x + a) -
 I*sin(b*x + a)) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*
c^3)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2
*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 4
*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a)
 + 1/2*I*sin(b*x + a) + 1/2) - 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 4*(b^3*d^3*
x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^
3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 4*(b^3*d^3*x^3 + 3*b^3*c*
d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(...
```

Sympy [F]

$$\int (c+dx)^3 \cos^2(a+bx) \cot(a+bx)\, dx = \int (c+dx)^3 \cos^2(a+bx) \cot(a+bx)\, dx$$

input

```
integrate((d*x+c)**3*cos(b*x+a)**2*cot(b*x+a),x)
```

output

```
Integral((c + d*x)**3*cos(a + b*x)**2*cot(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 979 vs. $2(211) = 422$.

Time = 0.20 (sec) , antiderivative size = 979, normalized size of antiderivative = 3.98

$$\int (c+dx)^3 \cos^2(a+bx) \cot(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

output

```
-1/16*(8*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*c^3 - 24*(sin(b*x + a)^2 -
 log(sin(b*x + a)^2))*a*c^2*d/b + 24*(sin(b*x + a)^2 - log(sin(b*x + a)^2)
)*a^2*c*d^2/b^2 - 8*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a^3*d^3/b^3 - (
-4*I*(b*x + a)^4*d^3 - 16*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^3 + 96*I*d^3*pol
ylog(4, -e^(I*b*x + I*a)) + 96*I*d^3*polylog(4, e^(I*b*x + I*a)) - 24*(I*b
^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*(b*x + a)^2 - 16*(-I*(b*x + a)^3*d^3
 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2
- I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 16*(I*(b
*x + a)^3*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I
*a*b*c*d^2 + I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1
) + 2*(2*(b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x
+ a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 - 1)*d^3)*(b*x + a))*cos(2*
b*x + 2*a) - 48*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d
^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*dilog(-e^(I*b*x + I*a)) - 48*(I*b^
2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I
*a*d^3)*(b*x + a))*dilog(e^(I*b*x + I*a)) + 8*((b*x + a)^3*d^3 + 3*(b*c*d^
2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*
log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + 8*((b*x + a)^3
*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + ...
```

Giac [F]

$$\int (c+dx)^3 \cos^2(a+bx)\cot(a+bx)\,dx = \int (dx+c)^3 \cos(bx+a)^2 \cot(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*cos(b*x + a)^2*cot(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \cos^2(a+bx)\cot(a+bx)\,dx = \int \cos(a+b\,x)^2 \cot(a+b\,x)\,(c+d\,x)^3\,dx$$

input
```
int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^3,x)
```

output
```
int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \cos^2(a+bx)\cot(a+bx)\,dx$$
$$= \frac{2\left(\int \cos(bx+a)^2 \cot(bx+a)\,x^3 dx\right) b\,d^3 + 6\left(\int \cos(bx+a)^2 \cot(bx+a)\,x^2 dx\right) bc\,d^2 + 6\left(\int \cos(bx+a)^2\right.}{2b}$$

input
```
int((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x)
```

output
```
(2*int(cos(a + b*x)**2*cot(a + b*x)*x**3,x)*b*d**3 + 6*int(cos(a + b*x)**2
*cot(a + b*x)*x**2,x)*b*c*d**2 + 6*int(cos(a + b*x)**2*cot(a + b*x)*x,x)*b
*c**2*d - 2*log(tan((a + b*x)/2)**2 + 1)*c**3 + 2*log(tan((a + b*x)/2))*c*
*3 - sin(a + b*x)**2*c**3)/(2*b)
```

3.166 $\int (c+dx)^2 \cos^2(a+bx)\cot(a+bx)\,dx$

Optimal result . . . 1510
Mathematica [B] (warning: unable to verify) . . . 1511
Rubi [A] (verified) . . . 1512
Maple [B] (verified) . . . 1516
Fricas [B] (verification not implemented) . . . 1516
Sympy [F] . . . 1517
Maxima [B] (verification not implemented) . . . 1517
Giac [F] . . . 1518
Mupad [F(-1)] . . . 1518
Reduce [F] . . . 1519

Optimal result

Integrand size = 22, antiderivative size = 172

$$\begin{aligned}\int (c+dx)^2 \cos^2(a+bx)\cot(a+bx)\,dx = {} & \frac{(c+dx)^2}{4b} - \frac{i(c+dx)^3}{3d} \\ & + \frac{(c+dx)^2 \log\left(1-e^{2i(a+bx)}\right)}{b} \\ & - \frac{id(c+dx)\,\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2} \\ & + \frac{d^2\,\mathrm{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3} \\ & - \frac{d(c+dx)\cos(a+bx)\sin(a+bx)}{2b^2} \\ & + \frac{d^2\sin^2(a+bx)}{4b^3} - \frac{(c+dx)^2\sin^2(a+bx)}{2b}\end{aligned}$$

output

```
1/4*(d*x+c)^2/b-1/3*I*(d*x+c)^3/d+(d*x+c)^2*ln(1-exp(2*I*(b*x+a)))/b-I*d*(
d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^2+1/2*d^2*polylog(3,exp(2*I*(b*x+a)))
/b^3-1/2*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2+1/4*d^2*sin(b*x+a)^2/b^3-1/2*
(d*x+c)^2*sin(b*x+a)^2/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 564 vs. $2(172) = 344$.

Time = 1.99 (sec) , antiderivative size = 564, normalized size of antiderivative = 3.28

$$\int (c+dx)^2 \cos^2(a+bx)\cot(a+bx)\,dx$$
$$= \frac{48ib^2cd\pi x + 16ib^3d^2x^3 - 96ib^2cdx\arctan(\tan(a)) + 48b^3cdx^2\cot(a) - 6bcd\cos(a+2bx)\csc(a) - 6bd^2x$$

input
```
Integrate[(c + d*x)^2*Cos[a + b*x]^2*Cot[a + b*x],x]
```

output
```
((48*I)*b^2*c*d*Pi*x + (16*I)*b^3*d^2*x^3 - (96*I)*b^2*c*d*x*ArcTan[Tan[a]
] + 48*b^3*c*d*x^2*Cot[a] - 6*b*c*d*Cos[a + 2*b*x]*Csc[a] - 6*b*d^2*x*Cos[
a + 2*b*x]*Csc[a] + 6*b*c*d*Cos[3*a + 2*b*x]*Csc[a] + 6*b*d^2*x*Cos[3*a +
2*b*x]*Csc[a] + 48*b*c*d*Pi*Log[1 + E^((-2*I)*b*x)] + 48*b^2*d^2*x^2*Log[1
 - E^((-I)*(a + b*x))] + 48*b^2*d^2*x^2*Log[1 + E^((-I)*(a + b*x))] + 96*b
^2*c*d*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + 96*b*c*d*ArcTan[Tan[a
]]*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] - 48*b*c*d*Pi*Log[Cos[b*x]] +
 48*b^2*c^2*Log[Sin[a + b*x]] - 96*b*c*d*ArcTan[Tan[a]]*Log[Sin[b*x + ArcT
an[Tan[a]]]] + (96*I)*b*d^2*x*PolyLog[2, -E^((-I)*(a + b*x))] + (96*I)*b*d
^2*x*PolyLog[2, E^((-I)*(a + b*x))] - (48*I)*b*c*d*PolyLog[2, E^((2*I)*(b*
x + ArcTan[Tan[a]]))] + 96*d^2*PolyLog[3, -E^((-I)*(a + b*x))] + 96*d^2*Po
lyLog[3, E^((-I)*(a + b*x))] - 48*b^3*c*d*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*
Sqrt[Sec[a]^2] - 6*b^2*c^2*Csc[a]*Sin[a + 2*b*x] + 3*d^2*Csc[a]*Sin[a + 2*
b*x] - 12*b^2*c*d*x*Csc[a]*Sin[a + 2*b*x] - 6*b^2*d^2*x^2*Csc[a]*Sin[a + 2
*b*x] + 6*b^2*c^2*Csc[a]*Sin[3*a + 2*b*x] - 3*d^2*Csc[a]*Sin[3*a + 2*b*x]
+ 12*b^2*c*d*x*Csc[a]*Sin[3*a + 2*b*x] + 6*b^2*d^2*x^2*Csc[a]*Sin[3*a + 2*
b*x])/(48*b^3)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.20, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {4908, 3042, 25, 4202, 2620, 3011, 2720, 4904, 3042, 3791, 17, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \cos^2(a+bx)\cot(a+bx)\,dx$$

$$\downarrow 4908$$

$$\int (c+dx)^2 \cot(a+bx)dx - \int (c+dx)^2 \cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 3042$$

$$\int -(c+dx)^2 \tan\left(a+bx+\frac{\pi}{2}\right)dx - \int (c+dx)^2 \cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 25$$

$$-\int (c+dx)^2 \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx - \int (c+dx)^2 \cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 4202$$

$$2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)^2}{1+e^{i(2a+2bx+\pi)}}dx - \int (c+dx)^2 \cos(a+bx)\sin(a+bx)dx - \frac{i(c+dx)^3}{3d}$$

$$\downarrow 2620$$

$$2i\left(\frac{id\int (c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{b} - \frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right) - \int (c+dx)^2\cos(a+bx)\sin(a+bx)dx - \frac{i(c+dx)^3}{3d}$$

$$\downarrow 3011$$

$$2i\left(\frac{id\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b} - \frac{id\int \text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b} - \frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right) - \int (c+dx)^2\cos(a+bx)\sin(a+bx)dx - \frac{i(c+dx)^3}{3d}$$

$$\downarrow 2720$$

$$2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i}\right.}{2b}$$

$$\int(c+dx)^2\cos(a+bx)\sin(a+bx)dx-\frac{i(c+dx)^3}{3d}$$

$$\downarrow 4904$$

$$2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i}\right.}{2b}$$

$$\frac{d\int(c+dx)\sin^2(a+bx)dx}{b}-\frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

$$\downarrow 3042$$

$$2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i}\right.}{2b}$$

$$\frac{d\int(c+dx)\sin(a+bx)^2dx}{b}-\frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

$$\downarrow 3791$$

$$2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i}\right.}{2b}$$

$$\frac{d\left(\frac{1}{2}\int(c+dx)dx+\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right)}{b}-\frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

$$\downarrow 17$$

$$2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i}\right.}{2b}$$

$$\frac{d\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{b}-\frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

$$\downarrow 7143$$

$$2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)+$$
$$\frac{d\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{b}-\frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]^2*Cot[a + b*x],x]
```

output
```
((-1/3*I)*(c + d*x)^3)/d + (2*I)*(((-1/2*I)*(c + d*x)^2*Log[1 + E^(I*(2*a
+ Pi + 2*b*x))])/b + (I*d*(((I/2)*(c + d*x)*PolyLog[2, -E^(I*(2*a + Pi + 2
*b*x))])/b - (d*PolyLog[3, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)))/b) - ((c
+ d*x)^2*Sin[a + b*x]^2)/(2*b) + (d*((c + d*x)^2/(4*d) - ((c + d*x)*Cos[a
+ b*x]*Sin[a + b*x])/(2*b) + (d*Sin[a + b*x]^2)/(4*b^2)))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4202

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4904

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4908

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(152) = 304$.

Time = 1.27 (sec) , antiderivative size = 544, normalized size of antiderivative = 3.16

method	result
risch	$\frac{2dc\ln(1-e^{i(bx+a)})x}{b} - \frac{2cda\ln(e^{i(bx+a)}-1)}{b^2} + \frac{4cda\ln(e^{i(bx+a)})}{b^2} - \frac{2id^2\,\mathrm{polylog}(2,e^{i(bx+a)})x}{b^2} - \frac{4idcax}{b} + \frac{2dc\ln(e^{i(bx+a)}$

input
```
int((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-4*I/b*d*c*a*x-2*I/b^2*d*c*polylog(2,-exp(I*(b*x+a)))-2*I/b^2*d*c*polylog(
2,exp(I*(b*x+a)))+2*I/b^2*d^2*a^2*x-2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))
*x-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x+1/b^3*d^2*a^2*ln(exp(I*(b*x+a))
-1)-2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+1/
b*d^2*ln(exp(I*(b*x+a))+1)*x^2+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2+4/3*I/b^3*
d^2*a^3+2/b^2*d*c*ln(1-exp(I*(b*x+a)))*a+2/b*d*c*ln(exp(I*(b*x+a))+1)*x+2/
b*d*c*ln(1-exp(I*(b*x+a)))*x-2/b^2*c*d*a*ln(exp(I*(b*x+a))-1)+4/b^2*c*d*a*
ln(exp(I*(b*x+a)))-2*I/b^2*d*c*a^2-I*d*c*x^2+1/8*(2*b^2*d^2*x^2+4*b^2*c*d*
x+2*b^2*c^2-d^2)/b^3*cos(2*b*x+2*a)-2/b*c^2*ln(exp(I*(b*x+a)))+1/b*c^2*ln(
exp(I*(b*x+a))-1)+1/b*c^2*ln(exp(I*(b*x+a))+1)+I*c^2*x+1/3*I/d*c^3-1/3*I*d
^2*x^3+2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I*(b*x+a))
)/b^3-1/4*d*(d*x+c)*sin(2*b*x+2*a)/b^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(149) = 298$.

Time = 0.12 (sec) , antiderivative size = 594, normalized size of antiderivative = 3.45

$$\int (c+dx)^2\cos^2(a+bx)\cot(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2
 - d^2)*cos(b*x + a)^2 - 4*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) -
 4*d^2*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 4*d^2*polylog(3, -cos(b
*x + a) + I*sin(b*x + a)) - 4*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a
)) + 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) + 4*(I*b*d^2*x + I*b*c*
d)*dilog(cos(b*x + a) + I*sin(b*x + a)) + 4*(-I*b*d^2*x - I*b*c*d)*dilog(c
os(b*x + a) - I*sin(b*x + a)) + 4*(-I*b*d^2*x - I*b*c*d)*dilog(-cos(b*x +
a) + I*sin(b*x + a)) + 4*(I*b*d^2*x + I*b*c*d)*dilog(-cos(b*x + a) - I*sin
(b*x + a)) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) + I*
sin(b*x + a) + 1) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x +
a) - I*sin(b*x + a) + 1) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(
b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*l
og(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 2*(b^2*d^2*x^2 + 2*b^2*
c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(
b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin
(b*x + a) + 1))/b^3
```

Sympy [F]

$$\int (c+dx)^2 \cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)^2 \cos^2(a+bx)\cot(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*cos(b*x+a)**2*cot(b*x+a),x)
```

output
```
Integral((c + d*x)**2*cos(a + b*x)**2*cot(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(149) = 298$.

Time $= 0.14$ (sec) , antiderivative size $= 529$, normalized size of antiderivative $= 3.08$

$$\int (c+dx)^2 \cos^2(a+bx)\cot(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

output
```
-1/24*(12*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*c^2 - 24*(sin(b*x + a)^2
- log(sin(b*x + a)^2))*a*c*d/b + 12*(sin(b*x + a)^2 - log(sin(b*x + a)^2))
*a^2*d^2/b^2 - (-8*I*(b*x + a)^3*d^2 - 24*(I*b*c*d - I*a*d^2)*(b*x + a)^2
+ 48*d^2*polylog(3, -e^(I*b*x + I*a)) + 48*d^2*polylog(3, e^(I*b*x + I*a))
 - 24*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan2(sin(
b*x + a), cos(b*x + a) + 1) - 24*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2
)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 3*(2*(b*x + a)^2*d
^2 + 4*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(2*b*x + 2*a) - 48*(I*b*c*d + I
*(b*x + a)*d^2 - I*a*d^2)*dilog(-e^(I*b*x + I*a)) - 48*(I*b*c*d + I*(b*x +
 a)*d^2 - I*a*d^2)*dilog(e^(I*b*x + I*a)) + 12*((b*x + a)^2*d^2 + 2*(b*c*d
 - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a)
+ 1) + 12*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 6*(b*c*d + (b*x + a)*d^2 - a*d
^2)*sin(2*b*x + 2*a))/b^2)/b
```

Giac [F]

$$\int (c+dx)^2 \cos^2(a+bx)\cot(a+bx)\,dx = \int (dx+c)^2 \cos\left(bx+a\right)^2 \cot\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*cos(b*x + a)^2*cot(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \cos^2(a+bx)\cot(a+bx)\,dx = \int \cos\left(a+b\,x\right)^2 \cot(a+b\,x)\,\left(c+d\,x\right)^2\,dx$$

input
```
int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^2,x)
```

output
```
int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \cos^2(a+bx)\cot(a+bx)\,dx$$
$$= \frac{2\left(\int \cos(bx+a)^2 \cot(bx+a)\, x^2 dx\right) b\, d^2 + 4\left(\int \cos(bx+a)^2 \cot(bx+a)\, x dx\right) bcd - 2\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right.}{2b}$$

input
```
int((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x)
```

output
```
(2*int(cos(a + b*x)**2*cot(a + b*x)*x**2,x)*b*d**2 + 4*int(cos(a + b*x)**2
*cot(a + b*x)*x,x)*b*c*d - 2*log(tan((a + b*x)/2)**2 + 1)*c**2 + 2*log(tan
((a + b*x)/2))*c**2 - sin(a + b*x)**2*c**2)/(2*b)
```

3.167 $\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx$

Optimal result . . . 1520
Mathematica [A] (verified) . . . 1521
Rubi [A] (verified) . . . 1521
Maple [B] (verified) . . . 1525
Fricas [B] (verification not implemented) . . . 1525
Sympy [F] . . . 1526
Maxima [B] (verification not implemented) . . . 1526
Giac [F] . . . 1527
Mupad [F(-1)] . . . 1527
Reduce [F] . . . 1528

Optimal result

Integrand size = 20, antiderivative size = 114

$$\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx = \frac{dx}{4b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id\,\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2} - \frac{d\cos(a+bx)\sin(a+bx)}{4b^2} - \frac{(c+dx)\sin^2(a+bx)}{2b}$$

output

```
1/4*d*x/b-1/2*I*(d*x+c)^2/d+(d*x+c)*ln(1-exp(2*I*(b*x+a)))/b-1/2*I*d*polyl
og(2,exp(2*I*(b*x+a)))/b^2-1/4*d*cos(b*x+a)*sin(b*x+a)/b^2-1/2*(d*x+c)*sin
(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.15

$$\begin{aligned}
&\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx \\
&= \frac{dx\cos(2(a+bx))}{4b} + \frac{c\log(\sin(a+bx))}{b} - \frac{ad\log(\sin(a+bx))}{b^2} \\
&\quad + \frac{d\left((a+bx)\log\left(1-e^{2i(a+bx)}\right) - \frac{1}{2}i\left((a+bx)^2 + \text{PolyLog}\left(2, e^{2i(a+bx)}\right)\right)\right)}{b^2} \\
&\quad - \frac{c\sin^2(a+bx)}{2b} - \frac{d\sin(2(a+bx))}{8b^2}
\end{aligned}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]^2*Cot[a + b*x],x]
```

output
```
(d*x*Cos[2*(a + b*x)])/(4*b) + (c*Log[Sin[a + b*x]])/b - (a*d*Log[Sin[a +
b*x]])/b^2 + (d*((a + b*x)*Log[1 - E^((2*I)*(a + b*x))] - (I/2)*((a + b*x)
^2 + PolyLog[2, E^((2*I)*(a + b*x))])))/b^2 - (c*Sin[a + b*x]^2)/(2*b) - (
d*Sin[2*(a + b*x)])/(8*b^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {4908, 3042, 25, 4202, 2620, 2715, 2838, 4904, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx$$

$$\downarrow 4908$$

$$\int (c+dx)\cot(a+bx)dx - \int (c+dx)\cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 3042$$

$$\int -\left((c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)\right)dx - \int (c+dx)\cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 25$$

$$-\int (c+dx)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx-\int (c+dx)\cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 4202$$

$$2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)}{1+e^{i(2a+2bx+\pi)}}dx-\int (c+dx)\cos(a+bx)\sin(a+bx)dx-\frac{i(c+dx)^2}{2d}$$

$$\downarrow 2620$$

$$2i\left(\frac{id\int \log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\int (c+dx)\cos(a+bx)\sin(a+bx)dx-\frac{i(c+dx)^2}{2d}$$

$$\downarrow 2715$$

$$2i\left(\frac{d\int e^{-i(2a+2bx+\pi)}\log\left(1+e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\int (c+dx)\cos(a+bx)\sin(a+bx)dx-\frac{i(c+dx)^2}{2d}$$

$$\downarrow 2838$$

$$-\int (c+dx)\cos(a+bx)\sin(a+bx)dx+2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{i(c+dx)^2}{2d}$$

$$\downarrow 4904$$

$$\frac{d\int \sin^2(a+bx)dx}{2b}+2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{(c+dx)\sin^2(a+bx)}{2b}-\frac{i(c+dx)^2}{2d}$$

$$\downarrow 3042$$

$$\frac{d\int \sin(a+bx)^2dx}{2b}+2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{(c+dx)\sin^2(a+bx)}{2b}-\frac{i(c+dx)^2}{2d}$$

$$\downarrow 3115$$

$$\frac{d\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b}+$$
$$2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-$$
$$\frac{(c+dx)\sin^2(a+bx)}{2b}-\frac{i(c+dx)^2}{2d}$$

$$\downarrow 24$$

$$2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-$$
$$\frac{(c+dx)\sin^2(a+bx)}{2b}+\frac{d\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b}-\frac{i(c+dx)^2}{2d}$$

input
```
Int[(c + d*x)*Cos[a + b*x]^2*Cot[a + b*x],x]
```

output
```
((-1/2*I)*(c + d*x)^2)/d + (2*I)*(((-1/2*I)*(c + d*x)*Log[1 + E^(I*(2*a +
Pi + 2*b*x))])/b - (d*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)) - ((
c + d*x)*Sin[a + b*x]^2)/(2*b) + (d*(x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*
b)))/(2*b)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(98) = 196$.

Time = 0.99 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.18

method	result
risch	$-\frac{id x^2}{2} + ixc - \frac{2c\ln\left(e^{i(bx+a)}\right)}{b} + \frac{c\ln\left(e^{i(bx+a)}-1\right)}{b} + \frac{c\ln\left(e^{i(bx+a)}+1\right)}{b} - \frac{id\,\mathrm{polylog}\left(2,-e^{i(bx+a)}\right)}{b^2} - \frac{2idax}{b} - \frac{id\,\mathrm{polyl}$

input
```
int((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*d*x^2+I*x*c-2/b*c*ln(exp(I*(b*x+a)))+1/b*c*ln(exp(I*(b*x+a))-1)+1/b
*c*ln(exp(I*(b*x+a))+1)-I*d*polylog(2,-exp(I*(b*x+a)))/b^2-2*I/b*d*a*x-I*d
*polylog(2,exp(I*(b*x+a)))/b^2+1/b*d*ln(exp(I*(b*x+a))+1)*x-I/b^2*d*a^2+1/
b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(b*x+a)))*a+2/b^2*d*a*ln(exp
(I*(b*x+a)))-1/b^2*d*a*ln(exp(I*(b*x+a))-1)+1/4*(d*x+c)*cos(2*b*x+2*a)/b-1
/8*d*sin(2*b*x+2*a)/b^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(95) = 190$.

Time = 0.10 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.56

$$\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx =$$

$$-\frac{bdx - 2\,(bdx+bc)\cos\left(bx+a\right)^2 + d\cos\left(bx+a\right)\sin\left(bx+a\right) + 2i\,d\mathrm{Li}_2(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)) -}{}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a)
 + 2*I*d*dilog(cos(b*x + a) + I*sin(b*x + a)) - 2*I*d*dilog(cos(b*x + a) -
 I*sin(b*x + a)) - 2*I*d*dilog(-cos(b*x + a) + I*sin(b*x + a)) + 2*I*d*dil
og(-cos(b*x + a) - I*sin(b*x + a)) - 2*(b*d*x + b*c)*log(cos(b*x + a) + I*
sin(b*x + a) + 1) - 2*(b*d*x + b*c)*log(cos(b*x + a) - I*sin(b*x + a) + 1)
 - 2*(b*c - a*d)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*(b*
c - a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 2*(b*d*x + a*
d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b*d*x + a*d)*log(-cos(b*x
+ a) - I*sin(b*x + a) + 1))/b^2
```

Sympy [F]

$$\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx$$

input
```
integrate((d*x+c)*cos(b*x+a)**2*cot(b*x+a),x)
```

output
```
Integral((c + d*x)*cos(a + b*x)**2*cot(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(95) = 190$.

Time = 0.14 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.96

$$\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx$$
$$= \frac{-4i\,b^2dx^2 - 8i\,b^2cx - 8i\,bdx\arctan(\sin(bx+a), -\cos(bx+a)+1) + 8i\,bc\arctan(\sin(bx+a), \cos(b$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

output
```
1/8*(-4*I*b^2*d*x^2 - 8*I*b^2*c*x - 8*I*b*d*x*arctan2(sin(b*x + a), -cos(b
*x + a) + 1) + 8*I*b*c*arctan2(sin(b*x + a), cos(b*x + a) - 1) - 8*(-I*b*d
*x - I*b*c)*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 2*(b*d*x + b*c)*cos(
2*b*x + 2*a) - 8*I*d*dilog(-e^(I*b*x + I*a)) - 8*I*d*dilog(e^(I*b*x + I*a)
) + 4*(b*d*x + b*c)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) +
 1) + 4*(b*d*x + b*c)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a)
 + 1) - d*sin(2*b*x + 2*a))/b^2
```

Giac [F]

$$\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx = \int (dx+c)\cos\left(bx+a\right)^2\cot\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*cos(b*x + a)^2*cot(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx = \int \cos\left(a+b\,x\right)^2\cot(a+b\,x)\,\left(c+d\,x\right)\,dx$$

input
```
int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x),x)
```

output
```
int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx$$
$$= \frac{2\left(\int \cos(bx+a)^2\cot(bx+a)\,x dx\right)bd - 2\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right)c + 2\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)c - \sin(bx+a$$
$$2b$$

input
```
int((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x)
```

output
```
(2*int(cos(a + b*x)**2*cot(a + b*x)*x,x)*b*d - 2*log(tan((a + b*x)/2)**2 +
 1)*c + 2*log(tan((a + b*x)/2))*c - sin(a + b*x)**2*c)/(2*b)
```

3.168 $\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$

Optimal result 1529
Mathematica [N/A] 1529
Rubi [N/A] 1530
Maple [N/A] 1533
Fricas [N/A] 1533
Sympy [N/A] 1534
Maxima [N/A] 1534
Giac [N/A] 1535
Mupad [N/A] 1535
Reduce [N/A] 1535

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx = -\frac{\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d} - \frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d} + \text{Int}\left(\frac{\cot(a+bx)}{c+dx},x\right)$$

output

```
-1/2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d-1/2*cos(2*a-2*b*c/d)*Si(2*b*c/d+
2*b*x)/d+Defer(Int)(cot(b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$$

input

```
Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x),x]
```

output
```
Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4908, 3042, 25, 4222, 4906, 27, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$$

$$\downarrow 4908$$

$$\int \frac{\cot(a+bx)}{c+dx}dx - \int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}dx$$

$$\downarrow 3042$$

$$\int -\frac{\tan\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx - \int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}dx$$

$$\downarrow 25$$

$$-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}dx - \int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}dx$$

$$\downarrow 4222$$

$$-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}dx - \int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}dx$$

$$\downarrow 4906$$

$$-\int \frac{\sin(2a+2bx)}{2(c+dx)}dx - \int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}dx$$

$$\downarrow 27$$

$$-\frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}dx - \int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}dx$$

$$\downarrow 3042$$

$$-\frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}dx - \int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}dx$$

$$\downarrow 3784$$

$$\frac{1}{2}\left(-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx - \cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right) - \int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}dx$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx - \cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right) - \int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}dx$$

$$\downarrow 3780$$

$$\frac{1}{2}\left(-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx - \frac{\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right) - \int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}dx$$

$$\downarrow 3783$$

$$\frac{1}{2}\left(-\frac{\sin\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d} - \frac{\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right) - \int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}dx$$

input
```
Int[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Maple [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cos(bx+a)^2 \cot(bx+a)}{dx+c} dx$$

input
```
int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x)
```

output
```
int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx} dx = \int \frac{\cos(bx+a)^2 \cot(bx+a)}{dx+c} dx$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^2*cot(b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)**2*cot(b*x+a)/(d*x+c),x)
```

output
```
Integral(cos(a + b*x)**2*cot(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 231, normalized size of antiderivative = 10.50

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)^2\cot(bx+a)}{dx+c}\,dx$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
-1/4*((I*exp_integral_e(1, 2*(-I*b*d*x - I*b*c)/d) - I*exp_integral_e(1, -
2*(-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 4*d*integrate(sin(b*x + a
)/((d*x + c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)
*cos(b*x + a) + c), x) - 4*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a
)^2 + (d*x + c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) -
 (exp_integral_e(1, 2*(-I*b*d*x - I*b*c)/d) + exp_integral_e(1, -2*(-I*b*d
*x - I*b*c)/d))*sin(-2*(b*c - a*d)/d))/d
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)^2\cot(bx+a)}{dx+c}\,dx$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2*cot(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 18.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+b\,x)^2\cot(a+b\,x)}{c+d\,x}\,dx$$

input
```
int((cos(a + b*x)^2*cot(a + b*x))/(c + d*x),x)
```

output
```
int((cos(a + b*x)^2*cot(a + b*x))/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)^2\cot(bx+a)}{dx+c}\,dx$$

input
```
int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x)
```

output
```
int((cos(a + b*x)**2*cot(a + b*x))/(c + d*x),x)
```

3.169 $\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$

Optimal result 1537
Mathematica [N/A] 1537
Rubi [N/A] 1538
Maple [N/A] 1541
Fricas [N/A] 1542
Sympy [N/A] 1542
Maxima [N/A] 1543
Giac [N/A] 1543
Mupad [N/A] 1544
Reduce [N/A] 1544

Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx &= -\frac{b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}\\ &\quad+\frac{\sin(2a+2bx)}{2d(c+dx)}+\frac{b\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}\\ &\quad+\operatorname{Int}\left(\frac{\cot(a+bx)}{(c+dx)^2},x\right)\end{aligned}$$

output
```
-b*cos(2*a-2*b*c/d)*Ci(2*b*c/d+2*b*x)/d^2+1/2*sin(2*b*x+2*a)/d/(d*x+c)+b*s
in(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^2+Defer(Int)(cot(b*x+a)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.80 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4908, 3042, 25, 4222, 4906, 27, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

$$\downarrow 4908$$

$$\int \frac{\cot(a+bx)}{(c+dx)^2}dx - \int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}dx$$

$$\downarrow 3042$$

$$\int -\frac{\tan\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx - \int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}dx$$

$$\downarrow 25$$

$$-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx - \int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}dx$$

$$\downarrow 4222$$

$$-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx - \int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}dx$$

$$\downarrow 4906$$

$$-\int \frac{\sin(2a+2bx)}{2(c+dx)^2}dx - \int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$

$$\downarrow 27$$

$$-\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}dx - \int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$

$$\downarrow 3042$$

$$-\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}dx - \int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$

$$\downarrow 3778$$

$$\frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)} - \frac{2b\int \frac{\cos(2a+2bx)}{c+dx}dx}{d}\right) - \int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)} - \frac{2b\int \frac{\sin\left(2a+2bx+\frac{\pi}{2}\right)}{c+dx}dx}{d}\right) - \int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$

$$\downarrow 3784$$

$$\frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)} - \frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx - \sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}\right) -$$

$$\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)} - \frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx - \sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}\right) -$$

$$\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$

$$\downarrow 3780$$

$$\frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)} - \frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx - \frac{\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}\right) -$$

$$\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$

$$\downarrow 3783$$

$$\frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)}-\frac{2b\left(\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}\right)-\int\frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$

input
```
Int[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Maple [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cos(bx+a)^2 \cot(bx+a)}{(dx+c)^2} dx$$

input
```
int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x)
```

output
```
int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(bx+a\right)^2\cot\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^2*cot(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos^2\left(a+bx\right)\cot\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(cos(b*x+a)**2*cot(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(cos(a + b*x)**2*cot(a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 343, normalized size of antiderivative = 15.59

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(bx+a)^2\cot(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/4*((I*exp_integral_e(2, 2*(-I*b*d*x - I*b*c)/d) - I*exp_integral_e(2, -
2*(-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 4*(d^2*x + c*d)*integrate
(sin(b*x + a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^
2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(b*x + a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x
+ c^2)*cos(b*x + a)), x) - 4*(d^2*x + c*d)*integrate(sin(b*x + a)/(d^2*x^2
 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x
 + c^2)*sin(b*x + a)^2 + c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)),
x) - (exp_integral_e(2, 2*(-I*b*d*x - I*b*c)/d) + exp_integral_e(2, -2*(-I
*b*d*x - I*b*c)/d))*sin(-2*(b*c - a*d)/d))/(d^2*x + c*d)
```

Giac [N/A]

Not integrable

Time = 2.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(bx+a)^2\cot(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2*cot(b*x + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 19.89 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(a+b\,x)^2\cot(a+b\,x)}{(c+d\,x)^2}\,dx$$

input
```
int((cos(a + b*x)^2*cot(a + b*x))/(c + d*x)^2,x)
```

output
```
int((cos(a + b*x)^2*cot(a + b*x))/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(bx+a)^2\cot(bx+a)}{d^2x^2+2cdx+c^2}dx$$

input
```
int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x)
```

output
```
int((cos(a + b*x)**2*cot(a + b*x))/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.170 $\int (c+dx)^m \cos(a+bx) \cot^2(a+bx)\, dx$

Optimal result 1545
Mathematica [N/A] 1546
Rubi [N/A] 1546
Maple [N/A] 1548
Fricas [N/A] 1549
Sympy [N/A] 1549
Maxima [N/A] 1549
Giac [N/A] 1550
Mupad [N/A] 1550
Reduce [N/A] 1551

Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}
&\int (c+dx)^m \cos(a+bx) \cot^2(a+bx)\, dx \\
&= \frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{ib(c+dx)}{d}\right)}{2b} \\
&\quad - \frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, \frac{ib(c+dx)}{d}\right)}{2b} \\
&\quad + \text{Int}((c+dx)^m \cot(a+bx) \csc(a+bx), x)
\end{aligned}$$

output

```
1/2*I*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c
)/d)^m)-1/2*I*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*
(d*x+c)/d)^m)+Defer(Int)((d*x+c)^m*cot(b*x+a)*csc(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 8.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx = \int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4908, 3042, 3788, 26, 2612, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cot^2(a+bx)(c+dx)^m\,dx$$

$$\downarrow 4908$$

$$\int (c+dx)^m \cot(a+bx)\csc(a+bx)dx - \int (c+dx)^m \cos(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)^m \cot(a+bx)\csc(a+bx)dx - \int (c+dx)^m \sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\downarrow 3788$$

$$-\frac{1}{2}i\int -ie^{-i(a+bx)}(c+dx)^m dx + \frac{1}{2}i\int ie^{i(a+bx)}(c+dx)^m dx + \int (c+dx)^m \cot(a+bx)\csc(a+bx)dx$$

$$\downarrow 26$$

$$-\frac{1}{2}\int e^{-i(a+bx)}(c+dx)^m dx - \frac{1}{2}\int e^{i(a+bx)}(c+dx)^m dx + \int (c+dx)^m \cot(a+bx)\csc(a+bx)dx$$

$$\downarrow 2612$$

$$\int (c+dx)^m \cot(a+bx)\csc(a+bx)dx + \frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b} - \frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}$$

$$\downarrow 7299$$

$$\int (c+dx)^m \cot(a+bx)\csc(a+bx)dx + \frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b} - \frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \cos(bx+a) \cot(bx+a)^2 \, dx$$

input
```
int((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cos(a+bx) \cot^2(a+bx)\, dx = \int (dx+c)^m \cos(bx+a) \cot(bx+a)^2\, dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*cos(b*x + a)*cot(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 20.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \cos(a+bx) \cot^2(a+bx)\, dx = \int (c+dx)^m \cos(a+bx) \cot^2(a+bx)\, dx$$

input
```
integrate((d*x+c)**m*cos(b*x+a)*cot(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*cos(a + b*x)*cot(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cos(a+bx) \cot^2(a+bx)\, dx = \int (dx+c)^m \cos(bx+a) \cot(bx+a)^2\, dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cos(a+bx) \cot^2(a+bx)\,dx = \int (dx+c)^m \cos(bx+a) \cot(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 19.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cos(a+bx) \cot^2(a+bx)\,dx = \int \cos(a+b\,x)\, \cot(a+b\,x)^2\, (c+d\,x)^m\,dx$$

input
```
int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cos(a+bx) \cot^2(a+bx)\, dx = \int (dx+c)^m \cos(bx+a) \cot(bx+a)^2\, dx$$

input
```
int((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x)
```

output
```
int((c + d*x)**m*cos(a + b*x)*cot(a + b*x)**2,x)
```

3.171 $\int (c+dx)^4 \cos(a+bx) \cot^2(a+bx)\, dx$

Optimal result . . . 1552
Mathematica [B] (verified) . . . 1553
Rubi [A] (verified) . . . 1554
Maple [B] (verified) . . . 1561
Fricas [B] (verification not implemented) . . . 1562
Sympy [F] . . . 1563
Maxima [B] (verification not implemented) . . . 1563
Giac [F] . . . 1564
Mupad [F(-1)] . . . 1565
Reduce [F] . . . 1565

Optimal result

Integrand size = 22, antiderivative size = 299

$$
\begin{aligned}
&\int (c+dx)^4 \cos(a+bx) \cot^2(a+bx)\, dx \\
&= -\frac{8d(c+dx)^3 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^3(c+dx)\cos(a+bx)}{b^4} \\
&\quad - \frac{4d(c+dx)^3\cos(a+bx)}{b^2} - \frac{(c+dx)^4 \csc(a+bx)}{b} \\
&\quad + \frac{12id^2(c+dx)^2 \operatorname{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} - \frac{12id^2(c+dx)^2 \operatorname{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3} \\
&\quad - \frac{24d^3(c+dx)\operatorname{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^4} + \frac{24d^3(c+dx)\operatorname{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^4} \\
&\quad - \frac{24id^4 \operatorname{PolyLog}\left(4, -e^{i(a+bx)}\right)}{b^5} + \frac{24id^4 \operatorname{PolyLog}\left(4, e^{i(a+bx)}\right)}{b^5} \\
&\quad - \frac{24d^4 \sin(a+bx)}{b^5} + \frac{12d^2(c+dx)^2\sin(a+bx)}{b^3} - \frac{(c+dx)^4 \sin(a+bx)}{b}
\end{aligned}
$$

output

```
-8*d*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b^2+24*d^3*(d*x+c)*cos(b*x+a)/b^4-4
*d*(d*x+c)^3*cos(b*x+a)/b^2-(d*x+c)^4*csc(b*x+a)/b+12*I*d^2*(d*x+c)^2*poly
log(2,-exp(I*(b*x+a)))/b^3-12*I*d^2*(d*x+c)^2*polylog(2,exp(I*(b*x+a)))/b^
3-24*d^3*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/b^4+24*d^3*(d*x+c)*polylog(3,e
xp(I*(b*x+a)))/b^4-24*I*d^4*polylog(4,-exp(I*(b*x+a)))/b^5+24*I*d^4*polylo
g(4,exp(I*(b*x+a)))/b^5-24*d^4*sin(b*x+a)/b^5+12*d^2*(d*x+c)^2*sin(b*x+a)/
b^3-(d*x+c)^4*sin(b*x+a)/b
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 798 vs. $2(299) = 598$.

Time = 1.22 (sec) , antiderivative size = 798, normalized size of antiderivative = 2.67

$$\int (c+dx)^4 \cos(a+bx) \cot^2(a+bx)\, dx$$
$$= \frac{\csc(a+bx)\left(-3b^4c^4 + 12b^2c^2d^2 - 24d^4 - 12b^4c^3dx + 24b^2cd^3x - 18b^4c^2d^2x^2 + 12b^2d^4x^2 - 12b^4cd^3x^3 - 3\right.}{}$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

output
```
(Csc[a + b*x]*(-3*b^4*c^4 + 12*b^2*c^2*d^2 - 24*d^4 - 12*b^4*c^3*d*x + 24*
b^2*c*d^3*x - 18*b^4*c^2*d^2*x^2 + 12*b^2*d^4*x^2 - 12*b^4*c*d^3*x^3 - 3*b
^4*d^4*x^4 + b^4*c^4*Cos[2*(a + b*x)] - 12*b^2*c^2*d^2*Cos[2*(a + b*x)] +
24*d^4*Cos[2*(a + b*x)] + 4*b^4*c^3*d*x*Cos[2*(a + b*x)] - 24*b^2*c*d^3*x*
Cos[2*(a + b*x)] + 6*b^4*c^2*d^2*x^2*Cos[2*(a + b*x)] - 12*b^2*d^4*x^2*Cos
[2*(a + b*x)] + 4*b^4*c*d^3*x^3*Cos[2*(a + b*x)] + b^4*d^4*x^4*Cos[2*(a +
b*x)] - 16*b^3*c^3*d*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] -
 48*b^3*c^2*d^2*x*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - 48
*b^3*c*d^3*x^2*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - 16*b^
3*d^4*x^3*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] + (24*I)*b^2
*d^2*(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]]*Sin[a + b*x] -
 (24*I)*b^2*d^2*(c + d*x)^2*PolyLog[2, Cos[a + b*x] + I*Sin[a + b*x]]*Sin[
a + b*x] - 48*b*c*d^3*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]]*Sin[a + b
*x] - 48*b*d^4*x*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]]*Sin[a + b*x] +
 48*b*c*d^3*PolyLog[3, Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] + 48*b*
d^4*x*PolyLog[3, Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - (48*I)*d^4*
PolyLog[4, -Cos[a + b*x] - I*Sin[a + b*x]]*Sin[a + b*x] + (48*I)*d^4*PolyL
og[4, Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - 4*b^3*c^3*d*Sin[2*(a +
 b*x)] + 24*b*c*d^3*Sin[2*(a + b*x)] - 12*b^3*c^2*d^2*x*Sin[2*(a + b*x)] +
 24*b*d^4*x*Sin[2*(a + b*x)] - 12*b^3*c*d^3*x^2*Sin[2*(a + b*x)] - 4*b^...
```

Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.10, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {4908, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117, 4910, 3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \cos(a+bx) \cot^2(a+bx)\, dx$$

$$\downarrow 4908$$

$$\int (c+dx)^4 \cot(a+bx) \csc(a+bx) dx - \int (c+dx)^4 \cos(a+bx) dx$$

$$\downarrow 3042$$

$$\int (c+dx)^4 \cot(a+bx) \csc(a+bx) dx - \int (c+dx)^4 \sin\left(a+bx+\frac{\pi}{2}\right) dx$$

$$\downarrow 3777$$

$$-\frac{4d\int -(c+dx)^3 \sin(a+bx) dx}{b} + \int (c+dx)^4 \cot(a+bx) \csc(a+bx) dx - \frac{(c+dx)^4 \sin(a+bx)}{b}$$

$$\downarrow 25$$

$$\frac{4d\int (c+dx)^3 \sin(a+bx) dx}{b} + \int (c+dx)^4 \cot(a+bx) \csc(a+bx) dx - \frac{(c+dx)^4 \sin(a+bx)}{b}$$

$$\downarrow 3042$$

$$\frac{4d\int (c+dx)^3 \sin(a+bx) dx}{b} + \int (c+dx)^4 \cot(a+bx) \csc(a+bx) dx - \frac{(c+dx)^4 \sin(a+bx)}{b}$$

$$\downarrow 3777$$

$$\frac{4d\left(\frac{3d\int (c+dx)^2 \cos(a+bx) dx}{b} - \frac{(c+dx)^3 \cos(a+bx)}{b}\right)}{b} + \int (c+dx)^4 \cot(a+bx) \csc(a+bx) dx - \frac{(c+dx)^4 \sin(a+bx)}{b}$$

$$\downarrow 3042$$

$$\frac{4d\left(\frac{3d\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}+\int (c+dx)^4\cot(a+bx)\csc(a+bx)dx-\frac{(c+dx)^4\sin(a+bx)}{b}$$

$$\downarrow 3777$$

$$\frac{4d\left(\frac{3d\left(\frac{2d\int -((c+dx)\sin(a+bx))dx}{b}+\frac{(c+dx)^2\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}+\int (c+dx)^4\cot(a+bx)\csc(a+bx)dx-\frac{(c+dx)^4\sin(a+bx)}{b}$$

$$\downarrow 25$$

$$\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int (c+dx)\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}+\int (c+dx)^4\cot(a+bx)\csc(a+bx)dx-\frac{(c+dx)^4\sin(a+bx)}{b}$$

$$\downarrow 3042$$

$$\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int (c+dx)\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}+\int (c+dx)^4\cot(a+bx)\csc(a+bx)dx-\frac{(c+dx)^4\sin(a+bx)}{b}$$

$$\downarrow 3777$$

$$\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int \cos(a+bx)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}+\int (c+dx)^4\cot(a+bx)\csc(a+bx)dx-\frac{(c+dx)^4\sin(a+bx)}{b}$$

$$\downarrow 3042$$

$$\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}+\int(c+dx)^4\cot(a+bx)\csc(a+bx)dx-\frac{(c+dx)^4\sin(a+bx)}{b}$$

$$\downarrow 3117$$

$$\int(c+dx)^4\cot(a+bx)\csc(a+bx)dx+\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^4\sin(a+bx)}{b}$$

$$\downarrow 4910$$

$$\frac{4d\int(c+dx)^3\csc(a+bx)dx}{b}+\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{(c+dx)^4\csc(a+bx)}{b}$$

$$\downarrow 3042$$

$$\frac{4d\int(c+dx)^3\csc(a+bx)dx}{b}+\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{(c+dx)^4\csc(a+bx)}{b}$$

$$\downarrow 4671$$

$$\frac{4d\left(-\frac{3d\int(c+dx)^2\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{3d\int(c+dx)^2\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)}{b}+$$

$$\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}-$$

$$\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{(c+dx)^4\csc(a+bx)}{b}$$

$$\downarrow 3011$$

$$\frac{4d\left(\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}\right)}{b}$$

$$\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}-$$

$$\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{(c+dx)^4\csc(a+bx)}{b}$$

$$\downarrow 7163$$

$$\frac{4d\left(\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}\right)}{b}$$

$$\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}-$$

$$\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{(c+dx)^4\csc(a+bx)}{b}$$

$$\downarrow 2720$$

$$4d\left(\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)dx}{b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2}{b}\right.}{b}$$

$$\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}-$$

$$\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{(c+dx)^4\csc(a+bx)}{b}$$

$$\downarrow 7143$$

$$4d\left(-\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\,\mathrm{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\right.$$

$$\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}-$$

$$\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{(c+dx)^4\csc(a+bx)}{b}$$

input
```
Int[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

output
```
-(((c + d*x)^4*Csc[a + b*x])/b) + (4*d*((-2*(c + d*x)^3*ArcTanh[E^(I*(a +
b*x))])/b + (3*d*((I*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b - ((2*I)*
d*(((-I)*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b + (d*PolyLog[4, -E^(I*(
a + b*x))])/b^2))/b))/b - (3*d*((I*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))]
)/b - ((2*I)*d*(((-I)*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b + (d*PolyLo
g[4, E^(I*(a + b*x))])/b^2))/b))/b))/b - ((c + d*x)^4*Sin[a + b*x])/b + (4
*d*(-(((c + d*x)^3*Cos[a + b*x])/b) + (3*d*(((c + d*x)^2*Sin[a + b*x])/b -
 (2*d*(-(((c + d*x)*Cos[a + b*x])/b) + (d*Sin[a + b*x])/b^2))/b))/b))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)  Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4671

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)  Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)  Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4908

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 4910

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))  Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))  Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1055 vs. $2(281) = 562$.

Time = 2.18 (sec) , antiderivative size = 1056, normalized size of antiderivative = 3.53

method	result	size
risch	Expression too large to display	1056

input
```
int((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(d^4*x^4*b^4+4*b^4*c*d^3*x^3+6*b^4*c^2*d^2*x^2+4*b^4*c^3*d*x-4*I*b^
3*d^4*x^3+b^4*c^4-12*b^2*d^4*x^2-12*I*b^3*c*d^3*x^2-24*b^2*c*d^3*x-12*I*b^
3*c^2*d^2*x-12*b^2*c^2*d^2-4*I*b^3*c^3*d+24*I*b*d^4*x+24*d^4+24*I*b*c*d^3)
/b^5*exp(-I*(b*x+a))+1/2*I*(d^4*x^4*b^4+4*b^4*c*d^3*x^3+6*b^4*c^2*d^2*x^2+
4*b^4*c^3*d*x+4*I*b^3*d^4*x^3+b^4*c^4-12*b^2*d^4*x^2+12*I*b^3*c*d^3*x^2-24
*b^2*c*d^3*x+12*I*b^3*c^2*d^2*x-12*b^2*c^2*d^2+4*I*b^3*c^3*d-24*I*b*d^4*x+
24*d^4-24*I*b*c*d^3)/b^5*exp(I*(b*x+a))+24*I*d^3/b^3*c*polylog(2,-exp(I*(b
*x+a)))*x-24*I*d^3/b^3*c*polylog(2,exp(I*(b*x+a)))*x-2*I*(d^4*x^4+4*c*d^3*
x^3+6*c^2*d^2*x^2+4*c^3*d*x+c^4)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)+24*
d^3/b^4*c*polylog(3,exp(I*(b*x+a)))+8*d^4/b^5*a^3*arctanh(exp(I*(b*x+a)))-
8*d/b^2*c^3*arctanh(exp(I*(b*x+a)))-24*d^4/b^4*polylog(3,-exp(I*(b*x+a)))*
x+24*d^4/b^4*polylog(3,exp(I*(b*x+a)))*x-4*d^4/b^5*ln(exp(I*(b*x+a))+1)*a^
3+4*d^4/b^5*ln(1-exp(I*(b*x+a)))*a^3-24*d^3/b^4*c*polylog(3,-exp(I*(b*x+a)
))-4*d^4/b^2*ln(exp(I*(b*x+a))+1)*x^3+4*d^4/b^2*ln(1-exp(I*(b*x+a)))*x^3+1
2*I*d^4/b^3*polylog(2,-exp(I*(b*x+a)))*x^2-12*I*d^4/b^3*polylog(2,exp(I*(b
*x+a)))*x^2+12*I*d^2/b^3*c^2*polylog(2,-exp(I*(b*x+a)))-12*I*d^2/b^3*c^2*p
olylog(2,exp(I*(b*x+a)))-12*d^3/b^2*c*ln(exp(I*(b*x+a))+1)*x^2+12*d^3/b^2*
c*ln(1-exp(I*(b*x+a)))*x^2-12*d^2/b^2*c^2*ln(exp(I*(b*x+a))+1)*x+12*d^2/b^
2*c^2*ln(1-exp(I*(b*x+a)))*x-24*d^3/b^4*c*a^2*arctanh(exp(I*(b*x+a)))+24*d
^2/b^3*c^2*a*arctanh(exp(I*(b*x+a)))-12*d^2/b^3*c^2*ln(exp(I*(b*x+a))+1...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1233 vs. $2(275) = 550$.

Time = 0.14 (sec) , antiderivative size = 1233, normalized size of antiderivative = 4.12

$$\int (c+dx)^4 \cos(a+bx) \cot^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 12*b^2*c^2*d^2 - 12*I*d^4*
polylog(4, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I*d^4*polylog(
4, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - 12*I*d^4*polylog(4, -cos(
b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I*d^4*polylog(4, -cos(b*x + a
) - I*sin(b*x + a))*sin(b*x + a) + 24*d^4 + 12*(b^4*c^2*d^2 - b^2*d^4)*x^2
 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 24*d^4 + 6*
(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a
)^2 + 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d - 6*b*c*d^3 + 3*(b^3*c^
2*d^2 - 2*b*d^4)*x)*cos(b*x + a)*sin(b*x + a) + 6*(I*b^2*d^4*x^2 + 2*I*b^2
*c*d^3*x + I*b^2*c^2*d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a
) + 6*(-I*b^2*d^4*x^2 - 2*I*b^2*c*d^3*x - I*b^2*c^2*d^2)*dilog(cos(b*x + a
) - I*sin(b*x + a))*sin(b*x + a) + 6*(I*b^2*d^4*x^2 + 2*I*b^2*c*d^3*x + I*
b^2*c^2*d^2)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*(-I*b^
2*d^4*x^2 - 2*I*b^2*c*d^3*x - I*b^2*c^2*d^2)*dilog(-cos(b*x + a) - I*sin(b
*x + a))*sin(b*x + a) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x
 + b^3*c^3*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 2*(b^3
*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*log(cos(b*x + a)
 - I*sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a
^2*b*c*d^3 - a^3*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*si
n(b*x + a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*...
```

Sympy [F]

$$\int (c+dx)^4 \cos(a+bx) \cot^2(a+bx)\, dx = \int (c+dx)^4 \cos(a+bx) \cot^2(a+bx)\, dx$$

input
```
integrate((d*x+c)**4*cos(b*x+a)*cot(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**4*cos(a + b*x)*cot(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 17589 vs. $2(275) = 550$.

Time = 4.43 (sec) , antiderivative size = 17589, normalized size of antiderivative = 58.83

$$\int (c+dx)^4 \cos(a+bx) \cot^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/2*(2*c^4*(1/sin(b*x + a) + sin(b*x + a)) - 8*a*c^3*d*(1/sin(b*x + a) +
sin(b*x + a))/b + 12*a^2*c^2*d^2*(1/sin(b*x + a) + sin(b*x + a))/b^2 - 8*a
^3*c*d^3*(1/sin(b*x + a) + sin(b*x + a))/b^3 + 2*a^4*d^4*(1/sin(b*x + a) +
 sin(b*x + a))/b^4 - 4*(((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1
)*cos(3*b*x + 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin(2*b*x +
 2*a))*sin(3*b*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^3 - 2*(4*(b*x + a)*co
s(b*x + a)*sin(2*b*x + 2*a) - (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*co
s(2*b*x + 2*a) + 3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a)
^2 - ((b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x
+ a)*cos(2*b*x + 2*a)*sin(b*x + a) + ((b*x + a)*sin(2*b*x + 2*a) - cos(2*b
*x + 2*a) + 1)*cos(3*b*x + 3*a) - 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x +
a))*sin(2*b*x + 2*a) - 8*(b*x + a)*sin(b*x + a))*sin(3*b*x + 3*a)^2 - ((b*
x + a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a)^2 + (12*(b*x + a)*cos
(b*x + a)*sin(b*x + a) - (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x
 + a)^2 + sin(b*x + a)^2 + 2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*a)^2 + cos(
b*x + a)^2 + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2)*sin(
2*b*x + 2*a) + sin(2*b*x + 2*a)^2 + sin(b*x + a)^2 + 1)*cos(3*b*x + 3*a) +
 2*(3*(b*x + a)*sin(b*x + a)^3 + (3*(b*x + a)*cos(b*x + a)^2 + b*x + a)*si
n(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a) - ((cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x +...
```

Giac [F]

$$\int (c+dx)^4 \cos(a+bx) \cot^2(a+bx)\,dx = \int (dx+c)^4 \cos\left(bx+a\right) \cot\left(bx+a\right)^2\,dx$$

input

```
integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

output

```
integrate((d*x + c)^4*cos(b*x + a)*cot(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \cos(a+bx) \cot^2(a+bx)\,dx = \int \cos(a+b\,x)\,\cot(a+b\,x)^2\,(c+d\,x)^4\,dx$$

input
```
int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^4,x)
```

output
```
int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^4, x)
```

Reduce [F]

$$\int (c+dx)^4 \cos(a+bx) \cot^2(a+bx)\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x)
```

output

```
( - 160*cos(a + b*x)*sin(a + b*x)*b**3*c**3*d - 540*cos(a + b*x)*sin(a + b
*x)*b**3*c**2*d**2*x - 540*cos(a + b*x)*sin(a + b*x)*b**3*c*d**3*x**2 - 18
0*cos(a + b*x)*sin(a + b*x)*b**3*d**4*x**3 + 1080*cos(a + b*x)*sin(a + b*x
)*b*c*d**3 + 1080*cos(a + b*x)*sin(a + b*x)*b*d**4*x + 120*cos(a + b*x)*b*
*4*c**2*d**2*x**2 + 80*cos(a + b*x)*b**4*c*d**3*x**3 + 20*cos(a + b*x)*b**
4*d**4*x**4 + 10*int(x**4/(tan((a + b*x)/2)**4 + tan((a + b*x)/2)**2),x)*s
in(a + b*x)*b**5*d**4 + 40*int(x**3/(tan((a + b*x)/2)**4 + tan((a + b*x)/2
)**2),x)*sin(a + b*x)*b**5*c*d**3 + 60*int(x**2/(tan((a + b*x)/2)**4 + tan
((a + b*x)/2)**2),x)*sin(a + b*x)*b**5*c**2*d**2 + 80*int((tan((a + b*x)/2
)**3*x**3)/(tan((a + b*x)/2)**2 + 1),x)*sin(a + b*x)*b**4*d**4 + 240*int((
tan((a + b*x)/2)**3*x**2)/(tan((a + b*x)/2)**2 + 1),x)*sin(a + b*x)*b**4*c
*d**3 + 240*int((tan((a + b*x)/2)**3*x)/(tan((a + b*x)/2)**2 + 1),x)*sin(a
 + b*x)*b**4*c**2*d**2 + 160*log(tan((a + b*x)/2))*sin(a + b*x)*b**3*c**3*
d - 40*sin(a + b*x)**2*b**4*c**4 - 160*sin(a + b*x)**2*b**4*c**3*d*x - 210
*sin(a + b*x)**2*b**4*c**2*d**2*x**2 - 140*sin(a + b*x)**2*b**4*c*d**3*x**
3 - 35*sin(a + b*x)**2*b**4*d**4*x**4 + 540*sin(a + b*x)**2*b**2*c**2*d**2
 + 1080*sin(a + b*x)**2*b**2*c*d**3*x + 540*sin(a + b*x)**2*b**2*d**4*x**2
 - 1080*sin(a + b*x)**2*d**4 + 30*sin(a + b*x)*b**5*c**2*d**2*x**3 + 15*si
n(a + b*x)*b**5*c*d**3*x**4 + 3*sin(a + b*x)*b**5*d**4*x**5 + 160*sin(a +
b*x)*b**3*c**3*d - 1080*sin(a + b*x)*b*c*d**3 - 40*b**4*c**4 - 160*b**4...
```

3.172 $\int (c+dx)^3 \cos(a+bx) \cot^2(a+bx)\, dx$

Optimal result . . . 1567
Mathematica [B] (verified) . . . 1568
Rubi [A] (verified) . . . 1568
Maple [B] (verified) . . . 1573
Fricas [B] (verification not implemented) . . . 1574
Sympy [F] . . . 1575
Maxima [B] (verification not implemented) . . . 1576
Giac [F] . . . 1577
Mupad [F(-1)] . . . 1577
Reduce [F] . . . 1577

Optimal result

Integrand size = 22, antiderivative size = 216

$$\begin{aligned}
&\int (c+dx)^3 \cos(a+bx) \cot^2(a+bx)\, dx \\
&= -\frac{6d(c+dx)^2 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6d^3 \cos(a+bx)}{b^4} - \frac{3d(c+dx)^2 \cos(a+bx)}{b^2} \\
&\quad - \frac{(c+dx)^3 \csc(a+bx)}{b} + \frac{6id^2(c+dx)\, \mathrm{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} \\
&\quad - \frac{6id^2(c+dx)\, \mathrm{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3} - \frac{6d^3\, \mathrm{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^4} \\
&\quad + \frac{6d^3\, \mathrm{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^4} + \frac{6d^2(c+dx)\sin(a+bx)}{b^3} - \frac{(c+dx)^3 \sin(a+bx)}{b}
\end{aligned}$$

output

```
-6*d*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b^2+6*d^3*cos(b*x+a)/b^4-3*d*(d*x+c
)^2*cos(b*x+a)/b^2-(d*x+c)^3*csc(b*x+a)/b+6*I*d^2*(d*x+c)*polylog(2,-exp(I
*(b*x+a)))/b^3-6*I*d^2*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^3-6*d^3*polylog
(3,-exp(I*(b*x+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4+6*d^2*(d*x+c)*
sin(b*x+a)/b^3-(d*x+c)^3*sin(b*x+a)/b
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 539 vs. $2(216) = 432$.

Time = 0.95 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.50

$$\int (c+dx)^3 \cos(a+bx) \cot^2(a+bx)\, dx$$
$$= \frac{\csc(a+bx)\left(-3b^3c^3 + 6bcd^2 - 9b^3c^2dx + 6bd^3x - 9b^3cd^2x^2 - 3b^3d^3x^3 + b^3c^3\cos(2(a+bx)) - 6bcd^2\cos(\right.}{}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

output
```
(Csc[a + b*x]*(-3*b^3*c^3 + 6*b*c*d^2 - 9*b^3*c^2*d*x + 6*b*d^3*x - 9*b^3*
c*d^2*x^2 - 3*b^3*d^3*x^3 + b^3*c^3*Cos[2*(a + b*x)] - 6*b*c*d^2*Cos[2*(a
+ b*x)] + 3*b^3*c^2*d*x*Cos[2*(a + b*x)] - 6*b*d^3*x*Cos[2*(a + b*x)] + 3*
b^3*c*d^2*x^2*Cos[2*(a + b*x)] + b^3*d^3*x^3*Cos[2*(a + b*x)] + 6*b^2*c^2*
d*Log[1 - E^(I*(a + b*x))]*Sin[a + b*x] + 12*b^2*c*d^2*x*Log[1 - E^(I*(a +
 b*x))]*Sin[a + b*x] + 6*b^2*d^3*x^2*Log[1 - E^(I*(a + b*x))]*Sin[a + b*x]
 - 6*b^2*c^2*d*Log[1 + E^(I*(a + b*x))]*Sin[a + b*x] - 12*b^2*c*d^2*x*Log[
1 + E^(I*(a + b*x))]*Sin[a + b*x] - 6*b^2*d^3*x^2*Log[1 + E^(I*(a + b*x))]
*Sin[a + b*x] + (12*I)*b*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))]*Sin[a
+ b*x] - (12*I)*b*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))]*Sin[a + b*x] -
 12*d^3*PolyLog[3, -E^(I*(a + b*x))]*Sin[a + b*x] + 12*d^3*PolyLog[3, E^(I
*(a + b*x))]*Sin[a + b*x] - 3*b^2*c^2*d*Sin[2*(a + b*x)] + 6*d^3*Sin[2*(a
+ b*x)] - 6*b^2*c*d^2*x*Sin[2*(a + b*x)] - 3*b^2*d^3*x^2*Sin[2*(a + b*x)])
)/(2*b^4)
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.07, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {4908, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118, 4910, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \cos(a+bx) \cot^2(a+bx)\, dx$$

$$\downarrow 4908$$

$$\int (c+dx)^3 \cot(a+bx) \csc(a+bx) dx - \int (c+dx)^3 \cos(a+bx) dx$$

$$\downarrow 3042$$

$$\int (c+dx)^3 \cot(a+bx) \csc(a+bx) dx - \int (c+dx)^3 \sin\left(a+bx+\frac{\pi}{2}\right) dx$$

$$\downarrow 3777$$

$$-\frac{3d\int -(c+dx)^2 \sin(a+bx) dx}{b} + \int (c+dx)^3 \cot(a+bx) \csc(a+bx) dx - \frac{(c+dx)^3 \sin(a+bx)}{b}$$

$$\downarrow 25$$

$$\frac{3d\int (c+dx)^2 \sin(a+bx) dx}{b} + \int (c+dx)^3 \cot(a+bx) \csc(a+bx) dx - \frac{(c+dx)^3 \sin(a+bx)}{b}$$

$$\downarrow 3042$$

$$\frac{3d\int (c+dx)^2 \sin(a+bx) dx}{b} + \int (c+dx)^3 \cot(a+bx) \csc(a+bx) dx - \frac{(c+dx)^3 \sin(a+bx)}{b}$$

$$\downarrow 3777$$

$$\frac{3d\left(\frac{2d\int (c+dx)\cos(a+bx) dx}{b} - \frac{(c+dx)^2 \cos(a+bx)}{b}\right)}{b} + \int (c+dx)^3 \cot(a+bx) \csc(a+bx) dx - \frac{(c+dx)^3 \sin(a+bx)}{b}$$

$$\downarrow 3042$$

$$\frac{3d\left(\frac{2d\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right) dx}{b} - \frac{(c+dx)^2 \cos(a+bx)}{b}\right)}{b} + \int (c+dx)^3 \cot(a+bx) \csc(a+bx) dx - \frac{(c+dx)^3 \sin(a+bx)}{b}$$

$$\downarrow 3777$$

$$\frac{3d\left(\frac{2d\left(\frac{d\int -\sin(a+bx) dx}{b} + \frac{(c+dx)\sin(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2 \cos(a+bx)}{b}\right)}{b} + \int (c+dx)^3 \cot(a+bx) \csc(a+bx) dx - \frac{(c+dx)^3 \sin(a+bx)}{b}$$

$$\downarrow 25$$

$$\frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}+\int(c+dx)^3\cot(a+bx)\csc(a+bx)dx-\frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\downarrow 3042$$

$$\frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}+\int(c+dx)^3\cot(a+bx)\csc(a+bx)dx-\frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\downarrow 3118$$

$$\int(c+dx)^3\cot(a+bx)\csc(a+bx)dx+\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\downarrow 4910$$

$$\frac{3d\int(c+dx)^2\csc(a+bx)dx}{b}+\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

$$\downarrow 3042$$

$$\frac{3d\int(c+dx)^2\csc(a+bx)dx}{b}+\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

$$\downarrow 4671$$

$$\frac{3d\left(-\frac{2d\int(c+dx)\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{2d\int(c+dx)\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)}{b}+\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

$$\downarrow 3011$$

$$3d\left(\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{id\int \text{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{id\int \text{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2(c}{b}\right)$$

$$3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)-\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

$$\downarrow 2720$$

$$3d\left(\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyL}}{b}\right.}{b}$$

$$3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)-\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

$$\downarrow 7143$$

$$3d\left(-\frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\,\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\,\text{P}}{b}\right.}{b}\right)$$

$$3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)-\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

output
```
-(((c + d*x)^3*Csc[a + b*x])/b) + (3*d*((-2*(c + d*x)^2*ArcTanh[E^(I*(a +
b*x))])/b + (2*d*((I*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b - (d*PolyLo
g[3, -E^(I*(a + b*x))])/b^2))/b - (2*d*((I*(c + d*x)*PolyLog[2, E^(I*(a +
b*x))])/b - (d*PolyLog[3, E^(I*(a + b*x))])/b^2))/b))/b - ((c + d*x)^3*Sin
[a + b*x])/b + (3*d*(-(((c + d*x)^2*Cos[a + b*x])/b) + (2*d*((d*Cos[a + b*
x])/b^2 + ((c + d*x)*Sin[a + b*x])/b))/b))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(204) = 408$.

Time = 1.14 (sec) , antiderivative size = 649, normalized size of antiderivative = 3.00

method	result
risch	$\frac{6id^2c\,\mathrm{polylog}(2,-e^{i(bx+a)})}{b^3} - \frac{2i(d^3x^3+3cd^2x^2+3c^2dx+c^3)e^{i(bx+a)}}{b(e^{2i(bx+a)}-1)} - \frac{6id^3\,\mathrm{polylog}(2,e^{i(bx+a)})x}{b^3} - \frac{6d^2c\ln(e^{i(bx+a)}+1)x}{b^2} +$

input
```
int((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
6*I*d^2/b^3*c*polylog(2,-exp(I*(b*x+a)))-2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*
x+c^3)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)-6*I*d^3/b^3*polylog(2,exp(I*(
b*x+a)))*x-6*d^2/b^2*c*ln(exp(I*(b*x+a))+1)*x+6*d^2/b^2*c*ln(1-exp(I*(b*x+
a)))*x+12*d^2/b^3*c*a*arctanh(exp(I*(b*x+a)))+3*d^3/b^4*ln(exp(I*(b*x+a))+
1)*a^2-3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a^2-6*I*d^2/b^3*c*polylog(2,exp(I*(b
*x+a)))+6*I*d^3/b^3*polylog(2,-exp(I*(b*x+a)))*x-6*d/b^2*c^2*arctanh(exp(I
*(b*x+a)))-6*d^3/b^4*a^2*arctanh(exp(I*(b*x+a)))-6*d^2/b^3*c*ln(exp(I*(b*x
+a))+1)*a+1/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2
*d^3*x^2-6*b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*ex
p(I*(b*x+a))+6*d^2/b^3*c*ln(1-exp(I*(b*x+a)))*a-1/2*I*(d^3*x^3*b^3+3*b^3*c
*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d^3*x-6*I*b^2*c*d^2*x-6
*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))+3*d^3/b^2*ln(1-exp(I*(
b*x+a)))*x^2+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4-3*d^3/b^2*ln(exp(I*(b*x+a
))+1)*x^2-6*d^3*polylog(3,-exp(I*(b*x+a)))/b^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 797 vs. $2(200) = 400$.

Time $= 0.13$ (sec) , antiderivative size $= 797$, normalized size of antiderivative $= 3.69$

$$\int (c + dx)^3 \cos(a + bx) \cot^2(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

output

```
-1/2*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 4*b^3*c^3 - 6*d^3*polylog(3, cos(
b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 6*d^3*polylog(3, cos(b*x + a) -
I*sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a) + I*sin(b*x
+ a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(
b*x + a) - 12*b*c*d^2 - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c
*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 6*(b^2*d^3*x^2 + 2*b^2*
c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a)*sin(b*x + a) + 6*(I*b*d^3*x + I*
b*c*d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*(-I*b*d^3*x
 - I*b*c*d^2)*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*(I*b*d
^3*x + I*b*c*d^2)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*(
-I*b*d^3*x - I*b*c*d^2)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a)
 + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) + I*sin(b*
x + a) + 1)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log
(cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x +
 a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*
sin(b*x + a) + 1/2)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*
c*d^2 - a^2*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - 3*
(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) -
I*sin(b*x + a) + 1)*sin(b*x + a) + 12*(b^3*c^2*d - b*d^3)*x)/(b^4*sin(b...
```

Sympy [F]

$$\int (c+dx)^3 \cos(a+bx) \cot^2(a+bx)\,dx = \int (c+dx)^3 \cos\left(a+bx\right) \cot^2\left(a+bx\right)\,dx$$

input

```
integrate((d*x+c)**3*cos(b*x+a)*cot(b*x+a)**2,x)
```

output

```
Integral((c + d*x)**3*cos(a + b*x)*cot(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 10994 vs. $2(200) = 400$.

Time = 1.10 (sec) , antiderivative size = 10994, normalized size of antiderivative = 50.90

$$\int (c+dx)^3 \cos(a+bx) \cot^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*c^3*(1/sin(b*x + a) + sin(b*x + a)) - 6*a*c^2*d*(1/sin(b*x + a) +
sin(b*x + a))/b + 6*a^2*c*d^2*(1/sin(b*x + a) + sin(b*x + a))/b^2 - 2*a^3*
d^3*(1/sin(b*x + a) + sin(b*x + a))/b^3 - 3*(((b*x + a)*sin(2*b*x + 2*a) -
 cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2
*a) + a - sin(2*b*x + 2*a))*sin(3*b*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^
3 - 2*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - (3*(b*x + a)*sin(b*x +
a) + cos(b*x + a))*cos(2*b*x + 2*a) + 3*(b*x + a)*sin(b*x + a) + cos(b*x +
 a))*cos(3*b*x + 3*a)^2 - ((b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*
x + 2*a)^2 + (8*(b*x + a)*cos(2*b*x + 2*a)*sin(b*x + a) + ((b*x + a)*sin(2
*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a) - 2*(3*(b*x + a)*cos(
b*x + a) - sin(b*x + a))*sin(2*b*x + 2*a) - 8*(b*x + a)*sin(b*x + a))*sin(
3*b*x + 3*a)^2 - ((b*x + a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a)^
2 + (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) - (12*(b*x + a)*cos(b*x + a)*s
in(b*x + a) + cos(b*x + a)^2 + sin(b*x + a)^2 + 2)*cos(2*b*x + 2*a) + cos(
2*b*x + 2*a)^2 + cos(b*x + a)^2 + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)
*sin(b*x + a)^2)*sin(2*b*x + 2*a) + sin(2*b*x + 2*a)^2 + sin(b*x + a)^2 +
1)*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*sin(b*x + a)^3 + (3*(b*x + a)*cos(b*x
 + a)^2 + b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a) - ((cos(2
*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3
*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b...
```

Giac [F]

$$\int (c+dx)^3 \cos(a+bx) \cot^2(a+bx)\,dx = \int (dx+c)^3 \cos(bx+a) \cot(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*cos(b*x + a)*cot(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \cos(a+bx) \cot^2(a+bx)\,dx = \int \cos(a+b\,x)\, \cot(a+b\,x)^2\, (c+d\,x)^3\,dx$$

input
```
int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^3,x)
```

output
```
int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \cos(a+bx) \cot^2(a+bx)\,dx$$

$$= \frac{48\cos(bx+a)\,b^3 c\,d^2x^2 - 96\cos(bx+a)\sin(bx+a)\,b^2c^2d - 108\cos(bx+a)\sin(bx+a)\,b^2d^3x^2 - 96\sin}{}$$

input
```
int((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x)
```

output

```
( - 96*cos(a + b*x)*sin(a + b*x)*b**2*c**2*d - 216*cos(a + b*x)*sin(a + b*
x)*b**2*c*d**2*x - 108*cos(a + b*x)*sin(a + b*x)*b**2*d**3*x**2 + 216*cos(
a + b*x)*sin(a + b*x)*d**3 + 48*cos(a + b*x)*b**3*c*d**2*x**2 + 16*cos(a +
 b*x)*b**3*d**3*x**3 + 8*int(x**3/(tan((a + b*x)/2)**4 + tan((a + b*x)/2)*
*2),x)*sin(a + b*x)*b**4*d**3 + 24*int(x**2/(tan((a + b*x)/2)**4 + tan((a
+ b*x)/2)**2),x)*sin(a + b*x)*b**4*c*d**2 + 48*int((tan((a + b*x)/2)**3*x*
*2)/(tan((a + b*x)/2)**2 + 1),x)*sin(a + b*x)*b**3*d**3 + 96*int((tan((a +
 b*x)/2)**3*x)/(tan((a + b*x)/2)**2 + 1),x)*sin(a + b*x)*b**3*c*d**2 + 96*
log(tan((a + b*x)/2))*sin(a + b*x)*b**2*c**2*d - 32*sin(a + b*x)**2*b**3*c
**3 - 96*sin(a + b*x)**2*b**3*c**2*d*x - 84*sin(a + b*x)**2*b**3*c*d**2*x*
*2 - 28*sin(a + b*x)**2*b**3*d**3*x**3 + 216*sin(a + b*x)**2*b*c*d**2 + 21
6*sin(a + b*x)**2*b*d**3*x + 12*sin(a + b*x)*b**4*c*d**2*x**3 + 3*sin(a +
b*x)*b**4*d**3*x**4 + 96*sin(a + b*x)*b**2*c**2*d - 216*sin(a + b*x)*d**3
- 32*b**3*c**3 - 96*b**3*c**2*d*x - 48*b**3*c*d**2*x**2 - 16*b**3*d**3*x**
3)/(32*sin(a + b*x)*b**4)
```

3.173 $\int (c+dx)^2 \cos(a+bx) \cot^2(a+bx)\, dx$

Optimal result . . . 1579
Mathematica [B] (warning: unable to verify) . . . 1580
Rubi [A] (verified) . . . 1580
Maple [B] (verified) . . . 1584
Fricas [B] (verification not implemented) . . . 1584
Sympy [F] . . . 1585
Maxima [B] (verification not implemented) . . . 1585
Giac [F] . . . 1586
Mupad [F(-1)] . . . 1587
Reduce [F] . . . 1587

Optimal result

Integrand size = 22, antiderivative size = 139

$$\begin{aligned}
&\int (c+dx)^2 \cos(a+bx) \cot^2(a+bx)\, dx \\
&= -\frac{4d(c+dx)\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d(c+dx)\cos(a+bx)}{b^2} \\
&\quad - \frac{(c+dx)^2 \csc(a+bx)}{b} + \frac{2id^2 \operatorname{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} \\
&\quad - \frac{2id^2 \operatorname{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3} + \frac{2d^2 \sin(a+bx)}{b^3} - \frac{(c+dx)^2 \sin(a+bx)}{b}
\end{aligned}$$

output

```
-4*d*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^2-2*d*(d*x+c)*cos(b*x+a)/b^2-(d*x+c
)^2*csc(b*x+a)/b+2*I*d^2*polylog(2,-exp(I*(b*x+a)))/b^3-2*I*d^2*polylog(2,
exp(I*(b*x+a)))/b^3+2*d^2*sin(b*x+a)/b^3-(d*x+c)^2*sin(b*x+a)/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 310 vs. $2(139) = 278$.

Time $= 2.79$ (sec) , antiderivative size $= 310$, normalized size of antiderivative $= 2.23$

$$\int (c+dx)^2 \cos(a+bx) \cot^2(a+bx)\, dx =$$

$$-\frac{8bcd\operatorname{arctanh}\left(\cos(a)-\sin(a)\tan\left(\frac{bx}{2}\right)\right)+2b^2(c+dx)^2\csc(a)-4d^2\Big(2\arctan(\tan(a))\operatorname{arctanh}\left(\cos(a)-\right.}{}$$

input

```
Integrate[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

output

```
-1/2*(8*b*c*d*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] + 2*b^2*(c + d*x)^2*Cs
c[a] - 4*d^2*(2*ArcTan[Tan[a]]*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] + (((
b*x + ArcTan[Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[Tan[a]]))] - Log[1 + E^(
I*(b*x + ArcTan[Tan[a]]))]) + I*PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]]))]
 - I*PolyLog[2, E^(I*(b*x + ArcTan[Tan[a]]))])*Sec[a])/Sqrt[Sec[a]^2]) + 2*
Cos[b*x]*(2*b*d*(c + d*x)*Cos[a] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a]) - b^
2*(c + d*x)^2*Csc[a/2]*Csc[(a + b*x)/2]*Sin[(b*x)/2] + b^2*(c + d*x)^2*Sec
[a/2]*Sec[(a + b*x)/2]*Sin[(b*x)/2] + 2*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a]
 - 2*b*d*(c + d*x)*Sin[a])*Sin[b*x])/b^3
```

Rubi [A] (verified)

Time $= 0.81$ (sec) , antiderivative size $= 144$, normalized size of antiderivative $= 1.04$, number of steps used $= 14$, number of rules used $= 13$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used $= \{4908, 3042, 3777, 25, 3042, 3777, 3042, 3117, 4910, 3042, 4671, 2715, 2838\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \cos(a+bx) \cot^2(a+bx)\, dx$$

$$\downarrow 4908$$

$$\int (c+dx)^2 \cot(a+bx)\csc(a+bx)dx - \int (c+dx)^2 \cos(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)^2 \cot(a+bx)\csc(a+bx)dx - \int (c+dx)^2 \sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\downarrow 3777$$

$$-\frac{2d\int -((c+dx)\sin(a+bx))dx}{b} + \int (c+dx)^2\cot(a+bx)\csc(a+bx)dx - \frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\downarrow 25$$

$$\frac{2d\int (c+dx)\sin(a+bx)dx}{b} + \int (c+dx)^2\cot(a+bx)\csc(a+bx)dx - \frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\downarrow 3042$$

$$\frac{2d\int (c+dx)\sin(a+bx)dx}{b} + \int (c+dx)^2\cot(a+bx)\csc(a+bx)dx - \frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\downarrow 3777$$

$$\frac{2d\left(\frac{d\int \cos(a+bx)dx}{b} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b} + \int (c+dx)^2\cot(a+bx)\csc(a+bx)dx - \frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\downarrow 3042$$

$$\frac{2d\left(\frac{d\int \sin\left(a+bx+\frac{\pi}{2}\right)dx}{b} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b} + \int (c+dx)^2\cot(a+bx)\csc(a+bx)dx - \frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\downarrow 3117$$

$$\int (c+dx)^2\cot(a+bx)\csc(a+bx)dx + \frac{2d\left(\frac{d\sin(a+bx)}{b^2} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\downarrow 4910$$

$$\frac{2d\int (c+dx)\csc(a+bx)dx}{b} + \frac{2d\left(\frac{d\sin(a+bx)}{b^2} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2\sin(a+bx)}{b} - \frac{(c+dx)^2\csc(a+bx)}{b}$$

$$\downarrow 3042$$

$$\frac{2d\int (c+dx)\csc(a+bx)dx}{b}+\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

$$\downarrow 4671$$

$$\frac{2d\left(-\frac{d\int\log(1-e^{i(a+bx)})dx}{b}+\frac{d\int\log(1+e^{i(a+bx)})dx}{b}-\frac{2(c+dx)\text{arctanh}(e^{i(a+bx)})}{b}\right)}{b}+\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

$$\downarrow 2715$$

$$\frac{2d\left(\frac{id\int e^{-i(a+bx)}\log(1-e^{i(a+bx)})de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log(1+e^{i(a+bx)})de^{i(a+bx)}}{b^2}-\frac{2(c+dx)\text{arctanh}(e^{i(a+bx)})}{b}\right)}{b}+\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

$$\downarrow 2838$$

$$\frac{2d\left(-\frac{2(c+dx)\text{arctanh}(e^{i(a+bx)})}{b}+\frac{id\,\text{PolyLog}(2,-e^{i(a+bx)})}{b^2}-\frac{id\,\text{PolyLog}(2,e^{i(a+bx)})}{b^2}\right)}{b}+\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

input

```
Int[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

output

```
-(((c + d*x)^2*Csc[a + b*x])/b) + (2*d*((-2*(c + d*x)*ArcTanh[E^(I*(a + b*
x))])/b + (I*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^(I*(a
 + b*x))])/b^2))/b - ((c + d*x)^2*Sin[a + b*x])/b + (2*d*(-(((c + d*x)*Cos
[a + b*x])/b) + (d*Sin[a + b*x])/b^2))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 4910

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(131) = 262$.

Time = 0.69 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.39

method	result
risch	$\frac{i(x^2d^2b^2+2b^2cdx+2ib\,d^2x+b^2c^2+2ibcd-2d^2)e^{i(bx+a)}}{2b^3} - \frac{i(x^2d^2b^2+2b^2cdx-2ib\,d^2x+b^2c^2-2ibcd-2d^2)e^{-i(bx+a)}}{2b^3} - \frac{2i(x^2d^2+}{b(e^{2}}$

input

```
int((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
1/2*I*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*ex
p(I*(b*x+a))-1/2*I*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*
b*c*d)/b^3*exp(-I*(b*x+a))-2*I*(d^2*x^2+2*c*d*x+c^2)*exp(I*(b*x+a))/b/(exp
(2*I*(b*x+a))-1)-4*d/b^2*c*arctanh(exp(I*(b*x+a)))-2*d^2/b^2*ln(exp(I*(b*x
+a))+1)*x-2*d^2/b^3*ln(exp(I*(b*x+a))+1)*a+2*I*d^2*polylog(2,-exp(I*(b*x+a
)))/b^3+2*d^2/b^2*ln(1-exp(I*(b*x+a)))*x+2*d^2/b^3*ln(1-exp(I*(b*x+a)))*a-
2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^3+4*d^2/b^3*a*arctanh(exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(127) = 254$.

Time = 0.10 (sec) , antiderivative size = 448, normalized size of antiderivative = 3.22

$$\int (c+dx)^2\cos(a+bx)\cot^2(a+bx)\,dx =$$

$$-\frac{2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2+i\,d^2\mathrm{Li}_2(\cos(bx+a)+i\,\sin(bx+a))\sin(bx+a)-i\,d^2\mathrm{Li}_2(\cos(bx+a)+}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + I*d^2*dilog(cos(b*x + a) + I*s
in(b*x + a))*sin(b*x + a) - I*d^2*dilog(cos(b*x + a) - I*sin(b*x + a))*sin
(b*x + a) + I*d^2*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - I*d
^2*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - (b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 + 2*(b*d^2*x + b*c*d)*cos(b*x +
 a)*sin(b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1
)*sin(b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x + a) - I*sin(b*x + a) + 1)*
sin(b*x + a) - (b*c*d - a*d^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a)
+ 1/2)*sin(b*x + a) - (b*c*d - a*d^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*
x + a) + 1/2)*sin(b*x + a) - (b*d^2*x + a*d^2)*log(-cos(b*x + a) + I*sin(b
*x + a) + 1)*sin(b*x + a) - (b*d^2*x + a*d^2)*log(-cos(b*x + a) - I*sin(b*
x + a) + 1)*sin(b*x + a) - 2*d^2)/(b^3*sin(b*x + a))
```

Sympy [F]

$$\int (c+dx)^2 \cos(a+bx) \cot^2(a+bx)\,dx = \int (c+dx)^2 \cos(a+bx) \cot^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*cos(b*x+a)*cot(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*cos(a + b*x)*cot(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3199 vs. $2(127) = 254$.

Time = 0.96 (sec) , antiderivative size = 3199, normalized size of antiderivative = 23.01

$$\int (c+dx)^2 \cos(a+bx) \cot^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

output

```
1/2*(b^2*d^2*x^2*(-I*cos(a) + sin(a)) + b^2*c^2*(-I*cos(a) + sin(a)) - 2*b
*c*d*(cos(a) + I*sin(a)) - 2*d^2*(-I*cos(a) + sin(a)) - 2*(b^2*c*d*(I*cos(
a) - sin(a)) + b*d^2*(cos(a) + I*sin(a)))*x - 4*((b*d^2*x*(-I*cos(a) + sin
(a)) + b*c*d*(-I*cos(a) + sin(a)) + (I*b*d^2*x + I*b*c*d)*cos(2*b*x + 3*a)
 - (b*d^2*x + b*c*d)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + ((-I*b*d^2*x - I
*b*c*d)*cos(b*x + a) + (b*d^2*x + b*c*d)*sin(b*x + a))*cos(2*b*x + 3*a) +
(b*d^2*x*(I*cos(a) - sin(a)) + b*c*d*(I*cos(a) - sin(a)))*cos(b*x + a) + (
b*d^2*x*(cos(a) + I*sin(a)) + b*c*d*(cos(a) + I*sin(a)) - (b*d^2*x + b*c*d
)*cos(2*b*x + 3*a) + (-I*b*d^2*x - I*b*c*d)*sin(2*b*x + 3*a))*sin(3*b*x +
3*a) + ((b*d^2*x + b*c*d)*cos(b*x + a) + (I*b*d^2*x + I*b*c*d)*sin(b*x + a
))*sin(2*b*x + 3*a) - (b*d^2*x*(cos(a) + I*sin(a)) + b*c*d*(cos(a) + I*sin
(a)))*sin(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 4*(b*c*d*(-I
*cos(a) + sin(a))*cos(b*x + a) + b*c*d*(cos(a) + I*sin(a))*sin(b*x + a) +
(b*c*d*(I*cos(a) - sin(a)) - I*b*c*d*cos(2*b*x + 3*a) + b*c*d*sin(2*b*x +
3*a))*cos(3*b*x + 3*a) + (I*b*c*d*cos(b*x + a) - b*c*d*sin(b*x + a))*cos(2
*b*x + 3*a) - (b*c*d*(cos(a) + I*sin(a)) - b*c*d*cos(2*b*x + 3*a) - I*b*c*
d*sin(2*b*x + 3*a))*sin(3*b*x + 3*a) - (b*c*d*cos(b*x + a) + I*b*c*d*sin(b
*x + a))*sin(2*b*x + 3*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) - 4*(b*
d^2*x*(I*cos(a) - sin(a))*cos(b*x + a) - b*d^2*x*(cos(a) + I*sin(a))*sin(b
*x + a) + (b*d^2*x*(-I*cos(a) + sin(a)) + I*b*d^2*x*cos(2*b*x + 3*a) - ...
```

Giac [F]

$$\int (c+dx)^2 \cos(a+bx) \cot^2(a+bx)\,dx = \int (dx+c)^2 \cos\left(bx+a\right) \cot\left(bx+a\right)^2\,dx$$

input

```
integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

output

```
integrate((d*x + c)^2*cos(b*x + a)*cot(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \cos(a+bx) \cot^2(a+bx)\,dx = \int \cos(a+b\,x)\,\cot(a+b\,x)^2\,(c+d\,x)^2\,dx$$

input
```
int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^2,x)
```

output
```
int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \cos(a+bx) \cot^2(a+bx)\,dx$$

$$= \frac{-16\cos(bx+a)\sin(bx+a)\,bcd - 18\cos(bx+a)\sin(bx+a)\,b\,d^2x + 4\cos(bx+a)\,b^2d^2x^2 + 2\left(\int \frac{}{\tan\left(\frac{ba}{2}\right.}\right.}{}$$

input
```
int((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x)
```

output
```
( - 16*cos(a + b*x)*sin(a + b*x)*b*c*d - 18*cos(a + b*x)*sin(a + b*x)*b*d*
*2*x + 4*cos(a + b*x)*b**2*d**2*x**2 + 2*int(x**2/(tan((a + b*x)/2)**4 + t
an((a + b*x)/2)**2),x)*sin(a + b*x)*b**3*d**2 + 8*int((tan((a + b*x)/2)**3
*x)/(tan((a + b*x)/2)**2 + 1),x)*sin(a + b*x)*b**2*d**2 + 16*log(tan((a +
b*x)/2))*sin(a + b*x)*b*c*d - 8*sin(a + b*x)**2*b**2*c**2 - 16*sin(a + b*x
)**2*b**2*c*d*x - 7*sin(a + b*x)**2*b**2*d**2*x**2 + 18*sin(a + b*x)**2*d*
*2 + sin(a + b*x)*b**3*d**2*x**3 + 16*sin(a + b*x)*b*c*d - 8*b**2*c**2 - 1
6*b**2*c*d*x - 4*b**2*d**2*x**2)/(8*sin(a + b*x)*b**3)
```

3.174 $\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx$

Optimal result . . . 1588
Mathematica [A] (verified) . . . 1588
Rubi [A] (verified) . . . 1589
Maple [C] (verified) . . . 1591
Fricas [A] (verification not implemented) . . . 1591
Sympy [F] . . . 1592
Maxima [B] (verification not implemented) . . . 1592
Giac [B] (verification not implemented) . . . 1593
Mupad [B] (verification not implemented) . . . 1594
Reduce [B] (verification not implemented) . . . 1595

Optimal result

Integrand size = 20, antiderivative size = 58

$$\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx = -\frac{d\operatorname{arctanh}(\cos(a+bx))}{b^2} - \frac{d\cos(a+bx)}{b^2} - \frac{(c+dx)\csc(a+bx)}{b} - \frac{(c+dx)\sin(a+bx)}{b}$$

output

```
-d*arctanh(cos(b*x+a))/b^2-d*cos(b*x+a)/b^2-(d*x+c)*csc(b*x+a)/b-(d*x+c)*s
in(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.79

$$\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx = -\frac{2d\cos(a+bx) + bdx\cot\left(\frac{1}{2}(a+bx)\right) + 2bc\csc(a+bx) + 2d\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right) - 2d\log\left(\sin\left(\frac{1}{2}(a+\right.\right.}{2b^2}$$

input

```
Integrate[(c + d*x)*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

output

```
-1/2*(2*d*Cos[a + b*x] + b*d*x*Cot[(a + b*x)/2] + 2*b*c*Csc[a + b*x] + 2*d
*Log[Cos[(a + b*x)/2]] - 2*d*Log[Sin[(a + b*x)/2]] + 2*b*c*Sin[a + b*x] +
2*b*d*x*Sin[a + b*x] + b*d*x*Tan[(a + b*x)/2])/b^2
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {4908, 3042, 3777, 25, 3042, 3118, 4910, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx$$

$$\downarrow 4908$$

$$\int (c+dx)\cot(a+bx)\csc(a+bx)dx - \int (c+dx)\cos(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)\cot(a+bx)\csc(a+bx)dx - \int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\downarrow 3777$$

$$\int (c+dx)\cot(a+bx)\csc(a+bx)dx - \frac{d\int -\sin(a+bx)dx}{b} - \frac{(c+dx)\sin(a+bx)}{b}$$

$$\downarrow 25$$

$$\int (c+dx)\cot(a+bx)\csc(a+bx)dx + \frac{d\int \sin(a+bx)dx}{b} - \frac{(c+dx)\sin(a+bx)}{b}$$

$$\downarrow 3042$$

$$\int (c+dx)\cot(a+bx)\csc(a+bx)dx + \frac{d\int \sin(a+bx)dx}{b} - \frac{(c+dx)\sin(a+bx)}{b}$$

$$\downarrow 3118$$

$$\int (c+dx)\cot(a+bx)\csc(a+bx)dx - \frac{d\cos(a+bx)}{b^2} - \frac{(c+dx)\sin(a+bx)}{b}$$

$$\downarrow 4910$$

$$\frac{d\int \csc(a+bx)dx}{b} - \frac{d\cos(a+bx)}{b^2} - \frac{(c+dx)\sin(a+bx)}{b} - \frac{(c+dx)\csc(a+bx)}{b}$$

$$\downarrow 3042$$

$$\frac{d\int \csc(a+bx)dx}{b} - \frac{d\cos(a+bx)}{b^2} - \frac{(c+dx)\sin(a+bx)}{b} - \frac{(c+dx)\csc(a+bx)}{b}$$

$$\downarrow 4257$$

$$-\frac{d\operatorname{arctanh}(\cos(a+bx))}{b^2} - \frac{d\cos(a+bx)}{b^2} - \frac{(c+dx)\sin(a+bx)}{b} - \frac{(c+dx)\csc(a+bx)}{b}$$

input
```
Int[(c + d*x)*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

output
```
-((d*ArcTanh[Cos[a + b*x]])/b^2) - (d*Cos[a + b*x])/b^2 - ((c + d*x)*Csc[a
 + b*x])/b - ((c + d*x)*Sin[a + b*x])/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.14

method	result	size
risch	$\frac{i(bdx+cb+id)e^{i(bx+a)}}{2b^2}-\frac{i(bdx+cb-id)e^{-i(bx+a)}}{2b^2}-\frac{2i(dx+c)e^{i(bx+a)}}{b(e^{2i(bx+a)}-1)}-\frac{d\ln(e^{i(bx+a)}+1)}{b^2}+\frac{d\ln(e^{i(bx+a)}-1)}{b^2}$	124

input
```
int((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(b*d*x+c*b+I*d)/b^2*exp(I*(b*x+a))-1/2*I*(b*d*x+c*b-I*d)/b^2*exp(-I*
(b*x+a))-2*I*(d*x+c)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)-d/b^2*ln(exp(I*
(b*x+a))+1)+d/b^2*ln(exp(I*(b*x+a))-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.64

$$\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx =$$
$$-\frac{4\,bdx-2\,(bdx+bc)\cos(bx+a)^2+2\,d\cos(bx+a)\sin(bx+a)+d\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)\sin(bx+}{2\,b^2\sin(bx+a)}$$

input
```
integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(4*b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + 2*d*cos(b*x + a)*sin(b*x
+ a) + d*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - d*log(-1/2*cos(b*x + a
) + 1/2)*sin(b*x + a) + 4*b*c)/(b^2*sin(b*x + a))
```

Sympy [F]

$$\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx = \int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx$$

input
```
integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*cos(a + b*x)*cot(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2110 vs. $2(58) = 116$.

Time $= 0.06$ (sec) , antiderivative size $= 2110$, normalized size of antiderivative $= 36.38$

$$\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/2*(2*c*(1/sin(b*x + a) + sin(b*x + a)) - 2*a*d*(1/sin(b*x + a) + sin(b*
x + a))/b - (((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*cos(3*b*x
 + 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin(2*b*x + 2*a))*sin(
3*b*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^3 - 2*(4*(b*x + a)*cos(b*x + a)*
sin(2*b*x + 2*a) - (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2
*a) + 3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a)^2 - ((b*x
 + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x + a)*cos(2*
b*x + 2*a)*sin(b*x + a) + ((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) +
 1)*cos(3*b*x + 3*a) - 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x + a))*sin(2*b
*x + 2*a) - 8*(b*x + a)*sin(b*x + a))*sin(3*b*x + 3*a)^2 - ((b*x + a)*sin(
b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a)^2 + (12*(b*x + a)*cos(b*x + a)*s
in(b*x + a) - (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x + a)^2 + s
in(b*x + a)^2 + 2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*a)^2 + cos(b*x + a)^2
 + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a
) + sin(2*b*x + 2*a)^2 + sin(b*x + a)^2 + 1)*cos(3*b*x + 3*a) + 2*(3*(b*x
 + a)*sin(b*x + a)^3 + (3*(b*x + a)*cos(b*x + a)^2 + b*x + a)*sin(b*x + a)
 + cos(b*x + a))*cos(2*b*x + 2*a) - ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)
^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*
x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 -
2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1759 vs. $2(58) = 116$.

Time $= 0.66$ (sec) , antiderivative size $= 1759$, normalized size of antiderivative $= 30.33$

$$\int (c + dx)\cos(a + bx)\cot^2(a + bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

output

```
1/2*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4 +
 6*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^3
 - d*log(4*(tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(
tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 1))*tan(1/2*
b*x)^4*tan(1/2*a)^3 + d*log(4*(tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a)
+ tan(1/2*a)^2)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)
^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^3 + 6*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4
 - d*log(4*(tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(
tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 1))*tan(1/2*
b*x)^3*tan(1/2*a)^4 + d*log(4*(tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a)
+ tan(1/2*a)^2)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)
^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a)^4 + 6*b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 +
 8*b*c*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + 6*b
*c*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + b*d*x*t
an(1/2*b*x)^4 - 8*b*d*x*tan(1/2*b*x)^3*tan(1/2*a) - d*log(4*(tan(1/2*b*x)^
2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*b*x)^2*tan(1/2*a)
^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a) + d*log
(4*(tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*b*
x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan
(1/2*a) - 12*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - 8*b*d*x*tan(1/2*b*x)*t...
```

Mupad [B] (verification not implemented)

Time = 19.06 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.79

$$\begin{aligned}\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx = {} & \mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\left(\frac{(b\,c+d\,1\mathrm{i})\;1\mathrm{i}}{2\,b^2}+\frac{d\,x\,1\mathrm{i}}{2\,b}\right)\\ & +\mathrm{e}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}\left(\frac{(-b\,c+d\,1\mathrm{i})\;1\mathrm{i}}{2\,b^2}-\frac{d\,x\,1\mathrm{i}}{2\,b}\right)\\ & -\frac{d\,\ln\left(\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}{b^2}\\ & +\frac{d\,\ln\left(d\,2\mathrm{i}-d\,\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,1\mathrm{i}}\,2\mathrm{i}\right)}{b^2}\\ & +\frac{2\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,(c+d\,x)}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}\end{aligned}$$

input

```
int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x),x)
```

output
```
exp(a*1i + b*x*1i)*(((d*1i + b*c)*1i)/(2*b^2) + (d*x*1i)/(2*b)) + exp(- a*
1i - b*x*1i)*(((d*1i - b*c)*1i)/(2*b^2) - (d*x*1i)/(2*b)) - (d*log(exp(a*1
i + b*x*1i)*1i + 1i))/b^2 + (d*log(d*2i - d*exp(a*1i)*exp(b*x*1i)*2i))/b^2
 + (2*exp(a*1i + b*x*1i)*(c + d*x))/(b*(exp(a*2i + b*x*2i)*1i - 1i))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.52

$$\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx$$
$$= \frac{-\cos(bx+a)\sin(bx+a)\,d+\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin(bx+a)\,d-\sin(bx+a)^2\,bc-\sin(bx+a)^2\,bdx+\mathrm{s}}{\sin(bx+a)\,b^2}$$

input
```
int((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x)
```

output
```
( - cos(a + b*x)*sin(a + b*x)*d + log(tan((a + b*x)/2))*sin(a + b*x)*d - s
in(a + b*x)**2*b*c - sin(a + b*x)**2*b*d*x + sin(a + b*x)*d - b*c - b*d*x)
/(sin(a + b*x)*b**2)
```

3.175 $\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$

Optimal result 1596
Mathematica [N/A] 1596
Rubi [N/A] 1597
Maple [N/A] 1599
Fricas [N/A] 1599
Sympy [N/A] 1600
Maxima [N/A] 1600
Giac [N/A] 1601
Mupad [N/A] 1602
Reduce [N/A] 1602

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx = -\frac{\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d} + \frac{\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{d} + \operatorname{Int}\left(\frac{\cot(a+bx)\csc(a+bx)}{c+dx},x\right)$$

output
```
-cos(a-b*c/d)*Ci(b*c/d+b*x)/d+sin(a-b*c/d)*Si(b*c/d+b*x)/d+Defer(Int)(cot(
b*x+a)*csc(b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 2.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x),x]
```

output
```
Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4908, 3042, 3784, 3042, 3780, 3783, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$$

$$\downarrow 4908$$

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx - \int \frac{\cos(a+bx)}{c+dx}\,dx$$

$$\downarrow 3042$$

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx - \int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{c+dx}\,dx$$

$$\downarrow 3784$$

$$\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx - \cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx + \int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$$

$$\downarrow 3042$$

$$\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx - \cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}\,dx + \int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$$

$$\downarrow 3780$$

$$-\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}\,dx + \int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx + \frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

$$\downarrow 3783$$

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}dx - \frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d} + \frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

$$\downarrow 7299$$

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}dx - \frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d} + \frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

input
```
Int[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cos(bx+a)\cot(bx+a)^2}{dx+c}dx$$

input
```
int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x)
```

output
```
int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)\cot(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*cot(b*x + a)^2/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx = \int \frac{\cos\left(a+bx\right)\cot^2\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(cos(a + b*x)*cot(a + b*x)**2/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 1438, normalized size of antiderivative = 65.36

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx = \int \frac{\cos\left(bx+a\right)\cot\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output

```
1/2*(b*c*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b
*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(I*exp_integral_e(1, (I*b*d*x
+ I*b*c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/
d) + (b*c*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*
b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(I*exp_integral_e(1, (I*b*d*x
 + I*b*c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)
/d) + (b*d*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I
*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(I*exp_integral_e(1, (I*b*d*
x + I*b*c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d
)/d))*x)*cos(2*b*x + 2*a)^2 - 4*d*cos(b*x + a)*sin(2*b*x + 2*a) + (b*c*(ex
p_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c
)/d))*cos(-(b*c - a*d)/d) - b*c*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d)
- I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d) + (b*d*(e
xp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*
c)/d))*cos(-(b*c - a*d)/d) - b*d*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d)
 - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x)*sin(
2*b*x + 2*a)^2 + (b*d*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integr
al_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(I*exp_integral_e
(1, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(
-(b*c - a*d)/d))*x - 2*(b*c*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + e...
```

Giac [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx = \int \frac{\cos\left(bx+a\right)\cot\left(bx+a\right)^2}{dx+c}\,dx$$

input

```
integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output

```
integrate(cos(b*x + a)*cot(b*x + a)^2/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 18.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+b\,x)\,\cot(a+b\,x)^2}{c+d\,x}\,dx$$

input
```
int((cos(a + b*x)*cot(a + b*x)^2)/(c + d*x),x)
```

output
```
int((cos(a + b*x)*cot(a + b*x)^2)/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)\cot(bx+a)^2}{dx+c}dx$$

input
```
int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x)
```

output
```
int((cos(a + b*x)*cot(a + b*x)**2)/(c + d*x),x)
```

3.176 $\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx$

Optimal result 1603
Mathematica [N/A] 1603
Rubi [N/A] 1604
Maple [N/A] 1607
Fricas [N/A] 1607
Sympy [N/A] 1607
Maxima [N/A] 1608
Giac [N/A] 1609
Mupad [N/A] 1609
Reduce [N/A] 1609

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx = \frac{\cos(a+bx)}{d(c+dx)} + \frac{b\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^2} + \frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2} + \text{Int}\left(\frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}, x\right)$$

output

```
cos(b*x+a)/d/(d*x+c)+b*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^2+b*cos(a-b*c/d)*Si(b*
c/d+b*x)/d^2+Defer(Int)(cot(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx$$

input

```
Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4908, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx$$

$$\downarrow 4908$$

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}dx - \int \frac{\cos(a+bx)}{(c+dx)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}dx - \int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx$$

$$\downarrow 3778$$

$$-\frac{b\int -\frac{\sin(a+bx)}{c+dx}dx}{d} + \int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}dx + \frac{\cos(a+bx)}{d(c+dx)}$$

$$\downarrow 25$$

$$\frac{b\int \frac{\sin(a+bx)}{c+dx}dx}{d} + \int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}dx + \frac{\cos(a+bx)}{d(c+dx)}$$

$$\downarrow 3042$$

$$\frac{b\int \frac{\sin(a+bx)}{c+dx}dx}{d} + \int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}dx + \frac{\cos(a+bx)}{d(c+dx)}$$

$$\downarrow 3784$$

$$\frac{b\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}+\int\frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}dx+\frac{\cos(a+bx)}{d(c+dx)}$$

$$\downarrow 3042$$

$$\frac{b\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}+\int\frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}dx+\frac{\cos(a+bx)}{d(c+dx)}$$

$$\downarrow 3780$$

$$\frac{b\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}+\int\frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}dx+\frac{\cos(a+bx)}{d(c+dx)}$$

$$\downarrow 3783$$

$$\int\frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}dx+\frac{b\left(\frac{\sin\left(a-\frac{bc}{d}\right)\,\mathrm{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}+\frac{\cos(a+bx)}{d(c+dx)}$$

$$\downarrow 7299$$

$$\int\frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}dx+\frac{b\left(\frac{\sin\left(a-\frac{bc}{d}\right)\,\mathrm{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}+\frac{\cos(a+bx)}{d(c+dx)}$$

input
```
Int[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cos(bx+a)\cot(bx+a)^2}{(dx+c)^2}dx$$

input
```
int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(bx+a)\cot(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*cot(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(cos(a + b*x)*cot(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 1678, normalized size of antiderivative = 76.27

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(bx+a\right)\cot\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*(b*c*(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b
*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(I*exp_integral_e(2, (I*b*d*x
+ I*b*c)/d) - I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/
d) + (b*c*(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*
b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(I*exp_integral_e(2, (I*b*d*x
 + I*b*c)/d) - I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)
/d) + (b*d*(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I
*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(I*exp_integral_e(2, (I*b*d*
x + I*b*c)/d) - I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d
)/d))*x)*cos(2*b*x + 2*a)^2 - 4*d*cos(b*x + a)*sin(2*b*x + 2*a) + (b*c*(ex
p_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c
)/d))*cos(-(b*c - a*d)/d) - b*c*(I*exp_integral_e(2, (I*b*d*x + I*b*c)/d)
- I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d) + (b*d*(e
xp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*
c)/d))*cos(-(b*c - a*d)/d) - b*d*(I*exp_integral_e(2, (I*b*d*x + I*b*c)/d)
 - I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x)*sin(
2*b*x + 2*a)^2 + (b*d*(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integr
al_e(2, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(I*exp_integral_e
(2, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(
-(b*c - a*d)/d))*x - 2*(b*c*(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + e...
```

Giac [N/A]

Not integrable

Time = 6.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(bx+a\right)\cot\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*cot(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 18.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+b\,x\right)\,\cot(a+b\,x)^2}{\left(c+d\,x\right)^2}\,dx$$

input
```
int((cos(a + b*x)*cot(a + b*x)^2)/(c + d*x)^2,x)
```

output
```
int((cos(a + b*x)*cot(a + b*x)^2)/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(bx+a\right)\cot\left(bx+a\right)^2}{d^2x^2+2cdx+c^2}dx$$

input
```
int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int((cos(a + b*x)*cot(a + b*x)**2)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.177 $\int (c+dx)^m \cot^3(a+bx)\,dx$

Optimal result 1611
Mathematica [N/A] 1611
Rubi [N/A] 1612
Maple [N/A] 1613
Fricas [N/A] 1613
Sympy [N/A] 1614
Maxima [N/A] 1614
Giac [N/A] 1614
Mupad [N/A] 1615
Reduce [N/A] 1615

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (c+dx)^m \cot^3(a+bx)\,dx = \text{Int}\big((c+dx)^m \cot^3(a+bx), x\big)$$

output
```
Defer(Int)((d*x+c)^m*cot(b*x+a)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 9.57 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \cot^3(a+bx)\,dx = \int (c+dx)^m \cot^3(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Cot[a + b*x]^3,x]
```

output
```
Integrate[(c + d*x)^m*Cot[a + b*x]^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 25, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(a+bx)(c+dx)^m\,dx$$

$$\downarrow 3042$$

$$\int \tan\left(a+bx+\frac{\pi}{2}\right)^3(-(c+dx)^m)\,dx$$

$$\downarrow 25$$

$$-\int (c+dx)^m \tan\left(\frac{1}{2}(2a+\pi)+bx\right)^3 dx$$

$$\downarrow 4222$$

$$-\int (c+dx)^m \tan^3\left(\frac{1}{2}(2a+\pi)+bx\right) dx$$

input
```
Int[(c + d*x)^m*Cot[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \cot(bx+a)^3 \, dx$$

input

```
int((d*x+c)^m*cot(b*x+a)^3,x)
```

output

```
int((d*x+c)^m*cot(b*x+a)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \cot^3(a+bx) \, dx = \int (dx+c)^m \cot(bx+a)^3 \, dx$$

input

```
integrate((d*x+c)^m*cot(b*x+a)^3,x, algorithm="fricas")
```

output

```
integral((d*x + c)^m*cot(b*x + a)^3, x)
```

Sympy [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (c+dx)^m \cot^3(a+bx)\,dx = \int (c+dx)^m \cot^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cot(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**m*cot(a + b*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \cot^3(a+bx)\,dx = \int (dx+c)^m \cot\left(bx+a\right)^3\,dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cot(b*x + a)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \cot^3(a+bx)\,dx = \int (dx+c)^m \cot\left(bx+a\right)^3\,dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cot(b*x + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 18.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \cot^3(a+bx)\,dx = \int \cot(a+b\,x)^3\,(c+d\,x)^m\,dx$$

input
```
int(cot(a + b*x)^3*(c + d*x)^m,x)
```

output
```
int(cot(a + b*x)^3*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \cot^3(a+bx)\,dx = \int (dx+c)^m \cot\left(bx+a\right)^3\,dx$$

input
```
int((d*x+c)^m*cot(b*x+a)^3,x)
```

output
```
int((c + d*x)**m*cot(a + b*x)**3,x)
```

3.178 $\int (c+dx)^4 \cot^3(a+bx)\,dx$

Optimal result . . . 1616
Mathematica [B] (warning: unable to verify) . . . 1617
Rubi [A] (verified) . . . 1618
Maple [B] (verified) . . . 1627
Fricas [B] (verification not implemented) . . . 1628
Sympy [F] . . . 1629
Maxima [B] (verification not implemented) . . . 1630
Giac [F] . . . 1631
Mupad [F(-1)] . . . 1631
Reduce [F] . . . 1631

Optimal result

Integrand size = 16, antiderivative size = 302

$$\begin{aligned}\int (c+dx)^4 \cot^3(a+bx)\,dx = &-\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} \\ &- \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\cot^2(a+bx)}{2b} \\ &+ \frac{6d^2(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^3} \\ &- \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} \\ &- \frac{6id^3(c+dx)\,\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^4} \\ &+ \frac{2id(c+dx)^3\,\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2} \\ &+ \frac{3d^4\,\mathrm{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^5} \\ &- \frac{3d^2(c+dx)^2\,\mathrm{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3} \\ &- \frac{3id^3(c+dx)\,\mathrm{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4} \\ &+ \frac{3d^4\,\mathrm{PolyLog}\left(5,e^{2i(a+bx)}\right)}{2b^5}\end{aligned}$$

output

```
-2*I*d*(d*x+c)^3/b^2-1/2*(d*x+c)^4/b+1/5*I*(d*x+c)^5/d-2*d*(d*x+c)^3*cot(b
*x+a)/b^2-1/2*(d*x+c)^4*cot(b*x+a)^2/b+6*d^2*(d*x+c)^2*ln(1-exp(2*I*(b*x+a
)))/b^3-(d*x+c)^4*ln(1-exp(2*I*(b*x+a)))/b-6*I*d^3*(d*x+c)*polylog(2,exp(2
*I*(b*x+a)))/b^4+2*I*d*(d*x+c)^3*polylog(2,exp(2*I*(b*x+a)))/b^2+3*d^4*pol
ylog(3,exp(2*I*(b*x+a)))/b^5-3*d^2*(d*x+c)^2*polylog(3,exp(2*I*(b*x+a)))/b
^3-3*I*d^3*(d*x+c)*polylog(4,exp(2*I*(b*x+a)))/b^4+3/2*d^4*polylog(5,exp(2
*I*(b*x+a)))/b^5
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1632 vs. $2(302) = 604$.

Time = 6.70 (sec) , antiderivative size = 1632, normalized size of antiderivative = 5.40

$$\int (c + dx)^4 \cot^3(a + bx)\, dx = \text{Too large to display}$$

input

```
Integrate[(c + d*x)^4*Cot[a + b*x]^3,x]
```

output

```
-1/5*(x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4)*Cot[
a]) - ((c + d*x)^4*Csc[a + b*x]^2)/(2*b) + (c^2*d^2*E^(I*a)*Csc[a]*((2*b^3
*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a +
b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - 6*
b*(1 - E^((-2*I)*a))*x*PolyLog[2, -E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I
)*a))*x*PolyLog[2, E^((-I)*(a + b*x))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[
3, -E^((-I)*(a + b*x))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, E^((-I)*(a +
 b*x))]))/b^3 - (d^4*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(
1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*
I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[
2, -E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, E^((-I)*(a +
 b*x))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^((-I)*(a + b*x))] + (6*I)
*(1 - E^((-2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]))/b^5 + (c*d^3*E^(I*a)*
Csc[a]*((b^4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E
^((-I)*(a + b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a +
 b*x))] - 6*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - 6
*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[2, E^((-I)*(a + b*x))] + (12*I)*b*(1 -
 E^((-2*I)*a))*x*PolyLog[3, -E^((-I)*(a + b*x))] + (12*I)*b*(1 - E^((-2*I)
*a))*x*PolyLog[3, E^((-I)*(a + b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, -
E^((-I)*(a + b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, E^((-I)*(a + b*x...
```

Rubi [A] (verified)

Time = 2.23 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.35, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.438$, Rules used = {3042, 25, 4203, 25, 3042, 25, 4202, 2620, 3011, 4203, 17, 25, 3042, 25, 4202, 2620, 3011, 2720, 7143, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \cot^3(a+bx)\,dx$$

$$\downarrow 3042$$

$$\int -(c+dx)^4 \tan\left(a+bx+\frac{\pi}{2}\right)^3 dx$$

$$\downarrow 25$$

$$-\int (c+dx)^4 \tan\left(\frac{1}{2}(2a+\pi)+bx\right)^3 dx$$

$$\downarrow 4203$$

$$\frac{2d\int (c+dx)^3 \cot^2(a+bx)dx}{b}+\int -(c+dx)^4 \cot(a+bx)dx-\frac{(c+dx)^4 \cot^2(a+bx)}{2b}$$

$$\downarrow 25$$

$$\frac{2d\int (c+dx)^3 \cot^2(a+bx)dx}{b}-\int (c+dx)^4 \cot(a+bx)dx-\frac{(c+dx)^4 \cot^2(a+bx)}{2b}$$

$$\downarrow 3042$$

$$-\int -(c+dx)^4 \tan\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2d\int (c+dx)^3 \tan\left(a+bx+\frac{\pi}{2}\right)^2 dx}{b}-\frac{(c+dx)^4 \cot^2(a+bx)}{2b}$$

$$\downarrow 25$$

$$\frac{2d\int (c+dx)^3 \tan\left(a+bx+\frac{\pi}{2}\right)^2 dx}{b}+\int (c+dx)^4 \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx-\frac{(c+dx)^4 \cot^2(a+bx)}{2b}$$

$$\downarrow 4202$$

$$-2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)^4}{1+e^{i(2a+2bx+\pi)}}dx+\frac{2d\int (c+dx)^3 \tan\left(a+bx+\frac{\pi}{2}\right)^2 dx}{b}-\frac{(c+dx)^4 \cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$$\downarrow 2620$$

$$-2i\left(\frac{2id\int (c+dx)^3 \log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{b}-\frac{i(c+dx)^4 \log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)+\frac{2d\int (c+dx)^3 \tan\left(a+bx+\frac{\pi}{2}\right)^2 dx}{b}-\frac{(c+dx)^4 \cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$$\downarrow 3011$$

$$-2i\left(\frac{2id\left(\frac{i(c+dx)^3 \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int (c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4 \log\left(1+e^{i(2a+2bx+\pi}\right.}{2b}\right.$$

$$\frac{2d\int (c+dx)^3 \tan\left(a+bx+\frac{\pi}{2}\right)^2 dx}{b}-\frac{(c+dx)^4 \cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$$\downarrow 4203$$

$$-2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi}\right.}{2b}\right.$$
$$\frac{2d\left(-\frac{3d\int-(c+dx)^2\cot(a+bx)dx}{b}-\int(c+dx)^3dx-\frac{(c+dx)^3\cot(a+bx)}{b}\right)}{b}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}+$$
$$\frac{i(c+dx)^5}{5d}$$

$$\downarrow 17$$

$$-2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi}\right.}{2b}\right.$$
$$\frac{2d\left(-\frac{3d\int-(c+dx)^2\cot(a+bx)dx}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}\right)}{b}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}+$$
$$\frac{i(c+dx)^5}{5d}$$

$$\downarrow 25$$

$$-2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi}\right.}{2b}\right.$$
$$\frac{2d\left(\frac{3d\int(c+dx)^2\cot(a+bx)dx}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}\right)}{b}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$$\downarrow 3042$$

$$-2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi}\right.}{2b}\right.$$
$$\frac{2d\left(\frac{3d\int-(c+dx)^2\tan\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}\right)}{b}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}+$$
$$\frac{i(c+dx)^5}{5d}$$

$$\downarrow 25$$

$$-2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi}\right.}{2b}\right.$$

$$\frac{2d\left(-\frac{3d\int(c+dx)^2\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}\right)}{b}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}+$$

$$\frac{i(c+dx)^5}{5d}$$

$$\downarrow 4202$$

$$-2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi}\right.}{2b}\right.$$

$$\frac{2d\left(-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\int\frac{e^{i(2a+2bx+\pi)}(c+dx)^2}{1+e^{i(2a+2bx+\pi)}}dx\right)}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}\right)}{b}-$$

$$\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$$\downarrow 2620$$

$$-2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi}\right.}{2b}\right.$$

$$\frac{2d\left(-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\int(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}\right)}{b}-$$

$$\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$$\downarrow 3011$$

$$\frac{-2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi}\right.}{2b}\right)}{b}$$

$$\frac{2d\left(-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c-}{}\right)}{b}$$

$$\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$$\downarrow 2720$$

$$\frac{2d\left(-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+\right.}{2}\right)}{b}$$

$$\frac{2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}$$

$$\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$$\downarrow 7143$$

$$\frac{-2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi}\right.}{2b}\right)}{b}$$

$$\frac{2d\left(-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)}{}\right)}{b}$$

$$\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$$\downarrow 7163$$

$$\frac{-2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\left(\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)dx}{b}-\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{2b}\right)}{b}-2d\left(-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)}{}\right)}{b}$$

$$\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$$\downarrow 7163$$

$$\frac{-2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\left(\frac{id\left(\frac{id\int\mathrm{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}-\frac{i(c+dx)}{}\right)}{2b}\right)}{b}-2d\left(-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)}{}\right)}{b}$$

$$\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

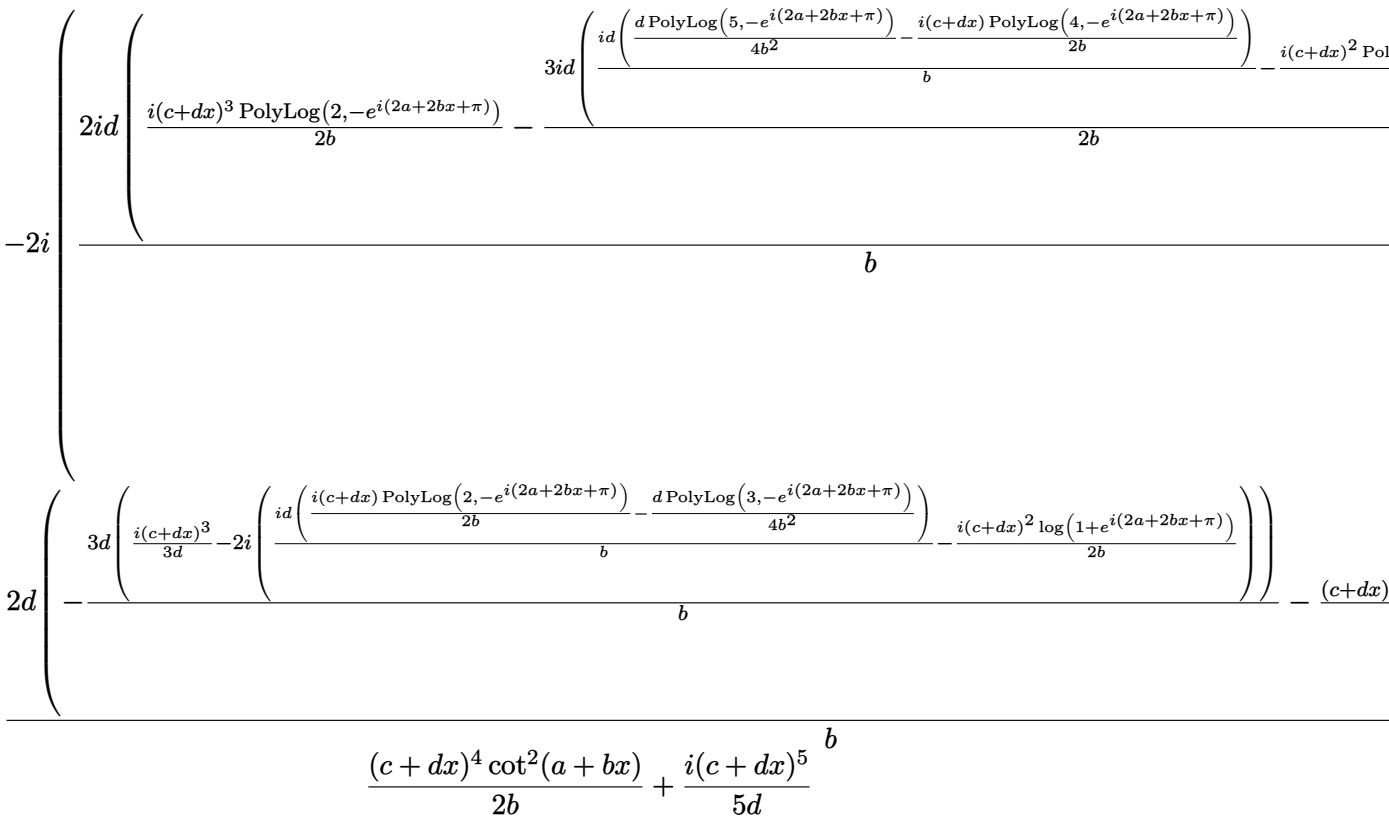
$$\downarrow 2720$$

$$-2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\left(\frac{id\left(\frac{d\int e^{-i(2a+2bx+\pi)}\,\mathrm{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(4,-\right.\right.}{2b}\right)}{b}}{2b}\right)}{b}\right.$$

$$\left.2d\left(-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)}{}\right.$$

$$\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}\Bigg)\Bigg/b$$

$$\downarrow 7143$$



input
```
Int[(c + d*x)^4*Cot[a + b*x]^3,x]
```

output
```
((I/5)*(c + d*x)^5)/d - ((c + d*x)^4*Cot[a + b*x]^2)/(2*b) + (2*d*(-1/4*(c
 + d*x)^4/d - ((c + d*x)^3*Cot[a + b*x])/b - (3*d*(((I/3)*(c + d*x)^3)/d -
 (2*I)*(((-1/2*I)*(c + d*x)^2*Log[1 + E^(I*(2*a + Pi + 2*b*x))])/b + (I*d*
(((I/2)*(c + d*x)*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/b - (d*PolyLog[3,
 -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)))/b))/b))/b - (2*I)*(((-1/2*I)*(c +
d*x)^4*Log[1 + E^(I*(2*a + Pi + 2*b*x))])/b + ((2*I)*d*(((I/2)*(c + d*x)^3
*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/b - (((3*I)/2)*d*(((-1/2*I)*(c + d
*x)^2*PolyLog[3, -E^(I*(2*a + Pi + 2*b*x))])/b + (I*d*(((-1/2*I)*(c + d*x)
*PolyLog[4, -E^(I*(2*a + Pi + 2*b*x))])/b + (d*PolyLog[5, -E^(I*(2*a + Pi
+ 2*b*x))])/(4*b^2)))/b))/b))/b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1876 vs. $2(273) = 546$.

Time = 0.47 (sec) , antiderivative size = 1877, normalized size of antiderivative = 6.22

method	result	size
risch	Expression too large to display	1877

input
```
int((d*x+c)^4*cot(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output

```
-24*I/b^3*d^3*c*a*x+12*I/b^2*c^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+12*I/b^2
*c^2*d^2*polylog(2,exp(I*(b*x+a)))*x-12*I/b^2*c^2*d^2*a^2*x+8*I/b^3*d^3*c*
a^3*x+12*I/b^2*d^3*c*polylog(2,-exp(I*(b*x+a)))*x^2+12*I/b^2*d^3*c*polylog
(2,exp(I*(b*x+a)))*x^2+8*I/b*d*c^3*a*x+2*I*d^2*c^2*x^3+2*I*d*c^3*x^2-I*c^4
*x-1/5*I/d*c^5+2*(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^3*exp(2*I*(b*x+a)
)+6*b*c^2*d^2*x^2*exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*x+a))-2*I*d^4*x^
3*exp(2*I*(b*x+a))+b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp(2*I*(b*x+a))-6
*I*c^2*d^2*x*exp(2*I*(b*x+a))+2*I*d^4*x^3-2*I*c^3*d*exp(2*I*(b*x+a))+6*I*c
*d^3*x^2+6*I*c^2*d^2*x+2*I*c^3*d)/b^2/(exp(2*I*(b*x+a))-1)^2-12/b^3*c^2*d^
2*ln(exp(I*(b*x+a)))+6/b^3*c^2*d^2*ln(exp(I*(b*x+a))-1)+6/b^3*c^2*d^2*ln(e
xp(I*(b*x+a))+1)+6/b^3*d^4*ln(1-exp(I*(b*x+a)))*x^2-12/b^5*a^2*d^4*ln(exp(
I*(b*x+a)))+6/b^5*a^2*d^4*ln(exp(I*(b*x+a))-1)-6/b^5*d^4*ln(1-exp(I*(b*x+a
)))*a^2+1/b^5*d^4*ln(1-exp(I*(b*x+a)))*a^4-1/b*d^4*ln(exp(I*(b*x+a))+1)*x^
4-1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4+2/b^5*a^4*d^4*ln(exp(I*(b*x+a)))-1/b^5*
a^4*d^4*ln(exp(I*(b*x+a))-1)-12/b^3*c^2*d^2*polylog(3,-exp(I*(b*x+a)))-12/
b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))-12/b^3*d^4*polylog(3,-exp(I*(b*x+a))
)*x^2-12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x^2+6/b^3*d^4*ln(exp(I*(b*x+a))
+1)*x^2-4*I/b^2*d^4*x^3+8*I/b^5*d^4*a^3-8/5*I/b^5*d^4*a^5+I*d^3*c*x^4-1/b*
c^4*ln(exp(I*(b*x+a))-1)-1/b*c^4*ln(exp(I*(b*x+a))+1)+2/b*c^4*ln(exp(I*(b*
x+a)))+1/5*I*d^4*x^5-4/b*d*c^3*ln(exp(I*(b*x+a))+1)*x+12/b^3*a^2*c^2*d^...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1751 vs. $2(266) = 532$.

Time $= 0.12$ (sec) , antiderivative size $= 1751$, normalized size of antiderivative $= 5.80$

$$\int (c + dx)^4 \cot^3(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^4*cot(b*x+a)^3,x, algorithm="fricas")
```

output

```
1/4*(4*b^4*d^4*x^4 + 16*b^4*c*d^3*x^3 + 24*b^4*c^2*d^2*x^2 + 16*b^4*c^3*d*
x + 4*b^4*c^4 - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + I*b^3*c^3*d - 3*I*b
*c*d^3 + 3*I*(b^3*c^2*d^2 - b*d^4)*x + (-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2
 - I*b^3*c^3*d + 3*I*b*c*d^3 - 3*I*(b^3*c^2*d^2 - b*d^4)*x)*cos(2*b*x + 2*
a))*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) - 4*(-I*b^3*d^4*x^3 - 3*I
*b^3*c*d^3*x^2 - I*b^3*c^3*d + 3*I*b*c*d^3 - 3*I*(b^3*c^2*d^2 - b*d^4)*x +
 (I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + I*b^3*c^3*d - 3*I*b*c*d^3 + 3*I*(b^3
*c^2*d^2 - b*d^4)*x)*cos(2*b*x + 2*a))*dilog(cos(2*b*x + 2*a) - I*sin(2*b*
x + 2*a)) + 2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 1)*b^2*c^2*d^2 - 4*(a^3
 - 3*a)*b*c*d^3 + (a^4 - 6*a^2)*d^4 - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 1
)*b^2*c^2*d^2 - 4*(a^3 - 3*a)*b*c*d^3 + (a^4 - 6*a^2)*d^4)*cos(2*b*x + 2*a
))*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b^4*c^4
 - 4*a*b^3*c^3*d + 6*(a^2 - 1)*b^2*c^2*d^2 - 4*(a^3 - 3*a)*b*c*d^3 + (a^4 -
 6*a^2)*d^4 - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 1)*b^2*c^2*d^2 - 4*(a^3
 - 3*a)*b*c*d^3 + (a^4 - 6*a^2)*d^4)*cos(2*b*x + 2*a))*log(-1/2*cos(2*b*x +
 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 +
 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 3*a)*b*c*d^3 - (a^4 - 6*a^2)
*d^4 + 6*(b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(b^4*c^3*d - 3*b^2*c*d^3)*x - (b^
4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 -
 3*a)*b*c*d^3 - (a^4 - 6*a^2)*d^4 + 6*(b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(...
```

Sympy [F]

$$\int (c+dx)^4 \cot^3(a+bx)\,dx = \int (c+dx)^4 \cot^3(a+bx)\,dx$$

input

```
integrate((d*x+c)**4*cot(b*x+a)**3,x)
```

output

```
Integral((c + d*x)**4*cot(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7158 vs. $2(266) = 532$.

Time = 3.26 (sec) , antiderivative size = 7158, normalized size of antiderivative = 23.70

$$\int (c+dx)^4 \cot^3(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^4*cot(b*x+a)^3,x, algorithm="maxima")
```

output

```
-1/2*(c^4*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) - 4*a*c^3*d*(1/sin(b*x
+ a)^2 + log(sin(b*x + a)^2))/b + 6*a^2*c^2*d^2*(1/sin(b*x + a)^2 + log(si
n(b*x + a)^2))/b^2 - 4*a^3*c*d^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/
b^3 + a^4*d^4*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/b^4 - 2*(2*(b*x + a
)^5*d^4 + 40*b^3*c^3*d - 120*a*b^2*c^2*d^2 + 120*a^2*b*c*d^3 - 40*a^3*d^4
+ 10*(b*c*d^3 - a*d^4)*(b*x + a)^4 + 20*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d
^4)*(b*x + a)^3 + 20*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^
4)*(b*x + a)^2 - 10*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 6*a^
2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (
a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b
*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a) + ((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 1
2*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2
 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d
^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) -
2*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3
 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x
 + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*
a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^4*d^4 + 6*I*b^2*c^2*d^
2 - 12*I*a*b*c*d^3 + 6*I*a^2*d^4 + 4*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 +
6*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 + (-I*a^2 + I)*d^4)*(b*x + a)^2 + 4*(...
```

Giac [F]

$$\int (c+dx)^4 \cot^3(a+bx)\,dx = \int (dx+c)^4 \cot(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^4*cot(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*cot(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \cot^3(a+bx)\,dx = \int \cot(a+b\,x)^3\,(c+d\,x)^4\,dx$$

input
```
int(cot(a + b*x)^3*(c + d*x)^4,x)
```

output
```
int(cot(a + b*x)^3*(c + d*x)^4, x)
```

Reduce [F]

$$\int (c+dx)^4 \cot^3(a+bx)\,dx$$
$$= \frac{4\left(\int \cot(bx+a)^3\,x^4 dx\right)\sin(bx+a)^2\,b\,d^4 + 16\left(\int \cot(bx+a)^3\,x^3 dx\right)\sin(bx+a)^2\,bc\,d^3 + 24\left(\int \cot(bx+\right.}{}$$

input
```
int((d*x+c)^4*cot(b*x+a)^3,x)
```

output

```
(4*int(cot(a + b*x)**3*x**4,x)*sin(a + b*x)**2*b*d**4 + 16*int(cot(a + b*x
)**3*x**3,x)*sin(a + b*x)**2*b*c*d**3 + 24*int(cot(a + b*x)**3*x**2,x)*sin
(a + b*x)**2*b*c**2*d**2 + 16*int(cot(a + b*x)**3*x,x)*sin(a + b*x)**2*b*c
**3*d + 4*log(tan((a + b*x)/2)**2 + 1)*sin(a + b*x)**2*c**4 - 4*log(tan((a
 + b*x)/2))*sin(a + b*x)**2*c**4 + sin(a + b*x)**2*c**4 - 2*c**4)/(4*sin(a
 + b*x)**2*b)
```

3.179 $\int (c+dx)^3 \cot^3(a+bx)\,dx$

Optimal result 1633
Mathematica [B] (warning: unable to verify) 1634
Rubi [A] (verified) 1635
Maple [B] (verified) 1643
Fricas [B] (verification not implemented) 1644
Sympy [F] 1645
Maxima [B] (verification not implemented) 1645
Giac [F] 1646
Mupad [F(-1)] 1647
Reduce [F] 1647

Optimal result

Integrand size = 16, antiderivative size = 256

$$\begin{aligned}\int (c+dx)^3 \cot^3(a+bx)\,dx = &-\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} + \frac{i(c+dx)^4}{4d} \\ &- \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\cot^2(a+bx)}{2b} \\ &+ \frac{3d^2(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^3} \\ &- \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} \\ &- \frac{3id^3\,\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^4} \\ &+ \frac{3id(c+dx)^2\,\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2} \\ &- \frac{3d^2(c+dx)\,\mathrm{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3} \\ &- \frac{3id^3\,\mathrm{PolyLog}\left(4,e^{2i(a+bx)}\right)}{4b^4}\end{aligned}$$

output

```
-3/2*I*d*(d*x+c)^2/b^2-1/2*(d*x+c)^3/b+1/4*I*(d*x+c)^4/d-3/2*d*(d*x+c)^2*c
ot(b*x+a)/b^2-1/2*(d*x+c)^3*cot(b*x+a)^2/b+3*d^2*(d*x+c)*ln(1-exp(2*I*(b*x
+a)))/b^3-(d*x+c)^3*ln(1-exp(2*I*(b*x+a)))/b-3/2*I*d^3*polylog(2,exp(2*I*(
b*x+a)))/b^4+3/2*I*d*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*
x+c)*polylog(3,exp(2*I*(b*x+a)))/b^3-3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))
/b^4
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1032 vs. $2(256) = 512$.

Time $= 6.57$ (sec) , antiderivative size $= 1032$, normalized size of antiderivative $= 4.03$

$$\int (c + dx)^3 \cot^3(a + bx)\, dx = \text{Too large to display}$$

input

```
Integrate[(c + d*x)^3*Cot[a + b*x]^3,x]
```

output

```
-1/4*(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Cot[a]) - ((c + d*x)^3
*Csc[a + b*x]^2)/(2*b) + (c*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) +
(3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(
1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))
*x*PolyLog[2, -E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, E
^((-I)*(a + b*x))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^((-I)*(a + b*x
))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]))/(2*b^3) +
(d^3*E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*
x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 +
 E^((-I)*(a + b*x))] - 6*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[2, -E^((-I)*(a
 + b*x))] - 6*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[2, E^((-I)*(a + b*x))] +
(12*I)*b*(1 - E^((-2*I)*a))*x*PolyLog[3, -E^((-I)*(a + b*x))] + (12*I)*b*(
1 - E^((-2*I)*a))*x*PolyLog[3, E^((-I)*(a + b*x))] + 12*(1 - E^((-2*I)*a))
*PolyLog[4, -E^((-I)*(a + b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, E^((-I
)*(a + b*x))]))/(4*b^4) - (c^3*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a]
 + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (3*c*d^2*Csc[a]*(
-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[
a]^2 + Sin[a]^2)) + (3*Csc[a]*Csc[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^2*x*Sin
[b*x] + d^3*x^2*Sin[b*x]))/(2*b^2) + (3*c^2*d*Csc[a]*Sec[a]*(b^2*E^(I*ArcT
an[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*...
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.25, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 1.312$, Rules used = {3042, 25, 4203, 25, 3042, 25, 4202, 2620, 3011, 4203, 17, 25, 3042, 25, 4202, 2620, 2715, 2838, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \cot^3(a+bx)\,dx$$

$$\downarrow 3042$$

$$\int -(c+dx)^3 \tan\left(a+bx+\frac{\pi}{2}\right)^3 dx$$

$$\downarrow 25$$

$$-\int (c+dx)^3 \tan\left(\frac{1}{2}(2a+\pi)+bx\right)^3 dx$$

$$\downarrow 4203$$

$$\frac{3d\int (c+dx)^2\cot^2(a+bx)dx}{2b}+\int -(c+dx)^3\cot(a+bx)dx-\frac{(c+dx)^3\cot^2(a+bx)}{2b}$$

$$\downarrow 25$$

$$\frac{3d\int (c+dx)^2\cot^2(a+bx)dx}{2b}-\int (c+dx)^3\cot(a+bx)dx-\frac{(c+dx)^3\cot^2(a+bx)}{2b}$$

$$\downarrow 3042$$

$$-\int -(c+dx)^3\tan\left(a+bx+\frac{\pi}{2}\right)dx+\frac{3d\int (c+dx)^2\tan\left(a+bx+\frac{\pi}{2}\right)^2 dx}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}$$

$$\downarrow 25$$

$$\frac{3d\int (c+dx)^2\tan\left(a+bx+\frac{\pi}{2}\right)^2 dx}{2b}+\int (c+dx)^3\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx-\frac{(c+dx)^3\cot^2(a+bx)}{2b}$$

$$\downarrow 4202$$

$$-2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)^3}{1+e^{i(2a+2bx+\pi)}}dx+\frac{3d\int (c+dx)^2\tan\left(a+bx+\frac{\pi}{2}\right)^2 dx}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$$\downarrow 2620$$

$$-2i\left(\frac{3id\int (c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)+\frac{3d\int (c+dx)^2\tan\left(a+bx+\frac{\pi}{2}\right)^2 dx}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$$\downarrow 3011$$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int (c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$

$$\left.\frac{3d\int (c+dx)^2\tan\left(a+bx+\frac{\pi}{2}\right)^2 dx}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}\right)$$

$$\downarrow 4203$$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\,\text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$
$$\left.\frac{3d\left(-\frac{2d\int-((c+dx)\cot(a+bx))dx}{b}-\int(c+dx)^2dx-\frac{(c+dx)^2\cot(a+bx)}{b}\right)}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\right.$$
$$\left.\frac{i(c+dx)^4}{4d}\right)$$

$$\downarrow 17$$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\,\text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$
$$\left.\frac{3d\left(-\frac{2d\int-((c+dx)\cot(a+bx))dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\right.$$
$$\left.\frac{i(c+dx)^4}{4d}\right)$$

$$\downarrow 25$$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\,\text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$
$$\left.\frac{3d\left(\frac{2d\int(c+dx)\cot(a+bx)dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}\right)$$

$$\downarrow 3042$$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\,\text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$
$$\left.\frac{3d\left(\frac{2d\int-((c+dx)\tan\left(a+bx+\frac{\pi}{2}\right))dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\right.$$
$$\left.\frac{i(c+dx)^4}{4d}\right)$$

$$\downarrow 25$$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$
$$\left.\frac{3d\left(-\frac{2d\int(c+dx)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\right.$$
$$\left.\frac{i(c+dx)^4}{4d}\right)$$

$$\downarrow 4202$$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$
$$\left.\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\int\frac{e^{i(2a+2bx+\pi)}(c+dx)}{1+e^{i(2a+2bx+\pi)}}dx\right)}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}-\right.$$
$$\left.\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}\right)$$

$$\downarrow 2620$$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$
$$\left.\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{id\int\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}-\right.$$
$$\left.\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}\right)$$

$$\downarrow 2715$$

$$3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{d\int e^{-i(2a+2bx+\pi)}\log\left(1+e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c\right.$$

$$\left.2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.\right.$$

$$\left.\left.\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}\right.\right)$$

$$\downarrow 2838$$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$

$$\left.3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)\right.-$$

$$\left.\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}\right.$$

$$\downarrow 7163$$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\left(\frac{id\int\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(c+\right.$$

$$\left.3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)\right.-$$

$$\left.\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}\right.$$

$$\downarrow 2720$$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\left(\frac{d\int e^{-i(2a+2bx+\pi)}\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)dx}{4b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}\right)}{2b}+\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}\right)-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$$\downarrow 7143$$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\left(\frac{d\,\mathrm{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(c+dx)}{}+\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}\right)-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

input
```
Int[(c + d*x)^3*Cot[a + b*x]^3,x]
```

output
```
((I/4)*(c + d*x)^4)/d - ((c + d*x)^3*Cot[a + b*x]^2)/(2*b) + (3*d*(-1/3*(c
 + d*x)^3/d - ((c + d*x)^2*Cot[a + b*x])/b - (2*d*(((I/2)*(c + d*x)^2)/d -
 (2*I)*(((-1/2*I)*(c + d*x)*Log[1 + E^(I*(2*a + Pi + 2*b*x))])/b - (d*Poly
Log[2, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2))))/b))/(2*b) - (2*I)*(((-1/2*I)
*(c + d*x)^3*Log[1 + E^(I*(2*a + Pi + 2*b*x))])/b + (((3*I)/2)*d*(((I/2)*(
c + d*x)^2*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/b - (I*d*(((-1/2*I)*(c +
 d*x)*PolyLog[3, -E^(I*(2*a + Pi + 2*b*x))])/b + (d*PolyLog[4, -E^(I*(2*a
+ Pi + 2*b*x))])/(4*b^2)))/b))/b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))  Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I  Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))  Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2  Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))  Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1202 vs. $2(221) = 442$.

Time = 0.43 (sec) , antiderivative size = 1203, normalized size of antiderivative = 4.70

method	result	size
risch	Expression too large to display	1203

input
```
int((d*x+c)^3*cot(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*I/b^3*d^3*a^3*x+3*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2+3*I/b^2*d^3*po
lylog(2,-exp(I*(b*x+a)))*x^2+3*I/b^2*d*c^2*a^2-4*I/b^3*c*d^2*a^3+3*I/b^2*d
*c^2*polylog(2,-exp(I*(b*x+a)))+3*I/b^2*d*c^2*polylog(2,exp(I*(b*x+a)))+3/
b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2+6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a)))+I*d^
2*c*x^3+(2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d
^2*x^2*exp(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(
b*x+a))-3*I*c^2*d*exp(2*I*(b*x+a))+3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))+6*
I*c*d^2*x+3*I*c^2*d)/b^2/(exp(2*I*(b*x+a))-1)^2+3/2*I/b^4*d^3*a^4-6*I/b^4*
d^3*polylog(4,exp(I*(b*x+a)))-6*I*d^3/b^3*a*x-I*c^3*x-1/4*I/d*c^4+3/2*I*d*
c^2*x^2-6*d^2/b^3*c*ln(exp(I*(b*x+a)))+3*d^2/b^3*c*ln(exp(I*(b*x+a))-1)+3*
d^2/b^3*c*ln(exp(I*(b*x+a))+1)+3*d^3/b^3*ln(exp(I*(b*x+a))+1)*x+3*d^3/b^3*
ln(1-exp(I*(b*x+a)))*x+3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a+6*d^3/b^4*a*ln(exp
(I*(b*x+a)))-3*d^3/b^4*a*ln(exp(I*(b*x+a))-1)-3*I*d^3/b^2*x^2-3*I*d^3/b^4*
a^2-3*I*d^3/b^4*polylog(2,-exp(I*(b*x+a)))+1/4*I*d^3*x^4-2/b^4*d^3*a^3*ln(
exp(I*(b*x+a)))+1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)-6/b^3*c*d^2*polylog(3,-
exp(I*(b*x+a)))-6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))-1/b*d^3*ln(1-exp(I*(
b*x+a)))*x^3-1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3-6/b^3*d^3*polylog(3,-exp(I*(
b*x+a)))*x-6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-1/b^4*d^3*ln(1-exp(I*(b*x
+a)))*a^3-3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)-6/b^2*c^2*d*a*ln(exp(I*(b*x
+a)))+3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)-3/b^2*d*c^2*ln(1-exp(I*(b*x+a)...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1139 vs. $2(214) = 428$.

Time = 0.10 (sec) , antiderivative size = 1139, normalized size of antiderivative = 4.45

$$\int (c+dx)^3 \cot^3(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*cot(b*x+a)^3,x, algorithm="fricas")
```

output

```
1/8*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 6*(I*
b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - I*d^3 + (-I*b^2*d^3*x^2 - 2*
I*b^2*c*d^2*x - I*b^2*c^2*d + I*d^3)*cos(2*b*x + 2*a))*dilog(cos(2*b*x + 2
*a) + I*sin(2*b*x + 2*a)) - 6*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^
2*d + I*d^3 + (I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - I*d^3)*cos(
2*b*x + 2*a))*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 4*(b^3*c^3 -
3*a*b^2*c^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3 - (b^3*c^3 - 3*a*b^2
*c^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3)*cos(2*b*x + 2*a))*log(-1/2
*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 4*(b^3*c^3 - 3*a*b^2*c
^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d +
3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3)*cos(2*b*x + 2*a))*log(-1/2*cos(2*b*
x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^
2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 3*a)*d^3 + 3*(b^3*c^2*d - b*d^3
)*x - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^
3 - 3*a)*d^3 + 3*(b^3*c^2*d - b*d^3)*x)*cos(2*b*x + 2*a))*log(-cos(2*b*x +
 2*a) + I*sin(2*b*x + 2*a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 3*a)*d^3 + 3*(b^3*c^2*d - b*d^3)*x - (b^
3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 3*a)*
d^3 + 3*(b^3*c^2*d - b*d^3)*x)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) - I
*sin(2*b*x + 2*a) + 1) - 3*(I*d^3*cos(2*b*x + 2*a) - I*d^3)*polylog(4, ...
```

Sympy [F]

$$\int (c+dx)^3 \cot^3(a+bx)\,dx = \int (c+dx)^3 \cot^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*cot(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**3*cot(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3958 vs. $2(214) = 428$.

Time = 0.96 (sec) , antiderivative size = 3958, normalized size of antiderivative = 15.46

$$\int (c+dx)^3 \cot^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cot(b*x+a)^3,x, algorithm="maxima")
```

output

```
-1/2*(c^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) - 3*a*c^2*d*(1/sin(b*x
+ a)^2 + log(sin(b*x + a)^2))/b + 3*a^2*c*d^2*(1/sin(b*x + a)^2 + log(sin(
b*x + a)^2))/b^2 - a^3*d^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/b^3 -
2*((b*x + a)^4*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^2*d^3 + 4*(b*c*d^2
 - a*d^3)*(b*x + a)^3 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a)^2
- 4*((b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)
^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a) + ((b*x + a)^3*
d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d
 - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 2*((b*x + a)
^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^
2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x
+ a)^3*d^3 + 3*I*b*c*d^2 - 3*I*a*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^
2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*a^2 + I)*d^3)*(b*x + a))*sin(4*b
*x + 4*a) - 2*(I*(b*x + a)^3*d^3 - 3*I*b*c*d^2 + 3*I*a*d^3 + 3*(I*b*c*d^2
- I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 - I)*d^3)
*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 12
*(b*c*d^2 - a*d^3 + (b*c*d^2 - a*d^3)*cos(4*b*x + 4*a) - 2*(b*c*d^2 - a*d^
3)*cos(2*b*x + 2*a) + (I*b*c*d^2 - I*a*d^3)*sin(4*b*x + 4*a) + 2*(-I*b*c*d
^2 + I*a*d^3)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) +
4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2...
```

Giac [F]

$$\int (c+dx)^3 \cot^3(a+bx)\,dx = \int (dx+c)^3 \cot(bx+a)^3\,dx$$

input

```
integrate((d*x+c)^3*cot(b*x+a)^3,x, algorithm="giac")
```

output

```
integrate((d*x + c)^3*cot(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \cot^3(a+bx)\,dx = \int \cot(a+b\,x)^3\,(c+d\,x)^3\,dx$$

input
```
int(cot(a + b*x)^3*(c + d*x)^3,x)
```

output
```
int(cot(a + b*x)^3*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \cot^3(a+bx)\,dx$$
$$= \frac{4\left(\int \cot(bx+a)^3\,x^3 dx\right)\sin(bx+a)^2\,b\,d^3 + 12\left(\int \cot(bx+a)^3\,x^2 dx\right)\sin(bx+a)^2\,bc\,d^2 + 12\left(\int \cot(bx+\right.}{}$$

input
```
int((d*x+c)^3*cot(b*x+a)^3,x)
```

output
```
(4*int(cot(a + b*x)**3*x**3,x)*sin(a + b*x)**2*b*d**3 + 12*int(cot(a + b*x
)**3*x**2,x)*sin(a + b*x)**2*b*c*d**2 + 12*int(cot(a + b*x)**3*x,x)*sin(a
 + b*x)**2*b*c**2*d + 4*log(tan((a + b*x)/2)**2 + 1)*sin(a + b*x)**2*c**3 -
 4*log(tan((a + b*x)/2))*sin(a + b*x)**2*c**3 + sin(a + b*x)**2*c**3 - 2*c
**3)/(4*sin(a + b*x)**2*b)
```

3.180 $\int (c+dx)^2 \cot^3(a+bx)\,dx$

Optimal result . . . 1648
Mathematica [B] (warning: unable to verify) . . . 1649
Rubi [A] (verified) . . . 1650
Maple [B] (verified) . . . 1655
Fricas [B] (verification not implemented) . . . 1655
Sympy [F] . . . 1656
Maxima [B] (verification not implemented) . . . 1656
Giac [F] . . . 1657
Mupad [F(-1)] . . . 1658
Reduce [F] . . . 1658

Optimal result

Integrand size = 16, antiderivative size = 161

$$\int (c+dx)^2 \cot^3(a+bx)\,dx = -\frac{(c+dx)^2}{2b} + \frac{i(c+dx)^3}{3d} - \frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b} - \frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b} + \frac{d^2\log(\sin(a+bx))}{b^3} + \frac{id(c+dx)\,\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2} - \frac{d^2\,\mathrm{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}$$

output

```
-1/2*(d*x+c)^2/b+1/3*I*(d*x+c)^3/d-d*(d*x+c)*cot(b*x+a)/b^2-1/2*(d*x+c)^2*
cot(b*x+a)^2/b-(d*x+c)^2*ln(1-exp(2*I*(b*x+a)))/b+d^2*ln(sin(b*x+a))/b^3+I
*d*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,exp(2*I*(b*x+
a)))/b^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 548 vs. $2(161) = 322$.

Time = 6.49 (sec) , antiderivative size = 548, normalized size of antiderivative = 3.40

$$\int (c+dx)^2 \cot^3(a+bx)\,dx = -\frac{1}{3}x\left(3c^2+3cdx+d^2x^2\right)\cot(a) - \frac{(c+dx)^2\csc^2(a+bx)}{2b}$$
$$+\frac{d^2e^{ia}\csc(a)\left(2b^3e^{-2ia}x^3+3ib^2(1-e^{-2ia})\,x^2\log\left(1-e^{-i(a+bx)}\right)+3ib^2(1-e^{-2ia})\,x^2\log\left(1+e^{-i(a+bx)}\right)\right.}{}$$
$$-\frac{c^2\csc(a)(-bx\cos(a)+\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a))}{b\left(\cos^2(a)+\sin^2(a)\right)}$$
$$+\frac{d^2\csc(a)(-bx\cos(a)+\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a))}{b^3\left(\cos^2(a)+\sin^2(a)\right)}$$
$$+\frac{\csc(a)\csc(a+bx)\left(cd\sin(bx)+d^2x\sin(bx)\right)}{b^2}$$
$$+\frac{cd\csc(a)\sec(a)\left(b^2e^{i\arctan(\tan(a))}x^2+\frac{(ibx(-\pi+2\arctan(\tan(a)))-\pi\log(1+e^{-2ibx})-2(bx+\arctan(\tan(a)))\log(1-e^{2i(bx+\arct}{}\right.}{}$$
$$+\frac{}{b^2\sqrt{\sec^2(a)\left(\cos^2(a)+\right.}}$$

input
```
Integrate[(c + d*x)^2*Cot[a + b*x]^3,x]
```

output
```
-1/3*(x*(3*c^2 + 3*c*d*x + d^2*x^2)*Cot[a]) - ((c + d*x)^2*Csc[a + b*x]^2)
/(2*b) + (d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^(
(-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*
x^2*Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, -E^(
(-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, E^((-I)*(a + b*x))]
 + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^((-I)*(a + b*x))] + (6*I)*(1 - E
^((-2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]))/(6*b^3) - (c^2*Csc[a]*(-(b*x
*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 +
Sin[a]^2)) + (d^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin
[b*x]]*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (Csc[a]*Csc[a + b*x]*(c*d*Si
n[b*x] + d^2*x*Sin[b*x]))/b^2 + (c*d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]
])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2
*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log
[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2
, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^2*Sqr
t[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.22, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.062$, Rules used = {3042, 25, 4203, 25, 3042, 25, 4202, 2620, 3011, 2720, 4203, 17, 25, 3042, 25, 3956, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \cot^3(a+bx)\,dx$$

$$\downarrow 3042$$

$$\int -(c+dx)^2 \tan\left(a+bx+\frac{\pi}{2}\right)^3 dx$$

$$\downarrow 25$$

$$-\int (c+dx)^2 \tan\left(\frac{1}{2}(2a+\pi)+bx\right)^3 dx$$

$$\downarrow 4203$$

$$\frac{d\int (c+dx)\cot^2(a+bx)dx}{b} + \int -(c+dx)^2\cot(a+bx)dx - \frac{(c+dx)^2\cot^2(a+bx)}{2b}$$

$$\downarrow 25$$

$$\frac{d\int (c+dx)\cot^2(a+bx)dx}{b} - \int (c+dx)^2\cot(a+bx)dx - \frac{(c+dx)^2\cot^2(a+bx)}{2b}$$

$$\downarrow 3042$$

$$-\int -(c+dx)^2\tan\left(a+bx+\frac{\pi}{2}\right)dx + \frac{d\int (c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)^2 dx}{b} - \frac{(c+dx)^2\cot^2(a+bx)}{2b}$$

$$\downarrow 25$$

$$\frac{d\int (c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)^2 dx}{b} + \int (c+dx)^2\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx - \frac{(c+dx)^2\cot^2(a+bx)}{2b}$$

$$\downarrow 4202$$

$$-2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)^2}{1+e^{i(2a+2bx+\pi)}}dx+\frac{d\int (c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)^2 dx}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\downarrow 2620$$

$$-2i\left(\frac{id\int (c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)+\frac{d\int (c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)^2 dx}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\downarrow 3011$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)+\frac{d\int (c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)^2 dx}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\downarrow 2720$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+\right.}{2b}\right)+\frac{d\int (c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)^2 dx}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\downarrow 4203$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+\right.}{2b}\right)+\frac{d\left(-\frac{d\int -\cot(a+bx)dx}{b}-\int (c+dx)dx-\frac{(c+dx)\cot(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\downarrow 17$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+\right.}{2b}\right.$$
$$\left.\frac{d\left(-\frac{d\int-\cot(a+bx)dx}{b}-\frac{(c+dx)\cot(a+bx)}{b}-\frac{(c+dx)^2}{2d}\right)}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}\right)$$

$$\downarrow 25$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+\right.}{2b}\right.$$
$$\left.\frac{d\left(\frac{d\int\cot(a+bx)dx}{b}-\frac{(c+dx)\cot(a+bx)}{b}-\frac{(c+dx)^2}{2d}\right)}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}\right)$$

$$\downarrow 3042$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+\right.}{2b}\right.$$
$$\left.\frac{d\left(\frac{d\int-\tan\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cot(a+bx)}{b}-\frac{(c+dx)^2}{2d}\right)}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}\right)$$

$$\downarrow 25$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+\right.}{2b}\right.$$
$$\left.\frac{d\left(-\frac{d\int\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx}{b}-\frac{(c+dx)\cot(a+bx)}{b}-\frac{(c+dx)^2}{2d}\right)}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}\right)$$

$$\downarrow 3956$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+\right.}{2b}\right.$$
$$\left.\frac{d\left(\frac{d\log(-\sin(a+bx))}{b^2}-\frac{(c+dx)\cot(a+bx)}{b}-\frac{(c+dx)^2}{2d}\right)}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}\right)$$

$$\downarrow 7143$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\,\mathrm{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)+\frac{d\left(\frac{d\log(-\sin(a+bx))}{b^2}-\frac{(c+dx)\cot(a+bx)}{b}-\frac{(c+dx)^2}{2d}\right)}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

input
```
Int[(c + d*x)^2*Cot[a + b*x]^3,x]
```

output
```
((I/3)*(c + d*x)^3)/d - ((c + d*x)^2*Cot[a + b*x]^2)/(2*b) + (d*(-1/2*(c +
 d*x)^2/d - ((c + d*x)*Cot[a + b*x])/b + (d*Log[-Sin[a + b*x]])/b^2))/b -
(2*I)*(((-1/2*I)*(c + d*x)^2*Log[1 + E^(I*(2*a + Pi + 2*b*x))])/b + (I*d*(
((I/2)*(c + d*x)*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/b - (d*PolyLog[3,
-E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)))/b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 643 vs. 2(145) = 290.

Time = 0.37 (sec) , antiderivative size = 644, normalized size of antiderivative = 4.00

method	result
risch	$\frac{2cda\ln\left(e^{i(bx+a)}-1\right)}{b^2}-\frac{4cda\ln\left(e^{i(bx+a)}\right)}{b^2}+\frac{2bd^2x^2e^{2i(bx+a)}+4bcdx\,e^{2i(bx+a)}+2bc^2e^{2i(bx+a)}-2id^2x\,e^{2i(bx+a)}-2icd\,e^{2i(bx+a)}}{b^2\left(e^{2i(bx+a)}-1\right)^2}$

input
```
int((d*x+c)^2*cot(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
4*I/b*c*d*a*x+2*(b*d^2*x^2*exp(2*I*(b*x+a))+2*b*c*d*x*exp(2*I*(b*x+a))+b*c
^2*exp(2*I*(b*x+a))-I*d^2*x*exp(2*I*(b*x+a))-I*c*d*exp(2*I*(b*x+a))+I*d^2*
x+I*d*c)/b^2/(exp(2*I*(b*x+a))-1)^2-2/b^2*c*d*ln(1-exp(I*(b*x+a)))*a+2*I/b
^2*c*d*a^2-2*I/b^2*d^2*a^2*x+2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+2*I/
b^2*d^2*polylog(2,exp(I*(b*x+a)))*x+2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))
+2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))-1/3*I/d*c^3+I*d*c*x^2+2/b^3*d^2*a^2
*ln(exp(I*(b*x+a)))-1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)-1/b*d^2*ln(exp(I*(b
*x+a))+1)*x^2-1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2+1/b^3*d^2*ln(1-exp(I*(b*x+a
)))*a^2-4/3*I/b^3*d^2*a^3-2/b*c*d*ln(exp(I*(b*x+a))+1)*x-4/b^2*c*d*a*ln(ex
p(I*(b*x+a)))+2/b^2*c*d*a*ln(exp(I*(b*x+a))-1)-2/b*c*d*ln(1-exp(I*(b*x+a))
)*x+2/b*c^2*ln(exp(I*(b*x+a)))-1/b*c^2*ln(exp(I*(b*x+a))-1)-1/b*c^2*ln(exp
(I*(b*x+a))+1)-I*c^2*x-2/b^3*d^2*ln(exp(I*(b*x+a)))+1/b^3*d^2*ln(exp(I*(b*
x+a))-1)+1/b^3*d^2*ln(exp(I*(b*x+a))+1)+1/3*I*d^2*x^3-2*d^2*polylog(3,-exp
(I*(b*x+a)))/b^3-2*d^2*polylog(3,exp(I*(b*x+a)))/b^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 657 vs. 2(142) = 284.

Time = 0.09 (sec) , antiderivative size = 657, normalized size of antiderivative = 4.08

$$\int (c+dx)^2\cot^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*cot(b*x+a)^3,x, algorithm="fricas")
```

output

```
1/4*(4*b^2*d^2*x^2 + 8*b^2*c*d*x + 4*b^2*c^2 - 2*(I*b*d^2*x + I*b*c*d + (-
I*b*d^2*x - I*b*c*d)*cos(2*b*x + 2*a))*dilog(cos(2*b*x + 2*a) + I*sin(2*b*
x + 2*a)) - 2*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(2*b*x + 2*
a))*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 2*(b^2*c^2 - 2*a*b*c*d
+ (a^2 - 1)*d^2 - (b^2*c^2 - 2*a*b*c*d + (a^2 - 1)*d^2)*cos(2*b*x + 2*a))*
log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b^2*c^2 - 2
*a*b*c*d + (a^2 - 1)*d^2 - (b^2*c^2 - 2*a*b*c*d + (a^2 - 1)*d^2)*cos(2*b*x
 + 2*a))*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b^
2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x
 + 2*a*b*c*d - a^2*d^2)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) + I*sin(2*
b*x + 2*a) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^
2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(2*b*x + 2*a))*log(-cos(
2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) - (d^2*cos(2*b*x + 2*a) - d^2)*poly
log(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) - (d^2*cos(2*b*x + 2*a) - d^
2)*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 4*(b*d^2*x + b*c*d)
*sin(2*b*x + 2*a))/(b^3*cos(2*b*x + 2*a) - b^3)
```

Sympy [F]

$$\int (c+dx)^2 \cot^3(a+bx)\,dx = \int (c+dx)^2 \cot^3(a+bx)\,dx$$

input

```
integrate((d*x+c)**2*cot(b*x+a)**3,x)
```

output

```
Integral((c + d*x)**2*cot(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1966 vs. $2(142) = 284$.

Time $= 0.30$ (sec) , antiderivative size $= 1966$, normalized size of antiderivative $= 12.21$

$$\int (c+dx)^2 \cot^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*cot(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(c^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) - 2*a*c*d*(1/sin(b*x +
a)^2 + log(sin(b*x + a)^2))/b + a^2*d^2*(1/sin(b*x + a)^2 + log(sin(b*x +
a)^2))/b^2 - 2*(2*(b*x + a)^3*d^2 + 6*(b*c*d - a*d^2)*(b*x + a)^2 + 12*b*c
*d - 12*a*d^2 - 6*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2 + (
(b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(4*b*x + 4*a) - 2*
((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(2*b*x + 2*a) - (
-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a) + I*d^2)*sin(4*b*x +
 4*a) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) - I*d^2)*si
n(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 6*(d^2*cos(4*b*x
 + 4*a) - 2*d^2*cos(2*b*x + 2*a) + I*d^2*sin(4*b*x + 4*a) - 2*I*d^2*sin(2*
b*x + 2*a) + d^2)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 6*((b*x + a)^2
*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*
(b*x + a))*cos(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
+ a))*cos(2*b*x + 2*a) + (I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x +
 a))*sin(4*b*x + 4*a) + 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*
x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 2*((b
*x + a)^3*d^2 + 3*(b*c*d - a*d^2)*(b*x + a)^2 - 6*(b*x + a)*d^2)*cos(4*b*x
 + 4*a) - 4*((b*x + a)^3*d^2 + 3*(b*c*d - (a - I)*d^2)*(b*x + a)^2 + 3*b*c
*d - 3*a*d^2 - 3*(-2*I*b*c*d + (2*I*a + 1)*d^2)*(b*x + a))*cos(2*b*x + 2*a
) + 12*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)...
```

Giac [F]

$$\int (c+dx)^2 \cot^3(a+bx)\,dx = \int (dx+c)^2 \cot(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^2*cot(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*cot(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \cot^3(a+bx)\,dx = \int \cot(a+b\,x)^3\,(c+d\,x)^2\,dx$$

input
```
int(cot(a + b*x)^3*(c + d*x)^2,x)
```

output
```
int(cot(a + b*x)^3*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \cot^3(a+bx)\,dx$$
$$= \frac{4\left(\int \cot(bx+a)^3\,x^2 dx\right)\sin(bx+a)^2\,b\,d^2 + 8\left(\int \cot(bx+a)^3\,x dx\right)\sin(bx+a)^2\,bcd + 4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}{4\sin(bx+a)^2\,b}$$

input
```
int((d*x+c)^2*cot(b*x+a)^3,x)
```

output
```
(4*int(cot(a + b*x)**3*x**2,x)*sin(a + b*x)**2*b*d**2 + 8*int(cot(a + b*x)
**3*x,x)*sin(a + b*x)**2*b*c*d + 4*log(tan((a + b*x)/2)**2 + 1)*sin(a + b*
x)**2*c**2 - 4*log(tan((a + b*x)/2))*sin(a + b*x)**2*c**2 + sin(a + b*x)**
2*c**2 - 2*c**2)/(4*sin(a + b*x)**2*b)
```

3.181 $\int (c+dx)\cot^3(a+bx)\,dx$

Optimal result . . . 1659
Mathematica [B] (warning: unable to verify) . . . 1659
Rubi [A] (verified) . . . 1660
Maple [B] (verified) . . . 1663
Fricas [B] (verification not implemented) . . . 1664
Sympy [F] . . . 1664
Maxima [B] (verification not implemented) . . . 1665
Giac [F] . . . 1666
Mupad [F(-1)] . . . 1666
Reduce [F] . . . 1666

Optimal result

Integrand size = 14, antiderivative size = 109

$$\int (c+dx)\cot^3(a+bx)\,dx = -\frac{dx}{2b} + \frac{i(c+dx)^2}{2d} - \frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} - \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} + \frac{id\,\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}$$

output

```
-1/2*d*x/b+1/2*I*(d*x+c)^2/d-1/2*d*cot(b*x+a)/b^2-1/2*(d*x+c)*cot(b*x+a)^2
/b-(d*x+c)*ln(1-exp(2*I*(b*x+a)))/b+1/2*I*d*polylog(2,exp(2*I*(b*x+a)))/b^
2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 234 vs. 2(109) = 218.

Time = 5.37 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.15

$$\int (c+dx)\cot^3(a+bx)\,dx = -\frac{1}{2}dx^2\cot(a) - \frac{c\csc^2(a+bx)}{2b} - \frac{dx\csc^2(a+bx)}{2b} - \frac{c\log(\sin(a+bx))}{b} + \frac{d\csc(a)\csc(a+bx)\sin(bx)}{2b^2} + \frac{d\csc(a)\sec(a)\left(b^2e^{i\arctan(\tan(a))}x^2 + \frac{(ibx(-\pi+2\arctan(\tan(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx+\arctan(\tan(a)))\log\left(1-e^{2i(bx+\arctan}}{2b^2\sqrt{\sec^2(a)\left(\cos^2(a)+\right.}}\right.$$

input
```
Integrate[(c + d*x)*Cot[a + b*x]^3,x]
```

output
```
-1/2*(d*x^2*Cot[a]) - (c*Csc[a + b*x]^2)/(2*b) - (d*x*Csc[a + b*x]^2)/(2*b
) - (c*Log[Sin[a + b*x]])/b + (d*Csc[a]*Csc[a + b*x]*Sin[b*x])/(2*b^2) + (
d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Ta
n[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^(
(2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[S
in[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))]
)*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)]
)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 25, 4203, 25, 3042, 25, 3954, 24, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cot^3(a+bx)\,dx$$

$$\downarrow 3042$$

$$\int -\left((c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)^3\right)dx$$

$$\downarrow 25$$

$$-\int (c+dx)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)^3 dx$$

$$\downarrow 4203$$

$$\int -((c+dx)\cot(a+bx))dx + \frac{d\int \cot^2(a+bx)dx}{2b} - \frac{(c+dx)\cot^2(a+bx)}{2b}$$

$$\downarrow 25$$

$$-\int (c+dx)\cot(a+bx)dx + \frac{d\int \cot^2(a+bx)dx}{2b} - \frac{(c+dx)\cot^2(a+bx)}{2b}$$

$$\downarrow 3042$$

$$-\int -\left((c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)\right)dx+\frac{d\int \tan\left(a+bx+\frac{\pi}{2}\right)^2 dx}{2b}-\frac{(c+dx)\cot^2(a+bx)}{2b}$$

$$\downarrow 25$$

$$\int (c+dx)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx+\frac{d\int \tan\left(a+bx+\frac{\pi}{2}\right)^2 dx}{2b}-\frac{(c+dx)\cot^2(a+bx)}{2b}$$

$$\downarrow 3954$$

$$\int (c+dx)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx+\frac{d\left(-\int 1dx-\frac{\cot(a+bx)}{b}\right)}{2b}-\frac{(c+dx)\cot^2(a+bx)}{2b}$$

$$\downarrow 24$$

$$\int (c+dx)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx-\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{d\left(-\frac{\cot(a+bx)}{b}-x\right)}{2b}$$

$$\downarrow 4202$$

$$-2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)}{1+e^{i(2a+2bx+\pi)}}dx-\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{d\left(-\frac{\cot(a+bx)}{b}-x\right)}{2b}+\frac{i(c+dx)^2}{2d}$$

$$\downarrow 2620$$

$$-2i\left(\frac{id\int \log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{d\left(-\frac{\cot(a+bx)}{b}-x\right)}{2b}+\frac{i(c+dx)^2}{2d}$$

$$\downarrow 2715$$

$$-2i\left(\frac{d\int e^{-i(2a+2bx+\pi)}\log\left(1+e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{d\left(-\frac{\cot(a+bx)}{b}-x\right)}{2b}+\frac{i(c+dx)^2}{2d}$$

$$\downarrow 2838$$

$$-2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{d\left(-\frac{\cot(a+bx)}{b}-x\right)}{2b}+\frac{i(c+dx)^2}{2d}$$

input
```
Int[(c + d*x)*Cot[a + b*x]^3,x]
```

output
```
((I/2)*(c + d*x)^2)/d - ((c + d*x)*Cot[a + b*x]^2)/(2*b) + (d*(-x - Cot[a
+ b*x]/b))/(2*b) - (2*I)*(((-1/2*I)*(c + d*x)*Log[1 + E^(I*(2*a + Pi + 2*b
*x))])/b - (d*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4202

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(93) = 186$.

Time = 0.26 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.58

method	result
risch	$\frac{idx^2}{2} - ixc + \frac{2bdx\,e^{2i(bx+a)} - id\,e^{2i(bx+a)} + 2bc\,e^{2i(bx+a)} + id}{b^2\left(e^{2i(bx+a)}-1\right)^2} + \frac{2c\ln\left(e^{i(bx+a)}\right)}{b} - \frac{c\ln\left(e^{i(bx+a)}-1\right)}{b} - \frac{c\ln\left(e^{i(bx+a)}+1\right)}{b} +$

input

```
int((d*x+c)*cot(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output

```
1/2*I*d*x^2-I*x*c+(2*b*d*x*exp(2*I*(b*x+a))-I*d*exp(2*I*(b*x+a))+2*b*c*exp
(2*I*(b*x+a))+I*d)/b^2/(exp(2*I*(b*x+a))-1)^2+2/b*c*ln(exp(I*(b*x+a)))-1/b
*c*ln(exp(I*(b*x+a))-1)-1/b*c*ln(exp(I*(b*x+a))+1)+I/b^2*d*polylog(2,exp(I
*(b*x+a)))+I*d*polylog(2,-exp(I*(b*x+a)))/b^2+I/b^2*d*a^2-1/b*d*ln(exp(I*(
b*x+a))+1)*x+2*I/b*d*a*x-1/b*d*ln(1-exp(I*(b*x+a)))*x-1/b^2*d*ln(1-exp(I*(
b*x+a)))*a-2/b^2*d*a*ln(exp(I*(b*x+a)))+1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(90) = 180$.

Time = 0.08 (sec) , antiderivative size = 339, normalized size of antiderivative = 3.11

$$\int (c+dx)\cot^3(a+bx)\,dx$$
$$= \frac{4\,bdx + 4\,bc + (i\,d\cos(2\,bx+2\,a) - i\,d)\mathrm{Li}_2(\cos(2\,bx+2\,a)+i\,\sin(2\,bx+2\,a)) + (-i\,d\cos(2\,bx+2\,a) +}{}$$

input
```
integrate((d*x+c)*cot(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(4*b*d*x + 4*b*c + (I*d*cos(2*b*x + 2*a) - I*d)*dilog(cos(2*b*x + 2*a)
 + I*sin(2*b*x + 2*a)) + (-I*d*cos(2*b*x + 2*a) + I*d)*dilog(cos(2*b*x + 2
*a) - I*sin(2*b*x + 2*a)) + 2*(b*c - a*d - (b*c - a*d)*cos(2*b*x + 2*a))*l
og(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b*c - a*d -
(b*c - a*d)*cos(2*b*x + 2*a))*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x
+ 2*a) + 1/2) + 2*(b*d*x + a*d - (b*d*x + a*d)*cos(2*b*x + 2*a))*log(-cos(
2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1) + 2*(b*d*x + a*d - (b*d*x + a*d)*co
s(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) + 2*d*sin(
2*b*x + 2*a))/(b^2*cos(2*b*x + 2*a) - b^2)
```

Sympy [F]

$$\int (c+dx)\cot^3(a+bx)\,dx = \int (c+dx)\cot^3(a+bx)\,dx$$

input
```
integrate((d*x+c)*cot(b*x+a)**3,x)
```

output
```
Integral((c + d*x)*cot(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(90) = 180$.

Time = 0.19 (sec) , antiderivative size = 830, normalized size of antiderivative = 7.61

$$\int (c + dx) \cot^3(a + bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*cot(b*x+a)^3,x, algorithm="maxima")
```

output
```
(b^2*d*x^2 + 2*b^2*c*x - 2*(b*d*x + b*c + (b*d*x + b*c)*cos(4*b*x + 4*a) -
 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (-I*b*d*x - I*b*c)*sin(4*b*x + 4*a) -
2*(I*b*d*x + I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) +
 1) - 2*(b*c*cos(4*b*x + 4*a) - 2*b*c*cos(2*b*x + 2*a) + I*b*c*sin(4*b*x +
 4*a) - 2*I*b*c*sin(2*b*x + 2*a) + b*c)*arctan2(sin(b*x + a), cos(b*x + a)
 - 1) + 2*(b*d*x*cos(4*b*x + 4*a) - 2*b*d*x*cos(2*b*x + 2*a) + I*b*d*x*sin
(4*b*x + 4*a) - 2*I*b*d*x*sin(2*b*x + 2*a) + b*d*x)*arctan2(sin(b*x + a),
-cos(b*x + a) + 1) + (b^2*d*x^2 + 2*b^2*c*x)*cos(4*b*x + 4*a) - 2*(b^2*d*x
^2 + 2*I*b*c + 2*(b^2*c + I*b*d)*x + d)*cos(2*b*x + 2*a) + 2*(d*cos(4*b*x
+ 4*a) - 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) - 2*I*d*sin(2*b*x + 2
*a) + d)*dilog(-e^(I*b*x + I*a)) + 2*(d*cos(4*b*x + 4*a) - 2*d*cos(2*b*x +
 2*a) + I*d*sin(4*b*x + 4*a) - 2*I*d*sin(2*b*x + 2*a) + d)*dilog(e^(I*b*x
+ I*a)) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(4*b*x + 4*a) - 2*(-I*
b*d*x - I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(4*b*x + 4*a) - 2*(b*d*
x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x
 + a) + 1) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(4*b*x + 4*a) - 2*(
-I*b*d*x - I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(4*b*x + 4*a) - 2*(b
*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(
b*x + a) + 1) - (-I*b^2*d*x^2 - 2*I*b^2*c*x)*sin(4*b*x + 4*a) + 2*(-I*b^2*
d*x^2 + 2*b*c + 2*(-I*b^2*c + b*d)*x - I*d)*sin(2*b*x + 2*a) + 2*d)/(-2...
```

Giac [F]

$$\int (c+dx)\cot^3(a+bx)\,dx = \int (dx+c)\cot\left(bx+a\right)^3\,dx$$

input
```
integrate((d*x+c)*cot(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*cot(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\cot^3(a+bx)\,dx = \int \cot(a+b\,x)^3\,(c+d\,x)\,dx$$

input
```
int(cot(a + b*x)^3*(c + d*x),x)
```

output
```
int(cot(a + b*x)^3*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)\cot^3(a+bx)\,dx$$
$$= \frac{4\left(\int \cot\left(bx+a\right)^3 x dx\right)\sin\left(bx+a\right)^2 bd + 4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right)\sin\left(bx+a\right)^2 c - 4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)$$
$$4\sin\left(bx+a\right)^2 b$$

input
```
int((d*x+c)*cot(b*x+a)^3,x)
```

output
```
(4*int(cot(a + b*x)**3*x,x)*sin(a + b*x)**2*b*d + 4*log(tan((a + b*x)/2)**
2 + 1)*sin(a + b*x)**2*c - 4*log(tan((a + b*x)/2))*sin(a + b*x)**2*c + sin
(a + b*x)**2*c - 2*c)/(4*sin(a + b*x)**2*b)
```

3.182 $\int \frac{\cot^3(a+bx)}{c+dx}\,dx$

Optimal result 1667
Mathematica [N/A] 1667
Rubi [N/A] 1668
Maple [N/A] 1669
Fricas [N/A] 1669
Sympy [N/A] 1670
Maxima [N/A] 1670
Giac [N/A] 1671
Mupad [N/A] 1672
Reduce [N/A] 1672

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\cot^3(a+bx)}{c+dx}, x\right)$$

output
```
Defer(Int)(cot(b*x+a)^3/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 6.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx = \int \frac{\cot^3(a+bx)}{c+dx}\,dx$$

input
```
Integrate[Cot[a + b*x]^3/(c + d*x),x]
```

output
```
Integrate[Cot[a + b*x]^3/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 25, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{\tan\left(a+bx+\frac{\pi}{2}\right)^3}{c+dx}\,dx$$
$$\downarrow 25$$
$$-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)^3}{c+dx}\,dx$$
$$\downarrow 4222$$
$$-\int \frac{\tan^3\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}\,dx$$

input
```
Int[Cot[a + b*x]^3/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cot\left(bx+a\right)^3}{dx+c}dx$$

input

```
int(cot(b*x+a)^3/(d*x+c),x)
```

output

```
int(cot(b*x+a)^3/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx = \int \frac{\cot\left(bx+a\right)^3}{dx+c}\,dx$$

input

```
integrate(cot(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output

```
integral(cot(b*x + a)^3/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx = \int \frac{\cot^3(a+bx)}{c+dx}\,dx$$

input
```
integrate(cot(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(cot(a + b*x)**3/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 1636, normalized size of antiderivative = 102.25

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx = \int \frac{\cot(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(cot(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output

```
-(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2
- (2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
 - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x
^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*
sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate((b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2 - d^2)*sin(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*
x + b^2*c^3)*cos(b*x + a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3)*sin(b*x + a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^
2*d*x + b^2*c^3)*cos(b*x + a)), x) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
+ (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*s
in(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d...
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx = \int \frac{\cot\left(bx+a\right)^3}{dx+c}\,dx$$

input

```
integrate(cot(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output

```
integrate(cot(b*x + a)^3/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 17.97 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+b\,x)^3}{c+d\,x}\,dx$$

input
```
int(cot(a + b*x)^3/(c + d*x),x)
```

output
```
int(cot(a + b*x)^3/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx = \int \frac{\cot\left(bx+a\right)^3}{dx+c}\,dx$$

input
```
int(cot(b*x+a)^3/(d*x+c),x)
```

output
```
int(cot(a + b*x)**3/(c + d*x),x)
```

3.183 $\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx$

Optimal result 1673
Mathematica [N/A] 1673
Rubi [N/A] 1674
Maple [N/A] 1675
Fricas [N/A] 1676
Sympy [N/A] 1676
Maxima [N/A] 1676
Giac [N/A] 1677
Mupad [N/A] 1678
Reduce [N/A] 1678

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\cot^3(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(cot(b*x+a)^3/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 7.50 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[Cot[a + b*x]^3/(c + d*x)^2,x]
```

output
```
Integrate[Cot[a + b*x]^3/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 25, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx$$

$$\downarrow 3042$$

$$\int -\frac{\tan\left(a+bx+\frac{\pi}{2}\right)^3}{(c+dx)^2}\,dx$$

$$\downarrow 25$$

$$-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)^3}{(c+dx)^2}\,dx$$

$$\downarrow 4222$$

$$-\int \frac{\tan^3\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}\,dx$$

input
```
Int[Cot[a + b*x]^3/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cot(bx+a)^3}{(dx+c)^2}dx$$

input

```
int(cot(b*x+a)^3/(d*x+c)^2,x)
```

output

```
int(cot(b*x+a)^3/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot\left(bx+a\right)^3}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(cot(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cot(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot^3\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(cot(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(cot(a + b*x)**3/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 11.38 (sec) , antiderivative size = 2124, normalized size of antiderivative = 132.75

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot\left(bx+a\right)^3}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(cot(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output

```
-(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2
- 2*((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
 - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^
2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*
c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 +
 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3
*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*
a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x
 + 2*a))*integrate((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 3*d^2)*sin(b*x +
 a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b
^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*
x + b^2*c^4)*cos(b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d
^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(b*x + a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*
c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x)
 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b...
```

Giac [N/A]

Not integrable

Time = 2.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot\left(bx+a\right)^3}{\left(dx+c\right)^2}\,dx$$

input

```
integrate(cot(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output

```
integrate(cot(b*x + a)^3/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 18.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(a+b\,x)^3}{(c+d\,x)^2}\,dx$$

input
```
int(cot(a + b*x)^3/(c + d*x)^2,x)
```

output
```
int(cot(a + b*x)^3/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot\left(bx+a\right)^3}{d^2x^2+2cdx+c^2}dx$$

input
```
int(cot(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int(cot(a + b*x)**3/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.184 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 1679
Mathematica [C] (verified) . . . 1680
Rubi [A] (verified) . . . 1681
Maple [A] (verified) . . . 1683
Fricas [A] (verification not implemented) . . . 1684
Sympy [F(-1)] . . . 1685
Maxima [C] (verification not implemented) . . . 1685
Giac [C] (verification not implemented) . . . 1686
Mupad [F(-1)] . . . 1687
Reduce [F] . . . 1687

Optimal result

Integrand size = 24, antiderivative size = 407

$$\begin{aligned}\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx &= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3} \\ &- \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} + \frac{15d^2\sqrt{c+dx}\cos(4a+4bx)}{2048b^3} \\ &- \frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b} \\ &- \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} \\ &- \frac{15d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}} \\ &+ \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{4096b^{7/2}} \\ &+ \frac{15d^{5/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{256b^{7/2}} \\ &+ \frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{32b^2} + \frac{5d(c+dx)^{3/2}\sin(4a+4bx)}{256b^2}\end{aligned}$$

output

```
15/128*d^2*(d*x+c)^(1/2)*cos(2*b*x+2*a)/b^3-1/8*(d*x+c)^(5/2)*cos(2*b*x+2*
a)/b+15/2048*d^2*(d*x+c)^(1/2)*cos(4*b*x+4*a)/b^3-1/32*(d*x+c)^(5/2)*cos(4
*b*x+4*a)/b-15/8192*d^(5/2)*2^(1/2)*Pi^(1/2)*cos(4*a-4*b*c/d)*FresnelC(2*b
^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-15/256*d^(5/2)*Pi^(
1/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b
^(7/2)+15/8192*d^(5/2)*2^(1/2)*Pi^(1/2)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2
)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)/b^(7/2)+15/256*d^(5/2)*Pi^(1/2)*
FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/b^(7/2
)+5/32*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2+5/256*d*(d*x+c)^(3/2)*sin(4*b*x+
4*a)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.84 (sec) , antiderivative size = 1332, normalized size of antiderivative = 3.27

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input

```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output

```
((1/128 + I/128)*c*Sqrt[d]*((2 + 2*I)*Sqrt[b]*Sqrt[d]*E^(((2*I)*b*c)/d)*Sq
rt[c + d*x]*(3 + (4*I)*b*x + E^((4*I)*(a + b*x))*(-3 + (4*I)*b*x)) + I*(4*
b*c + (3*I)*d)*E^(((2*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[((1 + I)*Sqrt[b]*S
qrt[c + d*x])/Sqrt[d]] + ((4*I)*b*c + 3*d)*E^((2*I)*(2*a + b*x))*Sqrt[Pi]*
Erfi[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^(((2*I)*(a*d +
b*(c + d*x)))/d)) + (c*Sqrt[d]*(-4*Sqrt[b]*Sqrt[d]*E^(((4*I)*b*c)/d)*Sqrt[
c + d*x]*(-3*I + 8*b*x + E^((8*I)*(a + b*x))*(3*I + 8*b*x)) + (-1)^(3/4)*(
8*b*c + (3*I)*d)*E^(((4*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[(2*(-1)^(1/4)*Sq
rt[b]*Sqrt[c + d*x])/Sqrt[d]] + (-1)^(1/4)*((8*I)*b*c + 3*d)*E^((4*I)*(2*a
 + b*x))*Sqrt[Pi]*Erfi[(2*(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(10
24*b^(5/2)*E^(((4*I)*(a*d + b*(c + d*x)))/d)) + (c^2*(-1/4*(E^((2*I)*(a -
(b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d])/(Sqrt[2]*b*Sqr
t[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma[3/2, ((2*I)*b*(c + d*x))/d
])/(4*Sqrt[2]*b*E^((2*I)*(a - (b*c)/d))*Sqrt[(I*b*(c + d*x))/d])))/4 + (c^
2*Sqrt[c + d*x]*(-((E^((8*I)*a)*Gamma[3/2, ((-4*I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((8*I)*b*c)/d)*Gamma[3/2, ((4*I)*b*(c + d*x))/
d])/Sqrt[(I*b*(c + d*x))/d]))/(128*b*E^(((4*I)*(b*c + a*d))/d)) + (Sqrt[d]
*((1 - I)*E^((2*I)*(a - (b*c)/d))*((2 + 2*I)*Sqrt[b]*Sqrt[d]*E^(((2*I)*b*(
c + d*x))/d)*Sqrt[c + d*x]*(15*d - 16*b^2*d*x^2 + (4*I)*b*(c - 5*d*x)) + (
16*b^2*c^2 - (24*I)*b*c*d - 15*d^2)*Sqrt[Pi]*Erfi[((1 + I)*Sqrt[b]*Sqrt...
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2} \sin(a+bx) \cos^3(a+bx)\, dx$$

$$\downarrow \text{4906}$$

$$\int \left(\frac{1}{4}(c+dx)^{5/2} \sin(2a+2bx) + \frac{1}{8}(c+dx)^{5/2} \sin(4a+4bx) \right) dx$$

$$\downarrow \text{2009}$$

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-$$
$$\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}+$$
$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+$$
$$\frac{15\sqrt{\pi}d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}+$$
$$\frac{15d^2\sqrt{c+dx}\cos(4a+4bx)}{2048b^3}+\frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{32b^2}+\frac{5d(c+dx)^{3/2}\sin(4a+4bx)}{256b^2}-$$
$$\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}$$

input

```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output

```
(15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2
*a + 2*b*x])/(8*b) + (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) -
((c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a
 - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(409
6*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]
*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2
]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)
/d])/(4096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*
x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*
x)^(3/2)*Sin[2*a + 2*b*x])/(32*b^2) + (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x
])/(256*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.15

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}\right.\right.}{\left.\left.\right.\right.}$
default	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}\right.\right.}{\left.\left.\right.\right.}$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+5/16/b*d*(1/
4/b*d*(d*x+c)^(3/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*
x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(
cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(
2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1/64/
b*d*(d*x+c)^(5/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+5/64/b*d*(1/8/b*d*(d*x+
c)^(3/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*
cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(co
s(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d
)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/
2)/d)))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.92

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(}{}\right.}$$

input

```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output

```
-1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_co
s(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d
))*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/
d) + 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*
x + c)*sqrt(b/(pi*d))) - 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x
+ c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(192*b^3*d^2*x^2 + 384*b^3*
c*d*x + 192*b^3*c^2 + 360*b*d^2*cos(b*x + a)^2 - 8*(64*b^3*d^2*x^2 + 128*b
^3*c*d*x + 64*b^3*c^2 - 15*b*d^2)*cos(b*x + a)^4 - 225*b*d^2 + 160*(2*(b^2
*d^2*x + b^2*c*d)*cos(b*x + a)^3 + 3*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*s
in(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.35

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/32768*(640*(d*x + c)^(3/2)*b^3*sin(4*((d*x + c)*b - b*c + a*d)/d) + 5120
*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 16*(64*(d*x + c)
^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(4*((d*x + c)*b - b*c + a*d)/d)
- 256*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c)
*b - b*c + a*d)/d) - 240*(-(I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2
)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^
2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 15*
(-(I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (
I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2
*sqrt(d*x + c)*sqrt(I*b/d)) - 15*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1
/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 240*((I + 1
)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (
I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^5
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 2412, normalized size of antiderivative = 5.93

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
1/16384*(512*(sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)) + 4*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)) + 4*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)))*c^3 - d^3*((sqrt(2)*sqrt(pi)*(512*b^3*c^3 - 192*I*b^2*c^2*d - 72*b*c*
d^2 + 15*I*d^3)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*
b^3) - 4*I*(64*I*(d*x + c)^(5/2)*b^2*d - 192*I*(d*x + c)^(3/2)*b^2*c*d + 1
92*I*sqrt(d*x + c)*b^2*c^2*d - 40*(d*x + c)^(3/2)*b*d^2 + 72*sqrt(d*x + c)
*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/
d)/b^3)/d^3 + (sqrt(2)*sqrt(pi)*(512*b^3*c^3 + 192*I*b^2*c^2*d - 72*b*c*d^
2 - 15*I*d^3)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*
b^3) - 4*I*(64*I*(d*x + c)^(5/2)*b^2*d - 192*I*(d*x + c)^(3/2)*b^2*c*d + 1
92*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x + c)
*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int \cos(a+b\,x)^3\,\sin(a+b\,x)\,(c+d\,x)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \left(\int \sqrt{dx+c}\,\cos(bx+a)^3\sin(bx+a)\,x^2dx\right)d^2$$
$$+2\left(\int \sqrt{dx+c}\,\cos(bx+a)^3\sin(bx+a)\,xdx\right)cd$$
$$+\left(\int \sqrt{dx+c}\,\cos(bx+a)^3\sin(bx+a)\,dx\right)c^2$$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**3*sin(a + b*x)*x**2,x)*d**2 + 2*int(sqrt(c
 + d*x)*cos(a + b*x)**3*sin(a + b*x)*x,x)*c*d + int(sqrt(c + d*x)*cos(a +
b*x)**3*sin(a + b*x),x)*c**2
```

3.185 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 1688
Mathematica [C] (verified) . . . 1689
Rubi [A] (verified) . . . 1690
Maple [A] (verified) . . . 1692
Fricas [A] (verification not implemented) . . . 1692
Sympy [F] . . . 1693
Maxima [C] (verification not implemented) . . . 1693
Giac [C] (verification not implemented) . . . 1694
Mupad [F(-1)] . . . 1695
Reduce [F] . . . 1696

Optimal result

Integrand size = 24, antiderivative size = 351

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} - \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{512b^{5/2}} - \frac{3d^{3/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{64b^{5/2}} + \frac{3d\sqrt{c+dx}\sin(2a+2bx)}{32b^2} + \frac{3d\sqrt{c+dx}\sin(4a+4bx)}{256b^2}$$

output

```
-1/8*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b-1/32*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b-3/
1024*d^(3/2)*2^(1/2)*Pi^(1/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/
Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)-3/64*d^(3/2)*Pi^(1/2)*cos(2*a-2*b*
c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b^(5/2)-3/1024*d^(
3/2)*2^(1/2)*Pi^(1/2)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*sin(4*a-4*b*c/d)/b^(5/2)-3/64*d^(3/2)*Pi^(1/2)*FresnelC(2*b^(1/2)*(
d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/b^(5/2)+3/32*d*(d*x+c)^(1/
2)*sin(2*b*x+2*a)/b^2+3/256*d*(d*x+c)^(1/2)*sin(4*b*x+4*a)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.97

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \frac{e^{-4ia}\left(8e^{2i\left(a-\frac{b(c+dx)}{d}\right)}\left(-4\sqrt{b}de^{\frac{2ibc}{d}}\sqrt{c+dx}\left(-3i+4bx+e^{4i(a+bx)}(3i+4bx)\right)-(1-i)(4bc+3id\right.\right.}{}$$

input

```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output

```
(8*E^((2*I)*(a - (b*(c + d*x))/d))*(-4*Sqrt[b]*d*E^(((2*I)*b*c)/d)*Sqrt[c
+ d*x]*(-3*I + 4*b*x + E^((4*I)*(a + b*x))*(3*I + 4*b*x)) - (1 - I)*(4*b*c
 + (3*I)*d)*Sqrt[d]*E^(((2*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[((1 + I)*Sqrt
[b]*Sqrt[c + d*x])/Sqrt[d]] + (1 + I)*Sqrt[d]*((4*I)*b*c + 3*d)*E^((2*I)*(
2*a + b*x))*Sqrt[Pi]*Erfi[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]) + (-4*
Sqrt[b]*d*E^(((4*I)*b*c)/d)*Sqrt[c + d*x]*(-3*I + 8*b*x + E^((8*I)*(a + b*
x))*(3*I + 8*b*x)) + (-1)^(1/4)*((8*I)*b*c - 3*d)*Sqrt[d]*E^(((4*I)*b*(2*c
 + d*x))/d)*Sqrt[Pi]*Erf[(2*(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + (
-1)^(1/4)*Sqrt[d]*((8*I)*b*c + 3*d)*E^((4*I)*(2*a + b*x))*Sqrt[Pi]*Erfi[(2
*(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/E^(((4*I)*b*(c + d*x))/d) - (
64*Sqrt[2]*b^(3/2)*c*E^((2*I)*(a - (b*c)/d))*Sqrt[c + d*x]*(E^((4*I)*a)*Sq
rt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d] + E^(((4*I)*b*c)/
d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((2*I)*b*(c + d*x))/d]))/Sqrt[(b^
2*(c + d*x)^2)/d^2] + (16*b^(3/2)*c*Sqrt[c + d*x]*(-((E^((8*I)*a)*Gamma[3/
2, ((-4*I)*b*(c + d*x))/d])/Sqrt[((-I)*b*(c + d*x))/d]) - (E^(((8*I)*b*c)/
d)*Gamma[3/2, ((4*I)*b*(c + d*x))/d])/Sqrt[(I*b*(c + d*x))/d]))/E^(((4*I)*
b*c)/d))/(2048*b^(5/2)*E^((4*I)*a))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin(a+bx)\cos^3(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx)+\frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-$$
$$\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}-$$
$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-$$
$$\frac{3\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(2a+2bx)}{32b^2}+$$
$$\frac{3d\sqrt{c+dx}\sin(4a+4bx)}{256b^2}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}$$

input

```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output

```
-1/8*((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/b - ((c + d*x)^(3/2)*Cos[4*a + 4*b
*x])/(32*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[
b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi]
)])/(64*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqr
t[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*
b*c)/d])/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) + (3
*d*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.07

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{8b\sqrt{\frac{b}{d}}}\right.}{8b}$
default	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{8b\sqrt{\frac{b}{d}}}\right.}{8b}$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+3/16/b*d*(1/
4/b*d*(d*x+c)^(1/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d
)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2
)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)
))-1/64/b*d*(d*x+c)^(3/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+3/64/b*d*(1/8/b
*d*(d*x+c)^(1/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2
)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*
b*(d*x+c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.84

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-}{d}\right.}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))
*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d)
 + 48*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x +
 c)*sqrt(b/(pi*d))) + 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)
*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x
+ a)^4 - 6*b^2*d*x - 6*b^2*c - 3*(2*b*d*cos(b*x + a)^3 + 3*b*d*cos(b*x + a
))*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F]

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)*cos(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.43

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output

```
-1/4096*(128*(d*x + c)^(3/2)*b^3*cos(4*((d*x + c)*b - b*c + a*d)/d)/d + 51
2*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 48*sqrt(d*x +
 c)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) - 384*sqrt(d*x + c)*b^2*sin(2*(
(d*x + c)*b - b*c + a*d)/d) + 24*((I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^
2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*
(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) +
3*((I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I
 - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sq
rt(d*x + c)*sqrt(I*b/d)) + 3*(-(I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4
)*cos(-4*(b*c - a*d)/d) + (I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin
(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + 24*(-(I - 1)*4^(1/
4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^
(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(-2*I*b/d)))*d/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 1490, normalized size of antiderivative = 4.25

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output

```
1/2048*(64*(sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) + 4*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)) + 4*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
))*c^2 - 16*(sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqr
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(I*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c
+ I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*sqrt(pi)*(4*b*c -
 I*d)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*
(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*sqrt(pi)*(4
*b*c + I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 16*sq
rt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 4*sqrt(d*x + c)
*d*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^(-2*(
-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4*(-I*(d*x...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)^{3/2}\,dx$$

input

```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(3/2),x)
```

output

```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c + dx)^{3/2} \cos^3(a+bx) \sin(a+bx)\, dx = \left(\int \sqrt{dx+c} \cos(bx+a)^3 \sin(bx+a)\, x dx \right) d + \left(\int \sqrt{dx+c} \cos(bx+a)^3 \sin(bx+a)\, dx \right) c$$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**3*sin(a + b*x)*x,x)*d + int(sqrt(c + d*x)*
cos(a + b*x)**3*sin(a + b*x),x)*c
```

3.186 $\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 1697
Mathematica [C] (verified) . . . 1698
Rubi [A] (verified) . . . 1699
Maple [A] (verified) . . . 1700
Fricas [A] (verification not implemented) . . . 1701
Sympy [F] . . . 1701
Maxima [C] (verification not implemented) . . . 1702
Giac [C] (verification not implemented) . . . 1702
Mupad [F(-1)] . . . 1703
Reduce [F] . . . 1704

Optimal result

Integrand size = 24, antiderivative size = 299

$$\begin{aligned}
&\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx \\
&= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} - \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{64b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{16b^{3/2}}
\end{aligned}$$

output
```
-1/8*(d*x+c)^(1/2)*cos(2*b*x+2*a)/b-1/32*(d*x+c)^(1/2)*cos(4*b*x+4*a)/b+1/
128*d^(1/2)*2^(1/2)*Pi^(1/2)*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+1/16*d^(1/2)*Pi^(1/2)*cos(2*a-2*b*c
/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b^(3/2)-1/128*d^(1/
2)*2^(1/2)*Pi^(1/2)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*sin(4*a-4*b*c/d)/b^(3/2)-1/16*d^(1/2)*Pi^(1/2)*FresnelS(2*b^(1/2)*(d*
x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.84

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{e^{-\frac{4i(bc+ad)}{d}}\sqrt{c+dx}\left(4\sqrt{2}e^{2i\left(3a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)+4\sqrt{2}e^{2i\left(a+\frac{3bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right.\right.}{128b\sqrt{\frac{b^2(c+dx)^2}{d^2}}}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
-1/128*(Sqrt[c + d*x]*(4*Sqrt[2]*E^((2*I)*(3*a + (b*c)/d))*Sqrt[(I*b*(c +
d*x))/d]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d] + 4*Sqrt[2]*E^((2*I)*(a + (3*b
*c)/d))*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((2*I)*b*(c + d*x))/d] + E^(
(8*I)*a)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-4*I)*b*(c + d*x))/d] + E^((
(8*I)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((4*I)*b*(c + d*x))/d]
))/(b*E^(((4*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin(a+bx)\cos^3(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}\sqrt{c+dx}\sin(2a+2bx)+\frac{1}{8}\sqrt{c+dx}\sin(4a+4bx)\right)dx$$

$$\downarrow 2009$$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}$$

input

```
Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output

```
-1/8*(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/b - (Sqrt[c + d*x]*Cos[4*a + 4*b*x])
/(32*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a
- (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b
^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x]
)/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS
[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b
^(3/2))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.96

method	result
derivativedivides	$-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d$
default	$-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d$

input

```
int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/32/b*d*Pi^
(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c
)^(1/2)/d))-1/64/b*d*(d*x+c)^(1/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+1/256/
b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi
^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2
)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.78

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx$$
$$= \frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - \sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_s
in(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 8*pi*d*
sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(p
i*d))) - 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))
*sin(-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 3*b)*sqrt(d*x + c))/b^2
```

Sympy [F]

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = \int \sqrt{c+dx}\sin(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.43

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{\left(\frac{16\sqrt{dx+c}b^2\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{64\sqrt{dx+c}b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d}-4\left(-(i-1)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(}{}\right.\right.\right.$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/512*(16*sqrt(d*x + c)*b^2*cos(4*((d*x + c)*b - b*c + a*d)/d)/d + 64*sqr
t(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 4*(-(I - 1)*4^(1/4)*
sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)
*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x +
c)*sqrt(2*I*b/d)) - (-(I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-4*(b
*c - a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*
d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - ((I + 1)*sqrt(2)*sqrt(pi)*b*(b^2
/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(
1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 4*((I + 1)
*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1
)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(-2*I*b/d)))*d/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 826, normalized size of antiderivative = 2.76

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output

```
-1/256*(sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(I*sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*(sqrt(2)*sqrt(pi)*d*er
f(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I
*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*
d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 4*sqrt(pi)*
d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 4*sqrt(pi)*d*erf(I*sq
rt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c + 8*sqrt(pi)*(4*b*c - I*d)*d
*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*sqrt(pi)*(4*b*c +
I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(
-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 16*sqrt(d*x
+ c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-
4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^(-2*(-I*(d*x
 + c)*b + I*b*c - I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4*(-I*(d*x + c)*b...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = \int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)\,\sqrt{c+d\,x}\,dx$$

input

```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(1/2),x)
```

output

```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = \int \sqrt{dx+c}\cos\left(bx+a\right)^3\sin\left(bx+a\right)dx$$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**3*sin(a + b*x),x)
```

3.187 $\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 1705
Mathematica [C] (verified) . . . 1706
Rubi [A] (verified) . . . 1707
Maple [A] (verified) . . . 1708
Fricas [A] (verification not implemented) . . . 1709
Sympy [F] . . . 1709
Maxima [C] (verification not implemented) . . . 1710
Giac [C] (verification not implemented) . . . 1710
Mupad [F(-1)] . . . 1711
Reduce [F] . . . 1712

Optimal result

Integrand size = 24, antiderivative size = 299

$$\begin{aligned}
&\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx \\
&= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} - \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{64b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{16b^{3/2}}
\end{aligned}$$

output
```
-1/8*(d*x+c)^(1/2)*cos(2*b*x+2*a)/b-1/32*(d*x+c)^(1/2)*cos(4*b*x+4*a)/b+1/
128*d^(1/2)*2^(1/2)*Pi^(1/2)*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+1/16*d^(1/2)*Pi^(1/2)*cos(2*a-2*b*c
/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b^(3/2)-1/128*d^(1/
2)*2^(1/2)*Pi^(1/2)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*sin(4*a-4*b*c/d)/b^(3/2)-1/16*d^(1/2)*Pi^(1/2)*FresnelS(2*b^(1/2)*(d*
x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.84

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{e^{-\frac{4i(bc+ad)}{d}}\sqrt{c+dx}\left(4\sqrt{2}e^{2i\left(3a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)+4\sqrt{2}e^{2i\left(a+\frac{3bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right.\right.}{128b\sqrt{\frac{b^2(c+dx)^2}{d^2}}}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
-1/128*(Sqrt[c + d*x]*(4*Sqrt[2]*E^((2*I)*(3*a + (b*c)/d))*Sqrt[(I*b*(c +
d*x))/d]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d] + 4*Sqrt[2]*E^((2*I)*(a + (3*b
*c)/d))*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((2*I)*b*(c + d*x))/d] + E^(
(8*I)*a)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-4*I)*b*(c + d*x))/d] + E^((
(8*I)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((4*I)*b*(c + d*x))/d]
))/(b*E^(((4*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin(a+bx)\cos^3(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}\sqrt{c+dx}\sin(2a+2bx)+\frac{1}{8}\sqrt{c+dx}\sin(4a+4bx)\right)dx$$

$$\downarrow 2009$$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+$$
$$\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}-$$
$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-$$
$$\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}$$

input

```
Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output

```
-1/8*(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/b - (Sqrt[c + d*x]*Cos[4*a + 4*b*x])
/(32*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a
- (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b
^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x]
)/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS
[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b
^(3/2))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.96

method	result
derivativedivides	$-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d$
default	$-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d$

input

```
int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/32/b*d*Pi^
(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c
)^(1/2)/d))-1/64/b*d*(d*x+c)^(1/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+1/256/
b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi
^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2
)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.78

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx$$

$$= \frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - \sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_s
in(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 8*pi*d*
sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(p
i*d))) - 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))
*sin(-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 3*b)*sqrt(d*x + c))/b^2
```

Sympy [F]

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = \int \sqrt{c+dx}\sin(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.43

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{\left(\frac{16\sqrt{dx+c}b^2\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{64\sqrt{dx+c}b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d}-4\left(-(i-1)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(}{}\right.\right.}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/512*(16*sqrt(d*x + c)*b^2*cos(4*((d*x + c)*b - b*c + a*d)/d)/d + 64*sqr
t(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 4*(-(I - 1)*4^(1/4)*
sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)
*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x +
c)*sqrt(2*I*b/d)) - (-(I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-4*(b
*c - a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*
d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - ((I + 1)*sqrt(2)*sqrt(pi)*b*(b^2
/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(
1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 4*((I + 1)
*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1
)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(-2*I*b/d)))*d/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 826, normalized size of antiderivative = 2.76

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output

```
-1/256*(sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(I*sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*(sqrt(2)*sqrt(pi)*d*er
f(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I
*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*
d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 4*sqrt(pi)*
d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 4*sqrt(pi)*d*erf(I*sq
rt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c + 8*sqrt(pi)*(4*b*c - I*d)*d
*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*sqrt(pi)*(4*b*c +
I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(
-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 16*sqrt(d*x
+ c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-
4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^(-2*(-I*(d*x
 + c)*b + I*b*c - I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4*(-I*(d*x + c)*b...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = \int \cos(a+b\,x)^3\,\sin(a+b\,x)\,\sqrt{c+d\,x}\,dx$$

input

```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(1/2),x)
```

output

```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = \int \sqrt{dx+c}\cos\left(bx+a\right)^3\sin\left(bx+a\right)dx$$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**3*sin(a + b*x),x)
```

3.188 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 1713
Mathematica [C] (verified) . . . 1714
Rubi [A] (verified) . . . 1715
Maple [A] (verified) . . . 1717
Fricas [A] (verification not implemented) . . . 1717
Sympy [F] . . . 1718
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Giac [C] (verification not implemented) . . . 1719
Mupad [F(-1)] . . . 1720
Reduce [F] . . . 1721

Optimal result

Integrand size = 24, antiderivative size = 351

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}$$
$$-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}$$
$$-\frac{3d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}$$
$$-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{512b^{5/2}}$$
$$-\frac{3d^{3/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{64b^{5/2}}$$
$$+\frac{3d\sqrt{c+dx}\sin(2a+2bx)}{32b^2} + \frac{3d\sqrt{c+dx}\sin(4a+4bx)}{256b^2}$$

output

```
-1/8*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b-1/32*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b-3/
1024*d^(3/2)*2^(1/2)*Pi^(1/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/
Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)-3/64*d^(3/2)*Pi^(1/2)*cos(2*a-2*b*
c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b^(5/2)-3/1024*d^(
3/2)*2^(1/2)*Pi^(1/2)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*sin(4*a-4*b*c/d)/b^(5/2)-3/64*d^(3/2)*Pi^(1/2)*FresnelC(2*b^(1/2)*(
d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/b^(5/2)+3/32*d*(d*x+c)^(1/
2)*sin(2*b*x+2*a)/b^2+3/256*d*(d*x+c)^(1/2)*sin(4*b*x+4*a)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.97

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \frac{e^{-4ia}\left(8e^{2i\left(a-\frac{b(c+dx)}{d}\right)}\left(-4\sqrt{b}de^{\frac{2ibc}{d}}\sqrt{c+dx}\left(-3i+4bx+e^{4i(a+bx)}(3i+4bx)\right)-(1-i)(4bc+3id\right.\right.}{}$$

input

```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output

```
(8*E^((2*I)*(a - (b*(c + d*x))/d))*(-4*Sqrt[b]*d*E^(((2*I)*b*c)/d)*Sqrt[c
+ d*x]*(-3*I + 4*b*x + E^((4*I)*(a + b*x))*(3*I + 4*b*x)) - (1 - I)*(4*b*c
 + (3*I)*d)*Sqrt[d]*E^(((2*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[((1 + I)*Sqrt
[b]*Sqrt[c + d*x])/Sqrt[d]] + (1 + I)*Sqrt[d]*((4*I)*b*c + 3*d)*E^((2*I)*(
2*a + b*x))*Sqrt[Pi]*Erfi[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]) + (-4*
Sqrt[b]*d*E^(((4*I)*b*c)/d)*Sqrt[c + d*x]*(-3*I + 8*b*x + E^((8*I)*(a + b*
x))*(3*I + 8*b*x)) + (-1)^(1/4)*((8*I)*b*c - 3*d)*Sqrt[d]*E^(((4*I)*b*(2*c
 + d*x))/d)*Sqrt[Pi]*Erf[(2*(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + (
-1)^(1/4)*Sqrt[d]*((8*I)*b*c + 3*d)*E^((4*I)*(2*a + b*x))*Sqrt[Pi]*Erfi[(2
*(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/E^(((4*I)*b*(c + d*x))/d) - (
64*Sqrt[2]*b^(3/2)*c*E^((2*I)*(a - (b*c)/d))*Sqrt[c + d*x]*(E^((4*I)*a)*Sq
rt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d] + E^(((4*I)*b*c)/
d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((2*I)*b*(c + d*x))/d]))/Sqrt[(b^
2*(c + d*x)^2)/d^2] + (16*b^(3/2)*c*Sqrt[c + d*x]*(-((E^((8*I)*a)*Gamma[3/
2, ((-4*I)*b*(c + d*x))/d])/Sqrt[((-I)*b*(c + d*x))/d]) - (E^(((8*I)*b*c)/
d)*Gamma[3/2, ((4*I)*b*(c + d*x))/d])/Sqrt[(I*b*(c + d*x))/d]))/E^(((4*I)*
b*c)/d))/(2048*b^(5/2)*E^((4*I)*a))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin(a+bx)\cos^3(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx)+\frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-$$
$$\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}-$$
$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-$$
$$\frac{3\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(2a+2bx)}{32b^2}+$$
$$\frac{3d\sqrt{c+dx}\sin(4a+4bx)}{256b^2}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}$$

input

```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output

```
-1/8*((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/b - ((c + d*x)^(3/2)*Cos[4*a + 4*b
*x])/(32*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[
b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi]
)])/(64*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqr
t[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*
b*c)/d])/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) + (3
*d*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.07

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{8b\sqrt{\frac{b}{d}}}\right.}{8b}$
default	$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{8b\sqrt{\frac{b}{d}}}\right.}{8b}$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+3/16/b*d*(1/
4/b*d*(d*x+c)^(1/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d
)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2
)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)
))-1/64/b*d*(d*x+c)^(3/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+3/64/b*d*(1/8/b
*d*(d*x+c)^(1/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2
)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*
b*(d*x+c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.84

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-}{d}\right.}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))
*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d)
 + 48*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x +
 c)*sqrt(b/(pi*d))) + 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)
*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x
+ a)^4 - 6*b^2*d*x - 6*b^2*c - 3*(2*b*d*cos(b*x + a)^3 + 3*b*d*cos(b*x + a
))*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F]

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)*cos(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.43

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output

```
-1/4096*(128*(d*x + c)^(3/2)*b^3*cos(4*((d*x + c)*b - b*c + a*d)/d)/d + 51
2*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 48*sqrt(d*x +
 c)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) - 384*sqrt(d*x + c)*b^2*sin(2*(
(d*x + c)*b - b*c + a*d)/d) + 24*((I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^
2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*
(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) +
3*((I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I
 - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sq
rt(d*x + c)*sqrt(I*b/d)) + 3*(-(I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4
)*cos(-4*(b*c - a*d)/d) + (I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin
(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + 24*(-(I - 1)*4^(1/
4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^
(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(-2*I*b/d)))*d/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 1490, normalized size of antiderivative = 4.25

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output

```
1/2048*(64*(sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) + 4*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)) + 4*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
))*c^2 - 16*(sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqr
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(I*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c
+ I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*sqrt(pi)*(4*b*c -
 I*d)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*
(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*sqrt(pi)*(4
*b*c + I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 16*sq
rt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 4*sqrt(d*x + c)
*d*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^(-2*(
-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4*(-I*(d*x...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)^{3/2}\,dx$$

input

```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(3/2),x)
```

output

```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c + dx)^{3/2} \cos^3(a+bx) \sin(a+bx)\, dx = \left(\int \sqrt{dx+c} \cos(bx+a)^3 \sin(bx+a)\, x dx \right) d + \left(\int \sqrt{dx+c} \cos(bx+a)^3 \sin(bx+a)\, dx \right) c$$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**3*sin(a + b*x)*x,x)*d + int(sqrt(c + d*x)*
cos(a + b*x)**3*sin(a + b*x),x)*c
```

3.189 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 1722
Mathematica [C] (verified) . . . 1723
Rubi [A] (verified) . . . 1724
Maple [A] (verified) . . . 1726
Fricas [A] (verification not implemented) . . . 1727
Sympy [F(-1)] . . . 1728
Maxima [C] (verification not implemented) . . . 1728
Giac [C] (verification not implemented) . . . 1729
Mupad [F(-1)] . . . 1730
Reduce [F] . . . 1730

Optimal result

Integrand size = 24, antiderivative size = 407

$$\begin{aligned}\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx &= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3} \\ &- \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} + \frac{15d^2\sqrt{c+dx}\cos(4a+4bx)}{2048b^3} \\ &- \frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b} \\ &- \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} \\ &- \frac{15d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}} \\ &+ \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{4096b^{7/2}} \\ &+ \frac{15d^{5/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{256b^{7/2}} \\ &+ \frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{32b^2} + \frac{5d(c+dx)^{3/2}\sin(4a+4bx)}{256b^2}\end{aligned}$$

output

```
15/128*d^2*(d*x+c)^(1/2)*cos(2*b*x+2*a)/b^3-1/8*(d*x+c)^(5/2)*cos(2*b*x+2*
a)/b+15/2048*d^2*(d*x+c)^(1/2)*cos(4*b*x+4*a)/b^3-1/32*(d*x+c)^(5/2)*cos(4
*b*x+4*a)/b-15/8192*d^(5/2)*2^(1/2)*Pi^(1/2)*cos(4*a-4*b*c/d)*FresnelC(2*b
^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-15/256*d^(5/2)*Pi^(
1/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b
^(7/2)+15/8192*d^(5/2)*2^(1/2)*Pi^(1/2)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2
)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)/b^(7/2)+15/256*d^(5/2)*Pi^(1/2)*
FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/b^(7/2
)+5/32*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2+5/256*d*(d*x+c)^(3/2)*sin(4*b*x+
4*a)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.01 (sec) , antiderivative size = 1332, normalized size of antiderivative = 3.27

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input

```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output

```
((1/128 + I/128)*c*Sqrt[d]*((2 + 2*I)*Sqrt[b]*Sqrt[d]*E^(((2*I)*b*c)/d)*Sq
rt[c + d*x]*(3 + (4*I)*b*x + E^((4*I)*(a + b*x))*(-3 + (4*I)*b*x)) + I*(4*
b*c + (3*I)*d)*E^(((2*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[((1 + I)*Sqrt[b]*S
qrt[c + d*x])/Sqrt[d]] + ((4*I)*b*c + 3*d)*E^((2*I)*(2*a + b*x))*Sqrt[Pi]*
Erfi[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^(((2*I)*(a*d +
b*(c + d*x)))/d)) + (c*Sqrt[d]*(-4*Sqrt[b]*Sqrt[d]*E^(((4*I)*b*c)/d)*Sqrt[
c + d*x]*(-3*I + 8*b*x + E^((8*I)*(a + b*x))*(3*I + 8*b*x)) + (-1)^(3/4)*(
8*b*c + (3*I)*d)*E^(((4*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[(2*(-1)^(1/4)*Sq
rt[b]*Sqrt[c + d*x])/Sqrt[d]] + (-1)^(1/4)*((8*I)*b*c + 3*d)*E^((4*I)*(2*a
 + b*x))*Sqrt[Pi]*Erfi[(2*(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(10
24*b^(5/2)*E^(((4*I)*(a*d + b*(c + d*x)))/d)) + (c^2*(-1/4*(E^((2*I)*(a -
(b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d])/(Sqrt[2]*b*Sqr
t[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma[3/2, ((2*I)*b*(c + d*x))/d
])/(4*Sqrt[2]*b*E^((2*I)*(a - (b*c)/d))*Sqrt[(I*b*(c + d*x))/d])))/4 + (c^
2*Sqrt[c + d*x]*(-((E^((8*I)*a)*Gamma[3/2, ((-4*I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((8*I)*b*c)/d)*Gamma[3/2, ((4*I)*b*(c + d*x))/
d])/Sqrt[(I*b*(c + d*x))/d]))/(128*b*E^(((4*I)*(b*c + a*d))/d)) + (Sqrt[d]
*((1 - I)*E^((2*I)*(a - (b*c)/d))*((2 + 2*I)*Sqrt[b]*Sqrt[d]*E^(((2*I)*b*(
c + d*x))/d)*Sqrt[c + d*x]*(15*d - 16*b^2*d*x^2 + (4*I)*b*(c - 5*d*x)) + (
16*b^2*c^2 - (24*I)*b*c*d - 15*d^2)*Sqrt[Pi]*Erfi[((1 + I)*Sqrt[b]*Sqrt...
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2} \sin(a+bx) \cos^3(a+bx)\, dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{4}(c+dx)^{5/2} \sin(2a+2bx) + \frac{1}{8}(c+dx)^{5/2} \sin(4a+4bx) \right) dx$$

$$\downarrow 2009$$

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-$$
$$\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}+$$
$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+$$
$$\frac{15\sqrt{\pi}d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}+$$
$$\frac{15d^2\sqrt{c+dx}\cos(4a+4bx)}{2048b^3}+\frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{32b^2}+\frac{5d(c+dx)^{3/2}\sin(4a+4bx)}{256b^2}-$$
$$\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}$$

input

```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output

```
(15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2
*a + 2*b*x])/(8*b) + (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) -
((c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a
 - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(409
6*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]
*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2
]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)
/d])/(4096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*
x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*
x)^(3/2)*Sin[2*a + 2*b*x])/(32*b^2) + (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x
])/(256*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.15

method	result
derivativedivides	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}\right.\right.}{\left.\left.\right.\right.}$
default	$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}\right.\right.}{\left.\left.\right.\right.}$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+5/16/b*d*(1/
4/b*d*(d*x+c)^(3/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*
x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(
cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(
2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1/64/
b*d*(d*x+c)^(5/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+5/64/b*d*(1/8/b*d*(d*x+
c)^(3/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*
cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(co
s(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d
)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/
2)/d)))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.92

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(}{}\right.}$$

input

```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output

```
-1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_co
s(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d
))*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/
d) + 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*
x + c)*sqrt(b/(pi*d))) - 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x
+ c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(192*b^3*d^2*x^2 + 384*b^3*
c*d*x + 192*b^3*c^2 + 360*b*d^2*cos(b*x + a)^2 - 8*(64*b^3*d^2*x^2 + 128*b
^3*c*d*x + 64*b^3*c^2 - 15*b*d^2)*cos(b*x + a)^4 - 225*b*d^2 + 160*(2*(b^2
*d^2*x + b^2*c*d)*cos(b*x + a)^3 + 3*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*s
in(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.35

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/32768*(640*(d*x + c)^(3/2)*b^3*sin(4*((d*x + c)*b - b*c + a*d)/d) + 5120
*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 16*(64*(d*x + c)
^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(4*((d*x + c)*b - b*c + a*d)/d)
- 256*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c)
*b - b*c + a*d)/d) - 240*(-(I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2
)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^
2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 15*
(-(I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (
I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2
*sqrt(d*x + c)*sqrt(I*b/d)) - 15*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1
/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 240*((I + 1
)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (
I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^5
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 2412, normalized size of antiderivative = 5.93

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
1/16384*(512*(sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)) + 4*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)) + 4*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)))*c^3 - d^3*((sqrt(2)*sqrt(pi)*(512*b^3*c^3 - 192*I*b^2*c^2*d - 72*b*c*
d^2 + 15*I*d^3)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*
b^3) - 4*I*(64*I*(d*x + c)^(5/2)*b^2*d - 192*I*(d*x + c)^(3/2)*b^2*c*d + 1
92*I*sqrt(d*x + c)*b^2*c^2*d - 40*(d*x + c)^(3/2)*b*d^2 + 72*sqrt(d*x + c)
*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/
d)/b^3)/d^3 + (sqrt(2)*sqrt(pi)*(512*b^3*c^3 + 192*I*b^2*c^2*d - 72*b*c*d^
2 - 15*I*d^3)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*
b^3) - 4*I*(64*I*(d*x + c)^(5/2)*b^2*d - 192*I*(d*x + c)^(3/2)*b^2*c*d + 1
92*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x + c)
*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int \cos(a+b\,x)^3\,\sin(a+b\,x)\,(c+d\,x)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \left(\int \sqrt{dx+c}\,\cos(bx+a)^3\sin(bx+a)\,x^2dx\right)d^2$$
$$+2\left(\int \sqrt{dx+c}\,\cos(bx+a)^3\sin(bx+a)\,xdx\right)cd$$
$$+\left(\int \sqrt{dx+c}\,\cos(bx+a)^3\sin(bx+a)\,dx\right)c^2$$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**3*sin(a + b*x)*x**2,x)*d**2 + 2*int(sqrt(c
 + d*x)*cos(a + b*x)**3*sin(a + b*x)*x,x)*c*d + int(sqrt(c + d*x)*cos(a +
b*x)**3*sin(a + b*x),x)*c**2
```

3.190 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal result 1732
Mathematica [C] (verified) 1733
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Optimal result

Integrand size = 26, antiderivative size = 615

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^{3/2}\cos(5a+5bx)}{160b^2} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}} - \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}} - \frac{3d^{5/2}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}} - \frac{3d^{5/2}\sqrt{\frac{\pi}{10}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{1600b^{7/2}} - \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{576b^{7/2}} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{32b^{7/2}} - \frac{15d^2\sqrt{c+dx}\sin(a+bx)}{32b^3} + \frac{(c+dx)^{5/2}\sin(a+bx)}{8b} + \frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{576b^3} - \frac{(c+dx)^{5/2}\sin(3a+3bx)}{48b} + \frac{3d^2\sqrt{c+dx}\sin(5a+5bx)}{1600b^3} - \frac{(c+dx)^{5/2}\sin(5a+5bx)}{80b}$$

output

```
5/16*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2-5/288*d*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b
^2-1/160*d*(d*x+c)^(3/2)*cos(5*b*x+5*a)/b^2+15/64*d^(5/2)*2^(1/2)*Pi^(1/2)
*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(
7/2)-5/3456*d^(5/2)*6^(1/2)*Pi^(1/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-3/16000*d^(5/2)*10^(1/2)*Pi^(
1/2)*cos(5*a-5*b*c/d)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))/b^(7/2)-3/16000*d^(5/2)*10^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*10^(1/2)/
Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)/b^(7/2)-5/3456*d^(5/2)*6^
(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*si
n(3*a-3*b*c/d)/b^(7/2)+15/64*d^(5/2)*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(7/2)-15/32*d^2*(d*x+c
)^(1/2)*sin(b*x+a)/b^3+1/8*(d*x+c)^(5/2)*sin(b*x+a)/b+5/576*d^2*(d*x+c)^(1
/2)*sin(3*b*x+3*a)/b^3-1/48*(d*x+c)^(5/2)*sin(3*b*x+3*a)/b+3/1600*d^2*(d*x
+c)^(1/2)*sin(5*b*x+5*a)/b^3-1/80*(d*x+c)^(5/2)*sin(5*b*x+5*a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.11 (sec) , antiderivative size = 2130, normalized size of antiderivative = 3.46

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Result too large to show}$$

input

```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output

```
-1/1152*(c*Sqrt[d]*(12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(1
+ (2*I)*b*x + E^((6*I)*(a + b*x))*(1 - (2*I)*b*x)) + (1 + I)*(2*b*c + I*d)
*E^(((3*I)*b*(2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqr
t[c + d*x])/Sqrt[d]] - (1 + I)*(2*b*c - I*d)*E^((3*I)*(2*a + b*x))*Sqrt[6*
Pi]*Erfi[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^(
((3*I)*(a*d + b*(c + d*x)))/d)) - (c*Sqrt[d]*(20*Sqrt[b]*Sqrt[d]*E^(((5*I)
*b*c)/d)*Sqrt[c + d*x]*(3 + (10*I)*b*x + E^((10*I)*(a + b*x))*(3 - (10*I)*
b*x)) + (1 + I)*(10*b*c + (3*I)*d)*E^(((5*I)*b*(2*c + d*x))/d)*Sqrt[10*Pi]
*Erf[((1 + I)*Sqrt[5/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - (1 + I)*(10*b*c
- (3*I)*d)*E^((5*I)*(2*a + b*x))*Sqrt[10*Pi]*Erfi[((1 + I)*Sqrt[5/2]*Sqrt[
b]*Sqrt[c + d*x])/Sqrt[d]]))/(16000*b^(5/2)*E^(((5*I)*(a*d + b*(c + d*x)))
/d)) + (c^2*d*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(
c + d*x))/d] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*(
c + d*x))/d]))/(16*b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x]) - (c^2*(c + d*
x)^(3/2)*(-((E^((6*I)*a)*Gamma[3/2, ((-3*I)*b*(c + d*x))/d])/(((-I)*b*(c +
 d*x))/d)^(3/2)) - (E^(((6*I)*b*c)/d)*Gamma[3/2, ((3*I)*b*(c + d*x))/d])/(
(I*b*(c + d*x))/d)^(3/2)))/(96*Sqrt[3]*d*E^(((3*I)*(b*c + a*d))/d)) - (c^2
*(c + d*x)^(3/2)*(-((E^((10*I)*a)*Gamma[3/2, ((-5*I)*b*(c + d*x))/d])/(((-
I)*b*(c + d*x))/d)^(3/2)) - (E^(((10*I)*b*c)/d)*Gamma[3/2, ((5*I)*b*(c + d
*x))/d])/((I*b*(c + d*x))/d)^(3/2)))/(160*Sqrt[5]*d*E^(((5*I)*(b*c + a*...
```

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2} \sin^2(a+bx) \cos^3(a+bx)\, dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8}(c+dx)^{5/2} \cos(a+bx) - \frac{1}{16}(c+dx)^{5/2} \cos(3a+3bx) - \frac{1}{16}(c+dx)^{5/2} \cos(5a+5bx) \right) dx$$

$$\downarrow 2009$$

$$-\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-$$
$$\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+$$
$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}+$$
$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-$$
$$\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}-$$
$$\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{15d^2\sqrt{c+dx}\sin(a+bx)}{32b^3}+$$
$$\frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{576b^3}+\frac{3d^2\sqrt{c+dx}\sin(5a+5bx)}{1600b^3}+\frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2}-$$
$$\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2}-\frac{d(c+dx)^{3/2}\cos(5a+5bx)}{160b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)}{8b}-$$
$$\frac{(c+dx)^{5/2}\sin(3a+3bx)}{48b}-\frac{(c+dx)^{5/2}\sin(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
(5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(16*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a
 + 3*b*x])/(288*b^2) - (d*(c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(160*b^2) + (1
5*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]])/(32*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d
]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) - (3
*d^(5/2)*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(1600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelC[(S
qrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(
7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/
Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(
7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(32*b^3) + ((c + d*x)^(5/2)*Si
n[a + b*x])/(8*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(576*b^3) - ((c
 + d*x)^(5/2)*Sin[3*a + 3*b*x])/(48*b) + (3*d^2*Sqrt[c + d*x]*Sin[5*a + 5*
b*x])/(1600*b^3) - ((c + d*x)^(5/2)*Sin[5*a + 5*b*x])/(80*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 9.49 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\sqrt{\pi}\left(\right.\right.}{8b}$
default	$\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\sqrt{\pi}\left(\right.\right.}{8b}$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/16/b*d*(d*x+c)^(5/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-5/16/b*d*(-1/2/b*
d*(d*x+c)^(3/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2
)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((
a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((
a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1/
96/b*d*(d*x+c)^(5/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+5/96/b*d*(-1/6/b*d*(
d*x+c)^(3/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/
2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d
)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*
b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b
/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1/160/b*d*(d*x+c)^(5/2)*sin(5*b/d*(d*x+c)+
5*(a*d-b*c)/d)+1/32/b*d*(-1/10/b*d*(d*x+c)^(3/2)*cos(5*b/d*(d*x+c)+5*(a*d-
b*c)/d)+3/10/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(5*b/d*(d*x+c)+5*(a*d-b*c)/d)-
1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*Fresnel
S(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(5*(a*d-b*c)/
d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 548, normalized size of antiderivative = 0.89

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{81\sqrt{10}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+625\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel
_sin(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 625*sqrt(6)*pi*d^3*sqrt(b/(p
i*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d
))) - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin
(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*
d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d)
+ 625*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt
(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fres
nel_cos(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480
*(90*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^5 - 50*(b^2*d^2*x + b^2*c*d)*cos(b
*x + a)^3 - 300*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) - (120*b^3*d^2*x^2 + 24
0*b^3*c*d*x + 120*b^3*c^2 - 9*(20*b^3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2
- 3*b*d^2)*cos(b*x + a)^4 - 428*b*d^2 + (60*b^3*d^2*x^2 + 120*b^3*c*d*x +
60*b^3*c^2 + 11*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.34

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/1728000*sqrt(2)*(5400*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(5*((d*x + c)*b -
b*c + a*d)/d)/d + 15000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(3*((d*x + c)*b - b
*c + a*d)/d)/d - 270000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(((d*x + c)*b - b*c
 + a*d)/d)/d - 81*(-(I + 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5
*(b*c - a*d)/d) + (I - 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-5*(
b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 625*(-(I + 1)*9^(1/4)*sq
rt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(
pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*
I*b/d)) - 101250*((I + 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/
d) - (I - 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(
d*x + c)*sqrt(I*b/d)) - 101250*(-(I - 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*co
s(-(b*c - a*d)/d) + (I + 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d
)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - 625*((I - 1)*9^(1/4)*sqrt(pi)*b^2*
d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(pi)*b^2*d*(
b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) -
81*((I - 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)
- (I + 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*e
rf(sqrt(d*x + c)*sqrt(-5*I*b/d)) + 540*(20*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2
 - 3*sqrt(2)*sqrt(d*x + c)*b^3)*sin(5*((d*x + c)*b - b*c + a*d)/d) + 1500*
(12*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 5*sqrt(2)*sqrt(d*x + c)*b^3)*sin(...
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.49 (sec) , antiderivative size = 3677, normalized size of antiderivative = 5.98

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/864000*(1800*(30*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt
(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/
(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*I
*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*
c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi
)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(
6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)) - 3*I*sqrt(10)*sqrt(pi)*d*erf(1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/s
qrt(b^2*d^2) + 1)))*c^3 + d^3*(6750*(-I*sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I
*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)*b^3) + 2*(-4*I*(d*x + c)^(5/2)*b^2*d + 12*I*(d*x + c
)^(3/2)*b^2*c*d - 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2
+ 18*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b +
I*b*c - I*a*d)/d)/b^3)/d^3 + 125*(-I*sqrt(6)*sqrt(pi)*(72*b^3*c^3 - 36*...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)^3\sin(a+b\,x)^2\,(c+d\,x)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\begin{aligned}&\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a\\&+bx)\,dx = \left(\int \sqrt{dx+c}\,\cos(bx+a)^3\sin(bx+a)^2\,x^2dx\right)d^2\\&+2\left(\int \sqrt{dx+c}\,\cos(bx+a)^3\sin(bx+a)^2\,xdx\right)cd\\&+\left(\int \sqrt{dx+c}\,\cos(bx+a)^3\sin(bx+a)^2\,dx\right)c^2\end{aligned}$$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**3*sin(a + b*x)**2*x**2,x)*d**2 + 2*int(sqr
t(c + d*x)*cos(a + b*x)**3*sin(a + b*x)**2*x,x)*c*d + int(sqrt(c + d*x)*co
s(a + b*x)**3*sin(a + b*x)**2,x)*c**2
```

3.191 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal result 1743
Mathematica [C] (verified) 1744
Rubi [A] (verified) 1745
Maple [A] (verified) 1747
Fricas [A] (verification not implemented) 1748
Sympy [F] 1749
Maxima [C] (verification not implemented) 1749
Giac [C] (verification not implemented) 1750
Mupad [F(-1)] 1751
Reduce [F] 1752

Optimal result

Integrand size = 26, antiderivative size = 534

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\cos(5a+5bx)}{800b^2} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}} + \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} + \frac{3d^{3/2}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}} - \frac{3d^{3/2}\sqrt{\frac{\pi}{10}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{800b^{5/2}} - \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{96b^{5/2}} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{16b^{5/2}} + \frac{(c+dx)^{3/2}\sin(a+bx)}{8b} - \frac{(c+dx)^{3/2}\sin(3a+3bx)}{48b} - \frac{(c+dx)^{3/2}\sin(5a+5bx)}{80b}$$

output

```
3/16*d*(d*x+c)^(1/2)*cos(b*x+a)/b^2-1/96*d*(d*x+c)^(1/2)*cos(3*b*x+3*a)/b^
2-3/800*d*(d*x+c)^(1/2)*cos(5*b*x+5*a)/b^2-3/32*d^(3/2)*2^(1/2)*Pi^(1/2)*c
os(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/
2)+1/576*d^(3/2)*6^(1/2)*Pi^(1/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2
)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)+3/8000*d^(3/2)*10^(1/2)*Pi^(1/2)
*cos(5*a-5*b*c/d)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)/b^(5/2)-3/8000*d^(3/2)*10^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)/b^(5/2)-1/576*d^(3/2)*6^(1/2)*
Pi^(1/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-
3*b*c/d)/b^(5/2)+3/32*d^(3/2)*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(5/2)+1/8*(d*x+c)^(3/2)*sin(b
*x+a)/b-1/48*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b-1/80*(d*x+c)^(3/2)*sin(5*b*x+5
*a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.81 (sec) , antiderivative size = 1088, normalized size of antiderivative = 2.04

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input

```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output

```
-1/2304*(Sqrt[d]*(12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(1 +
(2*I)*b*x + E^((6*I)*(a + b*x))*(1 - (2*I)*b*x)) + (1 + I)*(2*b*c + I*d)*E
^(((3*I)*b*(2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[
c + d*x])/Sqrt[d]] - (1 + I)*(2*b*c - I*d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi
]*Erfi[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^(((
3*I)*(a*d + b*(c + d*x)))/d)) - (Sqrt[d]*(20*Sqrt[b]*Sqrt[d]*E^(((5*I)*b*c
)/d)*Sqrt[c + d*x]*(3 + (10*I)*b*x + E^((10*I)*(a + b*x))*(3 - (10*I)*b*x)
) + (1 + I)*(10*b*c + (3*I)*d)*E^(((5*I)*b*(2*c + d*x))/d)*Sqrt[10*Pi]*Erf
[((1 + I)*Sqrt[5/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - (1 + I)*(10*b*c - (3
*I)*d)*E^((5*I)*(2*a + b*x))*Sqrt[10*Pi]*Erfi[((1 + I)*Sqrt[5/2]*Sqrt[b]*S
qrt[c + d*x])/Sqrt[d]]))/(32000*b^(5/2)*E^(((5*I)*(a*d + b*(c + d*x)))/d))
 + (c*d*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(c + d*
x))/d] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*
x))/d]))/(16*b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x]) - (c*(c + d*x)^(3/2)
*(-((E^((6*I)*a)*Gamma[3/2, ((-3*I)*b*(c + d*x))/d])/(((-I)*b*(c + d*x))/d
)^(3/2)) - (E^(((6*I)*b*c)/d)*Gamma[3/2, ((3*I)*b*(c + d*x))/d])/((I*b*(c
+ d*x))/d)^(3/2)))/(96*Sqrt[3]*d*E^(((3*I)*(b*c + a*d))/d)) - (c*(c + d*x)
^(3/2)*(-((E^((10*I)*a)*Gamma[3/2, ((-5*I)*b*(c + d*x))/d])/(((-I)*b*(c +
d*x))/d)^(3/2)) - (E^(((10*I)*b*c)/d)*Gamma[3/2, ((5*I)*b*(c + d*x))/d])/(
(I*b*(c + d*x))/d)^(3/2)))/(160*Sqrt[5]*d*E^(((5*I)*(b*c + a*d))/d)) + ...
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2} \sin^2(a+bx) \cos^3(a+bx)\, dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8}(c+dx)^{3/2} \cos(a+bx) - \frac{1}{16}(c+dx)^{3/2} \cos(3a+3bx) - \frac{1}{16}(c+dx)^{3/2} \cos(5a+5bx) \right) dx$$

$$\downarrow 2009$$

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}+\frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2}-\frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2}-\frac{3d\sqrt{c+dx}\cos(5a+5bx)}{800b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)}{8b}-\frac{(c+dx)^{3/2}\sin(3a+3bx)}{48b}-\frac{(c+dx)^{3/2}\sin(5a+5bx)}{80b}$$

input

```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output

```
(3*d*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b
*x])/(96*b^2) - (3*d*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(800*b^2) - (3*d^(3/2
)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/
Sqrt[d]])/(16*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC
[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sq
rt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x]
)/Sqrt[d]])/(800*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[
10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3
/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*
a - (3*b*c)/d])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + ((c + d*x
)^(3/2)*Sin[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(48*b) -
((c + d*x)^(3/2)*Sin[5*a + 5*b*x])/(80*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\right.}{4b\sqrt{\frac{b}{d}}}\right.}{8b}$
default	$\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\right.}{4b\sqrt{\frac{b}{d}}}\right.}{8b}$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
2/d*(1/16/b*d*(d*x+c)^(3/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-3/16/b*d*(-1/2/b*
d*(d*x+c)^(1/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^
(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^
(1/2)/d)))-1/96/b*d*(d*x+c)^(3/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/32/b*
d*(-1/6/b*d*(d*x+c)^(1/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2
)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/
2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1
/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))-1/160/b*d*(d*x+c)^(3
/2)*sin(5*b/d*(d*x+c)+5*(a*d-b*c)/d)+3/160/b*d*(-1/10/b*d*(d*x+c)^(1/2)*co
s(5*b/d*(d*x+c)+5*(a*d-b*c)/d)+1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1
/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^
(1/2)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.84

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \frac{27\sqrt{10}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+125\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{}\right.$$

input

```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_c
os(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 125*sqrt(6)*pi*d^2*sqrt(b/(pi*
d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))
) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fr
esnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*
sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi
*d)))*sin(-3*(b*c - a*d)/d) - 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_si
n(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(9*b*
d*cos(b*x + a)^5 - 5*b*d*cos(b*x + a)^3 - 30*b*d*cos(b*x + a) + 10*(3*(b^2
*d*x + b^2*c)*cos(b*x + a)^4 - 2*b^2*d*x - 2*b^2*c - (b^2*d*x + b^2*c)*cos
(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F]

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin^2(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 760, normalized size of antiderivative = 1.42

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/288000*sqrt(2)*(1800*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(5*((d*x + c)*b - b
*c + a*d)/d)/d^2 + 3000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(3*((d*x + c)*b - b
*c + a*d)/d)/d^2 - 18000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(((d*x + c)*b - b*
c + a*d)/d)/d^2 + 540*sqrt(2)*sqrt(d*x + c)*b^3*cos(5*((d*x + c)*b - b*c +
 a*d)/d)/d + 1500*sqrt(2)*sqrt(d*x + c)*b^3*cos(3*((d*x + c)*b - b*c + a*d
)/d)/d - 27000*sqrt(2)*sqrt(d*x + c)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/
d - 27*(-(I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/
d) - (I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*
erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 125*(-(I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2
/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)
^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 6750*((I
- 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(pi)*b
^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 6
750*(-(I + 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*s
qrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I
*b/d)) - 125*((I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a
*d)/d) + (I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - 27*((I + 1)*25^(1/4)*sqrt(pi)*b^2*(
b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/
d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^...
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 2278, normalized size of antiderivative = 4.27

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output

```
-1/144000*(300*(30*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I
*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(
sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*I*
sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi)
*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(6
)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
) - 3*I*sqrt(10)*sqrt(pi)*d*erf(1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)))*c^2 + 20*(-450*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1
/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b
*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*sqrt(6)*sqr
t(pi)*(6*b*c - I*d)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b) - 9*I*sqrt(10)*sqrt(pi)*(10*b*c - I*d)*d*erf(-1/2*I*sqrt(10)*sq
rt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)^3\sin(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

input

```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

output

```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c + dx)^{3/2} \cos^3(a+bx) \sin^2(a+bx)\, dx = \left(\int \sqrt{dx+c} \cos(bx+a)^3 \sin(bx+a)^2 x dx \right) d + \left(\int \sqrt{dx+c} \cos(bx+a)^3 \sin(bx+a)^2 dx \right) c$$

input

```
int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

output

```
int(sqrt(c + d*x)*cos(a + b*x)**3*sin(a + b*x)**2*x,x)*d + int(sqrt(c + d*
x)*cos(a + b*x)**3*sin(a + b*x)**2,x)*c
```

3.192 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal result . . . 1753
Mathematica [C] (verified) . . . 1754
Rubi [A] (verified) . . . 1755
Maple [A] (verified) . . . 1756
Fricas [A] (verification not implemented) . . . 1757
Sympy [F] . . . 1758
Maxima [C] (verification not implemented) . . . 1758
Giac [C] (verification not implemented) . . . 1759
Mupad [F(-1)] . . . 1760
Reduce [F] . . . 1761

Optimal result

Integrand size = 26, antiderivative size = 459

$$
\begin{aligned}
&\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx \\
&= -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{10}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{80b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{48b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{8b^{3/2}} + \frac{\sqrt{c+dx}\sin(a+bx)}{8b} \\
&\quad - \frac{\sqrt{c+dx}\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\sin(5a+5bx)}{80b}
\end{aligned}
$$

output
```
-1/16*d^(1/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+1/288*d^(1/2)*6^(1/2)*Pi^(1/2)*cos(3*a-
3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+
1/800*d^(1/2)*10^(1/2)*Pi^(1/2)*cos(5*a-5*b*c/d)*FresnelS(b^(1/2)*10^(1/2)
/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+1/800*d^(1/2)*10^(1/2)*Pi^(1/2)*F
resnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)/
b^(3/2)+1/288*d^(1/2)*6^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)/b^(3/2)-1/16*d^(1/2)*2^(1/2)*Pi^(1/
2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b
^(3/2)+1/8*(d*x+c)^(1/2)*sin(b*x+a)/b-1/48*(d*x+c)^(1/2)*sin(3*b*x+3*a)/b-
1/80*(d*x+c)^(1/2)*sin(5*b*x+5*a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.81

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx$$

$$= \frac{ie^{-\frac{5i(bc+ad)}{d}}\sqrt{c+dx}\left(-450e^{6ia+\frac{4ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+450e^{4ia+\frac{6ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)+25\right.}{}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
((I/7200)*Sqrt[c + d*x]*(-450*E^((6*I)*a + ((4*I)*b*c)/d)*Sqrt[(I*b*(c + d
*x))/d]*Gamma[3/2, ((-I)*b*(c + d*x))/d] + 450*E^((4*I)*a + ((6*I)*b*c)/d)
*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d] + 25*Sqrt[3]*E^(
(2*I)*(4*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-3*I)*b*(c + d
*x))/d] - 25*Sqrt[3]*E^((2*I)*a + ((8*I)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d
]*Gamma[3/2, ((3*I)*b*(c + d*x))/d] + 9*Sqrt[5]*E^((10*I)*a)*Sqrt[(I*b*(c
+ d*x))/d]*Gamma[3/2, ((-5*I)*b*(c + d*x))/d] - 9*Sqrt[5]*E^(((10*I)*b*c)/
d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((5*I)*b*(c + d*x))/d]))/(b*E^(((
5*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^2(a+bx)\cos^3(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8}\sqrt{c+dx}\cos(a+bx) - \frac{1}{16}\sqrt{c+dx}\cos(3a+3bx) - \frac{1}{16}\sqrt{c+dx}\cos(5a+5bx)\right)dx$$

$$\downarrow 2009$$

$$\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} + \frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}} + \frac{\sqrt{c+dx}\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\sin(5a+5bx)}{80b}$$

input

```
Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output

```
-1/8*(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqr
t[c + d*x])/Sqrt[d]])/b^(3/2) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*F
resnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) + (Sqrt[
d]*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(80*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqr
t[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(80*b^(3/2)) + (Sqr
t[d]*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3
*a - (3*b*c)/d])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt
[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(3/2)) + (Sqrt[c + d
*x]*Sin[a + b*x])/(8*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(48*b) - (Sqrt[
c + d*x]*Sin[5*a + 5*b*x])/(80*b)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 444, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b} - \frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}} - \frac{d\sqrt{}}{}$
default	$\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b} - \frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}} - \frac{d\sqrt{}}{}$

input

```
int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
2/d*(1/16/b*d*(d*x+c)^(1/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/32/b*d*2^(1/2)*
Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d))-1/96/b*d*(d*x+c)^(1/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c
)/d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fr
esnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-
b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))-
1/160/b*d*(d*x+c)^(1/2)*sin(5*b/d*(d*x+c)+5*(a*d-b*c)/d)+1/1600/b*d*2^(1/2
)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/
2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1
/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.80

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{9\sqrt{10}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+25\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{}{\pi}}\right.}$$

input

```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output

```
1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_sin(s
qrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos
(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450
*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(
d*x + c)*sqrt(b/(pi*d))) - 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*s
qrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*
c - a*d)/d) + 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(10)*sqrt(d*x
 + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^4 - b*
cos(b*x + a)^2 - 2*b)*sqrt(d*x + c)*sin(b*x + a))/b^2
```

Sympy [F]

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \sqrt{c+dx}\sin^2(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.48

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/28800*sqrt(2)*(180*sqrt(2)*sqrt(d*x + c)*b^3*sin(5*((d*x + c)*b - b*c +
 a*d)/d)/d^2 + 300*sqrt(2)*sqrt(d*x + c)*b^3*sin(3*((d*x + c)*b - b*c + a*
d)/d)/d^2 - 1800*sqrt(2)*sqrt(d*x + c)*b^3*sin(((d*x + c)*b - b*c + a*d)/d
)/d^2 - 9*((I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d
)/d)/d - (I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/
d)/d)*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 25*((I + 1)*9^(1/4)*sqrt(pi)*b^2*
(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d - (I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^
2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 4
50*(-(I + 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d + (I - 1)*
sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt
(I*b/d)) - 450*((I - 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d
 - (I + 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*
x + c)*sqrt(-I*b/d)) - 25*(-(I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*c
os(-3*(b*c - a*d)/d)/d + (I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(
-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - 9*(-(I - 1)*25^(1
/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)/d + (I + 1)*25^(1/4
)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*
sqrt(-5*I*b/d)))*d^2/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 1270, normalized size of antiderivative = 2.77

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output

```
-1/14400*(-450*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)
*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sq
rt(10)*sqrt(pi)*(10*b*c - I*d)*d*erf(-1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d
/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*I
*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c +
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)
*(6*b*c + I*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)*b) + 9*I*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(1/2*I*sqrt(10)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 30*(30*I*sqrt(2)*sqrt(pi)*d*erf
(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I
*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt
(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*
sqrt(10)*sqrt(pi)*d*erf(-1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)^3\sin(a+b\,x)^2\sqrt{c+d\,x}\,dx$$

input

```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

output

```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \sqrt{dx+c}\cos(bx+a)^3\sin(bx+a)^2\,dx$$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**3*sin(a + b*x)**2,x)
```

3.193 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal result . . . 1762
Mathematica [C] (verified) . . . 1763
Rubi [A] (verified) . . . 1764
Maple [A] (verified) . . . 1765
Fricas [A] (verification not implemented) . . . 1766
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Giac [C] (verification not implemented) . . . 1768
Mupad [F(-1)] . . . 1769
Reduce [F] . . . 1770

Optimal result

Integrand size = 26, antiderivative size = 459

$$
\begin{aligned}
&\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx \\
&= -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{10}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{80b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{48b^{3/2}} \\
&\quad - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{8b^{3/2}} + \frac{\sqrt{c+dx}\sin(a+bx)}{8b} \\
&\quad - \frac{\sqrt{c+dx}\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\sin(5a+5bx)}{80b}
\end{aligned}
$$

output

```
-1/16*d^(1/2)*2^(1/2)*Pi^(1/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+1/288*d^(1/2)*6^(1/2)*Pi^(1/2)*cos(3*a-
3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+
1/800*d^(1/2)*10^(1/2)*Pi^(1/2)*cos(5*a-5*b*c/d)*FresnelS(b^(1/2)*10^(1/2)
/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+1/800*d^(1/2)*10^(1/2)*Pi^(1/2)*F
resnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)/
b^(3/2)+1/288*d^(1/2)*6^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)/b^(3/2)-1/16*d^(1/2)*2^(1/2)*Pi^(1/
2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b
^(3/2)+1/8*(d*x+c)^(1/2)*sin(b*x+a)/b-1/48*(d*x+c)^(1/2)*sin(3*b*x+3*a)/b-
1/80*(d*x+c)^(1/2)*sin(5*b*x+5*a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.81

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx$$

$$= \frac{ie^{-\frac{5i(bc+ad)}{d}}\sqrt{c+dx}\left(-450e^{6ia+\frac{4ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+450e^{4ia+\frac{6ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)+25\right.}{}$$

input

```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output

```
((I/7200)*Sqrt[c + d*x]*(-450*E^((6*I)*a + ((4*I)*b*c)/d)*Sqrt[(I*b*(c + d
*x))/d]*Gamma[3/2, ((-I)*b*(c + d*x))/d] + 450*E^((4*I)*a + ((6*I)*b*c)/d)
*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d] + 25*Sqrt[3]*E^(
(2*I)*(4*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-3*I)*b*(c + d
*x))/d] - 25*Sqrt[3]*E^((2*I)*a + ((8*I)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d
]*Gamma[3/2, ((3*I)*b*(c + d*x))/d] + 9*Sqrt[5]*E^((10*I)*a)*Sqrt[(I*b*(c
+ d*x))/d]*Gamma[3/2, ((-5*I)*b*(c + d*x))/d] - 9*Sqrt[5]*E^(((10*I)*b*c)/
d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((5*I)*b*(c + d*x))/d]))/(b*E^(((
5*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^2(a+bx)\cos^3(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8}\sqrt{c+dx}\cos(a+bx) - \frac{1}{16}\sqrt{c+dx}\cos(3a+3bx) - \frac{1}{16}\sqrt{c+dx}\cos(5a+5bx)\right)dx$$

$$\downarrow 2009$$

$$\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} + \frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}} + \frac{\sqrt{c+dx}\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\sin(5a+5bx)}{80b}$$

input

```
Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output

```
-1/8*(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqr
t[c + d*x])/Sqrt[d]])/b^(3/2) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*F
resnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) + (Sqrt[
d]*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(80*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqr
t[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(80*b^(3/2)) + (Sqr
t[d]*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3
*a - (3*b*c)/d])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt
[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(3/2)) + (Sqrt[c + d
*x]*Sin[a + b*x])/(8*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(48*b) - (Sqrt[
c + d*x]*Sin[5*a + 5*b*x])/(80*b)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 444, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b} - \frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}} - \frac{d\sqrt{}}{}$
default	$\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b} - \frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}} - \frac{d\sqrt{}}{}$

input

```
int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/16/b*d*(d*x+c)^(1/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/32/b*d*2^(1/2)*
Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d))-1/96/b*d*(d*x+c)^(1/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c
)/d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fr
esnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-
b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))-
1/160/b*d*(d*x+c)^(1/2)*sin(5*b/d*(d*x+c)+5*(a*d-b*c)/d)+1/1600/b*d*2^(1/2
)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/
2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1
/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.80

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{9\sqrt{10}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+25\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{}{\pi}}\right.$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_sin(s
qrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos
(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450
*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(
d*x + c)*sqrt(b/(pi*d))) - 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*s
qrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*
c - a*d)/d) + 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(10)*sqrt(d*x
 + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^4 - b*
cos(b*x + a)^2 - 2*b)*sqrt(d*x + c)*sin(b*x + a))/b^2
```

Sympy [F]

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \sqrt{c+dx}\sin^2(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.48

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/28800*sqrt(2)*(180*sqrt(2)*sqrt(d*x + c)*b^3*sin(5*((d*x + c)*b - b*c +
 a*d)/d)/d^2 + 300*sqrt(2)*sqrt(d*x + c)*b^3*sin(3*((d*x + c)*b - b*c + a*
d)/d)/d^2 - 1800*sqrt(2)*sqrt(d*x + c)*b^3*sin(((d*x + c)*b - b*c + a*d)/d
)/d^2 - 9*((I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d
)/d)/d - (I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/
d)/d)*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 25*((I + 1)*9^(1/4)*sqrt(pi)*b^2*
(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d - (I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^
2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 4
50*(-(I + 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d + (I - 1)*
sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt
(I*b/d)) - 450*((I - 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d
 - (I + 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*
x + c)*sqrt(-I*b/d)) - 25*(-(I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*c
os(-3*(b*c - a*d)/d)/d + (I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(
-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - 9*(-(I - 1)*25^(1
/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)/d + (I + 1)*25^(1/4
)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*
sqrt(-5*I*b/d)))*d^2/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 1270, normalized size of antiderivative = 2.77

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output

```
-1/14400*(-450*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)
*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sq
rt(10)*sqrt(pi)*(10*b*c - I*d)*d*erf(-1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d
/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*I
*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c +
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)
*(6*b*c + I*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)*b) + 9*I*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(1/2*I*sqrt(10)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 30*(30*I*sqrt(2)*sqrt(pi)*d*erf
(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I
*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt
(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*
sqrt(10)*sqrt(pi)*d*erf(-1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)^3\sin(a+b\,x)^2\sqrt{c+d\,x}\,dx$$

input

```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

output

```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \sqrt{dx+c}\cos(bx+a)^3\sin(bx+a)^2\,dx$$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**3*sin(a + b*x)**2,x)
```

3.194 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal result 1772
Mathematica [C] (verified) 1773
Rubi [A] (verified) 1774
Maple [A] (verified) 1776
Fricas [A] (verification not implemented) 1777
Sympy [F] 1778
Maxima [C] (verification not implemented) 1778
Giac [C] (verification not implemented) 1779
Mupad [F(-1)] 1780
Reduce [F] 1781

Optimal result

Integrand size = 26, antiderivative size = 534

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\cos(5a+5bx)}{800b^2} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}} + \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} + \frac{3d^{3/2}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}} - \frac{3d^{3/2}\sqrt{\frac{\pi}{10}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{800b^{5/2}} - \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{96b^{5/2}} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{16b^{5/2}} + \frac{(c+dx)^{3/2}\sin(a+bx)}{8b} - \frac{(c+dx)^{3/2}\sin(3a+3bx)}{48b} - \frac{(c+dx)^{3/2}\sin(5a+5bx)}{80b}$$

output

```
3/16*d*(d*x+c)^(1/2)*cos(b*x+a)/b^2-1/96*d*(d*x+c)^(1/2)*cos(3*b*x+3*a)/b^
2-3/800*d*(d*x+c)^(1/2)*cos(5*b*x+5*a)/b^2-3/32*d^(3/2)*2^(1/2)*Pi^(1/2)*c
os(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/
2)+1/576*d^(3/2)*6^(1/2)*Pi^(1/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2
)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)+3/8000*d^(3/2)*10^(1/2)*Pi^(1/2)
*cos(5*a-5*b*c/d)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)/b^(5/2)-3/8000*d^(3/2)*10^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)/b^(5/2)-1/576*d^(3/2)*6^(1/2)*
Pi^(1/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-
3*b*c/d)/b^(5/2)+3/32*d^(3/2)*2^(1/2)*Pi^(1/2)*FresnelS(b^(1/2)*2^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(5/2)+1/8*(d*x+c)^(3/2)*sin(b
*x+a)/b-1/48*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b-1/80*(d*x+c)^(3/2)*sin(5*b*x+5
*a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 1088, normalized size of antiderivative = 2.04

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input

```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output

```
-1/2304*(Sqrt[d]*(12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(1 +
(2*I)*b*x + E^((6*I)*(a + b*x))*(1 - (2*I)*b*x)) + (1 + I)*(2*b*c + I*d)*E
^(((3*I)*b*(2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[
c + d*x])/Sqrt[d]] - (1 + I)*(2*b*c - I*d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi
]*Erfi[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^(((
3*I)*(a*d + b*(c + d*x)))/d)) - (Sqrt[d]*(20*Sqrt[b]*Sqrt[d]*E^(((5*I)*b*c
)/d)*Sqrt[c + d*x]*(3 + (10*I)*b*x + E^((10*I)*(a + b*x))*(3 - (10*I)*b*x)
) + (1 + I)*(10*b*c + (3*I)*d)*E^(((5*I)*b*(2*c + d*x))/d)*Sqrt[10*Pi]*Erf
[((1 + I)*Sqrt[5/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - (1 + I)*(10*b*c - (3
*I)*d)*E^((5*I)*(2*a + b*x))*Sqrt[10*Pi]*Erfi[((1 + I)*Sqrt[5/2]*Sqrt[b]*S
qrt[c + d*x])/Sqrt[d]]))/(32000*b^(5/2)*E^(((5*I)*(a*d + b*(c + d*x)))/d))
 + (c*d*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(c + d*
x))/d] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*
x))/d]))/(16*b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x]) - (c*(c + d*x)^(3/2)
*(-((E^((6*I)*a)*Gamma[3/2, ((-3*I)*b*(c + d*x))/d])/(((-I)*b*(c + d*x))/d
)^(3/2)) - (E^(((6*I)*b*c)/d)*Gamma[3/2, ((3*I)*b*(c + d*x))/d])/((I*b*(c
+ d*x))/d)^(3/2)))/(96*Sqrt[3]*d*E^(((3*I)*(b*c + a*d))/d)) - (c*(c + d*x)
^(3/2)*(-((E^((10*I)*a)*Gamma[3/2, ((-5*I)*b*(c + d*x))/d])/(((-I)*b*(c +
d*x))/d)^(3/2)) - (E^(((10*I)*b*c)/d)*Gamma[3/2, ((5*I)*b*(c + d*x))/d])/(
(I*b*(c + d*x))/d)^(3/2)))/(160*Sqrt[5]*d*E^(((5*I)*(b*c + a*d))/d)) + ...
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2} \sin^2(a+bx) \cos^3(a+bx)\, dx$$

$$\downarrow 4906$$

$$\int \left(\frac{1}{8}(c+dx)^{3/2} \cos(a+bx) - \frac{1}{16}(c+dx)^{3/2} \cos(3a+3bx) - \frac{1}{16}(c+dx)^{3/2} \cos(5a+5bx) \right) dx$$

$$\downarrow 2009$$

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}+\frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2}-\frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2}-\frac{3d\sqrt{c+dx}\cos(5a+5bx)}{800b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)}{8b}-\frac{(c+dx)^{3/2}\sin(3a+3bx)}{48b}-\frac{(c+dx)^{3/2}\sin(5a+5bx)}{80b}$$

input

```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output

```
(3*d*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b
*x])/(96*b^2) - (3*d*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(800*b^2) - (3*d^(3/2
)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/
Sqrt[d]])/(16*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC
[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sq
rt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x]
)/Sqrt[d]])/(800*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[
10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3
/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*
a - (3*b*c)/d])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + ((c + d*x
)^(3/2)*Sin[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(48*b) -
((c + d*x)^(3/2)*Sin[5*a + 5*b*x])/(80*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\right.}{4b\sqrt{\frac{b}{d}}}\right.}{8b}$
default	$\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\right.}{4b\sqrt{\frac{b}{d}}}\right.}{8b}$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
2/d*(1/16/b*d*(d*x+c)^(3/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-3/16/b*d*(-1/2/b*
d*(d*x+c)^(1/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^
(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^
(1/2)/d)))-1/96/b*d*(d*x+c)^(3/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/32/b*
d*(-1/6/b*d*(d*x+c)^(1/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2
)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/
2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1
/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))-1/160/b*d*(d*x+c)^(3
/2)*sin(5*b/d*(d*x+c)+5*(a*d-b*c)/d)+3/160/b*d*(-1/10/b*d*(d*x+c)^(1/2)*co
s(5*b/d*(d*x+c)+5*(a*d-b*c)/d)+1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1
/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^
(1/2)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.84

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \frac{27\sqrt{10}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+125\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{}\right.$$

input

```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_c
os(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 125*sqrt(6)*pi*d^2*sqrt(b/(pi*
d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))
) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fr
esnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*
sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi
*d)))*sin(-3*(b*c - a*d)/d) - 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_si
n(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(9*b*
d*cos(b*x + a)^5 - 5*b*d*cos(b*x + a)^3 - 30*b*d*cos(b*x + a) + 10*(3*(b^2
*d*x + b^2*c)*cos(b*x + a)^4 - 2*b^2*d*x - 2*b^2*c - (b^2*d*x + b^2*c)*cos
(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

Sympy [F]

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin^2(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 760, normalized size of antiderivative = 1.42

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/288000*sqrt(2)*(1800*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(5*((d*x + c)*b - b
*c + a*d)/d)/d^2 + 3000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(3*((d*x + c)*b - b
*c + a*d)/d)/d^2 - 18000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(((d*x + c)*b - b*
c + a*d)/d)/d^2 + 540*sqrt(2)*sqrt(d*x + c)*b^3*cos(5*((d*x + c)*b - b*c +
 a*d)/d)/d + 1500*sqrt(2)*sqrt(d*x + c)*b^3*cos(3*((d*x + c)*b - b*c + a*d
)/d)/d - 27000*sqrt(2)*sqrt(d*x + c)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/
d - 27*(-(I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/
d) - (I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*
erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 125*(-(I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2
/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)
^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 6750*((I
- 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(pi)*b
^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 6
750*(-(I + 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*s
qrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I
*b/d)) - 125*((I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a
*d)/d) + (I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - 27*((I + 1)*25^(1/4)*sqrt(pi)*b^2*(
b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/
d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^...
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 2278, normalized size of antiderivative = 4.27

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output

```
-1/144000*(300*(30*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I
*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(
sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*I*
sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi)
*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(6
)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
) - 3*I*sqrt(10)*sqrt(pi)*d*erf(1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)))*c^2 + 20*(-450*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1
/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b
*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*sqrt(6)*sqr
t(pi)*(6*b*c - I*d)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b) - 9*I*sqrt(10)*sqrt(pi)*(10*b*c - I*d)*d*erf(-1/2*I*sqrt(10)*sq
rt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)^3\sin(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

input

```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

output

```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c + dx)^{3/2} \cos^3(a+bx) \sin^2(a+bx)\, dx = \left(\int \sqrt{dx+c} \cos(bx+a)^3 \sin(bx+a)^2 x dx \right) d + \left(\int \sqrt{dx+c} \cos(bx+a)^3 \sin(bx+a)^2 dx \right) c$$

input

```
int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

output

```
int(sqrt(c + d*x)*cos(a + b*x)**3*sin(a + b*x)**2*x,x)*d + int(sqrt(c + d*
x)*cos(a + b*x)**3*sin(a + b*x)**2,x)*c
```

3.195 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal result 1783
Mathematica [C] (verified) 1784
Rubi [A] (verified) 1785
Maple [A] (verified) 1787
Fricas [A] (verification not implemented) 1789
Sympy [F(-1)] 1789
Maxima [C] (verification not implemented) 1790
Giac [C] (verification not implemented) 1791
Mupad [F(-1)] 1792
Reduce [F] 1792

Optimal result

Integrand size = 26, antiderivative size = 615

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^{3/2}\cos(5a+5bx)}{160b^2} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}} - \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}} - \frac{3d^{5/2}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}} - \frac{3d^{5/2}\sqrt{\frac{\pi}{10}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{1600b^{7/2}} - \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{576b^{7/2}} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{32b^{7/2}} - \frac{15d^2\sqrt{c+dx}\sin(a+bx)}{32b^3} + \frac{(c+dx)^{5/2}\sin(a+bx)}{8b} + \frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{576b^3} - \frac{(c+dx)^{5/2}\sin(3a+3bx)}{48b} + \frac{3d^2\sqrt{c+dx}\sin(5a+5bx)}{1600b^3} - \frac{(c+dx)^{5/2}\sin(5a+5bx)}{80b}$$

output

```
5/16*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2-5/288*d*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b
^2-1/160*d*(d*x+c)^(3/2)*cos(5*b*x+5*a)/b^2+15/64*d^(5/2)*2^(1/2)*Pi^(1/2)
*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(
7/2)-5/3456*d^(5/2)*6^(1/2)*Pi^(1/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-3/16000*d^(5/2)*10^(1/2)*Pi^(
1/2)*cos(5*a-5*b*c/d)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))/b^(7/2)-3/16000*d^(5/2)*10^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*10^(1/2)/
Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)/b^(7/2)-5/3456*d^(5/2)*6^
(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*si
n(3*a-3*b*c/d)/b^(7/2)+15/64*d^(5/2)*2^(1/2)*Pi^(1/2)*FresnelC(b^(1/2)*2^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)/b^(7/2)-15/32*d^2*(d*x+c
)^(1/2)*sin(b*x+a)/b^3+1/8*(d*x+c)^(5/2)*sin(b*x+a)/b+5/576*d^2*(d*x+c)^(1
/2)*sin(3*b*x+3*a)/b^3-1/48*(d*x+c)^(5/2)*sin(3*b*x+3*a)/b+3/1600*d^2*(d*x
+c)^(1/2)*sin(5*b*x+5*a)/b^3-1/80*(d*x+c)^(5/2)*sin(5*b*x+5*a)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.06 (sec) , antiderivative size = 2130, normalized size of antiderivative = 3.46

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Result too large to show}$$

input

```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output

```
-1/1152*(c*Sqrt[d]*(12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(1
+ (2*I)*b*x + E^((6*I)*(a + b*x))*(1 - (2*I)*b*x)) + (1 + I)*(2*b*c + I*d)
*E^(((3*I)*b*(2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqr
t[c + d*x])/Sqrt[d]] - (1 + I)*(2*b*c - I*d)*E^((3*I)*(2*a + b*x))*Sqrt[6*
Pi]*Erfi[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^(
((3*I)*(a*d + b*(c + d*x)))/d)) - (c*Sqrt[d]*(20*Sqrt[b]*Sqrt[d]*E^(((5*I)
*b*c)/d)*Sqrt[c + d*x]*(3 + (10*I)*b*x + E^((10*I)*(a + b*x))*(3 - (10*I)*
b*x)) + (1 + I)*(10*b*c + (3*I)*d)*E^(((5*I)*b*(2*c + d*x))/d)*Sqrt[10*Pi]
*Erf[((1 + I)*Sqrt[5/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - (1 + I)*(10*b*c
- (3*I)*d)*E^((5*I)*(2*a + b*x))*Sqrt[10*Pi]*Erfi[((1 + I)*Sqrt[5/2]*Sqrt[
b]*Sqrt[c + d*x])/Sqrt[d]]))/(16000*b^(5/2)*E^(((5*I)*(a*d + b*(c + d*x)))
/d)) + (c^2*d*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(
c + d*x))/d] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*(
c + d*x))/d]))/(16*b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x]) - (c^2*(c + d*
x)^(3/2)*(-((E^((6*I)*a)*Gamma[3/2, ((-3*I)*b*(c + d*x))/d])/(((-I)*b*(c +
 d*x))/d)^(3/2)) - (E^(((6*I)*b*c)/d)*Gamma[3/2, ((3*I)*b*(c + d*x))/d])/(
(I*b*(c + d*x))/d)^(3/2)))/(96*Sqrt[3]*d*E^(((3*I)*(b*c + a*d))/d)) - (c^2
*(c + d*x)^(3/2)*(-((E^((10*I)*a)*Gamma[3/2, ((-5*I)*b*(c + d*x))/d])/(((-
I)*b*(c + d*x))/d)^(3/2)) - (E^(((10*I)*b*c)/d)*Gamma[3/2, ((5*I)*b*(c + d
*x))/d])/((I*b*(c + d*x))/d)^(3/2)))/(160*Sqrt[5]*d*E^(((5*I)*(b*c + a*...
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2} \sin^2(a+bx) \cos^3(a+bx)\, dx$$

$$\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)^{5/2} \cos(a+bx) - \frac{1}{16}(c+dx)^{5/2} \cos(3a+3bx) - \frac{1}{16}(c+dx)^{5/2} \cos(5a+5bx) \right) dx$$

$$\downarrow \text{2009}$$

$$-\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-$$
$$\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+$$
$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}+$$
$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-$$
$$\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}-$$
$$\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{15d^2\sqrt{c+dx}\sin(a+bx)}{32b^3}+$$
$$\frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{576b^3}+\frac{3d^2\sqrt{c+dx}\sin(5a+5bx)}{1600b^3}+\frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2}-$$
$$\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2}-\frac{d(c+dx)^{3/2}\cos(5a+5bx)}{160b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)}{8b}-$$
$$\frac{(c+dx)^{5/2}\sin(3a+3bx)}{48b}-\frac{(c+dx)^{5/2}\sin(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
(5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(16*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a
 + 3*b*x])/(288*b^2) - (d*(c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(160*b^2) + (1
5*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]])/(32*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d
]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) - (3
*d^(5/2)*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(1600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelC[(S
qrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(
7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/
Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(
7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(32*b^3) + ((c + d*x)^(5/2)*Si
n[a + b*x])/(8*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(576*b^3) - ((c
 + d*x)^(5/2)*Sin[3*a + 3*b*x])/(48*b) + (3*d^2*Sqrt[c + d*x]*Sin[5*a + 5*
b*x])/(1600*b^3) - ((c + d*x)^(5/2)*Sin[5*a + 5*b*x])/(80*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 9.07 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\sqrt{\pi}\left(\right.\right.\right.}{8b}$
default	$\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\sqrt{\pi}\left(\right.\right.\right.}{8b}$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/16/b*d*(d*x+c)^(5/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-5/16/b*d*(-1/2/b*
d*(d*x+c)^(3/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2
)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((
a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((
a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1/
96/b*d*(d*x+c)^(5/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+5/96/b*d*(-1/6/b*d*(
d*x+c)^(3/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/
2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d
)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*
b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b
/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1/160/b*d*(d*x+c)^(5/2)*sin(5*b/d*(d*x+c)+
5*(a*d-b*c)/d)+1/32/b*d*(-1/10/b*d*(d*x+c)^(3/2)*cos(5*b/d*(d*x+c)+5*(a*d-
b*c)/d)+3/10/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(5*b/d*(d*x+c)+5*(a*d-b*c)/d)-
1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*Fresnel
S(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(5*(a*d-b*c)/
d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 548, normalized size of antiderivative = 0.89

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{81\sqrt{10}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+625\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\right.}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel
_sin(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 625*sqrt(6)*pi*d^3*sqrt(b/(p
i*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d
))) - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin
(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*
d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d)
+ 625*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt
(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fres
nel_cos(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480
*(90*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^5 - 50*(b^2*d^2*x + b^2*c*d)*cos(b
*x + a)^3 - 300*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) - (120*b^3*d^2*x^2 + 24
0*b^3*c*d*x + 120*b^3*c^2 - 9*(20*b^3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2
- 3*b*d^2)*cos(b*x + a)^4 - 428*b*d^2 + (60*b^3*d^2*x^2 + 120*b^3*c*d*x +
60*b^3*c^2 + 11*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.34

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/1728000*sqrt(2)*(5400*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(5*((d*x + c)*b -
b*c + a*d)/d)/d + 15000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(3*((d*x + c)*b - b
*c + a*d)/d)/d - 270000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(((d*x + c)*b - b*c
 + a*d)/d)/d - 81*(-(I + 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5
*(b*c - a*d)/d) + (I - 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-5*(
b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 625*(-(I + 1)*9^(1/4)*sq
rt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(
pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*
I*b/d)) - 101250*((I + 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/
d) - (I - 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(
d*x + c)*sqrt(I*b/d)) - 101250*(-(I - 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*co
s(-(b*c - a*d)/d) + (I + 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d
)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - 625*((I - 1)*9^(1/4)*sqrt(pi)*b^2*
d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(pi)*b^2*d*(
b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) -
81*((I - 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)
- (I + 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*e
rf(sqrt(d*x + c)*sqrt(-5*I*b/d)) + 540*(20*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2
 - 3*sqrt(2)*sqrt(d*x + c)*b^3)*sin(5*((d*x + c)*b - b*c + a*d)/d) + 1500*
(12*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 5*sqrt(2)*sqrt(d*x + c)*b^3)*sin(...
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.47 (sec) , antiderivative size = 3677, normalized size of antiderivative = 5.98

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/864000*(1800*(30*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt
(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/
(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*I
*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*
c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi
)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(
6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)) - 3*I*sqrt(10)*sqrt(pi)*d*erf(1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/s
qrt(b^2*d^2) + 1)))*c^3 + d^3*(6750*(-I*sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I
*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)*b^3) + 2*(-4*I*(d*x + c)^(5/2)*b^2*d + 12*I*(d*x + c
)^(3/2)*b^2*c*d - 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2
+ 18*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b +
I*b*c - I*a*d)/d)/b^3)/d^3 + 125*(-I*sqrt(6)*sqrt(pi)*(72*b^3*c^3 - 36*...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)^3\sin(a+b\,x)^2\,(c+d\,x)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\begin{aligned}&\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a\\&+bx)\,dx = \left(\int \sqrt{dx+c}\,\cos(bx+a)^3\sin(bx+a)^2\,x^2dx\right)d^2\\&+2\left(\int \sqrt{dx+c}\,\cos(bx+a)^3\sin(bx+a)^2\,xdx\right)cd\\&+\left(\int \sqrt{dx+c}\,\cos(bx+a)^3\sin(bx+a)^2\,dx\right)c^2\end{aligned}$$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**3*sin(a + b*x)**2*x**2,x)*d**2 + 2*int(sqr
t(c + d*x)*cos(a + b*x)**3*sin(a + b*x)**2*x,x)*c*d + int(sqrt(c + d*x)*co
s(a + b*x)**3*sin(a + b*x)**2,x)*c**2
```

3.196 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal result . . . 1793
Mathematica [C] (warning: unable to verify) . . . 1794
Rubi [A] (verified) . . . 1795
Maple [A] (verified) . . . 1796
Fricas [A] (verification not implemented) . . . 1798
Sympy [F(-1)] . . . 1798
Maxima [C] (verification not implemented) . . . 1799
Giac [C] (verification not implemented) . . . 1799
Mupad [F(-1)] . . . 1800
Reduce [F] . . . 1801

Optimal result

Integrand size = 26, antiderivative size = 407

$$\begin{aligned}\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx &= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3}\\ &- \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d^2\sqrt{c+dx}\cos(6a+6bx)}{9216b^3}\\ &+ \frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b} + \frac{5d^{5/2}\sqrt{\frac{\pi}{3}}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}}\\ &- \frac{45d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2048b^{7/2}}\\ &- \frac{5d^{5/2}\sqrt{\frac{\pi}{3}}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(6a-\frac{6bc}{d}\right)}{18432b^{7/2}}\\ &+ \frac{45d^{5/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{2048b^{7/2}}\\ &+ \frac{15d(c+dx)^{3/2}\sin(2a+2bx)}{256b^2} - \frac{5d(c+dx)^{3/2}\sin(6a+6bx)}{2304b^2}\end{aligned}$$

output

```
45/1024*d^2*(d*x+c)^(1/2)*cos(2*b*x+2*a)/b^3-3/64*(d*x+c)^(5/2)*cos(2*b*x+
2*a)/b-5/9216*d^2*(d*x+c)^(1/2)*cos(6*b*x+6*a)/b^3+1/192*(d*x+c)^(5/2)*cos
(6*b*x+6*a)/b+5/55296*d^(5/2)*3^(1/2)*Pi^(1/2)*cos(6*a-6*b*c/d)*FresnelC(2
*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-45/2048*d^(5/2)*P
i^(1/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2)
)/b^(7/2)-5/55296*d^(5/2)*3^(1/2)*Pi^(1/2)*FresnelS(2*b^(1/2)*3^(1/2)/Pi^(
1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)/b^(7/2)+45/2048*d^(5/2)*Pi^(1
/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/b^
(7/2)+15/256*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2-5/2304*d*(d*x+c)^(3/2)*sin
(6*b*x+6*a)/b^2
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.21 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.63

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{e^{-\frac{6i(bc+ad)}{d}}(c+dx)^{5/2}\left(243e^{4i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{2ib(c+dx)}{d}\right)+243e^{4ia+\frac{8ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{2ib(}{}\right.\right.}{41472\sqrt{2}b\left(\frac{b^2(c+dx)}{d^2}\right.}$$

input

```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output

```
((c + d*x)^(5/2)*(243*E^((4*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*Ga
mma[7/2, ((-2*I)*b*(c + d*x))/d] + 243*E^((4*I)*a + ((8*I)*b*c)/d)*Sqrt[((
-I)*b*(c + d*x))/d]*Gamma[7/2, ((2*I)*b*(c + d*x))/d] - Sqrt[3]*(E^((12*I)
*a)*Sqrt[(I*b*(c + d*x))/d]*Gamma[7/2, ((-6*I)*b*(c + d*x))/d] + E^(((12*I
)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[7/2, ((6*I)*b*(c + d*x))/d])))/
(41472*Sqrt[2]*b*E^(((6*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^(3/2))
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin^3(a+bx)\cos^3(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{3}{32}(c+dx)^{5/2}\sin(2a+2bx) - \frac{1}{32}(c+dx)^{5/2}\sin(6a+6bx)\right)dx$$

$$\downarrow 2009$$

$$\frac{5\sqrt{\frac{\pi}{3}}d^{5/2}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}} - \frac{45\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2048b^{7/2}} - \frac{5\sqrt{\frac{\pi}{3}}d^{5/2}\sin\left(6a-\frac{6bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}} + \frac{45\sqrt{\pi}d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2048b^{7/2}} + \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3} - \frac{5d^2\sqrt{c+dx}\cos(6a+6bx)}{9216b^3} + \frac{15d(c+dx)^{3/2}\sin(2a+2bx)}{256b^2} - \frac{5d(c+dx)^{3/2}\sin(6a+6bx)}{2304b^2} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(45*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(1024*b^3) - (3*(c + d*x)^(5/2)*Co
s[2*a + 2*b*x])/(64*b) - (5*d^2*Sqrt[c + d*x]*Cos[6*a + 6*b*x])/(9216*b^3)
 + ((c + d*x)^(5/2)*Cos[6*a + 6*b*x])/(192*b) + (5*d^(5/2)*Sqrt[Pi/3]*Cos[
6*a - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(
18432*b^(7/2)) - (45*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqr
t[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2048*b^(7/2)) - (5*d^(5/2)*Sqrt[
Pi/3]*FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[6*a - (6*
b*c)/d])/(18432*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c
 + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2048*b^(7/2)) + (15*d*
(c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(256*b^2) - (5*d*(c + d*x)^(3/2)*Sin[6*a
 + 6*b*x])/(2304*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 23.12 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.17

method	result
derivativedivides	$-\frac{3d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{15d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}\right.\right.}{\left.\left.\right.\right.}$
default	$-\frac{3d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{15d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}\right.\right.}{\left.\left.\right.\right.}$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-3/128/b*d*(d*x+c)^(5/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+15/128/b*d*
(1/4/b*d*(d*x+c)^(3/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*
(d*x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2
)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-s
in(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))+1/
384/b*d*(d*x+c)^(5/2)*cos(6*b/d*(d*x+c)+6*(a*d-b*c)/d)-5/384/b*d*(1/12/b*d
*(d*x+c)^(3/2)*sin(6*b/d*(d*x+c)+6*(a*d-b*c)/d)-1/4/b*d*(-1/12/b*d*(d*x+c)
^(1/2)*cos(6*b/d*(d*x+c)+6*(a*d-b*c)/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*6^(1/2)
/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(
1/2)*b*(d*x+c)^(1/2)/d)-sin(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/
2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.09

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{5\sqrt{3}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 5\sqrt{3}\pi d^3\sqrt{\frac{b}{\pi d}}\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/55296*(5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos
(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))
*fresnel_sin(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d)
 - 1215*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 1215*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x
+ c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 96*(24*b^3*d^2*x^2 + 2*(48*b^
3*d^2*x^2 + 96*b^3*c*d*x + 48*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^6 + 48*b^3*c
*d*x + 24*b^3*c^2 + 45*b*d^2*cos(b*x + a)^2 - 3*(48*b^3*d^2*x^2 + 96*b^3*c
*d*x + 48*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^4 - 25*b*d^2 - 20*(2*(b^2*d^2*x
+ b^2*c*d)*cos(b*x + a)^5 - 2*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3 - 3*(b^
2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.38

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/442368*(960*(d*x + c)^(3/2)*b^3*sin(6*((d*x + c)*b - b*c + a*d)/d) - 25
920*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 48*(48*(d*x +
 c)^(5/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*cos(6*((d*x + c)*b - b*c + a*d)/d
) + 1296*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x +
 c)*b - b*c + a*d)/d) - 5*(-(I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d
^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*
(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) -
1215*((I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c -
 a*d)/d) + (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(
b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 1215*(-(I + 1)*4^(1/4)*s
qrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1
/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(-2*I*b/d)) - 5*((I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^
2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) + (I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d
^2*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)
))*d/b^5
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.44 (sec) , antiderivative size = 2411, normalized size of antiderivative = 5.92

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output

```
-1/110592*(576*(sqrt(3)*sqrt(pi)*d*erf(-I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) + sqrt(3)*sqrt(pi)*d*erf(I*sqrt(3)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)) - 9*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)) - 9*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)))*c^3 - d^3*((sqrt(3)*sqrt(pi)*(576*b^3*c^3 - 144*I*b^2*c^2*d - 36*b*
c*d^2 + 5*I*d^3)*d*erf(-I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b^3) + 6*I*(-48*I*(d*x + c)^(5/2)*b^2*d + 144*I*(d*x + c)^(3/2)*b^2*c*d -
 144*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqrt(d*x +
c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^(-6*(-I*(d*x + c)*b + I*b*c - I*a*d)
/d)/b^3)/d^3 + (sqrt(3)*sqrt(pi)*(576*b^3*c^3 + 144*I*b^2*c^2*d - 36*b*c*d
^2 - 5*I*d^3)*d*erf(I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*
b^3) + 6*I*(-48*I*(d*x + c)^(5/2)*b^2*d + 144*I*(d*x + c)^(3/2)*b^2*c*d -
144*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c
)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^(-6*(I*(d*x + c)*b - I*b*c + I*a*d...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)^3\sin(a+b\,x)^3\,(c+d\,x)^{5/2}\,dx$$

input

```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

output

```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \left(\int \sqrt{dx+c}\cos(bx+a)^3\sin(bx+a)^3 x^2 dx\right)d^2 + 2\left(\int \sqrt{dx+c}\cos(bx+a)^3\sin(bx+a)^3 x\,dx\right)cd + \left(\int \sqrt{dx+c}\cos(bx+a)^3\sin(bx+a)^3\,dx\right)c^2$$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**3*sin(a + b*x)**3*x**2,x)*d**2 + 2*int(sqr
t(c + d*x)*cos(a + b*x)**3*sin(a + b*x)**3*x,x)*c*d + int(sqrt(c + d*x)*co
s(a + b*x)**3*sin(a + b*x)**3,x)*c**2
```

3.197 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal result . . . 1802
Mathematica [C] (warning: unable to verify) . . . 1803
Rubi [A] (verified) . . . 1804
Maple [A] (verified) . . . 1805
Fricas [A] (verification not implemented) . . . 1806
Sympy [F(-1)] . . . 1807
Maxima [C] (verification not implemented) . . . 1807
Giac [C] (verification not implemented) . . . 1808
Mupad [F(-1)] . . . 1809
Reduce [F] . . . 1809

Optimal result

Integrand size = 26, antiderivative size = 351

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b}$$
$$+\frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b}+\frac{d^{3/2}\sqrt{\frac{\pi}{3}}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}$$
$$-\frac{9d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{512b^{5/2}}$$
$$+\frac{d^{3/2}\sqrt{\frac{\pi}{3}}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(6a-\frac{6bc}{d}\right)}{1536b^{5/2}}$$
$$-\frac{9d^{3/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{512b^{5/2}}$$
$$+\frac{9d\sqrt{c+dx}\sin(2a+2bx)}{256b^2}-\frac{d\sqrt{c+dx}\sin(6a+6bx)}{768b^2}$$

output

```
-3/64*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b+1/192*(d*x+c)^(3/2)*cos(6*b*x+6*a)/b+
1/4608*d^(3/2)*3^(1/2)*Pi^(1/2)*cos(6*a-6*b*c/d)*FresnelS(2*b^(1/2)*3^(1/2
)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)-9/512*d^(3/2)*Pi^(1/2)*cos(2*a-2
*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b^(5/2)+1/4608*
d^(3/2)*3^(1/2)*Pi^(1/2)*FresnelC(2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)
/d^(1/2))*sin(6*a-6*b*c/d)/b^(5/2)-9/512*d^(3/2)*Pi^(1/2)*FresnelC(2*b^(1/
2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/b^(5/2)+9/256*d*(d*x+c
)^(1/2)*sin(2*b*x+2*a)/b^2-1/768*d*(d*x+c)^(1/2)*sin(6*b*x+6*a)/b^2
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.74

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{ie^{-\frac{6i(bc+ad)}{d}}(c+dx)^{5/2}\left(-81e^{4i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{5}{2},-\frac{2ib(c+dx)}{d}\right)+81e^{4ia+\frac{8ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{5}{2},\frac{2ib}{d}\right.\right.}{6912\sqrt{2}d\left(\frac{b^2(c+dx)}{d^2}\right.}$$

input

```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output

```
((I/6912)*(c + d*x)^(5/2)*(-81*E^((4*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*
x))/d]*Gamma[5/2, ((-2*I)*b*(c + d*x))/d] + 81*E^((4*I)*a + ((8*I)*b*c)/d)
*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[5/2, ((2*I)*b*(c + d*x))/d] + Sqrt[3]*(E
^((12*I)*a)*Sqrt[(I*b*(c + d*x))/d]*Gamma[5/2, ((-6*I)*b*(c + d*x))/d] - E
^(((12*I)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[5/2, ((6*I)*b*(c + d*x)
)/d])))/(Sqrt[2]*d*E^(((6*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^(3/2)
)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin^3(a+bx)\cos^3(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int\left(\frac{3}{32}(c+dx)^{3/2}\sin(2a+2bx)-\frac{1}{32}(c+dx)^{3/2}\sin(6a+6bx)\right)dx$$

$$\downarrow 2009$$

$$\frac{\sqrt{\frac{\pi}{3}}d^{3/2}\sin\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}-\frac{9\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{512b^{5/2}}+\frac{\sqrt{\frac{\pi}{3}}d^{3/2}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}-\frac{9\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{512b^{5/2}}+\frac{9d\sqrt{c+dx}\sin(2a+2bx)}{256b^2}-\frac{d\sqrt{c+dx}\sin(6a+6bx)}{768b^2}-\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b}+\frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b}$$

input

```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output

```
(-3*(c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)^(3/2)*Cos[6*a +
6*b*x])/(192*b) + (d^(3/2)*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelS[(2*Sqr
t[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[
Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
Pi])])/(512*b^(5/2)) + (d^(3/2)*Sqrt[Pi/3]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(1536*b^(5/2)) - (9*d^(3/2)*
Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a -
(2*b*c)/d])/(512*b^(5/2)) + (9*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(256*b^2)
 - (d*Sqrt[c + d*x]*Sin[6*a + 6*b*x])/(768*b^2)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.09

method	result
derivativedivides	$-\frac{3d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{9d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{8b\sqrt{}}\right.}{64b}$
default	$-\frac{3d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{9d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{8b\sqrt{}}\right.}{64b}$

input

```
int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output

```
2/d*(-3/128/b*d*(d*x+c)^(3/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+9/128/b*d*(
1/4/b*d*(d*x+c)^(1/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b
/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/
d)))+1/384/b*d*(d*x+c)^(3/2)*cos(6*b/d*(d*x+c)+6*(a*d-b*c)/d)-1/128/b*d*(1
/12/b*d*(d*x+c)^(1/2)*sin(6*b/d*(d*x+c)+6*(a*d-b*c)/d)-1/144/b*d*2^(1/2)*P
i^(1/2)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*
6^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(6*(a*d-b*c)/d)*FresnelC(2^(1/2)
/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.93

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{\sqrt{3}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+\sqrt{3}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-$$

input

```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output

```
1/4608*(sqrt(3)*pi*d^2*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_sin(2*
sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(3)*pi*d^2*sqrt(b/(pi*d))*fres
nel_cos(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 81
*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*s
qrt(b/(pi*d))) - 81*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt
(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 96*(8*(b^2*d*x + b^2*c)*cos(b*x + a)^6
 - 12*(b^2*d*x + b^2*c)*cos(b*x + a)^4 + 2*b^2*d*x + 2*b^2*c - (2*b*d*cos(
b*x + a)^5 - 2*b*d*cos(b*x + a)^3 - 3*b*d*cos(b*x + a))*sin(b*x + a))*sqrt
(d*x + c))/b^3
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.46

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/36864*(192*(d*x + c)^(3/2)*b^3*cos(6*((d*x + c)*b - b*c + a*d)/d)/d - 17
28*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 48*sqrt(d*x
+ c)*b^2*sin(6*((d*x + c)*b - b*c + a*d)/d) + 1296*sqrt(d*x + c)*b^2*sin(2
*((d*x + c)*b - b*c + a*d)/d) + ((I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^
2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d
*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) +
 81*(-(I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a
*d)/d) + (I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c
- a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + 81*((I - 1)*4^(1/4)*sqrt(2)*
sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(
2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(-2*I*b/d)) + (-(I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*c
os(-6*(b*c - a*d)/d) + (I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/
4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))*d/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 1489, normalized size of antiderivative = 4.24

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/9216*(48*(sqrt(3)*sqrt(pi)*d*erf(-I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) + sqrt(3)*sqrt(pi)*d*erf(I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)) - 9*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)) - 9*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)))*c^2 - 8*(sqrt(3)*sqrt(pi)*(12*b*c - I*d)*d*erf(-I*sqrt(3)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(3)*sqrt(pi)*(12*b*c + I*d)*d*erf(I*
sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(-I*b*
c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 27*sqrt(pi)*(4*b*
c - I*d)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 27*sqrt(pi
)*(4*b*c + I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 5
4*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*sqrt(d*x
+ c)*d*e^(-6*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 54*sqrt(d*x + c)*d*e^(
-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^(-6*(-I*...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)^3\sin(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c + dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \left(\int \sqrt{dx+c}\,\cos(bx+a)^3\sin(bx+a)^3\,x dx\right) d + \left(\int \sqrt{dx+c}\,\cos(bx+a)^3\sin(bx+a)^3\,dx\right) c$$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**3*sin(a + b*x)**3*x,x)*d + int(sqrt(c + d*
x)*cos(a + b*x)**3*sin(a + b*x)**3,x)*c
```

3.198 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal result . . . 1810
Mathematica [C] (verified) . . . 1811
Rubi [A] (verified) . . . 1812
Maple [A] (verified) . . . 1813
Fricas [A] (verification not implemented) . . . 1814
Sympy [F] . . . 1814
Maxima [C] (verification not implemented) . . . 1815
Giac [C] (verification not implemented) . . . 1815
Mupad [F(-1)] . . . 1816
Reduce [F] . . . 1817

Optimal result

Integrand size = 26, antiderivative size = 299

$$\begin{aligned}
&\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx \\
&= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} \\
&\quad - \frac{\sqrt{d}\sqrt{\frac{\pi}{3}}\cos\left(6a-\frac{6bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}} \\
&\quad + \frac{3\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{3}}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(6a-\frac{6bc}{d}\right)}{384b^{3/2}} \\
&\quad - \frac{3\sqrt{d}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{128b^{3/2}}
\end{aligned}$$

output
```
-3/64*(d*x+c)^(1/2)*cos(2*b*x+2*a)/b+1/192*(d*x+c)^(1/2)*cos(6*b*x+6*a)/b-
1/1152*d^(1/2)*3^(1/2)*Pi^(1/2)*cos(6*a-6*b*c/d)*FresnelC(2*b^(1/2)*3^(1/2
)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+3/128*d^(1/2)*Pi^(1/2)*cos(2*a-2
*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b^(3/2)+1/1152*
d^(1/2)*3^(1/2)*Pi^(1/2)*FresnelS(2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)
/d^(1/2))*sin(6*a-6*b*c/d)/b^(3/2)-3/128*d^(1/2)*Pi^(1/2)*FresnelS(2*b^(1/
2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.86

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx$$

$$= \frac{e^{-\frac{6i(bc+ad)}{d}}\sqrt{c+dx}\left(-27e^{4i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)-27e^{4ia+\frac{8ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)+\sqrt{}\right.}{1152\sqrt{2}b\sqrt{\frac{b^2(c+dx)^2}{d^2}}}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(Sqrt[c + d*x]*(-27*E^((4*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*Gamm
a[3/2, ((-2*I)*b*(c + d*x))/d] - 27*E^((4*I)*a + ((8*I)*b*c)/d)*Sqrt[((-I)
*b*(c + d*x))/d]*Gamma[3/2, ((2*I)*b*(c + d*x))/d] + Sqrt[3]*(E^((12*I)*a)
*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-6*I)*b*(c + d*x))/d] + E^(((12*I)*b
*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((6*I)*b*(c + d*x))/d])))/(11
52*Sqrt[2]*b*E^(((6*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^3(a+bx)\cos^3(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{3}{32}\sqrt{c+dx}\sin(2a+2bx) - \frac{1}{32}\sqrt{c+dx}\sin(6a+6bx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}} + \frac{3\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{3/2}} + \frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\sin\left(6a-\frac{6bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}} - \frac{3\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{3/2}} - \frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(-3*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b) + (Sqrt[c + d*x]*Cos[6*a + 6*b*
x])/(192*b) - (Sqrt[d]*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelC[(2*Sqrt[b]
*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(384*b^(3/2)) + (3*Sqrt[d]*Sqrt[Pi]*C
os[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]
)/(128*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/3]*FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[
c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(384*b^(3/2)) - (3*Sqrt[d]*Sqrt[P
i]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c
)/d])/(128*b^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.98

method	result
derivativedivides	$-\frac{3d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{3d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{128b\sqrt{\frac{b}{d}}}+$
default	$-\frac{3d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{3d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{128b\sqrt{\frac{b}{d}}}+$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-3/128/b*d*(d*x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+3/256/b*d*P
i^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*
(d*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x
+c)^(1/2)/d))+1/384/b*d*(d*x+c)^(1/2)*cos(6*b/d*(d*x+c)+6*(a*d-b*c)/d)-1/4
608/b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(
2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(6*(a*d-b*c)/d)
*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.81

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx =$$
$$-\frac{\sqrt{3}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{3}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right.}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/1152*(sqrt(3)*pi*d*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos(2*s
qrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(3)*pi*d*sqrt(b/(pi*d))*fresnel
_sin(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 27*pi
*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b
/(pi*d))) + 27*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*
d)))*sin(-2*(b*c - a*d)/d) - 48*(4*b*cos(b*x + a)^6 - 6*b*cos(b*x + a)^4 +
 b)*sqrt(d*x + c))/b^2
```

Sympy [F]

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \sqrt{c+dx}\sin^3(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.47

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/9216*(48*sqrt(d*x + c)*b^2*cos(6*((d*x + c)*b - b*c + a*d)/d)/d - 432*sq
rt(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - (-(I - 1)*36^(1/4)*
sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (I + 1)*36^(1/4
)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x +
 c)*sqrt(6*I*b/d)) - 27*((I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4
)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/
4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 27*(-(I + 1)*
4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)
*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(-2*I*b/d)) - ((I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2
)^(1/4)*cos(-6*(b*c - a*d)/d) + (I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d
^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))*d/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 826, normalized size of antiderivative = 2.76

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output

```
1/2304*(sqrt(3)*sqrt(pi)*(12*b*c - I*d)*d*erf(-I*sqrt(3)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(3)*sqrt(pi)*(12*b*c + I*d)*d*erf(I*sqrt(
3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 12*(sqrt(3)*sqrt(pi)*d
*erf(-I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6
*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(3)*sqrt(p
i)*d*erf(I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 9*sqrt(p
i)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*
b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 9*sqrt(pi)*d*erf(I
*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 27*sqrt(pi)*(4*b*c - I*
d)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*
b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 27*sqrt(pi)*(4*b
*c + I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 54*sqrt
(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*sqrt(d*x + c)*d
*e^(-6*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 54*sqrt(d*x + c)*d*e^(-2*(-I
*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^(-6*(-I*(d*x +...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)^3\sin(a+b\,x)^3\sqrt{c+d\,x}\,dx$$

input

```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

output

```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \sqrt{dx+c}\cos(bx+a)^3\sin(bx+a)^3\,dx$$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**3*sin(a + b*x)**3,x)
```

3.199 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal result . . . 1818
Mathematica [C] (verified) . . . 1819
Rubi [A] (verified) . . . 1820
Maple [A] (verified) . . . 1821
Fricas [A] (verification not implemented) . . . 1822
Sympy [F] . . . 1822
Maxima [C] (verification not implemented) . . . 1823
Giac [C] (verification not implemented) . . . 1823
Mupad [F(-1)] . . . 1824
Reduce [F] . . . 1825

Optimal result

Integrand size = 26, antiderivative size = 299

$$\begin{aligned}
&\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx \\
&= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} \\
&\quad - \frac{\sqrt{d}\sqrt{\frac{\pi}{3}}\cos\left(6a-\frac{6bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}} \\
&\quad + \frac{3\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{3/2}} \\
&\quad + \frac{\sqrt{d}\sqrt{\frac{\pi}{3}}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(6a-\frac{6bc}{d}\right)}{384b^{3/2}} \\
&\quad - \frac{3\sqrt{d}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{128b^{3/2}}
\end{aligned}$$

output
```
-3/64*(d*x+c)^(1/2)*cos(2*b*x+2*a)/b+1/192*(d*x+c)^(1/2)*cos(6*b*x+6*a)/b-
1/1152*d^(1/2)*3^(1/2)*Pi^(1/2)*cos(6*a-6*b*c/d)*FresnelC(2*b^(1/2)*3^(1/2
)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+3/128*d^(1/2)*Pi^(1/2)*cos(2*a-2
*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b^(3/2)+1/1152*
d^(1/2)*3^(1/2)*Pi^(1/2)*FresnelS(2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)
/d^(1/2))*sin(6*a-6*b*c/d)/b^(3/2)-3/128*d^(1/2)*Pi^(1/2)*FresnelS(2*b^(1/
2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.86

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx$$

$$= \frac{e^{-\frac{6i(bc+ad)}{d}}\sqrt{c+dx}\left(-27e^{4i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)-27e^{4ia+\frac{8ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)+\sqrt{}\right.}{1152\sqrt{2}b\sqrt{\frac{b^2(c+dx)^2}{d^2}}}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(Sqrt[c + d*x]*(-27*E^((4*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*Gamm
a[3/2, ((-2*I)*b*(c + d*x))/d] - 27*E^((4*I)*a + ((8*I)*b*c)/d)*Sqrt[((-I)
*b*(c + d*x))/d]*Gamma[3/2, ((2*I)*b*(c + d*x))/d] + Sqrt[3]*(E^((12*I)*a)
*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-6*I)*b*(c + d*x))/d] + E^(((12*I)*b
*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((6*I)*b*(c + d*x))/d])))/(11
52*Sqrt[2]*b*E^(((6*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^3(a+bx)\cos^3(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{3}{32}\sqrt{c+dx}\sin(2a+2bx) - \frac{1}{32}\sqrt{c+dx}\sin(6a+6bx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}} + \frac{3\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{3/2}} + \frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\sin\left(6a-\frac{6bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}} - \frac{3\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{3/2}} - \frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(-3*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b) + (Sqrt[c + d*x]*Cos[6*a + 6*b*
x])/(192*b) - (Sqrt[d]*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelC[(2*Sqrt[b]
*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(384*b^(3/2)) + (3*Sqrt[d]*Sqrt[Pi]*C
os[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]
)/(128*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/3]*FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[
c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(384*b^(3/2)) - (3*Sqrt[d]*Sqrt[P
i]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c
)/d])/(128*b^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.98

method	result
derivativedivides	$-\frac{3d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{3d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{128b\sqrt{\frac{b}{d}}}+$
default	$-\frac{3d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{3d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{128b\sqrt{\frac{b}{d}}}+$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-3/128/b*d*(d*x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+3/256/b*d*P
i^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*
(d*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x
+c)^(1/2)/d))+1/384/b*d*(d*x+c)^(1/2)*cos(6*b/d*(d*x+c)+6*(a*d-b*c)/d)-1/4
608/b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(
2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(6*(a*d-b*c)/d)
*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.81

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx =$$
$$-\frac{\sqrt{3}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{3}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right.}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/1152*(sqrt(3)*pi*d*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos(2*s
qrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(3)*pi*d*sqrt(b/(pi*d))*fresnel
_sin(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 27*pi
*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b
/(pi*d))) + 27*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*
d)))*sin(-2*(b*c - a*d)/d) - 48*(4*b*cos(b*x + a)^6 - 6*b*cos(b*x + a)^4 +
 b)*sqrt(d*x + c))/b^2
```

Sympy [F]

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \sqrt{c+dx}\sin^3(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.47

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/9216*(48*sqrt(d*x + c)*b^2*cos(6*((d*x + c)*b - b*c + a*d)/d)/d - 432*sq
rt(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - (-(I - 1)*36^(1/4)*
sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (I + 1)*36^(1/4
)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x +
 c)*sqrt(6*I*b/d)) - 27*((I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4
)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/
4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 27*(-(I + 1)*
4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)
*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(-2*I*b/d)) - ((I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2
)^(1/4)*cos(-6*(b*c - a*d)/d) + (I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d
^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))*d/b^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 826, normalized size of antiderivative = 2.76

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output

```
1/2304*(sqrt(3)*sqrt(pi)*(12*b*c - I*d)*d*erf(-I*sqrt(3)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(3)*sqrt(pi)*(12*b*c + I*d)*d*erf(I*sqrt(
3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 12*(sqrt(3)*sqrt(pi)*d
*erf(-I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6
*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(3)*sqrt(p
i)*d*erf(I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 9*sqrt(p
i)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*
b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 9*sqrt(pi)*d*erf(I
*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 27*sqrt(pi)*(4*b*c - I*
d)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*
b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 27*sqrt(pi)*(4*b
*c + I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 54*sqrt
(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*sqrt(d*x + c)*d
*e^(-6*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 54*sqrt(d*x + c)*d*e^(-2*(-I
*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^(-6*(-I*(d*x +...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)^3\sin(a+b\,x)^3\sqrt{c+d\,x}\,dx$$

input

```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

output

```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \sqrt{dx+c}\cos(bx+a)^3\sin(bx+a)^3\,dx$$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**3*sin(a + b*x)**3,x)
```

3.200 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal result . . . 1826
Mathematica [C] (warning: unable to verify) . . . 1827
Rubi [A] (verified) . . . 1828
Maple [A] (verified) . . . 1829
Fricas [A] (verification not implemented) . . . 1830
Sympy [F(-1)] . . . 1831
Maxima [C] (verification not implemented) . . . 1831
Giac [C] (verification not implemented) . . . 1832
Mupad [F(-1)] . . . 1833
Reduce [F] . . . 1833

Optimal result

Integrand size = 26, antiderivative size = 351

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{d^{3/2}\sqrt{\frac{\pi}{3}}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}} - \frac{9d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{512b^{5/2}} + \frac{d^{3/2}\sqrt{\frac{\pi}{3}}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(6a-\frac{6bc}{d}\right)}{1536b^{5/2}} - \frac{9d^{3/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{512b^{5/2}} + \frac{9d\sqrt{c+dx}\sin(2a+2bx)}{256b^2} - \frac{d\sqrt{c+dx}\sin(6a+6bx)}{768b^2}$$

output

```
-3/64*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b+1/192*(d*x+c)^(3/2)*cos(6*b*x+6*a)/b+
1/4608*d^(3/2)*3^(1/2)*Pi^(1/2)*cos(6*a-6*b*c/d)*FresnelS(2*b^(1/2)*3^(1/2
)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)-9/512*d^(3/2)*Pi^(1/2)*cos(2*a-2
*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))/b^(5/2)+1/4608*
d^(3/2)*3^(1/2)*Pi^(1/2)*FresnelC(2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)
/d^(1/2))*sin(6*a-6*b*c/d)/b^(5/2)-9/512*d^(3/2)*Pi^(1/2)*FresnelC(2*b^(1/
2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/b^(5/2)+9/256*d*(d*x+c
)^(1/2)*sin(2*b*x+2*a)/b^2-1/768*d*(d*x+c)^(1/2)*sin(6*b*x+6*a)/b^2
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.74

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{ie^{-\frac{6i(bc+ad)}{d}}(c+dx)^{5/2}\left(-81e^{4i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{5}{2},-\frac{2ib(c+dx)}{d}\right)+81e^{4ia+\frac{8ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{5}{2},\frac{2ib}{d}\right)\right.}{6912\sqrt{2}d\left(\frac{b^2(c+dx)}{d^2}\right.}$$

input

```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output

```
((I/6912)*(c + d*x)^(5/2)*(-81*E^((4*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*
x))/d]*Gamma[5/2, ((-2*I)*b*(c + d*x))/d] + 81*E^((4*I)*a + ((8*I)*b*c)/d)
*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[5/2, ((2*I)*b*(c + d*x))/d] + Sqrt[3]*(E
^((12*I)*a)*Sqrt[(I*b*(c + d*x))/d]*Gamma[5/2, ((-6*I)*b*(c + d*x))/d] - E
^(((12*I)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[5/2, ((6*I)*b*(c + d*x)
)/d])))/(Sqrt[2]*d*E^(((6*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^(3/2)
)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin^3(a+bx)\cos^3(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{3}{32}(c+dx)^{3/2}\sin(2a+2bx) - \frac{1}{32}(c+dx)^{3/2}\sin(6a+6bx)\right)dx$$

$$\downarrow 2009$$

$$\frac{\sqrt{\frac{\pi}{3}}d^{3/2}\sin\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}} - \frac{9\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{512b^{5/2}} + \frac{\sqrt{\frac{\pi}{3}}d^{3/2}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}} - \frac{9\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{512b^{5/2}} + \frac{9d\sqrt{c+dx}\sin(2a+2bx)}{256b^2} - \frac{d\sqrt{c+dx}\sin(6a+6bx)}{768b^2} - \frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b}$$

input

```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(-3*(c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)^(3/2)*Cos[6*a +
6*b*x])/(192*b) + (d^(3/2)*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelS[(2*Sqr
t[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[
Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
Pi])])/(512*b^(5/2)) + (d^(3/2)*Sqrt[Pi/3]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(1536*b^(5/2)) - (9*d^(3/2)*
Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a -
(2*b*c)/d])/(512*b^(5/2)) + (9*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(256*b^2)
 - (d*Sqrt[c + d*x]*Sin[6*a + 6*b*x])/(768*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.09

method	result
derivativedivides	$-\frac{3d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{9d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{8b\sqrt{}}\right.}{64b}$
default	$-\frac{3d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{9d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{8b\sqrt{}}\right.}{64b}$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-3/128/b*d*(d*x+c)^(3/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+9/128/b*d*(
1/4/b*d*(d*x+c)^(1/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b
/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/
d)))+1/384/b*d*(d*x+c)^(3/2)*cos(6*b/d*(d*x+c)+6*(a*d-b*c)/d)-1/128/b*d*(1
/12/b*d*(d*x+c)^(1/2)*sin(6*b/d*(d*x+c)+6*(a*d-b*c)/d)-1/144/b*d*2^(1/2)*P
i^(1/2)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*
6^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(6*(a*d-b*c)/d)*FresnelC(2^(1/2)
/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.93

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{\sqrt{3}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+\sqrt{3}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4608*(sqrt(3)*pi*d^2*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_sin(2*
sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(3)*pi*d^2*sqrt(b/(pi*d))*fres
nel_cos(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 81
*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*s
qrt(b/(pi*d))) - 81*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt
(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 96*(8*(b^2*d*x + b^2*c)*cos(b*x + a)^6
 - 12*(b^2*d*x + b^2*c)*cos(b*x + a)^4 + 2*b^2*d*x + 2*b^2*c - (2*b*d*cos(
b*x + a)^5 - 2*b*d*cos(b*x + a)^3 - 3*b*d*cos(b*x + a))*sin(b*x + a))*sqrt
(d*x + c))/b^3
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.46

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/36864*(192*(d*x + c)^(3/2)*b^3*cos(6*((d*x + c)*b - b*c + a*d)/d)/d - 17
28*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 48*sqrt(d*x
+ c)*b^2*sin(6*((d*x + c)*b - b*c + a*d)/d) + 1296*sqrt(d*x + c)*b^2*sin(2
*((d*x + c)*b - b*c + a*d)/d) + ((I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^
2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d
*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) +
 81*(-(I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a
*d)/d) + (I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c
- a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + 81*((I - 1)*4^(1/4)*sqrt(2)*
sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(
2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(-2*I*b/d)) + (-(I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*c
os(-6*(b*c - a*d)/d) + (I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/
4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))*d/b^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 1489, normalized size of antiderivative = 4.24

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/9216*(48*(sqrt(3)*sqrt(pi)*d*erf(-I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) + sqrt(3)*sqrt(pi)*d*erf(I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)) - 9*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)) - 9*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)))*c^2 - 8*(sqrt(3)*sqrt(pi)*(12*b*c - I*d)*d*erf(-I*sqrt(3)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(3)*sqrt(pi)*(12*b*c + I*d)*d*erf(I*
sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(-I*b*
c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 27*sqrt(pi)*(4*b*
c - I*d)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 27*sqrt(pi
)*(4*b*c + I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 5
4*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*sqrt(d*x
+ c)*d*e^(-6*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 54*sqrt(d*x + c)*d*e^(
-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^(-6*(-I*...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)^3\sin(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c + dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \left(\int \sqrt{dx+c}\cos(bx+a)^3\sin(bx+a)^3\,x dx\right)d + \left(\int \sqrt{dx+c}\cos(bx+a)^3\sin(bx+a)^3\,dx\right)c$$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**3*sin(a + b*x)**3*x,x)*d + int(sqrt(c + d*
x)*cos(a + b*x)**3*sin(a + b*x)**3,x)*c
```

3.201 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal result . . . 1834
Mathematica [C] (warning: unable to verify) . . . 1835
Rubi [A] (verified) . . . 1836
Maple [A] (verified) . . . 1837
Fricas [A] (verification not implemented) . . . 1839
Sympy [F(-1)] . . . 1839
Maxima [C] (verification not implemented) . . . 1840
Giac [C] (verification not implemented) . . . 1840
Mupad [F(-1)] . . . 1841
Reduce [F] . . . 1842

Optimal result

Integrand size = 26, antiderivative size = 407

$$\begin{aligned}\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx &= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3}\\ &- \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d^2\sqrt{c+dx}\cos(6a+6bx)}{9216b^3}\\ &+ \frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b} + \frac{5d^{5/2}\sqrt{\frac{\pi}{3}}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}}\\ &- \frac{45d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2048b^{7/2}}\\ &- \frac{5d^{5/2}\sqrt{\frac{\pi}{3}}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(6a-\frac{6bc}{d}\right)}{18432b^{7/2}}\\ &+ \frac{45d^{5/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{2048b^{7/2}}\\ &+ \frac{15d(c+dx)^{3/2}\sin(2a+2bx)}{256b^2} - \frac{5d(c+dx)^{3/2}\sin(6a+6bx)}{2304b^2}\end{aligned}$$

output

```
45/1024*d^2*(d*x+c)^(1/2)*cos(2*b*x+2*a)/b^3-3/64*(d*x+c)^(5/2)*cos(2*b*x+
2*a)/b-5/9216*d^2*(d*x+c)^(1/2)*cos(6*b*x+6*a)/b^3+1/192*(d*x+c)^(5/2)*cos
(6*b*x+6*a)/b+5/55296*d^(5/2)*3^(1/2)*Pi^(1/2)*cos(6*a-6*b*c/d)*FresnelC(2
*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-45/2048*d^(5/2)*P
i^(1/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2)
)/b^(7/2)-5/55296*d^(5/2)*3^(1/2)*Pi^(1/2)*FresnelS(2*b^(1/2)*3^(1/2)/Pi^(
1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)/b^(7/2)+45/2048*d^(5/2)*Pi^(1
/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)/b^
(7/2)+15/256*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2-5/2304*d*(d*x+c)^(3/2)*sin
(6*b*x+6*a)/b^2
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.63

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{e^{-\frac{6i(bc+ad)}{d}}(c+dx)^{5/2}\left(243e^{4i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{2ib(c+dx)}{d}\right)+243e^{4ia+\frac{8ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{2ib(}{}\right.\right.}{41472\sqrt{2}b\left(\frac{b^2(c+dx)}{d^2}\right.}$$

input

```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output

```
((c + d*x)^(5/2)*(243*E^((4*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*Ga
mma[7/2, ((-2*I)*b*(c + d*x))/d] + 243*E^((4*I)*a + ((8*I)*b*c)/d)*Sqrt[((
-I)*b*(c + d*x))/d]*Gamma[7/2, ((2*I)*b*(c + d*x))/d] - Sqrt[3]*(E^((12*I)
*a)*Sqrt[(I*b*(c + d*x))/d]*Gamma[7/2, ((-6*I)*b*(c + d*x))/d] + E^(((12*I
)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[7/2, ((6*I)*b*(c + d*x))/d])))/
(41472*Sqrt[2]*b*E^(((6*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin^3(a+bx)\cos^3(a+bx)\,dx$$

$$\downarrow 4906$$

$$\int \left(\frac{3}{32}(c+dx)^{5/2}\sin(2a+2bx) - \frac{1}{32}(c+dx)^{5/2}\sin(6a+6bx)\right)dx$$

$$\downarrow 2009$$

$$\frac{5\sqrt{\frac{\pi}{3}}d^{5/2}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}} - \frac{45\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2048b^{7/2}} - \frac{5\sqrt{\frac{\pi}{3}}d^{5/2}\sin\left(6a-\frac{6bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}} + \frac{45\sqrt{\pi}d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2048b^{7/2}} + \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3} - \frac{5d^2\sqrt{c+dx}\cos(6a+6bx)}{9216b^3} + \frac{15d(c+dx)^{3/2}\sin(2a+2bx)}{256b^2} - \frac{5d(c+dx)^{3/2}\sin(6a+6bx)}{2304b^2} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(45*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(1024*b^3) - (3*(c + d*x)^(5/2)*Co
s[2*a + 2*b*x])/(64*b) - (5*d^2*Sqrt[c + d*x]*Cos[6*a + 6*b*x])/(9216*b^3)
 + ((c + d*x)^(5/2)*Cos[6*a + 6*b*x])/(192*b) + (5*d^(5/2)*Sqrt[Pi/3]*Cos[
6*a - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(
18432*b^(7/2)) - (45*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqr
t[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2048*b^(7/2)) - (5*d^(5/2)*Sqrt[
Pi/3]*FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[6*a - (6*
b*c)/d])/(18432*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c
 + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2048*b^(7/2)) + (15*d*
(c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(256*b^2) - (5*d*(c + d*x)^(3/2)*Sin[6*a
 + 6*b*x])/(2304*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Maple [A] (verified)

Time = 22.67 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.17

method	result
derivativedivides	$-\frac{3d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{15d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}\right.\right.}{\left.\left.\right.\right.}$
default	$-\frac{3d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{15d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}\right.\right.}{\left.\left.\right.\right.}$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-3/128/b*d*(d*x+c)^(5/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+15/128/b*d*
(1/4/b*d*(d*x+c)^(3/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*
(d*x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2
)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-s
in(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))+1/
384/b*d*(d*x+c)^(5/2)*cos(6*b/d*(d*x+c)+6*(a*d-b*c)/d)-5/384/b*d*(1/12/b*d
*(d*x+c)^(3/2)*sin(6*b/d*(d*x+c)+6*(a*d-b*c)/d)-1/4/b*d*(-1/12/b*d*(d*x+c)
^(1/2)*cos(6*b/d*(d*x+c)+6*(a*d-b*c)/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*6^(1/2)
/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(
1/2)*b*(d*x+c)^(1/2)/d)-sin(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/
2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.09

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{5\sqrt{3}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 5\sqrt{3}\pi d^3\sqrt{\frac{b}{\pi d}}\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/55296*(5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos
(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))
*fresnel_sin(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d)
 - 1215*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 1215*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x
+ c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 96*(24*b^3*d^2*x^2 + 2*(48*b^
3*d^2*x^2 + 96*b^3*c*d*x + 48*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^6 + 48*b^3*c
*d*x + 24*b^3*c^2 + 45*b*d^2*cos(b*x + a)^2 - 3*(48*b^3*d^2*x^2 + 96*b^3*c
*d*x + 48*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^4 - 25*b*d^2 - 20*(2*(b^2*d^2*x
+ b^2*c*d)*cos(b*x + a)^5 - 2*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3 - 3*(b^
2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^4
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.38

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/442368*(960*(d*x + c)^(3/2)*b^3*sin(6*((d*x + c)*b - b*c + a*d)/d) - 25
920*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 48*(48*(d*x +
 c)^(5/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*cos(6*((d*x + c)*b - b*c + a*d)/d
) + 1296*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x +
 c)*b - b*c + a*d)/d) - 5*(-(I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d
^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*
(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) -
1215*((I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c -
 a*d)/d) + (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(
b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 1215*(-(I + 1)*4^(1/4)*s
qrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1
/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(-2*I*b/d)) - 5*((I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^
2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) + (I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d
^2*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)
))*d/b^5
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.44 (sec) , antiderivative size = 2411, normalized size of antiderivative = 5.92

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output

```
-1/110592*(576*(sqrt(3)*sqrt(pi)*d*erf(-I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) + sqrt(3)*sqrt(pi)*d*erf(I*sqrt(3)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)) - 9*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)) - 9*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)))*c^3 - d^3*((sqrt(3)*sqrt(pi)*(576*b^3*c^3 - 144*I*b^2*c^2*d - 36*b*
c*d^2 + 5*I*d^3)*d*erf(-I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b^3) + 6*I*(-48*I*(d*x + c)^(5/2)*b^2*d + 144*I*(d*x + c)^(3/2)*b^2*c*d -
 144*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqrt(d*x +
c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^(-6*(-I*(d*x + c)*b + I*b*c - I*a*d)
/d)/b^3)/d^3 + (sqrt(3)*sqrt(pi)*(576*b^3*c^3 + 144*I*b^2*c^2*d - 36*b*c*d
^2 - 5*I*d^3)*d*erf(I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*
b^3) + 6*I*(-48*I*(d*x + c)^(5/2)*b^2*d + 144*I*(d*x + c)^(3/2)*b^2*c*d -
144*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c
)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^(-6*(I*(d*x + c)*b - I*b*c + I*a*d...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)^3\sin(a+b\,x)^3\,(c+d\,x)^{5/2}\,dx$$

input

```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

output

```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \left(\int \sqrt{dx+c}\cos(bx+a)^3\sin(bx+a)^3 x^2 dx\right)d^2 + 2\left(\int \sqrt{dx+c}\cos(bx+a)^3\sin(bx+a)^3 x\,dx\right)cd + \left(\int \sqrt{dx+c}\cos(bx+a)^3\sin(bx+a)^3\,dx\right)c^2$$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*cos(a + b*x)**3*sin(a + b*x)**3*x**2,x)*d**2 + 2*int(sqr
t(c + d*x)*cos(a + b*x)**3*sin(a + b*x)**3*x,x)*c*d + int(sqrt(c + d*x)*co
s(a + b*x)**3*sin(a + b*x)**3,x)*c**2
```

3.202 $\int x^3 \cos^2(x) \cot^2(x)\, dx$

Optimal result . . . 1843
Mathematica [A] (verified) . . . 1843
Rubi [A] (verified) . . . 1844
Maple [A] (verified) . . . 1849
Fricas [B] (verification not implemented) . . . 1849
Sympy [F] . . . 1850
Maxima [F(-2)] . . . 1850
Giac [F] . . . 1850
Mupad [F(-1)] . . . 1851
Reduce [F] . . . 1851

Optimal result

Integrand size = 12, antiderivative size = 112

$$\begin{aligned}\int x^3 \cos^2(x) \cot^2(x)\, dx = {} & \frac{3x^2}{8} - ix^3 - \frac{3x^4}{8} + \frac{3\cos^2(x)}{8} - \frac{3}{4}x^2\cos^2(x) \\ & - x^3\cot(x) + 3x^2\log\left(1-e^{2ix}\right) - 3ix\,\text{PolyLog}\left(2,e^{2ix}\right) \\ & + \frac{3}{2}\,\text{PolyLog}\left(3,e^{2ix}\right) + \frac{3}{4}x\cos(x)\sin(x) - \frac{1}{2}x^3\cos(x)\sin(x)\end{aligned}$$

output

```
3/8*x^2-I*x^3-3/8*x^4+3/8*cos(x)^2-3/4*x^2*cos(x)^2-x^3*cot(x)+3*x^2*ln(1-
exp(2*I*x))-3*I*x*polylog(2,exp(2*I*x))+3/2*polylog(3,exp(2*I*x))+3/4*x*co
s(x)*sin(x)-1/2*x^3*cos(x)*sin(x)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.93

$$\begin{aligned}\int x^3 \cos^2(x) \cot^2(x)\, dx = \frac{1}{16}\big(& -2i\pi^3 + 16ix^3 - 6x^4 + 3\cos(2x) - 6x^2\cos(2x) - 16x^3\cot(x) \\ & + 48x^2\log\left(1-e^{-2ix}\right) + 48ix\,\text{PolyLog}\left(2,e^{-2ix}\right) \\ & + 24\,\text{PolyLog}\left(3,e^{-2ix}\right) + 6x\sin(2x) - 4x^3\sin(2x)\big)\end{aligned}$$

input

```
Integrate[x^3*Cos[x]^2*Cot[x]^2,x]
```

output

```
((-2*I)*Pi^3 + (16*I)*x^3 - 6*x^4 + 3*Cos[2*x] - 6*x^2*Cos[2*x] - 16*x^3*C
ot[x] + 48*x^2*Log[1 - E^((-2*I)*x)] + (48*I)*x*PolyLog[2, E^((-2*I)*x)] +
 24*PolyLog[3, E^((-2*I)*x)] + 6*x*Sin[2*x] - 4*x^3*Sin[2*x])/16
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.23, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {4908, 3042, 3792, 15, 3042, 3791, 15, 4203, 15, 25, 3042, 25, 4200, 25, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cos^2(x) \cot^2(x)\, dx$$

$$\downarrow 4908$$

$$\int x^3 \cot^2(x) dx - \int x^3 \cos^2(x) dx$$

$$\downarrow 3042$$

$$\int x^3 \tan\left(x + \frac{\pi}{2}\right)^2 dx - \int x^3 \sin\left(x + \frac{\pi}{2}\right)^2 dx$$

$$\downarrow 3792$$

$$-\frac{\int x^3 dx}{2} + \int x^3 \tan\left(x + \frac{\pi}{2}\right)^2 dx + \frac{3}{2}\int x \cos^2(x) dx - \frac{1}{2}x^3 \sin(x)\cos(x) - \frac{3}{4}x^2 \cos^2(x)$$

$$\downarrow 15$$

$$\int x^3 \tan\left(x + \frac{\pi}{2}\right)^2 dx + \frac{3}{2}\int x \cos^2(x) dx - \frac{x^4}{8} - \frac{1}{2}x^3 \sin(x)\cos(x) - \frac{3}{4}x^2 \cos^2(x)$$

$$\downarrow 3042$$

$$\int x^3 \tan\left(x + \frac{\pi}{2}\right)^2 dx + \frac{3}{2}\int x \sin\left(x + \frac{\pi}{2}\right)^2 dx - \frac{x^4}{8} - \frac{1}{2}x^3 \sin(x)\cos(x) - \frac{3}{4}x^2 \cos^2(x)$$

$$\downarrow 3791$$

$$\int x^3 \tan\left(x+\frac{\pi}{2}\right)^2 dx + \frac{3}{2}\left(\frac{\int x dx}{2} + \frac{\cos^2(x)}{4} + \frac{1}{2}x\sin(x)\cos(x)\right) - \frac{x^4}{8} - \frac{1}{2}x^3\sin(x)\cos(x) - \frac{3}{4}x^2\cos^2(x)$$

$$\downarrow 15$$

$$\int x^3 \tan\left(x+\frac{\pi}{2}\right)^2 dx - \frac{x^4}{8} - \frac{1}{2}x^3\sin(x)\cos(x) - \frac{3}{4}x^2\cos^2(x) + \frac{3}{2}\left(\frac{x^2}{4} + \frac{\cos^2(x)}{4} + \frac{1}{2}x\sin(x)\cos(x)\right)$$

$$\downarrow 4203$$

$$-\int x^3 dx - 3\int -x^2\cot(x)dx - \frac{x^4}{8} - x^3\cot(x) - \frac{1}{2}x^3\sin(x)\cos(x) - \frac{3}{4}x^2\cos^2(x) + \frac{3}{2}\left(\frac{x^2}{4} + \frac{\cos^2(x)}{4} + \frac{1}{2}x\sin(x)\cos(x)\right)$$

$$\downarrow 15$$

$$-3\int -x^2\cot(x)dx - \frac{3x^4}{8} - x^3\cot(x) - \frac{1}{2}x^3\sin(x)\cos(x) - \frac{3}{4}x^2\cos^2(x) + \frac{3}{2}\left(\frac{x^2}{4} + \frac{\cos^2(x)}{4} + \frac{1}{2}x\sin(x)\cos(x)\right)$$

$$\downarrow 25$$

$$3\int x^2\cot(x)dx - \frac{3x^4}{8} - x^3\cot(x) - \frac{1}{2}x^3\sin(x)\cos(x) - \frac{3}{4}x^2\cos^2(x) + \frac{3}{2}\left(\frac{x^2}{4} + \frac{\cos^2(x)}{4} + \frac{1}{2}x\sin(x)\cos(x)\right)$$

$$\downarrow 3042$$

$$3\int -x^2\tan\left(x+\frac{\pi}{2}\right)dx - \frac{3x^4}{8} - x^3\cot(x) - \frac{1}{2}x^3\sin(x)\cos(x) - \frac{3}{4}x^2\cos^2(x) + \frac{3}{2}\left(\frac{x^2}{4} + \frac{\cos^2(x)}{4} + \frac{1}{2}x\sin(x)\cos(x)\right)$$

$$\downarrow 25$$

$$-3\int x^2\tan\left(x+\frac{\pi}{2}\right)dx - \frac{3x^4}{8} - x^3\cot(x) - \frac{1}{2}x^3\sin(x)\cos(x) - \frac{3}{4}x^2\cos^2(x) + \frac{3}{2}\left(\frac{x^2}{4} + \frac{\cos^2(x)}{4} + \frac{1}{2}x\sin(x)\cos(x)\right)$$

$$\downarrow 4200$$

$$-3\left(\frac{ix^3}{3}-2i\int-\frac{e^{2ix}x^2}{1-e^{2ix}}dx\right)-\frac{3x^4}{8}-x^3\cot(x)-\frac{1}{2}x^3\sin(x)\cos(x)-\frac{3}{4}x^2\cos^2(x)+\frac{3}{2}\left(\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)\right)$$

$$\downarrow 25$$

$$-3\left(2i\int\frac{e^{2ix}x^2}{1-e^{2ix}}dx+\frac{ix^3}{3}\right)-\frac{3x^4}{8}-x^3\cot(x)-\frac{1}{2}x^3\sin(x)\cos(x)-\frac{3}{4}x^2\cos^2(x)+\frac{3}{2}\left(\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)\right)$$

$$\downarrow 2620$$

$$-3\left(2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\int x\log\left(1-e^{2ix}\right)dx\right)+\frac{ix^3}{3}\right)-\frac{3x^4}{8}-x^3\cot(x)-\frac{1}{2}x^3\sin(x)\cos(x)-\frac{3}{4}x^2\cos^2(x)+\frac{3}{2}\left(\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)\right)$$

$$\downarrow 3011$$

$$-3\left(2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\mathrm{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{2}i\int\mathrm{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)+\frac{ix^3}{3}\right)-\frac{3x^4}{8}-x^3\cot(x)-\frac{1}{2}x^3\sin(x)\cos(x)-\frac{3}{4}x^2\cos^2(x)+\frac{3}{2}\left(\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)\right)$$

$$\downarrow 2720$$

$$-3\left(2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\mathrm{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\mathrm{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)+\frac{ix^3}{3}\right)-\frac{3x^4}{8}-x^3\cot(x)-\frac{1}{2}x^3\sin(x)\cos(x)-\frac{3}{4}x^2\cos^2(x)+\frac{3}{2}\left(\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)\right)$$

$$\downarrow 7143$$

$$-3\left(2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\mathrm{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\,\mathrm{PolyLog}\left(3,e^{2ix}\right)\right)\right)+\frac{ix^3}{3}\right)-\frac{3x^4}{8}-x^3\cot(x)-\frac{1}{2}x^3\sin(x)\cos(x)-\frac{3}{4}x^2\cos^2(x)+\frac{3}{2}\left(\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)\right)$$

input `Int[x^3*Cos[x]^2*Cot[x]^2,x]`

output
```
(-3*x^4)/8 - (3*x^2*Cos[x]^2)/4 - x^3*Cot[x] - 3*((I/3)*x^3 + (2*I)*((I/2)
*x^2*Log[1 - E^((2*I)*x)] - I*((I/2)*x*PolyLog[2, E^((2*I)*x)] - PolyLog[3
, E^((2*I)*x)]/4))) - (x^3*Cos[x]*Sin[x])/2 + (3*(x^2/4 + Cos[x]^2/4 + (x*
Cos[x]*Sin[x])/2))/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4200

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 4203

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4908

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.34

method	result
risch	$-\frac{3x^4}{8}+\frac{i(4x^3+6ix^2-6x-3i)e^{2ix}}{32}-\frac{i(4x^3-6ix^2-6x+3i)e^{-2ix}}{32}-\frac{2ix^3}{e^{2ix}-1}-2ix^3+3x^2\ln(e^{ix}+1)-6ix\,\text{poly}$

input
```
int(x^3*cos(x)^2*cot(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/8*x^4+1/32*I*(6*I*x^2+4*x^3-3*I-6*x)*exp(2*I*x)-1/32*I*(-6*I*x^2+4*x^3+
3*I-6*x)*exp(-2*I*x)-2*I*x^3/(exp(2*I*x)-1)-2*I*x^3+3*x^2*ln(exp(I*x)+1)-6
*I*x*polylog(2,-exp(I*x))+6*polylog(3,-exp(I*x))+3*x^2*ln(1-exp(I*x))-6*I*
x*polylog(2,exp(I*x))+6*polylog(3,exp(I*x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(84) = 168$.

Time = 0.10 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.18

$$\int x^3\cos^2(x)\cot^2(x)\,dx$$
$$=\frac{4\,(2\,x^3-3\,x)\cos(x)^3+24\,x^2\log(\cos(x)+i\,\sin(x)+1)\sin(x)+24\,x^2\log(\cos(x)-i\,\sin(x)+1)\sin($$

input
```
integrate(x^3*cos(x)^2*cot(x)^2,x, algorithm="fricas")
```

output
```
1/16*(4*(2*x^3 - 3*x)*cos(x)^3 + 24*x^2*log(cos(x) + I*sin(x) + 1)*sin(x)
+ 24*x^2*log(cos(x) - I*sin(x) + 1)*sin(x) + 24*x^2*log(-cos(x) + I*sin(x)
 + 1)*sin(x) + 24*x^2*log(-cos(x) - I*sin(x) + 1)*sin(x) - 48*I*x*dilog(co
s(x) + I*sin(x))*sin(x) + 48*I*x*dilog(cos(x) - I*sin(x))*sin(x) + 48*I*x*
dilog(-cos(x) + I*sin(x))*sin(x) - 48*I*x*dilog(-cos(x) - I*sin(x))*sin(x)
 - 12*(2*x^3 - x)*cos(x) - 3*(2*x^4 + 2*(2*x^2 - 1)*cos(x)^2 - 2*x^2 + 1)*
sin(x) + 48*polylog(3, cos(x) + I*sin(x))*sin(x) + 48*polylog(3, cos(x) -
I*sin(x))*sin(x) + 48*polylog(3, -cos(x) + I*sin(x))*sin(x) + 48*polylog(3
, -cos(x) - I*sin(x))*sin(x))/sin(x)
```

Sympy [F]

$$\int x^3 \cos^2(x) \cot^2(x)\, dx = \int x^3 \cos^2(x) \cot^2(x)\, dx$$

input
```
integrate(x**3*cos(x)**2*cot(x)**2,x)
```

output
```
Integral(x**3*cos(x)**2*cot(x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^3 \cos^2(x) \cot^2(x)\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*cos(x)^2*cot(x)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int x^3 \cos^2(x) \cot^2(x)\, dx = \int x^3 \cos(x)^2 \cot(x)^2\, dx$$

input
```
integrate(x^3*cos(x)^2*cot(x)^2,x, algorithm="giac")
```

output
```
integrate(x^3*cos(x)^2*cot(x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \cos^2(x) \cot^2(x)\, dx = \int x^3 \cos(x)^2 \cot(x)^2\, dx$$

input
```
int(x^3*cos(x)^2*cot(x)^2,x)
```

output
```
int(x^3*cos(x)^2*cot(x)^2, x)
```

Reduce [F]

$$\int x^3 \cos^2(x) \cot^2(x)\, dx = \int \cos(x)^2 \cot(x)^2\, x^3 dx$$

input
```
int(x^3*cos(x)^2*cot(x)^2,x)
```

output
```
int(cos(x)**2*cot(x)**2*x**3,x)
```

3.203 $\int x^2 \cos^2(x) \cot^2(x)\, dx$

Optimal result . . . 1852
Mathematica [A] (verified) . . . 1852
Rubi [A] (verified) . . . 1853
Maple [A] (verified) . . . 1857
Fricas [B] (verification not implemented) . . . 1857
Sympy [F] . . . 1858
Maxima [F(-2)] . . . 1858
Giac [F] . . . 1859
Mupad [F(-1)] . . . 1859
Reduce [F] . . . 1859

Optimal result

Integrand size = 12, antiderivative size = 83

$$\int x^2 \cos^2(x) \cot^2(x)\, dx = \frac{x}{4} - ix^2 - \frac{x^3}{2} - \frac{1}{2}x\cos^2(x) - x^2\cot(x) + 2x\log\left(1-e^{2ix}\right) - i\,\text{PolyLog}\left(2, e^{2ix}\right) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x^2\cos(x)\sin(x)$$

output

```
1/4*x-I*x^2-1/2*x^3-1/2*x*cos(x)^2-x^2*cot(x)+2*x*ln(1-exp(2*I*x))-I*polyl
og(2,exp(2*I*x))+1/4*cos(x)*sin(x)-1/2*x^2*cos(x)*sin(x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int x^2 \cos^2(x) \cot^2(x)\, dx = \frac{1}{8}\left(-8ix^2 - 4x^3 - 2x\cos(2x) - 8x^2\cot(x) + 16x\log\left(1-e^{2ix}\right) - 8i\,\text{PolyLog}\left(2, e^{2ix}\right) + \sin(2x) - 2x^2\sin(2x)\right)$$

input

```
Integrate[x^2*Cos[x]^2*Cot[x]^2,x]
```

output

```
((-8*I)*x^2 - 4*x^3 - 2*x*Cos[2*x] - 8*x^2*Cot[x] + 16*x*Log[1 - E^((2*I)*
x)] - (8*I)*PolyLog[2, E^((2*I)*x)] + Sin[2*x] - 2*x^2*Sin[2*x])/8
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.417$, Rules used = {4908, 3042, 3792, 15, 3042, 3115, 24, 4203, 15, 25, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cos^2(x) \cot^2(x)\, dx$$

$$\downarrow 4908$$

$$\int x^2 \cot^2(x) dx - \int x^2 \cos^2(x) dx$$

$$\downarrow 3042$$

$$\int x^2 \tan\left(x+\frac{\pi}{2}\right)^2 dx - \int x^2 \sin\left(x+\frac{\pi}{2}\right)^2 dx$$

$$\downarrow 3792$$

$$-\frac{\int x^2 dx}{2} + \int x^2 \tan\left(x+\frac{\pi}{2}\right)^2 dx + \frac{1}{2}\int \cos^2(x) dx - \frac{1}{2}x^2 \sin(x)\cos(x) - \frac{1}{2}x\cos^2(x)$$

$$\downarrow 15$$

$$\int x^2 \tan\left(x+\frac{\pi}{2}\right)^2 dx + \frac{1}{2}\int \cos^2(x) dx - \frac{x^3}{6} - \frac{1}{2}x^2 \sin(x)\cos(x) - \frac{1}{2}x\cos^2(x)$$

$$\downarrow 3042$$

$$\int x^2 \tan\left(x+\frac{\pi}{2}\right)^2 dx + \frac{1}{2}\int \sin\left(x+\frac{\pi}{2}\right)^2 dx - \frac{x^3}{6} - \frac{1}{2}x^2 \sin(x)\cos(x) - \frac{1}{2}x\cos^2(x)$$

$$\downarrow 3115$$

$$\int x^2 \tan\left(x+\frac{\pi}{2}\right)^2 dx + \frac{1}{2}\left(\frac{\int 1 dx}{2} + \frac{1}{2}\sin(x)\cos(x)\right) - \frac{x^3}{6} - \frac{1}{2}x^2 \sin(x)\cos(x) - \frac{1}{2}x\cos^2(x)$$

$$\downarrow 24$$

$$\int x^2 \tan\left(x+\frac{\pi}{2}\right)^2 dx - \frac{x^3}{6} - \frac{1}{2}x^2 \sin(x)\cos(x) - \frac{1}{2}x\cos^2(x) + \frac{1}{2}\left(\frac{x}{2} + \frac{1}{2}\sin(x)\cos(x)\right)$$

$$\downarrow 4203$$

$$-\int x^2 dx - 2\int -x\cot(x)dx - \frac{x^3}{6} - x^2\cot(x) - \frac{1}{2}x^2\sin(x)\cos(x) - \frac{1}{2}x\cos^2(x) + \frac{1}{2}\left(\frac{x}{2} + \frac{1}{2}\sin(x)\cos(x)\right)$$

$$\downarrow 15$$

$$-2\int -x\cot(x)dx - \frac{x^3}{2} - x^2\cot(x) - \frac{1}{2}x^2\sin(x)\cos(x) - \frac{1}{2}x\cos^2(x) + \frac{1}{2}\left(\frac{x}{2} + \frac{1}{2}\sin(x)\cos(x)\right)$$

$$\downarrow 25$$

$$2\int x\cot(x)dx - \frac{x^3}{2} - x^2\cot(x) - \frac{1}{2}x^2\sin(x)\cos(x) - \frac{1}{2}x\cos^2(x) + \frac{1}{2}\left(\frac{x}{2} + \frac{1}{2}\sin(x)\cos(x)\right)$$

$$\downarrow 3042$$

$$2\int -x\tan\left(x + \frac{\pi}{2}\right)dx - \frac{x^3}{2} - x^2\cot(x) - \frac{1}{2}x^2\sin(x)\cos(x) - \frac{1}{2}x\cos^2(x) + \frac{1}{2}\left(\frac{x}{2} + \frac{1}{2}\sin(x)\cos(x)\right)$$

$$\downarrow 25$$

$$-2\int x\tan\left(x + \frac{\pi}{2}\right)dx - \frac{x^3}{2} - x^2\cot(x) - \frac{1}{2}x^2\sin(x)\cos(x) - \frac{1}{2}x\cos^2(x) + \frac{1}{2}\left(\frac{x}{2} + \frac{1}{2}\sin(x)\cos(x)\right)$$

$$\downarrow 4200$$

$$-2\left(\frac{ix^2}{2} - 2i\int -\frac{e^{2ix}x}{1 - e^{2ix}}dx\right) - \frac{x^3}{2} - x^2\cot(x) - \frac{1}{2}x^2\sin(x)\cos(x) - \frac{1}{2}x\cos^2(x) + \frac{1}{2}\left(\frac{x}{2} + \frac{1}{2}\sin(x)\cos(x)\right)$$

$$\downarrow 25$$

$$-2\left(2i\int \frac{e^{2ix}x}{1 - e^{2ix}}dx + \frac{ix^2}{2}\right) - \frac{x^3}{2} - x^2\cot(x) - \frac{1}{2}x^2\sin(x)\cos(x) - \frac{1}{2}x\cos^2(x) + \frac{1}{2}\left(\frac{x}{2} + \frac{1}{2}\sin(x)\cos(x)\right)$$

$$\downarrow 2620$$

$$-2\left(2i\left(\frac{1}{2}ix\log\left(1 - e^{2ix}\right) - \frac{1}{2}i\int \log\left(1 - e^{2ix}\right)dx\right) + \frac{ix^2}{2}\right) - \frac{x^3}{2} - x^2\cot(x) - \frac{1}{2}x^2\sin(x)\cos(x) - \frac{1}{2}x\cos^2(x) + \frac{1}{2}\left(\frac{x}{2} + \frac{1}{2}\sin(x)\cos(x)\right)$$

$$\downarrow 2715$$

$$-2\left(2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\log\left(1-e^{2ix}\right)de^{2ix}\right)+\frac{ix^2}{2}\right)-\frac{x^3}{2}-x^2\cot(x)-\frac{1}{2}x^2\sin(x)\cos(x)-\frac{1}{2}x\cos^2(x)+\frac{1}{2}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)$$

$$\downarrow 2838$$

$$-2\left(2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)-\frac{x^3}{2}-x^2\cot(x)-\frac{1}{2}x^2\sin(x)\cos(x)-\frac{1}{2}x\cos^2(x)+\frac{1}{2}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)$$

input
```
Int[x^2*Cos[x]^2*Cot[x]^2,x]
```

output
```
-1/2*x^3 - (x*Cos[x]^2)/2 - x^2*Cot[x] - 2*((I/2)*x^2 + (2*I)*((I/2)*x*Log
[1 - E^((2*I)*x)] + PolyLog[2, E^((2*I)*x)]/4)) - (x^2*Cos[x]*Sin[x])/2 +
(x/2 + (Cos[x]*Sin[x])/2)/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4908

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

method	result
risch	$-\frac{x^3}{2}+\frac{i(2x^2+2ix-1)e^{2ix}}{16}-\frac{i(2x^2-2ix-1)e^{-2ix}}{16}-\frac{2ix^2}{e^{2ix}-1}+2x\ln(e^{ix}+1)+2x\ln(1-e^{ix})-2ix^2-2i$

input

```
int(x^2*cos(x)^2*cot(x)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/2*x^3+1/16*I*(2*I*x+2*x^2-1)*exp(2*I*x)-1/16*I*(-2*I*x+2*x^2-1)*exp(-2*
I*x)-2*I*x^2/(exp(2*I*x)-1)+2*x*ln(exp(I*x)+1)+2*x*ln(1-exp(I*x))-2*I*x^2-
2*I*polylog(2,-exp(I*x))-2*I*polylog(2,exp(I*x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(62) = 124$.

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.95

$$\int x^2\cos^2(x)\cot^2(x)\,dx$$

$$=\frac{(2\,x^2-1)\cos(x)^3+4\,x\log(\cos(x)+i\,\sin(x)+1)\sin(x)+4\,x\log(\cos(x)-i\,\sin(x)+1)\sin(x)+4\,x$$

input

```
integrate(x^2*cos(x)^2*cot(x)^2,x, algorithm="fricas")
```

output
```
1/4*((2*x^2 - 1)*cos(x)^3 + 4*x*log(cos(x) + I*sin(x) + 1)*sin(x) + 4*x*lo
g(cos(x) - I*sin(x) + 1)*sin(x) + 4*x*log(-cos(x) + I*sin(x) + 1)*sin(x) +
 4*x*log(-cos(x) - I*sin(x) + 1)*sin(x) - (6*x^2 - 1)*cos(x) - (2*x^3 + 2*
x*cos(x)^2 - x)*sin(x) - 4*I*dilog(cos(x) + I*sin(x))*sin(x) + 4*I*dilog(c
os(x) - I*sin(x))*sin(x) + 4*I*dilog(-cos(x) + I*sin(x))*sin(x) - 4*I*dilo
g(-cos(x) - I*sin(x))*sin(x))/sin(x)
```

Sympy [F]

$$\int x^2 \cos^2(x) \cot^2(x)\, dx = \int x^2 \cos^2(x) \cot^2(x)\, dx$$

input
```
integrate(x**2*cos(x)**2*cot(x)**2,x)
```

output
```
Integral(x**2*cos(x)**2*cot(x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 \cos^2(x) \cot^2(x)\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*cos(x)^2*cot(x)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F]

$$\int x^2 \cos^2(x) \cot^2(x)\, dx = \int x^2 \cos(x)^2 \cot(x)^2\, dx$$

input
```
integrate(x^2*cos(x)^2*cot(x)^2,x, algorithm="giac")
```

output
```
integrate(x^2*cos(x)^2*cot(x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cos^2(x) \cot^2(x)\, dx = \int x^2 \cos(x)^2 \cot(x)^2\, dx$$

input
```
int(x^2*cos(x)^2*cot(x)^2,x)
```

output
```
int(x^2*cos(x)^2*cot(x)^2, x)
```

Reduce [F]

$$\int x^2 \cos^2(x) \cot^2(x)\, dx = \int \cos(x)^2 \cot(x)^2\, x^2 dx$$

input
```
int(x^2*cos(x)^2*cot(x)^2,x)
```

output
```
int(cos(x)**2*cot(x)**2*x**2,x)
```

3.204 $\int x \cos^2(x) \cot^2(x)\, dx$

Optimal result 1860
Mathematica [A] (verified) 1860
Rubi [A] (verified) 1861
Maple [C] (verified) 1863
Fricas [A] (verification not implemented) 1863
Sympy [F] 1864
Maxima [F(-2)] 1864
Giac [B] (verification not implemented) 1864
Mupad [B] (verification not implemented) 1865
Reduce [B] (verification not implemented) 1865

Optimal result

Integrand size = 10, antiderivative size = 33

$$\int x \cos^2(x) \cot^2(x)\, dx = -\frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x\cot(x) + \log(\sin(x)) - \frac{1}{2}x\cos(x)\sin(x)$$

output

```
-3/4*x^2-1/4*cos(x)^2-x*cot(x)+ln(sin(x))-1/2*x*cos(x)*sin(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x \cos^2(x) \cot^2(x)\, dx = -\frac{3x^2}{4} - \frac{1}{8}\cos(2x) - x\cot(x) + \log(\sin(x)) - \frac{1}{4}x\sin(2x)$$

input

```
Integrate[x*Cos[x]^2*Cot[x]^2,x]
```

output

```
(-3*x^2)/4 - Cos[2*x]/8 - x*Cot[x] + Log[Sin[x]] - (x*Sin[2*x])/4
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {4908, 3042, 3791, 15, 4203, 15, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cos^2(x) \cot^2(x)\, dx$$

$$\downarrow 4908$$

$$\int x \cot^2(x) dx - \int x \cos^2(x) dx$$

$$\downarrow 3042$$

$$\int x \tan\left(x + \frac{\pi}{2}\right)^2 dx - \int x \sin\left(x + \frac{\pi}{2}\right)^2 dx$$

$$\downarrow 3791$$

$$-\frac{\int x dx}{2} + \int x \tan\left(x + \frac{\pi}{2}\right)^2 dx - \frac{1}{4}\cos^2(x) - \frac{1}{2}x \sin(x)\cos(x)$$

$$\downarrow 15$$

$$\int x \tan\left(x + \frac{\pi}{2}\right)^2 dx - \frac{x^2}{4} - \frac{\cos^2(x)}{4} - \frac{1}{2}x \sin(x)\cos(x)$$

$$\downarrow 4203$$

$$-\int x dx - \int -\cot(x) dx - \frac{x^2}{4} - \frac{\cos^2(x)}{4} - x\cot(x) - \frac{1}{2}x \sin(x)\cos(x)$$

$$\downarrow 15$$

$$-\int -\cot(x) dx - \frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x\cot(x) - \frac{1}{2}x \sin(x)\cos(x)$$

$$\downarrow 25$$

$$\int \cot(x) dx - \frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x\cot(x) - \frac{1}{2}x \sin(x)\cos(x)$$

$$\downarrow 3042$$

$$\int -\tan\left(x+\frac{\pi}{2}\right)dx - \frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x\cot(x) - \frac{1}{2}x\sin(x)\cos(x)$$

$$\downarrow 25$$

$$-\int \tan\left(x+\frac{\pi}{2}\right)dx - \frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x\cot(x) - \frac{1}{2}x\sin(x)\cos(x)$$

$$\downarrow 3956$$

$$-\frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x\cot(x) + \log(\sin(x)) - \frac{1}{2}x\sin(x)\cos(x)$$

input
```
Int[x*Cos[x]^2*Cot[x]^2,x]
```

output
```
(-3*x^2)/4 - Cos[x]^2/4 - x*Cot[x] + Log[Sin[x]] - (x*Cos[x]*Sin[x])/2
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4908

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.82

method	result	size
risch	$-\frac{3x^2}{4}+\frac{i(2x+i)e^{2ix}}{16}-\frac{i(-i+2x)e^{-2ix}}{16}-2ix-\frac{2ix}{e^{2ix}-1}+\ln\left(e^{2ix}-1\right)$	60

input

```
int(x*cos(x)^2*cot(x)^2,x,method=_RETURNVERBOSE)
```

output

```
-3/4*x^2+1/16*I*(2*x+I)*exp(2*I*x)-1/16*I*(-I+2*x)*exp(-2*I*x)-2*I*x-2*I*x
/(exp(2*I*x)-1)+ln(exp(2*I*x)-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36

$$\int x\cos^2(x)\cot^2(x)\,dx$$
$$=\frac{4\,x\cos\left(x\right)^3-12\,x\cos\left(x\right)-\left(6\,x^2+2\,\cos\left(x\right)^2-1\right)\sin\left(x\right)+8\,\log\left(\frac{1}{2}\,\sin\left(x\right)\right)\sin\left(x\right)}{8\,\sin\left(x\right)}$$

input

```
integrate(x*cos(x)^2*cot(x)^2,x, algorithm="fricas")
```

output
```
1/8*(4*x*cos(x)^3 - 12*x*cos(x) - (6*x^2 + 2*cos(x)^2 - 1)*sin(x) + 8*log(
1/2*sin(x))*sin(x))/sin(x)
```

Sympy [F]

$$\int x \cos^2(x) \cot^2(x)\, dx = \int x \cos^2(x) \cot^2(x)\, dx$$

input
```
integrate(x*cos(x)**2*cot(x)**2,x)
```

output
```
Integral(x*cos(x)**2*cot(x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x \cos^2(x) \cot^2(x)\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*cos(x)^2*cot(x)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(27) = 54$.

Time $= 0.15$ (sec) , antiderivative size $= 206$, normalized size of antiderivative $= 6.24$

$$\int x \cos^2(x) \cot^2(x)\, dx =$$

$$-\frac{6\,x^2 \tan\left(\frac{1}{2}\,x\right)^5 - 4\,x \tan\left(\frac{1}{2}\,x\right)^6 - 4 \log\left(\frac{16 \tan\left(\frac{1}{2}\,x\right)^2}{\tan\left(\frac{1}{2}\,x\right)^4 + 2 \tan\left(\frac{1}{2}\,x\right)^2 + 1}\right) \tan\left(\frac{1}{2}\,x\right)^5 + 12\,x^2 \tan\left(\frac{1}{2}\,x\right)^3 - 12\,x \ta$$

input
```
integrate(x*cos(x)^2*cot(x)^2,x, algorithm="giac")
```

output
```
-1/8*(6*x^2*tan(1/2*x)^5 - 4*x*tan(1/2*x)^6 - 4*log(16*tan(1/2*x)^2/(tan(1
/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^5 + 12*x^2*tan(1/2*x)^3 - 12*x*t
an(1/2*x)^4 + tan(1/2*x)^5 - 8*log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1
/2*x)^2 + 1))*tan(1/2*x)^3 + 6*x^2*tan(1/2*x) + 12*x*tan(1/2*x)^2 - 6*tan(
1/2*x)^3 - 4*log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(
1/2*x) + 4*x + tan(1/2*x))/(tan(1/2*x)^5 + 2*tan(1/2*x)^3 + tan(1/2*x))
```

Mupad [B] (verification not implemented)

Time = 17.70 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int x\cos^2(x)\cot^2(x)\,dx = \ln\left(\mathrm{e}^{x\,2\mathrm{i}}-1\right) - \mathrm{e}^{-x\,2\mathrm{i}}\left(\frac{1}{16}+\frac{x\,1\mathrm{i}}{8}\right) + \mathrm{e}^{x\,2\mathrm{i}}\left(-\frac{1}{16}+\frac{x\,1\mathrm{i}}{8}\right) - \frac{3\,x^2}{4} - x\,2\mathrm{i} - \frac{x\,2\mathrm{i}}{\mathrm{e}^{x\,2\mathrm{i}}-1}$$

input
```
int(x*cos(x)^2*cot(x)^2,x)
```

output
```
log(exp(x*2i) - 1) - x*2i - exp(-x*2i)*((x*1i)/8 + 1/16) + exp(x*2i)*((x*1
i)/8 - 1/16) - (x*2i)/(exp(x*2i) - 1) - (3*x^2)/4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.76

$$\int x\cos^2(x)\cot^2(x)\,dx$$
$$= \frac{-2\cos(x)\sin(x)^2\,x - 4\cos(x)\,x - 4\log\left(\tan\left(\frac{x}{2}\right)^2+1\right)\sin(x) + 4\log\left(\tan\left(\frac{x}{2}\right)\right)\sin(x) + \sin(x)^3 - 3\sin}{4\sin(x)}$$

input
```
int(x*cos(x)^2*cot(x)^2,x)
```

output

```
( - 2*cos(x)*sin(x)**2*x - 4*cos(x)*x - 4*log(tan(x/2)**2 + 1)*sin(x) + 4*
log(tan(x/2))*sin(x) + sin(x)**3 - 3*sin(x)*x**2 - 2*sin(x))/(4*sin(x))
```

3.205 $\int x^3 \cos^2(x) \cot^3(x)\, dx$

Optimal result . . . 1867
Mathematica [A] (verified) . . . 1868
Rubi [F] . . . 1868
Maple [A] (verified) . . . 1876
Fricas [B] (verification not implemented) . . . 1876
Sympy [F] . . . 1877
Maxima [B] (verification not implemented) . . . 1877
Giac [F] . . . 1878
Mupad [F(-1)] . . . 1879
Reduce [F] . . . 1879

Optimal result

Integrand size = 12, antiderivative size = 180

$$\begin{aligned}\int x^3 \cos^2(x) \cot^3(x)\, dx = {} & \frac{3x}{8} - \frac{3ix^2}{2} - \frac{3x^3}{4} + \frac{ix^4}{2} - \frac{3}{2}x^2 \cot(x) - \frac{1}{2}x^3 \cot^2(x) \\ & + 3x \log\left(1 - e^{2ix}\right) - 2x^3 \log\left(1 - e^{2ix}\right) - \frac{3}{2}i \operatorname{PolyLog}\left(2, e^{2ix}\right) \\ & + 3ix^2 \operatorname{PolyLog}\left(2, e^{2ix}\right) - 3x \operatorname{PolyLog}\left(3, e^{2ix}\right) \\ & - \frac{3}{2}i \operatorname{PolyLog}\left(4, e^{2ix}\right) - \frac{3}{8}\cos(x)\sin(x) \\ & + \frac{3}{4}x^2 \cos(x)\sin(x) - \frac{3}{4}x \sin^2(x) + \frac{1}{2}x^3 \sin^2(x)\end{aligned}$$

output

```
3/8*x+3*I*x^2*polylog(2,exp(2*I*x))-3/4*x^3-3/2*I*polylog(2,exp(2*I*x))-3/
2*x^2*cot(x)-1/2*x^3*cot(x)^2+3*x*ln(1-exp(2*I*x))-2*x^3*ln(1-exp(2*I*x))-
3/2*I*polylog(4,exp(2*I*x))+1/2*I*x^4-3*x*polylog(3,exp(2*I*x))-3/2*I*x^2-
3/8*cos(x)*sin(x)+3/4*x^2*cos(x)*sin(x)-3/4*x*sin(x)^2+1/2*x^3*sin(x)^2
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.88

$$\int x^3 \cos^2(x) \cot^3(x)\, dx = \frac{1}{32}\left(i\pi^4 - 48ix^2 - 16ix^4 + 12x\cos(2x) - 8x^3\cos(2x) - 48x^2\cot(x) - 16x^3\csc^2(x) - 64x^3\log\left(1-e^{-2ix}\right) + 96x\log\left(1-e^{2ix}\right) - 96ix^2\,\text{PolyLog}\left(2,e^{-2ix}\right) - 48i\,\text{PolyLog}\left(2,e^{2ix}\right) - 96x\,\text{PolyLog}\left(3,e^{-2ix}\right) + 48i\,\text{PolyLog}\left(4,e^{-2ix}\right) - 6\sin(2x) + 12x^2\sin(2x)\right)$$

input
```
Integrate[x^3*Cos[x]^2*Cot[x]^3,x]
```

output
```
(I*Pi^4 - (48*I)*x^2 - (16*I)*x^4 + 12*x*Cos[2*x] - 8*x^3*Cos[2*x] - 48*x^
2*Cot[x] - 16*x^3*Csc[x]^2 - 64*x^3*Log[1 - E^((-2*I)*x)] + 96*x*Log[1 - E
^((2*I)*x)] - (96*I)*x^2*PolyLog[2, E^((-2*I)*x)] - (48*I)*PolyLog[2, E^((
2*I)*x)] - 96*x*PolyLog[3, E^((-2*I)*x)] + (48*I)*PolyLog[4, E^((-2*I)*x)]
 - 6*Sin[2*x] + 12*x^2*Sin[2*x])/32
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cos^2(x) \cot^3(x)\, dx$$

$$\downarrow 4908$$

$$\int x^3 \cot^3(x) dx - \int x^3 \cos^2(x) \cot(x) dx$$

$$\downarrow 3042$$

$$\int -x^3 \tan\left(x+\frac{\pi}{2}\right)^3 dx - \int x^3 \cos^2(x) \cot(x) dx$$

$$\downarrow 25$$

$$-\int x^3 \tan\left(x+\frac{\pi}{2}\right)^3 dx - \int x^3 \cos^2(x) \cot(x) dx$$

$$\downarrow 4203$$

$$\int -x^3 \cot(x) dx - \int x^3 \cos^2(x) \cot(x) dx + \frac{3}{2} \int x^2 \cot^2(x) dx - \frac{1}{2} x^3 \cot^2(x)$$

$$\downarrow 25$$

$$-\int x^3 \cot(x) dx - \int x^3 \cos^2(x) \cot(x) dx + \frac{3}{2} \int x^2 \cot^2(x) dx - \frac{1}{2} x^3 \cot^2(x)$$

$$\downarrow 3042$$

$$-\int -x^3 \tan\left(x + \frac{\pi}{2}\right) dx - \int x^3 \cos^2(x) \cot(x) dx + \frac{3}{2} \int x^2 \tan\left(x + \frac{\pi}{2}\right)^2 dx - \frac{1}{2} x^3 \cot^2(x)$$

$$\downarrow 25$$

$$\int x^3 \tan\left(x + \frac{\pi}{2}\right) dx - \int x^3 \cos^2(x) \cot(x) dx + \frac{3}{2} \int x^2 \tan\left(x + \frac{\pi}{2}\right)^2 dx - \frac{1}{2} x^3 \cot^2(x)$$

$$\downarrow 4200$$

$$-2i \int -\frac{e^{2ix} x^3}{1 - e^{2ix}} dx - \int x^3 \cos^2(x) \cot(x) dx + \frac{3}{2} \int x^2 \tan\left(x + \frac{\pi}{2}\right)^2 dx + \frac{ix^4}{4} - \frac{1}{2} x^3 \cot^2(x)$$

$$\downarrow 25$$

$$2i \int \frac{e^{2ix} x^3}{1 - e^{2ix}} dx - \int x^3 \cos^2(x) \cot(x) dx + \frac{3}{2} \int x^2 \tan\left(x + \frac{\pi}{2}\right)^2 dx + \frac{ix^4}{4} - \frac{1}{2} x^3 \cot^2(x)$$

$$\downarrow 2620$$

$$-\int x^3 \cos^2(x) \cot(x) dx + \frac{3}{2} \int x^2 \tan\left(x + \frac{\pi}{2}\right)^2 dx +$$
$$2i \left(\frac{1}{2} i x^3 \log\left(1 - e^{2ix}\right) - \frac{3}{2} i \int x^2 \log\left(1 - e^{2ix}\right) dx \right) + \frac{ix^4}{4} - \frac{1}{2} x^3 \cot^2(x)$$

$$\downarrow 3011$$

$$2i \left(\frac{1}{2} i x^3 \log\left(1 - e^{2ix}\right) - \frac{3}{2} i \left(\frac{1}{2} i x^2 \operatorname{PolyLog}\left(2, e^{2ix}\right) - i \int x \operatorname{PolyLog}\left(2, e^{2ix}\right) dx \right) \right) -$$
$$\int x^3 \cos^2(x) \cot(x) dx + \frac{3}{2} \int x^2 \tan\left(x + \frac{\pi}{2}\right)^2 dx + \frac{ix^4}{4} - \frac{1}{2} x^3 \cot^2(x)$$

$$\downarrow 4203$$

$$2i \left(\frac{1}{2} i x^3 \log\left(1 - e^{2ix}\right) - \frac{3}{2} i \left(\frac{1}{2} i x^2 \operatorname{PolyLog}\left(2, e^{2ix}\right) - i \int x \operatorname{PolyLog}\left(2, e^{2ix}\right) dx \right) \right) -$$
$$\int x^3 \cos^2(x) \cot(x) dx + \frac{3}{2} \left(-\int x^2 dx - 2 \int -x \cot(x) dx + x^2 (-\cot(x)) \right) + \frac{ix^4}{4} - \frac{1}{2} x^3 \cot^2(x)$$

$$\downarrow 15$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)-\int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\left(-2\int-x\cot(x)dx-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\downarrow 25$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)-\int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\left(2\int x\cot(x)dx-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\downarrow 3042$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)-\int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\left(2\int-x\tan\left(x+\frac{\pi}{2}\right)dx-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\downarrow 25$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)-\int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\left(-2\int x\tan\left(x+\frac{\pi}{2}\right)dx-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\downarrow 4200$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)-\int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\left(-2\left(\frac{ix^2}{2}-2i\int-\frac{e^{2ix}x}{1-e^{2ix}}dx\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\downarrow 25$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)-\int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\left(-2\left(2i\int\frac{e^{2ix}x}{1-e^{2ix}}dx+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\downarrow 2620$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)-\int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\left(-2\left(2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{2}i\int\log\left(1-e^{2ix}\right)dx\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\downarrow 2715$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)-\int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\left(-2\left(2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\log\left(1-e^{2ix}\right)de^{2ix}\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\downarrow 2838$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)-\int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\left(-2\left(2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\downarrow 4908$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)-\int x^3\cot(x)dx+\int x^3\cos(x)\sin(x)dx+\frac{3}{2}\left(-2\left(2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\downarrow 3042$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)-\int -x^3\tan\left(x+\frac{\pi}{2}\right)dx+\int x^3\cos(x)\sin(x)dx+\frac{3}{2}\left(-2\left(2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\downarrow 25$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)+\int x^3\tan\left(x+\frac{\pi}{2}\right)dx+\int x^3\cos(x)\sin(x)dx+\frac{3}{2}\left(-2\left(2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\downarrow 3924$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)+\int x^3\tan\left(x+\frac{\pi}{2}\right)dx-\frac{3}{2}\int x^2\sin^2(x)dx+\frac{3}{2}\left(-2\left(2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}+\frac{1}{2}x^3\sin^2(x)-\frac{1}{2}x^3\cot^2(x)$$

$$\downarrow 3042$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)+\int x^3\tan\left(x+\frac{\pi}{2}\right)dx-\frac{3}{2}\int x^2\sin(x)^2dx+\frac{3}{2}\left(-2\left(2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}+\frac{1}{2}x^3\sin^2(x)-\frac{1}{2}x^3\cot^2(x)$$

$$\downarrow 3792$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\,\text{PolyLog}\left(2,e^{2ix}\right)-i\int x\,\text{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)+$$
$$\int x^3\tan\left(x+\frac{\pi}{2}\right)dx-\frac{3}{2}\left(\frac{\int x^2dx}{2}-\frac{1}{2}\int\sin^2(x)dx-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\sin^2(x)\right)+$$
$$\frac{3}{2}\left(-2\left(2i\left(\frac{1}{4}\,\text{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}+$$
$$\frac{1}{2}x^3\sin^2(x)-\frac{1}{2}x^3\cot^2(x)$$

$$\downarrow 15$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\,\text{PolyLog}\left(2,e^{2ix}\right)-i\int x\,\text{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)+$$
$$\int x^3\tan\left(x+\frac{\pi}{2}\right)dx-\frac{3}{2}\left(-\frac{1}{2}\int\sin^2(x)dx+\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\sin^2(x)\right)+$$
$$\frac{3}{2}\left(-2\left(2i\left(\frac{1}{4}\,\text{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}+$$
$$\frac{1}{2}x^3\sin^2(x)-\frac{1}{2}x^3\cot^2(x)$$

$$\downarrow 3042$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\,\text{PolyLog}\left(2,e^{2ix}\right)-i\int x\,\text{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)+$$
$$\int x^3\tan\left(x+\frac{\pi}{2}\right)dx-\frac{3}{2}\left(-\frac{1}{2}\int\sin(x)^2dx+\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\sin^2(x)\right)+$$
$$\frac{3}{2}\left(-2\left(2i\left(\frac{1}{4}\,\text{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}+$$
$$\frac{1}{2}x^3\sin^2(x)-\frac{1}{2}x^3\cot^2(x)$$

$$\downarrow 3115$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\,\text{PolyLog}\left(2,e^{2ix}\right)-i\int x\,\text{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)+$$
$$\int x^3\tan\left(x+\frac{\pi}{2}\right)dx-$$
$$\frac{3}{2}\left(\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x)-\frac{\int 1dx}{2}\right)+\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\sin^2(x)\right)+$$
$$\frac{3}{2}\left(-2\left(2i\left(\frac{1}{4}\,\text{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}+$$
$$\frac{1}{2}x^3\sin^2(x)-\frac{1}{2}x^3\cot^2(x)$$

input
```
Int[x^3*Cos[x]^2*Cot[x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3924

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 4200

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 4203

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4908

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.33

method	result
risch	$-3i\,\mathrm{polylog}\left(2,-\mathrm{e}^{ix}\right)-\frac{\left(4x^3+6ix^2-6x-3i\right)\mathrm{e}^{2ix}}{32}-\frac{\left(4x^3-6ix^2-6x+3i\right)\mathrm{e}^{-2ix}}{32}+\frac{x^2\left(2\,\mathrm{e}^{2ix}x-3i\mathrm{e}^{2ix}+3i\right)}{\left(\mathrm{e}^{2ix}-1\right)^2}-2x^3\ln($

input
```
int(x^3*cos(x)^2*cot(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-3*I*polylog(2,-exp(I*x))-1/32*(6*I*x^2+4*x^3-3*I-6*x)*exp(2*I*x)-1/32*(-6
*I*x^2+4*x^3+3*I-6*x)*exp(-2*I*x)+x^2*(2*exp(2*I*x)*x-3*I*exp(2*I*x)+3*I)/
(exp(2*I*x)-1)^2-2*x^3*ln(exp(I*x)+1)-2*x^3*ln(1-exp(I*x))-3*I*x^2+3*x*ln(
exp(I*x)+1)+1/2*I*x^4+3*x*ln(1-exp(I*x))-3*I*polylog(2,exp(I*x))-12*x*poly
log(3,-exp(I*x))-12*x*polylog(3,exp(I*x))-12*I*polylog(4,exp(I*x))+6*I*pol
ylog(2,exp(I*x))*x^2-12*I*polylog(4,-exp(I*x))+6*I*polylog(2,-exp(I*x))*x^
2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 508 vs. 2(126) = 252.

Time = 0.10 (sec) , antiderivative size = 508, normalized size of antiderivative = 2.82

$$\int x^3\cos^2(x)\cot^3(x)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

output

```
-1/8*(2*(2*x^3 - 3*x)*cos(x)^4 - 2*x^3 - 3*(2*x^3 - 3*x)*cos(x)^2 + 12*((-
2*I*x^2 + I)*cos(x)^2 + 2*I*x^2 - I)*dilog(cos(x) + I*sin(x)) + 12*((2*I*x
^2 - I)*cos(x)^2 - 2*I*x^2 + I)*dilog(cos(x) - I*sin(x)) + 12*((2*I*x^2 -
I)*cos(x)^2 - 2*I*x^2 + I)*dilog(-cos(x) + I*sin(x)) + 12*((-2*I*x^2 + I)*
cos(x)^2 + 2*I*x^2 - I)*dilog(-cos(x) - I*sin(x)) - 4*(2*x^3 - (2*x^3 - 3*
x)*cos(x)^2 - 3*x)*log(cos(x) + I*sin(x) + 1) - 4*(2*x^3 - (2*x^3 - 3*x)*c
os(x)^2 - 3*x)*log(cos(x) - I*sin(x) + 1) - 4*(2*x^3 - (2*x^3 - 3*x)*cos(x
)^2 - 3*x)*log(-cos(x) + I*sin(x) + 1) - 4*(2*x^3 - (2*x^3 - 3*x)*cos(x)^2
 - 3*x)*log(-cos(x) - I*sin(x) + 1) + 48*(I*cos(x)^2 - I)*polylog(4, cos(x
) + I*sin(x)) + 48*(-I*cos(x)^2 + I)*polylog(4, cos(x) - I*sin(x)) + 48*(-
I*cos(x)^2 + I)*polylog(4, -cos(x) + I*sin(x)) + 48*(I*cos(x)^2 - I)*polyl
og(4, -cos(x) - I*sin(x)) + 48*(x*cos(x)^2 - x)*polylog(3, cos(x) + I*sin(
x)) + 48*(x*cos(x)^2 - x)*polylog(3, cos(x) - I*sin(x)) + 48*(x*cos(x)^2 -
 x)*polylog(3, -cos(x) + I*sin(x)) + 48*(x*cos(x)^2 - x)*polylog(3, -cos(x
) - I*sin(x)) - 3*((2*x^2 - 1)*cos(x)^3 + (2*x^2 + 1)*cos(x))*sin(x) - 3*x
)/(cos(x)^2 - 1)
```

Sympy [F]

$$\int x^3 \cos^2(x) \cot^3(x)\, dx = \int x^3 \cos^2(x) \cot^3(x)\, dx$$

input

```
integrate(x**3*cos(x)**2*cot(x)**3,x)
```

output

```
Integral(x**3*cos(x)**2*cot(x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3740 vs. $2(126) = 252$.

Time $= 0.44$ (sec) , antiderivative size $= 3740$, normalized size of antiderivative $= 20.78$

$$\int x^3 \cos^2(x) \cot^3(x)\, dx = \text{Too large to display}$$

input
```
integrate(x^3*cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

output
```
-1/32*(4*x^3 + (4*x^3 + 6*I*x^2 - 6*x - 3*I)*cos(6*x)^2 + 4*(8*I*x^4 + 4*x
^3 - 42*I*x^2 - 6*x - 3*I)*cos(4*x)^2 + 4*(8*I*x^4 - 14*x^3 - 24*I*x^2 - 3
*x)*cos(2*x)^2 - (4*x^3 + 6*I*x^2 - 6*x - 3*I)*sin(6*x)^2 + 4*(-8*I*x^4 -
4*x^3 + 42*I*x^2 + 6*x + 3*I)*sin(4*x)^2 + 4*(-8*I*x^4 + 14*x^3 + 24*I*x^2
 + 3*x)*sin(2*x)^2 - 6*I*x^2 + 32*(2*(-2*I*x^3 + 3*I*x)*cos(4*x)^2 + 2*(-2
*I*x^3 + 3*I*x)*cos(2*x)^2 + 2*(2*I*x^3 - 3*I*x)*sin(4*x)^2 + 2*(2*I*x^3 -
 3*I*x)*sin(2*x)^2 + (2*I*x^3 + (2*I*x^3 - 3*I*x)*cos(4*x) + 2*(-2*I*x^3 +
 3*I*x)*cos(2*x) - (2*x^3 - 3*x)*sin(4*x) + 2*(2*x^3 - 3*x)*sin(2*x) - 3*I
*x)*cos(6*x) + (-4*I*x^3 + 5*(2*I*x^3 - 3*I*x)*cos(2*x) - 5*(2*x^3 - 3*x)*
sin(2*x) + 6*I*x)*cos(4*x) + (2*I*x^3 - 3*I*x)*cos(2*x) - (2*x^3 + (2*x^3
- 3*x)*cos(4*x) - 2*(2*x^3 - 3*x)*cos(2*x) - (-2*I*x^3 + 3*I*x)*sin(4*x) -
 2*(2*I*x^3 - 3*I*x)*sin(2*x) - 3*x)*sin(6*x) + (4*x^3 + 4*(2*x^3 - 3*x)*c
os(4*x) - 5*(2*x^3 - 3*x)*cos(2*x) + 5*(-2*I*x^3 + 3*I*x)*sin(2*x) - 6*x)*
sin(4*x) - (2*x^3 - 4*(2*x^3 - 3*x)*cos(2*x) - 3*x)*sin(2*x))*arctan2(sin(
x), cos(x) + 1) + 32*(2*(2*I*x^3 - 3*I*x)*cos(4*x)^2 + 2*(2*I*x^3 - 3*I*x)
*cos(2*x)^2 + 2*(-2*I*x^3 + 3*I*x)*sin(4*x)^2 + 2*(-2*I*x^3 + 3*I*x)*sin(2
*x)^2 + (-2*I*x^3 + (-2*I*x^3 + 3*I*x)*cos(4*x) + 2*(2*I*x^3 - 3*I*x)*cos(
2*x) + (2*x^3 - 3*x)*sin(4*x) - 2*(2*x^3 - 3*x)*sin(2*x) + 3*I*x)*cos(6*x)
 + (4*I*x^3 + 5*(-2*I*x^3 + 3*I*x)*cos(2*x) + 5*(2*x^3 - 3*x)*sin(2*x) - 6
*I*x)*cos(4*x) + (-2*I*x^3 + 3*I*x)*cos(2*x) + (2*x^3 + (2*x^3 - 3*x)*c...
```

Giac [F]

$$\int x^3 \cos^2(x) \cot^3(x)\, dx = \int x^3 \cos(x)^2 \cot(x)^3\, dx$$

input
```
integrate(x^3*cos(x)^2*cot(x)^3,x, algorithm="giac")
```

output
```
integrate(x^3*cos(x)^2*cot(x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \cos^2(x) \cot^3(x)\, dx = \int x^3 \cos(x)^2 \cot(x)^3\, dx$$

input
```
int(x^3*cos(x)^2*cot(x)^3,x)
```

output
```
int(x^3*cos(x)^2*cot(x)^3, x)
```

Reduce [F]

$$\int x^3 \cos^2(x) \cot^3(x)\, dx = \int \cos(x)^2 \cot(x)^3\, x^3 dx$$

input
```
int(x^3*cos(x)^2*cot(x)^3,x)
```

output
```
int(cos(x)**2*cot(x)**3*x**3,x)
```

3.206 $\int x^2 \cos^2(x) \cot^3(x)\, dx$

Optimal result . . . 1880
Mathematica [A] (verified) . . . 1880
Rubi [F] . . . 1881
Maple [A] (verified) . . . 1887
Fricas [B] (verification not implemented) . . . 1888
Sympy [F] . . . 1888
Maxima [B] (verification not implemented) . . . 1889
Giac [F] . . . 1890
Mupad [F(-1)] . . . 1890
Reduce [F] . . . 1890

Optimal result

Integrand size = 12, antiderivative size = 106

$$\begin{aligned}\int x^2 \cos^2(x) \cot^3(x)\, dx = &-\frac{3x^2}{4} + \frac{2ix^3}{3} - x\cot(x) - \frac{1}{2}x^2\cot^2(x) - 2x^2\log\left(1-e^{2ix}\right) \\ &+ \log(\sin(x)) + 2ix\,\text{PolyLog}\left(2, e^{2ix}\right) - \text{PolyLog}\left(3, e^{2ix}\right) \\ &+ \frac{1}{2}x\cos(x)\sin(x) - \frac{\sin^2(x)}{4} + \frac{1}{2}x^2\sin^2(x)\end{aligned}$$

output

```
-3/4*x^2+2/3*I*x^3-x*cot(x)-1/2*x^2*cot(x)^2-2*x^2*ln(1-exp(2*I*x))+ln(sin
(x))+2*I*x*polylog(2,exp(2*I*x))-polylog(3,exp(2*I*x))+1/2*x*cos(x)*sin(x)
-1/4*sin(x)^2+1/2*x^2*sin(x)^2
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02

$$\begin{aligned}\int x^2 \cos^2(x) \cot^3(x)\, dx = &\frac{i\pi^3}{12} - \frac{2ix^3}{3} + \frac{1}{8}\cos(2x) - \frac{1}{4}x^2\cos(2x) - x\cot(x) \\ &- \frac{1}{2}x^2\csc^2(x) - 2x^2\log\left(1-e^{-2ix}\right) + \log(\sin(x)) \\ &- 2ix\,\text{PolyLog}\left(2, e^{-2ix}\right) - \text{PolyLog}\left(3, e^{-2ix}\right) + \frac{1}{4}x\sin(2x)\end{aligned}$$

input
```
Integrate[x^2*Cos[x]^2*Cot[x]^3,x]
```

output
```
(I/12)*Pi^3 - ((2*I)/3)*x^3 + Cos[2*x]/8 - (x^2*Cos[2*x])/4 - x*Cot[x] - (
x^2*Csc[x]^2)/2 - 2*x^2*Log[1 - E^((-2*I)*x)] + Log[Sin[x]] - (2*I)*x*Poly
Log[2, E^((-2*I)*x)] - PolyLog[3, E^((-2*I)*x)] + (x*Sin[2*x])/4
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cos^2(x) \cot^3(x)\, dx$$

$$\downarrow 4908$$

$$\int x^2 \cot^3(x) dx - \int x^2 \cos^2(x) \cot(x) dx$$

$$\downarrow 3042$$

$$\int -x^2 \tan\left(x + \frac{\pi}{2}\right)^3 dx - \int x^2 \cos^2(x) \cot(x) dx$$

$$\downarrow 25$$

$$-\int x^2 \tan\left(x + \frac{\pi}{2}\right)^3 dx - \int x^2 \cos^2(x) \cot(x) dx$$

$$\downarrow 4203$$

$$\int -x^2 \cot(x) dx - \int x^2 \cos^2(x) \cot(x) dx + \int x \cot^2(x) dx - \frac{1}{2} x^2 \cot^2(x)$$

$$\downarrow 25$$

$$-\int x^2 \cot(x) dx - \int x^2 \cos^2(x) \cot(x) dx + \int x \cot^2(x) dx - \frac{1}{2} x^2 \cot^2(x)$$

$$\downarrow 3042$$

$$-\int -x^2 \tan\left(x + \frac{\pi}{2}\right) dx - \int x^2 \cos^2(x) \cot(x) dx + \int x \tan\left(x + \frac{\pi}{2}\right)^2 dx - \frac{1}{2} x^2 \cot^2(x)$$

$$\downarrow 25$$

$$\int x^2 \tan\left(x+\frac{\pi}{2}\right) dx - \int x^2 \cos^2(x) \cot(x) dx + \int x \tan\left(x+\frac{\pi}{2}\right)^2 dx - \frac{1}{2} x^2 \cot^2(x)$$

$$\downarrow 4200$$

$$-2i \int -\frac{e^{2ix} x^2}{1-e^{2ix}} dx - \int x^2 \cos^2(x) \cot(x) dx + \int x \tan\left(x+\frac{\pi}{2}\right)^2 dx + \frac{ix^3}{3} - \frac{1}{2} x^2 \cot^2(x)$$

$$\downarrow 25$$

$$2i \int \frac{e^{2ix} x^2}{1-e^{2ix}} dx - \int x^2 \cos^2(x) \cot(x) dx + \int x \tan\left(x+\frac{\pi}{2}\right)^2 dx + \frac{ix^3}{3} - \frac{1}{2} x^2 \cot^2(x)$$

$$\downarrow 2620$$

$$2i \left(\frac{1}{2} i x^2 \log\left(1-e^{2ix}\right) - i \int x \log\left(1-e^{2ix}\right) dx\right) - \int x^2 \cos^2(x) \cot(x) dx + \int x \tan\left(x+\frac{\pi}{2}\right)^2 dx + \frac{ix^3}{3} - \frac{1}{2} x^2 \cot^2(x)$$

$$\downarrow 3011$$

$$2i \left(\frac{1}{2} i x^2 \log\left(1-e^{2ix}\right) - i \left(\frac{1}{2} i x \operatorname{PolyLog}\left(2, e^{2ix}\right) - \frac{1}{2} i \int \operatorname{PolyLog}\left(2, e^{2ix}\right) dx\right)\right) - \int x^2 \cos^2(x) \cot(x) dx + \int x \tan\left(x+\frac{\pi}{2}\right)^2 dx + \frac{ix^3}{3} - \frac{1}{2} x^2 \cot^2(x)$$

$$\downarrow 2720$$

$$2i \left(\frac{1}{2} i x^2 \log\left(1-e^{2ix}\right) - i \left(\frac{1}{2} i x \operatorname{PolyLog}\left(2, e^{2ix}\right) - \frac{1}{4} \int e^{-2ix} \operatorname{PolyLog}\left(2, e^{2ix}\right) de^{2ix}\right)\right) - \int x^2 \cos^2(x) \cot(x) dx + \int x \tan\left(x+\frac{\pi}{2}\right)^2 dx + \frac{ix^3}{3} - \frac{1}{2} x^2 \cot^2(x)$$

$$\downarrow 4203$$

$$2i \left(\frac{1}{2} i x^2 \log\left(1-e^{2ix}\right) - i \left(\frac{1}{2} i x \operatorname{PolyLog}\left(2, e^{2ix}\right) - \frac{1}{4} \int e^{-2ix} \operatorname{PolyLog}\left(2, e^{2ix}\right) de^{2ix}\right)\right) - \int x^2 \cos^2(x) \cot(x) dx - \int x dx - \int -\cot(x) dx + \frac{ix^3}{3} - \frac{1}{2} x^2 \cot^2(x) - x \cot(x)$$

$$\downarrow 15$$

$$2i \left(\frac{1}{2} i x^2 \log\left(1-e^{2ix}\right) - i \left(\frac{1}{2} i x \operatorname{PolyLog}\left(2, e^{2ix}\right) - \frac{1}{4} \int e^{-2ix} \operatorname{PolyLog}\left(2, e^{2ix}\right) de^{2ix}\right)\right) - \int x^2 \cos^2(x) \cot(x) dx - \int -\cot(x) dx + \frac{ix^3}{3} - \frac{x^2}{2} - \frac{1}{2} x^2 \cot^2(x) - x \cot(x)$$

$$\downarrow 25$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)-$$
$$\int x^2\cos^2(x)\cot(x)dx+\int\cot(x)dx+\frac{ix^3}{3}-\frac{x^2}{2}-\frac{1}{2}x^2\cot^2(x)-x\cot(x)$$

$$\downarrow 3042$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)-$$
$$\int x^2\cos^2(x)\cot(x)dx+\int-\tan\left(x+\frac{\pi}{2}\right)dx+\frac{ix^3}{3}-\frac{x^2}{2}-\frac{1}{2}x^2\cot^2(x)-x\cot(x)$$

$$\downarrow 25$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)-$$
$$\int x^2\cos^2(x)\cot(x)dx-\int\tan\left(x+\frac{\pi}{2}\right)dx+\frac{ix^3}{3}-\frac{x^2}{2}-\frac{1}{2}x^2\cot^2(x)-x\cot(x)$$

$$\downarrow 3956$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)-$$
$$\int x^2\cos^2(x)\cot(x)dx+\frac{ix^3}{3}-\frac{x^2}{2}-\frac{1}{2}x^2\cot^2(x)-x\cot(x)+\log(\sin(x))$$

$$\downarrow 4908$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)-$$
$$\int x^2\cot(x)dx+\int x^2\cos(x)\sin(x)dx+\frac{ix^3}{3}-\frac{x^2}{2}-\frac{1}{2}x^2\cot^2(x)-x\cot(x)+\log(\sin(x))$$

$$\downarrow 3042$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)-$$
$$\int-x^2\tan\left(x+\frac{\pi}{2}\right)dx+\int x^2\cos(x)\sin(x)dx+\frac{ix^3}{3}-\frac{x^2}{2}-\frac{1}{2}x^2\cot^2(x)-x\cot(x)+\log(\sin(x))$$

$$\downarrow 25$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)+$$
$$\int x^2\tan\left(x+\frac{\pi}{2}\right)dx+\int x^2\cos(x)\sin(x)dx+\frac{ix^3}{3}-\frac{x^2}{2}-\frac{1}{2}x^2\cot^2(x)-x\cot(x)+\log(\sin(x))$$

$$\downarrow 3924$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)+$$
$$\int x^2\tan\left(x+\frac{\pi}{2}\right)dx-\int x\sin^2(x)dx+\frac{ix^3}{3}-\frac{x^2}{2}+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-x\cot(x)+$$
$$\log(\sin(x))$$

$$\downarrow 3042$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)+$$
$$\int x^2\tan\left(x+\frac{\pi}{2}\right)dx-\int x\sin(x)^2dx+\frac{ix^3}{3}-\frac{x^2}{2}+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-x\cot(x)+$$
$$\log(\sin(x))$$

$$\downarrow 3791$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)+$$
$$\int x^2\tan\left(x+\frac{\pi}{2}\right)dx-\frac{\int xdx}{2}+\frac{ix^3}{3}-\frac{x^2}{2}+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-\frac{\sin^2(x)}{4}-x\cot(x)+$$
$$\log(\sin(x))+\frac{1}{2}x\sin(x)\cos(x)$$

$$\downarrow 15$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)+$$
$$\int x^2\tan\left(x+\frac{\pi}{2}\right)dx+\frac{ix^3}{3}-\frac{3x^2}{4}+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-\frac{\sin^2(x)}{4}-x\cot(x)+$$
$$\log(\sin(x))+\frac{1}{2}x\sin(x)\cos(x)$$

$$\downarrow 4200$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)-$$
$$2i\int-\frac{e^{2ix}x^2}{1-e^{2ix}}dx+\frac{2ix^3}{3}-\frac{3x^2}{4}+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-\frac{\sin^2(x)}{4}-x\cot(x)+$$
$$\log(\sin(x))+\frac{1}{2}x\sin(x)\cos(x)$$

$$\downarrow 25$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)+$$
$$2i\int\frac{e^{2ix}x^2}{1-e^{2ix}}dx+\frac{2ix^3}{3}-\frac{3x^2}{4}+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-\frac{\sin^2(x)}{4}-x\cot(x)+$$
$$\log(\sin(x))+\frac{1}{2}x\sin(x)\cos(x)$$

$$\downarrow 2620$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)+$$
$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\int x\log\left(1-e^{2ix}\right)dx\right)+\frac{2ix^3}{3}-\frac{3x^2}{4}+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-$$
$$\frac{\sin^2(x)}{4}-x\cot(x)+\log(\sin(x))+\frac{1}{2}x\sin(x)\cos(x)$$

$$\downarrow 3011$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{2}i\int\text{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)+$$
$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)+$$
$$\frac{2ix^3}{3}-\frac{3x^2}{4}+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-\frac{\sin^2(x)}{4}-x\cot(x)+\log(\sin(x))+\frac{1}{2}x\sin(x)\cos(x)$$

$$\downarrow 2720$$

$$4i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)+$$
$$\frac{2ix^3}{3}-\frac{3x^2}{4}+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-\frac{\sin^2(x)}{4}-x\cot(x)+\log(\sin(x))+\frac{1}{2}x\sin(x)\cos(x)$$

input
```
Int[x^2*Cos[x]^2*Cot[x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))  Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
  Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))  Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)  Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3924

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))  Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 3956

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4200

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 4203

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4908

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.60

method	result
risch	$\frac{2ix^3}{3}-\frac{(2x^2+2ix-1)e^{2ix}}{16}-\frac{(2x^2-2ix-1)e^{-2ix}}{16}+\frac{2x(e^{2ix}x-ie^{2ix}+i)}{(e^{2ix}-1)^2}+\ln(e^{ix}-1)+\ln(e^{ix}+1)-2\ln(e^{ix})$

input

```
int(x^2*cos(x)^2*cot(x)^3,x,method=_RETURNVERBOSE)
```

output

```
2/3*I*x^3-1/16*(2*I*x+2*x^2-1)*exp(2*I*x)-1/16*(-2*I*x+2*x^2-1)*exp(-2*I*x
)+2*x*(exp(2*I*x)*x-I*exp(2*I*x)+I)/(exp(2*I*x)-1)^2+ln(exp(I*x)-1)+ln(exp
(I*x)+1)-2*ln(exp(I*x))-2*x^2*ln(exp(I*x)+1)+4*I*x*polylog(2,-exp(I*x))-4*
polylog(3,-exp(I*x))-2*x^2*ln(1-exp(I*x))+4*I*x*polylog(2,exp(I*x))-4*poly
log(3,exp(I*x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(80) = 160$.

Time = 0.10 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.49

$$\int x^2 \cos^2(x) \cot^3(x)\, dx = \text{Too large to display}$$

input
```
integrate(x^2*cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

output
```
-1/8*(2*(2*x^2 - 1)*cos(x)^4 - 3*(2*x^2 - 1)*cos(x)^2 - 2*x^2 + 16*(-I*x*c
os(x)^2 + I*x)*dilog(cos(x) + I*sin(x)) + 16*(I*x*cos(x)^2 - I*x)*dilog(co
s(x) - I*sin(x)) + 16*(I*x*cos(x)^2 - I*x)*dilog(-cos(x) + I*sin(x)) + 16*
(-I*x*cos(x)^2 + I*x)*dilog(-cos(x) - I*sin(x)) + 4*((2*x^2 - 1)*cos(x)^2
 - 2*x^2 + 1)*log(cos(x) + I*sin(x) + 1) + 4*((2*x^2 - 1)*cos(x)^2 - 2*x^2
 + 1)*log(cos(x) - I*sin(x) + 1) - 4*(cos(x)^2 - 1)*log(-1/2*cos(x) + 1/2*I
*sin(x) + 1/2) - 4*(cos(x)^2 - 1)*log(-1/2*cos(x) - 1/2*I*sin(x) + 1/2) +
8*(x^2*cos(x)^2 - x^2)*log(-cos(x) + I*sin(x) + 1) + 8*(x^2*cos(x)^2 - x^2
)*log(-cos(x) - I*sin(x) + 1) + 16*(cos(x)^2 - 1)*polylog(3, cos(x) + I*si
n(x)) + 16*(cos(x)^2 - 1)*polylog(3, cos(x) - I*sin(x)) + 16*(cos(x)^2 - 1
)*polylog(3, -cos(x) + I*sin(x)) + 16*(cos(x)^2 - 1)*polylog(3, -cos(x) -
I*sin(x)) - 4*(x*cos(x)^3 + x*cos(x))*sin(x) - 1)/(cos(x)^2 - 1)
```

Sympy [F]

$$\int x^2 \cos^2(x) \cot^3(x)\, dx = \int x^2 \cos^2(x) \cot^3(x)\, dx$$

input
```
integrate(x**2*cos(x)**2*cot(x)**3,x)
```

output
```
Integral(x**2*cos(x)**2*cot(x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2842 vs. $2(80) = 160$.

Time = 0.32 (sec) , antiderivative size = 2842, normalized size of antiderivative = 26.81

$$\int x^2 \cos^2(x) \cot^3(x)\, dx = \text{Too large to display}$$

input

```
integrate(x^2*cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

output

```
-1/48*(3*(2*x^2 + 2*I*x - 1)*cos(6*x)^2 + 4*(16*I*x^3 + 6*x^2 - 42*I*x - 3
)*cos(4*x)^2 + 2*(32*I*x^3 - 42*x^2 - 48*I*x - 3)*cos(2*x)^2 - 3*(2*x^2 +
2*I*x - 1)*sin(6*x)^2 + 4*(-16*I*x^3 - 6*x^2 + 42*I*x + 3)*sin(4*x)^2 + 2*
(-32*I*x^3 + 42*x^2 + 48*I*x + 3)*sin(2*x)^2 + 6*x^2 + 48*(2*(-2*I*x^2 + I
)*cos(4*x)^2 + 2*(-2*I*x^2 + I)*cos(2*x)^2 + 2*(2*I*x^2 - I)*sin(4*x)^2 +
2*(2*I*x^2 - I)*sin(2*x)^2 + (2*I*x^2 + (2*I*x^2 - I)*cos(4*x) + 2*(-2*I*x
^2 + I)*cos(2*x) - (2*x^2 - 1)*sin(4*x) + 2*(2*x^2 - 1)*sin(2*x) - I)*cos(
6*x) + (-4*I*x^2 + 5*(2*I*x^2 - I)*cos(2*x) - 5*(2*x^2 - 1)*sin(2*x) + 2*I
)*cos(4*x) + (2*I*x^2 - I)*cos(2*x) - (2*x^2 + (2*x^2 - 1)*cos(4*x) - 2*(2
*x^2 - 1)*cos(2*x) - (-2*I*x^2 + I)*sin(4*x) - 2*(2*I*x^2 - I)*sin(2*x) -
1)*sin(6*x) + (4*x^2 + 4*(2*x^2 - 1)*cos(4*x) - 5*(2*x^2 - 1)*cos(2*x) + 5
*(-2*I*x^2 + I)*sin(2*x) - 2)*sin(4*x) - (2*x^2 - 4*(2*x^2 - 1)*cos(2*x) -
 1)*sin(2*x))*arctan2(sin(x), cos(x) + 1) + 48*((-I*cos(4*x) + 2*I*cos(2*x
) + sin(4*x) - 2*sin(2*x) - I)*cos(6*x) + (-5*I*cos(2*x) + 5*sin(2*x) + 2*
I)*cos(4*x) + 2*I*cos(4*x)^2 + 2*I*cos(2*x)^2 + (cos(4*x) - 2*cos(2*x) + I
*sin(4*x) - 2*I*sin(2*x) + 1)*sin(6*x) - (4*cos(4*x) - 5*cos(2*x) - 5*I*si
n(2*x) + 2)*sin(4*x) - 2*I*sin(4*x)^2 - (4*cos(2*x) - 1)*sin(2*x) - 2*I*si
n(2*x)^2 - I*cos(2*x))*arctan2(sin(x), cos(x) - 1) + 96*(2*I*x^2*cos(4*x)^
2 + 2*I*x^2*cos(2*x)^2 - 2*I*x^2*sin(4*x)^2 - 2*I*x^2*sin(2*x)^2 - I*x^2*c
os(2*x) + (-I*x^2*cos(4*x) + 2*I*x^2*cos(2*x) + x^2*sin(4*x) - 2*x^2*si...
```

Giac [F]

$$\int x^2 \cos^2(x) \cot^3(x)\, dx = \int x^2 \cos(x)^2 \cot(x)^3\, dx$$

input
```
integrate(x^2*cos(x)^2*cot(x)^3,x, algorithm="giac")
```

output
```
integrate(x^2*cos(x)^2*cot(x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cos^2(x) \cot^3(x)\, dx = \int x^2 \cos(x)^2 \cot(x)^3\, dx$$

input
```
int(x^2*cos(x)^2*cot(x)^3,x)
```

output
```
int(x^2*cos(x)^2*cot(x)^3, x)
```

Reduce [F]

$$\int x^2 \cos^2(x) \cot^3(x)\, dx = \int \cos(x)^2 \cot(x)^3\, x^2 dx$$

input
```
int(x^2*cos(x)^2*cot(x)^3,x)
```

output
```
int(cos(x)**2*cot(x)**3*x**2,x)
```

3.207 $\int x \cos^2(x) \cot^3(x)\, dx$

Optimal result . . . 1891
Mathematica [A] (verified) . . . 1891
Rubi [A] (verified) . . . 1892
Maple [A] (verified) . . . 1897
Fricas [B] (verification not implemented) . . . 1898
Sympy [F] . . . 1898
Maxima [B] (verification not implemented) . . . 1899
Giac [F] . . . 1900
Mupad [F(-1)] . . . 1900
Reduce [F] . . . 1900

Optimal result

Integrand size = 10, antiderivative size = 73

$$\int x \cos^2(x) \cot^3(x)\, dx = -\frac{3x}{4} + ix^2 - \frac{\cot(x)}{2} - \frac{1}{2}x\cot^2(x) - 2x\log\left(1-e^{2ix}\right) + i\,\text{PolyLog}\left(2, e^{2ix}\right) + \frac{1}{4}\cos(x)\sin(x) + \frac{1}{2}x\sin^2(x)$$

output

```
-3/4*x+I*x^2-1/2*cot(x)-1/2*x*cot(x)^2-2*x*ln(1-exp(2*I*x))+I*polylog(2,ex
p(2*I*x))+1/4*cos(x)*sin(x)+1/2*x*sin(x)^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int x \cos^2(x) \cot^3(x)\, dx = \frac{1}{8}\left(8ix^2 - 2x\cos(2x) - 4\cot(x) - 4x\csc^2(x) - 16x\log\left(1-e^{2ix}\right) + 8i\,\text{PolyLog}\left(2, e^{2ix}\right) + \sin(2x)\right)$$

input

```
Integrate[x*Cos[x]^2*Cot[x]^3,x]
```

output

```
((8*I)*x^2 - 2*x*Cos[2*x] - 4*Cot[x] - 4*x*Csc[x]^2 - 16*x*Log[1 - E^((2*I
)*x)] + (8*I)*PolyLog[2, E^((2*I)*x)] + Sin[2*x])/8
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 2.600$, Rules used = {4908, 3042, 25, 4203, 25, 3042, 25, 3954, 24, 4200, 25, 2620, 2715, 2838, 4908, 3042, 25, 3924, 3042, 3115, 24, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos^2(x)\cot^3(x)\,dx$$

$$\downarrow 4908$$

$$\int x\cot^3(x)dx - \int x\cos^2(x)\cot(x)dx$$

$$\downarrow 3042$$

$$\int -x\tan\left(x+\frac{\pi}{2}\right)^3 dx - \int x\cos^2(x)\cot(x)dx$$

$$\downarrow 25$$

$$-\int x\tan\left(x+\frac{\pi}{2}\right)^3 dx - \int x\cos^2(x)\cot(x)dx$$

$$\downarrow 4203$$

$$\frac{1}{2}\int \cot^2(x)dx + \int -x\cot(x)dx - \int x\cos^2(x)\cot(x)dx - \frac{1}{2}x\cot^2(x)$$

$$\downarrow 25$$

$$\frac{1}{2}\int \cot^2(x)dx - \int x\cot(x)dx - \int x\cos^2(x)\cot(x)dx - \frac{1}{2}x\cot^2(x)$$

$$\downarrow 3042$$

$$-\int -x\tan\left(x+\frac{\pi}{2}\right)dx + \frac{1}{2}\int \tan\left(x+\frac{\pi}{2}\right)^2 dx - \int x\cos^2(x)\cot(x)dx - \frac{1}{2}x\cot^2(x)$$

$$\downarrow 25$$

$$\int x\tan\left(x+\frac{\pi}{2}\right)dx + \frac{1}{2}\int \tan\left(x+\frac{\pi}{2}\right)^2 dx - \int x\cos^2(x)\cot(x)dx - \frac{1}{2}x\cot^2(x)$$

$$\downarrow 3954$$

$$\int x\tan\left(x+\frac{\pi}{2}\right)dx+\frac{1}{2}(-\int 1dx-\cot(x))-\int x\cos^2(x)\cot(x)dx-\frac{1}{2}x\cot^2(x)$$

$$\downarrow 24$$

$$\int x\tan\left(x+\frac{\pi}{2}\right)dx-\int x\cos^2(x)\cot(x)dx-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\downarrow 4200$$

$$-2i\int -\frac{e^{2ix}x}{1-e^{2ix}}dx-\int x\cos^2(x)\cot(x)dx+\frac{ix^2}{2}-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\downarrow 25$$

$$2i\int \frac{e^{2ix}x}{1-e^{2ix}}dx-\int x\cos^2(x)\cot(x)dx+\frac{ix^2}{2}-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\downarrow 2620$$

$$2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{2}i\int\log\left(1-e^{2ix}\right)dx\right)-\int x\cos^2(x)\cot(x)dx+\frac{ix^2}{2}-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\downarrow 2715$$

$$2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\log\left(1-e^{2ix}\right)de^{2ix}\right)-\int x\cos^2(x)\cot(x)dx+\frac{ix^2}{2}-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\downarrow 2838$$

$$-\int x\cos^2(x)\cot(x)dx+2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\downarrow 4908$$

$$-\int x\cot(x)dx+\int x\cos(x)\sin(x)dx+2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\downarrow 3042$$

$$-\int -x\tan\left(x+\frac{\pi}{2}\right)dx+\int x\cos(x)\sin(x)dx+2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\downarrow 25$$

$$\int x\tan\left(x+\frac{\pi}{2}\right)dx+\int x\cos(x)\sin(x)dx+2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\downarrow 3924$$

$$-\frac{1}{2}\int\sin^2(x)dx+\int x\tan\left(x+\frac{\pi}{2}\right)dx+2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\downarrow 3042$$

$$-\frac{1}{2}\int\sin(x)^2dx+\int x\tan\left(x+\frac{\pi}{2}\right)dx+2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\downarrow 3115$$

$$\int x\tan\left(x+\frac{\pi}{2}\right)dx+\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x)-\frac{\int 1dx}{2}\right)+2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\downarrow 24$$

$$\int x\tan\left(x+\frac{\pi}{2}\right)dx+2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))+\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x)-\frac{x}{2}\right)$$

$$\downarrow 4200$$

$$-2i\int-\frac{e^{2ix}x}{1-e^{2ix}}dx+2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+ix^2+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))+\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x)-\frac{x}{2}\right)$$

$$\downarrow 25$$

$$2i\int\frac{e^{2ix}x}{1-e^{2ix}}dx+2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+ix^2+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))+\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x)-\frac{x}{2}\right)$$

$$\downarrow 2620$$

$$2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{2}i\int\log\left(1-e^{2ix}\right)dx\right)+$$
$$2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+ix^2+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-$$
$$\cot(x))+\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x)-\frac{x}{2}\right)$$

$$\downarrow 2715$$

$$2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\log\left(1-e^{2ix}\right)de^{2ix}\right)+$$
$$2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+ix^2+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-$$
$$\cot(x))+\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x)-\frac{x}{2}\right)$$

$$\downarrow 2838$$

$$4i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+ix^2+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-$$
$$\cot(x))+\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x)-\frac{x}{2}\right)$$

input
```
Int[x*Cos[x]^2*Cot[x]^3,x]
```

output
```
I*x^2 + (-x - Cot[x])/2 - (x*Cot[x]^2)/2 + (4*I)*((I/2)*x*Log[1 - E^((2*I)
*x)] + PolyLog[2, E^((2*I)*x)]/4) + (x*Sin[x]^2)/2 + (-1/2*x + (Cos[x]*Sin
[x])/2)/2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))  Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])  Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)  Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))  Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2  Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4200

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 4203

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4908

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.49

method	result
risch	$ix^2 - \frac{(2x+i)e^{2ix}}{16} - \frac{(-i+2x)e^{-2ix}}{16} + \frac{2e^{2ix}x - ie^{2ix} + i}{(e^{2ix}-1)^2} - 2x\ln(e^{ix}+1) - 2x\ln(1-e^{ix}) + 2i\,\mathrm{polylog}(2, e$

input

```
int(x*cos(x)^2*cot(x)^3,x,method=_RETURNVERBOSE)
```

output

```
I*x^2-1/16*(2*x+I)*exp(2*I*x)-1/16*(-I+2*x)*exp(-2*I*x)+(2*exp(2*I*x)*x-I*
exp(2*I*x)+I)/(exp(2*I*x)-1)^2-2*x*ln(exp(I*x)+1)-2*x*ln(1-exp(I*x))+2*I*p
olylog(2,exp(I*x))+2*I*polylog(2,-exp(I*x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(52) = 104$.

Time = 0.09 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.78

$$\int x\cos^2(x)\cot^3(x)\,dx =$$

$$-\frac{2\,x\cos(x)^4 - 3\,x\cos(x)^2 + 4\left(-i\,\cos(x)^2 + i\right)\mathrm{Li}_2(\cos(x) + i\,\sin(x)) + 4\left(i\,\cos(x)^2 - i\right)\mathrm{Li}_2(\cos(x) -}{}$$

input
```
integrate(x*cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*x*cos(x)^4 - 3*x*cos(x)^2 + 4*(-I*cos(x)^2 + I)*dilog(cos(x) + I*s
in(x)) + 4*(I*cos(x)^2 - I)*dilog(cos(x) - I*sin(x)) + 4*(I*cos(x)^2 - I)*
dilog(-cos(x) + I*sin(x)) + 4*(-I*cos(x)^2 + I)*dilog(-cos(x) - I*sin(x))
+ 4*(x*cos(x)^2 - x)*log(cos(x) + I*sin(x) + 1) + 4*(x*cos(x)^2 - x)*log(c
os(x) - I*sin(x) + 1) + 4*(x*cos(x)^2 - x)*log(-cos(x) + I*sin(x) + 1) + 4
*(x*cos(x)^2 - x)*log(-cos(x) - I*sin(x) + 1) - (cos(x)^3 + cos(x))*sin(x)
 - x)/(cos(x)^2 - 1)
```

Sympy [F]

$$\int x\cos^2(x)\cot^3(x)\,dx = \int x\cos^2(x)\cot^3(x)\,dx$$

input
```
integrate(x*cos(x)**2*cot(x)**3,x)
```

output
```
Integral(x*cos(x)**2*cot(x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1718 vs. $2(52) = 104$.

Time = 0.18 (sec) , antiderivative size = 1718, normalized size of antiderivative = 23.53

$$\int x \cos^2(x) \cot^3(x)\, dx = \text{Too large to display}$$

input

```
integrate(x*cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

output

```
-1/16*((2*x + I)*cos(6*x)^2 + 4*(8*I*x^2 + 2*x + I)*cos(4*x)^2 + 4*(8*I*x^
2 - 7*x + 4*I)*cos(2*x)^2 - (2*x + I)*sin(6*x)^2 + 4*(-8*I*x^2 - 2*x - I)*
sin(4*x)^2 + 4*(-8*I*x^2 + 7*x - 4*I)*sin(2*x)^2 + 32*(-2*I*x*cos(4*x)^2 -
 2*I*x*cos(2*x)^2 + 2*I*x*sin(4*x)^2 + 2*I*x*sin(2*x)^2 + (I*x*cos(4*x) -
2*I*x*cos(2*x) - x*sin(4*x) + 2*x*sin(2*x) + I*x)*cos(6*x) + (5*I*x*cos(2*
x) - 5*x*sin(2*x) - 2*I*x)*cos(4*x) + I*x*cos(2*x) - (x*cos(4*x) - 2*x*cos
(2*x) + I*x*sin(4*x) - 2*I*x*sin(2*x) + x)*sin(6*x) + (4*x*cos(4*x) - 5*x*
cos(2*x) - 5*I*x*sin(2*x) + 2*x)*sin(4*x) + (4*x*cos(2*x) - x)*sin(2*x))*a
rctan2(sin(x), cos(x) + 1) + 32*(2*I*x*cos(4*x)^2 + 2*I*x*cos(2*x)^2 - 2*I
*x*sin(4*x)^2 - 2*I*x*sin(2*x)^2 + (-I*x*cos(4*x) + 2*I*x*cos(2*x) + x*sin
(4*x) - 2*x*sin(2*x) - I*x)*cos(6*x) + (-5*I*x*cos(2*x) + 5*x*sin(2*x) + 2
*I*x)*cos(4*x) - I*x*cos(2*x) + (x*cos(4*x) - 2*x*cos(2*x) + I*x*sin(4*x)
- 2*I*x*sin(2*x) + x)*sin(6*x) - (4*x*cos(4*x) - 5*x*cos(2*x) - 5*I*x*sin(
2*x) + 2*x)*sin(4*x) - (4*x*cos(2*x) - x)*sin(2*x))*arctan2(sin(x), -cos(x
) + 1) - (16*I*x^2 - 4*(-4*I*x^2 - 2*x - I)*cos(4*x) + (-32*I*x^2 + 26*x -
 17*I)*cos(2*x) - 4*(4*x^2 - 2*I*x + 1)*sin(4*x) + (32*x^2 + 26*I*x + 17)*
sin(2*x) + 4*x + 14*I)*cos(6*x) - (-32*I*x^2 - 2*(-40*I*x^2 + 26*x - 17*I)
*cos(2*x) - 2*(40*x^2 + 26*I*x + 17)*sin(2*x) - 10*x - 27*I)*cos(4*x) + 4*
(-4*I*x^2 - 2*x - 3*I)*cos(2*x) + 32*((-I*cos(4*x) + 2*I*cos(2*x) + sin(4*
x) - 2*sin(2*x) - I)*cos(6*x) + (-5*I*cos(2*x) + 5*sin(2*x) + 2*I)*cos(...
```

Giac [F]

$$\int x \cos^2(x) \cot^3(x)\, dx = \int x \cos(x)^2 \cot(x)^3\, dx$$

input
```
integrate(x*cos(x)^2*cot(x)^3,x, algorithm="giac")
```

output
```
integrate(x*cos(x)^2*cot(x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x \cos^2(x) \cot^3(x)\, dx = \int x \cos(x)^2 \cot(x)^3\, dx$$

input
```
int(x*cos(x)^2*cot(x)^3,x)
```

output
```
int(x*cos(x)^2*cot(x)^3, x)
```

Reduce [F]

$$\int x \cos^2(x) \cot^3(x)\, dx = \int \cos(x)^2 \cot(x)^3\, x dx$$

input
```
int(x*cos(x)^2*cot(x)^3,x)
```

output
```
int(cos(x)**2*cot(x)**3*x,x)
```

3.208 $\int (c+dx)^m \tan(a+bx)\,dx$

Optimal result 1901
Mathematica [N/A] 1901
Rubi [N/A] 1902
Maple [N/A] 1903
Fricas [N/A] 1903
Sympy [N/A] 1904
Maxima [N/A] 1904
Giac [N/A] 1905
Mupad [N/A] 1905
Reduce [N/A] 1905

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (c+dx)^m \tan(a+bx)\,dx = \text{Int}((c+dx)^m \tan(a+bx), x)$$

output
```
Defer(Int)((d*x+c)^m*tan(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 1.65 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \tan(a+bx)\,dx = \int (c+dx)^m \tan(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Tan[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Tan[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(a+bx)(c+dx)^m\,dx$$
$$\downarrow 3042$$
$$\int \tan(a+bx)(c+dx)^m dx$$
$$\downarrow 4222$$
$$\int \tan(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Tan[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \tan(bx+a)\,dx$$

input
```
int((d*x+c)^m*tan(b*x+a),x)
```

output
```
int((d*x+c)^m*tan(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \tan(a+bx)\,dx = \int (dx+c)^m \tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*tan(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*tan(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \tan(a+bx)\,dx = \int (c+dx)^m \tan\left(a+bx\right)\,dx$$

input
```
integrate((d*x+c)**m*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**m*tan(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \tan(a+bx)\,dx = \int (dx+c)^m \tan\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)^m*tan(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*tan(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \tan(a+bx)\,dx = \int (dx+c)^m \tan\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)^m*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*tan(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 17.60 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \tan(a+bx)\,dx = \int \tan(a+b\,x)\,\left(c+d\,x\right)^m\,dx$$

input
```
int(tan(a + b*x)*(c + d*x)^m,x)
```

output
```
int(tan(a + b*x)*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \tan(a+bx)\,dx = \int (dx+c)^m \tan\left(bx+a\right)\,dx$$

input
```
int((d*x+c)^m*tan(b*x+a),x)
```

output

```
int((c + d*x)**m*tan(a + b*x),x)
```

3.209 $\int (c+dx)^4 \tan(a+bx)\,dx$

Optimal result . . . 1907
Mathematica [A] (verified) . . . 1908
Rubi [A] (verified) . . . 1908
Maple [B] (verified) . . . 1912
Fricas [B] (verification not implemented) . . . 1913
Sympy [F] . . . 1913
Maxima [B] (verification not implemented) . . . 1914
Giac [F] . . . 1915
Mupad [F(-1)] . . . 1915
Reduce [F] . . . 1915

Optimal result

Integrand size = 14, antiderivative size = 158

$$\begin{aligned}\int (c+dx)^4 \tan(a+bx)\,dx = {} & \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4 \log\left(1+e^{2i(a+bx)}\right)}{b} \\ & + \frac{2id(c+dx)^3 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} \\ & - \frac{3d^2(c+dx)^2 \operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3} \\ & - \frac{3id^3(c+dx) \operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4} \\ & + \frac{3d^4 \operatorname{PolyLog}\left(5,-e^{2i(a+bx)}\right)}{2b^5}\end{aligned}$$

output

```
1/5*I*(d*x+c)^5/d-(d*x+c)^4*ln(1+exp(2*I*(b*x+a)))/b+2*I*d*(d*x+c)^3*polyl
og(2,-exp(2*I*(b*x+a)))/b^2-3*d^2*(d*x+c)^2*polylog(3,-exp(2*I*(b*x+a)))/b
^3-3*I*d^3*(d*x+c)*polylog(4,-exp(2*I*(b*x+a)))/b^4+3/2*d^4*polylog(5,-exp
(2*I*(b*x+a)))/b^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int (c+dx)^4 \tan(a+bx)\,dx$$
$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4 \log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}$$
$$- \frac{3d^2\left(2b^2(c+dx)^2 \operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right) + d\left(2ib(c+dx)\operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right) - d\operatorname{PolyLog}\left(5,-e^{2}\right.\right.\right.}{2b^5}$$

input
```
Integrate[(c + d*x)^4*Tan[a + b*x],x]
```

output
```
((I/5)*(c + d*x)^5)/d - ((c + d*x)^4*Log[1 + E^((2*I)*(a + b*x))])/b + ((2
*I)*d*(c + d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(2*b^2*(c
 + d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))] + d*((2*I)*b*(c + d*x)*PolyLog[
4, -E^((2*I)*(a + b*x))] - d*PolyLog[5, -E^((2*I)*(a + b*x))])))/(2*b^5)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.23, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4202, 2620, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \tan(a+bx)\,dx$$
$$\downarrow 3042$$
$$\int (c+dx)^4 \tan(a+bx)dx$$
$$\downarrow 4202$$
$$\frac{i(c+dx)^5}{5d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^4}{1+e^{2i(a+bx)}}dx$$
$$\downarrow 2620$$

$$\frac{i(c+dx)^5}{5d} - 2i\left(\frac{2id\int(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)dx}{b} - \frac{i(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

$$\downarrow 3011$$

$$\frac{i(c+dx)^5}{5d} - 2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b} - \frac{3id\int(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{2b}\right)}{b} - \frac{i(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

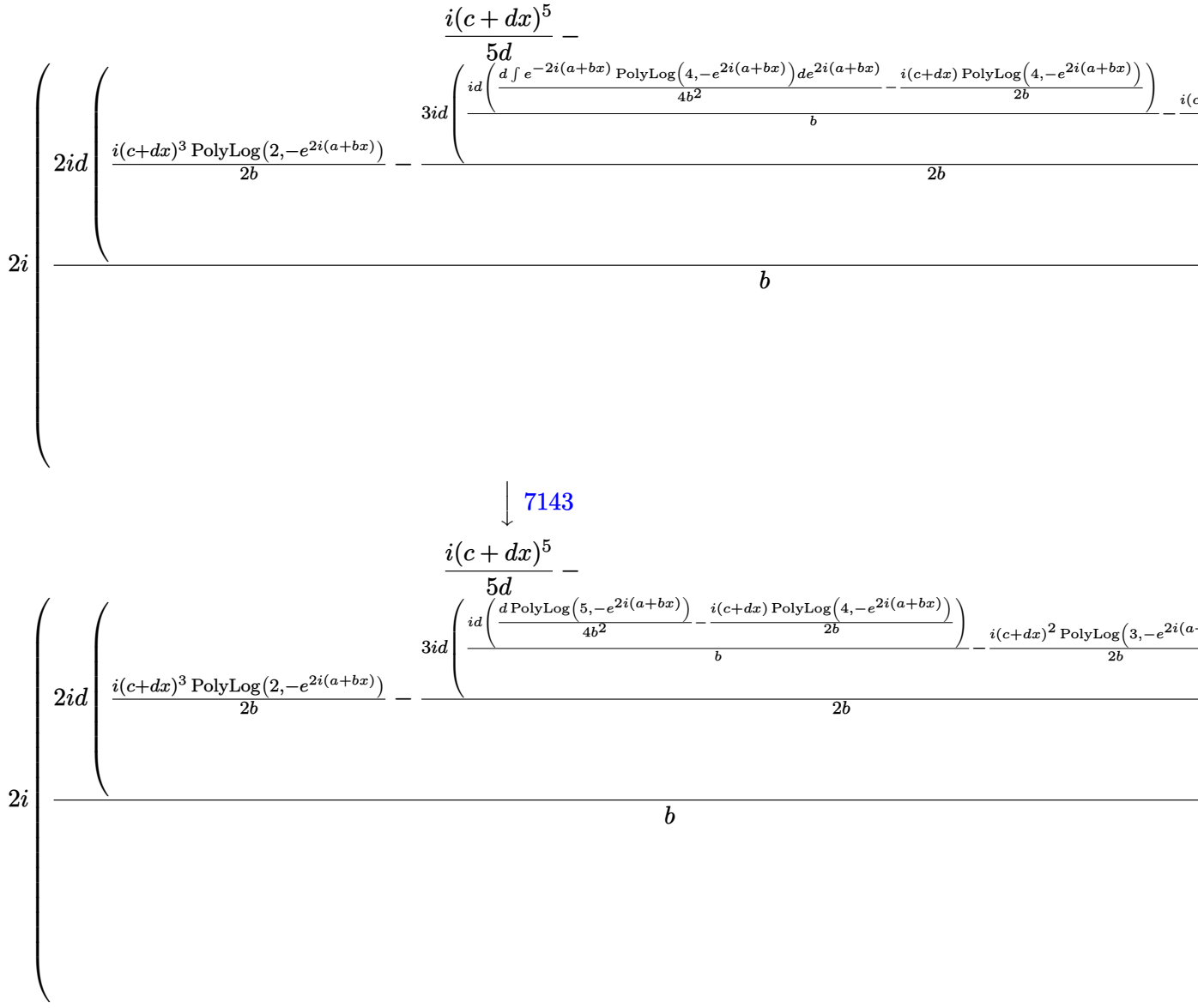
$$\downarrow 7163$$

$$\frac{i(c+dx)^5}{5d} - 2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b} - \frac{3id\left(\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)dx}{b} - \frac{i(c+dx)^2\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{2b}\right)}{b} - \frac{i(c+dx)}{}\right.$$

$$\downarrow 7163$$

$$\frac{i(c+dx)^5}{5d} - 2i\left(\frac{2id\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b} - \frac{3id\left(\frac{id\left(\frac{id\int\mathrm{PolyLog}\left(4,-e^{2i(a+bx)}\right)dx}{2b} - \frac{i(c+dx)\,\mathrm{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{2b}\right)}{b} - \frac{i(c+dx)^2\,\mathrm{PolyLog}\left(3,-e\right.}{2b}\right)}{2b}\right)}{b}\right.$$

$$\downarrow 2720$$



$$\downarrow 7143$$

input
```
Int[(c + d*x)^4*Tan[a + b*x],x]
```

output
```
((I/5)*(c + d*x)^5)/d - (2*I)*(((-1/2*I)*(c + d*x)^4*Log[1 + E^((2*I)*(a +
 b*x))])/b + ((2*I)*d*(((I/2)*(c + d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))]
)/b - (((3*I)/2)*d*(((-1/2*I)*(c + d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))]
)/b + (I*d*(((-1/2*I)*(c + d*x)*PolyLog[4, -E^((2*I)*(a + b*x))])/b + (d*P
olyLog[5, -E^((2*I)*(a + b*x))])/(4*b^2)))/b))/b))/b)
```

Defintions of rubi rules used

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(141) = 282$.

Time = 0.42 (sec) , antiderivative size = 625, normalized size of antiderivative = 3.96

method	result
risch	$\frac{12c^2d^2a^2\ln\left(e^{i(bx+a)}\right)}{b^3} - \frac{8id^2c^2a^3}{b^3} - \frac{2id^4a^4x}{b^4} - \frac{3id^4\,\mathrm{polylog}\left(4,-e^{2i(bx+a)}\right)x}{b^4} + \frac{4idc^3a^2}{b^2} + \frac{2idc^3\,\mathrm{polylog}\left(2,-e^{2i(bx+a)}\right)}{b^2}$

input

```
int((d*x+c)^4*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
-I*c^4*x-1/5*I/d*c^5-1/b*c^4*ln(exp(2*I*(b*x+a))+1)+2/b*c^4*ln(exp(I*(b*x+
a)))-6/b^3*d^3*c*polylog(3,-exp(2*I*(b*x+a)))*x-8/b^2*c^3*d*a*ln(exp(I*(b*
x+a)))-4/b*d*c^3*ln(exp(2*I*(b*x+a))+1)*x-6/b*d^2*c^2*ln(exp(2*I*(b*x+a))+
1)*x^2-4/b*d^3*c*ln(exp(2*I*(b*x+a))+1)*x^3-8/b^4*c*d^3*a^3*ln(exp(I*(b*x+
a)))+12/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a)))-8*I/b^3*d^2*c^2*a^3-2*I/b^4*d^4
*a^4*x-3*I/b^4*d^4*polylog(4,-exp(2*I*(b*x+a)))*x+4*I/b^2*d*c^3*a^2+2*I/b^
2*d*c^3*polylog(2,-exp(2*I*(b*x+a)))+6*I/b^4*d^3*c*a^4-3*I/b^4*d^3*c*polyl
og(4,-exp(2*I*(b*x+a)))+2*I/b^2*d^4*polylog(2,-exp(2*I*(b*x+a)))*x^3+1/5*I
*d^4*x^5+I*d^3*c*x^4+2*I*d^2*c^2*x^3+2*I*d*c^3*x^2-1/b*d^4*ln(exp(2*I*(b*x
+a))+1)*x^4+2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))-3/b^3*d^4*polylog(3,-exp(2*I*
(b*x+a)))*x^2-3/b^3*d^2*c^2*polylog(3,-exp(2*I*(b*x+a)))-8/5*I/b^5*d^4*a^5
-12*I/b^2*d^2*c^2*a^2*x+6*I/b^2*d^2*c^2*polylog(2,-exp(2*I*(b*x+a)))*x+6*I
/b^2*d^3*c*polylog(2,-exp(2*I*(b*x+a)))*x^2+8*I/b*d*c^3*a*x+8*I/b^3*d^3*c*
a^3*x+3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(137) = 274$.

Time = 0.08 (sec) , antiderivative size = 624, normalized size of antiderivative = 3.95

$$\int (c+dx)^4 \tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*tan(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(3*d^4*polylog(5, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a
)^2 + 1)) + 3*d^4*polylog(5, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(
b*x + a)^2 + 1)) - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*
x + I*b^3*c^3*d)*dilog(2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) -
4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*d
ilog(2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) - 2*(b^4*d^4*x^4 +
4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(-2*(I*t
an(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3
+ 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(-2*(-I*tan(b*x + a) - 1
)/(tan(b*x + a)^2 + 1)) - 6*(-I*b*d^4*x - I*b*c*d^3)*polylog(4, (tan(b*x +
 a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 6*(I*b*d^4*x + I*b*c
*d^3)*polylog(4, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 +
 1)) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, (tan(b*x +
 a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 6*(b^2*d^4*x^2 + 2*b
^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, (tan(b*x + a)^2 - 2*I*tan(b*x + a) -
1)/(tan(b*x + a)^2 + 1)))/b^5
```

Sympy [F]

$$\int (c+dx)^4 \tan(a+bx)\,dx = \int (c+dx)^4 \tan(a+bx)\,dx$$

input
```
integrate((d*x+c)**4*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**4*tan(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(137) = 274$.

Time = 0.22 (sec) , antiderivative size = 773, normalized size of antiderivative = 4.89

$$\int (c+dx)^4 \tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*tan(b*x+a),x, algorithm="maxima")
```

output
```
1/15*(15*c^4*log(sec(b*x + a)) - 60*a*c^3*d*log(sec(b*x + a))/b + 90*a^2*c
^2*d^2*log(sec(b*x + a))/b^2 - 60*a^3*c*d^3*log(sec(b*x + a))/b^3 + 15*a^4
*d^4*log(sec(b*x + a))/b^4 - (-3*I*(b*x + a)^5*d^4 - 15*(I*b*c*d^3 - I*a*d
^4)*(b*x + a)^4 - 45*d^4*polylog(5, -e^(2*I*b*x + 2*I*a)) - 30*(I*b^2*c^2*
d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a)^3 - 30*(I*b^3*c^3*d - 3*I*a*b^2
*c^2*d^2 + 3*I*a^2*b*c*d^3 - I*a^3*d^4)*(b*x + a)^2 - 10*(-3*I*(b*x + a)^4
*d^4 + 8*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 9*(-I*b^2*c^2*d^2 + 2*I*a*b*
c*d^3 - I*a^2*d^4)*(b*x + a)^2 + 6*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 - 3*I
*a^2*b*c*d^3 + I*a^3*d^4)*(b*x + a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x +
 2*a) + 1) - 30*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 + 2*I*(
b*x + a)^3*d^4 - I*a^3*d^4 + 4*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^
2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*
a)) + 5*(3*(b*x + a)^4*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 9*(b^2*c^2*
d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 6*(b^3*c^3*d - 3*a*b^2*c^2*d^2
+ 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x +
 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 30*(-2*I*b*c*d^3 - 3*I*(b*x + a)*d^4 +
 2*I*a*d^4)*polylog(4, -e^(2*I*b*x + 2*I*a)) + 15*(3*b^2*c^2*d^2 - 6*a*b*c
*d^3 + 6*(b*x + a)^2*d^4 + 3*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a))*poly
log(3, -e^(2*I*b*x + 2*I*a)))/b^4)/b
```

Giac [F]

$$\int (c+dx)^4 \tan(a+bx)\,dx = \int (dx+c)^4 \tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^4*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*tan(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \tan(a+bx)\,dx = \int \tan(a+b\,x)\,(c+d\,x)^4\,dx$$

input
```
int(tan(a + b*x)*(c + d*x)^4,x)
```

output
```
int(tan(a + b*x)*(c + d*x)^4, x)
```

Reduce [F]

$$\int (c+dx)^4 \tan(a+bx)\,dx$$
$$= \frac{2\left(\int \tan(bx+a)\,x^4dx\right) b\,d^4 + 8\left(\int \tan(bx+a)\,x^3dx\right) bc\,d^3 + 12\left(\int \tan(bx+a)\,x^2dx\right) b\,c^2d^2 + 8\left(\int \tan(b\right.}{2b}$$

input
```
int((d*x+c)^4*tan(b*x+a),x)
```

output
```
(2*int(tan(a + b*x)*x**4,x)*b*d**4 + 8*int(tan(a + b*x)*x**3,x)*b*c*d**3 +
 12*int(tan(a + b*x)*x**2,x)*b*c**2*d**2 + 8*int(tan(a + b*x)*x,x)*b*c**3*
d + log(tan(a + b*x)**2 + 1)*c**4)/(2*b)
```

3.210 $\int (c+dx)^3 \tan(a+bx)\,dx$

Optimal result . . . 1916
Mathematica [A] (verified) . . . 1917
Rubi [A] (verified) . . . 1917
Maple [B] (verified) . . . 1920
Fricas [B] (verification not implemented) . . . 1921
Sympy [F] . . . 1922
Maxima [B] (verification not implemented) . . . 1922
Giac [F] . . . 1923
Mupad [F(-1)] . . . 1923
Reduce [F] . . . 1923

Optimal result

Integrand size = 14, antiderivative size = 132

$$\begin{aligned}\int (c+dx)^3 \tan(a+bx)\,dx = {} & \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3 \log\left(1+e^{2i(a+bx)}\right)}{b} \\ & + \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} \\ & - \frac{3d^2(c+dx) \operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} \\ & - \frac{3id^3 \operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4}\end{aligned}$$

output

```
1/4*I*(d*x+c)^4/d-(d*x+c)^3*ln(1+exp(2*I*(b*x+a)))/b+3/2*I*d*(d*x+c)^2*pol
ylog(2,-exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+c)*polylog(3,-exp(2*I*(b*x+a)))
/b^3-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int (c+dx)^3 \tan(a+bx)\,dx = \frac{1}{4}i\left(\frac{(c+dx)^4}{d} + \frac{4i(c+dx)^3 \log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3d\left(2b^2(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right) + d\left(2ib(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right) - d\operatorname{PolyLog}\left(4,-e^{2i(}\right.\right.\right.}{b^4}$$

input
```
Integrate[(c + d*x)^3*Tan[a + b*x],x]
```

output
```
(I/4)*((c + d*x)^4/d + ((4*I)*(c + d*x)^3*Log[1 + E^((2*I)*(a + b*x))])/b
+ (3*d*(2*b^2*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))] + d*((2*I)*b*(c
 + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))] - d*PolyLog[4, -E^((2*I)*(a + b*x
))])))/b^4)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4202, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \tan(a+bx)\,dx$$
$$\downarrow 3042$$
$$\int (c+dx)^3 \tan(a+bx)dx$$
$$\downarrow 4202$$
$$\frac{i(c+dx)^4}{4d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1+e^{2i(a+bx)}}dx$$
$$\downarrow 2620$$

$$\frac{i(c+dx)^4}{4d} - 2i\left(\frac{3id\int(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)dx}{2b} - \frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

$$\downarrow 3011$$

$$\frac{i(c+dx)^4}{4d} - 2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b} - \frac{id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b} - \frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

$$\downarrow 7163$$

$$\frac{i(c+dx)^4}{4d} - 2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b} - \frac{id\left(\frac{id\int\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)dx}{2b} - \frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b} - \frac{i(c+dx)^3\log\left(1\right.}{2b}\right.$$

$$\downarrow 2720$$

$$\frac{i(c+dx)^4}{4d} - 2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b} - \frac{id\left(\frac{d\int e^{-2i(a+bx)}\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2} - \frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b} - \frac{i(}{}\right.$$

$$\downarrow 7143$$

$$\frac{i(c+dx)^4}{4d} - 2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b} - \frac{id\left(\frac{d\,\mathrm{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^2} - \frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b} - \frac{i(c+dx)^3\log\left(1+e\right.}{2b}\right.$$

input
```
Int[(c + d*x)^3*Tan[a + b*x],x]
```

output
```
((I/4)*(c + d*x)^4)/d - (2*I)*(((-1/2*I)*(c + d*x)^3*Log[1 + E^((2*I)*(a +
 b*x))])/b + (((3*I)/2)*d*(((I/2)*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*
x))])/b - (I*d*(((-1/2*I)*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/b +
(d*PolyLog[4, -E^((2*I)*(a + b*x))])/(4*b^2)))/b))/b)
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(113) = 226$.

Time = 0.37 (sec) , antiderivative size = 432, normalized size of antiderivative = 3.27

method	result
risch	$-\frac{3dc^2\ln(e^{2i(bx+a)}+1)x}{b}-\frac{3cd^2\ln(e^{2i(bx+a)}+1)x^2}{b}+\frac{3id^3\operatorname{polylog}(2,-e^{2i(bx+a)})x^2}{2b^2}+\frac{3idc^2\operatorname{polylog}(2,-e^{2i(bx+a)})}{2b^2}+\frac{3ic$

input
```
int((d*x+c)^3*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a)))+3*I/b^2*c*d^2*polylog(2,-exp(2*I*(b*x+a
)))*x-6*I/b^2*c*d^2*a^2*x+6*I/b*d*c^2*a*x-I*c^3*x-1/4*I/d*c^4-3/b*d*c^2*ln
(exp(2*I*(b*x+a))+1)*x-3/b*c*d^2*ln(exp(2*I*(b*x+a))+1)*x^2-4*I/b^3*c*d^2*
a^3+3*I/b^2*d*c^2*a^2+3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2+3/2*I
/b^2*d*c^2*polylog(2,-exp(2*I*(b*x+a)))+2*I/b^3*d^3*a^3*x+I*d^2*c*x^3-3/2/
b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x-3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b
*x+a)))+3/2*I/b^4*d^3*a^4-1/b*d^3*ln(exp(2*I*(b*x+a))+1)*x^3+3/2*I*d*c^2*x
^2+1/4*I*d^3*x^4-1/b*c^3*ln(exp(2*I*(b*x+a))+1)-2/b^4*d^3*a^3*ln(exp(I*(b*
x+a)))-6/b^2*c^2*d*a*ln(exp(I*(b*x+a)))+2/b*c^3*ln(exp(I*(b*x+a)))-3/4*I*d
^3*polylog(4,-exp(2*I*(b*x+a)))/b^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(109) = 218$.

Time $= 0.09$ (sec) , antiderivative size $= 432$, normalized size of antiderivative $= 3.27$

$$\int (c+dx)^3 \tan(a+bx)\,dx$$

$$= \frac{3i\,d^3\mathrm{polylog}\left(4, \frac{\tan(bx+a)^2+2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right) - 3i\,d^3\mathrm{polylog}\left(4, \frac{\tan(bx+a)^2-2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right) - 6\left(i\,b^2d^3x^2 + 2i\,b^2c\right.}{}$$

input
```
integrate((d*x+c)^3*tan(b*x+a),x, algorithm="fricas")
```

output
```
1/8*(3*I*d^3*polylog(4, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x +
 a)^2 + 1)) - 3*I*d^3*polylog(4, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(
tan(b*x + a)^2 + 1)) - 6*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*d
ilog(2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) - 6*(-I*b^2*d^3*x^2
 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(2*(-I*tan(b*x + a) - 1)/(tan(b*x +
 a)^2 + 1) + 1) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^
3)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 4*(b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(-2*(-I*tan(b*x + a) - 1)/(tan
(b*x + a)^2 + 1)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, (tan(b*x + a)^2 + 2*I
*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 6*(b*d^3*x + b*c*d^2)*polylog(3
, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)))/b^4
```

Sympy [F]

$$\int (c+dx)^3 \tan(a+bx)\,dx = \int (c+dx)^3 \tan(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**3*tan(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(109) = 218$.

Time = 0.20 (sec) , antiderivative size = 474, normalized size of antiderivative = 3.59

$$\int (c+dx)^3 \tan(a+bx)\,dx$$

$$= \frac{12\,c^3 \log\left(\sec\left(bx+a\right)\right) - \frac{36\,ac^2d\log(\sec(bx+a))}{b} + \frac{36\,a^2cd^2\log(\sec(bx+a))}{b^2} - \frac{12\,a^3d^3\log(\sec(bx+a))}{b^3} - \frac{-3i\,(bx+a)^4d^3-12\,(i\,bc$$

input
```
integrate((d*x+c)^3*tan(b*x+a),x, algorithm="maxima")
```

output
```
1/12*(12*c^3*log(sec(b*x + a)) - 36*a*c^2*d*log(sec(b*x + a))/b + 36*a^2*c
*d^2*log(sec(b*x + a))/b^2 - 12*a^3*d^3*log(sec(b*x + a))/b^3 - (-3*I*(b*x
 + a)^4*d^3 - 12*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^3 + 12*I*d^3*polylog(4, -
e^(2*I*b*x + 2*I*a)) - 18*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*(b*x +
 a)^2 - 4*(-4*I*(b*x + a)^3*d^3 + 9*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 9
*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*(b*x + a))*arctan2(sin(2*b*x +
 2*a), cos(2*b*x + 2*a) + 1) - 6*(3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 4*I*(b*x
 + a)^2*d^3 + 3*I*a^2*d^3 + 6*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*dilog(-e^(2
*I*b*x + 2*I*a)) + 2*(4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2
+ 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(2*b*x + 2*a)^2
+ sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 6*(3*b*c*d^2 + 4*(b*x + a
)*d^3 - 3*a*d^3)*polylog(3, -e^(2*I*b*x + 2*I*a)))/b^3)/b
```

Giac [F]

$$\int (c+dx)^3 \tan(a+bx)\,dx = \int (dx+c)^3 \tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*tan(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \tan(a+bx)\,dx = \int \tan(a+b\,x)\,(c+d\,x)^3\,dx$$

input
```
int(tan(a + b*x)*(c + d*x)^3,x)
```

output
```
int(tan(a + b*x)*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \tan(a+bx)\,dx$$
$$= \frac{2\left(\int \tan(bx+a)\,x^3dx\right) b\,d^3 + 6\left(\int \tan(bx+a)\,x^2dx\right) bc\,d^2 + 6\left(\int \tan(bx+a)\,xdx\right) b\,c^2d + \log\left(\tan(bx+\right.}{2b}$$

input
```
int((d*x+c)^3*tan(b*x+a),x)
```

output
```
(2*int(tan(a + b*x)*x**3,x)*b*d**3 + 6*int(tan(a + b*x)*x**2,x)*b*c*d**2 +
 6*int(tan(a + b*x)*x,x)*b*c**2*d + log(tan(a + b*x)**2 + 1)*c**3)/(2*b)
```

3.211 $\int (c+dx)^2 \tan(a+bx)\,dx$

Optimal result . . . 1924
Mathematica [A] (verified) . . . 1924
Rubi [A] (verified) . . . 1925
Maple [B] (verified) . . . 1927
Fricas [B] (verification not implemented) . . . 1928
Sympy [F] . . . 1929
Maxima [B] (verification not implemented) . . . 1929
Giac [F] . . . 1930
Mupad [F(-1)] . . . 1930
Reduce [F] . . . 1930

Optimal result

Integrand size = 14, antiderivative size = 96

$$\int (c+dx)^2 \tan(a+bx)\,dx = \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2 \log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} - \frac{d^2\,\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}$$

output

```
1/3*I*(d*x+c)^3/d-(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b+I*d*(d*x+c)*polylog(2
,-exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.04

$$\int (c+dx)^2 \tan(a+bx)\,dx = \frac{2ib^2(c+dx)^2\left(b(c+dx)+3id\log\left(1+e^{2i(a+bx)}\right)\right)+6ibd^2(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)-3d^3\,\text{PolyLog}}{6b^3d}$$

input

```
Integrate[(c + d*x)^2*Tan[a + b*x],x]
```

output

```
((2*I)*b^2*(c + d*x)^2*(b*(c + d*x) + (3*I)*d*Log[1 + E^((2*I)*(a + b*x))]
) + (6*I)*b*d^2*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))] - 3*d^3*PolyLog
[3, -E^((2*I)*(a + b*x))])/(6*b^3*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \tan(a+bx)\,dx$$

$$\downarrow 3042$$

$$\int (c+dx)^2 \tan(a+bx)dx$$

$$\downarrow 4202$$

$$\frac{i(c+dx)^3}{3d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}dx$$

$$\downarrow 2620$$

$$\frac{i(c+dx)^3}{3d} - 2i\left(\frac{id\int (c+dx)\log\left(1+e^{2i(a+bx)}\right)dx}{b} - \frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

$$\downarrow 3011$$

$$\frac{i(c+dx)^3}{3d} - 2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b} - \frac{id\int \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{2b}\right)}{b} - \frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

$$\downarrow 2720$$

$$\frac{i(c+dx)^3}{3d} - 2i\left(\frac{id\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b} - \frac{d\int e^{-2i(a+bx)}\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b} - \frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

$$\downarrow 7143$$

$$\frac{i(c+dx)^3}{3d} - 2i\left(\frac{id\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b} - \frac{d\,\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{4b^2}\right)}{b} - \frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

input
```
Int[(c + d*x)^2*Tan[a + b*x],x]
```

output
```
((I/3)*(c + d*x)^3)/d - (2*I)*(((-1/2*I)*(c + d*x)^2*Log[1 + E^((2*I)*(a +
 b*x))])/b + (I*d*(((I/2)*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b -
(d*PolyLog[3, -E^((2*I)*(a + b*x))])/(4*b^2)))/b)
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(84) = 168$.

Time = 0.34 (sec) , antiderivative size = 266, normalized size of antiderivative = 2.77

method	result
risch	$\frac{4icdax}{b} - \frac{4id^2a^3}{3b^3} + \frac{2icda^2}{b^2} - \frac{ic^3}{3d} - \frac{4cda\ln\left(e^{i(bx+a)}\right)}{b^2} - \frac{c^2\ln\left(e^{2i(bx+a)}+1\right)}{b} + \frac{2c^2\ln\left(e^{i(bx+a)}\right)}{b} - \frac{2id^2a^2x}{b^2} + \frac{id^2x^3}{3} +$

input
```
int((d*x+c)^2*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
4*I/b*c*d*a*x-4/3*I/b^3*d^2*a^3+2*I/b^2*c*d*a^2-1/3*I/d*c^3-4/b^2*c*d*a*ln
(exp(I*(b*x+a)))-1/b*c^2*ln(exp(2*I*(b*x+a))+1)+2/b*c^2*ln(exp(I*(b*x+a)))
-2*I/b^2*d^2*a^2*x+1/3*I*d^2*x^3+I*d*c*x^2-I*c^2*x-1/b*d^2*ln(exp(2*I*(b*x
+a))+1)*x^2-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+I/b^2*d*c*polylog(2,-
exp(2*I*(b*x+a)))-2/b*d*c*ln(exp(2*I*(b*x+a))+1)*x+I/b^2*d^2*polylog(2,-ex
p(2*I*(b*x+a)))*x+2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(81) = 162$.

Time $= 0.08$ (sec) , antiderivative size $= 270$, normalized size of antiderivative $= 2.81$

$$\int (c+dx)^2 \tan(a+bx)\,dx =$$

$$-\frac{d^2\text{polylog}\left(3, \frac{\tan(bx+a)^2+2i\tan(bx+a)-1}{\tan(bx+a)^2+1}\right) + d^2\text{polylog}\left(3, \frac{\tan(bx+a)^2-2i\tan(bx+a)-1}{\tan(bx+a)^2+1}\right) + 2\left(i\,bd^2x + i\,bcd\right)\text{Li}_2\left($$

input
```
integrate((d*x+c)^2*tan(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(d^2*polylog(3, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)
^2 + 1)) + d^2*polylog(3, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x
 + a)^2 + 1)) + 2*(I*b*d^2*x + I*b*c*d)*dilog(2*(I*tan(b*x + a) - 1)/(tan(
b*x + a)^2 + 1) + 1) + 2*(-I*b*d^2*x - I*b*c*d)*dilog(2*(-I*tan(b*x + a) -
 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*lo
g(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 2*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)))/b^3
```

Sympy [F]

$$\int (c+dx)^2 \tan(a+bx)\,dx = \int (c+dx)^2 \tan(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**2*tan(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(81) = 162$.

Time = 0.18 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.78

$$\int (c+dx)^2 \tan(a+bx)\,dx$$

$$= \frac{6\,c^2 \log\left(\sec\left(bx+a\right)\right) - \frac{12\,acd\log(\sec(bx+a))}{b} + \frac{6\,a^2d^2\log(\sec(bx+a))}{b^2} - \frac{-2i\,(bx+a)^3d^2 - 6\,(i\,bcd - i\,ad^2)(bx+a)^2 + 3\,d^2\mathrm{Li}_3(-e^{(2i\,b}}{}$$

input
```
integrate((d*x+c)^2*tan(b*x+a),x, algorithm="maxima")
```

output
```
1/6*(6*c^2*log(sec(b*x + a)) - 12*a*c*d*log(sec(b*x + a))/b + 6*a^2*d^2*lo
g(sec(b*x + a))/b^2 - (-2*I*(b*x + a)^3*d^2 - 6*(I*b*c*d - I*a*d^2)*(b*x +
 a)^2 + 3*d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) - 6*(-I*(b*x + a)^2*d^2 + 2
*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a
) + 1) - 6*(I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*dilog(-e^(2*I*b*x + 2*I*a
)) + 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(2*b*x + 2*a
)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1))/b^2)/b
```

Giac [F]

$$\int (c+dx)^2 \tan(a+bx)\,dx = \int (dx+c)^2 \tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*tan(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \tan(a+bx)\,dx = \int \tan(a+b\,x)\,(c+d\,x)^2\,dx$$

input
```
int(tan(a + b*x)*(c + d*x)^2,x)
```

output
```
int(tan(a + b*x)*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \tan(a+bx)\,dx$$
$$= \frac{2\left(\int \tan(bx+a)\,x^2 dx\right) b\,d^2 + 4\left(\int \tan(bx+a)\,x dx\right) bcd + \log\left(\tan(bx+a)^2+1\right) c^2}{2b}$$

input
```
int((d*x+c)^2*tan(b*x+a),x)
```

output
```
(2*int(tan(a + b*x)*x**2,x)*b*d**2 + 4*int(tan(a + b*x)*x,x)*b*c*d + log(t
an(a + b*x)**2 + 1)*c**2)/(2*b)
```

3.212 $\int (c+dx)\tan(a+bx)\,dx$

Optimal result . . . 1931
Mathematica [A] (verified) . . . 1931
Rubi [A] (verified) . . . 1932
Maple [B] (verified) . . . 1934
Fricas [B] (verification not implemented) . . . 1934
Sympy [F] . . . 1935
Maxima [B] (verification not implemented) . . . 1935
Giac [F] . . . 1936
Mupad [B] (verification not implemented) . . . 1936
Reduce [F] . . . 1936

Optimal result

Integrand size = 12, antiderivative size = 66

$$\int (c+dx)\tan(a+bx)\,dx = \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}$$

output

```
1/2*I*(d*x+c)^2/d-(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b+1/2*I*d*polylog(2,-exp(
2*I*(b*x+a)))/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int (c+dx)\tan(a+bx)\,dx = \frac{1}{2}idx^2 - \frac{dx\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{c\log(\cos(a+bx))}{b} + \frac{id\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}$$

input

```
Integrate[(c + d*x)*Tan[a + b*x],x]
```

output
```
(I/2)*d*x^2 - (d*x*Log[1 + E^((2*I)*(a + b*x))])/b - (c*Log[Cos[a + b*x]])
/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\tan(a+bx)\,dx$$

$$\downarrow 3042$$

$$\int (c+dx)\tan(a+bx)dx$$

$$\downarrow 4202$$

$$\frac{i(c+dx)^2}{2d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}dx$$

$$\downarrow 2620$$

$$\frac{i(c+dx)^2}{2d} - 2i\left(\frac{id\int \log\left(1+e^{2i(a+bx)}\right)dx}{2b} - \frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

$$\downarrow 2715$$

$$\frac{i(c+dx)^2}{2d} - 2i\left(\frac{d\int e^{-2i(a+bx)}\log\left(1+e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2} - \frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

$$\downarrow 2838$$

$$\frac{i(c+dx)^2}{2d} - 2i\left(-\frac{d\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2} - \frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

input
```
Int[(c + d*x)*Tan[a + b*x],x]
```

output
```
((I/2)*(c + d*x)^2)/d - (2*I)*(((-1/2*I)*(c + d*x)*Log[1 + E^((2*I)*(a + b
*x))])/b - (d*PolyLog[2, -E^((2*I)*(a + b*x))])/(4*b^2))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(56) = 112$.

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.86

method	result
risch	$\frac{id x^2}{2} - ixc - \frac{c \ln\left(e^{2i(bx+a)}+1\right)}{b} + \frac{2c \ln\left(e^{i(bx+a)}\right)}{b} + \frac{2idax}{b} + \frac{id a^2}{b^2} - \frac{d \ln\left(e^{2i(bx+a)}+1\right)x}{b} + \frac{id \operatorname{polylog}\left(2,-e^{2i(bx+a)}\right)}{2b^2}$
parts	$\frac{\ln\left(\tan(bx+a)^2+1\right)dx}{2b} + \frac{\ln\left(\tan(bx+a)^2+1\right)c}{2b} - \frac{d\left(-\frac{i\left(\ln(\tan(bx+a)-i)\ln\left(\tan(bx+a)^2+1\right)-\operatorname{dilog}\left(-\frac{i(\tan(bx+a)+i)}{2}\right)-\ln(\tan(bx+a)\right.}{2}\right.}{2}$

input
```
int((d*x+c)*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*d*x^2-I*x*c-1/b*c*ln(exp(2*I*(b*x+a))+1)+2/b*c*ln(exp(I*(b*x+a)))+2*
I/b*d*a*x+I/b^2*d*a^2-1/b*d*ln(exp(2*I*(b*x+a))+1)*x+1/2*I*d*polylog(2,-ex
p(2*I*(b*x+a)))/b^2-2/b^2*d*a*ln(exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(53) = 106$.

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.06

$$\int (c+dx)\tan(a+bx)\,dx$$
$$= \frac{-i\,d\mathrm{Li}_2\left(\frac{2\,(i\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right) + i\,d\mathrm{Li}_2\left(\frac{2\,(-i\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right) - 2\,(bdx+bc)\log\left(-\frac{2\,(i\tan(bx+a)-1)}{\tan(bx+a)^2+1}\right) - 2\,(bc}{4\,b^2}$$

input
```
integrate((d*x+c)*tan(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(-I*d*dilog(2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + I*d*dil
og(2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) - 2*(b*d*x + b*c)*log
(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 2*(b*d*x + b*c)*log(-2*(-
I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)))/b^2
```

Sympy [F]

$$\int (c+dx)\tan(a+bx)\,dx = \int (c+dx)\tan(a+bx)\,dx$$

input
```
integrate((d*x+c)*tan(b*x+a),x)
```

output
```
Integral((c + d*x)*tan(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(53) = 106$.

Time = 0.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.74

$$\int (c+dx)\tan(a+bx)\,dx =$$
$$-\frac{-i\,b^2dx^2 - 2i\,b^2cx - 2\left(-i\,bdx - i\,bc\right)\arctan\left(\sin\left(2\,bx+2\,a\right),\cos\left(2\,bx+2\,a\right)+1\right) - i\,d\mathrm{Li}_2\left(-e^{(2i\,bx+2}\right.}{2\,b^2}$$

input
```
integrate((d*x+c)*tan(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(-I*b^2*d*x^2 - 2*I*b^2*c*x - 2*(-I*b*d*x - I*b*c)*arctan2(sin(2*b*x
+ 2*a), cos(2*b*x + 2*a) + 1) - I*d*dilog(-e^(2*I*b*x + 2*I*a)) + (b*d*x +
 b*c)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1
))/b^2
```

Giac [F]

$$\int (c+dx)\tan(a+bx)\,dx = \int (dx+c)\tan\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*tan(b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 18.72 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.24

$$\int (c+dx)\tan(a+bx)\,dx = \frac{c\,\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b}$$
$$-\frac{d\left(\pi\,\ln\left(\cos\left(b\,x\right)\right)+\mathrm{polylog}\left(2,-\mathrm{e}^{-a\,2\mathrm{i}}\,\mathrm{e}^{-b\,x\,2\mathrm{i}}\right)\,1\mathrm{i}-\pi\,\ln\left(\mathrm{e}^{-a\,2\mathrm{i}}\,\mathrm{e}^{-b\,x\,2\mathrm{i}}+1\right)+2\,a\,\ln\left(\mathrm{e}^{-a\,2\mathrm{i}}\,\mathrm{e}^{-b\,x\,2\mathrm{i}}+1\right)\right.}{2\,b^2}$$

input
```
int(tan(a + b*x)*(c + d*x),x)
```

output
```
(c*log(tan(a + b*x)^2 + 1))/(2*b) - (d*(polylog(2, -exp(-a*2i)*exp(-b*x*2i
))*1i - pi*log(exp(b*x*2i) + 1) - pi*log(exp(-a*2i)*exp(-b*x*2i) + 1) + 2*
a*log(exp(-a*2i)*exp(-b*x*2i) + 1) + pi*log(cos(b*x)) + b^2*x^2*1i - log(c
os(a + b*x))*(2*a - pi) + 2*b*x*log(exp(-a*2i)*exp(-b*x*2i) + 1) + a*b*x*2
i))/(2*b^2)
```

Reduce [F]

$$\int (c+dx)\tan(a+bx)\,dx = \frac{2\left(\int \tan\left(bx+a\right)xdx\right)bd+\log\left(\tan\left(bx+a\right)^2+1\right)c}{2b}$$

input
```
int((d*x+c)*tan(b*x+a),x)
```

output
```
(2*int(tan(a + b*x)*x,x)*b*d + log(tan(a + b*x)**2 + 1)*c)/(2*b)
```

3.213 $\int \frac{\tan(a+bx)}{c+dx}\,dx$

Optimal result 1938
Mathematica [N/A] 1938
Rubi [N/A] 1939
Maple [N/A] 1940
Fricas [N/A] 1940
Sympy [N/A] 1941
Maxima [N/A] 1941
Giac [N/A] 1942
Mupad [N/A] 1942
Reduce [N/A] 1942

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\tan(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\tan(a+bx)}{c+dx}, x\right)$$

output
```
Defer(Int)(tan(b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 2.61 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tan(a+bx)}{c+dx}\,dx = \int \frac{\tan(a+bx)}{c+dx}\,dx$$

input
```
Integrate[Tan[a + b*x]/(c + d*x),x]
```

output
```
Integrate[Tan[a + b*x]/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(a+bx)}{c+dx}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\tan(a+bx)}{c+dx}dx$$
$$\downarrow 4222$$
$$\int \frac{\tan(a+bx)}{c+dx}dx$$

input
```
Int[Tan[a + b*x]/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\tan(bx+a)}{dx+c}dx$$

input

```
int(tan(b*x+a)/(d*x+c),x)
```

output

```
int(tan(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tan(a+bx)}{c+dx}\,dx = \int \frac{\tan(bx+a)}{dx+c}\,dx$$

input

```
integrate(tan(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(tan(b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\tan(a+bx)}{c+dx}\,dx = \int \frac{\tan\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(tan(b*x+a)/(d*x+c),x)
```

output
```
Integral(tan(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tan(a+bx)}{c+dx}\,dx = \int \frac{\tan\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(tan(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(tan(b*x + a)/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tan(a+bx)}{c+dx}\,dx = \int \frac{\tan(bx+a)}{dx+c}\,dx$$

input
```
integrate(tan(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(tan(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 18.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tan(a+bx)}{c+dx}\,dx = \int \frac{\tan(a+b\,x)}{c+d\,x}\,dx$$

input
```
int(tan(a + b*x)/(c + d*x),x)
```

output
```
int(tan(a + b*x)/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tan(a+bx)}{c+dx}\,dx = \int \frac{\tan(bx+a)}{dx+c}\,dx$$

input
```
int(tan(b*x+a)/(d*x+c),x)
```

output

```
int(tan(a + b*x)/(c + d*x),x)
```

3.214 $\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx$

Optimal result 1944
Mathematica [N/A] 1944
Rubi [N/A] 1945
Maple [N/A] 1946
Fricas [N/A] 1946
Sympy [N/A] 1947
Maxima [N/A] 1947
Giac [N/A] 1948
Mupad [N/A] 1948
Reduce [N/A] 1948

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\tan(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(tan(b*x+a)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 4.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[Tan[a + b*x]/(c + d*x)^2,x]
```

output
```
Integrate[Tan[a + b*x]/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\tan(a+bx)}{(c+dx)^2}dx$$
$$\downarrow 4222$$
$$\int \frac{\tan(a+bx)}{(c+dx)^2}dx$$

input
```
Int[Tan[a + b*x]/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\tan(bx+a)}{(dx+c)^2}dx$$

input

```
int(tan(b*x+a)/(d*x+c)^2,x)
```

output

```
int(tan(b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(bx+a)}{(dx+c)^2}\,dx$$

input

```
integrate(tan(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(tan(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(tan(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(tan(a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(tan(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(tan(b*x + a)/(d*x + c)^2, x)
```

Giac [N/A]

Not integrable

Time = 3.86 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(tan(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(tan(b*x + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 18.87 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(a+b\,x)}{(c+d\,x)^2}\,dx$$

input
```
int(tan(a + b*x)/(c + d*x)^2,x)
```

output
```
int(tan(a + b*x)/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(bx+a)}{d^2x^2+2cdx+c^2}dx$$

input
```
int(tan(b*x+a)/(d*x+c)^2,x)
```

output
```
int(tan(a + b*x)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.215 $\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx$

Optimal result 1950
Mathematica [N/A] 1950
Rubi [N/A] 1951
Maple [N/A] 1953
Fricas [N/A] 1953
Sympy [N/A] 1954
Maxima [N/A] 1954
Giac [N/A] 1954
Mupad [N/A] 1955
Reduce [N/A] 1955

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx$$
$$= \frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}$$
$$-\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{2b}+\text{Int}((c+dx)^m\sec(a+bx),x)$$

output

```
1/2*I*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c
)/d)^m)-1/2*I*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*
(d*x+c)/d)^m)+Defer(Int)((d*x+c)^m*sec(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 8.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx = \int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4907, 3042, 3788, 26, 2612, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\tan(a+bx)(c+dx)^m\,dx$$

$$\downarrow 4907$$

$$\int (c+dx)^m \sec(a+bx)dx - \int (c+dx)^m \cos(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)^m \csc\left(a+bx+\frac{\pi}{2}\right)dx - \int (c+dx)^m \sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\downarrow 3788$$

$$-\frac{1}{2}i\int -ie^{-i(a+bx)}(c+dx)^m dx + \frac{1}{2}i\int ie^{i(a+bx)}(c+dx)^m dx + \int (c+dx)^m \csc\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\downarrow 26$$

$$-\frac{1}{2}\int e^{-i(a+bx)}(c+dx)^m dx - \frac{1}{2}\int e^{i(a+bx)}(c+dx)^m dx + \int (c+dx)^m \csc\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\downarrow 2612$$

$$\int (c+dx)^m \csc\left(a+bx+\frac{\pi}{2}\right)dx + \frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b} - \frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}$$

$$\downarrow 4680$$

$$\int (c+dx)^m \sec(a+bx)dx + \frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b} - \frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}$$

input
```
Int[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 4680

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4907

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \sin(bx+a) \tan(bx+a)\, dx$$

input

```
int((d*x+c)^m*sin(b*x+a)*tan(b*x+a),x)
```

output

```
int((d*x+c)^m*sin(b*x+a)*tan(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \sin(a+bx) \tan(a+bx)\, dx = \int (dx+c)^m \sin(bx+a) \tan(bx+a)\ dx$$

input

```
integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

output

```
integral((d*x + c)^m*sin(b*x + a)*tan(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 13.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx = \int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*sin(b*x+a)*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**m*sin(a + b*x)*tan(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sin(bx+a)\tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*sin(b*x + a)*tan(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sin(bx+a)\tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sin(b*x + a)*tan(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 17.92 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \sin(a+bx) \tan(a+bx)\,dx = \int \sin(a+b\,x)\,\tan(a+b\,x)\,(c+d\,x)^m\,dx$$

input
```
int(sin(a + b*x)*tan(a + b*x)*(c + d*x)^m,x)
```

output
```
int(sin(a + b*x)*tan(a + b*x)*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \sin(a+bx) \tan(a+bx)\,dx = \int (dx+c)^m \sin(bx+a) \tan(bx+a)\,dx$$

input
```
int((d*x+c)^m*sin(b*x+a)*tan(b*x+a),x)
```

output
```
int((c + d*x)**m*sin(a + b*x)*tan(a + b*x),x)
```

3.216 $\int (c+dx)^3 \sin(a+bx) \tan(a+bx)\, dx$

Optimal result . . . 1956
Mathematica [B] (verified) . . . 1957
Rubi [A] (verified) . . . 1957
Maple [B] (verified) . . . 1963
Fricas [B] (verification not implemented) . . . 1964
Sympy [F] . . . 1965
Maxima [B] (verification not implemented) . . . 1965
Giac [F] . . . 1966
Mupad [F(-1)] . . . 1967
Reduce [F] . . . 1967

Optimal result

Integrand size = 20, antiderivative size = 275

$$
\begin{aligned}
&\int (c+dx)^3 \sin(a+bx) \tan(a+bx)\, dx \\
&= -\frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b} + \frac{6d^3 \cos(a+bx)}{b^4} - \frac{3d(c+dx)^2 \cos(a+bx)}{b^2} \\
&\quad + \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^2} - \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{6d^2(c+dx) \operatorname{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^3} + \frac{6d^2(c+dx) \operatorname{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^3} \\
&\quad - \frac{6id^3 \operatorname{PolyLog}\left(4, -ie^{i(a+bx)}\right)}{b^4} + \frac{6id^3 \operatorname{PolyLog}\left(4, ie^{i(a+bx)}\right)}{b^4} \\
&\quad + \frac{6d^2(c+dx) \sin(a+bx)}{b^3} - \frac{(c+dx)^3 \sin(a+bx)}{b}
\end{aligned}
$$

output

```
-2*I*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b+6*d^3*cos(b*x+a)/b^4-3*d*(d*x+c)^2
*cos(b*x+a)/b^2+3*I*d*(d*x+c)^2*polylog(2,-I*exp(I*(b*x+a)))/b^2-3*I*d*(d*
x+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)*polylog(3,-I*exp(I*(b
*x+a)))/b^3+6*d^2*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^3-6*I*d^3*polylog(
4,-I*exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4+6*d^2*(d*
x+c)*sin(b*x+a)/b^3-(d*x+c)^3*sin(b*x+a)/b
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 557 vs. $2(275) = 550$.

Time = 0.96 (sec) , antiderivative size = 557, normalized size of antiderivative = 2.03

$$\int (c+dx)^3 \sin(a+bx)\tan(a+bx)\,dx =$$
$$-\frac{2ib^3c^3\arctan\left(e^{i(a+bx)}\right) + 3b^2c^2d\cos(a+bx) - 6d^3\cos(a+bx) + 6b^2cd^2x\cos(a+bx) + 3b^2d^3x^2\cos(a}{}$$

input
```
Integrate[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x],x]
```

output
```
-(((2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^2*c^2*d*Cos[a + b*x] - 6*d^
3*Cos[a + b*x] + 6*b^2*c*d^2*x*Cos[a + b*x] + 3*b^2*d^3*x^2*Cos[a + b*x] -
 3*b^3*c^2*d*x*Log[1 - I*E^(I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 - I*E^(I
*(a + b*x))] - b^3*d^3*x^3*Log[1 - I*E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[
1 + I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + b^3*
d^3*x^3*Log[1 + I*E^(I*(a + b*x))] - (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-
I)*E^(I*(a + b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))
] + 6*b*c*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, (-I)
*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, I*E^(I*(a + b*x))] - 6*b*d^3*x*Po
lyLog[3, I*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] -
 (6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))] + b^3*c^3*Sin[a + b*x] - 6*b*c*d^
2*Sin[a + b*x] + 3*b^3*c^2*d*x*Sin[a + b*x] - 6*b*d^3*x*Sin[a + b*x] + 3*b
^3*c*d^2*x^2*Sin[a + b*x] + b^3*d^3*x^3*Sin[a + b*x])/b^4)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.09, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {4907, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118, 4669, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin(a+bx)\tan(a+bx)\,dx$$

$$\downarrow 4907$$

$$\int (c+dx)^3 \sec(a+bx)dx - \int (c+dx)^3 \cos(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right)dx - \int (c+dx)^3 \sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\downarrow 3777$$

$$-\frac{3d\int -(c+dx)^2\sin(a+bx)dx}{b} + \int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right)dx - \frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\downarrow 25$$

$$\frac{3d\int (c+dx)^2\sin(a+bx)dx}{b} + \int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right)dx - \frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\downarrow 3042$$

$$\frac{3d\int (c+dx)^2\sin(a+bx)dx}{b} + \int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right)dx - \frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\downarrow 3777$$

$$\frac{3d\left(\frac{2d\int (c+dx)\cos(a+bx)dx}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b} + \int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right)dx - \frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\downarrow 3042$$

$$\int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right)dx + \frac{3d\left(\frac{2d\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b} - \frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\downarrow 3777$$

$$\int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right)dx + \frac{3d\left(\frac{2d\left(\frac{d\int -\sin(a+bx)dx}{b} + \frac{(c+dx)\sin(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b} - \frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\downarrow 25$$

$$\int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right) dx + \frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b} - \frac{d\int \sin(a+bx)dx}{b}\right)}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b} - \frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\downarrow 3042$$

$$\int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right) dx + \frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b} - \frac{d\int \sin(a+bx)dx}{b}\right)}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b} - \frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\downarrow 3118$$

$$\int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right) dx + \frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2} + \frac{(c+dx)\sin(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b} - \frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\downarrow 4669$$

$$-\frac{3d\int (c+dx)^2 \log\left(1-ie^{i(a+bx)}\right) dx}{b} + \frac{3d\int (c+dx)^2 \log\left(1+ie^{i(a+bx)}\right) dx}{b} - \frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b} + \frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2} + \frac{(c+dx)\sin(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b} - \frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\downarrow 3011$$

$$\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b} - \frac{2id\int (c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b} - \frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b} - \frac{2id\int (c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)dx}{b}\right)}{b} - \frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b} + \frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2} + \frac{(c+dx)\sin(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b} - \frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\downarrow 7163$$

$$\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int \mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-$$
$$\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int \mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-$$
$$\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-$$
$$\frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\downarrow 2720$$

$$\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-$$
$$\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-$$
$$\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-$$
$$\frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\downarrow 7143$$

$$-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\operatorname{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\operatorname{PolyLog}\left(4,ie^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}$$

input
```
Int[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x],x]
```

output
```
((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + (3*d*((I*(c + d*x)^2*Poly
Log[2, (-I)*E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c + d*x)*PolyLog[3, (-I
)*E^(I*(a + b*x))])/b + (d*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^2))/b))/b -
 (3*d*((I*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(
c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b + (d*PolyLog[4, I*E^(I*(a + b*x)
)])/b^2))/b))/b - ((c + d*x)^3*Sin[a + b*x])/b + (3*d*(-(((c + d*x)^2*Cos[
a + b*x])/b) + (2*d*((d*Cos[a + b*x])/b^2 + ((c + d*x)*Sin[a + b*x])/b))/b
))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))  Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)  Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)  Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)  Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(250) = 500$.

Time = 1.12 (sec) , antiderivative size = 901, normalized size of antiderivative = 3.28

method	result	size
risch	Expression too large to display	901

input

```
int((d*x+c)^3*sin(b*x+a)*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
6*I/b^2*c^2*d*a*arctan(exp(I*(b*x+a)))-6*I/b^3*c*d^2*a^2*arctan(exp(I*(b*x
+a)))+6*I/b^2*d^2*c*polylog(2,-I*exp(I*(b*x+a)))*x-6*I/b^2*d^2*c*polylog(2
,I*exp(I*(b*x+a)))*x+1/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*
c^3+3*I*b^2*d^3*x^2-6*b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*
d^3)/b^4*exp(I*(b*x+a))+1/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3-1/b*d^3*ln(I*ex
p(I*(b*x+a))+1)*x^3-1/b^4*a^3*d^3*ln(I*exp(I*(b*x+a))+1)-6/b^3*d^2*c*polyl
og(3,-I*exp(I*(b*x+a)))+1/b^4*a^3*d^3*ln(1-I*exp(I*(b*x+a)))+6/b^3*d^2*c*p
olylog(3,I*exp(I*(b*x+a)))-6/b^3*d^3*polylog(3,-I*exp(I*(b*x+a)))*x+6/b^3*
d^3*polylog(3,I*exp(I*(b*x+a)))*x-2*I/b*c^3*arctan(exp(I*(b*x+a)))-1/2*I*(
d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d^3*
x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))+3*I
/b^2*c^2*d*polylog(2,-I*exp(I*(b*x+a)))-3*I/b^2*d^3*polylog(2,I*exp(I*(b*x
+a)))*x^2+3*I/b^2*d^3*polylog(2,-I*exp(I*(b*x+a)))*x^2+2*I/b^4*d^3*a^3*arc
tan(exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,I*exp(I*(b*x+a)))+3/b^3*a^2*c*
d^2*ln(I*exp(I*(b*x+a))+1)+3/b*c^2*d*ln(1-I*exp(I*(b*x+a)))*x+3/b^2*c^2*d*
ln(1-I*exp(I*(b*x+a)))*a-3/b^3*a^2*c*d^2*ln(1-I*exp(I*(b*x+a)))-3/b*c^2*d*
ln(I*exp(I*(b*x+a))+1)*x-3/b^2*c^2*d*ln(I*exp(I*(b*x+a))+1)*a-3/b*d^2*c*ln
(I*exp(I*(b*x+a))+1)*x^2+3/b*d^2*c*ln(1-I*exp(I*(b*x+a)))*x^2-6*I*d^3*poly
log(4,-I*exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1075 vs. $2(237) = 474$.

Time = 0.14 (sec) , antiderivative size = 1075, normalized size of antiderivative = 3.91

$$\int (c+dx)^3 \sin(a+bx) \tan(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

output

```
1/2*(6*I*d^3*polylog(4, I*cos(b*x + a) + sin(b*x + a)) + 6*I*d^3*polylog(4
, I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) + si
n(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 6*(b^2*
d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a) - 3*(I*b^2*d^3*x
^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(I*cos(b*x + a) + sin(b*x + a)) -
 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(I*cos(b*x + a) -
sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-
I*cos(b*x + a) + sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b
^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b^3*c^3 - 3*a*b^2*c^2*d
 + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*sin(b*
x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b^
3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^
2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3
*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(
-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b
^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a)
- sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
log(-cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*...
```

Sympy [F]

$$\int (c+dx)^3 \sin(a+bx) \tan(a+bx)\, dx = \int (c+dx)^3 \sin(a+bx) \tan(a+bx)\, dx$$

input
```
integrate((d*x+c)**3*sin(b*x+a)*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**3*sin(a + b*x)*tan(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 934 vs. $2(237) = 474$.

Time = 0.26 (sec) , antiderivative size = 934, normalized size of antiderivative = 3.40

$$\int (c+dx)^3 \sin(a+bx) \tan(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

output

```
1/2*(c^3*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))
- 3*a*c^2*d*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a
))/b + 3*a^2*c*d^2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(
b*x + a))/b^2 - a^3*d^3*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2
*sin(b*x + a))/b^3 + (12*I*d^3*polylog(4, I*e^(I*b*x + I*a)) - 12*I*d^3*po
lylog(4, -I*e^(I*b*x + I*a)) - 2*(I*(b*x + a)^3*d^3 + 3*(I*b*c*d^2 - I*a*d
^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*(b*x + a))*a
rctan2(cos(b*x + a), sin(b*x + a) + 1) - 2*(I*(b*x + a)^3*d^3 + 3*(I*b*c*d
^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*(b
*x + a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 6*(b^2*c^2*d - 2*a*b*c
*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*co
s(b*x + a) - 6*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^
3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*dilog(I*e^(I*b*x + I*a)) - 6*(-I*b^
2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 +
I*a*d^3)*(b*x + a))*dilog(-I*e^(I*b*x + I*a)) + ((b*x + a)^3*d^3 + 3*(b*c*
d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a)
)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - ((b*x + a)^3
*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1)
+ 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, I*e^(I*b*x + I*a)) - ...
```

Giac [F]

$$\int (c+dx)^3 \sin(a+bx) \tan(a+bx)\,dx = \int (dx+c)^3 \sin\left(bx+a\right) \tan\left(bx+a\right)\,dx$$

input

```
integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

output

```
integrate((d*x + c)^3*sin(b*x + a)*tan(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \sin(a+bx) \tan(a+bx)\,dx = \int \sin(a+b\,x)\,\tan(a+b\,x)\,(c+d\,x)^3\,dx$$

input
```
int(sin(a + b*x)*tan(a + b*x)*(c + d*x)^3,x)
```

output
```
int(sin(a + b*x)*tan(a + b*x)*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \sin(a+bx) \tan(a+bx)\,dx$$
$$= \frac{\left(\int \sin(bx+a)\tan(bx+a)\,x^3dx\right) b\,d^3 + 3\left(\int \sin(bx+a)\tan(bx+a)\,x^2dx\right) bc\,d^2 + 3\left(\int \sin(bx+a)\tan}{b}$$

input
```
int((d*x+c)^3*sin(b*x+a)*tan(b*x+a),x)
```

output
```
(int(sin(a + b*x)*tan(a + b*x)*x**3,x)*b*d**3 + 3*int(sin(a + b*x)*tan(a +
 b*x)*x**2,x)*b*c*d**2 + 3*int(sin(a + b*x)*tan(a + b*x)*x,x)*b*c**2*d - l
og(tan((a + b*x)/2) - 1)*c**3 + log(tan((a + b*x)/2) + 1)*c**3 - sin(a + b
*x)*c**3)/b
```

3.217 $\int (c+dx)^2 \sin(a+bx) \tan(a+bx)\, dx$

Optimal result . . . 1968
Mathematica [A] (verified) . . . 1969
Rubi [A] (verified) . . . 1969
Maple [B] (verified) . . . 1973
Fricas [B] (verification not implemented) . . . 1973
Sympy [F] . . . 1974
Maxima [B] (verification not implemented) . . . 1974
Giac [F] . . . 1975
Mupad [F(-1)] . . . 1975
Reduce [F] . . . 1976

Optimal result

Integrand size = 20, antiderivative size = 186

$$\begin{aligned}\int (c+dx)^2 \sin(a+bx) \tan(a+bx)\, dx = &-\frac{2i(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b} \\ &- \frac{2d(c+dx)\cos(a+bx)}{b^2} \\ &+ \frac{2id(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} \\ &- \frac{2id(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} \\ &- \frac{2d^2\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} \\ &+ \frac{2d^2\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} \\ &+ \frac{2d^2\sin(a+bx)}{b^3} - \frac{(c+dx)^2\sin(a+bx)}{b}\end{aligned}$$

output

```
-2*I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b-2*d*(d*x+c)*cos(b*x+a)/b^2+2*I*d*(
d*x+c)*polylog(2,-I*exp(I*(b*x+a)))/b^2-2*I*d*(d*x+c)*polylog(2,I*exp(I*(b
*x+a)))/b^2-2*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,I*exp(I
*(b*x+a)))/b^3+2*d^2*sin(b*x+a)/b^3-(d*x+c)^2*sin(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.69

$$\int (c+dx)^2 \sin(a+bx)\tan(a+bx)\,dx =$$

$$-\frac{2ib^2c^2\arctan\left(e^{i(a+bx)}\right)+2bcd\cos(a+bx)+2bd^2x\cos(a+bx)-2b^2cdx\log\left(1-ie^{i(a+bx)}\right)-b^2d^2x^2\,\text{lo}}{}$$

input

```
Integrate[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x],x]
```

output

```
-(((2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b*c*d*Cos[a + b*x] + 2*b*d^2*
x*Cos[a + b*x] - 2*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[
1 - I*E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x))] + b^2*d^2*
x^2*Log[1 + I*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*
(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] + 2*d^2*Po
lyLog[3, (-I)*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, I*E^(I*(a + b*x))] + b^2
*c^2*Sin[a + b*x] - 2*d^2*Sin[a + b*x] + 2*b^2*c*d*x*Sin[a + b*x] + b^2*d^
2*x^2*Sin[a + b*x])/b^3)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4907, 3042, 3777, 25, 3042, 3777, 3042, 3117, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin(a+bx)\tan(a+bx)\,dx$$

$$\downarrow 4907$$

$$\int (c+dx)^2 \sec(a+bx)dx - \int (c+dx)^2\cos(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)^2 \csc\left(a+bx+\frac{\pi}{2}\right)dx - \int (c+dx)^2 \sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\downarrow 3777$$

$$-\frac{2d\int -((c+dx)\sin(a+bx))dx}{b}+\int (c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx-\frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\downarrow 25$$

$$\frac{2d\int (c+dx)\sin(a+bx)dx}{b}+\int (c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx-\frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\downarrow 3042$$

$$\frac{2d\int (c+dx)\sin(a+bx)dx}{b}+\int (c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx-\frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\downarrow 3777$$

$$\frac{2d\left(\frac{d\int \cos(a+bx)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}+\int (c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx-\frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\downarrow 3042$$

$$\int (c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2d\left(\frac{d\int \sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\downarrow 3117$$

$$\int (c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\downarrow 4669$$

$$-\frac{2d\int (c+dx)\log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{2d\int (c+dx)\log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\downarrow 3011$$

$$\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{id\int \mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{id\int \mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\downarrow 2720$$

$$\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-$$
$$\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-$$
$$\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\downarrow 7143$$

$$-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\,\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^2}\right)}{b}-$$
$$\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{d\,\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^2}\right)}{b}+\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}-$$
$$\frac{(c+dx)^2\sin(a+bx)}{b}$$

input
```
Int[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x],x]
```

output
```
((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (2*d*((I*(c + d*x)*PolyLo
g[2, (-I)*E^(I*(a + b*x))])/b - (d*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^2))
/b - (2*d*((I*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b - (d*PolyLog[3, I
*E^(I*(a + b*x))])/b^2))/b - ((c + d*x)^2*Sin[a + b*x])/b + (2*d*(-(((c +
d*x)*Cos[a + b*x])/b) + (d*Sin[a + b*x])/b^2))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(169) = 338.

Time = 0.67 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.75

method	result
risch	$\frac{i\left(x^2d^2b^2+2b^2cdx+2ib\,d^2x+b^2c^2+2ibcd-2d^2\right)\mathrm{e}^{i(bx+a)}}{2b^3}-\frac{2id^2a^2\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b^3}+\frac{2id^2\,\mathrm{polylog}\left(2,-i\mathrm{e}^{i(bx+a)}\right)x}{b^2}-\frac{2cd\ln(ie$

input
```
int((d*x+c)^2*sin(b*x+a)*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*ex
p(I*(b*x+a))-2*I/b^3*d^2*a^2*arctan(exp(I*(b*x+a)))+2*I/b^2*d^2*polylog(2,
-I*exp(I*(b*x+a)))*x-2/b^2*c*d*ln(I*exp(I*(b*x+a))+1)*a+2/b*c*d*ln(1-I*exp
(I*(b*x+a)))*x+2*I/b^2*c*d*polylog(2,-I*exp(I*(b*x+a)))-2*I/b^2*c*d*polylo
g(2,I*exp(I*(b*x+a)))+1/b^3*a^2*d^2*ln(I*exp(I*(b*x+a))+1)+2*d^2*polylog(3
,I*exp(I*(b*x+a)))/b^3-2*I/b^2*d^2*polylog(2,I*exp(I*(b*x+a)))*x-2*I/b*c^2
*arctan(exp(I*(b*x+a)))-1/2*I*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x
-2*d^2-2*I*b*c*d)/b^3*exp(-I*(b*x+a))+4*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))
-2*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+1/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2
+2/b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a-1/b*d^2*ln(I*exp(I*(b*x+a))+1)*x^2-1/b
^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)))-2/b*c*d*ln(I*exp(I*(b*x+a))+1)*x
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 656 vs. 2(160) = 320.

Time = 0.12 (sec) , antiderivative size = 656, normalized size of antiderivative = 3.53

$$\int (c+dx)^2 \sin(a+bx)\tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*polylog(3, I
*cos(b*x + a) - sin(b*x + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) + sin(b*x
 + a)) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 4*(b*d^2*x + b
*c*d)*cos(b*x + a) + 2*(I*b*d^2*x + I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*
x + a)) + 2*(I*b*d^2*x + I*b*c*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) + 2
*(-I*b*d^2*x - I*b*c*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 2*(-I*b*d^
2*x - I*b*c*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^2*c^2 - 2*a*b*c*
d + a^2*d^2)*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*
c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2
*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) - sin(b*x
 + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(
b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^
2*d^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 2*(b^2*d^2*x^2 + 2*b^2*c*
d*x + b^2*c^2 - 2*d^2)*sin(b*x + a))/b^3
```

Sympy [F]

$$\int (c+dx)^2 \sin(a+bx) \tan(a+bx)\,dx = \int (c+dx)^2 \sin(a+bx) \tan(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*sin(b*x+a)*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**2*sin(a + b*x)*tan(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(160) = 320$.

Time = 0.20 (sec) , antiderivative size = 516, normalized size of antiderivative = 2.77

$$\int (c+dx)^2 \sin(a+bx) \tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(c^2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))
- 2*a*c*d*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))
/b + a^2*d^2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x +
a))/b^2 + (4*d^2*polylog(3, I*e^(I*b*x + I*a)) - 4*d^2*polylog(3, -I*e^(I*
b*x + I*a)) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*arct
an2(cos(b*x + a), sin(b*x + a) + 1) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d -
I*a*d^2)*(b*x + a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 4*(b*c*d +
(b*x + a)*d^2 - a*d^2)*cos(b*x + a) - 4*(I*b*c*d + I*(b*x + a)*d^2 - I*a*d
^2)*dilog(I*e^(I*b*x + I*a)) - 4*(-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*di
log(-I*e^(I*b*x + I*a)) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*
log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - ((b*x + a)^2*d
^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*
sin(b*x + a) + 1) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d
^2)*sin(b*x + a))/b^2)/b
```

Giac [F]

$$\int (c+dx)^2 \sin(a+bx) \tan(a+bx)\,dx = \int (dx+c)^2 \sin\left(bx+a\right) \tan\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*sin(b*x + a)*tan(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \sin(a+bx) \tan(a+bx)\,dx = \int \sin\left(a+b\,x\right) \tan(a+b\,x)\,\left(c+d\,x\right)^2\,dx$$

input
```
int(sin(a + b*x)*tan(a + b*x)*(c + d*x)^2,x)
```

output
```
int(sin(a + b*x)*tan(a + b*x)*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \sin(a+bx)\tan(a+bx)\,dx$$
$$= \frac{\left(\int \sin(bx+a)\tan(bx+a)\,x^2 dx\right) b\,d^2 + 2\left(\int \sin(bx+a)\tan(bx+a)\,x dx\right) bcd - \log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)$$
$$b$$

input
```
int((d*x+c)^2*sin(b*x+a)*tan(b*x+a),x)
```

output
```
(int(sin(a + b*x)*tan(a + b*x)*x**2,x)*b*d**2 + 2*int(sin(a + b*x)*tan(a +
 b*x)*x,x)*b*c*d - log(tan((a + b*x)/2) - 1)*c**2 + log(tan((a + b*x)/2) +
 1)*c**2 - sin(a + b*x)*c**2)/b
```

3.218 $\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx$

Optimal result 1977
Mathematica [B] (verified) 1977
Rubi [A] (verified) 1978
Maple [A] (verified) 1981
Fricas [B] (verification not implemented) 1981
Sympy [F] 1982
Maxima [F] 1982
Giac [F] 1983
Mupad [F(-1)] 1983
Reduce [F] 1983

Optimal result

Integrand size = 18, antiderivative size = 103

$$\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx = -\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)}{b^2} + \frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} - \frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)\sin(a+bx)}{b}$$

output

```
-2*I*(d*x+c)*arctan(exp(I*(b*x+a)))/b-d*cos(b*x+a)/b^2+I*d*polylog(2,-I*ex
p(I*(b*x+a)))/b^2-I*d*polylog(2,I*exp(I*(b*x+a)))/b^2-(d*x+c)*sin(b*x+a)/b
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 213 vs. 2(103) = 206.

Time = 0.31 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.07

$$\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx = \frac{c\operatorname{arctanh}(\sin(a+bx))}{b} + \frac{d\left(\left(-a+\frac{\pi}{2}-bx\right)\left(\log\left(1-e^{i\left(-a+\frac{\pi}{2}-bx\right)}\right)-\log\left(1+e^{i\left(-a+\frac{\pi}{2}-bx\right)}\right)\right)-\left(-a+\frac{\pi}{2}\right)\log\left(\tan\left(\frac{1}{2}\left(-a+\frac{\pi}{2}-\right.\right.\right.\right.}{b^2}$$

$$-\frac{d\cos(bx)(\cos(a)+bx\sin(a))}{b^2}-\frac{d(bx\cos(a)-\sin(a))\sin(bx)}{b^2}-\frac{c\sin(a+bx)}{b}$$

input
```
Integrate[(c + d*x)*Sin[a + b*x]*Tan[a + b*x],x]
```

output
```
(c*ArcTanh[Sin[a + b*x]])/b + (d*((-a + Pi/2 - b*x)*(Log[1 - E^(I*(-a + Pi
/2 - b*x))] - Log[1 + E^(I*(-a + Pi/2 - b*x))]) - (-a + Pi/2)*Log[Tan[(-a
+ Pi/2 - b*x)/2]] + I*(PolyLog[2, -E^(I*(-a + Pi/2 - b*x))] - PolyLog[2, E
^(I*(-a + Pi/2 - b*x))])))/b^2 - (d*Cos[b*x]*(Cos[a] + b*x*Sin[a]))/b^2 -
(d*(b*x*Cos[a] - Sin[a])*Sin[b*x])/b^2 - (c*Sin[a + b*x])/b
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4907, 3042, 3777, 25, 3042, 3118, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx$$

$$\downarrow 4907$$

$$\int (c+dx)\sec(a+bx)dx - \int (c+dx)\cos(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)dx - \int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\downarrow 3777$$

$$\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)dx - \frac{d\int -\sin(a+bx)dx}{b} - \frac{(c+dx)\sin(a+bx)}{b}$$

$$\downarrow 25$$

$$\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)dx + \frac{d\int \sin(a+bx)dx}{b} - \frac{(c+dx)\sin(a+bx)}{b}$$

$$\downarrow 3042$$

$$\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)dx + \frac{d\int \sin(a+bx)dx}{b} - \frac{(c+dx)\sin(a+bx)}{b}$$

$$\downarrow 3118$$

$$\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)dx - \frac{d\cos(a+bx)}{b^2} - \frac{(c+dx)\sin(a+bx)}{b}$$

$$\downarrow 4669$$

$$-\frac{d\int \log\left(1-ie^{i(a+bx)}\right)dx}{b} + \frac{d\int \log\left(1+ie^{i(a+bx)}\right)dx}{b} - \frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)}{b^2} - \frac{(c+dx)\sin(a+bx)}{b}$$

$$\downarrow 2715$$

$$\frac{id\int e^{-i(a+bx)}\log\left(1-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2} - \frac{id\int e^{-i(a+bx)}\log\left(1+ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2} - \frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)}{b^2} - \frac{(c+dx)\sin(a+bx)}{b}$$

$$\downarrow 2838$$

$$-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b} + \frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} - \frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} - \frac{d\cos(a+bx)}{b^2} - \frac{(c+dx)\sin(a+bx)}{b}$$

input

```
Int[(c + d*x)*Sin[a + b*x]*Tan[a + b*x],x]
```

output

```
((-2*I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b - (d*Cos[a + b*x])/b^2 + (I*d
*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))
])/b^2 - ((c + d*x)*Sin[a + b*x])/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.75

method	result
default	$\frac{da\sin(bx+a)}{b}-\sin(bx+a)c-\frac{d(\cos(bx+a)+(bx+a)\sin(bx+a))}{b}+\frac{-\frac{da\ln(\sec(bx+a)+\tan(bx+a))}{b}+c\ln(\sec(bx+a)+\tan(bx+a))+\frac{d\left(-(bx+$
risch	$\frac{i(bdx+cb+id)e^{i(bx+a)}}{2b^2}-\frac{i(bdx+cb-id)e^{-i(bx+a)}}{2b^2}-\frac{2ic\arctan\left(e^{i(bx+a)}\right)}{b}-\frac{d\ln\left(ie^{i(bx+a)}+1\right)x}{b}-\frac{d\ln\left(ie^{i(bx+a)}+1\right)a}{b^2}+\frac{d$

input
```
int((d*x+c)*sin(b*x+a)*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/b*d*a*sin(b*x+a)-sin(b*x+a)*c-1/b*d*(cos(b*x+a)+(b*x+a)*sin(b*x+a))
)+1/b*(-1/b*d*a*ln(sec(b*x+a)+tan(b*x+a))+c*ln(sec(b*x+a)+tan(b*x+a))+1/b*
d*(-(b*x+a)*ln(I*exp(I*(b*x+a))+1)+(b*x+a)*ln(1-I*exp(I*(b*x+a)))+I*dilog(
I*exp(I*(b*x+a))+1)-I*dilog(1-I*exp(I*(b*x+a)))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(88) = 176$.

Time = 0.11 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.21

$$\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx =$$
$$-\frac{2\,d\cos(bx+a)+i\,d\mathrm{Li}_2(i\,\cos(bx+a)+\sin(bx+a))+i\,d\mathrm{Li}_2(i\,\cos(bx+a)-\sin(bx+a))-i\,d\mathrm{Li}_2(}{}$$

input
```
integrate((d*x+c)*sin(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*d*cos(b*x + a) + I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*di
log(I*cos(b*x + a) - sin(b*x + a)) - I*d*dilog(-I*cos(b*x + a) + sin(b*x +
 a)) - I*d*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b*c - a*d)*log(cos(b*x
 + a) + I*sin(b*x + a) + I) + (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a
) + I) - (b*d*x + a*d)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x + a
*d)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b*d*x + a*d)*log(-I*cos(b*x
+ a) + sin(b*x + a) + 1) + (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a
) + 1) - (b*c - a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)
*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 2*(b*d*x + b*c)*sin(b*x + a))/b
^2
```

Sympy [F]

$$\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx = \int (c+dx)\sin(a+bx)\tan(a+bx)\,dx$$

input
```
integrate((d*x+c)*sin(b*x+a)*tan(b*x+a),x)
```

output
```
Integral((c + d*x)*sin(a + b*x)*tan(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx = \int (dx+c)\sin(bx+a)\tan(bx+a)\,dx$$

input
```
integrate((d*x+c)*sin(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(4*b^2*d*integrate((x*cos(2*b*x + 2*a)*cos(b*x + a) + x*sin(2*b*x + 2*
a)*sin(b*x + a) + x*cos(b*x + a))/(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
 + 2*cos(2*b*x + 2*a) + 1), x) + b*c*log(cos(b*x + a)^2 + sin(b*x + a)^2 +
 2*sin(b*x + a) + 1) - b*c*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x
 + a) + 1) - 2*d*cos(b*x + a) - 2*(b*d*x + b*c)*sin(b*x + a))/b^2
```

Giac [F]

$$\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx = \int (dx+c)\sin(bx+a)\tan(bx+a)\,dx$$

input
```
integrate((d*x+c)*sin(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*sin(b*x + a)*tan(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx = \int \sin(a+b\,x)\,\tan(a+b\,x)\,(c+d\,x)\,dx$$

input
```
int(sin(a + b*x)*tan(a + b*x)*(c + d*x),x)
```

output
```
int(sin(a + b*x)*tan(a + b*x)*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx$$
$$= \frac{\left(\int \sin(bx+a)\tan(bx+a)\,x dx\right) bd - \log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)c + \log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)c - \sin(bx+a)c}{b}$$

input
```
int((d*x+c)*sin(b*x+a)*tan(b*x+a),x)
```

output
```
(int(sin(a + b*x)*tan(a + b*x)*x,x)*b*d - log(tan((a + b*x)/2) - 1)*c + lo
g(tan((a + b*x)/2) + 1)*c - sin(a + b*x)*c)/b
```

3.219 $\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx$

Optimal result 1984
Mathematica [N/A] 1984
Rubi [N/A] 1985
Maple [N/A] 1987
Fricas [N/A] 1987
Sympy [N/A] 1988
Maxima [N/A] 1988
Giac [N/A] 1989
Mupad [N/A] 1989
Reduce [N/A] 1989

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx = -\frac{\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d} + \frac{\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{d} + \operatorname{Int}\left(\frac{\sec(a+bx)}{c+dx},x\right)$$

output
```
-cos(a-b*c/d)*Ci(b*c/d+b*x)/d+sin(a-b*c/d)*Si(b*c/d+b*x)/d+Defer(Int)(sec(
b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 4.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x),x]
```

output
```
Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4907, 3042, 3784, 3042, 3780, 3783, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx$$

$$\downarrow 4907$$

$$\int \frac{\sec(a+bx)}{c+dx}dx - \int \frac{\cos(a+bx)}{c+dx}dx$$

$$\downarrow 3042$$

$$\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx - \int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx$$

$$\downarrow 3784$$

$$\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx - \cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx + \int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx$$

$$\downarrow 3042$$

$$\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx + \int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx - \cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx$$

$$\downarrow 3780$$

$$\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx - \cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx + \frac{\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

$$\downarrow 3783$$

$$\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx - \frac{\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d} + \frac{\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

$$\downarrow 4680$$

$$\int \frac{\sec(a+bx)}{c+dx}dx - \frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d} + \frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

input
```
Int[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4907

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin(bx+a)\tan(bx+a)}{dx+c}dx$$

input

```
int(sin(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

output

```
int(sin(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sin(bx+a)\tan(bx+a)}{dx+c}\,dx$$

input

```
integrate(sin(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output

```
integral(sin(b*x + a)*tan(b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)*tan(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 210, normalized size of antiderivative = 10.50

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sin(bx+a)\tan(bx+a)}{dx+c}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*((exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x
 + I*b*c)/d))*cos(-(b*c - a*d)/d) + 4*d*integrate((cos(2*b*x + 2*a)*cos(b*
x + a) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/((d*x + c)*cos(2*b*
x + 2*a)^2 + (d*x + c)*sin(2*b*x + 2*a)^2 + d*x + 2*(d*x + c)*cos(2*b*x +
2*a) + c), x) + (-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*exp_integra
l_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))/d
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sin(bx+a)\tan(bx+a)}{dx+c}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)*tan(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 18.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sin(a+b\,x)\,\tan(a+b\,x)}{c+d\,x}\,dx$$

input
```
int((sin(a + b*x)*tan(a + b*x))/(c + d*x),x)
```

output
```
int((sin(a + b*x)*tan(a + b*x))/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sin(bx+a)\tan(bx+a)}{dx+c}dx$$

input
```
int(sin(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

output
```
int((sin(a + b*x)*tan(a + b*x))/(c + d*x),x)
```

3.220 $\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

Optimal result 1991
Mathematica [N/A] 1991
Rubi [N/A] 1992
Maple [N/A] 1995
Fricas [N/A] 1995
Sympy [N/A] 1996
Maxima [N/A] 1996
Giac [N/A] 1997
Mupad [N/A] 1997
Reduce [N/A] 1997

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \frac{\cos(a+bx)}{d(c+dx)} + \frac{b\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^2} + \frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2} + \text{Int}\left(\frac{\sec(a+bx)}{(c+dx)^2},x\right)$$

output

```
cos(b*x+a)/d/(d*x+c)+b*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^2+b*cos(a-b*c/d)*Si(b*
c/d+b*x)/d^2+Defer(Int)(sec(b*x+a)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 4.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

input

```
Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4907, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

$$\downarrow 4907$$

$$\int \frac{\sec(a+bx)}{(c+dx)^2}dx - \int \frac{\cos(a+bx)}{(c+dx)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx - \int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx$$

$$\downarrow 3778$$

$$-\frac{b\int -\frac{\sin(a+bx)}{c+dx}dx}{d} + \int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx + \frac{\cos(a+bx)}{d(c+dx)}$$

$$\downarrow 25$$

$$\frac{b\int \frac{\sin(a+bx)}{c+dx}dx}{d} + \int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx + \frac{\cos(a+bx)}{d(c+dx)}$$

$$\downarrow 3042$$

$$\frac{b\int \frac{\sin(a+bx)}{c+dx}dx}{d} + \int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx + \frac{\cos(a+bx)}{d(c+dx)}$$

$$\downarrow 3784$$

$$\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx + \frac{b\left(\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx + \cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d} + \frac{\cos(a+bx)}{d(c+dx)}$$

$$\downarrow 3042$$

$$\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx + \frac{b\left(\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx + \cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d} + \frac{\cos(a+bx)}{d(c+dx)}$$

$$\downarrow 3780$$

$$\frac{b\left(\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx + \frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d} + \int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx + \frac{\cos(a+bx)}{d(c+dx)}$$

$$\downarrow 3783$$

$$\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx + \frac{b\left(\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d} + \frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d} + \frac{\cos(a+bx)}{d(c+dx)}$$

$$\downarrow 4680$$

$$\int \frac{\sec(a+bx)}{(c+dx)^2}dx + \frac{b\left(\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d} + \frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d} + \frac{\cos(a+bx)}{d(c+dx)}$$

input
```
Int[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4907

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin(bx+a)\tan(bx+a)}{(dx+c)^2}dx$$

input

```
int(sin(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
```

output

```
int(sin(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(bx+a)\tan(bx+a)}{(dx+c)^2}\,dx$$

input

```
integrate(sin(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output

```
integral(sin(b*x + a)*tan(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin\left(a+bx\right)\tan\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)*tan(a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 270, normalized size of antiderivative = 13.50

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin\left(bx+a\right)\tan\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*((exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x
 + I*b*c)/d))*cos(-(b*c - a*d)/d) + 4*(d^2*x + c*d)*integrate((cos(2*b*x +
 2*a)*cos(b*x + a) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/(d^2*x^
2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x + 2*a)^2 + (d^2*x^2 + 2*
c*d*x + c^2)*sin(2*b*x + 2*a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(2*
b*x + 2*a)), x) + (-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*exp_integ
ral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))/(d^2*x + c*d)
```

Giac [N/A]

Not integrable

Time = 8.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(bx+a)\tan(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)*tan(b*x + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 18.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(a+b\,x)\,\tan(a+b\,x)}{(c+d\,x)^2}\,dx$$

input
```
int((sin(a + b*x)*tan(a + b*x))/(c + d*x)^2,x)
```

output
```
int((sin(a + b*x)*tan(a + b*x))/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(bx+a)\tan(bx+a)}{d^2x^2+2cdx+c^2}\,dx$$

input
```
int(sin(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
```

output
```
int((sin(a + b*x)*tan(a + b*x))/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.221 $\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx$

Optimal result 1999
Mathematica [N/A] 2000
Rubi [N/A] 2000
Maple [N/A] 2002
Fricas [N/A] 2003
Sympy [N/A] 2003
Maxima [N/A] 2004
Giac [N/A] 2004
Mupad [N/A] 2004
Reduce [N/A] 2005

Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}&\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx \\ &= \frac{2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} \\ &\quad + \frac{2^{-3-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2ib(c+dx)}{d}\right)}{b} \\ &\quad + \text{Int}((c+dx)^m\tan(a+bx),x)\end{aligned}$$

output

```
2^(-3-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*b
*(d*x+c)/d)^m)+2^(-3-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a-
b*c/d))/((I*b*(d*x+c)/d)^m)+Defer(Int)((d*x+c)^m*tan(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 9.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx = \int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Sin[a + b*x]^2*Tan[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Sin[a + b*x]^2*Tan[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4907, 3042, 4222, 4906, 27, 3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\tan(a+bx)(c+dx)^m\,dx$$

$$\downarrow 4907$$

$$\int (c+dx)^m \tan(a+bx)dx - \int (c+dx)^m \cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)^m \tan(a+bx)dx - \int (c+dx)^m \cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 4222$$

$$\int (c+dx)^m \tan(a+bx)dx - \int (c+dx)^m \cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 4906$$

$$\int (c+dx)^m \tan(a+bx)dx - \int \frac{1}{2}(c+dx)^m \sin(2a+2bx)dx$$

$$\downarrow 27$$

$$\int (c+dx)^m \tan(a+bx)dx - \frac{1}{2}\int (c+dx)^m \sin(2a+2bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)^m \tan(a+bx)dx - \frac{1}{2}\int (c+dx)^m \sin(2a+2bx)dx$$

$$\downarrow 3789$$

$$\int (c+dx)^m \tan(a+bx)dx + \frac{1}{2}\left(\frac{1}{2}i\int e^{2i(a+bx)}(c+dx)^m dx - \frac{1}{2}i\int e^{-2i(a+bx)}(c+dx)^m dx\right)$$

$$\downarrow 2612$$

$$\int (c+dx)^m \tan(a+bx)dx +$$

$$\frac{1}{2}\left(\frac{2^{-m-2}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b} + \frac{2^{-m-2}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}}{b}\right.$$

input
```
Int[(c + d*x)^m*Sin[a + b*x]^2*Tan[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \sin(bx+a)^2 \tan(bx+a)\,dx$$

input
```
int((d*x+c)^m*sin(b*x+a)^2*tan(b*x+a),x)
```

output
```
int((d*x+c)^m*sin(b*x+a)^2*tan(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sin\left(bx+a\right)^2 \tan\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)^m*sin(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*(d*x + c)^m*tan(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 41.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx = \int (c+dx)^m \sin^2\left(a+bx\right)\tan\left(a+bx\right)\,dx$$

input
```
integrate((d*x+c)**m*sin(b*x+a)**2*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**m*sin(a + b*x)**2*tan(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sin(bx+a)^2 \tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*sin(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*sin(b*x + a)^2*tan(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sin(bx+a)^2 \tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*sin(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sin(b*x + a)^2*tan(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 18.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx = \int \sin(a+b\,x)^2 \tan(a+b\,x)\,(c+d\,x)^m\,dx$$

input
```
int(sin(a + b*x)^2*tan(a + b*x)*(c + d*x)^m,x)
```

output
```
int(sin(a + b*x)^2*tan(a + b*x)*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 1558, normalized size of antiderivative = 70.82

$$\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^m*sin(b*x+a)^2*tan(b*x+a),x)
```

output
```
(4*((c + d*x)**m*tan((a + b*x)/2)**4 - int(((c + d*x)**m*tan((a + b*x)/2)*
*6)/(tan((a + b*x)/2)**6*c + tan((a + b*x)/2)**6*d*x + tan((a + b*x)/2)**4
*c + tan((a + b*x)/2)**4*d*x - tan((a + b*x)/2)**2*c - tan((a + b*x)/2)**2
*d*x - c - d*x),x)*tan((a + b*x)/2)**4*d*m - 2*int(((c + d*x)**m*tan((a +
b*x)/2)**6)/(tan((a + b*x)/2)**6*c + tan((a + b*x)/2)**6*d*x + tan((a + b*
x)/2)**4*c + tan((a + b*x)/2)**4*d*x - tan((a + b*x)/2)**2*c - tan((a + b*
x)/2)**2*d*x - c - d*x),x)*tan((a + b*x)/2)**2*d*m - int(((c + d*x)**m*tan
((a + b*x)/2)**6)/(tan((a + b*x)/2)**6*c + tan((a + b*x)/2)**6*d*x + tan((
a + b*x)/2)**4*c + tan((a + b*x)/2)**4*d*x - tan((a + b*x)/2)**2*c - tan((
a + b*x)/2)**2*d*x - c - d*x),x)*d*m - 2*int(((c + d*x)**m*tan((a + b*x)/2
)**5*x)/(tan((a + b*x)/2)**6*c + tan((a + b*x)/2)**6*d*x + tan((a + b*x)/2
)**4*c + tan((a + b*x)/2)**4*d*x - tan((a + b*x)/2)**2*c - tan((a + b*x)/2
)**2*d*x - c - d*x),x)*tan((a + b*x)/2)**4*b*d - 4*int(((c + d*x)**m*tan((
a + b*x)/2)**5*x)/(tan((a + b*x)/2)**6*c + tan((a + b*x)/2)**6*d*x + tan((
a + b*x)/2)**4*c + tan((a + b*x)/2)**4*d*x - tan((a + b*x)/2)**2*c - tan((
a + b*x)/2)**2*d*x - c - d*x),x)*tan((a + b*x)/2)**2*b*d - 2*int(((c + d*x
)**m*tan((a + b*x)/2)**5*x)/(tan((a + b*x)/2)**6*c + tan((a + b*x)/2)**6*d
*x + tan((a + b*x)/2)**4*c + tan((a + b*x)/2)**4*d*x - tan((a + b*x)/2)**2
*c - tan((a + b*x)/2)**2*d*x - c - d*x),x)*b*d - 2*int(((c + d*x)**m*tan((
a + b*x)/2)**5)/(tan((a + b*x)/2)**6*c + tan((a + b*x)/2)**6*d*x + tan(...
```

3.222 $\int (c+dx)^3 \sin^2(a+bx)\tan(a+bx)\,dx$

Optimal result . . . 2006
Mathematica [B] (warning: unable to verify) . . . 2007
Rubi [A] (verified) . . . 2008
Maple [B] (verified) . . . 2014
Fricas [B] (verification not implemented) . . . 2014
Sympy [F] . . . 2015
Maxima [B] (verification not implemented) . . . 2016
Giac [F] . . . 2016
Mupad [F(-1)] . . . 2017
Reduce [F] . . . 2017

Optimal result

Integrand size = 22, antiderivative size = 251

$$\begin{aligned}\int (c+dx)^3 \sin^2(a+bx)\tan(a+bx)\,dx = &-\frac{3d^3x}{8b^3}+\frac{(c+dx)^3}{4b}+\frac{i(c+dx)^4}{4d}\\ &-\frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b}\\ &+\frac{3id(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}\\ &-\frac{3d^2(c+dx)\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}\\ &-\frac{3id^3\,\mathrm{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4}\\ &+\frac{3d^3\cos(a+bx)\sin(a+bx)}{8b^4}\\ &-\frac{3d(c+dx)^2\cos(a+bx)\sin(a+bx)}{4b^2}\\ &+\frac{3d^2(c+dx)\sin^2(a+bx)}{4b^3}\\ &-\frac{(c+dx)^3\sin^2(a+bx)}{2b}\end{aligned}$$

output
```
-3/8*d^3*x/b^3+1/4*(d*x+c)^3/b+1/4*I*(d*x+c)^4/d-(d*x+c)^3*ln(1+exp(2*I*(b
*x+a)))/b+3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+
c)*polylog(3,-exp(2*I*(b*x+a)))/b^3-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))
/b^4+3/8*d^3*cos(b*x+a)*sin(b*x+a)/b^4-3/4*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+
a)/b^2+3/4*d^2*(d*x+c)*sin(b*x+a)^2/b^3-1/2*(d*x+c)^3*sin(b*x+a)^2/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1731 vs. $2(251) = 502$.

Time = 6.80 (sec) , antiderivative size = 1731, normalized size of antiderivative = 6.90

$$\int (c+dx)^3 \sin^2(a+bx) \tan(a+bx)\, dx = \text{Too large to display}$$

input
```
Integrate[(c + d*x)^3*Sin[a + b*x]^2*Tan[a + b*x],x]
```

output
```
((-1/4*I)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2
*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x)
)] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b
^3*E^(I*a)) - ((I/8)*d^3*E^(I*a)*((2*b^4*x^4)/E^((2*I)*a) - (4*I)*b^3*(1 +
 E^((-2*I)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x))] + 6*b^2*(1 + E^((-2*I)*a)
)*x^2*PolyLog[2, -E^((-2*I)*(a + b*x))] - (6*I)*b*(1 + E^((-2*I)*a))*x*Pol
yLog[3, -E^((-2*I)*(a + b*x))] - 3*(1 + E^((-2*I)*a))*PolyLog[4, -E^((-2*I
)*(a + b*x))])*Sec[a])/b^4 - (c^3*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin
[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (3*c^2*d*Csc[a]*(
(b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) -
 Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b
*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x -
 ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[
1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + Sec[
a]*(Cos[2*a + 2*b*x]/(64*b^4) - ((I/64)*Sin[2*a + 2*b*x])/b^4)*(8*b^3*c^3*
Cos[a] - (12*I)*b^2*c^2*d*Cos[a] - 12*b*c*d^2*Cos[a] + (6*I)*d^3*Cos[a] +
24*b^3*c^2*d*x*Cos[a] - (24*I)*b^2*c*d^2*x*Cos[a] - 12*b*d^3*x*Cos[a] + 24
*b^3*c*d^2*x^2*Cos[a] - (12*I)*b^2*d^3*x^2*Cos[a] + 8*b^3*d^3*x^3*Cos[a] +
 (32*I)*b^4*c^3*x*Cos[a + 2*b*x] + (48*I)*b^4*c^2*d*x^2*Cos[a + 2*b*x] + (
32*I)*b^4*c*d^2*x^3*Cos[a + 2*b*x] + (8*I)*b^4*d^3*x^4*Cos[a + 2*b*x] -...
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.12, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {4907, 3042, 4202, 2620, 3011, 4904, 3042, 3792, 17, 3042, 3115, 24, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin^2(a+bx)\tan(a+bx)\,dx$$

$$\downarrow 4907$$

$$\int (c+dx)^3 \tan(a+bx)dx - \int (c+dx)^3 \cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)^3 \tan(a+bx)dx - \int (c+dx)^3 \cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 4202$$

$$-2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1+e^{2i(a+bx)}}dx - \int (c+dx)^3 \cos(a+bx)\sin(a+bx)dx + \frac{i(c+dx)^4}{4d}$$

$$\downarrow 2620$$

$$-2i\left(\frac{3id\int (c+dx)^2 \log\left(1+e^{2i(a+bx)}\right)dx}{2b} - \frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right) - \int (c+dx)^3\cos(a+bx)\sin(a+bx)dx + \frac{i(c+dx)^4}{4d}$$

$$\downarrow 3011$$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b} - \frac{id\int (c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b} - \frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right) - \int (c+dx)^3\cos(a+bx)\sin(a+bx)dx + \frac{i(c+dx)^4}{4d}$$

$$\downarrow 4904$$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)+$$
$$\frac{3d\int(c+dx)^2\sin^2(a+bx)dx}{2b}-\frac{(c+dx)^3\sin^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$
$$\downarrow 3042$$
$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)+$$
$$\frac{3d\int(c+dx)^2\sin(a+bx)^2dx}{2b}-\frac{(c+dx)^3\sin^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$
$$\downarrow 3792$$
$$\frac{3d\left(-\frac{d^2\int\sin^2(a+bx)dx}{2b^2}+\frac{1}{2}\int(c+dx)^2dx+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b}-$$
$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-$$
$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$
$$\downarrow 17$$
$$\frac{3d\left(-\frac{d^2\int\sin^2(a+bx)dx}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}-$$
$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-$$
$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$
$$\downarrow 3042$$
$$\frac{3d\left(-\frac{d^2\int\sin(a+bx)^2dx}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}-$$
$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-$$
$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$$\downarrow 3115$$

$$\frac{3d\left(-\frac{d^2\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}-$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-$$

$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$$\downarrow 24$$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)+$$

$$\frac{3d\left(\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}-$$

$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$$\downarrow 7163$$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{id\int\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(c+dx)^3\log}{2}$$

$$\frac{3d\left(\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}-$$

$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$$\downarrow 2720$$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{d\int e^{-2i(a+bx)}\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{3d\left(\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}-\frac{(c+dx)^3\sin^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$$\downarrow 7143$$

$$\frac{3d\left(\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}-2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{d\,\mathrm{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e\right.}{2b}\right)$$

$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

input
```
Int[(c + d*x)^3*Sin[a + b*x]^2*Tan[a + b*x],x]
```

output
```
((I/4)*(c + d*x)^4)/d - (2*I)*(((-1/2*I)*(c + d*x)^3*Log[1 + E^((2*I)*(a +
 b*x))])/b + (((3*I)/2)*d*(((I/2)*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*
x))])/b - (I*d*(((-1/2*I)*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/b +
(d*PolyLog[4, -E^((2*I)*(a + b*x))])/(4*b^2)))/b))/b) - ((c + d*x)^3*Sin[a
 + b*x]^2)/(2*b) + (3*d*((c + d*x)^3/(6*d) - ((c + d*x)^2*Cos[a + b*x]*Sin
[a + b*x])/(2*b) + (d*(c + d*x)*Sin[a + b*x]^2)/(2*b^2) - (d^2*(x/2 - (Cos
[a + b*x]*Sin[a + b*x])/(2*b)))/(2*b^2)))/(2*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))  Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
  Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))  Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)  Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4202

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4904

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4907

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(220) = 440$.

Time = 1.54 (sec) , antiderivative size = 650, normalized size of antiderivative = 2.59

method	result
risch	$-\frac{3dc^2\ln(e^{2i(bx+a)}+1)x}{b}+\frac{3idc^2\operatorname{polylog}(2,-e^{2i(bx+a)})}{2b^2}+\frac{3icd^2\operatorname{polylog}(2,-e^{2i(bx+a)})x}{b^2}-\frac{3id^3\operatorname{polylog}(4,-e^{2i(bx+a)})}{4b^4}-$

input
```
int((d*x+c)^3*sin(b*x+a)^2*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a)))+1/32*(4*d^3*x^3*b^3+6*I*b^2*d^3*x^2+12*
b^3*c*d^2*x^2+12*I*b^2*c*d^2*x+12*b^3*c^2*d*x+6*I*b^2*c^2*d+4*b^3*c^3-6*b*
d^3*x-3*I*d^3-6*c*d^2*b)/b^4*exp(2*I*(b*x+a))+3*I/b^2*c*d^2*polylog(2,-exp
(2*I*(b*x+a)))*x-6*I/b^2*c*d^2*a^2*x+6*I/b*d*c^2*a*x-I*c^3*x-1/4*I/d*c^4-3
/b*d*c^2*ln(exp(2*I*(b*x+a))+1)*x-3/b*c*d^2*ln(exp(2*I*(b*x+a))+1)*x^2-4*I
/b^3*c*d^2*a^3+3*I/b^2*d*c^2*a^2+3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a))
)*x^2+3/2*I/b^2*d*c^2*polylog(2,-exp(2*I*(b*x+a)))+2*I/b^3*d^3*a^3*x+I*d^2
*c*x^3-3/2/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x-3/2/b^3*c*d^2*polylog(3,
-exp(2*I*(b*x+a)))+3/2*I/b^4*d^3*a^4-1/b*d^3*ln(exp(2*I*(b*x+a))+1)*x^3+3/
2*I*d*c^2*x^2+1/4*I*d^3*x^4-1/b*c^3*ln(exp(2*I*(b*x+a))+1)-2/b^4*d^3*a^3*l
n(exp(I*(b*x+a)))-6/b^2*c^2*d*a*ln(exp(I*(b*x+a)))+1/32*(4*d^3*x^3*b^3-6*I
*b^2*d^3*x^2+12*b^3*c*d^2*x^2-12*I*b^2*c*d^2*x+12*b^3*c^2*d*x-6*I*b^2*c^2*
d+4*b^3*c^3-6*b*d^3*x+3*I*d^3-6*c*d^2*b)/b^4*exp(-2*I*(b*x+a))+2/b*c^3*ln(
exp(I*(b*x+a)))-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(216) = 432$.

Time = 0.16 (sec) , antiderivative size = 1134, normalized size of antiderivative = 4.52

$$\int (c+dx)^3 \sin^2(a+bx)\tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sin(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

output

```
-1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 24*I*d^3*polylog(4, I*cos(b*x + a)
 + sin(b*x + a)) + 24*I*d^3*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 24
*I*d^3*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) - 24*I*d^3*polylog(4, -I
*cos(b*x + a) - sin(b*x + a)) - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3
*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b^2*d^
3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 3*(
2*b^3*c^2*d - b*d^3)*x + 12*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d
)*dilog(I*cos(b*x + a) + sin(b*x + a)) + 12*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^
2*x - I*b^2*c^2*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) + 12*(-I*b^2*d^3*x
^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a))
+ 12*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(-I*cos(b*x + a)
 - sin(b*x + a)) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*l
og(cos(b*x + a) + I*sin(b*x + a) + I) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*sin(b*x + a) + I) + 4*(b^3*d^3*x^
3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*
d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2
*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(
b*x + a) - sin(b*x + a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^
2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) + sin
(b*x + a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a...
```

Sympy [F]

$$\int (c+dx)^3 \sin^2(a+bx)\tan(a+bx)\,dx = \int (c+dx)^3 \sin^2(a+bx)\tan(a+bx)\,dx$$

input

```
integrate((d*x+c)**3*sin(b*x+a)**2*tan(b*x+a),x)
```

output

```
Integral((c + d*x)**3*sin(a + b*x)**2*tan(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(216) = 432$.

Time = 0.21 (sec) , antiderivative size = 692, normalized size of antiderivative = 2.76

$$\int (c+dx)^3 \sin^2(a+bx) \tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sin(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

output
```
-1/48*(24*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))*c^3 - 72*(sin(b*x + a
)^2 + log(sin(b*x + a)^2 - 1))*a*c^2*d/b + 72*(sin(b*x + a)^2 + log(sin(b*
x + a)^2 - 1))*a^2*c*d^2/b^2 - 24*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1
))*a^3*d^3/b^3 + (-12*I*(b*x + a)^4*d^3 - 48*(I*b*c*d^2 - I*a*d^3)*(b*x +
a)^3 + 48*I*d^3*polylog(4, -e^(2*I*b*x + 2*I*a)) - 72*(I*b^2*c^2*d - 2*I*a
*b*c*d^2 + I*a^2*d^3)*(b*x + a)^2 - 16*(-4*I*(b*x + a)^3*d^3 + 9*(-I*b*c*d
^2 + I*a*d^3)*(b*x + a)^2 + 9*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*(
b*x + a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*(2*(b*x + a)
^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*
c^2*d - 4*a*b*c*d^2 + (2*a^2 - 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 24*(3
*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 4*I*(b*x + a)^2*d^3 + 3*I*a^2*d^3 + 6*(I*b*
c*d^2 - I*a*d^3)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a)) + 8*(4*(b*x + a)^3
*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)*(b*x + a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
+ 2*a) + 1) + 24*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*polylog(3, -e^(2*
I*b*x + 2*I*a)) + 9*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^2*d^3 + (2*a^
2 - 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))/b^3)/b
```

Giac [F]

$$\int (c+dx)^3 \sin^2(a+bx) \tan(a+bx)\,dx = \int (dx+c)^3 \sin(bx+a)^2 \tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*sin(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*sin(b*x + a)^2*tan(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \sin^2(a+bx) \tan(a+bx)\,dx = \int \sin\left(a+b\,x\right)^2 \tan(a+b\,x)\,\left(c+d\,x\right)^3\,dx$$

input
```
int(sin(a + b*x)^2*tan(a + b*x)*(c + d*x)^3,x)
```

output
```
int(sin(a + b*x)^2*tan(a + b*x)*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \sin^2(a+bx) \tan(a+bx)\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^3*sin(b*x+a)^2*tan(b*x+a),x)
```

output

```
( - 3*cos(a + b*x)*sin(a + b*x)*c**2*d + 12*cos(a + b*x)*b*c**2*d*x + 12*c
os(a + b*x)*b*c*d**2*x**2 + 4*cos(a + b*x)*b*d**3*x**3 + 24*int(x**2/(tan(
(a + b*x)/2)**6 + tan((a + b*x)/2)**4 - tan((a + b*x)/2)**2 - 1),x)*b*d**3
 - 24*int((tan((a + b*x)/2)**2*x**2)/(tan((a + b*x)/2)**6 + tan((a + b*x)/
2)**4 - tan((a + b*x)/2)**2 - 1),x)*b*d**3 - 48*int((tan((a + b*x)/2)**2*x
)/(tan((a + b*x)/2)**6 + tan((a + b*x)/2)**4 - tan((a + b*x)/2)**2 - 1),x)
*b*c*d**2 - 16*int((tan((a + b*x)/2)*x**3)/(tan((a + b*x)/2)**6 + tan((a +
 b*x)/2)**4 - tan((a + b*x)/2)**2 - 1),x)*b**2*d**3 - 48*int((tan((a + b*x
)/2)*x**2)/(tan((a + b*x)/2)**6 + tan((a + b*x)/2)**4 - tan((a + b*x)/2)**
2 - 1),x)*b**2*c*d**2 - 48*int((tan((a + b*x)/2)*x)/(tan((a + b*x)/2)**6 +
 tan((a + b*x)/2)**4 - tan((a + b*x)/2)**2 - 1),x)*b**2*c**2*d + 48*int(x/
(tan((a + b*x)/2)**6 + tan((a + b*x)/2)**4 - tan((a + b*x)/2)**2 - 1),x)*b
*c*d**2 + 2*log(tan((a + b*x)/2)**2 + 1)*b*c**3 - 2*log(tan((a + b*x)/2) -
 1)*b*c**3 - 2*log(tan((a + b*x)/2) + 1)*b*c**3 - sin(a + b*x)**2*b*c**3 -
 6*sin(a + b*x)**2*b*c**2*d*x - 6*sin(a + b*x)**2*b*c*d**2*x**2 - 2*sin(a
+ b*x)**2*b*d**3*x**3 - 12*sin(a + b*x)*c**2*d + 15*a*c**2*d + 3*b*c**2*d*
x + 12*b*c*d**2*x**2 + 4*b*d**3*x**3)/(2*b**2)
```

3.223 $\int (c+dx)^2 \sin^2(a+bx)\tan(a+bx)\,dx$

Optimal result . . . 2019
Mathematica [B] (warning: unable to verify) . . . 2020
Rubi [A] (verified) . . . 2021
Maple [B] (verified) . . . 2025
Fricas [B] (verification not implemented) . . . 2025
Sympy [F] . . . 2026
Maxima [B] (verification not implemented) . . . 2027
Giac [F] . . . 2027
Mupad [F(-1)] . . . 2028
Reduce [F] . . . 2028

Optimal result

Integrand size = 22, antiderivative size = 175

$$\begin{aligned}\int (c+dx)^2 \sin^2(a+bx)\tan(a+bx)\,dx = {} & \frac{(c+dx)^2}{4b} + \frac{i(c+dx)^3}{3d} \\ & - \frac{(c+dx)^2 \log\left(1+e^{2i(a+bx)}\right)}{b} \\ & + \frac{id(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} \\ & - \frac{d^2\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} \\ & - \frac{d(c+dx)\cos(a+bx)\sin(a+bx)}{2b^2} \\ & + \frac{d^2\sin^2(a+bx)}{4b^3} - \frac{(c+dx)^2\sin^2(a+bx)}{2b}\end{aligned}$$

output

```
1/4*(d*x+c)^2/b+1/3*I*(d*x+c)^3/d-(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b+I*d*(
d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)
))/b^3-1/2*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2+1/4*d^2*sin(b*x+a)^2/b^3-1/
2*(d*x+c)^2*sin(b*x+a)^2/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 518 vs. $2(175) = 350$.

Time = 6.34 (sec) , antiderivative size = 518, normalized size of antiderivative = 2.96

$$\int (c+dx)^2 \sin^2(a+bx)\tan(a+bx)\,dx =$$

$$-\frac{id^2e^{-ia}\left(2b^2x^2\left(2bx-3i(1+e^{2ia})\log\left(1+e^{-2i(a+bx)}\right)\right)+6b(1+e^{2ia})\,x\,\mathrm{PolyLog}\left(2,-e^{-2i(a+bx)}\right)-3i(1\right.}{12b^3}$$

$$-\frac{c^2\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))}{b\left(\cos^2(a)+\sin^2(a)\right)}$$

$$-\frac{cd\csc(a)\left(b^2e^{-i\arctan(\cot(a))}x^2-\frac{\cot(a)(ibx(-\pi-2\arctan(\cot(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx-\arctan(\cot(a)))\log\left(1-e^{2i(bx-\arctan\right.}{b^2\sqrt{\csc^2(a)\left(\cos^2(a)+\right.}}$$

$$+\frac{\cos(2bx)\left(2b^2c^2\cos(2a)-d^2\cos(2a)+4b^2cdx\cos(2a)+2b^2d^2x^2\cos(2a)-2bcd\sin(2a)-2bd^2x\sin(2a\right.}{8b^3}$$

$$-\frac{(2bcd\cos(2a)+2bd^2x\cos(2a)+2b^2c^2\sin(2a)-d^2\sin(2a)+4b^2cdx\sin(2a)+2b^2d^2x^2\sin(2a))\sin(2bx}{8b^3}$$

$$+\frac{1}{3}x\left(3c^2+3cdx+d^2x^2\right)\tan(a)$$

input

```
Integrate[(c + d*x)^2*Sin[a + b*x]^2*Tan[a + b*x],x]
```

output

```
((-1/12*I)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*
I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))
] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^
3*E^(I*a)) - (c^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] +
b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcTa
n[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I
)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]])
)] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] +
I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a
])/(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Cos[2*b*x]*(2*b^2*c^2*Cos
[2*a] - d^2*Cos[2*a] + 4*b^2*c*d*x*Cos[2*a] + 2*b^2*d^2*x^2*Cos[2*a] - 2*b
*c*d*Sin[2*a] - 2*b*d^2*x*Sin[2*a]))/(8*b^3) - ((2*b*c*d*Cos[2*a] + 2*b*d^
2*x*Cos[2*a] + 2*b^2*c^2*Sin[2*a] - d^2*Sin[2*a] + 4*b^2*c*d*x*Sin[2*a] +
2*b^2*d^2*x^2*Sin[2*a])*Sin[2*b*x])/(8*b^3) + (x*(3*c^2 + 3*c*d*x + d^2*x^
2)*Tan[a])/3
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4907, 3042, 4202, 2620, 3011, 2720, 4904, 3042, 3791, 17, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin^2(a+bx)\tan(a+bx)\,dx$$

$$\downarrow 4907$$

$$\int (c+dx)^2 \tan(a+bx)dx - \int (c+dx)^2 \cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)^2 \tan(a+bx)dx - \int (c+dx)^2 \cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 4202$$

$$-2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}dx - \int (c+dx)^2\cos(a+bx)\sin(a+bx)dx + \frac{i(c+dx)^3}{3d}$$

$$\downarrow 2620$$

$$-2i\left(\frac{id\int (c+dx)\log\left(1+e^{2i(a+bx)}\right)dx}{b} - \frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right) - \int (c+dx)^2\cos(a+bx)\sin(a+bx)dx + \frac{i(c+dx)^3}{3d}$$

$$\downarrow 3011$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b} - \frac{id\int \text{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{2b}\right)}{b} - \frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right) - \int (c+dx)^2\cos(a+bx)\sin(a+bx)dx + \frac{i(c+dx)^3}{3d}$$

$$\downarrow 2720$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i(a+bx)}\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

$$\int(c+dx)^2\cos(a+bx)\sin(a+bx)dx+\frac{i(c+dx)^3}{3d}$$

$$\downarrow 4904$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i(a+bx)}\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

$$\frac{d\int(c+dx)\sin^2(a+bx)dx}{b}-\frac{(c+dx)^2\sin^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\downarrow 3042$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i(a+bx)}\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

$$\frac{d\int(c+dx)\sin(a+bx)^2dx}{b}-\frac{(c+dx)^2\sin^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\downarrow 3791$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i(a+bx)}\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

$$\frac{d\left(\frac{1}{2}\int(c+dx)dx+\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right)}{b}-\frac{(c+dx)^2\sin^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\downarrow 17$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i(a+bx)}\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

$$\frac{d\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{b}-\frac{(c+dx)^2\sin^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\downarrow 7143$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\,\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)+$$
$$\frac{d\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{b}-\frac{(c+dx)^2\sin^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

input
```
Int[(c + d*x)^2*Sin[a + b*x]^2*Tan[a + b*x],x]
```

output
```
((I/3)*(c + d*x)^3)/d - (2*I)*(((-1/2*I)*(c + d*x)^2*Log[1 + E^((2*I)*(a +
 b*x))])/b + (I*d*(((I/2)*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b -
(d*PolyLog[3, -E^((2*I)*(a + b*x))])/(4*b^2)))/b) - ((c + d*x)^2*Sin[a + b
*x]^2)/(2*b) + (d*((c + d*x)^2/(4*d) - ((c + d*x)*Cos[a + b*x]*Sin[a + b*x
])/(2*b) + (d*Sin[a + b*x]^2)/(4*b^2)))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4202

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4904

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4907

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(155) = 310$.

Time = 1.17 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.22

method	result
risch	$\frac{2icd\,a^2}{b^2} - \frac{2id^2a^2x}{b^2} - \frac{ic^3}{3d} + \frac{idc\,\mathrm{polylog}\left(2,-\mathrm{e}^{2i(bx+a)}\right)}{b^2} + \frac{\left(2x^2d^2b^2+4b^2cdx+2ib\,d^2x+2b^2c^2+2ibcd-d^2\right)\mathrm{e}^{2i(bx+a)}}{16b^3} + \frac{(2x^2d^2$

input
```
int((d*x+c)^2*sin(b*x+a)^2*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*I/b^2*c*d*a^2-2*I/b^2*d^2*a^2*x-1/3*I/d*c^3+I/b^2*d*c*polylog(2,-exp(2*I
*(b*x+a)))+1/16*(2*x^2*d^2*b^2+2*I*b*d^2*x+4*b^2*c*d*x+2*I*b*c*d+2*b^2*c^2
-d^2)/b^3*exp(2*I*(b*x+a))+1/16*(2*x^2*d^2*b^2-2*I*b*d^2*x+4*b^2*c*d*x-2*I
*b*c*d+2*b^2*c^2-d^2)/b^3*exp(-2*I*(b*x+a))-1/b*c^2*ln(exp(2*I*(b*x+a))+1)
+2/b*c^2*ln(exp(I*(b*x+a)))+1/3*I*d^2*x^3+2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))
-2/b*d*c*ln(exp(2*I*(b*x+a))+1)*x+I*d*c*x^2-I*c^2*x+I/b^2*d^2*polylog(2,-e
xp(2*I*(b*x+a)))*x-4/3*I/b^3*d^2*a^3+4*I/b*c*d*a*x-4/b^2*c*d*a*ln(exp(I*(b
*x+a)))-1/b*d^2*ln(exp(2*I*(b*x+a))+1)*x^2-1/2*d^2*polylog(3,-exp(2*I*(b*x
+a)))/b^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(152) = 304$.

Time = 0.13 (sec) , antiderivative size = 688, normalized size of antiderivative = 3.93

$$\int (c+dx)^2 \sin^2(a+bx)\tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*sin(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

output

```
-1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2
 - d^2)*cos(b*x + a)^2 + 4*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) +
 4*d^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 4*d^2*polylog(3, -I*cos
(b*x + a) + sin(b*x + a)) + 4*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a
)) + 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) + 4*(I*b*d^2*x + I*b*c*
d)*dilog(I*cos(b*x + a) + sin(b*x + a)) + 4*(-I*b*d^2*x - I*b*c*d)*dilog(I
*cos(b*x + a) - sin(b*x + a)) + 4*(-I*b*d^2*x - I*b*c*d)*dilog(-I*cos(b*x
+ a) + sin(b*x + a)) + 4*(I*b*d^2*x + I*b*c*d)*dilog(-I*cos(b*x + a) - sin
(b*x + a)) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*sin(b*
x + a) + I) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*sin(b
*x + a) + I) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*c
os(b*x + a) + sin(b*x + a) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d
 - a^2*d^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + 2*(b^2*d^2*x^2 + 2*b^
2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 2
*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - s
in(b*x + a) + 1) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I
*sin(b*x + a) + I) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) -
 I*sin(b*x + a) + I))/b^3
```

Sympy [F]

$$\int (c+dx)^2 \sin^2(a+bx)\tan(a+bx)\,dx = \int (c+dx)^2 \sin^2(a+bx)\tan(a+bx)\,dx$$

input

```
integrate((d*x+c)**2*sin(b*x+a)**2*tan(b*x+a),x)
```

output

```
Integral((c + d*x)**2*sin(a + b*x)**2*tan(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(152) = 304$.

Time = 0.19 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.19

$$\int (c+dx)^2 \sin^2(a+bx)\tan(a+bx)\,dx =$$

$$-\frac{12\left(\sin(bx+a)^2+\log\left(\sin(bx+a)^2-1\right)\right)c^2-\frac{24\left(\sin(bx+a)^2+\log\left(\sin(bx+a)^2-1\right)\right)acd}{b}+\frac{12\left(\sin(bx+a)^2+\log\left(\sin(b\right.\right.}{}$$

input
```
integrate((d*x+c)^2*sin(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

output
```
-1/24*(12*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))*c^2 - 24*(sin(b*x + a
)^2 + log(sin(b*x + a)^2 - 1))*a*c*d/b + 12*(sin(b*x + a)^2 + log(sin(b*x
+ a)^2 - 1))*a^2*d^2/b^2 + (-8*I*(b*x + a)^3*d^2 - 24*(I*b*c*d - I*a*d^2)*
(b*x + a)^2 + 12*d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) - 24*(-I*(b*x + a)^2
*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan2(sin(2*b*x + 2*a), cos(2*b
*x + 2*a) + 1) - 3*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - d^2)
*cos(2*b*x + 2*a) - 24*(I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*dilog(-e^(2*I
*b*x + 2*I*a)) + 12*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(co
s(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 6*(b*c*d
 + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))/b^2)/b
```

Giac [F]

$$\int (c+dx)^2 \sin^2(a+bx)\tan(a+bx)\,dx = \int (dx+c)^2 \sin(bx+a)^2 \tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*sin(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*sin(b*x + a)^2*tan(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \sin^2(a+bx)\tan(a+bx)\,dx = \int \sin(a+b\,x)^2 \tan(a+b\,x)\,(c+d\,x)^2\,dx$$

input
```
int(sin(a + b*x)^2*tan(a + b*x)*(c + d*x)^2,x)
```

output
```
int(sin(a + b*x)^2*tan(a + b*x)*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \sin^2(a+bx)\tan(a+bx)\,dx$$

$$= \frac{-2\cos(bx+a)\sin(bx+a)\,cd + 8\cos(bx+a)\,bcdx + 4\cos(bx+a)\,b\,d^2x^2 - 16\left(\int \frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}\right.}{}$$

input
```
int((d*x+c)^2*sin(b*x+a)^2*tan(b*x+a),x)
```

output
```
( - 2*cos(a + b*x)*sin(a + b*x)*c*d + 8*cos(a + b*x)*b*c*d*x + 4*cos(a + b
*x)*b*d**2*x**2 - 16*int((tan((a + b*x)/2)**2*x)/(tan((a + b*x)/2)**6 + ta
n((a + b*x)/2)**4 - tan((a + b*x)/2)**2 - 1),x)*b*d**2 - 16*int((tan((a +
b*x)/2)*x**2)/(tan((a + b*x)/2)**6 + tan((a + b*x)/2)**4 - tan((a + b*x)/2
)**2 - 1),x)*b**2*d**2 - 32*int((tan((a + b*x)/2)*x)/(tan((a + b*x)/2)**6
+ tan((a + b*x)/2)**4 - tan((a + b*x)/2)**2 - 1),x)*b**2*c*d + 16*int(x/(t
an((a + b*x)/2)**6 + tan((a + b*x)/2)**4 - tan((a + b*x)/2)**2 - 1),x)*b*d
**2 + 2*log(tan((a + b*x)/2)**2 + 1)*b*c**2 - 2*log(tan((a + b*x)/2) - 1)*
b*c**2 - 2*log(tan((a + b*x)/2) + 1)*b*c**2 - sin(a + b*x)**2*b*c**2 - 4*s
in(a + b*x)**2*b*c*d*x - 2*sin(a + b*x)**2*b*d**2*x**2 - 8*sin(a + b*x)*c*
d + 10*a*c*d + 2*b*c*d*x + 4*b*d**2*x**2)/(2*b**2)
```

3.224 $\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx$

Optimal result . . . 2029
Mathematica [A] (verified) . . . 2030
Rubi [A] (verified) . . . 2030
Maple [A] (verified) . . . 2033
Fricas [B] (verification not implemented) . . . 2034
Sympy [F] . . . 2035
Maxima [A] (verification not implemented) . . . 2035
Giac [F] . . . 2035
Mupad [F(-1)] . . . 2036
Reduce [F] . . . 2036

Optimal result

Integrand size = 20, antiderivative size = 115

$$\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx = \frac{dx}{4b} + \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} - \frac{d\cos(a+bx)\sin(a+bx)}{4b^2} - \frac{(c+dx)\sin^2(a+bx)}{2b}$$

output

```
1/4*d*x/b+1/2*I*(d*x+c)^2/d-(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b+1/2*I*d*polyl
og(2,-exp(2*I*(b*x+a)))/b^2-1/4*d*cos(b*x+a)*sin(b*x+a)/b^2-1/2*(d*x+c)*si
n(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.17

$$\begin{aligned}
&\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx \\
&= \frac{dx\cos(2(a+bx))}{4b} + \frac{ad\log(\cos(a+bx))}{b^2} - \frac{c\left(-\frac{1}{2}\cos^2(a+bx)+\log(\cos(a+bx))\right)}{b} \\
&\quad + \frac{d\left(\frac{1}{2}i(a+bx)^2-(a+bx)\log\left(1+e^{2i(a+bx)}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)\right)}{b^2} \\
&\quad - \frac{d\sin(2(a+bx))}{8b^2}
\end{aligned}$$

input
```
Integrate[(c + d*x)*Sin[a + b*x]^2*Tan[a + b*x],x]
```

output
```
(d*x*Cos[2*(a + b*x)])/(4*b) + (a*d*Log[Cos[a + b*x]])/b^2 - (c*(-1/2*Cos[
a + b*x]^2 + Log[Cos[a + b*x]]))/b + (d*((I/2)*(a + b*x)^2 - (a + b*x)*Log
[1 + E^((2*I)*(a + b*x))] + (I/2)*PolyLog[2, -E^((2*I)*(a + b*x))]))/b^2 -
 (d*Sin[2*(a + b*x)])/(8*b^2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4907, 3042, 4202, 2620, 2715, 2838, 4904, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx$$

$$\downarrow 4907$$

$$\int (c+dx)\tan(a+bx)dx - \int (c+dx)\cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)\tan(a+bx)dx - \int (c+dx)\cos(a+bx)\sin(a+bx)dx$$

$$\downarrow 4202$$

$$-2i\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}dx - \int (c+dx)\cos(a+bx)\sin(a+bx)dx + \frac{i(c+dx)^2}{2d}$$

$$\downarrow 2620$$

$$-2i\left(\frac{id\int \log\left(1+e^{2i(a+bx)}\right)dx}{2b} - \frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right) - \int (c+dx)\cos(a+bx)\sin(a+bx)dx + \frac{i(c+dx)^2}{2d}$$

$$\downarrow 2715$$

$$-2i\left(\frac{d\int e^{-2i(a+bx)}\log\left(1+e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2} - \frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right) - \int (c+dx)\cos(a+bx)\sin(a+bx)dx + \frac{i(c+dx)^2}{2d}$$

$$\downarrow 2838$$

$$-\int (c+dx)\cos(a+bx)\sin(a+bx)dx - 2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2} - \frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right) + \frac{i(c+dx)^2}{2d}$$

$$\downarrow 4904$$

$$\frac{d\int \sin^2(a+bx)dx}{2b} - 2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2} - \frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right) - \frac{(c+dx)\sin^2(a+bx)}{2b} + \frac{i(c+dx)^2}{2d}$$

$$\downarrow 3042$$

$$\frac{d\int \sin(a+bx)^2dx}{2b} - 2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2} - \frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right) - \frac{(c+dx)\sin^2(a+bx)}{2b} + \frac{i(c+dx)^2}{2d}$$

$$\downarrow 3115$$

$$\frac{d\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b}-$$
$$2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{(c+dx)\sin^2(a+bx)}{2b}+$$
$$\frac{i(c+dx)^2}{2d}$$
$$\downarrow 24$$
$$-2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{(c+dx)\sin^2(a+bx)}{2b}+$$
$$\frac{d\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b}+\frac{i(c+dx)^2}{2d}$$

input
```
Int[(c + d*x)*Sin[a + b*x]^2*Tan[a + b*x],x]
```

output
```
((I/2)*(c + d*x)^2)/d - (2*I)*(((-1/2*I)*(c + d*x)*Log[1 + E^((2*I)*(a + b
*x))])/b - (d*PolyLog[2, -E^((2*I)*(a + b*x))])/(4*b^2)) - ((c + d*x)*Sin[
a + b*x]^2)/(2*b) + (d*(x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)))/(2*b)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))  Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])  Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)  Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I  Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))  Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.56

method	result
risch	$\frac{id x^2}{2} - ixc + \frac{(2bdx+2cb+id)e^{2i(bx+a)}}{16b^2} + \frac{(2bdx+2cb-id)e^{-2i(bx+a)}}{16b^2} - \frac{c\ln(e^{2i(bx+a)}+1)}{b} + \frac{2c\ln(e^{i(bx+a)})}{b} + \frac{2idax}{b} +$

input
```
int((d*x+c)*sin(b*x+a)^2*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*d*x^2-I*x*c+1/16*(2*b*d*x+I*d+2*c*b)/b^2*exp(2*I*(b*x+a))+1/16*(2*b*
d*x-I*d+2*c*b)/b^2*exp(-2*I*(b*x+a))-1/b*c*ln(exp(2*I*(b*x+a))+1)+2/b*c*ln
(exp(I*(b*x+a)))+2*I/b*d*a*x+I/b^2*d*a^2-1/b*d*ln(exp(2*I*(b*x+a))+1)*x+1/
2*I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2-2/b^2*d*a*ln(exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(96) = 192$.

Time $= 0.11$ (sec) , antiderivative size $= 346$, normalized size of antiderivative $= 3.01$

$$\int (c + dx) \sin^2(a + bx) \tan(a + bx)\, dx =$$

$$-\frac{bdx - 2\,(bdx + bc)\cos\,(bx + a)^2 + d\cos\,(bx + a)\sin\,(bx + a) + 2i\,d\mathrm{Li}_2(i\,\cos\,(bx + a) + \sin\,(bx + a)) -}{}$$

input
```
integrate((d*x+c)*sin(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a)
 + 2*I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) - 2*I*d*dilog(I*cos(b*x + a)
 - sin(b*x + a)) - 2*I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 2*I*d*dil
og(-I*cos(b*x + a) - sin(b*x + a)) + 2*(b*c - a*d)*log(cos(b*x + a) + I*si
n(b*x + a) + I) + 2*(b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) + 2
*(b*d*x + a*d)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b*d*x + a*d)*lo
g(I*cos(b*x + a) - sin(b*x + a) + 1) + 2*(b*d*x + a*d)*log(-I*cos(b*x + a)
 + sin(b*x + a) + 1) + 2*(b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a)
+ 1) + 2*(b*c - a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + 2*(b*c - a*
d)*log(-cos(b*x + a) - I*sin(b*x + a) + I))/b^2
```

Sympy [F]

$$\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx = \int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx$$

input
```
integrate((d*x+c)*sin(b*x+a)**2*tan(b*x+a),x)
```

output
```
Integral((c + d*x)*sin(a + b*x)**2*tan(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.27

$$\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx =$$
$$-\frac{-4i\,b^2dx^2 - 8i\,b^2cx - 8\,(-i\,bdx - i\,bc)\arctan\left(\sin\left(2\,bx+2\,a\right),\cos\left(2\,bx+2\,a\right)+1\right) - 2\,(bdx+bc)\cos}{}$$

input
```
integrate((d*x+c)*sin(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

output
```
-1/8*(-4*I*b^2*d*x^2 - 8*I*b^2*c*x - 8*(-I*b*d*x - I*b*c)*arctan2(sin(2*b*
x + 2*a), cos(2*b*x + 2*a) + 1) - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - 4*I*d
*dilog(-e^(2*I*b*x + 2*I*a)) + 4*(b*d*x + b*c)*log(cos(2*b*x + 2*a)^2 + si
n(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + d*sin(2*b*x + 2*a))/b^2
```

Giac [F]

$$\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx = \int (dx+c)\sin\left(bx+a\right)^2\tan\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)*sin(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*sin(b*x + a)^2*tan(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx = \int \sin(a+b\,x)^2\tan(a+b\,x)\,(c+d\,x)\,dx$$

input
```
int(sin(a + b*x)^2*tan(a + b*x)*(c + d*x),x)
```

output
```
int(sin(a + b*x)^2*tan(a + b*x)*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx$$
$$= \frac{-\cos(bx+a)\sin(bx+a)\,d + 4\cos(bx+a)\,bdx - 16\left(\int \frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)x}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6+\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1}dx\right)b^2d +}{}$$

input
```
int((d*x+c)*sin(b*x+a)^2*tan(b*x+a),x)
```

output
```
( - cos(a + b*x)*sin(a + b*x)*d + 4*cos(a + b*x)*b*d*x - 16*int((tan((a +
b*x)/2)*x)/(tan((a + b*x)/2)**6 + tan((a + b*x)/2)**4 - tan((a + b*x)/2)**
2 - 1),x)*b**2*d + 2*log(tan((a + b*x)/2)**2 + 1)*b*c - 2*log(tan((a + b*x
)/2) - 1)*b*c - 2*log(tan((a + b*x)/2) + 1)*b*c - sin(a + b*x)**2*b*c - 2*
sin(a + b*x)**2*b*d*x - 4*sin(a + b*x)*d + 5*a*d + b*d*x)/(2*b**2)
```

3.225 $\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx$

Optimal result 2037
Mathematica [N/A] 2037
Rubi [N/A] 2038
Maple [N/A] 2041
Fricas [N/A] 2041
Sympy [N/A] 2041
Maxima [N/A] 2042
Giac [N/A] 2042
Mupad [N/A] 2043
Reduce [N/A] 2043

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx = -\frac{\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d} - \frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d} + \text{Int}\left(\frac{\tan(a+bx)}{c+dx},x\right)$$

output

```
-1/2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d-1/2*cos(2*a-2*b*c/d)*Si(2*b*c/d+
2*b*x)/d+Defer(Int)(tan(b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

input

```
Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
```

output
```
Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4907, 3042, 4222, 4906, 27, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

$$\downarrow 4907$$

$$\int \frac{\tan(a+bx)}{c+dx}dx - \int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(a+bx)}{c+dx}dx - \int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}dx$$

$$\downarrow 4222$$

$$\int \frac{\tan(a+bx)}{c+dx}dx - \int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}dx$$

$$\downarrow 4906$$

$$\int \frac{\tan(a+bx)}{c+dx}dx - \int \frac{\sin(2a+2bx)}{2(c+dx)}dx$$

$$\downarrow 27$$

$$\int \frac{\tan(a+bx)}{c+dx}dx - \frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(a+bx)}{c+dx}dx - \frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}dx$$

$$\downarrow 3784$$

$$\int \frac{\tan(a+bx)}{c+dx}dx + \frac{1}{2}\left(-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx - \cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)$$

$$\downarrow 3042$$

$$\int \frac{\tan(a+bx)}{c+dx}dx + \frac{1}{2}\left(-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx - \cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)$$

$$\downarrow 3780$$

$$\frac{1}{2}\left(-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx - \frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right) + \int \frac{\tan(a+bx)}{c+dx}dx$$

$$\downarrow 3783$$

$$\int \frac{\tan(a+bx)}{c+dx}dx + \frac{1}{2}\left(-\frac{\sin\left(2a-\frac{2bc}{d}\right)\mathrm{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d} - \frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)$$

input
```
Int[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Maple [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin(bx+a)^2 \tan(bx+a)}{dx+c} dx$$

input
```
int(sin(b*x+a)^2*tan(b*x+a)/(d*x+c),x)
```

output
```
int(sin(b*x+a)^2*tan(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx} dx = \int \frac{\sin(bx+a)^2 \tan(bx+a)}{dx+c} dx$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*tan(b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx} dx = \int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx} dx$$

input
```
integrate(sin(b*x+a)**2*tan(b*x+a)/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)**2*tan(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 182, normalized size of antiderivative = 8.27

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sin\left(bx+a\right)^2\tan\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
1/4*((-I*exp_integral_e(1, 2*(-I*b*d*x - I*b*c)/d) + I*exp_integral_e(1, -
2*(-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 8*d*integrate(sin(2*b*x +
 2*a)/((d*x + c)*cos(2*b*x + 2*a)^2 + (d*x + c)*sin(2*b*x + 2*a)^2 + d*x +
 2*(d*x + c)*cos(2*b*x + 2*a) + c), x) + (exp_integral_e(1, 2*(-I*b*d*x -
I*b*c)/d) + exp_integral_e(1, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b*c - a*d)
/d))/d
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sin\left(bx+a\right)^2\tan\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^2*tan(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 19.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sin(a+b\,x)^2\tan(a+b\,x)}{c+d\,x}\,dx$$

input
```
int((sin(a + b*x)^2*tan(a + b*x))/(c + d*x),x)
```

output
```
int((sin(a + b*x)^2*tan(a + b*x))/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sin(bx+a)^2\tan(bx+a)}{dx+c}dx$$

input
```
int(sin(b*x+a)^2*tan(b*x+a)/(d*x+c),x)
```

output
```
int((sin(a + b*x)**2*tan(a + b*x))/(c + d*x),x)
```

3.226 $\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 2044
Mathematica [N/A] . . . 2044
Rubi [N/A] . . . 2045
Maple [N/A] . . . 2048
Fricas [N/A] . . . 2049
Sympy [N/A] . . . 2049
Maxima [N/A] . . . 2049
Giac [N/A] . . . 2050
Mupad [N/A] . . . 2050
Reduce [N/A] . . . 2051

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = -\frac{b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2} + \frac{\sin(2a+2bx)}{2d(c+dx)} + \frac{b\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2} + \operatorname{Int}\left(\frac{\tan(a+bx)}{(c+dx)^2},x\right)$$

output
```
-b*cos(2*a-2*b*c/d)*Ci(2*b*c/d+2*b*x)/d^2+1/2*sin(2*b*x+2*a)/d/(d*x+c)+b*s
in(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^2+Defer(Int)(tan(b*x+a)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4907, 3042, 4222, 4906, 27, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

$$\downarrow 4907$$

$$\int \frac{\tan(a+bx)}{(c+dx)^2}dx - \int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(a+bx)}{(c+dx)^2}dx - \int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}dx$$

$$\downarrow 4222$$

$$\int \frac{\tan(a+bx)}{(c+dx)^2}dx - \int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}dx$$

$$\downarrow 4906$$

$$\int \frac{\tan(a+bx)}{(c+dx)^2}dx - \int \frac{\sin(2a+2bx)}{2(c+dx)^2}dx$$

$$\downarrow 27$$

$$\int \frac{\tan(a+bx)}{(c+dx)^2}dx - \frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\tan(a+bx)}{(c+dx)^2}dx - \frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}dx$$

$$\downarrow 3778$$

$$\int \frac{\tan(a+bx)}{(c+dx)^2}dx + \frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)} - \frac{2b\int \frac{\cos(2a+2bx)}{c+dx}dx}{d}\right)$$

$$\downarrow 3042$$

$$\frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)} - \frac{2b\int \frac{\sin(2a+2bx+\frac{\pi}{2})}{c+dx}dx}{d}\right) + \int \frac{\tan(a+bx)}{(c+dx)^2}dx$$

$$\downarrow 3784$$

$$\int \frac{\tan(a+bx)}{(c+dx)^2}dx + \frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)} - \frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx - \sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}\right)$$

$$\downarrow 3042$$

$$\int \frac{\tan(a+bx)}{(c+dx)^2}dx + \frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)} - \frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx - \sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}\right)$$

$$\downarrow 3780$$

$$\frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)} - \frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx - \frac{\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}\right) + \int \frac{\tan(a+bx)}{(c+dx)^2}dx$$

$$\downarrow 3783$$

$$\int \frac{\tan(a+bx)}{(c+dx)^2}dx + \frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)} - \frac{2b\left(\frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d} - \frac{\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}\right)$$

input
```
Int[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4222

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

rule 4906

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 4907

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Maple [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin(bx+a)^2 \tan(bx+a)}{(dx+c)^2} dx$$

input

```
int(sin(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x)
```

output

```
int(sin(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(bx+a)^2\tan(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*tan(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(sin(b*x+a)**2*tan(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)**2*tan(a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 242, normalized size of antiderivative = 11.00

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(bx+a)^2\tan(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/4*((-I*exp_integral_e(2, 2*(-I*b*d*x - I*b*c)/d) + I*exp_integral_e(2, -
2*(-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 8*(d^2*x + c*d)*integrate
(sin(2*b*x + 2*a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x
 + 2*a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(2*b*x + 2*a)^2 + c^2 + 2*(d^2*x^
2 + 2*c*d*x + c^2)*cos(2*b*x + 2*a)), x) + (exp_integral_e(2, 2*(-I*b*d*x
- I*b*c)/d) + exp_integral_e(2, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b*c - a*
d)/d))/(d^2*x + c*d)
```

Giac [N/A]

Not integrable

Time = 7.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin\left(bx+a\right)^2\tan\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(sin(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)^2*tan(b*x + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 18.87 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin\left(a+b\,x\right)^2\tan(a+b\,x)}{\left(c+d\,x\right)^2}\,dx$$

input
```
int((sin(a + b*x)^2*tan(a + b*x))/(c + d*x)^2,x)
```

output
```
int((sin(a + b*x)^2*tan(a + b*x))/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(bx+a)^2\tan(bx+a)}{d^2x^2+2cdx+c^2}dx$$

input
```
int(sin(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x)
```

output
```
int((sin(a + b*x)**2*tan(a + b*x))/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.227 $\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$

Optimal result . . . 2052
Mathematica [N/A] . . . 2052
Rubi [N/A] . . . 2053
Maple [N/A] . . . 2053
Fricas [N/A] . . . 2054
Sympy [N/A] . . . 2054
Maxima [N/A] . . . 2055
Giac [N/A] . . . 2055
Mupad [N/A] . . . 2055
Reduce [N/A] . . . 2056

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx = \text{Int}((c+dx)^m \csc(a+bx)\sec(a+bx), x)$$

output
```
Defer(Int)((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 5.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx = \int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\sec(a+bx)(c+dx)^m\,dx$$

$$\downarrow 7299$$

$$\int \csc(a+bx)\sec(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \csc(bx+a)\sec(bx+a)\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x)
```

output
```
int((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)\sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csc(b*x + a)*sec(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 35.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx = \int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*csc(b*x+a)*sec(b*x+a),x)
```

output
```
Integral((c + d*x)**m*csc(a + b*x)*sec(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)\sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)\sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 18.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\cos(a+b\,x)\,\sin(a+b\,x)}\,dx$$

input
```
int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)),x)
```

output
```
int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)\sec(bx+a)\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x)
```

output
```
int((c + d*x)**m*csc(a + b*x)*sec(a + b*x),x)
```

3.228 $\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx$

Optimal result . . . 2057
Mathematica [B] (verified) . . . 2058
Rubi [A] (verified) . . . 2059
Maple [B] (verified) . . . 2063
Fricas [B] (verification not implemented) . . . 2064
Sympy [F] . . . 2065
Maxima [B] (verification not implemented) . . . 2065
Giac [F] . . . 2066
Mupad [F(-1)] . . . 2067
Reduce [F] . . . 2067

Optimal result

Integrand size = 20, antiderivative size = 247

$$\begin{aligned}\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx = &-\frac{2(c+dx)^4 \operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b} \\ &+\frac{2id(c+dx)^3 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} \\ &-\frac{2id(c+dx)^3 \operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2} \\ &-\frac{3d^2(c+dx)^2 \operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3} \\ &+\frac{3d^2(c+dx)^2 \operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3} \\ &-\frac{3id^3(c+dx) \operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4} \\ &+\frac{3id^3(c+dx) \operatorname{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4} \\ &+\frac{3d^4 \operatorname{PolyLog}\left(5,-e^{2i(a+bx)}\right)}{2b^5} \\ &-\frac{3d^4 \operatorname{PolyLog}\left(5,e^{2i(a+bx)}\right)}{2b^5}\end{aligned}$$

output

```
-2*(d*x+c)^4*arctanh(exp(2*I*(b*x+a)))/b+2*I*d*(d*x+c)^3*polylog(2,-exp(2*
I*(b*x+a)))/b^2-2*I*d*(d*x+c)^3*polylog(2,exp(2*I*(b*x+a)))/b^2-3*d^2*(d*x
+c)^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+3*d^2*(d*x+c)^2*polylog(3,exp(2*I*(
b*x+a)))/b^3-3*I*d^3*(d*x+c)*polylog(4,-exp(2*I*(b*x+a)))/b^4+3*I*d^3*(d*x
+c)*polylog(4,exp(2*I*(b*x+a)))/b^4+3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b
^5-3/2*d^4*polylog(5,exp(2*I*(b*x+a)))/b^5
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 578 vs. $2(247) = 494$.

Time $= 0.72$ (sec) , antiderivative size $= 578$, normalized size of antiderivative $= 2.34$

$$\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx$$
$$= \frac{-4b^4c^4\text{arctanh}\left(e^{2i(a+bx)}\right) + 8b^4c^3dx\log\left(1-e^{2i(a+bx)}\right) + 12b^4c^2d^2x^2\log\left(1-e^{2i(a+bx)}\right) + 8b^4cd^3x^3\log\left(1\right.}{}$$

input

```
Integrate[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x],x]
```

output

```
(-4*b^4*c^4*ArcTanh[E^((2*I)*(a + b*x))] + 8*b^4*c^3*d*x*Log[1 - E^((2*I)*
(a + b*x))] + 12*b^4*c^2*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] + 8*b^4*c*d^
3*x^3*Log[1 - E^((2*I)*(a + b*x))] + 2*b^4*d^4*x^4*Log[1 - E^((2*I)*(a + b
*x))] - 8*b^4*c^3*d*x*Log[1 + E^((2*I)*(a + b*x))] - 12*b^4*c^2*d^2*x^2*Lo
g[1 + E^((2*I)*(a + b*x))] - 8*b^4*c*d^3*x^3*Log[1 + E^((2*I)*(a + b*x))]
- 2*b^4*d^4*x^4*Log[1 + E^((2*I)*(a + b*x))] + (4*I)*b^3*d*(c + d*x)^3*Pol
yLog[2, -E^((2*I)*(a + b*x))] - (4*I)*b^3*d*(c + d*x)^3*PolyLog[2, E^((2*I
)*(a + b*x))] - 6*b^2*c^2*d^2*PolyLog[3, -E^((2*I)*(a + b*x))] - 12*b^2*c*
d^3*x*PolyLog[3, -E^((2*I)*(a + b*x))] - 6*b^2*d^4*x^2*PolyLog[3, -E^((2*I
)*(a + b*x))] + 6*b^2*c^2*d^2*PolyLog[3, E^((2*I)*(a + b*x))] + 12*b^2*c*d
^3*x*PolyLog[3, E^((2*I)*(a + b*x))] + 6*b^2*d^4*x^2*PolyLog[3, E^((2*I)*(
a + b*x))] - (6*I)*b*c*d^3*PolyLog[4, -E^((2*I)*(a + b*x))] - (6*I)*b*d^4*
x*PolyLog[4, -E^((2*I)*(a + b*x))] + (6*I)*b*c*d^3*PolyLog[4, E^((2*I)*(a
+ b*x))] + (6*I)*b*d^4*x*PolyLog[4, E^((2*I)*(a + b*x))] + 3*d^4*PolyLog[5
, -E^((2*I)*(a + b*x))] - 3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4919, 3042, 4671, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx$$

$$\downarrow 4919$$

$$2\int (c+dx)^4 \csc(2a+2bx)dx$$

$$\downarrow 3042$$

$$2\int (c+dx)^4 \csc(2a+2bx)dx$$

$$\downarrow 4671$$

$$2\left(-\frac{2d\int (c+dx)^3 \log\left(1-e^{2i(a+bx)}\right)dx}{b}+\frac{2d\int (c+dx)^3 \log\left(1+e^{2i(a+bx)}\right)dx}{b}-\frac{(c+dx)^4 \text{arctanh}\left(e^{2i(a+bx)}\right)}{b}\right)$$

$$\downarrow 3011$$

$$2\left(\frac{2d\left(\frac{i(c+dx)^3 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{3id\int (c+dx)^2 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)^3 \operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b}-\frac{3id\int (c+d}{b}\right.$$

$$\downarrow 7163$$

$$2\left(\frac{2d\left(\frac{i(c+dx)^3 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{3id\left(\frac{id\int (c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)dx}{b}-\frac{i(c+dx)^2 \operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{2b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)}{}\right.\right.$$

$$\downarrow 7163$$

$$2\left(\frac{2d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{3id\left(\frac{id\left(\frac{\int \mathrm{PolyLog}\left(4,-e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{2b}\right)}{b}-\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(3,-e^{2i}\right.\right.}{2b}\right.}{2b}\right.}{b}$$

$$\downarrow 2720$$

$$2\left(\frac{2d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{3id\left(\frac{id\left(\frac{d\int e^{-2i(a+bx)}\,\mathrm{PolyLog}\left(4,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{2b}\right)}{b}-\frac{i(c+\right.}{2b}\right.}{2b}\right.}{b}$$

$$\downarrow 7143$$

$$2\left(-\frac{(c+dx)^4\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{3id\left(\frac{id\left(\frac{d\operatorname{PolyLog}\left(5,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,-\right.}{2b}\right.}{b}\right.}{2b}\right.}{b}\right.$$

input
```
Int[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x],x]
```

output
```
2*(-(((c + d*x)^4*ArcTanh[E^((2*I)*(a + b*x))])/b) + (2*d*(((I/2)*(c + d*x
)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d*(((-1/2*I)*(c + d*x
)^2*PolyLog[3, -E^((2*I)*(a + b*x))])/b + (I*d*(((-1/2*I)*(c + d*x)*PolyLo
g[4, -E^((2*I)*(a + b*x))])/b + (d*PolyLog[5, -E^((2*I)*(a + b*x))])/(4*b^
2)))/b))/b))/b - (2*d*(((I/2)*(c + d*x)^3*PolyLog[2, E^((2*I)*(a + b*x))])
/b - (((3*I)/2)*d*(((-1/2*I)*(c + d*x)^2*PolyLog[3, E^((2*I)*(a + b*x))])/
b + (I*d*(((-1/2*I)*(c + d*x)*PolyLog[4, E^((2*I)*(a + b*x))])/b + (d*Poly
Log[5, E^((2*I)*(a + b*x))])/(4*b^2)))/b))/b))/b)
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))  Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)  Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)  Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n  Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))  Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1241 vs. $2(221) = 442$.

Time = 0.41 (sec) , antiderivative size = 1242, normalized size of antiderivative = 5.03

method	result	size
risch	Expression too large to display	1242

input
```
int((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b*c^4*ln(exp(2*I*(b*x+a))+1)+1/b*c^4*ln(exp(I*(b*x+a))-1)+4/b^4*c*d^3*l
n(1-exp(I*(b*x+a)))*a^3+4/b^2*c^3*d*ln(1-exp(I*(b*x+a)))*a+6/b^3*c^2*d^2*a
^2*ln(exp(I*(b*x+a))-1)-4/b^2*c^3*d*a*ln(exp(I*(b*x+a))-1)-6/b^3*c*d^3*pol
ylog(3,-exp(2*I*(b*x+a)))*x-6/b*d^2*c^2*ln(exp(2*I*(b*x+a))+1)*x^2+6/b*d^2
*c^2*ln(exp(I*(b*x+a))+1)*x^2+6/b*d^2*c^2*ln(1-exp(I*(b*x+a)))*x^2-4/b*c*d
^3*ln(exp(2*I*(b*x+a))+1)*x^3+4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3+24/b^3*c*
d^3*polylog(3,-exp(I*(b*x+a)))*x+24/b^3*c*d^3*polylog(3,exp(I*(b*x+a)))*x-
6/b^3*d^2*c^2*ln(1-exp(I*(b*x+a)))*a^2+4/b*c*d^3*ln(1-exp(I*(b*x+a)))*x^3-
4/b*c^3*d*ln(exp(2*I*(b*x+a))+1)*x+4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x+4/b*c^
3*d*ln(1-exp(I*(b*x+a)))*x-4/b^4*c*d^3*a^3*ln(exp(I*(b*x+a))-1)-12*I/b^2*c
*d^3*polylog(2,exp(I*(b*x+a)))*x^2+6*I/b^2*d^2*c^2*polylog(2,-exp(2*I*(b*x
+a)))*x-12*I/b^2*d^2*c^2*polylog(2,-exp(I*(b*x+a)))*x-12*I/b^2*d^2*c^2*pol
ylog(2,exp(I*(b*x+a)))*x+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4+1/b*d^4*ln(exp(I
*(b*x+a))+1)*x^4-1/b*d^4*ln(exp(2*I*(b*x+a))+1)*x^4+12/b^3*d^4*polylog(3,e
xp(I*(b*x+a)))*x^2-3/b^3*d^4*polylog(3,-exp(2*I*(b*x+a)))*x^2+12/b^3*d^4*p
olylog(3,-exp(I*(b*x+a)))*x^2-1/b^5*d^4*ln(1-exp(I*(b*x+a)))*a^4-3/b^3*d^2
*c^2*polylog(3,-exp(2*I*(b*x+a)))+12/b^3*d^2*c^2*polylog(3,-exp(I*(b*x+a))
)+12/b^3*d^2*c^2*polylog(3,exp(I*(b*x+a)))+1/b^5*d^4*a^4*ln(exp(I*(b*x+a))
-1)+6*I/b^2*c*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2-12*I/b^2*c*d^3*polylog(
2,-exp(I*(b*x+a)))*x^2+2*I/b^2*d^4*polylog(2,-exp(2*I*(b*x+a)))*x^3-4*I...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2600 vs. $2(215) = 430$.

Time = 0.23 (sec) , antiderivative size = 2600, normalized size of antiderivative = 10.53

$$\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

output

```
-1/2*(24*d^4*polylog(5, cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*polylog(5,
 cos(b*x + a) - I*sin(b*x + a)) - 24*d^4*polylog(5, I*cos(b*x + a) + sin(b
*x + a)) - 24*d^4*polylog(5, I*cos(b*x + a) - sin(b*x + a)) - 24*d^4*polyl
og(5, -I*cos(b*x + a) + sin(b*x + a)) - 24*d^4*polylog(5, -I*cos(b*x + a)
- sin(b*x + a)) + 24*d^4*polylog(5, -cos(b*x + a) + I*sin(b*x + a)) + 24*d
^4*polylog(5, -cos(b*x + a) - I*sin(b*x + a)) + 4*(I*b^3*d^4*x^3 + 3*I*b^3
*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(cos(b*x + a) + I*sin(b
*x + a)) + 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*b
^3*c^3*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) + 4*(I*b^3*d^4*x^3 + 3*I*b^
3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(I*cos(b*x + a) + sin(
b*x + a)) + 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*
b^3*c^3*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) + 4*(-I*b^3*d^4*x^3 - 3*I*
b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*dilog(-I*cos(b*x + a) + s
in(b*x + a)) + 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x +
I*b^3*c^3*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 4*(-I*b^3*d^4*x^3 - 3
*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*dilog(-cos(b*x + a) +
I*sin(b*x + a)) + 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x
 + I*b^3*c^3*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^4*d^4*x^4 + 4*b
^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*x +
a) + I*sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2...
```

Sympy [F]

$$\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx = \int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**4*csc(b*x+a)*sec(b*x+a),x)
```

output
```
Integral((c + d*x)**4*csc(a + b*x)*sec(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1794 vs. $2(215) = 430$.

Time = 0.31 (sec) , antiderivative size = 1794, normalized size of antiderivative = 7.26

$$\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

output

```
-1/6*(3*c^4*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)) - 12*a*c^3*d*(
log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b + 18*a^2*c^2*d^2*(log(sin
(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - 12*a^3*c*d^3*(log(sin(b*x +
a)^2 - 1) - log(sin(b*x + a)^2))/b^3 + 3*a^4*d^4*(log(sin(b*x + a)^2 - 1)
- log(sin(b*x + a)^2))/b^4 - (18*d^4*polylog(5, -e^(2*I*b*x + 2*I*a)) - 14
4*d^4*polylog(5, -e^(I*b*x + I*a)) - 144*d^4*polylog(5, e^(I*b*x + I*a)) +
 4*(-3*I*(b*x + a)^4*d^4 + 8*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 9*(-I*b^
2*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a)^2 + 6*(-I*b^3*c^3*d + 3*I
*a*b^2*c^2*d^2 - 3*I*a^2*b*c*d^3 + I*a^3*d^4)*(b*x + a))*arctan2(sin(2*b*x
 + 2*a), cos(2*b*x + 2*a) + 1) + 6*(I*(b*x + a)^4*d^4 + 4*(I*b*c*d^3 - I*a
*d^4)*(b*x + a)^3 + 6*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a
)^2 + 4*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 - I*a^3*d^4)*(b
*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 6*(-I*(b*x + a)^4*d^4 +
 4*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 6*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3
- I*a^2*d^4)*(b*x + a)^2 + 4*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 - 3*I*a^2*b
*c*d^3 + I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) +
12*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 + 2*I*(b*x + a)^3*d^
4 - I*a^3*d^4 + 4*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2
*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a)) + 24*(-I*
b^3*c^3*d + 3*I*a*b^2*c^2*d^2 - 3*I*a^2*b*c*d^3 - I*(b*x + a)^3*d^4 + I...
```

Giac [F]

$$\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx = \int (dx+c)^4 \csc(bx+a)\sec(bx+a)\,dx$$

input

```
integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

output

```
integrate((d*x + c)^4*csc(b*x + a)*sec(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx = \int \frac{(c+d\,x)^4}{\cos(a+b\,x)\,\sin(a+b\,x)}\,dx$$

input
```
int((c + d*x)^4/(cos(a + b*x)*sin(a + b*x)),x)
```

output
```
int((c + d*x)^4/(cos(a + b*x)*sin(a + b*x)), x)
```

Reduce [F]

$$\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx$$
$$= \frac{\left(\int \csc(bx+a)\sec(bx+a)\,x^4dx\right)b\,d^4 + 4\left(\int \csc(bx+a)\sec(bx+a)\,x^3dx\right)bc\,d^3 + 6\left(\int \csc(bx+a)\sec(\right.}{}$$

input
```
int((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x)
```

output
```
(int(csc(a + b*x)*sec(a + b*x)*x**4,x)*b*d**4 + 4*int(csc(a + b*x)*sec(a +
 b*x)*x**3,x)*b*c*d**3 + 6*int(csc(a + b*x)*sec(a + b*x)*x**2,x)*b*c**2*d*
*2 + 4*int(csc(a + b*x)*sec(a + b*x)*x,x)*b*c**3*d - log(tan((a + b*x)/2)
- 1)*c**4 - log(tan((a + b*x)/2) + 1)*c**4 + log(tan((a + b*x)/2))*c**4)/b
```

3.229 $\int (c+dx)^3 \csc(a+bx)\sec(a+bx)\,dx$

Optimal result . . . 2068
Mathematica [A] (verified) . . . 2069
Rubi [A] (verified) . . . 2069
Maple [B] (verified) . . . 2072
Fricas [B] (verification not implemented) . . . 2073
Sympy [F] . . . 2074
Maxima [B] (verification not implemented) . . . 2075
Giac [F] . . . 2076
Mupad [F(-1)] . . . 2076
Reduce [F] . . . 2076

Optimal result

Integrand size = 20, antiderivative size = 197

$$\begin{aligned}\int (c+dx)^3 \csc(a+bx)\sec(a+bx)\,dx = &-\frac{2(c+dx)^3 \operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b} \\ &+\frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} \\ &-\frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2} \\ &-\frac{3d^2(c+dx) \operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} \\ &+\frac{3d^2(c+dx) \operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3} \\ &-\frac{3id^3 \operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4} \\ &+\frac{3id^3 \operatorname{PolyLog}\left(4,e^{2i(a+bx)}\right)}{4b^4}\end{aligned}$$

output

```
-2*(d*x+c)^3*arctanh(exp(2*I*(b*x+a)))/b+3/2*I*d*(d*x+c)^2*polylog(2,-exp(
2*I*(b*x+a)))/b^2-3/2*I*d*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2-3/2*d^
2*(d*x+c)*polylog(3,-exp(2*I*(b*x+a)))/b^3+3/2*d^2*(d*x+c)*polylog(3,exp(2
*I*(b*x+a)))/b^3-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+3/4*I*d^3*poly
log(4,exp(2*I*(b*x+a)))/b^4
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.78

$$\int (c+dx)^3 \csc(a+bx)\sec(a+bx)\,dx$$
$$= \frac{-8b^3c^3\text{arctanh}\left(e^{2i(a+bx)}\right) + 12b^3c^2dx\log\left(1-e^{2i(a+bx)}\right) + 12b^3cd^2x^2\log\left(1-e^{2i(a+bx)}\right) + 4b^3d^3x^3\log\left(1-\right.}{}$$

input
```
Integrate[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x],x]
```

output
```
(-8*b^3*c^3*ArcTanh[E^((2*I)*(a + b*x))] + 12*b^3*c^2*d*x*Log[1 - E^((2*I)
*(a + b*x))] + 12*b^3*c*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] + 4*b^3*d^3*x
^3*Log[1 - E^((2*I)*(a + b*x))] - 12*b^3*c^2*d*x*Log[1 + E^((2*I)*(a + b*x
))] - 12*b^3*c*d^2*x^2*Log[1 + E^((2*I)*(a + b*x))] - 4*b^3*d^3*x^3*Log[1
+ E^((2*I)*(a + b*x))] + (6*I)*b^2*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a +
 b*x))] - (6*I)*b^2*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))] - 6*b*d^
2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))] + 6*b*c*d^2*PolyLog[3, E^((2*
I)*(a + b*x))] + 6*b*d^3*x*PolyLog[3, E^((2*I)*(a + b*x))] - (3*I)*d^3*Pol
yLog[4, -E^((2*I)*(a + b*x))] + (3*I)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])
/(4*b^4)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {4919, 3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \csc(a+bx)\sec(a+bx)\,dx$$
$$\downarrow 4919$$
$$2\int (c+dx)^3 \csc(2a+2bx)dx$$
$$\downarrow 3042$$

$$2\int (c+dx)^3 \csc(2a+2bx)dx$$

$$\downarrow 4671$$

$$2\left(-\frac{3d\int (c+dx)^2 \log\left(1-e^{2i(a+bx)}\right)dx}{2b}+\frac{3d\int (c+dx)^2 \log\left(1+e^{2i(a+bx)}\right)dx}{2b}-\frac{(c+dx)^3 \operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b}\right)$$

$$\downarrow 3011$$

$$2\left(\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int (c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b}-\frac{id\int (c+dx)}{}\right.}{2b}\right.$$

$$\downarrow 7163$$

$$2\left(\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{id\int \operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLo}\right.}{2b}\right.$$

$$\downarrow 2720$$

$$2\left(\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{d\int e^{-2i(a+bx)}\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{3d\left(\right.}{}\right.$$

$$\downarrow 7143$$

$$2\left(-\frac{(c+dx)^3 \operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{d\operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a}\right.}{2b}\right.}{b}\right.}{2b}\right.$$

input
```
Int[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x],x]
```

output
```
2*(-(((c + d*x)^3*ArcTanh[E^((2*I)*(a + b*x))])/b) + (3*d*(((I/2)*(c + d*x
)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b - (I*d*(((-1/2*I)*(c + d*x)*PolyLo
g[3, -E^((2*I)*(a + b*x))])/b + (d*PolyLog[4, -E^((2*I)*(a + b*x))])/(4*b^
2)))/b))/(2*b) - (3*d*(((I/2)*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])
/b - (I*d*(((-1/2*I)*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/b + (d*Pol
yLog[4, E^((2*I)*(a + b*x))])/(4*b^2)))/b))/(2*b))
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))  Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)  Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)  Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 815 vs. $2(167) = 334$.

Time = 0.37 (sec) , antiderivative size = 816, normalized size of antiderivative = 4.14

method	result	size
risch	Expression too large to display	816

input
```
int((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
-3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2-6*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+
a)))*x-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x-1/b*d^3*ln(exp(2*I*(b*x+a
))+1)*x^3-3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))+3*I/b^2*c*d^2*polylog
(2,-exp(2*I*(b*x+a)))*x-1/b*c^3*ln(exp(2*I*(b*x+a))+1)-3/2/b^3*d^3*polylog
(3,-exp(2*I*(b*x+a)))*x-3/b*c^2*d*ln(exp(2*I*(b*x+a))+1)*x+6*I/b^4*d^3*pol
ylog(4,-exp(I*(b*x+a)))-3/b*c*d^2*ln(exp(2*I*(b*x+a))+1)*x^2+3/2*I/b^2*d^3
*polylog(2,-exp(2*I*(b*x+a)))*x^2+3/2*I/b^2*c^2*d*polylog(2,-exp(2*I*(b*x+
a)))-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+6/b^3*c*d^2*polylog(3,-exp(I*(b*x+
a)))+6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^
3+1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+
6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+3
/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)+3/b
^2*d*c^2*ln(1-exp(I*(b*x+a)))*a+3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b*c*d
^2*ln(1-exp(I*(b*x+a)))*x^2+3/b*d*c^2*ln(exp(I*(b*x+a))+1)*x+3/b*d*c^2*ln(
1-exp(I*(b*x+a)))*x-3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-3*I/b^2*d^3
*polylog(2,exp(I*(b*x+a)))*x^2-3*I/b^2*d*c^2*polylog(2,exp(I*(b*x+a)))-3*I
/b^2*d*c^2*polylog(2,-exp(I*(b*x+a)))+1/b*c^3*ln(exp(I*(b*x+a))-1)+1/b*c^3
*ln(exp(I*(b*x+a))+1)+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4-3/4*I*d^3*poly
log(4,-exp(2*I*(b*x+a)))/b^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1786 vs. $2(161) = 322$.

Time $= 0.18$ (sec) , antiderivative size $= 1786$, normalized size of antiderivative $= 9.07$

$$\int (c + dx)^3 \csc(a + bx) \sec(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(6*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4
, cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*polylog(4, I*cos(b*x + a) + sin
(b*x + a)) - 6*I*d^3*polylog(4, I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*p
olylog(4, -I*cos(b*x + a) + sin(b*x + a)) + 6*I*d^3*polylog(4, -I*cos(b*x
+ a) - sin(b*x + a)) - 6*I*d^3*polylog(4, -cos(b*x + a) + I*sin(b*x + a))
+ 6*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) - 3*(I*b^2*d^3*x^2 +
2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(cos(b*x + a) + I*sin(b*x + a)) - 3*(-
I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(cos(b*x + a) - I*sin(
b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(I*cos(
b*x + a) + sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2
*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d
^2*x - I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 3*(I*b^2*d^3*x
^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x + a))
- 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-cos(b*x + a) +
 I*sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog
(-cos(b*x + a) - I*sin(b*x + a)) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*
c^2*d*x + b^3*c^3)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) +
I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x
 + a) - I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 ...
```

Sympy [F]

$$\int (c+dx)^3 \csc(a+bx)\sec(a+bx)\,dx = \int (c+dx)^3 \csc(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*csc(b*x+a)*sec(b*x+a),x)
```

output
```
Integral((c + d*x)**3*csc(a + b*x)*sec(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1078 vs. $2(161) = 322$.

Time = 0.26 (sec) , antiderivative size = 1078, normalized size of antiderivative = 5.47

$$\int (c+dx)^3 \csc(a+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

output
```
-1/6*(3*c^3*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)) - 9*a*c^2*d*(l
og(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b + 9*a^2*c*d^2*(log(sin(b*x
 + a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - 3*a^3*d^3*(log(sin(b*x + a)^2 -
1) - log(sin(b*x + a)^2))/b^3 + (6*I*d^3*polylog(4, -e^(2*I*b*x + 2*I*a))
- 36*I*d^3*polylog(4, -e^(I*b*x + I*a)) - 36*I*d^3*polylog(4, e^(I*b*x + I
*a)) - 2*(-4*I*(b*x + a)^3*d^3 + 9*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 9*
(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*(b*x + a))*arctan2(sin(2*b*x +
2*a), cos(2*b*x + 2*a) + 1) - 6*(I*(b*x + a)^3*d^3 + 3*(I*b*c*d^2 - I*a*d^
3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*(b*x + a))*ar
ctan2(sin(b*x + a), cos(b*x + a) + 1) - 6*(-I*(b*x + a)^3*d^3 + 3*(-I*b*c*
d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*
(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 3*(3*I*b^2*c^2*d - 6
*I*a*b*c*d^2 + 4*I*(b*x + a)^2*d^3 + 3*I*a^2*d^3 + 6*(I*b*c*d^2 - I*a*d^3)
*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a)) - 18*(-I*b^2*c^2*d + 2*I*a*b*c*d^2
 - I*(b*x + a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*dil
og(-e^(I*b*x + I*a)) - 18*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^
3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*dilog(e^(I*b*x + I*a))
 + (4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2
*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)
^2 + 2*cos(2*b*x + 2*a) + 1) - 3*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)...
```

Giac [F]

$$\int (c+dx)^3 \csc(a+bx)\sec(a+bx)\,dx = \int (dx+c)^3 \csc(bx+a)\sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csc(b*x + a)*sec(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc(a+bx)\sec(a+bx)\,dx = \int \frac{(c+d\,x)^3}{\cos(a+b\,x)\,\sin(a+b\,x)}\,dx$$

input
```
int((c + d*x)^3/(cos(a + b*x)*sin(a + b*x)),x)
```

output
```
int((c + d*x)^3/(cos(a + b*x)*sin(a + b*x)), x)
```

Reduce [F]

$$\int (c+dx)^3 \csc(a+bx)\sec(a+bx)\,dx$$
$$= \frac{\left(\int \csc(bx+a)\sec(bx+a)\,x^3dx\right) b\,d^3 + 3\left(\int \csc(bx+a)\sec(bx+a)\,x^2dx\right) bc\,d^2 + 3\left(\int \csc(bx+a)\sec(\right.}{b}$$

input
```
int((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x)
```

output
```
(int(csc(a + b*x)*sec(a + b*x)*x**3,x)*b*d**3 + 3*int(csc(a + b*x)*sec(a +
 b*x)*x**2,x)*b*c*d**2 + 3*int(csc(a + b*x)*sec(a + b*x)*x,x)*b*c**2*d - l
og(tan((a + b*x)/2) - 1)*c**3 - log(tan((a + b*x)/2) + 1)*c**3 + log(tan((
a + b*x)/2))*c**3)/b
```

3.230 $\int (c+dx)^2 \csc(a+bx)\sec(a+bx)\,dx$

Optimal result . . . 2077
Mathematica [A] (verified) . . . 2078
Rubi [A] (verified) . . . 2078
Maple [B] (verified) . . . 2080
Fricas [B] (verification not implemented) . . . 2081
Sympy [F] . . . 2082
Maxima [B] (verification not implemented) . . . 2083
Giac [F] . . . 2083
Mupad [F(-1)] . . . 2084
Reduce [F] . . . 2084

Optimal result

Integrand size = 20, antiderivative size = 127

$$\begin{aligned}\int (c+dx)^2 \csc(a+bx)\sec(a+bx)\,dx = &-\frac{2(c+dx)^2 \operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b} \\ &+ \frac{id(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} \\ &- \frac{id(c+dx)\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2} \\ &- \frac{d^2\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} \\ &+ \frac{d^2\operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}\end{aligned}$$

output

```
-2*(d*x+c)^2*arctanh(exp(2*I*(b*x+a)))/b+I*d*(d*x+c)*polylog(2,-exp(2*I*(b
*x+a)))/b^2-I*d*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,
-exp(2*I*(b*x+a)))/b^3+1/2*d^2*polylog(3,exp(2*I*(b*x+a)))/b^3
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.68

$$\int (c+dx)^2 \csc(a+bx)\sec(a+bx)\,dx$$
$$= \frac{-4b^2c^2\text{arctanh}\left(e^{2i(a+bx)}\right) + 4b^2cdx\log\left(1-e^{2i(a+bx)}\right) + 2b^2d^2x^2\log\left(1-e^{2i(a+bx)}\right) - 4b^2cdx\log\left(1+e^{2i(}\right.}{}$$

input
```
Integrate[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x],x]
```

output
```
(-4*b^2*c^2*ArcTanh[E^((2*I)*(a + b*x))] + 4*b^2*c*d*x*Log[1 - E^((2*I)*(a
 + b*x))] + 2*b^2*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] - 4*b^2*c*d*x*Log[1
 + E^((2*I)*(a + b*x))] - 2*b^2*d^2*x^2*Log[1 + E^((2*I)*(a + b*x))] + (2*
I)*b*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))] - (2*I)*b*d*(c + d*x)*Po
lyLog[2, E^((2*I)*(a + b*x))] - d^2*PolyLog[3, -E^((2*I)*(a + b*x))] + d^2
*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4919, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \csc(a+bx)\sec(a+bx)\,dx$$
$$\downarrow 4919$$
$$2\int (c+dx)^2 \csc(2a+2bx)dx$$
$$\downarrow 3042$$
$$2\int (c+dx)^2 \csc(2a+2bx)dx$$
$$\downarrow 4671$$

$$2\left(-\frac{d\int(c+dx)\log\left(1-e^{2i(a+bx)}\right)dx}{b}+\frac{d\int(c+dx)\log\left(1+e^{2i(a+bx)}\right)dx}{b}-\frac{(c+dx)^2\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b}\right)$$

$$\downarrow 3011$$

$$2\left(\frac{d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b}-\frac{id\int\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right.\right.}{2b}\right.$$

$$\downarrow 2720$$

$$2\left(\frac{d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i(a+bx)}\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-}\right.}{b}\right.$$

$$\downarrow 7143$$

$$2\left(-\frac{(c+dx)^2\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b}+\frac{d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{4b^2}\right)}{b}-\frac{d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,e^{2}\right.\right.}{2b}\right.$$

input
```
Int[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x],x]
```

output
```
2*(-(((c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/b) + (d*(((I/2)*(c + d*x)*
PolyLog[2, -E^((2*I)*(a + b*x))])/b - (d*PolyLog[3, -E^((2*I)*(a + b*x))])
/(4*b^2)))/b - (d*(((I/2)*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b - (
d*PolyLog[3, E^((2*I)*(a + b*x))])/(4*b^2)))/b)
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(111) = 222$.

Time = 0.34 (sec) , antiderivative size = 469, normalized size of antiderivative = 3.69

method	result
risch	$\frac{2cd\ln(1-e^{i(bx+a)})a}{b^2}+\frac{idc\,\mathrm{polylog}(2,-e^{2i(bx+a)})}{b^2}-\frac{d^2\ln(e^{2i(bx+a)}+1)x^2}{b}-\frac{d^2\,\mathrm{polylog}(3,-e^{2i(bx+a)})}{2b^3}+\frac{d^2\ln(e^{i(bx+a)}+1)}{b}$

input
```
int((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/b^2*c*d*ln(1-exp(I*(b*x+a)))*a+I/b^2*d*c*polylog(2,-exp(2*I*(b*x+a)))-1/
b*d^2*ln(exp(2*I*(b*x+a))+1)*x^2-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+
1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+1/b*
d^2*ln(1-exp(I*(b*x+a)))*x^2+2*d^2*polylog(3,exp(I*(b*x+a)))/b^3+1/b^3*d^2
*a^2*ln(exp(I*(b*x+a))-1)-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2-2/b*d*c*ln(ex
p(2*I*(b*x+a))+1)*x+2/b*c*d*ln(exp(I*(b*x+a))+1)*x+2/b*c*d*ln(1-exp(I*(b*x
+a)))*x-2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-2*I/b^2*d^2*polylog(2,exp
(I*(b*x+a)))*x-2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))+1/b*c^2*ln(exp(I*(b*x
+a))+1)-1/b*c^2*ln(exp(2*I*(b*x+a))+1)+1/b*c^2*ln(exp(I*(b*x+a))-1)+I/b^2*
d^2*polylog(2,-exp(2*I*(b*x+a)))*x-2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-
2/b^2*c*d*a*ln(exp(I*(b*x+a))-1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1098 vs. $2(107) = 214$.

Time $= 0.15$ (sec) , antiderivative size $= 1098$, normalized size of antiderivative $= 8.65$

$$\int (c + dx)^2 \csc(a + bx) \sec(a + bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

output

```
1/2*(2*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3, co
s(b*x + a) - I*sin(b*x + a)) - 2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x +
 a)) - 2*d^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 2*d^2*polylog(3,
-I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b
*x + a)) + 2*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylo
g(3, -cos(b*x + a) - I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(cos(b
*x + a) + I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog(cos(b*x + a) -
I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*x +
 a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 2*(
-I*b*d^2*x - I*b*c*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 2*(I*b*d^2*x
 + I*b*c*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*
d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(-
cos(b*x + a) - I*sin(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log
(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(
cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*
log(cos(b*x + a) - I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*
b*c*d - a^2*d^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2
*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) -
 (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) ...
```

Sympy [F]

$$\int (c+dx)^2 \csc(a+bx)\sec(a+bx)\,dx = \int (c+dx)^2 \csc(a+bx)\sec(a+bx)\,dx$$

input

```
integrate((d*x+c)**2*csc(b*x+a)*sec(b*x+a),x)
```

output

```
Integral((c + d*x)**2*csc(a + b*x)*sec(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 599 vs. $2(107) = 214$.

Time $= 0.29$ (sec) , antiderivative size $= 599$, normalized size of antiderivative $= 4.72$

$$\int (c+dx)^2 \csc(a+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(c^2*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)) - 2*a*c*d*(log(s
in(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b + a^2*d^2*(log(sin(b*x + a)^2
- 1) - log(sin(b*x + a)^2))/b^2 + (d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) -
4*d^2*polylog(3, -e^(I*b*x + I*a)) - 4*d^2*polylog(3, e^(I*b*x + I*a)) - 2
*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan2(sin(2*b*x
 + 2*a), cos(2*b*x + 2*a) + 1) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d
^2)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(-I*(b*x + a)^2
*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x +
a) + 1) - 2*(I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*dilog(-e^(2*I*b*x + 2*I*
a)) - 4*(-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*dilog(-e^(I*b*x + I*a)) - 4
*(-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*dilog(e^(I*b*x + I*a)) + ((b*x + a
)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*
(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - ((b
*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x
+ a)^2 - 2*cos(b*x + a) + 1))/b^2)/b
```

Giac [F]

$$\int (c+dx)^2 \csc(a+bx)\sec(a+bx)\,dx = \int (dx+c)^2 \csc(bx+a)\sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csc(b*x + a)*sec(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc(a+bx)\sec(a+bx)\,dx = \int \frac{(c+d\,x)^2}{\cos(a+b\,x)\,\sin(a+b\,x)}\,dx$$

input
```
int((c + d*x)^2/(cos(a + b*x)*sin(a + b*x)),x)
```

output
```
int((c + d*x)^2/(cos(a + b*x)*sin(a + b*x)), x)
```

Reduce [F]

$$\int (c+dx)^2 \csc(a+bx)\sec(a+bx)\,dx$$
$$= \frac{\left(\int \csc(bx+a)\sec(bx+a)\,x^2dx\right) b\,d^2 + 2\left(\int \csc(bx+a)\sec(bx+a)\,xdx\right) bcd - \log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right) c}{b}$$

input
```
int((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x)
```

output
```
(int(csc(a + b*x)*sec(a + b*x)*x**2,x)*b*d**2 + 2*int(csc(a + b*x)*sec(a +
 b*x)*x,x)*b*c*d - log(tan((a + b*x)/2) - 1)*c**2 - log(tan((a + b*x)/2) +
 1)*c**2 + log(tan((a + b*x)/2))*c**2)/b
```

3.231 $\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx$

Optimal result . . . 2085
Mathematica [A] (verified) . . . 2085
Rubi [A] (verified) . . . 2086
Maple [B] (verified) . . . 2088
Fricas [B] (verification not implemented) . . . 2088
Sympy [F] . . . 2089
Maxima [B] (verification not implemented) . . . 2089
Giac [F] . . . 2090
Mupad [F(-1)] . . . 2090
Reduce [F] . . . 2091

Optimal result

Integrand size = 18, antiderivative size = 71

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx = -\frac{2(c+dx)\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b} + \frac{id\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} - \frac{id\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}$$

output

```
-2*(d*x+c)*arctanh(exp(2*I*(b*x+a)))/b+1/2*I*d*polylog(2,-exp(2*I*(b*x+a))
)/b^2-1/2*I*d*polylog(2,exp(2*I*(b*x+a)))/b^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.86

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx = -\frac{c\operatorname{arctanh}(\cos(2a+2bx))}{b} + \frac{d\left((2a+2bx)\left(\log\left(1-e^{i(2a+2bx)}\right)-\log\left(1+e^{i(2a+2bx)}\right)\right)-2a\log\left(\tan\left(\frac{1}{2}(2a+2bx)\right)\right)+i\left(\operatorname{PolyLog}\left(2\right.\right.\right.}{2b^2}$$

input

```
Integrate[(c + d*x)*Csc[a + b*x]*Sec[a + b*x],x]
```

output

```
-((c*ArcTanh[Cos[2*a + 2*b*x]])/b) + (d*((2*a + 2*b*x)*(Log[1 - E^(I*(2*a
+ 2*b*x))] - Log[1 + E^(I*(2*a + 2*b*x))]) - 2*a*Log[Tan[(2*a + 2*b*x)/2]]
 + I*(PolyLog[2, -E^(I*(2*a + 2*b*x))] - PolyLog[2, E^(I*(2*a + 2*b*x))]))
)/(2*b^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx$$

$$\downarrow 4919$$

$$2\int (c+dx)\csc(2a+2bx)dx$$

$$\downarrow 3042$$

$$2\int (c+dx)\csc(2a+2bx)dx$$

$$\downarrow 4671$$

$$2\left(-\frac{d\int \log\left(1-e^{2i(a+bx)}\right)dx}{2b}+\frac{d\int \log\left(1+e^{2i(a+bx)}\right)dx}{2b}-\frac{(c+dx)\text{arctanh}\left(e^{2i(a+bx)}\right)}{b}\right)$$

$$\downarrow 2715$$

$$2\left(\frac{id\int e^{-2i(a+bx)}\log\left(1-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{id\int e^{-2i(a+bx)}\log\left(1+e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{(c+dx)\text{arctanh}(}{b}\right.$$

$$\downarrow 2838$$

$$2\left(-\frac{(c+dx)\text{arctanh}\left(e^{2i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{id\,\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{4b^2}\right)$$

input
```
Int[(c + d*x)*Csc[a + b*x]*Sec[a + b*x],x]
```

output
```
2*(-(((c + d*x)*ArcTanh[E^((2*I)*(a + b*x))])/b) + ((I/4)*d*PolyLog[2, -E^
((2*I)*(a + b*x))])/b^2 - ((I/4)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2)
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(59) = 118$.

Time = 0.28 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.93

method	result
risch	$\frac{c\ln(e^{i(bx+a)}+1)}{b} - \frac{c\ln(e^{2i(bx+a)}+1)}{b} + \frac{c\ln(e^{i(bx+a)}-1)}{b} - \frac{d\ln(e^{2i(bx+a)}+1)x}{b} + \frac{id\,\mathrm{polylog}(2,-e^{2i(bx+a)})}{2b^2} + \frac{d\ln(e^{i(bx+}$

input
```
int((d*x+c)*csc(b*x+a)*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*c*ln(exp(I*(b*x+a))+1)-1/b*c*ln(exp(2*I*(b*x+a))+1)+1/b*c*ln(exp(I*(b*
x+a))-1)-1/b*d*ln(exp(2*I*(b*x+a))+1)*x+1/2*I*d*polylog(2,-exp(2*I*(b*x+a)
))/b^2+1/b*d*ln(exp(I*(b*x+a))+1)*x-I*d*polylog(2,-exp(I*(b*x+a)))/b^2+1/b
*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(b*x+a)))*a-I*d*polylog(2,exp
(I*(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(55) = 110$.

Time = 0.12 (sec) , antiderivative size = 554, normalized size of antiderivative = 7.80

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(-I*d*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*dilog(cos(b*x + a) -
I*sin(b*x + a)) - I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(I*c
os(b*x + a) - sin(b*x + a)) + I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) -
I*d*dilog(-I*cos(b*x + a) - sin(b*x + a)) + I*d*dilog(-cos(b*x + a) + I*si
n(b*x + a)) - I*d*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b*d*x + b*c)*lo
g(cos(b*x + a) + I*sin(b*x + a) + 1) - (b*c - a*d)*log(cos(b*x + a) + I*si
n(b*x + a) + I) + (b*d*x + b*c)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (
b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b*d*x + a*d)*log(I*co
s(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)*log(I*cos(b*x + a) - sin(b*
x + a) + 1) - (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d
*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d)*log(-1/2*c
os(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b*c - a*d)*log(-1/2*cos(b*x + a
) - 1/2*I*sin(b*x + a) + 1/2) + (b*d*x + a*d)*log(-cos(b*x + a) + I*sin(b*
x + a) + 1) - (b*c - a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*d*x
 + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - (b*c - a*d)*log(-cos(b*x
 + a) - I*sin(b*x + a) + I))/b^2
```

Sympy [F]

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx = \int (c+dx)\csc(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x)
```

output
```
Integral((c + d*x)*csc(a + b*x)*sec(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(55) = 110$.

Time $= 0.26$ (sec) , antiderivative size $= 269$, normalized size of antiderivative $= 3.79$

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx =$$

$$-\frac{2i\,bdx\arctan\left(\sin\left(bx+a\right),-\cos\left(bx+a\right)+1\right)-2i\,bc\arctan\left(\sin\left(bx+a\right),\cos\left(bx+a\right)-1\right)-2\left(-i\,b\right.}{}$$

input
```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(2*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*I*b*c*arctan2
(sin(b*x + a), cos(b*x + a) - 1) - 2*(-I*b*d*x - I*b*c)*arctan2(sin(2*b*x
+ 2*a), cos(2*b*x + 2*a) + 1) - 2*(I*b*d*x + I*b*c)*arctan2(sin(b*x + a),
cos(b*x + a) + 1) - I*d*dilog(-e^(2*I*b*x + 2*I*a)) + 2*I*d*dilog(-e^(I*b*
x + I*a)) + 2*I*d*dilog(e^(I*b*x + I*a)) + (b*d*x + b*c)*log(cos(2*b*x + 2
*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (b*d*x + b*c)*log(c
os(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (b*d*x + b*c)*log(c
os(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1))/b^2
```

Giac [F]

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx = \int (dx+c)\csc\left(bx+a\right)\sec\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*csc(b*x + a)*sec(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx = \int \frac{c+d\,x}{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)}\,dx$$

input
```
int((c + d*x)/(cos(a + b*x)*sin(a + b*x)),x)
```

output
```
int((c + d*x)/(cos(a + b*x)*sin(a + b*x)), x)
```

Reduce [F]

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx$$
$$= \frac{\left(\int \csc(bx+a)\sec(bx+a)\,x dx\right) bd - \log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right) c - \log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right) c + \log\left(\tan\left(\frac{bx}{2}+\right.\right.}{b}$$

input
```
int((d*x+c)*csc(b*x+a)*sec(b*x+a),x)
```

output
```
(int(csc(a + b*x)*sec(a + b*x)*x,x)*b*d - log(tan((a + b*x)/2) - 1)*c - lo
g(tan((a + b*x)/2) + 1)*c + log(tan((a + b*x)/2))*c)/b
```

3.232 $\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$

Optimal result . . . 2092
Mathematica [N/A] . . . 2092
Rubi [N/A] . . . 2093
Maple [N/A] . . . 2094
Fricas [N/A] . . . 2094
Sympy [N/A] . . . 2095
Maxima [N/A] . . . 2095
Giac [N/A] . . . 2095
Mupad [N/A] . . . 2096
Reduce [N/A] . . . 2096

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx = 2\text{Int}\left(\frac{\csc(2a+2bx)}{c+dx}, x\right)$$

output
```
2*Defer(Int)(csc(2*b*x+2*a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 2.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x),x]
```

output
```
Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4919, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$$
$$\downarrow 4919$$
$$2\int \frac{\csc(2a+2bx)}{c+dx}dx$$
$$\downarrow 3042$$
$$2\int \frac{\csc(2a+2bx)}{c+dx}dx$$
$$\downarrow 4680$$
$$2\int \frac{\csc(2a+2bx)}{c+dx}dx$$

input
```
Int[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4919

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\csc(bx+a)\sec(bx+a)}{dx+c}dx$$

input

```
int(csc(b*x+a)*sec(b*x+a)/(d*x+c),x)
```

output

```
int(csc(b*x+a)*sec(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)}{dx+c}\,dx$$

input

```
integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output

```
integral(csc(b*x + a)*sec(b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x)
```

output
```
Integral(csc(a + b*x)*sec(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 19.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos(a+b\,x)\,\sin(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(1/(cos(a + b*x)*sin(a + b*x)*(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)*sin(a + b*x)*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)}{dx+c}dx$$

input
```
int(csc(b*x+a)*sec(b*x+a)/(d*x+c),x)
```

output
```
int((csc(a + b*x)*sec(a + b*x))/(c + d*x),x)
```

3.233 $\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 2097
Mathematica [N/A] . . . 2097
Rubi [N/A] . . . 2098
Maple [N/A] . . . 2099
Fricas [N/A] . . . 2099
Sympy [N/A] . . . 2100
Maxima [N/A] . . . 2100
Giac [N/A] . . . 2100
Mupad [N/A] . . . 2101
Reduce [N/A] . . . 2101

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = 2\text{Int}\left(\frac{\csc(2a+2bx)}{(c+dx)^2}, x\right)$$

output
```
2*Defer(Int)(csc(2*b*x+2*a)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 3.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4919, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$
$$\downarrow 4919$$
$$2\int \frac{\csc(2a+2bx)}{(c+dx)^2}dx$$
$$\downarrow 3042$$
$$2\int \frac{\csc(2a+2bx)}{(c+dx)^2}dx$$
$$\downarrow 4680$$
$$2\int \frac{\csc(2a+2bx)}{(c+dx)^2}dx$$

input
```
Int[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\csc(bx+a)\sec(bx+a)}{(dx+c)^2}dx$$

input
```
int(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)*sec(a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c)^2, x)
```

Giac [N/A]

Not integrable

Time = 3.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 19.68 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\cos(a+b\,x)\,\sin(a+b\,x)\,(c+d\,x)^2}\,dx$$

input
```
int(1/(cos(a + b*x)*sin(a + b*x)*(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)*sin(a + b*x)*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)}{d^2x^2+2cdx+c^2}dx$$

input
```
int(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x)
```

output
```
int((csc(a + b*x)*sec(a + b*x))/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.234 $\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx$

Optimal result . . . 2102
Mathematica [N/A] . . . 2102
Rubi [N/A] . . . 2103
Maple [N/A] . . . 2103
Fricas [N/A] . . . 2104
Sympy [F(-1)] . . . 2104
Maxima [N/A] . . . 2104
Giac [N/A] . . . 2105
Mupad [N/A] . . . 2105
Reduce [N/A] . . . 2106

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx = \text{Int}\big((c+dx)^m \csc^2(a+bx)\sec(a+bx), x\big)$$

output
```
Defer(Int)((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 14.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx = \int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\sec(a+bx)(c+dx)^m\,dx$$

$$\downarrow 7299$$

$$\int \csc^2(a+bx)\sec(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x)
```

output
```
int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*csc(b*x+a)**2*sec(b*x+a),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx = \int (dx+c)^m \csc\left(bx+a\right)^2 \sec\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 19.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2}\,dx$$

input
```
int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)^2),x)
```

output
```
int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx = \int (dx+c)^m \csc\left(bx+a\right)^2 \sec\left(bx+a\right) dx$$

input
```
int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x)
```

output
```
int((c + d*x)**m*csc(a + b*x)**2*sec(a + b*x),x)
```

3.235 $\int (c+dx)^3 \csc^2(a+bx)\sec(a+bx)\,dx$

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Optimal result

Integrand size = 22, antiderivative size = 350

$$\begin{aligned}
&\int (c+dx)^3 \csc^2(a+bx)\sec(a+bx)\,dx \\
&= -\frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{(c+dx)^3 \csc(a+bx)}{b} + \frac{6id^2(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{3id(c+dx)^2\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{3id(c+dx)^2\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} - \frac{6id^2(c+dx)\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3} \\
&\quad - \frac{6d^3\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4} - \frac{6d^2(c+dx)\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{6d^2(c+dx)\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} + \frac{6d^3\,\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^4} \\
&\quad - \frac{6id^3\,\mathrm{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^4} + \frac{6id^3\,\mathrm{PolyLog}\left(4,ie^{i(a+bx)}\right)}{b^4}
\end{aligned}$$

output

```
-2*I*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b-6*d*(d*x+c)^2*arctanh(exp(I*(b*x+a
)))/b^2-(d*x+c)^3*csc(b*x+a)/b+6*I*d^2*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/
b^3+3*I*d*(d*x+c)^2*polylog(2,-I*exp(I*(b*x+a)))/b^2-3*I*d*(d*x+c)^2*polyl
og(2,I*exp(I*(b*x+a)))/b^2-6*I*d^2*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^3-6
*d^3*polylog(3,-exp(I*(b*x+a)))/b^4-6*d^2*(d*x+c)*polylog(3,-I*exp(I*(b*x+
a)))/b^3+6*d^2*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^3+6*d^3*polylog(3,exp
(I*(b*x+a)))/b^4-6*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(
4,I*exp(I*(b*x+a)))/b^4
```

Mathematica [A] (verified)

Time = 5.92 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.83

$$\int (c+dx)^3 \csc^2(a+bx)\sec(a+bx)\,dx$$
$$= \frac{-2b^3(c+dx)^3\csc(a) + 6d\big(b^2(c+dx)^2\log\left(1-e^{i(a+bx)}\right) - b^2(c+dx)^2\log\left(1+e^{i(a+bx)}\right) + 2ibd(c+dx)\mathrm{L}}{}$$

input

```
Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x],x]
```

output

```
(-2*b^3*(c + d*x)^3*Csc[a] + 6*d*(b^2*(c + d*x)^2*Log[1 - E^(I*(a + b*x))]
 - b^2*(c + d*x)^2*Log[1 + E^(I*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[
2, -E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] - 2
*d^2*PolyLog[3, -E^(I*(a + b*x))] + 2*d^2*PolyLog[3, E^(I*(a + b*x))]) + 2
*((-2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 - I*E^(I*(a
 + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1
 - I*E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] - 3*b^3*c
*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + I*E^(I*(a + b*x)
)] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))] - (3*I)*b^2*
d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, (-I)*E^
(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*c*d^2*Po
lyLog[3, I*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))] - (6
*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, I*E^(I*(a
+ b*x))]) + b^3*(c + d*x)^3*Csc[a/2]*Csc[(a + b*x)/2]*Sin[(b*x)/2] - b^3*(
c + d*x)^3*Sec[a/2]*Sec[(a + b*x)/2]*Sin[(b*x)/2])/(2*b^4)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4920, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \csc^2(a+bx)\sec(a+bx)\,dx$$

$$\downarrow 4920$$

$$-3d\int (c+dx)^2\left(\frac{\operatorname{arctanh}(\sin(a+bx))}{b}-\frac{\csc(a+bx)}{b}\right)dx+\frac{(c+dx)^3\operatorname{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

$$\downarrow 7292$$

$$-3d\int \frac{(c+dx)^2(\operatorname{arctanh}(\sin(a+bx))-\csc(a+bx))}{b}dx+\frac{(c+dx)^3\operatorname{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

$$\downarrow 27$$

$$-\frac{3d\int (c+dx)^2(\operatorname{arctanh}(\sin(a+bx))-\csc(a+bx))dx}{b}+\frac{(c+dx)^3\operatorname{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

$$\downarrow 7293$$

$$-\frac{3d\int \left((c+dx)^2\operatorname{arctanh}(\sin(a+bx))-(c+dx)^2\csc(a+bx)\right)dx}{b}+\frac{(c+dx)^3\operatorname{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

$$\downarrow 2009$$

$$-\frac{3d\left(\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{3d}+\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{(c+dx)^3\operatorname{arctanh}(\sin(a+bx))}{3d}+\frac{2d^2\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}-\frac{2d^2}{}\right.}{\frac{(c+dx)^3\operatorname{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}}$$

input
```
Int[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x],x]
```

output
```
((c + d*x)^3*ArcTanh[Sin[a + b*x]])/b - ((c + d*x)^3*Csc[a + b*x])/b - (3*
d*((((2*I)/3)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/d + (2*(c + d*x)^2*ArcT
anh[E^(I*(a + b*x))])/b + ((c + d*x)^3*ArcTanh[Sin[a + b*x]])/(3*d) - ((2*
I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (I*(c + d*x)^2*PolyLog[
2, (-I)*E^(I*(a + b*x))])/b + (I*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))]
)/b + ((2*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 + (2*d^2*PolyLog
[3, -E^(I*(a + b*x))])/b^3 + (2*d*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x)
)])/b^2 - (2*d*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^2 - (2*d^2*PolyL
og[3, E^(I*(a + b*x))])/b^3 + ((2*I)*d^2*PolyLog[4, (-I)*E^(I*(a + b*x))])
/b^3 - ((2*I)*d^2*PolyLog[4, I*E^(I*(a + b*x))])/b^3))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1157 vs. 2(313) = 626.

Time = 0.72 (sec) , antiderivative size = 1158, normalized size of antiderivative = 3.31

method	result	size
risch	Expression too large to display	1158

input
```
int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-6*I*d^3/b^3*polylog(2,exp(I*(b*x+a)))*x+6*I*d^3/b^3*polylog(2,-exp(I*(b*x
+a)))*x-2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*(b*x+a))/b/(exp(2*I*
(b*x+a))-1)+3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a^2+3*d^3/b^2*ln(1-exp(I*(b*x+a
)))*x^2-3*d^3/b^2*ln(exp(I*(b*x+a))+1)*x^2+6/b^3*d^2*c*polylog(3,I*exp(I*(
b*x+a)))+1/b^4*a^3*d^3*ln(1-I*exp(I*(b*x+a)))+3/b^4*d^3*a^2*ln(exp(I*(b*x+
a))-1)-6/b^3*d^2*c*polylog(3,-I*exp(I*(b*x+a)))+3/b^2*c^2*d*ln(exp(I*(b*x+
a))-1)-3/b^2*c^2*d*ln(exp(I*(b*x+a))+1)+1/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3
-1/b*d^3*ln(I*exp(I*(b*x+a))+1)*x^3-6/b^3*d^3*polylog(3,-I*exp(I*(b*x+a)))
*x+6/b^3*d^3*polylog(3,I*exp(I*(b*x+a)))*x-1/b^4*a^3*d^3*ln(I*exp(I*(b*x+a
))+1)-2*I/b*c^3*arctan(exp(I*(b*x+a)))-6*d^2/b^2*c*ln(exp(I*(b*x+a))+1)*x-
6*I/b^3*a^2*c*d^2*arctan(exp(I*(b*x+a)))+6*I/b^2*a*c^2*d*arctan(exp(I*(b*x
+a)))-6*I/b^2*d^2*c*polylog(2,I*exp(I*(b*x+a)))*x+6*I/b^2*d^2*c*polylog(2,
-I*exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-I*exp(I*(b*x+a)))*a-3/b^2*c^2*d*ln(I
*exp(I*(b*x+a))+1)*a-3/b*d^2*c*ln(I*exp(I*(b*x+a))+1)*x^2+6/b^3*d^3*ln(1-e
xp(I*(b*x+a)))*a*x-3/b^3*a^2*c*d^2*ln(1-I*exp(I*(b*x+a)))+3/b*c^2*d*ln(1-I
*exp(I*(b*x+a)))*x-3/b*c^2*d*ln(I*exp(I*(b*x+a))+1)*x+3/b*d^2*c*ln(1-I*exp
(I*(b*x+a)))*x^2-6/b^3*c*d^2*a*ln(exp(I*(b*x+a))-1)+3/b^3*a^2*c*d^2*ln(I*e
xp(I*(b*x+a))+1)+2*I/b^4*a^3*d^3*arctan(exp(I*(b*x+a)))-6*I/b^4*a*d^3*dilo
g(exp(I*(b*x+a))+1)+6*I/b^3*c*d^2*dilog(exp(I*(b*x+a)))-6*I/b^4*d^3*a*dilo
g(exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,I*exp(I*(b*x+a)))+6*I/b^3*c*d...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1753 vs. $2(296) = 592$.

Time = 0.19 (sec) , antiderivative size = 1753, normalized size of antiderivative = 5.01

$$\int (c+dx)^3 \csc^2(a+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 6*I*d^
3*polylog(4, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 6*I*d^3*polylog
(4, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + 6*I*d^3*polylog(4, -I*co
s(b*x + a) + sin(b*x + a))*sin(b*x + a) + 6*I*d^3*polylog(4, -I*cos(b*x +
a) - sin(b*x + a))*sin(b*x + a) - 6*d^3*polylog(3, cos(b*x + a) + I*sin(b*
x + a))*sin(b*x + a) - 6*d^3*polylog(3, cos(b*x + a) - I*sin(b*x + a))*sin
(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a)
+ 6*d^3*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*(I*b*d
^3*x + I*b*c*d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*(-
I*b*d^3*x - I*b*c*d^2)*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) +
 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(I*cos(b*x + a) +
sin(b*x + a))*sin(b*x + a) + 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^
2*d)*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + 3*(-I*b^2*d^3*x^2
 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a))*si
n(b*x + a) + 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-I*c
os(b*x + a) - sin(b*x + a))*sin(b*x + a) + 6*(I*b*d^3*x + I*b*c*d^2)*dilog
(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*(-I*b*d^3*x - I*b*c*d^2)
*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b
^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a
) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a...
```

Sympy [F]

$$\int (c+dx)^3 \csc^2(a+bx)\sec(a+bx)\,dx = \int (c+dx)^3 \csc^2(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*csc(b*x+a)**2*sec(b*x+a),x)
```

output
```
Integral((c + d*x)**3*csc(a + b*x)**2*sec(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3257 vs. $2(296) = 592$.

Time = 1.23 (sec) , antiderivative size = 3257, normalized size of antiderivative = 9.31

$$\int (c+dx)^3 \csc^2(a+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
```

output

```
-1/2*(c^3*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))
 - 3*a*c^2*d*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) -
1))/b + 3*a^2*c*d^2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x
+ a) - 1))/b^2 - a^3*d^3*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin
(b*x + a) - 1))/b^3 - 2*(2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a
)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a) - ((b*x + a)^3*d^3 +
 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(
b*x + a))*cos(2*b*x + 2*a) - (I*(b*x + a)^3*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*
(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*(b*x + a))*sin(2
*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + 2*((b*x + a)^3*d^3
+ 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*
(b*x + a) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^
2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (I*(b*x + a)^3*
d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2
 + I*a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x
+ a) + 1) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*
c*d^2 - a*d^3)*(b*x + a) - (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^
2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (I*b^2*c^2*d - 2
*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*
x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 6*(...
```

Giac [F]

$$\int (c+dx)^3 \csc^2(a+bx)\sec(a+bx)\,dx = \int (dx+c)^3 \csc\left(bx+a\right)^2 \sec\left(bx+a\right)\,dx$$

input

```
integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
```

output

```
integrate((d*x + c)^3*csc(b*x + a)^2*sec(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^2(a+bx)\sec(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)^3/(cos(a + b*x)*sin(a + b*x)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^3 \csc^2(a+bx)\sec(a+bx)\,dx$$

$$= \frac{24\cos(bx+a)\,b\,c^2dx - 12\left(\int \frac{x}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2}dx\right)\sin(bx+a)\,b^2c^2d + 2\left(\int \csc(bx+a)^2\sec(bx+\right.}{}$$

input
```
int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x)
```

output
```
(24*cos(a + b*x)*b*c**2*d*x - 12*int(x/(tan((a + b*x)/2)**4 - tan((a + b*x
)/2)**2),x)*sin(a + b*x)*b**2*c**2*d + 2*int(csc(a + b*x)**2*sec(a + b*x)*
x**3,x)*sin(a + b*x)*b**2*d**3 + 6*int(csc(a + b*x)**2*sec(a + b*x)*x**2,x
)*sin(a + b*x)*b**2*c*d**2 + 24*log(tan((a + b*x)/2)**2 + 1)*sin(a + b*x)*
c**2*d - 2*log(tan((a + b*x)/2) - 1)*sin(a + b*x)*b*c**3 + 2*log(tan((a +
b*x)/2) + 1)*sin(a + b*x)*b*c**3 - 42*log(tan((a + b*x)/2))*sin(a + b*x)*c
**2*d + 3*sin(a + b*x)*b**2*c**2*d*x**2 - 2*b*c**3 + 18*b*c**2*d*x)/(2*sin
(a + b*x)*b**2)
```

3.236 $\int (c+dx)^2 \csc^2(a+bx)\sec(a+bx)\,dx$

Optimal result . . . 2116
Mathematica [B] (warning: unable to verify) . . . 2117
Rubi [A] (verified) . . . 2117
Maple [B] (verified) . . . 2119
Fricas [B] (verification not implemented) . . . 2120
Sympy [F] . . . 2121
Maxima [B] (verification not implemented) . . . 2122
Giac [F(-2)] . . . 2123
Mupad [F(-1)] . . . 2123
Reduce [F] . . . 2123

Optimal result

Integrand size = 22, antiderivative size = 226

$$\begin{aligned}
&\int (c+dx)^2 \csc^2(a+bx)\sec(a+bx)\,dx \\
&= -\frac{2i(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b} - \frac{4d(c+dx)\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2\csc(a+bx)}{b} \\
&\quad + \frac{2id^2\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3} + \frac{2id(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{2id(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} - \frac{2id^2\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3} \\
&\quad - \frac{2d^2\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} + \frac{2d^2\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3}
\end{aligned}$$

output

```
-2*I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b-4*d*(d*x+c)*arctanh(exp(I*(b*x+a))
)/b^2-(d*x+c)^2*csc(b*x+a)/b+2*I*d^2*polylog(2,-exp(I*(b*x+a)))/b^3+2*I*d*
(d*x+c)*polylog(2,-I*exp(I*(b*x+a)))/b^2-2*I*d*(d*x+c)*polylog(2,I*exp(I*(
b*x+a)))/b^2-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^3-2*d^2*polylog(3,-I*exp(
I*(b*x+a)))/b^3+2*d^2*polylog(3,I*exp(I*(b*x+a)))/b^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 467 vs. $2(226) = 452$.

Time $= 5.36$ (sec) , antiderivative size $= 467$, normalized size of antiderivative $= 2.07$

$$\int (c+dx)^2 \csc^2(a+bx)\sec(a+bx)\,dx$$

$$= \frac{-4ib^2c^2\arctan\left(e^{i(a+bx)}\right) - 8bcd\text{arctanh}\left(\cos(a) - \sin(a)\tan\left(\frac{bx}{2}\right)\right) + 8d^2\arctan(\tan(a))\text{arctanh}\left(\cos(a) -$$

input
```
Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x],x]
```

output
```
((-4*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] - 8*b*c*d*ArcTanh[Cos[a] - Sin[a]*
Tan[(b*x)/2]] + 8*d^2*ArcTan[Tan[a]]*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]]
 - 2*b^2*(c + d*x)^2*Csc[a] + 4*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] + 2*b
^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - 4*b^2*c*d*x*Log[1 + I*E^(I*(a + b*
x))] - 2*b^2*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + (4*I)*b*d*(c + d*x)*Poly
Log[2, (-I)*E^(I*(a + b*x))] - (4*I)*b*d*(c + d*x)*PolyLog[2, I*E^(I*(a +
b*x))] - 4*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 4*d^2*PolyLog[3, I*E^(I*
(a + b*x))] + (4*d^2*((b*x + ArcTan[Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[T
an[a]]))] - Log[1 + E^(I*(b*x + ArcTan[Tan[a]]))]) + I*PolyLog[2, -E^(I*(b
*x + ArcTan[Tan[a]]))] - I*PolyLog[2, E^(I*(b*x + ArcTan[Tan[a]]))])*Sec[a
])/Sqrt[Sec[a]^2] + b^2*(c + d*x)^2*Csc[a/2]*Csc[(a + b*x)/2]*Sin[(b*x)/2]
 - b^2*(c + d*x)^2*Sec[a/2]*Sec[(a + b*x)/2]*Sin[(b*x)/2])/(2*b^3)
```

Rubi [A] (verified)

Time $= 0.73$ (sec) , antiderivative size $= 260$, normalized size of antiderivative $= 1.15$, number of steps used $= 5$, number of rules used $= 5$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used $= \{4920, 7292, 27, 7293, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \csc^2(a+bx)\sec(a+bx)\,dx$$

$$\downarrow 4920$$

$$-2d\int(c+dx)\left(\frac{\text{arctanh}(\sin(a+bx))}{b}-\frac{\csc(a+bx)}{b}\right)dx+\frac{(c+dx)^2\text{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

$$\downarrow 7292$$

$$-2d\int\frac{(c+dx)(\text{arctanh}(\sin(a+bx))-\csc(a+bx))}{b}dx+\frac{(c+dx)^2\text{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

$$\downarrow 27$$

$$-\frac{2d\int(c+dx)(\text{arctanh}(\sin(a+bx))-\csc(a+bx))dx}{b}+\frac{(c+dx)^2\text{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

$$\downarrow 7293$$

$$-\frac{2d\int((c+dx)\text{arctanh}(\sin(a+bx))-(c+dx)\csc(a+bx))dx}{b}+\frac{(c+dx)^2\text{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

$$\downarrow 2009$$

$$-\frac{2d\left(\frac{i(c+dx)^2\arctan(e^{i(a+bx)})}{d}+\frac{2(c+dx)\text{arctanh}(e^{i(a+bx)})}{b}+\frac{(c+dx)^2\text{arctanh}(\sin(a+bx))}{2d}-\frac{id\,\text{PolyLog}(2,-e^{i(a+bx)})}{b^2}+\frac{id\,\text{PolyL}}{}\right.}{\frac{(c+dx)^2\text{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)^2\csc(a+bx)}{b}}$$

input

```
Int[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x],x]
```

output

```
((c + d*x)^2*ArcTanh[Sin[a + b*x]])/b - ((c + d*x)^2*Csc[a + b*x])/b - (2*
d*((I*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/d + (2*(c + d*x)*ArcTanh[E^(I*(
a + b*x))])/b + ((c + d*x)^2*ArcTanh[Sin[a + b*x]])/(2*d) - (I*d*PolyLog[2
, -E^(I*(a + b*x))])/b^2 - (I*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/
b + (I*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b + (I*d*PolyLog[2, E^(I*(
a + b*x))])/b^2 + (d*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^2 - (d*PolyLog[3,
 I*E^(I*(a + b*x))])/b^2))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 555 vs. $2(201) = 402$.

Time = 0.53 (sec) , antiderivative size = 556, normalized size of antiderivative = 2.46

method	result
risch	$\frac{2icd\,\mathrm{polylog}\left(2,-ie^{i(bx+a)}\right)}{b^2}+\frac{2cd\ln\left(1-ie^{i(bx+a)}\right)x}{b}+\frac{2cd\ln\left(1-ie^{i(bx+a)}\right)a}{b^2}+\frac{2d^2\,\mathrm{polylog}\left(3,ie^{i(bx+a)}\right)}{b^3}-\frac{2d^2\ln\left(e^{i(bx+a)}+\right.}{b^2}$

input
```
int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
2*I/b^2*c*d*polylog(2,-I*exp(I*(b*x+a)))+2/b*c*d*ln(1-I*exp(I*(b*x+a)))*x+
2/b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a+2*d^2*polylog(3,I*exp(I*(b*x+a)))/b^3-2
*d^2/b^2*ln(exp(I*(b*x+a))+1)*x-2*I/b^2*d^2*polylog(2,I*exp(I*(b*x+a)))*x-
2*I/b^2*c*d*polylog(2,I*exp(I*(b*x+a)))-2*I*(d^2*x^2+2*c*d*x+c^2)*exp(I*(b
*x+a))/b/(exp(2*I*(b*x+a))-1)+1/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2-2*d^2*pol
ylog(3,-I*exp(I*(b*x+a)))/b^3+2*I/b^3*d^2*dilog(exp(I*(b*x+a)))+2*I/b^2*d^
2*polylog(2,-I*exp(I*(b*x+a)))*x-1/b^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)))-2*I/
b^3*d^2*a^2*arctan(exp(I*(b*x+a)))-2*d/b^2*c*ln(exp(I*(b*x+a))+1)-2/b*c*d*
ln(I*exp(I*(b*x+a))+1)*x-1/b*d^2*ln(I*exp(I*(b*x+a))+1)*x^2-2/b^2*c*d*ln(I
*exp(I*(b*x+a))+1)*a+2*d/b^2*c*ln(exp(I*(b*x+a))-1)+1/b^3*a^2*d^2*ln(I*exp
(I*(b*x+a))+1)-2*d^2/b^3*a*ln(exp(I*(b*x+a))-1)+4*I/b^2*c*d*a*arctan(exp(I
*(b*x+a)))+2*I/b^3*d^2*dilog(exp(I*(b*x+a))+1)-2*I/b*c^2*arctan(exp(I*(b*x
+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1067 vs. $2(188) = 376$.

Time $= 0.15$ (sec) , antiderivative size $= 1067$, normalized size of antiderivative $= 4.72$

$$\int (c + dx)^2 \csc^2(a + bx) \sec(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

output

```
-1/2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 2*I*d^2*dilog(cos(b*x + a)
 + I*sin(b*x + a))*sin(b*x + a) - 2*I*d^2*dilog(cos(b*x + a) - I*sin(b*x +
 a))*sin(b*x + a) + 2*I*d^2*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x
+ a) - 2*I*d^2*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 2*d^2*
polylog(3, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 2*d^2*polylog(3,
I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + 2*d^2*polylog(3, -I*cos(b*x
+ a) + sin(b*x + a))*sin(b*x + a) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin
(b*x + a))*sin(b*x + a) + 2*(I*b*d^2*x + I*b*c*d)*dilog(I*cos(b*x + a) + s
in(b*x + a))*sin(b*x + a) + 2*(I*b*d^2*x + I*b*c*d)*dilog(I*cos(b*x + a) -
 sin(b*x + a))*sin(b*x + a) + 2*(-I*b*d^2*x - I*b*c*d)*dilog(-I*cos(b*x +
a) + sin(b*x + a))*sin(b*x + a) + 2*(-I*b*d^2*x - I*b*c*d)*dilog(-I*cos(b*
x + a) - sin(b*x + a))*sin(b*x + a) + 2*(b*d^2*x + b*c*d)*log(cos(b*x + a)
 + I*sin(b*x + a) + 1)*sin(b*x + a) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(
cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 2*(b*d^2*x + b*c*d)*log(
cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + (b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) - (b^2*d^2*x^2
 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) + sin(b*x + a) +
1)*sin(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*
cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) - (b^2*d^2*x^2 + 2*b^2*c*d*x
 + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x...
```

Sympy [F]

$$\int (c+dx)^2 \csc^2(a+bx)\sec(a+bx)\,dx = \int (c+dx)^2 \csc^2(a+bx)\sec(a+bx)\,dx$$

input

```
integrate((d*x+c)**2*csc(b*x+a)**2*sec(b*x+a),x)
```

output

```
Integral((c + d*x)**2*csc(a + b*x)**2*sec(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1638 vs. $2(188) = 376$.

Time = 0.46 (sec) , antiderivative size = 1638, normalized size of antiderivative = 7.25

$$\int (c+dx)^2 \csc^2(a+bx) \sec(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
```

output

```
-1/2*(c^2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))
 - 2*a*c*d*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1)
)/b + a^2*d^2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) -
 1))/b^2 - 2*(2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - ((b*x + a
)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (I*(b*x + a)^2*d
^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x +
a), sin(b*x + a) + 1) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) -
 ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (I*(b*
x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(
cos(b*x + a), -sin(b*x + a) + 1) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2 - (b*c
*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (I*b*c*d + I*(b*x + a)*d^2
- I*a*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 4*(
b*c*d - a*d^2 - (b*c*d - a*d^2)*cos(2*b*x + 2*a) + (-I*b*c*d + I*a*d^2)*si
n(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) - 4*((b*x + a)*d^2
*cos(2*b*x + 2*a) + I*(b*x + a)*d^2*sin(2*b*x + 2*a) - (b*x + a)*d^2)*arct
an2(sin(b*x + a), -cos(b*x + a) + 1) - 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d
^2)*(b*x + a))*cos(b*x + a) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2 - (b*c*d +
(b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (I*b*c*d + I*(b*x + a)*d^2 - I*a
*d^2)*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - 4*(b*c*d + (b*x + a)*d^
2 - a*d^2 - (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (-I*b*c*...
```

Giac [F(-2)]

Exception generated.

$$\int (c+dx)^2 \csc^2(a+bx) \sec(a+bx)\, dx = \text{Exception raised: AttributeError}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc^2(a+bx) \sec(a+bx)\, dx = \text{Hanged}$$

input
```
int((c + d*x)^2/(cos(a + b*x)*sin(a + b*x)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^2 \csc^2(a+bx) \sec(a+bx)\, dx$$

$$= \frac{8\cos(bx+a)\, bcdx - 4\left(\int \frac{x}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4 - \tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2} dx\right) \sin(bx+a)\, b^2cd + \left(\int \csc(bx+a)^2 \sec(bx+a)\, x^2 d\right.}{}$$

input
```
int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x)
```

output

```
(8*cos(a + b*x)*b*c*d*x - 4*int(x/(tan((a + b*x)/2)**4 - tan((a + b*x)/2)*
**2),x)*sin(a + b*x)*b**2*c*d + int(csc(a + b*x)**2*sec(a + b*x)*x**2,x)*si
n(a + b*x)*b**2*d**2 + 8*log(tan((a + b*x)/2)**2 + 1)*sin(a + b*x)*c*d - l
og(tan((a + b*x)/2) - 1)*sin(a + b*x)*b*c**2 + log(tan((a + b*x)/2) + 1)*s
in(a + b*x)*b*c**2 - 14*log(tan((a + b*x)/2))*sin(a + b*x)*c*d + sin(a + b
*x)*b**2*c*d*x**2 - b*c**2 + 6*b*c*d*x)/(sin(a + b*x)*b**2)
```

3.237 $\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx$

Optimal result . . . 2125
Mathematica [C] (warning: unable to verify) . . . 2126
Rubi [A] (verified) . . . 2126
Maple [A] (verified) . . . 2128
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Sympy [F] . . . 2129
Maxima [F] . . . 2129
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Reduce [F] . . . 2131

Optimal result

Integrand size = 20, antiderivative size = 131

$$\begin{aligned}\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx = &-\frac{2idx\arctan\left(e^{i(a+bx)}\right)}{b}\\ &-\frac{d\operatorname{arctanh}(\cos(a+bx))}{b^2}\\ &-\frac{dx\operatorname{arctanh}(\sin(a+bx))}{b}\\ &+\frac{(c+dx)\operatorname{arctanh}(\sin(a+bx))}{b}\\ &-\frac{(c+dx)\csc(a+bx)}{b}\\ &+\frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}\\ &-\frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}\end{aligned}$$

output

```
-2*I*d*x*arctan(exp(I*(b*x+a)))/b-d*arctanh(cos(b*x+a))/b^2-d*x*arctanh(si
n(b*x+a))/b+(d*x+c)*arctanh(sin(b*x+a))/b-(d*x+c)*csc(b*x+a)/b+I*d*polylog
(2,-I*exp(I*(b*x+a)))/b^2-I*d*polylog(2,I*exp(I*(b*x+a)))/b^2
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.28 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.95

$$\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x],x]
```

output
```
(d*(a*Cos[(a + b*x)/2] - (a + b*x)*Cos[(a + b*x)/2])*Csc[(a + b*x)/2])/(2*
b^2) - (c*Csc[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[a + b*x]^2])/b
- (d*Log[Cos[(a + b*x)/2]])/b^2 + (d*Log[Sin[(a + b*x)/2]])/b^2 - (d*x*(a*
Log[1 - Tan[(a + b*x)/2]] - a*Log[1 + Tan[(a + b*x)/2]] - I*(Log[1 + I*Tan
[(a + b*x)/2]]*Log[(1/2 - I/2)*(1 + Tan[(a + b*x)/2])] + PolyLog[2, ((1 +
I) - (1 - I)*Tan[(a + b*x)/2])/2]) + I*(Log[1 - I*Tan[(a + b*x)/2]]*Log[(1
/2 + I/2)*(1 + Tan[(a + b*x)/2])] + PolyLog[2, (-1/2 - I/2)*(I + Tan[(a +
b*x)/2])]) - I*(Log[1 - I*Tan[(a + b*x)/2]]*Log[(-1/2 + I/2)*(-1 + Tan[(a
+ b*x)/2])] + PolyLog[2, ((1 + I) + (1 - I)*Tan[(a + b*x)/2])/2]) + I*(Log
[1 + I*Tan[(a + b*x)/2]]*Log[(-1/2 - I/2)*(-1 + Tan[(a + b*x)/2])] + PolyL
og[2, ((1 - I) + (1 + I)*Tan[(a + b*x)/2])/2])))/(b*(a - I*Log[1 - I*Tan[(
a + b*x)/2]] + I*Log[1 + I*Tan[(a + b*x)/2]])) + (d*Sec[(a + b*x)/2]*(a*Si
n[(a + b*x)/2] - (a + b*x)*Sin[(a + b*x)/2]))/(2*b^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4920, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx$$

$$\downarrow \text{4920}$$

$$-d\int\left(\frac{\operatorname{arctanh}(\sin(a+bx))}{b}-\frac{\csc(a+bx)}{b}\right)dx+\frac{(c+dx)\operatorname{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)\csc(a+bx)}{b}$$

$$\downarrow 2009$$

$$-d\left(\frac{2ix\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{\operatorname{arctanh}(\cos(a+bx))}{b^2}+\frac{x\operatorname{arctanh}(\sin(a+bx))}{b}-\frac{i\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}+\frac{i\operatorname{Poly}}{}\right.$$

$$\left.\frac{(c+dx)\operatorname{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)\csc(a+bx)}{b}\right)$$

input
```
Int[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x],x]
```

output
```
((c + d*x)*ArcTanh[Sin[a + b*x]])/b - ((c + d*x)*Csc[a + b*x])/b - d*(((2*
I)*x*ArcTan[E^(I*(a + b*x))])/b + ArcTanh[Cos[a + b*x]]/b^2 + (x*ArcTanh[S
in[a + b*x]])/b - (I*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 + (I*PolyLog[2,
 I*E^(I*(a + b*x))])/b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.79

method	result
risch	$-\frac{2i(dx+c)e^{i(bx+a)}}{b(e^{2i(bx+a)}-1)}+\frac{d\ln(e^{i(bx+a)}-1)}{b^2}-\frac{d\ln(e^{i(bx+a)}+1)}{b^2}-\frac{2ic\arctan(e^{i(bx+a)})}{b}-\frac{d\ln(ie^{i(bx+a)}+1)a}{b^2}-\frac{d\ln(ie^{i(bx+a}$

input
```
int((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2*I*(d*x+c)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)+d/b^2*ln(exp(I*(b*x+a))
-1)-d/b^2*ln(exp(I*(b*x+a))+1)-2*I/b*c*arctan(exp(I*(b*x+a)))-1/b^2*d*ln(I
*exp(I*(b*x+a))+1)*a-1/b*d*ln(I*exp(I*(b*x+a))+1)*x+1/b*d*ln(1-I*exp(I*(b*
x+a)))*x+1/b^2*d*ln(1-I*exp(I*(b*x+a)))*a+2*I/b^2*d*a*arctan(exp(I*(b*x+a)
))+I/b^2*d*dilog(I*exp(I*(b*x+a))+1)-I/b^2*d*dilog(1-I*exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(113) = 226.

Time = 0.10 (sec) , antiderivative size = 434, normalized size of antiderivative = 3.31

$$\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx =$$

$$-\frac{2\,bdx + i\,d\mathrm{Li}_2(i\,\cos(bx+a)+\sin(bx+a))\sin(bx+a) + i\,d\mathrm{Li}_2(i\,\cos(bx+a)-\sin(bx+a))\sin(bx$$

input
```
integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*b*d*x + I*d*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + I*
d*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - I*d*dilog(-I*cos(b*x
 + a) + sin(b*x + a))*sin(b*x + a) - I*d*dilog(-I*cos(b*x + a) - sin(b*x +
 a))*sin(b*x + a) - (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x + a) + I)*sin
(b*x + a) + (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a
) + d*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - (b*d*x + a*d)*log(I*cos(b
*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + (b*d*x + a*d)*log(I*cos(b*x + a
) - sin(b*x + a) + 1)*sin(b*x + a) - (b*d*x + a*d)*log(-I*cos(b*x + a) + s
in(b*x + a) + 1)*sin(b*x + a) + (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*
x + a) + 1)*sin(b*x + a) - d*log(-1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - (
b*c - a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + (b*c - a
*d)*log(-cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + 2*b*c)/(b^2*sin
(b*x + a))
```

Sympy [F]

$$\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx = \int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)**2*sec(b*x+a),x)
```

output
```
Integral((c + d*x)*csc(a + b*x)**2*sec(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx = \int (dx+c)\csc(bx+a)^2\sec(bx+a)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(4*(b*d*x + b*c)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*d*x + b*c)*cos(
2*b*x + 2*a)*sin(b*x + a) - 4*(b^2*d*cos(2*b*x + 2*a)^2 + b^2*d*sin(2*b*x
+ 2*a)^2 - 2*b^2*d*cos(2*b*x + 2*a) + b^2*d)*integrate((x*cos(2*b*x + 2*a)
*cos(b*x + a) + x*sin(2*b*x + 2*a)*sin(b*x + a) + x*cos(b*x + a))/(cos(2*b
*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1), x) - (b*c*cos(
2*b*x + 2*a)^2 + b*c*sin(2*b*x + 2*a)^2 - 2*b*c*cos(2*b*x + 2*a) + b*c)*lo
g(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) + (b*c*cos(2*b*x +
 2*a)^2 + b*c*sin(2*b*x + 2*a)^2 - 2*b*c*cos(2*b*x + 2*a) + b*c)*log(cos(b
*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + (d*cos(2*b*x + 2*a)^2 +
 d*sin(2*b*x + 2*a)^2 - 2*d*cos(2*b*x + 2*a) + d)*log(cos(b*x)^2 + 2*cos(b
*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (d*co
s(2*b*x + 2*a)^2 + d*sin(2*b*x + 2*a)^2 - 2*d*cos(2*b*x + 2*a) + d)*log(co
s(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) +
 sin(a)^2) + 4*(b*d*x + b*c)*sin(b*x + a))/(b^2*cos(2*b*x + 2*a)^2 + b^2*s
in(2*b*x + 2*a)^2 - 2*b^2*cos(2*b*x + 2*a) + b^2)
```

Giac [F]

$$\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx = \int (dx+c)\csc\left(bx+a\right)^2\sec\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*csc(b*x + a)^2*sec(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)/(cos(a + b*x)*sin(a + b*x)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx$$

$$= \frac{8\cos(bx+a)\,bdx - 4\left(\int \frac{x}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4 - \tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2} dx\right)\sin(bx+a)\,b^2d + 8\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right)\sin(bx+$$

input
```
int((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x)
```

output
```
(8*cos(a + b*x)*b*d*x - 4*int(x/(tan((a + b*x)/2)**4 - tan((a + b*x)/2)**2
),x)*sin(a + b*x)*b**2*d + 8*log(tan((a + b*x)/2)**2 + 1)*sin(a + b*x)*d -
 2*log(tan((a + b*x)/2) - 1)*sin(a + b*x)*b*c + 2*log(tan((a + b*x)/2) + 1
)*sin(a + b*x)*b*c - 14*log(tan((a + b*x)/2))*sin(a + b*x)*d + sin(a + b*x
)*b**2*d*x**2 - 2*b*c + 6*b*d*x)/(2*sin(a + b*x)*b**2)
```

3.238 $\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$

Optimal result . . . 2132
Mathematica [N/A] . . . 2132
Rubi [N/A] . . . 2133
Maple [N/A] . . . 2133
Fricas [N/A] . . . 2134
Sympy [N/A] . . . 2134
Maxima [N/A] . . . 2135
Giac [N/A] . . . 2135
Mupad [N/A] . . . 2136
Reduce [N/A] . . . 2136

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}, x\right)$$

output
```
Defer(Int)(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 17.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x),x]
```

output
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$
$$\downarrow 7299$$
$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}dx$$

input
```
Int[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)}{dx+c}dx$$

input
```
int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x)
```

output
```
int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2*sec(b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc^2\left(a+bx\right)\sec\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)**2*sec(b*x+a)/(d*x+c),x)
```

output
```
Integral(csc(a + b*x)**2*sec(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 677, normalized size of antiderivative = 30.77

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^2\sec(bx+a)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
(2*(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x + 2
*a)^2 + b*c - 2*(b*d*x + b*c)*cos(2*b*x + 2*a))*integrate((cos(2*b*x + 2*a
)*cos(b*x + a) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/((d*x + c)*
cos(2*b*x + 2*a)^2 + (d*x + c)*sin(2*b*x + 2*a)^2 + d*x + 2*(d*x + c)*cos(
2*b*x + 2*a) + c), x) - (b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2
*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^2*x + b*c*d)*cos(2*b
*x + 2*a))*integrate(sin(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*
x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*
sin(b*x + a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)), x) - (b*
d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*s
in(2*b*x + 2*a)^2 - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))*integrate(sin(b*
x + a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*co
s(b*x + a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 - 2*(b*d^2*x
^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)), x) - 2*cos(b*x + a)*sin(2*b*x + 2*a
) + 2*cos(2*b*x + 2*a)*sin(b*x + a) - 2*sin(b*x + a))/(b*d*x + (b*d*x + b*
c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x + 2*a)^2 + b*c - 2*(b*d*x
+ b*c)*cos(2*b*x + 2*a))
```

Giac [N/A]

Not integrable

Time = 5.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^2\sec(bx+a)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sec(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 18.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos(a+b\,x)\,\sin(a+b\,x)^2\,(c+d\,x)}\,dx$$

input
```
int(1/(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^2\sec(bx+a)}{dx+c}dx$$

input
```
int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x)
```

output
```
int((csc(a + b*x)**2*sec(a + b*x))/(c + d*x),x)
```

3.239 $\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$

Optimal result 2137
Mathematica [N/A] 2137
Rubi [N/A] 2138
Maple [N/A] 2138
Fricas [N/A] 2139
Sympy [N/A] 2139
Maxima [N/A] 2140
Giac [N/A] 2141
Mupad [N/A] 2141
Reduce [N/A] 2141

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 17.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$
$$\downarrow 7299$$
$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}dx$$

input
```
Int[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc(bx+a)^2\sec(bx+a)}{(dx+c)^2}dx$$

input
```
int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^2\left(a+bx\right)\sec\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)**2*sec(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)**2*sec(a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 1008, normalized size of antiderivative = 45.82

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input

```
integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output

```
2*((b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*
b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a)^2 - 2*(b*d
^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a))*integrate((cos(2*b*x + 2*a)*
cos(b*x + a) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/(d^2*x^2 + 2*
c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x + 2*a)^2 + (d^2*x^2 + 2*c*d*x
+ c^2)*sin(2*b*x + 2*a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x +
2*a)), x) - (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x
+ b*c^2*d)*cos(2*b*x + 2*a)^2 + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(2*
b*x + 2*a)^2 - 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a))*int
egrate(sin(b*x + a)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*
d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)^2 + (b*d^3*x^3
 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(b*x + a)^2 + 2*(b*d^3*x^3 + 3*
b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)), x) - (b*d^3*x^2 + 2*b*c*
d^2*x + b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a)^2 +
 (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^3*x^2 + 2
*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^3*x^3
+ 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c
^2*d*x + b*c^3)*cos(b*x + a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x
+ b*c^3)*sin(b*x + a)^2 - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c
^3)*cos(b*x + a)), x) - cos(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a...
```

Giac [N/A]

Not integrable

Time = 35.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sec(b*x + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 18.53 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)}{d^2x^2+2cdx+c^2}\,dx$$

input
```
int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x)
```

output
```
int((csc(a + b*x)**2*sec(a + b*x))/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.240 $\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx$

Optimal result . . . 2143
Mathematica [N/A] . . . 2143
Rubi [N/A] . . . 2144
Maple [N/A] . . . 2144
Fricas [N/A] . . . 2145
Sympy [F(-1)] . . . 2145
Maxima [N/A] . . . 2145
Giac [N/A] . . . 2146
Mupad [N/A] . . . 2146
Reduce [N/A] . . . 2147

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx = \text{Int}\big((c+dx)^m \csc^3(a+bx)\sec(a+bx), x\big)$$

output
```
Defer(Int)((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 18.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx = \int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\sec(a+bx)(c+dx)^m\,dx$$

$$\downarrow 7299$$

$$\int \csc^3(a+bx)\sec(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x)
```

output
```
int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*csc(b*x+a)**3*sec(b*x+a),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx = \int (dx+c)^m \csc\left(bx+a\right)^3 \sec\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 18.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^3}\,dx$$

input
```
int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)^3),x)
```

output
```
int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x)
```

output
```
int((c + d*x)**m*csc(a + b*x)**3*sec(a + b*x),x)
```

3.241 $\int (c+dx)^3 \csc^3(a+bx)\sec(a+bx)\,dx$

Optimal result . . . 2148
Mathematica [B] (warning: unable to verify) . . . 2149
Rubi [A] (verified) . . . 2150
Maple [B] (verified) . . . 2152
Fricas [B] (verification not implemented) . . . 2153
Sympy [F(-1)] . . . 2153
Maxima [B] (verification not implemented) . . . 2153
Giac [F] . . . 2154
Mupad [F(-1)] . . . 2155
Reduce [F] . . . 2155

Optimal result

Integrand size = 22, antiderivative size = 325

$$\begin{aligned}
&\int (c+dx)^3 \csc^3(a+bx)\sec(a+bx)\,dx \\
&= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3 \operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b} \\
&\quad - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\cot^2(a+bx)}{2b} \\
&\quad + \frac{3d^2(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^3} + \frac{3id(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} \\
&\quad - \frac{3id^3\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^4} - \frac{3id(c+dx)^2\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2} \\
&\quad - \frac{3d^2(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} + \frac{3d^2(c+dx)\operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3} \\
&\quad - \frac{3id^3\operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4} + \frac{3id^3\operatorname{PolyLog}\left(4,e^{2i(a+bx)}\right)}{4b^4}
\end{aligned}$$

output

```
-3/2*I*d*(d*x+c)^2/b^2-1/2*(d*x+c)^3/b-2*(d*x+c)^3*arctanh(exp(2*I*(b*x+a)
))/b-3/2*d*(d*x+c)^2*cot(b*x+a)/b^2-1/2*(d*x+c)^3*cot(b*x+a)^2/b+3*d^2*(d*
x+c)*ln(1-exp(2*I*(b*x+a)))/b^3+3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+
a)))/b^2-3/2*I*d^3*polylog(2,exp(2*I*(b*x+a)))/b^4-3/2*I*d*(d*x+c)^2*polyl
og(2,exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+c)*polylog(3,-exp(2*I*(b*x+a)))/b^
3+3/2*d^2*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^3-3/4*I*d^3*polylog(4,-exp
(2*I*(b*x+a)))/b^4+3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1526 vs. $2(325) = 650$.

Time = 6.63 (sec) , antiderivative size = 1526, normalized size of antiderivative = 4.70

$$\int (c+dx)^3 \csc^3(a+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input

```
Integrate[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x],x]
```

output

```
-1/2*((c + d*x)^3*Csc[a + b*x]^2)/b - (c*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E
^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))]
+ (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 -
E^((-2*I)*a))*x*PolyLog[2, -E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x
*PolyLog[2, E^((-I)*(a + b*x))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^(
(-I)*(a + b*x))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]
))/(2*b^3) - (d^3*E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E
^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a)
)*x^3*Log[1 + E^((-I)*(a + b*x))] - 6*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[2
, -E^((-I)*(a + b*x))] - 6*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[2, E^((-I)*(
a + b*x))] + (12*I)*b*(1 - E^((-2*I)*a))*x*PolyLog[3, -E^((-I)*(a + b*x))]
 + (12*I)*b*(1 - E^((-2*I)*a))*x*PolyLog[3, E^((-I)*(a + b*x))] + 12*(1 -
E^((-2*I)*a))*PolyLog[4, -E^((-I)*(a + b*x))] + 12*(1 - E^((-2*I)*a))*Poly
Log[4, E^((-I)*(a + b*x))]))/(4*b^4) + (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2
 + d^3*x^3)*Csc[a]*Sec[a])/4 - ((I/4)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 +
 E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*Pol
yLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-
2*I)*(a + b*x))])*Sec[a])/(b^3*E^(I*a)) - ((I/8)*d^3*E^(I*a)*((2*b^4*x^4)/
E^((2*I)*a) - (4*I)*b^3*(1 + E^((-2*I)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x)
)] + 6*b^2*(1 + E^((-2*I)*a))*x^2*PolyLog[2, -E^((-2*I)*(a + b*x))] - (...
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4920, 27, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \csc^3(a+bx)\sec(a+bx)\,dx$$

$$\downarrow 4920$$

$$-3d\int -\frac{1}{2}(c+dx)^2\left(\frac{\cot^2(a+bx)}{b}-\frac{2\log(\tan(a+bx))}{b}\right)dx-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{(c+dx)^3\log(\tan(a+bx))}{b}$$

$$\downarrow 27$$

$$\frac{3}{2}d\int (c+dx)^2\left(\frac{\cot^2(a+bx)}{b}-\frac{2\log(\tan(a+bx))}{b}\right)dx-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{(c+dx)^3\log(\tan(a+bx))}{b}$$

$$\downarrow 7292$$

$$\frac{3}{2}d\int \frac{(c+dx)^2\left(\cot^2(a+bx)-2\log(\tan(a+bx))\right)}{b}dx-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{(c+dx)^3\log(\tan(a+bx))}{b}$$

$$\downarrow 27$$

$$\frac{3d\int (c+dx)^2\left(\cot^2(a+bx)-2\log(\tan(a+bx))\right)dx}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{(c+dx)^3\log(\tan(a+bx))}{b}$$

$$\downarrow 7293$$

$$\frac{3d\int \left((c+dx)^2\cot^2(a+bx)-2(c+dx)^2\log(\tan(a+bx))\right)dx}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{(c+dx)^3\log(\tan(a+bx))}{b}$$

$$\downarrow 2009$$

$$\frac{3d\left(-\frac{4(c+dx)^3\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{3d}-\frac{id^2\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}-\frac{id^2\operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{2b^3}+\frac{id^2\operatorname{PolyLog}\left(4,e^{2i(a+bx)}\right)}{2b^3}-\frac{d(c+dx)}{}\right.}{}$$

$$\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{(c+dx)^3\log(\tan(a+bx))}{b}$$

input
```
Int[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x],x]
```

output
```
-1/2*((c + d*x)^3*Cot[a + b*x]^2)/b + ((c + d*x)^3*Log[Tan[a + b*x]])/b +
(3*d*(((-I)*(c + d*x)^2)/b - (c + d*x)^3/(3*d) - (4*(c + d*x)^3*ArcTanh[E^
((2*I)*(a + b*x))])/(3*d) - ((c + d*x)^2*Cot[a + b*x])/b + (2*d*(c + d*x)*
Log[1 - E^((2*I)*(a + b*x))])/b^2 - (2*(c + d*x)^3*Log[Tan[a + b*x]])/(3*d
) + (I*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b - (I*d^2*PolyLog[2,
 E^((2*I)*(a + b*x))])/b^3 - (I*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))
])/b - (d*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/b^2 + (d*(c + d*x)*P
olyLog[3, E^((2*I)*(a + b*x))])/b^2 - ((I/2)*d^2*PolyLog[4, -E^((2*I)*(a +
 b*x))])/b^3 + ((I/2)*d^2*PolyLog[4, E^((2*I)*(a + b*x))])/b^3))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1222 vs. 2(279) = 558.

Time = 0.63 (sec) , antiderivative size = 1223, normalized size of antiderivative = 3.76

method	result	size
risch	Expression too large to display	1223

input
```
int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2-6*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+
a)))*x-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x+3*I/b^2*c*d^2*polylog(2,-
exp(2*I*(b*x+a)))*x+(2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x
+a))+6*b*c*d^2*x^2*exp(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d
*x*exp(2*I*(b*x+a))-3*I*c^2*d*exp(2*I*(b*x+a))+3*I*d^3*x^2+2*b*c^3*exp(2*I
*(b*x+a))+6*I*c*d^2*x+3*I*c^2*d)/b^2/(exp(2*I*(b*x+a))-1)^2-6*I*d^3/b^3*a*
x-3/b*c^2*d*ln(exp(2*I*(b*x+a))+1)*x-3/b*c*d^2*ln(exp(2*I*(b*x+a))+1)*x^2+
3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2+3/2*I/b^2*c^2*d*polylog(2,-
exp(2*I*(b*x+a)))-6*d^2/b^3*c*ln(exp(I*(b*x+a)))+3*d^2/b^3*c*ln(exp(I*(b*x
+a))-1)+3*d^2/b^3*c*ln(exp(I*(b*x+a))+1)+3*d^3/b^3*ln(exp(I*(b*x+a))+1)*x+
3*d^3/b^3*ln(1-exp(I*(b*x+a)))*x+3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a+6*d^3/b^
4*a*ln(exp(I*(b*x+a)))-3*d^3/b^4*a*ln(exp(I*(b*x+a))-1)-3*I*d^3/b^2*x^2-3*
I*d^3/b^4*a^2-3*I*d^3/b^4*polylog(2,-exp(I*(b*x+a)))-3/2/b^3*c*d^2*polylog
(3,-exp(2*I*(b*x+a)))-3/2/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x+6*I/b^4*d
^3*polylog(4,-exp(I*(b*x+a)))-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+6/b^3*c*d
^2*polylog(3,-exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+1/b*d^
3*ln(1-exp(I*(b*x+a)))*x^3+1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+6/b^3*d^3*poly
log(3,-exp(I*(b*x+a)))*x+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x+1/b^4*d^3*l
n(1-exp(I*(b*x+a)))*a^3+3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)-3/b^2*c^2*d*a
*ln(exp(I*(b*x+a))-1)+3/b^2*d*c^2*ln(1-exp(I*(b*x+a)))*a+3/b*c*d^2*ln(e...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3475 vs. $2(270) = 540$.

Time = 0.23 (sec) , antiderivative size = 3475, normalized size of antiderivative = 10.69

$$\int (c+dx)^3 \csc^3(a+bx) \sec(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^3(a+bx) \sec(a+bx)\, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3*csc(b*x+a)**3*sec(b*x+a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5165 vs. $2(270) = 540$.

Time = 2.30 (sec) , antiderivative size = 5165, normalized size of antiderivative = 15.89

$$\int (c+dx)^3 \csc^3(a+bx) \sec(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

output

```
-1/2*(c^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2
)) - 3*a*c^2*d*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x +
 a)^2))/b + 3*a^2*c*d^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(
sin(b*x + a)^2))/b^2 - a^3*d^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1)
 - log(sin(b*x + a)^2))/b^3 - 2*(18*b^2*c^2*d - 36*a*b*c*d^2 + 18*a^2*d^3
- 2*(4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d -
2*a*b*c*d^2 + a^2*d^3)*(b*x + a) + (4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3
)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(4*b*x
 + 4*a) - 2*(4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*
c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-4*I*(b*x +
a)^3*d^3 + 9*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 9*(-I*b^2*c^2*d + 2*I*a*
b*c*d^2 - I*a^2*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(4*I*(b*x + a)^3*d^3
+ 9*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 9*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I
*a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x
 + 2*a) + 1) + 6*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d
^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a) +
((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 +
 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) -
 2*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^
2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(2*b*x + ...
```

Giac [F]

$$\int (c+dx)^3 \csc^3(a+bx)\sec(a+bx)\,dx = \int (dx+c)^3 \csc(bx+a)^3 \sec(bx+a)\,dx$$

input

```
integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

output

```
integrate((d*x + c)^3*csc(b*x + a)^3*sec(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^3(a+bx)\sec(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)^3/(cos(a + b*x)*sin(a + b*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^3 \csc^3(a+bx)\sec(a+bx)\,dx$$
$$= \frac{4\left(\int \csc(bx+a)^3\sec(bx+a)\,x^3dx\right)\sin(bx+a)^2\,b\,d^3 + 12\left(\int \csc(bx+a)^3\sec(bx+a)\,x^2dx\right)\sin(bx+a}{}$$

input
```
int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x)
```

output
```
(4*int(csc(a + b*x)**3*sec(a + b*x)*x**3,x)*sin(a + b*x)**2*b*d**3 + 12*in
t(csc(a + b*x)**3*sec(a + b*x)*x**2,x)*sin(a + b*x)**2*b*c*d**2 + 12*int(c
sc(a + b*x)**3*sec(a + b*x)*x,x)*sin(a + b*x)**2*b*c**2*d - 4*log(tan((a +
 b*x)/2) - 1)*sin(a + b*x)**2*c**3 - 4*log(tan((a + b*x)/2) + 1)*sin(a + b
*x)**2*c**3 + 4*log(tan((a + b*x)/2))*sin(a + b*x)**2*c**3 + sin(a + b*x)*
*2*c**3 - 2*c**3)/(4*sin(a + b*x)**2*b)
```

3.242 $\int (c+dx)^2 \csc^3(a+bx)\sec(a+bx)\,dx$

Optimal result . . . 2156
Mathematica [B] (warning: unable to verify) . . . 2157
Rubi [A] (verified) . . . 2158
Maple [B] (verified) . . . 2160
Fricas [B] (verification not implemented) . . . 2160
Sympy [F] . . . 2161
Maxima [B] (verification not implemented) . . . 2162
Giac [F] . . . 2163
Mupad [F(-1)] . . . 2163
Reduce [F] . . . 2163

Optimal result

Integrand size = 22, antiderivative size = 194

$$\begin{aligned}\int (c+dx)^2 \csc^3(a+bx)\sec(a+bx)\,dx = &-\frac{(c+dx)^2}{2b} - \frac{2(c+dx)^2 \operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b} \\ &- \frac{d(c+dx)\cot(a+bx)}{b^2} \\ &- \frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{d^2\log(\sin(a+bx))}{b^3} \\ &+ \frac{id(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} \\ &- \frac{id(c+dx)\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2} \\ &- \frac{d^2\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} \\ &+ \frac{d^2\operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}\end{aligned}$$

output

```
-1/2*(d*x+c)^2/b-2*(d*x+c)^2*arctanh(exp(2*I*(b*x+a)))/b-d*(d*x+c)*cot(b*x
+a)/b^2-1/2*(d*x+c)^2*cot(b*x+a)^2/b+d^2*ln(sin(b*x+a))/b^3+I*d*(d*x+c)*po
lylog(2,-exp(2*I*(b*x+a)))/b^2-I*d*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^2
-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+1/2*d^2*polylog(3,exp(2*I*(b*x+a
)))/b^3
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 880 vs. $2(194) = 388$.

Time = 6.52 (sec) , antiderivative size = 880, normalized size of antiderivative = 4.54

$$\int (c+dx)^2 \csc^3(a+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input

```
Integrate[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x],x]
```

output

```
-1/2*((c + d*x)^2*Csc[a + b*x]^2)/b - (d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^(
(2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] +
(3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 - E^
((-2*I)*a))*x*PolyLog[2, -E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*P
olyLog[2, E^((-I)*(a + b*x))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^((-
I)*(a + b*x))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]))
/(6*b^3) + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Csc[a]*Sec[a])/3 - ((I/12)*d^2*(
2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))])
+ 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E
^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(I*a)) - (c^
2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*
(Cos[a]^2 + Sin[a]^2)) + (c^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a]
+ Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (d^2*Csc[a]*(-(b*x
*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[a]^2
+ Sin[a]^2)) - (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*
x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[
Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*
ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x
 - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*Sqrt[Csc[a]^2*(Cos
[a]^2 + Sin[a]^2)]) + (Csc[a]*Csc[a + b*x]*(c*d*Sin[b*x] + d^2*x*Sin[b*...
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4920, 27, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \csc^3(a+bx)\sec(a+bx)\,dx$$

$$\downarrow 4920$$

$$-2d\int -\frac{1}{2}(c+dx)\left(\frac{\cot^2(a+bx)}{b}-\frac{2\log(\tan(a+bx))}{b}\right)dx-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}$$

$$\downarrow 27$$

$$d\int (c+dx)\left(\frac{\cot^2(a+bx)}{b}-\frac{2\log(\tan(a+bx))}{b}\right)dx-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}$$

$$\downarrow 7292$$

$$d\int \frac{(c+dx)\left(\cot^2(a+bx)-2\log(\tan(a+bx))\right)}{b}dx-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}$$

$$\downarrow 27$$

$$\frac{d\int (c+dx)\left(\cot^2(a+bx)-2\log(\tan(a+bx))\right)dx}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}$$

$$\downarrow 7293$$

$$\frac{d\int \left((c+dx)\cot^2(a+bx)-2(c+dx)\log(\tan(a+bx))\right)dx}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}$$

$$\downarrow 2009$$

$$\frac{d\left(-\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{d}-\frac{d\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^2}+\frac{d\operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^2}+\frac{d\log(\sin(a+bx))}{b^2}+\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{2}\right.}{b}\right.}{b}$$

$$\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}$$

input
```
Int[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x],x]
```

output
```
-1/2*((c + d*x)^2*Cot[a + b*x]^2)/b + ((c + d*x)^2*Log[Tan[a + b*x]])/b +
(d*(-1/2*(c + d*x)^2/d - (2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/d -
((c + d*x)*Cot[a + b*x])/b + (d*Log[Sin[a + b*x]])/b^2 - ((c + d*x)^2*Log[
Tan[a + b*x]])/d + (I*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b - (I*(
c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b - (d*PolyLog[3, -E^((2*I)*(a +
 b*x))])/(2*b^2) + (d*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^2)))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(174) = 348$.

Time = 0.53 (sec) , antiderivative size = 632, normalized size of antiderivative = 3.26

method	result
risch	$-\frac{2icd\,\mathrm{polylog}\left(2,-\mathrm{e}^{i(bx+a)}\right)}{b^2}-\frac{2cda\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b^2}+\frac{2cd\ln\left(1-\mathrm{e}^{i(bx+a)}\right)a}{b^2}-\frac{d^2\ln\left(\mathrm{e}^{2i(bx+a)}+1\right)x^2}{b}+\frac{d^2\ln\left(\mathrm{e}^{i(bx+a)}+1\right)x^2}{b}$

input
```
int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b*c^2*ln(exp(2*I*(b*x+a))+1)-2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-2
*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a
)))-2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))+2*(b*d^2*x^2*exp(2*I*(b*x+a))+2*
b*c*d*x*exp(2*I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^2*x*exp(2*I*(b*x+a))-I
*c*d*exp(2*I*(b*x+a))+I*d^2*x+I*d*c)/b^2/(exp(2*I*(b*x+a))-1)^2+2/b^2*c*d*
ln(1-exp(I*(b*x+a)))*a-2/b*d*c*ln(exp(2*I*(b*x+a))+1)*x-1/b*d^2*ln(exp(2*I
*(b*x+a))+1)*x^2+I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x+I/b^2*d*c*polylo
g(2,-exp(2*I*(b*x+a)))+1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+1/b*d^2*ln(exp(I
*(b*x+a))+1)*x^2+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d^2*ln(1-exp(I*(b*
x+a)))*a^2+2/b*c*d*ln(exp(I*(b*x+a))+1)*x-2/b^2*c*d*a*ln(exp(I*(b*x+a))-1)
+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+1/b*c^2*ln(exp(I*(b*x+a))-1)+1/b*c^2*ln(ex
p(I*(b*x+a))+1)-2/b^3*d^2*ln(exp(I*(b*x+a)))+1/b^3*d^2*ln(exp(I*(b*x+a))-1
)+1/b^3*d^2*ln(exp(I*(b*x+a))+1)+2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^
2*polylog(3,exp(I*(b*x+a)))/b^3-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1995 vs. $2(170) = 340$.

Time = 0.18 (sec) , antiderivative size = 1995, normalized size of antiderivative = 10.28

$$\int (c+dx)^2 \csc^3(a+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
```

output

```
1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a
)*sin(b*x + a) - 2*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(b*x +
 a)^2)*dilog(cos(b*x + a) + I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d + (-I
*b*d^2*x - I*b*c*d)*cos(b*x + a)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) -
 2*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilog(I*c
os(b*x + a) + sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c
*d)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 2*(I*b*d^2*x +
I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) + s
in(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(b*x + a
)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d + (-I*
b*d^2*x - I*b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a)) -
 2*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilog(-co
s(b*x + a) - I*sin(b*x + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + d^2)*cos(b*x + a)^2 + d^2)*log(cos(b*x
+ a) + I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + I)
 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2 + d^2)*cos(b*x + a)^2 + d^2)*log(cos(b*x + a) - I*sin(b*x + a) + 1) +
 (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(b*x
+ a)^2)*log(cos(b*x + a) - I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c...
```

Sympy [F]

$$\int (c+dx)^2 \csc^3(a+bx)\sec(a+bx)\,dx = \int (c+dx)^2 \csc^3(a+bx)\sec(a+bx)\,dx$$

input

```
integrate((d*x+c)**2*csc(b*x+a)**3*sec(b*x+a),x)
```

output

```
Integral((c + d*x)**2*csc(a + b*x)**3*sec(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2528 vs. $2(170) = 340$.

Time = 0.62 (sec) , antiderivative size = 2528, normalized size of antiderivative = 13.03

$$\int (c+dx)^2 \csc^3(a+bx) \sec(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

output

```
-1/2*(c^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2
)) - 2*a*c*d*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a
)^2))/b + a^2*d^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*
x + a)^2))/b^2 + 2*(4*(b*x + a)*d^2*cos(4*b*x + 4*a) + 4*I*(b*x + a)*d^2*s
in(4*b*x + 4*a) - 4*b*c*d + 4*a*d^2 + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^
2)*(b*x + a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x +
 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a)
 - (-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a
) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*sin(2*b*x + 2*
a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 2*((b*x + a)^2*d^2 +
 2*(b*c*d - a*d^2)*(b*x + a) + d^2 + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*
(b*x + a) + d^2)*cos(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)
*(b*x + a) + d^2)*cos(2*b*x + 2*a) + (I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a
*d^2)*(b*x + a) + I*d^2)*sin(4*b*x + 4*a) + 2*(-I*(b*x + a)^2*d^2 + 2*(-I*
b*c*d + I*a*d^2)*(b*x + a) - I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a)
, cos(b*x + a) + 1) - 2*(d^2*cos(4*b*x + 4*a) - 2*d^2*cos(2*b*x + 2*a) + I
*d^2*sin(4*b*x + 4*a) - 2*I*d^2*sin(2*b*x + 2*a) + d^2)*arctan2(sin(b*x +
a), cos(b*x + a) - 1) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) +
 ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a) - 2*((b*
x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x...
```

Giac [F]

$$\int (c+dx)^2 \csc^3(a+bx)\sec(a+bx)\,dx = \int (dx+c)^2 \csc(bx+a)^3 \sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csc(b*x + a)^3*sec(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc^3(a+bx)\sec(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)^2/(cos(a + b*x)*sin(a + b*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^2 \csc^3(a+bx)\sec(a+bx)\,dx$$
$$= \frac{4\left(\int \csc(bx+a)^3 \sec(bx+a)\,x^2 dx\right)\sin(bx+a)^2\,b\,d^2 + 8\left(\int \csc(bx+a)^3 \sec(bx+a)\,x dx\right)\sin(bx+a)^2}{}$$

input
```
int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x)
```

output
```
(4*int(csc(a + b*x)**3*sec(a + b*x)*x**2,x)*sin(a + b*x)**2*b*d**2 + 8*int
(csc(a + b*x)**3*sec(a + b*x)*x,x)*sin(a + b*x)**2*b*c*d - 4*log(tan((a +
b*x)/2) - 1)*sin(a + b*x)**2*c**2 - 4*log(tan((a + b*x)/2) + 1)*sin(a + b*
x)**2*c**2 + 4*log(tan((a + b*x)/2))*sin(a + b*x)**2*c**2 + sin(a + b*x)**
2*c**2 - 2*c**2)/(4*sin(a + b*x)**2*b)
```

3.243 $\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx$

Optimal result . . . 2164
Mathematica [A] (verified) . . . 2165
Rubi [A] (verified) . . . 2165
Maple [B] (verified) . . . 2167
Fricas [B] (verification not implemented) . . . 2167
Sympy [F] . . . 2168
Maxima [B] (verification not implemented) . . . 2169
Giac [F] . . . 2170
Mupad [F(-1)] . . . 2170
Reduce [F] . . . 2170

Optimal result

Integrand size = 20, antiderivative size = 141

$$\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx = -\frac{dx}{2b} - \frac{2dx\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} - \frac{dx\log(\tan(a+bx))}{b} + \frac{(c+dx)\log(\tan(a+bx))}{b} + \frac{id\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} - \frac{id\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}$$

output

```
-1/2*d*x/b-2*d*x*arctanh(exp(2*I*(b*x+a)))/b-1/2*d*cot(b*x+a)/b^2-1/2*(d*x
+c)*cot(b*x+a)^2/b-d*x*ln(tan(b*x+a))/b+(d*x+c)*ln(tan(b*x+a))/b+1/2*I*d*p
olylog(2,-exp(2*I*(b*x+a)))/b^2-1/2*I*d*polylog(2,exp(2*I*(b*x+a)))/b^2
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.53

$$\begin{aligned}
&\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx \\
&= -\frac{d\cot(a+bx)}{2b^2} - \frac{c\csc^2(a+bx)}{2b} - \frac{dx\csc^2(a+bx)}{2b} - \frac{c\log(\cos(a+bx))}{b} \\
&\quad + \frac{ad\log(\cos(a+bx))}{b^2} + \frac{c\log(\sin(a+bx))}{b} - \frac{ad\log(\sin(a+bx))}{b^2} \\
&\quad + \frac{d\left(\frac{1}{2}i(a+bx)^2 - (a+bx)\log\left(1+e^{2i(a+bx)}\right) + \frac{1}{2}i\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)\right)}{b^2} \\
&\quad + \frac{d\left((a+bx)\log\left(1-e^{2i(a+bx)}\right) - \frac{1}{2}i\left((a+bx)^2 + \mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)\right)\right)}{b^2}
\end{aligned}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x],x]
```

output
```
-1/2*(d*Cot[a + b*x])/b^2 - (c*Csc[a + b*x]^2)/(2*b) - (d*x*Csc[a + b*x]^2
)/(2*b) - (c*Log[Cos[a + b*x]])/b + (a*d*Log[Cos[a + b*x]])/b^2 + (c*Log[S
in[a + b*x]])/b - (a*d*Log[Sin[a + b*x]])/b^2 + (d*((I/2)*(a + b*x)^2 - (a
 + b*x)*Log[1 + E^((2*I)*(a + b*x))] + (I/2)*PolyLog[2, -E^((2*I)*(a + b*x
))]))/b^2 + (d*((a + b*x)*Log[1 - E^((2*I)*(a + b*x))] - (I/2)*((a + b*x)^
2 + PolyLog[2, E^((2*I)*(a + b*x))])))/b^2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4920, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx$$

$$\downarrow \text{4920}$$

$$-d\int\left(\frac{\log(\tan(a+bx))}{b}-\frac{\cot^2(a+bx)}{2b}\right)dx-\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{(c+dx)\log(\tan(a+bx))}{b}$$

$$\downarrow 2009$$

$$-d\left(\frac{2x\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b}-\frac{i\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}+\frac{i\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}+\frac{\cot(a+bx)}{2b^2}+\frac{x\log(\tan(a+\right.$$
$$\left.\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{(c+dx)\log(\tan(a+bx))}{b}\right)$$

input
```
Int[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x],x]
```

output
```
-1/2*((c + d*x)*Cot[a + b*x]^2)/b + ((c + d*x)*Log[Tan[a + b*x]])/b - d*(x
/(2*b) + (2*x*ArcTanh[E^((2*I)*(a + b*x))])/b + Cot[a + b*x]/(2*b^2) + (x*
Log[Tan[a + b*x]])/b - ((I/2)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + ((I/
2)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(123) = 246$.

Time = 0.38 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.91

method	result
risch	$\frac{2bdx\,e^{2i(bx+a)}-id\,e^{2i(bx+a)}+2bc\,e^{2i(bx+a)}+id}{b^2\left(e^{2i(bx+a)}-1\right)^2}+\frac{c\ln\left(e^{i(bx+a)}+1\right)}{b}-\frac{c\ln\left(e^{2i(bx+a)}+1\right)}{b}+\frac{c\ln\left(e^{i(bx+a)}-1\right)}{b}-\frac{d\ln\left(e^{2i(bx+a}\right.}{b}$

input
```
int((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(2*b*d*x*exp(2*I*(b*x+a))-I*d*exp(2*I*(b*x+a))+2*b*c*exp(2*I*(b*x+a))+I*d)
/b^2/(exp(2*I*(b*x+a))-1)^2+1/b*c*ln(exp(I*(b*x+a))+1)-1/b*c*ln(exp(2*I*(b
*x+a))+1)+1/b*c*ln(exp(I*(b*x+a))-1)-1/b*d*ln(exp(2*I*(b*x+a))+1)*x+1/2*I*
d*polylog(2,-exp(2*I*(b*x+a)))/b^2+1/b*d*ln(exp(I*(b*x+a))+1)*x-I*d*polylo
g(2,-exp(I*(b*x+a)))/b^2+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(
b*x+a)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1
)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 942 vs. $2(119) = 238$.

Time = 0.13 (sec) , antiderivative size = 942, normalized size of antiderivative = 6.68

$$\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
```

output

```
1/2*(b*d*x + d*cos(b*x + a)*sin(b*x + a) + b*c + (-I*d*cos(b*x + a)^2 + I*
d)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog
(cos(b*x + a) - I*sin(b*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(I*cos(
b*x + a) + sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(I*cos(b*x + a)
 - sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(-I*cos(b*x + a) + sin(
b*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(-I*cos(b*x + a) - sin(b*x +
a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (
-I*d*cos(b*x + a)^2 + I*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b*d*x
- (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a) + I*sin(b*x + a) +
1) - ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(cos(b*x + a) + I*sin(b*x
 + a) + I) - (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a)
 - I*sin(b*x + a) + 1) - ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(cos(
b*x + a) - I*sin(b*x + a) + I) + (b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a
*d)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x - (b*d*x + a*d)*cos(b*
x + a)^2 + a*d)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b*d*x - (b*d*x +
 a*d)*cos(b*x + a)^2 + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d
*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d)*log(-I*cos(b*x + a) - sin(b*x + a
) + 1) + ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(-1/2*cos(b*x + a) +
1/2*I*sin(b*x + a) + 1/2) + ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(-
1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - (b*d*x - (b*d*x + a*d)*c...
```

Sympy [F]

$$\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx = \int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx$$

input

```
integrate((d*x+c)*csc(b*x+a)**3*sec(b*x+a),x)
```

output

```
Integral((c + d*x)*csc(a + b*x)**3*sec(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(119) = 238$.

Time = 0.38 (sec) , antiderivative size = 1028, normalized size of antiderivative = 7.29

$$\int (c + dx) \csc^3(a + bx) \sec(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

output

```
-(2*(b*d*x + b*c + (b*d*x + b*c)*cos(4*b*x + 4*a) - 2*(b*d*x + b*c)*cos(2*
b*x + 2*a) - (-I*b*d*x - I*b*c)*sin(4*b*x + 4*a) - 2*(I*b*d*x + I*b*c)*sin
(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 2*(b*d*x
+ b*c + (b*d*x + b*c)*cos(4*b*x + 4*a) - 2*(b*d*x + b*c)*cos(2*b*x + 2*a)
+ (I*b*d*x + I*b*c)*sin(4*b*x + 4*a) + 2*(-I*b*d*x - I*b*c)*sin(2*b*x + 2*
a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(b*c*cos(4*b*x + 4*a) - 2*
b*c*cos(2*b*x + 2*a) + I*b*c*sin(4*b*x + 4*a) - 2*I*b*c*sin(2*b*x + 2*a) +
 b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 2*(b*d*x*cos(4*b*x + 4*a)
- 2*b*d*x*cos(2*b*x + 2*a) + I*b*d*x*sin(4*b*x + 4*a) - 2*I*b*d*x*sin(2*b*
x + 2*a) + b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*(-2*I*b*d*x
 - 2*I*b*c - d)*cos(2*b*x + 2*a) - (d*cos(4*b*x + 4*a) - 2*d*cos(2*b*x + 2
*a) + I*d*sin(4*b*x + 4*a) - 2*I*d*sin(2*b*x + 2*a) + d)*dilog(-e^(2*I*b*x
 + 2*I*a)) + 2*(d*cos(4*b*x + 4*a) - 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x
+ 4*a) - 2*I*d*sin(2*b*x + 2*a) + d)*dilog(-e^(I*b*x + I*a)) + 2*(d*cos(4*
b*x + 4*a) - 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) - 2*I*d*sin(2*b*x
 + 2*a) + d)*dilog(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*
c)*cos(4*b*x + 4*a) - 2*(-I*b*d*x - I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c
)*sin(4*b*x + 4*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a
)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (I*b*d*x + I*b*c + (I
*b*d*x + I*b*c)*cos(4*b*x + 4*a) - 2*(I*b*d*x + I*b*c)*cos(2*b*x + 2*a)...
```

Giac [F]

$$\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx = \int (dx+c)\csc(bx+a)^3\sec(bx+a)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*csc(b*x + a)^3*sec(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)/(cos(a + b*x)*sin(a + b*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx$$
$$= \frac{4\left(\int \csc(bx+a)^3\sec(bx+a)\,x dx\right)\sin(bx+a)^2 bd - 4\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)^2 c - 4\log(\tan}{4\sin(bx+a)^2 b}$$

input
```
int((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x)
```

output
```
(4*int(csc(a + b*x)**3*sec(a + b*x)*x,x)*sin(a + b*x)**2*b*d - 4*log(tan((
a + b*x)/2) - 1)*sin(a + b*x)**2*c - 4*log(tan((a + b*x)/2) + 1)*sin(a + b
*x)**2*c + 4*log(tan((a + b*x)/2))*sin(a + b*x)**2*c + sin(a + b*x)**2*c -
 2*c)/(4*sin(a + b*x)**2*b)
```

3.244 $\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$

Optimal result 2171
Mathematica [N/A] 2171
Rubi [N/A] 2172
Maple [N/A] 2172
Fricas [N/A] 2173
Sympy [N/A] 2173
Maxima [N/A] 2174
Giac [N/A] 2175
Mupad [N/A] 2175
Reduce [N/A] 2175

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}, x\right)$$

output

```
Defer(Int)(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 11.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$$

input

```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x),x]
```

output

```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$$
$$\downarrow 7299$$
$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}dx$$

input
```
Int[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc(bx+a)^3\sec(bx+a)}{dx+c}dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x)
```

output
```
int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc\left(bx+a\right)^3\sec\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^3*sec(b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc^3\left(a+bx\right)\sec\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)**3*sec(b*x+a)/(d*x+c),x)
```

output
```
Integral(csc(a + b*x)**3*sec(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 3.82 (sec) , antiderivative size = 2040, normalized size of antiderivative = 92.73

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc\left(bx+a\right)^3\sec\left(bx+a\right)}{dx+c}\,dx$$

input

```
integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output

```
-(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2
- (2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
 - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x
^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*
sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate((b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2 + d^2)*sin(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*
x + b^2*c^3)*cos(b*x + a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3)*sin(b*x + a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^
2*d*x + b^2*c^3)*cos(b*x + a)), x) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
+ (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*s
in(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d...
```

Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3*sec(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 19.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos(a+b\,x)\,\sin(a+b\,x)^3\,(c+d\,x)}\,dx$$

input
```
int(1/(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)}{dx+c}\,dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x)
```

output
```
int((csc(a + b*x)**3*sec(a + b*x))/(c + d*x),x)
```

3.245 $\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 2177
Mathematica [N/A] . . . 2177
Rubi [N/A] . . . 2178
Maple [N/A] . . . 2178
Fricas [N/A] . . . 2179
Sympy [N/A] . . . 2179
Maxima [N/A] . . . 2180
Giac [F(-1)] . . . 2181
Mupad [N/A] . . . 2181
Reduce [N/A] . . . 2181

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 10.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$
$$\downarrow 7299$$
$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}dx$$

input
```
Int[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc\left(bx+a\right)^3\sec\left(bx+a\right)}{\left(dx+c\right)^2}dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)^3\sec\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^3*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 5.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^3\left(a+bx\right)\sec\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)**3*sec(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)**3*sec(a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 14.71 (sec) , antiderivative size = 2702, normalized size of antiderivative = 122.82

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)^3\sec\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2
- 2*((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
 - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^
2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*
c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 +
 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3
*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*
a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x
 + 2*a))*integrate((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 3*d^2)*sin(b*x +
 a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b
^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*
x + b^2*c^4)*cos(b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d
^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(b*x + a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*
c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x)
 - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 19.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\cos(a+b\,x)\,\sin(a+b\,x)^3\,(c+d\,x)^2}\,dx$$

input
```
int(1/(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)}{d^2x^2+2cdx+c^2}dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x)
```

output
```
int((csc(a + b*x)**3*sec(a + b*x))/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.246 $\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx$

Optimal result . . . 2182
Mathematica [N/A] . . . 2182
Rubi [N/A] . . . 2183
Maple [N/A] . . . 2183
Fricas [N/A] . . . 2184
Sympy [N/A] . . . 2184
Maxima [N/A] . . . 2185
Giac [N/A] . . . 2185
Mupad [N/A] . . . 2185
Reduce [N/A] . . . 2186

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx = \text{Int}((c+dx)^m \sec(a+bx)\tan(a+bx), x)$$

output
```
Defer(Int)((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx = \int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(a+bx)\sec(a+bx)(c+dx)^m\,dx$$

$$\downarrow 7299$$

$$\int \tan(a+bx)\sec(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \sec(bx+a)\tan(bx+a)\,dx$$

input
```
int((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)
```

output
```
int((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*sec(b*x + a)*tan(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 4.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx = \int (c+dx)^m \tan(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*sec(b*x+a)*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**m*tan(a + b*x)*sec(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 19.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx = \int \frac{\tan(a+b\,x)\,(c+d\,x)^m}{\cos(a+b\,x)}\,dx$$

input
```
int((tan(a + b*x)*(c + d*x)^m)/cos(a + b*x),x)
```

output
```
int((tan(a + b*x)*(c + d*x)^m)/cos(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\tan(bx+a)\,dx$$

input
```
int((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)
```

output
```
int((c + d*x)**m*sec(a + b*x)*tan(a + b*x),x)
```

3.247 $\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx$

Optimal result . . . 2187
Mathematica [A] (verified) . . . 2188
Rubi [A] (verified) . . . 2189
Maple [B] (verified) . . . 2192
Fricas [B] (verification not implemented) . . . 2193
Sympy [F] . . . 2194
Maxima [B] (verification not implemented) . . . 2194
Giac [F] . . . 2195
Mupad [F(-1)] . . . 2196
Reduce [F] . . . 2196

Optimal result

Integrand size = 20, antiderivative size = 227

$$\begin{aligned}\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx = {} & \frac{8id(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b^2} \\ & - \frac{12id^2(c+dx)^2 \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} \\ & + \frac{12id^2(c+dx)^2 \operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} \\ & + \frac{24d^3(c+dx) \operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4} \\ & - \frac{24d^3(c+dx) \operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4} \\ & + \frac{24id^4 \operatorname{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^5} \\ & - \frac{24id^4 \operatorname{PolyLog}\left(4,ie^{i(a+bx)}\right)}{b^5} \\ & + \frac{(c+dx)^4 \sec(a+bx)}{b}\end{aligned}$$

output
```
8*I*d*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b^2-12*I*d^2*(d*x+c)^2*polylog(2,-I
*exp(I*(b*x+a)))/b^3+12*I*d^2*(d*x+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^3+24
*d^3*(d*x+c)*polylog(3,-I*exp(I*(b*x+a)))/b^4-24*d^3*(d*x+c)*polylog(3,I*e
xp(I*(b*x+a)))/b^4+24*I*d^4*polylog(4,-I*exp(I*(b*x+a)))/b^5-24*I*d^4*poly
log(4,I*exp(I*(b*x+a)))/b^5+(d*x+c)^4*sec(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.89

$$\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx =$$
$$-\frac{4d\left(-2ib^3c^3\arctan\left(e^{i(a+bx)}\right)+3b^3c^2dx\log\left(1-ie^{i(a+bx)}\right)+3b^3cd^2x^2\log\left(1-ie^{i(a+bx)}\right)+b^3d^3x^3\log\left(1\right.\right.}{}$$
$$+\frac{(c+dx)^4\sec(a+bx)}{b}$$

input
```
Integrate[(c + d*x)^4*Sec[a + b*x]*Tan[a + b*x],x]
```

output
```
(-4*d*((-2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 - I*E^
(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] + b^3*d^3*x^3*
Log[1 - I*E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] - 3*
b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + I*E^(I*(a +
 b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))] - (3*I)
*b^2*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, (-
I)*E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*c*d
^2*PolyLog[3, I*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))]
 - (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, I*E^(
I*(a + b*x))]))/b^5 + ((c + d*x)^4*Sec[a + b*x])/b
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {4909, 3042, 4669, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \tan(a+bx)\sec(a+bx)\,dx$$

$$\downarrow 4909$$

$$\frac{(c+dx)^4\sec(a+bx)}{b} - \frac{4d\int (c+dx)^3\sec(a+bx)dx}{b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^4\sec(a+bx)}{b} - \frac{4d\int (c+dx)^3\csc\left(a+bx+\frac{\pi}{2}\right)dx}{b}$$

$$\downarrow 4669$$

$$\frac{(c+dx)^4\sec(a+bx)}{b} - \frac{4d\left(-\frac{3d\int (c+dx)^2\log\left(1-ie^{i(a+bx)}\right)dx}{b} + \frac{3d\int (c+dx)^2\log\left(1+ie^{i(a+bx)}\right)dx}{b} - \frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}\right)}{b}$$

$$\downarrow 3011$$

$$\frac{(c+dx)^4\sec(a+bx)}{b} - \frac{4d\left(\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b} - \frac{2id\int (c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b} - \frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b} - \frac{2id\int (c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)dx}{b}\right)}{b}\right)}{b}$$

$$\downarrow 7163$$

$$\frac{(c+dx)^4\sec(a+bx)}{b} - 4d\left(\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b} - \frac{2id\left(\frac{id\int\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)dx}{b} - \frac{i(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b} - \frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,ie^{i(a}\right.\right.}{b}$$

$$\downarrow 2720$$

$$\frac{(c+dx)^4\sec(a+bx)}{b} - 4d\left(\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b} - \frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2} - \frac{i(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b} - \frac{3d\left(\frac{i(c+dx)}{b}\right.}{b}$$

$$\downarrow 7143$$

$$\frac{(c+dx)^4\sec(a+bx)}{b} - 4d\left(-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b} + \frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b} - \frac{2id\left(\frac{d\operatorname{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^2} - \frac{i(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}\right)}{b}$$

input
```
Int[(c + d*x)^4*Sec[a + b*x]*Tan[a + b*x],x]
```

output
```
(-4*d*(((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + (3*d*((I*(c + d*x)
^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c + d*x)*PolyLog
[3, (-I)*E^(I*(a + b*x))])/b + (d*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^2))/
b))/b - (3*d*((I*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b - ((2*I)*d*(
((-I)*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b + (d*PolyLog[4, I*E^(I*(a
 + b*x))])/b^2))/b))/b))/b + ((c + d*x)^4*Sec[a + b*x])/b
```

Defintions of rubi rules used

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4909

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 766 vs. $2(202) = 404$.

Time = 2.48 (sec) , antiderivative size = 767, normalized size of antiderivative = 3.38

method	result
risch	$\frac{12d^3c\ln\left(ie^{i(bx+a)}+1\right)x^2}{b^2}-\frac{12d^3a^2c\ln\left(ie^{i(bx+a)}+1\right)}{b^4}-\frac{12d^2c^2\ln\left(1-ie^{i(bx+a)}\right)x}{b^2}-\frac{12d^2c^2\ln\left(1-ie^{i(bx+a)}\right)a}{b^3}+\frac{12id^4\text{ polylo}}{}$

input

```
int((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
24*I/b^4*d^3*c*a^2*arctan(exp(I*(b*x+a)))-24*I/b^3*d^2*c^2*a*arctan(exp(I*
(b*x+a)))+24*I/b^3*d^3*c*polylog(2,I*exp(I*(b*x+a)))*x-24*I/b^3*d^3*c*poly
log(2,-I*exp(I*(b*x+a)))*x+2*exp(I*(b*x+a))*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2
*x^2+4*c^3*d*x+c^4)/b/(exp(2*I*(b*x+a))+1)+4/b^5*d^4*a^3*ln(I*exp(I*(b*x+a
))+1)-4/b^2*d^4*ln(1-I*exp(I*(b*x+a)))*x^3-24/b^4*d^3*c*polylog(3,I*exp(I*
(b*x+a)))+24/b^4*d^3*c*polylog(3,-I*exp(I*(b*x+a)))-24/b^4*d^4*polylog(3,I
*exp(I*(b*x+a)))*x-4/b^5*d^4*a^3*ln(1-I*exp(I*(b*x+a)))+4/b^2*d^4*ln(I*exp
(I*(b*x+a))+1)*x^3+24/b^4*d^4*polylog(3,-I*exp(I*(b*x+a)))*x-8*I/b^5*d^4*a
^3*arctan(exp(I*(b*x+a)))+12*I/b^3*d^2*c^2*polylog(2,I*exp(I*(b*x+a)))-12*
I/b^3*d^4*polylog(2,-I*exp(I*(b*x+a)))*x^2+8*I/b^2*d*c^3*arctan(exp(I*(b*x
+a)))+12/b^4*d^3*a^2*c*ln(1-I*exp(I*(b*x+a)))+12/b^2*d^2*c^2*ln(I*exp(I*(b
*x+a))+1)*x+12/b^3*d^2*c^2*ln(I*exp(I*(b*x+a))+1)*a-12/b^2*d^3*c*ln(1-I*ex
p(I*(b*x+a)))*x^2+12/b^2*d^3*c*ln(I*exp(I*(b*x+a))+1)*x^2-12/b^4*d^3*a^2*c
*ln(I*exp(I*(b*x+a))+1)-12/b^2*d^2*c^2*ln(1-I*exp(I*(b*x+a)))*x-12/b^3*d^2
*c^2*ln(1-I*exp(I*(b*x+a)))*a+12*I/b^3*d^4*polylog(2,I*exp(I*(b*x+a)))*x^2
-12*I/b^3*d^2*c^2*polylog(2,-I*exp(I*(b*x+a)))+24*I*d^4*polylog(4,-I*exp(I
*(b*x+a)))/b^5-24*I*d^4*polylog(4,I*exp(I*(b*x+a)))/b^5
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1190 vs. $2(189) = 378$.

Time = 0.13 (sec) , antiderivative size = 1190, normalized size of antiderivative = 5.24

$$\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

output

```
(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c
^4 - 12*I*d^4*cos(b*x + a)*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - 12*
I*d^4*cos(b*x + a)*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 12*I*d^4*co
s(b*x + a)*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) + 12*I*d^4*cos(b*x +
 a)*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 6*(-I*b^2*d^4*x^2 - 2*I*b
^2*c*d^3*x - I*b^2*c^2*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x +
a)) - 6*(-I*b^2*d^4*x^2 - 2*I*b^2*c*d^3*x - I*b^2*c^2*d^2)*cos(b*x + a)*di
log(I*cos(b*x + a) - sin(b*x + a)) - 6*(I*b^2*d^4*x^2 + 2*I*b^2*c*d^3*x +
I*b^2*c^2*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 6*(I*b
^2*d^4*x^2 + 2*I*b^2*c*d^3*x + I*b^2*c^2*d^2)*cos(b*x + a)*dilog(-I*cos(b*
x + a) - sin(b*x + a)) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 -
a^3*d^4)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) + 2*(b^3*c^3*
d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(cos(b*x +
a) - I*sin(b*x + a) + I) - 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^
2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(I*cos(b*
x + a) + sin(b*x + a) + 1) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*
d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(I*cos(
b*x + a) - sin(b*x + a) + 1) - 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^
2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(-I*c
os(b*x + a) + sin(b*x + a) + 1) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*...
```

Sympy [F]

$$\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx = \int (c+dx)^4 \tan(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**4*sec(b*x+a)*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**4*tan(a + b*x)*sec(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2944 vs. $2(189) = 378$.

Time = 0.58 (sec) , antiderivative size = 2944, normalized size of antiderivative = 12.97

$$\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

output

```
(2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x + 2*
a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2*b
*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2
+ 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2
*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1)
)*c^3*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1
)*b) - 6*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*
x + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + s
in(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x +
 a)^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2
*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a
) + 1))*a*c^2*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
+ 2*a) + 1)*b^2) + 6*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x +
 a)*sin(2*b*x + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x
+ 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2
 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2
*sin(b*x + a) + 1))*a^2*c*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 +
2*cos(2*b*x + 2*a) + 1)*b^3) - 2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a
) + 4*(b*x + a)*sin(2*b*x + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a...
```

Giac [F]

$$\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx = \int (dx+c)^4 \sec(bx+a)\tan(bx+a)\,dx$$

input

```
integrate((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

output

```
integrate((d*x + c)^4*sec(b*x + a)*tan(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx = \int \frac{\tan(a+b\,x)\,(c+d\,x)^4}{\cos(a+b\,x)}\,dx$$

input
```
int((tan(a + b*x)*(c + d*x)^4)/cos(a + b*x),x)
```

output
```
int((tan(a + b*x)*(c + d*x)^4)/cos(a + b*x), x)
```

Reduce [F]

$$\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx$$

$$= \frac{8\cos(bx+a)\left(\int \frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2 x^3}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1}dx\right) b\,d^4 + 24\cos(bx+a)\left(\int \frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2 x^2}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1}dx\right) bc\,d^3 + 24\cos(bx+a)\left(\int$$

input
```
int((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x)
```

output
```
(8*cos(a + b*x)*int((tan((a + b*x)/2)**2*x**3)/(tan((a + b*x)/2)**2 - 1),x
)*b*d**4 + 24*cos(a + b*x)*int((tan((a + b*x)/2)**2*x**2)/(tan((a + b*x)/2
)**2 - 1),x)*b*c*d**3 + 24*cos(a + b*x)*int((tan((a + b*x)/2)**2*x)/(tan((
a + b*x)/2)**2 - 1),x)*b*c**2*d**2 + 4*cos(a + b*x)*log(tan((a + b*x)/2) -
 1)*c**3*d - 4*cos(a + b*x)*log(tan((a + b*x)/2) + 1)*c**3*d - cos(a + b*x
)*b*c**4 - 6*cos(a + b*x)*b*c**2*d**2*x**2 - 4*cos(a + b*x)*b*c*d**3*x**3
 - cos(a + b*x)*b*d**4*x**4 + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 +
4*b*c*d**3*x**3 + b*d**4*x**4)/(cos(a + b*x)*b**2)
```

3.248 $\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx$

Optimal result . . . 2197
Mathematica [A] (verified) . . . 2198
Rubi [A] (verified) . . . 2198
Maple [B] (verified) . . . 2201
Fricas [B] (verification not implemented) . . . 2201
Sympy [F] . . . 2202
Maxima [B] (verification not implemented) . . . 2203
Giac [F] . . . 2204
Mupad [F(-1)] . . . 2204
Reduce [F] . . . 2204

Optimal result

Integrand size = 20, antiderivative size = 159

$$\begin{aligned}\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx = {} & \frac{6id(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b^2} \\ & - \frac{6id^2(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} \\ & + \frac{6id^2(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} \\ & + \frac{6d^3\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4} \\ & - \frac{6d^3\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4} \\ & + \frac{(c+dx)^3\sec(a+bx)}{b}\end{aligned}$$

output

```
6*I*d*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b^2-6*I*d^2*(d*x+c)*polylog(2,-I*ex
p(I*(b*x+a)))/b^3+6*I*d^2*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^3+6*d^3*po
lylog(3,-I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4+(d*x+
c)^3*sec(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.61

$$\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx =$$
$$-\frac{3d\left(-2ib^2c^2\arctan\left(e^{i(a+bx)}\right)+2b^2cdx\log\left(1-ie^{i(a+bx)}\right)+b^2d^2x^2\log\left(1-ie^{i(a+bx)}\right)-2b^2cdx\log\left(1+\right.\right.}{}$$
$$+\frac{(c+dx)^3\sec(a+bx)}{b}$$

input
```
Integrate[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x],x]
```

output
```
(-3*d*((-2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 - I*E^(I
*(a + b*x))] + b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - 2*b^2*c*d*x*Log[1
+ I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + (2*I)*b*d*
(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2
, I*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 2*d^2*Poly
Log[3, I*E^(I*(a + b*x))]))/b^4 + ((c + d*x)^3*Sec[a + b*x])/b
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4909, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\tan(a+bx)\sec(a+bx)\,dx$$
$$\downarrow 4909$$
$$\frac{(c+dx)^3\sec(a+bx)}{b}-\frac{3d\int(c+dx)^2\sec(a+bx)dx}{b}$$
$$\downarrow 3042$$
$$\frac{(c+dx)^3\sec(a+bx)}{b}-\frac{3d\int(c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx}{b}$$

$$\downarrow 4669$$

$$\frac{(c+dx)^3\sec(a+bx)}{b} - \frac{3d\left(-\frac{2d\int(c+dx)\log\left(1-ie^{i(a+bx)}\right)dx}{b} + \frac{2d\int(c+dx)\log\left(1+ie^{i(a+bx)}\right)dx}{b} - \frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}\right)}{b}$$

$$\downarrow 3011$$

$$\frac{(c+dx)^3\sec(a+bx)}{b} - \frac{3d\left(\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b} - \frac{id\int\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b} - \frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b} - \frac{id\int\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)dx}{b}\right)}{b} - \right.}{b}$$

$$\downarrow 2720$$

$$\frac{(c+dx)^3\sec(a+bx)}{b} - \frac{3d\left(\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b} - \frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b} - \frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b} - \frac{d\int e^{-i(a+bx)}\,\mathrm{Poly}\right.}{b}}{b}$$

$$\downarrow 7143$$

$$\frac{(c+dx)^3\sec(a+bx)}{b} - \frac{3d\left(-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b} + \frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b} - \frac{d\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^2}\right)}{b} - \frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b} - \frac{d\,\mathrm{P}\right.}{b}}{b}$$

input

```
Int[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x],x]
```

output

```
(-3*d*(((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (2*d*((I*(c + d*x)
*PolyLog[2, (-I)*E^(I*(a + b*x))])/b - (d*PolyLog[3, (-I)*E^(I*(a + b*x))]
)/b^2))/b - (2*d*((I*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b - (d*PolyL
og[3, I*E^(I*(a + b*x))])/b^2))/b))/b + ((c + d*x)^3*Sec[a + b*x])/b
```

Defintions of rubi rules used

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4909

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(142) = 284$.

Time = 1.04 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.91

method	result
risch	$\frac{2(d^3x^3+3cd^2x^2+3c^2dx+c^3)e^{i(bx+a)}}{b(e^{2i(bx+a)}+1)} - \frac{6id^3x\,\mathrm{polylog}(2,-ie^{i(bx+a)})}{b^3} - \frac{12id^2ca\arctan(e^{i(bx+a)})}{b^3} - \frac{6d^2c\ln(1-ie^{i(bx+a)})a}{b^3}$

input
```
int((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))+1
)-6*I*d^3*x*polylog(2,-I*exp(I*(b*x+a)))/b^3-12*I/b^3*d^2*c*a*arctan(exp(I
*(b*x+a)))-6/b^3*d^2*c*ln(1-I*exp(I*(b*x+a)))*a-6/b^2*d^2*c*ln(1-I*exp(I*(
b*x+a)))*x-6*I*c*d^2*polylog(2,-I*exp(I*(b*x+a)))/b^3+6/b^2*d^2*c*ln(I*exp
(I*(b*x+a))+1)*x+3/b^4*d^3*a^2*ln(1-I*exp(I*(b*x+a)))+6*d^3*polylog(3,-I*e
xp(I*(b*x+a)))/b^4+6*I*c*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3+6*I/b^2*d*c^2
*arctan(exp(I*(b*x+a)))+6/b^3*d^2*c*ln(I*exp(I*(b*x+a))+1)*a+3/b^2*d^3*ln(
I*exp(I*(b*x+a))+1)*x^2+6*I*d^3*x*polylog(2,I*exp(I*(b*x+a)))/b^3+6*I/b^4*
d^3*a^2*arctan(exp(I*(b*x+a)))-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4-3/b^2
*d^3*ln(1-I*exp(I*(b*x+a)))*x^2-3/b^4*d^3*a^2*ln(I*exp(I*(b*x+a))+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 783 vs. $2(133) = 266$.

Time = 0.11 (sec) , antiderivative size = 783, normalized size of antiderivative = 4.92

$$\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

output

```
1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*c
os(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)
*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3,
 -I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*
x + a) - sin(b*x + a)) - 6*(-I*b*d^3*x - I*b*c*d^2)*cos(b*x + a)*dilog(I*c
os(b*x + a) + sin(b*x + a)) - 6*(-I*b*d^3*x - I*b*c*d^2)*cos(b*x + a)*dilo
g(I*cos(b*x + a) - sin(b*x + a)) - 6*(I*b*d^3*x + I*b*c*d^2)*cos(b*x + a)*
dilog(-I*cos(b*x + a) + sin(b*x + a)) - 6*(I*b*d^3*x + I*b*c*d^2)*cos(b*x
+ a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*(b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) + 3*(b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a)
+ I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a
)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x
+ 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) +
1) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*
log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x +
 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) +
1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a)
+ I*sin(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)
*log(-cos(b*x + a) - I*sin(b*x + a) + I))/(b^4*cos(b*x + a))
```

Sympy [F]

$$\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx = \int (c+dx)^3 \tan(a+bx)\sec(a+bx)\,dx$$

input

```
integrate((d*x+c)**3*sec(b*x+a)*tan(b*x+a),x)
```

output

```
Integral((c + d*x)**3*tan(a + b*x)*sec(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1770 vs. $2(133) = 266$.

Time = 0.47 (sec) , antiderivative size = 1770, normalized size of antiderivative = 11.13

$$\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

output

```
1/2*(3*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x
+ 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin
(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a
)^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*c
os(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a)
+ 1))*c^2*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)
 + 1)*b) - 6*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(
2*b*x + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2
 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b
*x + a)^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x
 + a) + 1))*a*c*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*
x + 2*a) + 1)*b^2) + 3*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x
 + a)*sin(2*b*x + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*
x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)
^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*
x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 -
 2*sin(b*x + a) + 1))*a^2*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 +
2*cos(2*b*x + 2*a) + 1)*b^3) + 2*c^3/cos(b*x + a) - 6*a*c^2*d/(b*cos(b*x +
 a)) + 6*a^2*c*d^2/(b^2*cos(b*x + a)) - 2*a^3*d^3/(b^3*cos(b*x + a)) + ...
```

Giac [F]

$$\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx = \int (dx+c)^3 \sec(bx+a)\tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*sec(b*x + a)*tan(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx = \int \frac{\tan(a+b\,x)\,(c+d\,x)^3}{\cos(a+b\,x)}\,dx$$

input
```
int((tan(a + b*x)*(c + d*x)^3)/cos(a + b*x),x)
```

output
```
int((tan(a + b*x)*(c + d*x)^3)/cos(a + b*x), x)
```

Reduce [F]

$$\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx$$

$$= \frac{6\cos(bx+a)\left(\int \frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2 x^2}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1}dx\right) b\,d^3 + 12\cos(bx+a)\left(\int \frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2 x}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1}dx\right) bc\,d^2 + 3\cos(bx+a)\log($$

input
```
int((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x)
```

output

```
(6*cos(a + b*x)*int((tan((a + b*x)/2)**2*x**2)/(tan((a + b*x)/2)**2 - 1),x
)*b*d**3 + 12*cos(a + b*x)*int((tan((a + b*x)/2)**2*x)/(tan((a + b*x)/2)**
2 - 1),x)*b*c*d**2 + 3*cos(a + b*x)*log(tan((a + b*x)/2) - 1)*c**2*d - 3*c
os(a + b*x)*log(tan((a + b*x)/2) + 1)*c**2*d - cos(a + b*x)*b*c**3 - 3*cos
(a + b*x)*b*c*d**2*x**2 - cos(a + b*x)*b*d**3*x**3 + b*c**3 + 3*b*c**2*d*x
 + 3*b*c*d**2*x**2 + b*d**3*x**3)/(cos(a + b*x)*b**2)
```

3.249 $\int (c+dx)^2 \sec(a+bx)\tan(a+bx)\,dx$

Optimal result . . . 2206
Mathematica [A] (warning: unable to verify) . . . 2206
Rubi [A] (verified) . . . 2207
Maple [B] (verified) . . . 2209
Fricas [B] (verification not implemented) . . . 2210
Sympy [F] . . . 2210
Maxima [F] . . . 2211
Giac [F] . . . 2211
Mupad [F(-1)] . . . 2212
Reduce [F] . . . 2212

Optimal result

Integrand size = 20, antiderivative size = 97

$$\int (c+dx)^2 \sec(a+bx)\tan(a+bx)\,dx = \frac{4id(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^2} - \frac{2id^2\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} + \frac{2id^2\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^2\sec(a+bx)}{b}$$

output

```
4*I*d*(d*x+c)*arctan(exp(I*(b*x+a)))/b^2-2*I*d^2*polylog(2,-I*exp(I*(b*x+a
)))/b^3+2*I*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3+(d*x+c)^2*sec(b*x+a)/b
```

Mathematica [A] (warning: unable to verify)

Time = 1.15 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.79

$$\int (c+dx)^2 \sec(a+bx)\tan(a+bx)\,dx$$

$$= \frac{-4bcd\operatorname{arctanh}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right) - 4d^2\arctan(\cot(a))\operatorname{arctanh}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right) + \frac{2d^2\csc}{}}{}$$

input
```
Integrate[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x],x]
```

output
```
(-4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - 4*d^2*ArcTan[Cot[a]]*Arc
Tanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + (2*d^2*Csc[a]*((b*x - ArcTan[Cot[a]])
*(Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan[Cot[a
]]))]) + I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyLog[2, E^(I*(
b*x - ArcTan[Cot[a]]))]))/Sqrt[Csc[a]^2] + b^2*(c + d*x)^2*Sec[a + b*x])/b
^3
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4909, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \tan(a+bx)\sec(a+bx)\,dx$$

$$\downarrow 4909$$

$$\frac{(c+dx)^2\sec(a+bx)}{b} - \frac{2d\int (c+dx)\sec(a+bx)dx}{b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^2\sec(a+bx)}{b} - \frac{2d\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)dx}{b}$$

$$\downarrow 4669$$

$$\frac{(c+dx)^2\sec(a+bx)}{b} - \frac{2d\left(-\frac{d\int \log(1-ie^{i(a+bx)})dx}{b} + \frac{d\int \log(1+ie^{i(a+bx)})dx}{b} - \frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}\right)}{b}$$

$$\downarrow 2715$$

$$\frac{(c+dx)^2\sec(a+bx)}{b} - \frac{2d\left(\frac{id\int e^{-i(a+bx)}\log\left(1-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2} - \frac{id\int e^{-i(a+bx)}\log\left(1+ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2} - \frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}\right)}{b}$$

$$\downarrow 2838$$

$$\frac{(c+dx)^2\sec(a+bx)}{b} - \frac{2d\left(-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b} + \frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} - \frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}\right)}{b}$$

input
```
Int[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x],x]
```

output
```
(-2*d*(((-2*I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b + (I*d*PolyLog[2, (-I)
*E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2))/b + ((c
 + d*x)^2*Sec[a + b*x])/b
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4909

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(86) = 172.

Time = 0.52 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.34

method	result
risch	$\frac{2\,\mathrm{e}^{i(bx+a)}(x^2d^2+2cdx+c^2)}{b\left(\mathrm{e}^{2i(bx+a)}+1\right)}+\frac{4idc\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b^2}+\frac{2d^2\ln\left(i\mathrm{e}^{i(bx+a)}+1\right)x}{b^2}+\frac{2d^2\ln\left(i\mathrm{e}^{i(bx+a)}+1\right)a}{b^3}-\frac{2d^2\ln($
derivativedivides	$\frac{a^2d^2}{b^2\cos(bx+a)}-\frac{2acd}{b\cos(bx+a)}-\frac{2a\,d^2\left(\frac{bx+a}{\cos(bx+a)}-\ln(\sec(bx+a)+\tan(bx+a))\right)}{b^2}+\frac{c^2}{\cos(bx+a)}+\frac{2cd\left(\frac{bx+a}{\cos(bx+a)}-\ln(\sec(bx+a)+\tan(b$
default	$\frac{a^2d^2}{b^2\cos(bx+a)}-\frac{2acd}{b\cos(bx+a)}-\frac{2a\,d^2\left(\frac{bx+a}{\cos(bx+a)}-\ln(\sec(bx+a)+\tan(bx+a))\right)}{b^2}+\frac{c^2}{\cos(bx+a)}+\frac{2cd\left(\frac{bx+a}{\cos(bx+a)}-\ln(\sec(bx+a)+\tan(b$

input

```
int((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
2*exp(I*(b*x+a))*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x+a))+1)+4*I/b^2*d*c*
arctan(exp(I*(b*x+a)))+2/b^2*d^2*ln(I*exp(I*(b*x+a))+1)*x+2/b^3*d^2*ln(I*e
xp(I*(b*x+a))+1)*a-2/b^2*d^2*ln(1-I*exp(I*(b*x+a)))*x-2/b^3*d^2*ln(1-I*exp
(I*(b*x+a)))*a-2*I/b^3*d^2*dilog(I*exp(I*(b*x+a))+1)+2*I/b^3*d^2*dilog(1-I
*exp(I*(b*x+a)))-4*I/b^3*d^2*a*arctan(exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(79) = 158$.

Time $= 0.10$ (sec) , antiderivative size $= 446$, normalized size of antiderivative $= 4.60$

$$\int (c+dx)^2 \sec(a+bx)\tan(a+bx)\,dx$$
$$= \frac{b^2d^2x^2 + 2\,b^2cdx + b^2c^2 + i\,d^2\cos(bx+a)\,\mathrm{Li}_2(i\cos(bx+a) + \sin(bx+a)) + i\,d^2\cos(bx+a)\,\mathrm{Li}_2(i\cos}{}$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

output
```
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*cos(b*x + a)*dilog(I*cos(b*x
+ a) + sin(b*x + a)) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x +
 a)) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d^2*co
s(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b*c*d - a*d^2)*cos(b*x
 + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a
)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b*d^2*x + a*d^2)*cos(b*x + a)*
log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*lo
g(I*cos(b*x + a) - sin(b*x + a) + 1) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(
-I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-
I*cos(b*x + a) - sin(b*x + a) + 1) - (b*c*d - a*d^2)*cos(b*x + a)*log(-cos
(b*x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*
x + a) - I*sin(b*x + a) + I))/(b^3*cos(b*x + a))
```

Sympy [F]

$$\int (c+dx)^2 \sec(a+bx)\tan(a+bx)\,dx = \int (c+dx)^2 \tan(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*sec(b*x+a)*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**2*tan(a + b*x)*sec(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)^2 \sec(a+bx)\tan(a+bx)\,dx = \int (dx+c)^2 \sec(bx+a)\tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

output
```
(2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)*cos(b*x + a) + 2*(b*d^
2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a)*sin(b*x + a) + 2*(b*d^2*x^2 +
2*b*c*d*x + b*c^2)*cos(b*x + a) - 4*(b^2*d^2*cos(2*b*x + 2*a)^2 + b^2*d^2*
sin(2*b*x + 2*a)^2 + 2*b^2*d^2*cos(2*b*x + 2*a) + b^2*d^2)*integrate((x*co
s(2*b*x + 2*a)*cos(b*x + a) + x*sin(2*b*x + 2*a)*sin(b*x + a) + x*cos(b*x
+ a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 + 2*b*cos(2*b*x + 2*a)
+ b), x) - (c*d*cos(2*b*x + 2*a)^2 + c*d*sin(2*b*x + 2*a)^2 + 2*c*d*cos(2*
b*x + 2*a) + c*d)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1
) + (c*d*cos(2*b*x + 2*a)^2 + c*d*sin(2*b*x + 2*a)^2 + 2*c*d*cos(2*b*x + 2
*a) + c*d)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))/(b^2
*cos(2*b*x + 2*a)^2 + b^2*sin(2*b*x + 2*a)^2 + 2*b^2*cos(2*b*x + 2*a) + b^
2)
```

Giac [F]

$$\int (c+dx)^2 \sec(a+bx)\tan(a+bx)\,dx = \int (dx+c)^2 \sec(bx+a)\tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*sec(b*x + a)*tan(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \sec(a+bx)\tan(a+bx)\,dx = \int \frac{\tan(a+b\,x)\,(c+d\,x)^2}{\cos(a+b\,x)}\,dx$$

input
```
int((tan(a + b*x)*(c + d*x)^2)/cos(a + b*x),x)
```

output
```
int((tan(a + b*x)*(c + d*x)^2)/cos(a + b*x), x)
```

Reduce [F]

$$\int (c+dx)^2 \sec(a+bx)\tan(a+bx)\,dx$$

$$= \frac{4\cos(bx+a)\left(\int \frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2 x}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1}dx\right) b\,d^2 + 2\cos(bx+a)\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right) cd - 2\cos(bx+a)\log(\text{ta}}{\cos(bx+a)\,b^2}$$

input
```
int((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x)
```

output
```
(4*cos(a + b*x)*int((tan((a + b*x)/2)**2*x)/(tan((a + b*x)/2)**2 - 1),x)*b
*d**2 + 2*cos(a + b*x)*log(tan((a + b*x)/2) - 1)*c*d - 2*cos(a + b*x)*log(
tan((a + b*x)/2) + 1)*c*d - cos(a + b*x)*b*c**2 - cos(a + b*x)*b*d**2*x**2
 + b*c**2 + 2*b*c*d*x + b*d**2*x**2)/(cos(a + b*x)*b**2)
```

3.250 $\int (c+dx)\sec(a+bx)\tan(a+bx)\,dx$

Optimal result . . . 2213
Mathematica [B] (verified) . . . 2213
Rubi [A] (verified) . . . 2214
Maple [B] (verified) . . . 2215
Fricas [B] (verification not implemented) . . . 2216
Sympy [F] . . . 2216
Maxima [B] (verification not implemented) . . . 2216
Giac [B] (verification not implemented) . . . 2217
Mupad [B] (verification not implemented) . . . 2218
Reduce [B] (verification not implemented) . . . 2219

Optimal result

Integrand size = 18, antiderivative size = 29

$$\int (c+dx)\sec(a+bx)\tan(a+bx)\,dx = -\frac{d\operatorname{arctanh}(\sin(a+bx))}{b^2} + \frac{(c+dx)\sec(a+bx)}{b}$$

output
```
-d*arctanh(sin(b*x+a))/b^2+(d*x+c)*sec(b*x+a)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 93 vs. $2(29) = 58$.

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.21

$$\begin{aligned}\int (c+dx)\sec(a+bx)\tan(a+bx)\,dx = {} & \frac{d\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2} \\ & - \frac{d\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2} \\ & + \frac{c\sec(a+bx)}{b} + \frac{dx\sec(a+bx)}{b}\end{aligned}$$

input
```
Integrate[(c + d*x)*Sec[a + b*x]*Tan[a + b*x],x]
```

output
```
(d*Log[Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2]])/b^2 - (d*Log[Cos[a/2 + (b
*x)/2] + Sin[a/2 + (b*x)/2]])/b^2 + (c*Sec[a + b*x])/b + (d*x*Sec[a + b*x]
)/b
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4909, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\tan(a+bx)\sec(a+bx)\,dx$$

$$\downarrow 4909$$

$$\frac{(c+dx)\sec(a+bx)}{b} - \frac{d\int \sec(a+bx)dx}{b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)\sec(a+bx)}{b} - \frac{d\int \csc\left(a+bx+\frac{\pi}{2}\right)dx}{b}$$

$$\downarrow 4257$$

$$\frac{(c+dx)\sec(a+bx)}{b} - \frac{d\operatorname{arctanh}(\sin(a+bx))}{b^2}$$

input
```
Int[(c + d*x)*Sec[a + b*x]*Tan[a + b*x],x]
```

output
```
-((d*ArcTanh[Sin[a + b*x]])/b^2) + ((c + d*x)*Sec[a + b*x])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(29) = 58$.

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.31

method	result	size
derivativedivides	$\frac{-\frac{da}{b\cos(bx+a)}+\frac{c}{\cos(bx+a)}+\frac{d\left(\frac{bx+a}{\cos(bx+a)}-\ln(\sec(bx+a)+\tan(bx+a))\right)}{b}}{b}$	67
default	$\frac{-\frac{da}{b\cos(bx+a)}+\frac{c}{\cos(bx+a)}+\frac{d\left(\frac{bx+a}{\cos(bx+a)}-\ln(\sec(bx+a)+\tan(bx+a))\right)}{b}}{b}$	67
risch	$\frac{2e^{i(bx+a)}(dx+c)}{b\left(e^{2i(bx+a)}+1\right)}-\frac{d\ln\left(e^{i(bx+a)}+i\right)}{b^2}+\frac{d\ln\left(e^{i(bx+a)}-i\right)}{b^2}$	71

input
```
int((d*x+c)*sec(b*x+a)*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/b*d*a/cos(b*x+a)+c/cos(b*x+a)+1/b*d*((b*x+a)/cos(b*x+a)-ln(sec(b*x
+a)+tan(b*x+a))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(29) = 58$.

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.07

$$\int (c + dx) \sec(a + bx) \tan(a + bx)\, dx$$
$$= \frac{2\,bdx - d\cos(bx + a)\log(\sin(bx + a) + 1) + d\cos(bx + a)\log(-\sin(bx + a) + 1) + 2\,bc}{2\,b^2\cos(bx + a)}$$

input
```
integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*b*d*x - d*cos(b*x + a)*log(sin(b*x + a) + 1) + d*cos(b*x + a)*log(-
sin(b*x + a) + 1) + 2*b*c)/(b^2*cos(b*x + a))
```

Sympy [F]

$$\int (c + dx) \sec(a + bx) \tan(a + bx)\, dx = \int (c + dx) \tan(a + bx) \sec(a + bx)\, dx$$

input
```
integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x)
```

output
```
Integral((c + d*x)*tan(a + b*x)*sec(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(29) = 58$.

Time = 0.13 (sec) , antiderivative size = 259, normalized size of antiderivative = 8.93

$$\int (c + dx) \sec(a + bx) \tan(a + bx)\, dx$$
$$= \frac{\left(4\,(bx+a)\cos(2\,bx+2\,a)\cos(bx+a)+4\,(bx+a)\sin(2\,bx+2\,a)\sin(bx+a)+4\,(bx+a)\cos(bx+a)-\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)\right.\right.}{\left(\cos(2\,bx+2\,a)^2+\si\right.}$$

input
```
integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

output
```
1/2*((4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x +
2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^
2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos
(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) +
1))*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*
b) + 2*c/cos(b*x + a) - 2*a*d/(b*cos(b*x + a)))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1267 vs. $2(29) = 58$.

Time $= 0.42$ (sec) , antiderivative size $= 1267$, normalized size of antiderivative $= 43.69$

$$\int (c + dx)\sec(a + bx)\tan(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

output

```
1/2*(2*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2*tan(1/2*a)
^2 + d*log(2*(tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2*tan(1/2*a) +
2*tan(1/2*b*x)*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*
x) - 2*tan(1/2*a) + 1)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan
(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a)^2 - d*log(2*(tan(1/2*b*x)^2*tan(
1/2*a)^2 - 2*tan(1/2*b*x)^2*tan(1/2*a) - 2*tan(1/2*b*x)*tan(1/2*a)^2 + tan
(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*b
*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2*ta
n(1/2*a)^2 + 2*b*d*x*tan(1/2*b*x)^2 + 2*b*d*x*tan(1/2*a)^2 + 2*b*c*tan(1/2
*b*x)^2 - d*log(2*(tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2*tan(1/2*
a) + 2*tan(1/2*b*x)*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1
/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2
+ tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2 + d*log(2*(tan(1/2*b*x)^2*tan(1/2*a)^2
 - 2*tan(1/2*b*x)^2*tan(1/2*a) - 2*tan(1/2*b*x)*tan(1/2*a)^2 + tan(1/2*b*x
)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*b*x)^2*ta
n(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2 - 4*d*log(
2*(tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2*tan(1/2*a) + 2*tan(1/2*b
*x)*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(
1/2*a) + 1)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 +
 1))*tan(1/2*b*x)*tan(1/2*a) + 4*d*log(2*(tan(1/2*b*x)^2*tan(1/2*a)^2 -...
```

Mupad [B] (verification not implemented)

Time = 19.92 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.69

$$\int (c+dx)\sec(a+bx)\tan(a+bx)\,dx = \frac{d\,\ln\left(\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}-\mathrm{i}\right)}{b^2} - \frac{d\,\ln\left(\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}+1\mathrm{i}\right)}{b^2} + \frac{2\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,(c+d\,x)}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}$$

input

```
int((tan(a + b*x)*(c + d*x))/cos(a + b*x),x)
```

output

```
(d*log(exp(a*1i + b*x*1i) - 1i))/b^2 - (d*log(exp(a*1i + b*x*1i) + 1i))/b^
2 + (2*exp(a*1i + b*x*1i)*(c + d*x))/(b*(exp(a*2i + b*x*2i) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.45

$$\int (c+dx)\sec(a+bx)\tan(a+bx)\,dx$$
$$= \frac{\cos(bx+a)\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)d-\cos(bx+a)\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)d-\cos(bx+a)bc+bc+bdx}{\cos(bx+a)b^2}$$

input
```
int((d*x+c)*sec(b*x+a)*tan(b*x+a),x)
```

output
```
(cos(a + b*x)*log(tan((a + b*x)/2) - 1)*d - cos(a + b*x)*log(tan((a + b*x)
/2) + 1)*d - cos(a + b*x)*b*c + b*c + b*d*x)/(cos(a + b*x)*b**2)
```

3.251 $\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx$

Optimal result 2220
Mathematica [N/A] 2220
Rubi [N/A] 2221
Maple [N/A] 2221
Fricas [N/A] 2222
Sympy [N/A] 2222
Maxima [N/A] 2223
Giac [N/A] 2223
Mupad [N/A] 2224
Reduce [N/A] 2224

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\sec(a+bx)\tan(a+bx)}{c+dx}, x\right)$$

output
```
Defer(Int)(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 7.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x),x]
```

output
```
Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(a+bx)\sec(a+bx)}{c+dx}\,dx$$
$$\downarrow 7299$$
$$\int \frac{\tan(a+bx)\sec(a+bx)}{c+dx}\,dx$$

input
```
Int[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sec(bx+a)\tan(bx+a)}{dx+c}\,dx$$

input
```
int(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

output
```
int(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)\tan(bx+a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)*tan(b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\tan(a+bx)\sec(a+bx)}{c+dx}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

output
```
Integral(tan(a + b*x)*sec(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 349, normalized size of antiderivative = 17.45

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)\tan(bx+a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
2*(cos(2*b*x + 2*a)*cos(b*x + a) + (b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*co
s(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x + b*c
*d)*cos(2*b*x + 2*a))*integrate((cos(2*b*x + 2*a)*cos(b*x + a) + sin(2*b*x
 + 2*a)*sin(b*x + a) + cos(b*x + a))/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d
^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x +
b*c^2)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x +
2*a)), x) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/(b*d*x + (b*d*x
+ b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x + 2*a)^2 + b*c + 2*(b*
d*x + b*c)*cos(2*b*x + 2*a))
```

Giac [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)\tan(bx+a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)*tan(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 18.80 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\tan(a+b\,x)}{\cos(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(tan(a + b*x)/(cos(a + b*x)*(c + d*x)),x)
```

output
```
int(tan(a + b*x)/(cos(a + b*x)*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)\tan(bx+a)}{dx+c}dx$$

input
```
int(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

output
```
int((sec(a + b*x)*tan(a + b*x))/(c + d*x),x)
```

3.252 $\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 2225
Mathematica [N/A] . . . 2225
Rubi [N/A] . . . 2226
Maple [N/A] . . . 2226
Fricas [N/A] . . . 2227
Sympy [N/A] . . . 2227
Maxima [N/A] . . . 2228
Giac [N/A] . . . 2228
Mupad [N/A] . . . 2229
Reduce [N/A] . . . 2229

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 16.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$
$$\downarrow 7299$$
$$\int \frac{\tan(a+bx)\sec(a+bx)}{(c+dx)^2}dx$$

input
```
Int[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sec(bx+a)\tan(bx+a)}{(dx+c)^2}dx$$

input
```
int(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
```

output
```
int(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec\left(bx+a\right)\tan\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)*tan(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan\left(a+bx\right)\sec\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(tan(a + b*x)*sec(a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 494, normalized size of antiderivative = 24.70

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec(bx+a)\tan(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
2*(cos(2*b*x + 2*a)*cos(b*x + a) + 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d +
(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a)^2 + (b*d^3*x^2 + 2*b*
c*d^2*x + b*c^2*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2
*d)*cos(2*b*x + 2*a))*integrate((cos(2*b*x + 2*a)*cos(b*x + a) + sin(2*b*x
 + 2*a)*sin(b*x + a) + cos(b*x + a))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*
d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(2*b*x
+ 2*a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(2*b*x + 2
*a)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(2*b*x + 2*
a)), x) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/(b*d^2*x^2 + 2*b*c
*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2
*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x +
b*c^2)*cos(2*b*x + 2*a))
```

Giac [N/A]

Not integrable

Time = 8.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec(bx+a)\tan(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)*tan(b*x + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 18.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(a+b\,x)}{\cos(a+b\,x)\,(c+d\,x)^2}\,dx$$

input
```
int(tan(a + b*x)/(cos(a + b*x)*(c + d*x)^2),x)
```

output
```
int(tan(a + b*x)/(cos(a + b*x)*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 745, normalized size of antiderivative = 37.25

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
```

output

```
(2*cos(a + b*x)*int(sin(a + b*x)**2/(sin(a + b*x)**2*c**2 + 2*sin(a + b*x)
**2*c*d*x + sin(a + b*x)**2*d**2*x**2 - c**2 - 2*c*d*x - d**2*x**2),x)*b*c
**2*d + 2*cos(a + b*x)*int(sin(a + b*x)**2/(sin(a + b*x)**2*c**2 + 2*sin(a
 + b*x)**2*c*d*x + sin(a + b*x)**2*d**2*x**2 - c**2 - 2*c*d*x - d**2*x**2)
,x)*b*c*d**2*x - cos(a + b*x)*int(sin(a + b*x)/(sin(a + b*x)**2*c**2 + 2*s
in(a + b*x)**2*c*d*x + sin(a + b*x)**2*d**2*x**2 - c**2 - 2*c*d*x - d**2*x
**2),x)*c*d**2 - cos(a + b*x)*int(sin(a + b*x)/(sin(a + b*x)**2*c**2 + 2*s
in(a + b*x)**2*c*d*x + sin(a + b*x)**2*d**2*x**2 - c**2 - 2*c*d*x - d**2*x
**2),x)*d**3*x - 2*cos(a + b*x)*int((cos(a + b*x)*sin(a + b*x))/(sin(a + b
*x)**2*c**2 + 2*sin(a + b*x)**2*c*d*x + sin(a + b*x)**2*d**2*x**2 - c**2 -
 2*c*d*x - d**2*x**2),x)*c*d**2 - 2*cos(a + b*x)*int((cos(a + b*x)*sin(a +
 b*x))/(sin(a + b*x)**2*c**2 + 2*sin(a + b*x)**2*c*d*x + sin(a + b*x)**2*d
**2*x**2 - c**2 - 2*c*d*x - d**2*x**2),x)*d**3*x + 2*cos(a + b*x)*int((sin
(a + b*x)**2*x)/(sin(a + b*x)**2*c**2 + 2*sin(a + b*x)**2*c*d*x + sin(a +
b*x)**2*d**2*x**2 - c**2 - 2*c*d*x - d**2*x**2),x)*b*c*d**2 + 2*cos(a + b*
x)*int((sin(a + b*x)**2*x)/(sin(a + b*x)**2*c**2 + 2*sin(a + b*x)**2*c*d*x
 + sin(a + b*x)**2*d**2*x**2 - c**2 - 2*c*d*x - d**2*x**2),x)*b*d**3*x - 2
*cos(a + b*x)*log(c + d*x)*b*c - 2*cos(a + b*x)*log(c + d*x)*b*d*x + 2*sin
(a + b*x)*d)/(cos(a + b*x)*d**2*(c + d*x))
```

3.253 $\int (c+dx)^m \tan^2(a+bx)\,dx$

Optimal result 2231
Mathematica [N/A] 2231
Rubi [N/A] 2232
Maple [N/A] 2233
Fricas [N/A] 2233
Sympy [N/A] 2234
Maxima [N/A] 2234
Giac [N/A] 2235
Mupad [N/A] 2235
Reduce [N/A] 2235

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (c+dx)^m \tan^2(a+bx)\,dx = \text{Int}\big((c+dx)^m \tan^2(a+bx), x\big)$$

output
```
Defer(Int)((d*x+c)^m*tan(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.80 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \tan^2(a+bx)\,dx = \int (c+dx)^m \tan^2(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Tan[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Tan[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(a+bx)(c+dx)^m\,dx$$

$$\downarrow 3042$$

$$\int \tan(a+bx)^2(c+dx)^m dx$$

$$\downarrow 4222$$

$$\int \tan^2(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Tan[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \tan(bx+a)^2 \, dx$$

input

```
int((d*x+c)^m*tan(b*x+a)^2,x)
```

output

```
int((d*x+c)^m*tan(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \tan^2(a+bx) \, dx = \int (dx+c)^m \tan(bx+a)^2 \, dx$$

input

```
integrate((d*x+c)^m*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*tan(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (c+dx)^m \tan^2(a+bx)\,dx = \int (c+dx)^m \tan^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*tan(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*tan(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \tan^2(a+bx)\,dx = \int (dx+c)^m \tan\left(bx+a\right)^2\,dx$$

input
```
integrate((d*x+c)^m*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*tan(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \tan^2(a+bx)\,dx = \int (dx+c)^m \tan(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*tan(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 18.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \tan^2(a+bx)\,dx = \int \tan(a+b\,x)^2\,(c+d\,x)^m\,dx$$

input
```
int(tan(a + b*x)^2*(c + d*x)^m,x)
```

output
```
int(tan(a + b*x)^2*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 127, normalized size of antiderivative = 7.94

$$\int (c+dx)^m \tan^2(a+bx)\,dx$$
$$= \frac{(dx+c)^m \tan(bx+a)\,dm + (dx+c)^m \tan(bx+a)\,d - (dx+c)^m\,bc - (dx+c)^m\,bdx - \left(\int \frac{(dx+c)^m \tan(bx}{dx+c}\right.}{bd\,(m+1)}$$

input
```
int((d*x+c)^m*tan(b*x+a)^2,x)
```

output
```
((c + d*x)**m*tan(a + b*x)*d*m + (c + d*x)**m*tan(a + b*x)*d - (c + d*x)**
m*b*c - (c + d*x)**m*b*d*x - int(((c + d*x)**m*tan(a + b*x))/(c + d*x),x)*
d**2*m**2 - int(((c + d*x)**m*tan(a + b*x))/(c + d*x),x)*d**2*m)/(b*d*(m +
 1))
```

3.254 $\int (c+dx)^3 \tan^2(a+bx)\,dx$

Optimal result . . . 2237
Mathematica [B] (warning: unable to verify) . . . 2237
Rubi [A] (verified) . . . 2238
Maple [B] (verified) . . . 2242
Fricas [B] (verification not implemented) . . . 2242
Sympy [F] . . . 2243
Maxima [B] (verification not implemented) . . . 2243
Giac [F] . . . 2244
Mupad [F(-1)] . . . 2245
Reduce [F] . . . 2245

Optimal result

Integrand size = 16, antiderivative size = 128

$$\int (c+dx)^3 \tan^2(a+bx)\,dx = -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2 \log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3} + \frac{3d^3\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^4} + \frac{(c+dx)^3 \tan(a+bx)}{b}$$

output

```
-I*(d*x+c)^3/b-1/4*(d*x+c)^4/d+3*d*(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b^2-3*
I*d^2*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^3+3/2*d^3*polylog(3,-exp(2*I*
(b*x+a)))/b^4+(d*x+c)^3*tan(b*x+a)/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 424 vs. 2(128) = 256.

Time = 6.32 (sec) , antiderivative size = 424, normalized size of antiderivative = 3.31

$$\int (c+dx)^3 \tan^2(a+bx)\,dx = -\frac{1}{4}x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)$$
$$+\frac{id^3e^{-ia}\left(2b^2x^2\left(2bx-3i(1+e^{2ia})\log\left(1+e^{-2i(a+bx)}\right)\right)+6b(1+e^{2ia})\,x\,\mathrm{PolyLog}\left(2,-e^{-2i(a+bx)}\right)-3i(1\right.}{4b^4}$$
$$+\frac{3c^2d\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))}{b^2\left(\cos^2(a)+\sin^2(a)\right)}$$
$$+\frac{3cd^2\csc(a)\left(b^2e^{-i\arctan(\cot(a))}x^2-\frac{\cot(a)(ibx(-\pi-2\arctan(\cot(a)))-\pi\log(1+e^{-2ibx})-2(bx-\arctan(\cot(a)))\log(1-e^{2i(bx-ar}\right.}{b^3\sqrt{\csc^2(a)\left(\cos^2(a)-\right.}}$$
$$+\frac{\sec(a)\sec(a+bx)\left(c^3\sin(bx)+3c^2dx\sin(bx)+3cd^2x^2\sin(bx)+d^3x^3\sin(bx)\right)}{b}$$

input
```
Integrate[(c + d*x)^3*Tan[a + b*x]^2,x]
```

output
```
-1/4*(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)) + ((I/4)*d^3*(2*b^2*x
^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(
1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)
*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^4*E^(I*a)) + (3*c^2*d*S
ec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^2*(
Cos[a]^2 + Sin[a]^2)) + (3*c*d^2*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) -
(Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(
b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[C
os[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2,
E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^3*Sqrt[
Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Sec[a]*Sec[a + b*x]*(c^3*Sin[b*x] + 3*
c^2*d*x*Sin[b*x] + 3*c*d^2*x^2*Sin[b*x] + d^3*x^3*Sin[b*x]))/b
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 4203, 17, 3042, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \tan^2(a+bx)\,dx$$

$$\downarrow 3042$$

$$\int (c+dx)^3 \tan(a+bx)^2 dx$$

$$\downarrow 4203$$

$$-\frac{3d\int (c+dx)^2 \tan(a+bx)dx}{b} - \int (c+dx)^3 dx + \frac{(c+dx)^3 \tan(a+bx)}{b}$$

$$\downarrow 17$$

$$-\frac{3d\int (c+dx)^2 \tan(a+bx)dx}{b} + \frac{(c+dx)^3 \tan(a+bx)}{b} - \frac{(c+dx)^4}{4d}$$

$$\downarrow 3042$$

$$-\frac{3d\int (c+dx)^2 \tan(a+bx)dx}{b} + \frac{(c+dx)^3 \tan(a+bx)}{b} - \frac{(c+dx)^4}{4d}$$

$$\downarrow 4202$$

$$-\frac{3d\left(\frac{i(c+dx)^3}{3d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}dx\right)}{b} + \frac{(c+dx)^3 \tan(a+bx)}{b} - \frac{(c+dx)^4}{4d}$$

$$\downarrow 2620$$

$$-\frac{3d\left(\frac{i(c+dx)^3}{3d} - 2i\left(\frac{id\int (c+dx)\log\left(1+e^{2i(a+bx)}\right)dx}{b} - \frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b} + \frac{(c+dx)^3\tan(a+bx)}{b} - \frac{(c+dx)^4}{4d}$$

$$\downarrow 3011$$

$$-\frac{3d\left(\frac{i(c+dx)^3}{3d} - 2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b} - \frac{id\int \mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{2b}\right)}{b} - \frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b} + \frac{(c+dx)^3\tan(a+bx)}{b} - \frac{(c+dx)^4}{4d}$$

$$\downarrow 2720$$

$$-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i(a+bx)}\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)}{b}}{\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{(c+dx)^4}{4d}}$$

$$\downarrow 7143$$

$$-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}}{\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{(c+dx)^4}{4d}}+$$

input
```
Int[(c + d*x)^3*Tan[a + b*x]^2,x]
```

output
```
-1/4*(c + d*x)^4/d - (3*d*(((I/3)*(c + d*x)^3)/d - (2*I)*(((-1/2*I)*(c + d
*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b + (I*d*(((I/2)*(c + d*x)*PolyLog[2,
-E^((2*I)*(a + b*x))])/b - (d*PolyLog[3, -E^((2*I)*(a + b*x))])/(4*b^2)))/
b)))/b + ((c + d*x)^3*Tan[a + b*x])/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(116) = 232.

Time = 0.50 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.78

method	result
risch	$-\frac{d^3x^4}{4} - d^2c\,x^3 - \frac{3dc^2x^2}{2} - c^3x - \frac{c^4}{4d} - \frac{12id^2cax}{b^2} - \frac{2id^3x^3}{b} - \frac{6id^2ca^2}{b^3} + \frac{4id^3a^3}{b^4} - \frac{6id^2cx^2}{b} - \frac{3id^3\,\mathrm{polylog}(2,$

input
```
int((d*x+c)^3*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*d^3*x^4-d^2*c*x^3-3/2*d*c^2*x^2-c^3*x-1/4/d*c^4-12*I*d^2/b^2*c*a*x-2*
I*d^3/b*x^3-6*I*d^2/b^3*c*a^2+4*I*d^3/b^4*a^3-6*I*d^2/b*c*x^2-3*I*d^3/b^3*
polylog(2,-exp(2*I*(b*x+a)))*x+3*d/b^2*c^2*ln(exp(2*I*(b*x+a))+1)-6*d/b^2*
c^2*ln(exp(I*(b*x+a)))+6*I*d^3/b^3*a^2*x+2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*
x+c^3)/b/(exp(2*I*(b*x+a))+1)+12*d^2/b^3*c*a*ln(exp(I*(b*x+a)))+6*d^2/b^2*
c*ln(exp(2*I*(b*x+a))+1)*x-6*d^3/b^4*a^2*ln(exp(I*(b*x+a)))-3*I*d^2/b^3*c*
polylog(2,-exp(2*I*(b*x+a)))+3*d^3/b^2*ln(exp(2*I*(b*x+a))+1)*x^2+3/2*d^3*
polylog(3,-exp(2*I*(b*x+a)))/b^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(113) = 226.

Time = 0.08 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.91

$$\int (c+dx)^3 \tan^2(a+bx)\,dx =$$

$$-\frac{b^4d^3x^4 + 4\,b^4cd^2x^3 + 6\,b^4c^2dx^2 + 4\,b^4c^3x - 3\,d^3\mathrm{polylog}\left(3, \frac{\tan(bx+a)^2+2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right) - 3\,d^3\mathrm{polylog}\left(3, \frac{\mathrm{ta}}{}\right.$$

input
```
integrate((d*x+c)^3*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 6*b^4*c^2*d*x^2 + 4*b^4*c^3*x - 3*d^
3*polylog(3, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1))
 - 3*d^3*polylog(3, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^
2 + 1)) + 6*(-I*b*d^3*x - I*b*c*d^2)*dilog(2*(I*tan(b*x + a) - 1)/(tan(b*x
 + a)^2 + 1) + 1) + 6*(I*b*d^3*x + I*b*c*d^2)*dilog(2*(-I*tan(b*x + a) - 1
)/(tan(b*x + a)^2 + 1) + 1) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*
log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 6*(b^2*d^3*x^2 + 2*b^2
*c*d^2*x + b^2*c^2*d)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) -
 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*tan(b*x + a))
/b^4
```

Sympy [F]

$$\int (c+dx)^3 \tan^2(a+bx)\,dx = \int (c+dx)^3 \tan^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*tan(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**3*tan(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1363 vs. $2(113) = 226$.

Time $= 0.41$ (sec) , antiderivative size $= 1363$, normalized size of antiderivative $= 10.65$

$$\int (c+dx)^3 \tan^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*tan(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/2*(2*(b*x + a - tan(b*x + a))*c^3 - 6*(b*x + a - tan(b*x + a))*a*c^2*d/
b + 6*(b*x + a - tan(b*x + a))*a^2*c*d^2/b^2 - 2*(b*x + a - tan(b*x + a))*
a^3*d^3/b^3 + 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x +
2*a)^2 + 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^
2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 +
sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a
))*c^2*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) +
1)*b) - 6*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2
 + 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + si
n(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*
b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*a*c
*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b
^2) + 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 +
 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*
x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*d
^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b^3
) - 2*(I*(b*x + a)^4*d^3 - 4*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^3 + 12*((b*x
 + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) + ((b*x + a)^2*d^3 + 2*(b*c*d^
2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^3 + 2*(-I*b*...
```

Giac [F]

$$\int (c+dx)^3 \tan^2(a+bx)\,dx = \int (dx+c)^3 \tan(bx+a)^2\,dx$$

input

```
integrate((d*x+c)^3*tan(b*x+a)^2,x, algorithm="giac")
```

output

```
integrate((d*x + c)^3*tan(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \tan^2(a+bx)\,dx = \int \tan(a+b\,x)^2\,(c+d\,x)^3\,dx$$

input
```
int(tan(a + b*x)^2*(c + d*x)^3,x)
```

output
```
int(tan(a + b*x)^2*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \tan^2(a+bx)\,dx$$
$$= \frac{-12\left(\int \tan(bx+a)\,x^2dx\right)b\,d^3 - 24\left(\int \tan(bx+a)\,xdx\right)bc\,d^2 - 6\log\left(\tan(bx+a)^2+1\right)c^2d + 4\tan(bx}{}$$

input
```
int((d*x+c)^3*tan(b*x+a)^2,x)
```

output
```
( - 12*int(tan(a + b*x)*x**2,x)*b*d**3 - 24*int(tan(a + b*x)*x,x)*b*c*d**2
 - 6*log(tan(a + b*x)**2 + 1)*c**2*d + 4*tan(a + b*x)*b*c**3 + 12*tan(a +
b*x)*b*c**2*d*x + 12*tan(a + b*x)*b*c*d**2*x**2 + 4*tan(a + b*x)*b*d**3*x*
*3 - 4*b**2*c**3*x - 6*b**2*c**2*d*x**2 - 4*b**2*c*d**2*x**3 - b**2*d**3*x
**4)/(4*b**2)
```

3.255 $\int (c+dx)^2 \tan^2(a+bx)\,dx$

Optimal result . . . 2246
Mathematica [B] (verified) . . . 2246
Rubi [A] (verified) . . . 2247
Maple [B] (verified) . . . 2250
Fricas [B] (verification not implemented) . . . 2250
Sympy [F] . . . 2251
Maxima [B] (verification not implemented) . . . 2251
Giac [F] . . . 2252
Mupad [F(-1)] . . . 2252
Reduce [F] . . . 2253

Optimal result

Integrand size = 16, antiderivative size = 96

$$\int (c+dx)^2 \tan^2(a+bx)\,dx = -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{id^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)^2\tan(a+bx)}{b}$$

output

```
-I*(d*x+c)^2/b-1/3*(d*x+c)^3/d+2*d*(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b^2-I*d^
2*polylog(2,-exp(2*I*(b*x+a)))/b^3+(d*x+c)^2*tan(b*x+a)/b
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 203 vs. 2(96) = 192.

Time = 5.38 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.11

$$\int (c+dx)^2 \tan^2(a+bx)\,dx = -\frac{1}{3}x\left(3c^2+3cdx+d^2x^2\right) + \frac{(c+dx)^2\sec(a)\sec(a+bx)\sin(bx)}{b} + \frac{2cd(\log(\cos(a+bx))+bx\tan(a))}{b^2} + \frac{d^2\Big(ibx(\pi+2\arctan(\cot(a))) + \pi\log\left(1+e^{-2ibx}\right) + 2(bx-\arctan(\cot(a)))\log\left(1-e^{2i(bx-\arctan(\cot(a)))}\right)$$

input
```
Integrate[(c + d*x)^2*Tan[a + b*x]^2,x]
```

output
```
-1/3*(x*(3*c^2 + 3*c*d*x + d^2*x^2)) + ((c + d*x)^2*Sec[a]*Sec[a + b*x]*Si
n[b*x])/b + (2*c*d*(Log[Cos[a + b*x]] + b*x*Tan[a]))/b^2 + (d^2*(I*b*x*(Pi
 + 2*ArcTan[Cot[a]]) + Pi*Log[1 + E^((-2*I)*b*x)] + 2*(b*x - ArcTan[Cot[a]
])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] - Pi*Log[Cos[b*x]] + 2*ArcTan
[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] - I*PolyLog[2, E^((2*I)*(b*x - Arc
Tan[Cot[a]]))] + (b^2*x^2*Sqrt[Csc[a]^2]*Tan[a])/E^(I*ArcTan[Cot[a]])))/b^
3
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4203, 17, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \tan^2(a+bx)\,dx$$

$$\downarrow 3042$$

$$\int (c+dx)^2 \tan(a+bx)^2 dx$$

$$\downarrow 4203$$

$$-\frac{2d\int (c+dx)\tan(a+bx)dx}{b} - \int (c+dx)^2 dx + \frac{(c+dx)^2\tan(a+bx)}{b}$$

$$\downarrow 17$$

$$-\frac{2d\int (c+dx)\tan(a+bx)dx}{b} + \frac{(c+dx)^2\tan(a+bx)}{b} - \frac{(c+dx)^3}{3d}$$

$$\downarrow 3042$$

$$-\frac{2d\int (c+dx)\tan(a+bx)dx}{b} + \frac{(c+dx)^2\tan(a+bx)}{b} - \frac{(c+dx)^3}{3d}$$

$$\downarrow 4202$$

$$-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\int\frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}dx\right)}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

$$\downarrow 2620$$

$$-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{id\int\log\left(1+e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

$$\downarrow 2715$$

$$-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{d\int e^{-2i(a+bx)}\log\left(1+e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

$$\downarrow 2838$$

$$-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

input

```
Int[(c + d*x)^2*Tan[a + b*x]^2,x]
```

output

```
-1/3*(c + d*x)^3/d - (2*d*(((I/2)*(c + d*x)^2)/d - (2*I)*(((-1/2*I)*(c + d
*x)*Log[1 + E^((2*I)*(a + b*x))])/b - (d*PolyLog[2, -E^((2*I)*(a + b*x))])
/(4*b^2))))/b + ((c + d*x)^2*Tan[a + b*x])/b
```

Defintions of rubi rules used

rule 17

```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(88) = 176$.

Time = 0.49 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.07

method	result
risch	$-\frac{d^2x^3}{3} - cd\,x^2 - c^2x - \frac{c^3}{3d} + \frac{2i\left(x^2d^2+2cdx+c^2\right)}{b\left(\mathrm{e}^{2i(bx+a)}+1\right)} + \frac{2dc\ln\left(\mathrm{e}^{2i(bx+a)}+1\right)}{b^2} - \frac{4dc\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^2} - \frac{2id^2x^2}{b} - \frac{4id^2ax}{b^2} -$

input
```
int((d*x+c)^2*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*d^2*x^3-c*d*x^2-c^2*x-1/3/d*c^3+2*I*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*
(b*x+a))+1)+2*d/b^2*c*ln(exp(2*I*(b*x+a))+1)-4*d/b^2*c*ln(exp(I*(b*x+a)))-
2*I*d^2/b*x^2-4*I*d^2/b^2*a*x-2*I*d^2/b^3*a^2+2*d^2/b^2*ln(exp(2*I*(b*x+a)
)+1)*x-I*d^2*polylog(2,-exp(2*I*(b*x+a)))/b^3+4*d^2/b^3*a*ln(exp(I*(b*x+a)
))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(85) = 170$.

Time = 0.07 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.19

$$\int (c+dx)^2 \tan^2(a+bx)\,dx =$$

$$-\frac{2\,b^3d^2x^3 + 6\,b^3cdx^2 + 6\,b^3c^2x - 3i\,d^2\mathrm{Li}_2\left(\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right) + 3i\,d^2\mathrm{Li}_2\left(\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right) - 6\,(bd$$

input
```
integrate((d*x+c)^2*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/6*(2*b^3*d^2*x^3 + 6*b^3*c*d*x^2 + 6*b^3*c^2*x - 3*I*d^2*dilog(2*(I*tan
(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + 3*I*d^2*dilog(2*(-I*tan(b*x + a
) - 1)/(tan(b*x + a)^2 + 1) + 1) - 6*(b*d^2*x + b*c*d)*log(-2*(I*tan(b*x +
 a) - 1)/(tan(b*x + a)^2 + 1)) - 6*(b*d^2*x + b*c*d)*log(-2*(-I*tan(b*x +
a) - 1)/(tan(b*x + a)^2 + 1)) - 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*ta
n(b*x + a))/b^3
```

Sympy [F]

$$\int (c+dx)^2 \tan^2(a+bx)\,dx = \int (c+dx)^2 \tan^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*tan(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*tan(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(85) = 170$.

Time = 0.30 (sec) , antiderivative size = 418, normalized size of antiderivative = 4.35

$$\int (c+dx)^2 \tan^2(a+bx)\,dx$$
$$= \frac{i\,b^3d^2x^3 + 3i\,b^3cdx^2 + 3i\,b^3c^2x + 6\,b^2c^2 + 6\,(bd^2x + bcd + (bd^2x + bcd)\cos(2\,bx + 2\,a) - (-i\,bd^2x - i\,bc$$

input
```
integrate((d*x+c)^2*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
(I*b^3*d^2*x^3 + 3*I*b^3*c*d*x^2 + 3*I*b^3*c^2*x + 6*b^2*c^2 + 6*(b*d^2*x
+ b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a) - (-I*b*d^2*x - I*b*c*d)*sin(
2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + (I*b^3*d^2
*x^3 - 3*(-I*b^3*c*d + 2*b^2*d^2)*x^2 - 3*(-I*b^3*c^2 + 4*b^2*c*d)*x)*cos(
2*b*x + 2*a) - 3*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) + d^2)*dil
og(-e^(2*I*b*x + 2*I*a)) - 3*(I*b*d^2*x + I*b*c*d + (I*b*d^2*x + I*b*c*d)*
cos(2*b*x + 2*a) - (b*d^2*x + b*c*d)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a
)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (b^3*d^2*x^3 + 3*(b^3
*c*d + 2*I*b^2*d^2)*x^2 + 3*(b^3*c^2 + 4*I*b^2*c*d)*x)*sin(2*b*x + 2*a))/(
-3*I*b^3*cos(2*b*x + 2*a) + 3*b^3*sin(2*b*x + 2*a) - 3*I*b^3)
```

Giac [F]

$$\int (c+dx)^2 \tan^2(a+bx)\,dx = \int (dx+c)^2 \tan\left(bx+a\right)^2\,dx$$

input
```
integrate((d*x+c)^2*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*tan(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \tan^2(a+bx)\,dx = \int \tan(a+b\,x)^2\,(c+d\,x)^2\,dx$$

input
```
int(tan(a + b*x)^2*(c + d*x)^2,x)
```

output
```
int(tan(a + b*x)^2*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \tan^2(a+bx)\,dx$$

$$= \frac{-6\left(\int \tan(bx+a)\,x dx\right) b\,d^2 - 3\log\left(\tan(bx+a)^2+1\right) cd + 3\tan(bx+a)\,b\,c^2 + 6\tan(bx+a)\,bcdx + 3}{3b^2}$$

input

```
int((d*x+c)^2*tan(b*x+a)^2,x)
```

output

```
( - 6*int(tan(a + b*x)*x,x)*b*d**2 - 3*log(tan(a + b*x)**2 + 1)*c*d + 3*ta
n(a + b*x)*b*c**2 + 6*tan(a + b*x)*b*c*d*x + 3*tan(a + b*x)*b*d**2*x**2 -
3*b**2*c**2*x - 3*b**2*c*d*x**2 - b**2*d**2*x**3)/(3*b**2)
```

3.256 $\int (c+dx)\tan^2(a+bx)\,dx$

Optimal result 2254
Mathematica [A] (verified) 2254
Rubi [A] (verified) 2255
Maple [A] (verified) 2256
Fricas [A] (verification not implemented) 2257
Sympy [A] (verification not implemented) 2257
Maxima [B] (verification not implemented) 2258
Giac [B] (verification not implemented) 2258
Mupad [B] (verification not implemented) 2259
Reduce [B] (verification not implemented) 2259

Optimal result

Integrand size = 14, antiderivative size = 42

$$\int (c+dx)\tan^2(a+bx)\,dx = -\frac{(c+dx)^2}{2d} + \frac{d\log(\cos(a+bx))}{b^2} + \frac{(c+dx)\tan(a+bx)}{b}$$

output
```
-1/2*(d*x+c)^2/d+d*ln(cos(b*x+a))/b^2+(d*x+c)*tan(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.81

$$\begin{aligned}\int (c+dx)\tan^2(a+bx)\,dx = &-\frac{c\arctan(\tan(a+bx))}{b} + \frac{d\log(\cos(a+bx))}{b^2} \\ &- \frac{dx\sec(a)(bx\cos(a)-2\sin(a))}{2b} \\ &+ \frac{dx\sec(a)\sec(a+bx)\sin(bx)}{b} + \frac{c\tan(a+bx)}{b}\end{aligned}$$

input
```
Integrate[(c + d*x)*Tan[a + b*x]^2,x]
```

output
```
-((c*ArcTan[Tan[a + b*x]])/b) + (d*Log[Cos[a + b*x]])/b^2 - (d*x*Sec[a]*(b
*x*Cos[a] - 2*Sin[a]))/(2*b) + (d*x*Sec[a]*Sec[a + b*x]*Sin[b*x])/b + (c*T
an[a + b*x])/b
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4203, 17, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\tan^2(a+bx)\,dx$$

$$\downarrow 3042$$

$$\int (c+dx)\tan(a+bx)^2 dx$$

$$\downarrow 4203$$

$$-\frac{d\int \tan(a+bx)dx}{b} - \int (c+dx)dx + \frac{(c+dx)\tan(a+bx)}{b}$$

$$\downarrow 17$$

$$-\frac{d\int \tan(a+bx)dx}{b} + \frac{(c+dx)\tan(a+bx)}{b} - \frac{(c+dx)^2}{2d}$$

$$\downarrow 3042$$

$$-\frac{d\int \tan(a+bx)dx}{b} + \frac{(c+dx)\tan(a+bx)}{b} - \frac{(c+dx)^2}{2d}$$

$$\downarrow 3956$$

$$\frac{d\log(\cos(a+bx))}{b^2} + \frac{(c+dx)\tan(a+bx)}{b} - \frac{(c+dx)^2}{2d}$$

input
```
Int[(c + d*x)*Tan[a + b*x]^2,x]
```

output
```
-1/2*(c + d*x)^2/d + (d*Log[Cos[a + b*x]])/b^2 + ((c + d*x)*Tan[a + b*x])/
b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

method	result	size
norman	$\frac{c\tan(bx+a)}{b}+\frac{dx\tan(bx+a)}{b}-\frac{dx^2}{2}-xc-\frac{d\ln\left(\tan(bx+a)^2+1\right)}{2b^2}$	52
parallelrisch	$-\frac{dx^2b^2-2\tan(bx+a)bdx+2xcb^2-2\tan(bx+a)bc+d\ln\left(\tan(bx+a)^2+1\right)}{2b^2}$	56
default	$-\frac{dx^2}{2}-xc+\frac{-\frac{da\tan(bx+a)}{b}+c\tan(bx+a)+\frac{d((bx+a)\tan(bx+a)+\ln(\cos(bx+a)))}{b}}{b}$	63
risch	$-\frac{dx^2}{2}-xc-\frac{2idx}{b}-\frac{2ida}{b^2}+\frac{2i(dx+c)}{b\left(e^{2i(bx+a)}+1\right)}+\frac{d\ln\left(e^{2i(bx+a)}+1\right)}{b^2}$	69

input
```
int((d*x+c)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
c/b*tan(b*x+a)+d*x/b*tan(b*x+a)-1/2*d*x^2-x*c-1/2*d/b^2*ln(tan(b*x+a)^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int (c+dx)\tan^2(a+bx)\,dx = -\frac{b^2dx^2+2\,b^2cx-d\log\left(\frac{1}{\tan(bx+a)^2+1}\right)-2\,(bdx+bc)\tan(bx+a)}{2\,b^2}$$

input
```
integrate((d*x+c)*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(b^2*d*x^2 + 2*b^2*c*x - d*log(1/(tan(b*x + a)^2 + 1)) - 2*(b*d*x + b
*c)*tan(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.55

$$\int (c+dx)\tan^2(a+bx)\,dx = \begin{cases} -cx-\frac{dx^2}{2}+\frac{c\tan(a+bx)}{b}+\frac{dx\tan(a+bx)}{b}-\frac{d\log(\tan^2(a+bx)+1)}{2b^2} & \text{for } b\neq 0 \\ \left(cx+\frac{dx^2}{2}\right)\tan^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*tan(b*x+a)**2,x)
```

output
```
Piecewise((-c*x - d*x**2/2 + c*tan(a + b*x)/b + d*x*tan(a + b*x)/b - d*log
(tan(a + b*x)**2 + 1)/(2*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*tan(a)**2, Tr
ue))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(40) = 80$.

Time $= 0.14$ (sec) , antiderivative size $= 237$, normalized size of antiderivative $= 5.64$

$$\int (c+dx)\tan^2(a+bx)\,dx =$$

$$-\frac{2\,(bx+a-\tan(bx+a))c - \frac{2\,(bx+a-\tan(bx+a))ad}{b} + \frac{\left((bx+a)^2\cos(2\,bx+2\,a)^2+(bx+a)^2\sin(2\,bx+2\,a)^2+2\,(bx+a)^2\cos(2\,bx+\right.}{}}{}$$

input
```
integrate((d*x+c)*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(b*x + a - tan(b*x + a))*c - 2*(b*x + a - tan(b*x + a))*a*d/b + ((
b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 + 2*(b*x +
a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*
a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
 + 2*cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*d/((cos(2*b*x +
 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(40) = 80$.

Time $= 0.21$ (sec) , antiderivative size $= 203$, normalized size of antiderivative $= 4.83$

$$\int (c+dx)\tan^2(a+bx)\,dx =$$

$$-\frac{b^2dx^2\tan(bx)\tan(a) + 2\,b^2cx\tan(bx)\tan(a) - b^2dx^2 - 2\,b^2cx + 2\,bdx\tan(bx) + 2\,bdx\tan(a) - d\,\mathrm{lo}}{2}$$

input
```
integrate((d*x+c)*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(b^2*d*x^2*tan(b*x)*tan(a) + 2*b^2*c*x*tan(b*x)*tan(a) - b^2*d*x^2 -
2*b^2*c*x + 2*b*d*x*tan(b*x) + 2*b*d*x*tan(a) - d*log(4*(tan(b*x)^2*tan(a)
^2 - 2*tan(b*x)*tan(a) + 1)/(tan(b*x)^2*tan(a)^2 + tan(b*x)^2 + tan(a)^2 +
 1))*tan(b*x)*tan(a) + 2*b*c*tan(b*x) + 2*b*c*tan(a) + d*log(4*(tan(b*x)^2
*tan(a)^2 - 2*tan(b*x)*tan(a) + 1)/(tan(b*x)^2*tan(a)^2 + tan(b*x)^2 + tan
(a)^2 + 1)))/(b^2*tan(b*x)*tan(a) - b^2)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

$$\int (c+dx)\tan^2(a+bx)\,dx = -c\,x - \frac{d\,x^2}{2} - \frac{\frac{d\,\ln\left(\tan(a+b\,x)^2+1\right)}{2} - b\,(c\tan(a+b\,x)+d\,x\tan(a+b\,x))}{b^2}$$

input
```
int(tan(a + b*x)^2*(c + d*x),x)
```

output
```
- c*x - (d*x^2)/2 - ((d*log(tan(a + b*x)^2 + 1))/2 - b*(c*tan(a + b*x) + d
*x*tan(a + b*x)))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.36

$$\int (c+dx)\tan^2(a+bx)\,dx = \frac{-\log\left(\tan(bx+a)^2+1\right)d + 2\tan(bx+a)\,bc + 2\tan(bx+a)\,bdx - 2b^2cx - b^2d\,x^2}{2b^2}$$

input
```
int((d*x+c)*tan(b*x+a)^2,x)
```

output
```
( - log(tan(a + b*x)**2 + 1)*d + 2*tan(a + b*x)*b*c + 2*tan(a + b*x)*b*d*x
 - 2*b**2*c*x - b**2*d*x**2)/(2*b**2)
```

3.257 $\int \frac{\tan^2(a+bx)}{c+dx}\,dx$

Optimal result . . . 2260
Mathematica [N/A] . . . 2260
Rubi [N/A] . . . 2261
Maple [N/A] . . . 2262
Fricas [N/A] . . . 2262
Sympy [N/A] . . . 2263
Maxima [N/A] . . . 2263
Giac [N/A] . . . 2264
Mupad [N/A] . . . 2264
Reduce [N/A] . . . 2264

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\tan^2(a+bx)}{c+dx}, x\right)$$

output
```
Defer(Int)(tan(b*x+a)^2/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 4.97 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\tan^2(a+bx)}{c+dx}\,dx$$

input
```
Integrate[Tan[a + b*x]^2/(c + d*x),x]
```

output
```
Integrate[Tan[a + b*x]^2/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\tan(a+bx)^2}{c+dx}\,dx$$
$$\downarrow 4222$$
$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx$$

input
```
Int[Tan[a + b*x]^2/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\tan(bx+a)^2}{dx+c}dx$$

input

```
int(tan(b*x+a)^2/(d*x+c),x)
```

output

```
int(tan(b*x+a)^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\tan(bx+a)^2}{dx+c}\,dx$$

input

```
integrate(tan(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(tan(b*x + a)^2/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\tan^2(a+bx)}{c+dx}\,dx$$

input
```
integrate(tan(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(tan(a + b*x)**2/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 388, normalized size of antiderivative = 24.25

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\tan(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(tan(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
(2*(b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a)^2 + (b*d^3*x
+ b*c*d^2)*sin(2*b*x + 2*a)^2 + 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*in
tegrate(sin(2*b*x + 2*a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b
*c*d*x + b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2
*b*x + 2*a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)), x) -
(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x + 2*a)
^2 + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a))*log(d*x + c) + 2*d*sin(2*b*x
+ 2*a))/(b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x
 + b*c*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\tan(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(tan(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(tan(b*x + a)^2/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 18.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\tan(a+b\,x)^2}{c+d\,x}\,dx$$

input
```
int(tan(a + b*x)^2/(c + d*x),x)
```

output
```
int(tan(a + b*x)^2/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\tan(bx+a)^2}{dx+c}dx$$

input
```
int(tan(b*x+a)^2/(d*x+c),x)
```

output
```
int(tan(a + b*x)**2/(c + d*x),x)
```

3.258 $\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx$

Optimal result 2266
Mathematica [N/A] 2266
Rubi [N/A] 2267
Maple [N/A] 2268
Fricas [N/A] 2268
Sympy [N/A] 2269
Maxima [N/A] 2269
Giac [N/A] 2270
Mupad [N/A] 2270
Reduce [N/A] 2271

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\tan^2(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(tan(b*x+a)^2/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 3.78 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[Tan[a + b*x]^2/(c + d*x)^2,x]
```

output
```
Integrate[Tan[a + b*x]^2/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\tan(a+bx)^2}{(c+dx)^2}\,dx$$
$$\downarrow 4222$$
$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
Int[Tan[a + b*x]^2/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\tan(bx+a)^2}{(dx+c)^2}dx$$

input
```
int(tan(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(tan(b*x+a)^2/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(tan(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(tan(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(tan(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(tan(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 523, normalized size of antiderivative = 32.69

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(tan(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x + 2*a)
^2 + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + 4*(b*d^4*x^2 + 2*b*c*d^3*x +
 b*c^2*d^2 + (b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2)*cos(2*b*x + 2*a)^2 + (b
*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2)*sin(2*b*x + 2*a)^2 + 2*(b*d^4*x^2 + 2*
b*c*d^3*x + b*c^2*d^2)*cos(2*b*x + 2*a))*integrate(sin(2*b*x + 2*a)/(b*d^3
*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 +
3*b*c^2*d*x + b*c^3)*cos(2*b*x + 2*a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b
*c^2*d*x + b*c^3)*sin(2*b*x + 2*a)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*
c^2*d*x + b*c^3)*cos(2*b*x + 2*a)), x) + 2*d*sin(2*b*x + 2*a))/(b*d^3*x^2
+ 2*b*c*d^2*x + b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x +
2*a)^2 + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^3
*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a))
```

Giac [N/A]

Not integrable

Time = 7.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(tan(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(tan(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 19.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(a+b\,x)^2}{(c+d\,x)^2}\,dx$$

input
```
int(tan(a + b*x)^2/(c + d*x)^2,x)
```

output
```
int(tan(a + b*x)^2/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan\left(bx+a\right)^2}{d^2x^2+2cdx+c^2}dx$$

input
```
int(tan(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(tan(a + b*x)**2/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.259 $\int (c+dx)^m \sin(a+bx) \tan^2(a+bx)\, dx$

Optimal result . . . 2272
Mathematica [N/A] . . . 2273
Rubi [N/A] . . . 2273
Maple [N/A] . . . 2275
Fricas [N/A] . . . 2275
Sympy [N/A] . . . 2276
Maxima [N/A] . . . 2276
Giac [N/A] . . . 2276
Mupad [N/A] . . . 2277
Reduce [N/A] . . . 2277

Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}
&\int (c+dx)^m \sin(a+bx) \tan^2(a+bx)\, dx \\
&= \frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{ib(c+dx)}{d}\right)}{2b} \\
&\quad + \frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, \frac{ib(c+dx)}{d}\right)}{2b} \\
&\quad + \text{Int}((c+dx)^m \sec(a+bx) \tan(a+bx), x)
\end{aligned}$$

output

```
1/2*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/
d)^m)+1/2*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*x
+c)/d)^m)+Defer(Int)((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 16.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx = \int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4907, 3042, 3789, 2612, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\tan^2(a+bx)(c+dx)^m\,dx$$

$$\downarrow 4907$$

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)dx - \int (c+dx)^m \sin(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)dx - \int (c+dx)^m \sin(a+bx)dx$$

$$\downarrow 3789$$

$$-\frac{1}{2}i\int e^{-i(a+bx)}(c+dx)^m dx + \frac{1}{2}i\int e^{i(a+bx)}(c+dx)^m dx + \int (c+dx)^m \sec(a+bx)\tan(a+bx)dx$$

$$\downarrow 2612$$

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)dx + \frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b} + \frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}$$

$$\downarrow \text{ 7299}$$

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)dx + \frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b} + \frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}$$

input
```
Int[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \sin(bx+a) \tan(bx+a)^2 \, dx$$

input
```
int((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sin(a+bx) \tan^2(a+bx) \, dx = \int (dx+c)^m \sin(bx+a) \tan(bx+a)^2 \, dx$$

input
```
integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*sin(b*x + a)*tan(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 36.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \sin(a+bx) \tan^2(a+bx)\, dx = \int (c+dx)^m \sin(a+bx) \tan^2(a+bx)\, dx$$

input
```
integrate((d*x+c)**m*sin(b*x+a)*tan(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*sin(a + b*x)*tan(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sin(a+bx) \tan^2(a+bx)\, dx = \int (dx+c)^m \sin(bx+a) \tan(bx+a)^2\, dx$$

input
```
integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*sin(b*x + a)*tan(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sin(a+bx) \tan^2(a+bx)\, dx = \int (dx+c)^m \sin(bx+a) \tan(bx+a)^2\, dx$$

input
```
integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sin(b*x + a)*tan(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 19.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sin(a+bx) \tan^2(a+bx)\,dx = \int \sin(a+b\,x)\,\tan(a+b\,x)^2\,(c+d\,x)^m\,dx$$

input
```
int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^m,x)
```

output
```
int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sin(a+bx) \tan^2(a+bx)\,dx = \int (dx+c)^m \sin(bx+a) \tan(bx+a)^2\,dx$$

input
```
int((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x)
```

output
```
int((c + d*x)**m*sin(a + b*x)*tan(a + b*x)**2,x)
```

3.260 $\int (c+dx)^3 \sin(a+bx) \tan^2(a+bx)\, dx$

Optimal result . . . 2278
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Giac [F] . . . 2288
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Reduce [F] . . . 2288

Optimal result

Integrand size = 22, antiderivative size = 228

$$\begin{aligned}
&\int (c+dx)^3 \sin(a+bx) \tan^2(a+bx)\, dx \\
&= \frac{6id(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+dx)^3\cos(a+bx)}{b} \\
&\quad - \frac{6id^2(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} + \frac{6id^2(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{6d^3\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4} - \frac{6d^3\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4} \\
&\quad + \frac{(c+dx)^3\sec(a+bx)}{b} + \frac{6d^3\sin(a+bx)}{b^4} - \frac{3d(c+dx)^2\sin(a+bx)}{b^2}
\end{aligned}$$

output

```
6*I*d*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)*cos(b*x+a)/b^3+(d
*x+c)^3*cos(b*x+a)/b-6*I*d^2*(d*x+c)*polylog(2,-I*exp(I*(b*x+a)))/b^3+6*I*
d^2*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^3+6*d^3*polylog(3,-I*exp(I*(b*x+
a)))/b^4-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4+(d*x+c)^3*sec(b*x+a)/b+6*d^
3*sin(b*x+a)/b^4-3*d*(d*x+c)^2*sin(b*x+a)/b^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 532 vs. $2(228) = 456$.

Time = 1.01 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.33

$$\int (c+dx)^3 \sin(a+bx) \tan^2(a+bx)\, dx$$
$$= \frac{\sec(a+bx)\left(3b^3c^3 - 6bcd^2 + 9b^3c^2dx - 6bd^3x + 9b^3cd^2x^2 + 3b^3d^3x^3 + 12ib^2c^2d \arctan\left(e^{i(a+bx)}\right) \cos(a+\right.}{}$$

input
```
Integrate[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
(Sec[a + b*x]*(3*b^3*c^3 - 6*b*c*d^2 + 9*b^3*c^2*d*x - 6*b*d^3*x + 9*b^3*c
*d^2*x^2 + 3*b^3*d^3*x^3 + (12*I)*b^2*c^2*d*ArcTan[E^(I*(a + b*x))]*Cos[a
+ b*x] + b^3*c^3*Cos[2*(a + b*x)] - 6*b*c*d^2*Cos[2*(a + b*x)] + 3*b^3*c^2
*d*x*Cos[2*(a + b*x)] - 6*b*d^3*x*Cos[2*(a + b*x)] + 3*b^3*c*d^2*x^2*Cos[2
*(a + b*x)] + b^3*d^3*x^3*Cos[2*(a + b*x)] - 12*b^2*c*d^2*x*Cos[a + b*x]*L
og[1 - I*E^(I*(a + b*x))] - 6*b^2*d^3*x^2*Cos[a + b*x]*Log[1 - I*E^(I*(a +
 b*x))] + 12*b^2*c*d^2*x*Cos[a + b*x]*Log[1 + I*E^(I*(a + b*x))] + 6*b^2*d
^3*x^2*Cos[a + b*x]*Log[1 + I*E^(I*(a + b*x))] - (12*I)*b*d^2*(c + d*x)*Co
s[a + b*x]*PolyLog[2, (-I)*E^(I*(a + b*x))] + (12*I)*b*d^2*(c + d*x)*Cos[a
 + b*x]*PolyLog[2, I*E^(I*(a + b*x))] + 12*d^3*Cos[a + b*x]*PolyLog[3, (-I
)*E^(I*(a + b*x))] - 12*d^3*Cos[a + b*x]*PolyLog[3, I*E^(I*(a + b*x))] - 3
*b^2*c^2*d*Sin[2*(a + b*x)] + 6*d^3*Sin[2*(a + b*x)] - 6*b^2*c*d^2*x*Sin[2
*(a + b*x)] - 3*b^2*d^3*x^2*Sin[2*(a + b*x)]))/(2*b^4)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {4907, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117, 4909, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin(a+bx) \tan^2(a+bx)\,dx$$

$$\downarrow 4907$$

$$\int (c+dx)^3 \sec(a+bx) \tan(a+bx) dx - \int (c+dx)^3 \sin(a+bx) dx$$

$$\downarrow 3042$$

$$\int (c+dx)^3 \sec(a+bx) \tan(a+bx) dx - \int (c+dx)^3 \sin(a+bx) dx$$

$$\downarrow 3777$$

$$-\frac{3d \int (c+dx)^2 \cos(a+bx) dx}{b} + \int (c+dx)^3 \sec(a+bx) \tan(a+bx) dx + \frac{(c+dx)^3 \cos(a+bx)}{b}$$

$$\downarrow 3042$$

$$-\frac{3d \int (c+dx)^2 \sin\left(a+bx+\frac{\pi}{2}\right) dx}{b} + \int (c+dx)^3 \sec(a+bx) \tan(a+bx) dx + \frac{(c+dx)^3 \cos(a+bx)}{b}$$

$$\downarrow 3777$$

$$-\frac{3d\left(\frac{2d \int -((c+dx)\sin(a+bx)) dx}{b} + \frac{(c+dx)^2 \sin(a+bx)}{b}\right)}{b} + \int (c+dx)^3 \sec(a+bx) \tan(a+bx) dx + \frac{(c+dx)^3 \cos(a+bx)}{b}$$

$$\downarrow 25$$

$$-\frac{3d\left(\frac{(c+dx)^2 \sin(a+bx)}{b} - \frac{2d \int (c+dx)\sin(a+bx) dx}{b}\right)}{b} + \int (c+dx)^3 \sec(a+bx) \tan(a+bx) dx + \frac{(c+dx)^3 \cos(a+bx)}{b}$$

$$\downarrow 3042$$

$$-\frac{3d\left(\frac{(c+dx)^2 \sin(a+bx)}{b} - \frac{2d \int (c+dx)\sin(a+bx) dx}{b}\right)}{b} + \int (c+dx)^3 \sec(a+bx) \tan(a+bx) dx + \frac{(c+dx)^3 \cos(a+bx)}{b}$$

$$\downarrow 3777$$

$$-\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\cos(a+bx)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}+\int(c+dx)^3\sec(a+bx)\tan(a+bx)dx+\frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\downarrow 3042$$

$$-\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}+\int(c+dx)^3\sec(a+bx)\tan(a+bx)dx+\frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\downarrow 3117$$

$$\int(c+dx)^3\sec(a+bx)\tan(a+bx)dx-\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\downarrow 4909$$

$$-\frac{3d\int(c+dx)^2\sec(a+bx)dx}{b}-\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^3\cos(a+bx)}{b}+\frac{(c+dx)^3\sec(a+bx)}{b}$$

$$\downarrow 3042$$

$$-\frac{3d\int(c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^3\cos(a+bx)}{b}+\frac{(c+dx)^3\sec(a+bx)}{b}$$

$$\downarrow 4669$$

$$-\frac{3d\left(-\frac{2d\int(c+dx)\log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{2d\int(c+dx)\log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}\right)}{b}-$$

$$\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^3\cos(a+bx)}{b}+$$

$$\frac{(c+dx)^3\sec(a+bx)}{b}$$

$$\downarrow 3011$$

$$-\frac{3d\left(\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{id\int\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{id\int\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)dx}{b}\right)}{b}-\right.}{b}$$

$$\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^3\cos(a+bx)}{b}+$$

$$\frac{(c+dx)^3\sec(a+bx)}{b}$$

$$\downarrow 2720$$

$$-\frac{3d\left(\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{P}}{b}\right.}{b}$$

$$\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^3\cos(a+bx)}{b}+$$

$$\frac{(c+dx)^3\sec(a+bx)}{b}$$

$$\downarrow 7143$$

$$-\frac{3d\left(-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\right.}{b}$$

$$\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^3\cos(a+bx)}{b}+$$

$$\frac{(c+dx)^3\sec(a+bx)}{b}$$

input
```
Int[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
((c + d*x)^3*Cos[a + b*x])/b - (3*d*(((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a +
b*x))])/b + (2*d*((I*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b - (d*Po
lyLog[3, (-I)*E^(I*(a + b*x))])/b^2))/b - (2*d*((I*(c + d*x)*PolyLog[2, I*
E^(I*(a + b*x))])/b - (d*PolyLog[3, I*E^(I*(a + b*x))])/b^2))/b))/b + ((c
+ d*x)^3*Sec[a + b*x])/b - (3*d*(((c + d*x)^2*Sin[a + b*x])/b - (2*d*(-(((
c + d*x)*Cos[a + b*x])/b) + (d*Sin[a + b*x])/b^2))/b))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(211) = 422$.

Time = 1.68 (sec) , antiderivative size = 677, normalized size of antiderivative = 2.97

method	result
risch	$\frac{(d^3x^3b^3+3b^3cd^2x^2+3ib^2d^3x^2+3b^3c^2dx+6ib^2cd^2x+b^3c^3+3ib^2c^2d-6bd^3x-6cd^2b-6id^3)e^{i(bx+a)}}{2b^4}+\frac{(d^3x^3b^3+3b^3cd^2x^2-3ib^2d$

input
```
int((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b
*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))
+1/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*
b*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a
))+2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a)
)+1)+6*I/b^2*d*c^2*arctan(exp(I*(b*x+a)))+6*I/b^4*d^3*a^2*arctan(exp(I*(b*
x+a)))-6*I*c*d^2*polylog(2,-I*exp(I*(b*x+a)))/b^3+6/b^3*d^2*c*ln(I*exp(I*(
b*x+a))+1)*a+6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4+6*I*d^3*x*polylog(2,I*
exp(I*(b*x+a)))/b^3-6*I*d^3*x*polylog(2,-I*exp(I*(b*x+a)))/b^3+3/b^4*d^3*a
^2*ln(1-I*exp(I*(b*x+a)))-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4+6*I*c*d^2*
polylog(2,I*exp(I*(b*x+a)))/b^3-6/b^2*d^2*c*ln(1-I*exp(I*(b*x+a)))*x+3/b^2
*d^3*ln(I*exp(I*(b*x+a))+1)*x^2-12*I/b^3*d^2*c*a*arctan(exp(I*(b*x+a)))+6/
b^2*d^2*c*ln(I*exp(I*(b*x+a))+1)*x-6/b^3*d^2*c*ln(1-I*exp(I*(b*x+a)))*a-3/
b^2*d^3*ln(1-I*exp(I*(b*x+a)))*x^2-3/b^4*d^3*a^2*ln(I*exp(I*(b*x+a))+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(202) = 404$.

Time = 0.13 (sec) , antiderivative size = 896, normalized size of antiderivative = 3.93

$$\int (c+dx)^3 \sin(a+bx) \tan^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

output

```
1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*c
os(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)
*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3,
 -I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*
x + a) - sin(b*x + a)) + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*
c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 - 6*(-I*b*d^3*x - I*b*c*
d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) - 6*(-I*b*d^3*x - I
*b*c*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 6*(I*b*d^3*x
 + I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 6*(I*b*
d^3*x + I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*
(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(
b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos
(b*x + a) - I*sin(b*x + a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c
*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b
^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos
(b*x + a) - sin(b*x + a) + 1) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d
^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^
2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos
(b*x + a) - sin(b*x + a) + 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(
b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b...
```

Sympy [F]

$$\int (c+dx)^3 \sin(a+bx) \tan^2(a+bx)\,dx = \int (c+dx)^3 \sin\left(a+bx\right) \tan^2\left(a+bx\right)\,dx$$

input

```
integrate((d*x+c)**3*sin(b*x+a)*tan(b*x+a)**2,x)
```

output

```
Integral((c + d*x)**3*sin(a + b*x)*tan(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 11010 vs. $2(202) = 404$.

Time = 1.63 (sec) , antiderivative size = 11010, normalized size of antiderivative = 48.29

$$\int (c+dx)^3 \sin(a+bx) \tan^2(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

output

```
1/2*(2*c^3*(1/cos(b*x + a) + cos(b*x + a)) - 6*a*c^2*d*(1/cos(b*x + a) + c
os(b*x + a))/b + 6*a^2*c*d^2*(1/cos(b*x + a) + cos(b*x + a))/b^2 - 2*a^3*d
^3*(1/cos(b*x + a) + cos(b*x + a))/b^3 + 3*((b*x + (b*x + a)*cos(2*b*x + 2
*a) + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)^
3 + ((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a)^3
 + 6*(b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + 2*(4*(b*x + a)*cos(2*b*x + 2*
a)*cos(b*x + a) + 4*(b*x + a)*cos(b*x + a) + (3*(b*x + a)*sin(b*x + a) + c
os(b*x + a))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x + a
) - sin(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x + a)*sin(2*b*x + 2*a)*sin(b
*x + a) + (b*x + (b*x + a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(3*
b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a)
+ 6*(b*x + a)*cos(b*x + a) - 2*sin(b*x + a))*sin(3*b*x + 3*a)^2 + ((b*x +
a)*cos(b*x + a) - sin(b*x + a))*sin(2*b*x + 2*a)^2 + ((b*x + a)*cos(2*b*x
+ 2*a)^2 + 13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(2*b*x + 2*a)^2 + (b
*x + a)*sin(b*x + a)^2 + b*x + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*si
n(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*(b*x + a)*cos(b*x + a)*
sin(b*x + a) + cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a) + a)*cos(
3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a)^3 + 3*(b*x + a)*cos(b*x + a)*si
n(b*x + a)^2 + (b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a) + (
b*x + a)*cos(b*x + a) - ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*c...
```

Giac [F]

$$\int (c+dx)^3 \sin(a+bx) \tan^2(a+bx)\,dx = \int (dx+c)^3 \sin(bx+a) \tan(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*sin(b*x + a)*tan(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \sin(a+bx) \tan^2(a+bx)\,dx = \int \sin(a+b\,x)\, \tan(a+b\,x)^2\, (c+d\,x)^3\,dx$$

input
```
int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^3,x)
```

output
```
int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \sin(a+bx) \tan^2(a+bx)\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x)
```

output

```
( - 8*cos(a + b*x)*int(x**2/(tan((a + b*x)/2)**6 - tan((a + b*x)/2)**4 - t
an((a + b*x)/2)**2 + 1),x)*b**3*d**3 - 8*cos(a + b*x)*int((tan((a + b*x)/2
)*x)/(tan((a + b*x)/2)**6 - tan((a + b*x)/2)**4 - tan((a + b*x)/2)**2 + 1)
,x)*b**2*d**3 - 16*cos(a + b*x)*int(x/(tan((a + b*x)/2)**6 - tan((a + b*x)
/2)**4 - tan((a + b*x)/2)**2 + 1),x)*b**3*c*d**2 - 2*cos(a + b*x)*log(tan(
(a + b*x)/2)**2 + 1)*b*c*d**2 + 3*cos(a + b*x)*log(tan((a + b*x)/2) - 1)*b
**2*c**2*d + 2*cos(a + b*x)*log(tan((a + b*x)/2) - 1)*b*c*d**2 + cos(a + b
*x)*log(tan((a + b*x)/2) - 1)*d**3 - 3*cos(a + b*x)*log(tan((a + b*x)/2) +
 1)*b**2*c**2*d + 2*cos(a + b*x)*log(tan((a + b*x)/2) + 1)*b*c*d**2 - cos(
a + b*x)*log(tan((a + b*x)/2) + 1)*d**3 - 3*cos(a + b*x)*sin(a + b*x)*b**2
*c**2*d - 4*cos(a + b*x)*sin(a + b*x)*b**2*c*d**2*x - 2*cos(a + b*x)*sin(a
 + b*x)*b**2*d**3*x**2 + 5*cos(a + b*x)*sin(a + b*x)*d**3 - 6*cos(a + b*x)
*a*b**2*c**2*d + 4*cos(a + b*x)*a*d**3 - 2*cos(a + b*x)*b**3*c**3 + 3*cos(
a + b*x)*b**3*c*d**2*x**2 + cos(a + b*x)*b**3*d**3*x**3 + 4*cos(a + b*x)*b
*c*d**2 - sin(a + b*x)**2*b**3*c**3 - 3*sin(a + b*x)**2*b**3*c**2*d*x - 3*
sin(a + b*x)**2*b**3*c*d**2*x**2 - sin(a + b*x)**2*b**3*d**3*x**3 + 4*sin(
a + b*x)**2*b*c*d**2 + 5*sin(a + b*x)**2*b*d**3*x + 2*sin(a + b*x)*b**2*c*
d**2*x + sin(a + b*x)*b**2*d**3*x**2 + 2*b**3*c**3 + 6*b**3*c**2*d*x + 6*b
**3*c*d**2*x**2 + 2*b**3*d**3*x**3 - 4*b*c*d**2 - 4*b*d**3*x)/(cos(a + b*x
)*b**4)
```

3.261 $\int (c+dx)^2 \sin(a+bx) \tan^2(a+bx)\, dx$

Optimal result . . . 2290
Mathematica [B] (warning: unable to verify) . . . 2291
Rubi [A] (verified) . . . 2291
Maple [B] (verified) . . . 2295
Fricas [B] (verification not implemented) . . . 2295
Sympy [F] . . . 2296
Maxima [F] . . . 2296
Giac [F] . . . 2297
Mupad [F(-1)] . . . 2298
Reduce [F] . . . 2298

Optimal result

Integrand size = 22, antiderivative size = 145

$$\begin{aligned}
&\int (c+dx)^2 \sin(a+bx) \tan^2(a+bx)\, dx \\
&= \frac{4id(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d^2\cos(a+bx)}{b^3} + \frac{(c+dx)^2\cos(a+bx)}{b} \\
&\quad - \frac{2id^2\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} + \frac{2id^2\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{(c+dx)^2\sec(a+bx)}{b} - \frac{2d(c+dx)\sin(a+bx)}{b^2}
\end{aligned}$$

output

```
4*I*d*(d*x+c)*arctan(exp(I*(b*x+a)))/b^2-2*d^2*cos(b*x+a)/b^3+(d*x+c)^2*co
s(b*x+a)/b-2*I*d^2*polylog(2,-I*exp(I*(b*x+a)))/b^3+2*I*d^2*polylog(2,I*ex
p(I*(b*x+a)))/b^3+(d*x+c)^2*sec(b*x+a)/b-2*d*(d*x+c)*sin(b*x+a)/b^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 362 vs. $2(145) = 290$.

Time $= 2.12$ (sec) , antiderivative size $= 362$, normalized size of antiderivative $= 2.50$

$$\int (c+dx)^2 \sin(a+bx) \tan^2(a+bx)\, dx$$

$$= \frac{-4bcd\text{arctanh}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right) - 4d^2 \arctan(\cot(a))\text{arctanh}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right) + \frac{2d^2 \csc}{}}{}$$

input
```
Integrate[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
(-4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - 4*d^2*ArcTan[Cot[a]]*Arc
Tanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + (2*d^2*Csc[a]*((b*x - ArcTan[Cot[a]])
*(Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan[Cot[a
]]))]) + I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyLog[2, E^(I*(
b*x - ArcTan[Cot[a]]))]))/Sqrt[Csc[a]^2] + b^2*(c + d*x)^2*Sec[a] + Cos[b*
x]*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] - 2*b*d*(c + d*x)*Sin[a]) - (2*b*d*(
c + d*x)*Cos[a] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x] + (b^2*(c +
d*x)^2*Sin[(b*x)/2])/((Cos[a/2] - Sin[a/2])*(Cos[(a + b*x)/2] - Sin[(a + b
*x)/2])) - (b^2*(c + d*x)^2*Sin[(b*x)/2])/((Cos[a/2] + Sin[a/2])*(Cos[(a +
 b*x)/2] + Sin[(a + b*x)/2])))/b^3
```

Rubi [A] (verified)

Time $= 0.78$ (sec) , antiderivative size $= 149$, normalized size of antiderivative $= 1.03$, number of steps used $= 14$, number of rules used $= 13$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used $= \{4907, 3042, 3777, 3042, 3777, 25, 3042, 3118, 4909, 3042, 4669, 2715, 2838\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin(a+bx) \tan^2(a+bx)\, dx$$

$$\downarrow 4907$$

$$\int (c+dx)^2 \sec(a+bx)\tan(a+bx)dx - \int (c+dx)^2 \sin(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)^2 \sec(a+bx)\tan(a+bx)dx - \int (c+dx)^2 \sin(a+bx)dx$$

$$\downarrow 3777$$

$$-\frac{2d\int (c+dx)\cos(a+bx)dx}{b} + \int (c+dx)^2 \sec(a+bx)\tan(a+bx)dx + \frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\downarrow 3042$$

$$-\frac{2d\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b} + \int (c+dx)^2 \sec(a+bx)\tan(a+bx)dx + \frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\downarrow 3777$$

$$-\frac{2d\left(\frac{d\int -\sin(a+bx)dx}{b} + \frac{(c+dx)\sin(a+bx)}{b}\right)}{b} + \int (c+dx)^2 \sec(a+bx)\tan(a+bx)dx + \frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\downarrow 25$$

$$-\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b} - \frac{d\int \sin(a+bx)dx}{b}\right)}{b} + \int (c+dx)^2 \sec(a+bx)\tan(a+bx)dx + \frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\downarrow 3042$$

$$-\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b} - \frac{d\int \sin(a+bx)dx}{b}\right)}{b} + \int (c+dx)^2 \sec(a+bx)\tan(a+bx)dx + \frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\downarrow 3118$$

$$\int (c+dx)^2 \sec(a+bx)\tan(a+bx)dx - \frac{2d\left(\frac{d\cos(a+bx)}{b^2} + \frac{(c+dx)\sin(a+bx)}{b}\right)}{b} + \frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\downarrow 4909$$

$$-\frac{2d\int(c+dx)\sec(a+bx)dx}{b}-\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\frac{(c+dx)^2\cos(a+bx)}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

$$\downarrow 3042$$

$$-\frac{2d\int(c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\frac{(c+dx)^2\cos(a+bx)}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

$$\downarrow 4669$$

$$-\frac{2d\left(-\frac{d\int\log(1-ie^{i(a+bx)})dx}{b}+\frac{d\int\log(1+ie^{i(a+bx)})dx}{b}-\frac{2i(c+dx)\arctan(e^{i(a+bx)})}{b}\right)}{b}-\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\frac{(c+dx)^2\cos(a+bx)}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

$$\downarrow 2715$$

$$-\frac{2d\left(\frac{id\int e^{-i(a+bx)}\log(1-ie^{i(a+bx)})de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log(1+ie^{i(a+bx)})de^{i(a+bx)}}{b^2}-\frac{2i(c+dx)\arctan(e^{i(a+bx)})}{b}\right)}{b}-\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\frac{(c+dx)^2\cos(a+bx)}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

$$\downarrow 2838$$

$$-\frac{2d\left(-\frac{2i(c+dx)\arctan(e^{i(a+bx)})}{b}+\frac{id\,\mathrm{PolyLog}(2,-ie^{i(a+bx)})}{b^2}-\frac{id\,\mathrm{PolyLog}(2,ie^{i(a+bx)})}{b^2}\right)}{b}-\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\frac{(c+dx)^2\cos(a+bx)}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

input

```
Int[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

output

```
((c + d*x)^2*Cos[a + b*x])/b - (2*d*(((-2*I)*(c + d*x)*ArcTan[E^(I*(a + b*
x))])/b + (I*d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, I*E
^(I*(a + b*x))])/b^2))/b + ((c + d*x)^2*Sec[a + b*x])/b - (2*d*((d*Cos[a +
 b*x])/b^2 + ((c + d*x)*Sin[a + b*x])/b))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 4909

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(134) = 268$.

Time = 0.90 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.38

method	result
risch	$\frac{(x^2d^2b^2+2b^2cdx+2ib\,d^2x+b^2c^2+2ibcd-2d^2)e^{i(bx+a)}}{2b^3}+\frac{(x^2d^2b^2+2b^2cdx-2ib\,d^2x+b^2c^2-2ibcd-2d^2)e^{-i(bx+a)}}{2b^3}+\frac{2e^{i(bx+a)}(x$ $b(e^{2i($

input

```
int((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
1/2*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(
I*(b*x+a))+1/2*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*
d)/b^3*exp(-I*(b*x+a))+2*exp(I*(b*x+a))*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(
b*x+a))+1)+4*I/b^2*d*c*arctan(exp(I*(b*x+a)))+2/b^2*d^2*ln(I*exp(I*(b*x+a)
)+1)*x+2/b^3*d^2*ln(I*exp(I*(b*x+a))+1)*a-2/b^2*d^2*ln(1-I*exp(I*(b*x+a)))
*x-2/b^3*d^2*ln(1-I*exp(I*(b*x+a)))*a-2*I/b^3*d^2*dilog(I*exp(I*(b*x+a))+1
)+2*I/b^3*d^2*dilog(1-I*exp(I*(b*x+a)))-4*I/b^3*d^2*a*arctan(exp(I*(b*x+a)
))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(127) = 254$.

Time = 0.11 (sec) , antiderivative size = 511, normalized size of antiderivative = 3.52

$$\int (c+dx)^2 \sin(a+bx) \tan^2(a+bx)\,dx$$

$$= \frac{b^2d^2x^2 + 2\,b^2cdx + b^2c^2 + i\,d^2\cos(bx+a)\,\mathrm{Li}_2(i\,\cos(bx+a) + \sin(bx+a)) + i\,d^2\cos(bx+a)\,\mathrm{Li}_2(i\,\cos$$

input
```
integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*cos(b*x + a)*dilog(I*cos(b*x
+ a) + sin(b*x + a)) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x +
 a)) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d^2*co
s(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*
d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - (b*c*d - a*d^2)*cos(b*x + a)*log(c
os(b*x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b
*x + a) - I*sin(b*x + a) + I) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b
*x + a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x
 + a) - sin(b*x + a) + 1) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x
+ a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x +
 a) - sin(b*x + a) + 1) - (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) +
 I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*
sin(b*x + a) + I) - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3*co
s(b*x + a))
```

Sympy [F]

$$\int (c+dx)^2 \sin(a+bx) \tan^2(a+bx)\,dx = \int (c+dx)^2 \sin\left(a+bx\right) \tan^2\left(a+bx\right)\,dx$$

input
```
integrate((d*x+c)**2*sin(b*x+a)*tan(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*sin(a + b*x)*tan(a + b*x)**2, x)
```

Maxima [F]

$$\int (c+dx)^2 \sin(a+bx) \tan^2(a+bx)\,dx = \int \left(dx+c\right)^2 \sin\left(bx+a\right) \tan\left(bx+a\right)^2\,dx$$

input
```
integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

output

```
1/2*(2*((3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*cos(2*b*x + 3*a)
*cos(b*x + 2*a) + (3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*sin(2*
b*x + 3*a)*sin(b*x + 2*a) + (3*b^2*d^2*x^2*cos(a) + 6*b^2*c*d*x*cos(a) + 3
*b^2*c^2*cos(a) - 2*d^2*cos(a))*cos(b*x + 2*a) + (3*b^2*d^2*x^2*sin(a) + 6
*b^2*c*d*x*sin(a) + 3*b^2*c^2*sin(a) - 2*d^2*sin(a))*sin(b*x + 2*a))*cos(3
*b*x + 3*a)^2 + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a
) - 2*(b*d^2*x + b*c*d)*sin(b*x + a))*cos(2*b*x + 3*a)^2 + 2*((3*b^2*d^2*x
^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*cos(2*b*x + 3*a)*cos(b*x + 2*a) + (3
*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*sin(2*b*x + 3*a)*sin(b*x +
 2*a) + (3*b^2*d^2*x^2*cos(a) + 6*b^2*c*d*x*cos(a) + 3*b^2*c^2*cos(a) - 2*
d^2*cos(a))*cos(b*x + 2*a) + (3*b^2*d^2*x^2*sin(a) + 6*b^2*c*d*x*sin(a) +
3*b^2*c^2*sin(a) - 2*d^2*sin(a))*sin(b*x + 2*a))*sin(3*b*x + 3*a)^2 + ((b^
2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a) - 2*(b*d^2*x + b*c
*d)*sin(b*x + a))*sin(2*b*x + 3*a)^2 + ((b^2*d^2*x^2*cos(a) + b^2*c^2*cos(
a) + 2*b*c*d*sin(a) - 2*d^2*cos(a) + 2*(b^2*c*d*cos(a) + b*d^2*sin(a))*x +
 (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(2*b*x + 3*a) + 2*(b*d^2
*x + b*c*d)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a)^2 + (b^2*d^2*x^2*cos(a) + b
^2*c^2*cos(a) + 2*b*c*d*sin(a) - 2*d^2*cos(a) + 2*(b^2*c*d*cos(a) + b*d^2*
sin(a))*x)*cos(b*x + a)^2 + (b^2*d^2*x^2*cos(a) + b^2*c^2*cos(a) + 2*b*c*d
*sin(a) - 2*d^2*cos(a) + 2*(b^2*c*d*cos(a) + b*d^2*sin(a))*x + (b^2*d^2...
```

Giac [F]

$$\int (c+dx)^2 \sin(a+bx) \tan^2(a+bx)\,dx = \int (dx+c)^2 \sin\left(bx+a\right) \tan\left(bx+a\right)^2\,dx$$

input

```
integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

output

```
integrate((d*x + c)^2*sin(b*x + a)*tan(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \sin(a+bx) \tan^2(a+bx)\,dx = \int \sin(a+b\,x)\,\tan(a+b\,x)^2\,(c+d\,x)^2\,dx$$

input
```
int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^2,x)
```

output
```
int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \sin(a+bx) \tan^2(a+bx)\,dx$$
$$= \frac{-16\cos(bx+a)\left(\int \frac{x}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^6-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^4-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1}dx\right)b^2d^2 - 2\cos(bx+a)\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right.}{}$$

input
```
int((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x)
```

output
```
( - 16*cos(a + b*x)*int(x/(tan((a + b*x)/2)**6 - tan((a + b*x)/2)**4 - tan
((a + b*x)/2)**2 + 1),x)*b**2*d**2 - 2*cos(a + b*x)*log(tan((a + b*x)/2)**
2 + 1)*d**2 + 6*cos(a + b*x)*log(tan((a + b*x)/2) - 1)*b*c*d + 2*cos(a + b
*x)*log(tan((a + b*x)/2) - 1)*d**2 - 6*cos(a + b*x)*log(tan((a + b*x)/2) +
 1)*b*c*d + 2*cos(a + b*x)*log(tan((a + b*x)/2) + 1)*d**2 - 6*cos(a + b*x)
*sin(a + b*x)*b*c*d - 4*cos(a + b*x)*sin(a + b*x)*b*d**2*x - 12*cos(a + b*
x)*a*b*c*d - 6*cos(a + b*x)*b**2*c**2 + 3*cos(a + b*x)*b**2*d**2*x**2 + 4*
cos(a + b*x)*d**2 - 3*sin(a + b*x)**2*b**2*c**2 - 6*sin(a + b*x)**2*b**2*c
*d*x - 3*sin(a + b*x)**2*b**2*d**2*x**2 + 4*sin(a + b*x)**2*d**2 + 2*sin(a
 + b*x)*b*d**2*x + 6*b**2*c**2 + 12*b**2*c*d*x + 6*b**2*d**2*x**2 - 4*d**2
)/(3*cos(a + b*x)*b**3)
```

3.262 $\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx$

Optimal result . . . 2299
Mathematica [A] (verified) . . . 2299
Rubi [A] (verified) . . . 2300
Maple [C] (verified) . . . 2302
Fricas [A] (verification not implemented) . . . 2302
Sympy [F] . . . 2303
Maxima [B] (verification not implemented) . . . 2303
Giac [B] (verification not implemented) . . . 2304
Mupad [B] (verification not implemented) . . . 2305
Reduce [B] (verification not implemented) . . . 2306

Optimal result

Integrand size = 20, antiderivative size = 56

$$\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx = -\frac{d\operatorname{arctanh}(\sin(a+bx))}{b^2} + \frac{(c+dx)\cos(a+bx)}{b} + \frac{(c+dx)\sec(a+bx)}{b} - \frac{d\sin(a+bx)}{b^2}$$

output
```
-d*arctanh(sin(b*x+a))/b^2+(d*x+c)*cos(b*x+a)/b+(d*x+c)*sec(b*x+a)/b-d*sin
(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.91

$$\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx = \frac{\sec(a+bx)\left(3bc+3bdx+b(c+dx)\cos(2(a+bx))+2d\cos(a+bx)\left(\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right.\right.\right. - \sin\left(\frac{1}{2}(a+$$

$$2b^2$$

input
```
Integrate[(c + d*x)*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

output

```
(Sec[a + b*x]*(3*b*c + 3*b*d*x + b*(c + d*x)*Cos[2*(a + b*x)] + 2*d*Cos[a
+ b*x]*(Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]] - Log[Cos[(a + b*x)/2] +
Sin[(a + b*x)/2]]) - d*Sin[2*(a + b*x)]))/(2*b^2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4907, 3042, 3777, 3042, 3117, 4909, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx$$

$$\downarrow 4907$$

$$\int (c+dx)\sec(a+bx)\tan(a+bx)dx - \int (c+dx)\sin(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)\sec(a+bx)\tan(a+bx)dx - \int (c+dx)\sin(a+bx)dx$$

$$\downarrow 3777$$

$$\int (c+dx)\sec(a+bx)\tan(a+bx)dx - \frac{d\int \cos(a+bx)dx}{b} + \frac{(c+dx)\cos(a+bx)}{b}$$

$$\downarrow 3042$$

$$\int (c+dx)\sec(a+bx)\tan(a+bx)dx - \frac{d\int \sin\left(a+bx+\frac{\pi}{2}\right)dx}{b} + \frac{(c+dx)\cos(a+bx)}{b}$$

$$\downarrow 3117$$

$$\int (c+dx)\sec(a+bx)\tan(a+bx)dx - \frac{d\sin(a+bx)}{b^2} + \frac{(c+dx)\cos(a+bx)}{b}$$

$$\downarrow 4909$$

$$-\frac{d\int \sec(a+bx)dx}{b} - \frac{d\sin(a+bx)}{b^2} + \frac{(c+dx)\cos(a+bx)}{b} + \frac{(c+dx)\sec(a+bx)}{b}$$

$$\downarrow 3042$$

$$-\frac{d\int \csc\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{d\sin(a+bx)}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}+\frac{(c+dx)\sec(a+bx)}{b}$$

$$\downarrow 4257$$

$$-\frac{d\operatorname{arctanh}(\sin(a+bx))}{b^2}-\frac{d\sin(a+bx)}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}+\frac{(c+dx)\sec(a+bx)}{b}$$

input
```
Int[(c + d*x)*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
-((d*ArcTanh[Sin[a + b*x]])/b^2) + ((c + d*x)*Cos[a + b*x])/b + ((c + d*x)
*Sec[a + b*x])/b - (d*Sin[a + b*x])/b^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 4909

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.20

method	result	size
risch	$\frac{(bdx+cb+id)e^{i(bx+a)}}{2b^2}+\frac{(bdx+cb-id)e^{-i(bx+a)}}{2b^2}+\frac{2e^{i(bx+a)}(dx+c)}{b(e^{2i(bx+a)}+1)}-\frac{d\ln(e^{i(bx+a)}+i)}{b^2}+\frac{d\ln(e^{i(bx+a)}-i)}{b^2}$	123

input

```
int((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
1/2*(b*d*x+c*b+I*d)/b^2*exp(I*(b*x+a))+1/2*(b*d*x+c*b-I*d)/b^2*exp(-I*(b*x
+a))+2*exp(I*(b*x+a))*(d*x+c)/b/(exp(2*I*(b*x+a))+1)-d/b^2*ln(exp(I*(b*x+a
))+I)+d/b^2*ln(exp(I*(b*x+a))-I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.66

$$\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx$$
$$= \frac{2\,bdx+2\,(bdx+bc)\cos(bx+a)^2-d\cos(bx+a)\log(\sin(bx+a)+1)+d\cos(bx+a)\log(-\sin(bx+}{2\,b^2\cos(bx+a)}$$

input

```
integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

output

```
1/2*(2*b*d*x + 2*(b*d*x + b*c)*cos(b*x + a)^2 - d*cos(b*x + a)*log(sin(b*x
 + a) + 1) + d*cos(b*x + a)*log(-sin(b*x + a) + 1) - 2*d*cos(b*x + a)*sin(
b*x + a) + 2*b*c)/(b^2*cos(b*x + a))
```

Sympy [F]

$$\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx = \int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx$$

input
```
integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*sin(a + b*x)*tan(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2123 vs. $2(56) = 112$.

Time $= 0.20$ (sec) , antiderivative size $= 2123$, normalized size of antiderivative $= 37.91$

$$\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

output

```
1/2*(2*c*(1/cos(b*x + a) + cos(b*x + a)) - 2*a*d*(1/cos(b*x + a) + cos(b*x
 + a))/b + ((b*x + (b*x + a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(
3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)^3 + ((b*x + a)*sin(2*b*x + 2*a)
- cos(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)*sin(
b*x + a)^2 + 2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*co
s(b*x + a) + (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a))*c
os(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*
a)^2 + (8*(b*x + a)*sin(2*b*x + 2*a)*sin(b*x + a) + (b*x + (b*x + a)*cos(2
*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(
b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a) + 6*(b*x + a)*cos(b*x + a) - 2*s
in(b*x + a))*sin(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(b*x + a))*
sin(2*b*x + 2*a)^2 + ((b*x + a)*cos(2*b*x + 2*a)^2 + 13*(b*x + a)*cos(b*x
+ a)^2 + (b*x + a)*sin(2*b*x + 2*a)^2 + (b*x + a)*sin(b*x + a)^2 + b*x + (
13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(
2*b*x + 2*a) + (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x + a)^2 +
sin(b*x + a)^2)*sin(2*b*x + 2*a) + a)*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*co
s(b*x + a)^3 + 3*(b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + (b*x + a)*cos(b*x
 + a) - sin(b*x + a))*cos(2*b*x + 2*a) + (b*x + a)*cos(b*x + a) - ((cos(2*
b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*
a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2330 vs. $2(56) = 112$.

Time $= 0.78$ (sec) , antiderivative size $= 2330$, normalized size of antiderivative $= 41.61$

$$\int (c + dx)\sin(a + bx)\tan^2(a + bx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

output

```
1/2*(4*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b*c*tan(1/2*b*x)^4*tan(1/2*a)
^4 + d*log(2*(tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2*tan(1/2*a) +
2*tan(1/2*b*x)*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*
x) - 2*tan(1/2*a) + 1)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan
(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^4 - d*log(2*(tan(1/2*b*x)^2*tan(
1/2*a)^2 - 2*tan(1/2*b*x)^2*tan(1/2*a) - 2*tan(1/2*b*x)*tan(1/2*a)^2 + tan
(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*b
*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*ta
n(1/2*a)^4 - 16*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^3 - 16*b*c*tan(1/2*b*x)^3*
tan(1/2*a)^3 - 4*d*log(2*(tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2*t
an(1/2*a) + 2*tan(1/2*b*x)*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 -
2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*
b*x)^2 + tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a)^3 + 4*d*log(2*(tan(1
/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^2*tan(1/2*a) - 2*tan(1/2*b*x)*tan(
1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) +
 1)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 1))*tan
(1/2*b*x)^3*tan(1/2*a)^3 + 4*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + 4*d*tan(1/2*b
*x)^3*tan(1/2*a)^4 + 4*b*d*x*tan(1/2*b*x)^4 + 16*b*d*x*tan(1/2*b*x)^3*tan(
1/2*a) + 48*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 16*b*d*x*tan(1/2*b*x)*tan(
1/2*a)^3 + 4*b*d*x*tan(1/2*a)^4 + 4*b*c*tan(1/2*b*x)^4 - d*log(2*(tan(1...
```

Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.70

$$\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx = \mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\left(\frac{b\,c+d\,1\mathrm{i}}{2\,b^2}+\frac{d\,x}{2\,b}\right) - \mathrm{e}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}\left(\frac{-b\,c+d\,1\mathrm{i}}{2\,b^2}-\frac{d\,x}{2\,b}\right) + \frac{d\,\ln\left(\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}-\mathrm{i}\right)}{b^2} - \frac{d\,\ln\left(\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}+1\mathrm{i}\right)}{b^2} + \frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,(c+d\,x)\,2\mathrm{i}}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}$$

input

```
int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x),x)
```

output
```
exp(a*1i + b*x*1i)*((d*1i + b*c)/(2*b^2) + (d*x)/(2*b)) - exp(- a*1i - b*x
*1i)*((d*1i - b*c)/(2*b^2) - (d*x)/(2*b)) + (d*log(exp(a*1i + b*x*1i) - 1i
))/b^2 - (d*log(exp(a*1i + b*x*1i) + 1i))/b^2 + (exp(a*1i + b*x*1i)*(c + d
*x)*2i)/(b*(exp(a*2i + b*x*2i)*1i + 1i))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.20

$$\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx$$
$$= \frac{\cos(bx+a)\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)d - \cos(bx+a)\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)d - \cos(bx+a)\sin(bx+a)\,d}{\cos(bx+a)\,b^2}$$

input
```
int((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x)
```

output
```
(cos(a + b*x)*log(tan((a + b*x)/2) - 1)*d - cos(a + b*x)*log(tan((a + b*x)
/2) + 1)*d - cos(a + b*x)*sin(a + b*x)*d + 2*cos(a + b*x)*a*d - 2*cos(a +
b*x)*b*c - sin(a + b*x)**2*b*c - sin(a + b*x)**2*b*d*x + 2*b*c + 2*b*d*x)/
(cos(a + b*x)*b**2)
```

3.263 $\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$

Optimal result . . . 2307
Mathematica [N/A] . . . 2307
Rubi [N/A] . . . 2308
Maple [N/A] . . . 2310
Fricas [N/A] . . . 2310
Sympy [N/A] . . . 2311
Maxima [N/A] . . . 2311
Giac [N/A] . . . 2312
Mupad [N/A] . . . 2313
Reduce [N/A] . . . 2313

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx = -\frac{\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d} - \frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d} + \text{Int}\left(\frac{\sec(a+bx)\tan(a+bx)}{c+dx}, x\right)$$

output

```
-Ci(b*c/d+b*x)*sin(a-b*c/d)/d-cos(a-b*c/d)*Si(b*c/d+b*x)/d+Defer(Int)(sec(
b*x+a)*tan(b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

input

```
Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
```

output
```
Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4907, 3042, 3784, 3042, 3780, 3783, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

$$\downarrow 4907$$

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}dx - \int \frac{\sin(a+bx)}{c+dx}dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}dx - \int \frac{\sin(a+bx)}{c+dx}dx$$

$$\downarrow 3784$$

$$-\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx - \cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx + \int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}dx$$

$$\downarrow 3042$$

$$-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx - \cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx + \int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}dx$$

$$\downarrow 3780$$

$$-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx + \int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}dx - \frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

$$\downarrow 3783$$

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}dx - \frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d} - \frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

$$\downarrow 7299$$

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}dx - \frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d} - \frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

input
```
Int[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(bx+a\right)\tan\left(bx+a\right)^2}{dx+c}dx$$

input
```
int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

output
```
int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\sin\left(bx+a\right)\tan\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(sin(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\sin\left(a+bx\right)\tan^2\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)*tan(a + b*x)**2/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 1268, normalized size of antiderivative = 57.64

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\sin\left(bx+a\right)\tan\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output

```
1/2*(b*c*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(1, -
(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*c*(exp_integral_e(1, (I*b*d*
x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/
d) + (b*c*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(1,
-(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*c*(exp_integral_e(1, (I*b*d
*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)
/d) + (b*d*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(1,
 -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*d*(exp_integral_e(1, (I*b*
d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d
)/d))*x)*cos(2*b*x + 2*a)^2 + (b*c*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/
d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*c*
(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*
b*c)/d))*sin(-(b*c - a*d)/d) + (b*d*(I*exp_integral_e(1, (I*b*d*x + I*b*c)
/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*d
*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I
*b*c)/d))*sin(-(b*c - a*d)/d))*x)*sin(2*b*x + 2*a)^2 + 4*d*sin(2*b*x + 2*a
)*sin(b*x + a) + (b*d*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_in
tegral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*d*(exp_integral
_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(
-(b*c - a*d)/d))*x - 2*(b*c*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) ...
```

Giac [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\sin\left(bx+a\right)\tan\left(bx+a\right)^2}{dx+c}\,dx$$

input

```
integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output

```
integrate(sin(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 19.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\sin(a+b\,x)\,\tan(a+b\,x)^2}{c+d\,x}\,dx$$

input
```
int((sin(a + b*x)*tan(a + b*x)^2)/(c + d*x),x)
```

output
```
int((sin(a + b*x)*tan(a + b*x)^2)/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\sin(bx+a)\tan(bx+a)^2}{dx+c}dx$$

input
```
int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

output
```
int((sin(a + b*x)*tan(a + b*x)**2)/(c + d*x),x)
```

3.264 $\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$

Optimal result 2314
Mathematica [N/A] 2314
Rubi [N/A] 2315
Maple [N/A] 2318
Fricas [N/A] 2318
Sympy [N/A] 2318
Maxima [N/A] 2319
Giac [N/A] 2320
Mupad [N/A] 2320
Reduce [N/A] 2320

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = -\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2} + \frac{\sin(a+bx)}{d(c+dx)} + \frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2} + \text{Int}\left(\frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2},x\right)$$

output

```
-b*cos(a-b*c/d)*Ci(b*c/d+b*x)/d^2+sin(b*x+a)/d/(d*x+c)+b*sin(a-b*c/d)*Si(b
*c/d+b*x)/d^2+Defer(Int)(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

input

```
Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4907, 3042, 3778, 3042, 3784, 3042, 3780, 3783, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

$$\downarrow 4907$$

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}dx - \int \frac{\sin(a+bx)}{(c+dx)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}dx - \int \frac{\sin(a+bx)}{(c+dx)^2}dx$$

$$\downarrow 3778$$

$$-\frac{b\int \frac{\cos(a+bx)}{c+dx}dx}{d} + \int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}dx + \frac{\sin(a+bx)}{d(c+dx)}$$

$$\downarrow 3042$$

$$-\frac{b\int \frac{\sin(a+bx+\frac{\pi}{2})}{c+dx}dx}{d} + \int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}dx + \frac{\sin(a+bx)}{d(c+dx)}$$

$$\downarrow 3784$$

$$-\frac{b\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx - \sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d} + \int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}dx + \frac{\sin(a+bx)}{d(c+dx)}$$

$$\downarrow 3042$$

$$-\frac{b\left(\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}+$$
$$\int\frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}dx+\frac{\sin(a+bx)}{d(c+dx)}$$

$$\downarrow 3780$$

$$-\frac{b\left(\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx-\frac{\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}+\int\frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}dx+$$
$$\frac{\sin(a+bx)}{d(c+dx)}$$

$$\downarrow 3783$$

$$\int\frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}dx-\frac{b\left(\frac{\cos\left(a-\frac{bc}{d}\right)\mathrm{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}+$$
$$\frac{\sin(a+bx)}{d(c+dx)}$$

$$\downarrow 7299$$

$$\int\frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}dx-\frac{b\left(\frac{\cos\left(a-\frac{bc}{d}\right)\mathrm{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}+$$
$$\frac{\sin(a+bx)}{d(c+dx)}$$

input
```
Int[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin(bx+a)\tan(bx+a)^2}{(dx+c)^2}dx$$

input
```
int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(bx+a)\tan(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(sin(b*x + a)*tan(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)*tan(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 1412, normalized size of antiderivative = 64.18

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin\left(bx+a\right)\tan\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*(b*c*(I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(2, -
(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*c*(exp_integral_e(2, (I*b*d*
x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/
d) + (b*c*(I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(2,
-(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*c*(exp_integral_e(2, (I*b*d
*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)
/d) + (b*d*(I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(2,
 -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*d*(exp_integral_e(2, (I*b*
d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d
)/d))*x)*cos(2*b*x + 2*a)^2 + (b*c*(I*exp_integral_e(2, (I*b*d*x + I*b*c)/
d) - I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*c*
(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*
b*c)/d))*sin(-(b*c - a*d)/d) + (b*d*(I*exp_integral_e(2, (I*b*d*x + I*b*c)
/d) - I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*d
*(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I
*b*c)/d))*sin(-(b*c - a*d)/d))*x)*sin(2*b*x + 2*a)^2 + 4*d*sin(2*b*x + 2*a
)*sin(b*x + a) + (b*d*(I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - I*exp_in
tegral_e(2, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*d*(exp_integral
_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(
-(b*c - a*d)/d))*x - 2*(b*c*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) ...
```

Giac [N/A]

Not integrable

Time = 16.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(bx+a)\tan(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)*tan(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 20.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(a+b\,x)\,\tan(a+b\,x)^2}{(c+d\,x)^2}\,dx$$

input
```
int((sin(a + b*x)*tan(a + b*x)^2)/(c + d*x)^2,x)
```

output
```
int((sin(a + b*x)*tan(a + b*x)^2)/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(bx+a)\tan(bx+a)^2}{d^2x^2+2cdx+c^2}\,dx$$

input
```
int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int((sin(a + b*x)*tan(a + b*x)**2)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.265 $\int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx$

Optimal result 2322
Mathematica [N/A] 2322
Rubi [N/A] 2323
Maple [N/A] 2323
Fricas [N/A] 2324
Sympy [N/A] 2324
Maxima [N/A] 2325
Giac [N/A] 2325
Mupad [N/A] 2325
Reduce [N/A] 2326

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx = \text{Int}\big((c+dx)^m \csc(a+bx)\sec^2(a+bx), x\big)$$

output
```
Defer(Int)((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 14.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx = \int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\sec^2(a+bx)(c+dx)^m\,dx$$

$$\downarrow 7299$$

$$\int \csc(a+bx)\sec^2(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \csc(bx+a)\sec(bx+a)^2\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc(a+bx) \sec^2(a+bx)\, dx = \int (dx+c)^m \csc(bx+a) \sec(bx+a)^2\, dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 116.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \csc(a+bx) \sec^2(a+bx)\, dx = \int (c+dx)^m \csc(a+bx) \sec^2(a+bx)\, dx$$

input
```
integrate((d*x+c)**m*csc(b*x+a)*sec(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*csc(a + b*x)*sec(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)\sec(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)\sec(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 19.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\cos(a+b\,x)^2\sin(a+b\,x)}\,dx$$

input
```
int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)),x)
```

output
```
int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc(a+bx) \sec^2(a+bx)\, dx = \int (dx+c)^m \csc(bx+a) \sec(bx+a)^2\, dx$$

input
```
int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x)
```

output
```
int((c + d*x)**m*csc(a + b*x)*sec(a + b*x)**2,x)
```

3.266 $\int (c+dx)^4 \csc(a+bx)\sec^2(a+bx)\,dx$

Optimal result . . . 2327
Mathematica [A] (verified) . . . 2328
Rubi [A] (verified) . . . 2329
Maple [B] (verified) . . . 2332
Fricas [B] (verification not implemented) . . . 2333
Sympy [F(-1)] . . . 2334
Maxima [B] (verification not implemented) . . . 2334
Giac [F] . . . 2335
Mupad [F(-1)] . . . 2336
Reduce [F] . . . 2336

Optimal result

Integrand size = 22, antiderivative size = 469

$$
\begin{aligned}
&\int (c+dx)^4 \csc(a+bx)\sec^2(a+bx)\,dx \\
&= \frac{8id(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^4 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} \\
&\quad + \frac{4id(c+dx)^3 \operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} - \frac{12id^2(c+dx)^2 \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{12id^2(c+dx)^2 \operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} - \frac{4id(c+dx)^3 \operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{12d^2(c+dx)^2 \operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} + \frac{24d^3(c+dx) \operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4} \\
&\quad - \frac{24d^3(c+dx) \operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4} + \frac{12d^2(c+dx)^2 \operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} \\
&\quad - \frac{24id^3(c+dx) \operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4} + \frac{24id^4 \operatorname{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^5} \\
&\quad - \frac{24id^4 \operatorname{PolyLog}\left(4,ie^{i(a+bx)}\right)}{b^5} + \frac{24id^3(c+dx) \operatorname{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4} \\
&\quad + \frac{24d^4 \operatorname{PolyLog}\left(5,-e^{i(a+bx)}\right)}{b^5} - \frac{24d^4 \operatorname{PolyLog}\left(5,e^{i(a+bx)}\right)}{b^5} + \frac{(c+dx)^4 \sec(a+bx)}{b}
\end{aligned}
$$

output

```
-24*I*d^4*polylog(4,I*exp(I*(b*x+a)))/b^5-2*(d*x+c)^4*arctanh(exp(I*(b*x+a
)))/b+12*I*d^2*(d*x+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^3+8*I*d*(d*x+c)^3*a
rctan(exp(I*(b*x+a)))/b^2+24*I*d^3*(d*x+c)*polylog(4,exp(I*(b*x+a)))/b^4-2
4*I*d^3*(d*x+c)*polylog(4,-exp(I*(b*x+a)))/b^4-12*d^2*(d*x+c)^2*polylog(3,
-exp(I*(b*x+a)))/b^3+24*d^3*(d*x+c)*polylog(3,-I*exp(I*(b*x+a)))/b^4-24*d^
3*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^4+12*d^2*(d*x+c)^2*polylog(3,exp(I
*(b*x+a)))/b^3+24*I*d^4*polylog(4,-I*exp(I*(b*x+a)))/b^5-4*I*d*(d*x+c)^3*p
olylog(2,exp(I*(b*x+a)))/b^2+4*I*d*(d*x+c)^3*polylog(2,-exp(I*(b*x+a)))/b^
2-12*I*d^2*(d*x+c)^2*polylog(2,-I*exp(I*(b*x+a)))/b^3+24*d^4*polylog(5,-ex
p(I*(b*x+a)))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^5+(d*x+c)^4*sec(b*x+a
)/b
```

Mathematica [A] (verified)

Time = 2.16 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.48

$$\int (c+dx)^4 \csc(a+bx) \sec^2(a+bx)\, dx$$
$$= \frac{b^4(c+dx)^4 \log\left(1-e^{i(a+bx)}\right) - b^4(c+dx)^4 \log\left(1+e^{i(a+bx)}\right) - 4d\left(-2ib^3c^3 \arctan\left(e^{i(a+bx)}\right) + 3b^3c^2dx \log\right.}{}$$

input

```
Integrate[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

output

```
(b^4*(c + d*x)^4*Log[1 - E^(I*(a + b*x))] - b^4*(c + d*x)^4*Log[1 + E^(I*(
a + b*x))] - 4*d*((-2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^3*c^2*d*x*L
og[1 - I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] + b
^3*d^3*x^3*Log[1 - I*E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a +
b*x))] - 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 +
I*E^(I*(a + b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x
))] - (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] - 6*b*c*d^2*Po
lyLog[3, (-I)*E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))
] + 6*b*c*d^2*PolyLog[3, I*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, I*E^(I*
(a + b*x))] - (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] + (6*I)*d^3*PolyL
og[4, I*E^(I*(a + b*x))]) + (4*I)*d*(b^3*(c + d*x)^3*PolyLog[2, -E^(I*(a +
 b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[3, -E^(I*(a + b*x))] - 6*d^2*(b*
(c + d*x)*PolyLog[4, -E^(I*(a + b*x))] + I*d*PolyLog[5, -E^(I*(a + b*x))])
) - (4*I)*d*(b^3*(c + d*x)^3*PolyLog[2, E^(I*(a + b*x))] + (3*I)*b^2*d*(c
+ d*x)^2*PolyLog[3, E^(I*(a + b*x))] - 6*d^2*(b*(c + d*x)*PolyLog[4, E^(I*
(a + b*x))] + I*d*PolyLog[5, E^(I*(a + b*x))])) + b^4*(c + d*x)^4*Sec[a +
b*x])/b^5
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4920, 25, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \csc(a+bx) \sec^2(a+bx)\, dx$$

$$\downarrow 4920$$

$$-4d \int -(c+dx)^3 \left(\frac{\operatorname{arctanh}(\cos(a+bx))}{b} - \frac{\sec(a+bx)}{b} \right) dx - \frac{(c+dx)^4 \operatorname{arctanh}(\cos(a+bx))}{b} + \frac{(c+dx)^4 \sec(a+bx)}{b}$$

$$\downarrow 25$$

$$4d\int (c+dx)^3\left(\frac{\operatorname{arctanh}(\cos(a+bx))}{b}-\frac{\sec(a+bx)}{b}\right)dx-\frac{(c+dx)^4\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^4\sec(a+bx)}{b}$$

$$\downarrow 7292$$

$$4d\int\frac{(c+dx)^3(\operatorname{arctanh}(\cos(a+bx))-\sec(a+bx))}{b}dx-\frac{(c+dx)^4\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^4\sec(a+bx)}{b}$$

$$\downarrow 27$$

$$\frac{4d\int(c+dx)^3(\operatorname{arctanh}(\cos(a+bx))-\sec(a+bx))dx}{b}-\frac{(c+dx)^4\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^4\sec(a+bx)}{b}$$

$$\downarrow 7293$$

$$\frac{4d\int\left((c+dx)^3\operatorname{arctanh}(\cos(a+bx))-(c+dx)^3\sec(a+bx)\right)dx}{b}-\frac{(c+dx)^4\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^4\sec(a+bx)}{b}$$

$$\downarrow 2009$$

$$\frac{4d\left(\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{(c+dx)^4\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{2d}+\frac{(c+dx)^4\operatorname{arctanh}(\cos(a+bx))}{4d}+\frac{6id^3\operatorname{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^4}-\frac{6id^3\,\mathrm{I}}{}\right.}{\frac{(c+dx)^4\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^4\sec(a+bx)}{b}}$$

input

```
Int[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

output

```
-(((c + d*x)^4*ArcTanh[Cos[a + b*x]])/b) + (4*d*(((2*I)*(c + d*x)^3*ArcTan
[E^(I*(a + b*x))])/b - ((c + d*x)^4*ArcTanh[E^(I*(a + b*x))])/(2*d) + ((c
+ d*x)^4*ArcTanh[Cos[a + b*x]])/(4*d) + (I*(c + d*x)^3*PolyLog[2, -E^(I*(a
 + b*x))])/b - ((3*I)*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2
+ ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (I*(c + d*x)^3
*PolyLog[2, E^(I*(a + b*x))])/b - (3*d*(c + d*x)^2*PolyLog[3, -E^(I*(a + b
*x))])/b^2 + (6*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 - (6*d
^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + (3*d*(c + d*x)^2*PolyLog
[3, E^(I*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)*PolyLog[4, -E^(I*(a + b*x
))])/b^3 + ((6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^4 - ((6*I)*d^3*P
olyLog[4, I*E^(I*(a + b*x))])/b^4 + ((6*I)*d^2*(c + d*x)*PolyLog[4, E^(I*(
a + b*x))])/b^3 + (6*d^3*PolyLog[5, -E^(I*(a + b*x))])/b^4 - (6*d^3*PolyLo
g[5, E^(I*(a + b*x))])/b^4))/b + ((c + d*x)^4*Sec[a + b*x])/b
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1865 vs. $2(422) = 844$.

Time = 0.77 (sec) , antiderivative size = 1866, normalized size of antiderivative = 3.98

method	result	size
risch	Expression too large to display	1866

input
```
int((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
6/b^3*a^2*d^2*c^2*ln(exp(I*(b*x+a))-1)-12/b^3*d^2*c^2*polylog(3,-exp(I*(b*
x+a)))+4/b^2*d^4*ln(I*exp(I*(b*x+a))+1)*x^3-4/b^2*d^4*ln(1-I*exp(I*(b*x+a)
))*x^3-4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x-24/b^3*c*d^3*polylog(3,-exp(I*(b*x
+a)))*x+4/b*c^3*d*ln(1-exp(I*(b*x+a)))*x-4/b^2*a*c^3*d*ln(exp(I*(b*x+a))-1
)+12/b^3*d^2*c^2*polylog(3,exp(I*(b*x+a)))+1/b^5*a^4*d^4*ln(exp(I*(b*x+a))
-1)+24/b^4*c*d^3*polylog(3,-I*exp(I*(b*x+a)))+6/b*d^2*c^2*ln(1-exp(I*(b*x+
a)))*x^2+12*I/b^5*a^2*d^4*dilog(1-I*exp(I*(b*x+a)))-12*I/b^3*d^2*c^2*dilog
(I*exp(I*(b*x+a))+1)-24*I/b^4*c*d^3*polylog(4,-exp(I*(b*x+a)))-12*I/b^5*a^
2*d^4*polylog(2,I*exp(I*(b*x+a)))-12/b^4*c*d^3*ln(I*exp(I*(b*x+a))+1)*a^2-
1/b^5*a^4*d^4*ln(1-exp(I*(b*x+a)))-12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x
^2+24/b^4*d^4*polylog(3,-I*exp(I*(b*x+a)))*x+12/b^3*d^4*polylog(3,exp(I*(b
*x+a)))*x^2-1/b*d^4*ln(exp(I*(b*x+a))+1)*x^4-24/b^4*d^4*polylog(3,I*exp(I*
(b*x+a)))*x+4/b^5*a^3*d^4*ln(I*exp(I*(b*x+a))+1)-4/b^5*a^3*d^4*ln(1-I*exp(
I*(b*x+a)))+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4+2*exp(I*(b*x+a))*(d^4*x^4+4*c
*d^3*x^3+6*c^2*d^2*x^2+4*c^3*d*x+c^4)/b/(exp(2*I*(b*x+a))+1)-6/b^3*d^2*c^2
*ln(1-exp(I*(b*x+a)))*a^2+4/b^4*c*d^3*a^3*ln(1-exp(I*(b*x+a)))+4/b*c*d^3*l
n(1-exp(I*(b*x+a)))*x^3-4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3+12/b^4*c*d^3*ln
(1-I*exp(I*(b*x+a)))*a^2+12*I/b^3*d^2*c^2*dilog(1-I*exp(I*(b*x+a)))-24*I/b
^4*d^4*polylog(4,-exp(I*(b*x+a)))*x-4*I/b^2*c^3*d*polylog(2,exp(I*(b*x+a))
)+4*I/b^2*c^3*d*polylog(2,-exp(I*(b*x+a)))+12*I/b^3*d^4*polylog(2,I*exp...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2519 vs. $2(403) = 806$.

Time = 0.24 (sec) , antiderivative size = 2519, normalized size of antiderivative = 5.37

$$\int (c+dx)^4 \csc(a+bx) \sec^2(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

output

```
1/2*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x
+ 2*b^4*c^4 - 24*d^4*cos(b*x + a)*polylog(5, cos(b*x + a) + I*sin(b*x + a)
) - 24*d^4*cos(b*x + a)*polylog(5, cos(b*x + a) - I*sin(b*x + a)) + 24*d^4
*cos(b*x + a)*polylog(5, -cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*cos(b*x
+ a)*polylog(5, -cos(b*x + a) - I*sin(b*x + a)) - 24*I*d^4*cos(b*x + a)*po
lylog(4, I*cos(b*x + a) + sin(b*x + a)) - 24*I*d^4*cos(b*x + a)*polylog(4,
 I*cos(b*x + a) - sin(b*x + a)) + 24*I*d^4*cos(b*x + a)*polylog(4, -I*cos(
b*x + a) + sin(b*x + a)) + 24*I*d^4*cos(b*x + a)*polylog(4, -I*cos(b*x + a
) - sin(b*x + a)) - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2
*x + I*b^3*c^3*d)*cos(b*x + a)*dilog(cos(b*x + a) + I*sin(b*x + a)) - 4*(-
I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*cos(b
*x + a)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 12*(-I*b^2*d^4*x^2 - 2*I*b^
2*c*d^3*x - I*b^2*c^2*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a
)) - 12*(-I*b^2*d^4*x^2 - 2*I*b^2*c*d^3*x - I*b^2*c^2*d^2)*cos(b*x + a)*di
log(I*cos(b*x + a) - sin(b*x + a)) - 12*(I*b^2*d^4*x^2 + 2*I*b^2*c*d^3*x +
 I*b^2*c^2*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 12*(I
*b^2*d^4*x^2 + 2*I*b^2*c*d^3*x + I*b^2*c^2*d^2)*cos(b*x + a)*dilog(-I*cos(
b*x + a) - sin(b*x + a)) - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*
c^2*d^2*x + I*b^3*c^3*d)*cos(b*x + a)*dilog(-cos(b*x + a) + I*sin(b*x + a)
) - 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*b^3*c...
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^4 \csc(a+bx) \sec^2(a+bx)\, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**4*csc(b*x+a)*sec(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5709 vs. $2(403) = 806$.

Time = 2.67 (sec) , antiderivative size = 5709, normalized size of antiderivative = 12.17

$$\int (c+dx)^4 \csc(a+bx) \sec^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

output

```
1/2*(c^4*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
- 4*a*c^3*d*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1
))/b + 6*a^2*c^2*d^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x
 + a) - 1))/b^2 - 4*a^3*c*d^3*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + lo
g(cos(b*x + a) - 1))/b^3 + a^4*d^4*(2/cos(b*x + a) - log(cos(b*x + a) + 1)
 + log(cos(b*x + a) - 1))/b^4 + 2*(8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*
b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*
(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) + (b^3*c^3*d - 3*a*b^2*c^2
*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*
x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x +
2*a) + (I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 + I*(b*x + a)^3*
d^4 - I*a^3*d^4 + 3*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 -
 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x +
 a), sin(b*x + a) + 1) + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 +
(b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d
^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) + (b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a
^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 +
 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (I*
b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 + I*(b*x + a)^3*d^4 - I*a^
3*d^4 + 3*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*...
```

Giac [F]

$$\int (c+dx)^4 \csc(a+bx) \sec^2(a+bx)\,dx = \int (dx+c)^4 \csc\left(bx+a\right) \sec\left(bx+a\right)^2\,dx$$

input

```
integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

output

```
integrate((d*x + c)^4*csc(b*x + a)*sec(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \csc(a+bx) \sec^2(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)^4/(cos(a + b*x)^2*sin(a + b*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^4 \csc(a+bx) \sec^2(a+bx)\,dx$$
$$= \frac{\cos(bx+a)\left(\int \csc(bx+a)\sec(bx+a)^2 x^4 dx\right) b\,d^4 + 4\cos(bx+a)\left(\int \csc(bx+a)\sec(bx+a)^2 x^3 dx\right) b}{}$$

input
```
int((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x)
```

output
```
(cos(a + b*x)*int(csc(a + b*x)*sec(a + b*x)**2*x**4,x)*b*d**4 + 4*cos(a +
b*x)*int(csc(a + b*x)*sec(a + b*x)**2*x**3,x)*b*c*d**3 + 6*cos(a + b*x)*in
t(csc(a + b*x)*sec(a + b*x)**2*x**2,x)*b*c**2*d**2 + 4*cos(a + b*x)*int(cs
c(a + b*x)*sec(a + b*x)**2*x,x)*b*c**3*d + cos(a + b*x)*log(tan((a + b*x)/
2))*c**4 - cos(a + b*x)*c**4 + c**4)/(cos(a + b*x)*b)
```

3.267 $\int (c+dx)^3 \csc(a+bx) \sec^2(a+bx)\, dx$

Optimal result . . . 2337
Mathematica [A] (verified) . . . 2338
Rubi [A] (verified) . . . 2339
Maple [B] (verified) . . . 2341
Fricas [B] (verification not implemented) . . . 2342
Sympy [F] . . . 2343
Maxima [B] (verification not implemented) . . . 2344
Giac [F] . . . 2345
Mupad [F(-1)] . . . 2345
Reduce [F] . . . 2345

Optimal result

Integrand size = 22, antiderivative size = 343

$$
\begin{aligned}
&\int (c+dx)^3 \csc(a+bx) \sec^2(a+bx)\, dx \\
&= \frac{6id(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^3 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} \\
&\quad + \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^2} - \frac{6id^2(c+dx) \operatorname{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{6id^2(c+dx) \operatorname{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3} - \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{6d^2(c+dx) \operatorname{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^3} + \frac{6d^3 \operatorname{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^4} \\
&\quad - \frac{6d^3 \operatorname{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^4} + \frac{6d^2(c+dx) \operatorname{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3} \\
&\quad - \frac{6id^3 \operatorname{PolyLog}\left(4, -e^{i(a+bx)}\right)}{b^4} + \frac{6id^3 \operatorname{PolyLog}\left(4, e^{i(a+bx)}\right)}{b^4} + \frac{(c+dx)^3 \sec(a+bx)}{b}
\end{aligned}
$$

output
```
6*I*d*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b^2-2*(d*x+c)^3*arctanh(exp(I*(b*x+
a)))/b+3*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^2-6*I*d^2*(d*x+c)*poly
log(2,-I*exp(I*(b*x+a)))/b^3+6*I*d^2*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b
^3-3*I*d*(d*x+c)^2*polylog(2,exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)*polylog(3,-
exp(I*(b*x+a)))/b^3+6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3
,I*exp(I*(b*x+a)))/b^4+6*d^2*(d*x+c)*polylog(3,exp(I*(b*x+a)))/b^3-6*I*d^3
*polylog(4,-exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+(d*x
+c)^3*sec(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.38

$$\int (c+dx)^3 \csc(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{-2b^3(c+dx)^3\operatorname{arctanh}(\cos(a+bx)+i\sin(a+bx)) - 3d\left(-2ib^2c^2\arctan\left(e^{i(a+bx)}\right)+2b^2cdx\log\left(1-ie^{i(}\right.\right.}{}$$

input
```
Integrate[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

output
```
(-2*b^3*(c + d*x)^3*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]] - 3*d*((-2*I)*b
^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] +
b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - 2*b^2*c*d*x*Log[1 + I*E^(I*(a + b
*x))] - b^2*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyL
og[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b
*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 2*d^2*PolyLog[3, I*E^(I*(
a + b*x))]) + (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a
+ b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] -
 2*d^2*PolyLog[4, -Cos[a + b*x] - I*Sin[a + b*x]]) - (3*I)*d*(b^2*(c + d*x
)^2*PolyLog[2, Cos[a + b*x] + I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLo
g[3, Cos[a + b*x] + I*Sin[a + b*x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Si
n[a + b*x]]) + b^3*(c + d*x)^3*Sec[a + b*x])/b^4
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4920, 25, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \csc(a+bx) \sec^2(a+bx)\, dx$$

$$\downarrow 4920$$

$$-3d \int -(c+dx)^2 \left(\frac{\operatorname{arctanh}(\cos(a+bx))}{b} - \frac{\sec(a+bx)}{b} \right) dx - \frac{(c+dx)^3 \operatorname{arctanh}(\cos(a+bx))}{b} + \frac{(c+dx)^3 \sec(a+bx)}{b}$$

$$\downarrow 25$$

$$3d \int (c+dx)^2 \left(\frac{\operatorname{arctanh}(\cos(a+bx))}{b} - \frac{\sec(a+bx)}{b} \right) dx - \frac{(c+dx)^3 \operatorname{arctanh}(\cos(a+bx))}{b} + \frac{(c+dx)^3 \sec(a+bx)}{b}$$

$$\downarrow 7292$$

$$3d \int \frac{(c+dx)^2 (\operatorname{arctanh}(\cos(a+bx)) - \sec(a+bx))}{b} dx - \frac{(c+dx)^3 \operatorname{arctanh}(\cos(a+bx))}{b} + \frac{(c+dx)^3 \sec(a+bx)}{b}$$

$$\downarrow 27$$

$$\frac{3d \int (c+dx)^2 (\operatorname{arctanh}(\cos(a+bx)) - \sec(a+bx)) dx}{b} - \frac{(c+dx)^3 \operatorname{arctanh}(\cos(a+bx))}{b} + \frac{(c+dx)^3 \sec(a+bx)}{b}$$

$$\downarrow 7293$$

$$\frac{3d \int \left((c+dx)^2 \operatorname{arctanh}(\cos(a+bx)) - (c+dx)^2 \sec(a+bx) \right) dx}{b} - \frac{(c+dx)^3 \operatorname{arctanh}(\cos(a+bx))}{b} + \frac{(c+dx)^3 \sec(a+bx)}{b}$$

$$\downarrow 2009$$

$$3d\left(\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{3d}+\frac{(c+dx)^3\text{arctanh}(\cos(a+bx))}{3d}+\frac{2d^2\,\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}-\frac{2d^2\,\text{P}}{\text{---}}\right.$$

$$\frac{(c+dx)^3\text{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^3\sec(a+bx)}{b}$$

input
```
Int[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

output
```
-(((c + d*x)^3*ArcTanh[Cos[a + b*x]])/b) + (3*d*(((2*I)*(c + d*x)^2*ArcTan
[E^(I*(a + b*x))])/b - (2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/(3*d) + ((
c + d*x)^3*ArcTanh[Cos[a + b*x]])/(3*d) + (I*(c + d*x)^2*PolyLog[2, -E^(I*
(a + b*x))])/b - ((2*I)*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2
+ ((2*I)*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (I*(c + d*x)^2*P
olyLog[2, E^(I*(a + b*x))])/b - (2*d*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))
])/b^2 + (2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 - (2*d^2*PolyLog[3,
I*E^(I*(a + b*x))])/b^3 + (2*d*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^2
- ((2*I)*d^2*PolyLog[4, -E^(I*(a + b*x))])/b^3 + ((2*I)*d^2*PolyLog[4, E^(
I*(a + b*x))])/b^3))/b + ((c + d*x)^3*Sec[a + b*x])/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1151 vs. $2(308) = 616$.

Time = 0.67 (sec) , antiderivative size = 1152, normalized size of antiderivative = 3.36

method	result	size
risch	Expression too large to display	1152

input
```
int((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
-3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a
)))*x+3/b^4*a^2*d^3*ln(1-I*exp(I*(b*x+a)))+6/b^2*c*d^2*ln(I*exp(I*(b*x+a))
+1)*x-6/b^2*c*d^2*ln(1-I*exp(I*(b*x+a)))*x+6/b^3*c*d^2*ln(I*exp(I*(b*x+a))
+1)*a-6/b^3*c*d^2*ln(1-I*exp(I*(b*x+a)))*a+3*I/b^2*d^3*polylog(2,-exp(I*(b
*x+a)))*x^2+3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))+6*I/b^2*c^2*d*arctan(
exp(I*(b*x+a)))+6*I/b^4*a*d^3*dilog(I*exp(I*(b*x+a))+1)-6*I/b^4*a*d^3*dilo
g(1-I*exp(I*(b*x+a)))+6*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+a)))*x-12*I/b^3*
a*d^2*c*arctan(exp(I*(b*x+a)))-3/b^4*a^2*d^3*ln(I*exp(I*(b*x+a))+1)+3/b^2*
d^3*ln(I*exp(I*(b*x+a))+1)*x^2-3/b^2*d^3*ln(1-I*exp(I*(b*x+a)))*x^2-1/b^4*
d^3*a^3*ln(exp(I*(b*x+a))-1)-6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+6/b^3*
c*d^2*polylog(3,exp(I*(b*x+a)))+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3-1/b*d^3*l
n(exp(I*(b*x+a))+1)*x^3-6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+6/b^3*d^3*p
olylog(3,exp(I*(b*x+a)))*x+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+2*(d^3*x^3+3
*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))+1)+3/b^3*c*d^
2*a^2*ln(exp(I*(b*x+a))-1)-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)+3/b^2*d*c^2*
ln(1-exp(I*(b*x+a)))*a-3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b*c*d^2*ln(1-e
xp(I*(b*x+a)))*x^2-3/b*d*c^2*ln(exp(I*(b*x+a))+1)*x+3/b*d*c^2*ln(1-exp(I*(
b*x+a)))*x-3*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2-3*I/b^2*d*c^2*polylog
(2,exp(I*(b*x+a)))+6*I/b^4*d^3*polylog(2,I*exp(I*(b*x+a)))*a-6*I/b^4*d^3*p
olylog(2,-I*exp(I*(b*x+a)))*a+6*I/b^4*d^3*a^2*arctan(exp(I*(b*x+a)))-6*...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1705 vs. $2(293) = 586$.

Time $= 0.19$ (sec) , antiderivative size $= 1705$, normalized size of antiderivative $= 4.97$

$$\int (c + dx)^3 \csc(a + bx) \sec^2(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

output

```
1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*I*d^3
*cos(b*x + a)*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*cos(b*x
+ a)*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*cos(b*x + a)*poly
log(4, -cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*cos(b*x + a)*polylog(4, -
cos(b*x + a) - I*sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3, I*cos(b*x +
 a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, I*cos(b*x + a) - sin(b
*x + a)) + 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) -
 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b^2*
d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*cos(b*x + a)*dilog(cos(b*x + a) +
 I*sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*cos(
b*x + a)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 6*(-I*b*d^3*x - I*b*c*d^2)
*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) - 6*(-I*b*d^3*x - I*b*c
*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 6*(I*b*d^3*x + I
*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 6*(I*b*d^3*
x + I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b
^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*cos(b*x + a)*dilog(-cos(b*x +
a) + I*sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*
cos(b*x + a)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^3*d^3*x^3 + 3*b^3*
c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin
(b*x + a) + 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log...
```

Sympy [F]

$$\int (c+dx)^3 \csc(a+bx) \sec^2(a+bx)\,dx = \int (c+dx)^3 \csc\left(a+bx\right) \sec^2\left(a+bx\right)\,dx$$

input

```
integrate((d*x+c)**3*csc(b*x+a)*sec(b*x+a)**2,x)
```

output

```
Integral((c + d*x)**3*csc(a + b*x)*sec(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3202 vs. $2(293) = 586$.

Time = 1.01 (sec) , antiderivative size = 3202, normalized size of antiderivative = 9.34

$$\int (c+dx)^3 \csc(a+bx) \sec^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(c^3*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
- 3*a*c^2*d*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1
))/b + 3*a^2*c*d^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x +
 a) - 1))/b^2 - a^3*d^3*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(
b*x + a) - 1))/b^3 + 2*(6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2
*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) + (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a
)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (I*b
^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 -
I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) +
 1) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2
- a*d^3)*(b*x + a) + (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3
+ 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (I*b^2*c^2*d - 2*I*a*b
*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a)
)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 2*((b*x + a
)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*(b*x + a) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3
*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*
x + a)^3*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*
I*a*b*c*d^2 - I*a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a)
, cos(b*x + a) + 1) - 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a...
```

Giac [F]

$$\int (c+dx)^3 \csc(a+bx) \sec^2(a+bx)\, dx = \int (dx+c)^3 \csc(bx+a) \sec(bx+a)^2\, dx$$

input
```
integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csc(b*x + a)*sec(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc(a+bx) \sec^2(a+bx)\, dx = \text{Hanged}$$

input
```
int((c + d*x)^3/(cos(a + b*x)^2*sin(a + b*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^3 \csc(a+bx) \sec^2(a+bx)\, dx$$
$$= \frac{\cos(bx+a)\left(\int \csc(bx+a)\sec(bx+a)^2 x^3 dx\right) b\, d^3 + 3\cos(bx+a)\left(\int \csc(bx+a)\sec(bx+a)^2 x^2 dx\right) b}{c}$$

input
```
int((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x)
```

output
```
(cos(a + b*x)*int(csc(a + b*x)*sec(a + b*x)**2*x**3,x)*b*d**3 + 3*cos(a +
b*x)*int(csc(a + b*x)*sec(a + b*x)**2*x**2,x)*b*c*d**2 + 3*cos(a + b*x)*in
t(csc(a + b*x)*sec(a + b*x)**2*x,x)*b*c**2*d + cos(a + b*x)*log(tan((a + b
*x)/2))*c**3 - cos(a + b*x)*c**3 + c**3)/(cos(a + b*x)*b)
```

3.268 $\int (c+dx)^2 \csc(a+bx)\sec^2(a+bx)\,dx$

Optimal result . . . 2346
Mathematica [A] (warning: unable to verify) . . . 2347
Rubi [A] (verified) . . . 2347
Maple [B] (verified) . . . 2349
Fricas [B] (verification not implemented) . . . 2350
Sympy [F] . . . 2351
Maxima [B] (verification not implemented) . . . 2352
Giac [F(-2)] . . . 2353
Mupad [F(-1)] . . . 2353
Reduce [F] . . . 2353

Optimal result

Integrand size = 22, antiderivative size = 219

$$\begin{aligned}
&\int (c+dx)^2 \csc(a+bx)\sec^2(a+bx)\,dx \\
&= \frac{4id(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} \\
&\quad + \frac{2id(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} - \frac{2id^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{2id^2\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} - \frac{2id(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{2d^2\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} + \frac{2d^2\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^2\sec(a+bx)}{b}
\end{aligned}$$

output

```
4*I*d*(d*x+c)*arctan(exp(I*(b*x+a)))/b^2-2*(d*x+c)^2*arctanh(exp(I*(b*x+a)
))/b+2*I*d*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^2-2*I*d^2*polylog(2,-I*exp
(I*(b*x+a)))/b^3+2*I*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3-2*I*d*(d*x+c)*pol
ylog(2,exp(I*(b*x+a)))/b^2-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*poly
log(3,exp(I*(b*x+a)))/b^3+(d*x+c)^2*sec(b*x+a)/b
```

Mathematica [A] (warning: unable to verify)

Time = 1.66 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.45

$$\int (c+dx)^2 \csc(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{-4bcd\operatorname{arctanh}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right) - 4d^2\arctan(\cot(a))\operatorname{arctanh}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right) + b^2(c+$$

input
```
Integrate[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

output
```
(-4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - 4*d^2*ArcTan[Cot[a]]*Arc
Tanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + b^2*(c + d*x)^2*Log[1 - E^(I*(a + b*x
))] - b^2*(c + d*x)^2*Log[1 + E^(I*(a + b*x))] + (2*d^2*Csc[a]*((b*x - Arc
Tan[Cot[a]])*(Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x -
ArcTan[Cot[a]]))]) + I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyL
og[2, E^(I*(b*x - ArcTan[Cot[a]]))]))/Sqrt[Csc[a]^2] + (2*I)*d*(b*(c + d*x
)*PolyLog[2, -E^(I*(a + b*x))] + I*d*PolyLog[3, -E^(I*(a + b*x))]) + 2*d*(
(-I)*b*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] + d*PolyLog[3, E^(I*(a + b*x)
)]) + b^2*(c + d*x)^2*Sec[a + b*x])/b^3
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4920, 25, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \csc(a+bx)\sec^2(a+bx)\,dx$$
$$\downarrow 4920$$
$$-2d\int -\left((c+dx)\left(\frac{\operatorname{arctanh}(\cos(a+bx))}{b} - \frac{\sec(a+bx)}{b}\right)\right)dx - \frac{(c+dx)^2\operatorname{arctanh}(\cos(a+bx))}{b} + \frac{(c+dx)^2\sec(a+bx)}{b}$$

$$\downarrow 25$$

$$2d\int (c+dx)\left(\frac{\text{arctanh}(\cos(a+bx))}{b}-\frac{\sec(a+bx)}{b}\right)dx-\frac{(c+dx)^2\text{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

$$\downarrow 7292$$

$$2d\int \frac{(c+dx)(\text{arctanh}(\cos(a+bx))-\sec(a+bx))}{b}dx-\frac{(c+dx)^2\text{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

$$\downarrow 27$$

$$\frac{2d\int (c+dx)(\text{arctanh}(\cos(a+bx))-\sec(a+bx))dx}{b}-\frac{(c+dx)^2\text{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

$$\downarrow 7293$$

$$\frac{2d\int ((c+dx)\text{arctanh}(\cos(a+bx))-(c+dx)\sec(a+bx))dx}{b}-\frac{(c+dx)^2\text{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

$$\downarrow 2009$$

$$\frac{2d\left(\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{d}+\frac{(c+dx)^2\text{arctanh}(\cos(a+bx))}{2d}-\frac{id\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}+\frac{id\,\text{PolyL}}{b}\right.}{\frac{(c+dx)^2\text{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}}$$

input

```
Int[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

output

```
-(((c + d*x)^2*ArcTanh[Cos[a + b*x]])/b) + (2*d*(((2*I)*(c + d*x)*ArcTan[E
^(I*(a + b*x))])/b - ((c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/d + ((c + d*x)
^2*ArcTanh[Cos[a + b*x]])/(2*d) + (I*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))
])/b - (I*d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 + (I*d*PolyLog[2, I*E^(I
*(a + b*x))])/b^2 - (I*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b - (d*PolyL
og[3, -E^(I*(a + b*x))])/b^2 + (d*PolyLog[3, E^(I*(a + b*x))])/b^2))/b + (
(c + d*x)^2*Sec[a + b*x])/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(196) = 392$.

Time = 0.52 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.59

method	result
risch	$\frac{2icd\,\mathrm{polylog}\left(2,-\mathrm{e}^{i(bx+a)}\right)}{b^2}+\frac{2cd\ln\left(1-\mathrm{e}^{i(bx+a)}\right)a}{b^2}-\frac{d^2\ln\left(\mathrm{e}^{i(bx+a)}+1\right)x^2}{b}+\frac{d^2a^2\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b^3}-\frac{d^2\ln\left(1-\mathrm{e}^{i(bx+a)}\right)a^2}{b^3}+$

input
```
int((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b^2*c*d*polylog(2,exp(I*(b*x+
a)))+2/b^2*c*d*ln(1-exp(I*(b*x+a)))*a+2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)
))*x+2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-2*I/b^3*d^2*dilog(I*exp(I*(b*x
+a))+1)+2*I/b^3*d^2*dilog(1-I*exp(I*(b*x+a)))+2/b^2*d^2*ln(I*exp(I*(b*x+a)
)+1)*x+2/b^3*d^2*ln(I*exp(I*(b*x+a))+1)*a-2/b^2*d^2*ln(1-I*exp(I*(b*x+a)))
*x-2/b^3*d^2*ln(1-I*exp(I*(b*x+a)))*a+4*I/b^2*d*c*arctan(exp(I*(b*x+a)))-4
*I/b^3*d^2*a*arctan(exp(I*(b*x+a)))+2*exp(I*(b*x+a))*(d^2*x^2+2*c*d*x+c^2)
/b/(exp(2*I*(b*x+a))+1)+1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)-1/b*d^2*ln(exp(
I*(b*x+a))+1)*x^2+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d^2*ln(1-exp(I*(b
*x+a)))*a^2-2/b*c*d*ln(exp(I*(b*x+a))+1)*x-2/b^2*c*d*a*ln(exp(I*(b*x+a))-1
)+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+1/b*c^2*ln(exp(I*(b*x+a))-1)-1/b*c^2*ln(e
xp(I*(b*x+a))+1)-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(
I*(b*x+a)))/b^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1035 vs. $2(185) = 370$.

Time $= 0.14$ (sec) , antiderivative size $= 1035$, normalized size of antiderivative $= 4.73$

$$\int (c+dx)^2 \csc(a+bx) \sec^2(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

output

```
1/2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 2*I*d^2*cos(b*x + a)*dilog(
I*cos(b*x + a) + sin(b*x + a)) + 2*I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a)
 - sin(b*x + a)) - 2*I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x +
a)) - 2*I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*c
os(b*x + a)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*cos(b*x + a)
*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 2*d^2*cos(b*x + a)*polylog(3,
 -cos(b*x + a) + I*sin(b*x + a)) - 2*d^2*cos(b*x + a)*polylog(3, -cos(b*x
+ a) - I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*cos(b*x + a)*dilog(cos(b*
x + a) + I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*cos(b*x + a)*dilog(cos
(b*x + a) - I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*cos(b*x + a)*dilog(-
cos(b*x + a) + I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*cos(b*x + a)*dil
og(-cos(b*x + a) - I*sin(b*x + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b*c*d - a*d^2)*c
os(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*
c*d*x + b^2*c^2)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 2*(
b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + I) - 2*(b*
d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b*
d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 2*(b*
d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b
*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + ...
```

Sympy [F]

$$\int (c+dx)^2 \csc(a+bx) \sec^2(a+bx)\,dx = \int (c+dx)^2 \csc(a+bx) \sec^2(a+bx)\,dx$$

input

```
integrate((d*x+c)**2*csc(b*x+a)*sec(b*x+a)**2,x)
```

output

```
Integral((c + d*x)**2*csc(a + b*x)*sec(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1590 vs. $2(185) = 370$.

Time = 0.53 (sec) , antiderivative size = 1590, normalized size of antiderivative = 7.26

$$\int (c+dx)^2 \csc(a+bx) \sec^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(c^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
- 2*a*c*d*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b + a^2*d^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) -
1))/b^2 + 2*(4*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a
*d^2)*cos(2*b*x + 2*a) + (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(2*b*x +
 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + 4*(b*c*d + (b*x + a)*d^2
- a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (I*b*c*d + I*
(b*x + a)*d^2 - I*a*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x
+ a) + 1) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + ((b*x + a)^
2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^
2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x +
a), cos(b*x + a) + 1) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) +
 ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b
*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan
2(sin(b*x + a), -cos(b*x + a) + 1) + 4*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d +
 I*a*d^2)*(b*x + a))*cos(b*x + a) + 4*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*
b*x + 2*a) + d^2)*dilog(I*e^(I*b*x + I*a)) - 4*(d^2*cos(2*b*x + 2*a) + I*d
^2*sin(2*b*x + 2*a) + d^2)*dilog(-I*e^(I*b*x + I*a)) + 4*(b*c*d + (b*x + a
)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (I*b*c*
d + I*(b*x + a)*d^2 - I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)...
```

Giac [F(-2)]

Exception generated.

$$\int (c+dx)^2 \csc(a+bx) \sec^2(a+bx)\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc(a+bx) \sec^2(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)^2/(cos(a + b*x)^2*sin(a + b*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^2 \csc(a+bx) \sec^2(a+bx)\,dx$$
$$= \frac{\cos(bx+a)\left(\int \csc(bx+a)\sec(bx+a)^2 x^2 dx\right) b\,d^2 + 2\cos(bx+a)\left(\int \csc(bx+a)\sec(bx+a)^2 x dx\right) bc}{\cos(bx+a)\,b}$$

input
```
int((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x)
```

output
```
(cos(a + b*x)*int(csc(a + b*x)*sec(a + b*x)**2*x**2,x)*b*d**2 + 2*cos(a +
b*x)*int(csc(a + b*x)*sec(a + b*x)**2*x,x)*b*c*d + cos(a + b*x)*log(tan((a
 + b*x)/2))*c**2 - cos(a + b*x)*c**2 + c**2)/(cos(a + b*x)*b)
```

3.269 $\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx$

Optimal result . . . 2354
Mathematica [A] (verified) . . . 2355
Rubi [A] (verified) . . . 2355
Maple [A] (verified) . . . 2357
Fricas [B] (verification not implemented) . . . 2357
Sympy [F] . . . 2358
Maxima [B] (verification not implemented) . . . 2358
Giac [F] . . . 2359
Mupad [F(-1)] . . . 2360
Reduce [F] . . . 2360

Optimal result

Integrand size = 20, antiderivative size = 113

$$\begin{aligned}\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx = &-\frac{2dx\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}\\ &-\frac{c\operatorname{arctanh}(\cos(a+bx))}{b}\\ &-\frac{d\operatorname{arctanh}(\sin(a+bx))}{b^2}\\ &+\frac{id\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}\\ &-\frac{id\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}\\ &+\frac{c\sec(a+bx)}{b}+\frac{dx\sec(a+bx)}{b}\end{aligned}$$

output

```
-2*d*x*arctanh(exp(I*(b*x+a)))/b-c*arctanh(cos(b*x+a))/b-d*arctanh(sin(b*x
+a))/b^2+I*d*polylog(2,-exp(I*(b*x+a)))/b^2-I*d*polylog(2,exp(I*(b*x+a)))/
b^2+c*sec(b*x+a)/b+d*x*sec(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.88

$$\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx = -\frac{c\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{d\log\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} + \frac{c\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b} - \frac{d\log\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} - \frac{ad\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} + \frac{d\left((a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)+i\left(\text{PolyLog}\left(2,-e^{i(a+bx)}\right)-\text{PolyLog}\left(2,e^{i(a+bx)}\right)\right)\right)}{b^2} + \frac{c\sec(a+bx)}{b} + \frac{dx\sec(a+bx)}{b}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

output
```
-((c*Log[Cos[(a + b*x)/2]])/b) + (d*Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2
]])/b^2 + (c*Log[Sin[(a + b*x)/2]])/b - (d*Log[Cos[(a + b*x)/2] + Sin[(a +
 b*x)/2]])/b^2 - (a*d*Log[Tan[(a + b*x)/2]])/b^2 + (d*((a + b*x)*(Log[1 -
E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*(PolyLog[2, -E^(I*(a + b*
x))] - PolyLog[2, E^(I*(a + b*x))])))/b^2 + (c*Sec[a + b*x])/b + (d*x*Sec[
a + b*x])/b
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4920, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx$$

$$\downarrow \text{4920}$$

$$-d\int\left(\frac{\sec(a+bx)}{b}-\frac{\operatorname{arctanh}(\cos(a+bx))}{b}\right)dx-\frac{(c+dx)\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)\sec(a+bx)}{b}$$

$$\downarrow 2009$$

$$-d\left(\frac{\operatorname{arctanh}(\sin(a+bx))}{b^2}+\frac{2x\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{x\operatorname{arctanh}(\cos(a+bx))}{b}-\frac{i\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}+\frac{i\operatorname{Polyl}}{}\right.$$

$$\left.\frac{(c+dx)\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)\sec(a+bx)}{b}\right)$$

input
```
Int[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

output
```
-(((c + d*x)*ArcTanh[Cos[a + b*x]])/b) - d*((2*x*ArcTanh[E^(I*(a + b*x))])
/b - (x*ArcTanh[Cos[a + b*x]])/b + ArcTanh[Sin[a + b*x]]/b^2 - (I*PolyLog[
2, -E^(I*(a + b*x))])/b^2 + (I*PolyLog[2, E^(I*(a + b*x))])/b^2) + ((c + d
*x)*Sec[a + b*x])/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.42

method	result
risch	$\frac{2\,e^{i(bx+a)}(dx+c)}{b\left(e^{2i(bx+a)}+1\right)}+\frac{c\ln\left(e^{i(bx+a)}-1\right)}{b}-\frac{c\ln\left(e^{i(bx+a)}+1\right)}{b}-\frac{da\ln\left(e^{i(bx+a)}-1\right)}{b^2}+\frac{2id\arctan\left(e^{i(bx+a)}\right)}{b^2}+\frac{id\,\mathrm{dilog}\left(e^{i(bx+a)}\right.}{b^2}$

input
```
int((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*exp(I*(b*x+a))*(d*x+c)/b/(exp(2*I*(b*x+a))+1)+1/b*c*ln(exp(I*(b*x+a))-1)
-1/b*c*ln(exp(I*(b*x+a))+1)-1/b^2*d*a*ln(exp(I*(b*x+a))-1)+2*I/b^2*d*arcta
n(exp(I*(b*x+a)))+I/b^2*d*dilog(exp(I*(b*x+a))+1)+I/b^2*d*dilog(exp(I*(b*x
+a)))-1/b*d*ln(exp(I*(b*x+a))+1)*x
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(101) = 202$.

Time = 0.10 (sec) , antiderivative size = 366, normalized size of antiderivative = 3.24

$$\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{2\,bdx - i\,d\cos(bx+a)\,\mathrm{Li}_2(\cos(bx+a)+i\,\sin(bx+a)) + i\,d\cos(bx+a)\,\mathrm{Li}_2(\cos(bx+a)-i\,\sin(bx+}{}$$

input
```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(2*b*d*x - I*d*cos(b*x + a)*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d
*cos(b*x + a)*dilog(cos(b*x + a) - I*sin(b*x + a)) - I*d*cos(b*x + a)*dilo
g(-cos(b*x + a) + I*sin(b*x + a)) + I*d*cos(b*x + a)*dilog(-cos(b*x + a) -
 I*sin(b*x + a)) - (b*d*x + b*c)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x
 + a) + 1) - (b*d*x + b*c)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a)
+ 1) + (b*c - a*d)*cos(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a)
 + 1/2) + (b*c - a*d)*cos(b*x + a)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x +
 a) + 1/2) + (b*d*x + a*d)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a)
 + 1) + (b*d*x + a*d)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)
 - d*cos(b*x + a)*log(sin(b*x + a) + 1) + d*cos(b*x + a)*log(-sin(b*x + a)
 + 1) + 2*b*c)/(b^2*cos(b*x + a))
```

Sympy [F]

$$\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx = \int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*csc(a + b*x)*sec(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 800 vs. $2(101) = 202$.

Time $= 0.53$ (sec) , antiderivative size $= 800$, normalized size of antiderivative $= 7.08$

$$\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

output

```
-(2*(d*cos(2*b*x + 2*a) + I*d*sin(2*b*x + 2*a) + d)*arctan2(2*(cos(b*x + 2
*a)*cos(a) + sin(b*x + 2*a)*sin(a))/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a
)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2),
 (cos(b*x + 2*a)^2 - cos(a)^2 + sin(b*x + 2*a)^2 - sin(a)^2)/(cos(b*x + 2*
a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x +
 2*a)*sin(a) + sin(a)^2)) + 2*(b*d*x + b*c + (b*d*x + b*c)*cos(2*b*x + 2*a
) + (I*b*d*x + I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a)
 + 1) - 2*(b*c*cos(2*b*x + 2*a) + I*b*c*sin(2*b*x + 2*a) + b*c)*arctan2(si
n(b*x + a), cos(b*x + a) - 1) + 2*(b*d*x*cos(2*b*x + 2*a) + I*b*d*x*sin(2*
b*x + 2*a) + b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 4*(I*b*d*x
+ I*b*c)*cos(b*x + a) - 2*(d*cos(2*b*x + 2*a) + I*d*sin(2*b*x + 2*a) + d)*
dilog(-e^(I*b*x + I*a)) + 2*(d*cos(2*b*x + 2*a) + I*d*sin(2*b*x + 2*a) + d
)*dilog(e^(I*b*x + I*a)) - (I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(2*b*x
+ 2*a) - (b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)
^2 + 2*cos(b*x + a) + 1) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(2*b*
x + 2*a) + (b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x +
a)^2 - 2*cos(b*x + a) + 1) - (-I*d*cos(2*b*x + 2*a) + d*sin(2*b*x + 2*a) -
 I*d)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x
 + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)
^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(...
```

Giac [F]

$$\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)\csc\left(bx+a\right)\sec\left(bx+a\right)^2\,dx$$

input

```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

output

```
integrate((d*x + c)*csc(b*x + a)*sec(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)/(cos(a + b*x)^2*sin(a + b*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{\cos(bx+a)\left(\int \csc(bx+a)\sec(bx+a)^2\,xdx\right)bd + \cos(bx+a)\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)c - \cos(bx+a)\,c + c}{\cos(bx+a)\,b}$$

input
```
int((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x)
```

output
```
(cos(a + b*x)*int(csc(a + b*x)*sec(a + b*x)**2*x,x)*b*d + cos(a + b*x)*log
(tan((a + b*x)/2))*c - cos(a + b*x)*c + c)/(cos(a + b*x)*b)
```

3.270 $\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$

Optimal result 2361
Mathematica [N/A] 2361
Rubi [N/A] 2362
Maple [N/A] 2362
Fricas [N/A] 2363
Sympy [N/A] 2363
Maxima [N/A] 2364
Giac [N/A] 2364
Mupad [N/A] 2365
Reduce [N/A] 2365

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}, x\right)$$

output
```
Defer(Int)(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 16.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x),x]
```

output
```
Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$
$$\downarrow 7299$$
$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}dx$$

input
```
Int[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc(bx+a)\sec(bx+a)^2}{dx+c}dx$$

input
```
int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x)
```

output
```
int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc\left(bx+a\right)\sec\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)*sec(b*x + a)^2/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc\left(a+bx\right)\sec^2\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(csc(a + b*x)*sec(a + b*x)**2/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 603, normalized size of antiderivative = 27.41

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
(2*cos(2*b*x + 2*a)*cos(b*x + a) + 2*(b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*
cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x + b
*c*d)*cos(2*b*x + 2*a))*integrate((cos(2*b*x + 2*a)*cos(b*x + a) + sin(2*b
*x + 2*a)*sin(b*x + a) + cos(b*x + a))/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b
*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x
+ b*c^2)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x
+ 2*a)), x) + (b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*si
n(2*b*x + 2*a)^2 + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a))*integrate(sin(b
*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*
x + c)*cos(b*x + a) + c), x) + (b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 +
 (b*d*x + b*c)*sin(2*b*x + 2*a)^2 + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a)
)*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a
)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) + 2*sin(2*b*x + 2*a)*sin(b*x
 + a) + 2*cos(b*x + a))/(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x
 + b*c)*sin(2*b*x + 2*a)^2 + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a))
```

Giac [N/A]

Not integrable

Time = 5.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)*sec(b*x + a)^2/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 19.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos(a+b\,x)^2\,\sin(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(1/(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)^2}{dx+c}dx$$

input
```
int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x)
```

output
```
int((csc(a + b*x)*sec(a + b*x)**2)/(c + d*x),x)
```

3.271 $\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 2366
Mathematica [N/A] . . . 2366
Rubi [N/A] . . . 2367
Maple [N/A] . . . 2367
Fricas [N/A] . . . 2368
Sympy [N/A] . . . 2368
Maxima [N/A] . . . 2369
Giac [F(-2)] . . . 2370
Mupad [N/A] . . . 2370
Reduce [N/A] . . . 2370

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 16.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$
$$\downarrow 7299$$
$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}dx$$

input
```
Int[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc\left(bx+a\right)\sec\left(bx+a\right)^2}{\left(dx+c\right)^2}dx$$

input
```
int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)\sec\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)*sec(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.97 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(a+bx\right)\sec^2\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)*sec(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 931, normalized size of antiderivative = 42.32

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)\sec\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input

```
integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output

```
(2*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d +
(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a)^2 + (b*d^3*x^2 + 2*b*
c*d^2*x + b*c^2*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2
*d)*cos(2*b*x + 2*a))*integrate((cos(2*b*x + 2*a)*cos(b*x + a) + sin(2*b*x
 + 2*a)*sin(b*x + a) + cos(b*x + a))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*
d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(2*b*x
+ 2*a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(2*b*x + 2
*a)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(2*b*x + 2*
a)), x) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)
*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a)^2 +
 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a
)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2
 + 2*c*d*x + c^2)*sin(b*x + a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b
*x + a)), x) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b
*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a
)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a))*integrate(sin(b*
x + a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^
2*x^2 + 2*c*d*x + c^2)*sin(b*x + a)^2 + c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*
cos(b*x + a)), x) + 2*sin(2*b*x + 2*a)*sin(b*x + a) + 2*cos(b*x + a))/(b*d
^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x ...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

Mupad [N/A]

Not integrable

Time = 19.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\cos(a+b\,x)^2\,\sin(a+b\,x)\,(c+d\,x)^2}\,dx$$

input
```
int(1/(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)^2}{d^2x^2+2cdx+c^2}dx$$

input
```
int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int((csc(a + b*x)*sec(a + b*x)**2)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.272 $\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx$

Optimal result . . . 2371
Mathematica [N/A] . . . 2371
Rubi [N/A] . . . 2372
Maple [N/A] . . . 2372
Fricas [N/A] . . . 2373
Sympy [F(-1)] . . . 2373
Maxima [N/A] . . . 2373
Giac [N/A] . . . 2374
Mupad [N/A] . . . 2374
Reduce [N/A] . . . 2375

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx = \text{Int}\left((c+dx)^m \csc^2(a+bx)\sec^2(a+bx), x\right)$$

output
```
Defer(Int)((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx = \int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\sec^2(a+bx)(c+dx)^m\,dx$$

$$\downarrow 7299$$

$$\int \csc^2(a+bx)\sec^2(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^2\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*csc(b*x+a)**2*sec(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 18.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\cos(a+b\,x)^2 \sin(a+b\,x)^2}\,dx$$

input
```
int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)^2),x)
```

output
```
int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^2\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

output
```
int((c + d*x)**m*csc(a + b*x)**2*sec(a + b*x)**2,x)
```

3.273 $\int (c+dx)^3 \csc^2(a+bx)\sec^2(a+bx)\,dx$

Optimal result . . . 2376
Mathematica [B] (verified) . . . 2377
Rubi [A] (verified) . . . 2377
Maple [B] (verified) . . . 2381
Fricas [B] (verification not implemented) . . . 2381
Sympy [F(-1)] . . . 2382
Maxima [B] (verification not implemented) . . . 2383
Giac [F] . . . 2384
Mupad [F(-1)] . . . 2384
Reduce [F] . . . 2384

Optimal result

Integrand size = 24, antiderivative size = 118

$$\int (c+dx)^3 \csc^2(a+bx)\sec^2(a+bx)\,dx = -\frac{2i(c+dx)^3}{b} - \frac{2(c+dx)^3\cot(2a+2bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{4i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\,\mathrm{PolyLog}\left(2,e^{4i(a+bx)}\right)}{2b^3} + \frac{3d^3\,\mathrm{PolyLog}\left(3,e^{4i(a+bx)}\right)}{8b^4}$$

output

```
-2*I*(d*x+c)^3/b-2*(d*x+c)^3*cot(2*b*x+2*a)/b+3*d*(d*x+c)^2*ln(1-exp(4*I*(
b*x+a)))/b^2-3/2*I*d^2*(d*x+c)*polylog(2,exp(4*I*(b*x+a)))/b^3+3/8*d^3*pol
ylog(3,exp(4*I*(b*x+a)))/b^4
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 285 vs. $2(118) = 236$.

Time = 1.38 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.42

$$\int (c+dx)^3 \csc^2(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{-\frac{8ib^3(c+dx)^3}{-1+e^{4ia}} + 6b^2d(c+dx)^2\log\left(1-e^{-i(a+bx)}\right) + 6b^2d(c+dx)^2\log\left(1+e^{-i(a+bx)}\right) + 6b^2d(c+dx)^2\log\left(1}$$

input
```
Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

output
```
(((-8*I)*b^3*(c + d*x)^3)/(-1 + E^((4*I)*a)) + 6*b^2*d*(c + d*x)^2*Log[1 -
 E^((-I)*(a + b*x))] + 6*b^2*d*(c + d*x)^2*Log[1 + E^((-I)*(a + b*x))] + 6
*b^2*d*(c + d*x)^2*Log[1 + E^((-2*I)*(a + b*x))] + (12*I)*b*d^2*(c + d*x)*
PolyLog[2, -E^((-I)*(a + b*x))] + (12*I)*b*d^2*(c + d*x)*PolyLog[2, E^((-I
)*(a + b*x))] + (6*I)*b*d^2*(c + d*x)*PolyLog[2, -E^((-2*I)*(a + b*x))] +
12*d^3*PolyLog[3, -E^((-I)*(a + b*x))] + 12*d^3*PolyLog[3, E^((-I)*(a + b*
x))] + 3*d^3*PolyLog[3, -E^((-2*I)*(a + b*x))] + 4*b^3*(c + d*x)^3*Csc[2*a
]*Csc[2*(a + b*x)]*Sin[2*b*x])/(2*b^4)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.36, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {4919, 3042, 4672, 3042, 25, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \csc^2(a+bx)\sec^2(a+bx)\,dx$$
$$\downarrow 4919$$
$$4\int (c+dx)^3 \csc^2(2a+2bx)dx$$

$$\downarrow 3042$$

$$4\int (c+dx)^3\csc(2a+2bx)^2dx$$

$$\downarrow 4672$$

$$4\left(\frac{3d\int (c+dx)^2\cot(2a+2bx)dx}{2b}-\frac{(c+dx)^3\cot(2a+2bx)}{2b}\right)$$

$$\downarrow 3042$$

$$4\left(\frac{3d\int -(c+dx)^2\tan\left(2a+2bx+\frac{\pi}{2}\right)dx}{2b}-\frac{(c+dx)^3\cot(2a+2bx)}{2b}\right)$$

$$\downarrow 25$$

$$4\left(-\frac{3d\int (c+dx)^2\tan\left(\frac{1}{2}(4a+\pi)+2bx\right)dx}{2b}-\frac{(c+dx)^3\cot(2a+2bx)}{2b}\right)$$

$$\downarrow 4202$$

$$4\left(-\frac{(c+dx)^3\cot(2a+2bx)}{2b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\int\frac{e^{i(4a+4bx+\pi)}(c+dx)^2}{1+e^{i(4a+4bx+\pi)}}dx\right)}{2b}\right)$$

$$\downarrow 2620$$

$$4\left(-\frac{(c+dx)^3\cot(2a+2bx)}{2b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\int (c+dx)\log\left(1+e^{i(4a+4bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)^2\log\left(1+e^{i(4a+4bx+\pi)}\right)}{4b}\right)\right)}{2b}\right)$$

$$\downarrow 3011$$

$$4\left(-\frac{(c+dx)^3\cot(2a+2bx)}{2b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(4a+4bx+\pi)}\right)}{4b}-\frac{id\int\operatorname{PolyLog}\left(2,-e^{i(4a+4bx+\pi)}\right)dx}{4b}\right)}{2b}-\right.\right.\right.$$

$$\downarrow 2720$$

$$4\left(-\frac{(c+dx)^3\cot(2a+2bx)}{2b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{i(4a+4bx+\pi)}\right)}{4b}-\frac{d\int e^{-i(4a+4bx+\pi)}\,\text{PolyLog}\left(2,-e^{i(4a+4b\cdots}\right.}{16b^2}\right.}{2b}\right.\right.}{2b}\right.$$

$\downarrow$ 7143

$$4\left(-\frac{(c+dx)^3\cot(2a+2bx)}{2b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{i(4a+4bx+\pi)}\right)}{4b}-\frac{d\,\text{PolyLog}\left(3,-e^{i(4a+4bx+\pi)}\right)}{16b^2}\right)}{2b}-\frac{i(c+\cdots}{\cdots}\right.\right.}{2b}\right.$$

input
```
Int[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

output
```
4*(-1/2*((c + d*x)^3*Cot[2*a + 2*b*x])/b - (3*d*(((I/3)*(c + d*x)^3)/d - (
2*I)*(((-1/4*I)*(c + d*x)^2*Log[1 + E^(I*(4*a + Pi + 4*b*x))])/b + ((I/2)*
d*(((I/4)*(c + d*x)*PolyLog[2, -E^(I*(4*a + Pi + 4*b*x))])/b - (d*PolyLog[
3, -E^(I*(4*a + Pi + 4*b*x))])/(16*b^2)))/b)))/(2*b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4919

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(106) = 212$.

Time = 0.67 (sec) , antiderivative size = 687, normalized size of antiderivative = 5.82

method	result
risch	$-\frac{12id^2cx^2}{b}+\frac{6d^2c\ln\left(e^{i(bx+a)}+1\right)x}{b^2}+\frac{6d^2c\ln\left(1-e^{i(bx+a)}\right)x}{b^2}-\frac{4i\left(d^3x^3+3cd^2x^2+3c^2dx+c^3\right)}{b\left(e^{2i(bx+a)}+1\right)\left(e^{2i(bx+a)}-1\right)}-\frac{6d^2ca\ln\left(e^{i(bx+a)}-1\right)}{b^3}+$

input
```
int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
6*d^2/b^3*c*ln(1-exp(I*(b*x+a)))*a-6*I*d^2/b^3*c*polylog(2,exp(I*(b*x+a)))
-6*I*d^3/b^3*polylog(2,exp(I*(b*x+a)))*x-4*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*
x+c^3)/b/(exp(2*I*(b*x+a))+1)/(exp(2*I*(b*x+a))-1)-3*d^3/b^4*ln(1-exp(I*(b
*x+a)))*a^2+3*d^3/b^2*ln(1-exp(I*(b*x+a)))*x^2+3*d^3/b^2*ln(exp(I*(b*x+a))
+1)*x^2-6*d^2/b^3*c*a*ln(exp(I*(b*x+a))-1)+24*d^2/b^3*c*a*ln(exp(I*(b*x+a)
))+6*d^2/b^2*c*ln(exp(2*I*(b*x+a))+1)*x+12*I*d^3/b^3*a^2*x-3*I*d^3/b^3*pol
ylog(2,-exp(2*I*(b*x+a)))*x-12*I*d^2/b*c*x^2-12*I*d^2/b^3*c*a^2-3*I*d^2/b^
3*c*polylog(2,-exp(2*I*(b*x+a)))-6*I*d^2/b^3*c*polylog(2,-exp(I*(b*x+a)))-
6*I*d^3/b^3*polylog(2,-exp(I*(b*x+a)))*x-24*I*d^2/b^2*c*a*x+3*d^3/b^4*a^2*
ln(exp(I*(b*x+a))-1)-12*d^3/b^4*a^2*ln(exp(I*(b*x+a)))+3*d/b^2*c^2*ln(exp(
2*I*(b*x+a))+1)+3*d/b^2*c^2*ln(exp(I*(b*x+a))-1)-12*d/b^2*c^2*ln(exp(I*(b*
x+a)))+3*d/b^2*c^2*ln(exp(I*(b*x+a))+1)+3*d^3/b^2*ln(exp(2*I*(b*x+a))+1)*x
^2-4*I*d^3/b*x^3+8*I*d^3/b^4*a^3+6*d^2/b^2*c*ln(exp(I*(b*x+a))+1)*x+6*d^2/
b^2*c*ln(1-exp(I*(b*x+a)))*x+3/2*d^3*polylog(3,-exp(2*I*(b*x+a)))/b^4+6*d^
3*polylog(3,-exp(I*(b*x+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1635 vs. $2(103) = 206$.

Time = 0.17 (sec) , antiderivative size = 1635, normalized size of antiderivative = 13.86

$$\int (c+dx)^3 \csc^2(a+bx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*c
os(b*x + a)*polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*d^3
*cos(b*x + a)*polylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*d
^3*cos(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + 6
*d^3*cos(b*x + a)*polylog(3, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) +
 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a
) + 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x + a) - sin(b*x + a))*sin(b*x
+ a) + 6*d^3*cos(b*x + a)*polylog(3, -cos(b*x + a) + I*sin(b*x + a))*sin(b
*x + a) + 6*d^3*cos(b*x + a)*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*si
n(b*x + a) - 6*(I*b*d^3*x + I*b*c*d^2)*cos(b*x + a)*dilog(cos(b*x + a) + I
*sin(b*x + a))*sin(b*x + a) - 6*(-I*b*d^3*x - I*b*c*d^2)*cos(b*x + a)*dilo
g(cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - 6*(-I*b*d^3*x - I*b*c*d^2)
*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 6*(I*b*d
^3*x + I*b*c*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*
x + a) - 6*(I*b*d^3*x + I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + si
n(b*x + a))*sin(b*x + a) - 6*(-I*b*d^3*x - I*b*c*d^2)*cos(b*x + a)*dilog(-
I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - 6*(-I*b*d^3*x - I*b*c*d^2)*c
os(b*x + a)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 6*(I*b*d^
3*x + I*b*c*d^2)*cos(b*x + a)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*
x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*log(c...
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^2(a+bx) \sec^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3*csc(b*x+a)**2*sec(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2360 vs. $2(103) = 206$.

Time = 0.50 (sec) , antiderivative size = 2360, normalized size of antiderivative = 20.00

$$\int (c+dx)^3 \csc^2(a+bx) \sec^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*c^3*(1/tan(b*x + a) - tan(b*x + a)) - 6*a*c^2*d*(1/tan(b*x + a) -
tan(b*x + a))/b + 6*a^2*c*d^2*(1/tan(b*x + a) - tan(b*x + a))/b^2 - 2*a^3*
d^3*(1/tan(b*x + a) - tan(b*x + a))/b^3 - 3*((cos(4*b*x + 4*a)^2 + sin(4*b
*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x +
 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)
^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b
*x + a) + 1) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*
a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 8*(b*x
 + a)*sin(4*b*x + 4*a))*c^2*d/((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 -
2*cos(4*b*x + 4*a) + 1)*b) + 6*((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 -
 2*cos(4*b*x + 4*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*c
os(2*b*x + 2*a) + 1) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*
b*x + 4*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
+ (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(c
os(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 8*(b*x + a)*sin(4*b
*x + 4*a))*a*c*d^2/((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x
 + 4*a) + 1)*b^2) - 3*((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*
b*x + 4*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
+ 2*a) + 1) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a
) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos...
```

Giac [F]

$$\int (c+dx)^3 \csc^2(a+bx) \sec^2(a+bx)\, dx = \int (dx+c)^3 \csc(bx+a)^2 \sec(bx+a)^2\, dx$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^2(a+bx) \sec^2(a+bx)\, dx = \int \frac{(c+d\,x)^3}{\cos(a+b\,x)^2 \sin(a+b\,x)^2}\, dx$$

input
```
int((c + d*x)^3/(cos(a + b*x)^2*sin(a + b*x)^2),x)
```

output
```
int((c + d*x)^3/(cos(a + b*x)^2*sin(a + b*x)^2), x)
```

Reduce [F]

$$\int (c+dx)^3 \csc^2(a+bx) \sec^2(a+bx)\, dx = \text{Too large to display}$$

input
```
int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

output

```
(112*cos(a + b*x)*int(x**3/(tan((a + b*x)/2)**6 - 2*tan((a + b*x)/2)**4 +
tan((a + b*x)/2)**2),x)*sin(a + b*x)*b**4*d**3 - 224*cos(a + b*x)*int(x**3
/(tan((a + b*x)/2)**4 - 2*tan((a + b*x)/2)**2 + 1),x)*sin(a + b*x)*b**4*d*
*3 + 336*cos(a + b*x)*int(x**2/(tan((a + b*x)/2)**6 - 2*tan((a + b*x)/2)**
4 + tan((a + b*x)/2)**2),x)*sin(a + b*x)*b**4*c*d**2 - 552*cos(a + b*x)*in
t(x**2/(tan((a + b*x)/2)**5 - 2*tan((a + b*x)/2)**3 + tan((a + b*x)/2)),x)
*sin(a + b*x)*b**3*d**3 - 672*cos(a + b*x)*int(x**2/(tan((a + b*x)/2)**4 -
 2*tan((a + b*x)/2)**2 + 1),x)*sin(a + b*x)*b**4*c*d**2 - 120*cos(a + b*x)
*int((tan((a + b*x)/2)**5*x**2)/(tan((a + b*x)/2)**4 - 2*tan((a + b*x)/2)*
*2 + 1),x)*sin(a + b*x)*b**3*d**3 - 240*cos(a + b*x)*int((tan((a + b*x)/2)
**5*x)/(tan((a + b*x)/2)**4 - 2*tan((a + b*x)/2)**2 + 1),x)*sin(a + b*x)*b
**3*c*d**2 - 336*cos(a + b*x)*int(x/(tan((a + b*x)/2)**6 - 2*tan((a + b*x)
/2)**4 + tan((a + b*x)/2)**2),x)*sin(a + b*x)*b**2*d**3 - 1104*cos(a + b*x
)*int(x/(tan((a + b*x)/2)**5 - 2*tan((a + b*x)/2)**3 + tan((a + b*x)/2)),x
)*sin(a + b*x)*b**3*c*d**2 - 480*cos(a + b*x)*log(tan((a + b*x)/2)**2 + 1)
*sin(a + b*x)*b**2*c**2*d - 756*cos(a + b*x)*log(tan((a + b*x)/2)**2 + 1)*
sin(a + b*x)*d**3 + 240*cos(a + b*x)*log(tan((a + b*x)/2) - 1)*sin(a + b*x
)*b**2*c**2*d + 336*cos(a + b*x)*log(tan((a + b*x)/2) - 1)*sin(a + b*x)*b*
c*d**2 + 84*cos(a + b*x)*log(tan((a + b*x)/2) - 1)*sin(a + b*x)*d**3 + 240
*cos(a + b*x)*log(tan((a + b*x)/2) + 1)*sin(a + b*x)*b**2*c**2*d - 336*...
```

3.274 $\int (c+dx)^2 \csc^2(a+bx)\sec^2(a+bx)\,dx$

Optimal result . . . 2386
Mathematica [B] (verified) . . . 2386
Rubi [A] (verified) . . . 2387
Maple [B] (verified) . . . 2390
Fricas [B] (verification not implemented) . . . 2390
Sympy [F] . . . 2391
Maxima [B] (verification not implemented) . . . 2392
Giac [F] . . . 2393
Mupad [F(-1)] . . . 2393
Reduce [F] . . . 2393

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int (c+dx)^2 \csc^2(a+bx)\sec^2(a+bx)\,dx = -\frac{2i(c+dx)^2}{b} - \frac{2(c+dx)^2\cot(2a+2bx)}{b} + \frac{2d(c+dx)\log\left(1-e^{4i(a+bx)}\right)}{b^2} - \frac{id^2\,\mathrm{PolyLog}\left(2,e^{4i(a+bx)}\right)}{2b^3}$$

output

```
-2*I*(d*x+c)^2/b-2*(d*x+c)^2*cot(2*b*x+2*a)/b+2*d*(d*x+c)*ln(1-exp(4*I*(b*
x+a)))/b^2-1/2*I*d^2*polylog(2,exp(4*I*(b*x+a)))/b^3
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 277 vs. 2(88) = 176.

Time = 1.13 (sec) , antiderivative size = 277, normalized size of antiderivative = 3.15

$$\int (c+dx)^2 \csc^2(a+bx)\sec^2(a+bx)\,dx$$

$$= -\frac{ie^{4ia}\left(4b^2e^{-4ia}(c+dx)^2+2ibd\left(1-e^{-4ia}\right)(c+dx)\log\left(1-e^{-i(a+bx)}\right)+2ibd\left(1-e^{-4ia}\right)(c+dx)\log\left(1+e^{-i(a+bx)}\right)+2ibd\left(1-e^{-4ia}\right)(c+dx)\log\left(1+\right.\right.}{-1+e^{4i}}$$

input
```
Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

output
```
(((-I)*E^((4*I)*a)*((4*b^2*(c + d*x)^2)/E^((4*I)*a) + (2*I)*b*d*(1 - E^((-
4*I)*a))*(c + d*x)*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b*d*(1 - E^((-4*I)*
a))*(c + d*x)*Log[1 + E^((-I)*(a + b*x))] + (2*I)*b*d*(1 - E^((-4*I)*a))*(
c + d*x)*Log[1 + E^((-2*I)*(a + b*x))] - 2*d^2*(1 - E^((-4*I)*a))*PolyLog[
2, -E^((-I)*(a + b*x))] - 2*d^2*(1 - E^((-4*I)*a))*PolyLog[2, E^((-I)*(a +
 b*x))] - d^2*(1 - E^((-4*I)*a))*PolyLog[2, -E^((-2*I)*(a + b*x))]))/(-1 +
 E^((4*I)*a)) + 2*b^2*(c + d*x)^2*Csc[2*a]*Csc[2*(a + b*x)]*Sin[2*b*x])/b^
3
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4919, 3042, 4672, 3042, 25, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \csc^2(a+bx) \sec^2(a+bx)\, dx$$

$$\downarrow 4919$$

$$4\int (c+dx)^2 \csc^2(2a+2bx) dx$$

$$\downarrow 3042$$

$$4\int (c+dx)^2 \csc(2a+2bx)^2 dx$$

$$\downarrow 4672$$

$$4\left(\frac{d\int (c+dx)\cot(2a+2bx) dx}{b} - \frac{(c+dx)^2\cot(2a+2bx)}{2b}\right)$$

$$\downarrow 3042$$

$$4\left(\frac{d\int -\left((c+dx)\tan\left(2a+2bx+\frac{\pi}{2}\right)\right) dx}{b} - \frac{(c+dx)^2\cot(2a+2bx)}{2b}\right)$$

$$\downarrow 25$$

$$4\left(-\frac{d\int (c+dx)\tan\left(\frac{1}{2}(4a+\pi)+2bx\right)dx}{b}-\frac{(c+dx)^2\cot(2a+2bx)}{2b}\right)$$

$$\downarrow 4202$$

$$4\left(-\frac{(c+dx)^2\cot(2a+2bx)}{2b}-\frac{d\left(\frac{i(c+dx)^2}{2d}-2i\int\frac{e^{i(4a+4bx+\pi)}(c+dx)}{1+e^{i(4a+4bx+\pi)}}dx\right)}{b}\right)$$

$$\downarrow 2620$$

$$4\left(-\frac{(c+dx)^2\cot(2a+2bx)}{2b}-\frac{d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{id\int\log\left(1+e^{i(4a+4bx+\pi)}\right)dx}{4b}-\frac{i(c+dx)\log\left(1+e^{i(4a+4bx+\pi)}\right)}{4b}\right)\right)}{b}\right)$$

$$\downarrow 2715$$

$$4\left(-\frac{(c+dx)^2\cot(2a+2bx)}{2b}-\frac{d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{d\int e^{-i(4a+4bx+\pi)}\log\left(1+e^{i(4a+4bx+\pi)}\right)de^{i(4a+4bx+\pi)}}{16b^2}-\frac{i(c+dx)\log\left(1+e^{i(4a+4}\right.\right.\right.\right.}{4b}$$

$$\downarrow 2838$$

$$4\left(-\frac{(c+dx)^2\cot(2a+2bx)}{2b}-\frac{d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{i(4a+4bx+\pi)}\right)}{16b^2}-\frac{i(c+dx)\log\left(1+e^{i(4a+4bx+\pi)}\right)}{4b}\right)\right)}{b}\right)$$

input

```
Int[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

output

```
4*(-1/2*((c + d*x)^2*Cot[2*a + 2*b*x])/b - (d*(((I/2)*(c + d*x)^2)/d - (2*
I)*(((-1/4*I)*(c + d*x)*Log[1 + E^(I*(4*a + Pi + 4*b*x))])/b - (d*PolyLog[
2, -E^(I*(4*a + Pi + 4*b*x))])/(16*b^2))))/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(80) = 160$.

Time = 0.60 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.99

method	result
risch	$-\frac{4id^2x^2}{b} + \frac{2dc\ln(e^{2i(bx+a)}+1)}{b^2} + \frac{2dc\ln(e^{i(bx+a)}-1)}{b^2} - \frac{8dc\ln(e^{i(bx+a)})}{b^2} + \frac{2dc\ln(e^{i(bx+a)}+1)}{b^2} - \frac{4id^2a^2}{b^3} - \frac{2id^2}{\text{polylo}}$

input
```
int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-4*I*d^2/b*x^2+2*d/b^2*c*ln(exp(2*I*(b*x+a))+1)+2*d/b^2*c*ln(exp(I*(b*x+a)
)-1)-8*d/b^2*c*ln(exp(I*(b*x+a)))+2*d/b^2*c*ln(exp(I*(b*x+a))+1)-4*I*d^2/b
^3*a^2-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^3-2*I*d^2/b^3*polylog(2,-exp(I*
(b*x+a)))+2*d^2/b^2*ln(exp(2*I*(b*x+a))+1)*x-4*I*(d^2*x^2+2*c*d*x+c^2)/b/(
exp(2*I*(b*x+a))+1)/(exp(2*I*(b*x+a))-1)+2*d^2/b^2*ln(exp(I*(b*x+a))+1)*x-
8*I*d^2/b^2*a*x+2*d^2/b^2*ln(1-exp(I*(b*x+a)))*x+2*d^2/b^3*ln(1-exp(I*(b*x
+a)))*a-I*d^2*polylog(2,-exp(2*I*(b*x+a)))/b^3-2*d^2/b^3*a*ln(exp(I*(b*x+a
))-1)+8*d^2/b^3*a*ln(exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 950 vs. $2(77) = 154$.

Time = 0.15 (sec) , antiderivative size = 950, normalized size of antiderivative = 10.80

$$\int (c+dx)^2 \csc^2(a+bx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

output

```
(b^2*d^2*x^2 + 2*b^2*c*d*x - I*d^2*cos(b*x + a)*dilog(cos(b*x + a) + I*sin
(b*x + a))*sin(b*x + a) + I*d^2*cos(b*x + a)*dilog(cos(b*x + a) - I*sin(b*
x + a))*sin(b*x + a) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x +
 a))*sin(b*x + a) - I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a)
)*sin(b*x + a) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a))*
sin(b*x + a) + I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a))*si
n(b*x + a) + I*d^2*cos(b*x + a)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(
b*x + a) - I*d^2*cos(b*x + a)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*
x + a) + b^2*c^2 + (b*d^2*x + b*c*d)*cos(b*x + a)*log(cos(b*x + a) + I*sin
(b*x + a) + 1)*sin(b*x + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x + a
) + I*sin(b*x + a) + I)*sin(b*x + a) + (b*d^2*x + b*c*d)*cos(b*x + a)*log(
cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + (b*c*d - a*d^2)*cos(b*x
+ a)*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + (b*d^2*x + a*d^
2)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + (b*d
^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x
+ a) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) +
 1)*sin(b*x + a) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - si
n(b*x + a) + 1)*sin(b*x + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(-1/2*cos(b
*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + (b*c*d - a*d^2)*cos(b*x
 + a)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + ...
```

Sympy [F]

$$\int (c+dx)^2 \csc^2(a+bx) \sec^2(a+bx)\,dx = \int (c+dx)^2 \csc^2(a+bx) \sec^2(a+bx)\,dx$$

input

```
integrate((d*x+c)**2*csc(b*x+a)**2*sec(b*x+a)**2,x)
```

output

```
Integral((c + d*x)**2*csc(a + b*x)**2*sec(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(77) = 154$.

Time = 0.38 (sec) , antiderivative size = 772, normalized size of antiderivative = 8.77

$$\int (c+dx)^2 \csc^2(a+bx) \sec^2(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

output

```
-(4*b^2*c^2 + 2*(b*d^2*x + b*c*d - (b*d^2*x + b*c*d)*cos(4*b*x + 4*a) + (-
I*b*d^2*x - I*b*c*d)*sin(4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x
 + 2*a) + 1) + 2*(b*d^2*x + b*c*d - (b*d^2*x + b*c*d)*cos(4*b*x + 4*a) + (
-I*b*d^2*x - I*b*c*d)*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), cos(b*x + a)
 + 1) - 2*(b*c*d*cos(4*b*x + 4*a) + I*b*c*d*sin(4*b*x + 4*a) - b*c*d)*arct
an2(sin(b*x + a), cos(b*x + a) - 1) + 2*(b*d^2*x*cos(4*b*x + 4*a) + I*b*d^
2*x*sin(4*b*x + 4*a) - b*d^2*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) +
 4*(b^2*d^2*x^2 + 2*b^2*c*d*x)*cos(4*b*x + 4*a) + (d^2*cos(4*b*x + 4*a) +
I*d^2*sin(4*b*x + 4*a) - d^2)*dilog(-e^(2*I*b*x + 2*I*a)) + 2*(d^2*cos(4*b
*x + 4*a) + I*d^2*sin(4*b*x + 4*a) - d^2)*dilog(-e^(I*b*x + I*a)) + 2*(d^2
*cos(4*b*x + 4*a) + I*d^2*sin(4*b*x + 4*a) - d^2)*dilog(e^(I*b*x + I*a)) -
 (I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(4*b*x + 4*a) + (b*d^2*x
 + b*c*d)*sin(4*b*x + 4*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 +
2*cos(2*b*x + 2*a) + 1) - (I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*co
s(4*b*x + 4*a) + (b*d^2*x + b*c*d)*sin(4*b*x + 4*a))*log(cos(b*x + a)^2 +
sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (I*b*d^2*x + I*b*c*d + (-I*b*d^2*x
- I*b*c*d)*cos(4*b*x + 4*a) + (b*d^2*x + b*c*d)*sin(4*b*x + 4*a))*log(cos(
b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(I*b^2*d^2*x^2 + 2*I
*b^2*c*d*x)*sin(4*b*x + 4*a))/(-I*b^3*cos(4*b*x + 4*a) + b^3*sin(4*b*x + 4
*a) + I*b^3)
```

Giac [F]

$$\int (c+dx)^2 \csc^2(a+bx) \sec^2(a+bx)\, dx = \int (dx+c)^2 \csc(bx+a)^2 \sec(bx+a)^2\, dx$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc^2(a+bx) \sec^2(a+bx)\, dx = \int \frac{(c+d\,x)^2}{\cos(a+b\,x)^2 \sin(a+b\,x)^2}\, dx$$

input
```
int((c + d*x)^2/(cos(a + b*x)^2*sin(a + b*x)^2),x)
```

output
```
int((c + d*x)^2/(cos(a + b*x)^2*sin(a + b*x)^2), x)
```

Reduce [F]

$$\int (c+dx)^2 \csc^2(a+bx) \sec^2(a+bx)\, dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

output

```
(28*cos(a + b*x)*int(x**2/(tan((a + b*x)/2)**6 - 2*tan((a + b*x)/2)**4 + t
an((a + b*x)/2)**2),x)*sin(a + b*x)*b**3*d**2 - 56*cos(a + b*x)*int(x**2/(
tan((a + b*x)/2)**4 - 2*tan((a + b*x)/2)**2 + 1),x)*sin(a + b*x)*b**3*d**2
 - 20*cos(a + b*x)*int((tan((a + b*x)/2)**5*x)/(tan((a + b*x)/2)**4 - 2*ta
n((a + b*x)/2)**2 + 1),x)*sin(a + b*x)*b**2*d**2 - 92*cos(a + b*x)*int(x/(
tan((a + b*x)/2)**5 - 2*tan((a + b*x)/2)**3 + tan((a + b*x)/2)),x)*sin(a +
 b*x)*b**2*d**2 - 80*cos(a + b*x)*log(tan((a + b*x)/2)**2 + 1)*sin(a + b*x
)*b*c*d + 40*cos(a + b*x)*log(tan((a + b*x)/2) - 1)*sin(a + b*x)*b*c*d + 2
8*cos(a + b*x)*log(tan((a + b*x)/2) - 1)*sin(a + b*x)*d**2 + 40*cos(a + b*
x)*log(tan((a + b*x)/2) + 1)*sin(a + b*x)*b*c*d - 28*cos(a + b*x)*log(tan(
(a + b*x)/2) + 1)*sin(a + b*x)*d**2 + 40*cos(a + b*x)*log(tan((a + b*x)/2)
)*sin(a + b*x)*b*c*d + 7*cos(a + b*x)*sin(a + b*x)*b**3*d**2*x**3 + 56*cos
(a + b*x)*b**2*d**2*x**2 + 40*sin(a + b*x)**2*b**2*c**2 + 80*sin(a + b*x)*
*2*b**2*c*d*x - 9*sin(a + b*x)**2*b**2*d**2*x**2 + 28*sin(a + b*x)*b*d**2*
x - 20*b**2*c**2 - 40*b**2*c*d*x + 36*b**2*d**2*x**2)/(20*cos(a + b*x)*sin
(a + b*x)*b**3)
```

3.275 $\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx$

Optimal result . . . 2395
Mathematica [A] (verified) . . . 2395
Rubi [A] (verified) . . . 2396
Maple [C] (verified) . . . 2398
Fricas [B] (verification not implemented) . . . 2398
Sympy [F] . . . 2399
Maxima [B] (verification not implemented) . . . 2399
Giac [B] (verification not implemented) . . . 2400
Mupad [B] (verification not implemented) . . . 2401
Reduce [B] (verification not implemented) . . . 2401

Optimal result

Integrand size = 22, antiderivative size = 35

$$\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx = -\frac{2(c+dx)\cot(2a+2bx)}{b} + \frac{d\log(\sin(2a+2bx))}{b^2}$$

output
```
-2*(d*x+c)*cot(2*b*x+2*a)/b+d*ln(sin(2*b*x+2*a))/b^2
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx = \frac{-2b(c+dx)\cot(2(a+bx)) + d\log(\sin(2(a+bx)))}{b^2}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

output
```
(-2*b*(c + d*x)*Cot[2*(a + b*x)] + d*Log[Sin[2*(a + b*x)]])/b^2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4919, 3042, 4672, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx$$

$$\downarrow 4919$$

$$4\int (c+dx)\csc^2(2a+2bx)dx$$

$$\downarrow 3042$$

$$4\int (c+dx)\csc(2a+2bx)^2dx$$

$$\downarrow 4672$$

$$4\left(\frac{d\int \cot(2a+2bx)dx}{2b}-\frac{(c+dx)\cot(2a+2bx)}{2b}\right)$$

$$\downarrow 3042$$

$$4\left(\frac{d\int -\tan\left(2a+2bx+\frac{\pi}{2}\right)dx}{2b}-\frac{(c+dx)\cot(2a+2bx)}{2b}\right)$$

$$\downarrow 25$$

$$4\left(-\frac{d\int \tan\left(\frac{1}{2}(4a+\pi)+2bx\right)dx}{2b}-\frac{(c+dx)\cot(2a+2bx)}{2b}\right)$$

$$\downarrow 3956$$

$$4\left(\frac{d\log(-\sin(2a+2bx))}{4b^2}-\frac{(c+dx)\cot(2a+2bx)}{2b}\right)$$

input
```
Int[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

output
```
4*(-1/2*((c + d*x)*Cot[2*a + 2*b*x])/b + (d*Log[-Sin[2*a + 2*b*x]])/(4*b^2
))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.06

method	result
risch	$-\frac{4idx}{b}-\frac{4ida}{b^2}-\frac{4i(dx+c)}{b\left(e^{2i(bx+a)}+1\right)\left(e^{2i(bx+a)}-1\right)}+\frac{d\ln\left(e^{4i(bx+a)}-1\right)}{b^2}$
parallelrisch	$\frac{-4\cos(bx+a)\ln\left(\sec\left(\frac{a}{2}+\frac{bx}{2}\right)^2\right)d+2\cos(bx+a)\ln\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-1\right)d+2\cos(bx+a)\ln\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)d+2\cos(bx+a)\ln($
norman	$\frac{\frac{c}{2b}-\frac{3c\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{b}+\frac{c\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4}{2b}+\frac{dx}{2b}-\frac{3dx\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{b}+\frac{dx\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4}{2b}}{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-1\right)}+\frac{d\ln\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}+\frac{d\ln\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-\right.}{b^2}$

input
```
int((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-4*I*d/b*x-4*I*d/b^2*a-4*I*(d*x+c)/b/(exp(2*I*(b*x+a))+1)/(exp(2*I*(b*x+a)
)-1)+d/b^2*ln(exp(4*I*(b*x+a))-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(35) = 70.

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.14

$$\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx$$
$$=\frac{d\cos(bx+a)\log\left(-\frac{1}{2}\cos(bx+a)\sin(bx+a)\right)\sin(bx+a)+bdx-2(bdx+bc)\cos(bx+a)^2+bc}{b^2\cos(bx+a)\sin(bx+a)}$$

input
```
integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
(d*cos(b*x + a)*log(-1/2*cos(b*x + a)*sin(b*x + a))*sin(b*x + a) + b*d*x -
 2*(b*d*x + b*c)*cos(b*x + a)^2 + b*c)/(b^2*cos(b*x + a)*sin(b*x + a))
```

Sympy [F]

$$\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx = \int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)**2*sec(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*csc(a + b*x)**2*sec(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(35) = 70$.

Time $= 0.16$ (sec) , antiderivative size $= 308$, normalized size of antiderivative $= 8.80$

$$\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx =$$

$$-\frac{2c\left(\frac{1}{\tan(bx+a)} - \tan(bx+a)\right) - \frac{2ad\left(\frac{1}{\tan(bx+a)} - \tan(bx+a)\right)}{b} - \frac{\left(\left(\cos(4bx+4a)^2+\sin(4bx+4a)^2-2\cos(4bx+4a)+1\right)\log\left(c\right.\right.}{}$$

input
```
integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*c*(1/tan(b*x + a) - tan(b*x + a)) - 2*a*d*(1/tan(b*x + a) - tan(b*
x + a))/b - ((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a)
 + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1
) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log
(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(4*b*x + 4*a)
^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(b*x + a)^2 + sin
(b*x + a)^2 - 2*cos(b*x + a) + 1) - 8*(b*x + a)*sin(4*b*x + 4*a))*d/((cos(
4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*b))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10271 vs. $2(35) = 70$.

Time = 2.42 (sec) , antiderivative size = 10271, normalized size of antiderivative = 293.46

$$\int (c + dx) \csc^2(a + bx) \sec^2(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

output

```
1/2*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4 -
 6*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 16*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^
3 + d*log(64*(tan(1/2*b*x)^8*tan(1/2*a)^6 + 2*tan(1/2*b*x)^7*tan(1/2*a)^7
+ tan(1/2*b*x)^6*tan(1/2*a)^8 - 2*tan(1/2*b*x)^8*tan(1/2*a)^4 - 14*tan(1/2
*b*x)^7*tan(1/2*a)^5 - 24*tan(1/2*b*x)^6*tan(1/2*a)^6 - 14*tan(1/2*b*x)^5*
tan(1/2*a)^7 - 2*tan(1/2*b*x)^4*tan(1/2*a)^8 + tan(1/2*b*x)^8*tan(1/2*a)^2
 + 14*tan(1/2*b*x)^7*tan(1/2*a)^3 + 62*tan(1/2*b*x)^6*tan(1/2*a)^4 + 98*ta
n(1/2*b*x)^5*tan(1/2*a)^5 + 62*tan(1/2*b*x)^4*tan(1/2*a)^6 + 14*tan(1/2*b*
x)^3*tan(1/2*a)^7 + tan(1/2*b*x)^2*tan(1/2*a)^8 - 2*tan(1/2*b*x)^7*tan(1/2
*a) - 24*tan(1/2*b*x)^6*tan(1/2*a)^2 - 98*tan(1/2*b*x)^5*tan(1/2*a)^3 - 15
2*tan(1/2*b*x)^4*tan(1/2*a)^4 - 98*tan(1/2*b*x)^3*tan(1/2*a)^5 - 24*tan(1/
2*b*x)^2*tan(1/2*a)^6 - 2*tan(1/2*b*x)*tan(1/2*a)^7 + tan(1/2*b*x)^6 + 14*
tan(1/2*b*x)^5*tan(1/2*a) + 62*tan(1/2*b*x)^4*tan(1/2*a)^2 + 98*tan(1/2*b*
x)^3*tan(1/2*a)^3 + 62*tan(1/2*b*x)^2*tan(1/2*a)^4 + 14*tan(1/2*b*x)*tan(1
/2*a)^5 + tan(1/2*a)^6 - 2*tan(1/2*b*x)^4 - 14*tan(1/2*b*x)^3*tan(1/2*a) -
 24*tan(1/2*b*x)^2*tan(1/2*a)^2 - 14*tan(1/2*b*x)*tan(1/2*a)^3 - 2*tan(1/2
*a)^4 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/
2*b*x)^8*tan(1/2*a)^8 + 4*tan(1/2*b*x)^8*tan(1/2*a)^6 + 4*tan(1/2*b*x)^6*t
an(1/2*a)^8 + 6*tan(1/2*b*x)^8*tan(1/2*a)^4 + 16*tan(1/2*b*x)^6*tan(1/2*a)
^6 + 6*tan(1/2*b*x)^4*tan(1/2*a)^8 + 4*tan(1/2*b*x)^8*tan(1/2*a)^2 + 24...
```

Mupad [B] (verification not implemented)

Time = 21.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57

$$\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx = \frac{d\,\ln\left(e^{a\,4i}\,e^{b\,x\,4i}-1\right)}{b^2} - \frac{(c+d\,x)\;4i}{b\;\left(e^{a\,4i+b\,x\,4i}-1\right)} - \frac{d\,x\,4i}{b}$$

input
```
int((c + d*x)/(cos(a + b*x)^2*sin(a + b*x)^2),x)
```

output
```
(d*log(exp(a*4i)*exp(b*x*4i) - 1))/b^2 - ((c + d*x)*4i)/(b*(exp(a*4i + b*x
*4i) - 1)) - (d*x*4i)/b
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 160, normalized size of antiderivative = 4.57

$$\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{-2\cos(bx+a)\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1\right)\sin(bx+a)\,d+\cos(bx+a)\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\sin(bx+a)\,d}{}$$

input
```
int((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

output
```
( - 2*cos(a + b*x)*log(tan((a + b*x)/2)**2 + 1)*sin(a + b*x)*d + cos(a + b
*x)*log(tan((a + b*x)/2) - 1)*sin(a + b*x)*d + cos(a + b*x)*log(tan((a + b
*x)/2) + 1)*sin(a + b*x)*d + cos(a + b*x)*log(tan((a + b*x)/2))*sin(a + b*
x)*d + 2*sin(a + b*x)**2*b*c + 2*sin(a + b*x)**2*b*d*x - b*c - b*d*x)/(cos
(a + b*x)*sin(a + b*x)*b**2)
```

3.276 $\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$

Optimal result . . . 2402
Mathematica [N/A] . . . 2402
Rubi [N/A] . . . 2403
Maple [N/A] . . . 2404
Fricas [N/A] . . . 2404
Sympy [N/A] . . . 2405
Maxima [N/A] . . . 2405
Giac [N/A] . . . 2406
Mupad [N/A] . . . 2407
Reduce [N/A] . . . 2407

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx = 4\text{Int}\left(\frac{\csc^2(2a+2bx)}{c+dx}, x\right)$$

output
```
4*Defer(Int)(csc(2*b*x+2*a)^2/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 5.74 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x),x]
```

output
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4919, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$
$$\downarrow 4919$$
$$4\int \frac{\csc^2(2a+2bx)}{c+dx}dx$$
$$\downarrow 3042$$
$$4\int \frac{\csc(2a+2bx)^2}{c+dx}dx$$
$$\downarrow 4680$$
$$4\int \frac{\csc^2(2a+2bx)}{c+dx}dx$$

input
```
Int[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4919

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc(bx+a)^2 \sec(bx+a)^2}{dx+c} dx$$

input

```
int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x)
```

output

```
int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^2 \sec(bx+a)^2}{dx+c}\,dx$$

input

```
integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output

```
integral(csc(b*x + a)^2*sec(b*x + a)^2/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)**2*sec(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(csc(a + b*x)**2*sec(a + b*x)**2/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 687, normalized size of antiderivative = 28.62

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^2\sec(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
(2*(b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(4*b*x + 4*a)^2 + (b*d^2*x + b*
c*d)*sin(4*b*x + 4*a)^2 - 2*(b*d^2*x + b*c*d)*cos(4*b*x + 4*a))*integrate(
sin(2*b*x + 2*a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x +
 b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2
*a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)), x) + (b*d^2*x
 + b*c*d + (b*d^2*x + b*c*d)*cos(4*b*x + 4*a)^2 + (b*d^2*x + b*c*d)*sin(4*
b*x + 4*a)^2 - 2*(b*d^2*x + b*c*d)*cos(4*b*x + 4*a))*integrate(sin(b*x + a
)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x
 + a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 + 2*(b*d^2*x^2 +
2*b*c*d*x + b*c^2)*cos(b*x + a)), x) - (b*d^2*x + b*c*d + (b*d^2*x + b*c*d
)*cos(4*b*x + 4*a)^2 + (b*d^2*x + b*c*d)*sin(4*b*x + 4*a)^2 - 2*(b*d^2*x +
 b*c*d)*cos(4*b*x + 4*a))*integrate(sin(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x +
b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2 + (b*d^2*x^2 + 2*b*
c*d*x + b*c^2)*sin(b*x + a)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x
+ a)), x) - 4*sin(4*b*x + 4*a))/(b*d*x + (b*d*x + b*c)*cos(4*b*x + 4*a)^2
+ (b*d*x + b*c)*sin(4*b*x + 4*a)^2 + b*c - 2*(b*d*x + b*c)*cos(4*b*x + 4*a
))
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sec(b*x + a)^2/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 19.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos(a+b\,x)^2\sin(a+b\,x)^2\,(c+d\,x)}\,dx$$

input
```
int(1/(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^2\sec(bx+a)^2}{dx+c}dx$$

input
```
int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x)
```

output
```
int((csc(a + b*x)**2*sec(a + b*x)**2)/(c + d*x),x)
```

3.277 $\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$

Optimal result 2408
Mathematica [N/A] 2408
Rubi [N/A] 2409
Maple [N/A] 2410
Fricas [N/A] 2410
Sympy [N/A] 2411
Maxima [N/A] 2411
Giac [N/A] 2412
Mupad [N/A] 2413
Reduce [N/A] 2413

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = 4\text{Int}\left(\frac{\csc^2(2a+2bx)}{(c+dx)^2}, x\right)$$

output
```
4*Defer(Int)(csc(2*b*x+2*a)^2/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 5.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4919, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

$$\downarrow 4919$$

$$4\int \frac{\csc^2(2a+2bx)}{(c+dx)^2}dx$$

$$\downarrow 3042$$

$$4\int \frac{\csc(2a+2bx)^2}{(c+dx)^2}dx$$

$$\downarrow 4680$$

$$4\int \frac{\csc^2(2a+2bx)}{(c+dx)^2}dx$$

input
```
Int[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4919

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc(bx+a)^2 \sec(bx+a)^2}{(dx+c)^2} dx$$

input

```
int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x)
```

output

```
int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\, dx = \int \frac{\csc(bx+a)^2 \sec(bx+a)^2}{(dx+c)^2}\, dx$$

input

```
integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output

```
integral(csc(b*x + a)^2*sec(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 5.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(csc(b*x+a)**2*sec(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)**2*sec(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 3.31 (sec) , antiderivative size = 1024, normalized size of antiderivative = 42.67

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)^2\sec(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
2*(2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2
*d)*cos(4*b*x + 4*a)^2 + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(4*b*x + 4
*a)^2 - 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(4*b*x + 4*a))*integrate(
sin(2*b*x + 2*a)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3
*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(2*b*x + 2*a)^2 + (b*d^3*x^
3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(2*b*x + 2*a)^2 + 2*(b*d^3*x^3
 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(2*b*x + 2*a)), x) + (b*d^3*x^2
 + 2*b*c*d^2*x + b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(4*b*x +
 4*a)^2 + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(4*b*x + 4*a)^2 - 2*(b*d^
3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(4*b*x + 4*a))*integrate(sin(b*x + a)/(b
*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^
2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b
*c^2*d*x + b*c^3)*sin(b*x + a)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*
d*x + b*c^3)*cos(b*x + a)), x) - (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d + (b*d
^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(4*b*x + 4*a)^2 + (b*d^3*x^2 + 2*b*c*d^
2*x + b*c^2*d)*sin(4*b*x + 4*a)^2 - 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*
cos(4*b*x + 4*a))*integrate(sin(b*x + a)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*
c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*
x + a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(b*x + a)^
2 - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)), ...
```

Giac [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sec(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 20.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\cos(a+b\,x)^2\sin(a+b\,x)^2(c+d\,x)^2}\,dx$$

input
```
int(1/(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)^2\sec(bx+a)^2}{d^2x^2+2cdx+c^2}dx$$

input
```
int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int((csc(a + b*x)**2*sec(a + b*x)**2)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.278 $\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal result . . . 2414
Mathematica [N/A] . . . 2414
Rubi [N/A] . . . 2415
Maple [N/A] . . . 2415
Fricas [N/A] . . . 2416
Sympy [F(-1)] . . . 2416
Maxima [N/A] . . . 2416
Giac [N/A] . . . 2417
Mupad [N/A] . . . 2417
Reduce [N/A] . . . 2418

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \text{Int}\left((c+dx)^m \csc^3(a+bx)\sec^2(a+bx), x\right)$$

output
```
Defer(Int)((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 17.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\sec^2(a+bx)(c+dx)^m\,dx$$

$$\downarrow 7299$$

$$\int \csc^3(a+bx)\sec^2(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^2\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 19.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\cos(a+b\,x)^2 \sin(a+b\,x)^3}\,dx$$

input
```
int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

output
```
int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^2\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

output
```
int((c + d*x)**m*csc(a + b*x)**3*sec(a + b*x)**2,x)
```

3.279 $\int (c+dx)^3 \csc^3(a+bx) \sec^2(a+bx)\, dx$

Optimal result . . . 2420
Mathematica [A] (verified) . . . 2421
Rubi [A] (verified) . . . 2422
Maple [B] (verified) . . . 2425
Fricas [B] (verification not implemented) . . . 2426
Sympy [F(-1)] . . . 2426
Maxima [B] (verification not implemented) . . . 2426
Giac [F] . . . 2427
Mupad [F(-1)] . . . 2428
Reduce [F] . . . 2428

Optimal result

Integrand size = 24, antiderivative size = 601

$$
\begin{aligned}
&\int (c+dx)^3 \csc^3(a+bx) \sec^2(a+bx)\, dx \\
&= \frac{12icd^2 x \arctan\left(e^{i(a+bx)}\right)}{b^2} + \frac{6id^3 x^2 \arctan\left(e^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{6d^3 x \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3(c+dx)^3 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} \\
&\quad - \frac{3cd^2 \operatorname{arctanh}(\cos(a+bx))}{b^3} - \frac{3c^2 d \operatorname{arctanh}(\sin(a+bx))}{b^2} \\
&\quad - \frac{3c^2 d \csc(a+bx)}{2b^2} - \frac{3cd^2 x \csc(a+bx)}{b^2} - \frac{3d^3 x^2 \csc(a+bx)}{2b^2} \\
&\quad + \frac{3id^3 \operatorname{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^4} + \frac{9id(c+dx)^2 \operatorname{PolyLog}\left(2, -e^{i(a+bx)}\right)}{2b^2} \\
&\quad - \frac{6icd^2 \operatorname{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} - \frac{6id^3 x \operatorname{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{6icd^2 \operatorname{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3} + \frac{6id^3 x \operatorname{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3} \\
&\quad - \frac{3id^3 \operatorname{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^4} - \frac{9id(c+dx)^2 \operatorname{PolyLog}\left(2, e^{i(a+bx)}\right)}{2b^2} \\
&\quad - \frac{9d^2(c+dx) \operatorname{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^3} + \frac{6d^3 \operatorname{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^4} \\
&\quad - \frac{6d^3 \operatorname{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^4} + \frac{9d^2(c+dx) \operatorname{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3} \\
&\quad - \frac{9id^3 \operatorname{PolyLog}\left(4, -e^{i(a+bx)}\right)}{b^4} + \frac{9id^3 \operatorname{PolyLog}\left(4, e^{i(a+bx)}\right)}{b^4} \\
&\quad + \frac{3(c+dx)^3 \sec(a+bx)}{2b} - \frac{(c+dx)^3 \csc^2(a+bx) \sec(a+bx)}{2b}
\end{aligned}
$$

output

```
6*I*c*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3+6*I*d^3*x*polylog(2,I*exp(I*(b*x
+a)))/b^3+6*I*d^3*x^2*arctan(exp(I*(b*x+a)))/b^2-9*d^2*(d*x+c)*polylog(3,-
exp(I*(b*x+a)))/b^3+9*d^2*(d*x+c)*polylog(3,exp(I*(b*x+a)))/b^3-3*c^2*d*ar
ctanh(sin(b*x+a))/b^2-3*c*d^2*arctanh(cos(b*x+a))/b^3-1/2*(d*x+c)^3*csc(b*
x+a)^2*sec(b*x+a)/b-3/2*c^2*d*csc(b*x+a)/b^2-3/2*d^3*x^2*csc(b*x+a)/b^2-6*
d^3*x*arctanh(exp(I*(b*x+a)))/b^3-9*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4+3
*I*d^3*polylog(2,-exp(I*(b*x+a)))/b^4+9*I*d^3*polylog(4,exp(I*(b*x+a)))/b^
4+12*I*c*d^2*x*arctan(exp(I*(b*x+a)))/b^2+9/2*I*d*(d*x+c)^2*polylog(2,-exp
(I*(b*x+a)))/b^2-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4+3/2*(d*x+c)^3*sec(b
*x+a)/b-3*c*d^2*x*csc(b*x+a)/b^2-6*I*c*d^2*polylog(2,-I*exp(I*(b*x+a)))/b^
3-6*I*d^3*x*polylog(2,-I*exp(I*(b*x+a)))/b^3-9/2*I*d*(d*x+c)^2*polylog(2,e
xp(I*(b*x+a)))/b^2+6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,
I*exp(I*(b*x+a)))/b^4-3*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b
```

Mathematica [A] (verified)

Time = 7.37 (sec) , antiderivative size = 907, normalized size of antiderivative = 1.51

$$\int (c+dx)^3 \csc^3(a+bx) \sec^2(a+bx)\, dx =$$

$$-\frac{3d\left(-2ib^2c^2 \arctan\left(e^{i(a+bx)}\right) + 2b^2cdx \log\left(1-ie^{i(a+bx)}\right) + b^2d^2x^2 \log\left(1-ie^{i(a+bx)}\right) - 2b^2cdx \log\left(1+\right.\right.}{}$$

$$+\frac{3\left(b^3c^3 \log\left(1-e^{i(a+bx)}\right) + 2bcd^2 \log\left(1-e^{i(a+bx)}\right) + 3b^3c^2dx \log\left(1-e^{i(a+bx)}\right) + 2bd^3x \log\left(1-e^{i(a+bx)}\right.\right.}{}$$

$$-\frac{\csc^2(a+bx)\sec(a+bx)\left(-bc^3 - 3bc^2dx - 3bcd^2x^2 - bd^3x^3 + 3bc^3\cos(2a+2bx) + 9bc^2dx\cos(2a+2\right.}{}$$

input

```
Integrate[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output

```
(-3*d*((-2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 - I*E^(I
*(a + b*x))] + b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - 2*b^2*c*d*x*Log[1
+ I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + (2*I)*b*d*
(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2
, I*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 2*d^2*Poly
Log[3, I*E^(I*(a + b*x))]))/b^4 + (3*(b^3*c^3*Log[1 - E^(I*(a + b*x))] + 2
*b*c*d^2*Log[1 - E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 - E^(I*(a + b*x))]
 + 2*b*d^3*x*Log[1 - E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - E^(I*(a +
b*x))] + b^3*d^3*x^3*Log[1 - E^(I*(a + b*x))] - b^3*c^3*Log[1 + E^(I*(a +
b*x))] - 2*b*c*d^2*Log[1 + E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + E^(I*(
a + b*x))] - 2*b*d^3*x*Log[1 + E^(I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 +
E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + E^(I*(a + b*x))] + I*d*(2*d^2 + 3*b
^2*(c + d*x)^2)*PolyLog[2, -E^(I*(a + b*x))] - I*d*(2*d^2 + 3*b^2*(c + d*x
)^2)*PolyLog[2, E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, -E^(I*(a + b*x))]
- 6*b*d^3*x*PolyLog[3, -E^(I*(a + b*x))] + 6*b*c*d^2*PolyLog[3, E^(I*(a +
b*x))] + 6*b*d^3*x*PolyLog[3, E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, -E^(
I*(a + b*x))] + (6*I)*d^3*PolyLog[4, E^(I*(a + b*x))]))/(2*b^4) - (Csc[a +
 b*x]^2*Sec[a + b*x]*(-(b*c^3) - 3*b*c^2*d*x - 3*b*c*d^2*x^2 - b*d^3*x^3 +
 3*b*c^3*Cos[2*a + 2*b*x] + 9*b*c^2*d*x*Cos[2*a + 2*b*x] + 9*b*c*d^2*x^2*C
os[2*a + 2*b*x] + 3*b*d^3*x^3*Cos[2*a + 2*b*x] + 3*c^2*d*Sin[2*a + 2*b*...
```

Rubi [A] (verified)

Time = 2.37 (sec) , antiderivative size = 623, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4920, 27, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \csc^3(a+bx) \sec^2(a+bx)\, dx$$

$$\downarrow 4920$$

$$-3d \int -\frac{1}{2}(c+dx)^2 \left(\frac{\sec(a+bx)\csc^2(a+bx)}{b} + \frac{3\operatorname{arctanh}(\cos(a+bx))}{b} - \frac{3\sec(a+bx)}{b} \right) dx - \frac{3(c+dx)^3 \operatorname{arctanh}(\cos(a+bx))}{2b} + \frac{3(c+dx)^3 \sec(a+bx)}{2b} - \frac{(c+dx)^3 \csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 27$$

$$\frac{\frac{3}{2}d\int(c+dx)^2\left(\frac{\sec(a+bx)\csc^2(a+bx)}{b}+\frac{3\text{arctanh}(\cos(a+bx))}{b}-\frac{3\sec(a+bx)}{b}\right)dx-3(c+dx)^3\text{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^3\sec(a+bx)}{2b}-\frac{(c+dx)^3\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 7292$$

$$\frac{\frac{3}{2}d\int\frac{(c+dx)^2\left(\sec(a+bx)\csc^2(a+bx)+3\text{arctanh}(\cos(a+bx))-3\sec(a+bx)\right)}{b}dx-3(c+dx)^3\text{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^3\sec(a+bx)}{2b}-\frac{(c+dx)^3\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 27$$

$$\frac{3d\int(c+dx)^2\left(\sec(a+bx)\csc^2(a+bx)+3\text{arctanh}(\cos(a+bx))-3\sec(a+bx)\right)dx}{2b}-\frac{3(c+dx)^3\text{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^3\sec(a+bx)}{2b}-\frac{(c+dx)^3\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 7293$$

$$\frac{3d\int\left(3\text{arctanh}(\cos(a+bx))(c+dx)^2+\left(\csc^2(a+bx)-3\right)\sec(a+bx)(c+dx)^2\right)dx}{2b}-\frac{3(c+dx)^3\text{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^3\sec(a+bx)}{2b}-\frac{(c+dx)^3\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 2009$$

$$\frac{3d\left(\frac{8icdx\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{4id^2x^2\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{2cd\text{arctanh}(\cos(a+bx))}{b^2}-\frac{4d^2x\text{arctanh}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2c^2\text{arctanh}(\sin(a+b}{b}\right.}{\frac{3(c+dx)^3\text{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^3\sec(a+bx)}{2b}-\frac{(c+dx)^3\csc^2(a+bx)\sec(a+bx)}{2b}}$$

input

```
Int[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
(-3*(c + d*x)^3*ArcTanh[Cos[a + b*x]])/(2*b) + (3*d*(((8*I)*c*d*x*ArcTan[E
^(I*(a + b*x))])/b + ((4*I)*d^2*x^2*ArcTan[E^(I*(a + b*x))])/b - (4*d^2*x*
ArcTanh[E^(I*(a + b*x))])/b^2 - (2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/d
 - (2*c*d*ArcTanh[Cos[a + b*x]])/b^2 + ((c + d*x)^3*ArcTanh[Cos[a + b*x]])
/d - (2*c^2*ArcTanh[Sin[a + b*x]])/b - (c^2*Csc[a + b*x])/b - (2*c*d*x*Csc
[a + b*x])/b - (d^2*x^2*Csc[a + b*x])/b + ((2*I)*d^2*PolyLog[2, -E^(I*(a +
 b*x))])/b^3 + ((3*I)*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b - ((4*I)
*c*d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((4*I)*d^2*x*PolyLog[2, (-I)*
E^(I*(a + b*x))])/b^2 + ((4*I)*c*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 + ((
4*I)*d^2*x*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - ((2*I)*d^2*PolyLog[2, E^(I
*(a + b*x))])/b^3 - ((3*I)*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b - (6
*d*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^2 + (4*d^2*PolyLog[3, (-I)*E^
(I*(a + b*x))])/b^3 - (4*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + (6*d*(c
+ d*x)*PolyLog[3, E^(I*(a + b*x))])/b^2 - ((6*I)*d^2*PolyLog[4, -E^(I*(a +
 b*x))])/b^3 + ((6*I)*d^2*PolyLog[4, E^(I*(a + b*x))])/b^3))/(2*b) + (3*(c
 + d*x)^3*Sec[a + b*x])/(2*b) - ((c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x])/
(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1612 vs. 2(535) = 1070.

Time = 1.06 (sec) , antiderivative size = 1613, normalized size of antiderivative = 2.68

method	result	size
risch	Expression too large to display	1613

input
```
int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-9/2/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2+3/b^4*a^2*d^3*ln(1-I*exp(I*(b*x+a)
))+1/b^2/(exp(2*I*(b*x+a))-1)^2/(exp(2*I*(b*x+a))+1)*(3*d^3*x^3*b*exp(5*I*
(b*x+a))+9*c*d^2*x^2*b*exp(5*I*(b*x+a))+9*c^2*d*x*b*exp(5*I*(b*x+a))-2*d^3
*x^3*b*exp(3*I*(b*x+a))+3*b*c^3*exp(5*I*(b*x+a))-6*c*d^2*x^2*b*exp(3*I*(b*
x+a))+3*I*d^3*x^2*exp(I*(b*x+a))-6*c^2*d*x*b*exp(3*I*(b*x+a))+3*d^3*x^3*b*
exp(I*(b*x+a))+3*I*c^2*d*exp(I*(b*x+a))-2*b*c^3*exp(3*I*(b*x+a))+9*c*d^2*x
^2*b*exp(I*(b*x+a))-3*I*c^2*d*exp(5*I*(b*x+a))+9*c^2*d*x*b*exp(I*(b*x+a))+
3*b*c^3*exp(I*(b*x+a))-3*I*d^3*x^2*exp(5*I*(b*x+a))+6*I*c*d^2*x*exp(I*(b*x
+a))-6*I*c*d^2*x*exp(5*I*(b*x+a)))+3*d^2/b^3*c*ln(exp(I*(b*x+a))-1)-3*d^2/
b^3*c*ln(exp(I*(b*x+a))+1)-3*d^3/b^3*ln(exp(I*(b*x+a))+1)*x+3*d^3/b^3*ln(1
-exp(I*(b*x+a)))*x+3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a-3*d^3/b^4*a*ln(exp(I*(
b*x+a))-1)+6/b^2*c*d^2*ln(I*exp(I*(b*x+a))+1)*x-6/b^2*c*d^2*ln(1-I*exp(I*(
b*x+a)))*x+6/b^3*c*d^2*ln(I*exp(I*(b*x+a))+1)*a-6/b^3*c*d^2*ln(1-I*exp(I*(
b*x+a)))*a+6*I/b^2*c^2*d*arctan(exp(I*(b*x+a)))+6*I/b^4*a*d^3*dilog(I*exp(
I*(b*x+a))+1)-6*I/b^4*a*d^3*dilog(1-I*exp(I*(b*x+a)))+9/2*I/b^2*d^3*polylo
g(2,-exp(I*(b*x+a)))*x^2+9/2*I/b^2*d*c^2*polylog(2,-exp(I*(b*x+a)))-9/2*I/
b^2*d*c^2*polylog(2,exp(I*(b*x+a)))-9/2*I/b^2*d^3*polylog(2,exp(I*(b*x+a))
)*x^2-12*I/b^3*a*d^2*c*arctan(exp(I*(b*x+a)))-3/b^4*a^2*d^3*ln(I*exp(I*(b*
x+a))+1)+3/b^2*d^3*ln(I*exp(I*(b*x+a))+1)*x^2-3/b^2*d^3*ln(1-I*exp(I*(b*x+
a)))*x^2-3/2/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)-9/b^3*c*d^2*polylog(3,-ex...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3173 vs. $2(509) = 1018$.

Time = 0.23 (sec) , antiderivative size = 3173, normalized size of antiderivative = 5.28

$$\int (c+dx)^3 \csc^3(a+bx) \sec^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^3(a+bx) \sec^2(a+bx)\, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8032 vs. $2(509) = 1018$.

Time = 4.88 (sec) , antiderivative size = 8032, normalized size of antiderivative = 13.36

$$\int (c+dx)^3 \csc^3(a+bx) \sec^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

output

```
1/4*(c^3*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log
(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) - 3*a*c^2*d*(2*(3*cos(b*x +
a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*lo
g(cos(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x +
a)^3 - cos(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1))/
b^2 - a^3*d^3*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) -
3*log(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1))/b^3 + 4*(12*(b^2*c^2*d
- 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a)
+ (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^
3)*(b*x + a))*cos(6*b*x + 6*a) - (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^
3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - (b^2*c^2*d
 - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a)
)*cos(2*b*x + 2*a) + (I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*
a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(6*b*x + 6*a) + (-I*b^2*c^
2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*
d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x
+ a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*sin(2*b*x + 2
*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + 12*(b^2*c^2*d - 2*a*b*c*d^2
 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) + (b^2*c^2*d
- 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + ...
```

Giac [F]

$$\int (c+dx)^3 \csc^3(a+bx) \sec^2(a+bx)\,dx = \int (dx+c)^3 \csc(bx+a)^3 \sec(bx+a)^2\,dx$$

input

```
integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

output

```
integrate((d*x + c)^3*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^3(a+bx) \sec^2(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)^3/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^3 \csc^3(a+bx) \sec^2(a+bx)\,dx$$
$$= \frac{8\cos(bx+a)\left(\int \csc(bx+a)^3 \sec(bx+a)^2 x^3 dx\right)\sin(bx+a)^2 b\,d^3 + 24\cos(bx+a)\left(\int \csc(bx+a)^3 \sec}{}$$

input
```
int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

output
```
(8*cos(a + b*x)*int(csc(a + b*x)**3*sec(a + b*x)**2*x**3,x)*sin(a + b*x)**
2*b*d**3 + 24*cos(a + b*x)*int(csc(a + b*x)**3*sec(a + b*x)**2*x**2,x)*sin
(a + b*x)**2*b*c*d**2 + 24*cos(a + b*x)*int(csc(a + b*x)**3*sec(a + b*x)**
2*x,x)*sin(a + b*x)**2*b*c**2*d + 12*cos(a + b*x)*log(tan((a + b*x)/2))*si
n(a + b*x)**2*c**3 - 9*cos(a + b*x)*sin(a + b*x)**2*c**3 + 12*sin(a + b*x)
**2*c**3 - 4*c**3)/(8*cos(a + b*x)*sin(a + b*x)**2*b)
```

3.280 $\int (c+dx)^2 \csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal result . . . 2429
Mathematica [B] (warning: unable to verify) . . . 2430
Rubi [A] (verified) . . . 2431
Maple [B] (verified) . . . 2433
Fricas [B] (verification not implemented) . . . 2434
Sympy [F(-1)] . . . 2435
Maxima [B] (verification not implemented) . . . 2435
Giac [F] . . . 2436
Mupad [F(-1)] . . . 2437
Reduce [F] . . . 2437

Optimal result

Integrand size = 24, antiderivative size = 305

$$\begin{aligned}
&\int (c+dx)^2 \csc^3(a+bx)\sec^2(a+bx)\,dx \\
&= \frac{4id^2x\arctan\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\operatorname{arctanh}(\cos(a+bx))}{b^3} \\
&\quad - \frac{2cd\operatorname{arctanh}(\sin(a+bx))}{b^2} - \frac{cd\csc(a+bx)}{b^2} - \frac{d^2x\csc(a+bx)}{b^2} \\
&\quad + \frac{3id(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} - \frac{2id^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{2id^2\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} - \frac{3id(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{3d^2\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} + \frac{3d^2\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{3(c+dx)^2\sec(a+bx)}{2b} - \frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b}
\end{aligned}$$

output

```
4*I*d^2*x*arctan(exp(I*(b*x+a)))/b^2-3*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b
-d^2*arctanh(cos(b*x+a))/b^3-2*c*d*arctanh(sin(b*x+a))/b^2-c*d*csc(b*x+a)/
b^2-d^2*x*csc(b*x+a)/b^2+3*I*d*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^2-2*I*
d^2*polylog(2,-I*exp(I*(b*x+a)))/b^3+2*I*d^2*polylog(2,I*exp(I*(b*x+a)))/b
^3-3*I*d*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^2-3*d^2*polylog(3,-exp(I*(b*x
+a)))/b^3+3*d^2*polylog(3,exp(I*(b*x+a)))/b^3+3/2*(d*x+c)^2*sec(b*x+a)/b-1
/2*(d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 889 vs. $2(305) = 610$.

Time = 7.23 (sec) , antiderivative size = 889, normalized size of antiderivative = 2.91

$$\int (c+dx)^2 \csc^3(a+bx) \sec^2(a+bx)\, dx = \text{Too large to display}$$

input

```
Integrate[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output

```
((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a/2 + (b*x)/2]^2)/(8*b) + (3*b^2*c^2*Log[1
 - E^(I*(a + b*x))] + 2*d^2*Log[1 - E^(I*(a + b*x))] + 6*b^2*c*d*x*Log[1 -
 E^(I*(a + b*x))] + 3*b^2*d^2*x^2*Log[1 - E^(I*(a + b*x))] - 3*b^2*c^2*Log
[1 + E^(I*(a + b*x))] - 2*d^2*Log[1 + E^(I*(a + b*x))] - 6*b^2*c*d*x*Log[1
 + E^(I*(a + b*x))] - 3*b^2*d^2*x^2*Log[1 + E^(I*(a + b*x))] + (6*I)*b*d*(
c + d*x)*PolyLog[2, -E^(I*(a + b*x))] - (6*I)*b*d*(c + d*x)*PolyLog[2, E^(
I*(a + b*x))] - 6*d^2*PolyLog[3, -E^(I*(a + b*x))] + 6*d^2*PolyLog[3, E^(I
*(a + b*x))])/(2*b^3) + ((c^2 + 2*c*d*x + d^2*x^2)*Sec[a/2 + (b*x)/2]^2)/(
8*b) + ((c + d*x)*Csc[a]*Sec[a]*(-(d*Cos[a]) + b*c*Sin[a] + b*d*x*Sin[a]))
/b^2 - ((4*I)*c*d*ArcTan[((-I)*Sin[a] - I*Cos[a]*Tan[(b*x)/2])/Sqrt[Cos[a]
^2 + Sin[a]^2]])/(b^2*Sqrt[Cos[a]^2 + Sin[a]^2]) - (2*d^2*(-((Csc[a]*((b*x
 - ArcTan[Cot[a]])*(Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(
b*x - ArcTan[Cot[a]]))]) + I*(PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] -
PolyLog[2, E^(I*(b*x - ArcTan[Cot[a]]))])))/Sqrt[1 + Cot[a]^2]) + (2*ArcTa
n[Cot[a]]*ArcTanh[(Sin[a] + Cos[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]
])/Sqrt[Cos[a]^2 + Sin[a]^2]))/b^3 + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c*d*S
in[(b*x)/2]) - d^2*x*Sin[(b*x)/2]))/(2*b^2) + (Csc[a/2]*Csc[a/2 + (b*x)/2]
*(c*d*Sin[(b*x)/2] + d^2*x*Sin[(b*x)/2]))/(2*b^2) + (c^2*Sin[(b*x)/2] + 2*
c*d*x*Sin[(b*x)/2] + d^2*x^2*Sin[(b*x)/2])/(b*(Cos[a/2] - Sin[a/2])*(Cos[a
/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) + (-(c^2*Sin[(b*x)/2]) - 2*c*d*x*S...
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4920, 27, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \csc^3(a+bx)\sec^2(a+bx)\,dx$$

$$\downarrow 4920$$

$$-2d\int -\frac{1}{2}(c+dx)\left(\frac{\sec(a+bx)\csc^2(a+bx)}{b}+\frac{3\operatorname{arctanh}(\cos(a+bx))}{b}-\frac{3\sec(a+bx)}{b}\right)dx-\frac{3(c+dx)^2\operatorname{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^2\sec(a+bx)}{2b}-\frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 27$$

$$d\int (c+dx)\left(\frac{\sec(a+bx)\csc^2(a+bx)}{b}+\frac{3\operatorname{arctanh}(\cos(a+bx))}{b}-\frac{3\sec(a+bx)}{b}\right)dx-\frac{3(c+dx)^2\operatorname{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^2\sec(a+bx)}{2b}-\frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 7292$$

$$d\int \frac{(c+dx)\left(\sec(a+bx)\csc^2(a+bx)+3\operatorname{arctanh}(\cos(a+bx))-3\sec(a+bx)\right)}{b}dx-\frac{3(c+dx)^2\operatorname{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^2\sec(a+bx)}{2b}-\frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 27$$

$$\frac{d\int (c+dx)\left(\sec(a+bx)\csc^2(a+bx)+3\operatorname{arctanh}(\cos(a+bx))-3\sec(a+bx)\right)dx}{b}-\frac{3(c+dx)^2\operatorname{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^2\sec(a+bx)}{2b}-\frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 7293$$

$$\frac{d\int \left(3(c+dx)\operatorname{arctanh}(\cos(a+bx))+(c+dx)\left(\csc^2(a+bx)-3\right)\sec(a+bx)\right)dx}{b}-\frac{3(c+dx)^2\operatorname{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^2\sec(a+bx)}{2b}-\frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 2009$$

$$\frac{d\left(\frac{4idx\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{d\text{arctanh}(\cos(a+bx))}{b^2}-\frac{3(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{d}+\frac{3(c+dx)^2\text{arctanh}(\cos(a+bx))}{2d}-\frac{2c\text{arctanh}(\text{si}}{b}\right.}{}$$

$$\frac{3(c+dx)^2\text{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^2\sec(a+bx)}{2b}-\frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

input
```
Int[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
(-3*(c + d*x)^2*ArcTanh[Cos[a + b*x]])/(2*b) + (d*(((4*I)*d*x*ArcTan[E^(I*
(a + b*x))])/b - (3*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/d - (d*ArcTanh[C
os[a + b*x]])/b^2 + (3*(c + d*x)^2*ArcTanh[Cos[a + b*x]])/(2*d) - (2*c*Arc
Tanh[Sin[a + b*x]])/b - (c*Csc[a + b*x])/b - (d*x*Csc[a + b*x])/b + ((3*I)
*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b - ((2*I)*d*PolyLog[2, (-I)*E^(I
*(a + b*x))])/b^2 + ((2*I)*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - ((3*I)*(
c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b - (3*d*PolyLog[3, -E^(I*(a + b*x))
])/b^2 + (3*d*PolyLog[3, E^(I*(a + b*x))])/b^2))/b + (3*(c + d*x)^2*Sec[a
+ b*x])/(2*b) - ((c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 801 vs. 2(278) = 556.

Time = 0.77 (sec) , antiderivative size = 802, normalized size of antiderivative = 2.63

method	result
risch	$\frac{3icd\,\mathrm{polylog}\left(2,-\mathrm{e}^{i(bx+a)}\right)}{b^2}+\frac{3d^2x^2b\,\mathrm{e}^{5i(bx+a)}+6cdxb\,\mathrm{e}^{5i(bx+a)}+3c^2b\,\mathrm{e}^{5i(bx+a)}-2d^2x^2b\,\mathrm{e}^{3i(bx+a)}-4cdxb\,\mathrm{e}^{3i(bx+a)}-2id^2x\,\mathrm{e}^{5i(bx}}{b^2\left(\mathrm{e}^{2i(b}\right.}$

input
```
int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3/b^2*c*d*ln(1-exp(I*(b*x+a)))*a+1/b^2/(exp(2*I*(b*x+a))-1)^2/(exp(2*I*(b*
x+a))+1)*(3*d^2*x^2*b*exp(5*I*(b*x+a))+6*c*d*x*b*exp(5*I*(b*x+a))+3*c^2*b*
exp(5*I*(b*x+a))-2*d^2*x^2*b*exp(3*I*(b*x+a))-4*c*d*x*b*exp(3*I*(b*x+a))-2
*I*d^2*x*exp(5*I*(b*x+a))-2*c^2*b*exp(3*I*(b*x+a))+3*d^2*x^2*b*exp(I*(b*x+
a))-2*I*c*d*exp(5*I*(b*x+a))+6*c*d*x*b*exp(I*(b*x+a))+3*c^2*b*exp(I*(b*x+a
))+2*I*d^2*x*exp(I*(b*x+a))+2*I*c*d*exp(I*(b*x+a)))-2*I/b^3*d^2*dilog(I*ex
p(I*(b*x+a))+1)+2*I/b^3*d^2*dilog(1-I*exp(I*(b*x+a)))+2/b^2*d^2*ln(I*exp(I
*(b*x+a))+1)*x+2/b^3*d^2*ln(I*exp(I*(b*x+a))+1)*a-2/b^2*d^2*ln(1-I*exp(I*(
b*x+a)))*x-2/b^3*d^2*ln(1-I*exp(I*(b*x+a)))*a+4*I/b^2*d*c*arctan(exp(I*(b*
x+a)))-4*I/b^3*d^2*a*arctan(exp(I*(b*x+a)))+3/2/b^3*d^2*a^2*ln(exp(I*(b*x+
a))-1)-3/2/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/2/b*d^2*ln(1-exp(I*(b*x+a)))*x
^2-3/2/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2-3/b*c*d*ln(exp(I*(b*x+a))+1)*x-3/b
^2*c*d*a*ln(exp(I*(b*x+a))-1)+3/b*c*d*ln(1-exp(I*(b*x+a)))*x+3/2/b*c^2*ln(
exp(I*(b*x+a))-1)-3/2/b*c^2*ln(exp(I*(b*x+a))+1)+1/b^3*d^2*ln(exp(I*(b*x+a
))-1)-1/b^3*d^2*ln(exp(I*(b*x+a))+1)-3*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))
*x+3*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-3*I/b^2*c*d*polylog(2,exp(I*(b
*x+a)))+3*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-3*d^2*polylog(3,-exp(I*(b*x
+a)))/b^3+3*d^2*polylog(3,exp(I*(b*x+a)))/b^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1801 vs. $2(267) = 534$.

Time = 0.17 (sec) , antiderivative size = 1801, normalized size of antiderivative = 5.90

$$\int (c+dx)^2 \csc^3(a+bx) \sec^2(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

output

```
-1/4*(4*b^2*d^2*x^2 + 8*b^2*c*d*x + 4*b^2*c^2 - 6*(b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2)*cos(b*x + a)^2 - 4*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x +
a) + 6*((I*b*d^2*x + I*b*c*d)*cos(b*x + a)^3 + (-I*b*d^2*x - I*b*c*d)*cos(
b*x + a))*dilog(cos(b*x + a) + I*sin(b*x + a)) + 6*((-I*b*d^2*x - I*b*c*d)
*cos(b*x + a)^3 + (I*b*d^2*x + I*b*c*d)*cos(b*x + a))*dilog(cos(b*x + a) -
 I*sin(b*x + a)) + 4*(-I*d^2*cos(b*x + a)^3 + I*d^2*cos(b*x + a))*dilog(I*
cos(b*x + a) + sin(b*x + a)) + 4*(-I*d^2*cos(b*x + a)^3 + I*d^2*cos(b*x +
a))*dilog(I*cos(b*x + a) - sin(b*x + a)) + 4*(I*d^2*cos(b*x + a)^3 - I*d^2
*cos(b*x + a))*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 4*(I*d^2*cos(b*x +
a)^3 - I*d^2*cos(b*x + a))*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 6*((I*b
*d^2*x + I*b*c*d)*cos(b*x + a)^3 + (-I*b*d^2*x - I*b*c*d)*cos(b*x + a))*di
log(-cos(b*x + a) + I*sin(b*x + a)) + 6*((-I*b*d^2*x - I*b*c*d)*cos(b*x +
a)^3 + (I*b*d^2*x + I*b*c*d)*cos(b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x
 + a)) + ((3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*cos(b*x + a)^3
 - (3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*cos(b*x + a))*log(cos
(b*x + a) + I*sin(b*x + a) + 1) + 4*((b*c*d - a*d^2)*cos(b*x + a)^3 - (b*c
*d - a*d^2)*cos(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + I) + ((3*b^2
*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*cos(b*x + a)^3 - (3*b^2*d^2*x^
2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*cos(b*x + a))*log(cos(b*x + a) - I*si
n(b*x + a) + 1) - 4*((b*c*d - a*d^2)*cos(b*x + a)^3 - (b*c*d - a*d^2)*c...
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc^3(a+bx) \sec^2(a+bx)\, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3814 vs. $2(267) = 534$.

Time = 1.12 (sec) , antiderivative size = 3814, normalized size of antiderivative = 12.50

$$\int (c+dx)^2 \csc^3(a+bx) \sec^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

output

```
1/4*(c^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log
(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) - 2*a*c*d*(2*(3*cos(b*x + a)
^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*log(
cos(b*x + a) - 1))/b + a^2*d^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 -
 cos(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1))/b^2 +
4*(8*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(
6*b*x + 6*a) - (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - (b*c*d +
 (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (I*b*c*d + I*(b*x + a)*d^2 - I*
a*d^2)*sin(6*b*x + 6*a) + (-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(4*b*x
 + 4*a) + (-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(2*b*x + 2*a))*arctan2
(cos(b*x + a), sin(b*x + a) + 1) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c
*d + (b*x + a)*d^2 - a*d^2)*cos(6*b*x + 6*a) - (b*c*d + (b*x + a)*d^2 - a*
d^2)*cos(4*b*x + 4*a) - (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) +
 (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(6*b*x + 6*a) + (-I*b*c*d - I*(b
*x + a)*d^2 + I*a*d^2)*sin(4*b*x + 4*a) + (-I*b*c*d - I*(b*x + a)*d^2 + I*
a*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 2*(3*(
b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + (3*(b*x + a)^2*d^2
+ 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(6*b*x + 6*a) - (3*(b*x + a)^2*d
^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) - (3*(b*x + a)^
2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*a) - (-3*I*(...
```

Giac [F]

$$\int (c+dx)^2 \csc^3(a+bx) \sec^2(a+bx)\,dx = \int (dx+c)^2 \csc(bx+a)^3 \sec(bx+a)^2\,dx$$

input

```
integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

output

```
integrate((d*x + c)^2*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc^3(a+bx) \sec^2(a+bx)\, dx = \text{Hanged}$$

input
```
int((c + d*x)^2/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^2 \csc^3(a+bx) \sec^2(a+bx)\, dx$$
$$= \frac{8\cos(bx+a)\left(\int \csc(bx+a)^3 \sec(bx+a)^2 x^2 dx\right)\sin(bx+a)^2 b\, d^2 + 16\cos(bx+a)\left(\int \csc(bx+a)^3 \sec}{}$$

input
```
int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

output
```
(8*cos(a + b*x)*int(csc(a + b*x)**3*sec(a + b*x)**2*x**2,x)*sin(a + b*x)**
2*b*d**2 + 16*cos(a + b*x)*int(csc(a + b*x)**3*sec(a + b*x)**2*x,x)*sin(a
 + b*x)**2*b*c*d + 12*cos(a + b*x)*log(tan((a + b*x)/2))*sin(a + b*x)**2*c*
*2 - 9*cos(a + b*x)*sin(a + b*x)**2*c**2 + 12*sin(a + b*x)**2*c**2 - 4*c**
2)/(8*cos(a + b*x)*sin(a + b*x)**2*b)
```

3.281 $\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal result . . . 2438
Mathematica [B] (verified) . . . 2439
Rubi [A] (verified) . . . 2440
Maple [A] (verified) . . . 2441
Fricas [B] (verification not implemented) . . . 2442
Sympy [F] . . . 2443
Maxima [B] (verification not implemented) . . . 2443
Giac [F] . . . 2444
Mupad [F(-1)] . . . 2445
Reduce [F] . . . 2445

Optimal result

Integrand size = 22, antiderivative size = 154

$$\begin{aligned}
&\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx \\
&= -\frac{3dx\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{3c\operatorname{arctanh}(\cos(a+bx))}{2b} - \frac{d\operatorname{arctanh}(\sin(a+bx))}{b^2} \\
&\quad - \frac{d\csc(a+bx)}{2b^2} + \frac{3id\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2} - \frac{3id\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2} \\
&\quad + \frac{3(c+dx)\sec(a+bx)}{2b} - \frac{(c+dx)\csc^2(a+bx)\sec(a+bx)}{2b}
\end{aligned}$$

output

```
-3*d*x*arctanh(exp(I*(b*x+a)))/b-3/2*c*arctanh(cos(b*x+a))/b-d*arctanh(sin
(b*x+a))/b^2-1/2*d*csc(b*x+a)/b^2+3/2*I*d*polylog(2,-exp(I*(b*x+a)))/b^2-3
/2*I*d*polylog(2,exp(I*(b*x+a)))/b^2+3/2*(d*x+c)*sec(b*x+a)/b-1/2*(d*x+c)*
csc(b*x+a)^2*sec(b*x+a)/b
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 520 vs. $2(154) = 308$.

Time = 3.94 (sec) , antiderivative size = 520, normalized size of antiderivative = 3.38

$$\begin{aligned}
\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx &= \frac{dx}{b} - \frac{d\cot\left(\frac{1}{2}(a+bx)\right)}{4b^2} \\
&- \frac{c\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{dx\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{3c\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b} \\
&+ \frac{d\log\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} + \frac{3c\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b} \\
&- \frac{d\log\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} - \frac{3ad\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)}{2b^2} \\
&+ \frac{3d\big((a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)+i\big(\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)-\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)\big)}{2b^2} \\
&+ \frac{c\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{dx\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b} \\
&+ \frac{c\sin\left(\frac{1}{2}(a+bx)\right)}{b\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)} - \frac{c\sin\left(\frac{1}{2}(a+bx)\right)}{b\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)} \\
&+ \frac{d\big(a\sin\left(\frac{1}{2}(a+bx)\right)-(a+bx)\sin\left(\frac{1}{2}(a+bx)\right)\big)}{b^2\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)} \\
&+ \frac{d\big(-a\sin\left(\frac{1}{2}(a+bx)\right)+(a+bx)\sin\left(\frac{1}{2}(a+bx)\right)\big)}{b^2\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)} - \frac{d\tan\left(\frac{1}{2}(a+bx)\right)}{4b^2}
\end{aligned}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output

```
(d*x)/b - (d*Cot[(a + b*x)/2])/(4*b^2) - (c*Csc[(a + b*x)/2]^2)/(8*b) - (d
*x*Csc[(a + b*x)/2]^2)/(8*b) - (3*c*Log[Cos[(a + b*x)/2]])/(2*b) + (d*Log[
Cos[(a + b*x)/2] - Sin[(a + b*x)/2]])/b^2 + (3*c*Log[Sin[(a + b*x)/2]])/(2
*b) - (d*Log[Cos[(a + b*x)/2] + Sin[(a + b*x)/2]])/b^2 - (3*a*d*Log[Tan[(a
 + b*x)/2]])/(2*b^2) + (3*d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 +
 E^(I*(a + b*x))]) + I*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a
+ b*x))])))/(2*b^2) + (c*Sec[(a + b*x)/2]^2)/(8*b) + (d*x*Sec[(a + b*x)/2]
^2)/(8*b) + (c*Sin[(a + b*x)/2])/(b*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2]))
 - (c*Sin[(a + b*x)/2])/(b*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])) + (d*(a*
Sin[(a + b*x)/2] - (a + b*x)*Sin[(a + b*x)/2]))/(b^2*(Cos[(a + b*x)/2] + S
in[(a + b*x)/2])) + (d*(-(a*Sin[(a + b*x)/2]) + (a + b*x)*Sin[(a + b*x)/2]
))/(b^2*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])) - (d*Tan[(a + b*x)/2])/(4*b
^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4920, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx$$

$$\downarrow 4920$$

$$-d\int\left(-\frac{\sec(a+bx)\csc^2(a+bx)}{2b}-\frac{3\operatorname{arctanh}(\cos(a+bx))}{2b}+\frac{3\sec(a+bx)}{2b}\right)dx-\frac{3(c+dx)\operatorname{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)\sec(a+bx)}{2b}-\frac{(c+dx)\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 2009$$

$$-d\left(\frac{\operatorname{arctanh}(\sin(a+bx))}{b^2}+\frac{3x\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{3x\operatorname{arctanh}(\cos(a+bx))}{2b}-\frac{3i\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}+\frac{3i\,\mathrm{P}}{}\right.$$

$$\frac{3(c+dx)\operatorname{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)\sec(a+bx)}{2b}-\frac{(c+dx)\csc^2(a+bx)\sec(a+bx)}{2b}$$

input
```
Int[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
(-3*(c + d*x)*ArcTanh[Cos[a + b*x]])/(2*b) - d*((3*x*ArcTanh[E^(I*(a + b*x
))])/b - (3*x*ArcTanh[Cos[a + b*x]])/(2*b) + ArcTanh[Sin[a + b*x]]/b^2 + C
sc[a + b*x]/(2*b^2) - (((3*I)/2)*PolyLog[2, -E^(I*(a + b*x))])/b^2 + (((3*
I)/2)*PolyLog[2, E^(I*(a + b*x))])/b^2) + (3*(c + d*x)*Sec[a + b*x])/(2*b)
 - ((c + d*x)*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.73

method	result
risch	$\frac{3bdx\,e^{5i(bx+a)}+3cb\,e^{5i(bx+a)}-2bdx\,e^{3i(bx+a)}-2cb\,e^{3i(bx+a)}-id\,e^{5i(bx+a)}+3bdx\,e^{i(bx+a)}+3bc\,e^{i(bx+a)}+id\,e^{i(bx+a)}}{b^2\left(e^{2i(bx+a)}-1\right)^2\left(e^{2i(bx+a)}+1\right)}+\frac{3id\,\mathrm{dilog}(e}{2}$

input
```
int((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^2/(exp(2*I*(b*x+a))-1)^2/(exp(2*I*(b*x+a))+1)*(3*b*d*x*exp(5*I*(b*x+a)
)+3*c*b*exp(5*I*(b*x+a))-2*b*d*x*exp(3*I*(b*x+a))-2*c*b*exp(3*I*(b*x+a))-I
*d*exp(5*I*(b*x+a))+3*b*d*x*exp(I*(b*x+a))+3*b*c*exp(I*(b*x+a))+I*d*exp(I*
(b*x+a)))+3/2*I/b^2*d*dilog(exp(I*(b*x+a))+1)+3/2*I/b^2*d*dilog(exp(I*(b*x
+a)))-3/2/b*d*ln(exp(I*(b*x+a))+1)*x-3/2/b^2*d*a*ln(exp(I*(b*x+a))-1)+2*I/
b^2*d*arctan(exp(I*(b*x+a)))+3/2/b*c*ln(exp(I*(b*x+a))-1)-3/2/b*c*ln(exp(I
*(b*x+a))+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(130) = 260$.

Time $= 0.11$ (sec) , antiderivative size $= 621$, normalized size of antiderivative $= 4.03$

$$\int (c + dx) \csc^3(a + bx) \sec^2(a + bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*b*d*x - 6*(b*d*x + b*c)*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x
+ a) + 4*b*c + 3*(I*d*cos(b*x + a)^3 - I*d*cos(b*x + a))*dilog(cos(b*x + a
) + I*sin(b*x + a)) + 3*(-I*d*cos(b*x + a)^3 + I*d*cos(b*x + a))*dilog(cos
(b*x + a) - I*sin(b*x + a)) + 3*(I*d*cos(b*x + a)^3 - I*d*cos(b*x + a))*di
log(-cos(b*x + a) + I*sin(b*x + a)) + 3*(-I*d*cos(b*x + a)^3 + I*d*cos(b*x
 + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) + 3*((b*d*x + b*c)*cos(b*x +
a)^3 - (b*d*x + b*c)*cos(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + 1)
+ 3*((b*d*x + b*c)*cos(b*x + a)^3 - (b*d*x + b*c)*cos(b*x + a))*log(cos(b*
x + a) - I*sin(b*x + a) + 1) - 3*((b*c - a*d)*cos(b*x + a)^3 - (b*c - a*d)
*cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 3*((b*c
 - a*d)*cos(b*x + a)^3 - (b*c - a*d)*cos(b*x + a))*log(-1/2*cos(b*x + a) -
 1/2*I*sin(b*x + a) + 1/2) - 3*((b*d*x + a*d)*cos(b*x + a)^3 - (b*d*x + a*
d)*cos(b*x + a))*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 3*((b*d*x + a*d
)*cos(b*x + a)^3 - (b*d*x + a*d)*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b
*x + a) + 1) + 2*(d*cos(b*x + a)^3 - d*cos(b*x + a))*log(sin(b*x + a) + 1)
 - 2*(d*cos(b*x + a)^3 - d*cos(b*x + a))*log(-sin(b*x + a) + 1))/(b^2*cos(
b*x + a)^3 - b^2*cos(b*x + a))
```

Sympy [F]

$$\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx = \int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*csc(a + b*x)**3*sec(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1481 vs. $2(130) = 260$.

Time = 0.82 (sec) , antiderivative size = 1481, normalized size of antiderivative = 9.62

$$\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

output

```
-(4*(d*cos(6*b*x + 6*a) - d*cos(4*b*x + 4*a) - d*cos(2*b*x + 2*a) + I*d*si
n(6*b*x + 6*a) - I*d*sin(4*b*x + 4*a) - I*d*sin(2*b*x + 2*a) + d)*arctan2(
2*(cos(b*x + 2*a)*cos(a) + sin(b*x + 2*a)*sin(a))/(cos(b*x + 2*a)^2 + cos(
a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a
) + sin(a)^2), (cos(b*x + 2*a)^2 - cos(a)^2 + sin(b*x + 2*a)^2 - sin(a)^2)
/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2
 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) + 6*(b*d*x + b*c + (b*d*x + b*c)*c
os(6*b*x + 6*a) - (b*d*x + b*c)*cos(4*b*x + 4*a) - (b*d*x + b*c)*cos(2*b*x
 + 2*a) + (I*b*d*x + I*b*c)*sin(6*b*x + 6*a) + (-I*b*d*x - I*b*c)*sin(4*b*
x + 4*a) + (-I*b*d*x - I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(
b*x + a) + 1) - 6*(b*c*cos(6*b*x + 6*a) - b*c*cos(4*b*x + 4*a) - b*c*cos(2
*b*x + 2*a) + I*b*c*sin(6*b*x + 6*a) - I*b*c*sin(4*b*x + 4*a) - I*b*c*sin(
2*b*x + 2*a) + b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 6*(b*d*x*cos
(6*b*x + 6*a) - b*d*x*cos(4*b*x + 4*a) - b*d*x*cos(2*b*x + 2*a) + I*b*d*x*
sin(6*b*x + 6*a) - I*b*d*x*sin(4*b*x + 4*a) - I*b*d*x*sin(2*b*x + 2*a) + b
*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 4*(3*I*b*d*x + 3*I*b*c +
d)*cos(5*b*x + 5*a) + 8*(-I*b*d*x - I*b*c)*cos(3*b*x + 3*a) + 4*(3*I*b*d*x
 + 3*I*b*c - d)*cos(b*x + a) - 6*(d*cos(6*b*x + 6*a) - d*cos(4*b*x + 4*a)
- d*cos(2*b*x + 2*a) + I*d*sin(6*b*x + 6*a) - I*d*sin(4*b*x + 4*a) - I*d*s
in(2*b*x + 2*a) + d)*dilog(-e^(I*b*x + I*a)) + 6*(d*cos(6*b*x + 6*a) - ...
```

Giac [F]

$$\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)\csc\left(bx+a\right)^3\sec\left(bx+a\right)^2\,dx$$

input

```
integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

output

```
integrate((d*x + c)*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{8\cos(bx+a)\left(\int \csc(bx+a)^3\sec(bx+a)^2\,xdx\right)\sin(bx+a)^2\,bd + 12\cos(bx+a)\log\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sin($$
$$8\cos(bx+a)\sin(bx+a)^2\,b$$

input
```
int((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

output
```
(8*cos(a + b*x)*int(csc(a + b*x)**3*sec(a + b*x)**2*x,x)*sin(a + b*x)**2*b
*d + 12*cos(a + b*x)*log(tan((a + b*x)/2))*sin(a + b*x)**2*c - 9*cos(a + b
*x)*sin(a + b*x)**2*c + 12*sin(a + b*x)**2*c - 4*c)/(8*cos(a + b*x)*sin(a
+ b*x)**2*b)
```

3.282 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$

Optimal result . . . 2446
Mathematica [N/A] . . . 2446
Rubi [N/A] . . . 2447
Maple [N/A] . . . 2447
Fricas [N/A] . . . 2448
Sympy [N/A] . . . 2448
Maxima [N/A] . . . 2449
Giac [N/A] . . . 2450
Mupad [N/A] . . . 2450
Reduce [N/A] . . . 2450

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}, x\right)$$

output
```
Defer(Int)(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 15.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x),x]
```

output
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$
$$\downarrow 7299$$
$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}dx$$

input
```
Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc\left(bx+a\right)^3\sec\left(bx+a\right)^2}{dx+c}dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x)
```

output
```
int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc\left(bx+a\right)^3\sec\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^3*sec(b*x + a)^2/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 7.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc^3\left(a+bx\right)\sec^2\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)**3*sec(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(csc(a + b*x)**3*sec(a + b*x)**2/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 7.90 (sec) , antiderivative size = 3697, normalized size of antiderivative = 154.04

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
(2*(b*d*x + b*c)*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) + (3*(b*d*x + b*c)*cos(
5*b*x + 5*a) - 2*(b*d*x + b*c)*cos(3*b*x + 3*a) + 3*(b*d*x + b*c)*cos(b*x
+ a) - d*sin(5*b*x + 5*a) + d*sin(b*x + a))*cos(6*b*x + 6*a) + (3*b*d*x +
3*b*c - 3*(b*d*x + b*c)*cos(4*b*x + 4*a) - 3*(b*d*x + b*c)*cos(2*b*x + 2*a
) - d*sin(4*b*x + 4*a) - d*sin(2*b*x + 2*a))*cos(5*b*x + 5*a) + (2*(b*d*x
+ b*c)*cos(3*b*x + 3*a) - 3*(b*d*x + b*c)*cos(b*x + a) - d*sin(b*x + a))*c
os(4*b*x + 4*a) - 2*(b*d*x + b*c - (b*d*x + b*c)*cos(2*b*x + 2*a))*cos(3*b
*x + 3*a) - (3*(b*d*x + b*c)*cos(b*x + a) + d*sin(b*x + a))*cos(2*b*x + 2*
a) + 3*(b*d*x + b*c)*cos(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 +
 (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(6*b*x + 6*a)^2 + (b^2*d^2*x^2 +
 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(6*
b*x + 6*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 +
2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a)
+ (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*
x + 4*a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(6*b
*x + 6*a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^2*d^2*x^2 + 2*b^2*
c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + ...
```

Giac [N/A]

Not integrable

Time = 120.90 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3*sec(b*x + a)^2/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 20.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos(a+b\,x)^2\sin(a+b\,x)^3\,(c+d\,x)}\,dx$$

input
```
int(1/(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^2}{dx+c}\,dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x)
```

output
```
int((csc(a + b*x)**3*sec(a + b*x)**2)/(c + d*x),x)
```

3.283 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 2452
Mathematica [N/A] . . . 2452
Rubi [N/A] . . . 2453
Maple [N/A] . . . 2453
Fricas [N/A] . . . 2454
Sympy [N/A] . . . 2454
Maxima [N/A] . . . 2455
Giac [F(-1)] . . . 2456
Mupad [N/A] . . . 2456
Reduce [N/A] . . . 2456

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 19.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$
$$\downarrow 7299$$
$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc(bx+a)^3\sec(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)^3\sec\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^3*sec(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 14.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^3\left(a+bx\right)\sec^2\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)**3*sec(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)**3*sec(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 21.66 (sec) , antiderivative size = 4757, normalized size of antiderivative = 198.21

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
(2*(b*d*x + b*c)*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) + (3*(b*d*x + b*c)*cos(
5*b*x + 5*a) - 2*(b*d*x + b*c)*cos(3*b*x + 3*a) + 3*(b*d*x + b*c)*cos(b*x
+ a) - 2*d*sin(5*b*x + 5*a) + 2*d*sin(b*x + a))*cos(6*b*x + 6*a) + (3*b*d*
x + 3*b*c - 3*(b*d*x + b*c)*cos(4*b*x + 4*a) - 3*(b*d*x + b*c)*cos(2*b*x +
 2*a) - 2*d*sin(4*b*x + 4*a) - 2*d*sin(2*b*x + 2*a))*cos(5*b*x + 5*a) + (2
*(b*d*x + b*c)*cos(3*b*x + 3*a) - 3*(b*d*x + b*c)*cos(b*x + a) - 2*d*sin(b
*x + a))*cos(4*b*x + 4*a) - 2*(b*d*x + b*c - (b*d*x + b*c)*cos(2*b*x + 2*a
))*cos(3*b*x + 3*a) - (3*(b*d*x + b*c)*cos(b*x + a) + 2*d*sin(b*x + a))*co
s(2*b*x + 2*a) + 3*(b*d*x + b*c)*cos(b*x + a) + (b^2*d^3*x^3 + 3*b^2*c*d^2
*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^
2*d*x + b^2*c^3)*cos(6*b*x + 6*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
+ 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2
*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(6*b*x + 6*a)^2 + (b^2*d^3*x^3 + 3*b^2*
c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 + 2*(b^2*d^3*x^3 +
 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2
*a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x
+ 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - (b
^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a) -
 (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + ...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 20.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\cos(a+b\,x)^2\sin(a+b\,x)^3(c+d\,x)^2}\,dx$$

input
```
int(1/(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^2}{d^2x^2+2cdx+c^2}dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int((csc(a + b*x)**3*sec(a + b*x)**2)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.284 $\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal result . . . 2457
Mathematica [N/A] . . . 2457
Rubi [N/A] . . . 2458
Maple [N/A] . . . 2458
Fricas [N/A] . . . 2459
Sympy [F(-1)] . . . 2459
Maxima [N/A] . . . 2459
Giac [N/A] . . . 2460
Mupad [N/A] . . . 2460
Reduce [N/A] . . . 2461

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \text{Int}\left(x^m \csc^3(a+bx)\sec^2(a+bx), x\right)$$

output
```
Defer(Int)(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 26.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx$$

input
```
Integrate[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
Integrate[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx$$

$$\downarrow 7299$$

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)dx$$

input
```
Int[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \csc\left(bx+a\right)^3 \sec\left(bx+a\right)^2 dx$$

input
```
int(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

output
```
int(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \int x^m \csc(bx+a)^3 \sec(bx+a)^2\,dx$$

input
```
integrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x**m*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \int x^m \csc(bx+a)^3 \sec(bx+a)^2\,dx$$

input
```
integrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(x^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \int x^m \csc\left(bx+a\right)^3 \sec\left(bx+a\right)^2\,dx$$

input
```
integrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 20.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \int \frac{x^m}{\cos\left(a+b\,x\right)^2 \sin\left(a+b\,x\right)^3}\,dx$$

input
```
int(x^m/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

output
```
int(x^m/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \int x^m \csc(bx+a)^3 \sec(bx+a)^2\,dx$$

input
```
int(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

output
```
int(x**m*csc(a + b*x)**3*sec(a + b*x)**2,x)
```

3.285 $\int x^3 \csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal result . . . 2463
Mathematica [A] (warning: unable to verify) . . . 2464
Rubi [A] (verified) . . . 2465
Maple [F] . . . 2467
Fricas [B] (verification not implemented) . . . 2467
Sympy [F(-1)] . . . 2468
Maxima [B] (verification not implemented) . . . 2469
Giac [F] . . . 2470
Mupad [F(-1)] . . . 2470
Reduce [F] . . . 2470

Optimal result

Integrand size = 20, antiderivative size = 387

$$\begin{aligned}\int x^3 \csc^3(a+bx)\sec^2(a+bx)\,dx = {} & \frac{6ix^2\arctan\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b^3} \\ & - \frac{3x^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{3x^2\csc(a+bx)}{2b^2} \\ & + \frac{3i\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^4} \\ & + \frac{9ix^2\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2} \\ & - \frac{6ix\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} \\ & + \frac{6ix\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} \\ & - \frac{3i\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^4} \\ & - \frac{9ix^2\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2} \\ & - \frac{9x\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} \\ & + \frac{6\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4} \\ & - \frac{6\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4} + \frac{9x\,\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} \\ & - \frac{9i\,\mathrm{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4} \\ & + \frac{9i\,\mathrm{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4} + \frac{3x^3\sec(a+bx)}{2b} \\ & - \frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b}\end{aligned}$$

output

```
3*I*polylog(2,-exp(I*(b*x+a)))/b^4-6*x*arctanh(exp(I*(b*x+a)))/b^3-3*x^3*a
rctanh(exp(I*(b*x+a)))/b-3/2*x^2*csc(b*x+a)/b^2-3*I*polylog(2,exp(I*(b*x+a
)))/b^4+9/2*I*x^2*polylog(2,-exp(I*(b*x+a)))/b^2+6*I*x*polylog(2,I*exp(I*(
b*x+a)))/b^3-9*I*polylog(4,-exp(I*(b*x+a)))/b^4+9*I*polylog(4,exp(I*(b*x+a
)))/b^4-9/2*I*x^2*polylog(2,exp(I*(b*x+a)))/b^2-9*x*polylog(3,-exp(I*(b*x+
a)))/b^3+6*polylog(3,-I*exp(I*(b*x+a)))/b^4-6*polylog(3,I*exp(I*(b*x+a)))/
b^4+9*x*polylog(3,exp(I*(b*x+a)))/b^3-6*I*x*polylog(2,-I*exp(I*(b*x+a)))/b
^3+6*I*x^2*arctan(exp(I*(b*x+a)))/b^2+3/2*x^3*sec(b*x+a)/b-1/2*x^3*csc(b*x
+a)^2*sec(b*x+a)/b
```

Mathematica [A] (warning: unable to verify)

Time = 6.74 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.74

$$\int x^3 \csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{x^3 \csc^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}$$
$$+\frac{6(ib^2x^2 \arctan(\cos(a+bx)+i\sin(a+bx)) + ibx\,\mathrm{PolyLog}(2, i\cos(a+bx)-\sin(a+bx)) - ibx\,\mathrm{PolyL}}{}$$
$$+\frac{3(2bx\log(1-\cos(a+bx)-i\sin(a+bx)) + b^3x^3\log(1-\cos(a+bx)-i\sin(a+bx)) - 2bx\log(1+c}{}$$
$$+\frac{x^3\sec^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} + \frac{x^2\csc(a)\sec(a)(-3\cos(a)+2bx\sin(a))}{2b^2}$$
$$+\frac{3x^2\csc\left(\frac{a}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{bx}{2}\right)}{4b^2} - \frac{3x^2\sec\left(\frac{a}{2}\right)\sec\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{bx}{2}\right)}{4b^2}$$
$$+\frac{x^3\sin\left(\frac{bx}{2}\right)}{b\left(\cos\left(\frac{a}{2}\right)-\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}$$
$$-\frac{x^3\sin\left(\frac{bx}{2}\right)}{b\left(\cos\left(\frac{a}{2}\right)+\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}$$

input

```
Integrate[x^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output

```
-1/8*(x^3*Csc[a/2 + (b*x)/2]^2)/b + (6*(I*b^2*x^2*ArcTan[Cos[a + b*x] + I*
Sin[a + b*x]] + I*b*x*PolyLog[2, I*Cos[a + b*x] - Sin[a + b*x]] - I*b*x*Po
lyLog[2, (-I)*Cos[a + b*x] + Sin[a + b*x]] - PolyLog[3, I*Cos[a + b*x] - S
in[a + b*x]] + PolyLog[3, (-I)*Cos[a + b*x] + Sin[a + b*x]]))/b^4 + (3*(2*
b*x*Log[1 - Cos[a + b*x] - I*Sin[a + b*x]] + b^3*x^3*Log[1 - Cos[a + b*x]
 - I*Sin[a + b*x]] - 2*b*x*Log[1 + Cos[a + b*x] + I*Sin[a + b*x]] - b^3*x^3
*Log[1 + Cos[a + b*x] + I*Sin[a + b*x]] + I*(2 + 3*b^2*x^2)*PolyLog[2, -Co
s[a + b*x] - I*Sin[a + b*x]] - I*(2 + 3*b^2*x^2)*PolyLog[2, Cos[a + b*x] +
 I*Sin[a + b*x]] - 6*b*x*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] + 6*b*
x*PolyLog[3, Cos[a + b*x] + I*Sin[a + b*x]] - (6*I)*PolyLog[4, -Cos[a + b*
x] - I*Sin[a + b*x]] + (6*I)*PolyLog[4, Cos[a + b*x] + I*Sin[a + b*x]]))/(
2*b^4) + (x^3*Sec[a/2 + (b*x)/2]^2)/(8*b) + (x^2*Csc[a]*Sec[a]*(-3*Cos[a]
 + 2*b*x*Sin[a]))/(2*b^2) + (3*x^2*Csc[a/2]*Csc[a/2 + (b*x)/2]*Sin[(b*x)/2]
)/(4*b^2) - (3*x^2*Sec[a/2]*Sec[a/2 + (b*x)/2]*Sin[(b*x)/2])/(4*b^2) + (x^
3*Sin[(b*x)/2])/(b*(Cos[a/2] - Sin[a/2])*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (
b*x)/2])) - (x^3*Sin[(b*x)/2])/(b*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2
] + Sin[a/2 + (b*x)/2]))
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4920, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \csc^3(a+bx)\sec^2(a+bx)\,dx$$

$$\downarrow 4920$$

$$-3\int -\frac{1}{2}x^2\left(\frac{\sec(a+bx)\csc^2(a+bx)}{b}+\frac{3\operatorname{arctanh}(\cos(a+bx))}{b}-\frac{3\sec(a+bx)}{b}\right)dx-\frac{3x^3\operatorname{arctanh}(\cos(a+bx))}{2b}+\frac{3x^3\sec(a+bx)}{2b}-\frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 27$$

$$\frac{3}{2}\int x^2\left(\frac{\sec(a+bx)\csc^2(a+bx)}{b}+\frac{3\operatorname{arctanh}(\cos(a+bx))}{b}-\frac{3\sec(a+bx)}{b}\right)dx-\frac{3x^3\operatorname{arctanh}(\cos(a+bx))}{2b}+\frac{3x^3\sec(a+bx)}{2b}-\frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 2010$$

$$\frac{3}{2}\int\left(\frac{3\operatorname{arctanh}(\cos(a+bx))x^2}{b}+\frac{(\csc^2(a+bx)-3)\sec(a+bx)x^2}{b}\right)dx-\frac{3x^3\operatorname{arctanh}(\cos(a+bx))}{2b}+\frac{3x^3\sec(a+bx)}{2b}-\frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 2009$$

$$\frac{3}{2}\left(\frac{4ix^2\arctan\left(e^{i(a+bx)}\right)}{b^2}-\frac{4x\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b^3}-\frac{2x^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{x^3\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{2i\,\mathrm{PolyL}}{}\right.$$
$$\left.-\frac{3x^3\operatorname{arctanh}(\cos(a+bx))}{2b}+\frac{3x^3\sec(a+bx)}{2b}-\frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b}\right)$$

input

```
Int[x^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output

```
(-3*x^3*ArcTanh[Cos[a + b*x]])/(2*b) + (3*(((4*I)*x^2*ArcTan[E^(I*(a + b*x
))])/b^2 - (4*x*ArcTanh[E^(I*(a + b*x))])/b^3 - (2*x^3*ArcTanh[E^(I*(a + b
*x))])/b + (x^3*ArcTanh[Cos[a + b*x]])/b - (x^2*Csc[a + b*x])/b^2 + ((2*I)
*PolyLog[2, -E^(I*(a + b*x))])/b^4 + ((3*I)*x^2*PolyLog[2, -E^(I*(a + b*x)
)])/b^2 - ((4*I)*x*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((4*I)*x*PolyLo
g[2, I*E^(I*(a + b*x))])/b^3 - ((2*I)*PolyLog[2, E^(I*(a + b*x))])/b^4 - (
(3*I)*x^2*PolyLog[2, E^(I*(a + b*x))])/b^2 - (6*x*PolyLog[3, -E^(I*(a + b*
x))])/b^3 + (4*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (4*PolyLog[3, I*E^(
I*(a + b*x))])/b^4 + (6*x*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((6*I)*PolyLo
g[4, -E^(I*(a + b*x))])/b^4 + ((6*I)*PolyLog[4, E^(I*(a + b*x))])/b^4))/2
+ (3*x^3*Sec[a + b*x])/(2*b) - (x^3*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Maple [F]

$$\int x^3 \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

input
```
int(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

output
```
int(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1747 vs. $2(315) = 630$.

Time = 0.17 (sec) , antiderivative size = 1747, normalized size of antiderivative = 4.51

$$\int x^3 \csc^3(a+bx) \sec^2(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(6*b^3*x^3*cos(b*x + a)^2 - 4*b^3*x^3 + 6*b^2*x^2*cos(b*x + a)*sin(b*x
 + a) - 3*((3*I*b^2*x^2 + 2*I)*cos(b*x + a)^3 + (-3*I*b^2*x^2 - 2*I)*cos(b
*x + a))*dilog(cos(b*x + a) + I*sin(b*x + a)) - 3*((-3*I*b^2*x^2 - 2*I)*co
s(b*x + a)^3 + (3*I*b^2*x^2 + 2*I)*cos(b*x + a))*dilog(cos(b*x + a) - I*si
n(b*x + a)) - 12*(-I*b*x*cos(b*x + a)^3 + I*b*x*cos(b*x + a))*dilog(I*cos(
b*x + a) + sin(b*x + a)) - 12*(-I*b*x*cos(b*x + a)^3 + I*b*x*cos(b*x + a))
*dilog(I*cos(b*x + a) - sin(b*x + a)) - 12*(I*b*x*cos(b*x + a)^3 - I*b*x*c
os(b*x + a))*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 12*(I*b*x*cos(b*x + a
)^3 - I*b*x*cos(b*x + a))*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*((3*I*
b^2*x^2 + 2*I)*cos(b*x + a)^3 + (-3*I*b^2*x^2 - 2*I)*cos(b*x + a))*dilog(-
cos(b*x + a) + I*sin(b*x + a)) - 3*((-3*I*b^2*x^2 - 2*I)*cos(b*x + a)^3 +
(3*I*b^2*x^2 + 2*I)*cos(b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) -
3*((b^3*x^3 + 2*b*x)*cos(b*x + a)^3 - (b^3*x^3 + 2*b*x)*cos(b*x + a))*log(
cos(b*x + a) + I*sin(b*x + a) + 1) - 6*(a^2*cos(b*x + a)^3 - a^2*cos(b*x +
 a))*log(cos(b*x + a) + I*sin(b*x + a) + I) - 3*((b^3*x^3 + 2*b*x)*cos(b*x
 + a)^3 - (b^3*x^3 + 2*b*x)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a
) + 1) + 6*(a^2*cos(b*x + a)^3 - a^2*cos(b*x + a))*log(cos(b*x + a) - I*si
n(b*x + a) + I) - 6*((b^2*x^2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 - a^2)*cos(
b*x + a))*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 6*((b^2*x^2 - a^2)*cos(
b*x + a)^3 - (b^2*x^2 - a^2)*cos(b*x + a))*log(I*cos(b*x + a) - sin(b*x...
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \csc^3(a+bx) \sec^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x**3*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3940 vs. $2(315) = 630$.

Time = 0.81 (sec) , antiderivative size = 3940, normalized size of antiderivative = 10.18

$$\int x^3 \csc^3(a+bx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/4*(a^3*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*lo
g(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) - 4*(12*((b*x + a)^2 - 2*(b
*x + a)*a + a^2 + ((b*x + a)^2 - 2*(b*x + a)*a + a^2)*cos(6*b*x + 6*a) - (
(b*x + a)^2 - 2*(b*x + a)*a + a^2)*cos(4*b*x + 4*a) - ((b*x + a)^2 - 2*(b*
x + a)*a + a^2)*cos(2*b*x + 2*a) + (I*(b*x + a)^2 - 2*I*(b*x + a)*a + I*a^
2)*sin(6*b*x + 6*a) + (-I*(b*x + a)^2 + 2*I*(b*x + a)*a - I*a^2)*sin(4*b*x
 + 4*a) + (-I*(b*x + a)^2 + 2*I*(b*x + a)*a - I*a^2)*sin(2*b*x + 2*a))*arc
tan2(cos(b*x + a), sin(b*x + a) + 1) + 12*((b*x + a)^2 - 2*(b*x + a)*a + a
^2 + ((b*x + a)^2 - 2*(b*x + a)*a + a^2)*cos(6*b*x + 6*a) - ((b*x + a)^2 -
 2*(b*x + a)*a + a^2)*cos(4*b*x + 4*a) - ((b*x + a)^2 - 2*(b*x + a)*a + a^
2)*cos(2*b*x + 2*a) + (I*(b*x + a)^2 - 2*I*(b*x + a)*a + I*a^2)*sin(6*b*x
+ 6*a) + (-I*(b*x + a)^2 + 2*I*(b*x + a)*a - I*a^2)*sin(4*b*x + 4*a) + (-I
*(b*x + a)^2 + 2*I*(b*x + a)*a - I*a^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x
+ a), -sin(b*x + a) + 1) - 6*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*
(b*x + a) + ((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a) - 2*a)*
cos(6*b*x + 6*a) - ((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a)
- 2*a)*cos(4*b*x + 4*a) - ((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*
x + a) - 2*a)*cos(2*b*x + 2*a) - (-I*(b*x + a)^3 + 3*I*(b*x + a)^2*a + (-3
*I*a^2 - 2*I)*(b*x + a) + 2*I*a)*sin(6*b*x + 6*a) - (I*(b*x + a)^3 - 3*I*(
b*x + a)^2*a + (3*I*a^2 + 2*I)*(b*x + a) - 2*I*a)*sin(4*b*x + 4*a) - (I...
```

Giac [F]

$$\int x^3 \csc^3(a+bx)\sec^2(a+bx)\,dx = \int x^3 \csc(bx+a)^3 \sec(bx+a)^2\,dx$$

input
```
integrate(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^3*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \csc^3(a+bx)\sec^2(a+bx)\,dx = \int \frac{x^3}{\cos(a+b\,x)^2 \sin(a+b\,x)^3}\,dx$$

input
```
int(x^3/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

output
```
int(x^3/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

Reduce [F]

$$\int x^3 \csc^3(a+bx)\sec^2(a+bx)\,dx = \int \csc(bx+a)^3 \sec(bx+a)^2\,x^3 dx$$

input
```
int(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

output
```
int(csc(a + b*x)**3*sec(a + b*x)**2*x**3,x)
```

3.286 $\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal result . . . 2471
Mathematica [B] (warning: unable to verify) . . . 2472
Rubi [A] (verified) . . . 2473
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Giac [F] . . . 2478
Mupad [F(-1)] . . . 2479
Reduce [F] . . . 2479

Optimal result

Integrand size = 20, antiderivative size = 235

$$\begin{aligned}\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx = {} & \frac{4ix\arctan\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} \\ & - \frac{\operatorname{arctanh}(\cos(a+bx))}{b^3} - \frac{x\csc(a+bx)}{b^2} \\ & + \frac{3ix\,\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} \\ & - \frac{2i\,\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} \\ & + \frac{2i\,\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} \\ & - \frac{3ix\,\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} \\ & - \frac{3\,\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} + \frac{3\,\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} \\ & + \frac{3x^2\sec(a+bx)}{2b} - \frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b}\end{aligned}$$

output

```
4*I*x*arctan(exp(I*(b*x+a)))/b^2-3*x^2*arctanh(exp(I*(b*x+a)))/b-arctanh(c
os(b*x+a))/b^3-x*csc(b*x+a)/b^2+3*I*x*polylog(2,-exp(I*(b*x+a)))/b^2-2*I*p
olylog(2,-I*exp(I*(b*x+a)))/b^3+2*I*polylog(2,I*exp(I*(b*x+a)))/b^3-3*I*x*
polylog(2,exp(I*(b*x+a)))/b^2-3*polylog(3,-exp(I*(b*x+a)))/b^3+3*polylog(3
,exp(I*(b*x+a)))/b^3+3/2*x^2*sec(b*x+a)/b-1/2*x^2*csc(b*x+a)^2*sec(b*x+a)/
b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 613 vs. $2(235) = 470$.

Time = 6.52 (sec) , antiderivative size = 613, normalized size of antiderivative = 2.61

$$\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{x^2\csc^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}$$
$$-\frac{2\Big(\left(-a+\frac{\pi}{2}-bx\right)\left(\log\left(1-e^{i\left(-a+\frac{\pi}{2}-bx\right)}\right)-\log\left(1+e^{i\left(-a+\frac{\pi}{2}-bx\right)}\right)\right)-\left(-a+\frac{\pi}{2}\right)\log\left(\tan\left(\frac{1}{2}\left(-a+\frac{\pi}{2}-\right.\right.\right.}{b^3}$$
$$+\frac{2\log(1-\cos(a+bx)-i\sin(a+bx))+3b^2x^2\log(1-\cos(a+bx)-i\sin(a+bx))-2\log(1+\cos(a-}{}$$
$$+\frac{x^2\sec^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}+\frac{x\csc(a)\sec(a)(-\cos(a)+bx\sin(a))}{b^2}$$
$$+\frac{x\csc\left(\frac{a}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{bx}{2}\right)}{2b^2}-\frac{x\sec\left(\frac{a}{2}\right)\sec\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{bx}{2}\right)}{2b^2}$$
$$+\frac{x^2\sin\left(\frac{bx}{2}\right)}{b\left(\cos\left(\frac{a}{2}\right)-\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}$$
$$-\frac{x^2\sin\left(\frac{bx}{2}\right)}{b\left(\cos\left(\frac{a}{2}\right)+\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}$$

input

```
Integrate[x^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output

```
-1/8*(x^2*Csc[a/2 + (b*x)/2]^2)/b - (2*((-a + Pi/2 - b*x)*(Log[1 - E^(I*(-
a + Pi/2 - b*x))] - Log[1 + E^(I*(-a + Pi/2 - b*x))]) - (-a + Pi/2)*Log[Ta
n[(-a + Pi/2 - b*x)/2]] + I*(PolyLog[2, -E^(I*(-a + Pi/2 - b*x))] - PolyLo
g[2, E^(I*(-a + Pi/2 - b*x))])))/b^3 + (2*Log[1 - Cos[a + b*x] - I*Sin[a +
 b*x]] + 3*b^2*x^2*Log[1 - Cos[a + b*x] - I*Sin[a + b*x]] - 2*Log[1 + Cos[
a + b*x] + I*Sin[a + b*x]] - 3*b^2*x^2*Log[1 + Cos[a + b*x] + I*Sin[a + b*
x]] + (6*I)*b*x*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]] - (6*I)*b*x*Pol
yLog[2, Cos[a + b*x] + I*Sin[a + b*x]] - 6*PolyLog[3, -Cos[a + b*x] - I*Si
n[a + b*x]] + 6*PolyLog[3, Cos[a + b*x] + I*Sin[a + b*x]])/(2*b^3) + (x^2*
Sec[a/2 + (b*x)/2]^2)/(8*b) + (x*Csc[a]*Sec[a]*(-Cos[a] + b*x*Sin[a]))/b^2
 + (x*Csc[a/2]*Csc[a/2 + (b*x)/2]*Sin[(b*x)/2])/(2*b^2) - (x*Sec[a/2]*Sec[
a/2 + (b*x)/2]*Sin[(b*x)/2])/(2*b^2) + (x^2*Sin[(b*x)/2])/(b*(Cos[a/2] - S
in[a/2])*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) - (x^2*Sin[(b*x)/2])/(
b*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2]))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4920, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx$$

$$\downarrow 4920$$

$$-2\int -\frac{1}{2}x\left(\frac{\sec(a+bx)\csc^2(a+bx)}{b}+\frac{3\operatorname{arctanh}(\cos(a+bx))}{b}-\frac{3\sec(a+bx)}{b}\right)dx - \frac{3x^2\operatorname{arctanh}(\cos(a+bx))}{2b}+\frac{3x^2\sec(a+bx)}{2b}-\frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 27$$

$$\int x\left(\frac{\sec(a+bx)\csc^2(a+bx)}{b}+\frac{3\operatorname{arctanh}(\cos(a+bx))}{b}-\frac{3\sec(a+bx)}{b}\right)dx - \frac{3x^2\operatorname{arctanh}(\cos(a+bx))}{2b}+\frac{3x^2\sec(a+bx)}{2b}-\frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 2010$$

$$\int \left(\frac{3x\text{arctanh}(\cos(a+bx))}{b} + \frac{x\left(\csc^2(a+bx)-3\right)\sec(a+bx)}{b} \right) dx - \frac{3x^2\text{arctanh}(\cos(a+bx))}{2b} + \frac{3x^2\sec(a+bx)}{2b} - \frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow \text{2009}$$

$$\frac{4ix\arctan\left(e^{i(a+bx)}\right)}{b^2} - \frac{\text{arctanh}(\cos(a+bx))}{b^3} - \frac{3x^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{2i\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} + \frac{2i\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} - \frac{3\,\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} + \frac{3\,\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} + \frac{3ix\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} - \frac{3ix\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} - \frac{x\csc(a+bx)}{b^2} + \frac{3x^2\sec(a+bx)}{2b} - \frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

input
```
Int[x^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
((4*I)*x*ArcTan[E^(I*(a + b*x))])/b^2 - (3*x^2*ArcTanh[E^(I*(a + b*x))])/b
 - ArcTanh[Cos[a + b*x]]/b^3 - (x*Csc[a + b*x])/b^2 + ((3*I)*x*PolyLog[2,
-E^(I*(a + b*x))])/b^2 - ((2*I)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((
2*I)*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*x*PolyLog[2, E^(I*(a + b*
x))])/b^2 - (3*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (3*PolyLog[3, E^(I*(a +
 b*x))])/b^3 + (3*x^2*Sec[a + b*x])/(2*b) - (x^2*Csc[a + b*x]^2*Sec[a + b*
x])/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4920

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(208) = 416$.

Time = 0.58 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.83

method	result
risch	$\frac{x\left(3bx\,\mathrm{e}^{5i(bx+a)}-2bx\,\mathrm{e}^{3i(bx+a)}-2i\mathrm{e}^{5i(bx+a)}+3bx\,\mathrm{e}^{i(bx+a)}+2i\mathrm{e}^{i(bx+a)}\right)}{b^2\left(\mathrm{e}^{2i(bx+a)}-1\right)^2\left(\mathrm{e}^{2i(bx+a)}+1\right)}+\frac{\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b^3}-\frac{3a^2\ln\left(1-\mathrm{e}^{i(bx+a)}\right)}{2b^3}-\frac{2\ln\left(1-ie^{i}\right.}{b^3}$

input

```
int(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
x/b^2/(exp(2*I*(b*x+a))-1)^2/(exp(2*I*(b*x+a))+1)*(3*b*x*exp(5*I*(b*x+a))-
2*b*x*exp(3*I*(b*x+a))-2*I*exp(5*I*(b*x+a))+3*b*x*exp(I*(b*x+a))+2*I*exp(I
*(b*x+a)))+1/b^3*ln(exp(I*(b*x+a))-1)-3/2/b^3*a^2*ln(1-exp(I*(b*x+a)))-2/b
^3*ln(1-I*exp(I*(b*x+a)))*a+2/b^3*ln(I*exp(I*(b*x+a))+1)*a-3*polylog(3,-ex
p(I*(b*x+a)))/b^3+3*polylog(3,exp(I*(b*x+a)))/b^3-1/b^3*ln(exp(I*(b*x+a))+
1)+2*I/b^3*dilog(1-I*exp(I*(b*x+a)))-2*I/b^3*dilog(I*exp(I*(b*x+a))+1)+3/2
/b^3*a^2*ln(exp(I*(b*x+a))-1)-2/b^2*ln(1-I*exp(I*(b*x+a)))*x-3/2/b*ln(exp(
I*(b*x+a))+1)*x^2+3*I*x*polylog(2,-exp(I*(b*x+a)))/b^2+3/2/b*ln(1-exp(I*(b
*x+a)))*x^2-3*I*x*polylog(2,exp(I*(b*x+a)))/b^2+2/b^2*ln(I*exp(I*(b*x+a))+
1)*x-4*I/b^3*a*arctan(exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1237 vs. $2(197) = 394$.

Time = 0.15 (sec) , antiderivative size = 1237, normalized size of antiderivative = 5.26

$$\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input

```
integrate(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

output

```
1/4*(6*b^2*x^2*cos(b*x + a)^2 - 4*b^2*x^2 + 4*b*x*cos(b*x + a)*sin(b*x + a
) - 6*(I*b*x*cos(b*x + a)^3 - I*b*x*cos(b*x + a))*dilog(cos(b*x + a) + I*s
in(b*x + a)) - 6*(-I*b*x*cos(b*x + a)^3 + I*b*x*cos(b*x + a))*dilog(cos(b*
x + a) - I*sin(b*x + a)) - 4*(-I*cos(b*x + a)^3 + I*cos(b*x + a))*dilog(I*
cos(b*x + a) + sin(b*x + a)) - 4*(-I*cos(b*x + a)^3 + I*cos(b*x + a))*dilo
g(I*cos(b*x + a) - sin(b*x + a)) - 4*(I*cos(b*x + a)^3 - I*cos(b*x + a))*d
ilog(-I*cos(b*x + a) + sin(b*x + a)) - 4*(I*cos(b*x + a)^3 - I*cos(b*x + a
))*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 6*(I*b*x*cos(b*x + a)^3 - I*b*x
*cos(b*x + a))*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 6*(-I*b*x*cos(b*x +
 a)^3 + I*b*x*cos(b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) - ((3*b^
2*x^2 + 2)*cos(b*x + a)^3 - (3*b^2*x^2 + 2)*cos(b*x + a))*log(cos(b*x + a)
 + I*sin(b*x + a) + 1) + 4*(a*cos(b*x + a)^3 - a*cos(b*x + a))*log(cos(b*x
 + a) + I*sin(b*x + a) + I) - ((3*b^2*x^2 + 2)*cos(b*x + a)^3 - (3*b^2*x^2
 + 2)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 4*(a*cos(b*x
+ a)^3 - a*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + I) - 4*((b*x
+ a)*cos(b*x + a)^3 - (b*x + a)*cos(b*x + a))*log(I*cos(b*x + a) + sin(b*x
 + a) + 1) + 4*((b*x + a)*cos(b*x + a)^3 - (b*x + a)*cos(b*x + a))*log(I*c
os(b*x + a) - sin(b*x + a) + 1) - 4*((b*x + a)*cos(b*x + a)^3 - (b*x + a)*
cos(b*x + a))*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 4*((b*x + a)*cos(b
*x + a)^3 - (b*x + a)*cos(b*x + a))*log(-I*cos(b*x + a) - sin(b*x + a) ...
```

Sympy [F]

$$\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx = \int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx$$

input
```
integrate(x**2*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

output
```
Integral(x**2*csc(a + b*x)**3*sec(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2205 vs. $2(197) = 394$.

Time = 0.55 (sec) , antiderivative size = 2205, normalized size of antiderivative = 9.38

$$\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

output

```
1/4*(a^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log
(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) + 4*(8*(b*x*cos(6*b*x + 6*a)
 - b*x*cos(4*b*x + 4*a) - b*x*cos(2*b*x + 2*a) + I*b*x*sin(6*b*x + 6*a) -
I*b*x*sin(4*b*x + 4*a) - I*b*x*sin(2*b*x + 2*a) + b*x)*arctan2(cos(b*x + a
), sin(b*x + a) + 1) + 8*(b*x*cos(6*b*x + 6*a) - b*x*cos(4*b*x + 4*a) - b*
x*cos(2*b*x + 2*a) + I*b*x*sin(6*b*x + 6*a) - I*b*x*sin(4*b*x + 4*a) - I*b
*x*sin(2*b*x + 2*a) + b*x)*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 2*(3
*(b*x + a)^2 - 6*(b*x + a)*a + (3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*cos(6*b
*x + 6*a) - (3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*cos(4*b*x + 4*a) - (3*(b*x
 + a)^2 - 6*(b*x + a)*a + 2)*cos(2*b*x + 2*a) - (-3*I*(b*x + a)^2 + 6*I*(b
*x + a)*a - 2*I)*sin(6*b*x + 6*a) - (3*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 2
*I)*sin(4*b*x + 4*a) - (3*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 2*I)*sin(2*b*x
 + 2*a) + 2)*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 4*(cos(6*b*x + 6*a)
 - cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + I*sin(6*b*x + 6*a) - I*sin(4*b*x
+ 4*a) - I*sin(2*b*x + 2*a) + 1)*arctan2(sin(b*x + a), cos(b*x + a) - 1) -
 6*((b*x + a)^2 - 2*(b*x + a)*a + ((b*x + a)^2 - 2*(b*x + a)*a)*cos(6*b*x
+ 6*a) - ((b*x + a)^2 - 2*(b*x + a)*a)*cos(4*b*x + 4*a) - ((b*x + a)^2 - 2
*(b*x + a)*a)*cos(2*b*x + 2*a) - (-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*sin(6*
b*x + 6*a) - (I*(b*x + a)^2 - 2*I*(b*x + a)*a)*sin(4*b*x + 4*a) - (I*(b*x
+ a)^2 - 2*I*(b*x + a)*a)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(...
```

Giac [F]

$$\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx = \int x^2 \csc\left(bx+a\right)^3 \sec\left(bx+a\right)^2\,dx$$

input

```
integrate(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

output

```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx = \int \frac{x^2}{\cos(a+b\,x)^2 \sin(a+b\,x)^3}\,dx$$

input
```
int(x^2/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

output
```
int(x^2/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

Reduce [F]

$$\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx = \int \csc(bx+a)^3 \sec(bx+a)^2\, x^2 dx$$

input
```
int(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

output
```
int(csc(a + b*x)**3*sec(a + b*x)**2*x**2,x)
```

3.287 $\int x \csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal result . . . 2480
Mathematica [B] (verified) . . . 2481
Rubi [A] (verified) . . . 2481
Maple [B] (verified) . . . 2482
Fricas [B] (verification not implemented) . . . 2483
Sympy [F] . . . 2484
Maxima [B] (verification not implemented) . . . 2484
Giac [F] . . . 2485
Mupad [F(-1)] . . . 2486
Reduce [F] . . . 2486

Optimal result

Integrand size = 18, antiderivative size = 126

$$\begin{aligned}\int x\csc^3(a+bx)\sec^2(a+bx)\,dx = &-\frac{3x\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{\operatorname{arctanh}(\sin(a+bx))}{b^2} \\ &- \frac{\csc(a+bx)}{2b^2} + \frac{3i\,\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2} \\ &- \frac{3i\,\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2} + \frac{3x\sec(a+bx)}{2b} \\ &- \frac{x\csc^2(a+bx)\sec(a+bx)}{2b}\end{aligned}$$

output

```
-3*x*arctanh(exp(I*(b*x+a)))/b-arctanh(sin(b*x+a))/b^2-1/2*csc(b*x+a)/b^2+
3/2*I*polylog(2,-exp(I*(b*x+a)))/b^2-3/2*I*polylog(2,exp(I*(b*x+a)))/b^2+3
/2*x*sec(b*x+a)/b-1/2*x*csc(b*x+a)^2*sec(b*x+a)/b
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 282 vs. $2(126) = 252$.

Time = 2.17 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.24

$$\int x \csc^3(a+bx)\sec^2(a+bx)\,dx$$

$$= \frac{8bx - 2\cot\left(\frac{1}{2}(a+bx)\right) - bx\csc^2\left(\frac{1}{2}(a+bx)\right) + 12(a+bx)\left(\log\left(1-e^{i(a+bx)}\right) - \log\left(1+e^{i(a+bx)}\right)\right) + 8l}{}$$

input

```
Integrate[x*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output

```
(8*b*x - 2*Cot[(a + b*x)/2] - b*x*Csc[(a + b*x)/2]^2 + 12*(a + b*x)*(Log[1
 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + 8*Log[Cos[(a + b*x)/2] -
 Sin[(a + b*x)/2]] - 8*Log[Cos[(a + b*x)/2] + Sin[(a + b*x)/2]] - 12*a*Log
[Tan[(a + b*x)/2]] + (12*I)*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(
I*(a + b*x))]) + b*x*Sec[(a + b*x)/2]^2 + (8*b*x*Sin[(a + b*x)/2])/(Cos[(a
 + b*x)/2] - Sin[(a + b*x)/2]) - (8*b*x*Sin[(a + b*x)/2])/(Cos[(a + b*x)/2
] + Sin[(a + b*x)/2]) - 2*Tan[(a + b*x)/2])/(8*b^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4920, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \csc^3(a+bx)\sec^2(a+bx)\,dx$$

$$\downarrow 4920$$

$$-\int\left(-\frac{\sec(a+bx)\csc^2(a+bx)}{2b}-\frac{3\operatorname{arctanh}(\cos(a+bx))}{2b}+\frac{3\sec(a+bx)}{2b}\right)dx-\frac{3x\operatorname{arctanh}(\cos(a+bx))}{2b}+\frac{3x\sec(a+bx)}{2b}-\frac{x\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 2009$$

$$-\frac{\operatorname{arctanh}(\sin(a+bx))}{b^2}-\frac{3x\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{3i\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}-\frac{3i\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}-\frac{\csc(a+bx)}{2b^2}+\frac{3x\sec(a+bx)}{2b}-\frac{x\csc^2(a+bx)\sec(a+bx)}{2b}$$

input
```
Int[x*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
(-3*x*ArcTanh[E^(I*(a + b*x))])/b - ArcTanh[Sin[a + b*x]]/b^2 - Csc[a + b*
x]/(2*b^2) + (((3*I)/2)*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (((3*I)/2)*Pol
yLog[2, E^(I*(a + b*x))])/b^2 + (3*x*Sec[a + b*x])/(2*b) - (x*Csc[a + b*x]
^2*Sec[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(108) = 216$.

Time = 0.48 (sec) , antiderivative size = 519, normalized size of antiderivative = 4.12

method	result
risch	$\frac{3\ln(e^{i(bx+a)}+1)e^{4i(bx+a)}bx-3\ln(e^{i(bx+a)}+1)e^{6i(bx+a)}bx+3\ln(e^{i(bx+a)}+1)e^{2i(bx+a)}bx+3i\,\mathrm{dilog}(e^{i(bx+a)}+1)+3i\,\mathrm{dilog}(e^{i(bx+a}$

input
```
int(x*csc(b*x+a)^3*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(3*ln(exp(I*(b*x+a))+1)*exp(4*I*(b*x+a))*b*x+3*ln(exp(I*(b*x+a))-1)*ex
p(4*I*(b*x+a))*a-2*I*exp(5*I*(b*x+a))+2*I*exp(I*(b*x+a))-3*ln(exp(I*(b*x+a
))+1)*exp(6*I*(b*x+a))*b*x-3*I*dilog(exp(I*(b*x+a))+1)*exp(2*I*(b*x+a))-3*
I*dilog(exp(I*(b*x+a)))*exp(2*I*(b*x+a))-4*I*arctan(exp(I*(b*x+a)))*exp(2*
I*(b*x+a))+3*I*dilog(exp(I*(b*x+a))+1)+3*I*dilog(exp(I*(b*x+a)))+4*I*arcta
n(exp(I*(b*x+a)))+3*ln(exp(I*(b*x+a))+1)*exp(2*I*(b*x+a))*b*x+4*I*arctan(e
xp(I*(b*x+a)))*exp(6*I*(b*x+a))+6*b*x*exp(5*I*(b*x+a))-4*b*x*exp(3*I*(b*x+
a))+6*b*x*exp(I*(b*x+a))+3*I*dilog(exp(I*(b*x+a)))*exp(6*I*(b*x+a))-3*a*ln
(exp(I*(b*x+a))-1)-3*I*dilog(exp(I*(b*x+a))+1)*exp(4*I*(b*x+a))-3*ln(exp(I
*(b*x+a))+1)*b*x-4*I*arctan(exp(I*(b*x+a)))*exp(4*I*(b*x+a))-3*I*dilog(exp
(I*(b*x+a)))*exp(4*I*(b*x+a))+3*I*dilog(exp(I*(b*x+a))+1)*exp(6*I*(b*x+a))
-3*ln(exp(I*(b*x+a))-1)*exp(6*I*(b*x+a))*a+3*ln(exp(I*(b*x+a))-1)*exp(2*I*
(b*x+a))*a)/b^2/(exp(2*I*(b*x+a))-1)^2/(exp(2*I*(b*x+a))+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(104) = 208$.

Time = 0.10 (sec) , antiderivative size = 531, normalized size of antiderivative = 4.21

$$\int x\csc^3(a+bx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(6*b*x*cos(b*x + a)^2 - 4*b*x - 3*(I*cos(b*x + a)^3 - I*cos(b*x + a))*
dilog(cos(b*x + a) + I*sin(b*x + a)) - 3*(-I*cos(b*x + a)^3 + I*cos(b*x +
a))*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*(I*cos(b*x + a)^3 - I*cos(b*x
 + a))*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 3*(-I*cos(b*x + a)^3 + I*co
s(b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*(b*x*cos(b*x + a)^3
- b*x*cos(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b*x*cos(b*
x + a)^3 - b*x*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 3*(a
*cos(b*x + a)^3 - a*cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x +
a) + 1/2) - 3*(a*cos(b*x + a)^3 - a*cos(b*x + a))*log(-1/2*cos(b*x + a) -
1/2*I*sin(b*x + a) + 1/2) + 3*((b*x + a)*cos(b*x + a)^3 - (b*x + a)*cos(b*
x + a))*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*((b*x + a)*cos(b*x + a
)^3 - (b*x + a)*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 2*
(cos(b*x + a)^3 - cos(b*x + a))*log(sin(b*x + a) + 1) + 2*(cos(b*x + a)^3
- cos(b*x + a))*log(-sin(b*x + a) + 1) + 2*cos(b*x + a)*sin(b*x + a))/(b^2
*cos(b*x + a)^3 - b^2*cos(b*x + a))
```

Sympy [F]

$$\int x \csc^3(a+bx) \sec^2(a+bx)\,dx = \int x \csc^3(a+bx) \sec^2(a+bx)\,dx$$

input
```
integrate(x*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

output
```
Integral(x*csc(a + b*x)**3*sec(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1173 vs. $2(104) = 208$.

Time = 0.30 (sec) , antiderivative size = 1173, normalized size of antiderivative = 9.31

$$\int x \csc^3(a+bx) \sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

output

```
(8*I*b*x*cos(3*b*x + 3*a) - 8*b*x*sin(3*b*x + 3*a) - 4*(cos(6*b*x + 6*a) -
 cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + I*sin(6*b*x + 6*a) - I*sin(4*b*x +
4*a) - I*sin(2*b*x + 2*a) + 1)*arctan2(2*(cos(b*x + 2*a)*cos(a) + sin(b*x
+ 2*a)*sin(a))/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + si
n(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2), (cos(b*x + 2*a)^2 -
cos(a)^2 + sin(b*x + 2*a)^2 - sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*c
os(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)
^2)) - 6*(b*x*cos(6*b*x + 6*a) - b*x*cos(4*b*x + 4*a) - b*x*cos(2*b*x + 2*
a) + I*b*x*sin(6*b*x + 6*a) - I*b*x*sin(4*b*x + 4*a) - I*b*x*sin(2*b*x + 2
*a) + b*x)*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 6*(b*x*cos(6*b*x + 6*
a) - b*x*cos(4*b*x + 4*a) - b*x*cos(2*b*x + 2*a) + I*b*x*sin(6*b*x + 6*a)
- I*b*x*sin(4*b*x + 4*a) - I*b*x*sin(2*b*x + 2*a) + b*x)*arctan2(sin(b*x +
 a), -cos(b*x + a) + 1) - 4*(3*I*b*x + 1)*cos(5*b*x + 5*a) - 4*(3*I*b*x -
1)*cos(b*x + a) + 6*(cos(6*b*x + 6*a) - cos(4*b*x + 4*a) - cos(2*b*x + 2*a
) + I*sin(6*b*x + 6*a) - I*sin(4*b*x + 4*a) - I*sin(2*b*x + 2*a) + 1)*dilo
g(-e^(I*b*x + I*a)) - 6*(cos(6*b*x + 6*a) - cos(4*b*x + 4*a) - cos(2*b*x +
 2*a) + I*sin(6*b*x + 6*a) - I*sin(4*b*x + 4*a) - I*sin(2*b*x + 2*a) + 1)*
dilog(e^(I*b*x + I*a)) - 3*(-I*b*x*cos(6*b*x + 6*a) + I*b*x*cos(4*b*x + 4*
a) + I*b*x*cos(2*b*x + 2*a) + b*x*sin(6*b*x + 6*a) - b*x*sin(4*b*x + 4*a)
- b*x*sin(2*b*x + 2*a) - I*b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2...
```

Giac [F]

$$\int x \csc^3(a+bx)\sec^2(a+bx)\,dx = \int x \csc\left(bx+a\right)^3 \sec\left(bx+a\right)^2\,dx$$

input

```
integrate(x*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

output

```
integrate(x*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x \csc^3(a+bx) \sec^2(a+bx)\, dx = \int \frac{x}{\cos(a+b\,x)^2 \sin(a+b\,x)^3}\, dx$$

input
```
int(x/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

output
```
int(x/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

Reduce [F]

$$\int x \csc^3(a+bx) \sec^2(a+bx)\, dx = \int \csc(bx+a)^3 \sec(bx+a)^2\, x dx$$

input
```
int(x*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

output
```
int(csc(a + b*x)**3*sec(a + b*x)**2*x,x)
```

3.288 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx$

Optimal result . . . 2487
Mathematica [N/A] . . . 2487
Rubi [N/A] . . . 2488
Maple [N/A] . . . 2488
Fricas [N/A] . . . 2489
Sympy [N/A] . . . 2489
Maxima [N/A] . . . 2490
Giac [N/A] . . . 2491
Mupad [N/A] . . . 2491
Reduce [N/A] . . . 2491

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx = \text{Int}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{x}, x\right)$$

output
```
Defer(Int)(csc(b*x+a)^3*sec(b*x+a)^2/x,x)
```

Mathematica [N/A]

Not integrable

Time = 34.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx = \int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx$$

input
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x,x]
```

output
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx$$
$$\downarrow 7299$$
$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}dx$$

input
```
Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\csc\left(bx+a\right)^3\sec\left(bx+a\right)^2}{x}dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^2/x,x)
```

output
```
int(csc(b*x+a)^3*sec(b*x+a)^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx = \int \frac{\csc\left(bx+a\right)^3\sec\left(bx+a\right)^2}{x}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^3*sec(b*x + a)^2/x, x)
```

Sympy [N/A]

Not integrable

Time = 8.63 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx = \int \frac{\csc^3\left(a+bx\right)\sec^2\left(a+bx\right)}{x}\,dx$$

input
```
integrate(csc(b*x+a)**3*sec(b*x+a)**2/x,x)
```

output
```
Integral(csc(a + b*x)**3*sec(a + b*x)**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 1872, normalized size of antiderivative = 93.60

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^2}{x}\,dx$$

input

```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x, algorithm="maxima")
```

output

```
(2*b*x*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) + 3*b*x*cos(b*x + a) + (3*b*x*cos
(5*b*x + 5*a) - 2*b*x*cos(3*b*x + 3*a) + 3*b*x*cos(b*x + a) - sin(5*b*x +
5*a) + sin(b*x + a))*cos(6*b*x + 6*a) - (3*b*x*cos(4*b*x + 4*a) + 3*b*x*co
s(2*b*x + 2*a) - 3*b*x + sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*cos(5*b*x +
5*a) + (2*b*x*cos(3*b*x + 3*a) - 3*b*x*cos(b*x + a) - sin(b*x + a))*cos(4*
b*x + 4*a) + 2*(b*x*cos(2*b*x + 2*a) - b*x)*cos(3*b*x + 3*a) - (3*b*x*cos(
b*x + a) + sin(b*x + a))*cos(2*b*x + 2*a) + (b^2*x^2*cos(6*b*x + 6*a)^2 +
b^2*x^2*cos(4*b*x + 4*a)^2 + b^2*x^2*cos(2*b*x + 2*a)^2 + b^2*x^2*sin(6*b*
x + 6*a)^2 + b^2*x^2*sin(4*b*x + 4*a)^2 + 2*b^2*x^2*sin(4*b*x + 4*a)*sin(2
*b*x + 2*a) + b^2*x^2*sin(2*b*x + 2*a)^2 - 2*b^2*x^2*cos(2*b*x + 2*a) + b^
2*x^2 - 2*(b^2*x^2*cos(4*b*x + 4*a) + b^2*x^2*cos(2*b*x + 2*a) - b^2*x^2)*
cos(6*b*x + 6*a) + 2*(b^2*x^2*cos(2*b*x + 2*a) - b^2*x^2)*cos(4*b*x + 4*a)
 - 2*(b^2*x^2*sin(4*b*x + 4*a) + b^2*x^2*sin(2*b*x + 2*a))*sin(6*b*x + 6*a
))*integrate(1/2*(3*b^2*x^2 + 2)*sin(b*x + a)/(b^2*x^3*cos(b*x + a)^2 + b^
2*x^3*sin(b*x + a)^2 + 2*b^2*x^3*cos(b*x + a) + b^2*x^3), x) + (b^2*x^2*co
s(6*b*x + 6*a)^2 + b^2*x^2*cos(4*b*x + 4*a)^2 + b^2*x^2*cos(2*b*x + 2*a)^2
 + b^2*x^2*sin(6*b*x + 6*a)^2 + b^2*x^2*sin(4*b*x + 4*a)^2 + 2*b^2*x^2*sin
(4*b*x + 4*a)*sin(2*b*x + 2*a) + b^2*x^2*sin(2*b*x + 2*a)^2 - 2*b^2*x^2*co
s(2*b*x + 2*a) + b^2*x^2 - 2*(b^2*x^2*cos(4*b*x + 4*a) + b^2*x^2*cos(2*b*x
 + 2*a) - b^2*x^2)*cos(6*b*x + 6*a) + 2*(b^2*x^2*cos(2*b*x + 2*a) - b^2...
```

Giac [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^2}{x}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3*sec(b*x + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 20.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx = \int \frac{1}{x\cos(a+b\,x)^2\sin(a+b\,x)^3}\,dx$$

input
```
int(1/(x*cos(a + b*x)^2*sin(a + b*x)^3),x)
```

output
```
int(1/(x*cos(a + b*x)^2*sin(a + b*x)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^2}{x}dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^2/x,x)
```

output
```
int((csc(a + b*x)**3*sec(a + b*x)**2)/x,x)
```

3.289 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$

Optimal result . . . 2493
Mathematica [N/A] . . . 2493
Rubi [N/A] . . . 2494
Maple [N/A] . . . 2494
Fricas [N/A] . . . 2495
Sympy [N/A] . . . 2495
Maxima [N/A] . . . 2496
Giac [N/A] . . . 2497
Mupad [N/A] . . . 2497
Reduce [N/A] . . . 2497

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx = \text{Int}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}, x\right)$$

output
```
Defer(Int)(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 17.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx = \int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$$

input
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2,x]
```

output
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$$
$$\downarrow 7299$$
$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}dx$$

input
```
Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\csc\left(bx+a\right)^3\sec\left(bx+a\right)^2}{x^2}dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x)
```

output
```
int(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx = \int \frac{\csc\left(bx+a\right)^3\sec\left(bx+a\right)^2}{x^2}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^3*sec(b*x + a)^2/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 11.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx = \int \frac{\csc^3\left(a+bx\right)\sec^2\left(a+bx\right)}{x^2}\,dx$$

input
```
integrate(csc(b*x+a)**3*sec(b*x+a)**2/x**2,x)
```

output
```
Integral(csc(a + b*x)**3*sec(a + b*x)**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 1884, normalized size of antiderivative = 94.20

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx = \int \frac{\csc\left(bx+a\right)^3\sec\left(bx+a\right)^2}{x^2}\,dx$$

input

```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x, algorithm="maxima")
```

output

```
(2*b*x*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) + 3*b*x*cos(b*x + a) + (3*b*x*cos
(5*b*x + 5*a) - 2*b*x*cos(3*b*x + 3*a) + 3*b*x*cos(b*x + a) - 2*sin(5*b*x
 + 5*a) + 2*sin(b*x + a))*cos(6*b*x + 6*a) - (3*b*x*cos(4*b*x + 4*a) + 3*b*
x*cos(2*b*x + 2*a) - 3*b*x + 2*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(
5*b*x + 5*a) + (2*b*x*cos(3*b*x + 3*a) - 3*b*x*cos(b*x + a) - 2*sin(b*x + 
a))*cos(4*b*x + 4*a) + 2*(b*x*cos(2*b*x + 2*a) - b*x)*cos(3*b*x + 3*a) - (
3*b*x*cos(b*x + a) + 2*sin(b*x + a))*cos(2*b*x + 2*a) + (b^2*x^3*cos(6*b*x
 + 6*a)^2 + b^2*x^3*cos(4*b*x + 4*a)^2 + b^2*x^3*cos(2*b*x + 2*a)^2 + b^2*
x^3*sin(6*b*x + 6*a)^2 + b^2*x^3*sin(4*b*x + 4*a)^2 + 2*b^2*x^3*sin(4*b*x
 + 4*a)*sin(2*b*x + 2*a) + b^2*x^3*sin(2*b*x + 2*a)^2 - 2*b^2*x^3*cos(2*b*x
 + 2*a) + b^2*x^3 - 2*(b^2*x^3*cos(4*b*x + 4*a) + b^2*x^3*cos(2*b*x + 2*a)
 - b^2*x^3)*cos(6*b*x + 6*a) + 2*(b^2*x^3*cos(2*b*x + 2*a) - b^2*x^3)*cos(
4*b*x + 4*a) - 2*(b^2*x^3*sin(4*b*x + 4*a) + b^2*x^3*sin(2*b*x + 2*a))*sin
(6*b*x + 6*a))*integrate(3/2*(b^2*x^2 + 2)*sin(b*x + a)/(b^2*x^4*cos(b*x + 
 a)^2 + b^2*x^4*sin(b*x + a)^2 + 2*b^2*x^4*cos(b*x + a) + b^2*x^4), x) + (
b^2*x^3*cos(6*b*x + 6*a)^2 + b^2*x^3*cos(4*b*x + 4*a)^2 + b^2*x^3*cos(2*b*
x + 2*a)^2 + b^2*x^3*sin(6*b*x + 6*a)^2 + b^2*x^3*sin(4*b*x + 4*a)^2 + 2*b
^2*x^3*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b^2*x^3*sin(2*b*x + 2*a)^2 - 2*
b^2*x^3*cos(2*b*x + 2*a) + b^2*x^3 - 2*(b^2*x^3*cos(4*b*x + 4*a) + b^2*x^3
*cos(2*b*x + 2*a) - b^2*x^3)*cos(6*b*x + 6*a) + 2*(b^2*x^3*cos(2*b*x + ...
```

Giac [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^2}{x^2}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3*sec(b*x + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 19.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx = \int \frac{1}{x^2\cos(a+b\,x)^2\sin(a+b\,x)^3}\,dx$$

input
```
int(1/(x^2*cos(a + b*x)^2*sin(a + b*x)^3),x)
```

output
```
int(1/(x^2*cos(a + b*x)^2*sin(a + b*x)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^2}{x^2}dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x)
```

output

```
int((csc(a + b*x)**3*sec(a + b*x)**2)/x**2,x)
```

3.290 $\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx$

Optimal result . . . 2499
Mathematica [N/A] . . . 2499
Rubi [N/A] . . . 2500
Maple [N/A] . . . 2500
Fricas [N/A] . . . 2501
Sympy [N/A] . . . 2501
Maxima [N/A] . . . 2502
Giac [N/A] . . . 2502
Mupad [N/A] . . . 2502
Reduce [N/A] . . . 2503

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx = \text{Int}\big((c+dx)^m \sec^2(a+bx)\tan(a+bx), x\big)$$

output
```
Defer(Int)((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 4.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx = \int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(a+bx)\sec^2(a+bx)(c+dx)^m\,dx$$

$$\downarrow 7299$$

$$\int \tan(a+bx)\sec^2(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \sec(bx+a)^2 \tan(bx+a)\,dx$$

input
```
int((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x)
```

output
```
int((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)^2 \tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*sec(b*x + a)^2*tan(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 9.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx = \int (c+dx)^m \tan(a+bx)\sec^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*sec(b*x+a)**2*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**m*tan(a + b*x)*sec(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)^2 \tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*sec(b*x + a)^2*tan(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)^2 \tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sec(b*x + a)^2*tan(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 19.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx = \int \frac{\tan(a+b\,x)\,(c+d\,x)^m}{\cos(a+b\,x)^2}\,dx$$

input
```
int((tan(a + b*x)*(c + d*x)^m)/cos(a + b*x)^2,x)
```

output
```
int((tan(a + b*x)*(c + d*x)^m)/cos(a + b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)^2 \tan(bx+a)\,dx$$

input
```
int((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x)
```

output
```
int((c + d*x)**m*sec(a + b*x)**2*tan(a + b*x),x)
```

3.291 $\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx$

Optimal result . . . 2504
Mathematica [B] (warning: unable to verify) . . . 2505
Rubi [A] (verified) . . . 2505
Maple [B] (verified) . . . 2509
Fricas [B] (verification not implemented) . . . 2510
Sympy [F] . . . 2511
Maxima [B] (verification not implemented) . . . 2511
Giac [F] . . . 2512
Mupad [F(-1)] . . . 2513
Reduce [F] . . . 2513

Optimal result

Integrand size = 22, antiderivative size = 139

$$\begin{aligned}\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx = {} & \frac{2id(c+dx)^3}{b^2} - \frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3} \\ & + \frac{6id^3(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^4} \\ & - \frac{3d^4\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^5} \\ & + \frac{(c+dx)^4\sec^2(a+bx)}{2b} \\ & - \frac{2d(c+dx)^3\tan(a+bx)}{b^2}\end{aligned}$$

output

```
2*I*d*(d*x+c)^3/b^2-6*d^2*(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b^3+6*I*d^3*(d*
x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^4-3*d^4*polylog(3,-exp(2*I*(b*x+a)))/b
^5+1/2*(d*x+c)^4*sec(b*x+a)^2/b-2*d*(d*x+c)^3*tan(b*x+a)/b^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 324 vs. $2(139) = 278$.

Time = 6.28 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.33

$$\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx$$

$$= \frac{-d^4 e^{-ia}\left(2b^2x^2\left(2ibx + 3(1+e^{2ia})\log\left(1+e^{-2i(a+bx)}\right)\right) + 6ib(1+e^{2ia})\,x\,\text{PolyLog}\left(2,-e^{-2i(a+bx)}\right) + 3(1 -$$

input
```
Integrate[(c + d*x)^4*Sec[a + b*x]^2*Tan[a + b*x],x]
```

output
```
(-((d^4*(2*b^2*x^2*((2*I)*b*x + 3*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a +
 b*x))]) + (6*I)*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] +
 3*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/E^(I*a)) +
 b^4*(c + d*x)^4*Sec[a + b*x]^2 - 4*b^3*d*(c + d*x)^3*Sec[a]*Sec[a + b*x]*
Sin[b*x] - 12*b^2*c^2*d^2*(Log[Cos[a + b*x]] + b*x*Tan[a]) + 12*b*c*d^3*((
-I)*b*x*(Pi + 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - Ar
cTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]]
 - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)
*(b*x - ArcTan[Cot[a]]))] - (b^2*x^2*Sqrt[Csc[a]^2]*Tan[a])/E^(I*ArcTan[Co
t[a]])))/(2*b^5)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {4909, 3042, 4672, 25, 3042, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \tan(a+bx)\sec^2(a+bx)\,dx$$

$$\downarrow \text{4909}$$

$$\frac{(c+dx)^4\sec^2(a+bx)}{2b}-\frac{2d\int(c+dx)^3\sec^2(a+bx)dx}{b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^4\sec^2(a+bx)}{2b}-\frac{2d\int(c+dx)^3\csc\left(a+bx+\frac{\pi}{2}\right)^2dx}{b}$$

$$\downarrow 4672$$

$$\frac{(c+dx)^4\sec^2(a+bx)}{2b}-\frac{2d\left(\frac{3d\int-(c+dx)^2\tan(a+bx)dx}{b}+\frac{(c+dx)^3\tan(a+bx)}{b}\right)}{b}$$

$$\downarrow 25$$

$$\frac{(c+dx)^4\sec^2(a+bx)}{2b}-\frac{2d\left(\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{3d\int(c+dx)^2\tan(a+bx)dx}{b}\right)}{b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^4\sec^2(a+bx)}{2b}-\frac{2d\left(\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{3d\int(c+dx)^2\tan(a+bx)dx}{b}\right)}{b}$$

$$\downarrow 4202$$

$$\frac{(c+dx)^4\sec^2(a+bx)}{2b}-\frac{2d\left(\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\int\frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}dx\right)}{b}\right)}{b}$$

$$\downarrow 2620$$

$$\frac{(c+dx)^4\sec^2(a+bx)}{2b}-\frac{2d\left(\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\int(c+dx)\log\left(1+e^{2i(a+bx)}\right)dx}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}\right)}{b}$$

$$\downarrow 3011$$

$$\frac{(c+dx)^4\sec^2(a+bx)}{2b} - \frac{2d\left(\frac{(c+dx)^3\tan(a+bx)}{b} - \frac{3d\left(\frac{i(c+dx)^3}{3d} - 2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b} - \frac{id\int \mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{2b}\right)}{b} - \frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}\right)}{b}$$

$$\downarrow 2720$$

$$\frac{(c+dx)^4\sec^2(a+bx)}{2b} - \frac{2d\left(\frac{(c+dx)^3\tan(a+bx)}{b} - \frac{3d\left(\frac{i(c+dx)^3}{3d} - 2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b} - \frac{d\int e^{-2i(a+bx)}\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b} - \frac{i(c+dx)^2}{}\right.\right.\right.}{b}\right)}{b}$$

$$\downarrow 7143$$

$$\frac{(c+dx)^4\sec^2(a+bx)}{2b} - \frac{2d\left(\frac{(c+dx)^3\tan(a+bx)}{b} - \frac{3d\left(\frac{i(c+dx)^3}{3d} - 2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b} - \frac{d\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{4b^2}\right)}{b} - \frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}\right)}{b}$$

input

```
Int[(c + d*x)^4*Sec[a + b*x]^2*Tan[a + b*x],x]
```

output

```
((c + d*x)^4*Sec[a + b*x]^2)/(2*b) - (2*d*((-3*d*(((I/3)*(c + d*x)^3)/d -
(2*I)*(((-1/2*I)*(c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b + (I*d*(((I/2
)*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b - (d*PolyLog[3, -E^((2*I)*
(a + b*x))])/(4*b^2)))/b)))/b + ((c + d*x)^3*Tan[a + b*x])/b))/b
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4909

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(129) = 258$.

Time = 4.75 (sec) , antiderivative size = 489, normalized size of antiderivative = 3.52

method	result
risch	$\frac{2b\,d^4x^4e^{2i(bx+a)}+8bc\,d^3x^3e^{2i(bx+a)}+12b\,c^2d^2x^2e^{2i(bx+a)}+8b\,c^3dx\,e^{2i(bx+a)}-4id^4x^3e^{2i(bx+a)}+2b\,c^4e^{2i(bx+a)}-12ic\,d^3x^2e^{2i(bx+a)}}{b^2\left(e^{2i(bx+a)}+1\right)^2}$

input

```
int((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
2*(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^3*exp(2*I*(b*x+a))+6*b*c^2*d^2*x
^2*exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*x+a))-2*I*d^4*x^3*exp(2*I*(b*x+
a))+b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp(2*I*(b*x+a))-6*I*c^2*d^2*x*ex
p(2*I*(b*x+a))-2*I*d^4*x^3-2*I*c^3*d*exp(2*I*(b*x+a))-6*I*c*d^3*x^2-6*I*c^
2*d^2*x-2*I*c^3*d)/b^2/(exp(2*I*(b*x+a))+1)^2-6/b^3*d^2*c^2*ln(exp(2*I*(b*
x+a))+1)+12/b^3*d^2*c^2*ln(exp(I*(b*x+a)))-8*I/b^5*d^4*a^3+24*I/b^3*d^3*c*
a*x-24/b^4*d^3*c*a*ln(exp(I*(b*x+a)))-6/b^3*d^4*ln(exp(2*I*(b*x+a))+1)*x^2
-3*d^4*polylog(3,-exp(2*I*(b*x+a)))/b^5+4*I/b^2*d^4*x^3+6*I/b^4*d^3*c*poly
log(2,-exp(2*I*(b*x+a)))+12/b^5*d^4*a^2*ln(exp(I*(b*x+a)))-12/b^3*d^3*c*ln
(exp(2*I*(b*x+a))+1)*x+12*I/b^4*d^3*c*a^2-12*I/b^4*d^4*a^2*x+6*I/b^4*d^4*p
olylog(2,-exp(2*I*(b*x+a)))*x+12*I/b^2*d^3*c*x^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 892 vs. $2(126) = 252$.

Time = 0.12 (sec) , antiderivative size = 892, normalized size of antiderivative = 6.42

$$\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b
^4*c^4 - 12*d^4*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) -
 12*d^4*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 12*d^4*
cos(b*x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 12*d^4*cos(b*x
 + a)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 12*(I*b*d^4*x + I*b*c
*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) - 12*(-I*b*d^4*x
 - I*b*c*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - 12*(-I
*b*d^4*x - I*b*c*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a))
 - 12*(I*b*d^4*x + I*b*c*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b
*x + a)) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2*log(cos(
b*x + a) + I*sin(b*x + a) + I) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*c
os(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b^2*d^4*x^2 + 2*
b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*cos(b*x + a)^2*log(I*cos(b*x + a) + s
in(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)
*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 6*(b^2*d^4*x^2 +
2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*cos(b*x + a)^2*log(-I*cos(b*x + a)
+ sin(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d
^4)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - 6*(b^2*c^2*d^
2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x +
a) + I) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2*log(-c...
```

Sympy [F]

$$\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx = \int (c+dx)^4 \tan(a+bx)\sec^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**4*sec(b*x+a)**2*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**4*tan(a + b*x)*sec(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3446 vs. $2(126) = 252$.

Time $= 0.29$ (sec) , antiderivative size $= 3446$, normalized size of antiderivative $= 24.79$

$$\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

output

```
1/2*(c^4*tan(b*x + a)^2 - 4*a*c^3*d*tan(b*x + a)^2/b + 6*a^2*c^2*d^2*tan(b
*x + a)^2/b^2 - 4*a^3*c*d^3*tan(b*x + a)^2/b^3 + a^4*d^4*tan(b*x + a)^2/b^
4 + 8*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (
2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x
 + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)
- 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*c^3*d/((2*(2*cos(2*b*x + 2*a) +
1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*
x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 +
4*cos(2*b*x + 2*a) + 1)*b) - 24*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x +
 a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))
*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x
+ 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*a*c^2*
d^2/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4
*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x +
2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2) + 24*(4*(b*x +
a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(
2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x
+ 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x +
 4*a) - sin(2*b*x + 2*a))*a^2*c*d^3/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x
 + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^...
```

Giac [F]

$$\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx = \int (dx+c)^4 \sec(bx+a)^2 \tan(bx+a)\,dx$$

input

```
integrate((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

output

```
integrate((d*x + c)^4*sec(b*x + a)^2*tan(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx = \int \frac{\tan(a+b\,x)\,(c+d\,x)^4}{\cos(a+b\,x)^2}\,dx$$

input
```
int((tan(a + b*x)*(c + d*x)^4)/cos(a + b*x)^2,x)
```

output
```
int((tan(a + b*x)*(c + d*x)^4)/cos(a + b*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx = \text{too large to display}$$

input
```
int((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x)
```

output

```
( - 14*cos(a + b*x)*sin(a + b*x)*tan((a + b*x)/2)**4*b**4*c*d**3*x**4 + 12
*cos(a + b*x)*sin(a + b*x)*tan((a + b*x)/2)**4*b**2*c**3*d - 24*cos(a + b*
x)*sin(a + b*x)*tan((a + b*x)/2)**4*b**2*c**2*d**2*x - 24*cos(a + b*x)*sin
(a + b*x)*tan((a + b*x)/2)**4*b**2*c*d**3*x**2 - 8*cos(a + b*x)*sin(a + b*
x)*tan((a + b*x)/2)**4*b**2*d**4*x**3 + 24*cos(a + b*x)*sin(a + b*x)*tan((
a + b*x)/2)**4*c*d**3 + 28*cos(a + b*x)*sin(a + b*x)*tan((a + b*x)/2)**2*b
**4*c*d**3*x**4 - 24*cos(a + b*x)*sin(a + b*x)*tan((a + b*x)/2)**2*b**2*c*
*3*d + 48*cos(a + b*x)*sin(a + b*x)*tan((a + b*x)/2)**2*b**2*c**2*d**2*x +
 48*cos(a + b*x)*sin(a + b*x)*tan((a + b*x)/2)**2*b**2*c*d**3*x**2 + 16*co
s(a + b*x)*sin(a + b*x)*tan((a + b*x)/2)**2*b**2*d**4*x**3 - 48*cos(a + b*
x)*sin(a + b*x)*tan((a + b*x)/2)**2*c*d**3 - 14*cos(a + b*x)*sin(a + b*x)*
b**4*c*d**3*x**4 + 12*cos(a + b*x)*sin(a + b*x)*b**2*c**3*d - 24*cos(a + b
*x)*sin(a + b*x)*b**2*c**2*d**2*x - 24*cos(a + b*x)*sin(a + b*x)*b**2*c*d*
*3*x**2 - 8*cos(a + b*x)*sin(a + b*x)*b**2*d**4*x**3 + 24*cos(a + b*x)*sin
(a + b*x)*c*d**3 - 56*cos(a + b*x)*tan((a + b*x)/2)**4*b**3*c*d**3*x**3 -
14*cos(a + b*x)*tan((a + b*x)/2)**4*b**3*d**4*x**4 + 24*cos(a + b*x)*tan((
a + b*x)/2)**4*b*c**2*d**2 - 48*cos(a + b*x)*tan((a + b*x)/2)**4*b*c*d**3*
x - 24*cos(a + b*x)*tan((a + b*x)/2)**4*b*d**4*x**2 + 112*cos(a + b*x)*tan
((a + b*x)/2)**2*b**3*c*d**3*x**3 + 28*cos(a + b*x)*tan((a + b*x)/2)**2*b*
*3*d**4*x**4 - 48*cos(a + b*x)*tan((a + b*x)/2)**2*b*c**2*d**2 + 96*cos...
```

3.292 $\int (c+dx)^3 \sec^2(a+bx)\tan(a+bx)\,dx$

Optimal result . . . 2515
Mathematica [B] (warning: unable to verify) . . . 2516
Rubi [A] (verified) . . . 2516
Maple [B] (verified) . . . 2519
Fricas [B] (verification not implemented) . . . 2520
Sympy [F] . . . 2521
Maxima [B] (verification not implemented) . . . 2521
Giac [F] . . . 2522
Mupad [F(-1)] . . . 2522
Reduce [F] . . . 2522

Optimal result

Integrand size = 22, antiderivative size = 115

$$\int (c+dx)^3 \sec^2(a+bx)\tan(a+bx)\,dx = \frac{3id(c+dx)^2}{2b^2} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{3id^3\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^4} + \frac{(c+dx)^3\sec^2(a+bx)}{2b} - \frac{3d(c+dx)^2\tan(a+bx)}{2b^2}$$

output

```
3/2*I*d*(d*x+c)^2/b^2-3*d^2*(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b^3+3/2*I*d^3*p
olylog(2,-exp(2*I*(b*x+a)))/b^4+1/2*(d*x+c)^3*sec(b*x+a)^2/b-3/2*d*(d*x+c)
^2*tan(b*x+a)/b^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 286 vs. $2(115) = 230$.

Time = 6.27 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.49

$$\int (c+dx)^3 \sec^2(a+bx)\tan(a+bx)\,dx = \frac{(c+dx)^3\sec^2(a+bx)}{2b}$$
$$-\frac{3cd^2\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))}{b^3(\cos^2(a)+\sin^2(a))}$$
$$-\frac{3d^3\csc(a)\left(b^2e^{-i\arctan(\cot(a))}x^2-\frac{\cot(a)(ibx(-\pi-2\arctan(\cot(a)))-\pi\log(1+e^{-2ibx})-2(bx-\arctan(\cot(a)))\log(1-e^{2i(bx-\arct}{2b^4\sqrt{\csc^2(a)(\cos^2(a)+}}$$
$$-\frac{3\sec(a)\sec(a+bx)(c^2d\sin(bx)+2cd^2x\sin(bx)+d^3x^2\sin(bx))}{2b^2}$$

input
```
Integrate[(c + d*x)^3*Sec[a + b*x]^2*Tan[a + b*x],x]
```

output
```
((c + d*x)^3*Sec[a + b*x]^2)/(2*b) - (3*c*d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Co
s[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) - (3*
d^3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTa
n[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 -
 E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*L
og[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]
]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^4*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]
^2)]) - (3*Sec[a]*Sec[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3*
x^2*Sin[b*x]))/(2*b^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {4909, 3042, 4672, 25, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \tan(a+bx) \sec^2(a+bx)\,dx$$

$$\downarrow 4909$$

$$\frac{(c+dx)^3 \sec^2(a+bx)}{2b} - \frac{3d\int (c+dx)^2 \sec^2(a+bx)dx}{2b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^3 \sec^2(a+bx)}{2b} - \frac{3d\int (c+dx)^2 \csc\left(a+bx+\frac{\pi}{2}\right)^2 dx}{2b}$$

$$\downarrow 4672$$

$$\frac{(c+dx)^3 \sec^2(a+bx)}{2b} - \frac{3d\left(\frac{2d\int -((c+dx)\tan(a+bx))dx}{b} + \frac{(c+dx)^2\tan(a+bx)}{b}\right)}{2b}$$

$$\downarrow 25$$

$$\frac{(c+dx)^3 \sec^2(a+bx)}{2b} - \frac{3d\left(\frac{(c+dx)^2\tan(a+bx)}{b} - \frac{2d\int (c+dx)\tan(a+bx)dx}{b}\right)}{2b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^3 \sec^2(a+bx)}{2b} - \frac{3d\left(\frac{(c+dx)^2\tan(a+bx)}{b} - \frac{2d\int (c+dx)\tan(a+bx)dx}{b}\right)}{2b}$$

$$\downarrow 4202$$

$$\frac{(c+dx)^3 \sec^2(a+bx)}{2b} - \frac{3d\left(\frac{(c+dx)^2\tan(a+bx)}{b} - \frac{2d\left(\frac{i(c+dx)^2}{2d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}dx\right)}{b}\right)}{2b}$$

$$\downarrow 2620$$

$$\frac{(c+dx)^3 \sec^2(a+bx)}{2b} - \frac{3d\left(\frac{(c+dx)^2\tan(a+bx)}{b} - \frac{2d\left(\frac{i(c+dx)^2}{2d} - 2i\left(\frac{id\int \log\left(1+e^{2i(a+bx)}\right)dx}{2b} - \frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}\right)}{2b}$$

$$\downarrow 2715$$

$$\frac{(c+dx)^3\sec^2(a+bx)}{2b} - \frac{3d\left(\frac{(c+dx)^2\tan(a+bx)}{b} - \frac{2d\left(\frac{i(c+dx)^2}{2d} - 2i\left(\frac{d\int e^{-2i(a+bx)}\log\left(1+e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2} - \frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}\right)}{2b}$$

$$\downarrow 2838$$

$$\frac{(c+dx)^3\sec^2(a+bx)}{2b} - \frac{3d\left(\frac{(c+dx)^2\tan(a+bx)}{b} - \frac{2d\left(\frac{i(c+dx)^2}{2d} - 2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2} - \frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}\right)}{2b}$$

input
```
Int[(c + d*x)^3*Sec[a + b*x]^2*Tan[a + b*x],x]
```

output
```
((c + d*x)^3*Sec[a + b*x]^2)/(2*b) - (3*d*((-2*d*(((I/2)*(c + d*x)^2)/d -
(2*I)*(((-1/2*I)*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b - (d*PolyLog[2,
 -E^((2*I)*(a + b*x))])/(4*b^2))))/b + ((c + d*x)^2*Tan[a + b*x])/b))/(2*b
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(101) = 202$.

Time = 2.54 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.62

method	result
risch	$\frac{2b\,d^3x^3e^{2i(bx+a)}-3id^3x^2e^{2i(bx+a)}+6bc\,d^2x^2e^{2i(bx+a)}-6ic\,d^2x\,e^{2i(bx+a)}+6b\,c^2dx\,e^{2i(bx+a)}-3ic^2d\,e^{2i(bx+a)}-3id^3x^2+2b\,c^3e^{2i(bx}$ $\overline{b^2\left(e^{2i(bx+a)}+1\right)^2}$

input
```
int((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*e
xp(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-
3*I*c^2*d*exp(2*I*(b*x+a))-3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))-6*I*c*d^2*
x-3*I*c^2*d)/b^2/(exp(2*I*(b*x+a))+1)^2-3/b^3*d^2*c*ln(exp(2*I*(b*x+a))+1)
+6*d^2/b^3*c*ln(exp(I*(b*x+a)))+3*I/b^2*d^3*x^2+6*I/b^3*d^3*a*x+3*I/b^4*d^
3*a^2-3/b^3*d^3*ln(exp(2*I*(b*x+a))+1)*x+3/2*I*d^3*polylog(2,-exp(2*I*(b*x
+a)))/b^4-6*d^3/b^4*a*ln(exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(98) = 196$.

Time $= 0.11$ (sec) , antiderivative size $= 540$, normalized size of antiderivative $= 4.70$

$$\int (c+dx)^3 \sec^2(a+bx)\tan(a+bx)\,dx$$
$$= \frac{b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3 - 3i\,d^3\cos(bx+a)^2\,\mathrm{Li}_2(i\cos(bx+a)+\sin(bx+a)) + 3i\,d^3\cos(b}{}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 3*I*d^3*cos
(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + 3*I*d^3*cos(b*x + a)^2*
dilog(I*cos(b*x + a) - sin(b*x + a)) + 3*I*d^3*cos(b*x + a)^2*dilog(-I*cos
(b*x + a) + sin(b*x + a)) - 3*I*d^3*cos(b*x + a)^2*dilog(-I*cos(b*x + a) -
 sin(b*x + a)) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(cos(b*x + a) + I*s
in(b*x + a) + I) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I
*sin(b*x + a) + I) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(I*cos(b*x + a)
 + sin(b*x + a) + 1) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(I*cos(b*x +
a) - sin(b*x + a) + 1) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(-I*cos(b*x
 + a) + sin(b*x + a) + 1) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(-I*cos(
b*x + a) - sin(b*x + a) + 1) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(-cos
(b*x + a) + I*sin(b*x + a) + I) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(-
cos(b*x + a) - I*sin(b*x + a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*
c^2*d)*cos(b*x + a)*sin(b*x + a))/(b^4*cos(b*x + a)^2)
```

Sympy [F]

$$\int (c+dx)^3 \sec^2(a+bx)\tan(a+bx)\,dx = \int (c+dx)^3 \tan(a+bx)\sec^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*sec(b*x+a)**2*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**3*tan(a + b*x)*sec(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 668 vs. $2(98) = 196$.

Time = 0.28 (sec) , antiderivative size = 668, normalized size of antiderivative = 5.81

$$\int (c+dx)^3 \sec^2(a+bx)\tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

output
```
-(6*b^2*c^2*d + 6*(b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*cos(4*b*x + 4*a
) + 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a) - (-I*b*d^3*x - I*b*c*d^2)*sin(
4*b*x + 4*a) - 2*(-I*b*d^3*x - I*b*c*d^2)*sin(2*b*x + 2*a))*arctan2(sin(2*
b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x)*cos(4*
b*x + 4*a) - 2*(-2*I*b^3*d^3*x^3 - 2*I*b^3*c^3 - 3*b^2*c^2*d + 3*(-2*I*b^3
*c*d^2 + b^2*d^3)*x^2 + 6*(-I*b^3*c^2*d + b^2*c*d^2)*x)*cos(2*b*x + 2*a) -
 3*(d^3*cos(4*b*x + 4*a) + 2*d^3*cos(2*b*x + 2*a) + I*d^3*sin(4*b*x + 4*a)
 + 2*I*d^3*sin(2*b*x + 2*a) + d^3)*dilog(-e^(2*I*b*x + 2*I*a)) - 3*(I*b*d^
3*x + I*b*c*d^2 + (I*b*d^3*x + I*b*c*d^2)*cos(4*b*x + 4*a) + 2*(I*b*d^3*x
+ I*b*c*d^2)*cos(2*b*x + 2*a) - (b*d^3*x + b*c*d^2)*sin(4*b*x + 4*a) - 2*(
b*d^3*x + b*c*d^2)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x +
2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 6*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x)*sin
(4*b*x + 4*a) - 2*(2*b^3*d^3*x^3 + 2*b^3*c^3 - 3*I*b^2*c^2*d + 3*(2*b^3*c*
d^2 + I*b^2*d^3)*x^2 + 6*(b^3*c^2*d + I*b^2*c*d^2)*x)*sin(2*b*x + 2*a))/(-
2*I*b^4*cos(4*b*x + 4*a) - 4*I*b^4*cos(2*b*x + 2*a) + 2*b^4*sin(4*b*x + 4*
a) + 4*b^4*sin(2*b*x + 2*a) - 2*I*b^4)
```

Giac [F]

$$\int (c+dx)^3 \sec^2(a+bx)\tan(a+bx)\,dx = \int (dx+c)^3 \sec(bx+a)^2 \tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*sec(b*x + a)^2*tan(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \sec^2(a+bx)\tan(a+bx)\,dx = \int \frac{\tan(a+b\,x)\,(c+d\,x)^3}{\cos(a+b\,x)^2}\,dx$$

input
```
int((tan(a + b*x)*(c + d*x)^3)/cos(a + b*x)^2,x)
```

output
```
int((tan(a + b*x)*(c + d*x)^3)/cos(a + b*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^3 \sec^2(a+bx)\tan(a+bx)\,dx = \text{too large to display}$$

input
```
int((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x)
```

output

```
( - 14*cos(a + b*x)*sin(a + b*x)*tan((a + b*x)/2)**4*b**4*c*d**2*x**3 + 9*
cos(a + b*x)*sin(a + b*x)*tan((a + b*x)/2)**4*b**2*c**2*d - 12*cos(a + b*x
)*sin(a + b*x)*tan((a + b*x)/2)**4*b**2*c*d**2*x - 6*cos(a + b*x)*sin(a +
b*x)*tan((a + b*x)/2)**4*b**2*d**3*x**2 + 6*cos(a + b*x)*sin(a + b*x)*tan(
(a + b*x)/2)**4*d**3 + 28*cos(a + b*x)*sin(a + b*x)*tan((a + b*x)/2)**2*b*
*4*c*d**2*x**3 - 18*cos(a + b*x)*sin(a + b*x)*tan((a + b*x)/2)**2*b**2*c**
2*d + 24*cos(a + b*x)*sin(a + b*x)*tan((a + b*x)/2)**2*b**2*c*d**2*x + 12*
cos(a + b*x)*sin(a + b*x)*tan((a + b*x)/2)**2*b**2*d**3*x**2 - 12*cos(a +
b*x)*sin(a + b*x)*tan((a + b*x)/2)**2*d**3 - 14*cos(a + b*x)*sin(a + b*x)*
b**4*c*d**2*x**3 + 9*cos(a + b*x)*sin(a + b*x)*b**2*c**2*d - 12*cos(a + b*
x)*sin(a + b*x)*b**2*c*d**2*x - 6*cos(a + b*x)*sin(a + b*x)*b**2*d**3*x**2
 + 6*cos(a + b*x)*sin(a + b*x)*d**3 - 42*cos(a + b*x)*tan((a + b*x)/2)**4*
b**3*c*d**2*x**2 - 14*cos(a + b*x)*tan((a + b*x)/2)**4*b**3*d**3*x**3 + 12
*cos(a + b*x)*tan((a + b*x)/2)**4*b*c*d**2 - 12*cos(a + b*x)*tan((a + b*x)
/2)**4*b*d**3*x + 84*cos(a + b*x)*tan((a + b*x)/2)**2*b**3*c*d**2*x**2 + 2
8*cos(a + b*x)*tan((a + b*x)/2)**2*b**3*d**3*x**3 - 24*cos(a + b*x)*tan((a
 + b*x)/2)**2*b*c*d**2 + 24*cos(a + b*x)*tan((a + b*x)/2)**2*b*d**3*x - 42
*cos(a + b*x)*b**3*c*d**2*x**2 - 14*cos(a + b*x)*b**3*d**3*x**3 + 12*cos(a
 + b*x)*b*c*d**2 - 12*cos(a + b*x)*b*d**3*x + 12*int(x**2/(tan((a + b*x)/2
)**6 - 3*tan((a + b*x)/2)**4 + 3*tan((a + b*x)/2)**2 - 1),x)*sin(a + b*...
```

3.293 $\int (c+dx)^2 \sec^2(a+bx)\tan(a+bx)\,dx$

Optimal result . . . 2524
Mathematica [A] (verified) . . . 2524
Rubi [A] (verified) . . . 2525
Maple [C] (verified) . . . 2526
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Giac [B] (verification not implemented) . . . 2529
Mupad [B] (verification not implemented) . . . 2530
Reduce [F] . . . 2531

Optimal result

Integrand size = 22, antiderivative size = 55

$$\int (c+dx)^2 \sec^2(a+bx)\tan(a+bx)\,dx = -\frac{d^2\log(\cos(a+bx))}{b^3} + \frac{(c+dx)^2\sec^2(a+bx)}{2b} - \frac{d(c+dx)\tan(a+bx)}{b^2}$$

output

```
-d^2*ln(cos(b*x+a))/b^3+1/2*(d*x+c)^2*sec(b*x+a)^2/b-d*(d*x+c)*tan(b*x+a)/
b^2
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

$$\int (c+dx)^2 \sec^2(a+bx)\tan(a+bx)\,dx = \frac{b^2(c+dx)^2\sec^2(a+bx) - 2bd(c+dx)\sec(a)\sec(a+bx)\sin(bx) - 2d^2(\log(\cos(a+bx)) + bx\tan(a))}{2b^3}$$

input

```
Integrate[(c + d*x)^2*Sec[a + b*x]^2*Tan[a + b*x],x]
```

output
```
(b^2*(c + d*x)^2*Sec[a + b*x]^2 - 2*b*d*(c + d*x)*Sec[a]*Sec[a + b*x]*Sin[
b*x] - 2*d^2*(Log[Cos[a + b*x]] + b*x*Tan[a]))/(2*b^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4909, 3042, 4672, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \tan(a+bx) \sec^2(a+bx)\, dx$$

$$\downarrow 4909$$

$$\frac{(c+dx)^2 \sec^2(a+bx)}{2b} - \frac{d\int (c+dx)\sec^2(a+bx)dx}{b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^2 \sec^2(a+bx)}{2b} - \frac{d\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)^2 dx}{b}$$

$$\downarrow 4672$$

$$\frac{(c+dx)^2 \sec^2(a+bx)}{2b} - \frac{d\left(\frac{d\int -\tan(a+bx)dx}{b} + \frac{(c+dx)\tan(a+bx)}{b}\right)}{b}$$

$$\downarrow 25$$

$$\frac{(c+dx)^2 \sec^2(a+bx)}{2b} - \frac{d\left(\frac{(c+dx)\tan(a+bx)}{b} - \frac{d\int \tan(a+bx)dx}{b}\right)}{b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)^2 \sec^2(a+bx)}{2b} - \frac{d\left(\frac{(c+dx)\tan(a+bx)}{b} - \frac{d\int \tan(a+bx)dx}{b}\right)}{b}$$

$$\downarrow 3956$$

$$\frac{(c+dx)^2 \sec^2(a+bx)}{2b} - \frac{d\left(\frac{d\log(\cos(a+bx))}{b^2} + \frac{(c+dx)\tan(a+bx)}{b}\right)}{b}$$

input
```
Int[(c + d*x)^2*Sec[a + b*x]^2*Tan[a + b*x],x]
```

output
```
((c + d*x)^2*Sec[a + b*x]^2)/(2*b) - (d*((d*Log[Cos[a + b*x]])/b^2 + ((c +
 d*x)*Tan[a + b*x])/b))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.71

method	result
risch	$\frac{2id^2x}{b^2}+\frac{2id^2a}{b^3}+\frac{2b\,d^2x^2e^{2i(bx+a)}+4bcdx\,e^{2i(bx+a)}+2b\,c^2e^{2i(bx+a)}-2id^2x\,e^{2i(bx+a)}-2icd\,e^{2i(bx+a)}-2id^2x-2idc}{b^2\left(e^{2i(bx+a)}+1\right)^2}$
derivativedivides	$\frac{\frac{a^2d^2}{2b^2\cos(bx+a)^2}-\frac{acd}{b\cos(bx+a)^2}-\frac{2a\,d^2\left(\frac{bx+a}{2\cos(bx+a)^2}-\frac{\tan(bx+a)}{2}\right)}{b^2}+\frac{c^2}{2\cos(bx+a)^2}+\frac{2cd\left(\frac{bx+a}{2\cos(bx+a)^2}-\frac{\tan(bx+a)}{2}\right)}{b}+\frac{d^2\left(\frac{}{2c}\right.}{b}$
default	$\frac{\frac{a^2d^2}{2b^2\cos(bx+a)^2}-\frac{acd}{b\cos(bx+a)^2}-\frac{2a\,d^2\left(\frac{bx+a}{2\cos(bx+a)^2}-\frac{\tan(bx+a)}{2}\right)}{b^2}+\frac{c^2}{2\cos(bx+a)^2}+\frac{2cd\left(\frac{bx+a}{2\cos(bx+a)^2}-\frac{\tan(bx+a)}{2}\right)}{b}+\frac{d^2\left(\frac{}{2c}\right.}{b}$

input
```
int((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*I*d^2/b^2*x+2*I*d^2/b^3*a+2*(b*d^2*x^2*exp(2*I*(b*x+a))+2*b*c*d*x*exp(2*
I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^2*x*exp(2*I*(b*x+a))-I*c*d*exp(2*I*(
b*x+a))-I*d^2*x-I*d*c)/b^2/(exp(2*I*(b*x+a))+1)^2-d^2/b^3*ln(exp(2*I*(b*x+
a))+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.56

$$\int (c+dx)^2\sec^2(a+bx)\tan(a+bx)\,dx$$
$$=\frac{b^2d^2x^2+2\,b^2cdx-2\,d^2\cos(bx+a)^2\log(-\cos(bx+a))+b^2c^2-2\,(bd^2x+bcd)\cos(bx+a)\sin(bx+a}{2\,b^3\cos(bx+a)^2}$$

input
```
integrate((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x - 2*d^2*cos(b*x + a)^2*log(-cos(b*x + a)) +
 b^2*c^2 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3*cos(b*x + a
)^2)
```

Sympy [F]

$$\int (c+dx)^2 \sec^2(a+bx)\tan(a+bx)\,dx = \int (c+dx)^2 \tan(a+bx)\sec^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*sec(b*x+a)**2*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**2*tan(a + b*x)*sec(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 988 vs. $2(53) = 106$.

Time = 0.14 (sec) , antiderivative size = 988, normalized size of antiderivative = 17.96

$$\int (c+dx)^2 \sec^2(a+bx)\tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

output

```
1/2*(c^2*tan(b*x + a)^2 - 2*a*c*d*tan(b*x + a)^2/b + a^2*d^2*tan(b*x + a)^
2/b^2 + 4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2
 + (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*
(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2
*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*c*d/((2*(2*cos(2*b*x + 2*a)
+ 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*
b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2
+ 4*cos(2*b*x + 2*a) + 1)*b) - 4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x
+ a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)
)*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x
 + 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*a*d^2
/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*co
s(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a
) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2) + (8*(b*x + a)^2*c
os(2*b*x + 2*a)^2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 + 4*(b*x + a)^2*cos(2
*b*x + 2*a) + 4*((b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a)
)*cos(4*b*x + 4*a) - (2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*
b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4
*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*(...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4331 vs. $2(53) = 106$.

Time $= 0.86$ (sec) , antiderivative size $= 4331$, normalized size of antiderivative $= 78.75$

$$\int (c + dx)^2 \sec^2(a + bx) \tan(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

output

```
1/2*(b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4*
tan(1/2*a)^4 + 2*b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*d^2*x^2*t
an(1/2*b*x)^2*tan(1/2*a)^4 + b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b^2*c
*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 4*b*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^3 +
 4*b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 + 4*b*d^2*x*tan(1/2*b*x)^3*tan(1/
2*a)^4 - d^2*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^4 - 2*tan(1/2*b*x)^4*tan(1/2
*a)^2 - 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^2*tan(1/2*a)^4 + ta
n(1/2*b*x)^4 + 8*tan(1/2*b*x)^3*tan(1/2*a) + 20*tan(1/2*b*x)^2*tan(1/2*a)^
2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 2*tan(1/2*b*x)^2 - 8*tan(
1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^4 + 2
*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2*tan(1/2*a)^4 + tan(1/2*b*x
)^4 + 4*tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*a)^4 + 2*tan(1/2*b*x)^2 + 2*
tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^4 + b^2*d^2*x^2*tan(1/2*b*x)^
4 + 4*b^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b^2*c^2*tan(1/2*b*x)^4*t
an(1/2*a)^2 + 4*b*c*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + b^2*d^2*x^2*tan(1/2*a)
^4 + 2*b^2*c^2*tan(1/2*b*x)^2*tan(1/2*a)^4 + 4*b*c*d*tan(1/2*b*x)^3*tan(1/
2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4 - 4*b*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)
+ 8*b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - 24*b*d^2*x*tan(1/2*b*x)^3*tan(
1/2*a)^2 + 2*d^2*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^4 - 2*tan(1/2*b*x)^4*tan
(1/2*a)^2 - 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^2*tan(1/2*a)...
```

Mupad [B] (verification not implemented)

Time = 22.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.73

$$\int (c+dx)^2 \sec^2(a+bx)\tan(a+bx)\,dx = -\frac{\frac{(c+dx)^2}{b} - \frac{e^{a\,2i+b\,x\,2i}\,(c+dx)^2}{b}}{2\,e^{a\,2i+b\,x\,2i} + e^{a\,4i+b\,x\,4i} + 1} + \frac{d^2\,x\,2i}{b^2} + \frac{b\,c^2 + 2\,b\,c\,d\,x - c\,d\,2i + b\,d^2\,x^2 - d^2\,x\,2i}{b^2\,\left(e^{a\,2i+b\,x\,2i} + 1\right)} - \frac{d^2\,\ln\left(e^{a\,2i}\,e^{b\,x\,2i} + 1\right)}{b^3}$$

input

```
int((tan(a + b*x)*(c + d*x)^2)/cos(a + b*x)^2,x)
```

output

```
(d^2*x*2i)/b^2 - ((c + d*x)^2/b - (exp(a*2i + b*x*2i)*(c + d*x)^2)/b)/(2*e
xp(a*2i + b*x*2i) + exp(a*4i + b*x*4i) + 1) + (b*c^2 - c*d*2i - d^2*x*2i +
 b*d^2*x^2 + 2*b*c*d*x)/(b^2*(exp(a*2i + b*x*2i) + 1)) - (d^2*log(exp(a*2i
)*exp(b*x*2i) + 1))/b^3
```

Reduce [F]

$$\int (c+dx)^2 \sec^2(a+bx)\tan(a+bx)\,dx = \text{too large to display}$$

input
```
int((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x)
```

output
```
(6*cos(a + b*x)*sin(a + b*x)*tan((a + b*x)/2)**4*b*c*d - 4*cos(a + b*x)*si
n(a + b*x)*tan((a + b*x)/2)**4*b*d**2*x - 12*cos(a + b*x)*sin(a + b*x)*tan
((a + b*x)/2)**2*b*c*d + 8*cos(a + b*x)*sin(a + b*x)*tan((a + b*x)/2)**2*b
*d**2*x + 6*cos(a + b*x)*sin(a + b*x)*b*c*d - 4*cos(a + b*x)*sin(a + b*x)*
b*d**2*x - 14*cos(a + b*x)*tan((a + b*x)/2)**4*b**2*d**2*x**2 + 4*cos(a +
b*x)*tan((a + b*x)/2)**4*d**2 + 28*cos(a + b*x)*tan((a + b*x)/2)**2*b**2*d
**2*x**2 - 8*cos(a + b*x)*tan((a + b*x)/2)**2*d**2 - 14*cos(a + b*x)*b**2*
d**2*x**2 + 4*cos(a + b*x)*d**2 + 56*int((tan((a + b*x)/2)**2*x)/(tan((a +
 b*x)/2)**6 - 3*tan((a + b*x)/2)**4 + 3*tan((a + b*x)/2)**2 - 1),x)*sin(a
+ b*x)**2*tan((a + b*x)/2)**4*b**2*d**2 - 112*int((tan((a + b*x)/2)**2*x)/
(tan((a + b*x)/2)**6 - 3*tan((a + b*x)/2)**4 + 3*tan((a + b*x)/2)**2 - 1),
x)*sin(a + b*x)**2*tan((a + b*x)/2)**2*b**2*d**2 + 56*int((tan((a + b*x)/2
)**2*x)/(tan((a + b*x)/2)**6 - 3*tan((a + b*x)/2)**4 + 3*tan((a + b*x)/2)*
*2 - 1),x)*sin(a + b*x)**2*b**2*d**2 - 56*int((tan((a + b*x)/2)**2*x)/(tan
((a + b*x)/2)**6 - 3*tan((a + b*x)/2)**4 + 3*tan((a + b*x)/2)**2 - 1),x)*t
an((a + b*x)/2)**4*b**2*d**2 + 112*int((tan((a + b*x)/2)**2*x)/(tan((a + b
*x)/2)**6 - 3*tan((a + b*x)/2)**4 + 3*tan((a + b*x)/2)**2 - 1),x)*tan((a +
 b*x)/2)**2*b**2*d**2 - 56*int((tan((a + b*x)/2)**2*x)/(tan((a + b*x)/2)**
6 - 3*tan((a + b*x)/2)**4 + 3*tan((a + b*x)/2)**2 - 1),x)*b**2*d**2 + 8*in
t(x/(tan((a + b*x)/2)**6 - 3*tan((a + b*x)/2)**4 + 3*tan((a + b*x)/2)**...
```

3.294 $\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx$

Optimal result . . . 2532
Mathematica [A] (verified) . . . 2532
Rubi [A] (verified) . . . 2533
Maple [A] (verified) . . . 2534
Fricas [A] (verification not implemented) . . . 2535
Sympy [F] . . . 2535
Maxima [B] (verification not implemented) . . . 2535
Giac [B] (verification not implemented) . . . 2536
Mupad [B] (verification not implemented) . . . 2537
Reduce [B] (verification not implemented) . . . 2537

Optimal result

Integrand size = 20, antiderivative size = 35

$$\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx = \frac{(c+dx)\sec^2(a+bx)}{2b} - \frac{d\tan(a+bx)}{2b^2}$$

output
```
1/2*(d*x+c)*sec(b*x+a)^2/b-1/2*d*tan(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx = \frac{c\sec^2(a+bx)}{2b} + \frac{dx\sec^2(a+bx)}{2b} - \frac{d\tan(a+bx)}{2b^2}$$

input
```
Integrate[(c + d*x)*Sec[a + b*x]^2*Tan[a + b*x],x]
```

output
```
(c*Sec[a + b*x]^2)/(2*b) + (d*x*Sec[a + b*x]^2)/(2*b) - (d*Tan[a + b*x])/(
2*b^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4909, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\tan(a+bx)\sec^2(a+bx)\,dx$$

$$\downarrow 4909$$

$$\frac{(c+dx)\sec^2(a+bx)}{2b} - \frac{d\int \sec^2(a+bx)dx}{2b}$$

$$\downarrow 3042$$

$$\frac{(c+dx)\sec^2(a+bx)}{2b} - \frac{d\int \csc\left(a+bx+\frac{\pi}{2}\right)^2 dx}{2b}$$

$$\downarrow 4254$$

$$\frac{d\int 1d(-\tan(a+bx))}{2b^2} + \frac{(c+dx)\sec^2(a+bx)}{2b}$$

$$\downarrow 24$$

$$\frac{(c+dx)\sec^2(a+bx)}{2b} - \frac{d\tan(a+bx)}{2b^2}$$

input
```
Int[(c + d*x)*Sec[a + b*x]^2*Tan[a + b*x],x]
```

output
```
((c + d*x)*Sec[a + b*x]^2)/(2*b) - (d*Tan[a + b*x])/(2*b^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

method	result	size
derivativedivides	$\frac{-\frac{da}{2b\cos(bx+a)^2}+\frac{c}{2\cos(bx+a)^2}+\frac{d\left(\frac{bx+a}{2\cos(bx+a)^2}-\frac{\tan(bx+a)}{2}\right)}{b}}{b}$	61
default	$\frac{-\frac{da}{2b\cos(bx+a)^2}+\frac{c}{2\cos(bx+a)^2}+\frac{d\left(\frac{bx+a}{2\cos(bx+a)^2}-\frac{\tan(bx+a)}{2}\right)}{b}}{b}$	61
risch	$\frac{2bdx\,e^{2i(bx+a)}-id\,e^{2i(bx+a)}+2bc\,e^{2i(bx+a)}-id}{b^2\left(e^{2i(bx+a)}+1\right)^2}$	63

input
```
int((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/b*d*a/cos(b*x+a)^2+1/2*c/cos(b*x+a)^2+1/b*d*(1/2*(b*x+a)/cos(b*x
+a)^2-1/2*tan(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx = \frac{bdx - d\cos(bx+a)\sin(bx+a) + bc}{2\,b^2\cos(bx+a)^2}$$

input
```
integrate((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b*d*x - d*cos(b*x + a)*sin(b*x + a) + b*c)/(b^2*cos(b*x + a)^2)
```

Sympy [F]

$$\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx = \int (c+dx)\tan(a+bx)\sec^2(a+bx)\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a)**2*tan(b*x+a),x)
```

output
```
Integral((c + d*x)*tan(a + b*x)*sec(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(31) = 62.

Time = 0.04 (sec) , antiderivative size = 283, normalized size of antiderivative = 8.09

$$\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx$$
$$= \frac{c\tan(bx+a)^2 - \frac{ad\tan(bx+a)^2}{b} + \frac{2\left(4\,(bx+a)\cos(2\,bx+2\,a)^2+4\,(bx+a)\sin(2\,bx+2\,a)^2+(2\,(bx+a)\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a))\cos(4\right.}{\left(2\,(2\,\cos(2\,bx+2\,a)+1)\cos(4\,bx+4\,a)+\cos(4\,bx+4\,a)^2+4\,\cos(2\,bx+2\,a)^2+s\right.}}{2\,b}$$

input
```
integrate((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(c*tan(b*x + a)^2 - a*d*tan(b*x + a)^2/b + 2*(4*(b*x + a)*cos(2*b*x +
2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) +
sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*
x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b
*x + 2*a))*d/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4
*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(31) = 62$.

Time $= 0.22$ (sec) , antiderivative size $= 571$, normalized size of antiderivative $= 16.31$

$$\int (c + dx) \sec^2(a + bx) \tan(a + bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

output
```
1/2*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4 +
 2*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4
 + 2*b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + 2
*b*c*tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + b*d*x
*tan(1/2*b*x)^4 + 4*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*d*x*tan(1/2*a)^4
 + b*c*tan(1/2*b*x)^4 - 2*d*tan(1/2*b*x)^4*tan(1/2*a) + 4*b*c*tan(1/2*b*x)
^2*tan(1/2*a)^2 - 12*d*tan(1/2*b*x)^3*tan(1/2*a)^2 - 12*d*tan(1/2*b*x)^2*t
an(1/2*a)^3 + b*c*tan(1/2*a)^4 - 2*d*tan(1/2*b*x)*tan(1/2*a)^4 + 2*b*d*x*t
an(1/2*b*x)^2 + 2*b*d*x*tan(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2 + 2*d*tan(1/2*
b*x)^3 + 12*d*tan(1/2*b*x)^2*tan(1/2*a) + 2*b*c*tan(1/2*a)^2 + 12*d*tan(1/
2*b*x)*tan(1/2*a)^2 + 2*d*tan(1/2*a)^3 + b*d*x + b*c - 2*d*tan(1/2*b*x) -
2*d*tan(1/2*a))/(b^2*tan(1/2*b*x)^4*tan(1/2*a)^4 - 2*b^2*tan(1/2*b*x)^4*ta
n(1/2*a)^2 - 8*b^2*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*b^2*tan(1/2*b*x)^2*tan(
1/2*a)^4 + b^2*tan(1/2*b*x)^4 + 8*b^2*tan(1/2*b*x)^3*tan(1/2*a) + 20*b^2*t
an(1/2*b*x)^2*tan(1/2*a)^2 + 8*b^2*tan(1/2*b*x)*tan(1/2*a)^3 + b^2*tan(1/2
*a)^4 - 2*b^2*tan(1/2*b*x)^2 - 8*b^2*tan(1/2*b*x)*tan(1/2*a) - 2*b^2*tan(1
/2*a)^2 + b^2)
```

Mupad [B] (verification not implemented)

Time = 19.81 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx = -\frac{d\,1\mathrm{i} + \mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\left(-b\left(2\,c+2\,d\,x\right)+d\,1\mathrm{i}\right)}{b^2\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)^2}$$

input
```
int((tan(a + b*x)*(c + d*x))/cos(a + b*x)^2,x)
```

output
```
-(d*1i + exp(a*2i + b*x*2i)*(d*1i - b*(2*c + 2*d*x)))/(b^2*(exp(a*2i + b*x
*2i) + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.40

$$\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx = \frac{\cos\left(bx+a\right)\sin\left(bx+a\right)d - \sin\left(bx+a\right)^2 bc - bdx}{2b^2\left(\sin\left(bx+a\right)^2-1\right)}$$

input
```
int((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x)
```

output
```
(cos(a + b*x)*sin(a + b*x)*d - sin(a + b*x)**2*b*c - b*d*x)/(2*b**2*(sin(a
 + b*x)**2 - 1))
```

3.295 $\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$

Optimal result . . . 2538
Mathematica [N/A] . . . 2538
Rubi [N/A] . . . 2539
Maple [N/A] . . . 2539
Fricas [N/A] . . . 2540
Sympy [N/A] . . . 2540
Maxima [N/A] . . . 2541
Giac [N/A] . . . 2542
Mupad [N/A] . . . 2542
Reduce [N/A] . . . 2542

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}, x\right)$$

output
```
Defer(Int)(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 5.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
```

output
```
Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$
$$\downarrow 7299$$
$$\int \frac{\tan(a+bx)\sec^2(a+bx)}{c+dx}dx$$

input
```
Int[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sec(bx+a)^2\tan(bx+a)}{dx+c}dx$$

input
```
int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x)
```

output
```
int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec\left(bx+a\right)^2\tan\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)^2*tan(b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\tan\left(a+bx\right)\sec^2\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(sec(b*x+a)**2*tan(b*x+a)/(d*x+c),x)
```

output
```
Integral(tan(a + b*x)*sec(a + b*x)**2/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 1090, normalized size of antiderivative = 49.55

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec\left(bx+a\right)^2\tan\left(bx+a\right)}{dx+c}\,dx$$

input

```
integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output

```
(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
 (2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
+ 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + 2*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*
c^2*d^2 + (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(4*b*x + 4*a)^2 +
 4*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(2*b*x + 2*a)^2 + (b^2*d
^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^4*x^2
+ 2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*
d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^4*x^2
 + 2*b^2*c*d^3*x + b^2*c^2*d^2 + 2*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*
d^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^4*x^2 + 2*b^2*c*d^3*x +
 b^2*c^2*d^2)*cos(2*b*x + 2*a))*integrate(sin(2*b*x + 2*a)/(b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^
2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b
^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)), x) + (d*cos(2*b
*x + 2*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a) + d)*sin(4*b*x + 4*a) + d*sin
(2*b*x + 2*a))/(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*
c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x
+ 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin...
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)^2\tan(bx+a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^2*tan(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 19.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\tan(a+b\,x)}{\cos(a+b\,x)^2\,(c+d\,x)}\,dx$$

input
```
int(tan(a + b*x)/(cos(a + b*x)^2*(c + d*x)),x)
```

output
```
int(tan(a + b*x)/(cos(a + b*x)^2*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)^2\tan(bx+a)}{dx+c}dx$$

input
```
int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x)
```

output
```
int((sec(a + b*x)**2*tan(a + b*x))/(c + d*x),x)
```

3.296 $\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 2544
Mathematica [N/A] . . . 2544
Rubi [N/A] . . . 2545
Maple [N/A] . . . 2545
Fricas [N/A] . . . 2546
Sympy [N/A] . . . 2546
Maxima [N/A] . . . 2547
Giac [N/A] . . . 2548
Mupad [N/A] . . . 2548
Reduce [N/A] . . . 2548

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 7.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$
$$\downarrow 7299$$
$$\int \frac{\tan(a+bx)\sec^2(a+bx)}{(c+dx)^2}dx$$

input
```
Int[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sec(bx+a)^2\tan(bx+a)}{(dx+c)^2}dx$$

input
```
int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x)
```

output
```
int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec\left(bx+a\right)^2\tan\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)^2*tan(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan\left(a+bx\right)\sec^2\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(sec(b*x+a)**2*tan(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(tan(a + b*x)*sec(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 1396, normalized size of antiderivative = 63.45

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec\left(bx+a\right)^2\tan\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input

```
integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output

```
2*(2*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 2*(b*d*x + b*c)*sin(2*b*x + 2*a)^2
 + ((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
+ (b*d*x + b*c)*cos(2*b*x + 2*a) + 3*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^
2*c^2*d^3*x + b^2*c^3*d^2 + (b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3
*x + b^2*c^3*d^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 +
3*b^2*c^2*d^3*x + b^2*c^3*d^2)*cos(2*b*x + 2*a)^2 + (b^2*d^5*x^3 + 3*b^2*c
*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^5*
x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*sin(4*b*x + 4*a)*si
n(2*b*x + 2*a) + 4*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*
c^3*d^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2
*d^3*x + b^2*c^3*d^2 + 2*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x
+ b^2*c^3*d^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^5*x^3 + 3*b^2
*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*cos(2*b*x + 2*a))*integrate(si
n(2*b*x + 2*a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*
c^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4
*b^2*c^3*d*x + b^2*c^4)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^
3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(2*b*x + 2*a)^2 + 2*(b
^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4
)*cos(2*b*x + 2*a)), x) + (d*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(2*b*x +
2*a) + d)*sin(4*b*x + 4*a) + d*sin(2*b*x + 2*a))/(b^2*d^3*x^3 + 3*b^2*c...
```

Giac [N/A]

Not integrable

Time = 6.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec\left(bx+a\right)^2\tan\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^2*tan(b*x + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 21.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(a+b\,x)}{\cos\left(a+b\,x\right)^2\left(c+d\,x\right)^2}\,dx$$

input
```
int(tan(a + b*x)/(cos(a + b*x)^2*(c + d*x)^2),x)
```

output
```
int(tan(a + b*x)/(cos(a + b*x)^2*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 5187, normalized size of antiderivative = 235.77

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x)
```

output

```
( - 5*cos(a + b*x)**2*sin(a + b*x)*d + 5*cos(a + b*x)*int(sin(a + b*x)**3/
(cos(a + b*x)*sin(a + b*x)**2*c**2 + 2*cos(a + b*x)*sin(a + b*x)**2*c*d*x
+ cos(a + b*x)*sin(a + b*x)**2*d**2*x**2 - cos(a + b*x)*c**2 - 2*cos(a + b
*x)*c*d*x - cos(a + b*x)*d**2*x**2),x)*sin(a + b*x)**2*c*d**2 + 5*cos(a +
b*x)*int(sin(a + b*x)**3/(cos(a + b*x)*sin(a + b*x)**2*c**2 + 2*cos(a + b*
x)*sin(a + b*x)**2*c*d*x + cos(a + b*x)*sin(a + b*x)**2*d**2*x**2 - cos(a
+ b*x)*c**2 - 2*cos(a + b*x)*c*d*x - cos(a + b*x)*d**2*x**2),x)*sin(a + b*
x)**2*d**3*x - 5*cos(a + b*x)*int(sin(a + b*x)**3/(cos(a + b*x)*sin(a + b*
x)**2*c**2 + 2*cos(a + b*x)*sin(a + b*x)**2*c*d*x + cos(a + b*x)*sin(a + b
*x)**2*d**2*x**2 - cos(a + b*x)*c**2 - 2*cos(a + b*x)*c*d*x - cos(a + b*x)
*d**2*x**2),x)*c*d**2 - 5*cos(a + b*x)*int(sin(a + b*x)**3/(cos(a + b*x)*s
in(a + b*x)**2*c**2 + 2*cos(a + b*x)*sin(a + b*x)**2*c*d*x + cos(a + b*x)*
sin(a + b*x)**2*d**2*x**2 - cos(a + b*x)*c**2 - 2*cos(a + b*x)*c*d*x - cos
(a + b*x)*d**2*x**2),x)*d**3*x + 8*cos(a + b*x)*int(sin(a + b*x)**2/(cos(a
 + b*x)*sin(a + b*x)**2*c**2 + 2*cos(a + b*x)*sin(a + b*x)**2*c*d*x + cos(
a + b*x)*sin(a + b*x)**2*d**2*x**2 - cos(a + b*x)*c**2 - 2*cos(a + b*x)*c*
d*x - cos(a + b*x)*d**2*x**2),x)*sin(a + b*x)**2*b*c**2*d + 8*cos(a + b*x)
*int(sin(a + b*x)**2/(cos(a + b*x)*sin(a + b*x)**2*c**2 + 2*cos(a + b*x)*s
in(a + b*x)**2*c*d*x + cos(a + b*x)*sin(a + b*x)**2*d**2*x**2 - cos(a + b*
x)*c**2 - 2*cos(a + b*x)*c*d*x - cos(a + b*x)*d**2*x**2),x)*sin(a + b*x...
```

3.297 $\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx$

Optimal result . . . 2550
Mathematica [N/A] . . . 2550
Rubi [N/A] . . . 2551
Maple [N/A] . . . 2552
Fricas [N/A] . . . 2552
Sympy [N/A] . . . 2553
Maxima [N/A] . . . 2553
Giac [N/A] . . . 2553
Mupad [N/A] . . . 2554
Reduce [N/A] . . . 2554

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx = -\text{Int}((c+dx)^m \sec(a+bx), x) + \text{Int}\left((c+dx)^m \sec^3(a+bx), x\right)$$

output
```
-Defer(Int)((d*x+c)^m*sec(b*x+a),x)+Defer(Int)((d*x+c)^m*sec(b*x+a)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 34.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx = \int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4913, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(a+bx)\sec(a+bx)(c+dx)^m\,dx$$

$$\downarrow 4913$$

$$\int (c+dx)^m \sec^3(a+bx)dx - \int (c+dx)^m \sec(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)^m \csc\left(a+bx+\frac{\pi}{2}\right)^3 dx - \int (c+dx)^m \csc\left(a+bx+\frac{\pi}{2}\right) dx$$

$$\downarrow 4680$$

$$\int (c+dx)^m \sec^3(a+bx)dx - \int (c+dx)^m \sec(a+bx)dx$$

input
```
Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4913

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \sec(bx+a) \tan(bx+a)^2 \, dx$$

input

```
int((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x)
```

output

```
int((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sec(a+bx) \tan^2(a+bx) \, dx = \int (dx+c)^m \sec(bx+a) \tan(bx+a)^2 \, dx$$

input

```
integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

output

```
integral((d*x + c)^m*sec(b*x + a)*tan(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 9.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx = \int (c+dx)^m \tan^2(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*sec(b*x+a)*tan(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*tan(a + b*x)**2*sec(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\tan(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\tan(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 20.53 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (c+dx)^m \sec(a+bx) \tan^2(a+bx)\, dx = \int \frac{\tan(a+b\,x)^2\,(c+d\,x)^m}{\cos(a+b\,x)}\, dx$$

input
```
int((tan(a + b*x)^2*(c + d*x)^m)/cos(a + b*x),x)
```

output
```
int((tan(a + b*x)^2*(c + d*x)^m)/cos(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sec(a+bx) \tan^2(a+bx)\, dx = \int (dx+c)^m \sec(bx+a) \tan(bx+a)^2\, dx$$

input
```
int((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x)
```

output
```
int((c + d*x)**m*sec(a + b*x)*tan(a + b*x)**2,x)
```

3.298 $\int (c+dx)^3 \sec(a+bx) \tan^2(a+bx)\, dx$

Optimal result . . . 2555
Mathematica [A] (verified) . . . 2556
Rubi [A] (verified) . . . 2556
Maple [B] (verified) . . . 2563
Fricas [B] (verification not implemented) . . . 2564
Sympy [F] . . . 2565
Maxima [B] (verification not implemented) . . . 2566
Giac [F] . . . 2567
Mupad [F(-1)] . . . 2567
Reduce [F] . . . 2567

Optimal result

Integrand size = 22, antiderivative size = 337

$$
\begin{aligned}
&\int (c+dx)^3 \sec(a+bx) \tan^2(a+bx)\, dx \\
&= -\frac{6id^2(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^3} + \frac{i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b} \\
&\quad + \frac{3id^3 \operatorname{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^4} - \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{2b^2} \\
&\quad - \frac{3id^3 \operatorname{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^4} + \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2, ie^{i(a+bx)}\right)}{2b^2} \\
&\quad + \frac{3d^2(c+dx) \operatorname{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^3} - \frac{3d^2(c+dx) \operatorname{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{3id^3 \operatorname{PolyLog}\left(4, -ie^{i(a+bx)}\right)}{b^4} - \frac{3id^3 \operatorname{PolyLog}\left(4, ie^{i(a+bx)}\right)}{b^4} \\
&\quad - \frac{3d(c+dx)^2 \sec(a+bx)}{2b^2} + \frac{(c+dx)^3 \sec(a+bx)\tan(a+bx)}{2b}
\end{aligned}
$$

output

```
-6*I*d^2*(d*x+c)*arctan(exp(I*(b*x+a)))/b^3+I*(d*x+c)^3*arctan(exp(I*(b*x+
a)))/b+3*I*d^3*polylog(2,-I*exp(I*(b*x+a)))/b^4-3/2*I*d*(d*x+c)^2*polylog(
2,-I*exp(I*(b*x+a)))/b^2-3*I*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4+3/2*I*d*(
d*x+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^2+3*d^2*(d*x+c)*polylog(3,-I*exp(I*
(b*x+a)))/b^3-3*d^2*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^3+3*I*d^3*polylo
g(4,-I*exp(I*(b*x+a)))/b^4-3*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-3/2*d*(
d*x+c)^2*sec(b*x+a)/b^2+1/2*(d*x+c)^3*sec(b*x+a)*tan(b*x+a)/b
```

Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.57

$$\int (c+dx)^3 \sec(a+bx)\tan^2(a+bx)\,dx$$
$$= \frac{2ib^3c^3 \arctan\left(e^{i(a+bx)}\right) - 12ibcd^2 \arctan\left(e^{i(a+bx)}\right) - 3b^3c^2dx \log\left(1-ie^{i(a+bx)}\right) + 6bd^3x \log\left(1-ie^{i(a+bx)}\right)}{}$$

input
```
Integrate[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
((2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] - (12*I)*b*c*d^2*ArcTan[E^(I*(a + b
*x))] - 3*b^3*c^2*d*x*Log[1 - I*E^(I*(a + b*x))] + 6*b*d^3*x*Log[1 - I*E^(
I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - b^3*d^3*x^3*L
og[1 - I*E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] - 6*b
*d^3*x*Log[1 + I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*
x))] + b^3*d^3*x^3*Log[1 + I*E^(I*(a + b*x))] - (3*I)*d*(-2*d^2 + b^2*(c +
 d*x)^2)*PolyLog[2, (-I)*E^(I*(a + b*x))] + (3*I)*d*(-2*d^2 + b^2*(c + d*x
)^2)*PolyLog[2, I*E^(I*(a + b*x))] + 6*b*c*d^2*PolyLog[3, (-I)*E^(I*(a + b
*x))] + 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3,
I*E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))] + (6*I)*d^3*P
olyLog[4, (-I)*E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))]
+ b^2*(c + d*x)^2*Sec[a + b*x]*(-3*d + b*(c + d*x)*Tan[a + b*x]))/(2*b^4)
```

Rubi [A] (verified)

Time = 2.09 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.72, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {4913, 3042, 4669, 3011, 4674, 3042, 4669, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \tan^2(a+bx)\sec(a+bx)\,dx$$
$$\downarrow \text{4913}$$

$$\int (c+dx)^3 \sec^3(a+bx)dx - \int (c+dx)^3 \sec(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right)^3 dx - \int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right) dx$$

$$\downarrow 4669$$

$$\frac{3d\int (c+dx)^2 \log\left(1-ie^{i(a+bx)}\right) dx}{b} - \frac{3d\int (c+dx)^2 \log\left(1+ie^{i(a+bx)}\right) dx}{b} + \int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right)^3 dx + \frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\downarrow 3011$$

$$-\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b} - \frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b} + \frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b} - \frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)dx}{b}\right)}{b} + \int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right)^3 dx + \frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\downarrow 4674$$

$$\frac{3d^2\int (c+dx)\sec(a+bx)dx}{b^2} - \frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b} - \frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b} + \frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b} - \frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)dx}{b}\right)}{b} + \frac{1}{2}\int (c+dx)^3 \sec(a+bx)dx + \frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2 \sec(a+bx)}{2b^2} + \frac{(c+dx)^3 \tan(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 3042$$

$$\frac{3d^2\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)dx}{b^2} - \frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b} - \frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b} + \frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b} - \frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)dx}{b}\right)}{b} + \frac{1}{2}\int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right)dx + \frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2 \sec(a+bx)}{2b^2} + \frac{(c+dx)^3 \tan(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 4669$$

$$\frac{3d^2\left(-\frac{d\int \log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{d\int \log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}\right)}{b^2}+$$
$$\frac{1}{2}\left(-\frac{3d\int (c+dx)^2\log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{3d\int (c+dx)^2\log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}\right)$$
$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b}+$$
$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)dx}{b}\right)}{b}+$$
$$\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 2715$$

$$\frac{3d^2\left(\frac{id\int e^{-i(a+bx)}\log\left(1-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log\left(1+ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}\right)}{b^2}+$$
$$\frac{1}{2}\left(-\frac{3d\int (c+dx)^2\log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{3d\int (c+dx)^2\log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}\right)$$
$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b}+$$
$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)dx}{b}\right)}{b}+$$
$$\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 2838$$

$$\frac{1}{2}\left(-\frac{3d\int (c+dx)^2\log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{3d\int (c+dx)^2\log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}\right)$$
$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b}+$$
$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)dx}{b}\right)}{b}+$$
$$\frac{3d^2\left(-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{id\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}\right)}{b^2}+$$
$$\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 3011$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)}{}\right.\right.$$

$$\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b}+$$

$$\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)dx}{b}\right)}{b}+$$

$$\frac{3d^2\left(-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}\right)}{b^2}+$$

$$\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 7163$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyL}}{b}\right.}{}\right.$$

$$\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+$$

$$\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+$$

$$\frac{3d^2\left(-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}\right)}{b^2}+$$

$$\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 2720$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+\frac{3d^2\left(-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}\right)}{b^2}+\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 7143$$

$$\frac{3d^2\left(-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}\right)}{b^2}+$$

$$\frac{1}{2}\left(-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\,\text{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\,\text{PolyLog}\left(3,-ie^{i}\right.}{b}\right.}{b}\right.}{b}\right.$$

$$\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}-$$

$$\frac{3d\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\,\text{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\,\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+$$

$$\frac{3d\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\,\text{PolyLog}\left(4,ie^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\,\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-$$

$$\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

input
```
Int[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
((2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + (3*d^2*(((-2*I)*(c + d*x)*
ArcTan[E^(I*(a + b*x))])/b + (I*d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 -
(I*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2))/b^2 - (3*d*((I*(c + d*x)^2*PolyL
og[2, (-I)*E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c + d*x)*PolyLog[3, (-I)
*E^(I*(a + b*x))])/b + (d*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^2))/b))/b +
(3*d*((I*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c
 + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b + (d*PolyLog[4, I*E^(I*(a + b*x))
])/b^2))/b))/b + (((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + (3*d*((
I*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c + d
*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b + (d*PolyLog[4, (-I)*E^(I*(a + b*x
))])/b^2))/b))/b - (3*d*((I*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b -
 ((2*I)*d*(((-I)*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b + (d*PolyLog[4
, I*E^(I*(a + b*x))])/b^2))/b))/b)/2 - (3*d*(c + d*x)^2*Sec[a + b*x])/(2*b
^2) + ((c + d*x)^3*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4913
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1126 vs. $2(293) = 586$.

Time = 1.91 (sec) , antiderivative size = 1127, normalized size of antiderivative = 3.34

method	result	size
risch	Expression too large to display	1127

input
```
int((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
-3/2/b^3*a^2*c*d^2*ln(I*exp(I*(b*x+a))+1)+3/2/b*c*d^2*ln(I*exp(I*(b*x+a))+
1)*x^2-3/2/b*c*d^2*ln(1-I*exp(I*(b*x+a)))*x^2-3/2/b*c^2*d*ln(1-I*exp(I*(b*
x+a)))*x-3/2/b^2*c^2*d*ln(1-I*exp(I*(b*x+a)))*a+3/2/b*c^2*d*ln(I*exp(I*(b*
x+a))+1)*x+3*I/b^3*c*d^2*a^2*arctan(exp(I*(b*x+a)))+3*I/b^2*c*d^2*polylog(
2,I*exp(I*(b*x+a)))*x-3*I/b^2*c*d^2*polylog(2,-I*exp(I*(b*x+a)))*x-3*I/b^2
*c^2*d*a*arctan(exp(I*(b*x+a)))+3/b^3*d^3*polylog(3,-I*exp(I*(b*x+a)))*x+1
/2/b^4*a^3*d^3*ln(I*exp(I*(b*x+a))+1)-1/2/b^4*a^3*d^3*ln(1-I*exp(I*(b*x+a)
))-3/b^3*c*d^2*polylog(3,I*exp(I*(b*x+a)))+3/b^3*c*d^2*polylog(3,-I*exp(I*
(b*x+a)))-3/b^3*d^3*ln(I*exp(I*(b*x+a))+1)*x-3/b^4*d^3*ln(I*exp(I*(b*x+a))
+1)*a-3/b^3*d^3*polylog(3,I*exp(I*(b*x+a)))*x+3/b^3*d^3*ln(1-I*exp(I*(b*x+
a)))*x+3/b^4*d^3*ln(1-I*exp(I*(b*x+a)))*a+1/2/b*d^3*ln(I*exp(I*(b*x+a))+1)
*x^3-1/2/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3+I/b*c^3*arctan(exp(I*(b*x+a)))+3
/2/b^2*c^2*d*ln(I*exp(I*(b*x+a))+1)*a+3/2/b^3*a^2*c*d^2*ln(1-I*exp(I*(b*x+
a)))-6*I/b^3*c*d^2*arctan(exp(I*(b*x+a)))+3/2*I/b^2*d^3*polylog(2,I*exp(I*
(b*x+a)))*x^2-I/b^4*d^3*a^3*arctan(exp(I*(b*x+a)))+6*I/b^4*d^3*a*arctan(ex
p(I*(b*x+a)))-3/2*I/b^2*d^3*polylog(2,-I*exp(I*(b*x+a)))*x^2-3/2*I/b^2*c^2
*d*polylog(2,-I*exp(I*(b*x+a)))+3/2*I/b^2*c^2*d*polylog(2,I*exp(I*(b*x+a))
)-I/b^2/(exp(2*I*(b*x+a))+1)^2*(d^3*x^3*b*exp(3*I*(b*x+a))+3*c*d^2*x^2*b*e
xp(3*I*(b*x+a))+3*c^2*d*x*b*exp(3*I*(b*x+a))-d^3*x^3*b*exp(I*(b*x+a))+b*c^
3*exp(3*I*(b*x+a))-3*c*d^2*x^2*b*exp(I*(b*x+a))-3*I*d^3*x^2*exp(3*I*(b*...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1315 vs. $2(273) = 546$.

Time $= 0.15$ (sec) , antiderivative size $= 1315$, normalized size of antiderivative $= 3.90$

$$\int (c + dx)^3 \sec(a + bx) \tan^2(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(-6*I*d^3*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - 6
*I*d^3*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 6*I*d^3*
cos(b*x + a)^2*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) + 6*I*d^3*cos(b*
x + a)^2*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 -
2*I*b^2*c*d^2*x - I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x +
a) + sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d + 2
*I*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b^2*d^3
*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - 2*I*d^3)*cos(b*x + a)^2*dilog(-I*co
s(b*x + a) + sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^
2*d - 2*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^3
*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)
^2*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(
a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I*si
n(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2*log(I
*cos(b*x + a) + sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*co
s(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3
*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*
d - 2*b*d^3)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) ...
```

Sympy [F]

$$\int (c+dx)^3 \sec(a+bx) \tan^2(a+bx)\,dx = \int (c+dx)^3 \tan^2(a+bx) \sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*sec(b*x+a)*tan(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**3*tan(a + b*x)**2*sec(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3831 vs. $2(273) = 546$.

Time = 1.15 (sec) , antiderivative size = 3831, normalized size of antiderivative = 11.37

$$\int (c+dx)^3 \sec(a+bx) \tan^2(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/4*(c^3*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - l
og(sin(b*x + a) - 1)) - 3*a*c^2*d*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + l
og(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*sin(b*x +
 a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/
b^2 - a^3*d^3*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1)
 - log(sin(b*x + a) - 1))/b^3 - 4*(2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^
3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 -
2)*d^3)*(b*x + a) + ((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 -
a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a)
)*cos(4*b*x + 4*a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2
 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x +
 a))*cos(2*b*x + 2*a) + (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + 3*(
I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2
 - 2*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*(I*(b*x + a)^3*d^3 - 6*I*b*c*
d^2 + 6*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2
*I*a*b*c*d^2 + (I*a^2 - 2*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos
(b*x + a), sin(b*x + a) + 1) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 +
3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d
^3)*(b*x + a) + ((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^
3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))...
```

Giac [F]

$$\int (c+dx)^3 \sec(a+bx) \tan^2(a+bx)\, dx = \int (dx+c)^3 \sec(bx+a) \tan(bx+a)^2\, dx$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*sec(b*x + a)*tan(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \sec(a+bx) \tan^2(a+bx)\, dx = \text{Hanged}$$

input
```
int((tan(a + b*x)^2*(c + d*x)^3)/cos(a + b*x),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^3 \sec(a+bx) \tan^2(a+bx)\, dx$$
$$= \frac{2\left(\int \sec(bx+a) \tan(bx+a)^2 x^3 dx\right) \sin(bx+a)^2 b\, d^3 - 2\left(\int \sec(bx+a) \tan(bx+a)^2 x^3 dx\right) b\, d^3 + 6\left(\int s\right.}{}$$

input
```
int((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x)
```

output

```
(2*int(sec(a + b*x)*tan(a + b*x)**2*x**3,x)*sin(a + b*x)**2*b*d**3 - 2*int
(sec(a + b*x)*tan(a + b*x)**2*x**3,x)*b*d**3 + 6*int(sec(a + b*x)*tan(a +
b*x)**2*x**2,x)*sin(a + b*x)**2*b*c*d**2 - 6*int(sec(a + b*x)*tan(a + b*x)
**2*x**2,x)*b*c*d**2 + 6*int(sec(a + b*x)*tan(a + b*x)**2*x,x)*sin(a + b*x
)**2*b*c**2*d - 6*int(sec(a + b*x)*tan(a + b*x)**2*x,x)*b*c**2*d + log(tan
((a + b*x)/2) - 1)*sin(a + b*x)**2*c**3 - log(tan((a + b*x)/2) - 1)*c**3 -
 log(tan((a + b*x)/2) + 1)*sin(a + b*x)**2*c**3 + log(tan((a + b*x)/2) + 1
)*c**3 - sin(a + b*x)*c**3)/(2*b*(sin(a + b*x)**2 - 1))
```

3.299 $\int (c+dx)^2 \sec(a+bx)\tan^2(a+bx)\,dx$

Optimal result . . . 2569
Mathematica [B] (verified) . . . 2570
Rubi [A] (verified) . . . 2570
Maple [B] (verified) . . . 2575
Fricas [B] (verification not implemented) . . . 2576
Sympy [F] . . . 2577
Maxima [B] (verification not implemented) . . . 2578
Giac [F] . . . 2579
Mupad [F(-1)] . . . 2579
Reduce [F] . . . 2579

Optimal result

Integrand size = 22, antiderivative size = 193

$$\begin{aligned}
&\int (c+dx)^2 \sec(a+bx)\tan^2(a+bx)\,dx \\
&= \frac{i(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b} + \frac{d^2 \operatorname{arctanh}(\sin(a+bx))}{b^3} \\
&\quad - \frac{id(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} + \frac{id(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} \\
&\quad + \frac{d^2\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} - \frac{d^2\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} \\
&\quad - \frac{d(c+dx)\sec(a+bx)}{b^2} + \frac{(c+dx)^2 \sec(a+bx)\tan(a+bx)}{2b}
\end{aligned}$$

output

```
I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b+d^2*arctanh(sin(b*x+a))/b^3-I*d*(d*x+
c)*polylog(2,-I*exp(I*(b*x+a)))/b^2+I*d*(d*x+c)*polylog(2,I*exp(I*(b*x+a))
)/b^2+d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3-d^2*polylog(3,I*exp(I*(b*x+a)))
/b^3-d*(d*x+c)*sec(b*x+a)/b^2+1/2*(d*x+c)^2*sec(b*x+a)*tan(b*x+a)/b
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 443 vs. $2(193) = 386$.

Time = 6.75 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.30

$$\int (c+dx)^2 \sec(a+bx) \tan^2(a+bx)\, dx$$

$$= \frac{4ibc^2 \arctan\left(e^{i(a+bx)}\right) - \frac{8id^2 \arctan\left(e^{i(a+bx)}\right)}{b} - 4bcdx \log\left(1 - ie^{i(a+bx)}\right) - 2bd^2x^2 \log\left(1 - ie^{i(a+bx)}\right) + 4bcdx$$

input
```
Integrate[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
((4*I)*b*c^2*ArcTan[E^(I*(a + b*x))] - ((8*I)*d^2*ArcTan[E^(I*(a + b*x))])
/b - 4*b*c*d*x*Log[1 - I*E^(I*(a + b*x))] - 2*b*d^2*x^2*Log[1 - I*E^(I*(a
+ b*x))] + 4*b*c*d*x*Log[1 + I*E^(I*(a + b*x))] + 2*b*d^2*x^2*Log[1 + I*E^
(I*(a + b*x))] - (4*I)*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] + (4*I
)*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] + (4*d^2*PolyLog[3, (-I)*E^(I*
(a + b*x))])/b - (4*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b - 4*d*(c + d*x)*S
ec[a] + (b*(c + d*x)^2)/(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])^2 - (4*d*(c
+ d*x)*Sin[(b*x)/2])/((Cos[a/2] - Sin[a/2])*(Cos[(a + b*x)/2] - Sin[(a + b
*x)/2])) - (b*(c + d*x)^2)/(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])^2 + (4*d*
(c + d*x)*Sin[(b*x)/2])/((Cos[a/2] + Sin[a/2])*(Cos[(a + b*x)/2] + Sin[(a
+ b*x)/2])))/(4*b^2)
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.82, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {4913, 3042, 4669, 3011, 2720, 4674, 3042, 4257, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \tan^2(a+bx) \sec(a+bx)\, dx$$

$$\downarrow 4913$$

$$\int (c+dx)^2 \sec^3(a+bx)dx - \int (c+dx)^2 \sec(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)^2 \csc\left(a+bx+\frac{\pi}{2}\right)^3 dx - \int (c+dx)^2 \csc\left(a+bx+\frac{\pi}{2}\right) dx$$

$$\downarrow 4669$$

$$\frac{2d\int (c+dx)\log\left(1-ie^{i(a+bx)}\right)dx}{b} - \frac{2d\int (c+dx)\log\left(1+ie^{i(a+bx)}\right)dx}{b} + \int (c+dx)^2 \csc\left(a+bx+\frac{\pi}{2}\right)^3 dx + \frac{2i(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\downarrow 3011$$

$$-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b} - \frac{id\int \mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b} + \frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b} - \frac{id\int \mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)dx}{b}\right)}{b} + \int (c+dx)^2 \csc\left(a+bx+\frac{\pi}{2}\right)^3 dx + \frac{2i(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\downarrow 2720$$

$$-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b} - \frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b} + \frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b} - \frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b} + \int (c+dx)^2 \csc\left(a+bx+\frac{\pi}{2}\right)^3 dx + \frac{2i(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\downarrow 4674$$

$$-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b} - \frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b} + \frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b} - \frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b} + \frac{d^2\int \sec(a+bx)dx}{b^2} + \frac{1}{2}\int (c+dx)^2 \sec(a+bx)dx + \frac{2i(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b} - \frac{d(c+dx)\sec(a+bx)}{b^2} + \frac{(c+dx)^2 \tan(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 3042$$

$$-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+$$
$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+$$
$$\frac{d^2\int\csc\left(a+bx+\frac{\pi}{2}\right)dx}{b^2}+\frac{1}{2}\int(c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx+$$
$$\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{d(c+dx)\sec(a+bx)}{b^2}+\frac{(c+dx)^2\tan(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 4257$$

$$-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+$$
$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+\frac{1}{2}\int(c+$$
$$dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{d^2\operatorname{arctanh}(\sin(a+bx))}{b^3}-$$
$$\frac{d(c+dx)\sec(a+bx)}{b^2}+\frac{(c+dx)^2\tan(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 4669$$

$$\frac{1}{2}\left(-\frac{2d\int(c+dx)\log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{2d\int(c+dx)\log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}\right)-$$
$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+$$
$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+$$
$$\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{d^2\operatorname{arctanh}(\sin(a+bx))}{b^3}-\frac{d(c+dx)\sec(a+bx)}{b^2}+$$
$$\frac{(c+dx)^2\tan(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 3011$$

$$\frac{1}{2}\left(\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{id\int \mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{id\int \mathrm{PolyLog}\left(2,ie^{i(a+\right.}\right.}{b}$$

$$\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+$$

$$\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+$$

$$\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{d^2\mathrm{arctanh}(\sin(a+bx))}{b^3}-\frac{d(c+dx)\sec(a+bx)}{b^2}+$$

$$\frac{(c+dx)^2\tan(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 2720$$

$$\frac{1}{2}\left(\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{}\right.}{b}\right.$$

$$\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+$$

$$\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+$$

$$\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{d^2\mathrm{arctanh}(\sin(a+bx))}{b^3}-\frac{d(c+dx)\sec(a+bx)}{b^2}+$$

$$\frac{(c+dx)^2\tan(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 7143$$

$$\frac{1}{2}\left(-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}(\right.}{b}\right.$$

$$\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{d^2\mathrm{arctanh}(\sin(a+bx))}{b^3}-$$

$$\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^2}\right)}{b}+$$

$$\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{d\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{d(c+dx)\sec(a+bx)}{b^2}+$$

$$\frac{(c+dx)^2\tan(a+bx)\sec(a+bx)}{2b}$$

input
```
Int[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
((2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (d^2*ArcTanh[Sin[a + b*x]]
)/b^3 - (2*d*((I*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b - (d*PolyLo
g[3, (-I)*E^(I*(a + b*x))])/b^2))/b + (2*d*((I*(c + d*x)*PolyLog[2, I*E^(I
*(a + b*x))])/b - (d*PolyLog[3, I*E^(I*(a + b*x))])/b^2))/b + (((-2*I)*(c
+ d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (2*d*((I*(c + d*x)*PolyLog[2, (-I)*E
^(I*(a + b*x))])/b - (d*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^2))/b - (2*d*(
(I*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b - (d*PolyLog[3, I*E^(I*(a +
b*x))])/b^2))/b)/2 - (d*(c + d*x)*Sec[a + b*x])/b^2 + ((c + d*x)^2*Sec[a +
 b*x]*Tan[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))  Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
 - 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4913
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(174) = 348$.

Time = 1.02 (sec) , antiderivative size = 584, normalized size of antiderivative = 3.03

method	result
risch	$-\frac{id^2\,\mathrm{polylog}\left(2,-ie^{i(bx+a)}\right)x}{b^2} - \frac{i\left(d^2x^2be^{3i(bx+a)}+2cdxbe^{3i(bx+a)}+c^2be^{3i(bx+a)}-d^2x^2be^{i(bx+a)}-2cdxbe^{i(bx+a)}-2id^2xe^{3i(bx}{b^2\left(e^{2i(bx+a)}+1\right)^2}$

input
```
int((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output

```
-I/b^2*d^2*polylog(2,-I*exp(I*(b*x+a)))*x-I/b^2/(exp(2*I*(b*x+a))+1)^2*(d^
2*x^2*b*exp(3*I*(b*x+a))+2*c*d*x*b*exp(3*I*(b*x+a))+c^2*b*exp(3*I*(b*x+a))
-d^2*x^2*b*exp(I*(b*x+a))-2*c*d*x*b*exp(I*(b*x+a))-2*I*d^2*x*exp(3*I*(b*x+
a))-c^2*b*exp(I*(b*x+a))-2*I*c*d*exp(3*I*(b*x+a))-2*I*d^2*x*exp(I*(b*x+a))
-2*I*c*d*exp(I*(b*x+a)))+I/b^2*c*d*polylog(2,I*exp(I*(b*x+a)))-I/b^2*c*d*p
olylog(2,-I*exp(I*(b*x+a)))-1/2/b^3*a^2*d^2*ln(I*exp(I*(b*x+a))+1)-1/b^2*c
*d*ln(1-I*exp(I*(b*x+a)))*a-d^2*polylog(3,I*exp(I*(b*x+a)))/b^3+I/b^3*d^2*
a^2*arctan(exp(I*(b*x+a)))+1/2/b^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)))+d^2*poly
log(3,-I*exp(I*(b*x+a)))/b^3+1/2/b*d^2*ln(I*exp(I*(b*x+a))+1)*x^2+I/b^2*d^
2*polylog(2,I*exp(I*(b*x+a)))*x+I/b*c^2*arctan(exp(I*(b*x+a)))+1/b^2*c*d*l
n(I*exp(I*(b*x+a))+1)*a-1/2/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2-1/b*c*d*ln(1-
I*exp(I*(b*x+a)))*x+1/b*c*d*ln(I*exp(I*(b*x+a))+1)*x-2*I/b^2*c*d*a*arctan(
exp(I*(b*x+a)))-2*I/b^3*d^2*arctan(exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(165) = 330$.

Time = 0.13 (sec) , antiderivative size = 795, normalized size of antiderivative = 4.12

$$\int (c + dx)^2 \sec(a + bx) \tan^2(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*d^
2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*cos(b*x
 + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*cos(b*x + a)^2*
polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*cos(
b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)
*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - 2*(I*b*d^2*x + I*b*
c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 2*(I*b*d^2*x +
 I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^2*c^2
- 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x +
 a) + I) + (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2*log(cos(b*
x + a) - I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^
2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^
2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) -
 sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos
(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2
*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x
 + a) + 1) - (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2*log(-cos
(b*x + a) + I*sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*co
s(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 4*(b*d^2*x + b*c*d)
*cos(b*x + a) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(b*x + a))/(...
```

Sympy [F]

$$\int (c+dx)^2 \sec(a+bx) \tan^2(a+bx)\,dx = \int (c+dx)^2 \tan^2(a+bx) \sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*sec(b*x+a)*tan(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*tan(a + b*x)**2*sec(a + b*x), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1891 vs. $2(165) = 330$.

Time = 0.70 (sec) , antiderivative size = 1891, normalized size of antiderivative = 9.80

$$\int (c+dx)^2 \sec(a+bx) \tan^2(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/4*(c^2*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - l
og(sin(b*x + a) - 1)) - 2*a*c*d*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log
(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b + a^2*d^2*(2*sin(b*x + a)/(s
in(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b^2 -
4*(2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2 + ((b*x + a)^2
*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(4*b*x + 4*a) + 2*((b*x + a
)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(2*b*x + 2*a) + (I*(b*x
+ a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(4*b*x + 4*a) +
 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(2*b
*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + 2*((b*x + a)^2*d^2 +
2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2 + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)
*(b*x + a) - 2*d^2)*cos(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d
^2)*(b*x + a) - 2*d^2)*cos(2*b*x + 2*a) + (I*(b*x + a)^2*d^2 + 2*(I*b*c*d
- I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(4*b*x + 4*a) + 2*(I*(b*x + a)^2*d^2 +
2*(I*b*c*d - I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b
*x + a), -sin(b*x + a) + 1) - 4*((b*x + a)^2*d^2 - 2*I*b*c*d + 2*I*a*d^2 +
 2*(b*c*d - (a + I)*d^2)*(b*x + a))*cos(3*b*x + 3*a) + 4*((b*x + a)^2*d^2
+ 2*I*b*c*d - 2*I*a*d^2 + 2*(b*c*d - (a - I)*d^2)*(b*x + a))*cos(b*x + a)
+ 4*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4
*b*x + 4*a) + 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (I*b...
```

Giac [F]

$$\int (c+dx)^2 \sec(a+bx) \tan^2(a+bx)\, dx = \int (dx+c)^2 \sec(bx+a) \tan(bx+a)^2\, dx$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*sec(b*x + a)*tan(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \sec(a+bx) \tan^2(a+bx)\, dx = \text{Hanged}$$

input
```
int((tan(a + b*x)^2*(c + d*x)^2)/cos(a + b*x),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^2 \sec(a+bx) \tan^2(a+bx)\, dx$$
$$= \frac{2\left(\int \sec(bx+a) \tan(bx+a)^2 x^2 dx\right) \sin(bx+a)^2 b\, d^2 - 2\left(\int \sec(bx+a) \tan(bx+a)^2 x^2 dx\right) b\, d^2 + 4\left(\int s}{}$$

input
```
int((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x)
```

output

```
(2*int(sec(a + b*x)*tan(a + b*x)**2*x**2,x)*sin(a + b*x)**2*b*d**2 - 2*int
(sec(a + b*x)*tan(a + b*x)**2*x**2,x)*b*d**2 + 4*int(sec(a + b*x)*tan(a +
b*x)**2*x,x)*sin(a + b*x)**2*b*c*d - 4*int(sec(a + b*x)*tan(a + b*x)**2*x,
x)*b*c*d + log(tan((a + b*x)/2) - 1)*sin(a + b*x)**2*c**2 - log(tan((a + b
*x)/2) - 1)*c**2 - log(tan((a + b*x)/2) + 1)*sin(a + b*x)**2*c**2 + log(ta
n((a + b*x)/2) + 1)*c**2 - sin(a + b*x)*c**2)/(2*b*(sin(a + b*x)**2 - 1))
```

3.300 $\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx$

Optimal result . . . 2581
Mathematica [B] (warning: unable to verify) . . . 2582
Rubi [A] (verified) . . . 2582
Maple [B] (verified) . . . 2586
Fricas [B] (verification not implemented) . . . 2586
Sympy [F] . . . 2587
Maxima [F] . . . 2587
Giac [F] . . . 2588
Mupad [F(-1)] . . . 2589
Reduce [F] . . . 2589

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx = \frac{i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b} - \frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2} + \frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2} - \frac{d\sec(a+bx)}{2b^2} + \frac{(c+dx)\sec(a+bx)\tan(a+bx)}{2b}$$

output

```
I*(d*x+c)*arctan(exp(I*(b*x+a)))/b-1/2*I*d*polylog(2,-I*exp(I*(b*x+a)))/b^
2+1/2*I*d*polylog(2,I*exp(I*(b*x+a)))/b^2-1/2*d*sec(b*x+a)/b^2+1/2*(d*x+c)
*sec(b*x+a)*tan(b*x+a)/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 555 vs. $2(117) = 234$.

Time = 6.56 (sec) , antiderivative size = 555, normalized size of antiderivative = 4.74

$$\int (c + dx) \sec(a + bx) \tan^2(a + bx)\, dx = \text{Too large to display}$$

input
```
Integrate[(c + d*x)*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
-1/2*(c*ArcTanh[Sin[a + b*x]])/b + (d*x*(a*Log[1 - Tan[(a + b*x)/2]] + I*L
og[1 + I*Tan[(a + b*x)/2]]*Log[(-1/2 - I/2)*(-1 + Tan[(a + b*x)/2])] - I*L
og[1 - I*Tan[(a + b*x)/2]]*Log[(-1/2 + I/2)*(-1 + Tan[(a + b*x)/2])] - I*L
og[1 + I*Tan[(a + b*x)/2]]*Log[(1/2 - I/2)*(1 + Tan[(a + b*x)/2])] + I*Log
[1 - I*Tan[(a + b*x)/2]]*Log[(1/2 + I/2)*(1 + Tan[(a + b*x)/2])] - a*Log[1
 + Tan[(a + b*x)/2]] - I*PolyLog[2, ((1 + I) - (1 - I)*Tan[(a + b*x)/2])/2
] + I*PolyLog[2, (-1/2 - I/2)*(I + Tan[(a + b*x)/2])] - I*PolyLog[2, ((1 +
 I) + (1 - I)*Tan[(a + b*x)/2])/2] + I*PolyLog[2, ((1 - I) + (1 + I)*Tan[(
a + b*x)/2])/2]))/(2*b*(a - I*Log[1 - I*Tan[(a + b*x)/2]] + I*Log[1 + I*Ta
n[(a + b*x)/2]])) + (d*x)/(4*b*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])^2) -
(d*Sin[(a + b*x)/2])/(2*b^2*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])) - (d*x)
/(4*b*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])^2) + (d*Sin[(a + b*x)/2])/(2*b
^2*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])) + (c*Sec[a + b*x]*Tan[a + b*x])/
(2*b)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.64, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4913, 3042, 4669, 2715, 2838, 4673, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c + dx) \tan^2(a + bx) \sec(a + bx)\, dx$$

$$\downarrow 4913$$

$$\int (c+dx)\sec^3(a+bx)dx - \int (c+dx)\sec(a+bx)dx$$

$$\downarrow 3042$$

$$\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)^3 dx - \int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right) dx$$

$$\downarrow 4669$$

$$\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)^3 dx + \frac{d\int \log\left(1-ie^{i(a+bx)}\right) dx}{b} - \frac{d\int \log\left(1+ie^{i(a+bx)}\right) dx}{b} + \frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\downarrow 2715$$

$$-\frac{id\int e^{-i(a+bx)}\log\left(1-ie^{i(a+bx)}\right) de^{i(a+bx)}}{b^2} + \frac{id\int e^{-i(a+bx)}\log\left(1+ie^{i(a+bx)}\right) de^{i(a+bx)}}{b^2} + \int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)^3 dx + \frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\downarrow 2838$$

$$\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)^3 dx + \frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b} - \frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} + \frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}$$

$$\downarrow 4673$$

$$\frac{1}{2}\int (c+dx)\sec(a+bx)dx + \frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b} - \frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} + \frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} - \frac{d\sec(a+bx)}{2b^2} + \frac{(c+dx)\tan(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 3042$$

$$\frac{1}{2}\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right) dx + \frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b} - \frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} + \frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} - \frac{d\sec(a+bx)}{2b^2} + \frac{(c+dx)\tan(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 4669$$

$$\frac{1}{2}\left(-\frac{d\int \log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{d\int \log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}\right)+\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}+\frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\tan(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 2715$$

$$\frac{1}{2}\left(\frac{id\int e^{-i(a+bx)}\log\left(1-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log\left(1+ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{2i(c+dx)\arctan\left(e^{i}\right.}{b}\right.$$

$$\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}+\frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\tan(a+bx)\sec(a+bx)}{2b}$$

$$\downarrow 2838$$

$$\frac{1}{2}\left(-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}\right)+\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}+\frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\tan(a+bx)\sec(a+bx)}{2b}$$

input

```
Int[(c + d*x)*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

output

```
((2*I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b - (I*d*PolyLog[2, (-I)*E^(I*(a
 + b*x))])/b^2 + (I*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 + (((-2*I)*(c + d
*x)*ArcTan[E^(I*(a + b*x))])/b + (I*d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^
2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2)/2 - (d*Sec[a + b*x])/(2*b^2)
+ ((c + d*x)*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4673

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4913

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(98) = 196$.

Time $= 0.52$ (sec) , antiderivative size $= 267$, normalized size of antiderivative $= 2.28$

method	result
risch	$-\frac{i\left(bdx\,e^{3i(bx+a)}-id\,e^{3i(bx+a)}+cb\,e^{3i(bx+a)}-bdx\,e^{i(bx+a)}-id\,e^{i(bx+a)}-bc\,e^{i(bx+a)}\right)}{b^2\left(e^{2i(bx+a)}+1\right)^2}+\frac{ic\arctan\left(e^{i(bx+a)}\right)}{b}+\frac{d\ln\left(ie^{i(bx+a)}+1\right.}{2b}$

input
```
int((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-I/b^2/(exp(2*I*(b*x+a))+1)^2*(b*d*x*exp(3*I*(b*x+a))-I*d*exp(3*I*(b*x+a))
+c*b*exp(3*I*(b*x+a))-b*d*x*exp(I*(b*x+a))-I*d*exp(I*(b*x+a))-b*c*exp(I*(b
*x+a)))+I/b*c*arctan(exp(I*(b*x+a)))+1/2/b*d*ln(I*exp(I*(b*x+a))+1)*x+1/2/
b^2*d*ln(I*exp(I*(b*x+a))+1)*a-1/2/b*d*ln(1-I*exp(I*(b*x+a)))*x-1/2/b^2*d*
ln(1-I*exp(I*(b*x+a)))*a-1/2*I/b^2*d*dilog(I*exp(I*(b*x+a))+1)+1/2*I/b^2*d
*dilog(1-I*exp(I*(b*x+a)))-I/b^2*d*a*arctan(exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(93) = 186$.

Time $= 0.10$ (sec) , antiderivative size $= 435$, normalized size of antiderivative $= 3.72$

$$\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx$$
$$= \frac{i\,d\cos(bx+a)^2\,\mathrm{Li}_2(i\,\cos(bx+a)+\sin(bx+a))+i\,d\cos(bx+a)^2\,\mathrm{Li}_2(i\,\cos(bx+a)-\sin(bx+a))-}{}$$

input
```
integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*cos(b*x
 + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(-I
*cos(b*x + a) + sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) -
 sin(b*x + a)) - (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x +
 a) + I) + (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) +
I) - (b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) +
 (b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b*
d*x + a*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x
 + a*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b*c - a*
d)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)*co
s(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 2*d*cos(b*x + a) +
2*(b*d*x + b*c)*sin(b*x + a))/(b^2*cos(b*x + a)^2)
```

Sympy [F]

$$\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx = \int (c+dx)\tan^2(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*tan(a + b*x)**2*sec(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx = \int (dx+c)\sec(bx+a)\tan(bx+a)^2\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/4*(4*(d*cos(3*b*x + 3*a) + d*cos(b*x + a) - (b*d*x + b*c)*sin(3*b*x + 3
*a) + (b*d*x + b*c)*sin(b*x + a))*cos(4*b*x + 4*a) + 4*(2*d*cos(2*b*x + 2*
a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a) + d)*cos(3*b*x + 3*a) + 8*(d*cos(b*x
 + a) + (b*d*x + b*c)*sin(b*x + a))*cos(2*b*x + 2*a) + 4*d*cos(b*x + a) +
4*(b^2*d*cos(4*b*x + 4*a)^2 + 4*b^2*d*cos(2*b*x + 2*a)^2 + b^2*d*sin(4*b*x
 + 4*a)^2 + 4*b^2*d*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b^2*d*sin(2*b*x
+ 2*a)^2 + 4*b^2*d*cos(2*b*x + 2*a) + b^2*d + 2*(2*b^2*d*cos(2*b*x + 2*a)
+ b^2*d)*cos(4*b*x + 4*a))*integrate((x*cos(2*b*x + 2*a)*cos(b*x + a) + x*
sin(2*b*x + 2*a)*sin(b*x + a) + x*cos(b*x + a))/(cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1), x) + (b*c*cos(4*b*x + 4*a)^2 + 4
*b*c*cos(2*b*x + 2*a)^2 + b*c*sin(4*b*x + 4*a)^2 + 4*b*c*sin(4*b*x + 4*a)*
sin(2*b*x + 2*a) + 4*b*c*sin(2*b*x + 2*a)^2 + 4*b*c*cos(2*b*x + 2*a) + b*c
 + 2*(2*b*c*cos(2*b*x + 2*a) + b*c)*cos(4*b*x + 4*a))*log(cos(b*x + a)^2 +
 sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (b*c*cos(4*b*x + 4*a)^2 + 4*b*c*co
s(2*b*x + 2*a)^2 + b*c*sin(4*b*x + 4*a)^2 + 4*b*c*sin(4*b*x + 4*a)*sin(2*b
*x + 2*a) + 4*b*c*sin(2*b*x + 2*a)^2 + 4*b*c*cos(2*b*x + 2*a) + b*c + 2*(2
*b*c*cos(2*b*x + 2*a) + b*c)*cos(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*
x + a)^2 - 2*sin(b*x + a) + 1) + 4*((b*d*x + b*c)*cos(3*b*x + 3*a) - (b*d*
x + b*c)*cos(b*x + a) + d*sin(3*b*x + 3*a) + d*sin(b*x + a))*sin(4*b*x + 4
*a) - 4*(b*d*x + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - 2*d*sin(2*b*x...
```

Giac [F]

$$\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx = \int (dx+c)\sec\left(bx+a\right)\tan\left(bx+a\right)^2\,dx$$

input

```
integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

output

```
integrate((d*x + c)*sec(b*x + a)*tan(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx = \text{Hanged}$$

input
```
int((tan(a + b*x)^2*(c + d*x))/cos(a + b*x),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx$$
$$= \frac{2\left(\int \sec(bx+a)\tan(bx+a)^2\,x dx\right)\sin(bx+a)^2\,bd - 2\left(\int \sec(bx+a)\tan(bx+a)^2\,x dx\right)bd + \log\left(\tan\left(\right.\right.}{}$$

input
```
int((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x)
```

output
```
(2*int(sec(a + b*x)*tan(a + b*x)**2*x,x)*sin(a + b*x)**2*b*d - 2*int(sec(a
 + b*x)*tan(a + b*x)**2*x,x)*b*d + log(tan((a + b*x)/2) - 1)*sin(a + b*x)*
*2*c - log(tan((a + b*x)/2) - 1)*c - log(tan((a + b*x)/2) + 1)*sin(a + b*x
)**2*c + log(tan((a + b*x)/2) + 1)*c - sin(a + b*x)*c)/(2*b*(sin(a + b*x)*
*2 - 1))
```

3.301 $\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx$

Optimal result 2590
Mathematica [N/A] 2590
Rubi [N/A] 2591
Maple [N/A] 2592
Fricas [N/A] 2592
Sympy [N/A] 2593
Maxima [N/A] 2593
Giac [N/A] 2594
Mupad [N/A] 2595
Reduce [N/A] 2595

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx = -\text{Int}\left(\frac{\sec(a+bx)}{c+dx},x\right) + \text{Int}\left(\frac{\sec^3(a+bx)}{c+dx},x\right)$$

output
```
-Defer(Int)(sec(b*x+a)/(d*x+c),x)+Defer(Int)(sec(b*x+a)^3/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 24.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
```

output
```
Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4913, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

$$\downarrow 4913$$

$$\int \frac{\sec^3(a+bx)}{c+dx}dx - \int \frac{\sec(a+bx)}{c+dx}dx$$

$$\downarrow 3042$$

$$\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)^3}{c+dx}dx - \int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx$$

$$\downarrow 4680$$

$$\int \frac{\sec^3(a+bx)}{c+dx}dx - \int \frac{\sec(a+bx)}{c+dx}dx$$

input
```
Int[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4913

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sec(bx+a)\tan(bx+a)^2}{dx+c}dx$$

input

```
int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

output

```
int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)\tan(bx+a)^2}{dx+c}\,dx$$

input

```
integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\tan^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(tan(a + b*x)**2*sec(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 1331, normalized size of antiderivative = 60.50

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)\tan(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
((d*cos(3*b*x + 3*a) + d*cos(b*x + a) + (b*d*x + b*c)*sin(3*b*x + 3*a) - (
b*d*x + b*c)*sin(b*x + a))*cos(4*b*x + 4*a) + (2*d*cos(2*b*x + 2*a) - 2*(b
*d*x + b*c)*sin(2*b*x + 2*a) + d)*cos(3*b*x + 3*a) + 2*(d*cos(b*x + a) - (
b*d*x + b*c)*sin(b*x + a))*cos(2*b*x + 2*a) + d*cos(b*x + a) - (b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*
x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 +
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*c
os(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)
)*integrate(((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(2*b*x + 2*a
)*cos(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(2*b*x +
 2*a)*sin(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x
 + a))/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3
*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2
+ 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x +
2*a)), x) - ((b*d*x + b*c)*cos(3*b*x + 3*a) - (b*d*x + b*c)*cos(b*x + a) -
 d*sin(3*b*x + 3*a) - d*sin(b*x + a))*sin(4*b*x + 4*a) + (b*d*x + b*c +...
```

Giac [N/A]

Not integrable

Time = 4.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\sec\left(bx+a\right)\tan\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 19.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\tan(a+b\,x)^2}{\cos(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(tan(a + b*x)^2/(cos(a + b*x)*(c + d*x)),x)
```

output
```
int(tan(a + b*x)^2/(cos(a + b*x)*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)\tan(bx+a)^2}{dx+c}dx$$

input
```
int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

output
```
int((sec(a + b*x)*tan(a + b*x)**2)/(c + d*x),x)
```

3.302 $\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$

Optimal result 2596
Mathematica [N/A] 2596
Rubi [N/A] 2597
Maple [N/A] 2598
Fricas [N/A] 2598
Sympy [N/A] 2599
Maxima [N/A] 2599
Giac [N/A] 2600
Mupad [N/A] 2601
Reduce [N/A] 2601

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = -\text{Int}\left(\frac{\sec(a+bx)}{(c+dx)^2},x\right) + \text{Int}\left(\frac{\sec^3(a+bx)}{(c+dx)^2},x\right)$$

output
```
-Defer(Int)(sec(b*x+a)/(d*x+c)^2,x)+Defer(Int)(sec(b*x+a)^3/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 27.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4913, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

$$\downarrow 4913$$

$$\int \frac{\sec^3(a+bx)}{(c+dx)^2}dx - \int \frac{\sec(a+bx)}{(c+dx)^2}dx$$

$$\downarrow 3042$$

$$\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)^3}{(c+dx)^2}dx - \int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx$$

$$\downarrow 4680$$

$$\int \frac{\sec^3(a+bx)}{(c+dx)^2}dx - \int \frac{\sec(a+bx)}{(c+dx)^2}dx$$

input
```
Int[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4913
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sec(bx+a)\tan(bx+a)^2}{(dx+c)^2}dx$$

input
```
int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec(bx+a)\tan(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)*tan(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(tan(a + b*x)**2*sec(a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 5.21 (sec) , antiderivative size = 1645, normalized size of antiderivative = 74.77

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec(bx+a)\tan(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output

```
((2*d*cos(3*b*x + 3*a) + 2*d*cos(b*x + a) + (b*d*x + b*c)*sin(3*b*x + 3*a)
 - (b*d*x + b*c)*sin(b*x + a))*cos(4*b*x + 4*a) + 2*(2*d*cos(2*b*x + 2*a)
- (b*d*x + b*c)*sin(2*b*x + 2*a) + d)*cos(3*b*x + 3*a) + 2*(2*d*cos(b*x +
a) - (b*d*x + b*c)*sin(b*x + a))*cos(2*b*x + 2*a) + 2*d*cos(b*x + a) - (b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b
^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^
3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d
^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 + 4
*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a
)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^
2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)
*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*integrate(((b^2*d^2*x^2 + 2*b^2*
c*d*x + b^2*c^2 - 6*d^2)*cos(2*b*x + 2*a)*cos(b*x + a) + (b^2*d^2*x^2 + 2*
b^2*c*d*x + b^2*c^2 - 6*d^2)*sin(2*b*x + 2*a)*sin(b*x + a) + (b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2 - 6*d^2)*cos(b*x + a))/(b^2*d^4*x^4 + 4*b^2*c*d^3*
x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c
*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(2*b*x + 2*a)^2
 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x +...
```

Giac [N/A]

Not integrable

Time = 27.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec\left(bx+a\right)\tan\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input

```
integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output

```
integrate(sec(b*x + a)*tan(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 20.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(a+b\,x)^2}{\cos(a+b\,x)\,(c+d\,x)^2}\,dx$$

input
```
int(tan(a + b*x)^2/(cos(a + b*x)*(c + d*x)^2),x)
```

output
```
int(tan(a + b*x)^2/(cos(a + b*x)*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec(bx+a)\tan(bx+a)^2}{d^2x^2+2cdx+c^2}dx$$

input
```
int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int((sec(a + b*x)*tan(a + b*x)**2)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.303 $\int (c+dx)^m \tan^3(a+bx)\,dx$

Optimal result . . . 2602
Mathematica [N/A] . . . 2602
Rubi [N/A] . . . 2603
Maple [N/A] . . . 2604
Fricas [N/A] . . . 2604
Sympy [N/A] . . . 2605
Maxima [N/A] . . . 2605
Giac [N/A] . . . 2606
Mupad [N/A] . . . 2606
Reduce [N/A] . . . 2606

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (c+dx)^m \tan^3(a+bx)\,dx = \text{Int}\big((c+dx)^m \tan^3(a+bx), x\big)$$

output
```
Defer(Int)((d*x+c)^m*tan(b*x+a)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 6.80 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \tan^3(a+bx)\,dx = \int (c+dx)^m \tan^3(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Tan[a + b*x]^3,x]
```

output
```
Integrate[(c + d*x)^m*Tan[a + b*x]^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(a+bx)(c+dx)^m\,dx$$

$$\downarrow 3042$$

$$\int \tan(a+bx)^3(c+dx)^m dx$$

$$\downarrow 4222$$

$$\int \tan^3(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Tan[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \tan(bx+a)^3 \, dx$$

input

```
int((d*x+c)^m*tan(b*x+a)^3,x)
```

output

```
int((d*x+c)^m*tan(b*x+a)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \tan^3(a+bx) \, dx = \int (dx+c)^m \tan(bx+a)^3 \, dx$$

input

```
integrate((d*x+c)^m*tan(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*tan(b*x + a)^3, x)
```

Sympy [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (c+dx)^m \tan^3(a+bx)\,dx = \int (c+dx)^m \tan^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*tan(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**m*tan(a + b*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \tan^3(a+bx)\,dx = \int (dx+c)^m \tan\left(bx+a\right)^3\,dx$$

input
```
integrate((d*x+c)^m*tan(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*tan(b*x + a)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \tan^3(a+bx)\,dx = \int (dx+c)^m \tan\left(bx+a\right)^3\,dx$$

input
```
integrate((d*x+c)^m*tan(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*tan(b*x + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 20.69 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \tan^3(a+bx)\,dx = \int \tan(a+b\,x)^3\,(c+d\,x)^m\,dx$$

input
```
int(tan(a + b*x)^3*(c + d*x)^m,x)
```

output
```
int(tan(a + b*x)^3*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \tan^3(a+bx)\,dx = \int (dx+c)^m \tan\left(bx+a\right)^3\,dx$$

input
```
int((d*x+c)^m*tan(b*x+a)^3,x)
```

output
```
int((c + d*x)**m*tan(a + b*x)**3,x)
```

3.304 $\int (c+dx)^3 \tan^3(a+bx)\,dx$

Optimal result . . . 2608
Mathematica [B] (warning: unable to verify) . . . 2609
Rubi [A] (verified) . . . 2610
Maple [B] (verified) . . . 2617
Fricas [B] (verification not implemented) . . . 2618
Sympy [F] . . . 2618
Maxima [B] (verification not implemented) . . . 2619
Giac [F] . . . 2620
Mupad [F(-1)] . . . 2620
Reduce [F] . . . 2620

Optimal result

Integrand size = 16, antiderivative size = 259

$$\begin{aligned}\int (c+dx)^3 \tan^3(a+bx)\,dx = {} & \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{i(c+dx)^4}{4d} \\ & - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} \\ & + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} \\ & + \frac{3id^3\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^4} \\ & - \frac{3id(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} \\ & + \frac{3d^2(c+dx)\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} \\ & + \frac{3id^3\,\mathrm{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4} \\ & - \frac{3d(c+dx)^2\tan(a+bx)}{2b^2} + \frac{(c+dx)^3\tan^2(a+bx)}{2b}\end{aligned}$$

output
```
3/2*I*d*(d*x+c)^2/b^2+1/2*(d*x+c)^3/b-1/4*I*(d*x+c)^4/d-3*d^2*(d*x+c)*ln(1
+exp(2*I*(b*x+a)))/b^3+(d*x+c)^3*ln(1+exp(2*I*(b*x+a)))/b+3/2*I*d^3*polylo
g(2,-exp(2*I*(b*x+a)))/b^4-3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a)))/
b^2+3/2*d^2*(d*x+c)*polylog(3,-exp(2*I*(b*x+a)))/b^3+3/4*I*d^3*polylog(4,-
exp(2*I*(b*x+a)))/b^4-3/2*d*(d*x+c)^2*tan(b*x+a)/b^2+1/2*(d*x+c)^3*tan(b*x
+a)^2/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 814 vs. $2(259) = 518$.

Time $= 6.59$ (sec) , antiderivative size $= 814$, normalized size of antiderivative $= 3.14$

$$\int (c + dx)^3 \tan^3(a + bx)\, dx = \text{Too large to display}$$

input
```
Integrate[(c + d*x)^3*Tan[a + b*x]^3,x]
```

output

```
((I/4)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)
*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))]
- (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*
E^(I*a)) + ((I/8)*d^3*E^(I*a)*((2*b^4*x^4)/E^((2*I)*a) - (4*I)*b^3*(1 + E^
((-2*I)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x))] + 6*b^2*(1 + E^((-2*I)*a))*x
^2*PolyLog[2, -E^((-2*I)*(a + b*x))] - (6*I)*b*(1 + E^((-2*I)*a))*x*PolyLo
g[3, -E^((-2*I)*(a + b*x))] - 3*(1 + E^((-2*I)*a))*PolyLog[4, -E^((-2*I)*(
a + b*x))])*Sec[a])/b^4 + ((c + d*x)^3*Sec[a + b*x]^2)/(2*b) + (c^3*Sec[a]
*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^
2 + Sin[a]^2)) - (3*c*d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[
b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (3*c^2*d*Csc[a]*((b^2*x
^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Lo
g[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - A
rcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTa
n[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Co
t[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (3*d^3*Csc
[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a
]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*
I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[
b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]...
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.14, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4203, 3042, 4202, 2620, 3011, 4203, 17, 3042, 4202, 2620, 2715, 2838, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \tan^3(a+bx)\,dx$$

$$\downarrow 3042$$

$$\int (c+dx)^3 \tan(a+bx)^3 dx$$

$$\downarrow 4203$$

$$-\frac{3d\int(c+dx)^2\tan^2(a+bx)dx}{2b}-\int(c+dx)^3\tan(a+bx)dx+\frac{(c+dx)^3\tan^2(a+bx)}{2b}$$

$$\downarrow 3042$$

$$-\int(c+dx)^3\tan(a+bx)dx-\frac{3d\int(c+dx)^2\tan(a+bx)^2dx}{2b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}$$

$$\downarrow 4202$$

$$2i\int\frac{e^{2i(a+bx)}(c+dx)^3}{1+e^{2i(a+bx)}}dx-\frac{3d\int(c+dx)^2\tan(a+bx)^2dx}{2b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\downarrow 2620$$

$$2i\left(\frac{3id\int(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{3d\int(c+dx)^2\tan(a+bx)^2dx}{2b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\downarrow 3011$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{3d\int(c+dx)^2\tan(a+bx)^2dx}{2b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\downarrow 4203$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{3d\left(-\frac{2d\int(c+dx)\tan(a+bx)dx}{b}-\int(c+dx)^2dx+\frac{(c+dx)^2\tan(a+bx)}{b}\right)}{2b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\downarrow 17$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-$$
$$\frac{3d\left(-\frac{2d\int(c+dx)\tan(a+bx)dx}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\downarrow 3042$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-$$
$$\frac{3d\left(-\frac{2d\int(c+dx)\tan(a+bx)dx}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\downarrow 4202$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-$$
$$\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\int\frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}dx\right)}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}+$$
$$\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\downarrow 2620$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-$$
$$\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{id\int\log\left(1+e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}+$$
$$\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\downarrow 2715$$

$$\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{d\int e^{-2i(a+bx)}\log\left(1+e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}+$$
$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)+$$
$$\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\downarrow 2838$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-$$
$$\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}+$$
$$\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\downarrow 7163$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{id\int\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1\right.}{2b}$$
$$\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}+$$
$$\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\downarrow 2720$$

$$\frac{2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{d\int e^{-2i(a+bx)}\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(}{}$$

$$\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}+$$

$$\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\downarrow 7143$$

$$\frac{2i\left(\frac{3id\left(\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{d\,\mathrm{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e\right.}{2b}$$

$$\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}+$$

$$\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

input
```
Int[(c + d*x)^3*Tan[a + b*x]^3,x]
```

output
```
((-1/4*I)*(c + d*x)^4)/d + (2*I)*(((-1/2*I)*(c + d*x)^3*Log[1 + E^((2*I)*(
a + b*x))])/b + (((3*I)/2)*d*(((I/2)*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a +
 b*x))])/b - (I*d*(((-1/2*I)*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/b
 + (d*PolyLog[4, -E^((2*I)*(a + b*x))])/(4*b^2)))/b))/b) + ((c + d*x)^3*Ta
n[a + b*x]^2)/(2*b) - (3*d*(-1/3*(c + d*x)^3/d - (2*d*(((I/2)*(c + d*x)^2)
/d - (2*I)*(((-1/2*I)*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b - (d*PolyL
og[2, -E^((2*I)*(a + b*x))])/(4*b^2))))/b + ((c + d*x)^2*Tan[a + b*x])/b))
/(2*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))  Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])  Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
  Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))  Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I  Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))  Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2  Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))  Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(224) = 448$.

Time = 0.40 (sec) , antiderivative size = 729, normalized size of antiderivative = 2.81

method	result
risch	$-\frac{3id\,c^2\,\mathrm{polylog}\left(2,-\mathrm{e}^{2i(bx+a)}\right)}{2b^2}+\frac{6id^3ax}{b^3}+\frac{d^3\ln\left(\mathrm{e}^{2i(bx+a)}+1\right)x^3}{b}+\frac{3id^3a^2}{b^4}+\frac{6d^2c\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^3}-\frac{6d^3a\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^4}+$

input
```
int((d*x+c)^3*tan(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a)))-6*I/b*d*c^2*a*x+6*I/b^2*c*d^2*a^2*x+1/
b*d^3*ln(exp(2*I*(b*x+a))+1)*x^3+(2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2
*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*exp(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+
a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*I*c^2*d*exp(2*I*(b*x+a))-3*I*d^3*x^2+2*
b*c^3*exp(2*I*(b*x+a))-6*I*c*d^2*x-3*I*c^2*d)/b^2/(exp(2*I*(b*x+a))+1)^2+3
/2/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x+3/2/b^3*c*d^2*polylog(3,-exp(2*I
*(b*x+a)))-3/b^3*d^2*c*ln(exp(2*I*(b*x+a))+1)-3/b^3*d^3*ln(exp(2*I*(b*x+a)
)+1)*x+3*I/b^2*d^3*x^2+3*I/b^4*d^3*a^2-3*I/b^2*c*d^2*polylog(2,-exp(2*I*(b
*x+a)))*x+6*I/b^3*d^3*a*x-3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2-3
/2*I/b^2*d*c^2*polylog(2,-exp(2*I*(b*x+a)))+6*d^2/b^3*c*ln(exp(I*(b*x+a)))
-6*d^3/b^4*a*ln(exp(I*(b*x+a)))+I*c^3*x+1/4*I/d*c^4-3/2*I/b^4*d^3*a^4-I*d^
2*c*x^3-3/2*I*d*c^2*x^2+2/b^4*d^3*a^3*ln(exp(I*(b*x+a)))+3/b*d*c^2*ln(exp(
2*I*(b*x+a))+1)*x+3/b*c*d^2*ln(exp(2*I*(b*x+a))+1)*x^2+6/b^2*c^2*d*a*ln(ex
p(I*(b*x+a)))-3*I/b^2*d*c^2*a^2-2*I/b^3*d^3*a^3*x+4*I/b^3*c*d^2*a^3+1/b*c^
3*ln(exp(2*I*(b*x+a))+1)-1/4*I*d^3*x^4-2/b*c^3*ln(exp(I*(b*x+a)))+3/4*I*d^
3*polylog(4,-exp(2*I*(b*x+a)))/b^4+3/2*I*d^3*polylog(2,-exp(2*I*(b*x+a)))/
b^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 592 vs. 2(217) = 434.

Time = 0.09 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.29

$$\int (c+dx)^3 \tan^3(a+bx)\,dx$$

$$= \frac{4\,b^3d^3x^3 + 12\,b^3cd^2x^2 + 12\,b^3c^2dx - 3i\,d^3\mathrm{polylog}\left(4, \frac{\tan(bx+a)^2+2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right) + 3i\,d^3\mathrm{polylog}\left(4, \frac{\tan(bx+a)^2-}{\tan(}\right.$$

input
```
integrate((d*x+c)^3*tan(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/8*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x - 3*I*d^3*polylog(4
, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 3*I*d^3*
polylog(4, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) +
 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*tan(b*x + a)^
2 - 6*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d + I*d^3)*dilog(2*(I*
tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) - 6*(I*b^2*d^3*x^2 + 2*I*b^2*c
*d^2*x + I*b^2*c^2*d - I*d^3)*dilog(2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^
2 + 1) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 3*b*c*d^2 + 3*(
b^3*c^2*d - b*d^3)*x)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) +
4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 3*b*c*d^2 + 3*(b^3*c^2*d - b*
d^3)*x)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 6*(b*d^3*x +
b*c*d^2)*polylog(3, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^
2 + 1)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, (tan(b*x + a)^2 - 2*I*tan(b*x +
 a) - 1)/(tan(b*x + a)^2 + 1)) - 12*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2
*d)*tan(b*x + a))/b^4
```

Sympy [F]

$$\int (c+dx)^3 \tan^3(a+bx)\,dx = \int (c+dx)^3 \tan^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*tan(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**3*tan(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2413 vs. $2(217) = 434$.

Time $= 0.72$ (sec) , antiderivative size $= 2413$, normalized size of antiderivative $= 9.32$

$$\int (c + dx)^3 \tan^3(a + bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*tan(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(c^3*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1)) - 3*a*c^2*d*(
1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b + 3*a^2*c*d^2*(1/(sin(
b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b^2 - a^3*d^3*(1/(sin(b*x + a)^
2 - 1) - log(sin(b*x + a)^2 - 1))/b^3 + 2*(3*(b*x + a)^4*d^3 + 36*b^2*c^2*
d - 72*a*b*c*d^2 + 36*a^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a)^3 + 18*(b^2
*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a)^2 - 4*(4*(b*x + a)^3*d^3 - 9*b*c
*d^2 + 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*
d^2 + (a^2 - 1)*d^3)*(b*x + a) + (4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a*d^3
+ 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)
*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 2*(4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a
*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2
 - 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-4*I*(b*x + a)^3*d^3 + 9*I*b*c*d
^2 - 9*I*a*d^3 + 9*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 9*(-I*b^2*c^2*d +
2*I*a*b*c*d^2 + (-I*a^2 + I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(-4*I*(b
*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + 9*(-I*b*c*d^2 + I*a*d^3)*(b*x +
a)^2 + 9*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*a^2 + I)*d^3)*(b*x + a))*sin(
2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + 3*((b*x +
a)^4*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a)^3 + 6*(b^2*c^2*d - 2*a*b*c*d^2 +
(a^2 - 2)*d^3)*(b*x + a)^2 - 24*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4
*a) + 6*((b*x + a)^4*d^3 + 6*b^2*c^2*d - 12*a*b*c*d^2 + 6*a^2*d^3 + 4*(...
```

Giac [F]

$$\int (c+dx)^3 \tan^3(a+bx)\,dx = \int (dx+c)^3 \tan(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^3*tan(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*tan(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \tan^3(a+bx)\,dx = \int \tan(a+b\,x)^3\,(c+d\,x)^3\,dx$$

input
```
int(tan(a + b*x)^3*(c + d*x)^3,x)
```

output
```
int(tan(a + b*x)^3*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \tan^3(a+bx)\,dx$$
$$= \frac{2\left(\int \tan(bx+a)^3\,x^3dx\right) b\,d^3 + 6\left(\int \tan(bx+a)^3\,x^2dx\right) bc\,d^2 + 6\left(\int \tan(bx+a)^3\,xdx\right) b\,c^2d - \log\left(\tan(bx\right.}{2b}$$

input
```
int((d*x+c)^3*tan(b*x+a)^3,x)
```

output
```
(2*int(tan(a + b*x)**3*x**3,x)*b*d**3 + 6*int(tan(a + b*x)**3*x**2,x)*b*c*
d**2 + 6*int(tan(a + b*x)**3*x,x)*b*c**2*d - log(tan(a + b*x)**2 + 1)*c**3
 + tan(a + b*x)**2*c**3)/(2*b)
```

3.305 $\int (c+dx)^2 \tan^3(a+bx)\,dx$

Optimal result . . . 2621
Mathematica [B] (warning: unable to verify) . . . 2622
Rubi [A] (verified) . . . 2623
Maple [B] (verified) . . . 2627
Fricas [B] (verification not implemented) . . . 2627
Sympy [F] . . . 2628
Maxima [B] (verification not implemented) . . . 2628
Giac [F] . . . 2629
Mupad [F(-1)] . . . 2630
Reduce [F] . . . 2630

Optimal result

Integrand size = 16, antiderivative size = 163

$$\begin{aligned}\int (c+dx)^2 \tan^3(a+bx)\,dx = {} & \frac{(c+dx)^2}{2b} - \frac{i(c+dx)^3}{3d} \\ & + \frac{(c+dx)^2 \log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d^2 \log(\cos(a+bx))}{b^3} \\ & - \frac{id(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} \\ & + \frac{d^2\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} \\ & - \frac{d(c+dx)\tan(a+bx)}{b^2} + \frac{(c+dx)^2\tan^2(a+bx)}{2b}\end{aligned}$$

output

```
1/2*(d*x+c)^2/b-1/3*I*(d*x+c)^3/d+(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b-d^2*l
n(cos(b*x+a))/b^3-I*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2+1/2*d^2*pol
ylog(3,-exp(2*I*(b*x+a)))/b^3-d*(d*x+c)*tan(b*x+a)/b^2+1/2*(d*x+c)^2*tan(b
*x+a)^2/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 454 vs. $2(163) = 326$.

Time = 6.44 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.79

$$\int (c+dx)^2 \tan^3(a+bx)\,dx$$

$$= \frac{id^2 e^{-ia}\left(2b^2x^2\left(2bx - 3i(1+e^{2ia})\log\left(1+e^{-2i(a+bx)}\right)\right) + 6b(1+e^{2ia})\,x\,\mathrm{PolyLog}\left(2,-e^{-2i(a+bx)}\right) - 3i(1+\right.}{12b^3}$$

$$+ \frac{(c+dx)^2\sec^2(a+bx)}{2b}$$

$$+ \frac{c^2\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx)) + bx\sin(a))}{b\left(\cos^2(a)+\sin^2(a)\right)}$$

$$- \frac{d^2\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx)) + bx\sin(a))}{b^3\left(\cos^2(a)+\sin^2(a)\right)}$$

$$+ \frac{cd\csc(a)\left(b^2e^{-i\arctan(\cot(a))}x^2 - \frac{\cot(a)(ibx(-\pi-2\arctan(\cot(a))) - \pi\log(1+e^{-2ibx}) - 2(bx-\arctan(\cot(a)))\log(1-e^{2i(bx-\arctan}}{}\right.}{b^2\sqrt{\csc^2(a)\left(\cos^2(a)+\right.}$$

$$+ \frac{\sec(a)\sec(a+bx)\left(-cd\sin(bx) - d^2x\sin(bx)\right)}{b^2} - \frac{1}{3}x\left(3c^2+3cdx+d^2x^2\right)\tan(a)$$

input

```
Integrate[(c + d*x)^2*Tan[a + b*x]^3,x]
```

output

```
((I/12)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*
(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] -
 (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E
^(I*a)) + ((c + d*x)^2*Sec[a + b*x]^2)/(2*b) + (c^2*Sec[a]*(Cos[a]*Log[Cos
[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) -
 (d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))
/(b^3*(Cos[a]^2 + Sin[a]^2)) + (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]])
 - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] -
2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Lo
g[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[
2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*Sq
rt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Sec[a]*Sec[a + b*x]*(-(c*d*Sin[b*x]
) - d^2*x*Sin[b*x]))/b^2 - (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Tan[a])/3
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 4203, 3042, 4202, 2620, 3011, 2720, 4203, 17, 3042, 3956, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \tan^3(a+bx)\,dx$$

$$\downarrow 3042$$

$$\int (c+dx)^2 \tan(a+bx)^3 dx$$

$$\downarrow 4203$$

$$-\frac{d\int (c+dx)\tan^2(a+bx)dx}{b} - \int (c+dx)^2\tan(a+bx)dx + \frac{(c+dx)^2\tan^2(a+bx)}{2b}$$

$$\downarrow 3042$$

$$-\int (c+dx)^2\tan(a+bx)dx - \frac{d\int (c+dx)\tan(a+bx)^2 dx}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b}$$

$$\downarrow 4202$$

$$2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}dx - \frac{d\int (c+dx)\tan(a+bx)^2 dx}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - \frac{i(c+dx)^3}{3d}$$

$$\downarrow 2620$$

$$2i\left(\frac{id\int (c+dx)\log\left(1+e^{2i(a+bx)}\right)dx}{b} - \frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right) - \frac{d\int (c+dx)\tan(a+bx)^2 dx}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - \frac{i(c+dx)^3}{3d}$$

$$\downarrow 3011$$

$$2i\left(\frac{id\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b} - \frac{id\int \text{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{2b}\right)}{b} - \frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right) - \frac{d\int (c+dx)\tan(a+bx)^2 dx}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - \frac{i(c+dx)^3}{3d}$$

$$\downarrow 2720$$

$$2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i(a+bx)}\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-$$
$$\frac{d\int(c+dx)\tan(a+bx)^2dx}{b}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

$$\downarrow 4203$$

$$2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i(a+bx)}\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-$$
$$\frac{d\left(-\frac{d\int\tan(a+bx)dx}{b}-\int(c+dx)dx+\frac{(c+dx)\tan(a+bx)}{b}\right)}{b}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

$$\downarrow 17$$

$$2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i(a+bx)}\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-$$
$$\frac{d\left(-\frac{d\int\tan(a+bx)dx}{b}+\frac{(c+dx)\tan(a+bx)}{b}-\frac{(c+dx)^2}{2d}\right)}{b}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

$$\downarrow 3042$$

$$2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i(a+bx)}\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-$$
$$\frac{d\left(-\frac{d\int\tan(a+bx)dx}{b}+\frac{(c+dx)\tan(a+bx)}{b}-\frac{(c+dx)^2}{2d}\right)}{b}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

$$\downarrow 3956$$

$$2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i(a+bx)}\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-$$
$$\frac{d\left(\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}-\frac{(c+dx)^2}{2d}\right)}{b}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

$$\downarrow 7143$$

$$2i\left(\frac{id\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{d\left(\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}-\frac{(c+dx)^2}{2d}\right)}{b}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

input
```
Int[(c + d*x)^2*Tan[a + b*x]^3,x]
```

output
```
((-1/3*I)*(c + d*x)^3)/d + (2*I)*(((-1/2*I)*(c + d*x)^2*Log[1 + E^((2*I)*(
a + b*x))])/b + (I*d*(((I/2)*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b
 - (d*PolyLog[3, -E^((2*I)*(a + b*x))])/(4*b^2)))/b) + ((c + d*x)^2*Tan[a
+ b*x]^2)/(2*b) - (d*(-1/2*(c + d*x)^2/d + (d*Log[Cos[a + b*x]])/b^2 + ((c
 + d*x)*Tan[a + b*x])/b))/b
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4202

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(147) = 294$.

Time = 0.34 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.51

method	result
risch	$-\frac{id^2\,\mathrm{polylog}\left(2,-\mathrm{e}^{2i(bx+a)}\right)x}{b^2}-\frac{idc\,\mathrm{polylog}\left(2,-\mathrm{e}^{2i(bx+a)}\right)}{b^2}-\frac{2icd\,a^2}{b^2}+\frac{ic^3}{3d}+\frac{2b\,d^2x^2\mathrm{e}^{2i(bx+a)}+4bcdx\,\mathrm{e}^{2i(bx+a)}+2b\,c^2\mathrm{e}^{2i(b}}{b^2(\mathrm{e}^{2i}}$

input
```
int((d*x+c)^2*tan(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-I/b^2*d*c*polylog(2,-exp(2*I*(b*
x+a)))-2*I/b^2*c*d*a^2+1/3*I/d*c^3+2*(b*d^2*x^2*exp(2*I*(b*x+a))+2*b*c*d*x
*exp(2*I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^2*x*exp(2*I*(b*x+a))-I*c*d*ex
p(2*I*(b*x+a))-I*d^2*x-I*d*c)/b^2/(exp(2*I*(b*x+a))+1)^2-1/3*I*d^2*x^3-I*d
*c*x^2+I*c^2*x-4*I/b*c*d*a*x+1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-d^2/
b^3*ln(exp(2*I*(b*x+a))+1)+2/b^3*d^2*ln(exp(I*(b*x+a)))+2*I/b^2*d^2*a^2*x+
1/b*d^2*ln(exp(2*I*(b*x+a))+1)*x^2+4/3*I/b^3*d^2*a^3+4/b^2*c*d*a*ln(exp(I*
(b*x+a)))+2/b*d*c*ln(exp(2*I*(b*x+a))+1)*x+1/b*c^2*ln(exp(2*I*(b*x+a))+1)-
2/b*c^2*ln(exp(I*(b*x+a)))-2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(144) = 288$.

Time = 0.09 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.17

$$\int (c+dx)^2\tan^3(a+bx)\,dx$$
$$=\frac{2\,b^2d^2x^2+4\,b^2cdx+d^2\mathrm{polylog}\left(3,\frac{\tan(bx+a)^2+2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)+d^2\mathrm{polylog}\left(3,\frac{\tan(bx+a)^2-2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)+2\,(}{}$$

input
```
integrate((d*x+c)^2*tan(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + d^2*polylog(3, (tan(b*x + a)^2 + 2*I*ta
n(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + d^2*polylog(3, (tan(b*x + a)^2 - 2
*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*tan(b*x + a)^2 - 2*(-I*b*d^2*x - I*b*c*d)*dilog(2*(I*tan(b*x +
a) - 1)/(tan(b*x + a)^2 + 1) + 1) - 2*(I*b*d^2*x + I*b*c*d)*dilog(2*(-I*ta
n(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2 - d^2)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 2*(b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - d^2)*log(-2*(-I*tan(b*x + a) - 1)/(tan(
b*x + a)^2 + 1)) - 4*(b*d^2*x + b*c*d)*tan(b*x + a))/b^3
```

Sympy [F]

$$\int (c+dx)^2 \tan^3(a+bx)\,dx = \int (c+dx)^2 \tan^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*tan(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**2*tan(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1228 vs. $2(144) = 288$.

Time $= 0.33$ (sec) , antiderivative size $= 1228$, normalized size of antiderivative $= 7.53$

$$\int (c+dx)^2 \tan^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*tan(b*x+a)^3,x, algorithm="maxima")
```

output

```
-1/2*(c^2*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1)) - 2*a*c*d*(1/
(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b + a^2*d^2*(1/(sin(b*x +
a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b^2 + 2*(2*(b*x + a)^3*d^2 + 6*(b*c*d
 - a*d^2)*(b*x + a)^2 + 12*b*c*d - 12*a*d^2 - 6*((b*x + a)^2*d^2 + 2*(b*c*
d - a*d^2)*(b*x + a) - d^2 + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a
) - d^2)*cos(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x +
a) - d^2)*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*
(b*x + a) + I*d^2)*sin(4*b*x + 4*a) - 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d
+ I*a*d^2)*(b*x + a) + I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a),
cos(2*b*x + 2*a) + 1) + 2*((b*x + a)^3*d^2 + 3*(b*c*d - a*d^2)*(b*x + a)^2
 - 6*(b*x + a)*d^2)*cos(4*b*x + 4*a) + 4*((b*x + a)^3*d^2 + 3*(b*c*d - (a
- I)*d^2)*(b*x + a)^2 + 3*b*c*d - 3*a*d^2 + 3*(2*I*b*c*d + (-2*I*a - 1)*d^
2)*(b*x + a))*cos(2*b*x + 2*a) + 6*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d
 + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + 2*(b*c*d + (b*x + a)*d^2 - a*
d^2)*cos(2*b*x + 2*a) + (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(4*b*x +
4*a) + 2*(I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^
(2*I*b*x + 2*I*a)) + 3*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a
) - I*d^2 + (I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) - I*d^2)*
cos(4*b*x + 4*a) + 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a)
- I*d^2)*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x +...
```

Giac [F]

$$\int (c+dx)^2 \tan^3(a+bx)\,dx = \int (dx+c)^2 \tan(bx+a)^3\,dx$$

input

```
integrate((d*x+c)^2*tan(b*x+a)^3,x, algorithm="giac")
```

output

```
integrate((d*x + c)^2*tan(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \tan^3(a+bx)\,dx = \int \tan(a+b\,x)^3\,(c+d\,x)^2\,dx$$

input
```
int(tan(a + b*x)^3*(c + d*x)^2,x)
```

output
```
int(tan(a + b*x)^3*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \tan^3(a+bx)\,dx$$
$$= \frac{2\left(\int \tan(bx+a)^3\,x^2dx\right)b\,d^2 + 4\left(\int \tan(bx+a)^3\,xdx\right)bcd - \log\left(\tan(bx+a)^2+1\right)c^2 + \tan(bx+a)^2\,c^2}{2b}$$

input
```
int((d*x+c)^2*tan(b*x+a)^3,x)
```

output
```
(2*int(tan(a + b*x)**3*x**2,x)*b*d**2 + 4*int(tan(a + b*x)**3*x,x)*b*c*d -
 log(tan(a + b*x)**2 + 1)*c**2 + tan(a + b*x)**2*c**2)/(2*b)
```

3.306 $\int (c+dx)\tan^3(a+bx)\,dx$

Optimal result . . . 2631
Mathematica [B] (warning: unable to verify) . . . 2631
Rubi [A] (verified) . . . 2632
Maple [A] (verified) . . . 2635
Fricas [A] (verification not implemented) . . . 2635
Sympy [F] . . . 2636
Maxima [B] (verification not implemented) . . . 2636
Giac [F] . . . 2637
Mupad [F(-1)] . . . 2637
Reduce [F] . . . 2638

Optimal result

Integrand size = 14, antiderivative size = 108

$$\int (c+dx)\tan^3(a+bx)\,dx = \frac{dx}{2b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{id\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} - \frac{d\tan(a+bx)}{2b^2} + \frac{(c+dx)\tan^2(a+bx)}{2b}$$

output

```
1/2*d*x/b-1/2*I*(d*x+c)^2/d+(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b-1/2*I*d*polyl
og(2,-exp(2*I*(b*x+a)))/b^2-1/2*d*tan(b*x+a)/b^2+1/2*(d*x+c)*tan(b*x+a)^2/
b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 240 vs. 2(108) = 216.

Time = 6.16 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.22

$$\int (c+dx)\tan^3(a+bx)\,dx = \frac{dx\sec^2(a+bx)}{2b} + \frac{c(2\log(\cos(a+bx)) + \sec^2(a+bx))}{2b}$$
$$+ \frac{d\csc(a)\left(b^2 e^{-i\arctan(\cot(a))}x^2 - \frac{\cot(a)(ibx(-\pi-2\arctan(\cot(a)))-\pi\log(1+e^{-2ibx})-2(bx-\arctan(\cot(a)))\log(1-e^{2i(bx-\arctan(}}{}\right.}{2b^2\sqrt{\csc^2(a)\,(\cos^2(a)+}}$$
$$- \frac{d\sec(a)\sec(a+bx)\sin(bx)}{2b^2} - \frac{1}{2}dx^2\tan(a)$$

input
```
Integrate[(c + d*x)*Tan[a + b*x]^3,x]
```

output
```
(d*x*Sec[a + b*x]^2)/(2*b) + (c*(2*Log[Cos[a + b*x]] + Sec[a + b*x]^2))/(2
*b) + (d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*
ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Lo
g[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[
a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[C
ot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + S
in[a]^2)]) - (d*Sec[a]*Sec[a + b*x]*Sin[b*x])/(2*b^2) - (d*x^2*Tan[a])/2
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3042, 4203, 3042, 3954, 24, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\tan^3(a+bx)\,dx$$
$$\downarrow 3042$$
$$\int (c+dx)\tan(a+bx)^3 dx$$
$$\downarrow 4203$$

$$-\int (c+dx)\tan(a+bx)dx - \frac{d\int \tan^2(a+bx)dx}{2b} + \frac{(c+dx)\tan^2(a+bx)}{2b}$$

$$\downarrow 3042$$

$$-\int (c+dx)\tan(a+bx)dx - \frac{d\int \tan(a+bx)^2dx}{2b} + \frac{(c+dx)\tan^2(a+bx)}{2b}$$

$$\downarrow 3954$$

$$-\int (c+dx)\tan(a+bx)dx - \frac{d\left(\frac{\tan(a+bx)}{b} - \int 1dx\right)}{2b} + \frac{(c+dx)\tan^2(a+bx)}{2b}$$

$$\downarrow 24$$

$$-\int (c+dx)\tan(a+bx)dx + \frac{(c+dx)\tan^2(a+bx)}{2b} - \frac{d\left(\frac{\tan(a+bx)}{b} - x\right)}{2b}$$

$$\downarrow 4202$$

$$2i\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}dx + \frac{(c+dx)\tan^2(a+bx)}{2b} - \frac{d\left(\frac{\tan(a+bx)}{b} - x\right)}{2b} - \frac{i(c+dx)^2}{2d}$$

$$\downarrow 2620$$

$$2i\left(\frac{id\int \log\left(1+e^{2i(a+bx)}\right)dx}{2b} - \frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right) + \frac{(c+dx)\tan^2(a+bx)}{2b} - \frac{d\left(\frac{\tan(a+bx)}{b} - x\right)}{2b} - \frac{i(c+dx)^2}{2d}$$

$$\downarrow 2715$$

$$2i\left(\frac{d\int e^{-2i(a+bx)}\log\left(1+e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2} - \frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right) + \frac{(c+dx)\tan^2(a+bx)}{2b} - \frac{d\left(\frac{\tan(a+bx)}{b} - x\right)}{2b} - \frac{i(c+dx)^2}{2d}$$

$$\downarrow 2838$$

$$2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2} - \frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right) + \frac{(c+dx)\tan^2(a+bx)}{2b} - \frac{d\left(\frac{\tan(a+bx)}{b} - x\right)}{2b} - \frac{i(c+dx)^2}{2d}$$

input
```
Int[(c + d*x)*Tan[a + b*x]^3,x]
```

output
```
((-1/2*I)*(c + d*x)^2)/d + (2*I)*(((-1/2*I)*(c + d*x)*Log[1 + E^((2*I)*(a
+ b*x))])/b - (d*PolyLog[2, -E^((2*I)*(a + b*x))])/(4*b^2)) + ((c + d*x)*T
an[a + b*x]^2)/(2*b) - (d*(-x + Tan[a + b*x]/b))/(2*b)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))  Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])  Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2  Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4202

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.69

method	result
risch	$-\frac{id x^2}{2}+ixc+\frac{2bdx\,e^{2i(bx+a)}-id\,e^{2i(bx+a)}+2bc\,e^{2i(bx+a)}-id}{b^2\left(e^{2i(bx+a)}+1\right)^2}+\frac{c\ln\left(e^{2i(bx+a)}+1\right)}{b}-\frac{2c\ln\left(e^{i(bx+a)}\right)}{b}-\frac{2idax}{b}-\frac{id\,a^2}{b^2}$

input

```
int((d*x+c)*tan(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output

```
-1/2*I*d*x^2+I*x*c+(2*b*d*x*exp(2*I*(b*x+a))-I*d*exp(2*I*(b*x+a))+2*b*c*ex
p(2*I*(b*x+a))-I*d)/b^2/(exp(2*I*(b*x+a))+1)^2+1/b*c*ln(exp(2*I*(b*x+a))+1
)-2/b*c*ln(exp(I*(b*x+a)))-2*I/b*d*a*x-I/b^2*d*a^2+1/b*d*ln(exp(2*I*(b*x+a
))+1)*x-1/2*I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+2/b^2*d*a*ln(exp(I*(b*x+a
)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.56

$$\int (c+dx)\tan^3(a+bx)\,dx$$

$$=\frac{2\,bdx+2\,(bdx+bc)\tan(bx+a)^2+i\,d\mathrm{Li}_2\left(\frac{2\,(i\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)-i\,d\mathrm{Li}_2\left(\frac{2\,(-i\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)+2\,(bdx+$$

$$4\,b^2$$

input
```
integrate((d*x+c)*tan(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(2*b*d*x + 2*(b*d*x + b*c)*tan(b*x + a)^2 + I*d*dilog(2*(I*tan(b*x + a
) - 1)/(tan(b*x + a)^2 + 1) + 1) - I*d*dilog(2*(-I*tan(b*x + a) - 1)/(tan(
b*x + a)^2 + 1) + 1) + 2*(b*d*x + b*c)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*
x + a)^2 + 1)) + 2*(b*d*x + b*c)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a
)^2 + 1)) - 2*d*tan(b*x + a))/b^2
```

Sympy [F]

$$\int (c+dx)\tan^3(a+bx)\,dx = \int (c+dx)\tan^3(a+bx)\,dx$$

input
```
integrate((d*x+c)*tan(b*x+a)**3,x)
```

output
```
Integral((c + d*x)*tan(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(89) = 178$.

Time $= 0.24$ (sec) , antiderivative size $= 517$, normalized size of antiderivative $= 4.79$

$$\int (c+dx)\tan^3(a+bx)\,dx =$$
$$-\frac{b^2dx^2 + 2\,b^2cx - 2\,(bdx + bc + (bdx + bc)\cos(4\,bx + 4\,a) + 2\,(bdx + bc)\cos(2\,bx + 2\,a) - (-i\,bdx -$$

input
```
integrate((d*x+c)*tan(b*x+a)^3,x, algorithm="maxima")
```

output
```
-(b^2*d*x^2 + 2*b^2*c*x - 2*(b*d*x + b*c + (b*d*x + b*c)*cos(4*b*x + 4*a)
+ 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (-I*b*d*x - I*b*c)*sin(4*b*x + 4*a) -
 2*(-I*b*d*x - I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*
x + 2*a) + 1) + (b^2*d*x^2 + 2*b^2*c*x)*cos(4*b*x + 4*a) + 2*(b^2*d*x^2 +
2*I*b*c + 2*(b^2*c + I*b*d)*x + d)*cos(2*b*x + 2*a) + (d*cos(4*b*x + 4*a)
+ 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) + 2*I*d*sin(2*b*x + 2*a) + d
)*dilog(-e^(2*I*b*x + 2*I*a)) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos
(4*b*x + 4*a) - 2*(I*b*d*x + I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(4
*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + s
in(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (-I*b^2*d*x^2 - 2*I*b^2*c*x)
*sin(4*b*x + 4*a) + 2*(I*b^2*d*x^2 - 2*b*c + 2*(I*b^2*c - b*d)*x + I*d)*si
n(2*b*x + 2*a) + 2*d)/(-2*I*b^2*cos(4*b*x + 4*a) - 4*I*b^2*cos(2*b*x + 2*a
) + 2*b^2*sin(4*b*x + 4*a) + 4*b^2*sin(2*b*x + 2*a) - 2*I*b^2)
```

Giac [F]

$$\int (c+dx)\tan^3(a+bx)\,dx = \int (dx+c)\tan(bx+a)^3\,dx$$

input
```
integrate((d*x+c)*tan(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*tan(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\tan^3(a+bx)\,dx = \int \tan(a+b\,x)^3\,(c+d\,x)\,dx$$

input
```
int(tan(a + b*x)^3*(c + d*x),x)
```

output
```
int(tan(a + b*x)^3*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)\tan^3(a+bx)\,dx = \frac{2\left(\int \tan(bx+a)^3\,x dx\right)bd - \log\left(\tan(bx+a)^2+1\right)c + \tan(bx+a)^2 c}{2b}$$

input
```
int((d*x+c)*tan(b*x+a)^3,x)
```

output
```
(2*int(tan(a + b*x)**3*x,x)*b*d - log(tan(a + b*x)**2 + 1)*c + tan(a + b*x
)**2*c)/(2*b)
```

3.307 $\int \frac{\tan^3(a+bx)}{c+dx}\,dx$

Optimal result 2639
Mathematica [N/A] 2639
Rubi [N/A] 2640
Maple [N/A] 2641
Fricas [N/A] 2641
Sympy [N/A] 2642
Maxima [N/A] 2642
Giac [N/A] 2643
Mupad [N/A] 2644
Reduce [N/A] 2644

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\tan^3(a+bx)}{c+dx}, x\right)$$

output
```
Defer(Int)(tan(b*x+a)^3/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 5.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx = \int \frac{\tan^3(a+bx)}{c+dx}\,dx$$

input
```
Integrate[Tan[a + b*x]^3/(c + d*x),x]
```

output
```
Integrate[Tan[a + b*x]^3/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\tan(a+bx)^3}{c+dx}\,dx$$
$$\downarrow 4222$$
$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx$$

input
```
Int[Tan[a + b*x]^3/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\tan(bx+a)^3}{dx+c}dx$$

input

```
int(tan(b*x+a)^3/(d*x+c),x)
```

output

```
int(tan(b*x+a)^3/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx = \int \frac{\tan(bx+a)^3}{dx+c}\,dx$$

input

```
integrate(tan(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
integral(tan(b*x + a)^3/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx = \int \frac{\tan^3(a+bx)}{c+dx}\,dx$$

input
```
integrate(tan(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(tan(a + b*x)**3/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 1077, normalized size of antiderivative = 67.31

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx = \int \frac{\tan(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(tan(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output

```
(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
 (2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
+ 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
+ (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*s
in(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*
x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate(2*(b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2 - d^2)*sin(2*b*x + 2*a)/(b^2*d^3*x^3 + 3*b^2*c*d^
2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x
^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)), x) + (d*cos(2*b*x + 2*a)
+ 2*(b*d*x + b*c)*sin(2*b*x + 2*a) + d)*sin(4*b*x + 4*a) + d*sin(2*b*x + 2
*a))/(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2
*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 +
 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2...
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx = \int \frac{\tan\left(bx+a\right)^3}{dx+c}\,dx$$

input

```
integrate(tan(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output

```
integrate(tan(b*x + a)^3/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 21.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx = \int \frac{\tan(a+b\,x)^3}{c+d\,x}\,dx$$

input
```
int(tan(a + b*x)^3/(c + d*x),x)
```

output
```
int(tan(a + b*x)^3/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx = \int \frac{\tan(bx+a)^3}{dx+c}dx$$

input
```
int(tan(b*x+a)^3/(d*x+c),x)
```

output
```
int(tan(a + b*x)**3/(c + d*x),x)
```

3.308 $\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 2645
Mathematica [N/A] . . . 2645
Rubi [N/A] . . . 2646
Maple [N/A] . . . 2647
Fricas [N/A] . . . 2647
Sympy [N/A] . . . 2648
Maxima [N/A] . . . 2648
Giac [N/A] . . . 2649
Mupad [N/A] . . . 2650
Reduce [N/A] . . . 2650

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\tan^3(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(tan(b*x+a)^3/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 5.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[Tan[a + b*x]^3/(c + d*x)^2,x]
```

output
```
Integrate[Tan[a + b*x]^3/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\tan(a+bx)^3}{(c+dx)^2}\,dx$$
$$\downarrow 4222$$
$$\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx$$

input
```
Int[Tan[a + b*x]^3/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\tan(bx+a)^3}{(dx+c)^2}dx$$

input

```
int(tan(b*x+a)^3/(d*x+c)^2,x)
```

output

```
int(tan(b*x+a)^3/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(bx+a)^3}{(dx+c)^2}\,dx$$

input

```
integrate(tan(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(tan(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(tan(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(tan(a + b*x)**3/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 6.63 (sec) , antiderivative size = 1386, normalized size of antiderivative = 86.62

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(bx+a)^3}{(dx+c)^2}\,dx$$

input
```
integrate(tan(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
 2*((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
+ 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2
*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*
x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 +
3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^
3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2
*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*
b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a
) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x
+ 2*a))*integrate(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 3*d^2)*sin(2*b*
x + 2*a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*
x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c
^3*d*x + b^2*c^4)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*
b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^4
*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(
2*b*x + 2*a)), x) + 2*(d*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(2*b*x + 2*a)
 + d)*sin(4*b*x + 4*a) + 2*d*sin(2*b*x + 2*a))/(b^2*d^3*x^3 + 3*b^2*c*d...
```

Giac [N/A]

Not integrable

Time = 11.52 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan\left(bx+a\right)^3}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(tan(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(tan(b*x + a)^3/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 20.63 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(a+b\,x)^3}{(c+d\,x)^2}\,dx$$

input
```
int(tan(a + b*x)^3/(c + d*x)^2,x)
```

output
```
int(tan(a + b*x)^3/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(bx+a)^3}{d^2x^2+2cdx+c^2}dx$$

input
```
int(tan(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int(tan(a + b*x)**3/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.309 $\int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx$

Optimal result . . . 2651
Mathematica [N/A] . . . 2651
Rubi [N/A] . . . 2652
Maple [N/A] . . . 2652
Fricas [N/A] . . . 2653
Sympy [F(-1)] . . . 2653
Maxima [N/A] . . . 2653
Giac [N/A] . . . 2654
Mupad [N/A] . . . 2654
Reduce [N/A] . . . 2655

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx = \text{Int}\big((c+dx)^m \csc(a+bx)\sec^3(a+bx), x\big)$$

output
```
Defer(Int)((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 14.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx = \int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

output
```
Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\sec^3(a+bx)(c+dx)^m\,dx$$

$$\downarrow 7299$$

$$\int \csc(a+bx)\sec^3(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \csc(bx+a)\sec(bx+a)^3\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x)
```

output
```
int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)\sec(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*csc(b*x+a)*sec(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)\sec(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc(a+bx) \sec^3(a+bx)\,dx = \int (dx+c)^m \csc\left(bx+a\right) \sec\left(bx+a\right)^3\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 19.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (c+dx)^m \csc(a+bx) \sec^3(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\cos\left(a+b\,x\right)^3 \sin\left(a+b\,x\right)}\,dx$$

input
```
int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)),x)
```

output
```
int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)\sec(bx+a)^3\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x)
```

output
```
int((c + d*x)**m*csc(a + b*x)*sec(a + b*x)**3,x)
```

3.310 $\int (c+dx)^4 \csc(a+bx)\sec^3(a+bx)\,dx$

Optimal result . . . 2656
Mathematica [B] (warning: unable to verify) . . . 2657
Rubi [A] (verified) . . . 2658
Maple [B] (verified) . . . 2661
Fricas [B] (verification not implemented) . . . 2662
Sympy [F(-1)] . . . 2662
Maxima [B] (verification not implemented) . . . 2662
Giac [F] . . . 2663
Mupad [F(-1)] . . . 2664
Reduce [F] . . . 2664

Optimal result

Integrand size = 22, antiderivative size = 399

$$
\begin{aligned}
&\int (c+dx)^4 \csc(a+bx)\sec^3(a+bx)\,dx \\
&= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4 \operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b} \\
&\quad - \frac{6d^2(c+dx)^2 \log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{6id^3(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^4} \\
&\quad + \frac{2id(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)^3\,\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2} \\
&\quad - \frac{3d^4\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^5} - \frac{3d^2(c+dx)^2\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3} \\
&\quad + \frac{3d^2(c+dx)^2\,\mathrm{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3} - \frac{3id^3(c+dx)\,\mathrm{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4} \\
&\quad + \frac{3id^3(c+dx)\,\mathrm{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4} + \frac{3d^4\,\mathrm{PolyLog}\left(5,-e^{2i(a+bx)}\right)}{2b^5} \\
&\quad - \frac{3d^4\,\mathrm{PolyLog}\left(5,e^{2i(a+bx)}\right)}{2b^5} - \frac{2d(c+dx)^3\tan(a+bx)}{b^2} + \frac{(c+dx)^4\tan^2(a+bx)}{2b}
\end{aligned}
$$

output

```
-2*I*d*(d*x+c)^3*polylog(2,exp(2*I*(b*x+a)))/b^2+1/2*(d*x+c)^4/b-2*(d*x+c)
^4*arctanh(exp(2*I*(b*x+a)))/b-6*d^2*(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b^3+
3*I*d^3*(d*x+c)*polylog(4,exp(2*I*(b*x+a)))/b^4+2*I*d*(d*x+c)^3/b^2+6*I*d^
3*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^4-3*d^4*polylog(3,-exp(2*I*(b*x+a
)))/b^5-3*d^2*(d*x+c)^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+3*d^2*(d*x+c)^2*p
olylog(3,exp(2*I*(b*x+a)))/b^3-3*I*d^3*(d*x+c)*polylog(4,-exp(2*I*(b*x+a))
)/b^4+2*I*d*(d*x+c)^3*polylog(2,-exp(2*I*(b*x+a)))/b^2+3/2*d^4*polylog(5,-
exp(2*I*(b*x+a)))/b^5-3/2*d^4*polylog(5,exp(2*I*(b*x+a)))/b^5-2*d*(d*x+c)^
3*tan(b*x+a)/b^2+1/2*(d*x+c)^4*tan(b*x+a)^2/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2227 vs. $2(399) = 798$.

Time = 7.01 (sec) , antiderivative size = 2227, normalized size of antiderivative = 5.58

$$\int (c+dx)^4 \csc(a+bx) \sec^3(a+bx)\, dx = \text{Result too large to show}$$

input

```
Integrate[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

output

```
-((c^2*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2
*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2
*Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, -E^((-I
)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, E^((-I)*(a + b*x))] +
(6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^((-I)*(a + b*x))] + (6*I)*(1 - E^((
-2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]))/b^3) - (c*d^3*E^(I*a)*Csc[a]*((
b^4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a
 + b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] -
 6*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - 6*b^2*(1 -
 E^((-2*I)*a))*x^2*PolyLog[2, E^((-I)*(a + b*x))] + (12*I)*b*(1 - E^((-2*I
)*a))*x*PolyLog[3, -E^((-I)*(a + b*x))] + (12*I)*b*(1 - E^((-2*I)*a))*x*Po
lyLog[3, E^((-I)*(a + b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, -E^((-I)*(
a + b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, E^((-I)*(a + b*x))]))/b^4 -
(d^4*E^(I*a)*Csc[a]*((2*b^5*x^5)/E^((2*I)*a) + (5*I)*b^4*(1 - E^((-2*I)*a)
)*x^4*Log[1 - E^((-I)*(a + b*x))] + (5*I)*b^4*(1 - E^((-2*I)*a))*x^4*Log[1
 + E^((-I)*(a + b*x))] - 20*b^3*(1 - E^((-2*I)*a))*x^3*PolyLog[2, -E^((-I)
*(a + b*x))] - 20*b^3*(1 - E^((-2*I)*a))*x^3*PolyLog[2, E^((-I)*(a + b*x))
] + (60*I)*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[3, -E^((-I)*(a + b*x))] + (6
0*I)*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[3, E^((-I)*(a + b*x))] + 120*b*(1
- E^((-2*I)*a))*x*PolyLog[4, -E^((-I)*(a + b*x))] + 120*b*(1 - E^((-2*I...
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4920, 27, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \csc(a+bx) \sec^3(a+bx)\, dx$$

$$\downarrow 4920$$

$$-4d \int \frac{1}{2}(c+dx)^3 \left(\frac{\tan^2(a+bx)}{b} + \frac{2\log(\tan(a+bx))}{b}\right) dx + \frac{(c+dx)^4 \tan^2(a+bx)}{2b} + \frac{(c+dx)^4 \log(\tan(a+bx))}{b}$$

$$\downarrow 27$$

$$-2d\int (c+dx)^3\left(\frac{\tan^2(a+bx)}{b}+\frac{2\log(\tan(a+bx))}{b}\right)dx+\frac{(c+dx)^4\tan^2(a+bx)}{2b}+\frac{(c+dx)^4\log(\tan(a+bx))}{b}$$

$$\downarrow 7292$$

$$-2d\int \frac{(c+dx)^3\left(\tan^2(a+bx)+2\log(\tan(a+bx))\right)}{b}dx+\frac{(c+dx)^4\tan^2(a+bx)}{2b}+\frac{(c+dx)^4\log(\tan(a+bx))}{b}$$

$$\downarrow 27$$

$$-\frac{2d\int (c+dx)^3\left(\tan^2(a+bx)+2\log(\tan(a+bx))\right)dx}{b}+\frac{(c+dx)^4\tan^2(a+bx)}{2b}+\frac{(c+dx)^4\log(\tan(a+bx))}{b}$$

$$\downarrow 7293$$

$$-\frac{2d\int \left(\tan^2(a+bx)(c+dx)^3+2\log(\tan(a+bx))(c+dx)^3\right)dx}{b}+\frac{(c+dx)^4\tan^2(a+bx)}{2b}+\frac{(c+dx)^4\log(\tan(a+bx))}{b}$$

$$\downarrow 2009$$

$$-\frac{2d\left(\frac{(c+dx)^4\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{d}+\frac{3d^3\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^4}-\frac{3d^3\operatorname{PolyLog}\left(5,-e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^3\operatorname{PolyLog}\left(5,e^{2i(a+bx)}\right)}{4b^4}-\frac{3id^2(c+}{}\right.}{\frac{(c+dx)^4\tan^2(a+bx)}{2b}+\frac{(c+dx)^4\log(\tan(a+bx))}{b}}$$

input

```
Int[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

output
```
((c + d*x)^4*Log[Tan[a + b*x]])/b + ((c + d*x)^4*Tan[a + b*x]^2)/(2*b) - (
2*d*(((-I)*(c + d*x)^3)/b - (c + d*x)^4/(4*d) + ((c + d*x)^4*ArcTanh[E^((2
*I)*(a + b*x))])/d + (3*d*(c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b^2 +
((c + d*x)^4*Log[Tan[a + b*x]])/(2*d) - ((3*I)*d^2*(c + d*x)*PolyLog[2, -E
^((2*I)*(a + b*x))])/b^3 - (I*(c + d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))]
)/b + (I*(c + d*x)^3*PolyLog[2, E^((2*I)*(a + b*x))])/b + (3*d^3*PolyLog[3
, -E^((2*I)*(a + b*x))])/(2*b^4) + (3*d*(c + d*x)^2*PolyLog[3, -E^((2*I)*(
a + b*x))])/(2*b^2) - (3*d*(c + d*x)^2*PolyLog[3, E^((2*I)*(a + b*x))])/(2
*b^2) + (((3*I)/2)*d^2*(c + d*x)*PolyLog[4, -E^((2*I)*(a + b*x))])/b^3 - (
((3*I)/2)*d^2*(c + d*x)*PolyLog[4, E^((2*I)*(a + b*x))])/b^3 - (3*d^3*Poly
Log[5, -E^((2*I)*(a + b*x))])/(4*b^4) + (3*d^3*PolyLog[5, E^((2*I)*(a + b*
x))])/(4*b^4) + ((c + d*x)^3*Tan[a + b*x])/b))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1728 vs. 2(361) = 722.

Time = 0.68 (sec) , antiderivative size = 1729, normalized size of antiderivative = 4.33

method	result	size
risch	Expression too large to display	1729

input
```
int((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^3*exp(2*I*(b*x+a))+6*b*c^2*d^2*x
^2*exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*x+a))-2*I*d^4*x^3*exp(2*I*(b*x+
a))+b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp(2*I*(b*x+a))-6*I*c^2*d^2*x*ex
p(2*I*(b*x+a))-2*I*d^4*x^3-2*I*c^3*d*exp(2*I*(b*x+a))-6*I*c*d^3*x^2-6*I*c^
2*d^2*x-2*I*c^3*d)/b^2/(exp(2*I*(b*x+a))+1)^2+12/b^3*c^2*d^2*ln(exp(I*(b*x
+a)))+12/b^5*a^2*d^4*ln(exp(I*(b*x+a)))+1/b*d^4*ln(exp(I*(b*x+a))+1)*x^4+1
/b*d^4*ln(1-exp(I*(b*x+a)))*x^4-1/b^5*a^4*d^4*ln(1-exp(I*(b*x+a)))-1/b*d^4
*ln(exp(2*I*(b*x+a))+1)*x^4-6/b^3*c^2*d^2*ln(exp(2*I*(b*x+a))+1)+1/b^5*a^4
*d^4*ln(exp(I*(b*x+a))-1)-6/b^3*d^4*ln(exp(2*I*(b*x+a))+1)*x^2+12/b^3*d^4*
polylog(3,exp(I*(b*x+a)))*x^2-3/b^3*d^4*polylog(3,-exp(2*I*(b*x+a)))*x^2+1
2/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2-3/b^3*c^2*d^2*polylog(3,-exp(2*I*
(b*x+a)))+12/b^3*c^2*d^2*polylog(3,-exp(I*(b*x+a)))+12/b^3*c^2*d^2*polylog
(3,exp(I*(b*x+a)))-8*I/b^5*a^3*d^4+4*I/b^2*d^4*x^3+6*I/b^2*c^2*d^2*polylog
(2,-exp(2*I*(b*x+a)))*x-12*I/b^2*c^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-12*I
/b^2*c^2*d^2*polylog(2,exp(I*(b*x+a)))*x+24*I/b^3*c*d^3*a*x+6*I/b^2*c*d^3*
polylog(2,-exp(2*I*(b*x+a)))*x^2-12*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))
*x^2-12*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2-4*I/b^2*c^3*d*polylog(2,
-exp(I*(b*x+a)))-4*I/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))+6*I/b^4*c*d^3*pol
ylog(2,-exp(2*I*(b*x+a)))+6*I/b^4*d^4*polylog(2,-exp(2*I*(b*x+a)))*x-12*I/
b^4*a^2*d^4*x-3*I/b^4*c*d^3*polylog(4,-exp(2*I*(b*x+a)))+24*I/b^4*c*d^3...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3324 vs. $2(352) = 704$.

Time = 0.34 (sec) , antiderivative size = 3324, normalized size of antiderivative = 8.33

$$\int (c+dx)^4 \csc(a+bx) \sec^3(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^4 \csc(a+bx) \sec^3(a+bx)\, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**4*csc(b*x+a)*sec(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8853 vs. $2(352) = 704$.

Time = 4.20 (sec) , antiderivative size = 8853, normalized size of antiderivative = 22.19

$$\int (c+dx)^4 \csc(a+bx) \sec^3(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

output

```
-1/2*(c^4*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x
+ a)^2)) - 4*a*c^3*d*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - l
og(sin(b*x + a)^2))/b + 6*a^2*c^2*d^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*
x + a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - 4*a^3*c*d^3*(1/(sin(b*x + a)^2
 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^3 + a^4*d^4*(1/(si
n(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^4 + 2
*(24*b^3*c^3*d - 72*a*b^2*c^2*d^2 + 72*a^2*b*c*d^3 - 24*a^3*d^4 + 4*(3*(b*
x + a)^4*d^4 + 9*b^2*c^2*d^2 - 18*a*b*c*d^3 + 9*a^2*d^4 + 8*(b*c*d^3 - a*d
^4)*(b*x + a)^3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a)^
2 + 6*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4
)*(b*x + a) + (3*(b*x + a)^4*d^4 + 9*b^2*c^2*d^2 - 18*a*b*c*d^3 + 9*a^2*d^
4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2
 + 1)*d^4)*(b*x + a)^2 + 6*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d
^3 - (a^3 + 3*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + 2*(3*(b*x + a)^4*d^4 +
 9*b^2*c^2*d^2 - 18*a*b*c*d^3 + 9*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^
3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a)^2 + 6*(b^3*c^3
*d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4)*(b*x + a))*c
os(2*b*x + 2*a) - (-3*I*(b*x + a)^4*d^4 - 9*I*b^2*c^2*d^2 + 18*I*a*b*c*d^3
 - 9*I*a^2*d^4 + 8*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 9*(-I*b^2*c^2*d^2
+ 2*I*a*b*c*d^3 + (-I*a^2 - I)*d^4)*(b*x + a)^2 + 6*(-I*b^3*c^3*d + 3*I...
```

Giac [F]

$$\int (c+dx)^4 \csc(a+bx) \sec^3(a+bx)\,dx = \int (dx+c)^4 \csc\left(bx+a\right) \sec\left(bx+a\right)^3\,dx$$

input

```
integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

output

```
integrate((d*x + c)^4*csc(b*x + a)*sec(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \csc(a+bx) \sec^3(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)^4/(cos(a + b*x)^3*sin(a + b*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^4 \csc(a+bx) \sec^3(a+bx)\,dx$$
$$= \frac{2\left(\int \csc(bx+a)\sec(bx+a)^3 x^4 dx\right)\sin(bx+a)^2 b d^4 - 2\left(\int \csc(bx+a)\sec(bx+a)^3 x^4 dx\right) b d^4 + 8\left(\int \text{cs}\right.}{}$$

input
```
int((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x)
```

output
```
(2*int(csc(a + b*x)*sec(a + b*x)**3*x**4,x)*sin(a + b*x)**2*b*d**4 - 2*int
(csc(a + b*x)*sec(a + b*x)**3*x**4,x)*b*d**4 + 8*int(csc(a + b*x)*sec(a +
b*x)**3*x**3,x)*sin(a + b*x)**2*b*c*d**3 - 8*int(csc(a + b*x)*sec(a + b*x)
**3*x**3,x)*b*c*d**3 + 12*int(csc(a + b*x)*sec(a + b*x)**3*x**2,x)*sin(a +
 b*x)**2*b*c**2*d**2 - 12*int(csc(a + b*x)*sec(a + b*x)**3*x**2,x)*b*c**2*
d**2 + 8*int(csc(a + b*x)*sec(a + b*x)**3*x,x)*sin(a + b*x)**2*b*c**3*d -
8*int(csc(a + b*x)*sec(a + b*x)**3*x,x)*b*c**3*d - 2*log(tan((a + b*x)/2)
- 1)*sin(a + b*x)**2*c**4 + 2*log(tan((a + b*x)/2) - 1)*c**4 - 2*log(tan((
a + b*x)/2) + 1)*sin(a + b*x)**2*c**4 + 2*log(tan((a + b*x)/2) + 1)*c**4 +
 2*log(tan((a + b*x)/2))*sin(a + b*x)**2*c**4 - 2*log(tan((a + b*x)/2))*c*
*4 - sin(a + b*x)**2*c**4)/(2*b*(sin(a + b*x)**2 - 1))
```

3.311 $\int (c+dx)^3 \csc(a+bx) \sec^3(a+bx)\, dx$

Optimal result . . . 2665
Mathematica [B] (warning: unable to verify) . . . 2666
Rubi [A] (verified) . . . 2667
Maple [B] (verified) . . . 2669
Fricas [B] (verification not implemented) . . . 2670
Sympy [F(-1)] . . . 2671
Maxima [B] (verification not implemented) . . . 2671
Giac [F] . . . 2672
Mupad [F(-1)] . . . 2673
Reduce [F] . . . 2673

Optimal result

Integrand size = 22, antiderivative size = 325

$$
\begin{aligned}
&\int (c+dx)^3 \csc(a+bx) \sec^3(a+bx)\, dx \\
&= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3 \operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b} \\
&\quad - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{3id^3 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^4} \\
&\quad + \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} - \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2} \\
&\quad - \frac{3d^2(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} + \frac{3d^2(c+dx)\operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3} \\
&\quad - \frac{3id^3 \operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4} + \frac{3id^3 \operatorname{PolyLog}\left(4,e^{2i(a+bx)}\right)}{4b^4} \\
&\quad - \frac{3d(c+dx)^2 \tan(a+bx)}{2b^2} + \frac{(c+dx)^3 \tan^2(a+bx)}{2b}
\end{aligned}
$$

output

```
3/2*I*d*(d*x+c)^2/b^2+1/2*(d*x+c)^3/b-2*(d*x+c)^3*arctanh(exp(2*I*(b*x+a))
)/b-3*d^2*(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b^3+3/2*I*d^3*polylog(2,-exp(2*I*
(b*x+a)))/b^4+3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a)))/b^2-3/2*I*d*(
d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+c)*polylog(3,-exp(2*
I*(b*x+a)))/b^3+3/2*d^2*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^3-3/4*I*d^3*
polylog(4,-exp(2*I*(b*x+a)))/b^4+3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4
-3/2*d*(d*x+c)^2*tan(b*x+a)/b^2+1/2*(d*x+c)^3*tan(b*x+a)^2/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1535 vs. $2(325) = 650$.

Time = 6.59 (sec) , antiderivative size = 1535, normalized size of antiderivative = 4.72

$$\int (c+dx)^3 \csc(a+bx) \sec^3(a+bx)\, dx = \text{Too large to display}$$

input

```
Integrate[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

output

```
-1/2*(c*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-
2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^
2*Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, -E^((-
I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, E^((-I)*(a + b*x))] +
 (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^((-I)*(a + b*x))] + (6*I)*(1 - E^(
(-2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]))/b^3 - (d^3*E^(I*a)*Csc[a]*((b^
4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a +
 b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - 6
*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - 6*b^2*(1 - E
^((-2*I)*a))*x^2*PolyLog[2, E^((-I)*(a + b*x))] + (12*I)*b*(1 - E^((-2*I)*
a))*x*PolyLog[3, -E^((-I)*(a + b*x))] + (12*I)*b*(1 - E^((-2*I)*a))*x*Poly
Log[3, E^((-I)*(a + b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, -E^((-I)*(a
+ b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, E^((-I)*(a + b*x))]))/(4*b^4)
+ (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Csc[a]*Sec[a])/4 - ((I/4)
*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a +
b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I
)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(I*a
)) - ((I/8)*d^3*E^(I*a)*((2*b^4*x^4)/E^((2*I)*a) - (4*I)*b^3*(1 + E^((-2*I
)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x))] + 6*b^2*(1 + E^((-2*I)*a))*x^2*Pol
yLog[2, -E^((-2*I)*(a + b*x))] - (6*I)*b*(1 + E^((-2*I)*a))*x*PolyLog[3...
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4920, 27, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \csc(a+bx) \sec^3(a+bx)\, dx$$

$$\downarrow 4920$$

$$-3d \int \frac{1}{2}(c+dx)^2 \left(\frac{\tan^2(a+bx)}{b} + \frac{2\log(\tan(a+bx))}{b} \right) dx + \frac{(c+dx)^3 \tan^2(a+bx)}{2b} + \frac{(c+dx)^3 \log(\tan(a+bx))}{b}$$

$$\downarrow 27$$

$$-\frac{3}{2}d \int (c+dx)^2 \left(\frac{\tan^2(a+bx)}{b} + \frac{2\log(\tan(a+bx))}{b} \right) dx + \frac{(c+dx)^3 \tan^2(a+bx)}{2b} + \frac{(c+dx)^3 \log(\tan(a+bx))}{b}$$

$$\downarrow 7292$$

$$-\frac{3}{2}d \int \frac{(c+dx)^2 \left(\tan^2(a+bx) + 2\log(\tan(a+bx))\right)}{b} dx + \frac{(c+dx)^3 \tan^2(a+bx)}{2b} + \frac{(c+dx)^3 \log(\tan(a+bx))}{b}$$

$$\downarrow 27$$

$$-\frac{3d \int (c+dx)^2 \left(\tan^2(a+bx) + 2\log(\tan(a+bx))\right) dx}{2b} + \frac{(c+dx)^3 \tan^2(a+bx)}{2b} + \frac{(c+dx)^3 \log(\tan(a+bx))}{b}$$

$$\downarrow 7293$$

$$-\frac{3d \int \left(\tan^2(a+bx)(c+dx)^2 + 2\log(\tan(a+bx))(c+dx)^2\right) dx}{2b} + \frac{(c+dx)^3 \tan^2(a+bx)}{2b} + \frac{(c+dx)^3 \log(\tan(a+bx))}{b}$$

$$\downarrow 2009$$

$$-\frac{3d\left(\frac{4(c+dx)^3\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{3d}-\frac{id^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3}+\frac{id^2\operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{2b^3}-\frac{id^2\operatorname{PolyLog}\left(4,e^{2i(a+bx)}\right)}{2b^3}+\frac{d(c+dx)}{}\right.}{\frac{(c+dx)^3\tan^2(a+bx)}{2b}+\frac{(c+dx)^3\log(\tan(a+bx))}{b}}$$

input
```
Int[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

output
```
((c + d*x)^3*Log[Tan[a + b*x]])/b + ((c + d*x)^3*Tan[a + b*x]^2)/(2*b) - (
3*d*(((-I)*(c + d*x)^2)/b - (c + d*x)^3/(3*d) + (4*(c + d*x)^3*ArcTanh[E^(
(2*I)*(a + b*x))])/(3*d) + (2*d*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b^
2 + (2*(c + d*x)^3*Log[Tan[a + b*x]])/(3*d) - (I*d^2*PolyLog[2, -E^((2*I)*
(a + b*x))])/b^3 - (I*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b + (I
*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b + (d*(c + d*x)*PolyLog[3,
-E^((2*I)*(a + b*x))])/b^2 - (d*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])
/b^2 + ((I/2)*d^2*PolyLog[4, -E^((2*I)*(a + b*x))])/b^3 - ((I/2)*d^2*PolyL
og[4, E^((2*I)*(a + b*x))])/b^3 + ((c + d*x)^2*Tan[a + b*x])/b))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1114 vs. 2(279) = 558.

Time = 0.56 (sec) , antiderivative size = 1115, normalized size of antiderivative = 3.43

method	result	size
risch	Expression too large to display	1115

input
```
int((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2-6*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+
a)))*x-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x+(2*b*d^3*x^3*exp(2*I*(b*x
+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*exp(2*I*(b*x+a))-6*I*c*d^2
*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*I*c^2*d*exp(2*I*(b*x+a)
)-3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))-6*I*c*d^2*x-3*I*c^2*d)/b^2/(exp(2*I
*(b*x+a))+1)^2-3/b^3*d^2*c*ln(exp(2*I*(b*x+a))+1)-3/b^3*d^3*ln(exp(2*I*(b*
x+a))+1)*x+3*I/b^2*d^3*x^2+3*I/b^4*d^3*a^2+3/2*I/b^2*d^3*polylog(2,-exp(2*
I*(b*x+a)))*x^2+3/2*I/b^2*c^2*d*polylog(2,-exp(2*I*(b*x+a)))-1/b*d^3*ln(ex
p(2*I*(b*x+a))+1)*x^3-3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))-3/2/b^3*d
^3*polylog(3,-exp(2*I*(b*x+a)))*x-3/b*c^2*d*ln(exp(2*I*(b*x+a))+1)*x-3/b*c
*d^2*ln(exp(2*I*(b*x+a))+1)*x^2+6*I/b^3*d^3*a*x+6*d^2/b^3*c*ln(exp(I*(b*x+
a)))-6*d^3/b^4*a*ln(exp(I*(b*x+a)))+6*I/b^4*d^3*polylog(4,-exp(I*(b*x+a)))
+3*I/b^2*c*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-1/b^4*d^3*a^3*ln(exp(I*(b*x+
a))-1)+6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,exp(I*
(b*x+a)))+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b*d^3*ln(exp(I*(b*x+a))+1)*x^
3+6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+6/b^3*d^3*polylog(3,exp(I*(b*x+a)
))*x+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-
1)-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)+3/b^2*d*c^2*ln(1-exp(I*(b*x+a)))*a+3
/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2+3/b*d
*c^2*ln(exp(I*(b*x+a))+1)*x+3/b*d*c^2*ln(1-exp(I*(b*x+a)))*x-3*I/b^2*d^...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2268 vs. $2(270) = 540$.

Time = 0.24 (sec) , antiderivative size = 2268, normalized size of antiderivative = 6.98

$$\int (c+dx)^3 \csc(a+bx) \sec^3(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 + 6*I*d^3*cos
(b*x + a)^2*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*cos(b*x +
a)^2*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*cos(b*x + a)^2*po
lylog(4, I*cos(b*x + a) + sin(b*x + a)) - 6*I*d^3*cos(b*x + a)^2*polylog(4
, I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*cos(b*x + a)^2*polylog(4, -I*co
s(b*x + a) + sin(b*x + a)) + 6*I*d^3*cos(b*x + a)^2*polylog(4, -I*cos(b*x
+ a) - sin(b*x + a)) - 6*I*d^3*cos(b*x + a)^2*polylog(4, -cos(b*x + a) + I
*sin(b*x + a)) + 6*I*d^3*cos(b*x + a)^2*polylog(4, -cos(b*x + a) - I*sin(b
*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*cos(b*x + a)^
2*dilog(cos(b*x + a) + I*sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2
*x - I*b^2*c^2*d)*cos(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*
(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d + I*d^3)*cos(b*x + a)^2*dil
og(I*cos(b*x + a) + sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x -
I*b^2*c^2*d - I*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) -
 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d - I*d^3)*cos(b*x + a)^2
*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*
x + I*b^2*c^2*d + I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x +
a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*cos(b*x + a)^2*di
log(-cos(b*x + a) + I*sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x +
 I*b^2*c^2*d)*cos(b*x + a)^2*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b...
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc(a+bx) \sec^3(a+bx)\, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3*csc(b*x+a)*sec(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4954 vs. $2(270) = 540$.

Time = 1.24 (sec) , antiderivative size = 4954, normalized size of antiderivative = 15.24

$$\int (c+dx)^3 \csc(a+bx) \sec^3(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

output

```
-1/2*(c^3*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x
+ a)^2)) - 3*a*c^2*d*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - l
og(sin(b*x + a)^2))/b + 3*a^2*c*d^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x
+ a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - a^3*d^3*(1/(sin(b*x + a)^2 - 1) +
 log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^3 + 2*(18*b^2*c^2*d - 36
*a*b*c*d^2 + 18*a^2*d^3 + 2*(4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 9*(
b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)
*(b*x + a) + (4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 9*(b*c*d^2 - a*d^3
)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos
(4*b*x + 4*a) + 2*(4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 9*(b*c*d^2 -
a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a)
)*cos(2*b*x + 2*a) - (-4*I*(b*x + a)^3*d^3 - 9*I*b*c*d^2 + 9*I*a*d^3 + 9*(
-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 9*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*
a^2 - I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(-4*I*(b*x + a)^3*d^3 - 9*I*
b*c*d^2 + 9*I*a*d^3 + 9*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 9*(-I*b^2*c^2
*d + 2*I*a*b*c*d^2 + (-I*a^2 - I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan
2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*((b*x + a)^3*d^3 + 3*(b*c*d^
2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a) +
 ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b
*c*d^2 + a^2*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 2*((b*x + a)^3*d^3 + 3*...
```

Giac [F]

$$\int (c+dx)^3 \csc(a+bx) \sec^3(a+bx)\, dx = \int (dx+c)^3 \csc(bx+a) \sec(bx+a)^3\, dx$$

input

```
integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

output

```
integrate((d*x + c)^3*csc(b*x + a)*sec(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc(a+bx) \sec^3(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)^3/(cos(a + b*x)^3*sin(a + b*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^3 \csc(a+bx) \sec^3(a+bx)\,dx$$
$$= \frac{2\left(\int \csc(bx+a)\sec(bx+a)^3 x^3 dx\right)\sin(bx+a)^2 b d^3 - 2\left(\int \csc(bx+a)\sec(bx+a)^3 x^3 dx\right) b d^3 + 6\left(\int \csc}{}$$

input
```
int((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x)
```

output
```
(2*int(csc(a + b*x)*sec(a + b*x)**3*x**3,x)*sin(a + b*x)**2*b*d**3 - 2*int
(csc(a + b*x)*sec(a + b*x)**3*x**3,x)*b*d**3 + 6*int(csc(a + b*x)*sec(a +
b*x)**3*x**2,x)*sin(a + b*x)**2*b*c*d**2 - 6*int(csc(a + b*x)*sec(a + b*x)
**3*x**2,x)*b*c*d**2 + 6*int(csc(a + b*x)*sec(a + b*x)**3*x,x)*sin(a + b*x
)**2*b*c**2*d - 6*int(csc(a + b*x)*sec(a + b*x)**3*x,x)*b*c**2*d - 2*log(t
an((a + b*x)/2) - 1)*sin(a + b*x)**2*c**3 + 2*log(tan((a + b*x)/2) - 1)*c*
*3 - 2*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**2*c**3 + 2*log(tan((a + b*x
)/2) + 1)*c**3 + 2*log(tan((a + b*x)/2))*sin(a + b*x)**2*c**3 - 2*log(tan(
(a + b*x)/2))*c**3 - sin(a + b*x)**2*c**3)/(2*b*(sin(a + b*x)**2 - 1))
```

3.312 $\int (c+dx)^2 \csc(a+bx)\sec^3(a+bx)\,dx$

Optimal result . . . 2674
Mathematica [B] (warning: unable to verify) . . . 2675
Rubi [A] (verified) . . . 2676
Maple [B] (verified) . . . 2678
Fricas [B] (verification not implemented) . . . 2678
Sympy [F] . . . 2679
Maxima [B] (verification not implemented) . . . 2680
Giac [F] . . . 2681
Mupad [F(-1)] . . . 2681
Reduce [F] . . . 2681

Optimal result

Integrand size = 22, antiderivative size = 195

$$\begin{aligned}
&\int (c+dx)^2 \csc(a+bx)\sec^3(a+bx)\,dx \\
&= \frac{(c+dx)^2}{2b} - \frac{2(c+dx)^2 \operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2 \log(\cos(a+bx))}{b^3} \\
&\quad + \frac{id(c+dx)\,\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} - \frac{id(c+dx)\,\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2} \\
&\quad - \frac{d^2\,\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} + \frac{d^2\,\operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3} \\
&\quad - \frac{d(c+dx)\tan(a+bx)}{b^2} + \frac{(c+dx)^2\tan^2(a+bx)}{2b}
\end{aligned}$$

output

```
1/2*(d*x+c)^2/b-2*(d*x+c)^2*arctanh(exp(2*I*(b*x+a)))/b-d^2*ln(cos(b*x+a))
/b^3+I*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2-I*d*(d*x+c)*polylog(2,ex
p(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+1/2*d^2*polyl
og(3,exp(2*I*(b*x+a)))/b^3-d*(d*x+c)*tan(b*x+a)/b^2+1/2*(d*x+c)^2*tan(b*x+
a)^2/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 883 vs. $2(195) = 390$.

Time = 6.54 (sec) , antiderivative size = 883, normalized size of antiderivative = 4.53

$$\int (c+dx)^2 \csc(a+bx) \sec^3(a+bx)\, dx = \text{Too large to display}$$

input

```
Integrate[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

output

```
-1/6*(d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*
I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*
Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, -E^((-I)
*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, E^((-I)*(a + b*x))] + (
6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^((-I)*(a + b*x))] + (6*I)*(1 - E^((-
2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]))/b^3 + (x*(3*c^2 + 3*c*d*x + d^2*
x^2)*Csc[a]*Sec[a])/3 - ((I/12)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I
)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2,
-E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a
+ b*x))])*Sec[a])/(b^3*E^(I*a)) + ((c + d*x)^2*Sec[a + b*x]^2)/(2*b) - (c^
2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*
(Cos[a]^2 + Sin[a]^2)) - (d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*
Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (c^2*Csc[a]*(-(b*x*
Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + S
in[a]^2)) - (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(
-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot
[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*Arc
Tan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x -
ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*Sqrt[Csc[a]^2*(Cos[a]
^2 + Sin[a]^2)]) + (Sec[a]*Sec[a + b*x]*(-(c*d*Sin[b*x]) - d^2*x*Sin[b*...
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4920, 27, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \csc(a+bx) \sec^3(a+bx)\, dx$$

$$\downarrow 4920$$

$$-2d \int \frac{1}{2}(c+dx)\left(\frac{\tan^2(a+bx)}{b} + \frac{2\log(\tan(a+bx))}{b}\right) dx + \frac{(c+dx)^2 \tan^2(a+bx)}{2b} + \frac{(c+dx)^2 \log(\tan(a+bx))}{b}$$

$$\downarrow 27$$

$$-d \int (c+dx)\left(\frac{\tan^2(a+bx)}{b} + \frac{2\log(\tan(a+bx))}{b}\right) dx + \frac{(c+dx)^2 \tan^2(a+bx)}{2b} + \frac{(c+dx)^2 \log(\tan(a+bx))}{b}$$

$$\downarrow 7292$$

$$-d \int \frac{(c+dx)\left(\tan^2(a+bx) + 2\log(\tan(a+bx))\right)}{b} dx + \frac{(c+dx)^2 \tan^2(a+bx)}{2b} + \frac{(c+dx)^2 \log(\tan(a+bx))}{b}$$

$$\downarrow 27$$

$$-\frac{d\int (c+dx)\left(\tan^2(a+bx) + 2\log(\tan(a+bx))\right) dx}{b} + \frac{(c+dx)^2 \tan^2(a+bx)}{2b} + \frac{(c+dx)^2 \log(\tan(a+bx))}{b}$$

$$\downarrow 7293$$

$$-\frac{d\int \left((c+dx)\tan^2(a+bx) + 2(c+dx)\log(\tan(a+bx))\right) dx}{b} + \frac{(c+dx)^2 \tan^2(a+bx)}{2b} + \frac{(c+dx)^2 \log(\tan(a+bx))}{b}$$

$$\downarrow 2009$$

$$-\frac{d\left(\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{d}+\frac{d\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^2}-\frac{d\operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^2}+\frac{d\log(\cos(a+bx))}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{2}\right.}{b}\right.}{b}$$

$$\frac{(c+dx)^2\tan^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}$$

input
```
Int[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

output
```
((c + d*x)^2*Log[Tan[a + b*x]])/b + ((c + d*x)^2*Tan[a + b*x]^2)/(2*b) - (
d*(-1/2*(c + d*x)^2/d + (2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/d + (
d*Log[Cos[a + b*x]])/b^2 + ((c + d*x)^2*Log[Tan[a + b*x]])/d - (I*(c + d*x
)*PolyLog[2, -E^((2*I)*(a + b*x))])/b + (I*(c + d*x)*PolyLog[2, E^((2*I)*(
a + b*x))])/b + (d*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^2) - (d*PolyLog[
3, E^((2*I)*(a + b*x))])/(2*b^2) + ((c + d*x)*Tan[a + b*x])/b))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(175) = 350$.

Time = 0.47 (sec) , antiderivative size = 614, normalized size of antiderivative = 3.15

method	result
risch	$-\frac{2icd\,\mathrm{polylog}\left(2,-e^{i(bx+a)}\right)}{b^2}+\frac{id^2\,\mathrm{polylog}\left(2,-e^{2i(bx+a)}\right)x}{b^2}+\frac{2cd\ln\left(1-e^{i(bx+a)}\right)a}{b^2}-\frac{2cda\ln\left(e^{i(bx+a)}-1\right)}{b^2}+\frac{2cd\ln\left(1-e^{i(bx}\right.}{b}$

input
```
int((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b*c^2*ln(exp(2*I*(b*x+a))+1)-2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-2
*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a
)))-2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))+2/b^2*c*d*ln(1-exp(I*(b*x+a)))*a
+2*(b*d^2*x^2*exp(2*I*(b*x+a))+2*b*c*d*x*exp(2*I*(b*x+a))+b*c^2*exp(2*I*(b
*x+a))-I*d^2*x*exp(2*I*(b*x+a))-I*c*d*exp(2*I*(b*x+a))-I*d^2*x-I*d*c)/b^2/
(exp(2*I*(b*x+a))+1)^2-2/b*d*c*ln(exp(2*I*(b*x+a))+1)*x-1/b*d^2*ln(exp(2*I
*(b*x+a))+1)*x^2+I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x+I/b^2*d*c*polylo
g(2,-exp(2*I*(b*x+a)))+1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+1/b*d^2*ln(exp(I
*(b*x+a))+1)*x^2+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d^2*ln(1-exp(I*(b*
x+a)))*a^2+2/b*c*d*ln(exp(I*(b*x+a))+1)*x-2/b^2*c*d*a*ln(exp(I*(b*x+a))-1)
+2/b*c*d*ln(1-exp(I*(b*x+a)))*x-d^2/b^3*ln(exp(2*I*(b*x+a))+1)+1/b*c^2*ln(
exp(I*(b*x+a))-1)+1/b*c^2*ln(exp(I*(b*x+a))+1)+2/b^3*d^2*ln(exp(I*(b*x+a))
)+2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I*(b*x+a)))/b^3
-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1404 vs. $2(171) = 342$.

Time = 0.18 (sec) , antiderivative size = 1404, normalized size of antiderivative = 7.20

$$\int (c+dx)^2 \csc(a+bx)\sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

output

```
1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*d^2*cos(b*x + a)^2*polylog(3, cos(b*x +
 a) + I*sin(b*x + a)) + 2*d^2*cos(b*x + a)^2*polylog(3, cos(b*x + a) - I*s
in(b*x + a)) - 2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x +
a)) - 2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 2*d
^2*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*cos(b
*x + a)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*cos(b*x + a)^
2*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*cos(b*x + a)^2*polylo
g(3, -cos(b*x + a) - I*sin(b*x + a)) + b^2*c^2 - 2*(I*b*d^2*x + I*b*c*d)*c
os(b*x + a)^2*dilog(cos(b*x + a) + I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c
*d)*cos(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) - 2*(I*b*d^2*x + I
*b*c*d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) - 2*(-I*b*d^2*
x - I*b*c*d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - 2*(-I*b
*d^2*x - I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 2
*(I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)
) - 2*(-I*b*d^2*x - I*b*c*d)*cos(b*x + a)^2*dilog(-cos(b*x + a) + I*sin(b*
x + a)) - 2*(I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2*dilog(-cos(b*x + a) - I*s
in(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a)^2*log(co
s(b*x + a) + I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*c
os(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^
2*c*d*x + b^2*c^2)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + 1...
```

Sympy [F]

$$\int (c+dx)^2 \csc(a+bx) \sec^3(a+bx)\, dx = \int (c+dx)^2 \csc(a+bx) \sec^3(a+bx)\, dx$$

input

```
integrate((d*x+c)**2*csc(b*x+a)*sec(b*x+a)**3,x)
```

output

```
Integral((c + d*x)**2*csc(a + b*x)*sec(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2447 vs. $2(171) = 342$.

Time = 0.39 (sec) , antiderivative size = 2447, normalized size of antiderivative = 12.55

$$\int (c+dx)^2 \csc(a+bx) \sec^3(a+bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

output

```
-1/2*(c^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x
+ a)^2)) - 2*a*c*d*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log
(sin(b*x + a)^2))/b + a^2*d^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2
 - 1) - log(sin(b*x + a)^2))/b^2 - 2*(4*(b*x + a)*d^2*cos(4*b*x + 4*a) + 4
*I*(b*x + a)*d^2*sin(4*b*x + 4*a) - 4*b*c*d + 4*a*d^2 - 2*((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2 + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2
)*(b*x + a) + d^2)*cos(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^
2)*(b*x + a) + d^2)*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d +
 I*a*d^2)*(b*x + a) - I*d^2)*sin(4*b*x + 4*a) - 2*(-I*(b*x + a)^2*d^2 + 2*
(-I*b*c*d + I*a*d^2)*(b*x + a) - I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(2*b*
x + 2*a), cos(2*b*x + 2*a) + 1) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(
b*x + a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a
) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (
I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a) + 2*
(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*ar
ctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*
d^2)*(b*x + a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x
 + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*
a) - (-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*sin(4*b*x + 4
*a) - 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*sin(2*b...
```

Giac [F]

$$\int (c+dx)^2 \csc(a+bx) \sec^3(a+bx)\, dx = \int (dx+c)^2 \csc(bx+a) \sec(bx+a)^3\, dx$$

input
```
integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csc(b*x + a)*sec(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc(a+bx) \sec^3(a+bx)\, dx = \text{Hanged}$$

input
```
int((c + d*x)^2/(cos(a + b*x)^3*sin(a + b*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^2 \csc(a+bx) \sec^3(a+bx)\, dx$$
$$= \frac{2\left(\int \csc(bx+a) \sec(bx+a)^3 x^2 dx\right) \sin(bx+a)^2 b\, d^2 - 2\left(\int \csc(bx+a) \sec(bx+a)^3 x^2 dx\right) b\, d^2 + 4\left(\int \mathrm{cs}\right.}{}$$

input
```
int((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x)
```

output

```
(2*int(csc(a + b*x)*sec(a + b*x)**3*x**2,x)*sin(a + b*x)**2*b*d**2 - 2*int
(csc(a + b*x)*sec(a + b*x)**3*x**2,x)*b*d**2 + 4*int(csc(a + b*x)*sec(a +
b*x)**3*x,x)*sin(a + b*x)**2*b*c*d - 4*int(csc(a + b*x)*sec(a + b*x)**3*x,
x)*b*c*d - 2*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**2*c**2 + 2*log(tan((a
 + b*x)/2) - 1)*c**2 - 2*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**2*c**2 +
2*log(tan((a + b*x)/2) + 1)*c**2 + 2*log(tan((a + b*x)/2))*sin(a + b*x)**2
*c**2 - 2*log(tan((a + b*x)/2))*c**2 - sin(a + b*x)**2*c**2)/(2*b*(sin(a +
 b*x)**2 - 1))
```

3.313 $\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx$

Optimal result . . . 2683
Mathematica [A] (verified) . . . 2684
Rubi [A] (verified) . . . 2684
Maple [B] (verified) . . . 2686
Fricas [B] (verification not implemented) . . . 2686
Sympy [F] . . . 2687
Maxima [B] (verification not implemented) . . . 2688
Giac [F] . . . 2689
Mupad [F(-1)] . . . 2689
Reduce [F] . . . 2689

Optimal result

Integrand size = 20, antiderivative size = 139

$$\begin{aligned}\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx = {} & \frac{dx}{2b} - \frac{2dx\operatorname{arctanh}\left(e^{2ia+2ibx}\right)}{b} \\ & + \frac{c\log(\tan(a+bx))}{b} \\ & + \frac{id\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} \\ & - \frac{id\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2} - \frac{d\tan(a+bx)}{2b^2} \\ & + \frac{c\tan^2(a+bx)}{2b} + \frac{dx\tan^2(a+bx)}{2b}\end{aligned}$$

output

```
1/2*d*x/b-2*d*x*arctanh(exp(2*I*a+2*I*b*x))/b+c*ln(tan(b*x+a))/b+1/2*I*d*p
olylog(2,-exp(2*I*(b*x+a)))/b^2-1/2*I*d*polylog(2,exp(2*I*(b*x+a)))/b^2-1/
2*d*tan(b*x+a)/b^2+1/2*c*tan(b*x+a)^2/b+1/2*d*x*tan(b*x+a)^2/b
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.55

$$\begin{aligned}
&\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx \\
&= -\frac{c\log(\cos(a+bx))}{b} + \frac{ad\log(\cos(a+bx))}{b^2} + \frac{c\log(\sin(a+bx))}{b} - \frac{ad\log(\sin(a+bx))}{b^2} \\
&\quad + \frac{d\left(\frac{1}{2}i(a+bx)^2 - (a+bx)\log\left(1+e^{2i(a+bx)}\right) + \frac{1}{2}i\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)\right)}{b^2} \\
&\quad + \frac{d\left((a+bx)\log\left(1-e^{2i(a+bx)}\right) - \frac{1}{2}i\left((a+bx)^2 + \mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)\right)\right)}{b^2} \\
&\quad + \frac{c\sec^2(a+bx)}{2b} + \frac{dx\sec^2(a+bx)}{2b} - \frac{d\tan(a+bx)}{2b^2}
\end{aligned}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

output
```
-((c*Log[Cos[a + b*x]])/b) + (a*d*Log[Cos[a + b*x]])/b^2 + (c*Log[Sin[a +
b*x]])/b - (a*d*Log[Sin[a + b*x]])/b^2 + (d*((I/2)*(a + b*x)^2 - (a + b*x)
*Log[1 + E^((2*I)*(a + b*x))] + (I/2)*PolyLog[2, -E^((2*I)*(a + b*x))]))/b
^2 + (d*((a + b*x)*Log[1 - E^((2*I)*(a + b*x))] - (I/2)*((a + b*x)^2 + Pol
yLog[2, E^((2*I)*(a + b*x))])))/b^2 + (c*Sec[a + b*x]^2)/(2*b) + (d*x*Sec[
a + b*x]^2)/(2*b) - (d*Tan[a + b*x])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4920, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx$$

$$\downarrow \text{4920}$$

$$-d\int\left(\frac{\tan^2(a+bx)}{2b}+\frac{\log(\tan(a+bx))}{b}\right)dx+\frac{(c+dx)\tan^2(a+bx)}{2b}+\frac{(c+dx)\log(\tan(a+bx))}{b}$$

$$\downarrow 2009$$

$$-d\left(\frac{2x\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b}-\frac{i\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}+\frac{i\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}+\frac{\tan(a+bx)}{2b^2}+\frac{x\log(\tan(a+\right.$$
$$\left.\frac{(c+dx)\tan^2(a+bx)}{2b}+\frac{(c+dx)\log(\tan(a+bx))}{b}\right)$$

input
```
Int[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

output
```
((c + d*x)*Log[Tan[a + b*x]])/b + ((c + d*x)*Tan[a + b*x]^2)/(2*b) - d*(-1
/2*x/b + (2*x*ArcTanh[E^((2*I)*(a + b*x))])/b + (x*Log[Tan[a + b*x]])/b -
((I/2)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + ((I/2)*PolyLog[2, E^((2*I)*
(a + b*x))])/b^2 + Tan[a + b*x]/(2*b^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(118) = 236$.

Time = 0.36 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.94

method	result
risch	$\frac{2bdx\,e^{2i(bx+a)}-id\,e^{2i(bx+a)}+2bc\,e^{2i(bx+a)}-id}{b^2\left(e^{2i(bx+a)}+1\right)^2}+\frac{c\ln\left(e^{i(bx+a)}+1\right)}{b}-\frac{c\ln\left(e^{2i(bx+a)}+1\right)}{b}+\frac{c\ln\left(e^{i(bx+a)}-1\right)}{b}-\frac{d\ln\left(e^{2i(bx+a}\right.}{b}$

input
```
int((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(2*b*d*x*exp(2*I*(b*x+a))-I*d*exp(2*I*(b*x+a))+2*b*c*exp(2*I*(b*x+a))-I*d)
/b^2/(exp(2*I*(b*x+a))+1)^2+1/b*c*ln(exp(I*(b*x+a))+1)-1/b*c*ln(exp(2*I*(b
*x+a))+1)+1/b*c*ln(exp(I*(b*x+a))-1)-1/b*d*ln(exp(2*I*(b*x+a))+1)*x+1/2*I*
d*polylog(2,-exp(2*I*(b*x+a)))/b^2+1/b*d*ln(exp(I*(b*x+a))+1)*x-I*d*polylo
g(2,-exp(I*(b*x+a)))/b^2+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(
b*x+a)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1
)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(112) = 224$.

Time = 0.14 (sec) , antiderivative size = 760, normalized size of antiderivative = 5.47

$$\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

output

```
1/2*(-I*d*cos(b*x + a)^2*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*cos(b*
x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(I
*cos(b*x + a) + sin(b*x + a)) + I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) -
sin(b*x + a)) + I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) -
 I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) + I*d*cos(b*x +
a)^2*dilog(-cos(b*x + a) + I*sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(-cos
(b*x + a) - I*sin(b*x + a)) + (b*d*x + b*c)*cos(b*x + a)^2*log(cos(b*x + a
) + I*sin(b*x + a) + 1) - (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*
sin(b*x + a) + I) + (b*d*x + b*c)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(
b*x + a) + 1) - (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x +
a) + I) - (b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) +
 1) - (b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1)
- (b*d*x + a*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (
b*d*x + a*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c
 - a*d)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) +
 (b*c - a*d)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1
/2) + (b*d*x + a*d)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + 1)
 - (b*c - a*d)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b
*d*x + a*d)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - (b*c
- a*d)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) + b*d*x -...
```

Sympy [F]

$$\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx = \int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx$$

input

```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)**3,x)
```

output

```
Integral((c + d*x)*csc(a + b*x)*sec(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(112) = 224$.

Time = 0.26 (sec) , antiderivative size = 1028, normalized size of antiderivative = 7.40

$$\int (c + dx) \csc(a + bx) \sec^3(a + bx)\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

output

```
-(2*(b*d*x + b*c + (b*d*x + b*c)*cos(4*b*x + 4*a) + 2*(b*d*x + b*c)*cos(2*
b*x + 2*a) - (-I*b*d*x - I*b*c)*sin(4*b*x + 4*a) - 2*(-I*b*d*x - I*b*c)*si
n(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 2*(b*d*x
 + b*c + (b*d*x + b*c)*cos(4*b*x + 4*a) + 2*(b*d*x + b*c)*cos(2*b*x + 2*a)
 + (I*b*d*x + I*b*c)*sin(4*b*x + 4*a) + 2*(I*b*d*x + I*b*c)*sin(2*b*x + 2*
a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(b*c*cos(4*b*x + 4*a) + 2*
b*c*cos(2*b*x + 2*a) + I*b*c*sin(4*b*x + 4*a) + 2*I*b*c*sin(2*b*x + 2*a) +
 b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 2*(b*d*x*cos(4*b*x + 4*a)
+ 2*b*d*x*cos(2*b*x + 2*a) + I*b*d*x*sin(4*b*x + 4*a) + 2*I*b*d*x*sin(2*b*
x + 2*a) + b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*(-2*I*b*d*x
 - 2*I*b*c - d)*cos(2*b*x + 2*a) - (d*cos(4*b*x + 4*a) + 2*d*cos(2*b*x + 2
*a) + I*d*sin(4*b*x + 4*a) + 2*I*d*sin(2*b*x + 2*a) + d)*dilog(-e^(2*I*b*x
 + 2*I*a)) + 2*(d*cos(4*b*x + 4*a) + 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x
+ 4*a) + 2*I*d*sin(2*b*x + 2*a) + d)*dilog(-e^(I*b*x + I*a)) + 2*(d*cos(4*
b*x + 4*a) + 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) + 2*I*d*sin(2*b*x
 + 2*a) + d)*dilog(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*
c)*cos(4*b*x + 4*a) - 2*(I*b*d*x + I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)
*sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)
^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (I*b*d*x + I*b*c + (I*
b*d*x + I*b*c)*cos(4*b*x + 4*a) - 2*(-I*b*d*x - I*b*c)*cos(2*b*x + 2*a)...
```

Giac [F]

$$\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx = \int (dx+c)\csc(bx+a)\sec(bx+a)^3\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*csc(b*x + a)*sec(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)/(cos(a + b*x)^3*sin(a + b*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx$$
$$= \frac{2\left(\int \csc(bx+a)\sec(bx+a)^3\,xdx\right)\sin(bx+a)^2\,bd - 2\left(\int \csc(bx+a)\sec(bx+a)^3\,xdx\right)bd - 2\log(\tan($$

input
```
int((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x)
```

output

```
(2*int(csc(a + b*x)*sec(a + b*x)**3*x,x)*sin(a + b*x)**2*b*d - 2*int(csc(a
 + b*x)*sec(a + b*x)**3*x,x)*b*d - 2*log(tan((a + b*x)/2) - 1)*sin(a + b*x
)**2*c + 2*log(tan((a + b*x)/2) - 1)*c - 2*log(tan((a + b*x)/2) + 1)*sin(a
 + b*x)**2*c + 2*log(tan((a + b*x)/2) + 1)*c + 2*log(tan((a + b*x)/2))*sin
(a + b*x)**2*c - 2*log(tan((a + b*x)/2))*c - sin(a + b*x)**2*c)/(2*b*(sin(
a + b*x)**2 - 1))
```

3.314 $\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$

Optimal result 2691
Mathematica [N/A] 2691
Rubi [N/A] 2692
Maple [N/A] 2692
Fricas [N/A] 2693
Sympy [N/A] 2693
Maxima [N/A] 2694
Giac [N/A] 2695
Mupad [N/A] 2695
Reduce [N/A] 2695

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}, x\right)$$

output
```
Defer(Int)(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 6.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x),x]
```

output
```
Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$
$$\downarrow 7299$$
$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}dx$$

input
```
Int[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc(bx+a)\sec(bx+a)^3}{dx+c}dx$$

input
```
int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x)
```

output
```
int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc\left(bx+a\right)\sec\left(bx+a\right)^3}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)*sec(b*x + a)^3/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc\left(a+bx\right)\sec^3\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(csc(a + b*x)*sec(a + b*x)**3/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 1865, normalized size of antiderivative = 84.77

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc\left(bx+a\right)\sec\left(bx+a\right)^3}{dx+c}\,dx$$

input

```
integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output

```
(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
 (2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
+ 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
+ (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*s
in(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*
x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate(2*(b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2 + d^2)*sin(2*b*x + 2*a)/(b^2*d^3*x^3 + 3*b^2*c*d^
2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x
^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)), x) - (b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^
2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*
x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b...
```

Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)*sec(b*x + a)^3/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 20.84 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos(a+b\,x)^3\,\sin(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(1/(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)^3}{dx+c}\,dx$$

input
```
int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x)
```

output
```
int((csc(a + b*x)*sec(a + b*x)**3)/(c + d*x),x)
```

3.315 $\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 2697
Mathematica [N/A] . . . 2697
Rubi [N/A] . . . 2698
Maple [N/A] . . . 2698
Fricas [N/A] . . . 2699
Sympy [N/A] . . . 2699
Maxima [N/A] . . . 2700
Giac [F(-1)] . . . 2701
Mupad [N/A] . . . 2701
Reduce [N/A] . . . 2701

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 4.91 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$
$$\downarrow 7299$$
$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}dx$$

input
```
Int[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc\left(bx+a\right)\sec\left(bx+a\right)^3}{\left(dx+c\right)^2}dx$$

input
```
int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)\sec\left(bx+a\right)^3}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)*sec(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 5.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(a+bx\right)\sec^3\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)*sec(a + b*x)**3/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 5.33 (sec) , antiderivative size = 2516, normalized size of antiderivative = 114.36

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)\sec\left(bx+a\right)^3}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
 2*((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
+ 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2
*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*
x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 +
3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^
3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2
*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*
b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a
) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x
+ 2*a))*integrate(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 3*d^2)*sin(2*b*
x + 2*a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*
x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c
^3*d*x + b^2*c^4)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*
b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^4
*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(
2*b*x + 2*a)), x) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c
^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 21.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\cos(a+b\,x)^3\,\sin(a+b\,x)\,(c+d\,x)^2}\,dx$$

input
```
int(1/(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)^3}{d^2x^2+2cdx+c^2}dx$$

input
```
int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int((csc(a + b*x)*sec(a + b*x)**3)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.316 $\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx$

Optimal result . . . 2702
Mathematica [N/A] . . . 2702
Rubi [N/A] . . . 2703
Maple [N/A] . . . 2703
Fricas [N/A] . . . 2704
Sympy [F(-1)] . . . 2704
Maxima [N/A] . . . 2704
Giac [N/A] . . . 2705
Mupad [N/A] . . . 2705
Reduce [N/A] . . . 2706

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx = \text{Int}\left((c+dx)^m \csc^2(a+bx)\sec^3(a+bx), x\right)$$

output
```
Defer(Int)((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 21.63 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx = \int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

output
```
Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\sec^3(a+bx)(c+dx)^m\,dx$$

$$\downarrow 7299$$

$$\int \csc^2(a+bx)\sec^3(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^3\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

output
```
int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^2(a+bx) \sec^3(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m \csc^2(a+bx) \sec^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^2(a+bx) \sec^3(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^2(a+bx) \sec^3(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 22.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^2(a+bx) \sec^3(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\cos(a+b\,x)^3 \sin(a+b\,x)^2}\,dx$$

input
```
int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)^2),x)
```

output
```
int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^3\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

output
```
int((c + d*x)**m*csc(a + b*x)**2*sec(a + b*x)**3,x)
```

3.317 $\int (c+dx)^3 \csc^2(a+bx) \sec^3(a+bx)\, dx$

Optimal result . . . 2707
Mathematica [A] (verified) . . . 2708
Rubi [A] (verified) . . . 2709
Maple [B] (verified) . . . 2712
Fricas [B] (verification not implemented) . . . 2713
Sympy [F(-1)] . . . 2714
Maxima [B] (verification not implemented) . . . 2714
Giac [F] . . . 2715
Mupad [F(-1)] . . . 2716
Reduce [F] . . . 2716

Optimal result

Integrand size = 24, antiderivative size = 486

$$
\begin{aligned}
&\int (c+dx)^3 \csc^2(a+bx) \sec^3(a+bx)\, dx \\
&= -\frac{6id^2(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b} \\
&\quad - \frac{6d(c+dx)^2 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^3 \csc(a+bx)}{2b} \\
&\quad + \frac{6id^2(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3} + \frac{3id^3\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^4} \\
&\quad + \frac{9id(c+dx)^2\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2} - \frac{3id^3\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^4} \\
&\quad - \frac{9id(c+dx)^2\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2} - \frac{6id^2(c+dx)\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3} \\
&\quad - \frac{6d^3\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4} - \frac{9d^2(c+dx)\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{9d^2(c+dx)\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} + \frac{6d^3\,\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^4} \\
&\quad - \frac{9id^3\,\mathrm{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^4} + \frac{9id^3\,\mathrm{PolyLog}\left(4,ie^{i(a+bx)}\right)}{b^4} \\
&\quad - \frac{3d(c+dx)^2 \sec(a+bx)}{2b^2} + \frac{(c+dx)^3 \csc(a+bx)\sec^2(a+bx)}{2b}
\end{aligned}
$$

output

```
-6*I*d^2*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^3-9/2*I*d*(d*x+c)^2*polylog(2
,I*exp(I*(b*x+a)))/b^2-6*d*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b^2-3/2*(d*x+
c)^3*csc(b*x+a)/b+9/2*I*d*(d*x+c)^2*polylog(2,-I*exp(I*(b*x+a)))/b^2+3*I*d
^3*polylog(2,-I*exp(I*(b*x+a)))/b^4+9*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^
4-6*I*d^2*(d*x+c)*arctan(exp(I*(b*x+a)))/b^3+6*I*d^2*(d*x+c)*polylog(2,-ex
p(I*(b*x+a)))/b^3-9*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3
,-exp(I*(b*x+a)))/b^4-9*d^2*(d*x+c)*polylog(3,-I*exp(I*(b*x+a)))/b^3+9*d^2
*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^3+6*d^3*polylog(3,exp(I*(b*x+a)))/b
^4-3*I*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b-3*I*d^3*polylog(2,I*exp(I*(b*x+a
)))/b^4-3/2*d*(d*x+c)^2*sec(b*x+a)/b^2+1/2*(d*x+c)^3*csc(b*x+a)*sec(b*x+a)
^2/b
```

Mathematica [A] (verified)

Time = 7.20 (sec) , antiderivative size = 819, normalized size of antiderivative = 1.69

$$\int (c+dx)^3 \csc^2(a+bx)\sec^3(a+bx)\,dx$$

$$= \frac{3d\Big((c+dx)^2\log\left(1-e^{i(a+bx)}\right)-(c+dx)^2\log\left(1+e^{i(a+bx)}\right)+\frac{2id\left(b(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)+id\operatorname{PolyLog}\left(3,-e^{i(}\right.\right.}{b^2}$$

$$-\frac{3\left(2ib^3c^3\arctan\left(e^{i(a+bx)}\right)+4ibcd^2\arctan\left(e^{i(a+bx)}\right)-3b^3c^2dx\log\left(1-ie^{i(a+bx)}\right)-2bd^3x\log\left(1-ie^{i(a-}\right.\right.$$

$$-\frac{\csc(a+bx)\sec^2(a+bx)\left(bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3+3bc^3\cos(2a+2bx)+9bc^2dx\cos(2a+2bx\right.}{}$$

input

```
Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

output

```
(3*d*((c + d*x)^2*Log[1 - E^(I*(a + b*x))] - (c + d*x)^2*Log[1 + E^(I*(a +
 b*x))] + ((2*I)*d*(b*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] + I*d*PolyLog
[3, -E^(I*(a + b*x))]))/b^2 + (2*d*((-I)*b*(c + d*x)*PolyLog[2, E^(I*(a +
b*x))] + d*PolyLog[3, E^(I*(a + b*x))]))/b^2))/b^2 - (3*((2*I)*b^3*c^3*Arc
Tan[E^(I*(a + b*x))] + (4*I)*b*c*d^2*ArcTan[E^(I*(a + b*x))] - 3*b^3*c^2*d
*x*Log[1 - I*E^(I*(a + b*x))] - 2*b*d^3*x*Log[1 - I*E^(I*(a + b*x))] - 3*b
^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 - I*E^(I*(a +
b*x))] + 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] + 2*b*d^3*x*Log[1 + I*E^
(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + b^3*d^3*x^3*
Log[1 + I*E^(I*(a + b*x))] - I*d*(2*d^2 + 3*b^2*(c + d*x)^2)*PolyLog[2, (-
I)*E^(I*(a + b*x))] + I*d*(2*d^2 + 3*b^2*(c + d*x)^2)*PolyLog[2, I*E^(I*(a
 + b*x))] + 6*b*c*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog
[3, (-I)*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, I*E^(I*(a + b*x))] - 6*b*
d^3*x*PolyLog[3, I*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a +
b*x))] - (6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))]))/(2*b^4) - (Csc[a + b*x]
*Sec[a + b*x]^2*(b*c^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2 + b*d^3*x^3 + 3*b*c^3
*Cos[2*a + 2*b*x] + 9*b*c^2*d*x*Cos[2*a + 2*b*x] + 9*b*c*d^2*x^2*Cos[2*a +
 2*b*x] + 3*b*d^3*x^3*Cos[2*a + 2*b*x] + 3*c^2*d*Sin[2*a + 2*b*x] + 6*c*d^
2*x*Sin[2*a + 2*b*x] + 3*d^3*x^2*Sin[2*a + 2*b*x]))/(4*b^2)
```

Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4920, 27, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \csc^2(a+bx) \sec^3(a+bx)\, dx$$

$$\downarrow 4920$$

$$-3d \int \frac{1}{2}(c+dx)^2 \left(\frac{\csc(a+bx)\sec^2(a+bx)}{b} + \frac{3\operatorname{arctanh}(\sin(a+bx))}{b} - \frac{3\csc(a+bx)}{b} \right) dx + \frac{3(c+dx)^3 \operatorname{arctanh}(\sin(a+bx))}{2b} - \frac{3(c+dx)^3 \csc(a+bx)}{2b} + \frac{(c+dx)^3 \csc(a+bx)\sec^2(a+bx)}{2b}$$

$$\downarrow 27$$

$$-\frac{3}{2}d\int(c+dx)^2\left(\frac{\csc(a+bx)\sec^2(a+bx)}{b}+\frac{3\text{arctanh}(\sin(a+bx))}{b}-\frac{3\csc(a+bx)}{b}\right)dx+\frac{3(c+dx)^3\text{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)^3\csc(a+bx)}{2b}+\frac{(c+dx)^3\csc(a+bx)\sec^2(a+bx)}{2b}$$

$$\downarrow 7292$$

$$-\frac{3}{2}d\int\frac{(c+dx)^2\left(\csc(a+bx)\sec^2(a+bx)+3\text{arctanh}(\sin(a+bx))-3\csc(a+bx)\right)}{b}dx+\frac{3(c+dx)^3\text{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)^3\csc(a+bx)}{2b}+\frac{(c+dx)^3\csc(a+bx)\sec^2(a+bx)}{2b}$$

$$\downarrow 27$$

$$-\frac{3d\int(c+dx)^2\left(\csc(a+bx)\sec^2(a+bx)+3\text{arctanh}(\sin(a+bx))-3\csc(a+bx)\right)dx}{2b}+\frac{3(c+dx)^3\text{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)^3\csc(a+bx)}{2b}+\frac{(c+dx)^3\csc(a+bx)\sec^2(a+bx)}{2b}$$

$$\downarrow 7293$$

$$-\frac{3d\int\left(\csc(a+bx)\sec^2(a+bx)(c+dx)^2+3(\text{arctanh}(\sin(a+bx))-\csc(a+bx))(c+dx)^2\right)dx}{2b}+\frac{3(c+dx)^3\text{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)^3\csc(a+bx)}{2b}+\frac{(c+dx)^3\csc(a+bx)\sec^2(a+bx)}{2b}$$

$$\downarrow 2009$$

$$-\frac{3d\left(\frac{4id(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^2}+\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{d}+\frac{4(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{(c+dx)^3\text{arctanh}(\sin(a+bx))}{d}-\frac{2i}{}\right.}{}$$

$$\frac{3(c+dx)^3\text{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)^3\csc(a+bx)}{2b}+\frac{(c+dx)^3\csc(a+bx)\sec^2(a+bx)}{2b}$$

input
```
Int[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

output
```
(3*(c + d*x)^3*ArcTanh[Sin[a + b*x]])/(2*b) - (3*(c + d*x)^3*Csc[a + b*x])
/(2*b) + ((c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^2)/(2*b) - (3*d*(((4*I)*d*
(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 + ((2*I)*(c + d*x)^3*ArcTan[E^(I*(a
 + b*x))])/d + (4*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b + ((c + d*x)^3*A
rcTanh[Sin[a + b*x]])/d - ((4*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])
/b^2 - ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 - ((3*I)*(c + d*x)
^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b + ((2*I)*d^2*PolyLog[2, I*E^(I*(a +
 b*x))])/b^3 + ((3*I)*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b + ((4*I
)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 + (4*d^2*PolyLog[3, -E^(I*(
a + b*x))])/b^3 + (6*d*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^2 - (
6*d*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^2 - (4*d^2*PolyLog[3, E^(I*
(a + b*x))])/b^3 + ((6*I)*d^2*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^3 - ((6*
I)*d^2*PolyLog[4, I*E^(I*(a + b*x))])/b^3 + ((c + d*x)^2*Sec[a + b*x])/b))
/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1628 vs. $2(428) = 856$.

Time = 1.00 (sec) , antiderivative size = 1629, normalized size of antiderivative = 3.35

method	result	size
risch	Expression too large to display	1629

input
```
int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-6*I*d^3/b^3*polylog(2,exp(I*(b*x+a)))*x+6*I*d^3/b^3*polylog(2,-exp(I*(b*x
+a)))*x-3*I/b*c^3*arctan(exp(I*(b*x+a)))+3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a^
2+3*d^3/b^2*ln(1-exp(I*(b*x+a)))*x^2-3*d^3/b^2*ln(exp(I*(b*x+a))+1)*x^2-3/
b^3*d^3*ln(I*exp(I*(b*x+a))+1)*x-3/b^4*d^3*ln(I*exp(I*(b*x+a))+1)*a-9/b^3*
d^3*polylog(3,-I*exp(I*(b*x+a)))*x+9/b^3*c*d^2*polylog(3,I*exp(I*(b*x+a)))
-9/b^3*c*d^2*polylog(3,-I*exp(I*(b*x+a)))+3/b^4*d^3*a^2*ln(exp(I*(b*x+a))-
1)+3/b^3*d^3*ln(1-I*exp(I*(b*x+a)))*x+3/b^4*d^3*ln(1-I*exp(I*(b*x+a)))*a+9
/b^3*d^3*polylog(3,I*exp(I*(b*x+a)))*x-3/2/b*d^3*ln(I*exp(I*(b*x+a))+1)*x^
3+3/2/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3+3/b^2*c^2*d*ln(exp(I*(b*x+a))-1)-3/
b^2*c^2*d*ln(exp(I*(b*x+a))+1)+3/2/b^4*a^3*d^3*ln(1-I*exp(I*(b*x+a)))-3/2/
b^4*a^3*d^3*ln(I*exp(I*(b*x+a))+1)-9*I/b^3*a^2*c*d^2*arctan(exp(I*(b*x+a))
)-9*I/b^2*c*d^2*polylog(2,I*exp(I*(b*x+a)))*x+9*I/b^2*c*d^2*polylog(2,-I*e
xp(I*(b*x+a)))*x+9*I/b^2*a*c^2*d*arctan(exp(I*(b*x+a)))-6*d^2/b^2*c*ln(exp
(I*(b*x+a))+1)*x-I/b^2/(exp(2*I*(b*x+a))+1)^2/(exp(2*I*(b*x+a))-1)*(3*d^3*
x^3*b*exp(5*I*(b*x+a))+9*c*d^2*x^2*b*exp(5*I*(b*x+a))+9*c^2*d*x*b*exp(5*I*
(b*x+a))+2*d^3*x^3*b*exp(3*I*(b*x+a))+3*b*c^3*exp(5*I*(b*x+a))+6*c*d^2*x^2
*b*exp(3*I*(b*x+a))-3*I*d^3*x^2*exp(5*I*(b*x+a))+6*c^2*d*x*b*exp(3*I*(b*x+
a))+3*d^3*x^3*b*exp(I*(b*x+a))+6*I*c*d^2*x*exp(I*(b*x+a))+2*b*c^3*exp(3*I*
(b*x+a))+9*c*d^2*x^2*b*exp(I*(b*x+a))-3*I*c^2*d*exp(5*I*(b*x+a))+9*c^2*d*x
*b*exp(I*(b*x+a))+3*b*c^3*exp(I*(b*x+a))+3*I*c^2*d*exp(I*(b*x+a))-6*I*c...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2226 vs. $2(404) = 808$.

Time = 0.24 (sec) , antiderivative size = 2226, normalized size of antiderivative = 4.58

$$\int (c+dx)^3 \csc^2(a+bx) \sec^3(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 18*I*d^3*cos(b*x +
a)^2*polylog(4, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + 18*I*d^3*cos
(b*x + a)^2*polylog(4, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - 18*I*
d^3*cos(b*x + a)^2*polylog(4, -I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a)
 - 18*I*d^3*cos(b*x + a)^2*polylog(4, -I*cos(b*x + a) - sin(b*x + a))*sin(
b*x + a) + 12*d^3*cos(b*x + a)^2*polylog(3, cos(b*x + a) + I*sin(b*x + a))
*sin(b*x + a) + 12*d^3*cos(b*x + a)^2*polylog(3, cos(b*x + a) - I*sin(b*x
+ a))*sin(b*x + a) - 12*d^3*cos(b*x + a)^2*polylog(3, -cos(b*x + a) + I*si
n(b*x + a))*sin(b*x + a) - 12*d^3*cos(b*x + a)^2*polylog(3, -cos(b*x + a)
- I*sin(b*x + a))*sin(b*x + a) + 2*b^3*c^3 - 12*(I*b*d^3*x + I*b*c*d^2)*co
s(b*x + a)^2*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 12*(-I*b*
d^3*x - I*b*c*d^2)*cos(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a))*sin
(b*x + a) - 3*(3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 2*I*d^3
)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 3*(3*
I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a)^2*
dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - 3*(-3*I*b^2*d^3*x^2 -
6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 2*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x
 + a) + sin(b*x + a))*sin(b*x + a) - 3*(-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x
 - 3*I*b^2*c^2*d - 2*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x
 + a))*sin(b*x + a) - 12*(I*b*d^3*x + I*b*c*d^2)*cos(b*x + a)^2*dilog(-...
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^2(a+bx) \sec^3(a+bx)\, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8046 vs. $2(404) = 808$.

Time = 3.91 (sec) , antiderivative size = 8046, normalized size of antiderivative = 16.56

$$\int (c+dx)^3 \csc^2(a+bx) \sec^3(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

output

```
-1/4*(c^3*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*lo
g(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1)) - 3*a*c^2*d*(2*(3*sin(b*x +
 a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*l
og(sin(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x +
 a)^3 - sin(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))
/b^2 - a^3*d^3*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) -
 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b^3 - 4*(6*((b*x + a)^
3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2
*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a) - ((b*x + a)^3*d^3 + 2*b*c*d
^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^
2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(6*b*x + 6*a) - ((b*x + a)^3*d^3 + 2*b*
c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c
*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + ((b*x + a)^3*d^3 + 2
*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*
b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (I*(b*x + a)^3*d^
3 + 2*I*b*c*d^2 - 2*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + (3*I*b
^2*c^2*d - 6*I*a*b*c*d^2 + (3*I*a^2 + 2*I)*d^3)*(b*x + a))*sin(6*b*x + 6*a
) - (I*(b*x + a)^3*d^3 + 2*I*b*c*d^2 - 2*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)
*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + (3*I*a^2 + 2*I)*d^3)*(b*x
 + a))*sin(4*b*x + 4*a) - (-I*(b*x + a)^3*d^3 - 2*I*b*c*d^2 + 2*I*a*d^3 ...
```

Giac [F]

$$\int (c+dx)^3 \csc^2(a+bx) \sec^3(a+bx)\, dx = \int (dx+c)^3 \csc(bx+a)^2 \sec(bx+a)^3\, dx$$

input

```
integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

output

```
integrate((d*x + c)^3*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^2(a+bx) \sec^3(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)^3/(cos(a + b*x)^3*sin(a + b*x)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^3 \csc^2(a+bx) \sec^3(a+bx)\,dx = \text{too large to display}$$

input
```
int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

output

```
(24192*cos(a + b*x)*sin(a + b*x)**2*b**3*c**2*d*x - 2484*cos(a + b*x)*sin(
a + b*x)**2*b**3*c*d**2*x**2 - 828*cos(a + b*x)*sin(a + b*x)**2*b**3*d**3*
x**3 - 2484*cos(a + b*x)*sin(a + b*x)**2*b*c*d**2 + 53964*cos(a + b*x)*sin
(a + b*x)**2*b*d**3*x + 3840*cos(a + b*x)*sin(a + b*x)*b**2*c**2*d + 30912
*cos(a + b*x)*sin(a + b*x)*b**2*c*d**2*x + 15456*cos(a + b*x)*sin(a + b*x)
*b**2*d**3*x**2 - 1344*cos(a + b*x)*sin(a + b*x)*d**3 - 18432*cos(a + b*x)
*b**3*c**2*d*x + 19872*cos(a + b*x)*b**3*c*d**2*x**2 + 6624*cos(a + b*x)*b
**3*d**3*x**3 - 43008*cos(a + b*x)*b*d**3*x + 3312*int(x**3/(tan((a + b*x)
/2)**8 - 3*tan((a + b*x)/2)**6 + 3*tan((a + b*x)/2)**4 - tan((a + b*x)/2)*
*2),x)*sin(a + b*x)**3*b**4*d**3 - 3312*int(x**3/(tan((a + b*x)/2)**8 - 3*
tan((a + b*x)/2)**6 + 3*tan((a + b*x)/2)**4 - tan((a + b*x)/2)**2),x)*sin(
a + b*x)*b**4*d**3 - 20976*int(x**3/(tan((a + b*x)/2)**6 - 3*tan((a + b*x)
/2)**4 + 3*tan((a + b*x)/2)**2 - 1),x)*sin(a + b*x)**3*b**4*d**3 + 20976*i
nt(x**3/(tan((a + b*x)/2)**6 - 3*tan((a + b*x)/2)**4 + 3*tan((a + b*x)/2)*
*2 - 1),x)*sin(a + b*x)*b**4*d**3 + 9936*int(x**2/(tan((a + b*x)/2)**8 - 3
*tan((a + b*x)/2)**6 + 3*tan((a + b*x)/2)**4 - tan((a + b*x)/2)**2),x)*sin
(a + b*x)**3*b**4*c*d**2 - 9936*int(x**2/(tan((a + b*x)/2)**8 - 3*tan((a +
 b*x)/2)**6 + 3*tan((a + b*x)/2)**4 - tan((a + b*x)/2)**2),x)*sin(a + b*x)
*b**4*c*d**2 - 24288*int(x**2/(tan((a + b*x)/2)**7 - 3*tan((a + b*x)/2)**5
 + 3*tan((a + b*x)/2)**3 - tan((a + b*x)/2)),x)*sin(a + b*x)**3*b**3*d*...
```

3.318 $\int (c+dx)^2 \csc^2(a+bx)\sec^3(a+bx)\,dx$

Optimal result . . . 2718
Mathematica [B] (warning: unable to verify) . . . 2719
Rubi [A] (verified) . . . 2720
Maple [B] (verified) . . . 2723
Fricas [B] (verification not implemented) . . . 2724
Sympy [F(-1)] . . . 2725
Maxima [B] (verification not implemented) . . . 2725
Giac [F] . . . 2726
Mupad [F(-1)] . . . 2727
Reduce [F] . . . 2727

Optimal result

Integrand size = 24, antiderivative size = 341

$$
\begin{aligned}
&\int (c+dx)^2 \csc^2(a+bx)\sec^3(a+bx)\,dx \\
&= -\frac{3i(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b} + \frac{2d^2 x \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{6d(c+dx)\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b^2} - \frac{d^2 x \operatorname{arctanh}(\cos(a+bx))}{b^2} \\
&\quad + \frac{d(c+dx)\operatorname{arctanh}(\cos(a+bx))}{b^2} + \frac{d^2 \operatorname{arctanh}(\sin(a+bx))}{b^3} - \frac{3(c+dx)^2 \csc(a+bx)}{2b} \\
&\quad + \frac{2id^2 \operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3} + \frac{3id(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{3id(c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} - \frac{2id^2 \operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3} \\
&\quad - \frac{3d^2 \operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} + \frac{3d^2 \operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} \\
&\quad - \frac{d(c+dx)\sec(a+bx)}{b^2} + \frac{(c+dx)^2 \csc(a+bx)\sec^2(a+bx)}{2b}
\end{aligned}
$$

output
```
-3*I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b+2*d^2*x*arctanh(exp(I*(b*x+a)))/b^
2-6*d*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^2-d^2*x*arctanh(cos(b*x+a))/b^2+d*
(d*x+c)*arctanh(cos(b*x+a))/b^2+d^2*arctanh(sin(b*x+a))/b^3-3/2*(d*x+c)^2*
csc(b*x+a)/b+2*I*d^2*polylog(2,-exp(I*(b*x+a)))/b^3+3*I*d*(d*x+c)*polylog(
2,-I*exp(I*(b*x+a)))/b^2-3*I*d*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^2-2*I
*d^2*polylog(2,exp(I*(b*x+a)))/b^3-3*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+
3*d^2*polylog(3,I*exp(I*(b*x+a)))/b^3-d*(d*x+c)*sec(b*x+a)/b^2+1/2*(d*x+c)
^2*csc(b*x+a)*sec(b*x+a)^2/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 889 vs. $2(341) = 682$.

Time = 7.08 (sec) , antiderivative size = 889, normalized size of antiderivative = 2.61

$$\int (c+dx)^2 \csc^2(a+bx) \sec^3(a+bx)\, dx = \text{Too large to display}$$

input
```
Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

output

```
-1/2*((6*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + (4*I)*d^2*ArcTan[E^(I*(a + b
*x))] - 6*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] - 3*b^2*d^2*x^2*Log[1 - I*E
^(I*(a + b*x))] + 6*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x))] + 3*b^2*d^2*x^2*L
og[1 + I*E^(I*(a + b*x))] - (6*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a +
b*x))] + (6*I)*b*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] + 6*d^2*PolyLog
[3, (-I)*E^(I*(a + b*x))] - 6*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - ((c
 + d*x)*Csc[a]*Sec[a]*(b*c*Cos[a] + b*d*x*Cos[a] + d*Sin[a]))/b^2 + ((4*I)
*c*d*ArcTan[(I*Cos[a] - I*Sin[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]])
/(b^2*Sqrt[Cos[a]^2 + Sin[a]^2]) + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c^2*Sin
[(b*x)/2]) - 2*c*d*x*Sin[(b*x)/2] - d^2*x^2*Sin[(b*x)/2]))/(2*b) + (Csc[a/
2]*Csc[a/2 + (b*x)/2]*(c^2*Sin[(b*x)/2] + 2*c*d*x*Sin[(b*x)/2] + d^2*x^2*S
in[(b*x)/2]))/(2*b) + (c^2 + 2*c*d*x + d^2*x^2)/(4*b*(Cos[a/2 + (b*x)/2] -
 Sin[a/2 + (b*x)/2])^2) + (-(c*d*Sin[(b*x)/2]) - d^2*x*Sin[(b*x)/2])/(b^2*
(Cos[a/2] - Sin[a/2])*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) + (-c^2 -
 2*c*d*x - d^2*x^2)/(4*b*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2])^2) + (c
*d*Sin[(b*x)/2] + d^2*x*Sin[(b*x)/2])/(b^2*(Cos[a/2] + Sin[a/2])*(Cos[a/2
+ (b*x)/2] + Sin[a/2 + (b*x)/2])) + (2*d^2*((-2*ArcTan[Tan[a]]*ArcTanh[(-C
os[a] + Sin[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]])/Sqrt[Cos[a]^2 + S
in[a]^2] + (((b*x + ArcTan[Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[Tan[a]]))]
 - Log[1 + E^(I*(b*x + ArcTan[Tan[a]]))]) + I*(PolyLog[2, -E^(I*(b*x + ...
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4920, 27, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \csc^2(a+bx) \sec^3(a+bx)\, dx$$

$$\downarrow 4920$$

$$-2d \int \frac{1}{2}(c+dx)\left(\frac{\csc(a+bx)\sec^2(a+bx)}{b} + \frac{3\operatorname{arctanh}(\sin(a+bx))}{b} - \frac{3\csc(a+bx)}{b}\right) dx + \frac{3(c+dx)^2 \operatorname{arctanh}(\sin(a+bx))}{2b} - \frac{3(c+dx)^2\csc(a+bx)}{2b} + \frac{(c+dx)^2\csc(a+bx)\sec^2(a+bx)}{2b}$$

$$\downarrow 27$$

$$-d\int(c+dx)\left(\frac{\csc(a+bx)\sec^2(a+bx)}{b}+\frac{3\operatorname{arctanh}(\sin(a+bx))}{b}-\frac{3\csc(a+bx)}{b}\right)dx+\frac{3(c+dx)^2\operatorname{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)^2\csc(a+bx)}{2b}+\frac{(c+dx)^2\csc(a+bx)\sec^2(a+bx)}{2b}$$

$$\downarrow 7292$$

$$-d\int\frac{(c+dx)\left(\csc(a+bx)\sec^2(a+bx)+3\operatorname{arctanh}(\sin(a+bx))-3\csc(a+bx)\right)}{b}dx+\frac{3(c+dx)^2\operatorname{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)^2\csc(a+bx)}{2b}+\frac{(c+dx)^2\csc(a+bx)\sec^2(a+bx)}{2b}$$

$$\downarrow 27$$

$$-\frac{d\int(c+dx)\left(\csc(a+bx)\sec^2(a+bx)+3\operatorname{arctanh}(\sin(a+bx))-3\csc(a+bx)\right)dx}{b}+\frac{3(c+dx)^2\operatorname{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)^2\csc(a+bx)}{2b}+\frac{(c+dx)^2\csc(a+bx)\sec^2(a+bx)}{2b}$$

$$\downarrow 7293$$

$$-\frac{d\int\left((c+dx)\csc(a+bx)\sec^2(a+bx)+3(c+dx)(\operatorname{arctanh}(\sin(a+bx))-\csc(a+bx))\right)dx}{b}+\frac{3(c+dx)^2\operatorname{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)^2\csc(a+bx)}{2b}+\frac{(c+dx)^2\csc(a+bx)\sec^2(a+bx)}{2b}$$

$$\downarrow 2009$$

$$-\frac{d\left(\frac{3i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{d}-\frac{d\operatorname{arctanh}(\sin(a+bx))}{b^2}+\frac{6(c+dx)\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{3(c+dx)^2\operatorname{arctanh}(\sin(a+bx))}{2d}-\frac{(c+dx)a}{}\right.}{\frac{3(c+dx)^2\operatorname{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)^2\csc(a+bx)}{2b}+\frac{(c+dx)^2\csc(a+bx)\sec^2(a+bx)}{2b}}$$

input

```
Int[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

output
```
(3*(c + d*x)^2*ArcTanh[Sin[a + b*x]])/(2*b) - (3*(c + d*x)^2*Csc[a + b*x])
/(2*b) + ((c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^2)/(2*b) - (d*(((3*I)*(c +
 d*x)^2*ArcTan[E^(I*(a + b*x))])/d - (2*d*x*ArcTanh[E^(I*(a + b*x))])/b +
(6*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + (d*x*ArcTanh[Cos[a + b*x]])/b -
 ((c + d*x)*ArcTanh[Cos[a + b*x]])/b - (d*ArcTanh[Sin[a + b*x]])/b^2 + (3*
(c + d*x)^2*ArcTanh[Sin[a + b*x]])/(2*d) - ((2*I)*d*PolyLog[2, -E^(I*(a +
b*x))])/b^2 - ((3*I)*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b + ((3*I
)*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b + ((2*I)*d*PolyLog[2, E^(I*(a
 + b*x))])/b^2 + (3*d*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^2 - (3*d*PolyLog
[3, I*E^(I*(a + b*x))])/b^2 + ((c + d*x)*Sec[a + b*x])/b))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 769 vs. 2(310) = 620.

Time = 0.71 (sec) , antiderivative size = 770, normalized size of antiderivative = 2.26

method	result
risch	$-\frac{3id^2\,\mathrm{polylog}\left(2,ie^{i(bx+a)}\right)x}{b^2}-\frac{i\left(3d^2x^2be^{5i(bx+a)}+6cdxbe^{5i(bx+a)}+3c^2be^{5i(bx+a)}+2d^2x^2be^{3i(bx+a)}+4cdxbe^{3i(bx+a)}-2id^2xe\right.}{b^2(e}$

input
```
int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/b^2*c*d*ln(I*exp(I*(b*x+a))+1)*a+3/b*c*d*ln(1-I*exp(I*(b*x+a)))*x+6*I/b
^2*c*d*a*arctan(exp(I*(b*x+a)))+3/2/b^3*a^2*d^2*ln(I*exp(I*(b*x+a))+1)+3/2
/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2-3/2/b*d^2*ln(I*exp(I*(b*x+a))+1)*x^2-3/2
/b^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)))-2*I/b^3*d^2*arctan(exp(I*(b*x+a)))+2*I
/b^3*dilog(exp(I*(b*x+a))+1)*d^2+2*I/b^3*d^2*dilog(exp(I*(b*x+a)))-3*I/b*c
^2*arctan(exp(I*(b*x+a)))+3/b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a-3/b*c*d*ln(I*
exp(I*(b*x+a))+1)*x+2*d/b^2*c*ln(exp(I*(b*x+a))-1)-2*d/b^2*c*ln(exp(I*(b*x
+a))+1)-2*d^2/b^2*ln(exp(I*(b*x+a))+1)*x-2*d^2/b^3*a*ln(exp(I*(b*x+a))-1)-
3*I/b^3*d^2*a^2*arctan(exp(I*(b*x+a)))-3*I/b^2*c*d*polylog(2,I*exp(I*(b*x+
a)))+3*I/b^2*c*d*polylog(2,-I*exp(I*(b*x+a)))-3*I/b^2*d^2*polylog(2,I*exp(
I*(b*x+a)))*x+3*I/b^2*d^2*polylog(2,-I*exp(I*(b*x+a)))*x-I/b^2/(exp(2*I*(b
*x+a))+1)^2/(exp(2*I*(b*x+a))-1)*(3*d^2*x^2*b*exp(5*I*(b*x+a))+6*c*d*x*b*e
xp(5*I*(b*x+a))+3*c^2*b*exp(5*I*(b*x+a))+2*d^2*x^2*b*exp(3*I*(b*x+a))+4*c*
d*x*b*exp(3*I*(b*x+a))-2*I*d^2*x*exp(5*I*(b*x+a))+2*c^2*b*exp(3*I*(b*x+a))
+3*d^2*x^2*b*exp(I*(b*x+a))-2*I*c*d*exp(5*I*(b*x+a))+6*c*d*x*b*exp(I*(b*x+
a))+3*c^2*b*exp(I*(b*x+a))+2*I*d^2*x*exp(I*(b*x+a))+2*I*c*d*exp(I*(b*x+a))
)-3*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+3*d^2*polylog(3,I*exp(I*(b*x+a)))
/b^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1366 vs. $2(297) = 594$.

Time = 0.18 (sec) , antiderivative size = 1366, normalized size of antiderivative = 4.01

$$\int (c+dx)^2 \csc^2(a+bx) \sec^3(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(2*b^2*d^2*x^2 - 4*I*d^2*cos(b*x + a)^2*dilog(cos(b*x + a) + I*sin(b*x
 + a))*sin(b*x + a) + 4*I*d^2*cos(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*
x + a))*sin(b*x + a) - 4*I*d^2*cos(b*x + a)^2*dilog(-cos(b*x + a) + I*sin(
b*x + a))*sin(b*x + a) + 4*I*d^2*cos(b*x + a)^2*dilog(-cos(b*x + a) - I*si
n(b*x + a))*sin(b*x + a) - 6*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a)
+ sin(b*x + a))*sin(b*x + a) + 6*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x +
 a) - sin(b*x + a))*sin(b*x + a) - 6*d^2*cos(b*x + a)^2*polylog(3, -I*cos(
b*x + a) + sin(b*x + a))*sin(b*x + a) + 6*d^2*cos(b*x + a)^2*polylog(3, -I
*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + 4*b^2*c*d*x - 6*(I*b*d^2*x +
I*b*c*d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a)
- 6*(I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x +
a))*sin(b*x + a) - 6*(-I*b*d^2*x - I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*
x + a) + sin(b*x + a))*sin(b*x + a) - 6*(-I*b*d^2*x - I*b*c*d)*cos(b*x + a
)^2*dilog(-I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - 4*(b*d^2*x + b*c*
d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + (3
*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) +
I*sin(b*x + a) + I)*sin(b*x + a) - 4*(b*d^2*x + b*c*d)*cos(b*x + a)^2*log(
cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - (3*b^2*c^2 - 6*a*b*c*d +
 (3*a^2 + 2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I)*si
n(b*x + a) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*...
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc^2(a+bx) \sec^3(a+bx)\, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3828 vs. $2(297) = 594$.

Time = 0.79 (sec) , antiderivative size = 3828, normalized size of antiderivative = 11.23

$$\int (c+dx)^2 \csc^2(a+bx) \sec^3(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

output

```
-1/4*(c^2*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*lo
g(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1)) - 2*a*c*d*(2*(3*sin(b*x + a
)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*log
(sin(b*x + a) - 1))/b + a^2*d^2*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3
 - sin(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b^2 -
 4*(2*(3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 - (3*(b*x +
 a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(6*b*x + 6*a) - (3*(b*
x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) + (3*
(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*a) -
(3*I*(b*x + a)^2*d^2 + 6*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(6*b*
x + 6*a) - (3*I*(b*x + a)^2*d^2 + 6*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^
2)*sin(4*b*x + 4*a) - (-3*I*(b*x + a)^2*d^2 + 6*(-I*b*c*d + I*a*d^2)*(b*x
+ a) - 2*I*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1)
+ 2*(3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 - (3*(b*x + a
)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(6*b*x + 6*a) - (3*(b*x
+ a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) + (3*(b
*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*a) - (3
*I*(b*x + a)^2*d^2 + 6*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(6*b*x
+ 6*a) - (3*I*(b*x + a)^2*d^2 + 6*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)
*sin(4*b*x + 4*a) - (-3*I*(b*x + a)^2*d^2 + 6*(-I*b*c*d + I*a*d^2)*(b*x...
```

Giac [F]

$$\int (c+dx)^2 \csc^2(a+bx) \sec^3(a+bx)\,dx = \int (dx+c)^2 \csc(bx+a)^2 \sec(bx+a)^3\,dx$$

input

```
integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

output

```
integrate((d*x + c)^2*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc^2(a+bx) \sec^3(a+bx)\, dx = \text{Hanged}$$

input
```
int((c + d*x)^2/(cos(a + b*x)^3*sin(a + b*x)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^2 \csc^2(a+bx) \sec^3(a+bx)\, dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

output

```
(4032*cos(a + b*x)*sin(a + b*x)**2*b**2*c*d*x - 207*cos(a + b*x)*sin(a + b
*x)**2*b**2*d**2*x**2 - 207*cos(a + b*x)*sin(a + b*x)**2*d**2 + 640*cos(a
+ b*x)*sin(a + b*x)*b*c*d + 2576*cos(a + b*x)*sin(a + b*x)*b*d**2*x - 3072
*cos(a + b*x)*b**2*c*d*x + 1656*cos(a + b*x)*b**2*d**2*x**2 + 828*int(x**2
/(tan((a + b*x)/2)**8 - 3*tan((a + b*x)/2)**6 + 3*tan((a + b*x)/2)**4 - ta
n((a + b*x)/2)**2),x)*sin(a + b*x)**3*b**3*d**2 - 828*int(x**2/(tan((a + b
*x)/2)**8 - 3*tan((a + b*x)/2)**6 + 3*tan((a + b*x)/2)**4 - tan((a + b*x)/
2)**2),x)*sin(a + b*x)*b**3*d**2 - 5244*int(x**2/(tan((a + b*x)/2)**6 - 3*
tan((a + b*x)/2)**4 + 3*tan((a + b*x)/2)**2 - 1),x)*sin(a + b*x)**3*b**3*d
**2 + 5244*int(x**2/(tan((a + b*x)/2)**6 - 3*tan((a + b*x)/2)**4 + 3*tan((
a + b*x)/2)**2 - 1),x)*sin(a + b*x)*b**3*d**2 + 736*int((tan((a + b*x)/2)*
*7*x)/(tan((a + b*x)/2)**6 - 3*tan((a + b*x)/2)**4 + 3*tan((a + b*x)/2)**2
 - 1),x)*sin(a + b*x)**3*b**2*d**2 - 736*int((tan((a + b*x)/2)**7*x)/(tan(
(a + b*x)/2)**6 - 3*tan((a + b*x)/2)**4 + 3*tan((a + b*x)/2)**2 - 1),x)*si
n(a + b*x)*b**2*d**2 - 1536*int(x/(tan((a + b*x)/2)**8 - 3*tan((a + b*x)/2
)**6 + 3*tan((a + b*x)/2)**4 - tan((a + b*x)/2)**2),x)*sin(a + b*x)**3*b**
3*c*d + 1536*int(x/(tan((a + b*x)/2)**8 - 3*tan((a + b*x)/2)**6 + 3*tan((a
 + b*x)/2)**4 - tan((a + b*x)/2)**2),x)*sin(a + b*x)*b**3*c*d - 4048*int(x
/(tan((a + b*x)/2)**7 - 3*tan((a + b*x)/2)**5 + 3*tan((a + b*x)/2)**3 - ta
n((a + b*x)/2)),x)*sin(a + b*x)**3*b**2*d**2 + 4048*int(x/(tan((a + b*x...
```

3.319 $\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx$

Optimal result . . . 2729
Mathematica [C] (warning: unable to verify) . . . 2730
Rubi [A] (verified) . . . 2730
Maple [B] (verified) . . . 2732
Fricas [B] (verification not implemented) . . . 2732
Sympy [F] . . . 2733
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Reduce [F] . . . 2735

Optimal result

Integrand size = 22, antiderivative size = 162

$$\begin{aligned}&\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx\\ &= -\frac{3idx\arctan\left(e^{i(a+bx)}\right)}{b} - \frac{d\operatorname{arctanh}(\cos(a+bx))}{b^2} + \frac{3c\operatorname{arctanh}(\sin(a+bx))}{2b}\\ &\quad - \frac{3(c+dx)\csc(a+bx)}{2b} + \frac{3id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2}\\ &\quad - \frac{3id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2} - \frac{d\sec(a+bx)}{2b^2} + \frac{(c+dx)\csc(a+bx)\sec^2(a+bx)}{2b}\end{aligned}$$

output

```
-3*I*d*x*arctan(exp(I*(b*x+a)))/b-d*arctanh(cos(b*x+a))/b^2+3/2*c*arctanh(
sin(b*x+a))/b-3/2*(d*x+c)*csc(b*x+a)/b+3/2*I*d*polylog(2,-I*exp(I*(b*x+a))
)/b^2-3/2*I*d*polylog(2,I*exp(I*(b*x+a)))/b^2-1/2*d*sec(b*x+a)/b^2+1/2*(d*
x+c)*csc(b*x+a)*sec(b*x+a)^2/b
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.75 (sec) , antiderivative size = 669, normalized size of antiderivative = 4.13

$$\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

output
```
(d*(a*Cos[(a + b*x)/2] - (a + b*x)*Cos[(a + b*x)/2])*Csc[(a + b*x)/2])/(2*
b^2) - (c*Csc[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, Sin[a + b*x]^2])/b
- (d*Log[Cos[(a + b*x)/2]])/b^2 + (d*Log[Sin[(a + b*x)/2]])/b^2 - (3*d*x*(
a*Log[1 - Tan[(a + b*x)/2]] - a*Log[1 + Tan[(a + b*x)/2]] - I*(Log[1 + I*T
an[(a + b*x)/2]]*Log[(1/2 - I/2)*(1 + Tan[(a + b*x)/2])] + PolyLog[2, ((1
+ I) - (1 - I)*Tan[(a + b*x)/2])/2]) + I*(Log[1 - I*Tan[(a + b*x)/2]]*Log[
(1/2 + I/2)*(1 + Tan[(a + b*x)/2])] + PolyLog[2, (-1/2 - I/2)*(I + Tan[(a
+ b*x)/2])]) - I*(Log[1 - I*Tan[(a + b*x)/2]]*Log[(-1/2 + I/2)*(-1 + Tan[(
a + b*x)/2])] + PolyLog[2, ((1 + I) + (1 - I)*Tan[(a + b*x)/2])/2]) + I*(L
og[1 + I*Tan[(a + b*x)/2]]*Log[(-1/2 - I/2)*(-1 + Tan[(a + b*x)/2])] + Pol
yLog[2, ((1 - I) + (1 + I)*Tan[(a + b*x)/2])/2])))/(2*b*(a - I*Log[1 - I*T
an[(a + b*x)/2]] + I*Log[1 + I*Tan[(a + b*x)/2]])) + (d*x)/(4*b*(Cos[(a +
b*x)/2] - Sin[(a + b*x)/2])^2) - (d*Sin[(a + b*x)/2])/(2*b^2*(Cos[(a + b*x
)/2] - Sin[(a + b*x)/2])) - (d*x)/(4*b*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2
])^2) + (d*Sin[(a + b*x)/2])/(2*b^2*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2]))
 + (d*Sec[(a + b*x)/2]*(a*Sin[(a + b*x)/2] - (a + b*x)*Sin[(a + b*x)/2]))/
(2*b^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4920, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx$$

$$\downarrow 4920$$

$$-d\int\left(\frac{\csc(a+bx)\sec^2(a+bx)}{2b}+\frac{3\text{arctanh}(\sin(a+bx))}{2b}-\frac{3\csc(a+bx)}{2b}\right)dx+$$
$$\frac{3(c+dx)\text{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)\csc(a+bx)}{2b}+\frac{(c+dx)\csc(a+bx)\sec^2(a+bx)}{2b}$$

$$\downarrow 2009$$

$$-d\left(\frac{3ix\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{\text{arctanh}(\cos(a+bx))}{b^2}+\frac{3x\text{arctanh}(\sin(a+bx))}{2b}-\frac{3i\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2}+\frac{3i}{}\right.$$
$$\frac{3(c+dx)\text{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)\csc(a+bx)}{2b}+\frac{(c+dx)\csc(a+bx)\sec^2(a+bx)}{2b}$$

input
```
Int[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

output
```
(3*(c + d*x)*ArcTanh[Sin[a + b*x]])/(2*b) - (3*(c + d*x)*Csc[a + b*x])/(2*
b) + ((c + d*x)*Csc[a + b*x]*Sec[a + b*x]^2)/(2*b) - d*(((3*I)*x*ArcTan[E^
(I*(a + b*x))])/b + ArcTanh[Cos[a + b*x]]/b^2 + (3*x*ArcTanh[Sin[a + b*x]]
)/(2*b) - (((3*I)/2)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 + (((3*I)/2)*Po
lyLog[2, I*E^(I*(a + b*x))])/b^2 + Sec[a + b*x]/(2*b^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(139) = 278$.

Time = 0.57 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.12

method	result
risch	$-\frac{i\left(3bdx\,e^{5i(bx+a)}+3cb\,e^{5i(bx+a)}+2bdx\,e^{3i(bx+a)}+2cb\,e^{3i(bx+a)}-id\,e^{5i(bx+a)}+3bdx\,e^{i(bx+a)}+3bc\,e^{i(bx+a)}+id\,e^{i(bx+a)}\right)}{b^2\left(e^{2i(bx+a)}+1\right)^2\left(e^{2i(bx+a)}-1\right)} - \frac{3d\ln$

input
```
int((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-I/b^2/(exp(2*I*(b*x+a))+1)^2/(exp(2*I*(b*x+a))-1)*(3*b*d*x*exp(5*I*(b*x+a
))+3*c*b*exp(5*I*(b*x+a))+2*b*d*x*exp(3*I*(b*x+a))+2*c*b*exp(3*I*(b*x+a))-
I*d*exp(5*I*(b*x+a))+3*b*d*x*exp(I*(b*x+a))+3*b*c*exp(I*(b*x+a))+I*d*exp(I
*(b*x+a)))-3/2/b*d*ln(I*exp(I*(b*x+a))+1)*x+3/2/b*d*ln(1-I*exp(I*(b*x+a)))
*x-3/2/b^2*d*ln(I*exp(I*(b*x+a))+1)*a+3/2/b^2*d*ln(1-I*exp(I*(b*x+a)))*a+3
/2*I/b^2*d*dilog(I*exp(I*(b*x+a))+1)-3/2*I/b^2*d*dilog(1-I*exp(I*(b*x+a)))
-3*I/b*c*arctan(exp(I*(b*x+a)))+3*I/b^2*a*d*arctan(exp(I*(b*x+a)))+d/b^2*l
n(exp(I*(b*x+a))-1)-d/b^2*ln(exp(I*(b*x+a))+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(132) = 264$.

Time = 0.11 (sec) , antiderivative size = 592, normalized size of antiderivative = 3.65

$$\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(-3*I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x +
a) - 3*I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a
) + 3*I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a
) + 3*I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a
) + 3*(b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I)*si
n(b*x + a) - 3*(b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a
) + I)*sin(b*x + a) - 2*d*cos(b*x + a)^2*log(1/2*cos(b*x + a) + 1/2)*sin(b
*x + a) + 3*(b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a)
 + 1)*sin(b*x + a) - 3*(b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - s
in(b*x + a) + 1)*sin(b*x + a) + 3*(b*d*x + a*d)*cos(b*x + a)^2*log(-I*cos(
b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) - 3*(b*d*x + a*d)*cos(b*x + a)^2
*log(-I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) + 2*d*cos(b*x + a)^2
*log(-1/2*cos(b*x + a) + 1/2)*sin(b*x + a) + 3*(b*c - a*d)*cos(b*x + a)^2*
log(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) - 3*(b*c - a*d)*cos(b
*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + 2*b*d*x -
 6*(b*d*x + b*c)*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x + a) + 2*b*c)/(
b^2*cos(b*x + a)^2*sin(b*x + a))
```

Sympy [F]

$$\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx = \int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

output
```
Integral((c + d*x)*csc(a + b*x)**2*sec(a + b*x)**3, x)
```

Maxima [F]

$$\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx = \int (dx+c)\csc(bx+a)^2\sec(bx+a)^3\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

output

```
1/4*(8*(b*d*x + b*c)*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) - 4*(d*cos(5*b*x +
5*a) - d*cos(b*x + a) - 3*(b*d*x + b*c)*sin(5*b*x + 5*a) - 2*(b*d*x + b*c)
*sin(3*b*x + 3*a) - 3*(b*d*x + b*c)*sin(b*x + a))*cos(6*b*x + 6*a) - 4*(d*
cos(4*b*x + 4*a) - d*cos(2*b*x + 2*a) + 3*(b*d*x + b*c)*sin(4*b*x + 4*a) -
 3*(b*d*x + b*c)*sin(2*b*x + 2*a) - d)*cos(5*b*x + 5*a) + 4*(d*cos(b*x + a
) + 2*(b*d*x + b*c)*sin(3*b*x + 3*a) + 3*(b*d*x + b*c)*sin(b*x + a))*cos(4
*b*x + 4*a) - 4*(d*cos(b*x + a) + 3*(b*d*x + b*c)*sin(b*x + a))*cos(2*b*x
+ 2*a) - 4*d*cos(b*x + a) + 12*(b^2*d*cos(6*b*x + 6*a)^2 + b^2*d*cos(4*b*x
 + 4*a)^2 + b^2*d*cos(2*b*x + 2*a)^2 + b^2*d*sin(6*b*x + 6*a)^2 + b^2*d*si
n(4*b*x + 4*a)^2 - 2*b^2*d*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b^2*d*sin(2
*b*x + 2*a)^2 + 2*b^2*d*cos(2*b*x + 2*a) + b^2*d + 2*(b^2*d*cos(4*b*x + 4*
a) - b^2*d*cos(2*b*x + 2*a) - b^2*d)*cos(6*b*x + 6*a) - 2*(b^2*d*cos(2*b*x
 + 2*a) + b^2*d)*cos(4*b*x + 4*a) + 2*(b^2*d*sin(4*b*x + 4*a) - b^2*d*sin(
2*b*x + 2*a))*sin(6*b*x + 6*a))*integrate((x*cos(2*b*x + 2*a)*cos(b*x + a)
 + x*sin(2*b*x + 2*a)*sin(b*x + a) + x*cos(b*x + a))/(cos(2*b*x + 2*a)^2 +
 sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1), x) + 3*(b*c*cos(6*b*x + 6*a
)^2 + b*c*cos(4*b*x + 4*a)^2 + b*c*cos(2*b*x + 2*a)^2 + b*c*sin(6*b*x + 6*
a)^2 + b*c*sin(4*b*x + 4*a)^2 - 2*b*c*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) +
b*c*sin(2*b*x + 2*a)^2 + 2*b*c*cos(2*b*x + 2*a) + b*c + 2*(b*c*cos(4*b*x +
 4*a) - b*c*cos(2*b*x + 2*a) - b*c)*cos(6*b*x + 6*a) - 2*(b*c*cos(2*b*x...
```

Giac [F]

$$\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx = \int (dx+c)\csc\left(bx+a\right)^2\sec\left(bx+a\right)^3\,dx$$

input

```
integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

output

```
integrate((d*x + c)*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)/(cos(a + b*x)^3*sin(a + b*x)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
int((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

output
```
(126*cos(a + b*x)*sin(a + b*x)**2*b*d*x + 20*cos(a + b*x)*sin(a + b*x)*d -
 96*cos(a + b*x)*b*d*x - 48*int(x/(tan((a + b*x)/2)**8 - 3*tan((a + b*x)/2
)**6 + 3*tan((a + b*x)/2)**4 - tan((a + b*x)/2)**2),x)*sin(a + b*x)**3*b**
2*d + 48*int(x/(tan((a + b*x)/2)**8 - 3*tan((a + b*x)/2)**6 + 3*tan((a + b
*x)/2)**4 - tan((a + b*x)/2)**2),x)*sin(a + b*x)*b**2*d + 126*log(tan((a +
 b*x)/2)**2 + 1)*sin(a + b*x)**3*d - 126*log(tan((a + b*x)/2)**2 + 1)*sin(
a + b*x)*d - 69*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**3*b*c - 30*log(tan
((a + b*x)/2) - 1)*sin(a + b*x)**3*d + 69*log(tan((a + b*x)/2) - 1)*sin(a
+ b*x)*b*c + 30*log(tan((a + b*x)/2) - 1)*sin(a + b*x)*d + 69*log(tan((a +
 b*x)/2) + 1)*sin(a + b*x)**3*b*c - 30*log(tan((a + b*x)/2) + 1)*sin(a + b
*x)**3*d - 69*log(tan((a + b*x)/2) + 1)*sin(a + b*x)*b*c + 30*log(tan((a +
 b*x)/2) + 1)*sin(a + b*x)*d - 146*log(tan((a + b*x)/2))*sin(a + b*x)**3*d
 + 146*log(tan((a + b*x)/2))*sin(a + b*x)*d + 3*sin(a + b*x)**3*b**2*d*x**
2 - 69*sin(a + b*x)**2*b*c + 30*sin(a + b*x)**2*b*d*x - 3*sin(a + b*x)*b**
2*d*x**2 + 46*b*c - 50*b*d*x)/(46*sin(a + b*x)*b**2*(sin(a + b*x)**2 - 1))
```

3.320 $\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$

Optimal result . . . 2736
Mathematica [N/A] . . . 2736
Rubi [N/A] . . . 2737
Maple [N/A] . . . 2737
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Sympy [N/A] . . . 2738
Maxima [N/A] . . . 2739
Giac [N/A] . . . 2740
Mupad [N/A] . . . 2740
Reduce [N/A] . . . 2740

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}, x\right)$$

output
```
Defer(Int)(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 12.84 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x),x]
```

output
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$
$$\downarrow 7299$$
$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

input
```
Int[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)^3}{dx+c}\,dx$$

input
```
int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x)
```

output
```
int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)^3}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2*sec(b*x + a)^3/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 7.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc^2\left(a+bx\right)\sec^3\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)**2*sec(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(csc(a + b*x)**2*sec(a + b*x)**3/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 3695, normalized size of antiderivative = 153.96

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^2\sec(bx+a)^3}{dx+c}\,dx$$

input

```
integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output

```
(2*(b*d*x + b*c)*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) + (d*cos(5*b*x + 5*a) -
 d*cos(b*x + a) + 3*(b*d*x + b*c)*sin(5*b*x + 5*a) + 2*(b*d*x + b*c)*sin(3
*b*x + 3*a) + 3*(b*d*x + b*c)*sin(b*x + a))*cos(6*b*x + 6*a) + (d*cos(4*b*
x + 4*a) - d*cos(2*b*x + 2*a) - 3*(b*d*x + b*c)*sin(4*b*x + 4*a) + 3*(b*d*
x + b*c)*sin(2*b*x + 2*a) - d)*cos(5*b*x + 5*a) - (d*cos(b*x + a) - 2*(b*d
*x + b*c)*sin(3*b*x + 3*a) - 3*(b*d*x + b*c)*sin(b*x + a))*cos(4*b*x + 4*a
) + (d*cos(b*x + a) - 3*(b*d*x + b*c)*sin(b*x + a))*cos(2*b*x + 2*a) + d*c
os(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*
c*d*x + b^2*c^2)*cos(6*b*x + 6*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*cos(4*b*x + 4*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2
*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(6*b*x + 6*a)^2 + (b^2*d^
2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 2*(b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + (b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*
c^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a) + (b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(6*b*x + 6*a) - 2*(b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2
*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*co
s(2*b*x + 2*a) + 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)
 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a))*sin(6*b*x + ...
```

Giac [N/A]

Not integrable

Time = 115.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^2\sec(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sec(b*x + a)^3/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 21.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos(a+b\,x)^3\sin(a+b\,x)^2\,(c+d\,x)}\,dx$$

input
```
int(1/(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^2\sec(bx+a)^3}{dx+c}dx$$

input
```
int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x)
```

output
```
int((csc(a + b*x)**2*sec(a + b*x)**3)/(c + d*x),x)
```

3.321 $\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 2742
Mathematica [N/A] . . . 2742
Rubi [N/A] . . . 2743
Maple [N/A] . . . 2743
Fricas [N/A] . . . 2744
Sympy [N/A] . . . 2744
Maxima [N/A] . . . 2745
Giac [F(-1)] . . . 2746
Mupad [N/A] . . . 2746
Reduce [N/A] . . . 2746

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 16.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$
$$\downarrow 7299$$
$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

input
```
Int[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)^3}{\left(dx+c\right)^2}\,dx$$

input
```
int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)^3}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2*sec(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 15.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^2\left(a+bx\right)\sec^3\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)**2*sec(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)**2*sec(a + b*x)**3/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 7.23 (sec) , antiderivative size = 4747, normalized size of antiderivative = 197.79

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)^3}{\left(dx+c\right)^2}\,dx$$

input

```
integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output

```
(2*(b*d*x + b*c)*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) + (2*d*cos(5*b*x + 5*a)
 - 2*d*cos(b*x + a) + 3*(b*d*x + b*c)*sin(5*b*x + 5*a) + 2*(b*d*x + b*c)*s
in(3*b*x + 3*a) + 3*(b*d*x + b*c)*sin(b*x + a))*cos(6*b*x + 6*a) + (2*d*co
s(4*b*x + 4*a) - 2*d*cos(2*b*x + 2*a) - 3*(b*d*x + b*c)*sin(4*b*x + 4*a) +
 3*(b*d*x + b*c)*sin(2*b*x + 2*a) - 2*d)*cos(5*b*x + 5*a) - (2*d*cos(b*x +
 a) - 2*(b*d*x + b*c)*sin(3*b*x + 3*a) - 3*(b*d*x + b*c)*sin(b*x + a))*cos
(4*b*x + 4*a) + (2*d*cos(b*x + a) - 3*(b*d*x + b*c)*sin(b*x + a))*cos(2*b*
x + 2*a) + 2*d*cos(b*x + a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*c
os(6*b*x + 6*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c
^3)*cos(4*b*x + 4*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3)*sin(6*b*x + 6*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*
c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 +
 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + (b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 - 2*(b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - (b^2*d^3*x^3 + 3*b^2
*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a) + (b^2*d^3*x^3 + 3*
b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(6*b*x + 6*a
) - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [N/A]

Not integrable

Time = 20.93 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\cos(a+b\,x)^3\sin(a+b\,x)^2(c+d\,x)^2}\,dx$$

input
```
int(1/(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)^2\sec(bx+a)^3}{d^2x^2+2cdx+c^2}dx$$

input
```
int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int((csc(a + b*x)**2*sec(a + b*x)**3)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.322 $\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx$

Optimal result . . . 2747
Mathematica [N/A] . . . 2747
Rubi [N/A] . . . 2748
Maple [N/A] . . . 2748
Fricas [N/A] . . . 2749
Sympy [F(-1)] . . . 2749
Maxima [N/A] . . . 2749
Giac [N/A] . . . 2750
Mupad [N/A] . . . 2750
Reduce [N/A] . . . 2751

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx = \text{Int}\left((c+dx)^m \csc^3(a+bx)\sec^3(a+bx), x\right)$$

output
```
Defer(Int)((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 26.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx = \int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

output
```
Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\sec^3(a+bx)(c+dx)^m\,dx$$

$$\downarrow 7299$$

$$\int \csc^3(a+bx)\sec^3(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^3\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

output
```
int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*csc(b*x+a)**3*sec(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 20.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\cos(a+b\,x)^3 \sin(a+b\,x)^3}\,dx$$

input
```
int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)^3),x)
```

output
```
int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^3\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

output
```
int((c + d*x)**m*csc(a + b*x)**3*sec(a + b*x)**3,x)
```

3.323 $\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx)\, dx$

Optimal result . . . 2752
Mathematica [A] (verified) . . . 2753
Rubi [A] (verified) . . . 2754
Maple [B] (verified) . . . 2758
Fricas [B] (verification not implemented) . . . 2759
Sympy [F(-1)] . . . 2759
Maxima [B] (verification not implemented) . . . 2759
Giac [F] . . . 2760
Mupad [F(-1)] . . . 2761
Reduce [F] . . . 2761

Optimal result

Integrand size = 24, antiderivative size = 318

$$\begin{aligned}
&\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx)\, dx \\
&= -\frac{6d^2(c+dx)\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b^3} - \frac{4(c+dx)^3\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b} \\
&\quad - \frac{3d(c+dx)^2 \csc(2a+2bx)}{b^2} - \frac{2(c+dx)^3 \cot(2a+2bx)\csc(2a+2bx)}{b} \\
&\quad + \frac{3id^3 \operatorname{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{2b^4} + \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2, -e^{2i(a+bx)}\right)}{b^2} \\
&\quad - \frac{3id^3 \operatorname{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^4} - \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^2} \\
&\quad - \frac{3d^2(c+dx) \operatorname{PolyLog}\left(3, -e^{2i(a+bx)}\right)}{b^3} + \frac{3d^2(c+dx) \operatorname{PolyLog}\left(3, e^{2i(a+bx)}\right)}{b^3} \\
&\quad - \frac{3id^3 \operatorname{PolyLog}\left(4, -e^{2i(a+bx)}\right)}{2b^4} + \frac{3id^3 \operatorname{PolyLog}\left(4, e^{2i(a+bx)}\right)}{2b^4}
\end{aligned}$$

output
```
-6*d^2*(d*x+c)*arctanh(exp(2*I*(b*x+a)))/b^3-4*(d*x+c)^3*arctanh(exp(2*I*(
b*x+a)))/b-3*d*(d*x+c)^2*csc(2*b*x+2*a)/b^2-2*(d*x+c)^3*cot(2*b*x+2*a)*csc
(2*b*x+2*a)/b+3/2*I*d^3*polylog(2,-exp(2*I*(b*x+a)))/b^4+3*I*d*(d*x+c)^2*p
olylog(2,-exp(2*I*(b*x+a)))/b^2-3/2*I*d^3*polylog(2,exp(2*I*(b*x+a)))/b^4-
3*I*d*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2-3*d^2*(d*x+c)*polylog(3,-e
xp(2*I*(b*x+a)))/b^3+3*d^2*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^3-3/2*I*d
^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+3/2*I*d^3*polylog(4,exp(2*I*(b*x+a)))/
b^4
```

Mathematica [A] (verified)

Time = 6.01 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.52

$$\int (c+dx)^3 \csc^3(a+bx)\sec^3(a+bx)\,dx =$$
$$-\frac{8b^3c^3\operatorname{arctanh}\left(e^{2i(a+bx)}\right)+12bcd^2\operatorname{arctanh}\left(e^{2i(a+bx)}\right)+2b^2(c+dx)^2(3d+2b(c+dx)\cot(2(a+bx)))\csc$$

input
```
Integrate[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

output
```
-1/2*(8*b^3*c^3*ArcTanh[E^((2*I)*(a + b*x))] + 12*b*c*d^2*ArcTanh[E^((2*I)
*(a + b*x))] + 2*b^2*(c + d*x)^2*(3*d + 2*b*(c + d*x)*Cot[2*(a + b*x)])*Cs
c[2*(a + b*x)] - 12*b^3*c^2*d*x*Log[1 - E^((2*I)*(a + b*x))] - 6*b*d^3*x*L
og[1 - E^((2*I)*(a + b*x))] - 12*b^3*c*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))
] - 4*b^3*d^3*x^3*Log[1 - E^((2*I)*(a + b*x))] + 12*b^3*c^2*d*x*Log[1 + E^
((2*I)*(a + b*x))] + 6*b*d^3*x*Log[1 + E^((2*I)*(a + b*x))] + 12*b^3*c*d^2
*x^2*Log[1 + E^((2*I)*(a + b*x))] + 4*b^3*d^3*x^3*Log[1 + E^((2*I)*(a + b*
x))] - (3*I)*d*(d^2 + 2*b^2*(c + d*x)^2)*PolyLog[2, -E^((2*I)*(a + b*x))]
+ (3*I)*d*(d^2 + 2*b^2*(c + d*x)^2)*PolyLog[2, E^((2*I)*(a + b*x))] + 6*b*
c*d^2*PolyLog[3, -E^((2*I)*(a + b*x))] + 6*b*d^3*x*PolyLog[3, -E^((2*I)*(a
 + b*x))] - 6*b*c*d^2*PolyLog[3, E^((2*I)*(a + b*x))] - 6*b*d^3*x*PolyLog[
3, E^((2*I)*(a + b*x))] + (3*I)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))] - (3*
I)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {4919, 3042, 4674, 3042, 4671, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx)\, dx$$

$$\Big\downarrow \text{4919}$$

$$8\int (c+dx)^3 \csc^3(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$8\int (c+dx)^3 \csc(2a+2bx)^3dx$$

$$\Big\downarrow \text{4674}$$

$$8\left(\frac{3d^2\int (c+dx)\csc(2a+2bx)dx}{4b^2}+\frac{1}{2}\int (c+dx)^3\csc(2a+2bx)dx-\frac{3d(c+dx)^2\csc(2a+2bx)}{8b^2}-\frac{(c+dx)^3\cot}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$8\left(\frac{3d^2\int (c+dx)\csc(2a+2bx)dx}{4b^2}+\frac{1}{2}\int (c+dx)^3\csc(2a+2bx)dx-\frac{3d(c+dx)^2\csc(2a+2bx)}{8b^2}-\frac{(c+dx)^3\cot}{}\right.$$

$$\Big\downarrow \text{4671}$$

$$8\left(\frac{3d^2\left(-\frac{d\int \log\left(1-e^{2i(a+bx)}\right)dx}{2b}+\frac{d\int \log\left(1+e^{2i(a+bx)}\right)dx}{2b}-\frac{(c+dx)\text{arctanh}\left(e^{2i(a+bx)}\right)}{b}\right)}{4b^2}+\frac{1}{2}\left(-\frac{3d\int (c+dx)^2\log\left(1-e^{2}\right.}{2b}\right.\right.$$

$$\Big\downarrow \text{2715}$$

$$8\left(\frac{3d^2\left(\frac{id\int e^{-2i(a+bx)}\log\left(1-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{id\int e^{-2i(a+bx)}\log\left(1+e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{(c+dx)\text{arctanh}\left(e^{2i(a+bx)}\right)}{b}\right)}{4b^2}+\frac{1}{2}\right.$$

$$\downarrow 2838$$

$$8\left(\frac{1}{2}\left(-\frac{3d\int(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)dx}{2b}+\frac{3d\int(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)dx}{2b}-\frac{(c+dx)^3\text{arctanh}\left(e^{2i(a+bx)}\right.}{b}\right.\right.$$

$$\downarrow 3011$$

$$8\left(\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{3d\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+\right.}{2b}\right.\right.$$

$$\downarrow 7163$$

$$8\left(\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{id\int\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)\,\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{3d\left(\frac{i(c+dx)^2\,\text{Po}\right.}{2b}\right.\right.$$

$$\downarrow 2720$$

$$8\left(\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{d\int e^{-2i(a+bx)}\,\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(c+dx)\,\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\right.\right.$$

$$\downarrow 7143$$

$$8\left(\frac{3d^2\left(-\frac{(c+dx)\text{arctanh}\left(e^{2i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{id\,\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{4b^2}\right)}{4b^2}+\frac{1}{2}\left(-\frac{(c+dx)^3\text{arctanh}\left(e^{2i(a\right.}{b}\right.\right.$$

input
```
Int[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

output
```
8*((-3*d*(c + d*x)^2*Csc[2*a + 2*b*x])/(8*b^2) - ((c + d*x)^3*Cot[2*a + 2*
b*x]*Csc[2*a + 2*b*x])/(4*b) + (3*d^2*(-(((c + d*x)*ArcTanh[E^((2*I)*(a +
b*x))])/b) + ((I/4)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((I/4)*d*Pol
yLog[2, E^((2*I)*(a + b*x))])/b^2))/(4*b^2) + (-(((c + d*x)^3*ArcTanh[E^((
2*I)*(a + b*x))])/b) + (3*d*(((I/2)*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a +
b*x))])/b - (I*d*(((-1/2*I)*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/b
+ (d*PolyLog[4, -E^((2*I)*(a + b*x))])/(4*b^2)))/b))/(2*b) - (3*d*(((I/2)*
(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b - (I*d*(((-1/2*I)*(c + d*x)
*PolyLog[3, E^((2*I)*(a + b*x))])/b + (d*PolyLog[4, E^((2*I)*(a + b*x))])/
(4*b^2)))/b))/(2*b))/2)
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)  Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)  Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))  Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n  Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))  Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1328 vs. $2(284) = 568$.

Time = 0.91 (sec) , antiderivative size = 1329, normalized size of antiderivative = 4.18

method	result	size
risch	Expression too large to display	1329

input
```
int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-6/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2-3/b^3*d^2*c*ln(exp(2*I*(b*x+a))+1)-3
/b^3*d^3*ln(exp(2*I*(b*x+a))+1)*x-2/b*d^3*ln(exp(2*I*(b*x+a))+1)*x^3-3/b^3
*c*d^2*polylog(3,-exp(2*I*(b*x+a)))-3/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))
*x-6/b*c^2*d*ln(exp(2*I*(b*x+a))+1)*x-6/b*c*d^2*ln(exp(2*I*(b*x+a))+1)*x^2
+12*I/b^4*d^3*polylog(4,-exp(I*(b*x+a)))+12*I/b^4*d^3*polylog(4,exp(I*(b*x
+a)))+3*d^2/b^3*c*ln(exp(I*(b*x+a))-1)+3*d^2/b^3*c*ln(exp(I*(b*x+a))+1)+3*
d^3/b^3*ln(exp(I*(b*x+a))+1)*x+3*d^3/b^3*ln(1-exp(I*(b*x+a)))*x+3*d^3/b^4*
ln(1-exp(I*(b*x+a)))*a-3*d^3/b^4*a*ln(exp(I*(b*x+a))-1)-3*I*d^3/b^4*polylo
g(2,-exp(I*(b*x+a)))-6*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-6*I/b^2*d^
3*polylog(2,exp(I*(b*x+a)))*x^2+3*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x
^2+3*I/b^2*c^2*d*polylog(2,-exp(2*I*(b*x+a)))-6*I/b^2*c^2*d*polylog(2,-exp
(I*(b*x+a)))-6*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))+2/b^2/(exp(2*I*(b*x+a
))+1)^2/(exp(2*I*(b*x+a))-1)^2*(2*d^3*x^3*b*exp(6*I*(b*x+a))+6*c*d^2*x^2*b
*exp(6*I*(b*x+a))+6*c^2*d*x*b*exp(6*I*(b*x+a))+2*b*c^3*exp(6*I*(b*x+a))-3*
I*d^3*x^2*exp(6*I*(b*x+a))+2*b*d^3*x^3*exp(2*I*(b*x+a))-6*I*c*d^2*x*exp(6*
I*(b*x+a))+6*b*c*d^2*x^2*exp(2*I*(b*x+a))-3*I*c^2*d*exp(6*I*(b*x+a))+6*b*c
^2*d*x*exp(2*I*(b*x+a))+2*b*c^3*exp(2*I*(b*x+a))+3*I*d^3*x^2*exp(2*I*(b*x+
a))+6*I*c*d^2*x*exp(2*I*(b*x+a))+3*I*c^2*d*exp(2*I*(b*x+a)))-2/b^4*d^3*a^3
*ln(exp(I*(b*x+a))-1)+12/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+12/b^3*c*d^2
*polylog(3,exp(I*(b*x+a)))+2/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+2/b*d^3*ln(...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4193 vs. $2(274) = 548$.

Time = 0.29 (sec) , antiderivative size = 4193, normalized size of antiderivative = 13.19

$$\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx)\, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3*csc(b*x+a)**3*sec(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5646 vs. $2(274) = 548$.

Time = 3.18 (sec) , antiderivative size = 5646, normalized size of antiderivative = 17.75

$$\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

output

```
-1/2*(c^3*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*lo
g(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2)) - 3*a*c^2*d*((2*sin(b*x + a
)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2
*log(sin(b*x + a)^2))/b + 3*a^2*c*d^2*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a
)^4 - sin(b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))
/b^2 - a^3*d^3*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) +
 2*log(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b^3 + 2*(2*(8*(b*x + a
)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(2*b^
2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a) + (8*(b*x + a)^3*d^3 +
9*b*c*d^2 - 9*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(2*b^2*c^2*d -
4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*cos(8*b*x + 8*a) - 2*(8*(b*x + a
)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(2*b^
2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - (-8
*I*(b*x + a)^3*d^3 - 9*I*b*c*d^2 + 9*I*a*d^3 + 18*(-I*b*c*d^2 + I*a*d^3)*(
b*x + a)^2 + 9*(-2*I*b^2*c^2*d + 4*I*a*b*c*d^2 + (-2*I*a^2 - I)*d^3)*(b*x
+ a))*sin(8*b*x + 8*a) - 2*(8*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3
+ 18*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 9*(2*I*b^2*c^2*d - 4*I*a*b*c*d^2
+ (2*I*a^2 + I)*d^3)*(b*x + a))*sin(4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a)
, cos(2*b*x + 2*a) + 1) - 6*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*(
b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + ...
```

Giac [F]

$$\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx)\,dx = \int (dx+c)^3 \csc(bx+a)^3 \sec(bx+a)^3\,dx$$

input

```
integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

output

```
integrate((d*x + c)^3*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx)\, dx = \text{Hanged}$$

input
```
int((c + d*x)^3/(cos(a + b*x)^3*sin(a + b*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx)\, dx$$
$$= \frac{2\left(\int \csc(bx+a)^3 \sec(bx+a)^3 x^3 dx\right) \sin(bx+a)^4 b\, d^3 - 2\left(\int \csc(bx+a)^3 \sec(bx+a)^3 x^3 dx\right) \sin(bx+a}{}$$

input
```
int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

output
```
(2*int(csc(a + b*x)**3*sec(a + b*x)**3*x**3,x)*sin(a + b*x)**4*b*d**3 - 2*
int(csc(a + b*x)**3*sec(a + b*x)**3*x**3,x)*sin(a + b*x)**2*b*d**3 + 6*int
(csc(a + b*x)**3*sec(a + b*x)**3*x**2,x)*sin(a + b*x)**4*b*c*d**2 - 6*int(
csc(a + b*x)**3*sec(a + b*x)**3*x**2,x)*sin(a + b*x)**2*b*c*d**2 + 6*int(c
sc(a + b*x)**3*sec(a + b*x)**3*x,x)*sin(a + b*x)**4*b*c**2*d - 6*int(csc(a
 + b*x)**3*sec(a + b*x)**3*x,x)*sin(a + b*x)**2*b*c**2*d - 4*log(tan((a +
b*x)/2) - 1)*sin(a + b*x)**4*c**3 + 4*log(tan((a + b*x)/2) - 1)*sin(a + b*
x)**2*c**3 - 4*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**4*c**3 + 4*log(tan(
(a + b*x)/2) + 1)*sin(a + b*x)**2*c**3 + 4*log(tan((a + b*x)/2))*sin(a + b
*x)**4*c**3 - 4*log(tan((a + b*x)/2))*sin(a + b*x)**2*c**3 - sin(a + b*x)*
*4*c**3 - sin(a + b*x)**2*c**3 + c**3)/(2*sin(a + b*x)**2*b*(sin(a + b*x)*
*2 - 1))
```

3.324 $\int (c+dx)^2 \csc^3(a+bx)\sec^3(a+bx)\,dx$

Optimal result . . . 2762
Mathematica [B] (verified) . . . 2763
Rubi [A] (verified) . . . 2764
Maple [B] (verified) . . . 2767
Fricas [B] (verification not implemented) . . . 2767
Sympy [F(-1)] . . . 2768
Maxima [B] (verification not implemented) . . . 2769
Giac [F] . . . 2770
Mupad [F(-1)] . . . 2770
Reduce [F] . . . 2770

Optimal result

Integrand size = 24, antiderivative size = 190

$$\begin{aligned}
&\int (c+dx)^2 \csc^3(a+bx)\sec^3(a+bx)\,dx \\
&= -\frac{4(c+dx)^2 \operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2 \operatorname{arctanh}(\cos(2a+2bx))}{b^3} \\
&\quad - \frac{2d(c+dx)\csc(2a+2bx)}{b^2} - \frac{2(c+dx)^2\cot(2a+2bx)\csc(2a+2bx)}{b} \\
&\quad + \frac{2id(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2} \\
&\quad - \frac{d^2\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3} + \frac{d^2\operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}
\end{aligned}$$

output

```
-4*(d*x+c)^2*arctanh(exp(2*I*(b*x+a)))/b-d^2*arctanh(cos(2*b*x+2*a))/b^3-2
*d*(d*x+c)*csc(2*b*x+2*a)/b^2-2*(d*x+c)^2*cot(2*b*x+2*a)*csc(2*b*x+2*a)/b+
2*I*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2-2*I*d*(d*x+c)*polylog(2,exp
(2*I*(b*x+a)))/b^2-d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+d^2*polylog(3,exp(
2*I*(b*x+a)))/b^3
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 381 vs. $2(190) = 380$.

Time = 7.44 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.01

$$\int (c+dx)^2 \csc^3(a+bx)\sec^3(a+bx)\,dx$$
$$= 8\left(-\frac{d(c+dx)\csc(2a)}{4b^2} + \frac{(-c^2-2cdx-d^2x^2)\csc^2(a+bx)}{16b}\right.$$
$$-\frac{4b^2c^2\text{arctanh}\left(e^{2i(a+bx)}\right) + 2d^2\text{arctanh}\left(e^{2i(a+bx)}\right) - 4b^2cdx\log\left(1-e^{2i(a+bx)}\right) - 2b^2d^2x^2\log\left(1-e^{2i(a+b}\right.$$
$$+\frac{(c^2+2cdx+d^2x^2)\sec^2(a+bx)}{16b} + \frac{\sec(a)\sec(a+bx)\left(-cd\sin(bx)-d^2x\sin(bx)\right)}{8b^2}$$
$$\left.\left.+\frac{\csc(a)\csc(a+bx)\left(cd\sin(bx)+d^2x\sin(bx)\right)}{8b^2}\right)\right)$$

input
```
Integrate[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

output
```
8*(-1/4*(d*(c + d*x)*Csc[2*a])/b^2 + ((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a + b
*x]^2)/(16*b) - (4*b^2*c^2*ArcTanh[E^((2*I)*(a + b*x))] + 2*d^2*ArcTanh[E^
((2*I)*(a + b*x))] - 4*b^2*c*d*x*Log[1 - E^((2*I)*(a + b*x))] - 2*b^2*d^2*
x^2*Log[1 - E^((2*I)*(a + b*x))] + 4*b^2*c*d*x*Log[1 + E^((2*I)*(a + b*x))
] + 2*b^2*d^2*x^2*Log[1 + E^((2*I)*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyL
og[2, -E^((2*I)*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, E^((2*I)*(a +
 b*x))] + d^2*PolyLog[3, -E^((2*I)*(a + b*x))] - d^2*PolyLog[3, E^((2*I)*(
a + b*x))])/(8*b^3) + ((c^2 + 2*c*d*x + d^2*x^2)*Sec[a + b*x]^2)/(16*b) +
(Sec[a]*Sec[a + b*x]*(-(c*d*Sin[b*x]) - d^2*x*Sin[b*x]))/(8*b^2) + (Csc[a]
*Csc[a + b*x]*(c*d*Sin[b*x] + d^2*x*Sin[b*x]))/(8*b^2))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4919, 3042, 4674, 3042, 4257, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \csc^3(a+bx)\sec^3(a+bx)\,dx$$

$$\downarrow 4919$$

$$8\int (c+dx)^2 \csc^3(2a+2bx)dx$$

$$\downarrow 3042$$

$$8\int (c+dx)^2 \csc(2a+2bx)^3 dx$$

$$\downarrow 4674$$

$$8\left(\frac{d^2\int \csc(2a+2bx)dx}{4b^2}+\frac{1}{2}\int (c+dx)^2\csc(2a+2bx)dx-\frac{d(c+dx)\csc(2a+2bx)}{4b^2}-\frac{(c+dx)^2\cot(2a+2bx)}{4b}\right.$$

$$\downarrow 3042$$

$$8\left(\frac{d^2\int \csc(2a+2bx)dx}{4b^2}+\frac{1}{2}\int (c+dx)^2\csc(2a+2bx)dx-\frac{d(c+dx)\csc(2a+2bx)}{4b^2}-\frac{(c+dx)^2\cot(2a+2bx)}{4b}\right.$$

$$\downarrow 4257$$

$$8\left(\frac{1}{2}\int (c+dx)^2\csc(2a+2bx)dx-\frac{d^2\text{arctanh}(\cos(2a+2bx))}{8b^3}-\frac{d(c+dx)\csc(2a+2bx)}{4b^2}-\frac{(c+dx)^2\cot(2a+2bx)}{4b}\right.$$

$$\downarrow 4671$$

$$8\left(\frac{1}{2}\left(-\frac{d\int (c+dx)\log\left(1-e^{2i(a+bx)}\right)dx}{b}+\frac{d\int (c+dx)\log\left(1+e^{2i(a+bx)}\right)dx}{b}-\frac{(c+dx)^2\text{arctanh}\left(e^{2i(a+bx)}\right)}{b}\right)\right.$$

$$\downarrow 3011$$

$$8\left(\frac{1}{2}\left(\frac{d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int \mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b}-\frac{id\int \mathrm{PolyLog}\left(2,e^{2i(\right.}\right.}\right.$$

$$\downarrow 2720$$

$$8\left(\frac{1}{2}\left(\frac{d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i(a+bx)}\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b}-\frac{d}{}\right.}\right.}\right.$$

$$\downarrow 7143$$

$$8\left(-\frac{d^2\mathrm{arctanh}(\cos(2a+2bx))}{8b^3}+\frac{1}{2}\left(-\frac{(c+dx)^2\mathrm{arctanh}\left(e^{2i(a+bx)}\right)}{b}+\frac{d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\,\mathrm{PolyLog}\left(3,\right.}{4b}\right.}{b}\right.\right.\right.$$

input

```
Int[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

output

```
8*(-1/8*(d^2*ArcTanh[Cos[2*a + 2*b*x]])/b^3 - (d*(c + d*x)*Csc[2*a + 2*b*x
])/(4*b^2) - ((c + d*x)^2*Cot[2*a + 2*b*x]*Csc[2*a + 2*b*x])/(4*b) + (-(((
c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/b) + (d*(((I/2)*(c + d*x)*PolyLog
[2, -E^((2*I)*(a + b*x))])/b - (d*PolyLog[3, -E^((2*I)*(a + b*x))])/(4*b^2
)))/b - (d*(((I/2)*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b - (d*PolyL
og[3, E^((2*I)*(a + b*x))])/(4*b^2)))/b)/2)
```

Defintions of rubi rules used

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))  Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4671

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)  Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)  Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4674

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))  Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4919

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n  Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 715 vs. $2(178) = 356$.

Time = 0.73 (sec) , antiderivative size = 716, normalized size of antiderivative = 3.77

method	result
risch	$-\frac{4icd\,\mathrm{polylog}\left(2,-\mathrm{e}^{i(bx+a)}\right)}{b^2}+\frac{2id^2\,\mathrm{polylog}\left(2,-\mathrm{e}^{2i(bx+a)}\right)x}{b^2}+\frac{4cd\ln\left(1-\mathrm{e}^{i(bx+a)}\right)a}{b^2}-\frac{4cda\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b^2}+\frac{4cd\ln\left(1-\mathrm{e}^{i(}\right.}{b}$

input
```
int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/b*c^2*ln(exp(2*I*(b*x+a))+1)+4/b^2/(exp(2*I*(b*x+a))+1)^2/(exp(2*I*(b*x
+a))-1)^2*(d^2*x^2*b*exp(6*I*(b*x+a))+2*c*d*x*b*exp(6*I*(b*x+a))+c^2*b*exp
(6*I*(b*x+a))-I*d^2*x*exp(6*I*(b*x+a))+b*d^2*x^2*exp(2*I*(b*x+a))-I*c*d*ex
p(6*I*(b*x+a))+2*b*c*d*x*exp(2*I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))+I*d^2*x*e
xp(2*I*(b*x+a))+I*c*d*exp(2*I*(b*x+a)))+4/b^2*c*d*ln(1-exp(I*(b*x+a)))*a-4
/b*d*c*ln(exp(2*I*(b*x+a))+1)*x-2/b*d^2*ln(exp(2*I*(b*x+a))+1)*x^2+2/b^3*d
^2*a^2*ln(exp(I*(b*x+a))-1)+2/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+2/b*d^2*ln(1-
exp(I*(b*x+a)))*x^2-2/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+4/b*c*d*ln(exp(I*(b
*x+a))+1)*x-4/b^2*c*d*a*ln(exp(I*(b*x+a))-1)+4/b*c*d*ln(1-exp(I*(b*x+a)))*
x+2*I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-4*I/b^2*d^2*polylog(2,-exp(I*
(b*x+a)))*x-4*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x+2*I/b^2*d*c*polylog(2,
-exp(2*I*(b*x+a)))-4*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-d^2/b^3*ln(exp(2
*I*(b*x+a))+1)+2/b*c^2*ln(exp(I*(b*x+a))-1)+2/b*c^2*ln(exp(I*(b*x+a))+1)-4
*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))+1/b^3*d^2*ln(exp(I*(b*x+a))-1)+1/b^3*
d^2*ln(exp(I*(b*x+a))+1)+4*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+4*d^2*polylo
g(3,exp(I*(b*x+a)))/b^3-d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2387 vs. $2(174) = 348$.

Time = 0.20 (sec) , antiderivative size = 2387, normalized size of antiderivative = 12.56

$$\int (c+dx)^2 \csc^3(a+bx)\sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*cos(b*x + a)^2 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) +
 4*((I*b*d^2*x + I*b*c*d)*cos(b*x + a)^4 + (-I*b*d^2*x - I*b*c*d)*cos(b*x
+ a)^2)*dilog(cos(b*x + a) + I*sin(b*x + a)) + 4*((-I*b*d^2*x - I*b*c*d)*c
os(b*x + a)^4 + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilog(cos(b*x + a) -
 I*sin(b*x + a)) + 4*((I*b*d^2*x + I*b*c*d)*cos(b*x + a)^4 + (-I*b*d^2*x -
 I*b*c*d)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) + sin(b*x + a)) + 4*((-I*b*
d^2*x - I*b*c*d)*cos(b*x + a)^4 + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*di
log(I*cos(b*x + a) - sin(b*x + a)) + 4*((-I*b*d^2*x - I*b*c*d)*cos(b*x + a
)^4 + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) + sin(b*
x + a)) + 4*((I*b*d^2*x + I*b*c*d)*cos(b*x + a)^4 + (-I*b*d^2*x - I*b*c*d)
*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 4*((-I*b*d^2*x -
I*b*c*d)*cos(b*x + a)^4 + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilog(-cos
(b*x + a) + I*sin(b*x + a)) + 4*((I*b*d^2*x + I*b*c*d)*cos(b*x + a)^4 + (-
I*b*d^2*x - I*b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) - I*sin(b*x + a))
 - ((2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*cos(b*x + a)^4 - (2*b^
2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*cos(b*x + a)^2)*log(cos(b*x + a
) + I*sin(b*x + a) + 1) + ((2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b
*x + a)^4 - (2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^2)*log(
cos(b*x + a) + I*sin(b*x + a) + I) - ((2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*...
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc^3(a+bx) \sec^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*csc(b*x+a)**3*sec(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2728 vs. $2(174) = 348$.

Time = 1.30 (sec) , antiderivative size = 2728, normalized size of antiderivative = 14.36

$$\int (c+dx)^2 \csc^3(a+bx) \sec^3(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(c^2*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*lo
g(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2)) - 2*a*c*d*((2*sin(b*x + a)^
2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*l
og(sin(b*x + a)^2))/b + a^2*d^2*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 -
sin(b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b^2 +
 2*(2*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2 + (2*(b*x + a
)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*cos(8*b*x + 8*a) - 2*(2*(b*x
+ a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*cos(4*b*x + 4*a) - (-2*I*(
b*x + a)^2*d^2 + 4*(-I*b*c*d + I*a*d^2)*(b*x + a) - I*d^2)*sin(8*b*x + 8*a
) - 2*(2*I*(b*x + a)^2*d^2 + 4*(I*b*c*d - I*a*d^2)*(b*x + a) + I*d^2)*sin(
4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 2*(2*(b*x
+ a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2 + (2*(b*x + a)^2*d^2 + 4*(b
*c*d - a*d^2)*(b*x + a) + d^2)*cos(8*b*x + 8*a) - 2*(2*(b*x + a)^2*d^2 + 4
*(b*c*d - a*d^2)*(b*x + a) + d^2)*cos(4*b*x + 4*a) + (2*I*(b*x + a)^2*d^2
+ 4*(I*b*c*d - I*a*d^2)*(b*x + a) + I*d^2)*sin(8*b*x + 8*a) + 2*(-2*I*(b*x
 + a)^2*d^2 + 4*(-I*b*c*d + I*a*d^2)*(b*x + a) - I*d^2)*sin(4*b*x + 4*a))*
arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(d^2*cos(8*b*x + 8*a) - 2*d^2*
cos(4*b*x + 4*a) + I*d^2*sin(8*b*x + 8*a) - 2*I*d^2*sin(4*b*x + 4*a) + d^2
)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 4*((b*x + a)^2*d^2 + 2*(b*c*d
- a*d^2)*(b*x + a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*co...
```

Giac [F]

$$\int (c+dx)^2 \csc^3(a+bx) \sec^3(a+bx)\, dx = \int (dx+c)^2 \csc(bx+a)^3 \sec(bx+a)^3\, dx$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc^3(a+bx) \sec^3(a+bx)\, dx = \text{Hanged}$$

input
```
int((c + d*x)^2/(cos(a + b*x)^3*sin(a + b*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^2 \csc^3(a+bx) \sec^3(a+bx)\, dx$$
$$= \frac{2\left(\int \csc(bx+a)^3 \sec(bx+a)^3 x^2 dx\right) \sin(bx+a)^4 b\, d^2 - 2\left(\int \csc(bx+a)^3 \sec(bx+a)^3 x^2 dx\right) \sin(bx+a)}{}$$

input
```
int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

output

```
(2*int(csc(a + b*x)**3*sec(a + b*x)**3*x**2,x)*sin(a + b*x)**4*b*d**2 - 2*
int(csc(a + b*x)**3*sec(a + b*x)**3*x**2,x)*sin(a + b*x)**2*b*d**2 + 4*int
(csc(a + b*x)**3*sec(a + b*x)**3*x,x)*sin(a + b*x)**4*b*c*d - 4*int(csc(a
+ b*x)**3*sec(a + b*x)**3*x,x)*sin(a + b*x)**2*b*c*d - 4*log(tan((a + b*x)
/2) - 1)*sin(a + b*x)**4*c**2 + 4*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**
2*c**2 - 4*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**4*c**2 + 4*log(tan((a +
 b*x)/2) + 1)*sin(a + b*x)**2*c**2 + 4*log(tan((a + b*x)/2))*sin(a + b*x)*
*4*c**2 - 4*log(tan((a + b*x)/2))*sin(a + b*x)**2*c**2 - sin(a + b*x)**4*c
**2 - sin(a + b*x)**2*c**2 + c**2)/(2*sin(a + b*x)**2*b*(sin(a + b*x)**2 -
 1))
```

3.325 $\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx$

Optimal result . . . 2772
Mathematica [B] (verified) . . . 2773
Rubi [A] (verified) . . . 2773
Maple [B] (verified) . . . 2776
Fricas [B] (verification not implemented) . . . 2776
Sympy [F(-1)] . . . 2777
Maxima [B] (verification not implemented) . . . 2778
Giac [F] . . . 2779
Mupad [F(-1)] . . . 2779
Reduce [F] . . . 2779

Optimal result

Integrand size = 22, antiderivative size = 110

$$\begin{aligned}&\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx\\&= -\frac{4(c+dx)\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\csc(2a+2bx)}{b^2}\\&\quad - \frac{2(c+dx)\cot(2a+2bx)\csc(2a+2bx)}{b}\\&\quad + \frac{id\,\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} - \frac{id\,\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}\end{aligned}$$

output

```
-4*(d*x+c)*arctanh(exp(2*I*(b*x+a)))/b-d*csc(2*b*x+2*a)/b^2-2*(d*x+c)*cot(
2*b*x+2*a)*csc(2*b*x+2*a)/b+I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2-I*d*polyl
og(2,exp(2*I*(b*x+a)))/b^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 236 vs. $2(110) = 220$.

Time = 1.73 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.15

$$\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx$$
$$= -\frac{d\cot(a+bx)}{2b^2} - \frac{c\csc^2(a+bx)}{2b} + \frac{d(2a-2(a+bx))\csc^2(a+bx)}{4b^2}$$
$$- \frac{2c\log(\cos(a+bx))}{b} + \frac{2c\log(\sin(a+bx))}{b} - \frac{2ad\log(\tan(a+bx))}{b^2}$$
$$+ \frac{d\big(2(a+bx)\left(\log\left(1-e^{2i(a+bx)}\right) - \log\left(1+e^{2i(a+bx)}\right)\right) + i\big(\text{PolyLog}\left(2,-e^{2i(a+bx)}\right) - \text{PolyLog}\left(2,e^{2i(a+}\right.}{b^2}$$
$$+ \frac{c\sec^2(a+bx)}{2b} + \frac{d(-2a+2(a+bx))\sec^2(a+bx)}{4b^2} - \frac{d\tan(a+bx)}{2b^2}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

output
```
-1/2*(d*Cot[a + b*x])/b^2 - (c*Csc[a + b*x]^2)/(2*b) + (d*(2*a - 2*(a + b*
x))*Csc[a + b*x]^2)/(4*b^2) - (2*c*Log[Cos[a + b*x]])/b + (2*c*Log[Sin[a +
 b*x]])/b - (2*a*d*Log[Tan[a + b*x]])/b^2 + (d*(2*(a + b*x)*(Log[1 - E^((2
*I)*(a + b*x))] - Log[1 + E^((2*I)*(a + b*x))]) + I*(PolyLog[2, -E^((2*I)*
(a + b*x))] - PolyLog[2, E^((2*I)*(a + b*x))])))/b^2 + (c*Sec[a + b*x]^2)/
(2*b) + (d*(-2*a + 2*(a + b*x))*Sec[a + b*x]^2)/(4*b^2) - (d*Tan[a + b*x])
/(2*b^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {4919, 3042, 4673, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx$$

$$\downarrow 4919$$

$$8\int (c+dx)\csc^3(2a+2bx)dx$$

$$\downarrow 3042$$

$$8\int (c+dx)\csc(2a+2bx)^3dx$$

$$\downarrow 4673$$

$$8\left(\frac{1}{2}\int (c+dx)\csc(2a+2bx)dx - \frac{d\csc(2a+2bx)}{8b^2} - \frac{(c+dx)\cot(2a+2bx)\csc(2a+2bx)}{4b}\right)$$

$$\downarrow 3042$$

$$8\left(\frac{1}{2}\int (c+dx)\csc(2a+2bx)dx - \frac{d\csc(2a+2bx)}{8b^2} - \frac{(c+dx)\cot(2a+2bx)\csc(2a+2bx)}{4b}\right)$$

$$\downarrow 4671$$

$$8\left(\frac{1}{2}\left(-\frac{d\int \log\left(1-e^{2i(a+bx)}\right)dx}{2b} + \frac{d\int \log\left(1+e^{2i(a+bx)}\right)dx}{2b} - \frac{(c+dx)\text{arctanh}\left(e^{2i(a+bx)}\right)}{b}\right) - \frac{d\csc(2a+2bx)}{8b^2}$$

$$\downarrow 2715$$

$$8\left(\frac{1}{2}\left(\frac{id\int e^{-2i(a+bx)}\log\left(1-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2} - \frac{id\int e^{-2i(a+bx)}\log\left(1+e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2} - \frac{(c+dx)\text{arcta}}{b}\right.\right.$$

$$\downarrow 2838$$

$$8\left(\frac{1}{2}\left(-\frac{(c+dx)\text{arctanh}\left(e^{2i(a+bx)}\right)}{b} + \frac{id\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2} - \frac{id\,\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{4b^2}\right) - \frac{d\csc(2a+2bx)}{8b^2}\right.$$

input
```
Int[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

output
```
8*(-1/8*(d*Csc[2*a + 2*b*x])/b^2 - ((c + d*x)*Cot[2*a + 2*b*x]*Csc[2*a + 2
*b*x])/(4*b) + (-(((c + d*x)*ArcTanh[E^((2*I)*(a + b*x))])/b) + ((I/4)*d*P
olyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((I/4)*d*PolyLog[2, E^((2*I)*(a + b
*x))])/b^2)/2)
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(102) = 204$.

Time = 0.60 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.95

method	result
risch	$\frac{4bdx\,e^{6i(bx+a)}+4cb\,e^{6i(bx+a)}-2id\,e^{6i(bx+a)}+4bdx\,e^{2i(bx+a)}+4bc\,e^{2i(bx+a)}+2id\,e^{2i(bx+a)}}{b^2\left(e^{2i(bx+a)}-1\right)^2\left(e^{2i(bx+a)}+1\right)^2}+\frac{2c\ln\left(e^{i(bx+a)}+1\right)}{b}-\frac{2c\ln\left(e^{2i(bx+}\right.}{b}$

input
```
int((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/b^2/(exp(2*I*(b*x+a))-1)^2/(exp(2*I*(b*x+a))+1)^2*(2*b*d*x*exp(6*I*(b*x+
a))+2*c*b*exp(6*I*(b*x+a))-I*d*exp(6*I*(b*x+a))+2*b*d*x*exp(2*I*(b*x+a))+2
*b*c*exp(2*I*(b*x+a))+I*d*exp(2*I*(b*x+a)))+2/b*c*ln(exp(I*(b*x+a))+1)-2/b
*c*ln(exp(2*I*(b*x+a))+1)+2/b*c*ln(exp(I*(b*x+a))-1)-2/b*d*ln(exp(2*I*(b*x
+a))+1)*x+I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+2/b*d*ln(exp(I*(b*x+a))+1)*
x-2*I*d*polylog(2,-exp(I*(b*x+a)))/b^2+2/b*d*ln(1-exp(I*(b*x+a)))*x+2/b^2*
d*ln(1-exp(I*(b*x+a)))*a-2*I/b^2*d*polylog(2,exp(I*(b*x+a)))-2/b^2*d*a*ln(
exp(I*(b*x+a))-1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1193 vs. $2(98) = 196$.

Time = 0.15 (sec) , antiderivative size = 1193, normalized size of antiderivative = 10.85

$$\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

output

```
-1/2*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 - d*cos(b*x + a)*sin(b*x + a)
 + b*c + 2*(I*d*cos(b*x + a)^4 - I*d*cos(b*x + a)^2)*dilog(cos(b*x + a) +
I*sin(b*x + a)) + 2*(-I*d*cos(b*x + a)^4 + I*d*cos(b*x + a)^2)*dilog(cos(b
*x + a) - I*sin(b*x + a)) + 2*(I*d*cos(b*x + a)^4 - I*d*cos(b*x + a)^2)*di
log(I*cos(b*x + a) + sin(b*x + a)) + 2*(-I*d*cos(b*x + a)^4 + I*d*cos(b*x
+ a)^2)*dilog(I*cos(b*x + a) - sin(b*x + a)) + 2*(-I*d*cos(b*x + a)^4 + I*
d*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 2*(I*d*cos(b*x +
 a)^4 - I*d*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 2*(-I*
d*cos(b*x + a)^4 + I*d*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a
)) + 2*(I*d*cos(b*x + a)^4 - I*d*cos(b*x + a)^2)*dilog(-cos(b*x + a) - I*s
in(b*x + a)) - 2*((b*d*x + b*c)*cos(b*x + a)^4 - (b*d*x + b*c)*cos(b*x + a
)^2)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + 2*((b*c - a*d)*cos(b*x + a)^
4 - (b*c - a*d)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + I) - 2
*((b*d*x + b*c)*cos(b*x + a)^4 - (b*d*x + b*c)*cos(b*x + a)^2)*log(cos(b*x
 + a) - I*sin(b*x + a) + 1) + 2*((b*c - a*d)*cos(b*x + a)^4 - (b*c - a*d)*
cos(b*x + a)^2)*log(cos(b*x + a) - I*sin(b*x + a) + I) + 2*((b*d*x + a*d)*
cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)^2)*log(I*cos(b*x + a) + sin(b*
x + a) + 1) + 2*((b*d*x + a*d)*cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)
^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + 2*((b*d*x + a*d)*cos(b*x + a)
^4 - (b*d*x + a*d)*cos(b*x + a)^2)*log(-I*cos(b*x + a) + sin(b*x + a) +...
```

Sympy [F(-1)]

Timed out.

$$\int (c + dx) \csc^3(a + bx) \sec^3(a + bx)\, dx = \text{Timed out}$$

input

```
integrate((d*x+c)*csc(b*x+a)**3*sec(b*x+a)**3,x)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1070 vs. $2(98) = 196$.

Time = 0.74 (sec) , antiderivative size = 1070, normalized size of antiderivative = 9.73

$$\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
-(2*(b*d*x + b*c + (b*d*x + b*c)*cos(8*b*x + 8*a) - 2*(b*d*x + b*c)*cos(4*
b*x + 4*a) - (-I*b*d*x - I*b*c)*sin(8*b*x + 8*a) - 2*(I*b*d*x + I*b*c)*sin
(4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 2*(b*d*x
+ b*c + (b*d*x + b*c)*cos(8*b*x + 8*a) - 2*(b*d*x + b*c)*cos(4*b*x + 4*a)
+ (I*b*d*x + I*b*c)*sin(8*b*x + 8*a) + 2*(-I*b*d*x - I*b*c)*sin(4*b*x + 4*
a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(b*c*cos(8*b*x + 8*a) - 2*
b*c*cos(4*b*x + 4*a) + I*b*c*sin(8*b*x + 8*a) - 2*I*b*c*sin(4*b*x + 4*a) +
 b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 2*(b*d*x*cos(8*b*x + 8*a)
- 2*b*d*x*cos(4*b*x + 4*a) + I*b*d*x*sin(8*b*x + 8*a) - 2*I*b*d*x*sin(4*b*
x + 4*a) + b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*(-2*I*b*d*x
 - 2*I*b*c - d)*cos(6*b*x + 6*a) - 2*(-2*I*b*d*x - 2*I*b*c + d)*cos(2*b*x
+ 2*a) - (d*cos(8*b*x + 8*a) - 2*d*cos(4*b*x + 4*a) + I*d*sin(8*b*x + 8*a)
 - 2*I*d*sin(4*b*x + 4*a) + d)*dilog(-e^(2*I*b*x + 2*I*a)) + 2*(d*cos(8*b*
x + 8*a) - 2*d*cos(4*b*x + 4*a) + I*d*sin(8*b*x + 8*a) - 2*I*d*sin(4*b*x +
 4*a) + d)*dilog(-e^(I*b*x + I*a)) + 2*(d*cos(8*b*x + 8*a) - 2*d*cos(4*b*x
 + 4*a) + I*d*sin(8*b*x + 8*a) - 2*I*d*sin(4*b*x + 4*a) + d)*dilog(e^(I*b*
x + I*a)) + (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(8*b*x + 8*a) - 2*(-
I*b*d*x - I*b*c)*cos(4*b*x + 4*a) + (b*d*x + b*c)*sin(8*b*x + 8*a) - 2*(b*
d*x + b*c)*sin(4*b*x + 4*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 +
 2*cos(2*b*x + 2*a) + 1) + (I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(8*b...
```

Giac [F]

$$\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx = \int (dx+c)\csc(bx+a)^3\sec(bx+a)^3\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)/(cos(a + b*x)^3*sin(a + b*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx$$
$$= \frac{2\left(\int \csc(bx+a)^3\sec(bx+a)^3\,x dx\right)\sin(bx+a)^4\,bd - 2\left(\int \csc(bx+a)^3\sec(bx+a)^3\,x dx\right)\sin(bx+a)^2\,b}{}$$

input
```
int((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

output

```
(2*int(csc(a + b*x)**3*sec(a + b*x)**3*x,x)*sin(a + b*x)**4*b*d - 2*int(cs
c(a + b*x)**3*sec(a + b*x)**3*x,x)*sin(a + b*x)**2*b*d - 4*log(tan((a + b*
x)/2) - 1)*sin(a + b*x)**4*c + 4*log(tan((a + b*x)/2) - 1)*sin(a + b*x)**2
*c - 4*log(tan((a + b*x)/2) + 1)*sin(a + b*x)**4*c + 4*log(tan((a + b*x)/2
) + 1)*sin(a + b*x)**2*c + 4*log(tan((a + b*x)/2))*sin(a + b*x)**4*c - 4*l
og(tan((a + b*x)/2))*sin(a + b*x)**2*c - sin(a + b*x)**4*c - sin(a + b*x)*
*2*c + c)/(2*sin(a + b*x)**2*b*(sin(a + b*x)**2 - 1))
```

3.326 $\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$

Optimal result . . . 2781
Mathematica [N/A] . . . 2781
Rubi [N/A] . . . 2782
Maple [N/A] . . . 2783
Fricas [N/A] . . . 2783
Sympy [N/A] . . . 2784
Maxima [N/A] . . . 2784
Giac [F(-2)] . . . 2785
Mupad [N/A] . . . 2786
Reduce [N/A] . . . 2786

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx = 8\text{Int}\left(\frac{\csc^3(2a+2bx)}{c+dx}, x\right)$$

output
```
8*Defer(Int)(csc(2*b*x+2*a)^3/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 20.47 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x),x]
```

output
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4919, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

$$\downarrow 4919$$

$$8\int \frac{\csc^3(2a+2bx)}{c+dx}dx$$

$$\downarrow 3042$$

$$8\int \frac{\csc(2a+2bx)^3}{c+dx}dx$$

$$\downarrow 4680$$

$$8\int \frac{\csc^3(2a+2bx)}{c+dx}dx$$

input
```
Int[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4919

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc(bx+a)^3 \sec(bx+a)^3}{dx+c} dx$$

input

```
int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x)
```

output

```
int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^3 \sec(bx+a)^3}{dx+c}\,dx$$

input

```
integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output

```
integral(csc(b*x + a)^3*sec(b*x + a)^3/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 20.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)**3*sec(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(csc(a + b*x)**3*sec(a + b*x)**3/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 6.96 (sec) , antiderivative size = 2410, normalized size of antiderivative = 100.42

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output

```
(2*(2*(b*d*x + b*c)*cos(6*b*x + 6*a) + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) -
d*sin(6*b*x + 6*a) + d*sin(2*b*x + 2*a))*cos(8*b*x + 8*a) + 4*(b*d*x + b*c
 - 2*(b*d*x + b*c)*cos(4*b*x + 4*a) - d*sin(4*b*x + 4*a))*cos(6*b*x + 6*a)
 - 4*(2*(b*d*x + b*c)*cos(2*b*x + 2*a) + d*sin(2*b*x + 2*a))*cos(4*b*x + 4
*a) + 4*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*
c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(8*b*x + 8*a)^2 + 4*(b^2*d^
2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + (b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*sin(8*b*x + 8*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2)*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*
c^2)*sin(4*b*x + 4*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a))*cos(8*b*x + 8*a) - 4*(b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a))*integrate(2*(2*b^2*d
^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*sin(2*b*x + 2*a)/(b^2*d^3*x^3 + 3*
b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 +
 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2
*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)), x) - (b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(8*b*x
 + 8*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(8*b*x + 8*a)^2 - 4*(b^2*d^2*x^...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \text{Exception raised: RuntimeError}$$

input

```
integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output

```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Not invertible Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 20.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos(a+b\,x)^3\sin(a+b\,x)^3\,(c+d\,x)}\,dx$$

input
```
int(1/(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^3}{dx+c}dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x)
```

output
```
int((csc(a + b*x)**3*sec(a + b*x)**3)/(c + d*x),x)
```

3.327 $\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$

Optimal result . . . 2787
Mathematica [N/A] . . . 2787
Rubi [N/A] . . . 2788
Maple [N/A] . . . 2789
Fricas [N/A] . . . 2789
Sympy [N/A] . . . 2790
Maxima [N/A] . . . 2790
Giac [F(-1)] . . . 2791
Mupad [N/A] . . . 2792
Reduce [N/A] . . . 2792

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = 8\text{Int}\left(\frac{\csc^3(2a+2bx)}{(c+dx)^2}, x\right)$$

output
```
8*Defer(Int)(csc(2*b*x+2*a)^3/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 23.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4919, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

$$\downarrow 4919$$

$$8\int \frac{\csc^3(2a+2bx)}{(c+dx)^2}dx$$

$$\downarrow 3042$$

$$8\int \frac{\csc(2a+2bx)^3}{(c+dx)^2}dx$$

$$\downarrow 4680$$

$$8\int \frac{\csc^3(2a+2bx)}{(c+dx)^2}dx$$

input
```
Int[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc(bx+a)^3 \sec(bx+a)^3}{(dx+c)^2} dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\, dx = \int \frac{\csc(bx+a)^3 \sec(bx+a)^3}{(dx+c)^2}\, dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^3*sec(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 40.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(csc(b*x+a)**3*sec(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)**3*sec(a + b*x)**3/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 20.79 (sec) , antiderivative size = 3083, normalized size of antiderivative = 128.46

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^3}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output

```
(4*((b*d*x + b*c)*cos(6*b*x + 6*a) + (b*d*x + b*c)*cos(2*b*x + 2*a) - d*si
n(6*b*x + 6*a) + d*sin(2*b*x + 2*a))*cos(8*b*x + 8*a) + 4*(b*d*x + b*c - 2
*(b*d*x + b*c)*cos(4*b*x + 4*a) - 2*d*sin(4*b*x + 4*a))*cos(6*b*x + 6*a) -
 8*((b*d*x + b*c)*cos(2*b*x + 2*a) + d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
+ 4*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2
*c^3)*cos(8*b*x + 8*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*
x + b^2*c^3)*cos(4*b*x + 4*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*sin(8*b*x + 8*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 +
3*b^2*c^2*d*x + b^2*c^3)*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + 4*(b^2*d^3*x^
3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 + 2*(b^2
*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*
b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a))*cos(8*b*x + 8*a
) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x
+ 4*a))*integrate(2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 3*d^2)*sin(
2*b*x + 2*a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^
3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b
^2*c^3*d*x + b^2*c^4)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3
+ 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(2*b*x + 2*a)^2 + 2*(b^2
*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input

```
integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output

```
Timed out
```

Mupad [N/A]

Not integrable

Time = 20.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\cos(a+bx)^3\sin(a+bx)^3(c+dx)^2}\,dx$$

input
```
int(1/(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^3}{d^2x^2+2cdx+c^2}dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int((csc(a + b*x)**3*sec(a + b*x)**3)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.328 $\int x \cos^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 2793
Mathematica [A] (verified) . . . 2793
Rubi [A] (verified) . . . 2794
Maple [F] . . . 2796
Fricas [F(-2)] . . . 2796
Sympy [F(-1)] . . . 2796
Maxima [F] . . . 2797
Giac [F] . . . 2797
Mupad [F(-1)] . . . 2797
Reduce [F] . . . 2798

Optimal result

Integrand size = 18, antiderivative size = 83

$$\int x \cos^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx = -\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b} + \frac{20\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{147b^2} + \frac{20\sqrt{\cos(a+bx)}\sin(a+bx)}{147b^2} + \frac{4\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)}{49b^2}$$

output

```
-2/7*x*cos(b*x+a)^(7/2)/b+20/147*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))/b^
2+20/147*cos(b*x+a)^(1/2)*sin(b*x+a)/b^2+4/49*cos(b*x+a)^(5/2)*sin(b*x+a)/
b^2
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int x \cos^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx$$

$$= \frac{40\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right) + \sqrt{\cos(a+bx)}(-63bx\cos(a+bx) - 21bx\cos(3(a+bx)) + 46\sin(a+bx) +}{294b^2}$$

input
```
Integrate[x*Cos[a + b*x]^(5/2)*Sin[a + b*x],x]
```

output
```
(40*EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*(-63*b*x*Cos[a + b*x] -
 21*b*x*Cos[3*(a + b*x)] + 46*Sin[a + b*x] + 6*Sin[3*(a + b*x)]))/(294*b^2
)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3925, 3042, 3115, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin(a+bx) \cos^{\frac{5}{2}}(a+bx)\, dx$$

$$\downarrow 3925$$

$$\frac{2\int \cos^{\frac{7}{2}}(a+bx)dx}{7b} - \frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}$$

$$\downarrow 3042$$

$$\frac{2\int \sin\left(a+bx+\frac{\pi}{2}\right)^{7/2} dx}{7b} - \frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}$$

$$\downarrow 3115$$

$$\frac{2\left(\frac{5}{7}\int \cos^{\frac{3}{2}}(a+bx)dx + \frac{2\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{7b}\right)}{7b} - \frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}$$

$$\downarrow 3042$$

$$\frac{2\left(\frac{5}{7}\int \sin\left(a+bx+\frac{\pi}{2}\right)^{3/2} dx + \frac{2\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{7b}\right)}{7b} - \frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}$$

$$\downarrow 3115$$

$$\frac{2\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\cos(a+bx)}}dx+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}\right)+\frac{2\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{7b}\right)}{7b}-\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}$$

$$\downarrow 3042$$

$$\frac{2\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}\right)+\frac{2\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{7b}\right)}{7b}-\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}$$

$$\downarrow 3120$$

$$\frac{2\left(\frac{2\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{7b}+\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b}+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}\right)\right)}{7b}-\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}$$

input
```
Int[x*Cos[a + b*x]^(5/2)*Sin[a + b*x],x]
```

output
```
(-2*x*Cos[a + b*x]^(7/2))/(7*b) + (2*((2*Cos[a + b*x]^(5/2)*Sin[a + b*x])/
(7*b) + (5*((2*EllipticF[(a + b*x)/2, 2])/(3*b) + (2*Sqrt[Cos[a + b*x]]*Si
n[a + b*x])/(3*b)))/7))/(7*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)  Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3925

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cos[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Maple [F]

$$\int x \cos(bx+a)^{\frac{5}{2}} \sin(bx+a)\, dx$$

input

```
int(x*cos(b*x+a)^(5/2)*sin(b*x+a),x)
```

output

```
int(x*cos(b*x+a)^(5/2)*sin(b*x+a),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x \cos^{\frac{5}{2}}(a+bx) \sin(a+bx)\, dx = \text{Exception raised: TypeError}$$

input

```
integrate(x*cos(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int x \cos^{\frac{5}{2}}(a+bx) \sin(a+bx)\, dx = \text{Timed out}$$

input

```
integrate(x*cos(b*x+a)**(5/2)*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x \cos^{\frac{5}{2}}(a+bx) \sin(a+bx)\, dx = \int x \cos{(bx+a)}^{\frac{5}{2}} \sin{(bx+a)}\, dx$$

input
```
integrate(x*cos(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)^(5/2)*sin(b*x + a), x)
```

Giac [F]

$$\int x \cos^{\frac{5}{2}}(a+bx) \sin(a+bx)\, dx = \int x \cos{(bx+a)}^{\frac{5}{2}} \sin{(bx+a)}\, dx$$

input
```
integrate(x*cos(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)^(5/2)*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \cos^{\frac{5}{2}}(a+bx) \sin(a+bx)\, dx = \int x \cos{(a+b\,x)}^{5/2} \sin{(a+b\,x)}\, dx$$

input
```
int(x*cos(a + b*x)^(5/2)*sin(a + b*x),x)
```

output
```
int(x*cos(a + b*x)^(5/2)*sin(a + b*x), x)
```

Reduce [F]

$$\int x \cos^{\frac{5}{2}}(a+bx) \sin(a+bx)\, dx = \int \sqrt{\cos(bx+a)}\, \cos(bx+a)^2 \sin(bx+a)\, x dx$$

input
```
int(x*cos(b*x+a)^(5/2)*sin(b*x+a),x)
```

output
```
int(sqrt(cos(a + b*x))*cos(a + b*x)**2*sin(a + b*x)*x,x)
```

3.329 $\int x \cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 2799
Mathematica [A] (verified) . . . 2799
Rubi [A] (verified) . . . 2800
Maple [F] . . . 2801
Fricas [F(-2)] . . . 2802
Sympy [F(-1)] . . . 2802
Maxima [F] . . . 2802
Giac [F] . . . 2803
Mupad [F(-1)] . . . 2803
Reduce [F] . . . 2803

Optimal result

Integrand size = 18, antiderivative size = 60

$$\int x \cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = -\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b} + \frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2} + \frac{4\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)}{25b^2}$$

output

```
-2/5*x*cos(b*x+a)^(5/2)/b+12/25*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b^2+
4/25*cos(b*x+a)^(3/2)*sin(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int x \cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = -\frac{2\left(-6E\left(\frac{1}{2}(a+bx)\middle|2\right) + \cos^{\frac{3}{2}}(a+bx)(5bx\cos(a+bx) - 2\sin(a+bx))\right)}{25b^2}$$

input

```
Integrate[x*Cos[a + b*x]^(3/2)*Sin[a + b*x],x]
```

output
```
(-2*(-6*EllipticE[(a + b*x)/2, 2] + Cos[a + b*x]^(3/2)*(5*b*x*Cos[a + b*x]
 - 2*Sin[a + b*x])))/(25*b^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3925, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin(a+bx) \cos^{\frac{3}{2}}(a+bx)\, dx$$

$$\downarrow 3925$$

$$\frac{2\int \cos^{\frac{5}{2}}(a+bx)dx}{5b} - \frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}$$

$$\downarrow 3042$$

$$\frac{2\int \sin\left(a+bx+\frac{\pi}{2}\right)^{5/2} dx}{5b} - \frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}$$

$$\downarrow 3115$$

$$\frac{2\left(\frac{3}{5}\int \sqrt{\cos(a+bx)}dx + \frac{2\sin(a+bx)\cos^{\frac{3}{2}}(a+bx)}{5b}\right)}{5b} - \frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}$$

$$\downarrow 3042$$

$$\frac{2\left(\frac{3}{5}\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx + \frac{2\sin(a+bx)\cos^{\frac{3}{2}}(a+bx)}{5b}\right)}{5b} - \frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}$$

$$\downarrow 3119$$

$$\frac{2\left(\frac{6E\left(\frac{1}{2}(a+bx)|2\right)}{5b} + \frac{2\sin(a+bx)\cos^{\frac{3}{2}}(a+bx)}{5b}\right)}{5b} - \frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}$$

input
```
Int[x*Cos[a + b*x]^(3/2)*Sin[a + b*x],x]
```

output
```
(-2*x*Cos[a + b*x]^(5/2))/(5*b) + (2*((6*EllipticE[(a + b*x)/2, 2])/(5*b)
+ (2*Cos[a + b*x]^(3/2)*Sin[a + b*x])/(5*b)))/(5*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3925
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cos[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Maple [F]

$$\int x \cos(bx+a)^{\frac{3}{2}} \sin(bx+a)\, dx$$

input
```
int(x*cos(b*x+a)^(3/2)*sin(b*x+a),x)
```

output
```
int(x*cos(b*x+a)^(3/2)*sin(b*x+a),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x \cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int x \cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x*cos(b*x+a)**(3/2)*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x \cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = \int x \cos\left(bx+a\right)^{\frac{3}{2}} \sin\left(bx+a\right)\,dx$$

input
```
integrate(x*cos(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)^(3/2)*sin(b*x + a), x)
```

Giac [F]

$$\int x \cos^{\frac{3}{2}}(a+bx) \sin(a+bx)\, dx = \int x \cos\left(bx+a\right)^{\frac{3}{2}} \sin\left(bx+a\right)\, dx$$

input
```
integrate(x*cos(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)^(3/2)*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \cos^{\frac{3}{2}}(a+bx) \sin(a+bx)\, dx = \int x \cos\left(a+b\,x\right)^{3/2} \sin\left(a+b\,x\right)\, dx$$

input
```
int(x*cos(a + b*x)^(3/2)*sin(a + b*x),x)
```

output
```
int(x*cos(a + b*x)^(3/2)*sin(a + b*x), x)
```

Reduce [F]

$$\int x \cos^{\frac{3}{2}}(a+bx) \sin(a+bx)\, dx = \int \sqrt{\cos\left(bx+a\right)}\, \cos\left(bx+a\right) \sin\left(bx+a\right) x dx$$

input
```
int(x*cos(b*x+a)^(3/2)*sin(b*x+a),x)
```

output
```
int(sqrt(cos(a + b*x))*cos(a + b*x)*sin(a + b*x)*x,x)
```

3.330 $\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx$

Optimal result . . . 2804
Mathematica [A] (verified) . . . 2804
Rubi [A] (verified) . . . 2805
Maple [F] . . . 2806
Fricas [F(-2)] . . . 2807
Sympy [F] . . . 2807
Maxima [F] . . . 2807
Giac [F] . . . 2808
Mupad [F(-1)] . . . 2808
Reduce [F] . . . 2808

Optimal result

Integrand size = 18, antiderivative size = 60

$$\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx = -\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b} + \frac{4\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{9b^2} + \frac{4\sqrt{\cos(a+bx)}\sin(a+bx)}{9b^2}$$

output

```
-2/3*x*cos(b*x+a)^(3/2)/b+4/9*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))/b^2+4
/9*cos(b*x+a)^(1/2)*sin(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx = \frac{4\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right) + 2\sqrt{\cos(a+bx)}(-3bx\cos(a+bx)+2\sin(a+bx))}{9b^2}$$

input

```
Integrate[x*Sqrt[Cos[a + b*x]]*Sin[a + b*x],x]
```

output
```
(4*EllipticF[(a + b*x)/2, 2] + 2*Sqrt[Cos[a + b*x]]*(-3*b*x*Cos[a + b*x] +
 2*Sin[a + b*x]))/(9*b^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3925, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin(a+bx)\sqrt{\cos(a+bx)}\,dx$$

$$\downarrow 3925$$

$$\frac{2\int \cos^{\frac{3}{2}}(a+bx)dx}{3b} - \frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}$$

$$\downarrow 3042$$

$$\frac{2\int \sin\left(a+bx+\frac{\pi}{2}\right)^{3/2}dx}{3b} - \frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}$$

$$\downarrow 3115$$

$$\frac{2\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(a+bx)}}dx + \frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}\right)}{3b} - \frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}$$

$$\downarrow 3042$$

$$\frac{2\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx + \frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}\right)}{3b} - \frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}$$

$$\downarrow 3120$$

$$\frac{2\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b} + \frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}\right)}{3b} - \frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}$$

input
```
Int[x*Sqrt[Cos[a + b*x]]*Sin[a + b*x],x]
```

output
```
(-2*x*Cos[a + b*x]^(3/2))/(3*b) + (2*((2*EllipticF[(a + b*x)/2, 2])/(3*b)
+ (2*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(3*b)))/(3*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3925
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cos[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Maple [F]

$$\int x\sqrt{\cos\left(bx+a\right)}\,\sin\left(bx+a\right)\,dx$$

input
```
int(x*cos(b*x+a)^(1/2)*sin(b*x+a),x)
```

output
```
int(x*cos(b*x+a)^(1/2)*sin(b*x+a),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)^(1/2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx = \int x\sin(a+bx)\sqrt{\cos(a+bx)}\,dx$$

input
```
integrate(x*cos(b*x+a)**(1/2)*sin(b*x+a),x)
```

output
```
Integral(x*sin(a + b*x)*sqrt(cos(a + b*x)), x)
```

Maxima [F]

$$\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx = \int x\sqrt{\cos(bx+a)}\sin(bx+a)\,dx$$

input
```
integrate(x*cos(b*x+a)^(1/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*sqrt(cos(b*x + a))*sin(b*x + a), x)
```

Giac [F]

$$\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx = \int x\sqrt{\cos\left(bx+a\right)}\sin\left(bx+a\right)\,dx$$

input
```
integrate(x*cos(b*x+a)^(1/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*sqrt(cos(b*x + a))*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx = \int x\,\sqrt{\cos\left(a+b\,x\right)}\,\sin\left(a+b\,x\right)\,dx$$

input
```
int(x*cos(a + b*x)^(1/2)*sin(a + b*x),x)
```

output
```
int(x*cos(a + b*x)^(1/2)*sin(a + b*x), x)
```

Reduce [F]

$$\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx = \int \sqrt{\cos\left(bx+a\right)}\,\sin\left(bx+a\right)x dx$$

input
```
int(x*cos(b*x+a)^(1/2)*sin(b*x+a),x)
```

output
```
int(sqrt(cos(a + b*x))*sin(a + b*x)*x,x)
```

3.331 $\int \frac{x \sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx$

Optimal result . . . 2809
Mathematica [C] (verified) . . . 2809
Rubi [A] (verified) . . . 2810
Maple [C] (verified) . . . 2811
Fricas [F(-2)] . . . 2812
Sympy [F] . . . 2812
Maxima [F] . . . 2812
Giac [F] . . . 2813
Mupad [F(-1)] . . . 2813
Reduce [F] . . . 2813

Optimal result

Integrand size = 18, antiderivative size = 33

$$\int \frac{x \sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx = -\frac{2x\sqrt{\cos(a+bx)}}{b} + \frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

output
```
-2*x*cos(b*x+a)^(1/2)/b+4*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.67 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int \frac{x \sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx = -\frac{2\sqrt{\cos(a+bx)}\left(bx - 2\tan(a+bx) + \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\tan^2(a+bx)\right)\sqrt[4]{\sec^2(a+bx)}\tan\right.}{b^2}$$

input
```
Integrate[(x*Sin[a + b*x])/Sqrt[Cos[a + b*x]],x]
```

output
```
(-2*Sqrt[Cos[a + b*x]]*(b*x - 2*Tan[a + b*x] + Hypergeometric2F1[1/4, 1/2,
 3/2, -Tan[a + b*x]^2]*(Sec[a + b*x]^2)^(1/4)*Tan[a + b*x]))/b^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3925, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx$$

$$\downarrow 3925$$

$$\frac{2\int \sqrt{\cos(a+bx)}dx}{b} - \frac{2x\sqrt{\cos(a+bx)}}{b}$$

$$\downarrow 3042$$

$$\frac{2\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{b} - \frac{2x\sqrt{\cos(a+bx)}}{b}$$

$$\downarrow 3119$$

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{b^2} - \frac{2x\sqrt{\cos(a+bx)}}{b}$$

input
```
Int[(x*Sin[a + b*x])/Sqrt[Cos[a + b*x]],x]
```

output
```
(-2*x*Sqrt[Cos[a + b*x]])/b + (4*EllipticE[(a + b*x)/2, 2])/b^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3925
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cos[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 310, normalized size of antiderivative = 9.39

method	result
risch	$-\frac{(bx+2i)\left(e^{2i(bx+a)}+1\right)\sqrt{2}\,e^{-i(bx+a)}}{b^2\sqrt{e^{-i(bx+a)}\left(e^{2i(bx+a)}+1\right)}} - \frac{2i\left(-\frac{2\left(e^{2i(bx+a)}+1\right)}{\sqrt{e^{i(bx+a)}\left(e^{2i(bx+a)}+1\right)}}+\frac{i\sqrt{-i\left(e^{i(bx+a)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(bx+a)}-i\right)}\,\sqrt{ie^{i(bx+a)}}\left(-2i\,\mathrm{E}\right.}{\sqrt{e^{3i}}}\right.$ … $b^2\sqrt$

input
```
int(x*sin(b*x+a)/cos(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x+2*I)*(exp(I*(b*x+a))^2+1)/b^2*2^(1/2)/(1/exp(I*(b*x+a))*(exp(I*(b*x+
a))^2+1))^(1/2)/exp(I*(b*x+a))-2*I/b^2*(-2*(exp(I*(b*x+a))^2+1)/(exp(I*(b*
x+a))*(exp(I*(b*x+a))^2+1))^(1/2)+I*(-I*(exp(I*(b*x+a))+I))^(1/2)*2^(1/2)*
(I*(exp(I*(b*x+a))-I))^(1/2)*(I*exp(I*(b*x+a)))^(1/2)/(exp(I*(b*x+a))^3+ex
p(I*(b*x+a)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(b*x+a))+I))^(1/2),1/2*2^(1
/2))+I*EllipticF((-I*(exp(I*(b*x+a))+I))^(1/2),1/2*2^(1/2)))*2^(1/2)/(1/e
xp(I*(b*x+a))*(exp(I*(b*x+a))^2+1))^(1/2)*(exp(I*(b*x+a))*(exp(I*(b*x+a))^
2+1))^(1/2)/exp(I*(b*x+a))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{x \sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{x \sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)**(1/2),x)
```

output
```
Integral(x*sin(a + b*x)/sqrt(cos(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x \sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{x \sin(bx+a)}{\sqrt{\cos(bx+a)}}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*sin(b*x + a)/sqrt(cos(b*x + a)), x)
```

Giac [F]

$$\int \frac{x \sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{x \sin(bx+a)}{\sqrt{\cos(bx+a)}}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*sin(b*x + a)/sqrt(cos(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{x \sin(a+b\,x)}{\sqrt{\cos(a+b\,x)}}\,dx$$

input
```
int((x*sin(a + b*x))/cos(a + b*x)^(1/2),x)
```

output
```
int((x*sin(a + b*x))/cos(a + b*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x \sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{\sqrt{\cos(bx+a)}\,\sin(bx+a)\,x}{\cos(bx+a)}\,dx$$

input
```
int(x*sin(b*x+a)/cos(b*x+a)^(1/2),x)
```

output
```
int((sqrt(cos(a + b*x))*sin(a + b*x)*x)/cos(a + b*x),x)
```

3.332 $\int \frac{x \sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result . . . 2814
Mathematica [A] (verified) . . . 2814
Rubi [A] (verified) . . . 2815
Maple [F] . . . 2816
Fricas [F(-2)] . . . 2816
Sympy [F] . . . 2817
Maxima [F] . . . 2817
Giac [F] . . . 2817
Mupad [F(-1)] . . . 2818
Reduce [F] . . . 2818

Optimal result

Integrand size = 18, antiderivative size = 33

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{4\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b^2}$$

output

```
2*x/b/cos(b*x+a)^(1/2)-4*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))/b^2
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{4\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b^2}$$

input

```
Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(3/2),x]
```

output

```
(2*x)/(b*Sqrt[Cos[a + b*x]]) - (4*EllipticF[(a + b*x)/2, 2])/b^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3925, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx$$

$$\downarrow 3925$$

$$\frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{2\int \frac{1}{\sqrt{\cos(a+bx)}}dx}{b}$$

$$\downarrow 3042$$

$$\frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{2\int \frac{1}{\sqrt{\sin(a+bx+\frac{\pi}{2})}}dx}{b}$$

$$\downarrow 3120$$

$$\frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{4\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b^2}$$

input
```
Int[(x*Sin[a + b*x])/Cos[a + b*x]^(3/2),x]
```

output
```
(2*x)/(b*Sqrt[Cos[a + b*x]]) - (4*EllipticF[(a + b*x)/2, 2])/b^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3925

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cos[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Maple [F]

$$\int \frac{x \sin(bx+a)}{\cos(bx+a)^{\frac{3}{2}}} dx$$

input

```
int(x*sin(b*x+a)/cos(b*x+a)^(3/2),x)
```

output

```
int(x*sin(b*x+a)/cos(b*x+a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)} dx = \text{Exception raised: TypeError}$$

input

```
integrate(x*sin(b*x+a)/cos(b*x+a)^(3/2),x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x \sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)**(3/2),x)
```

output
```
Integral(x*sin(a + b*x)/cos(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x \sin(bx+a)}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x*sin(b*x + a)/cos(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x \sin(bx+a)}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*sin(b*x + a)/cos(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\,\sin(a+b\,x)}{\cos(a+b\,x)^{3/2}}\,dx$$

input
```
int((x*sin(a + b*x))/cos(a + b*x)^(3/2),x)
```

output
```
int((x*sin(a + b*x))/cos(a + b*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \frac{-2\cos(bx+a)\left(\int \frac{\sqrt{\cos(bx+a)}}{\cos(bx+a)}dx\right) + 2\sqrt{\cos(bx+a)}\,x}{\cos(bx+a)\,b}$$

input
```
int(x*sin(b*x+a)/cos(b*x+a)^(3/2),x)
```

output
```
(2*( - cos(a + b*x)*int(sqrt(cos(a + b*x))/cos(a + b*x),x) + sqrt(cos(a +
b*x))*x))/(cos(a + b*x)*b)
```

3.333 $\int \frac{x \sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx$

Optimal result . . . 2819
Mathematica [A] (verified) . . . 2819
Rubi [A] (verified) . . . 2820
Maple [F] . . . 2821
Fricas [F(-2)] . . . 2822
Sympy [F(-1)] . . . 2822
Maxima [F] . . . 2822
Giac [F] . . . 2823
Mupad [F(-1)] . . . 2823
Reduce [F] . . . 2823

Optimal result

Integrand size = 18, antiderivative size = 60

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx = \frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)} + \frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b^2} - \frac{4\sin(a+bx)}{3b^2\sqrt{\cos(a+bx)}}$$

output

```
2/3*x/b/cos(b*x+a)^(3/2)+4/3*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b^2-4/3
*sin(b*x+a)/b^2/cos(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx = \frac{2\left(bx + 2\cos^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right) - \sin(2(a+bx))\right)}{3b^2\cos^{\frac{3}{2}}(a+bx)}$$

input

```
Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(5/2),x]
```

output

```
(2*(b*x + 2*Cos[a + b*x]^(3/2)*EllipticE[(a + b*x)/2, 2] - Sin[2*(a + b*x)
]))/(3*b^2*Cos[a + b*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3925, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx$$

$$\downarrow 3925$$

$$\frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)} - \frac{2\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}dx}{3b}$$

$$\downarrow 3042$$

$$\frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)} - \frac{2\int \frac{1}{\sin(a+bx+\frac{\pi}{2})^{3/2}}dx}{3b}$$

$$\downarrow 3116$$

$$\frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)} - \frac{2\left(\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}} - \int \sqrt{\cos(a+bx)}dx\right)}{3b}$$

$$\downarrow 3042$$

$$\frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)} - \frac{2\left(\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}} - \int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx\right)}{3b}$$

$$\downarrow 3119$$

$$\frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)} - \frac{2\left(\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}} - \frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}\right)}{3b}$$

input
```
Int[(x*Sin[a + b*x])/Cos[a + b*x]^(5/2),x]
```

output
```
(2*x)/(3*b*Cos[a + b*x]^(3/2)) - (2*((-2*EllipticE[(a + b*x)/2, 2])/b + (2
*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])))/(3*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3925
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cos[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Maple [F]

$$\int \frac{x \sin(bx+a)}{\cos(bx+a)^{\frac{5}{2}}} dx$$

input
```
int(x*sin(b*x+a)/cos(b*x+a)^(5/2),x)
```

output
```
int(x*sin(b*x+a)/cos(b*x+a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x \sin(bx+a)}{\cos(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*sin(b*x + a)/cos(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x \sin(bx+a)}{\cos(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(x*sin(b*x + a)/cos(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x \sin(a+b\,x)}{\cos(a+b\,x)^{5/2}}\,dx$$

input
```
int((x*sin(a + b*x))/cos(a + b*x)^(5/2),x)
```

output
```
int((x*sin(a + b*x))/cos(a + b*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx = \frac{-\frac{2\cos(bx+a)^2\left(\int \frac{\sqrt{\cos(bx+a)}}{\cos(bx+a)^2}\,dx\right)}{3} + \frac{2\sqrt{\cos(bx+a)}\,x}{3}}{\cos(bx+a)^2 b}$$

input
```
int(x*sin(b*x+a)/cos(b*x+a)^(5/2),x)
```

output
```
(2*( - cos(a + b*x)**2*int(sqrt(cos(a + b*x))/cos(a + b*x)**2,x) + sqrt(co
s(a + b*x))*x))/(3*cos(a + b*x)**2*b)
```

3.334 $\int \frac{x \sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx$

Optimal result . . . 2824
Mathematica [A] (verified) . . . 2824
Rubi [A] (verified) . . . 2825
Maple [F] . . . 2826
Fricas [F(-2)] . . . 2827
Sympy [F(-1)] . . . 2827
Maxima [F] . . . 2827
Giac [F] . . . 2828
Mupad [F(-1)] . . . 2828
Reduce [F] . . . 2828

Optimal result

Integrand size = 18, antiderivative size = 60

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)} - \frac{4\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{15b^2} - \frac{4\sin(a+bx)}{15b^2\cos^{\frac{3}{2}}(a+bx)}$$

output

```
2/5*x/b/cos(b*x+a)^(5/2)-4/15*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))/b^2-4
/15*sin(b*x+a)/b^2/cos(b*x+a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2\left(-3bx + 2\cos^{\frac{5}{2}}(a+bx)\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right) + \sin(2(a+bx))\right)}{15b^2\cos^{\frac{5}{2}}(a+bx)}$$

input

```
Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(7/2),x]
```

output
```
(-2*(-3*b*x + 2*Cos[a + b*x]^(5/2)*EllipticF[(a + b*x)/2, 2] + Sin[2*(a +
b*x)]))/(15*b^2*Cos[a + b*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3925, 3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx$$

$$\downarrow 3925$$

$$\frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)} - \frac{2\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)}dx}{5b}$$

$$\downarrow 3042$$

$$\frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)} - \frac{2\int \frac{1}{\sin\left(a+bx+\frac{\pi}{2}\right)^{5/2}}dx}{5b}$$

$$\downarrow 3116$$

$$\frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)} - \frac{2\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(a+bx)}}dx + \frac{2\sin(a+bx)}{3b\cos^{\frac{3}{2}}(a+bx)}\right)}{5b}$$

$$\downarrow 3042$$

$$\frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)} - \frac{2\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx + \frac{2\sin(a+bx)}{3b\cos^{\frac{3}{2}}(a+bx)}\right)}{5b}$$

$$\downarrow 3120$$

$$\frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)} - \frac{2\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b} + \frac{2\sin(a+bx)}{3b\cos^{\frac{3}{2}}(a+bx)}\right)}{5b}$$

input
```
Int[(x*Sin[a + b*x])/Cos[a + b*x]^(7/2),x]
```

output
```
(2*x)/(5*b*Cos[a + b*x]^(5/2)) - (2*((2*EllipticF[(a + b*x)/2, 2])/(3*b) +
 (2*Sin[a + b*x])/(3*b*Cos[a + b*x]^(3/2))))/(5*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3925
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cos[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Maple [F]

$$\int \frac{x \sin(bx+a)}{\cos(bx+a)^{\frac{7}{2}}} dx$$

input
```
int(x*sin(b*x+a)/cos(b*x+a)^(7/2),x)
```

output
```
int(x*sin(b*x+a)/cos(b*x+a)^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{x \sin(bx+a)}{\cos(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(x*sin(b*x + a)/cos(b*x + a)^(7/2), x)
```

Giac [F]

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{x \sin(bx+a)}{\cos(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(x*sin(b*x + a)/cos(b*x + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{x \sin(a+b\,x)}{\cos(a+b\,x)^{7/2}}\,dx$$

input
```
int((x*sin(a + b*x))/cos(a + b*x)^(7/2),x)
```

output
```
int((x*sin(a + b*x))/cos(a + b*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \frac{-\frac{2\cos(bx+a)^3\left(\int \frac{\sqrt{\cos(bx+a)}}{\cos(bx+a)^3}\,dx\right)}{5} + \frac{2\sqrt{\cos(bx+a)}\,x}{5}}{\cos(bx+a)^3 b}$$

input
```
int(x*sin(b*x+a)/cos(b*x+a)^(7/2),x)
```

output
```
(2*( - cos(a + b*x)**3*int(sqrt(cos(a + b*x))/cos(a + b*x)**3,x) + sqrt(co
s(a + b*x))*x))/(5*cos(a + b*x)**3*b)
```

3.335 $\int \frac{x \sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx$

Optimal result . . . 2829
Mathematica [A] (verified) . . . 2829
Rubi [A] (verified) . . . 2830
Maple [F] . . . 2832
Fricas [F(-2)] . . . 2832
Sympy [F(-1)] . . . 2832
Maxima [F] . . . 2833
Giac [F] . . . 2833
Mupad [F(-1)] . . . 2833
Reduce [F] . . . 2834

Optimal result

Integrand size = 18, antiderivative size = 83

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx = \frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} + \frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2} - \frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)} - \frac{12\sin(a+bx)}{35b^2\sqrt{\cos(a+bx)}}$$

output

```
2/7*x/b/cos(b*x+a)^(7/2)+12/35*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b^2-4
/35*sin(b*x+a)/b^2/cos(b*x+a)^(5/2)-12/35*sin(b*x+a)/b^2/cos(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx = \frac{20bx + 24\cos^{\frac{7}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right) - 10\sin(2(a+bx)) - 3\sin(4(a+bx))}{70b^2\cos^{\frac{7}{2}}(a+bx)}$$

input

```
Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(9/2),x]
```

output
```
(20*b*x + 24*Cos[a + b*x]^(7/2)*EllipticE[(a + b*x)/2, 2] - 10*Sin[2*(a +
b*x)] - 3*Sin[4*(a + b*x)])/(70*b^2*Cos[a + b*x]^(7/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3925, 3042, 3116, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx$$

$$\downarrow 3925$$

$$\frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} - \frac{2\int \frac{1}{\cos^{\frac{7}{2}}(a+bx)}dx}{7b}$$

$$\downarrow 3042$$

$$\frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} - \frac{2\int \frac{1}{\sin\left(a+bx+\frac{\pi}{2}\right)^{7/2}}dx}{7b}$$

$$\downarrow 3116$$

$$\frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} - \frac{2\left(\frac{3}{5}\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}dx + \frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\downarrow 3042$$

$$\frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} - \frac{2\left(\frac{3}{5}\int \frac{1}{\sin\left(a+bx+\frac{\pi}{2}\right)^{3/2}}dx + \frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\downarrow 3116$$

$$\frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} - \frac{2\left(\frac{3}{5}\left(\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}} - \int \sqrt{\cos(a+bx)}dx\right) + \frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\downarrow 3042$$

$$\frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} - \frac{2\left(\frac{3}{5}\left(\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}} - \int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx\right) + \frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\downarrow 3119$$

$$\frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} - \frac{2\left(\frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)} + \frac{3}{5}\left(\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}} - \frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}\right)\right)}{7b}$$

input
```
Int[(x*Sin[a + b*x])/Cos[a + b*x]^(9/2),x]
```

output
```
(2*x)/(7*b*Cos[a + b*x]^(7/2)) - (2*((2*Sin[a + b*x])/(5*b*Cos[a + b*x]^(5
/2)) + (3*((-2*EllipticE[(a + b*x)/2, 2])/b + (2*Sin[a + b*x])/(b*Sqrt[Cos
[a + b*x]])))/5))/(7*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3925
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cos[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Maple [F]

$$\int \frac{x \sin(bx+a)}{\cos(bx+a)^{\frac{9}{2}}} dx$$

input
```
int(x*sin(b*x+a)/cos(b*x+a)^(9/2),x)
```

output
```
int(x*sin(b*x+a)/cos(b*x+a)^(9/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(9/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)} dx = \text{Timed out}$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x \sin(bx+a)}{\cos(bx+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x*sin(b*x + a)/cos(b*x + a)^(9/2), x)
```

Giac [F]

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x \sin(bx+a)}{\cos(bx+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(9/2),x, algorithm="giac")
```

output
```
integrate(x*sin(b*x + a)/cos(b*x + a)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x\,\sin(a+b\,x)}{\cos(a+b\,x)^{9/2}}\,dx$$

input
```
int((x*sin(a + b*x))/cos(a + b*x)^(9/2),x)
```

output
```
int((x*sin(a + b*x))/cos(a + b*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx = \frac{-\frac{2\cos(bx+a)^4\left(\int \frac{\sqrt{\cos(bx+a)}}{\cos(bx+a)^4}\,dx\right)}{7} + \frac{2\sqrt{\cos(bx+a)}\,x}{7}}{\cos(bx+a)^4 b}$$

input

```
int(x*sin(b*x+a)/cos(b*x+a)^(9/2),x)
```

output

```
(2*( - cos(a + b*x)**4*int(sqrt(cos(a + b*x))/cos(a + b*x)**4,x) + sqrt(co
s(a + b*x))*x))/(7*cos(a + b*x)**4*b)
```

3.336 $\int x \sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 2835
Mathematica [A] (verified) . . . 2835
Rubi [A] (verified) . . . 2836
Maple [F] . . . 2838
Fricas [F(-2)] . . . 2839
Sympy [F(-1)] . . . 2839
Maxima [F] . . . 2839
Giac [F] . . . 2840
Mupad [F(-1)] . . . 2840
Reduce [F] . . . 2840

Optimal result

Integrand size = 18, antiderivative size = 103

$$\int x \sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx = \frac{12\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{35b^2} + \frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{12\sqrt{\sec(a+bx)}\sin(a+bx)}{35b^2} - \frac{4\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)}{35b^2}$$

output

```
12/35*cos(b*x+a)^(1/2)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*sec(b*x+a)^(1
/2)/b^2+2/7*x*sec(b*x+a)^(7/2)/b-12/35*sec(b*x+a)^(1/2)*sin(b*x+a)/b^2-4/3
5*sec(b*x+a)^(5/2)*sin(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.63

$$\int x \sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx = \frac{\sec^{\frac{7}{2}}(a+bx)\left(20bx + 24\cos^{\frac{7}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right) - 10\sin(2(a+bx)) - 3\sin(4(a+bx))\right)}{70b^2}$$

input
```
Integrate[x*Sec[a + b*x]^(9/2)*Sin[a + b*x],x]
```

output
```
(Sec[a + b*x]^(7/2)*(20*b*x + 24*Cos[a + b*x]^(7/2)*EllipticE[(a + b*x)/2,
 2] - 10*Sin[2*(a + b*x)] - 3*Sin[4*(a + b*x)]))/(70*b^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4700, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin(a+bx) \sec^{\frac{9}{2}}(a+bx)\, dx$$

$$\downarrow 4700$$

$$\frac{2x \sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\int \sec^{\frac{7}{2}}(a+bx)dx}{7b}$$

$$\downarrow 3042$$

$$\frac{2x \sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\int \csc\left(a+bx+\frac{\pi}{2}\right)^{7/2} dx}{7b}$$

$$\downarrow 4255$$

$$\frac{2x \sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\left(\frac{3}{5}\int \sec^{\frac{3}{2}}(a+bx)dx + \frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}$$

$$\downarrow 3042$$

$$\frac{2x \sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\left(\frac{3}{5}\int \csc\left(a+bx+\frac{\pi}{2}\right)^{3/2} dx + \frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}$$

$$\downarrow 4255$$

$$\frac{2x \sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\left(\frac{3}{5}\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b} - \int \frac{1}{\sqrt{\sec(a+bx)}}dx\right) + \frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}$$

$$\downarrow 3042$$

$$\frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\left(\frac{3}{5}\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b} - \int \frac{1}{\sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}}dx\right) + \frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}$$

$$\downarrow 4258$$

$$\frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\left(\frac{3}{5}\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b} - \sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \sqrt{\cos(a+bx)}dx\right) + \frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}$$

$$\downarrow 3042$$

$$\frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\left(\frac{3}{5}\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b} - \sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx\right) + \frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}$$

$$\downarrow 3119$$

$$\frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\left(\frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b} + \frac{3}{5}\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b} - \frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}\right)\right)}{7b}$$

input

```
Int[x*Sec[a + b*x]^(9/2)*Sin[a + b*x],x]
```

output

```
(2*x*Sec[a + b*x]^(7/2))/(7*b) - (2*((2*Sec[a + b*x]^(5/2)*Sin[a + b*x])/(
5*b) + (3*((-2*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b
*x]])/b + (2*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/b))/5))/(7*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4700
```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sec[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Maple [F]

$$\int x \sec(bx+a)^{\frac{9}{2}} \sin(bx+a)\, dx$$

input
```
int(x*sec(b*x+a)^(9/2)*sin(b*x+a),x)
```

output
```
int(x*sec(b*x+a)^(9/2)*sin(b*x+a),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x \sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sec(b*x+a)^(9/2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x \sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x*sec(b*x+a)**(9/2)*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x \sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx = \int x \sec\left(bx+a\right)^{\frac{9}{2}} \sin\left(bx+a\right)\,dx$$

input
```
integrate(x*sec(b*x+a)^(9/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*sec(b*x + a)^(9/2)*sin(b*x + a), x)
```

Giac [F]

$$\int x \sec^{\frac{9}{2}}(a+bx) \sin(a+bx)\, dx = \int x \sec{(bx+a)}^{\frac{9}{2}} \sin{(bx+a)}\ dx$$

input
```
integrate(x*sec(b*x+a)^(9/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*sec(b*x + a)^(9/2)*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \sec^{\frac{9}{2}}(a+bx) \sin(a+bx)\, dx = \int x\ \sin{(a+b\,x)}\ \left(\frac{1}{\cos{(a+b\,x)}}\right)^{9/2} dx$$

input
```
int(x*sin(a + b*x)*(1/cos(a + b*x))^(9/2),x)
```

output
```
int(x*sin(a + b*x)*(1/cos(a + b*x))^(9/2), x)
```

Reduce [F]

$$\int x \sec^{\frac{9}{2}}(a+bx) \sin(a+bx)\, dx = \int \sqrt{\sec{(bx+a)}}\ \sec{(bx+a)}^4 \sin{(bx+a)}\, x dx$$

input
```
int(x*sec(b*x+a)^(9/2)*sin(b*x+a),x)
```

output
```
int(sqrt(sec(a + b*x))*sec(a + b*x)**4*sin(a + b*x)*x,x)
```

3.337 $\int x \sec^{\frac{7}{2}}(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 2841
Mathematica [A] (verified) . . . 2841
Rubi [A] (verified) . . . 2842
Maple [F] . . . 2844
Fricas [F(-2)] . . . 2844
Sympy [F(-1)] . . . 2845
Maxima [F] . . . 2845
Giac [F] . . . 2845
Mupad [F(-1)] . . . 2846
Reduce [F] . . . 2846

Optimal result

Integrand size = 18, antiderivative size = 80

$$\begin{aligned}&\int x \sec^{\frac{7}{2}}(a+bx)\sin(a+bx)\,dx \\ &= -\frac{4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{\sec(a+bx)}}{15b^2} \\ &\quad + \frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b} - \frac{4\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)}{15b^2}\end{aligned}$$

output

```
-4/15*cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*a+1/2*b*x,2^(1/2))*sec(b*x+a)^(
1/2)/b^2+2/5*x*sec(b*x+a)^(5/2)/b-4/15*sec(b*x+a)^(3/2)*sin(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\begin{aligned}&\int x \sec^{\frac{7}{2}}(a+bx)\sin(a+bx)\,dx \\ &= \frac{2\sqrt{\sec(a+bx)}\left(-2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right) + 3bx\sec^2(a+bx) - 2\tan(a+bx)\right)}{15b^2}\end{aligned}$$

input

```
Integrate[x*Sec[a + b*x]^(7/2)*Sin[a + b*x],x]
```

output

```
(2*Sqrt[Sec[a + b*x]]*(-2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 3
*b*x*Sec[a + b*x]^2 - 2*Tan[a + b*x]))/(15*b^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {4700, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin(a+bx) \sec^{\frac{7}{2}}(a+bx)\, dx$$

$$\downarrow 4700$$

$$\frac{2x \sec^{\frac{5}{2}}(a+bx)}{5b} - \frac{2\int \sec^{\frac{5}{2}}(a+bx)dx}{5b}$$

$$\downarrow 3042$$

$$\frac{2x \sec^{\frac{5}{2}}(a+bx)}{5b} - \frac{2\int \csc\left(a+bx+\frac{\pi}{2}\right)^{5/2} dx}{5b}$$

$$\downarrow 4255$$

$$\frac{2x \sec^{\frac{5}{2}}(a+bx)}{5b} - \frac{2\left(\frac{1}{3}\int \sqrt{\sec(a+bx)}dx + \frac{2\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{3b}\right)}{5b}$$

$$\downarrow 3042$$

$$\frac{2x \sec^{\frac{5}{2}}(a+bx)}{5b} - \frac{2\left(\frac{1}{3}\int \sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}dx + \frac{2\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{3b}\right)}{5b}$$

$$\downarrow 4258$$

$$\frac{2x \sec^{\frac{5}{2}}(a+bx)}{5b} - \frac{2\left(\frac{1}{3}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx + \frac{2\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{3b}\right)}{5b}$$

$$\downarrow 3042$$

$$\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b} - \frac{2\left(\frac{1}{3}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int\frac{1}{\sqrt{\sin(a+bx+\frac{\pi}{2})}}dx + \frac{2\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{3b}\right)}{5b}$$

$$\downarrow 3120$$

$$\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b} - \frac{2\left(\frac{2\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{3b} + \frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b}\right)}{5b}$$

input
```
Int[x*Sec[a + b*x]^(7/2)*Sin[a + b*x],x]
```

output
```
(2*x*Sec[a + b*x]^(5/2))/(5*b) - (2*((2*Sqrt[Cos[a + b*x]]*EllipticF[(a +
b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(3*b) + (2*Sec[a + b*x]^(3/2)*Sin[a + b*x])
/(3*b)))/(5*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4700

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sec[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Maple [F]

$$\int x \sec(bx+a)^{\frac{7}{2}} \sin(bx+a)\, dx$$

input

```
int(x*sec(b*x+a)^(7/2)*sin(b*x+a),x)
```

output

```
int(x*sec(b*x+a)^(7/2)*sin(b*x+a),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x \sec^{\frac{7}{2}}(a+bx) \sin(a+bx)\, dx = \text{Exception raised: TypeError}$$

input

```
integrate(x*sec(b*x+a)^(7/2)*sin(b*x+a),x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x \sec^{\frac{7}{2}}(a+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x*sec(b*x+a)**(7/2)*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x \sec^{\frac{7}{2}}(a+bx)\sin(a+bx)\,dx = \int x \sec\left(bx+a\right)^{\frac{7}{2}} \sin\left(bx+a\right)\,dx$$

input
```
integrate(x*sec(b*x+a)^(7/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*sec(b*x + a)^(7/2)*sin(b*x + a), x)
```

Giac [F]

$$\int x \sec^{\frac{7}{2}}(a+bx)\sin(a+bx)\,dx = \int x \sec\left(bx+a\right)^{\frac{7}{2}} \sin\left(bx+a\right)\,dx$$

input
```
integrate(x*sec(b*x+a)^(7/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*sec(b*x + a)^(7/2)*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \sec^{\frac{7}{2}}(a+bx)\sin(a+bx)\,dx = \int x\,\sin(a+b\,x)\left(\frac{1}{\cos(a+b\,x)}\right)^{7/2} dx$$

input
```
int(x*sin(a + b*x)*(1/cos(a + b*x))^(7/2),x)
```

output
```
int(x*sin(a + b*x)*(1/cos(a + b*x))^(7/2), x)
```

Reduce [F]

$$\int x \sec^{\frac{7}{2}}(a+bx)\sin(a+bx)\,dx = \int \sqrt{\sec(bx+a)}\,\sec(bx+a)^3\sin(bx+a)\,x dx$$

input
```
int(x*sec(b*x+a)^(7/2)*sin(b*x+a),x)
```

output
```
int(sqrt(sec(a + b*x))*sec(a + b*x)**3*sin(a + b*x)*x,x)
```

3.338 $\int x \sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 2847
Mathematica [A] (verified) . . . 2847
Rubi [A] (verified) . . . 2848
Maple [F] . . . 2850
Fricas [F(-2)] . . . 2850
Sympy [F(-1)] . . . 2851
Maxima [F] . . . 2851
Giac [F] . . . 2851
Mupad [F(-1)] . . . 2852
Reduce [F] . . . 2852

Optimal result

Integrand size = 18, antiderivative size = 80

$$\int x \sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx = \frac{4\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{3b^2} + \frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b} - \frac{4\sqrt{\sec(a+bx)}\sin(a+bx)}{3b^2}$$

output
```
4/3*cos(b*x+a)^(1/2)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*sec(b*x+a)^(1/2
)/b^2+2/3*x*sec(b*x+a)^(3/2)/b-4/3*sec(b*x+a)^(1/2)*sin(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int x \sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx = \frac{2\sec^{\frac{3}{2}}(a+bx)\left(bx + 2\cos^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right) - \sin(2(a+bx))\right)}{3b^2}$$

input
```
Integrate[x*Sec[a + b*x]^(5/2)*Sin[a + b*x],x]
```

output
```
(2*Sec[a + b*x]^(3/2)*(b*x + 2*Cos[a + b*x]^(3/2)*EllipticE[(a + b*x)/2, 2
] - Sin[2*(a + b*x)]))/(3*b^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {4700, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin(a+bx) \sec^{\frac{5}{2}}(a+bx)\, dx$$

$$\downarrow 4700$$

$$\frac{2x \sec^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\int \sec^{\frac{3}{2}}(a+bx)dx}{3b}$$

$$\downarrow 3042$$

$$\frac{2x \sec^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\int \csc\left(a+bx+\frac{\pi}{2}\right)^{3/2} dx}{3b}$$

$$\downarrow 4255$$

$$\frac{2x \sec^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b} - \int \frac{1}{\sqrt{\sec(a+bx)}} dx\right)}{3b}$$

$$\downarrow 3042$$

$$\frac{2x \sec^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b} - \int \frac{1}{\sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}} dx\right)}{3b}$$

$$\downarrow 4258$$

$$\frac{2x \sec^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b} - \sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \sqrt{\cos(a+bx)}dx\right)}{3b}$$

$$\downarrow 3042$$

$$\frac{\frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b} - \sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx\right)}{3b}}{\downarrow\ 3119}$$

$$\frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b} - \frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)|2\right)}{b}\right)}{3b}$$

input
```
Int[x*Sec[a + b*x]^(5/2)*Sin[a + b*x],x]
```

output
```
(2*x*Sec[a + b*x]^(3/2))/(3*b) - (2*((-2*Sqrt[Cos[a + b*x]]*EllipticE[(a +
 b*x)/2, 2]*Sqrt[Sec[a + b*x]])/b + (2*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/b)
)/(3*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4700

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sec[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Maple [F]

$$\int x \sec(bx+a)^{\frac{5}{2}} \sin(bx+a)\, dx$$

input

```
int(x*sec(b*x+a)^(5/2)*sin(b*x+a),x)
```

output

```
int(x*sec(b*x+a)^(5/2)*sin(b*x+a),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x \sec^{\frac{5}{2}}(a+bx) \sin(a+bx)\, dx = \text{Exception raised: TypeError}$$

input

```
integrate(x*sec(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x \sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x*sec(b*x+a)**(5/2)*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x \sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx = \int x \sec\left(bx+a\right)^{\frac{5}{2}} \sin\left(bx+a\right)\,dx$$

input
```
integrate(x*sec(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*sec(b*x + a)^(5/2)*sin(b*x + a), x)
```

Giac [F]

$$\int x \sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx = \int x \sec\left(bx+a\right)^{\frac{5}{2}} \sin\left(bx+a\right)\,dx$$

input
```
integrate(x*sec(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*sec(b*x + a)^(5/2)*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx = \int x\,\sin(a+b\,x)\left(\frac{1}{\cos(a+b\,x)}\right)^{5/2} dx$$

input
```
int(x*sin(a + b*x)*(1/cos(a + b*x))^(5/2),x)
```

output
```
int(x*sin(a + b*x)*(1/cos(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int x \sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx = \int \sqrt{\sec(bx+a)}\,\sec(bx+a)^2\sin(bx+a)\,x dx$$

input
```
int(x*sec(b*x+a)^(5/2)*sin(b*x+a),x)
```

output
```
int(sqrt(sec(a + b*x))*sec(a + b*x)**2*sin(a + b*x)*x,x)
```

3.339 $\int x \sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx$

Optimal result . . . 2853
Mathematica [A] (verified) . . . 2853
Rubi [A] (verified) . . . 2854
Maple [F] . . . 2855
Fricas [F(-2)] . . . 2856
Sympy [F(-1)] . . . 2856
Maxima [F] . . . 2856
Giac [F] . . . 2857
Mupad [F(-1)] . . . 2857
Reduce [F] . . . 2857

Optimal result

Integrand size = 18, antiderivative size = 53

$$\int x \sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = \frac{2x\sqrt{\sec(a+bx)}}{b} - \frac{4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{\sec(a+bx)}}{b^2}$$

output

```
2*x*sec(b*x+a)^(1/2)/b-4*cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*a+1/2*b*x,2^
(1/2))*sec(b*x+a)^(1/2)/b^2
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int x \sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = \frac{2\left(bx - 2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\right)\sqrt{\sec(a+bx)}}{b^2}$$

input

```
Integrate[x*Sec[a + b*x]^(3/2)*Sin[a + b*x],x]
```

output
```
(2*(b*x - 2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])*Sqrt[Sec[a + b*x
]])/b^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {4700, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin(a+bx) \sec^{\frac{3}{2}}(a+bx)\, dx$$

$$\downarrow 4700$$

$$\frac{2x\sqrt{\sec(a+bx)}}{b} - \frac{2\int \sqrt{\sec(a+bx)}dx}{b}$$

$$\downarrow 3042$$

$$\frac{2x\sqrt{\sec(a+bx)}}{b} - \frac{2\int \sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}dx}{b}$$

$$\downarrow 4258$$

$$\frac{2x\sqrt{\sec(a+bx)}}{b} - \frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx}{b}$$

$$\downarrow 3042$$

$$\frac{2x\sqrt{\sec(a+bx)}}{b} - \frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\downarrow 3120$$

$$\frac{2x\sqrt{\sec(a+bx)}}{b} - \frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b^2}$$

input
```
Int[x*Sec[a + b*x]^(3/2)*Sin[a + b*x],x]
```

output
```
(2*x*Sqrt[Sec[a + b*x]])/b - (4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2,
2]*Sqrt[Sec[a + b*x]])/b^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4700
```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sec[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Maple [F]

$$\int x \sec(bx+a)^{\frac{3}{2}} \sin(bx+a)\, dx$$

input
```
int(x*sec(b*x+a)^(3/2)*sin(b*x+a),x)
```

output
```
int(x*sec(b*x+a)^(3/2)*sin(b*x+a),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x \sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sec(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x \sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x*sec(b*x+a)**(3/2)*sin(b*x+a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x \sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = \int x \sec\left(bx+a\right)^{\frac{3}{2}} \sin\left(bx+a\right)\,dx$$

input
```
integrate(x*sec(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*sec(b*x + a)^(3/2)*sin(b*x + a), x)
```

Giac [F]

$$\int x \sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = \int x \sec{(bx+a)}^{\frac{3}{2}} \sin{(bx+a)}\,dx$$

input
```
integrate(x*sec(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*sec(b*x + a)^(3/2)*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = \int x\,\sin{(a+b\,x)}\left(\frac{1}{\cos{(a+b\,x)}}\right)^{3/2} dx$$

input
```
int(x*sin(a + b*x)*(1/cos(a + b*x))^(3/2),x)
```

output
```
int(x*sin(a + b*x)*(1/cos(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int x \sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = \int \sqrt{\sec{(bx+a)}}\,\sec{(bx+a)}\sin{(bx+a)}\,x dx$$

input
```
int(x*sec(b*x+a)^(3/2)*sin(b*x+a),x)
```

output
```
int(sqrt(sec(a + b*x))*sec(a + b*x)*sin(a + b*x)*x,x)
```

3.340 $\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx$

Optimal result . . . 2858
Mathematica [C] (verified) . . . 2858
Rubi [A] (verified) . . . 2859
Maple [C] (verified) . . . 2860
Fricas [F(-2)] . . . 2861
Sympy [F] . . . 2861
Maxima [F] . . . 2862
Giac [F] . . . 2862
Mupad [F(-1)] . . . 2862
Reduce [F] . . . 2863

Optimal result

Integrand size = 18, antiderivative size = 53

$$\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx = -\frac{2x}{b\sqrt{\sec(a+bx)}} + \frac{4\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b^2}$$

output

```
-2*x/b/sec(b*x+a)^(1/2)+4*cos(b*x+a)^(1/2)*EllipticE(sin(1/2*a+1/2*b*x),2^
(1/2))*sec(b*x+a)^(1/2)/b^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.54 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx = -\frac{2\left(bx - 2\tan(a+bx) + \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\tan^2(a+bx)\right)\sqrt[4]{\sec^2(a+bx)}\tan(a+bx)\right)}{b^2\sqrt{\sec(a+bx)}}$$

input

```
Integrate[x*Sqrt[Sec[a + b*x]]*Sin[a + b*x],x]
```

output
```
(-2*(b*x - 2*Tan[a + b*x] + Hypergeometric2F1[1/4, 1/2, 3/2, -Tan[a + b*x]
^2]*(Sec[a + b*x]^2)^(1/4)*Tan[a + b*x]))/(b^2*Sqrt[Sec[a + b*x]])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {4700, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin(a+bx)\sqrt{\sec(a+bx)}\,dx$$

$$\downarrow 4700$$

$$\frac{2\int \frac{1}{\sqrt{\sec(a+bx)}}dx}{b} - \frac{2x}{b\sqrt{\sec(a+bx)}}$$

$$\downarrow 3042$$

$$\frac{2\int \frac{1}{\sqrt{\csc(a+bx+\frac{\pi}{2})}}dx}{b} - \frac{2x}{b\sqrt{\sec(a+bx)}}$$

$$\downarrow 4258$$

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \sqrt{\cos(a+bx)}dx}{b} - \frac{2x}{b\sqrt{\sec(a+bx)}}$$

$$\downarrow 3042$$

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{b} - \frac{2x}{b\sqrt{\sec(a+bx)}}$$

$$\downarrow 3119$$

$$\frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2} - \frac{2x}{b\sqrt{\sec(a+bx)}}$$

input
```
Int[x*Sqrt[Sec[a + b*x]]*Sin[a + b*x],x]
```

output
```
(-2*x)/(b*Sqrt[Sec[a + b*x]]) + (4*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/
2, 2]*Sqrt[Sec[a + b*x]])/b^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4700
```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sec[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 310, normalized size of antiderivative = 5.85

method	result
risch	$-\frac{(bx+2i)\left(e^{2i(bx+a)}+1\right)\sqrt{2}\sqrt{\frac{e^{i(bx+a)}}{e^{2i(bx+a)}+1}}\,e^{-i(bx+a)}}{b^2} - \frac{2i\left(-\frac{2\left(e^{2i(bx+a)}+1\right)}{\sqrt{e^{i(bx+a)}\left(e^{2i(bx+a)}+1\right)}}+\frac{i\sqrt{-i\left(e^{i(bx+a)}+i\right)}\,\sqrt{2}\sqrt{i\left(e^{i(bx+a)}-i\right)}\,\sqrt{}$

input
```
int(x*sec(b*x+a)^(1/2)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-(b*x+2*I)*(exp(I*(b*x+a))^2+1)/b^2*2^(1/2)*(exp(I*(b*x+a))/(exp(I*(b*x+a)
)^2+1))^(1/2)/exp(I*(b*x+a))-2*I/b^2*(-2*(exp(I*(b*x+a))^2+1)/(exp(I*(b*x+
a))*(exp(I*(b*x+a))^2+1))^(1/2)+I*(-I*(exp(I*(b*x+a))+I))^(1/2)*2^(1/2)*(I
*(exp(I*(b*x+a))-I))^(1/2)*(I*exp(I*(b*x+a)))^(1/2)/(exp(I*(b*x+a))^3+exp(
I*(b*x+a)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(b*x+a))+I))^(1/2),1/2*2^(1/2
))+I*EllipticF((-I*(exp(I*(b*x+a))+I))^(1/2),1/2*2^(1/2)))*2^(1/2)*(exp(I
*(b*x+a))/(exp(I*(b*x+a))^2+1))^(1/2)*(exp(I*(b*x+a))*(exp(I*(b*x+a))^2+1)
)^(1/2)/exp(I*(b*x+a))
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sec(b*x+a)^(1/2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx = \int x\sin\left(a+bx\right)\sqrt{\sec\left(a+bx\right)}\,dx$$

input
```
integrate(x*sec(b*x+a)**(1/2)*sin(b*x+a),x)
```

output
```
Integral(x*sin(a + b*x)*sqrt(sec(a + b*x)), x)
```

Maxima [F]

$$\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx = \int x\sqrt{\sec(bx+a)}\sin(bx+a)\,dx$$

input
```
integrate(x*sec(b*x+a)^(1/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*sqrt(sec(b*x + a))*sin(b*x + a), x)
```

Giac [F]

$$\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx = \int x\sqrt{\sec(bx+a)}\sin(bx+a)\,dx$$

input
```
integrate(x*sec(b*x+a)^(1/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*sqrt(sec(b*x + a))*sin(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx = \int x\,\sin(a+b\,x)\,\sqrt{\frac{1}{\cos(a+b\,x)}}\,dx$$

input
```
int(x*sin(a + b*x)*(1/cos(a + b*x))^(1/2),x)
```

output
```
int(x*sin(a + b*x)*(1/cos(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx = \int \sqrt{\sec(bx+a)}\,\sin(bx+a)\,x dx$$

input
```
int(x*sec(b*x+a)^(1/2)*sin(b*x+a),x)
```

output
```
int(sqrt(sec(a + b*x))*sin(a + b*x)*x,x)
```

3.341 $\int \frac{x \sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx$

Optimal result . . . 2864
Mathematica [A] (verified) . . . 2864
Rubi [A] (verified) . . . 2865
Maple [F] . . . 2867
Fricas [F(-2)] . . . 2867
Sympy [F] . . . 2868
Maxima [F] . . . 2868
Giac [F] . . . 2868
Mupad [F(-1)] . . . 2869
Reduce [F] . . . 2869

Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{x \sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx = -\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)} + \frac{4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{\sec(a+bx)}}{9b^2} + \frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}}$$

output

```
-2/3*x/b/sec(b*x+a)^(3/2)+4/9*cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*a+1/2*b
*x,2^(1/2))*sec(b*x+a)^(1/2)/b^2+4/9*sin(b*x+a)/b^2/sec(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int \frac{x \sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx = \frac{\sqrt{\sec(a+bx)}\left(-6bx\cos^2(a+bx) + 4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right) + 2\sin(2(a+bx))\right)}{9b^2}$$

input
```
Integrate[(x*Sin[a + b*x])/Sqrt[Sec[a + b*x]],x]
```

output
```
(Sqrt[Sec[a + b*x]]*(-6*b*x*Cos[a + b*x]^2 + 4*Sqrt[Cos[a + b*x]]*Elliptic
F[(a + b*x)/2, 2] + 2*Sin[2*(a + b*x)]))/(9*b^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {4700, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx$$

$$\downarrow 4700$$

$$\frac{2\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}dx}{3b} - \frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

$$\downarrow 3042$$

$$\frac{2\int \frac{1}{\csc\left(a+bx+\frac{\pi}{2}\right)^{3/2}}dx}{3b} - \frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

$$\downarrow 4256$$

$$\frac{2\left(\frac{1}{3}\int \sqrt{\sec(a+bx)}dx + \frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}\right)}{3b} - \frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

$$\downarrow 3042$$

$$\frac{2\left(\frac{1}{3}\int \sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}dx + \frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}\right)}{3b} - \frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

$$\downarrow 4258$$

$$\frac{2\left(\frac{1}{3}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}}dx+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}\right)}{3b}-\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

$$\downarrow 3042$$

$$\frac{2\left(\frac{1}{3}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int\frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}\right)}{3b}-\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

$$\downarrow 3120$$

$$\frac{2\left(\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}+\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b}\right)}{3b}-\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

input
```
Int[(x*Sin[a + b*x])/Sqrt[Sec[a + b*x]],x]
```

output
```
(-2*x)/(3*b*Sec[a + b*x]^(3/2)) + (2*((2*Sqrt[Cos[a + b*x]]*EllipticF[(a +
 b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(3*b) + (2*Sin[a + b*x])/(3*b*Sqrt[Sec[a +
 b*x]])))/(3*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4700

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sec[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Maple [F]

$$\int \frac{x \sin(bx+a)}{\sqrt{\sec(bx+a)}} dx$$

input

```
int(x*sin(b*x+a)/sec(b*x+a)^(1/2),x)
```

output

```
int(x*sin(b*x+a)/sec(b*x+a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sin(a+bx)}{\sqrt{\sec(a+bx)}} dx = \text{Exception raised: TypeError}$$

input

```
integrate(x*sin(b*x+a)/sec(b*x+a)^(1/2),x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{x \sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx = \int \frac{x \sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx$$

input
```
integrate(x*sin(b*x+a)/sec(b*x+a)**(1/2),x)
```

output
```
Integral(x*sin(a + b*x)/sqrt(sec(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x \sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx = \int \frac{x \sin(bx+a)}{\sqrt{\sec(bx+a)}}\,dx$$

input
```
integrate(x*sin(b*x+a)/sec(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*sin(b*x + a)/sqrt(sec(b*x + a)), x)
```

Giac [F]

$$\int \frac{x \sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx = \int \frac{x \sin(bx+a)}{\sqrt{\sec(bx+a)}}\,dx$$

input
```
integrate(x*sin(b*x+a)/sec(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*sin(b*x + a)/sqrt(sec(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx = \int \frac{x\,\sin(a+b\,x)}{\sqrt{\frac{1}{\cos(a+b\,x)}}}\,dx$$

input
```
int((x*sin(a + b*x))/(1/cos(a + b*x))^(1/2),x)
```

output
```
int((x*sin(a + b*x))/(1/cos(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x \sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx = \int \frac{\sqrt{\sec(bx+a)}\,\sin(bx+a)\,x}{\sec(bx+a)}dx$$

input
```
int(x*sin(b*x+a)/sec(b*x+a)^(1/2),x)
```

output
```
int((sqrt(sec(a + b*x))*sin(a + b*x)*x)/sec(a + b*x),x)
```

3.342 $\int \frac{x \sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result . . . 2870
Mathematica [C] (verified) . . . 2870
Rubi [A] (verified) . . . 2871
Maple [F] . . . 2873
Fricas [F(-2)] . . . 2873
Sympy [F] . . . 2874
Maxima [F] . . . 2874
Giac [F] . . . 2874
Mupad [F(-1)] . . . 2875
Reduce [F] . . . 2875

Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{x \sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)} + \frac{12\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{25b^2} + \frac{4\sin(a+bx)}{25b^2\sec^{\frac{3}{2}}(a+bx)}$$

output

```
-2/5*x/b/sec(b*x+a)^(5/2)+12/25*cos(b*x+a)^(1/2)*EllipticE(sin(1/2*a+1/2*b
*x),2^(1/2))*sec(b*x+a)^(1/2)/b^2+4/25*sin(b*x+a)/b^2/sec(b*x+a)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.80 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.21

$$\int \frac{x \sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx = \frac{-5bx - 5bx\cos(2(a+bx)) + \sec(a+bx)\sin(3(a+bx)) + 13\tan(a+bx) - 6\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\right.}{25b^2\sqrt{\sec(a+bx)}}$$

input
```
Integrate[(x*Sin[a + b*x])/Sec[a + b*x]^(3/2),x]
```

output
```
(-5*b*x - 5*b*x*Cos[2*(a + b*x)] + Sec[a + b*x]*Sin[3*(a + b*x)] + 13*Tan[
a + b*x] - 6*Hypergeometric2F1[1/4, 1/2, 3/2, -Tan[a + b*x]^2]*(Sec[a + b*
x]^2)^(1/4)*Tan[a + b*x])/(25*b^2*Sqrt[Sec[a + b*x]])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {4700, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx$$

$$\downarrow 4700$$

$$\frac{2\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}dx}{5b} - \frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}$$

$$\downarrow 3042$$

$$\frac{2\int \frac{1}{\csc\left(a+bx+\frac{\pi}{2}\right)^{5/2}}dx}{5b} - \frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}$$

$$\downarrow 4256$$

$$\frac{2\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(a+bx)}}dx + \frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}\right)}{5b} - \frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}$$

$$\downarrow 3042$$

$$\frac{2\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}}dx + \frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}\right)}{5b} - \frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}$$

$$\downarrow 4258$$

$$\frac{2\left(\frac{3}{5}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int\sqrt{\cos(a+bx)}dx+\frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}\right)}{5b}-\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}$$

$$\downarrow 3042$$

$$\frac{2\left(\frac{3}{5}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx+\frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}\right)}{5b}-\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}$$

$$\downarrow 3119$$

$$\frac{2\left(\frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}+\frac{6\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}\right)}{5b}-\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}$$

input
```
Int[(x*Sin[a + b*x])/Sec[a + b*x]^(3/2),x]
```

output
```
(-2*x)/(5*b*Sec[a + b*x]^(5/2)) + (2*((6*Sqrt[Cos[a + b*x]]*EllipticE[(a +
 b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(5*b) + (2*Sin[a + b*x])/(5*b*Sec[a + b*x]
^(3/2))))/(5*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4700

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sec[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Maple [F]

$$\int \frac{x \sin(bx+a)}{\sec(bx+a)^{\frac{3}{2}}} dx$$

input

```
int(x*sin(b*x+a)/sec(b*x+a)^(3/2),x)
```

output

```
int(x*sin(b*x+a)/sec(b*x+a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)} dx = \text{Exception raised: TypeError}$$

input

```
integrate(x*sin(b*x+a)/sec(b*x+a)^(3/2),x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{x \sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x \sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(x*sin(b*x+a)/sec(b*x+a)**(3/2),x)
```

output
```
Integral(x*sin(a + b*x)/sec(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{x \sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x \sin(bx+a)}{\sec(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/sec(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x*sin(b*x + a)/sec(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x \sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x \sin(bx+a)}{\sec(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/sec(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*sin(b*x + a)/sec(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\,\sin(a+b\,x)}{\left(\frac{1}{\cos(a+b\,x)}\right)^{3/2}}\,dx$$

input
```
int((x*sin(a + b*x))/(1/cos(a + b*x))^(3/2),x)
```

output
```
int((x*sin(a + b*x))/(1/cos(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x \sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\sec(bx+a)}\,\sin(bx+a)\,x}{\sec(bx+a)^2}dx$$

input
```
int(x*sin(b*x+a)/sec(b*x+a)^(3/2),x)
```

output
```
int((sqrt(sec(a + b*x))*sin(a + b*x)*x)/sec(a + b*x)**2,x)
```

3.343 $\int \frac{x \sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx$

Optimal result . . . 2876
Mathematica [A] (verified) . . . 2876
Rubi [A] (verified) . . . 2877
Maple [F] . . . 2879
Fricas [F(-2)] . . . 2880
Sympy [F] . . . 2880
Maxima [F] . . . 2880
Giac [F] . . . 2881
Mupad [F(-1)] . . . 2881
Reduce [F] . . . 2881

Optimal result

Integrand size = 18, antiderivative size = 103

$$\int \frac{x \sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)} + \frac{20\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{\sec(a+bx)}}{147b^2} + \frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)} + \frac{20\sin(a+bx)}{147b^2\sqrt{\sec(a+bx)}}$$

output

```
-2/7*x/b/sec(b*x+a)^(7/2)+20/147*cos(b*x+a)^(1/2)*InverseJacobiAM(1/2*a+1/
2*b*x,2^(1/2))*sec(b*x+a)^(1/2)/b^2+4/49*sin(b*x+a)/b^2/sec(b*x+a)^(5/2)+2
0/147*sin(b*x+a)/b^2/sec(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int \frac{x \sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx = \frac{\sqrt{\sec(a+bx)}\left(-63bx - 84bx\cos(2(a+bx)) - 21bx\cos(4(a+bx)) + 80\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+\right.}{588b^2}$$

input
```
Integrate[(x*Sin[a + b*x])/Sec[a + b*x]^(5/2),x]
```

output
```
(Sqrt[Sec[a + b*x]]*(-63*b*x - 84*b*x*Cos[2*(a + b*x)] - 21*b*x*Cos[4*(a +
 b*x)] + 80*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 52*Sin[2*(a + b
*x)] + 6*Sin[4*(a + b*x)]))/(588*b^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4700, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx$$

$$\downarrow 4700$$

$$\frac{2\int \frac{1}{\sec^{\frac{7}{2}}(a+bx)}dx}{7b} - \frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

$$\downarrow 3042$$

$$\frac{2\int \frac{1}{\csc\left(a+bx+\frac{\pi}{2}\right)^{7/2}}dx}{7b} - \frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

$$\downarrow 4256$$

$$\frac{2\left(\frac{5}{7}\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}dx + \frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}\right)}{7b} - \frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

$$\downarrow 3042$$

$$\frac{2\left(\frac{5}{7}\int \frac{1}{\csc\left(a+bx+\frac{\pi}{2}\right)^{3/2}}dx + \frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}\right)}{7b} - \frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

$$\downarrow 4256$$

$$\frac{2\left(\frac{5}{7}\left(\frac{1}{3}\int\sqrt{\sec(a+bx)}dx+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}\right)+\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}\right)}{7b}-\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

$$\downarrow 3042$$

$$\frac{2\left(\frac{5}{7}\left(\frac{1}{3}\int\sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}dx+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}\right)+\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}\right)}{7b}-\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

$$\downarrow 4258$$

$$\frac{2\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}}dx+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}\right)+\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}\right)}{7b}-\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

$$\downarrow 3042$$

$$\frac{2\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int\frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}\right)+\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}\right)}{7b}-\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

$$\downarrow 3120$$

$$\frac{2\left(\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}+\frac{5}{7}\left(\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}+\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b}\right)\right)}{7b}-\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

input

```
Int[(x*Sin[a + b*x])/Sec[a + b*x]^(5/2),x]
```

output

```
(-2*x)/(7*b*Sec[a + b*x]^(7/2)) + (2*((2*Sin[a + b*x])/(7*b*Sec[a + b*x]^(
5/2)) + (5*((2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b
*x]])/(3*b) + (2*Sin[a + b*x])/(3*b*Sqrt[Sec[a + b*x]])))/7))/(7*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4700
```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sec[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Maple [F]

$$\int \frac{x \sin(bx+a)}{\sec(bx+a)^{\frac{5}{2}}} dx$$

input
```
int(x*sin(b*x+a)/sec(b*x+a)^(5/2),x)
```

output
```
int(x*sin(b*x+a)/sec(b*x+a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sin(b*x+a)/sec(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{x \sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x \sin\left(a+bx\right)}{\sec^{\frac{5}{2}}\left(a+bx\right)}\,dx$$

input
```
integrate(x*sin(b*x+a)/sec(b*x+a)**(5/2),x)
```

output
```
Integral(x*sin(a + b*x)/sec(a + b*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{x \sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x \sin\left(bx+a\right)}{\sec\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/sec(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*sin(b*x + a)/sec(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x \sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x \sin(bx+a)}{\sec(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/sec(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(x*sin(b*x + a)/sec(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x \sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x \sin(a+b\,x)}{\left(\frac{1}{\cos(a+b\,x)}\right)^{5/2}}\,dx$$

input
```
int((x*sin(a + b*x))/(1/cos(a + b*x))^(5/2),x)
```

output
```
int((x*sin(a + b*x))/(1/cos(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{x \sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\sec(bx+a)}\,\sin(bx+a)\,x}{\sec(bx+a)^3}\,dx$$

input
```
int(x*sin(b*x+a)/sec(b*x+a)^(5/2),x)
```

output
```
int((sqrt(sec(a + b*x))*sin(a + b*x)*x)/sec(a + b*x)**3,x)
```

3.344 $\int x \cos(a+bx) \sin^{\frac{5}{2}}(a+bx)\, dx$

Optimal result . . . 2882
Mathematica [A] (verified) . . . 2882
Rubi [A] (verified) . . . 2883
Maple [F] . . . 2885
Fricas [F(-2)] . . . 2885
Sympy [F(-1)] . . . 2885
Maxima [F] . . . 2886
Giac [F] . . . 2886
Mupad [F(-1)] . . . 2886
Reduce [F] . . . 2887

Optimal result

Integrand size = 18, antiderivative size = 88

$$\int x \cos(a+bx) \sin^{\frac{5}{2}}(a+bx)\, dx = -\frac{20 \operatorname{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)}{147b^2} + \frac{20\cos(a+bx)\sqrt{\sin(a+bx)}}{147b^2} + \frac{4\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{49b^2} + \frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}$$

output

```
-20/147*InverseJacobiAM(1/2*a-1/4*Pi+1/2*b*x,2^(1/2))/b^2+20/147*cos(b*x+a
)*sin(b*x+a)^(1/2)/b^2+4/49*cos(b*x+a)*sin(b*x+a)^(5/2)/b^2+2/7*x*sin(b*x+
a)^(7/2)/b
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.76

$$\int x \cos(a+bx) \sin^{\frac{5}{2}}(a+bx)\, dx$$
$$= \frac{40 \operatorname{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right) + \sqrt{\sin(a+bx)}(46\cos(a+bx) - 6\cos(3(a+bx)) + 84bx\sin^3(a+b}{294b^2}$$

input
```
Integrate[x*Cos[a + b*x]*Sin[a + b*x]^(5/2),x]
```

output
```
(40*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + Sqrt[Sin[a + b*x]]*(46*Cos[a + b
*x] - 6*Cos[3*(a + b*x)] + 84*b*x*Sin[a + b*x]^3))/(294*b^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3924, 3042, 3115, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin^{\frac{5}{2}}(a+bx)\cos(a+bx)\,dx$$

$$\downarrow 3924$$

$$\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\int \sin^{\frac{7}{2}}(a+bx)dx}{7b}$$

$$\downarrow 3042$$

$$\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\int \sin(a+bx)^{7/2}dx}{7b}$$

$$\downarrow 3115$$

$$\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\left(\frac{5}{7}\int \sin^{\frac{3}{2}}(a+bx)dx - \frac{2\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{7b}\right)}{7b}$$

$$\downarrow 3042$$

$$\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\left(\frac{5}{7}\int \sin(a+bx)^{3/2}dx - \frac{2\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{7b}\right)}{7b}$$

$$\downarrow 3115$$

$$\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+bx)}}dx - \frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}\right) - \frac{2\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{7b}\right)}{7b}$$

$$\downarrow 3042$$

$$\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+bx)}}dx - \frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}\right) - \frac{2\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{7b}\right)}{7b}$$

$$\downarrow 3120$$

$$\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\left(\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx-\frac{\pi}{2}),2\right)}{3b} - \frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}\right) - \frac{2\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{7b}\right)}{7b}$$

input
```
Int[x*Cos[a + b*x]*Sin[a + b*x]^(5/2),x]
```

output
```
(2*x*Sin[a + b*x]^(7/2))/(7*b) - (2*((5*((2*EllipticF[(a - Pi/2 + b*x)/2,
2])/(3*b) - (2*Cos[a + b*x]*Sqrt[Sin[a + b*x]])/(3*b)))/7 - (2*Cos[a + b*x
]*Sin[a + b*x]^(5/2))/(7*b)))/(7*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Maple [F]

$$\int x \cos(bx+a) \sin(bx+a)^{\frac{5}{2}}\, dx$$

input
```
int(x*cos(b*x+a)*sin(b*x+a)^(5/2),x)
```

output
```
int(x*cos(b*x+a)*sin(b*x+a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x \cos(a+bx) \sin^{\frac{5}{2}}(a+bx)\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int x \cos(a+bx) \sin^{\frac{5}{2}}(a+bx)\, dx = \text{Timed out}$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x \cos(a+bx) \sin^{\frac{5}{2}}(a+bx)\, dx = \int x \cos(bx+a) \sin(bx+a)^{\frac{5}{2}}\, dx$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)*sin(b*x + a)^(5/2), x)
```

Giac [F]

$$\int x \cos(a+bx) \sin^{\frac{5}{2}}(a+bx)\, dx = \int x \cos(bx+a) \sin(bx+a)^{\frac{5}{2}}\, dx$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)*sin(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \cos(a+bx) \sin^{\frac{5}{2}}(a+bx)\, dx = \int x\, \cos(a+b\,x)\, \sin(a+b\,x)^{5/2}\, dx$$

input
```
int(x*cos(a + b*x)*sin(a + b*x)^(5/2),x)
```

output
```
int(x*cos(a + b*x)*sin(a + b*x)^(5/2), x)
```

Reduce [F]

$$\int x\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)\,dx = \int \sqrt{\sin(bx+a)}\,\cos(bx+a)\sin(bx+a)^2\,xdx$$

input
```
int(x*cos(b*x+a)*sin(b*x+a)^(5/2),x)
```

output
```
int(sqrt(sin(a + b*x))*cos(a + b*x)*sin(a + b*x)**2*x,x)
```

3.345 $\int x\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)\,dx$

Optimal result . . . 2888
Mathematica [C] (verified) . . . 2888
Rubi [A] (verified) . . . 2889
Maple [F] . . . 2890
Fricas [F(-2)] . . . 2891
Sympy [F] . . . 2891
Maxima [F] . . . 2891
Giac [F] . . . 2892
Mupad [F(-1)] . . . 2892
Reduce [F] . . . 2892

Optimal result

Integrand size = 18, antiderivative size = 65

$$\int x\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)\,dx = -\frac{12E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{25b^2} + \frac{4\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)}{25b^2} + \frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}$$

output

```
12/25*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2+4/25*cos(b*x+a)*sin
(b*x+a)^(3/2)/b^2+2/5*x*sin(b*x+a)^(5/2)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.66

$$\int x\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)\,dx = \frac{\sqrt{\sin(a+bx)}\left(5bx - 5bx\cos(2(a+bx)) + 2\sin(2(a+bx)) - 12\tan\left(\frac{1}{2}(a+bx)\right) + 4\,\text{Hypergeometric2F}\right.}{25b^2}$$

input

```
Integrate[x*Cos[a + b*x]*Sin[a + b*x]^(3/2),x]
```

output

```
(Sqrt[Sin[a + b*x]]*(5*b*x - 5*b*x*Cos[2*(a + b*x)] + 2*Sin[2*(a + b*x)] -
 12*Tan[(a + b*x)/2] + 4*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[(a + b*x)/2
]^2]*Sqrt[Sec[(a + b*x)/2]^2]*Tan[(a + b*x)/2]))/(25*b^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3924, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin^{\frac{3}{2}}(a+bx)\cos(a+bx)\,dx$$

$$\downarrow 3924$$

$$\frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b} - \frac{2\int \sin^{\frac{5}{2}}(a+bx)dx}{5b}$$

$$\downarrow 3042$$

$$\frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b} - \frac{2\int \sin(a+bx)^{5/2}dx}{5b}$$

$$\downarrow 3115$$

$$\frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b} - \frac{2\left(\frac{3}{5}\int \sqrt{\sin(a+bx)}dx - \frac{2\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{5b}\right)}{5b}$$

$$\downarrow 3042$$

$$\frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b} - \frac{2\left(\frac{3}{5}\int \sqrt{\sin(a+bx)}dx - \frac{2\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{5b}\right)}{5b}$$

$$\downarrow 3119$$

$$\frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b} - \frac{2\left(\frac{6E\left(\frac{1}{2}(a+bx-\frac{\pi}{2})|2\right)}{5b} - \frac{2\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{5b}\right)}{5b}$$

input
```
Int[x*Cos[a + b*x]*Sin[a + b*x]^(3/2),x]
```

output
```
(2*x*Sin[a + b*x]^(5/2))/(5*b) - (2*((6*EllipticE[(a - Pi/2 + b*x)/2, 2])/
(5*b) - (2*Cos[a + b*x]*Sin[a + b*x]^(3/2))/(5*b)))/(5*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Maple [F]

$$\int x \cos(bx+a) \sin(bx+a)^{\frac{3}{2}}\, dx$$

input
```
int(x*cos(b*x+a)*sin(b*x+a)^(3/2),x)
```

output
```
int(x*cos(b*x+a)*sin(b*x+a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x \cos(a+bx) \sin^{\frac{3}{2}}(a+bx)\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x \cos(a+bx) \sin^{\frac{3}{2}}(a+bx)\, dx = \int x \sin^{\frac{3}{2}}(a+bx) \cos(a+bx)\, dx$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)**(3/2),x)
```

output
```
Integral(x*sin(a + b*x)**(3/2)*cos(a + b*x), x)
```

Maxima [F]

$$\int x \cos(a+bx) \sin^{\frac{3}{2}}(a+bx)\, dx = \int x \cos(bx+a) \sin(bx+a)^{\frac{3}{2}}\, dx$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)*sin(b*x + a)^(3/2), x)
```

Giac [F]

$$\int x\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)\,dx = \int x\cos\left(bx+a\right)\sin\left(bx+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)*sin(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)\,dx = \int x\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^{3/2}\,dx$$

input
```
int(x*cos(a + b*x)*sin(a + b*x)^(3/2),x)
```

output
```
int(x*cos(a + b*x)*sin(a + b*x)^(3/2), x)
```

Reduce [F]

$$\int x\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)\,dx = \int \sqrt{\sin\left(bx+a\right)}\,\cos\left(bx+a\right)\sin\left(bx+a\right)x dx$$

input
```
int(x*cos(b*x+a)*sin(b*x+a)^(3/2),x)
```

output
```
int(sqrt(sin(a + b*x))*cos(a + b*x)*sin(a + b*x)*x,x)
```

3.346 $\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx$

Optimal result . . . 2893
Mathematica [A] (verified) . . . 2893
Rubi [A] (verified) . . . 2894
Maple [F] . . . 2895
Fricas [F(-2)] . . . 2896
Sympy [F] . . . 2896
Maxima [F] . . . 2896
Giac [F] . . . 2897
Mupad [F(-1)] . . . 2897
Reduce [F] . . . 2897

Optimal result

Integrand size = 18, antiderivative size = 65

$$\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx = -\frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)}{9b^2} + \frac{4\cos(a+bx)\sqrt{\sin(a+bx)}}{9b^2} + \frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b}$$

output

```
-4/9*InverseJacobiAM(1/2*a-1/4*Pi+1/2*b*x,2^(1/2))/b^2+4/9*cos(b*x+a)*sin(
b*x+a)^(1/2)/b^2+2/3*x*sin(b*x+a)^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx = \frac{4\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)+2\sqrt{\sin(a+bx)}(2\cos(a+bx)+3bx\sin(a+bx))}{9b^2}$$

input

```
Integrate[x*Cos[a + b*x]*Sqrt[Sin[a + b*x]],x]
```

output
```
(4*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + 2*Sqrt[Sin[a + b*x]]*(2*Cos[a + b
*x] + 3*b*x*Sin[a + b*x]))/(9*b^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3924, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\sin(a+bx)}\cos(a+bx)\,dx$$

$$\downarrow 3924$$

$$\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\int \sin^{\frac{3}{2}}(a+bx)dx}{3b}$$

$$\downarrow 3042$$

$$\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\int \sin(a+bx)^{3/2}dx}{3b}$$

$$\downarrow 3115$$

$$\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+bx)}}dx - \frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}\right)}{3b}$$

$$\downarrow 3042$$

$$\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+bx)}}dx - \frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}\right)}{3b}$$

$$\downarrow 3120$$

$$\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx-\frac{\pi}{2}),2\right)}{3b} - \frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}\right)}{3b}$$

input
```
Int[x*Cos[a + b*x]*Sqrt[Sin[a + b*x]],x]
```

output
```
(-2*((2*EllipticF[(a - Pi/2 + b*x)/2, 2])/(3*b) - (2*Cos[a + b*x]*Sqrt[Sin
[a + b*x]])/(3*b)))/(3*b) + (2*x*Sin[a + b*x]^(3/2))/(3*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Maple [F]

$$\int x \cos(bx+a)\sqrt{\sin(bx+a)}dx$$

input
```
int(x*cos(b*x+a)*sin(b*x+a)^(1/2),x)
```

output
```
int(x*cos(b*x+a)*sin(b*x+a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x \cos(a+bx)\sqrt{\sin(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x \cos(a+bx)\sqrt{\sin(a+bx)}\,dx = \int x\sqrt{\sin(a+bx)}\cos(a+bx)\,dx$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)**(1/2),x)
```

output
```
Integral(x*sqrt(sin(a + b*x))*cos(a + b*x), x)
```

Maxima [F]

$$\int x \cos(a+bx)\sqrt{\sin(a+bx)}\,dx = \int x\cos(bx+a)\sqrt{\sin(bx+a)}\,dx$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)*sqrt(sin(b*x + a)), x)
```

Giac [F]

$$\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx = \int x\cos\left(bx+a\right)\sqrt{\sin\left(bx+a\right)}\,dx$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)*sqrt(sin(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx = \int x\,\cos\left(a+b\,x\right)\,\sqrt{\sin\left(a+b\,x\right)}\,dx$$

input
```
int(x*cos(a + b*x)*sin(a + b*x)^(1/2),x)
```

output
```
int(x*cos(a + b*x)*sin(a + b*x)^(1/2), x)
```

Reduce [F]

$$\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx = \int \sqrt{\sin\left(bx+a\right)}\,\cos\left(bx+a\right)x dx$$

input
```
int(x*cos(b*x+a)*sin(b*x+a)^(1/2),x)
```

output
```
int(sqrt(sin(a + b*x))*cos(a + b*x)*x,x)
```

3.347 $\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx$

Optimal result . . . 2898
Mathematica [C] (verified) . . . 2898
Rubi [A] (verified) . . . 2899
Maple [C] (verified) . . . 2900
Fricas [F(-2)] . . . 2901
Sympy [F] . . . 2901
Maxima [F] . . . 2901
Giac [F] . . . 2902
Mupad [F(-1)] . . . 2902
Reduce [F] . . . 2902

Optimal result

Integrand size = 18, antiderivative size = 38

$$\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx = -\frac{4E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|\,2\right)}{b^2}+\frac{2x\sqrt{\sin(a+bx)}}{b}$$

output
```
4*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2+2*x*sin(b*x+a)^(1/2)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.70 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.26

$$\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx$$
$$= \frac{2\sqrt{\sin(a+bx)}\left(3bx-6\tan\left(\frac{1}{2}(a+bx)\right)+2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)\sqrt{\sec^2\left(\frac{1}{2}\right.}\right.}{3b^2}$$

input
```
Integrate[(x*Cos[a + b*x])/Sqrt[Sin[a + b*x]],x]
```

output
```
(2*Sqrt[Sin[a + b*x]]*(3*b*x - 6*Tan[(a + b*x)/2] + 2*Hypergeometric2F1[1/
2, 3/4, 7/4, -Tan[(a + b*x)/2]^2]*Sqrt[Sec[(a + b*x)/2]^2]*Tan[(a + b*x)/2
]))/(3*b^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3924, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx$$

$$\downarrow 3924$$

$$\frac{2x\sqrt{\sin(a+bx)}}{b} - \frac{2\int \sqrt{\sin(a+bx)}dx}{b}$$

$$\downarrow 3042$$

$$\frac{2x\sqrt{\sin(a+bx)}}{b} - \frac{2\int \sqrt{\sin(a+bx)}dx}{b}$$

$$\downarrow 3119$$

$$\frac{2x\sqrt{\sin(a+bx)}}{b} - \frac{4E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|\,2\right)}{b^2}$$

input
```
Int[(x*Cos[a + b*x])/Sqrt[Sin[a + b*x]],x]
```

output
```
(-4*EllipticE[(a - Pi/2 + b*x)/2, 2])/b^2 + (2*x*Sqrt[Sin[a + b*x]])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 308, normalized size of antiderivative = 8.11

method	result
risch	$-\frac{i(bx+2i)\left(e^{2i(bx+a)}-1\right)\sqrt{2}\,e^{-i(bx+a)}}{b^2\sqrt{-i\left(e^{2i(bx+a)}-1\right)e^{-i(bx+a)}}} - \frac{2\left(\frac{2i\left(-ie^{2i(bx+a)}+i\right)}{\sqrt{e^{i(bx+a)}\left(-ie^{2i(bx+a)}+i\right)}} - \frac{\sqrt{e^{i(bx+a)}+1}\sqrt{-2e^{i(bx+a)}+2}\sqrt{-e^{i(bx+a)}}\left(-2\,\text{EllipticE}\right.}{\sqrt{-ie^{3i(bx+a)}+}}\right.}{b^2\sqrt{-i\left(e^{2i(b}}$

input
```
int(x*cos(b*x+a)/sin(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*(b*x+2*I)*(exp(I*(b*x+a))^2-1)/b^2*2^(1/2)/(-I*(exp(I*(b*x+a))^2-1)/exp
(I*(b*x+a)))^(1/2)/exp(I*(b*x+a))-2/b^2*(2*I*(I-I*exp(I*(b*x+a))^2)/(exp(I
*(b*x+a))*(I-I*exp(I*(b*x+a))^2))^(1/2)-(exp(I*(b*x+a))+1)^(1/2)*(-2*exp(I
*(b*x+a))+2)^(1/2)*(-exp(I*(b*x+a)))^(1/2)/(-I*exp(I*(b*x+a))^3+I*exp(I*(b
*x+a)))^(1/2)*(-2*EllipticE((exp(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))+Elliptic
F((exp(I*(b*x+a))+1)^(1/2),1/2*2^(1/2)))*2^(1/2)/(-I*(exp(I*(b*x+a))^2-1)
/exp(I*(b*x+a)))^(1/2)*(-I*(exp(I*(b*x+a))^2-1)*exp(I*(b*x+a)))^(1/2)/exp(
I*(b*x+a))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx = \int \frac{x\cos\left(a+bx\right)}{\sqrt{\sin\left(a+bx\right)}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)**(1/2),x)
```

output
```
Integral(x*cos(a + b*x)/sqrt(sin(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx = \int \frac{x\cos\left(bx+a\right)}{\sqrt{\sin\left(bx+a\right)}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)/sqrt(sin(b*x + a)), x)
```

Giac [F]

$$\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx = \int \frac{x\cos(bx+a)}{\sqrt{\sin(bx+a)}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)/sqrt(sin(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx = \int \frac{x\,\cos(a+b\,x)}{\sqrt{\sin(a+b\,x)}}\,dx$$

input
```
int((x*cos(a + b*x))/sin(a + b*x)^(1/2),x)
```

output
```
int((x*cos(a + b*x))/sin(a + b*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx = \int \frac{\sqrt{\sin(bx+a)}\,\cos(bx+a)\,x}{\sin(bx+a)}dx$$

input
```
int(x*cos(b*x+a)/sin(b*x+a)^(1/2),x)
```

output
```
int((sqrt(sin(a + b*x))*cos(a + b*x)*x)/sin(a + b*x),x)
```

3.348 $\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result . . . 2903
Mathematica [A] (verified) . . . 2903
Rubi [A] (verified) . . . 2904
Maple [F] . . . 2905
Fricas [F(-2)] . . . 2905
Sympy [F] . . . 2906
Maxima [F] . . . 2906
Giac [F] . . . 2906
Mupad [F(-1)] . . . 2907
Reduce [F] . . . 2907

Optimal result

Integrand size = 18, antiderivative size = 38

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)}{b^2} - \frac{2x}{b\sqrt{\sin(a+bx)}}$$

output
```
4*InverseJacobiAM(1/2*a-1/4*Pi+1/2*b*x,2^(1/2))/b^2-2*x/b/sin(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\left(-2\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right) - \frac{bx}{\sqrt{\sin(a+bx)}}\right)}{b^2}$$

input
```
Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(3/2),x]
```

output
```
(2*(-2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] - (b*x)/Sqrt[Sin[a + b*x]]))/b^
2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3924, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$$

$$\downarrow 3924$$

$$\frac{2\int \frac{1}{\sqrt{\sin(a+bx)}}dx}{b} - \frac{2x}{b\sqrt{\sin(a+bx)}}$$

$$\downarrow 3042$$

$$\frac{2\int \frac{1}{\sqrt{\sin(a+bx)}}dx}{b} - \frac{2x}{b\sqrt{\sin(a+bx)}}$$

$$\downarrow 3120$$

$$\frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{b^2} - \frac{2x}{b\sqrt{\sin(a+bx)}}$$

input
```
Int[(x*Cos[a + b*x])/Sin[a + b*x]^(3/2),x]
```

output
```
(4*EllipticF[(a - Pi/2 + b*x)/2, 2])/b^2 - (2*x)/(b*Sqrt[Sin[a + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3924

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Maple [F]

$$\int \frac{x \cos(bx+a)}{\sin(bx+a)^{\frac{3}{2}}} dx$$

input

```
int(x*cos(b*x+a)/sin(b*x+a)^(3/2),x)
```

output

```
int(x*cos(b*x+a)/sin(b*x+a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x \cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)} dx = \text{Exception raised: TypeError}$$

input

```
integrate(x*cos(b*x+a)/sin(b*x+a)^(3/2),x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x \cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x \cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)**(3/2),x)
```

output
```
Integral(x*cos(a + b*x)/sin(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{x \cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x \cos(bx+a)}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)/sin(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x \cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x \cos(bx+a)}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)/sin(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\,\cos\left(a+b\,x\right)}{\sin\left(a+b\,x\right)^{3/2}}\,dx$$

input
```
int((x*cos(a + b*x))/sin(a + b*x)^(3/2),x)
```

output
```
int((x*cos(a + b*x))/sin(a + b*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \frac{-2\sqrt{\sin\left(bx+a\right)}\,x + 2\left(\int \frac{\sqrt{\sin(bx+a)}}{\sin(bx+a)}dx\right)\sin\left(bx+a\right)}{\sin\left(bx+a\right)b}$$

input
```
int(x*cos(b*x+a)/sin(b*x+a)^(3/2),x)
```

output
```
(2*( - sqrt(sin(a + b*x))*x + int(sqrt(sin(a + b*x))/sin(a + b*x),x)*sin(a
 + b*x)))/(sin(a + b*x)*b)
```

3.349 $\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx$

Optimal result . . . 2908
Mathematica [A] (verified) . . . 2908
Rubi [A] (verified) . . . 2909
Maple [F] . . . 2910
Fricas [F(-2)] . . . 2911
Sympy [F] . . . 2911
Maxima [F] . . . 2911
Giac [F] . . . 2912
Mupad [F(-1)] . . . 2912
Reduce [F] . . . 2912

Optimal result

Integrand size = 18, antiderivative size = 65

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = -\frac{4E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|\,2\right)}{3b^2} - \frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)} - \frac{4\cos(a+bx)}{3b^2\sqrt{\sin(a+bx)}}$$

output

```
4/3*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2-2/3*x/b/sin(b*x+a)^(3
/2)-4/3*cos(b*x+a)/b^2/sin(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2\left(bx - 2E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|\,2\right)\sin^{\frac{3}{2}}(a+bx) + \sin(2(a+bx))\right)}{3b^2\sin^{\frac{3}{2}}(a+bx)}$$

input

```
Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(5/2),x]
```

output
```
(-2*(b*x - 2*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(3/2) + Sin[
2*(a + b*x)]))/(3*b^2*Sin[a + b*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3924, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx$$

$$\downarrow 3924$$

$$\frac{2\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}dx}{3b} - \frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}$$

$$\downarrow 3042$$

$$\frac{2\int \frac{1}{\sin(a+bx)^{3/2}}dx}{3b} - \frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}$$

$$\downarrow 3116$$

$$\frac{2\left(-\int \sqrt{\sin(a+bx)}dx - \frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{3b} - \frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}$$

$$\downarrow 3042$$

$$\frac{2\left(-\int \sqrt{\sin(a+bx)}dx - \frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{3b} - \frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}$$

$$\downarrow 3119$$

$$\frac{2\left(-\frac{2E\left(\frac{1}{2}(a+bx-\frac{\pi}{2})|2\right)}{b} - \frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{3b} - \frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}$$

input
```
Int[(x*Cos[a + b*x])/Sin[a + b*x]^(5/2),x]
```

output
```
(2*((-2*EllipticE[(a - Pi/2 + b*x)/2, 2])/b - (2*Cos[a + b*x])/(b*Sqrt[Sin
[a + b*x]])))/(3*b) - (2*x)/(3*b*Sin[a + b*x]^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Maple [F]

$$\int \frac{x \cos(bx+a)}{\sin(bx+a)^{\frac{5}{2}}} dx$$

input
```
int(x*cos(b*x+a)/sin(b*x+a)^(5/2),x)
```

output
```
int(x*cos(b*x+a)/sin(b*x+a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cos\left(a+bx\right)}{\sin^{\frac{5}{2}}\left(a+bx\right)}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)**(5/2),x)
```

output
```
Integral(x*cos(a + b*x)/sin(a + b*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cos\left(bx+a\right)}{\sin\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)/sin(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)/sin(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\,\cos(a+b\,x)}{\sin(a+b\,x)^{5/2}}\,dx$$

input
```
int((x*cos(a + b*x))/sin(a + b*x)^(5/2),x)
```

output
```
int((x*cos(a + b*x))/sin(a + b*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \frac{-\frac{2\sqrt{\sin(bx+a)}\,x}{3} + \frac{2\left(\int \frac{\sqrt{\sin(bx+a)}}{\sin(bx+a)^2}\,dx\right)\sin(bx+a)^2}{3}}{\sin(bx+a)^2\,b}$$

input
```
int(x*cos(b*x+a)/sin(b*x+a)^(5/2),x)
```

output
```
(2*( - sqrt(sin(a + b*x))*x + int(sqrt(sin(a + b*x))/sin(a + b*x)**2,x)*si
n(a + b*x)**2))/(3*sin(a + b*x)**2*b)
```

3.350 $\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx$

Optimal result . . . 2913
Mathematica [A] (verified) . . . 2913
Rubi [A] (verified) . . . 2914
Maple [F] . . . 2915
Fricas [F(-2)] . . . 2916
Sympy [F] . . . 2916
Maxima [F] . . . 2916
Giac [F] . . . 2917
Mupad [F(-1)] . . . 2917
Reduce [F] . . . 2917

Optimal result

Integrand size = 18, antiderivative size = 65

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)}{15b^2} - \frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)} - \frac{4\cos(a+bx)}{15b^2\sin^{\frac{3}{2}}(a+bx)}$$

output
```
4/15*InverseJacobiAM(1/2*a-1/4*Pi+1/2*b*x,2^(1/2))/b^2-2/5*x/b/sin(b*x+a)^
(5/2)-4/15*cos(b*x+a)/b^2/sin(b*x+a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2\left(3bx+2\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)\sin^{\frac{5}{2}}(a+bx)+\sin(2(a+bx))\right)}{15b^2\sin^{\frac{5}{2}}(a+bx)}$$

input
```
Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(7/2),x]
```

output
```
(-2*(3*b*x + 2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(5/2) + Si
n[2*(a + b*x)]))/(15*b^2*Sin[a + b*x]^(5/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3924, 3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx$$

$$\downarrow 3924$$

$$\frac{2\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}dx}{5b} - \frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)}$$

$$\downarrow 3042$$

$$\frac{2\int \frac{1}{\sin(a+bx)^{5/2}}dx}{5b} - \frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)}$$

$$\downarrow 3116$$

$$\frac{2\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+bx)}}dx - \frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}\right)}{5b} - \frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)}$$

$$\downarrow 3042$$

$$\frac{2\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+bx)}}dx - \frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}\right)}{5b} - \frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)}$$

$$\downarrow 3120$$

$$\frac{2\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx-\frac{\pi}{2}),2\right)}{3b} - \frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}\right)}{5b} - \frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)}$$

input
```
Int[(x*Cos[a + b*x])/Sin[a + b*x]^(7/2),x]
```

output
```
(2*((2*EllipticF[(a - Pi/2 + b*x)/2, 2])/(3*b) - (2*Cos[a + b*x])/(3*b*Sin
[a + b*x]^(3/2))))/(5*b) - (2*x)/(5*b*Sin[a + b*x]^(5/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Maple [F]

$$\int \frac{x \cos(bx+a)}{\sin(bx+a)^{\frac{7}{2}}} dx$$

input
```
int(x*cos(b*x+a)/sin(b*x+a)^(7/2),x)
```

output
```
int(x*cos(b*x+a)/sin(b*x+a)^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x \cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x \cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{x \cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)**(7/2),x)
```

output
```
Integral(x*cos(a + b*x)/sin(a + b*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{x \cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{x \cos(bx+a)}{\sin(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)/sin(b*x + a)^(7/2), x)
```

Giac [F]

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)/sin(b*x + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{x\,\cos(a+b\,x)}{\sin(a+b\,x)^{7/2}}\,dx$$

input
```
int((x*cos(a + b*x))/sin(a + b*x)^(7/2),x)
```

output
```
int((x*cos(a + b*x))/sin(a + b*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \frac{-\frac{2\sqrt{\sin(bx+a)}\,x}{5} + \frac{2\left(\int \frac{\sqrt{\sin(bx+a)}}{\sin(bx+a)^3}\,dx\right)\sin(bx+a)^3}{5}}{\sin(bx+a)^3\,b}$$

input
```
int(x*cos(b*x+a)/sin(b*x+a)^(7/2),x)
```

output
```
(2*( - sqrt(sin(a + b*x))*x + int(sqrt(sin(a + b*x))/sin(a + b*x)**3,x)*si
n(a + b*x)**3))/(5*sin(a + b*x)**3*b)
```

3.351 $\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx$

Optimal result . . . 2918
Mathematica [A] (verified) . . . 2918
Rubi [A] (verified) . . . 2919
Maple [F] . . . 2921
Fricas [F(-2)] . . . 2921
Sympy [F(-1)] . . . 2922
Maxima [F] . . . 2922
Giac [F] . . . 2922
Mupad [F(-1)] . . . 2923
Reduce [F] . . . 2923

Optimal result

Integrand size = 18, antiderivative size = 88

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx = -\frac{12E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|\,2\right)}{35b^2} - \frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)} - \frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)} - \frac{12\cos(a+bx)}{35b^2\sqrt{\sin(a+bx)}}$$

output

```
12/35*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2-2/7*x/b/sin(b*x+a)^
(7/2)-4/35*cos(b*x+a)/b^2/sin(b*x+a)^(5/2)-12/35*cos(b*x+a)/b^2/sin(b*x+a)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx = -\frac{2\left(5bx+6\cos(a+bx)\sin^3(a+bx)-6E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|\,2\right)\sin^{\frac{7}{2}}(a+bx)+\sin(2(a+bx))\right)}{35b^2\sin^{\frac{7}{2}}(a+bx)}$$

input
```
Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(9/2),x]
```

output
```
(-2*(5*b*x + 6*Cos[a + b*x]*Sin[a + b*x]^3 - 6*EllipticE[(-2*a + Pi - 2*b*
x)/4, 2]*Sin[a + b*x]^(7/2) + Sin[2*(a + b*x)]))/(35*b^2*Sin[a + b*x]^(7/2
))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3924, 3042, 3116, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx$$

$$\downarrow 3924$$

$$\frac{2\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}dx}{7b} - \frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

$$\downarrow 3042$$

$$\frac{2\int \frac{1}{\sin(a+bx)^{7/2}}dx}{7b} - \frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

$$\downarrow 3116$$

$$\frac{2\left(\frac{3}{5}\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}dx - \frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}\right)}{7b} - \frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

$$\downarrow 3042$$

$$\frac{2\left(\frac{3}{5}\int \frac{1}{\sin(a+bx)^{3/2}}dx - \frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}\right)}{7b} - \frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

$$\downarrow 3116$$

$$\frac{2\left(\frac{3}{5}\left(-\int\sqrt{\sin(a+bx)}dx-\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}\right)-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}\right)}{7b}-\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

$$\downarrow 3042$$

$$\frac{2\left(\frac{3}{5}\left(-\int\sqrt{\sin(a+bx)}dx-\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}\right)-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}\right)}{7b}-\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

$$\downarrow 3119$$

$$\frac{2\left(\frac{3}{5}\left(-\frac{2E\left(\frac{1}{2}(a+bx-\frac{\pi}{2})|2\right)}{b}-\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}\right)-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}\right)}{7b}-\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

input
```
Int[(x*Cos[a + b*x])/Sin[a + b*x]^(9/2),x]
```

output
```
(2*((3*((-2*EllipticE[(a - Pi/2 + b*x)/2, 2])/b - (2*Cos[a + b*x])/(b*Sqrt
[Sin[a + b*x]])))/5 - (2*Cos[a + b*x])/(5*b*Sin[a + b*x]^(5/2))))/(7*b) -
(2*x)/(7*b*Sin[a + b*x]^(7/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3924

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Maple [F]

$$\int \frac{x \cos(bx+a)}{\sin(bx+a)^{\frac{9}{2}}} dx$$

input

```
int(x*cos(b*x+a)/sin(b*x+a)^(9/2),x)
```

output

```
int(x*cos(b*x+a)/sin(b*x+a)^(9/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x \cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)} dx = \text{Exception raised: TypeError}$$

input

```
integrate(x*cos(b*x+a)/sin(b*x+a)^(9/2),x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x \cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x \cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x \cos(bx+a)}{\sin(bx+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)/sin(b*x + a)^(9/2), x)
```

Giac [F]

$$\int \frac{x \cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x \cos(bx+a)}{\sin(bx+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(9/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)/sin(b*x + a)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x\,\cos\left(a+b\,x\right)}{\sin\left(a+b\,x\right)^{9/2}}\,dx$$

input
```
int((x*cos(a + b*x))/sin(a + b*x)^(9/2),x)
```

output
```
int((x*cos(a + b*x))/sin(a + b*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx = \frac{-\frac{2\sqrt{\sin(bx+a)}\,x}{7} + \frac{2\left(\int \frac{\sqrt{\sin(bx+a)}}{\sin(bx+a)^4}\,dx\right)\sin(bx+a)^4}{7}}{\sin\left(bx+a\right)^4 b}$$

input
```
int(x*cos(b*x+a)/sin(b*x+a)^(9/2),x)
```

output
```
(2*( - sqrt(sin(a + b*x))*x + int(sqrt(sin(a + b*x))/sin(a + b*x)**4,x)*si
n(a + b*x)**4))/(7*sin(a + b*x)**4*b)
```

3.352 $\int x \cos(a+bx) \csc^{\frac{9}{2}}(a+bx)\, dx$

Optimal result . . . 2924
Mathematica [A] (verified) . . . 2924
Rubi [A] (verified) . . . 2925
Maple [F] . . . 2927
Fricas [F(-2)] . . . 2928
Sympy [F(-1)] . . . 2928
Maxima [F] . . . 2928
Giac [F] . . . 2929
Mupad [F(-1)] . . . 2929
Reduce [F] . . . 2929

Optimal result

Integrand size = 18, antiderivative size = 108

$$\int x \cos(a+bx) \csc^{\frac{9}{2}}(a+bx)\, dx = -\frac{12\cos(a+bx)\sqrt{\csc(a+bx)}}{35b^2} - \frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2} - \frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b} - \frac{12\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{35b^2}$$

output

```
-12/35*cos(b*x+a)*csc(b*x+a)^(1/2)/b^2-4/35*cos(b*x+a)*csc(b*x+a)^(5/2)/b^
2-2/7*x*csc(b*x+a)^(7/2)/b+12/35*csc(b*x+a)^(1/2)*EllipticE(cos(1/2*a+1/4*
Pi+1/2*b*x),2^(1/2))*sin(b*x+a)^(1/2)/b^2
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68

$$\int x \cos(a+bx) \csc^{\frac{9}{2}}(a+bx)\, dx = -\frac{2\csc^{\frac{7}{2}}(a+bx)\left(5bx+6\cos(a+bx)\sin^3(a+bx)-6E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sin^{\frac{7}{2}}(a+bx)+\sin(2(a+\right.}{35b^2}$$

input
```
Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(9/2),x]
```

output
```
(-2*Csc[a + b*x]^(7/2)*(5*b*x + 6*Cos[a + b*x]*Sin[a + b*x]^3 - 6*Elliptic
E[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(7/2) + Sin[2*(a + b*x)]))/(35*b^
2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4701, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos(a+bx)\csc^{\frac{9}{2}}(a+bx)\,dx$$

$$\downarrow 4701$$

$$\frac{2\int \csc^{\frac{7}{2}}(a+bx)dx}{7b} - \frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}$$

$$\downarrow 3042$$

$$\frac{2\int \csc(a+bx)^{7/2}dx}{7b} - \frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}$$

$$\downarrow 4255$$

$$\frac{2\left(\frac{3}{5}\int \csc^{\frac{3}{2}}(a+bx)dx - \frac{2\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b} - \frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}$$

$$\downarrow 3042$$

$$\frac{2\left(\frac{3}{5}\int \csc(a+bx)^{3/2}dx - \frac{2\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b} - \frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}$$

$$\downarrow 4255$$

$$\frac{2\left(\frac{3}{5}\left(-\int \frac{1}{\sqrt{\csc(a+bx)}}dx - \frac{2\cos(a+bx)\sqrt{\csc(a+bx)}}{b}\right) - \frac{2\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b} - \frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}$$

$$\downarrow 3042$$

$$\frac{2\left(\frac{3}{5}\left(-\int \frac{1}{\sqrt{\csc(a+bx)}}dx - \frac{2\cos(a+bx)\sqrt{\csc(a+bx)}}{b}\right) - \frac{2\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b} - \frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}$$

$$\downarrow 4258$$

$$\frac{2\left(\frac{3}{5}\left(-\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \sqrt{\sin(a+bx)}dx - \frac{2\cos(a+bx)\sqrt{\csc(a+bx)}}{b}\right) - \frac{2\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b} - \frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}$$

$$\downarrow 3042$$

$$\frac{2\left(\frac{3}{5}\left(-\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \sqrt{\sin(a+bx)}dx - \frac{2\cos(a+bx)\sqrt{\csc(a+bx)}}{b}\right) - \frac{2\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b} - \frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}$$

$$\downarrow 3119$$

$$\frac{2\left(\frac{3}{5}\left(-\frac{2\cos(a+bx)\sqrt{\csc(a+bx)}}{b} - \frac{2\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}(a+bx-\frac{\pi}{2})|2\right)}{b}\right) - \frac{2\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b} - \frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}$$

input

```
Int[x*Cos[a + b*x]*Csc[a + b*x]^(9/2),x]
```

output

```
(-2*x*Csc[a + b*x]^(7/2))/(7*b) + (2*((-2*Cos[a + b*x]*Csc[a + b*x]^(5/2))
/(5*b) + (3*((-2*Cos[a + b*x]*Sqrt[Csc[a + b*x]])/b - (2*Sqrt[Csc[a + b*x]
]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b))/5))/(7*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4701
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csc[a + b*x^n]^(p
 - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ
[p, 1]
```

Maple [F]

$$\int x \cos(bx+a) \csc(bx+a)^{\frac{9}{2}}\, dx$$

input
```
int(x*cos(b*x+a)*csc(b*x+a)^(9/2),x)
```

output
```
int(x*cos(b*x+a)*csc(b*x+a)^(9/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x \cos(a+bx) \csc^{\frac{9}{2}}(a+bx)\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(9/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x \cos(a+bx) \csc^{\frac{9}{2}}(a+bx)\, dx = \text{Timed out}$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x \cos(a+bx) \csc^{\frac{9}{2}}(a+bx)\, dx = \int x \cos\left(bx+a\right) \csc\left(bx+a\right)^{\frac{9}{2}}\, dx$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)*csc(b*x + a)^(9/2), x)
```

Giac [F]

$$\int x \cos(a+bx) \csc^{\frac{9}{2}}(a+bx)\, dx = \int x \cos(bx+a) \csc(bx+a)^{\frac{9}{2}}\, dx$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(9/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)*csc(b*x + a)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \cos(a+bx) \csc^{\frac{9}{2}}(a+bx)\, dx = \int x \cos(a+b\,x) \left(\frac{1}{\sin(a+b\,x)}\right)^{9/2} dx$$

input
```
int(x*cos(a + b*x)*(1/sin(a + b*x))^(9/2),x)
```

output
```
int(x*cos(a + b*x)*(1/sin(a + b*x))^(9/2), x)
```

Reduce [F]

$$\int x \cos(a+bx) \csc^{\frac{9}{2}}(a+bx)\, dx = \int \sqrt{\csc(bx+a)} \cos(bx+a) \csc(bx+a)^4\, x dx$$

input
```
int(x*cos(b*x+a)*csc(b*x+a)^(9/2),x)
```

output
```
int(sqrt(csc(a + b*x))*cos(a + b*x)*csc(a + b*x)**4*x,x)
```

3.353 $\int x \cos(a+bx) \csc^{\frac{7}{2}}(a+bx)\, dx$

Optimal result . . . 2930
Mathematica [A] (verified) . . . 2930
Rubi [A] (verified) . . . 2931
Maple [F] . . . 2933
Fricas [F(-2)] . . . 2933
Sympy [F(-1)] . . . 2934
Maxima [F] . . . 2934
Giac [F] . . . 2934
Mupad [F(-1)] . . . 2935
Reduce [F] . . . 2935

Optimal result

Integrand size = 18, antiderivative size = 85

$$\begin{aligned}&\int x \cos(a+bx) \csc^{\frac{7}{2}}(a+bx)\, dx \\ &= -\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2} - \frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b} \\ &\quad + \frac{4\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{15b^2}\end{aligned}$$

output

```
-4/15*cos(b*x+a)*csc(b*x+a)^(3/2)/b^2-2/5*x*csc(b*x+a)^(5/2)/b+4/15*csc(b*
x+a)^(1/2)*InverseJacobiAM(1/2*a-1/4*Pi+1/2*b*x,2^(1/2))*sin(b*x+a)^(1/2)/
b^2
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int x \cos(a+bx) \csc^{\frac{7}{2}}(a+bx)\, dx = -\frac{2\sqrt{\csc(a+bx)}\left(2\cot(a+bx)+3bx\csc^2(a+bx)+2\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)\sqrt{\sin(a+bx)}\right)}{15b^2}$$

input
```
Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(7/2),x]
```

output
```
(-2*Sqrt[Csc[a + b*x]]*(2*Cot[a + b*x] + 3*b*x*Csc[a + b*x]^2 + 2*Elliptic
F[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]]))/(15*b^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {4701, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos(a+bx)\csc^{\frac{7}{2}}(a+bx)\,dx$$

$$\downarrow 4701$$

$$\frac{2\int \csc^{\frac{5}{2}}(a+bx)dx}{5b} - \frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

$$\downarrow 3042$$

$$\frac{2\int \csc(a+bx)^{5/2}dx}{5b} - \frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

$$\downarrow 4255$$

$$\frac{2\left(\frac{1}{3}\int \sqrt{\csc(a+bx)}dx - \frac{2\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{3b}\right)}{5b} - \frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

$$\downarrow 3042$$

$$\frac{2\left(\frac{1}{3}\int \sqrt{\csc(a+bx)}dx - \frac{2\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{3b}\right)}{5b} - \frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

$$\downarrow 4258$$

$$\frac{2\left(\frac{1}{3}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}dx - \frac{2\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{3b}\right)}{5b} - \frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

$$\downarrow 3042$$

$$\frac{2\left(\frac{1}{3}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int\frac{1}{\sqrt{\sin(a+bx)}}dx-\frac{2\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{3b}\right)}{5b}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

$$\downarrow 3120$$

$$\frac{2\left(\frac{2\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx-\frac{\pi}{2}),2\right)}{3b}-\frac{2\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{3b}\right)}{5b}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

input
```
Int[x*Cos[a + b*x]*Csc[a + b*x]^(7/2),x]
```

output
```
(-2*x*Csc[a + b*x]^(5/2))/(5*b) + (2*((-2*Cos[a + b*x]*Csc[a + b*x]^(3/2))
/(3*b) + (2*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a
 + b*x]])/(3*b)))/(5*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4701

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csc[a + b*x^n]^(p
 - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ
[p, 1]
```

Maple [F]

$$\int x \cos(bx+a) \csc(bx+a)^{\frac{7}{2}}\, dx$$

input

```
int(x*cos(b*x+a)*csc(b*x+a)^(7/2),x)
```

output

```
int(x*cos(b*x+a)*csc(b*x+a)^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x \cos(a+bx) \csc^{\frac{7}{2}}(a+bx)\, dx = \text{Exception raised: TypeError}$$

input

```
integrate(x*cos(b*x+a)*csc(b*x+a)^(7/2),x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x \cos(a+bx) \csc^{\frac{7}{2}}(a+bx)\, dx = \text{Timed out}$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x \cos(a+bx) \csc^{\frac{7}{2}}(a+bx)\, dx = \int x \cos\left(bx+a\right) \csc\left(bx+a\right)^{\frac{7}{2}}\, dx$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)*csc(b*x + a)^(7/2), x)
```

Giac [F]

$$\int x \cos(a+bx) \csc^{\frac{7}{2}}(a+bx)\, dx = \int x \cos\left(bx+a\right) \csc\left(bx+a\right)^{\frac{7}{2}}\, dx$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)*csc(b*x + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \cos(a+bx) \csc^{\frac{7}{2}}(a+bx)\, dx = \int x\, \cos(a+b\,x)\, \left(\frac{1}{\sin(a+b\,x)}\right)^{7/2} dx$$

input
```
int(x*cos(a + b*x)*(1/sin(a + b*x))^(7/2),x)
```

output
```
int(x*cos(a + b*x)*(1/sin(a + b*x))^(7/2), x)
```

Reduce [F]

$$\int x \cos(a+bx) \csc^{\frac{7}{2}}(a+bx)\, dx = \int \sqrt{\csc(bx+a)}\, \cos(bx+a) \csc(bx+a)^3\, x dx$$

input
```
int(x*cos(b*x+a)*csc(b*x+a)^(7/2),x)
```

output
```
int(sqrt(csc(a + b*x))*cos(a + b*x)*csc(a + b*x)**3*x,x)
```

3.354 $\int x \cos(a+bx) \csc^{\frac{5}{2}}(a+bx)\, dx$

Optimal result . . . 2936
Mathematica [A] (verified) . . . 2936
Rubi [A] (verified) . . . 2937
Maple [F] . . . 2939
Fricas [F(-2)] . . . 2939
Sympy [F(-1)] . . . 2939
Maxima [F] . . . 2940
Giac [F] . . . 2940
Mupad [F(-1)] . . . 2940
Reduce [F] . . . 2941

Optimal result

Integrand size = 18, antiderivative size = 85

$$\int x \cos(a+bx) \csc^{\frac{5}{2}}(a+bx)\, dx = -\frac{4\cos(a+bx)\sqrt{\csc(a+bx)}}{3b^2} - \frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b} - \frac{4\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{3b^2}$$

output

```
-4/3*cos(b*x+a)*csc(b*x+a)^(1/2)/b^2-2/3*x*csc(b*x+a)^(3/2)/b+4/3*csc(b*x+
a)^(1/2)*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*sin(b*x+a)^(1/2)/b^2
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.66

$$\int x \cos(a+bx) \csc^{\frac{5}{2}}(a+bx)\, dx = -\frac{2\csc^{\frac{3}{2}}(a+bx)\left(bx - 2E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sin^{\frac{3}{2}}(a+bx) + \sin(2(a+bx))\right)}{3b^2}$$

input

```
Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(5/2),x]
```

output
```
(-2*Csc[a + b*x]^(3/2)*(b*x - 2*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a
+ b*x]^(3/2) + Sin[2*(a + b*x)]))/(3*b^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {4701, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx$$

$$\downarrow 4701$$

$$\frac{2\int \csc^{\frac{3}{2}}(a+bx)dx}{3b} - \frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}$$

$$\downarrow 3042$$

$$\frac{2\int \csc(a+bx)^{3/2}dx}{3b} - \frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}$$

$$\downarrow 4255$$

$$\frac{2\left(-\int \frac{1}{\sqrt{\csc(a+bx)}}dx - \frac{2\cos(a+bx)\sqrt{\csc(a+bx)}}{b}\right)}{3b} - \frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}$$

$$\downarrow 3042$$

$$\frac{2\left(-\int \frac{1}{\sqrt{\csc(a+bx)}}dx - \frac{2\cos(a+bx)\sqrt{\csc(a+bx)}}{b}\right)}{3b} - \frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}$$

$$\downarrow 4258$$

$$\frac{2\left(-\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \sqrt{\sin(a+bx)}dx - \frac{2\cos(a+bx)\sqrt{\csc(a+bx)}}{b}\right)}{3b} - \frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}$$

$$\downarrow 3042$$

$$\frac{2\left(-\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \sqrt{\sin(a+bx)}dx - \frac{2\cos(a+bx)\sqrt{\csc(a+bx)}}{b}\right)}{3b} - \frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}$$

$$\downarrow 3119$$

$$\frac{2\left(-\frac{2\cos(a+bx)\sqrt{\csc(a+bx)}}{b}-\frac{2\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}(a+bx-\frac{\pi}{2})|2\right)}{b}\right)}{3b}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}$$

input
```
Int[x*Cos[a + b*x]*Csc[a + b*x]^(5/2),x]
```

output
```
(-2*x*Csc[a + b*x]^(3/2))/(3*b) + (2*((-2*Cos[a + b*x]*Sqrt[Csc[a + b*x]])
/b - (2*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b
*x]])/b))/(3*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4701
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csc[a + b*x^n]^(p
 - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ
[p, 1]
```

Maple [F]

$$\int x \cos(bx+a) \csc(bx+a)^{\frac{5}{2}}\, dx$$

input
```
int(x*cos(b*x+a)*csc(b*x+a)^(5/2),x)
```

output
```
int(x*cos(b*x+a)*csc(b*x+a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x \cos(a+bx) \csc^{\frac{5}{2}}(a+bx)\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x \cos(a+bx) \csc^{\frac{5}{2}}(a+bx)\, dx = \text{Timed out}$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx = \int x\cos\left(bx+a\right)\csc\left(bx+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)*csc(b*x + a)^(5/2), x)
```

Giac [F]

$$\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx = \int x\cos\left(bx+a\right)\csc\left(bx+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)*csc(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx = \int x\,\cos\left(a+b\,x\right)\left(\frac{1}{\sin\left(a+b\,x\right)}\right)^{5/2}dx$$

input
```
int(x*cos(a + b*x)*(1/sin(a + b*x))^(5/2),x)
```

output
```
int(x*cos(a + b*x)*(1/sin(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx = \int \sqrt{\csc(bx+a)}\,\cos(bx+a)\csc(bx+a)^2\,x dx$$

input
```
int(x*cos(b*x+a)*csc(b*x+a)^(5/2),x)
```

output
```
int(sqrt(csc(a + b*x))*cos(a + b*x)*csc(a + b*x)**2*x,x)
```

3.355 $\int x \cos(a+bx) \csc^{\frac{3}{2}}(a+bx)\, dx$

Optimal result . . . 2942
Mathematica [A] (verified) . . . 2942
Rubi [A] (verified) . . . 2943
Maple [F] . . . 2944
Fricas [F(-2)] . . . 2945
Sympy [F(-1)] . . . 2945
Maxima [F] . . . 2945
Giac [F] . . . 2946
Mupad [F(-1)] . . . 2946
Reduce [F] . . . 2946

Optimal result

Integrand size = 18, antiderivative size = 58

$$\int x \cos(a+bx) \csc^{\frac{3}{2}}(a+bx)\, dx = -\frac{2x\sqrt{\csc(a+bx)}}{b} + \frac{4\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{b^2}$$

output

```
-2*x*csc(b*x+a)^(1/2)/b+4*csc(b*x+a)^(1/2)*InverseJacobiAM(1/2*a-1/4*Pi+1/
2*b*x,2^(1/2))*sin(b*x+a)^(1/2)/b^2
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int x \cos(a+bx) \csc^{\frac{3}{2}}(a+bx)\, dx = -\frac{2\sqrt{\csc(a+bx)}\left(bx+2\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)\sqrt{\sin(a+bx)}\right)}{b^2}$$

input

```
Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(3/2),x]
```

output
```
(-2*Sqrt[Csc[a + b*x]]*(b*x + 2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[S
in[a + b*x]]))/b^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {4701, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cos(a+bx) \csc^{\frac{3}{2}}(a+bx)\, dx$$

$$\downarrow 4701$$

$$\frac{2\int \sqrt{\csc(a+bx)}dx}{b} - \frac{2x\sqrt{\csc(a+bx)}}{b}$$

$$\downarrow 3042$$

$$\frac{2\int \sqrt{\csc(a+bx)}dx}{b} - \frac{2x\sqrt{\csc(a+bx)}}{b}$$

$$\downarrow 4258$$

$$\frac{2\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}dx}{b} - \frac{2x\sqrt{\csc(a+bx)}}{b}$$

$$\downarrow 3042$$

$$\frac{2\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}dx}{b} - \frac{2x\sqrt{\csc(a+bx)}}{b}$$

$$\downarrow 3120$$

$$\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{b^2} - \frac{2x\sqrt{\csc(a+bx)}}{b}$$

input
```
Int[x*Cos[a + b*x]*Csc[a + b*x]^(3/2),x]
```

output
```
(-2*x*Sqrt[Csc[a + b*x]])/b + (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 +
b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4701
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csc[a + b*x^n]^(p
 - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ
[p, 1]
```

Maple [F]

$$\int x \cos(bx+a) \csc(bx+a)^{\frac{3}{2}}\, dx$$

input
```
int(x*cos(b*x+a)*csc(b*x+a)^(3/2),x)
```

output
```
int(x*cos(b*x+a)*csc(b*x+a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x \cos(a+bx) \csc^{\frac{3}{2}}(a+bx)\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x \cos(a+bx) \csc^{\frac{3}{2}}(a+bx)\, dx = \text{Timed out}$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x \cos(a+bx) \csc^{\frac{3}{2}}(a+bx)\, dx = \int x \cos\left(bx+a\right) \csc\left(bx+a\right)^{\frac{3}{2}}\, dx$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)*csc(b*x + a)^(3/2), x)
```

Giac [F]

$$\int x\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)\,dx = \int x\cos\left(bx+a\right)\csc\left(bx+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)*csc(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)\,dx = \int x\,\cos\left(a+b\,x\right)\left(\frac{1}{\sin\left(a+b\,x\right)}\right)^{3/2}dx$$

input
```
int(x*cos(a + b*x)*(1/sin(a + b*x))^(3/2),x)
```

output
```
int(x*cos(a + b*x)*(1/sin(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int x\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)\,dx = \int \sqrt{\csc\left(bx+a\right)}\,\cos\left(bx+a\right)\csc\left(bx+a\right)x dx$$

input
```
int(x*cos(b*x+a)*csc(b*x+a)^(3/2),x)
```

output
```
int(sqrt(csc(a + b*x))*cos(a + b*x)*csc(a + b*x)*x,x)
```

3.356 $\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx$

Optimal result . . . 2947
Mathematica [C] (verified) . . . 2947
Rubi [A] (verified) . . . 2948
Maple [C] (verified) . . . 2949
Fricas [F(-2)] . . . 2950
Sympy [F] . . . 2950
Maxima [F] . . . 2951
Giac [F] . . . 2951
Mupad [F(-1)] . . . 2951
Reduce [F] . . . 2952

Optimal result

Integrand size = 18, antiderivative size = 58

$$\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx = \frac{2x}{b\sqrt{\csc(a+bx)}} - \frac{4\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{b^2}$$

output
```
2*x/b/csc(b*x+a)^(1/2)+4*csc(b*x+a)^(1/2)*EllipticE(cos(1/2*a+1/4*Pi+1/2*b
*x),2^(1/2))*sin(b*x+a)^(1/2)/b^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.81 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.83

$$\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx = \frac{4\cos\left(\frac{1}{2}(a+bx)\right)\sqrt{\csc(a+bx)}\sin\left(\frac{1}{2}(a+bx)\right)\left(3bx-6\tan\left(\frac{1}{2}(a+bx)\right)+2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\right.\right.}{3b^2}$$

input
```
Integrate[x*Cos[a + b*x]*Sqrt[Csc[a + b*x]],x]
```

output
```
(4*Cos[(a + b*x)/2]*Sqrt[Csc[a + b*x]]*Sin[(a + b*x)/2]*(3*b*x - 6*Tan[(a
+ b*x)/2] + 2*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[(a + b*x)/2]^2]*Sqrt[S
ec[(a + b*x)/2]^2]*Tan[(a + b*x)/2]))/(3*b^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {4701, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx$$

$$\downarrow 4701$$

$$\frac{2x}{b\sqrt{\csc(a+bx)}} - \frac{2\int \frac{1}{\sqrt{\csc(a+bx)}}dx}{b}$$

$$\downarrow 3042$$

$$\frac{2x}{b\sqrt{\csc(a+bx)}} - \frac{2\int \frac{1}{\sqrt{\csc(a+bx)}}dx}{b}$$

$$\downarrow 4258$$

$$\frac{2x}{b\sqrt{\csc(a+bx)}} - \frac{2\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \sqrt{\sin(a+bx)}dx}{b}$$

$$\downarrow 3042$$

$$\frac{2x}{b\sqrt{\csc(a+bx)}} - \frac{2\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \sqrt{\sin(a+bx)}dx}{b}$$

$$\downarrow 3119$$

$$\frac{2x}{b\sqrt{\csc(a+bx)}} - \frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|\,2\right)}{b^2}$$

input
```
Int[x*Cos[a + b*x]*Sqrt[Csc[a + b*x]],x]
```

output
```
(2*x)/(b*Sqrt[Csc[a + b*x]]) - (4*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 +
 b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4701
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csc[a + b*x^n]^(p
 - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ
[p, 1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 308, normalized size of antiderivative = 5.31

method	result
risch	$-\frac{i(bx+2i)\left(e^{2i(bx+a)}-1\right)\sqrt{2}\sqrt{\frac{ie^{i(bx+a)}}{e^{2i(bx+a)}-1}}\,e^{-i(bx+a)}}{b^2} - \frac{2\left(-\frac{2i\left(ie^{2i(bx+a)}-i\right)}{\sqrt{e^{i(bx+a)}\left(ie^{2i(bx+a)}-i\right)}} - \frac{\sqrt{e^{i(bx+a)}+1}\sqrt{-2e^{i(bx+a)}+2}\sqrt{-e^{i(bx+}}$

input
```
int(x*cos(b*x+a)*csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*(b*x+2*I)*(exp(I*(b*x+a))^2-1)/b^2*2^(1/2)*(I*exp(I*(b*x+a))/(exp(I*(b*
x+a))^2-1))^(1/2)/exp(I*(b*x+a))-2/b^2*(-2*I*(-I+I*exp(I*(b*x+a))^2)/(exp(
I*(b*x+a))*(-I+I*exp(I*(b*x+a))^2))^(1/2)-(exp(I*(b*x+a))+1)^(1/2)*(-2*exp
(I*(b*x+a))+2)^(1/2)*(-exp(I*(b*x+a)))^(1/2)/(I*exp(I*(b*x+a))^3-I*exp(I*(
b*x+a)))^(1/2)*(-2*EllipticE((exp(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))+Ellipti
cF((exp(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))))*2^(1/2)*(I*exp(I*(b*x+a))/(exp(
I*(b*x+a))^2-1))^(1/2)*(I*(exp(I*(b*x+a))^2-1)*exp(I*(b*x+a)))^(1/2)/exp(I
*(b*x+a))
```

Fricas [F(-2)]

Exception generated.

$$\int x \cos(a+bx)\sqrt{\csc(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x \cos(a+bx)\sqrt{\csc(a+bx)}\,dx = \int x \cos\left(a+bx\right)\sqrt{\csc\left(a+bx\right)}\,dx$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)**(1/2),x)
```

output
```
Integral(x*cos(a + b*x)*sqrt(csc(a + b*x)), x)
```

Maxima [F]

$$\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx = \int x\cos\left(bx+a\right)\sqrt{\csc\left(bx+a\right)}\,dx$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)*sqrt(csc(b*x + a)), x)
```

Giac [F]

$$\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx = \int x\cos\left(bx+a\right)\sqrt{\csc\left(bx+a\right)}\,dx$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)*sqrt(csc(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx = \int x\,\cos\left(a+b\,x\right)\sqrt{\frac{1}{\sin\left(a+b\,x\right)}}\,dx$$

input
```
int(x*cos(a + b*x)*(1/sin(a + b*x))^(1/2),x)
```

output
```
int(x*cos(a + b*x)*(1/sin(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx = \int \sqrt{\csc(bx+a)}\,\cos(bx+a)\,x dx$$

input
```
int(x*cos(b*x+a)*csc(b*x+a)^(1/2),x)
```

output
```
int(sqrt(csc(a + b*x))*cos(a + b*x)*x,x)
```

3.357 $\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

Optimal result . . . 2953
Mathematica [A] (verified) . . . 2953
Rubi [A] (verified) . . . 2954
Maple [F] . . . 2956
Fricas [F(-2)] . . . 2956
Sympy [F] . . . 2957
Maxima [F] . . . 2957
Giac [F] . . . 2957
Mupad [F(-1)] . . . 2958
Reduce [F] . . . 2958

Optimal result

Integrand size = 18, antiderivative size = 85

$$\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} + \frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}} - \frac{4\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{9b^2}$$

output

```
2/3*x/b/csc(b*x+a)^(3/2)+4/9*cos(b*x+a)/b^2/csc(b*x+a)^(1/2)-4/9*csc(b*x+a
)^(1/2)*InverseJacobiAM(1/2*a-1/4*Pi+1/2*b*x,2^(1/2))*sin(b*x+a)^(1/2)/b^2
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{2\sqrt{\csc(a+bx)}\left(2\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)\sqrt{\sin(a+bx)}+3bx\sin^2(a+bx)+\sin(2(a+bx))\right)}{9b^2}$$

input

```
Integrate[(x*Cos[a + b*x])/Sqrt[Csc[a + b*x]],x]
```

output
```
(2*Sqrt[Csc[a + b*x]]*(2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a +
b*x]] + 3*b*x*Sin[a + b*x]^2 + Sin[2*(a + b*x)]))/(9*b^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {4701, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$

$$\downarrow 4701$$

$$\frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} - \frac{2\int \frac{1}{\csc^{\frac{3}{2}}(a+bx)}dx}{3b}$$

$$\downarrow 3042$$

$$\frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} - \frac{2\int \frac{1}{\csc(a+bx)^{3/2}}dx}{3b}$$

$$\downarrow 4256$$

$$\frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} - \frac{2\left(\frac{1}{3}\int \sqrt{\csc(a+bx)}dx - \frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right)}{3b}$$

$$\downarrow 3042$$

$$\frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} - \frac{2\left(\frac{1}{3}\int \sqrt{\csc(a+bx)}dx - \frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right)}{3b}$$

$$\downarrow 4258$$

$$\frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} - \frac{2\left(\frac{1}{3}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}dx - \frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right)}{3b}$$

$$\downarrow 3042$$

$$\frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} - \frac{2\left(\frac{1}{3}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int\frac{1}{\sqrt{\sin(a+bx)}}dx - \frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right)}{3b}$$

$$\downarrow 3120$$

$$\frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} - \frac{2\left(\frac{2\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx-\frac{\pi}{2}),2\right)}{3b} - \frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right)}{3b}$$

input
```
Int[(x*Cos[a + b*x])/Sqrt[Csc[a + b*x]],x]
```

output
```
(2*x)/(3*b*Csc[a + b*x]^(3/2)) - (2*((-2*Cos[a + b*x])/(3*b*Sqrt[Csc[a + b
*x]]) + (2*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a
+ b*x]])/(3*b)))/(3*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4701

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csc[a + b*x^n]^(p
 - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ
[p, 1]
```

Maple [F]

$$\int \frac{x \cos(bx+a)}{\sqrt{\csc(bx+a)}} dx$$

input

```
int(x*cos(b*x+a)/csc(b*x+a)^(1/2),x)
```

output

```
int(x*cos(b*x+a)/csc(b*x+a)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x \cos(a+bx)}{\sqrt{\csc(a+bx)}} dx = \text{Exception raised: TypeError}$$

input

```
integrate(x*cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{x \cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{x \cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$

input
```
integrate(x*cos(b*x+a)/csc(b*x+a)**(1/2),x)
```

output
```
Integral(x*cos(a + b*x)/sqrt(csc(a + b*x)), x)
```

Maxima [F]

$$\int \frac{x \cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{x \cos(bx+a)}{\sqrt{\csc(bx+a)}}\,dx$$

input
```
integrate(x*cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)/sqrt(csc(b*x + a)), x)
```

Giac [F]

$$\int \frac{x \cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{x \cos(bx+a)}{\sqrt{\csc(bx+a)}}\,dx$$

input
```
integrate(x*cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)/sqrt(csc(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{x\,\cos(a+b\,x)}{\sqrt{\frac{1}{\sin(a+b\,x)}}}\,dx$$

input
```
int((x*cos(a + b*x))/(1/sin(a + b*x))^(1/2),x)
```

output
```
int((x*cos(a + b*x))/(1/sin(a + b*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{\sqrt{\csc(bx+a)}\,\cos(bx+a)\,x}{\csc(bx+a)}dx$$

input
```
int(x*cos(b*x+a)/csc(b*x+a)^(1/2),x)
```

output
```
int((sqrt(csc(a + b*x))*cos(a + b*x)*x)/csc(a + b*x),x)
```

3.358 $\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result . . . 2959
Mathematica [C] (verified) . . . 2959
Rubi [A] (verified) . . . 2960
Maple [F] . . . 2962
Fricas [F(-2)] . . . 2962
Sympy [F] . . . 2963
Maxima [F] . . . 2963
Giac [F] . . . 2963
Mupad [F(-1)] . . . 2964
Reduce [F] . . . 2964

Optimal result

Integrand size = 18, antiderivative size = 85

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx = \frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} + \frac{4\cos(a+bx)}{25b^2\csc^{\frac{3}{2}}(a+bx)} - \frac{12\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{25b^2}$$

output

```
2/5*x/b/csc(b*x+a)^(5/2)+4/25*cos(b*x+a)/b^2/csc(b*x+a)^(3/2)+12/25*csc(b*
x+a)^(1/2)*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*sin(b*x+a)^(1/2)/b
^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx = \frac{\left(-10+4\cos(a+bx)+2\cos(2(a+bx))+4\sqrt{2}\sqrt{\frac{1}{1+\cos(a+bx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\tan^2\left(\frac{1}{2}(a+\right.\right.}{25b^2\sqrt{\csc(a+bx)}}$$

input
```
Integrate[(x*Cos[a + b*x])/Csc[a + b*x]^(3/2),x]
```

output
```
((-10 + 4*Cos[a + b*x] + 2*Cos[2*(a + b*x)] + 4*Sqrt[2]*Sqrt[(1 + Cos[a +
b*x])^(-1)]*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[(a + b*x)/2]^2] + 10*b*x
*Sin[a + b*x] + 5*b*x*Sin[2*(a + b*x)])*Tan[(a + b*x)/2])/(25*b^2*Sqrt[Csc
[a + b*x]])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {4701, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx$$

$$\downarrow 4701$$

$$\frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} - \frac{2\int \frac{1}{\csc^{\frac{5}{2}}(a+bx)}dx}{5b}$$

$$\downarrow 3042$$

$$\frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} - \frac{2\int \frac{1}{\csc(a+bx)^{5/2}}dx}{5b}$$

$$\downarrow 4256$$

$$\frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} - \frac{2\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc(a+bx)}}dx - \frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}\right)}{5b}$$

$$\downarrow 3042$$

$$\frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} - \frac{2\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc(a+bx)}}dx - \frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}\right)}{5b}$$

$$\downarrow 4258$$

$$\frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} - \frac{2\left(\frac{3}{5}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int\sqrt{\sin(a+bx)}dx - \frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}\right)}{5b}$$

$$\downarrow 3042$$

$$\frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} - \frac{2\left(\frac{3}{5}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int\sqrt{\sin(a+bx)}dx - \frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}\right)}{5b}$$

$$\downarrow 3119$$

$$\frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} - \frac{2\left(\frac{6\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}(a+bx-\frac{\pi}{2})|2\right)}{5b} - \frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}\right)}{5b}$$

input
```
Int[(x*Cos[a + b*x])/Csc[a + b*x]^(3/2),x]
```

output
```
(2*x)/(5*b*Csc[a + b*x]^(5/2)) - (2*((-2*Cos[a + b*x])/(5*b*Csc[a + b*x]^(
3/2)) + (6*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a
+ b*x]])/(5*b)))/(5*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4701

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csc[a + b*x^n]^(p
 - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ
[p, 1]
```

Maple [F]

$$\int \frac{x \cos(bx+a)}{\csc(bx+a)^{\frac{3}{2}}} dx$$

input

```
int(x*cos(b*x+a)/csc(b*x+a)^(3/2),x)
```

output

```
int(x*cos(b*x+a)/csc(b*x+a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x \cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)} dx = \text{Exception raised: TypeError}$$

input

```
integrate(x*cos(b*x+a)/csc(b*x+a)^(3/2),x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{x \cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x \cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(x*cos(b*x+a)/csc(b*x+a)**(3/2),x)
```

output
```
Integral(x*cos(a + b*x)/csc(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{x \cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x \cos(bx+a)}{\csc(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/csc(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)/csc(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x \cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x \cos(bx+a)}{\csc(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/csc(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)/csc(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\,\cos(a+b\,x)}{\left(\frac{1}{\sin(a+b\,x)}\right)^{3/2}}\,dx$$

input
```
int((x*cos(a + b*x))/(1/sin(a + b*x))^(3/2),x)
```

output
```
int((x*cos(a + b*x))/(1/sin(a + b*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\csc(bx+a)}\,\cos(bx+a)\,x}{\csc(bx+a)^2}\,dx$$

input
```
int(x*cos(b*x+a)/csc(b*x+a)^(3/2),x)
```

output
```
int((sqrt(csc(a + b*x))*cos(a + b*x)*x)/csc(a + b*x)**2,x)
```

3.359 $\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx$

Optimal result . . . 2965
Mathematica [A] (verified) . . . 2965
Rubi [A] (verified) . . . 2966
Maple [F] . . . 2968
Fricas [F(-2)] . . . 2969
Sympy [F] . . . 2969
Maxima [F] . . . 2969
Giac [F] . . . 2970
Mupad [F(-1)] . . . 2970
Reduce [F] . . . 2970

Optimal result

Integrand size = 18, antiderivative size = 108

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx = \frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} + \frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)} + \frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}} - \frac{20\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{147b^2}$$

output

```
2/7*x/b/csc(b*x+a)^(7/2)+4/49*cos(b*x+a)/b^2/csc(b*x+a)^(5/2)+20/147*cos(b
*x+a)/b^2/csc(b*x+a)^(1/2)-20/147*csc(b*x+a)^(1/2)*InverseJacobiAM(1/2*a-1
/4*Pi+1/2*b*x,2^(1/2))*sin(b*x+a)^(1/2)/b^2
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.86

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx = \frac{\sqrt{\csc(a+bx)}\left(63bx - 84bx\cos(2(a+bx)) + 21bx\cos(4(a+bx)) + 80\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)\right.}{588b^2}$$

input

```
Integrate[(x*Cos[a + b*x])/Csc[a + b*x]^(5/2),x]
```

output

```
(Sqrt[Csc[a + b*x]]*(63*b*x - 84*b*x*Cos[2*(a + b*x)] + 21*b*x*Cos[4*(a +
b*x)] + 80*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]] + 52*Sin
[2*(a + b*x)] - 6*Sin[4*(a + b*x)]))/(588*b^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4701, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx$$

$$\downarrow 4701$$

$$\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} - \frac{2\int \frac{1}{\csc^{\frac{7}{2}}(a+bx)}dx}{7b}$$

$$\downarrow 3042$$

$$\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} - \frac{2\int \frac{1}{\csc(a+bx)^{7/2}}dx}{7b}$$

$$\downarrow 4256$$

$$\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} - \frac{2\left(\frac{5}{7}\int \frac{1}{\csc^{\frac{3}{2}}(a+bx)}dx - \frac{2\cos(a+bx)}{7b\csc^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\downarrow 3042$$

$$\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} - \frac{2\left(\frac{5}{7}\int \frac{1}{\csc(a+bx)^{3/2}}dx - \frac{2\cos(a+bx)}{7b\csc^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\downarrow 4256$$

$$\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} - \frac{2\left(\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\csc(a+bx)}dx - \frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right) - \frac{2\cos(a+bx)}{7b\csc^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\downarrow 3042$$

$$\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} - \frac{2\left(\frac{5}{7}\left(\frac{1}{3}\int\sqrt{\csc(a+bx)}dx - \frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right) - \frac{2\cos(a+bx)}{7b\csc^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\downarrow 4258$$

$$\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} - \frac{2\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int\frac{1}{\sqrt{\sin(a+bx)}}dx - \frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right) - \frac{2\cos(a+bx)}{7b\csc^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\downarrow 3042$$

$$\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} - \frac{2\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int\frac{1}{\sqrt{\sin(a+bx)}}dx - \frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right) - \frac{2\cos(a+bx)}{7b\csc^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\downarrow 3120$$

$$\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} - \frac{2\left(\frac{5}{7}\left(\frac{2\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx-\frac{\pi}{2}),2\right)}{3b} - \frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right) - \frac{2\cos(a+bx)}{7b\csc^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

input

```
Int[(x*Cos[a + b*x])/Csc[a + b*x]^(5/2),x]
```

output

```
(2*x)/(7*b*Csc[a + b*x]^(7/2)) - (2*((-2*Cos[a + b*x])/(7*b*Csc[a + b*x]^(
5/2)) + (5*((-2*Cos[a + b*x])/(3*b*Sqrt[Csc[a + b*x]]) + (2*Sqrt[Csc[a + b
*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(3*b)))/7))/(7*b
)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4701
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csc[a + b*x^n]^(p
 - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ
[p, 1]
```

Maple [F]

$$\int \frac{x \cos(bx+a)}{\csc(bx+a)^{\frac{5}{2}}} dx$$

input
```
int(x*cos(b*x+a)/csc(b*x+a)^(5/2),x)
```

output
```
int(x*cos(b*x+a)/csc(b*x+a)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)/csc(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cos\left(a+bx\right)}{\csc^{\frac{5}{2}}\left(a+bx\right)}\,dx$$

input
```
integrate(x*cos(b*x+a)/csc(b*x+a)**(5/2),x)
```

output
```
Integral(x*cos(a + b*x)/csc(a + b*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cos\left(bx+a\right)}{\csc\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/csc(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)/csc(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cos(bx+a)}{\csc(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/csc(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)/csc(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\,\cos(a+b\,x)}{\left(\frac{1}{\sin(a+b\,x)}\right)^{5/2}}\,dx$$

input
```
int((x*cos(a + b*x))/(1/sin(a + b*x))^(5/2),x)
```

output
```
int((x*cos(a + b*x))/(1/sin(a + b*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\csc(bx+a)}\,\cos(bx+a)\,x}{\csc(bx+a)^3}\,dx$$

input
```
int(x*cos(b*x+a)/csc(b*x+a)^(5/2),x)
```

output
```
int((sqrt(csc(a + b*x))*cos(a + b*x)*x)/csc(a + b*x)**3,x)
```

3.360 $\int x \csc(x) \sin(3x)\, dx$

Optimal result 2971
Mathematica [A] (verified) 2971
Rubi [A] (verified) 2972
Maple [A] (verified) 2973
Fricas [A] (verification not implemented) 2973
Sympy [A] (verification not implemented) 2973
Maxima [A] (verification not implemented) 2974
Giac [A] (verification not implemented) 2974
Mupad [B] (verification not implemented) 2974
Reduce [F] 2975

Optimal result

Integrand size = 8, antiderivative size = 31

$$\int x \csc(x) \sin(3x)\, dx = \frac{x^2}{2} + \frac{3\cos^2(x)}{4} + 2x\cos(x)\sin(x) - \frac{\sin^2(x)}{4}$$

output
```
1/2*x^2+3/4*cos(x)^2+2*x*cos(x)*sin(x)-1/4*sin(x)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int x \csc(x) \sin(3x)\, dx = \frac{x^2}{2} + \frac{1}{2}\cos(2x) + x\sin(2x)$$

input
```
Integrate[x*Csc[x]*Sin[3*x],x]
```

output
```
x^2/2 + Cos[2*x]/2 + x*Sin[2*x]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin(3x) \csc(x)\, dx$$

$$\downarrow 4931$$

$$\int \left(3x \cos^2(x) - x \sin^2(x)\right) dx$$

$$\downarrow 2009$$

$$\frac{x^2}{2} - \frac{\sin^2(x)}{4} + \frac{3\cos^2(x)}{4} + 2x \sin(x) \cos(x)$$

input
```
Int[x*Csc[x]*Sin[3*x],x]
```

output
```
x^2/2 + (3*Cos[x]^2)/4 + 2*x*Cos[x]*Sin[x] - Sin[x]^2/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{x^2}{2} + \frac{\cos(2x)}{2} + x\sin(2x)$	19
risch	$\frac{x^2}{2} + \frac{\cos(2x)}{2} + x\sin(2x)$	19

input
```
int(x*csc(x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+1/2*cos(2*x)+x*sin(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int x\csc(x)\sin(3x)\,dx = 2\,x\cos(x)\sin(x) + \frac{1}{2}\,x^2 + \cos(x)^2$$

input
```
integrate(x*csc(x)*sin(3*x),x, algorithm="fricas")
```

output
```
2*x*cos(x)*sin(x) + 1/2*x^2 + cos(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int x\csc(x)\sin(3x)\,dx = -\frac{x^2}{2} + x(x + \sin(2x)) + \frac{\cos(2x)}{2}$$

input
```
integrate(x*csc(x)*sin(3*x),x)
```

output
```
-x**2/2 + x*(x + sin(2*x)) + cos(2*x)/2
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

$$\int x \csc(x) \sin(3x)\, dx = \frac{1}{2} x^2 + x \sin(2x) + \frac{1}{2} \cos(2x)$$

input
```
integrate(x*csc(x)*sin(3*x),x, algorithm="maxima")
```

output
```
1/2*x^2 + x*sin(2*x) + 1/2*cos(2*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

$$\int x \csc(x) \sin(3x)\, dx = \frac{1}{2} x^2 + x \sin(2x) + \frac{1}{2} \cos(2x)$$

input
```
integrate(x*csc(x)*sin(3*x),x, algorithm="giac")
```

output
```
1/2*x^2 + x*sin(2*x) + 1/2*cos(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

$$\int x \csc(x) \sin(3x)\, dx = \frac{\cos(2x)}{2} + x \sin(2x) + \frac{x^2}{2}$$

input
```
int((x*sin(3*x))/sin(x),x)
```

output
```
cos(2*x)/2 + x*sin(2*x) + x^2/2
```

Reduce [F]

$$\int x \csc(x) \sin(3x)\, dx = \int \csc(x) \sin(3x)\, x dx$$

input
```
int(x*csc(x)*sin(3*x),x)
```

output
```
int(csc(x)*sin(3*x)*x,x)
```

3.361 $\int (c+dx)^4 \csc(x) \sin(3x)\, dx$

Optimal result . . . 2976
Mathematica [A] (verified) . . . 2976
Rubi [A] (verified) . . . 2977
Maple [A] (verified) . . . 2978
Fricas [A] (verification not implemented) . . . 2978
Sympy [A] (verification not implemented) . . . 2979
Maxima [A] (verification not implemented) . . . 2980
Giac [A] (verification not implemented) . . . 2980
Mupad [B] (verification not implemented) . . . 2981
Reduce [F] . . . 2982

Optimal result

Integrand size = 14, antiderivative size = 131

$$\begin{aligned}\int (c+dx)^4 \csc(x) \sin(3x)\, dx = {} & \frac{3d^4x}{2} - d(c+dx)^3 + \frac{(c+dx)^5}{5d} - \frac{9}{2}d^3(c+dx)\cos^2(x) \\ & + 3d(c+dx)^3\cos^2(x) + 3d^4\cos(x)\sin(x) \\ & - 6d^2(c+dx)^2\cos(x)\sin(x) + 2(c+dx)^4\cos(x)\sin(x) \\ & + \frac{3}{2}d^3(c+dx)\sin^2(x) - d(c+dx)^3\sin^2(x)\end{aligned}$$

output

```
3/2*d^4*x-d*(d*x+c)^3+1/5*(d*x+c)^5/d-9/2*d^3*(d*x+c)*cos(x)^2+3*d*(d*x+c)
^3*cos(x)^2+3*d^4*cos(x)*sin(x)-6*d^2*(d*x+c)^2*cos(x)*sin(x)+2*(d*x+c)^4*
cos(x)*sin(x)+3/2*d^3*(d*x+c)*sin(x)^2-d*(d*x+c)^3*sin(x)^2
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.18

$$\begin{aligned}\int (c+dx)^4 \csc(x) \sin(3x)\, dx = {} & c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5} + d\big(2c^3 + 6c^2dx \\ & + d^3x\big(-3+2x^2\big) + 3cd^2\big(-1+2x^2\big)\big)\cos(2x) \\ & + \frac{1}{2}\big(2c^4 + 8c^3dx + 4cd^3x\big(-3+2x^2\big) + 6c^2d^2\big(-1+2x^2\big) \\ & + d^4\big(3-6x^2+2x^4\big)\big)\sin(2x)\end{aligned}$$

input
```
Integrate[(c + d*x)^4*Csc[x]*Sin[3*x],x]
```

output
```
c^4*x + 2*c^3*d*x^2 + 2*c^2*d^2*x^3 + c*d^3*x^4 + (d^4*x^5)/5 + d*(2*c^3 +
 6*c^2*d*x + d^3*x*(-3 + 2*x^2) + 3*c*d^2*(-1 + 2*x^2))*Cos[2*x] + ((2*c^4
 + 8*c^3*d*x + 4*c*d^3*x*(-3 + 2*x^2) + 6*c^2*d^2*(-1 + 2*x^2) + d^4*(3 -
6*x^2 + 2*x^4))*Sin[2*x])/2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(3x)\csc(x)(c+dx)^4\,dx$$

$$\downarrow 4931$$

$$\int \left(3\cos^2(x)(c+dx)^4 - \sin^2(x)(c+dx)^4\right)dx$$

$$\downarrow 2009$$

$$\frac{3}{2}d^3\sin^2(x)(c+dx) - \frac{9}{2}d^3\cos^2(x)(c+dx) - 6d^2\sin(x)\cos(x)(c+dx)^2 + \frac{(c+dx)^5}{5d} - d(c+dx)^3 - d\sin^2(x)(c+dx)^3 + 3d\cos^2(x)(c+dx)^3 + 2\sin(x)\cos(x)(c+dx)^4 + \frac{3d^4x}{2} + 3d^4\sin(x)\cos(x)$$

input
```
Int[(c + d*x)^4*Csc[x]*Sin[3*x],x]
```

output
```
(3*d^4*x)/2 - d*(c + d*x)^3 + (c + d*x)^5/(5*d) - (9*d^3*(c + d*x)*Cos[x]^
2)/2 + 3*d*(c + d*x)^3*Cos[x]^2 + 3*d^4*Cos[x]*Sin[x] - 6*d^2*(c + d*x)^2*
Cos[x]*Sin[x] + 2*(c + d*x)^4*Cos[x]*Sin[x] + (3*d^3*(c + d*x)*Sin[x]^2)/2
 - d*(c + d*x)^3*Sin[x]^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.33

method	result
risch	$\frac{d^4x^5}{5} + d^3c\,x^4 + 2c^2d^2x^3 + 2c^3d\,x^2 + c^4x + \frac{c^5}{5d} + d(2d^3x^3 + 6c\,d^2x^2 + 6c^2dx - 3d^3x + 2c^3 - 3c\,d$
default	$4d^4\left(x^4\left(\frac{\sin(x)\cos(x)}{2} + \frac{x}{2}\right) + x^3\cos(x)^2 - 3x^2\left(\frac{\sin(x)\cos(x)}{2} + \frac{x}{2}\right) - \frac{3x\cos(x)^2}{2} + \frac{3\sin(x)\cos(x)}{4} + \frac{3x}{4} + x$

input
```
int((d*x+c)^4*csc(x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/5*d^4*x^5+d^3*c*x^4+2*c^2*d^2*x^3+2*c^3*d*x^2+c^4*x+1/5/d*c^5+d*(2*d^3*x
^3+6*c*d^2*x^2+6*c^2*d*x-3*d^3*x+2*c^3-3*c*d^2)*cos(2*x)+1/2*(2*d^4*x^4+8*
c*d^3*x^3+12*c^2*d^2*x^2-6*d^4*x^2+8*c^3*d*x-12*c*d^3*x+2*c^4-6*c^2*d^2+3*
d^4)*sin(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.53

$$\int (c+dx)^4 \csc(x)\sin(3x)\,dx = \frac{1}{5}d^4x^5 + cd^3x^4 + 2\left(c^2d^2 - d^4\right)x^3 + 2\left(c^3d - 3\,cd^3\right)x^2 + 2\left(2\,d^4x^3 + 6\,cd^3x^2 + 2\,c^3d - 3\,cd^3 + 3\left(2\,c^2d^2 - d^4\right)x\right)\cos(x)^2 + \left(2\,d^4x^4 + 8\,cd^3x^3 + 2\,c^4 - 6\,c^2d^2 + 3\,d^4 + 6\left(2\,c^2d^2 - d^4\right)x^2 + 4\left(2\,c^3d - 3\,cd^3\right)x\right)\cos(x)\sin(x) + \left(c^4 - 6\,c^2d^2 + 3\,d^4\right)x$$

input
```
integrate((d*x+c)^4*csc(x)*sin(3*x),x, algorithm="fricas")
```

output
```
1/5*d^4*x^5 + c*d^3*x^4 + 2*(c^2*d^2 - d^4)*x^3 + 2*(c^3*d - 3*c*d^3)*x^2
+ 2*(2*d^4*x^3 + 6*c*d^3*x^2 + 2*c^3*d - 3*c*d^3 + 3*(2*c^2*d^2 - d^4)*x)*
cos(x)^2 + (2*d^4*x^4 + 8*c*d^3*x^3 + 2*c^4 - 6*c^2*d^2 + 3*d^4 + 6*(2*c^2
*d^2 - d^4)*x^2 + 4*(2*c^3*d - 3*c*d^3)*x)*cos(x)*sin(x) + (c^4 - 6*c^2*d^
2 + 3*d^4)*x
```

Sympy [A] (verification not implemented)

Time = 15.79 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.70

$$\begin{aligned}\int (c+dx)^4 \csc(x)\sin(3x)\,dx = {} & c^4(x+\sin(2x)) + 4c^3 d\left(-\frac{x^2}{2} + x(x+\sin(2x)) + \frac{\cos(2x)}{2}\right) \\ & + 6c^2 d^2\left(\frac{x^3}{3} + x^2(x+\sin(2x)) - 2x\left(\frac{x^2}{2} - \frac{\cos(2x)}{2}\right) - \frac{\sin(2x)}{2}\right) \\ & + 4cd^3\left(-\frac{x^4}{4} + x^3(x+\sin(2x)) - 3x^2\left(\frac{x^2}{2} - \frac{\cos(2x)}{2}\right) + 3x\left(\frac{x^3}{3} - \frac{\sin(2x)}{2}\right) - \frac{3\cos(2x)}{4}\right) \\ & + d^4\left(\frac{x^5}{5} + x^4(x+\sin(2x)) - 4x^3\left(\frac{x^2}{2} - \frac{\cos(2x)}{2}\right) + 6x^2\left(\frac{x^3}{3} - \frac{\sin(2x)}{2}\right) - 2x\left(\frac{x^4}{2} + \frac{3\cos(2x)}{2}\right) + \frac{3\sin(2x)}{2}\right)\end{aligned}$$

input
```
integrate((d*x+c)**4*csc(x)*sin(3*x),x)
```

output
```
c**4*(x + sin(2*x)) + 4*c**3*d*(-x**2/2 + x*(x + sin(2*x)) + cos(2*x)/2) +
 6*c**2*d**2*(x**3/3 + x**2*(x + sin(2*x)) - 2*x*(x**2/2 - cos(2*x)/2) - s
in(2*x)/2) + 4*c*d**3*(-x**4/4 + x**3*(x + sin(2*x)) - 3*x**2*(x**2/2 - co
s(2*x)/2) + 3*x*(x**3/3 - sin(2*x)/2) - 3*cos(2*x)/4) + d**4*(x**5/5 + x**
4*(x + sin(2*x)) - 4*x**3*(x**2/2 - cos(2*x)/2) + 6*x**2*(x**3/3 - sin(2*x
)/2) - 2*x*(x**4/2 + 3*cos(2*x)/2) + 3*sin(2*x)/2)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

$$\begin{aligned}
&\int (c+dx)^4 \csc(x)\sin(3x)\,dx \\
&= 2\left(x^2 + 2x\sin(2x) + \cos(2x)\right)c^3 d \\
&\quad + \left(2x^3 + 6x\cos(2x) + 3\left(2x^2-1\right)\sin(2x)\right)c^2 d^2 \\
&\quad + \left(x^4 + 3\left(2x^2-1\right)\cos(2x) + 2\left(2x^3 - 3x\right)\sin(2x)\right)cd^3 \\
&\quad + \frac{1}{10}\left(2x^5 + 10\left(2x^3-3x\right)\cos(2x) + 5\left(2x^4 - 6x^2 + 3\right)\sin(2x)\right)d^4 \\
&\quad + c^4(x + \sin(2x))
\end{aligned}$$

input
```
integrate((d*x+c)^4*csc(x)*sin(3*x),x, algorithm="maxima")
```

output
```
2*(x^2 + 2*x*sin(2*x) + cos(2*x))*c^3*d + (2*x^3 + 6*x*cos(2*x) + 3*(2*x^2
 - 1)*sin(2*x))*c^2*d^2 + (x^4 + 3*(2*x^2 - 1)*cos(2*x) + 2*(2*x^3 - 3*x)*
sin(2*x))*c*d^3 + 1/10*(2*x^5 + 10*(2*x^3 - 3*x)*cos(2*x) + 5*(2*x^4 - 6*x
^2 + 3)*sin(2*x))*d^4 + c^4*(x + sin(2*x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.27

$$\begin{aligned}
&\int (c+dx)^4 \csc(x)\sin(3x)\,dx = \frac{1}{5}d^4x^5 + cd^3x^4 + 2c^2d^2x^3 + 2c^3dx^2 \\
&\quad + c^4x + \left(2d^4x^3 + 6cd^3x^2 + 6c^2d^2x - 3d^4x + 2c^3d - 3cd^3\right)\cos(2x) \\
&\quad + \frac{1}{2}\left(2d^4x^4 + 8cd^3x^3 + 12c^2d^2x^2 - 6d^4x^2 + 8c^3dx - 12cd^3x + 2c^4 - 6c^2d^2 + 3d^4\right)\sin(2x)
\end{aligned}$$

input
```
integrate((d*x+c)^4*csc(x)*sin(3*x),x, algorithm="giac")
```

output
```
1/5*d^4*x^5 + c*d^3*x^4 + 2*c^2*d^2*x^3 + 2*c^3*d*x^2 + c^4*x + (2*d^4*x^3
 + 6*c*d^3*x^2 + 6*c^2*d^2*x - 3*d^4*x + 2*c^3*d - 3*c*d^3)*cos(2*x) + 1/2
*(2*d^4*x^4 + 8*c*d^3*x^3 + 12*c^2*d^2*x^2 - 6*d^4*x^2 + 8*c^3*d*x - 12*c*
d^3*x + 2*c^4 - 6*c^2*d^2 + 3*d^4)*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 19.27 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.62

$$\int (c+dx)^4 \csc(x)\sin(3x)\,dx = c^4\sin(2x) + \frac{3d^4\sin(2x)}{2} + c^4 x + \frac{d^4 x^5}{5} - 3c^2 d^2 \sin(2x) + 2d^4 x^3\cos(2x) - 3d^4 x^2 \sin(2x) + d^4 x^4 \sin(2x) + 2c^3 d x^2 + c d^3 x^4 + 2c^2 d^2 x^3 - 3c d^3 \cos(2x) + 2c^3 d\cos(2x) - 3d^4 x\cos(2x) + 6c^2 d^2 x^2 \sin(2x) - 6c d^3 x \sin(2x) + 4c^3 d x \sin(2x) + 6c^2 d^2 x\cos(2x) + 6c d^3 x^2\cos(2x) + 4c d^3 x^3 \sin(2x)$$

input
```
int((sin(3*x)*(c + d*x)^4)/sin(x),x)
```

output
```
c^4*sin(2*x) + (3*d^4*sin(2*x))/2 + c^4*x + (d^4*x^5)/5 - 3*c^2*d^2*sin(2*
x) + 2*d^4*x^3*cos(2*x) - 3*d^4*x^2*sin(2*x) + d^4*x^4*sin(2*x) + 2*c^3*d*
x^2 + c*d^3*x^4 + 2*c^2*d^2*x^3 - 3*c*d^3*cos(2*x) + 2*c^3*d*cos(2*x) - 3*
d^4*x*cos(2*x) + 6*c^2*d^2*x^2*sin(2*x) - 6*c*d^3*x*sin(2*x) + 4*c^3*d*x*s
in(2*x) + 6*c^2*d^2*x*cos(2*x) + 6*c*d^3*x^2*cos(2*x) + 4*c*d^3*x^3*sin(2*
x)
```

Reduce [F]

$$\begin{aligned}\int (c+dx)^4 \csc(x)\sin(3x)\,dx = &\left(\int \csc(x)\sin(3x)\,x^4 dx\right) d^4 \\ &+ 4\left(\int \csc(x)\sin(3x)\,x^3 dx\right) c\,d^3 \\ &+ 6\left(\int \csc(x)\sin(3x)\,x^2 dx\right) c^2 d^2 \\ &+ 4\left(\int \csc(x)\sin(3x)\,x dx\right) c^3 d \\ &+ \left(\int \csc(x)\sin(3x)\,dx\right) c^4\end{aligned}$$

input
```
int((d*x+c)^4*csc(x)*sin(3*x),x)
```

output
```
int(csc(x)*sin(3*x)*x**4,x)*d**4 + 4*int(csc(x)*sin(3*x)*x**3,x)*c*d**3 +
6*int(csc(x)*sin(3*x)*x**2,x)*c**2*d**2 + 4*int(csc(x)*sin(3*x)*x,x)*c**3*
d + int(csc(x)*sin(3*x),x)*c**4
```

3.362 $\int (c+dx)^3 \csc(x) \sin(3x)\, dx$

Optimal result . . . 2983
Mathematica [A] (verified) . . . 2983
Rubi [A] (verified) . . . 2984
Maple [A] (verified) . . . 2985
Fricas [A] (verification not implemented) . . . 2985
Sympy [A] (verification not implemented) . . . 2986
Maxima [A] (verification not implemented) . . . 2987
Giac [A] (verification not implemented) . . . 2987
Mupad [B] (verification not implemented) . . . 2988
Reduce [F] . . . 2988

Optimal result

Integrand size = 14, antiderivative size = 108

$$\begin{aligned}\int (c+dx)^3 \csc(x) \sin(3x)\, dx = &-\frac{3}{4}d(c+dx)^2 + \frac{(c+dx)^4}{4d} \\ &- \frac{9}{8}d^3\cos^2(x) + \frac{9}{4}d(c+dx)^2\cos^2(x) \\ &- 3d^2(c+dx)\cos(x)\sin(x) + 2(c+dx)^3\cos(x)\sin(x) \\ &+ \frac{3}{8}d^3\sin^2(x) - \frac{3}{4}d(c+dx)^2\sin^2(x)\end{aligned}$$

output

```
-3/4*d*(d*x+c)^2+1/4*(d*x+c)^4/d-9/8*d^3*cos(x)^2+9/4*d*(d*x+c)^2*cos(x)^2
-3*d^2*(d*x+c)*cos(x)*sin(x)+2*(d*x+c)^3*cos(x)*sin(x)+3/8*d^3*sin(x)^2-3/
4*d*(d*x+c)^2*sin(x)^2
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.01

$$\begin{aligned}\int (c+dx)^3 \csc(x) \sin(3x)\, dx = \frac{1}{4}\big(&x\left(4c^3 + 6c^2dx + 4cd^2x^2 + d^3x^3\right) \\ &+ 3d\left(2c^2 + 4cdx + d^2\left(-1+2x^2\right)\right)\cos(2x) + 2\big(2c^3 \\ &+ 6c^2dx + d^3x\left(-3+2x^2\right) + 3cd^2\left(-1+2x^2\right)\big)\sin(2x)\big)\end{aligned}$$

input
```
Integrate[(c + d*x)^3*Csc[x]*Sin[3*x],x]
```

output
```
(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 3*d*(2*c^2 + 4*c*d*x + d^
2*(-1 + 2*x^2))*Cos[2*x] + 2*(2*c^3 + 6*c^2*d*x + d^3*x*(-3 + 2*x^2) + 3*c
*d^2*(-1 + 2*x^2))*Sin[2*x])/4
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(3x)\csc(x)(c+dx)^3\,dx$$

$$\downarrow 4931$$

$$\int \left(3\cos^2(x)(c+dx)^3 - \sin^2(x)(c+dx)^3\right)dx$$

$$\downarrow 2009$$

$$-3d^2\sin(x)\cos(x)(c+dx) + \frac{(c+dx)^4}{4d} - \frac{3}{4}d(c+dx)^2 - \frac{3}{4}d\sin^2(x)(c+dx)^2 + \frac{9}{4}d\cos^2(x)(c+dx)^2 + 2\sin(x)\cos(x)(c+dx)^3 + \frac{3}{8}d^3\sin^2(x) - \frac{9}{8}d^3\cos^2(x)$$

input
```
Int[(c + d*x)^3*Csc[x]*Sin[3*x],x]
```

output
```
(-3*d*(c + d*x)^2)/4 + (c + d*x)^4/(4*d) - (9*d^3*Cos[x]^2)/8 + (9*d*(c +
d*x)^2*Cos[x]^2)/4 - 3*d^2*(c + d*x)*Cos[x]*Sin[x] + 2*(c + d*x)^3*Cos[x]*
Sin[x] + (3*d^3*Sin[x]^2)/8 - (3*d*(c + d*x)^2*Sin[x]^2)/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10

method	result
risch	$\frac{d^3x^4}{4}+d^2cx^3+\frac{3dc^2x^2}{2}+c^3x+\frac{c^4}{4d}+\frac{3d\left(2x^2d^2+4cdx+2c^2-d^2\right)\cos(2x)}{4}+\frac{\left(2d^3x^3+6cd^2x^2+6c^2dx-3d^3x+2c^3-3cd\right.}{2}$
default	$4d^3\left(x^3\left(\frac{\sin(x)\cos(x)}{2}+\frac{x}{2}\right)+\frac{3x^2\cos(x)^2}{4}-\frac{3x\left(\frac{\sin(x)\cos(x)}{2}+\frac{x}{2}\right)}{2}+\frac{3x^2}{8}+\frac{3\sin(x)^2}{8}-\frac{3x^4}{8}\right)+12cd^2\left(x^2\left(\frac{\sin}{}\right.\right.$

input
```
int((d*x+c)^3*csc(x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*d^3*x^4+d^2*c*x^3+3/2*d*c^2*x^2+c^3*x+1/4/d*c^4+3/4*d*(2*d^2*x^2+4*c*d
*x+2*c^2-d^2)*cos(2*x)+1/2*(2*d^3*x^3+6*c*d^2*x^2+6*c^2*d*x-3*d^3*x+2*c^3-
3*c*d^2)*sin(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.18

$$\int (c+dx)^3\csc(x)\sin(3x)\,dx$$
$$=\frac{1}{4}d^3x^4+cd^2x^3+\frac{3}{2}\left(c^2d-d^3\right)x^2+\frac{3}{2}\left(2d^3x^2+4cd^2x+2c^2d-d^3\right)\cos(x)^2$$
$$+\left(2d^3x^3+6cd^2x^2+2c^3-3cd^2+3\left(2c^2d-d^3\right)x\right)\cos(x)\sin(x)+\left(c^3-3cd^2\right)x$$

input
```
integrate((d*x+c)^3*csc(x)*sin(3*x),x, algorithm="fricas")
```

output
```
1/4*d^3*x^4 + c*d^2*x^3 + 3/2*(c^2*d - d^3)*x^2 + 3/2*(2*d^3*x^2 + 4*c*d^2
*x + 2*c^2*d - d^3)*cos(x)^2 + (2*d^3*x^3 + 6*c*d^2*x^2 + 2*c^3 - 3*c*d^2
+ 3*(2*c^2*d - d^3)*x)*cos(x)*sin(x) + (c^3 - 3*c*d^2)*x
```

Sympy [A] (verification not implemented)

Time = 8.45 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.29

$$\int (c+dx)^3 \csc(x)\sin(3x)\,dx = c^3(x+\sin(2x)) + 3c^2 d\left(-\frac{x^2}{2} + x(x+\sin(2x)) + \frac{\cos(2x)}{2}\right) + 3cd^2\left(\frac{x^3}{3} + x^2(x+\sin(2x)) - 2x\left(\frac{x^2}{2} - \frac{\cos(2x)}{2}\right) - \frac{\sin(2x)}{2}\right) + d^3\left(-\frac{x^4}{4} + x^3(x+\sin(2x)) - 3x^2\left(\frac{x^2}{2} - \frac{\cos(2x)}{2}\right) + 3x\left(\frac{x^3}{3} - \frac{\sin(2x)}{2}\right) - \frac{3\cos(2x)}{4}\right)$$

input
```
integrate((d*x+c)**3*csc(x)*sin(3*x),x)
```

output
```
c**3*(x + sin(2*x)) + 3*c**2*d*(-x**2/2 + x*(x + sin(2*x)) + cos(2*x)/2) +
 3*c*d**2*(x**3/3 + x**2*(x + sin(2*x)) - 2*x*(x**2/2 - cos(2*x)/2) - sin(
2*x)/2) + d**3*(-x**4/4 + x**3*(x + sin(2*x)) - 3*x**2*(x**2/2 - cos(2*x)/
2) + 3*x*(x**3/3 - sin(2*x)/2) - 3*cos(2*x)/4)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\begin{aligned}
&\int (c+dx)^3 \csc(x)\sin(3x)\,dx \\
&= \frac{3}{2}\left(x^2 + 2x\sin(2x) + \cos(2x)\right)c^2 d \\
&\quad + \frac{1}{2}\left(2x^3 + 6x\cos(2x) + 3\left(2x^2-1\right)\sin(2x)\right)cd^2 \\
&\quad + \frac{1}{4}\left(x^4 + 3\left(2x^2-1\right)\cos(2x) + 2\left(2x^3 - 3x\right)\sin(2x)\right)d^3 + c^3(x+\sin(2x))
\end{aligned}$$

input
```
integrate((d*x+c)^3*csc(x)*sin(3*x),x, algorithm="maxima")
```

output
```
3/2*(x^2 + 2*x*sin(2*x) + cos(2*x))*c^2*d + 1/2*(2*x^3 + 6*x*cos(2*x) + 3*
(2*x^2 - 1)*sin(2*x))*c*d^2 + 1/4*(x^4 + 3*(2*x^2 - 1)*cos(2*x) + 2*(2*x^3
 - 3*x)*sin(2*x))*d^3 + c^3*(x + sin(2*x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.04

$$\begin{aligned}
&\int (c+dx)^3 \csc(x)\sin(3x)\,dx \\
&= \frac{1}{4}d^3x^4 + cd^2x^3 + \frac{3}{2}c^2dx^2 + c^3x + \frac{3}{4}\left(2d^3x^2 + 4cd^2x + 2c^2d - d^3\right)\cos(2x) \\
&\quad + \frac{1}{2}\left(2d^3x^3 + 6cd^2x^2 + 6c^2dx - 3d^3x + 2c^3 - 3cd^2\right)\sin(2x)
\end{aligned}$$

input
```
integrate((d*x+c)^3*csc(x)*sin(3*x),x, algorithm="giac")
```

output
```
1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + c^3*x + 3/4*(2*d^3*x^2 + 4*c*d^2
*x + 2*c^2*d - d^3)*cos(2*x) + 1/2*(2*d^3*x^3 + 6*c*d^2*x^2 + 6*c^2*d*x -
3*d^3*x + 2*c^3 - 3*c*d^2)*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.26

$$\int (c+dx)^3 \csc(x)\sin(3x)\,dx = c^3\sin(2x) - \frac{3d^3\cos(2x)}{4} + c^3 x + \frac{d^3x^4}{4} + \frac{3d^3x^2\cos(2x)}{2} + d^3x^3\sin(2x) + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{3c^2d\cos(2x)}{2} - \frac{3cd^2\sin(2x)}{2} - \frac{3d^3x\sin(2x)}{2} + 3cd^2x\cos(2x) + 3c^2dx\sin(2x) + 3cd^2x^2\sin(2x)$$

input
```
int((sin(3*x)*(c + d*x)^3)/sin(x),x)
```

output
```
c^3*sin(2*x) - (3*d^3*cos(2*x))/4 + c^3*x + (d^3*x^4)/4 + (3*d^3*x^2*cos(2
*x))/2 + d^3*x^3*sin(2*x) + (3*c^2*d*x^2)/2 + c*d^2*x^3 + (3*c^2*d*cos(2*x
))/2 - (3*c*d^2*sin(2*x))/2 - (3*d^3*x*sin(2*x))/2 + 3*c*d^2*x*cos(2*x) +
3*c^2*d*x*sin(2*x) + 3*c*d^2*x^2*sin(2*x)
```

Reduce [F]

$$\int (c+dx)^3 \csc(x)\sin(3x)\,dx = \left(\int \csc(x)\sin(3x)\,x^3dx\right)d^3 + 3\left(\int \csc(x)\sin(3x)\,x^2dx\right)cd^2 + 3\left(\int \csc(x)\sin(3x)\,xdx\right)c^2d + \left(\int \csc(x)\sin(3x)\,dx\right)c^3$$

input
```
int((d*x+c)^3*csc(x)*sin(3*x),x)
```

output
```
int(csc(x)*sin(3*x)*x**3,x)*d**3 + 3*int(csc(x)*sin(3*x)*x**2,x)*c*d**2 +
3*int(csc(x)*sin(3*x)*x,x)*c**2*d + int(csc(x)*sin(3*x),x)*c**3
```

3.363 $\int (c+dx)^2 \csc(x) \sin(3x)\, dx$

Optimal result . . . 2989
Mathematica [A] (verified) . . . 2989
Rubi [A] (verified) . . . 2990
Maple [A] (verified) . . . 2991
Fricas [A] (verification not implemented) . . . 2991
Sympy [A] (verification not implemented) . . . 2992
Maxima [A] (verification not implemented) . . . 2992
Giac [A] (verification not implemented) . . . 2993
Mupad [B] (verification not implemented) . . . 2993
Reduce [F] . . . 2993

Optimal result

Integrand size = 14, antiderivative size = 73

$$\int (c+dx)^2 \csc(x) \sin(3x)\, dx = -\frac{d^2 x}{2} + \frac{(c+dx)^3}{3d} + \frac{3}{2} d(c+dx)\cos^2(x) - d^2 \cos(x)\sin(x) + 2(c+dx)^2 \cos(x)\sin(x) - \frac{1}{2} d(c+dx)\sin^2(x)$$

output

```
-1/2*d^2*x+1/3*(d*x+c)^3/d+3/2*d*(d*x+c)*cos(x)^2-d^2*cos(x)*sin(x)+2*(d*x
+c)^2*cos(x)*sin(x)-1/2*d*(d*x+c)*sin(x)^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int (c+dx)^2 \csc(x) \sin(3x)\, dx = c^2 x + cdx^2 + \frac{d^2 x^3}{3} + d(c+dx)\cos(2x) + \left(2c^2 + 4cdx + d^2\left(-1+2x^2\right)\right)\cos(x)\sin(x)$$

input

```
Integrate[(c + d*x)^2*Csc[x]*Sin[3*x],x]
```

output

```
c^2*x + c*d*x^2 + (d^2*x^3)/3 + d*(c + d*x)*Cos[2*x] + (2*c^2 + 4*c*d*x +
d^2*(-1 + 2*x^2))*Cos[x]*Sin[x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(3x)\csc(x)(c+dx)^2\,dx$$

$$\downarrow 4931$$

$$\int \left(3\cos^2(x)(c+dx)^2 - \sin^2(x)(c+dx)^2\right)dx$$

$$\downarrow 2009$$

$$\frac{(c+dx)^3}{3d} - \frac{1}{2}d\sin^2(x)(c+dx) + \frac{3}{2}d\cos^2(x)(c+dx) + 2\sin(x)\cos(x)(c+dx)^2 - \frac{d^2x}{2} - d^2\sin(x)\cos(x)$$

input
```
Int[(c + d*x)^2*Csc[x]*Sin[3*x],x]
```

output
```
-1/2*(d^2*x) + (c + d*x)^3/(3*d) + (3*d*(c + d*x)*Cos[x]^2)/2 - d^2*Cos[x]
*Sin[x] + 2*(c + d*x)^2*Cos[x]*Sin[x] - (d*(c + d*x)*Sin[x]^2)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

method	result
risch	$\frac{d^2x^3}{3}+cd\,x^2+c^2x+\frac{c^3}{3d}+d(dx+c)\cos(2x)+\frac{(2x^2d^2+4cdx+2c^2-d^2)\sin(2x)}{2}$
default	$4d^2\left(x^2\left(\frac{\sin(x)\cos(x)}{2}+\frac{x}{2}\right)+\frac{x\cos(x)^2}{2}-\frac{\sin(x)\cos(x)}{4}-\frac{x}{4}-\frac{x^3}{3}\right)+8cd\left(x\left(\frac{\sin(x)\cos(x)}{2}+\frac{x}{2}\right)-\frac{x^2}{4}-\sin$

input
```
int((d*x+c)^2*csc(x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*d^2*x^3+c*d*x^2+c^2*x+1/3/d*c^3+d*(d*x+c)*cos(2*x)+1/2*(2*d^2*x^2+4*c*
d*x+2*c^2-d^2)*sin(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int (c+dx)^2\csc(x)\sin(3x)\,dx = \frac{1}{3}\,d^2x^3+cdx^2+2\left(d^2x+cd\right)\cos(x)^2 + \left(2\,d^2x^2+4\,cdx+2\,c^2-d^2\right)\cos(x)\sin(x) + \left(c^2-d^2\right)x$$

input
```
integrate((d*x+c)^2*csc(x)*sin(3*x),x, algorithm="fricas")
```

output
```
1/3*d^2*x^3 + c*d*x^2 + 2*(d^2*x + c*d)*cos(x)^2 + (2*d^2*x^2 + 4*c*d*x +
2*c^2 - d^2)*cos(x)*sin(x) + (c^2 - d^2)*x
```

Sympy [A] (verification not implemented)

Time = 4.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int (c+dx)^2 \csc(x)\sin(3x)\,dx = c^2(x+\sin(2x))+2cd\left(-\frac{x^2}{2}+x(x+\sin(2x))+\frac{\cos(2x)}{2}\right) + d^2\left(\frac{x^3}{3}+x^2(x+\sin(2x))-2x\left(\frac{x^2}{2}-\frac{\cos(2x)}{2}\right)-\frac{\sin(2x)}{2}\right)$$

input
```
integrate((d*x+c)**2*csc(x)*sin(3*x),x)
```

output
```
c**2*(x + sin(2*x)) + 2*c*d*(-x**2/2 + x*(x + sin(2*x)) + cos(2*x)/2) + d*
*2*(x**3/3 + x**2*(x + sin(2*x)) - 2*x*(x**2/2 - cos(2*x)/2) - sin(2*x)/2)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int (c+dx)^2 \csc(x)\sin(3x)\,dx = \left(x^2+2\,x\sin(2\,x)+\cos(2\,x)\right)cd + \frac{1}{6}\left(2\,x^3+6\,x\cos(2\,x)+3\left(2\,x^2-1\right)\sin(2\,x)\right)d^2 + c^2(x+\sin(2\,x))$$

input
```
integrate((d*x+c)^2*csc(x)*sin(3*x),x, algorithm="maxima")
```

output
```
(x^2 + 2*x*sin(2*x) + cos(2*x))*c*d + 1/6*(2*x^3 + 6*x*cos(2*x) + 3*(2*x^2
 - 1)*sin(2*x))*d^2 + c^2*(x + sin(2*x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int (c+dx)^2 \csc(x) \sin(3x)\, dx = \frac{1}{3} d^2 x^3 + cdx^2 + c^2 x + \left(d^2 x + cd\right) \cos(2x) + \frac{1}{2}\left(2 d^2 x^2 + 4 cdx + 2 c^2 - d^2\right) \sin(2x)$$

input
```
integrate((d*x+c)^2*csc(x)*sin(3*x),x, algorithm="giac")
```

output
```
1/3*d^2*x^3 + c*d*x^2 + c^2*x + (d^2*x + c*d)*cos(2*x) + 1/2*(2*d^2*x^2 +
4*c*d*x + 2*c^2 - d^2)*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int (c+dx)^2 \csc(x) \sin(3x)\, dx = c^2 \sin(2x) - \frac{d^2 \sin(2x)}{2} + c^2 x + \frac{d^2 x^3}{3} + d^2 x^2 \sin(2x) + c\,d \cos(2x) + d^2 x \cos(2x) + c\,d\,x^2 + 2\,c\,d\,x \sin(2x)$$

input
```
int((sin(3*x)*(c + d*x)^2)/sin(x),x)
```

output
```
c^2*sin(2*x) - (d^2*sin(2*x))/2 + c^2*x + (d^2*x^3)/3 + d^2*x^2*sin(2*x) +
 c*d*cos(2*x) + d^2*x*cos(2*x) + c*d*x^2 + 2*c*d*x*sin(2*x)
```

Reduce [F]

$$\int (c+dx)^2 \csc(x) \sin(3x)\, dx = \left(\int \csc(x) \sin(3x)\, x^2 dx\right) d^2 + 2\left(\int \csc(x) \sin(3x)\, x dx\right) cd + \left(\int \csc(x) \sin(3x)\, dx\right) c^2$$

input
```
int((d*x+c)^2*csc(x)*sin(3*x),x)
```

output
```
int(csc(x)*sin(3*x)*x**2,x)*d**2 + 2*int(csc(x)*sin(3*x)*x,x)*c*d + int(cs
c(x)*sin(3*x),x)*c**2
```

3.364 $\int (c+dx)\csc(x)\sin(3x)\,dx$

Optimal result . . . 2995
Mathematica [A] (verified) . . . 2995
Rubi [A] (verified) . . . 2996
Maple [A] (verified) . . . 2997
Fricas [A] (verification not implemented) . . . 2997
Sympy [A] (verification not implemented) . . . 2997
Maxima [A] (verification not implemented) . . . 2998
Giac [A] (verification not implemented) . . . 2998
Mupad [B] (verification not implemented) . . . 2998
Reduce [F] . . . 2999

Optimal result

Integrand size = 12, antiderivative size = 44

$$\int (c+dx)\csc(x)\sin(3x)\,dx = \frac{(c+dx)^2}{2d} + \frac{3}{4}d\cos^2(x) + 2(c+dx)\cos(x)\sin(x) - \frac{1}{4}d\sin^2(x)$$

output

```
1/2*(d*x+c)^2/d+3/4*d*cos(x)^2+2*(d*x+c)*cos(x)*sin(x)-1/4*d*sin(x)^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int (c+dx)\csc(x)\sin(3x)\,dx = cx + \frac{dx^2}{2} + \frac{1}{2}d\cos(2x) + c\sin(2x) + dx\sin(2x)$$

input

```
Integrate[(c + d*x)*Csc[x]*Sin[3*x],x]
```

output

```
c*x + (d*x^2)/2 + (d*Cos[2*x])/2 + c*Sin[2*x] + d*x*Sin[2*x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(3x)\csc(x)(c+dx)\,dx$$

$$\downarrow 4931$$

$$\int \left(3\cos^2(x)(c+dx) - \sin^2(x)(c+dx)\right)dx$$

$$\downarrow 2009$$

$$\frac{(c+dx)^2}{2d} + 2\sin(x)\cos(x)(c+dx) - \frac{1}{4}d\sin^2(x) + \frac{3}{4}d\cos^2(x)$$

input
```
Int[(c + d*x)*Csc[x]*Sin[3*x],x]
```

output
```
(c + d*x)^2/(2*d) + (3*d*Cos[x]^2)/4 + 2*(c + d*x)*Cos[x]*Sin[x] - (d*Sin[
x]^2)/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{dx^2}{2}+xc+\frac{d\cos(2x)}{2}+(dx+c)\sin(2x)$	28
default	$4d\left(x\left(\frac{\sin(x)\cos(x)}{2}+\frac{x}{2}\right)-\frac{x^2}{4}-\frac{\sin(x)^2}{4}\right)+4c\left(\frac{\sin(x)\cos(x)}{2}+\frac{x}{2}\right)-\frac{dx^2}{2}-xc$	52

input
```
int((d*x+c)*csc(x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*d*x^2+x*c+1/2*d*cos(2*x)+(d*x+c)*sin(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int (c+dx)\csc(x)\sin(3x)\,dx=\frac{1}{2}\,dx^2+d\cos(x)^2+2\,(dx+c)\cos(x)\sin(x)+cx$$

input
```
integrate((d*x+c)*csc(x)*sin(3*x),x, algorithm="fricas")
```

output
```
1/2*d*x^2 + d*cos(x)^2 + 2*(d*x + c)*cos(x)*sin(x) + c*x
```

Sympy [A] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int (c+dx)\csc(x)\sin(3x)\,dx=c(x+\sin(2x))+d\left(-\frac{x^2}{2}+x(x+\sin(2x))+\frac{\cos(2x)}{2}\right)$$

input
```
integrate((d*x+c)*csc(x)*sin(3*x),x)
```

output
```
c*(x + sin(2*x)) + d*(-x**2/2 + x*(x + sin(2*x)) + cos(2*x)/2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int (c+dx)\csc(x)\sin(3x)\,dx = \frac{1}{2}\left(x^2+2\,x\sin(2\,x)+\cos(2\,x)\right)d + c(x+\sin(2\,x))$$

input
```
integrate((d*x+c)*csc(x)*sin(3*x),x, algorithm="maxima")
```

output
```
1/2*(x^2 + 2*x*sin(2*x) + cos(2*x))*d + c*(x + sin(2*x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int (c+dx)\csc(x)\sin(3x)\,dx = \frac{1}{2}\,dx^2 + cx + \frac{1}{2}\,d\cos(2\,x) + (dx+c)\sin(2\,x)$$

input
```
integrate((d*x+c)*csc(x)*sin(3*x),x, algorithm="giac")
```

output
```
1/2*d*x^2 + c*x + 1/2*d*cos(2*x) + (d*x + c)*sin(2*x)
```

Mupad [B] (verification not implemented)

Time = 19.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

$$\int (c+dx)\csc(x)\sin(3x)\,dx = c\,\sin(2\,x) + c\,x + \frac{d\,x^2}{2} + \frac{d\,\cos(2\,x)}{2} + d\,x\,\sin(2\,x)$$

input
```
int((sin(3*x)*(c + d*x))/sin(x),x)
```

output
```
c*sin(2*x) + c*x + (d*x^2)/2 + (d*cos(2*x))/2 + d*x*sin(2*x)
```

Reduce [F]

$$\int (c+dx)\csc(x)\sin(3x)\,dx = \left(\int \csc(x)\sin(3x)\,x dx\right) d + \left(\int \csc(x)\sin(3x)\,dx\right) c$$

input
```
int((d*x+c)*csc(x)*sin(3*x),x)
```

output
```
int(csc(x)*sin(3*x)*x,x)*d + int(csc(x)*sin(3*x),x)*c
```

3.365 $\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx$

Optimal result . . . 3000
Mathematica [A] (verified) . . . 3000
Rubi [A] (verified) . . . 3001
Maple [A] (verified) . . . 3002
Fricas [A] (verification not implemented) . . . 3002
Sympy [F] . . . 3003
Maxima [C] (verification not implemented) . . . 3003
Giac [A] (verification not implemented) . . . 3003
Mupad [F(-1)] . . . 3004
Reduce [F] . . . 3004

Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx = \frac{2\cos\left(\frac{2c}{d}\right)\operatorname{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d} + \frac{\log(c+dx)}{d} + \frac{2\sin\left(\frac{2c}{d}\right)\operatorname{Si}\left(\frac{2c}{d}+2x\right)}{d}$$

output
```
2*cos(2*c/d)*Ci(2*c/d+2*x)/d+ln(d*x+c)/d+2*sin(2*c/d)*Si(2*c/d+2*x)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx = \frac{2\cos\left(\frac{2c}{d}\right)\operatorname{CosIntegral}\left(2\left(\frac{c}{d}+x\right)\right) + \log(c+dx) + 2\sin\left(\frac{2c}{d}\right)\operatorname{Si}\left(2\left(\frac{c}{d}+x\right)\right)}{d}$$

input
```
Integrate[(Csc[x]*Sin[3*x])/(c + d*x),x]
```

output
```
(2*Cos[(2*c)/d]*CosIntegral[2*(c/d + x)] + Log[c + d*x] + 2*Sin[(2*c)/d]*S
inIntegral[2*(c/d + x)])/d
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3x)\csc(x)}{c+dx}\,dx$$
$$\downarrow 4931$$
$$\int \left(\frac{3\cos^2(x)}{c+dx} - \frac{\sin^2(x)}{c+dx}\right)dx$$
$$\downarrow 2009$$
$$\frac{2\cos\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d} + \frac{2\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d} + \frac{\log(c+dx)}{d}$$

input
```
Int[(Csc[x]*Sin[3*x])/(c + d*x),x]
```

output
```
(2*Cos[(2*c)/d]*CosIntegral[(2*c)/d + 2*x])/d + Log[c + d*x]/d + (2*Sin[(2
*c)/d]*SinIntegral[(2*c)/d + 2*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{\ln(2dx+2c)}{d}+\frac{2\sin(\frac{2c}{d})\,\mathrm{Si}(\frac{2c}{d}+2x)}{d}+\frac{2\cos(\frac{2c}{d})\,\mathrm{Ci}(\frac{2c}{d}+2x)}{d}$	61
risch	$\frac{\ln(dx+c)}{d}-\frac{e^{\frac{2ic}{d}}\,\mathrm{expIntegral}_1(2ix+\frac{2ic}{d})}{d}-\frac{e^{-\frac{2ic}{d}}\,\mathrm{expIntegral}_1(-2ix-\frac{2ic}{d})}{d}$	66

input

```
int(csc(x)*sin(3*x)/(d*x+c),x,method=_RETURNVERBOSE)
```

output

```
ln(2*d*x+2*c)/d+2*sin(2*c/d)*Si(2*c/d+2*x)/d+2*cos(2*c/d)*Ci(2*c/d+2*x)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx = \frac{2\cos\left(\frac{2c}{d}\right)\mathrm{Ci}\left(\frac{2(dx+c)}{d}\right)+2\sin\left(\frac{2c}{d}\right)\mathrm{Si}\left(\frac{2(dx+c)}{d}\right)+\log(dx+c)}{d}$$

input

```
integrate(csc(x)*sin(3*x)/(d*x+c),x, algorithm="fricas")
```

output

```
(2*cos(2*c/d)*cos_integral(2*(d*x + c)/d) + 2*sin(2*c/d)*sin_integral(2*(d
*x + c)/d) + log(d*x + c))/d
```

Sympy [F]

$$\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx = \int \frac{\sin(3x)\csc(x)}{c+dx}\,dx$$

input
```
integrate(csc(x)*sin(3*x)/(d*x+c),x)
```

output
```
Integral(sin(3*x)*csc(x)/(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.70

$$\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx = -\frac{\left(E_1\left(\frac{2(-i\,dx-i\,c)}{d}\right)+E_1\left(-\frac{2(-i\,dx-i\,c)}{d}\right)\right)\cos\left(\frac{2c}{d}\right)-\left(i\,E_1\left(\frac{2(-i\,dx-i\,c)}{d}\right)-i\,E_1\left(-\frac{2(-i\,dx-i\,c)}{d}\right)\right)\sin\left(\frac{2c}{d}\right)-}{d}$$

input
```
integrate(csc(x)*sin(3*x)/(d*x+c),x, algorithm="maxima")
```

output
```
-((exp_integral_e(1, 2*(-I*d*x - I*c)/d) + exp_integral_e(1, -2*(-I*d*x -
I*c)/d))*cos(2*c/d) - (I*exp_integral_e(1, 2*(-I*d*x - I*c)/d) - I*exp_int
egral_e(1, -2*(-I*d*x - I*c)/d))*sin(2*c/d) - log(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx = \frac{2\cos\left(\frac{2c}{d}\right)\mathrm{Ci}\left(\frac{2(dx+c)}{d}\right)+2\sin\left(\frac{2c}{d}\right)\mathrm{Si}\left(\frac{2(dx+c)}{d}\right)+\log(dx+c)}{d}$$

input
```
integrate(csc(x)*sin(3*x)/(d*x+c),x, algorithm="giac")
```

output
```
(2*cos(2*c/d)*cos_integral(2*(d*x + c)/d) + 2*sin(2*c/d)*sin_integral(2*(d
*x + c)/d) + log(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx = \int \frac{\sin(3\,x)}{\sin(x)\,(c+d\,x)}\,dx$$

input
```
int(sin(3*x)/(sin(x)*(c + d*x)),x)
```

output
```
int(sin(3*x)/(sin(x)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx = \int \frac{\csc(x)\sin(3x)}{dx+c}dx$$

input
```
int(csc(x)*sin(3*x)/(d*x+c),x)
```

output
```
int((csc(x)*sin(3*x))/(c + d*x),x)
```

3.366 $\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx$

Optimal result . . . 3005
Mathematica [A] (verified) . . . 3005
Rubi [A] (verified) . . . 3006
Maple [A] (verified) . . . 3007
Fricas [A] (verification not implemented) . . . 3007
Sympy [F] . . . 3008
Maxima [C] (verification not implemented) . . . 3008
Giac [A] (verification not implemented) . . . 3009
Mupad [F(-1)] . . . 3009
Reduce [F] . . . 3010

Optimal result

Integrand size = 14, antiderivative size = 78

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx = -\frac{3\cos^2(x)}{d(c+dx)} + \frac{4\,\text{CosIntegral}\left(\frac{2c}{d}+2x\right)\sin\left(\frac{2c}{d}\right)}{d^2} + \frac{\sin^2(x)}{d(c+dx)} - \frac{4\cos\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^2}$$

output

```
-3*cos(x)^2/d/(d*x+c)+4*Ci(2*c/d+2*x)*sin(2*c/d)/d^2+sin(x)^2/d/(d*x+c)-4*
cos(2*c/d)*Si(2*c/d+2*x)/d^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx = \frac{-\frac{d(1+2\cos(2x))}{c+dx} + 4\,\text{CosIntegral}\left(2\left(\frac{c}{d}+x\right)\right)\sin\left(\frac{2c}{d}\right) - 4\cos\left(\frac{2c}{d}\right)\text{Si}\left(2\left(\frac{c}{d}+x\right)\right)}{d^2}$$

input

```
Integrate[(Csc[x]*Sin[3*x])/(c + d*x)^2,x]
```

output
```
(-((d*(1 + 2*Cos[2*x]))/(c + d*x)) + 4*CosIntegral[2*(c/d + x)]*Sin[(2*c)/
d] - 4*Cos[(2*c)/d]*SinIntegral[2*(c/d + x)])/d^2
```

Rubi [A] (verified)

Time $= 0.43$ (sec) , antiderivative size $= 78$, normalized size of antiderivative $= 1.00$, number of steps used $= 2$, number of rules used $= 2$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used $= \{4931, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3x)\csc(x)}{(c+dx)^2}\,dx$$

$$\downarrow 4931$$

$$\int \left(\frac{3\cos^2(x)}{(c+dx)^2} - \frac{\sin^2(x)}{(c+dx)^2}\right) dx$$

$$\downarrow 2009$$

$$\frac{4\sin\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d^2} - \frac{4\cos\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^2} + \frac{\sin^2(x)}{d(c+dx)} - \frac{3\cos^2(x)}{d(c+dx)}$$

input
```
Int[(Csc[x]*Sin[3*x])/(c + d*x)^2,x]
```

output
```
(-3*Cos[x]^2)/(d*(c + d*x)) + (4*CosIntegral[(2*c)/d + 2*x]*Sin[(2*c)/d])/
d^2 + Sin[x]^2/(d*(c + d*x)) - (4*Cos[(2*c)/d]*SinIntegral[(2*c)/d + 2*x])
/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12

method	result	size
default	$-\frac{2}{(2dx+2c)d}-\frac{4\cos(2x)}{(2dx+2c)d}-\frac{4\left(\frac{\mathrm{Si}\left(\frac{2c}{d}+2x\right)\cos\left(\frac{2c}{d}\right)}{d}-\frac{\mathrm{Ci}\left(\frac{2c}{d}+2x\right)\sin\left(\frac{2c}{d}\right)}{d}\right)}{d}$	87
risch	$-\frac{1}{d(dx+c)}+\frac{2ie^{\frac{2ic}{d}}\,\mathrm{expIntegral}_1\left(2ix+\frac{2ic}{d}\right)}{d^2}-\frac{2ie^{-\frac{2ic}{d}}\,\mathrm{expIntegral}_1\left(-2ix-\frac{2ic}{d}\right)}{d^2}-\frac{2i\cos(2x)}{d^2\left(ix+\frac{ic}{d}\right)}$	94

input
```
int(csc(x)*sin(3*x)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/(2*d*x+2*c)/d-4*cos(2*x)/(2*d*x+2*c)/d-4*(Si(2*c/d+2*x)*cos(2*c/d)/d-Ci
(2*c/d+2*x)*sin(2*c/d)/d)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx$$
$$= -\frac{4\,d\cos(x)^2-4\,(dx+c)\,\mathrm{Ci}\left(\frac{2\,(dx+c)}{d}\right)\sin\left(\frac{2\,c}{d}\right)+4\,(dx+c)\cos\left(\frac{2\,c}{d}\right)\mathrm{Si}\left(\frac{2\,(dx+c)}{d}\right)-d}{d^3x+cd^2}$$

input
```
integrate(csc(x)*sin(3*x)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-(4*d*cos(x)^2 - 4*(d*x + c)*cos_integral(2*(d*x + c)/d)*sin(2*c/d) + 4*(d
*x + c)*cos(2*c/d)*sin_integral(2*(d*x + c)/d) - d)/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx = \int \frac{\sin(3x)\csc(x)}{(c+dx)^2}\,dx$$

input
```
integrate(csc(x)*sin(3*x)/(d*x+c)**2,x)
```

output
```
Integral(sin(3*x)*csc(x)/(c + d*x)**2, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 330, normalized size of antiderivative = 4.23

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx =$$
$$-\frac{\left(E_2\left(\frac{2(-i\,dx-i\,c)}{d}\right)+E_2\left(-\frac{2(-i\,dx-i\,c)}{d}\right)\right)\cos\left(\frac{2c}{d}\right)^3+\left(-i\,E_2\left(\frac{2(-i\,dx-i\,c)}{d}\right)+i\,E_2\left(-\frac{2(-i\,dx-i\,c)}{d}\right)\right)\sin\left(\frac{2c}{d}\right.$$

input
```
integrate(csc(x)*sin(3*x)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*((exp_integral_e(2, 2*(-I*d*x - I*c)/d) + exp_integral_e(2, -2*(-I*d*
x - I*c)/d))*cos(2*c/d)^3 + (-I*exp_integral_e(2, 2*(-I*d*x - I*c)/d) + I*
exp_integral_e(2, -2*(-I*d*x - I*c)/d))*sin(2*c/d)^3 + ((exp_integral_e(2,
 2*(-I*d*x - I*c)/d) + exp_integral_e(2, -2*(-I*d*x - I*c)/d))*cos(2*c/d)
+ 2)*sin(2*c/d)^2 + (exp_integral_e(2, 2*(-I*d*x - I*c)/d) + exp_integral_
e(2, -2*(-I*d*x - I*c)/d))*cos(2*c/d) + 2*cos(2*c/d)^2 + ((-I*exp_integral
_e(2, 2*(-I*d*x - I*c)/d) + I*exp_integral_e(2, -2*(-I*d*x - I*c)/d))*cos(
2*c/d)^2 - I*exp_integral_e(2, 2*(-I*d*x - I*c)/d) + I*exp_integral_e(2, -
2*(-I*d*x - I*c)/d))*sin(2*c/d))/((cos(2*c/d)^2 + sin(2*c/d)^2)*d^2*x + (c
*cos(2*c/d)^2 + c*sin(2*c/d)^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.42

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx$$
$$= \frac{4\,dx\,\mathrm{Ci}\left(\frac{2\,(dx+c)}{d}\right)\sin\left(\frac{2\,c}{d}\right) - 4\,dx\cos\left(\frac{2\,c}{d}\right)\mathrm{Si}\left(\frac{2\,(dx+c)}{d}\right) + 4\,c\,\mathrm{Ci}\left(\frac{2\,(dx+c)}{d}\right)\sin\left(\frac{2\,c}{d}\right) - 4\,c\cos\left(\frac{2\,c}{d}\right)\mathrm{Si}\left(\frac{2\,(dx+}{d}\right.}{d^3x + cd^2}$$

input
```
integrate(csc(x)*sin(3*x)/(d*x+c)^2,x, algorithm="giac")
```

output
```
(4*d*x*cos_integral(2*(d*x + c)/d)*sin(2*c/d) - 4*d*x*cos(2*c/d)*sin_integ
ral(2*(d*x + c)/d) + 4*c*cos_integral(2*(d*x + c)/d)*sin(2*c/d) - 4*c*cos(
2*c/d)*sin_integral(2*(d*x + c)/d) - 2*d*cos(2*x) - d)/(d^3*x + c*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx = \int \frac{\sin(3\,x)}{\sin(x)\,(c+d\,x)^2}\,dx$$

input
```
int(sin(3*x)/(sin(x)*(c + d*x)^2),x)
```

output
```
int(sin(3*x)/(sin(x)*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx = \int \frac{\csc(x)\sin(3x)}{(dx+c)^2}\,dx$$

input
```
int(csc(x)*sin(3*x)/(d*x+c)^2,x)
```

output
```
int(csc(x)*sin(3*x)/(d*x+c)^2,x)
```

3.367 $\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx$

Optimal result . . . 3011
Mathematica [A] (verified) . . . 3011
Rubi [A] (verified) . . . 3012
Maple [A] (verified) . . . 3013
Fricas [A] (verification not implemented) . . . 3013
Sympy [F] . . . 3014
Maxima [C] (verification not implemented) . . . 3014
Giac [B] (verification not implemented) . . . 3015
Mupad [F(-1)] . . . 3016
Reduce [F] . . . 3016

Optimal result

Integrand size = 14, antiderivative size = 99

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx = -\frac{3\cos^2(x)}{2d(c+dx)^2} - \frac{4\cos\left(\frac{2c}{d}\right)\operatorname{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d^3} + \frac{4\cos(x)\sin(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} - \frac{4\sin\left(\frac{2c}{d}\right)\operatorname{Si}\left(\frac{2c}{d}+2x\right)}{d^3}$$

output
```
-3/2*cos(x)^2/d/(d*x+c)^2-4*cos(2*c/d)*Ci(2*c/d+2*x)/d^3+4*cos(x)*sin(x)/d
^2/(d*x+c)+1/2*sin(x)^2/d/(d*x+c)^2-4*sin(2*c/d)*Si(2*c/d+2*x)/d^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.78

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx = \frac{-8\cos\left(\frac{2c}{d}\right)\operatorname{CosIntegral}\left(2\left(\frac{c}{d}+x\right)\right) + \frac{d(-d-2d\cos(2x)+4(c+dx)\sin(2x))}{(c+dx)^2} - 8\sin\left(\frac{2c}{d}\right)\operatorname{Si}\left(2\left(\frac{c}{d}+x\right)\right)}{2d^3}$$

input
```
Integrate[(Csc[x]*Sin[3*x])/(c + d*x)^3,x]
```

output
```
(-8*Cos[(2*c)/d]*CosIntegral[2*(c/d + x)] + (d*(-d - 2*d*Cos[2*x] + 4*(c +
 d*x)*Sin[2*x]))/(c + d*x)^2 - 8*Sin[(2*c)/d]*SinIntegral[2*(c/d + x)])/(2
*d^3)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3x)\csc(x)}{(c+dx)^3}\,dx$$

$$\downarrow 4931$$

$$\int \left(\frac{3\cos^2(x)}{(c+dx)^3} - \frac{\sin^2(x)}{(c+dx)^3}\right) dx$$

$$\downarrow 2009$$

$$-\frac{4\cos\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d^3} - \frac{4\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^3} + \frac{4\sin(x)\cos(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} - \frac{3\cos^2(x)}{2d(c+dx)^2}$$

input
```
Int[(Csc[x]*Sin[3*x])/(c + d*x)^3,x]
```

output
```
(-3*Cos[x]^2)/(2*d*(c + d*x)^2) - (4*Cos[(2*c)/d]*CosIntegral[(2*c)/d + 2*
x])/d^3 + (4*Cos[x]*Sin[x])/(d^2*(c + d*x)) + Sin[x]^2/(2*d*(c + d*x)^2) -
 (4*Sin[(2*c)/d]*SinIntegral[(2*c)/d + 2*x])/d^3
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.11

method	result
default	$-\frac{2}{(2dx+2c)^2 d}-\frac{4\cos(2x)}{(2dx+2c)^2 d}-\frac{4\left(-\frac{\sin(2x)}{(2dx+2c)d}+\frac{\frac{\sin\left(\frac{2c}{d}\right)\operatorname{Si}\left(\frac{2c}{d}+2x\right)}{d}+\frac{\cos\left(\frac{2c}{d}\right)\operatorname{Ci}\left(\frac{2c}{d}+2x\right)}{d}}{d}\right)}{d}$
risch	$-\frac{1}{2d(dx+c)^2}+\frac{2\mathrm{e}^{\frac{2ic}{d}}\operatorname{expIntegral}_1\left(2ix+\frac{2ic}{d}\right)}{d^3}+\frac{2\mathrm{e}^{-\frac{2ic}{d}}\operatorname{expIntegral}_1\left(-2ix-\frac{2ic}{d}\right)}{d^3}-\frac{\cos(2x)}{d(dx+c)^2}+\frac{i(-4idx-4ic)\sin(2x)}{2(dx+c)^2 d^2}$

input

```
int(csc(x)*sin(3*x)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output

```
-2/(2*d*x+2*c)^2/d-4*cos(2*x)/(2*d*x+2*c)^2/d-4*(-sin(2*x)/(2*d*x+2*c)/d+(
sin(2*c/d)*Si(2*c/d+2*x)/d+cos(2*c/d)*Ci(2*c/d+2*x)/d)/d)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.29

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx =$$

$$-\frac{4\,d^2\cos(x)^2+8\left(d^2x^2+2\,cdx+c^2\right)\cos\left(\frac{2\,c}{d}\right)\operatorname{Ci}\left(\frac{2\,(dx+c)}{d}\right)-8\left(d^2x+cd\right)\cos(x)\sin(x)+8\left(d^2x^2+2\,c\right.}{2\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

input
```
integrate(csc(x)*sin(3*x)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/2*(4*d^2*cos(x)^2 + 8*(d^2*x^2 + 2*c*d*x + c^2)*cos(2*c/d)*cos_integral
(2*(d*x + c)/d) - 8*(d^2*x + c*d)*cos(x)*sin(x) + 8*(d^2*x^2 + 2*c*d*x + c
^2)*sin(2*c/d)*sin_integral(2*(d*x + c)/d) - d^2)/(d^5*x^2 + 2*c*d^4*x + c
^2*d^3)
```

Sympy [F]

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx = \int \frac{\sin(3x)\csc(x)}{(c+dx)^3}\,dx$$

input
```
integrate(csc(x)*sin(3*x)/(d*x+c)**3,x)
```

output
```
Integral(sin(3*x)*csc(x)/(c + d*x)**3, x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 365, normalized size of antiderivative = 3.69

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx =$$
$$-\frac{\left(E_3\left(\frac{2(-i\,dx-i\,c)}{d}\right)+E_3\left(-\frac{2(-i\,dx-i\,c)}{d}\right)\right)\cos\left(\frac{2c}{d}\right)^3-\left(i\,E_3\left(\frac{2(-i\,dx-i\,c)}{d}\right)-i\,E_3\left(-\frac{2(-i\,dx-i\,c)}{d}\right)\right)\sin\left(\frac{2c}{d}\right)^3}{}$$

input
```
integrate(csc(x)*sin(3*x)/(d*x+c)^3,x, algorithm="maxima")
```

output

```
-1/2*((exp_integral_e(3, 2*(-I*d*x - I*c)/d) + exp_integral_e(3, -2*(-I*d*
x - I*c)/d))*cos(2*c/d)^3 - (I*exp_integral_e(3, 2*(-I*d*x - I*c)/d) - I*e
xp_integral_e(3, -2*(-I*d*x - I*c)/d))*sin(2*c/d)^3 + ((exp_integral_e(3,
2*(-I*d*x - I*c)/d) + exp_integral_e(3, -2*(-I*d*x - I*c)/d))*cos(2*c/d) +
 1)*sin(2*c/d)^2 + (exp_integral_e(3, 2*(-I*d*x - I*c)/d) + exp_integral_e
(3, -2*(-I*d*x - I*c)/d))*cos(2*c/d) + cos(2*c/d)^2 - ((I*exp_integral_e(3
, 2*(-I*d*x - I*c)/d) - I*exp_integral_e(3, -2*(-I*d*x - I*c)/d))*cos(2*c/
d)^2 + I*exp_integral_e(3, 2*(-I*d*x - I*c)/d) - I*exp_integral_e(3, -2*(-
I*d*x - I*c)/d))*sin(2*c/d))/((cos(2*c/d)^2 + sin(2*c/d)^2)*d^3*x^2 + 2*(c
*cos(2*c/d)^2 + c*sin(2*c/d)^2)*d^2*x + (c^2*cos(2*c/d)^2 + c^2*sin(2*c/d)
^2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(95) = 190$.

Time $= 0.12$ (sec) , antiderivative size $= 201$, normalized size of antiderivative $= 2.03$

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx = -\frac{8\,d^2x^2\cos\left(\frac{2c}{d}\right)\operatorname{Ci}\left(\frac{2(dx+c)}{d}\right) + 8\,d^2x^2\sin\left(\frac{2c}{d}\right)\operatorname{Si}\left(\frac{2(dx+c)}{d}\right) + 16\,cdx\cos\left(\frac{2c}{d}\right)\operatorname{Ci}\left(\frac{2(dx+c)}{d}\right) + 16\,cdx\sin$$

input

```
integrate(csc(x)*sin(3*x)/(d*x+c)^3,x, algorithm="giac")
```

output

```
-1/2*(8*d^2*x^2*cos(2*c/d)*cos_integral(2*(d*x + c)/d) + 8*d^2*x^2*sin(2*c
/d)*sin_integral(2*(d*x + c)/d) + 16*c*d*x*cos(2*c/d)*cos_integral(2*(d*x
+ c)/d) + 16*c*d*x*sin(2*c/d)*sin_integral(2*(d*x + c)/d) + 8*c^2*cos(2*c/
d)*cos_integral(2*(d*x + c)/d) - 4*d^2*x*sin(2*x) + 8*c^2*sin(2*c/d)*sin_i
ntegral(2*(d*x + c)/d) + 2*d^2*cos(2*x) - 4*c*d*sin(2*x) + d^2)/(d^5*x^2 +
 2*c*d^4*x + c^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx = \int \frac{\sin(3x)}{\sin(x)\,(c+dx)^3}\,dx$$

input
```
int(sin(3*x)/(sin(x)*(c + d*x)^3),x)
```

output
```
int(sin(3*x)/(sin(x)*(c + d*x)^3), x)
```

Reduce [F]

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx = \int \frac{\csc(x)\sin(3x)}{(dx+c)^3}\,dx$$

input
```
int(csc(x)*sin(3*x)/(d*x+c)^3,x)
```

output
```
int(csc(x)*sin(3*x)/(d*x+c)^3,x)
```

3.368 $\int (c+dx)^4 \csc(a+bx)\sin(3a+3bx)\,dx$

Optimal result . . . 3017
Mathematica [A] (verified) . . . 3018
Rubi [A] (verified) . . . 3018
Maple [A] (verified) . . . 3019
Fricas [A] (verification not implemented) . . . 3020
Sympy [F(-1)] . . . 3020
Maxima [A] (verification not implemented) . . . 3021
Giac [B] (verification not implemented) . . . 3021
Mupad [B] (verification not implemented) . . . 3022
Reduce [F] . . . 3023

Optimal result

Integrand size = 23, antiderivative size = 198

$$\begin{aligned}
&\int (c+dx)^4 \csc(a+bx)\sin(3a+3bx)\,dx \\
&= \frac{3d^4x}{2b^4} - \frac{d(c+dx)^3}{b^2} + \frac{(c+dx)^5}{5d} - \frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4} \\
&\quad + \frac{3d(c+dx)^3\cos^2(a+bx)}{b^2} + \frac{3d^4\cos(a+bx)\sin(a+bx)}{b^5} \\
&\quad - \frac{6d^2(c+dx)^2\cos(a+bx)\sin(a+bx)}{b^3} + \frac{2(c+dx)^4\cos(a+bx)\sin(a+bx)}{b} \\
&\quad + \frac{3d^3(c+dx)\sin^2(a+bx)}{2b^4} - \frac{d(c+dx)^3\sin^2(a+bx)}{b^2}
\end{aligned}$$

output

```
3/2*d^4*x/b^4-d*(d*x+c)^3/b^2+1/5*(d*x+c)^5/d-9/2*d^3*(d*x+c)*cos(b*x+a)^2
/b^4+3*d*(d*x+c)^3*cos(b*x+a)^2/b^2+3*d^4*cos(b*x+a)*sin(b*x+a)/b^5-6*d^2*
(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^3+2*(d*x+c)^4*cos(b*x+a)*sin(b*x+a)/b+3/
2*d^3*(d*x+c)*sin(b*x+a)^2/b^4-d*(d*x+c)^3*sin(b*x+a)^2/b^2
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.65

$$\begin{aligned}
&\int (c+dx)^4 \csc(a+bx)\sin(3a+3bx)\,dx \\
&= c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5} \\
&\quad + \frac{d(c+dx)\left(-3d^2 + 2b^2(c+dx)^2\right)\cos(2(a+bx))}{b^4} \\
&\quad + \frac{\left(3d^4 - 6b^2d^2(c+dx)^2 + 2b^4(c+dx)^4\right)\sin(2(a+bx))}{2b^5}
\end{aligned}$$

input
```
Integrate[(c + d*x)^4*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
c^4*x + 2*c^3*d*x^2 + 2*c^2*d^2*x^3 + c*d^3*x^4 + (d^4*x^5)/5 + (d*(c + d*
x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])/b^4 + ((3*d^4 - 6*b^2*d^
2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Sin[2*(a + b*x)])/(2*b^5)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sin(3a+3bx)\csc(a+bx)\,dx$$

$$\downarrow \text{4931}$$

$$\int \left(3(c+dx)^4\cos^2(a+bx) - (c+dx)^4\sin^2(a+bx)\right)dx$$

$$\downarrow \text{2009}$$

$$\frac{3d^4\sin(a+bx)\cos(a+bx)}{b^5}+\frac{3d^3(c+dx)\sin^2(a+bx)}{2b^4}-\frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4}-\frac{6d^2(c+dx)^2\sin(a+bx)\cos(a+bx)}{b^3}-\frac{d(c+dx)^3\sin^2(a+bx)}{b^2}+\frac{3d(c+dx)^3\cos^2(a+bx)}{b^2}+\frac{2(c+dx)^4\sin(a+bx)\cos(a+bx)}{b}+\frac{3d^4x}{2b^4}-\frac{d(c+dx)^3}{b^2}+\frac{(c+dx)^5}{5d}$$

input
```
Int[(c + d*x)^4*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
(3*d^4*x)/(2*b^4) - (d*(c + d*x)^3)/b^2 + (c + d*x)^5/(5*d) - (9*d^3*(c +
d*x)*Cos[a + b*x]^2)/(2*b^4) + (3*d*(c + d*x)^3*Cos[a + b*x]^2)/b^2 + (3*d
^4*Cos[a + b*x]*Sin[a + b*x])/b^5 - (6*d^2*(c + d*x)^2*Cos[a + b*x]*Sin[a
+ b*x])/b^3 + (2*(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x])/b + (3*d^3*(c + d*
x)*Sin[a + b*x]^2)/(2*b^4) - (d*(c + d*x)^3*Sin[a + b*x]^2)/b^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.14

method	result
risch	$\frac{d^4x^5}{5}+d^3cx^4+2c^2d^2x^3+2c^3dx^2+c^4x+\frac{c^5}{5d}+\frac{d(2b^2d^3x^3+6b^2cd^2x^2+6b^2c^2dx+2b^2c^3-3d^3x-3cd^2)\cos(2bx+2}{b^4}$
default	Expression too large to display

input
```
int((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
1/5*d^4*x^5+d^3*c*x^4+2*c^2*d^2*x^3+2*c^3*d*x^2+c^4*x+1/5/d*c^5+1/b^4*d*(2
*b^2*d^3*x^3+6*b^2*c*d^2*x^2+6*b^2*c^2*d*x+2*b^2*c^3-3*d^3*x-3*c*d^2)*cos(
2*b*x+2*a)+1/2*(2*b^4*d^4*x^4+8*b^4*c*d^3*x^3+12*b^4*c^2*d^2*x^2+8*b^4*c^3
*d*x+2*b^4*c^4-6*b^2*d^4*x^2-12*b^2*c*d^3*x-6*b^2*c^2*d^2+3*d^4)/b^5*sin(2
*b*x+2*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.43

$$\int (c+dx)^4 \csc(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{b^5d^4x^5 + 5\,b^5cd^3x^4 + 10\,(b^5c^2d^2 - b^3d^4)x^3 + 10\,(b^5c^3d - 3\,b^3cd^3)x^2 + 10\,(2\,b^3d^4x^3 + 6\,b^3cd^3x^2 + 2\,b^3c^3d -$$

input
```
integrate((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
1/5*(b^5*d^4*x^5 + 5*b^5*c*d^3*x^4 + 10*(b^5*c^2*d^2 - b^3*d^4)*x^3 + 10*(
b^5*c^3*d - 3*b^3*c*d^3)*x^2 + 10*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3
*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^2 + 5*(2*b^
4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4
*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)*si
n(b*x + a) + 5*(b^5*c^4 - 6*b^3*c^2*d^2 + 3*b*d^4)*x)/b^5
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^4 \csc(a+bx)\sin(3a+3bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**4*csc(b*x+a)*sin(3*b*x+3*a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.23

$$\begin{aligned}
&\int (c+dx)^4 \csc(a+bx)\sin(3a+3bx)\,dx \\
&= \frac{(bx+\sin(2\,bx+2\,a))c^4}{b} + \frac{2\,(b^2x^2+2\,bx\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a))c^3d}{b^2} \\
&\quad + \frac{(2\,b^3x^3+6\,bx\cos(2\,bx+2\,a)+3\,(2\,b^2x^2-1)\sin(2\,bx+2\,a))c^2d^2}{b^3} \\
&\quad + \frac{(b^4x^4+3\,(2\,b^2x^2-1)\cos(2\,bx+2\,a)+2\,(2\,b^3x^3-3\,bx)\sin(2\,bx+2\,a))cd^3}{b^4} \\
&\quad + \frac{(2\,b^5x^5+10\,(2\,b^3x^3-3\,bx)\cos(2\,bx+2\,a)+5\,(2\,b^4x^4-6\,b^2x^2+3)\sin(2\,bx+2\,a))d^4}{10\,b^5}
\end{aligned}$$

input
```
integrate((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
(b*x + sin(2*b*x + 2*a))*c^4/b + 2*(b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + cos
(2*b*x + 2*a))*c^3*d/b^2 + (2*b^3*x^3 + 6*b*x*cos(2*b*x + 2*a) + 3*(2*b^2*
x^2 - 1)*sin(2*b*x + 2*a))*c^2*d^2/b^3 + (b^4*x^4 + 3*(2*b^2*x^2 - 1)*cos(
2*b*x + 2*a) + 2*(2*b^3*x^3 - 3*b*x)*sin(2*b*x + 2*a))*c*d^3/b^4 + 1/10*(2
*b^5*x^5 + 10*(2*b^3*x^3 - 3*b*x)*cos(2*b*x + 2*a) + 5*(2*b^4*x^4 - 6*b^2*
x^2 + 3)*sin(2*b*x + 2*a))*d^4/b^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1255 vs. 2(190) = 380.

Time = 0.27 (sec) , antiderivative size = 1255, normalized size of antiderivative = 6.34

$$\int (c+dx)^4 \csc(a+bx)\sin(3a+3bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

output

```
1/5*(10*b^4*c^4*cos(b*x + a)*sin(b*x + a) + 40*(b*x + a)*b^3*c^3*d*cos(b*x
 + a)*sin(b*x + a) - 40*a*b^3*c^3*d*cos(b*x + a)*sin(b*x + a) + 60*(b*x +
a)^2*b^2*c^2*d^2*cos(b*x + a)*sin(b*x + a) - 120*(b*x + a)*a*b^2*c^2*d^2*c
os(b*x + a)*sin(b*x + a) + 60*a^2*b^2*c^2*d^2*cos(b*x + a)*sin(b*x + a) +
40*(b*x + a)^3*b*c*d^3*cos(b*x + a)*sin(b*x + a) - 120*(b*x + a)^2*a*b*c*d
^3*cos(b*x + a)*sin(b*x + a) + 120*(b*x + a)*a^2*b*c*d^3*cos(b*x + a)*sin(
b*x + a) - 40*a^3*b*c*d^3*cos(b*x + a)*sin(b*x + a) + 10*(b*x + a)^4*d^4*c
os(b*x + a)*sin(b*x + a) - 40*(b*x + a)^3*a*d^4*cos(b*x + a)*sin(b*x + a)
+ 60*(b*x + a)^2*a^2*d^4*cos(b*x + a)*sin(b*x + a) - 40*(b*x + a)*a^3*d^4*
cos(b*x + a)*sin(b*x + a) + 10*a^4*d^4*cos(b*x + a)*sin(b*x + a) + 5*(b*x
+ a)*b^4*c^4 + 10*(b*x + a)^2*b^3*c^3*d - 20*(b*x + a)*a*b^3*c^3*d + 10*(b
*x + a)^3*b^2*c^2*d^2 - 30*(b*x + a)^2*a*b^2*c^2*d^2 + 30*(b*x + a)*a^2*b^
2*c^2*d^2 + 5*(b*x + a)^4*b*c*d^3 - 20*(b*x + a)^3*a*b*c*d^3 + 30*(b*x + a
)^2*a^2*b*c*d^3 - 20*(b*x + a)*a^3*b*c*d^3 + (b*x + a)^5*d^4 - 5*(b*x + a)
^4*a*d^4 + 10*(b*x + a)^3*a^2*d^4 - 10*(b*x + a)^2*a^3*d^4 + 5*(b*x + a)*a
^4*d^4 + 10*b^3*c^3*d*cos(b*x + a)^2 + 30*(b*x + a)*b^2*c^2*d^2*cos(b*x +
a)^2 - 30*a*b^2*c^2*d^2*cos(b*x + a)^2 + 30*(b*x + a)^2*b*c*d^3*cos(b*x +
a)^2 - 60*(b*x + a)*a*b*c*d^3*cos(b*x + a)^2 + 30*a^2*b*c*d^3*cos(b*x + a)
^2 + 10*(b*x + a)^3*d^4*cos(b*x + a)^2 - 30*(b*x + a)^2*a*d^4*cos(b*x + a)
^2 + 30*(b*x + a)*a^2*d^4*cos(b*x + a)^2 - 10*a^3*d^4*cos(b*x + a)^2 - ...
```

Mupad [B] (verification not implemented)

Time = 19.02 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.74

$$\int (c+dx)^4 \csc(a+bx)\sin(3a+3bx)\,dx$$

$$= \frac{\frac{3\,d^4\,\sin(2\,a+2\,b\,x)}{2} + b^5\,c^4\,x + b^4\,c^4\,\sin(2\,a+2\,b\,x) + \frac{b^5\,d^4\,x^5}{5} + 2\,b^3\,c^3\,d\,\cos(2\,a+2\,b\,x) + 2\,b^5\,c^3\,d\,x^2 + b^5\,c\,d^3}{}$$

input

```
int((sin(3*a + 3*b*x)*(c + d*x)^4)/sin(a + b*x),x)
```

output

```
((3*d^4*sin(2*a + 2*b*x))/2 + b^5*c^4*x + b^4*c^4*sin(2*a + 2*b*x) + (b^5*
d^4*x^5)/5 + 2*b^3*c^3*d*cos(2*a + 2*b*x) + 2*b^5*c^3*d*x^2 + b^5*c*d^3*x^
4 - 3*b^2*c^2*d^2*sin(2*a + 2*b*x) + 2*b^3*d^4*x^3*cos(2*a + 2*b*x) + 2*b^
5*c^2*d^2*x^3 - 3*b^2*d^4*x^2*sin(2*a + 2*b*x) + b^4*d^4*x^4*sin(2*a + 2*b
*x) - 3*b*c*d^3*cos(2*a + 2*b*x) - 3*b*d^4*x*cos(2*a + 2*b*x) + 6*b^4*c^2*
d^2*x^2*sin(2*a + 2*b*x) - 6*b^2*c*d^3*x*sin(2*a + 2*b*x) + 4*b^4*c^3*d*x*
sin(2*a + 2*b*x) + 6*b^3*c^2*d^2*x*cos(2*a + 2*b*x) + 6*b^3*c*d^3*x^2*cos(
2*a + 2*b*x) + 4*b^4*c*d^3*x^3*sin(2*a + 2*b*x))/b^5
```

Reduce [F]

$$\int (c+dx)^4 \csc(a+bx)\sin(3a+3bx)\,dx = \left(\int \csc(bx+a)\sin(3bx+3a)\,x^4 dx\right) d^4 + 4\left(\int \csc(bx+a)\sin(3bx+3a)\,x^3 dx\right) c\,d^3 + 6\left(\int \csc(bx+a)\sin(3bx+3a)\,x^2 dx\right) c^2 d^2 + 4\left(\int \csc(bx+a)\sin(3bx+3a)\,x dx\right) c^3 d + \left(\int \csc(bx+a)\sin(3bx+3a)\,dx\right) c^4$$

input

```
int((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x)
```

output

```
int(csc(a + b*x)*sin(3*a + 3*b*x)*x**4,x)*d**4 + 4*int(csc(a + b*x)*sin(3*
a + 3*b*x)*x**3,x)*c*d**3 + 6*int(csc(a + b*x)*sin(3*a + 3*b*x)*x**2,x)*c*
*2*d**2 + 4*int(csc(a + b*x)*sin(3*a + 3*b*x)*x,x)*c**3*d + int(csc(a + b*
x)*sin(3*a + 3*b*x),x)*c**4
```

3.369 $\int (c+dx)^3 \csc(a+bx)\sin(3a+3bx)\,dx$

Optimal result . . . 3024
Mathematica [A] (verified) . . . 3025
Rubi [A] (verified) . . . 3025
Maple [A] (verified) . . . 3026
Fricas [A] (verification not implemented) . . . 3027
Sympy [F(-1)] . . . 3027
Maxima [A] (verification not implemented) . . . 3027
Giac [B] (verification not implemented) . . . 3028
Mupad [B] (verification not implemented) . . . 3029
Reduce [F] . . . 3029

Optimal result

Integrand size = 23, antiderivative size = 161

$$\begin{aligned}\int (c+dx)^3 \csc(a+bx)\sin(3a+3bx)\,dx = &-\frac{3d(c+dx)^2}{4b^2}+\frac{(c+dx)^4}{4d}-\frac{9d^3\cos^2(a+bx)}{8b^4}\\&+\frac{9d(c+dx)^2\cos^2(a+bx)}{4b^2}\\&-\frac{3d^2(c+dx)\cos(a+bx)\sin(a+bx)}{b^3}\\&+\frac{2(c+dx)^3\cos(a+bx)\sin(a+bx)}{b}\\&+\frac{3d^3\sin^2(a+bx)}{8b^4}\\&-\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}\end{aligned}$$

output

```
-3/4*d*(d*x+c)^2/b^2+1/4*(d*x+c)^4/d-9/8*d^3*cos(b*x+a)^2/b^4+9/4*d*(d*x+c
)^2*cos(b*x+a)^2/b^2-3*d^2*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^3+2*(d*x+c)^3*c
os(b*x+a)*sin(b*x+a)/b+3/8*d^3*sin(b*x+a)^2/b^4-3/4*d*(d*x+c)^2*sin(b*x+a)
^2/b^2
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.65

$$\int (c+dx)^3 \csc(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{b^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)+3d(-d^2+2b^2(c+dx)^2)\cos(2(a+bx))+2b(c+dx)\left(-3d^2+2b^2(c\right.}{4b^4}$$

input
```
Integrate[(c + d*x)^3*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
(b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 3*d*(-d^2 + 2*b^2*(c
+ d*x)^2)*Cos[2*(a + b*x)] + 2*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Si
n[2*(a + b*x)])/(4*b^4)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin(3a+3bx)\csc(a+bx)\,dx$$
$$\downarrow 4931$$
$$\int \left(3(c+dx)^3\cos^2(a+bx)-(c+dx)^3\sin^2(a+bx)\right)dx$$
$$\downarrow 2009$$
$$\frac{3d^3\sin^2(a+bx)}{8b^4}-\frac{9d^3\cos^2(a+bx)}{8b^4}-\frac{3d^2(c+dx)\sin(a+bx)\cos(a+bx)}{b^3}-$$
$$\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}+\frac{9d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{2(c+dx)^3\sin(a+bx)\cos(a+bx)}{b}-$$
$$\frac{3d(c+dx)^2}{4b^2}+\frac{(c+dx)^4}{4d}$$

input
```
Int[(c + d*x)^3*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
(-3*d*(c + d*x)^2)/(4*b^2) + (c + d*x)^4/(4*d) - (9*d^3*Cos[a + b*x]^2)/(8
*b^4) + (9*d*(c + d*x)^2*Cos[a + b*x]^2)/(4*b^2) - (3*d^2*(c + d*x)*Cos[a
+ b*x]*Sin[a + b*x])/b^3 + (2*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/b + (
3*d^3*Sin[a + b*x]^2)/(8*b^4) - (3*d*(c + d*x)^2*Sin[a + b*x]^2)/(4*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.97

method	result
risch	$\frac{d^3x^4}{4}+d^2cx^3+\frac{3dc^2x^2}{2}+c^3x+\frac{c^4}{4d}+\frac{3d\left(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2\right)\cos(2bx+2a)}{4b^4}+\frac{(2b^2d^3x^3+6b^2cd^2x^2+6b^2c^2d$
default	$-c^3x+\frac{4c^3\left(\frac{\sin(bx+a)\cos(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{d^3x^4}{4}-d^2cx^3-\frac{3dc^2x^2}{2}+\frac{4d^3\left((bx+a)^3\left(\frac{\sin(bx+a)\cos(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)+\frac{3}{2}\right.}$

input
```
int((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
1/4*d^3*x^4+d^2*c*x^3+3/2*d*c^2*x^2+c^3*x+1/4/d*c^4+3/4*d*(2*b^2*d^2*x^2+4
*b^2*c*d*x+2*b^2*c^2-d^2)/b^4*cos(2*b*x+2*a)+1/2/b^3*(2*b^2*d^3*x^3+6*b^2*
c*d^2*x^2+6*b^2*c^2*d*x+2*b^2*c^3-3*d^3*x-3*c*d^2)*sin(2*b*x+2*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.17

$$\int (c+dx)^3 \csc(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{b^4d^3x^4 + 4\,b^4cd^2x^3 + 6\,(b^4c^2d - b^2d^3)x^2 + 6\,(2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d - d^3)\cos(bx+a)^2 + 4\,(2\,b^3d^3}{4\,b^4}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
1/4*(b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 6*(b^4*c^2*d - b^2*d^3)*x^2 + 6*(2*b^
2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 4*(2*b^3*d
^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)
*x)*cos(b*x + a)*sin(b*x + a) + 4*(b^4*c^3 - 3*b^2*c*d^2)*x)/b^4
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc(a+bx)\sin(3a+3bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3*csc(b*x+a)*sin(3*b*x+3*a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.07

$$\int (c+dx)^3 \csc(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{(bx + \sin(2\,bx+2\,a))c^3}{b} + \frac{3\,(b^2x^2 + 2\,bx\sin(2\,bx+2\,a) + \cos(2\,bx+2\,a))c^2d}{2\,b^2}$$
$$+ \frac{(2\,b^3x^3 + 6\,bx\cos(2\,bx+2\,a) + 3\,(2\,b^2x^2 - 1)\sin(2\,bx+2\,a))cd^2}{2\,b^3}$$
$$+ \frac{(b^4x^4 + 3\,(2\,b^2x^2 - 1)\cos(2\,bx+2\,a) + 2\,(2\,b^3x^3 - 3\,bx)\sin(2\,bx+2\,a))d^3}{4\,b^4}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
(b*x + sin(2*b*x + 2*a))*c^3/b + 3/2*(b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + c
os(2*b*x + 2*a))*c^2*d/b^2 + 1/2*(2*b^3*x^3 + 6*b*x*cos(2*b*x + 2*a) + 3*(
2*b^2*x^2 - 1)*sin(2*b*x + 2*a))*c*d^2/b^3 + 1/4*(b^4*x^4 + 3*(2*b^2*x^2 -
 1)*cos(2*b*x + 2*a) + 2*(2*b^3*x^3 - 3*b*x)*sin(2*b*x + 2*a))*d^3/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(149) = 298$.

Time $= 0.22$ (sec) , antiderivative size $= 682$, normalized size of antiderivative $= 4.24$

$$\int (c+dx)^3 \csc(a+bx)\sin(3a+3bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
1/4*(8*b^3*c^3*cos(b*x + a)*sin(b*x + a) + 24*(b*x + a)*b^2*c^2*d*cos(b*x
+ a)*sin(b*x + a) - 24*a*b^2*c^2*d*cos(b*x + a)*sin(b*x + a) + 24*(b*x + a
)^2*b*c*d^2*cos(b*x + a)*sin(b*x + a) - 48*(b*x + a)*a*b*c*d^2*cos(b*x + a
)*sin(b*x + a) + 24*a^2*b*c*d^2*cos(b*x + a)*sin(b*x + a) + 8*(b*x + a)^3*
d^3*cos(b*x + a)*sin(b*x + a) - 24*(b*x + a)^2*a*d^3*cos(b*x + a)*sin(b*x
+ a) + 24*(b*x + a)*a^2*d^3*cos(b*x + a)*sin(b*x + a) - 8*a^3*d^3*cos(b*x
+ a)*sin(b*x + a) + 4*(b*x + a)*b^3*c^3 + 6*(b*x + a)^2*b^2*c^2*d - 12*(b*
x + a)*a*b^2*c^2*d + 4*(b*x + a)^3*b*c*d^2 - 12*(b*x + a)^2*a*b*c*d^2 + 12
*(b*x + a)*a^2*b*c*d^2 + (b*x + a)^4*d^3 - 4*(b*x + a)^3*a*d^3 + 6*(b*x +
a)^2*a^2*d^3 - 4*(b*x + a)*a^3*d^3 + 6*b^2*c^2*d*cos(b*x + a)^2 + 12*(b*x
+ a)*b*c*d^2*cos(b*x + a)^2 - 12*a*b*c*d^2*cos(b*x + a)^2 + 6*(b*x + a)^2*
d^3*cos(b*x + a)^2 - 12*(b*x + a)*a*d^3*cos(b*x + a)^2 + 6*a^2*d^3*cos(b*x
 + a)^2 - 6*b^2*c^2*d*sin(b*x + a)^2 - 12*(b*x + a)*b*c*d^2*sin(b*x + a)^2
 + 12*a*b*c*d^2*sin(b*x + a)^2 - 6*(b*x + a)^2*d^3*sin(b*x + a)^2 + 12*(b*
x + a)*a*d^3*sin(b*x + a)^2 - 6*a^2*d^3*sin(b*x + a)^2 - 12*b*c*d^2*cos(b*
x + a)*sin(b*x + a) - 12*(b*x + a)*d^3*cos(b*x + a)*sin(b*x + a) + 12*a*d^
3*cos(b*x + a)*sin(b*x + a) - 3*d^3*cos(b*x + a)^2 + 3*d^3*sin(b*x + a)^2)
/b^4
```

Mupad [B] (verification not implemented)

Time = 18.91 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.34

$$\begin{aligned}
&\int (c+dx)^3 \csc(a+bx)\sin(3a+3bx)\,dx \\
&= c^3 x + \frac{d^3 x^4}{4} + \frac{3c^2 d x^2}{2} + c d^2 x^3 - \frac{3d^3\cos(2a+2bx)}{4b^4} \\
&\quad + \frac{c^3\sin(2a+2bx)}{b} + \frac{3c^2 d\cos(2a+2bx)}{2b^2} - \frac{3cd^2\sin(2a+2bx)}{2b^3} \\
&\quad - \frac{3d^3 x\sin(2a+2bx)}{2b^3} + \frac{3d^3 x^2\cos(2a+2bx)}{2b^2} + \frac{d^3 x^3\sin(2a+2bx)}{b} \\
&\quad + \frac{3cd^2 x\cos(2a+2bx)}{b^2} + \frac{3c^2 d x\sin(2a+2bx)}{b} + \frac{3cd^2 x^2\sin(2a+2bx)}{b}
\end{aligned}$$

input

```
int((sin(3*a + 3*b*x)*(c + d*x)^3)/sin(a + b*x),x)
```

output

```
c^3*x + (d^3*x^4)/4 + (3*c^2*d*x^2)/2 + c*d^2*x^3 - (3*d^3*cos(2*a + 2*b*x
))/(4*b^4) + (c^3*sin(2*a + 2*b*x))/b + (3*c^2*d*cos(2*a + 2*b*x))/(2*b^2)
 - (3*c*d^2*sin(2*a + 2*b*x))/(2*b^3) - (3*d^3*x*sin(2*a + 2*b*x))/(2*b^3)
 + (3*d^3*x^2*cos(2*a + 2*b*x))/(2*b^2) + (d^3*x^3*sin(2*a + 2*b*x))/b + (
3*c*d^2*x*cos(2*a + 2*b*x))/b^2 + (3*c^2*d*x*sin(2*a + 2*b*x))/b + (3*c*d^
2*x^2*sin(2*a + 2*b*x))/b
```

Reduce [F]

$$\begin{aligned}
\int (c+dx)^3 \csc(a+bx)\sin(3a+3bx)\,dx = &\left(\int \csc(bx+a)\sin(3bx+3a)\,x^3 dx\right) d^3 \\
&+ 3\left(\int \csc(bx+a)\sin(3bx+3a)\,x^2 dx\right) c\,d^2 \\
&+ 3\left(\int \csc(bx+a)\sin(3bx+3a)\,x\,dx\right) c^2 d \\
&+ \left(\int \csc(bx+a)\sin(3bx+3a)\,dx\right) c^3
\end{aligned}$$

input

```
int((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x)
```

output

```
int(csc(a + b*x)*sin(3*a + 3*b*x)*x**3,x)*d**3 + 3*int(csc(a + b*x)*sin(3*
a + 3*b*x)*x**2,x)*c*d**2 + 3*int(csc(a + b*x)*sin(3*a + 3*b*x)*x,x)*c**2*
d + int(csc(a + b*x)*sin(3*a + 3*b*x),x)*c**3
```

3.370 $\int (c+dx)^2 \csc(a+bx)\sin(3a+3bx)\,dx$

Optimal result . . . 3031
Mathematica [A] (verified) . . . 3032
Rubi [A] (verified) . . . 3032
Maple [A] (verified) . . . 3033
Fricas [A] (verification not implemented) . . . 3034
Sympy [F(-1)] . . . 3034
Maxima [A] (verification not implemented) . . . 3034
Giac [B] (verification not implemented) . . . 3035
Mupad [B] (verification not implemented) . . . 3035
Reduce [F] . . . 3036

Optimal result

Integrand size = 23, antiderivative size = 112

$$\int (c+dx)^2 \csc(a+bx)\sin(3a+3bx)\,dx = -\frac{d^2 x}{2b^2} + \frac{(c+dx)^3}{3d} + \frac{3d(c+dx)\cos^2(a+bx)}{2b^2} - \frac{d^2\cos(a+bx)\sin(a+bx)}{b^3} + \frac{2(c+dx)^2\cos(a+bx)\sin(a+bx)}{b} - \frac{d(c+dx)\sin^2(a+bx)}{2b^2}$$

output

```
-1/2*d^2*x/b^2+1/3*(d*x+c)^3/d+3/2*d*(d*x+c)*cos(b*x+a)^2/b^2-d^2*cos(b*x+
a)*sin(b*x+a)/b^3+2*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b-1/2*d*(d*x+c)*sin(b*
x+a)^2/b^2
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.65

$$\int (c+dx)^2 \csc(a+bx)\sin(3a+3bx)\,dx = c^2x + cdx^2 + \frac{d^2x^3}{3} + \frac{d(c+dx)\cos(2(a+bx))}{b^2} + \frac{(-d^2+2b^2(c+dx)^2)\sin(2(a+bx))}{2b^3}$$

input
```
Integrate[(c + d*x)^2*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
c^2*x + c*d*x^2 + (d^2*x^3)/3 + (d*(c + d*x)*Cos[2*(a + b*x)])/b^2 + ((-d^
2 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(2*b^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin(3a+3bx)\csc(a+bx)\,dx$$

$$\downarrow 4931$$

$$\int \left(3(c+dx)^2\cos^2(a+bx) - (c+dx)^2\sin^2(a+bx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{d^2\sin(a+bx)\cos(a+bx)}{b^3} - \frac{d(c+dx)\sin^2(a+bx)}{2b^2} + \frac{3d(c+dx)\cos^2(a+bx)}{2b^2} + \frac{2(c+dx)^2\sin(a+bx)\cos(a+bx)}{b} - \frac{d^2x}{2b^2} + \frac{(c+dx)^3}{3d}$$

input
```
Int[(c + d*x)^2*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
-1/2*(d^2*x)/b^2 + (c + d*x)^3/(3*d) + (3*d*(c + d*x)*Cos[a + b*x]^2)/(2*b
^2) - (d^2*Cos[a + b*x]*Sin[a + b*x])/b^3 + (2*(c + d*x)^2*Cos[a + b*x]*Si
n[a + b*x])/b - (d*(c + d*x)*Sin[a + b*x]^2)/(2*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.85

method	result
risch	$\frac{d^2x^3}{3}+cd\,x^2+c^2x+\frac{c^3}{3d}+\frac{d(dx+c)\cos(2bx+2a)}{b^2}+\frac{(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2)\sin(2bx+2a)}{2b^3}$
default	$-c^2x-\frac{d^2x^3}{3}+\frac{4c^2\left(\frac{\sin(bx+a)\cos(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}-cd\,x^2+\frac{4d^2\left((bx+a)^2\left(\frac{\sin(bx+a)\cos(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)+\frac{(bx+a)\cos(bx+a)}{2}$

input
```
int((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
1/3*d^2*x^3+c*d*x^2+c^2*x+1/3/d*c^3+1/b^2*d*(d*x+c)*cos(2*b*x+2*a)+1/2*(2*
b^2*d^2*x^2+4*b^2*c*d*x+2*b^2*c^2-d^2)/b^3*sin(2*b*x+2*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.99

$$\int (c+dx)^2 \csc(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{b^3d^2x^3 + 3\,b^3cdx^2 + 6\,(bd^2x+bcd)\cos(bx+a)^2 + 3\,(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - d^2)\cos(bx+a)\sin(b}{3\,b^3}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
1/3*(b^3*d^2*x^3 + 3*b^3*c*d*x^2 + 6*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 3*
(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(b*x + a)*sin(b*x + a)
+ 3*(b^3*c^2 - b*d^2)*x)/b^3
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc(a+bx)\sin(3a+3bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*csc(b*x+a)*sin(3*b*x+3*a),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

$$\int (c+dx)^2 \csc(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{(bx+\sin(2\,bx+2\,a))c^2}{b} + \frac{(b^2x^2 + 2\,bx\sin(2\,bx+2\,a) + \cos(2\,bx+2\,a))cd}{b^2}$$
$$+ \frac{(2\,b^3x^3 + 6\,bx\cos(2\,bx+2\,a) + 3\,(2\,b^2x^2-1)\sin(2\,bx+2\,a))d^2}{6\,b^3}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
(b*x + sin(2*b*x + 2*a))*c^2/b + (b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + cos(2
*b*x + 2*a))*c*d/b^2 + 1/6*(2*b^3*x^3 + 6*b*x*cos(2*b*x + 2*a) + 3*(2*b^2*
x^2 - 1)*sin(2*b*x + 2*a))*d^2/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(104) = 208$.

Time $= 0.18$ (sec) , antiderivative size $= 313$, normalized size of antiderivative $= 2.79$

$$\int (c+dx)^2 \csc(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{6\,b^2c^2\cos(bx+a)\sin(bx+a) + 12\,(bx+a)bcd\cos(bx+a)\sin(bx+a) - 12\,abcd\cos(bx+a)\sin(bx+$$

input
```
integrate((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
1/3*(6*b^2*c^2*cos(b*x + a)*sin(b*x + a) + 12*(b*x + a)*b*c*d*cos(b*x + a)
*sin(b*x + a) - 12*a*b*c*d*cos(b*x + a)*sin(b*x + a) + 6*(b*x + a)^2*d^2*c
os(b*x + a)*sin(b*x + a) - 12*(b*x + a)*a*d^2*cos(b*x + a)*sin(b*x + a) +
6*a^2*d^2*cos(b*x + a)*sin(b*x + a) + 3*(b*x + a)*b^2*c^2 + 3*(b*x + a)^2*
b*c*d - 6*(b*x + a)*a*b*c*d + (b*x + a)^3*d^2 - 3*(b*x + a)^2*a*d^2 + 3*(b
*x + a)*a^2*d^2 + 3*b*c*d*cos(b*x + a)^2 + 3*(b*x + a)*d^2*cos(b*x + a)^2
- 3*a*d^2*cos(b*x + a)^2 - 3*b*c*d*sin(b*x + a)^2 - 3*(b*x + a)*d^2*sin(b*
x + a)^2 + 3*a*d^2*sin(b*x + a)^2 - 3*d^2*cos(b*x + a)*sin(b*x + a))/b^3
```

Mupad [B] (verification not implemented)

Time $= 18.72$ (sec) , antiderivative size $= 121$, normalized size of antiderivative $= 1.08$

$$\int (c+dx)^2 \csc(a+bx)\sin(3a+3bx)\,dx$$
$$= c^2 x + \frac{d^2 x^3}{3} + \frac{c^2\,\sin(2\,a+2\,b\,x)}{b} - \frac{d^2\,\sin(2\,a+2\,b\,x)}{2\,b^3} + c\,d\,x^2 + \frac{d^2\,x\,\cos(2\,a+2\,b\,x)}{b^2}$$
$$+ \frac{d^2\,x^2\,\sin(2\,a+2\,b\,x)}{b} + \frac{c\,d\,\cos(2\,a+2\,b\,x)}{b^2} + \frac{2\,c\,d\,x\,\sin(2\,a+2\,b\,x)}{b}$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x)^2)/sin(a + b*x),x)
```

output
```
c^2*x + (d^2*x^3)/3 + (c^2*sin(2*a + 2*b*x))/b - (d^2*sin(2*a + 2*b*x))/(2
*b^3) + c*d*x^2 + (d^2*x*cos(2*a + 2*b*x))/b^2 + (d^2*x^2*sin(2*a + 2*b*x)
)/b + (c*d*cos(2*a + 2*b*x))/b^2 + (2*c*d*x*sin(2*a + 2*b*x))/b
```

Reduce [F]

$$\int (c+dx)^2 \csc(a+bx)\sin(3a+3bx)\,dx = \left(\int \csc(bx+a)\sin(3bx+3a)\,x^2 dx\right) d^2 + 2\left(\int \csc(bx+a)\sin(3bx+3a)\,x dx\right) cd + \left(\int \csc(bx+a)\sin(3bx+3a)\,dx\right) c^2$$

input
```
int((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x)
```

output
```
int(csc(a + b*x)*sin(3*a + 3*b*x)*x**2,x)*d**2 + 2*int(csc(a + b*x)*sin(3*
a + 3*b*x)*x,x)*c*d + int(csc(a + b*x)*sin(3*a + 3*b*x),x)*c**2
```

3.371 $\int (c+dx)\csc(a+bx)\sin(3a+3bx)\,dx$

Optimal result . . . 3037
Mathematica [A] (verified) . . . 3037
Rubi [A] (verified) . . . 3038
Maple [A] (verified) . . . 3039
Fricas [A] (verification not implemented) . . . 3039
Sympy [F] . . . 3040
Maxima [A] (verification not implemented) . . . 3040
Giac [A] (verification not implemented) . . . 3040
Mupad [B] (verification not implemented) . . . 3041
Reduce [F] . . . 3041

Optimal result

Integrand size = 21, antiderivative size = 69

$$\int (c+dx)\csc(a+bx)\sin(3a+3bx)\,dx = \frac{(c+dx)^2}{2d} + \frac{3d\cos^2(a+bx)}{4b^2} + \frac{2(c+dx)\cos(a+bx)\sin(a+bx)}{b} - \frac{d\sin^2(a+bx)}{4b^2}$$

output
```
1/2*(d*x+c)^2/d+3/4*d*cos(b*x+a)^2/b^2+2*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b-1
/4*d*sin(b*x+a)^2/b^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

$$\int (c+dx)\csc(a+bx)\sin(3a+3bx)\,dx = \frac{d\cos(2(a+bx)) + b(bx(2c+dx) + 2(c+dx)\sin(2(a+bx)))}{2b^2}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
(d*Cos[2*(a + b*x)] + b*(b*x*(2*c + d*x) + 2*(c + d*x)*Sin[2*(a + b*x)]))/
(2*b^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin(3a+3bx)\csc(a+bx)\,dx$$

$$\downarrow 4931$$

$$\int \left(3(c+dx)\cos^2(a+bx) - (c+dx)\sin^2(a+bx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{d\sin^2(a+bx)}{4b^2} + \frac{3d\cos^2(a+bx)}{4b^2} + \frac{2(c+dx)\sin(a+bx)\cos(a+bx)}{b} + \frac{(c+dx)^2}{2d}$$

input
```
Int[(c + d*x)*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
(c + d*x)^2/(2*d) + (3*d*Cos[a + b*x]^2)/(4*b^2) + (2*(c + d*x)*Cos[a + b*
x]*Sin[a + b*x])/b - (d*Sin[a + b*x]^2)/(4*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

method	result
risch	$\frac{d x^2}{2}+xc+\frac{d\cos(2bx+2a)}{2b^2}+\frac{(dx+c)\sin(2bx+2a)}{b}$
default	$-\frac{d x^2}{2}-xc+\frac{4c\left(\frac{\sin(bx+a)\cos(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{4d\left((bx+a)\left(\frac{\sin(bx+a)\cos(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^2}{4}-\frac{\sin(bx+a)^2}{4}-a\left(\frac{\sin(bx+$

input

```
int((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output

```
1/2*d*x^2+x*c+1/2*d/b^2*cos(2*b*x+2*a)+(d*x+c)/b*sin(2*b*x+2*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

$$\int (c+dx)\csc(a+bx)\sin(3a+3bx)\,dx = \frac{b^2dx^2+2\,b^2cx+2\,d\cos(bx+a)^2+4\,(bdx+bc)\cos(bx+a)\sin(bx+a)}{2\,b^2}$$

input

```
integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

output

```
1/2*(b^2*d*x^2 + 2*b^2*c*x + 2*d*cos(b*x + a)^2 + 4*(b*d*x + b*c)*cos(b*x
+ a)*sin(b*x + a))/b^2
```

Sympy [F]

$$\int (c+dx)\csc(a+bx)\sin(3a+3bx)\,dx = \int (c+dx)\sin(3a+3bx)\csc(a+bx)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x)
```

output
```
Integral((c + d*x)*sin(3*a + 3*b*x)*csc(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int (c+dx)\csc(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{(bx+\sin(2\,bx+2\,a))c}{b} + \frac{(b^2x^2+2\,bx\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a))d}{2\,b^2}$$

input
```
integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
(b*x + sin(2*b*x + 2*a))*c/b + 1/2*(b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + cos
(2*b*x + 2*a))*d/b^2
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.54

$$\int (c+dx)\csc(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{4\,bc\cos(bx+a)\sin(bx+a)+4\,(bx+a)d\cos(bx+a)\sin(bx+a)-4\,ad\cos(bx+a)\sin(bx+a)+2\,(b}{2\,b^2}$$

input
```
integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
1/2*(4*b*c*cos(b*x + a)*sin(b*x + a) + 4*(b*x + a)*d*cos(b*x + a)*sin(b*x
+ a) - 4*a*d*cos(b*x + a)*sin(b*x + a) + 2*(b*x + a)*b*c + (b*x + a)^2*d -
 2*(b*x + a)*a*d + d*cos(b*x + a)^2 - d*sin(b*x + a)^2)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

$$\int (c+dx)\csc(a+bx)\sin(3a+3bx)\,dx = c\,x+\frac{d\,x^2}{2}+\frac{\frac{d\cos(2\,a+2\,b\,x)}{2}+b\,(c\,\sin(2\,a+2\,b\,x)+d\,x\,\sin(2\,a+2\,b\,x))}{b^2}$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x))/sin(a + b*x),x)
```

output
```
c*x + (d*x^2)/2 + ((d*cos(2*a + 2*b*x))/2 + b*(c*sin(2*a + 2*b*x) + d*x*si
n(2*a + 2*b*x)))/b^2
```

Reduce [F]

$$\int (c+dx)\csc(a+bx)\sin(3a+3bx)\,dx = \left(\int \csc(bx+a)\sin(3bx+3a)\,x dx\right)d + \left(\int \csc(bx+a)\sin(3bx+3a)\,dx\right)c$$

input
```
int((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x)
```

output
```
int(csc(a + b*x)*sin(3*a + 3*b*x)*x,x)*d + int(csc(a + b*x)*sin(3*a + 3*b*
x),x)*c
```

3.372 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

Optimal result . . . 3042
Mathematica [A] (verified) . . . 3042
Rubi [A] (verified) . . . 3043
Maple [C] (verified) . . . 3044
Fricas [A] (verification not implemented) . . . 3044
Sympy [F] . . . 3045
Maxima [C] (verification not implemented) . . . 3045
Giac [A] (verification not implemented) . . . 3046
Mupad [F(-1)] . . . 3046
Reduce [F] . . . 3046

Optimal result

Integrand size = 23, antiderivative size = 71

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \frac{2\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d} + \frac{\log(c+dx)}{d} - \frac{2\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{d}$$

output
```
2*cos(2*a-2*b*c/d)*Ci(2*b*c/d+2*b*x)/d+ln(d*x+c)/d-2*sin(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \frac{2\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)+\log(c+dx)-2\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b(c+dx)}{d}\right)}{d}$$

input
```
Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

output
```
(2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + Log[c + d*x] - 2*
Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\csc(a+bx)}{c+dx}\,dx$$

$$\downarrow 4931$$

$$\int \left(\frac{3\cos^2(a+bx)}{c+dx} - \frac{\sin^2(a+bx)}{c+dx}\right)dx$$

$$\downarrow 2009$$

$$\frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d} - \frac{2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d} + \frac{\log(c+dx)}{d}$$

input
```
Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

output
```
(2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d + Log[c + d*x]/d
 - (2*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.49

method	result
risch	$\frac{\ln(dx+c)}{d} - \frac{e^{-\frac{2i(ad-cb)}{d}}\,\text{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{d} - \frac{e^{\frac{2i(ad-cb)}{d}}\,\text{expIntegral}_1\left(-2ibx-2ia-\frac{2(-iad+icb)}{d}\right)}{d}$
default	$-\frac{\ln(dx+c)}{d} + \frac{2\,\text{Si}\left(2bx+2a+\frac{-2ad+2cb}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d} + \frac{2\,\text{Ci}\left(2bx+2a+\frac{-2ad+2cb}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d} + \frac{2\ln(-ad+cb+d(bx$

input

```
int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output

```
ln(d*x+c)/d-1/d*exp(-2*I*(a*d-b*c)/d)*Ei(1,2*I*b*x+2*I*a-2*I*(a*d-b*c)/d)-
1/d*exp(2*I*(a*d-b*c)/d)*Ei(1,-2*I*b*x-2*I*a-2*(-I*a*d+I*c*b)/d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \frac{2\cos\left(-\frac{2(bc-ad)}{d}\right)\text{Ci}\left(\frac{2(bdx+bc)}{d}\right) - 2\sin\left(-\frac{2(bc-ad)}{d}\right)\text{Si}\left(\frac{2(bdx+bc)}{d}\right) + \log(dx+c)}{d}$$

input

```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
```

output
```
(2*cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*d*x + b*c)/d) - 2*sin(-2*(b*c -
 a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) + log(d*x + c))/d
```

Sympy [F]

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sin(3a+3bx)\csc(a+bx)}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

output
```
Integral(sin(3*a + 3*b*x)*csc(a + b*x)/(c + d*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.68

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx = -\frac{\left(E_1\left(\frac{2(-i\,bdx-i\,bc)}{d}\right)+E_1\left(-\frac{2(-i\,bdx-i\,bc)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)-\left(-i\,E_1\left(\frac{2(-i\,bdx-i\,bc)}{d}\right)+i\,E_1\left(-\frac{2(-i\,bdx-i}{d}\right.\right.}{d}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

output
```
-((exp_integral_e(1, 2*(-I*b*d*x - I*b*c)/d) + exp_integral_e(1, -2*(-I*b*
d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) - (-I*exp_integral_e(1, 2*(-I*b*d*x
 - I*b*c)/d) + I*exp_integral_e(1, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b*c -
 a*d)/d) - log(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.44

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$
$$= \frac{2\,b\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{2\,(bc+(bx+a)d-ad)}{d}\right)+2\,b\sin\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(-\frac{2\,(bc+(bx+a)d-ad)}{d}\right)+b\log\left(bc+(bx+\right.}{bd}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
```

output
```
(2*b*cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d) + 2
*b*sin(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)/d) + b*
log(b*c + (b*x + a)*d - a*d))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sin(3\,a+3\,b\,x)}{\sin(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)),x)
```

output
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)\sin(3bx+3a)}{dx+c}\,dx$$

input
```
int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

output
```
int((csc(a + b*x)*sin(3*a + 3*b*x))/(c + d*x),x)
```

3.373 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$

Optimal result . . . 3047
Mathematica [A] (verified) . . . 3047
Rubi [A] (verified) . . . 3048
Maple [C] (verified) . . . 3049
Fricas [A] (verification not implemented) . . . 3050
Sympy [F(-1)] . . . 3050
Maxima [C] (verification not implemented) . . . 3050
Giac [B] (verification not implemented) . . . 3051
Mupad [F(-1)] . . . 3052
Reduce [F] . . . 3052

Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = -\frac{3\cos^2(a+bx)}{d(c+dx)} - \frac{4b\,\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^2} + \frac{\sin^2(a+bx)}{d(c+dx)} - \frac{4b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}$$

output

```
-3*cos(b*x+a)^2/d/(d*x+c)-4*b*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^2+sin(b
*x+a)^2/d/(d*x+c)-4*b*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^2
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = -\frac{\frac{d(1+2\cos(2(a+bx)))}{c+dx} + 4b\,\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right) + 4b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)}{d^2}$$

input
```
Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

output
```
-(((d*(1 + 2*Cos[2*(a + b*x)]))/(c + d*x) + 4*b*CosIntegral[(2*b*(c + d*x)
)/d]*Sin[2*a - (2*b*c)/d] + 4*b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c +
 d*x))/d])/d^2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

$$\downarrow 4931$$

$$\int \left(\frac{3\cos^2(a+bx)}{(c+dx)^2} - \frac{\sin^2(a+bx)}{(c+dx)^2}\right)dx$$

$$\downarrow 2009$$

$$-\frac{4b\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2} - \frac{4b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2} + \frac{\sin^2(a+bx)}{d(c+dx)} - \frac{3\cos^2(a+bx)}{d(c+dx)}$$

input
```
Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

output
```
(-3*Cos[a + b*x]^2)/(d*(c + d*x)) - (4*b*CosIntegral[(2*b*c)/d + 2*b*x]*Si
n[2*a - (2*b*c)/d])/d^2 + Sin[a + b*x]^2/(d*(c + d*x)) - (4*b*Cos[2*a - (2
*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^2
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.52

method	result
risch	$-\frac{1}{d(dx+c)}+\frac{2ib\,\mathrm{e}^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{d^2}-\frac{2ib\,\mathrm{e}^{\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(-2ibx-2ia-\frac{2(-iad+icb)}{d}\right)}{d^2}-$
default	$\frac{1}{d(dx+c)}+\frac{b^2\left(-\frac{2\cos(2bx+2a)}{(-ad+cb+d(bx+a))d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(2bx+2a+\frac{-2ad+2cb}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2ad+2cb}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}\right)}{d}\right)}{b}-$

input

```
int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/d/(d*x+c)+2*I*b/d^2*exp(-2*I*(a*d-b*c)/d)*Ei(1,2*I*b*x+2*I*a-2*I*(a*d-b
*c)/d)-2*I*b/d^2*exp(2*I*(a*d-b*c)/d)*Ei(1,-2*I*b*x-2*I*a-2*(-I*a*d+I*c*b)
/d)-1/d*(-2*b*d*x-2*b*c)/(-b*d*x-b*c)/(d*x+c)*cos(2*b*x+2*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = -\frac{4\,d\cos(bx+a)^2 + 4\,(bdx+bc)\,\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right) + 4\,(bdx+bc)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx}{d}\right.}{d^3x+cd^2}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-(4*d*cos(b*x + a)^2 + 4*(b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d)*sin
(-2*(b*c - a*d)/d) + 4*(b*d*x + b*c)*cos(-2*(b*c - a*d)/d)*sin_integral(2*
(b*d*x + b*c)/d) - d)/(d^3*x + c*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.18

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = -\frac{\left(E_2\left(\frac{2\,(-i\,bdx-i\,bc)}{d}\right)+E_2\left(-\frac{2\,(-i\,bdx-i\,bc)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right) - \left(-i\,E_2\left(\frac{2\,(-i\,bdx-i\,bc)}{d}\right)+i\,E_2\left(-\frac{2\,(-i\,bdx-i}{d}\right.\right.}{d^2x+cd}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-((exp_integral_e(2, 2*(-I*b*d*x - I*b*c)/d) + exp_integral_e(2, -2*(-I*b*
d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) - (-I*exp_integral_e(2, 2*(-I*b*d*x
 - I*b*c)/d) + I*exp_integral_e(2, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b*c -
 a*d)/d) + 1)/(d^2*x + c*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(102) = 204$.

Time $= 0.17$ (sec) , antiderivative size $= 308$, normalized size of antiderivative $= 3.02$

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx =$$
$$-\frac{4\,b^3c\,\mathrm{Ci}\left(\frac{2\,(bc+(bx+a)d-ad)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+4\,(bx+a)b^2d\,\mathrm{Ci}\left(\frac{2\,(bc+(bx+a)d-ad)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-4\,ab$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
-(4*b^3*c*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(b*c - a*d)/d
) + 4*(b*x + a)*b^2*d*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(
b*c - a*d)/d) - 4*a*b^2*d*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(
-2*(b*c - a*d)/d) - 4*b^3*c*cos(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (
b*x + a)*d - a*d)/d) - 4*(b*x + a)*b^2*d*cos(-2*(b*c - a*d)/d)*sin_integra
l(-2*(b*c + (b*x + a)*d - a*d)/d) + 4*a*b^2*d*cos(-2*(b*c - a*d)/d)*sin_in
tegral(-2*(b*c + (b*x + a)*d - a*d)/d) + 2*b^2*d*cos(2*b*x + 2*a) + b^2*d)
/((b*c*d^2 + (b*x + a)*d^3 - a*d^3)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\sin(3a+3bx)}{\sin(a+bx)\,(c+dx)^2}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^2),x)
```

output
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)\sin(3bx+3a)}{(dx+c)^2}\,dx$$

input
```
int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

3.374 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

Optimal result . . . 3053
Mathematica [A] (verified) . . . 3054
Rubi [A] (verified) . . . 3054
Maple [A] (verified) . . . 3055
Fricas [A] (verification not implemented) . . . 3056
Sympy [F(-1)] . . . 3056
Maxima [C] (verification not implemented) . . . 3057
Giac [B] (verification not implemented) . . . 3057
Mupad [F(-1)] . . . 3058
Reduce [F] . . . 3059

Optimal result

Integrand size = 23, antiderivative size = 136

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = -\frac{3\cos^2(a+bx)}{2d(c+dx)^2} - \frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3} + \frac{4b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+bx)}{2d(c+dx)^2} + \frac{4b^2\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}$$

output

```
-3/2*cos(b*x+a)^2/d/(d*x+c)^2-4*b^2*cos(2*a-2*b*c/d)*Ci(2*b*c/d+2*b*x)/d^3
+4*b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c)+1/2*sin(b*x+a)^2/d/(d*x+c)^2+4*b^2*
sin(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^3
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.76

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = -\frac{8b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)+\frac{d(d+2d\cos(2(a+bx))-4b(c+dx)\sin(2(a+bx)))}{(c+dx)^2}-8b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}(}{2d^3}$$

input
```
Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

output
```
-1/2*(8*b^2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + (d*(d +
2*d*Cos[2*(a + b*x)] - 4*b*(c + d*x)*Sin[2*(a + b*x)]))/(c + d*x)^2 - 8*b^
2*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d^3
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\csc(a+bx)}{(c+dx)^3}\,dx$$

$$\downarrow 4931$$

$$\int \left(\frac{3\cos^2(a+bx)}{(c+dx)^3}-\frac{\sin^2(a+bx)}{(c+dx)^3}\right)dx$$

$$\downarrow 2009$$

$$-\frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{4b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{4b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)}+\frac{\sin^2(a+bx)}{2d(c+dx)^2}-\frac{3\cos^2(a+bx)}{2d(c+dx)^2}$$

input
```
Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

output
```
(-3*Cos[a + b*x]^2)/(2*d*(c + d*x)^2) - (4*b^2*Cos[2*a - (2*b*c)/d]*CosInt
egral[(2*b*c)/d + 2*b*x])/d^3 + (4*b*Cos[a + b*x]*Sin[a + b*x])/(d^2*(c +
d*x)) + Sin[a + b*x]^2/(2*d*(c + d*x)^2) + (4*b^2*Sin[2*a - (2*b*c)/d]*Sin
Integral[(2*b*c)/d + 2*b*x])/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.52

method	result
default	$\frac{1}{2d(dx+c)^2}+\frac{b^3\left(-\frac{\cos(2bx+2a)}{(-ad+cb+d(bx+a))^2d}-\frac{-\frac{2\sin(2bx+2a)}{(-ad+cb+d(bx+a))d}+\frac{4\,\mathrm{Si}\left(2bx+2a+\frac{-2ad+2cb}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(2bx+2a+\frac{-2ad+2}{d}\right.}{d}}{d}\right.}{b}$
risch	$-\frac{1}{2d(dx+c)^2}+\frac{2b^2\mathrm{e}^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{d^3}+\frac{2b^2\mathrm{e}^{\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(-2ibx-2ia-\frac{2(-iad+icb)}{d}\right)}{d^3}$

input
```
int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/d/(d*x+c)^2+4/b*(1/4*b^3*(-cos(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2/d-(-2
*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*s
in(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)
/d)/d)-1/4*b^3/(-a*d+c*b+d*(b*x+a))^2/d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.34

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx =$$

$$-\frac{4\,d^2\cos(bx+a)^2+8\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)-8\left(bd^2x+bcd\right)\cos(bx}{2\left(d^5x^2+2\,cd^4x+c^2d\right.}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/2*(4*d^2*cos(b*x + a)^2 + 8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-
2*(b*c - a*d)/d)*cos_integral(2*(b*d*x + b*c)/d) - 8*(b*d^2*x + b*c*d)*cos
(b*x + a)*sin(b*x + a) - 8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-2*(b
*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - d^2)/(d^5*x^2 + 2*c*d^4*x +
 c^2*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.97

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = -\frac{2\left(E_3\left(\frac{2(-i\,bdx-i\,bc)}{d}\right)+E_3\left(-\frac{2(-i\,bdx-i\,bc)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+2\left(i\,E_3\left(\frac{2(-i\,bdx-i\,bc)}{d}\right)-i\,E_3\left(-\frac{2(-i\,bdx-}{d}\right.\right.}{2\left(d^3x^2+2\,cd^2x+c^2d\right)}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/2*(2*(exp_integral_e(3, 2*(-I*b*d*x - I*b*c)/d) + exp_integral_e(3, -2*
(-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 2*(I*exp_integral_e(3, 2*(-
I*b*d*x - I*b*c)/d) - I*exp_integral_e(3, -2*(-I*b*d*x - I*b*c)/d))*sin(-2
*(b*c - a*d)/d) + 1)/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 704 vs. $2(132) = 264$.

Time = 0.20 (sec) , antiderivative size = 704, normalized size of antiderivative = 5.18

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

output

```
-1/2*(8*b^5*c^2*cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*c + (b*x + a)*d -
a*d)/d) + 16*(b*x + a)*b^4*c*d*cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*c +
 (b*x + a)*d - a*d)/d) - 16*a*b^4*c*d*cos(-2*(b*c - a*d)/d)*cos_integral(2
*(b*c + (b*x + a)*d - a*d)/d) + 8*(b*x + a)^2*b^3*d^2*cos(-2*(b*c - a*d)/d
)*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d) - 16*(b*x + a)*a*b^3*d^2*cos
(-2*(b*c - a*d)/d)*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d) + 8*a^2*b^3
*d^2*cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d) + 8
*b^5*c^2*sin(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)/d
) + 16*(b*x + a)*b^4*c*d*sin(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x
 + a)*d - a*d)/d) - 16*a*b^4*c*d*sin(-2*(b*c - a*d)/d)*sin_integral(-2*(b*
c + (b*x + a)*d - a*d)/d) + 8*(b*x + a)^2*b^3*d^2*sin(-2*(b*c - a*d)/d)*si
n_integral(-2*(b*c + (b*x + a)*d - a*d)/d) - 16*(b*x + a)*a*b^3*d^2*sin(-2
*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)/d) + 8*a^2*b^3*d
^2*sin(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)/d) - 4*
b^4*c*d*sin(2*b*x + 2*a) - 4*(b*x + a)*b^3*d^2*sin(2*b*x + 2*a) + 4*a*b^3*
d^2*sin(2*b*x + 2*a) + 2*b^3*d^2*cos(2*b*x + 2*a) + b^3*d^2)/((b^2*c^2*d^3
 + 2*(b*x + a)*b*c*d^4 - 2*a*b*c*d^4 + (b*x + a)^2*d^5 - 2*(b*x + a)*a*d^5
 + a^2*d^5)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\sin(3\,a+3\,b\,x)}{\sin(a+b\,x)\,(c+d\,x)^3}\,dx$$

input

```
int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^3),x)
```

output

```
int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^3), x)
```

Reduce [F]

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\csc(bx+a)\sin(3bx+3a)}{(dx+c)^3}\,dx$$

input
```
int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

output
```
int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

3.375 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx$

Optimal result . . . 3060
Mathematica [A] (verified) . . . 3061
Rubi [A] (verified) . . . 3061
Maple [A] (verified) . . . 3063
Fricas [A] (verification not implemented) . . . 3063
Sympy [F(-1)] . . . 3064
Maxima [C] (verification not implemented) . . . 3064
Giac [B] (verification not implemented) . . . 3065
Mupad [F(-1)] . . . 3066
Reduce [F] . . . 3066

Optimal result

Integrand size = 23, antiderivative size = 205

$$\begin{aligned}\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx = &-\frac{2b^2}{3d^3(c+dx)} - \frac{\cos^2(a+bx)}{d(c+dx)^3} + \frac{2b^2\cos^2(a+bx)}{d^3(c+dx)} \\ &+ \frac{8b^3\,\mathrm{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{3d^4} \\ &+ \frac{4b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2} + \frac{\sin^2(a+bx)}{3d(c+dx)^3} \\ &- \frac{2b^2\sin^2(a+bx)}{3d^3(c+dx)} + \frac{8b^3\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}\end{aligned}$$

output

```
-2/3*b^2/d^3/(d*x+c)-cos(b*x+a)^2/d/(d*x+c)^3+2*b^2*cos(b*x+a)^2/d^3/(d*x+
c)+8/3*b^3*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^4+4/3*b*cos(b*x+a)*sin(b*x
+a)/d^2/(d*x+c)^2+1/3*sin(b*x+a)^2/d/(d*x+c)^3-2/3*b^2*sin(b*x+a)^2/d^3/(d
*x+c)+8/3*b^3*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^4
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.61

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx$$
$$= \frac{8b^3\,\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right)+\frac{d\left(\left(-2d^2+4b^2(c+dx)^2\right)\cos(2(a+bx))+d(-d+2b(c+dx)\sin(2(a+bx))\right)}{(c+dx)^3}+8b^3\,\text{co}}{3d^4}$$

input
```
Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^4,x]
```

output
```
(8*b^3*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*((-2*d^2 +
 4*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + d*(-d + 2*b*(c + d*x)*Sin[2*(a + b*
x)])))/(c + d*x)^3 + 8*b^3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x)
)/d])/(3*d^4)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\csc(a+bx)}{(c+dx)^4}\,dx$$
$$\downarrow \text{4931}$$
$$\int \left(\frac{3\cos^2(a+bx)}{(c+dx)^4}-\frac{\sin^2(a+bx)}{(c+dx)^4}\right)dx$$
$$\downarrow \text{2009}$$

$$\frac{8b^3 \sin\left(2a-\frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{8b^3 \cos\left(2a-\frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{2b^2 \sin^2(a+bx)}{3d^3(c+dx)}+\frac{2b^2 \cos^2(a+bx)}{d^3(c+dx)}+\frac{4b \sin(a+bx)\cos(a+bx)}{3d^2(c+dx)^2}+\frac{\sin^2(a+bx)}{3d(c+dx)^3}-\frac{\cos^2(a+bx)}{d(c+dx)^3}-\frac{2b^2}{3d^3(c+dx)}$$

input

```
Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^4,x]
```

output

```
(-2*b^2)/(3*d^3*(c + d*x)) - Cos[a + b*x]^2/(d*(c + d*x)^3) + (2*b^2*Cos[a
 + b*x]^2)/(d^3*(c + d*x)) + (8*b^3*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a
 - (2*b*c)/d])/(3*d^4) + (4*b*Cos[a + b*x]*Sin[a + b*x])/(3*d^2*(c + d*x)^
2) + Sin[a + b*x]^2/(3*d*(c + d*x)^3) - (2*b^2*Sin[a + b*x]^2)/(3*d^3*(c +
 d*x)) + (8*b^3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(3*d^
4)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.19

method	result
default	$\frac{1}{3d(dx+c)^3}+\frac{b^4\left(-\frac{2\cos(2bx+2a)}{3(-ad+cb+d(bx+a))^3d}-\frac{2\left(-\frac{\sin(2bx+2a)}{(-ad+cb+d(bx+a))^2d}+\frac{-\frac{2\cos(2bx+2a)}{(-ad+cb+d(bx+a))d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(2bx+2a+\frac{-2ad+2cb}{d}\right)\cos\left(\frac{-2c}{d}\right.\right.}{d}}{d}\right)}{3d}\right)}{b}$
risch	$-\frac{1}{3d(dx+c)^3}-\frac{4ib^3e^{-\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{3d^4}+\frac{4ib^3e^{\frac{2i(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(-2ibx-2ia-\frac{2(-iad+icb)}{d}\right.}{3d^4}$

input
```
int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3/d/(d*x+c)^3+4/b*(1/4*b^4*(-2/3*cos(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^3/d
-2/3*(-sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2/d+(-2*cos(2*b*x+2*a)/(-a*d+c*
b+d*(b*x+a))/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*C
i(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d)/d)-1/6*b^4/(-a*d+
c*b+d*(b*x+a))^3/d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.40

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx =$$
$$-\frac{4\,b^2d^3x^2+8\,b^2cd^2x+4\,b^2c^2d-d^3-4\left(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3\right)\cos\left(bx+a\right)^2-4\left(bd^3x+b\right.}{}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/3*(4*b^2*d^3*x^2 + 8*b^2*c*d^2*x + 4*b^2*c^2*d - d^3 - 4*(2*b^2*d^3*x^2
 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 - 4*(b*d^3*x + b*c*d^
2)*cos(b*x + a)*sin(b*x + a) - 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^
2*d*x + b^3*c^3)*cos_integral(2*(b*d*x + b*c)/d)*sin(-2*(b*c - a*d)/d) - 8
*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-2*(b*c - a
*d)/d)*sin_integral(2*(b*d*x + b*c)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5
*x + c^3*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.70

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx = -\frac{3\left(E_4\left(\frac{2(-i\,bdx-i\,bc)}{d}\right)+E_4\left(-\frac{2(-i\,bdx-i\,bc)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+3\left(i\,E_4\left(\frac{2(-i\,bdx-i\,bc)}{d}\right)-i\,E_4\left(-\frac{2(-i\,bdx-}{d}\right.\right.}{3\left(d^4x^3+3\,cd^3x^2+3\,c^2d^2x+c^3d\right)}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/3*(3*(exp_integral_e(4, 2*(-I*b*d*x - I*b*c)/d) + exp_integral_e(4, -2*
(-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 3*(I*exp_integral_e(4, 2*(-
I*b*d*x - I*b*c)/d) - I*exp_integral_e(4, -2*(-I*b*d*x - I*b*c)/d))*sin(-2
*(b*c - a*d)/d) + 1)/(d^4*x^3 + 3*c*d^3*x^2 + 3*c^2*d^2*x + c^3*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1305 vs. $2(193) = 386$.

Time $= 0.23$ (sec) , antiderivative size $= 1305$, normalized size of antiderivative $= 6.37$

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x, algorithm="giac")
```

output
```
1/3*(8*b^7*c^3*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(b*c - a
*d)/d) + 24*(b*x + a)*b^6*c^2*d*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d
)*sin(-2*(b*c - a*d)/d) - 24*a*b^6*c^2*d*cos_integral(2*(b*c + (b*x + a)*d
 - a*d)/d)*sin(-2*(b*c - a*d)/d) + 24*(b*x + a)^2*b^5*c*d^2*cos_integral(2
*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(b*c - a*d)/d) - 48*(b*x + a)*a*b^5*c
*d^2*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(b*c - a*d)/d) + 2
4*a^2*b^5*c*d^2*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(b*c -
a*d)/d) + 8*(b*x + a)^3*b^4*d^3*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d
)*sin(-2*(b*c - a*d)/d) - 24*(b*x + a)^2*a*b^4*d^3*cos_integral(2*(b*c + (
b*x + a)*d - a*d)/d)*sin(-2*(b*c - a*d)/d) + 24*(b*x + a)*a^2*b^4*d^3*cos_
integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(b*c - a*d)/d) - 8*a^3*b^4*
d^3*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(b*c - a*d)/d) - 8*
b^7*c^3*cos(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)/d)
 - 24*(b*x + a)*b^6*c^2*d*cos(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*
x + a)*d - a*d)/d) + 24*a*b^6*c^2*d*cos(-2*(b*c - a*d)/d)*sin_integral(-2*
(b*c + (b*x + a)*d - a*d)/d) - 24*(b*x + a)^2*b^5*c*d^2*cos(-2*(b*c - a*d)
/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)/d) + 48*(b*x + a)*a*b^5*c*d^
2*cos(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)/d) - 24*
a^2*b^5*c*d^2*cos(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a
*d)/d) - 8*(b*x + a)^3*b^4*d^3*cos(-2*(b*c - a*d)/d)*sin_integral(-2*(b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx = \int \frac{\sin(3a+3bx)}{\sin(a+bx)\,(c+dx)^4}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^4),x)
```

output
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^4), x)
```

Reduce [F]

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx = \int \frac{\csc(bx+a)\sin(3bx+3a)}{(dx+c)^4}dx$$

input
```
int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x)
```

output
```
int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x)
```

3.376 $\int (c+dx)^3 \csc^2(a+bx)\sin(3a+3bx)\,dx$

Optimal result . . . 3067
Mathematica [A] (verified) . . . 3068
Rubi [A] (verified) . . . 3068
Maple [B] (verified) . . . 3070
Fricas [B] (verification not implemented) . . . 3071
Sympy [F(-1)] . . . 3072
Maxima [B] (verification not implemented) . . . 3072
Giac [F] . . . 3073
Mupad [F(-1)] . . . 3073
Reduce [F] . . . 3073

Optimal result

Integrand size = 25, antiderivative size = 255

$$\begin{aligned}
&\int (c+dx)^3 \csc^2(a+bx)\sin(3a+3bx)\,dx \\
&= -\frac{6(c+dx)^3 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{24d^2(c+dx)\cos(a+bx)}{b^3} \\
&\quad + \frac{4(c+dx)^3\cos(a+bx)}{b} + \frac{9id(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{9id(c+dx)^2\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} - \frac{18d^2(c+dx)\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{18d^2(c+dx)\,\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} - \frac{18id^3\,\mathrm{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4} \\
&\quad + \frac{18id^3\,\mathrm{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4} + \frac{24d^3\sin(a+bx)}{b^4} - \frac{12d(c+dx)^2\sin(a+bx)}{b^2}
\end{aligned}$$

output

```
-6*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b-24*d^2*(d*x+c)*cos(b*x+a)/b^3+4*(d*
x+c)^3*cos(b*x+a)/b+9*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^2-9*I*d*(
d*x+c)^2*polylog(2,exp(I*(b*x+a)))/b^2-18*d^2*(d*x+c)*polylog(3,-exp(I*(b*
x+a)))/b^3+18*d^2*(d*x+c)*polylog(3,exp(I*(b*x+a)))/b^3-18*I*d^3*polylog(4
,-exp(I*(b*x+a)))/b^4+18*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+24*d^3*sin(b*
x+a)/b^4-12*d*(d*x+c)^2*sin(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.80

$$\int (c+dx)^3 \csc^2(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{3(-2b^3(c+dx)^3\operatorname{arctanh}(\cos(a+bx)+i\sin(a+bx))+3id(b^2(c+dx)^2\operatorname{PolyLog}(2,-\cos(a+bx)-i\sin$$
$$+\frac{4\cos(bx)\left(b^3c^3\cos(a)-6bcd^2\cos(a)+3b^3c^2dx\cos(a)-6bd^3x\cos(a)+3b^3cd^2x^2\cos(a)+b^3d^3x^3\cos(a\right.}{b^4}$$
$$-\frac{4(3b^2c^2d\cos(a)-6d^3\cos(a)+6b^2cd^2x\cos(a)+3b^2d^3x^2\cos(a)+b^3c^3\sin(a)-6bcd^2\sin(a)+3b^3c^2dx}{b^4}$$

input
```
Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output
```
(3*(-2*b^3*(c + d*x)^3*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]] + (3*I)*d*(b
^2*(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]] + (2*I)*b*d*(c +
 d*x)*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] - 2*d^2*PolyLog[4, -Cos[a
 + b*x] - I*Sin[a + b*x]]) - (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, Cos[a + b
*x] + I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, Cos[a + b*x] + I*Si
n[a + b*x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Sin[a + b*x]])))/b^4 + (4*
Cos[b*x]*(b^3*c^3*Cos[a] - 6*b*c*d^2*Cos[a] + 3*b^3*c^2*d*x*Cos[a] - 6*b*d
^3*x*Cos[a] + 3*b^3*c*d^2*x^2*Cos[a] + b^3*d^3*x^3*Cos[a] - 3*b^2*c^2*d*Si
n[a] + 6*d^3*Sin[a] - 6*b^2*c*d^2*x*Sin[a] - 3*b^2*d^3*x^2*Sin[a]))/b^4 -
(4*(3*b^2*c^2*d*Cos[a] - 6*d^3*Cos[a] + 6*b^2*c*d^2*x*Cos[a] + 3*b^2*d^3*x
^2*Cos[a] + b^3*c^3*Sin[a] - 6*b*c*d^2*Sin[a] + 3*b^3*c^2*d*x*Sin[a] - 6*b
*d^3*x*Sin[a] + 3*b^3*c*d^2*x^2*Sin[a] + b^3*d^3*x^3*Sin[a])*Sin[b*x])/b^4
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin(3a+3bx) \csc^2(a+bx)\, dx$$

$$\downarrow 4931$$

$$\int \left(3(c+dx)^3 \cos(a+bx) \cot(a+bx) - (c+dx)^3 \sin(a+bx)\right) dx$$

$$\downarrow 2009$$

$$-\frac{6(c+dx)^3 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{18id^3 \operatorname{PolyLog}\left(4, -e^{i(a+bx)}\right)}{b^4} + \frac{18id^3 \operatorname{PolyLog}\left(4, e^{i(a+bx)}\right)}{b^4} + \frac{24d^3 \sin(a+bx)}{b^4} - \frac{18d^2(c+dx) \operatorname{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^3} + \frac{18d^2(c+dx) \operatorname{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3} - \frac{24d^2(c+dx) \cos(a+bx)}{b^3} + \frac{9id(c+dx)^2 \operatorname{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^2} - \frac{9id(c+dx)^2 \operatorname{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^2} - \frac{12d(c+dx)^2 \sin(a+bx)}{b^2} + \frac{4(c+dx)^3 \cos(a+bx)}{b}$$

input

```
Int[(c + d*x)^3*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output

```
(-6*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b - (24*d^2*(c + d*x)*Cos[a + b*
x])/b^3 + (4*(c + d*x)^3*Cos[a + b*x])/b + ((9*I)*d*(c + d*x)^2*PolyLog[2,
 -E^(I*(a + b*x))])/b^2 - ((9*I)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))]
)/b^2 - (18*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (18*d^2*(c +
 d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((18*I)*d^3*PolyLog[4, -E^(I*(a +
 b*x))])/b^4 + ((18*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4 + (24*d^3*Sin[
a + b*x])/b^4 - (12*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 848 vs. 2(237) = 474.

Time = 2.71 (sec) , antiderivative size = 849, normalized size of antiderivative = 3.33

method	result	size
risch	Expression too large to display	849

input
```
int((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
-9/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2+2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3
*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b
^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))+2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*
c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^
2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))-3/b^4*d^3*ln(exp(I*(b*x+a))+1)*a^3+6/
b^4*d^3*a^3*arctanh(exp(I*(b*x+a)))-18/b^3*c*d^2*a^2*arctanh(exp(I*(b*x+a)
))+18/b^2*c^2*d*a*arctanh(exp(I*(b*x+a)))+9/b^3*d^2*c*ln(exp(I*(b*x+a))+1)
*a^2+9*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-9*I/b^2*d^3*polylog(2,exp(
I*(b*x+a)))*x^2-9*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))+9*I/b^2*c^2*d*poly
log(2,-exp(I*(b*x+a)))-9/b^2*c^2*d*ln(exp(I*(b*x+a))+1)*a+18*I/b^2*d^2*c*p
olylog(2,-exp(I*(b*x+a)))*x-18*I/b^2*d^2*c*polylog(2,exp(I*(b*x+a)))*x-18/
b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+18/b^3*c*d^2*polylog(3,exp(I*(b*x+a))
)+3/b*d^3*ln(1-exp(I*(b*x+a)))*x^3-3/b*d^3*ln(exp(I*(b*x+a))+1)*x^3-18/b^3
*d^3*polylog(3,-exp(I*(b*x+a)))*x+18/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x+3
/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+9/b^2*d*c^2*ln(1-exp(I*(b*x+a)))*a-9/b*c
*d^2*ln(exp(I*(b*x+a))+1)*x^2+9/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-9/b*d*c^2
*ln(exp(I*(b*x+a))+1)*x+9/b*d*c^2*ln(1-exp(I*(b*x+a)))*x-6/b*c^3*arctanh(e
xp(I*(b*x+a)))+18*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4-18*I*d^3*polylog(4,-
exp(I*(b*x+a)))/b^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 929 vs. $2(231) = 462$.

Time = 0.13 (sec) , antiderivative size = 929, normalized size of antiderivative = 3.64

$$\int (c+dx)^3 \csc^2(a+bx) \sin(3a+3bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
1/2*(18*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 18*I*d^3*polylog
(4, cos(b*x + a) - I*sin(b*x + a)) + 18*I*d^3*polylog(4, -cos(b*x + a) + I
*sin(b*x + a)) - 18*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 8*(
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d
^3)*x)*cos(b*x + a) - 9*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*di
log(cos(b*x + a) + I*sin(b*x + a)) - 9*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x -
 I*b^2*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 9*(I*b^2*d^3*x^2 + 2*
I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 9*(-I
*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*sin(
b*x + a)) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*lo
g(cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 3*(b^3*c
^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*
I*sin(b*x + a) + 1/2) + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + 3*(b^3*d^3*x^3 + 3
*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*
log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x +
 a) - I*sin(b*x + a) + 1) + 18*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a)
 + I*sin(b*x + a)) + 18*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) - I...
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^2(a+bx) \sin(3a+3bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3*csc(b*x+a)**2*sin(3*b*x+3*a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(231) = 462$.

Time = 0.28 (sec) , antiderivative size = 606, normalized size of antiderivative = 2.38

$$\int (c+dx)^3 \csc^2(a+bx) \sin(3a+3bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
1/2*c^3*(8*cos(b*x + a) - 3*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2
+ sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*log(cos(b*x)^2 - 2*cos(b*
x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b - 1/2
*(36*I*d^3*polylog(4, -e^(I*b*x + I*a)) - 36*I*d^3*polylog(4, e^(I*b*x + I
*a)) - 6*(-I*b^3*d^3*x^3 - 3*I*b^3*c*d^2*x^2 - 3*I*b^3*c^2*d*x)*arctan2(si
n(b*x + a), cos(b*x + a) + 1) - 6*(-I*b^3*d^3*x^3 - 3*I*b^3*c*d^2*x^2 - 3*
I*b^3*c^2*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 8*(b^3*d^3*x^3 +
 3*b^3*c*d^2*x^2 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a) - 1
8*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(-e^(I*b*x + I*a))
- 18*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(e^(I*b*x + I*a
)) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x)*log(cos(b*x + a)^2
+ sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
+ 3*b^3*c^2*d*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1)
 + 36*(b*d^3*x + b*c*d^2)*polylog(3, -e^(I*b*x + I*a)) - 36*(b*d^3*x + b*c
*d^2)*polylog(3, e^(I*b*x + I*a)) + 24*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*
c^2*d - 2*d^3)*sin(b*x + a))/b^4
```

Giac [F]

$$\int (c+dx)^3 \csc^2(a+bx) \sin(3a+3bx)\,dx = \int (dx+c)^3 \csc(bx+a)^2 \sin(3bx+3a)\,dx$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csc(b*x + a)^2*sin(3*b*x + 3*a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^2(a+bx) \sin(3a+3bx)\,dx = \text{Hanged}$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x)^3)/sin(a + b*x)^2,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\begin{aligned}\int (c+dx)^3 \csc^2(a+bx) \sin(3a+3bx)\,dx = &\left(\int \csc(bx+a)^2 \sin(3bx+3a)\, x^3 dx\right) d^3 \\ &+3\left(\int \csc(bx+a)^2 \sin(3bx+3a)\, x^2 dx\right) c\, d^2 \\ &+3\left(\int \csc(bx+a)^2 \sin(3bx+3a)\, x dx\right) c^2 d \\ &+\left(\int \csc(bx+a)^2 \sin(3bx+3a)\, dx\right) c^3\end{aligned}$$

input
```
int((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x)
```

output

```
int(csc(a + b*x)**2*sin(3*a + 3*b*x)*x**3,x)*d**3 + 3*int(csc(a + b*x)**2*
sin(3*a + 3*b*x)*x**2,x)*c*d**2 + 3*int(csc(a + b*x)**2*sin(3*a + 3*b*x)*x
,x)*c**2*d + int(csc(a + b*x)**2*sin(3*a + 3*b*x),x)*c**3
```

3.377 $\int (c+dx)^2 \csc^2(a+bx)\sin(3a+3bx)\,dx$

Optimal result . . . 3075
Mathematica [A] (verified) . . . 3076
Rubi [A] (verified) . . . 3076
Maple [B] (verified) . . . 3077
Fricas [B] (verification not implemented) . . . 3078
Sympy [F(-1)] . . . 3079
Maxima [B] (verification not implemented) . . . 3080
Giac [F] . . . 3080
Mupad [F(-1)] . . . 3081
Reduce [F] . . . 3081

Optimal result

Integrand size = 25, antiderivative size = 172

$$\begin{aligned}
&\int (c+dx)^2 \csc^2(a+bx)\sin(3a+3bx)\,dx \\
&= -\frac{6(c+dx)^2 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{8d^2\cos(a+bx)}{b^3} + \frac{4(c+dx)^2\cos(a+bx)}{b} \\
&\quad + \frac{6id(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} - \frac{6id(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{6d^2\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} + \frac{6d^2\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} - \frac{8d(c+dx)\sin(a+bx)}{b^2}
\end{aligned}$$

output

```
-6*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b-8*d^2*cos(b*x+a)/b^3+4*(d*x+c)^2*co
s(b*x+a)/b+6*I*d*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^2-6*I*d*(d*x+c)*poly
log(2,exp(I*(b*x+a)))/b^2-6*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+6*d^2*polyl
og(3,exp(I*(b*x+a)))/b^3-8*d*(d*x+c)*sin(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.30

$$\int (c+dx)^2 \csc^2(a+bx) \sin(3a+3bx)\, dx$$
$$= \frac{3b^2(c+dx)^2 \log\left(1-e^{i(a+bx)}\right) - 3b^2(c+dx)^2 \log\left(1+e^{i(a+bx)}\right) + 6ibd(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right) - (}{}$$

input
```
Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output
```
(3*b^2*(c + d*x)^2*Log[1 - E^(I*(a + b*x))] - 3*b^2*(c + d*x)^2*Log[1 + E^
(I*(a + b*x))] + (6*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] - (6*I)*
b*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] - 6*d^2*PolyLog[3, -E^(I*(a + b*
x))] + 6*d^2*PolyLog[3, E^(I*(a + b*x))] + 4*Cos[b*x]*((-2*d^2 + b^2*(c +
d*x)^2)*Cos[a] - 2*b*d*(c + d*x)*Sin[a]) - 4*(2*b*d*(c + d*x)*Cos[a] + (-2
*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x])/b^3
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin(3a+3bx) \csc^2(a+bx)\, dx$$
$$\downarrow \text{4931}$$
$$\int \left(3(c+dx)^2 \cos(a+bx)\cot(a+bx) - (c+dx)^2 \sin(a+bx)\right) dx$$
$$\downarrow \text{2009}$$

$$-\frac{6(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{6d^2\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}-\frac{8d^2\cos(a+bx)}{b^3}+\frac{6id(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{6id(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{8d(c+dx)\sin(a+bx)}{b^2}+\frac{4(c+dx)^2\cos(a+bx)}{b}$$

input
```
Int[(c + d*x)^2*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output
```
(-6*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (8*d^2*Cos[a + b*x])/b^3 + (
4*(c + d*x)^2*Cos[a + b*x])/b + ((6*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b
*x))])/b^2 - ((6*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (6*d^2*
PolyLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^2*PolyLog[3, E^(I*(a + b*x))])/b^
3 - (8*d*(c + d*x)*Sin[a + b*x])/b^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(160) = 320$.

Time = 1.98 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.80

method	result
risch	$\frac{2(x^2d^2b^2+2b^2cdx+2ib\,d^2x+b^2c^2+2ibcd-2d^2)e^{i(bx+a)}}{b^3}+\frac{2(x^2d^2b^2+2b^2cdx-2ib\,d^2x+b^2c^2-2ibcd-2d^2)e^{-i(bx+a)}}{b^3}-\frac{6cd\ln(e^{i(}$

input
```
int((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
2*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(I*
(b*x+a))+2*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)/b
^3*exp(-I*(b*x+a))-6/b^2*c*d*ln(exp(I*(b*x+a))+1)*a+6*I/b^2*d^2*polylog(2,
-exp(I*(b*x+a)))*x+6/b^2*c*d*ln(1-exp(I*(b*x+a)))*a+6*I/b^2*c*d*polylog(2,
-exp(I*(b*x+a)))-6/b*c*d*ln(exp(I*(b*x+a))+1)*x+6/b*c*d*ln(1-exp(I*(b*x+a)
))*x-6*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))-6*I/b^2*d^2*polylog(2,exp(I*(b*
x+a)))*x+3/b^3*d^2*ln(exp(I*(b*x+a))+1)*a^2-3/b^3*d^2*ln(1-exp(I*(b*x+a)))
*a^2-3/b*d^2*ln(exp(I*(b*x+a))+1)*x^2-6*d^2*polylog(3,-exp(I*(b*x+a)))/b^3
+3/b*d^2*ln(1-exp(I*(b*x+a)))*x^2+6*d^2*polylog(3,exp(I*(b*x+a)))/b^3-6/b^
3*d^2*a^2*arctanh(exp(I*(b*x+a)))+12/b^2*c*d*a*arctanh(exp(I*(b*x+a)))-6/b
*c^2*arctanh(exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(156) = 312$.

Time $= 0.12$ (sec) , antiderivative size $= 566$, normalized size of antiderivative $= 3.29$

$$\int (c + dx)^2 \csc^2(a + bx) \sin(3a + 3bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

output

```
1/2*(6*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*d^2*polylog(3, co
s(b*x + a) - I*sin(b*x + a)) - 6*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x
+ a)) - 6*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 8*(b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a) - 6*(I*b*d^2*x + I*b*c*d)*di
log(cos(b*x + a) + I*sin(b*x + a)) - 6*(-I*b*d^2*x - I*b*c*d)*dilog(cos(b*
x + a) - I*sin(b*x + a)) - 6*(I*b*d^2*x + I*b*c*d)*dilog(-cos(b*x + a) + I
*sin(b*x + a)) - 6*(-I*b*d^2*x - I*b*c*d)*dilog(-cos(b*x + a) - I*sin(b*x
+ a)) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) + I*sin(b
*x + a) + 1) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) -
I*sin(b*x + a) + 1) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x +
 a) + 1/2*I*sin(b*x + a) + 1/2) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1
/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*(b^2*d
^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x
+ a) + 1) - 16*(b*d^2*x + b*c*d)*sin(b*x + a))/b^3
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc^2(a+bx) \sin(3a+3bx)\,dx = \text{Timed out}$$

input

```
integrate((d*x+c)**2*csc(b*x+a)**2*sin(3*b*x+3*a),x)
```

output

```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(156) = 312$.

Time $= 0.21$ (sec) , antiderivative size $= 413$, normalized size of antiderivative $= 2.40$

$$\int (c+dx)^2 \csc^2(a+bx)\sin(3a+3bx)\,dx$$

$$= \frac{c^2\left(8\cos(bx+a) - 3\log\left(\cos(bx)^2 + 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 - 2\sin(bx)\sin(a) + \sin(a\right.\right.}{2b}$$

$$- \frac{12\,d^2\mathrm{Li}_3(-e^{(i\,bx+i\,a)}) - 12\,d^2\mathrm{Li}_3(e^{(i\,bx+i\,a)}) - 6\left(-i\,b^2d^2x^2 - 2i\,b^2cdx\right)\arctan\left(\sin(bx+a), \cos(bx+a)\right.}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
1/2*c^2*(8*cos(b*x + a) - 3*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2
+ sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*log(cos(b*x)^2 - 2*cos(b*
x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b - 1/2
*(12*d^2*polylog(3, -e^(I*b*x + I*a)) - 12*d^2*polylog(3, e^(I*b*x + I*a))
 - 6*(-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x)*arctan2(sin(b*x + a), cos(b*x + a) +
 1) - 6*(-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x)*arctan2(sin(b*x + a), -cos(b*x +
a) + 1) - 8*(b^2*d^2*x^2 + 2*b^2*c*d*x - 2*d^2)*cos(b*x + a) - 12*(I*b*d^2
*x + I*b*c*d)*dilog(-e^(I*b*x + I*a)) - 12*(-I*b*d^2*x - I*b*c*d)*dilog(e^
(I*b*x + I*a)) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x)*log(cos(b*x + a)^2 + sin(b*
x + a)^2 + 2*cos(b*x + a) + 1) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x)*log(cos(b*x
 + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 16*(b*d^2*x + b*c*d)*sin(
b*x + a))/b^3
```

Giac [F]

$$\int (c+dx)^2 \csc^2(a+bx)\sin(3a+3bx)\,dx = \int (dx+c)^2 \csc(bx+a)^2 \sin(3\,bx+3\,a)\,dx$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csc(b*x + a)^2*sin(3*b*x + 3*a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc^2(a+bx) \sin(3a+3bx)\,dx = \text{Hanged}$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x)^2)/sin(a + b*x)^2,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\begin{aligned}\int (c+dx)^2 \csc^2(a+bx) \sin(3a+3bx)\,dx = &\left(\int \csc(bx+a)^2 \sin(3bx+3a)\, x^2 dx\right) d^2 \\ &+ 2\left(\int \csc(bx+a)^2 \sin(3bx+3a)\, x dx\right) cd \\ &+ \left(\int \csc(bx+a)^2 \sin(3bx+3a)\, dx\right) c^2\end{aligned}$$

input
```
int((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x)
```

output
```
int(csc(a + b*x)**2*sin(3*a + 3*b*x)*x**2,x)*d**2 + 2*int(csc(a + b*x)**2*
sin(3*a + 3*b*x)*x,x)*c*d + int(csc(a + b*x)**2*sin(3*a + 3*b*x),x)*c**2
```

3.378 $\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx$

Optimal result . . . 3082
Mathematica [A] (verified) . . . 3083
Rubi [A] (verified) . . . 3083
Maple [B] (verified) . . . 3084
Fricas [B] (verification not implemented) . . . 3085
Sympy [F(-1)] . . . 3086
Maxima [F] . . . 3086
Giac [F] . . . 3086
Mupad [F(-1)] . . . 3087
Reduce [F] . . . 3087

Optimal result

Integrand size = 23, antiderivative size = 95

$$\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx = -\frac{6(c+dx)\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} + \frac{4(c+dx)\cos(a+bx)}{b} + \frac{3id\,\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} - \frac{3id\,\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} - \frac{4d\sin(a+bx)}{b^2}$$

output

```
-6*(d*x+c)*arctanh(exp(I*(b*x+a)))/b+4*(d*x+c)*cos(b*x+a)/b+3*I*d*polylog(
2,-exp(I*(b*x+a)))/b^2-3*I*d*polylog(2,exp(I*(b*x+a)))/b^2-4*d*sin(b*x+a)/
b^2
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.80

$$\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx = \frac{4c\cos(a+bx)}{b} + \frac{4dx\cos(a+bx)}{b} - \frac{3c\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{3c\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b} - \frac{3ad\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} + \frac{3d\left((a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)+i\left(\text{PolyLog}\left(2,-e^{i(a+bx)}\right)-\text{PolyLog}\left(2,e^{i(a+bx)}\right)\right)\right.}{b^2} - \frac{4d\sin(a+bx)}{b^2}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output
```
(4*c*Cos[a + b*x])/b + (4*d*x*Cos[a + b*x])/b - (3*c*Log[Cos[(a + b*x)/2]]
)/b + (3*c*Log[Sin[(a + b*x)/2]])/b - (3*a*d*Log[Tan[(a + b*x)/2]])/b^2 +
(3*d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*
(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a + b*x))])))/b^2 - (4*d*
Sin[a + b*x])/b^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin(3a+3bx)\csc^2(a+bx)\,dx$$

$$\downarrow \text{4931}$$

$$\int (3(c+dx)\cos(a+bx)\cot(a+bx) - (c+dx)\sin(a+bx))dx$$

$$\downarrow \text{2009}$$

$$-\frac{6(c+dx)\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{3id\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3id\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{4d\sin(a+bx)}{b^2}+\frac{4(c+dx)\cos(a+bx)}{b}$$

input
```
Int[(c + d*x)*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output
```
(-6*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + (4*(c + d*x)*Cos[a + b*x])/b +
 ((3*I)*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((3*I)*d*PolyLog[2, E^(I*(a
+ b*x))])/b^2 - (4*d*Sin[a + b*x])/b^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(87) = 174$.

Time = 0.89 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.16

method	result
risch	$\frac{2(bdx+cb+id)e^{i(bx+a)}}{b^2}+\frac{2(bdx+cb-id)e^{-i(bx+a)}}{b^2}-\frac{6c\,\operatorname{arctanh}\left(e^{i(bx+a)}\right)}{b}-\frac{3d\ln\left(e^{i(bx+a)}+1\right)x}{b}-\frac{3d\ln\left(e^{i(bx+a)}+1\right)a}{b^2}+$
default	$-\frac{c\ln(\csc(bx+a)-\cot(bx+a))}{b}-\frac{d\left((bx+a)\ln\left(1-e^{i(bx+a)}\right)-(bx+a)\ln\left(e^{i(bx+a)}+1\right)+i\operatorname{dilog}\left(e^{i(bx+a)}+1\right)-i\operatorname{dilog}\left(1-e^{i(bx+a)}\right)-\right.}{b^2}$

input
```
int((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
2*(b*d*x+c*b+I*d)/b^2*exp(I*(b*x+a))+2*(b*d*x+c*b-I*d)/b^2*exp(-I*(b*x+a))
-6/b*c*arctanh(exp(I*(b*x+a)))-3/b*d*ln(exp(I*(b*x+a))+1)*x-3/b^2*d*ln(exp
(I*(b*x+a))+1)*a+3*I*d*polylog(2,-exp(I*(b*x+a)))/b^2+3/b*d*ln(1-exp(I*(b*
x+a)))*x+3/b^2*d*ln(1-exp(I*(b*x+a)))*a-3*I*d*polylog(2,exp(I*(b*x+a)))/b^
2+6/b^2*d*a*arctanh(exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(83) = 166$.

Time $= 0.10$ (sec) , antiderivative size $= 281$, normalized size of antiderivative $= 2.96$

$$\int (c + dx) \csc^2(a + bx) \sin(3a + 3bx)\, dx$$
$$= \frac{8\,(bdx + bc)\cos(bx + a) - 3i\,d\mathrm{Li}_2(\cos(bx + a) + i\,\sin(bx + a)) + 3i\,d\mathrm{Li}_2(\cos(bx + a) - i\,\sin(bx + a))}{}$$

input
```
integrate((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
1/2*(8*(b*d*x + b*c)*cos(b*x + a) - 3*I*d*dilog(cos(b*x + a) + I*sin(b*x +
 a)) + 3*I*d*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*I*d*dilog(-cos(b*x +
 a) + I*sin(b*x + a)) + 3*I*d*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*(b
*d*x + b*c)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b*d*x + b*c)*log(c
os(b*x + a) - I*sin(b*x + a) + 1) + 3*(b*c - a*d)*log(-1/2*cos(b*x + a) +
1/2*I*sin(b*x + a) + 1/2) + 3*(b*c - a*d)*log(-1/2*cos(b*x + a) - 1/2*I*si
n(b*x + a) + 1/2) + 3*(b*d*x + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1
) + 3*(b*d*x + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 8*d*sin(b*x
+ a))/b^2
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*csc(b*x+a)**2*sin(3*b*x+3*a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx = \int (dx+c)\csc(bx+a)^2\sin(3\,bx+3\,a)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
1/2*c*(8*cos(b*x + a) - 3*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 +
sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*log(cos(b*x)^2 - 2*cos(b*x)
*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b + (4*b*
x*cos(b*x + a) + 3*b^2*integrate(x*sin(b*x + a)/(cos(b*x + a)^2 + sin(b*x
+ a)^2 + 2*cos(b*x + a) + 1), x) + 3*b^2*integrate(x*sin(b*x + a)/(cos(b*x
 + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1), x) - 4*sin(b*x + a))*d/b^2
```

Giac [F]

$$\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx = \int (dx+c)\csc(bx+a)^2\sin(3\,bx+3\,a)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*csc(b*x + a)^2*sin(3*b*x + 3*a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx = \text{Hanged}$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x))/sin(a + b*x)^2,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx = \left(\int \csc(bx+a)^2\sin(3bx+3a)\,x dx\right)d + \left(\int \csc(bx+a)^2\sin(3bx+3a)\,dx\right)c$$

input
```
int((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x)
```

output
```
int(csc(a + b*x)**2*sin(3*a + 3*b*x)*x,x)*d + int(csc(a + b*x)**2*sin(3*a
+ 3*b*x),x)*c
```

3.379 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

Optimal result . . . 3088
Mathematica [N/A] . . . 3088
Rubi [N/A] . . . 3089
Maple [N/A] . . . 3090
Fricas [N/A] . . . 3090
Sympy [F(-1)] . . . 3091
Maxima [N/A] . . . 3091
Giac [N/A] . . . 3092
Mupad [N/A] . . . 3092
Reduce [N/A] . . . 3092

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = -\frac{4\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d} - \frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d} + 3\text{Int}\left(\frac{\csc(a+bx)}{c+dx},x\right)$$

output

```
-4*Ci(b*c/d+b*x)*sin(a-b*c/d)/d-4*cos(a-b*c/d)*Si(b*c/d+b*x)/d+3*Defer(Int
)(csc(b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 4.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

input

```
Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
```

output
```
Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\csc^2(a+bx)}{c+dx}\,dx$$

$$\downarrow 4931$$

$$\int \left(\frac{3\cos(a+bx)\cot(a+bx)}{c+dx} - \frac{\sin(a+bx)}{c+dx}\right)dx$$

$$\downarrow 2009$$

$$3\int \frac{\csc(a+bx)}{c+dx}dx - \frac{4\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d} - \frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

input
```
Int[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\csc(bx+a)^2 \sin(3bx+3a)}{dx+c} dx$$

input

```
int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

output

```
int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx} dx = \int \frac{\csc(bx+a)^2 \sin(3\,bx+3\,a)}{dx+c} dx$$

input

```
integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
```

output

```
integral(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 230, normalized size of antiderivative = 9.20

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\csc\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

output
```
-(2*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(1, -(I*b
*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - 3*d*integrate(sin(b*x + a)/((d*x +
 c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)*cos(b*x
+ a) + c), x) - 3*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*
x + c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) - 2*(exp_i
ntegral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d
))*sin(-(b*c - a*d)/d))/d
```

Giac [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\csc\left(bx+a\right)^2 \sin\left(3\,bx+3\,a\right)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 23.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sin\left(3\,a+3\,b\,x\right)}{\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)),x)
```

output
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\csc\left(bx+a\right)^2 \sin\left(3bx+3a\right)}{dx+c}\,dx$$

input
```
int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

output
```
int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

3.380 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$

Optimal result . . . 3094
Mathematica [N/A] . . . 3094
Rubi [N/A] . . . 3095
Maple [N/A] . . . 3096
Fricas [N/A] . . . 3096
Sympy [F(-1)] . . . 3097
Maxima [N/A] . . . 3097
Giac [N/A] . . . 3098
Mupad [N/A] . . . 3098
Reduce [N/A] . . . 3098

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = -\frac{4b\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{4\sin(a+bx)}{d(c+dx)} + \frac{4b\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+3\operatorname{Int}\left(\frac{\csc(a+bx)}{(c+dx)^2},x\right)$$

output

```
-4*b*cos(a-b*c/d)*Ci(b*c/d+b*x)/d^2+4*sin(b*x+a)/d/(d*x+c)+4*b*sin(a-b*c/d
)*Si(b*c/d+b*x)/d^2+3*Defer(Int)(csc(b*x+a)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 4.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

input

```
Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\csc^2(a+bx)}{(c+dx)^2}\,dx$$

$$\downarrow 4931$$

$$\int \left(\frac{3\cos(a+bx)\cot(a+bx)}{(c+dx)^2} - \frac{\sin(a+bx)}{(c+dx)^2}\right)dx$$

$$\downarrow 2009$$

$$3\int \frac{\csc(a+bx)}{(c+dx)^2}dx - \frac{4b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2} + \frac{4b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2} + \frac{4\sin(a+bx)}{d(c+dx)}$$

input
```
Int[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\csc(bx+a)^2 \sin(3bx+3a)}{(dx+c)^2} dx$$

input
```
int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2} dx = \int \frac{\csc(bx+a)^2 \sin(3\,bx+3\,a)}{(dx+c)^2} dx$$

input
```
integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 342, normalized size of antiderivative = 13.68

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-(2*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(2, -(I*b
*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - 3*(d^2*x + c*d)*integrate(sin(b*x
+ a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*
x^2 + 2*c*d*x + c^2)*sin(b*x + a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*co
s(b*x + a)), x) - 3*(d^2*x + c*d)*integrate(sin(b*x + a)/(d^2*x^2 + 2*c*d*
x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*s
in(b*x + a)^2 + c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)), x) - 2*(e
xp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*
c)/d))*sin(-(b*c - a*d)/d))/(d^2*x + c*d)
```

Giac [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 24.84 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\sin\left(3\,a+3\,b\,x\right)}{\sin\left(a+b\,x\right)^2\left(c+d\,x\right)^2}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)^2),x)
```

output
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)^2\sin\left(3bx+3a\right)}{\left(dx+c\right)^2}dx$$

input
```
int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

3.381 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

Optimal result . . . 3100
Mathematica [N/A] . . . 3101
Rubi [N/A] . . . 3101
Maple [N/A] . . . 3102
Fricas [N/A] . . . 3102
Sympy [F(-1)] . . . 3103
Maxima [N/A] . . . 3103
Giac [N/A] . . . 3104
Mupad [N/A] . . . 3104
Reduce [N/A] . . . 3104

Optimal result

Integrand size = 25, antiderivative size = 25

$$\begin{aligned}\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = &\frac{2b\cos(a+bx)}{d^2(c+dx)}\\ &+\frac{2b^2\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^3}\\ &+\frac{2\sin(a+bx)}{d(c+dx)^2}+\frac{2b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^3}\\ &+3\text{Int}\left(\frac{\csc(a+bx)}{(c+dx)^3},x\right)\end{aligned}$$

output

```
2*b*cos(b*x+a)/d^2/(d*x+c)+2*b^2*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^3+2*sin(b*x+
a)/d/(d*x+c)^2+2*b^2*cos(a-b*c/d)*Si(b*c/d+b*x)/d^3+3*Defer(Int)(csc(b*x+a
)/(d*x+c)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 5.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

input
```
Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

output
```
Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\csc^2(a+bx)}{(c+dx)^3}\,dx$$

$$\downarrow 4931$$

$$\int \left(\frac{3\cos(a+bx)\cot(a+bx)}{(c+dx)^3} - \frac{\sin(a+bx)}{(c+dx)^3}\right)dx$$

$$\downarrow 2009$$

$$3\int \frac{\csc(a+bx)}{(c+dx)^3}\,dx + \frac{2b^2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^3} + \frac{2b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^3} + \frac{2b\cos(a+bx)}{d^2(c+dx)} + \frac{2\sin(a+bx)}{d(c+dx)^2}$$

input
```
Int[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\csc(bx+a)^2 \sin(3bx+3a)}{(dx+c)^3} dx$$

input
```
int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

output
```
int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3} dx = \int \frac{\csc(bx+a)^2 \sin(3\,bx+3\,a)}{(dx+c)^3} dx$$

input
```
integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*
x + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 463, normalized size of antiderivative = 18.52

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\csc\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)}{\left(dx+c\right)^3}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-(2*(-I*exp_integral_e(3, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(3, -(I*b
*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - 3*(d^3*x^2 + 2*c*d^2*x + c^2*d)*in
tegrate(sin(b*x + a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3 + (d^3*x^3 +
 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*cos(b*x + a)^2 + (d^3*x^3 + 3*c*d^2*x^2 +
3*c^2*d*x + c^3)*sin(b*x + a)^2 + 2*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c
^3)*cos(b*x + a)), x) - 3*(d^3*x^2 + 2*c*d^2*x + c^2*d)*integrate(sin(b*x
+ a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3 + (d^3*x^3 + 3*c*d^2*x^2 + 3
*c^2*d*x + c^3)*cos(b*x + a)^2 + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)
*sin(b*x + a)^2 - 2*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*cos(b*x + a)
), x) - 2*(exp_integral_e(3, (I*b*d*x + I*b*c)/d) + exp_integral_e(3, -(I*
b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Giac [N/A]

Not integrable

Time = 3.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\csc\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)}{\left(dx+c\right)^3}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 26.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\sin\left(3\,a+3\,b\,x\right)}{\sin\left(a+b\,x\right)^2\left(c+d\,x\right)^3}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)^3),x)
```

output
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)^3), x)
```

Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\csc\left(bx+a\right)^2\sin\left(3bx+3a\right)}{\left(dx+c\right)^3}dx$$

input
```
int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

output
```
int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

3.382 $\int (c+dx)^4 \sec(a+bx)\sin(3a+3bx)\,dx$

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Optimal result

Integrand size = 23, antiderivative size = 293

$$\begin{aligned}\int (c+dx)^4 \sec(a+bx)\sin(3a+3bx)\,dx = {} & \frac{3d^2(c+dx)^2}{b^3} - \frac{(c+dx)^4}{b} - \frac{i(c+dx)^5}{5d} \\ & + \frac{(c+dx)^4 \log\left(1+e^{2i(a+bx)}\right)}{b} \\ & - \frac{2id(c+dx)^3 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} \\ & + \frac{3d^2(c+dx)^2 \operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3} \\ & + \frac{3id^3(c+dx) \operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4} \\ & - \frac{3d^4 \operatorname{PolyLog}\left(5,-e^{2i(a+bx)}\right)}{2b^5} \\ & - \frac{6d^3(c+dx)\cos(a+bx)\sin(a+bx)}{b^4} \\ & + \frac{4d(c+dx)^3\cos(a+bx)\sin(a+bx)}{b^2} \\ & + \frac{3d^4\sin^2(a+bx)}{b^5} \\ & - \frac{6d^2(c+dx)^2\sin^2(a+bx)}{b^3} \\ & + \frac{2(c+dx)^4\sin^2(a+bx)}{b}\end{aligned}$$

output

```
3*d^2*(d*x+c)^2/b^3-(d*x+c)^4/b-1/5*I*(d*x+c)^5/d+(d*x+c)^4*ln(1+exp(2*I*(
b*x+a)))/b-2*I*d*(d*x+c)^3*polylog(2,-exp(2*I*(b*x+a)))/b^2+3*d^2*(d*x+c)^
2*polylog(3,-exp(2*I*(b*x+a)))/b^3+3*I*d^3*(d*x+c)*polylog(4,-exp(2*I*(b*x
+a)))/b^4-3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5-6*d^3*(d*x+c)*cos(b*x+a
)*sin(b*x+a)/b^4+4*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2+3*d^4*sin(b*x+a)^
2/b^5-6*d^2*(d*x+c)^2*sin(b*x+a)^2/b^3+2*(d*x+c)^4*sin(b*x+a)^2/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2529 vs. $2(293) = 586$.

Time $= 6.38$ (sec) , antiderivative size $= 2529$, normalized size of antiderivative $= 8.63$

$$\int (c + dx)^4 \sec(a + bx) \sin(3a + 3bx)\, dx = \text{Result too large to show}$$

input

```
Integrate[(c + d*x)^4*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

output

```
((I/2)*c^2*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*
I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))
] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^
3*E^(I*a)) + ((I/2)*c*d^3*E^(I*a)*((2*b^4*x^4)/E^((2*I)*a) - (4*I)*b^3*(1
 + E^((-2*I)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x))] + 6*b^2*(1 + E^((-2*I)*a
))*x^2*PolyLog[2, -E^((-2*I)*(a + b*x))] - (6*I)*b*(1 + E^((-2*I)*a))*x*Po
lyLog[3, -E^((-2*I)*(a + b*x))] - 3*(1 + E^((-2*I)*a))*PolyLog[4, -E^((-2*
I)*(a + b*x))])*Sec[a])/b^4 + ((I/20)*d^4*E^(I*a)*((4*b^5*x^5)/E^((2*I)*a)
 - (10*I)*b^4*(1 + E^((-2*I)*a))*x^4*Log[1 + E^((-2*I)*(a + b*x))] + 20*b^
3*(1 + E^((-2*I)*a))*x^3*PolyLog[2, -E^((-2*I)*(a + b*x))] - (30*I)*b^2*(1
 + E^((-2*I)*a))*x^2*PolyLog[3, -E^((-2*I)*(a + b*x))] - 30*b*(1 + E^((-2*
I)*a))*x*PolyLog[4, -E^((-2*I)*(a + b*x))] + (15*I)*(1 + E^((-2*I)*a))*Pol
yLog[5, -E^((-2*I)*(a + b*x))])*Sec[a])/b^5 + (c^4*Sec[a]*(Cos[a]*Log[Cos[
a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) +
(2*c^3*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*
ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Lo
g[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[
a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[C
ot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin
[a]^2)]) + Sec[a]*(Cos[2*a + 2*b*x]/(40*b^5) - ((I/40)*Sin[2*a + 2*b*x]...
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sin(3a+3bx)\sec(a+bx)\,dx$$

$$\downarrow 4931$$

$$\int \left(3(c+dx)^4 \sin(a+bx)\cos(a+bx) - (c+dx)^4 \sin^2(a+bx)\tan(a+bx)\right) dx$$

$$\downarrow 2009$$

$$-\frac{3d^4\,\mathrm{PolyLog}\left(5,-e^{2i(a+bx)}\right)}{2b^5}+\frac{3d^4\sin^2(a+bx)}{b^5}+\frac{3id^3(c+dx)\,\mathrm{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4}-\frac{6d^3(c+dx)\sin(a+bx)\cos(a+bx)}{b^4}+\frac{3d^2(c+dx)^2\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}-\frac{6d^2(c+dx)^2\sin^2(a+bx)}{b^3}-\frac{2id(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}+\frac{4d(c+dx)^3\sin(a+bx)\cos(a+bx)}{b^2}+\frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{2(c+dx)^4\sin^2(a+bx)}{b}+\frac{3d^2(c+dx)^2}{b^3}-\frac{(c+dx)^4}{b}-\frac{i(c+dx)^5}{5d}$$

input

```
Int[(c + d*x)^4*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

output

```
(3*d^2*(c + d*x)^2)/b^3 - (c + d*x)^4/b - ((I/5)*(c + d*x)^5)/d + ((c + d*
x)^4*Log[1 + E^((2*I)*(a + b*x))])/b - ((2*I)*d*(c + d*x)^3*PolyLog[2, -E^
((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x
))])/b^3 + ((3*I)*d^3*(c + d*x)*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 - (3
*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/(2*b^5) - (6*d^3*(c + d*x)*Cos[a +
b*x]*Sin[a + b*x])/b^4 + (4*d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/b^2 +
 (3*d^4*Sin[a + b*x]^2)/b^5 - (6*d^2*(c + d*x)^2*Sin[a + b*x]^2)/b^3 + (2*
(c + d*x)^4*Sin[a + b*x]^2)/b
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 964 vs. 2(276) = 552.

Time = 6.35 (sec) , antiderivative size = 965, normalized size of antiderivative = 3.29

method	result
risch	$-\frac{8id^3ca^3x}{b^3}-\frac{6id^3c\,\mathrm{polylog}\left(2,-e^{2i(bx+a)}\right)x^2}{b^2}-\frac{2c^4\ln\left(e^{i(bx+a)}\right)}{b}-\frac{(2d^4x^4b^4+8b^4cd^3x^3-4ib^3d^4x^3+12b^4c^2d^2x^2-12ib^3cd^3x$

input
```
int((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
-2/b*c^4*ln(exp(I*(b*x+a)))+12*I/b^2*d^2*c^2*a^2*x-8*I/b*d*c^3*a*x-1/4*(2*
d^4*x^4*b^4-4*I*b^3*d^4*x^3+8*b^4*c*d^3*x^3-12*I*b^3*c*d^3*x^2+12*b^4*c^2*
d^2*x^2-12*I*b^3*c^2*d^2*x+8*b^4*c^3*d*x-4*I*b^3*c^3*d+2*b^4*c^4-6*b^2*d^4
*x^2+6*I*b*d^4*x-12*b^2*c*d^3*x+6*I*b*c*d^3-6*b^2*c^2*d^2+3*d^4)/b^5*exp(-
2*I*(b*x+a))-6*I/b^2*d^2*c^2*polylog(2,-exp(2*I*(b*x+a)))*x-6*I/b^2*d^3*c*
polylog(2,-exp(2*I*(b*x+a)))*x^2-8*I/b^3*d^3*c*a^3*x-I*d^3*c*x^4-2*I*d^2*c
^2*x^3-2*I*d*c^3*x^2+3*I/b^4*d^4*polylog(4,-exp(2*I*(b*x+a)))*x+8*I/b^3*d^
2*c^2*a^3-2*I/b^2*d^4*polylog(2,-exp(2*I*(b*x+a)))*x^3-6*I/b^4*d^3*c*a^4+3
*I/b^4*d^3*c*polylog(4,-exp(2*I*(b*x+a)))+2*I/b^4*d^4*a^4*x-4*I/b^2*d*c^3*
a^2-2*I/b^2*d*c^3*polylog(2,-exp(2*I*(b*x+a)))-2/b^5*d^4*a^4*ln(exp(I*(b*x
+a)))+3/b^3*d^4*polylog(3,-exp(2*I*(b*x+a)))*x^2+3/b^3*d^2*c^2*polylog(3,-
exp(2*I*(b*x+a)))+1/b*d^4*ln(exp(2*I*(b*x+a))+1)*x^4+8/5*I/b^5*d^4*a^5+8/b
^4*a^3*d^3*c*ln(exp(I*(b*x+a)))-12/b^3*a^2*d^2*c^2*ln(exp(I*(b*x+a)))+6/b^
3*d^3*c*polylog(3,-exp(2*I*(b*x+a)))*x+4/b*d^3*c*ln(exp(2*I*(b*x+a))+1)*x^
3+4/b*d*c^3*ln(exp(2*I*(b*x+a))+1)*x+6/b*d^2*c^2*ln(exp(2*I*(b*x+a))+1)*x^
2+8/b^2*a*d*c^3*ln(exp(I*(b*x+a)))-1/4*(2*d^4*x^4*b^4+4*I*b^3*d^4*x^3+8*b^
4*c*d^3*x^3+12*I*b^3*c*d^3*x^2+12*b^4*c^2*d^2*x^2+12*I*b^3*c^2*d^2*x+8*b^4
*c^3*d*x+4*I*b^3*c^3*d+2*b^4*c^4-6*b^2*d^4*x^2-6*I*b*d^4*x-12*b^2*c*d^3*x-
6*I*b*c*d^3-6*b^2*c^2*d^2+3*d^4)/b^5*exp(2*I*(b*x+a))-1/5*I*d^4*x^5+I*c^4*
x+1/5*I/d*c^5+1/b*c^4*ln(exp(2*I*(b*x+a))+1)-3/2*d^4*polylog(5,-exp(2*I...
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1652 vs. $2(272) = 544$.

Time = 0.18 (sec) , antiderivative size = 1652, normalized size of antiderivative = 5.64

$$\int (c+dx)^4 \sec(a+bx)\sin(3a+3bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
1/2*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 - 24*d^4*polylog(5, I*cos(b*x + a) +
sin(b*x + a)) - 24*d^4*polylog(5, I*cos(b*x + a) - sin(b*x + a)) - 24*d^4*
polylog(5, -I*cos(b*x + a) + sin(b*x + a)) - 24*d^4*polylog(5, -I*cos(b*x
+ a) - sin(b*x + a)) + 6*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 - 2*(2*b^4*d^4*x^4
+ 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 -
 b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4*(2*b^3
*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 -
b*d^4)*x)*cos(b*x + a)*sin(b*x + a) + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x - 4*
(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*dil
og(I*cos(b*x + a) + sin(b*x + a)) - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 +
 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 4
*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dil
og(-I*cos(b*x + a) + sin(b*x + a)) - 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2
 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*dilog(-I*cos(b*x + a) - sin(b*x + a))
+ (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*
log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*
b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(cos(b*x + a) - I*sin(b*x + a) +
 I) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x +
 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(I*cos(b*
x + a) + sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2...
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^4 \sec(a+bx) \sin(3a+3bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**4*sec(b*x+a)*sin(3*b*x+3*a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(272) = 544$.

Time = 0.31 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.08

$$\int (c+dx)^4 \sec(a+bx) \sin(3a+3bx)\,dx =$$

$$-\frac{c^4\left(2\cos(2bx+2a) - \log\left(\cos(2bx)^2 + 2\cos(2bx)\cos(2a) + \cos(2a)^2 + \sin(2bx)^2 - 2\sin(2bx)\right.\right.}{2b}$$

$$+\frac{-6ib^5d^4x^5 - 30ib^5cd^3x^4 - 60ib^5c^2d^2x^3 - 60ib^5c^3dx^2 - 90d^4\mathrm{Li}_5(-e^{(2ibx+2ia)}) - 20\left(-3ib^4d^4x^4 - 8ib\right.}{}$$

input
```
integrate((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
-1/2*c^4*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) +
cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b + 1/30*
(-6*I*b^5*d^4*x^5 - 30*I*b^5*c*d^3*x^4 - 60*I*b^5*c^2*d^2*x^3 - 60*I*b^5*c
^3*d*x^2 - 90*d^4*polylog(5, -e^(2*I*b*x + 2*I*a)) - 20*(-3*I*b^4*d^4*x^4
- 8*I*b^4*c*d^3*x^3 - 9*I*b^4*c^2*d^2*x^2 - 6*I*b^4*c^3*d*x)*arctan2(sin(2
*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 15*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 -
 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d
- 3*b^2*c*d^3)*x)*cos(2*b*x + 2*a) - 60*(2*I*b^3*d^4*x^3 + 4*I*b^3*c*d^3*x
^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(-e^(2*I*b*x + 2*I*a)) + 10*(3*
b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 9*b^4*c^2*d^2*x^2 + 6*b^4*c^3*d*x)*log(cos
(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 60*(-3*I*
b*d^4*x - 2*I*b*c*d^3)*polylog(4, -e^(2*I*b*x + 2*I*a)) + 30*(6*b^2*d^4*x^
2 + 8*b^2*c*d^3*x + 3*b^2*c^2*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) + 30*(
2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d
^2 - b*d^4)*x)*sin(2*b*x + 2*a))/b^5
```

Giac [F]

$$\int (c+dx)^4 \sec(a+bx) \sin(3a+3bx)\,dx = \int (dx+c)^4 \sec(bx+a) \sin(3\,bx+3\,a)\,dx$$

input
```
integrate((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \sec(a+bx) \sin(3a+3bx)\,dx = \int \frac{\sin(3\,a+3\,b\,x)\,(c+d\,x)^4}{\cos(a+b\,x)}\,dx$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x)^4)/cos(a + b*x),x)
```

output
```
int((sin(3*a + 3*b*x)*(c + d*x)^4)/cos(a + b*x), x)
```

Reduce [F]

$$\int (c+dx)^4 \sec(a+bx)\sin(3a+3bx)\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x)
```

output
```
(27*cos(3*a + 3*b*x)*b**4*c**4 + 108*cos(3*a + 3*b*x)*b**4*c**3*d*x + 162*
cos(3*a + 3*b*x)*b**4*c**2*d**2*x**2 + 108*cos(3*a + 3*b*x)*b**4*c*d**3*x*
*3 + 27*cos(3*a + 3*b*x)*b**4*d**4*x**4 - 36*cos(3*a + 3*b*x)*b**2*c**2*d*
*2 - 72*cos(3*a + 3*b*x)*b**2*c*d**3*x - 36*cos(3*a + 3*b*x)*b**2*d**4*x**
2 + 8*cos(3*a + 3*b*x)*d**4 - 324*int(tan((3*a + 3*b*x)/2)/(tan((3*a + 3*b
*x)/2)**2*tan((a + b*x)/2)**2 - tan((3*a + 3*b*x)/2)**2 + tan((a + b*x)/2)
**2 - 1),x)*b**5*c**4 - 324*int((tan((3*a + 3*b*x)/2)*x**4)/(tan((3*a + 3*
b*x)/2)**2*tan((a + b*x)/2)**2 - tan((3*a + 3*b*x)/2)**2 + tan((a + b*x)/2
)**2 - 1),x)*b**5*d**4 - 1296*int((tan((3*a + 3*b*x)/2)*x**3)/(tan((3*a +
3*b*x)/2)**2*tan((a + b*x)/2)**2 - tan((3*a + 3*b*x)/2)**2 + tan((a + b*x)
/2)**2 - 1),x)*b**5*c*d**3 - 1944*int((tan((3*a + 3*b*x)/2)*x**2)/(tan((3*
a + 3*b*x)/2)**2*tan((a + b*x)/2)**2 - tan((3*a + 3*b*x)/2)**2 + tan((a +
b*x)/2)**2 - 1),x)*b**5*c**2*d**2 - 1296*int((tan((3*a + 3*b*x)/2)*x)/(tan
((3*a + 3*b*x)/2)**2*tan((a + b*x)/2)**2 - tan((3*a + 3*b*x)/2)**2 + tan((
a + b*x)/2)**2 - 1),x)*b**5*c**3*d - 36*sin(3*a + 3*b*x)*b**3*c**3*d - 108
*sin(3*a + 3*b*x)*b**3*c**2*d**2*x - 108*sin(3*a + 3*b*x)*b**3*c*d**3*x**2
 - 36*sin(3*a + 3*b*x)*b**3*d**4*x**3 + 24*sin(3*a + 3*b*x)*b*c*d**3 + 24*
sin(3*a + 3*b*x)*b*d**4*x + 108*a*b**3*c**3*d - 72*a*b*c*d**3 - 27*b**4*c*
*4 + 36*b**2*c**2*d**2 - 8*d**4)/(81*b**5)
```

3.383 $\int (c+dx)^3 \sec(a+bx)\sin(3a+3bx)\,dx$

Optimal result . . . 3115
Mathematica [B] (warning: unable to verify) . . . 3116
Rubi [A] (verified) . . . 3117
Maple [B] (verified) . . . 3118
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Sympy [F(-1)] . . . 3120
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Giac [F] . . . 3121
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Reduce [F] . . . 3121

Optimal result

Integrand size = 23, antiderivative size = 242

$$\begin{aligned}\int (c+dx)^3 \sec(a+bx)\sin(3a+3bx)\,dx = {} & \frac{3d^3x}{2b^3} - \frac{(c+dx)^3}{b} - \frac{i(c+dx)^4}{4d} \\ & + \frac{(c+dx)^3 \log\left(1+e^{2i(a+bx)}\right)}{b} \\ & - \frac{3id(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} \\ & + \frac{3d^2(c+dx)\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} \\ & + \frac{3id^3\,\mathrm{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4} \\ & - \frac{3d^3\cos(a+bx)\sin(a+bx)}{2b^4} \\ & + \frac{3d(c+dx)^2\cos(a+bx)\sin(a+bx)}{b^2} \\ & - \frac{3d^2(c+dx)\sin^2(a+bx)}{b^3} \\ & + \frac{2(c+dx)^3\sin^2(a+bx)}{b}\end{aligned}$$

output
```
3/2*d^3*x/b^3-(d*x+c)^3/b-1/4*I*(d*x+c)^4/d+(d*x+c)^3*ln(1+exp(2*I*(b*x+a)
))/b-3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a)))/b^2+3/2*d^2*(d*x+c)*po
lylog(3,-exp(2*I*(b*x+a)))/b^3+3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4-
3/2*d^3*cos(b*x+a)*sin(b*x+a)/b^4+3*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^2-
3*d^2*(d*x+c)*sin(b*x+a)^2/b^3+2*(d*x+c)^3*sin(b*x+a)^2/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1730 vs. $2(242) = 484$.

Time = 6.33 (sec) , antiderivative size = 1730, normalized size of antiderivative = 7.15

$$\int (c+dx)^3 \sec(a+bx)\sin(3a+3bx)\,dx = \text{Too large to display}$$

input
```
Integrate[(c + d*x)^3*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
((I/4)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)
*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))]
- (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*
E^(I*a)) + ((I/8)*d^3*E^(I*a)*((2*b^4*x^4)/E^((2*I)*a) - (4*I)*b^3*(1 + E^
((-2*I)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x))] + 6*b^2*(1 + E^((-2*I)*a))*x
^2*PolyLog[2, -E^((-2*I)*(a + b*x))] - (6*I)*b*(1 + E^((-2*I)*a))*x*PolyLo
g[3, -E^((-2*I)*(a + b*x))] - 3*(1 + E^((-2*I)*a))*PolyLog[4, -E^((-2*I)*(
a + b*x))])*Sec[a])/b^4 + (c^3*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]
*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (3*c^2*d*Csc[a]*((b^
2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi
*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x
- ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - Ar
cTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 +
 Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + Sec[a]*
(Cos[2*a + 2*b*x]/(16*b^4) - ((I/16)*Sin[2*a + 2*b*x])/b^4)*(-8*b^3*c^3*Co
s[a] + (12*I)*b^2*c^2*d*Cos[a] + 12*b*c*d^2*Cos[a] - (6*I)*d^3*Cos[a] - 24
*b^3*c^2*d*x*Cos[a] + (24*I)*b^2*c*d^2*x*Cos[a] + 12*b*d^3*x*Cos[a] - 24*b
^3*c*d^2*x^2*Cos[a] + (12*I)*b^2*d^3*x^2*Cos[a] - 8*b^3*d^3*x^3*Cos[a] - (
8*I)*b^4*c^3*x*Cos[a + 2*b*x] - (12*I)*b^4*c^2*d*x^2*Cos[a + 2*b*x] - (8*I
)*b^4*c*d^2*x^3*Cos[a + 2*b*x] - (2*I)*b^4*d^3*x^4*Cos[a + 2*b*x] + (8*...
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin(3a+3bx)\sec(a+bx)\,dx$$

$$\downarrow 4931$$

$$\int \left(3(c+dx)^3 \sin(a+bx)\cos(a+bx) - (c+dx)^3 \sin^2(a+bx)\tan(a+bx)\right) dx$$

$$\downarrow 2009$$

$$\frac{3id^3 \operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4} - \frac{3d^3 \sin(a+bx)\cos(a+bx)}{2b^4} + \frac{3d^2(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} - \frac{3d^2(c+dx)\sin^2(a+bx)}{b^3} - \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} + \frac{3d(c+dx)^2 \sin(a+bx)\cos(a+bx)}{b^2} + \frac{(c+dx)^3 \log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2(c+dx)^3 \sin^2(a+bx)}{b} + \frac{3d^3x}{2b^3} - \frac{(c+dx)^3}{b} - \frac{i(c+dx)^4}{4d}$$

input

```
Int[(c + d*x)^3*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

output

```
(3*d^3*x)/(2*b^3) - (c + d*x)^3/b - ((I/4)*(c + d*x)^4)/d + ((c + d*x)^3*L
og[1 + E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((
2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])
/(2*b^3) + (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 - (3*d^3*C
os[a + b*x]*Sin[a + b*x])/(2*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a +
b*x])/b^2 - (3*d^2*(c + d*x)*Sin[a + b*x]^2)/b^3 + (2*(c + d*x)^3*Sin[a +
b*x]^2)/b
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 647 vs. $2(219) = 438$.

Time = 2.81 (sec) , antiderivative size = 648, normalized size of antiderivative = 2.68

method	result
risch	$\frac{3cd^2\ln\left(e^{2i(bx+a)}+1\right)x^2}{b}+\frac{3d^3\operatorname{polylog}\left(3,-e^{2i(bx+a)}\right)x}{2b^3}-\frac{6idc^2ax}{b}+\frac{6icd^2a^2x}{b^2}-\frac{3icd^2\operatorname{polylog}\left(2,-e^{2i(bx+a)}\right)x}{b^2}-\frac{3id^3a^4}{2b^4}$

input

```
int((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output

```
-6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a)))-6*I/b*d*c^2*a*x+6*I/b^2*c*d^2*a^2*x-3/
2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2-3/2*I/b^2*c^2*d*polylog(2,-ex
p(2*I*(b*x+a)))-1/8*(4*d^3*x^3*b^3-6*I*b^2*d^3*x^2+12*b^3*c*d^2*x^2-12*I*b
^2*c*d^2*x+12*b^3*c^2*d*x-6*I*b^2*c^2*d+4*b^3*c^3-6*b*d^3*x+3*I*d^3-6*c*d^
2*b)/b^4*exp(-2*I*(b*x+a))-3*I/b^2*d^2*c*polylog(2,-exp(2*I*(b*x+a)))*x+1/
b*d^3*ln(exp(2*I*(b*x+a))+1)*x^3+3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a))
)+3/2/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x+3/b*c^2*d*ln(exp(2*I*(b*x+a))
+1)*x+3/b*c*d^2*ln(exp(2*I*(b*x+a))+1)*x^2+I*c^3*x+1/4*I/d*c^4-3/2*I/b^4*d
^3*a^4-I*d^2*c*x^3-3/2*I*d*c^2*x^2+2/b^4*d^3*a^3*ln(exp(I*(b*x+a)))+6/b^2*
c^2*d*a*ln(exp(I*(b*x+a)))-3*I/b^2*d*c^2*a^2-2*I/b^3*d^3*a^3*x+4*I/b^3*c*d
^2*a^3-1/8*(4*d^3*x^3*b^3+6*I*b^2*d^3*x^2+12*b^3*c*d^2*x^2+12*I*b^2*c*d^2*
x+12*b^3*c^2*d*x+6*I*b^2*c^2*d+4*b^3*c^3-6*b*d^3*x-3*I*d^3-6*c*d^2*b)/b^4*
exp(2*I*(b*x+a))+1/b*c^3*ln(exp(2*I*(b*x+a))+1)-1/4*I*d^3*x^4-2/b*c^3*ln(e
xp(I*(b*x+a)))+3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1126 vs. $2(215) = 430$.

Time = 0.15 (sec) , antiderivative size = 1126, normalized size of antiderivative = 4.65

$$\int (c+dx)^3 \sec(a+bx) \sin(3a+3bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 6*I*d^3*polylog(4, I*cos(b*x + a) +
 sin(b*x + a)) + 6*I*d^3*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 6*I*d
^3*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(
b*x + a) - sin(b*x + a)) - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3
 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b^2*d^3*x^2
 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 3*(2*b^3
*c^2*d - b*d^3)*x - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dil
og(I*cos(b*x + a) + sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I
*b^2*c^2*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*
b^2*c*d^2*x + I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 3*(-I*b
^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*
x + a)) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x
 + a) + I*sin(b*x + a) + I) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*log(cos(b*x + a) - I*sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2
*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(
b*x + a) + sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*
d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*
x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b
^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*...
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^3 \sec(a+bx) \sin(3a+3bx)\, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3*sec(b*x+a)*sin(3*b*x+3*a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(215) = 430.

Time = 0.25 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.83

$$\int (c+dx)^3 \sec(a+bx) \sin(3a+3bx)\, dx =$$
$$-\frac{c^3\left(2\cos(2bx+2a) - \log\left(\cos(2bx)^2 + 2\cos(2bx)\cos(2a) + \cos(2a)^2 + \sin(2bx)^2 - 2\sin(2bx)\right.\right.}{2b}$$
$$+\frac{-3ib^4d^3x^4 - 12ib^4cd^2x^3 - 18ib^4c^2dx^2 + 12id^3\mathrm{Li}_4(-e^{(2ibx+2ia)}) - 4\left(-4ib^3d^3x^3 - 9ib^3cd^2x^2 - 9ib^3c\right.}{}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
-1/2*c^3*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) +
cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b + 1/12*
(-3*I*b^4*d^3*x^4 - 12*I*b^4*c*d^2*x^3 - 18*I*b^4*c^2*d*x^2 + 12*I*d^3*pol
ylog(4, -e^(2*I*b*x + 2*I*a)) - 4*(-4*I*b^3*d^3*x^3 - 9*I*b^3*c*d^2*x^2 -
9*I*b^3*c^2*d*x)*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*(2*b^
3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(2
*b*x + 2*a) - 6*(4*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(
-e^(2*I*b*x + 2*I*a)) + 2*(4*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x
)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) +
6*(4*b*d^3*x + 3*b*c*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) + 9*(2*b^2*d^3*
x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*sin(2*b*x + 2*a))/b^4
```

Giac [F]

$$\int (c+dx)^3 \sec(a+bx) \sin(3a+3bx)\,dx = \int (dx+c)^3 \sec(bx+a) \sin(3bx+3a)\,dx$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \sec(a+bx) \sin(3a+3bx)\,dx = \int \frac{\sin(3a+3bx)\,(c+dx)^3}{\cos(a+bx)}\,dx$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x)^3)/cos(a + b*x),x)
```

output
```
int((sin(3*a + 3*b*x)*(c + d*x)^3)/cos(a + b*x), x)
```

Reduce [F]

$$\int (c+dx)^3 \sec(a+bx) \sin(3a+3bx)\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x)
```

output

```
(9*cos(3*a + 3*b*x)*tan((3*a + 3*b*x)/2)**2*b**3*c**3 + 27*cos(3*a + 3*b*x
)*tan((3*a + 3*b*x)/2)**2*b**3*c**2*d*x + 27*cos(3*a + 3*b*x)*tan((3*a + 3
*b*x)/2)**2*b**3*c*d**2*x**2 + 9*cos(3*a + 3*b*x)*tan((3*a + 3*b*x)/2)**2*
b**3*d**3*x**3 - 6*cos(3*a + 3*b*x)*tan((3*a + 3*b*x)/2)**2*b*c*d**2 - 6*c
os(3*a + 3*b*x)*tan((3*a + 3*b*x)/2)**2*b*d**3*x + 9*cos(3*a + 3*b*x)*b**3
*c**3 + 27*cos(3*a + 3*b*x)*b**3*c**2*d*x + 27*cos(3*a + 3*b*x)*b**3*c*d**
2*x**2 + 9*cos(3*a + 3*b*x)*b**3*d**3*x**3 - 6*cos(3*a + 3*b*x)*b*c*d**2 -
 6*cos(3*a + 3*b*x)*b*d**3*x - 108*int(tan((3*a + 3*b*x)/2)/(tan((3*a + 3*
b*x)/2)**2*tan((a + b*x)/2)**2 - tan((3*a + 3*b*x)/2)**2 + tan((a + b*x)/2
)**2 - 1),x)*tan((3*a + 3*b*x)/2)**2*b**4*c**3 - 108*int(tan((3*a + 3*b*x)
/2)/(tan((3*a + 3*b*x)/2)**2*tan((a + b*x)/2)**2 - tan((3*a + 3*b*x)/2)**2
 + tan((a + b*x)/2)**2 - 1),x)*b**4*c**3 - 324*int((tan((3*a + 3*b*x)/2)*t
an((a + b*x)/2)**2*x**2)/(tan((3*a + 3*b*x)/2)**2*tan((a + b*x)/2)**2 - ta
n((3*a + 3*b*x)/2)**2 + tan((a + b*x)/2)**2 - 1),x)*tan((3*a + 3*b*x)/2)**
2*b**4*c*d**2 - 324*int((tan((3*a + 3*b*x)/2)*tan((a + b*x)/2)**2*x**2)/(t
an((3*a + 3*b*x)/2)**2*tan((a + b*x)/2)**2 - tan((3*a + 3*b*x)/2)**2 + tan
((a + b*x)/2)**2 - 1),x)*b**4*c*d**2 - 108*int((tan((3*a + 3*b*x)/2)*x**3)
/(tan((3*a + 3*b*x)/2)**2*tan((a + b*x)/2)**2 - tan((3*a + 3*b*x)/2)**2 +
tan((a + b*x)/2)**2 - 1),x)*tan((3*a + 3*b*x)/2)**2*b**4*d**3 - 108*int((t
an((3*a + 3*b*x)/2)*x**3)/(tan((3*a + 3*b*x)/2)**2*tan((a + b*x)/2)**2 ...
```

3.384 $\int (c+dx)^2 \sec(a+bx)\sin(3a+3bx)\,dx$

Optimal result . . . 3123
Mathematica [B] (warning: unable to verify) . . . 3124
Rubi [A] (verified) . . . 3125
Maple [B] (verified) . . . 3126
Fricas [B] (verification not implemented) . . . 3127
Sympy [F(-1)] . . . 3127
Maxima [B] (verification not implemented) . . . 3128
Giac [F] . . . 3128
Mupad [F(-1)] . . . 3129
Reduce [F] . . . 3129

Optimal result

Integrand size = 23, antiderivative size = 166

$$\begin{aligned}\int (c+dx)^2 \sec(a+bx)\sin(3a+3bx)\,dx = &-\frac{(c+dx)^2}{b} - \frac{i(c+dx)^3}{3d} \\ &+ \frac{(c+dx)^2 \log\left(1+e^{2i(a+bx)}\right)}{b} \\ &- \frac{id(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} \\ &+ \frac{d^2\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} \\ &+ \frac{2d(c+dx)\cos(a+bx)\sin(a+bx)}{b^2} \\ &- \frac{d^2\sin^2(a+bx)}{b^3} + \frac{2(c+dx)^2\sin^2(a+bx)}{b}\end{aligned}$$

output

```
-(d*x+c)^2/b-1/3*I*(d*x+c)^3/d+(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b-I*d*(d*x
+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2+1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/
b^3+2*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2-d^2*sin(b*x+a)^2/b^3+2*(d*x+c)^2
*sin(b*x+a)^2/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 516 vs. $2(166) = 332$.

Time = 6.32 (sec) , antiderivative size = 516, normalized size of antiderivative = 3.11

$$\int (c+dx)^2 \sec(a+bx)\sin(3a+3bx)\,dx$$

$$= \frac{id^2 e^{-ia}\left(2b^2x^2\left(2bx - 3i(1+e^{2ia})\log\left(1+e^{-2i(a+bx)}\right)\right) + 6b(1+e^{2ia})\,x\,\mathrm{PolyLog}\left(2,-e^{-2i(a+bx)}\right) - 3i(1+\right.}{12b^3}$$

$$+ \frac{c^2\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx)) + bx\sin(a))}{b\left(\cos^2(a)+\sin^2(a)\right)}$$

$$+ \frac{cd\csc(a)\left(b^2 e^{-i\arctan(\cot(a))}x^2 - \frac{\cot(a)(ibx(-\pi-2\arctan(\cot(a)))-\pi\log(1+e^{-2ibx})-2(bx-\arctan(\cot(a)))\log(1-e^{2i(bx-\arctan}}{}\right.}{b^2\sqrt{\csc^2(a)\left(\cos^2(a)+\right.}$$

$$- \frac{\cos(2bx)\left(2b^2c^2\cos(2a) - d^2\cos(2a) + 4b^2cdx\cos(2a) + 2b^2d^2x^2\cos(2a) - 2bcd\sin(2a) - 2bd^2x\sin(2a\right.}{2b^3}$$

$$+ \frac{(2bcd\cos(2a) + 2bd^2x\cos(2a) + 2b^2c^2\sin(2a) - d^2\sin(2a) + 4b^2cdx\sin(2a) + 2b^2d^2x^2\sin(2a))\sin(2bx\right.}{2b^3}$$

$$- \frac{1}{3}x\left(3c^2 + 3cdx + d^2x^2\right)\tan(a)$$

input

```
Integrate[(c + d*x)^2*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

output

```
((I/12)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*
(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] -
 (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E
^(I*a)) + (c^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x
*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[C
ot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b
*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))]
+ Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*P
olyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/
(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (Cos[2*b*x]*(2*b^2*c^2*Cos[2*
a] - d^2*Cos[2*a] + 4*b^2*c*d*x*Cos[2*a] + 2*b^2*d^2*x^2*Cos[2*a] - 2*b*c*
d*Sin[2*a] - 2*b*d^2*x*Sin[2*a]))/(2*b^3) + ((2*b*c*d*Cos[2*a] + 2*b*d^2*x
*Cos[2*a] + 2*b^2*c^2*Sin[2*a] - d^2*Sin[2*a] + 4*b^2*c*d*x*Sin[2*a] + 2*b
^2*d^2*x^2*Sin[2*a])*Sin[2*b*x])/(2*b^3) - (x*(3*c^2 + 3*c*d*x + d^2*x^2)*
Tan[a])/3
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin(3a+3bx)\sec(a+bx)\,dx$$

$$\downarrow 4931$$

$$\int \left(3(c+dx)^2 \sin(a+bx)\cos(a+bx) - (c+dx)^2 \sin^2(a+bx)\tan(a+bx)\right) dx$$

$$\downarrow 2009$$

$$\frac{d^2 \operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} - \frac{d^2 \sin^2(a+bx)}{b^3} - \frac{id(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} + \frac{2d(c+dx)\sin(a+bx)\cos(a+bx)}{b^2} + \frac{(c+dx)^2 \log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2(c+dx)^2 \sin^2(a+bx)}{b} - \frac{(c+dx)^2}{b} - \frac{i(c+dx)^3}{3d}$$

input

```
Int[(c + d*x)^2*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

output

```
-((c + d*x)^2/b) - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[1 + E^((2*I)*(
a + b*x))])/b - (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + (d^
2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (2*d*(c + d*x)*Cos[a + b*x]*
Sin[a + b*x])/b^2 - (d^2*Sin[a + b*x]^2)/b^3 + (2*(c + d*x)^2*Sin[a + b*x]
^2)/b
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(154) = 308$.

Time = 1.22 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.33

method	result
risch	$-\frac{4icdax}{b} - \frac{idc\,\mathrm{polylog}\left(2,-\mathrm{e}^{2i(bx+a)}\right)}{b^2} + \frac{ic^3}{3d} + \frac{2id^2a^2x}{b^2} - \frac{\left(2x^2d^2b^2+4b^2cdx+2ib\,d^2x+2b^2c^2+2ibcd-d^2\right)\mathrm{e}^{2i(bx+a)}}{4b^3} - \frac{(2x^2$

input

```
int((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output

```
-4*I/b*c*d*a*x-I/b^2*d*c*polylog(2,-exp(2*I*(b*x+a)))+1/3*I/d*c^3+2*I/b^2*
d^2*a^2*x-1/4*(2*x^2*d^2*b^2+2*I*b*d^2*x+4*b^2*c*d*x+2*I*b*c*d+2*b^2*c^2-d
^2)/b^3*exp(2*I*(b*x+a))-1/4*(2*x^2*d^2*b^2-2*I*b*d^2*x+4*b^2*c*d*x-2*I*b*
c*d+2*b^2*c^2-d^2)/b^3*exp(-2*I*(b*x+a))-2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))-
1/3*I*d^2*x^3+4/b^2*c*d*a*ln(exp(I*(b*x+a)))-I*d*c*x^2+I*c^2*x-I/b^2*d^2*p
olylog(2,-exp(2*I*(b*x+a)))*x-2*I/b^2*c*d*a^2+4/3*I/b^3*d^2*a^3+2/b*d*c*ln
(exp(2*I*(b*x+a))+1)*x+1/b*d^2*ln(exp(2*I*(b*x+a))+1)*x^2+1/2*d^2*polylog(
3,-exp(2*I*(b*x+a)))/b^3+1/b*c^2*ln(exp(2*I*(b*x+a))+1)-2/b*c^2*ln(exp(I*(
b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 681 vs. $2(151) = 302$.

Time = 0.12 (sec) , antiderivative size = 681, normalized size of antiderivative = 4.10

$$\int (c+dx)^2 \sec(a+bx) \sin(3a+3bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
1/2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x - 2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*
c^2 - d^2)*cos(b*x + a)^2 + 2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)
) + 2*d^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*polylog(3, -I*
cos(b*x + a) + sin(b*x + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x
+ a)) + 4*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) - 2*(-I*b*d^2*x - I*
b*c*d)*dilog(I*cos(b*x + a) + sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilo
g(I*cos(b*x + a) - sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(-I*cos(b*
x + a) + sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog(-I*cos(b*x + a) -
sin(b*x + a)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*sin(b
*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*sin(b*
x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(
b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^
2*d^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x
 + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2
*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x +
a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x +
a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x +
a) + I))/b^3
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^2 \sec(a+bx) \sin(3a+3bx)\, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*sec(b*x+a)*sin(3*b*x+3*a),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(151) = 302$.

Time = 0.22 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.83

$$\int (c+dx)^2 \sec(a+bx)\sin(3a+3bx)\,dx =$$
$$-\frac{c^2\left(2\cos(2bx+2a) - \log\left(\cos(2bx)^2 + 2\cos(2bx)\cos(2a) + \cos(2a)^2 + \sin(2bx)^2 - 2\sin(2bx)\right.\right.}{2b}$$
$$+\frac{-2ib^3d^2x^3 - 6ib^3cdx^2 + 3d^2\mathrm{Li}_3(-e^{(2ibx+2ia)}) - 6\left(-ib^2d^2x^2 - 2ib^2cdx\right)\arctan\left(\sin(2bx+2a), \cos(\right.}{}$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
-1/2*c^2*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) +
cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b + 1/6*(
-2*I*b^3*d^2*x^3 - 6*I*b^3*c*d*x^2 + 3*d^2*polylog(3, -e^(2*I*b*x + 2*I*a)
) - 6*(-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x)*arctan2(sin(2*b*x + 2*a), cos(2*b*x
 + 2*a) + 1) - 3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x - d^2)*cos(2*b*x + 2*a) - 6*
(I*b*d^2*x + I*b*c*d)*dilog(-e^(2*I*b*x + 2*I*a)) + 3*(b^2*d^2*x^2 + 2*b^2
*c*d*x)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) +
 1) + 6*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a))/b^3
```

Giac [F]

$$\int (c+dx)^2 \sec(a+bx)\sin(3a+3bx)\,dx = \int (dx+c)^2 \sec(bx+a)\sin(3bx+3a)\,dx$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \sec(a+bx) \sin(3a+3bx)\,dx = \int \frac{\sin(3a+3bx)\,(c+dx)^2}{\cos(a+bx)}\,dx$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x)^2)/cos(a + b*x),x)
```

output
```
int((sin(3*a + 3*b*x)*(c + d*x)^2)/cos(a + b*x), x)
```

Reduce [F]

$$\int (c+dx)^2 \sec(a+bx) \sin(3a+3bx)\,dx$$

$$= \frac{9\cos(3bx+3a)\,b^2c^2 + 18\cos(3bx+3a)\,b^2cdx + 9\cos(3bx+3a)\,b^2d^2x^2 - 2\cos(3bx+3a)\,d^2 - 108\left(\int$$

input
```
int((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x)
```

output
```
(9*cos(3*a + 3*b*x)*b**2*c**2 + 18*cos(3*a + 3*b*x)*b**2*c*d*x + 9*cos(3*a
 + 3*b*x)*b**2*d**2*x**2 - 2*cos(3*a + 3*b*x)*d**2 - 108*int(tan((3*a + 3*
b*x)/2)/(tan((3*a + 3*b*x)/2)**2*tan((a + b*x)/2)**2 - tan((3*a + 3*b*x)/2
)**2 + tan((a + b*x)/2)**2 - 1),x)*b**3*c**2 - 108*int((tan((3*a + 3*b*x)/
2)*x**2)/(tan((3*a + 3*b*x)/2)**2*tan((a + b*x)/2)**2 - tan((3*a + 3*b*x)/
2)**2 + tan((a + b*x)/2)**2 - 1),x)*b**3*d**2 - 216*int((tan((3*a + 3*b*x)
/2)*x)/(tan((3*a + 3*b*x)/2)**2*tan((a + b*x)/2)**2 - tan((3*a + 3*b*x)/2)
**2 + tan((a + b*x)/2)**2 - 1),x)*b**3*c*d - 6*sin(3*a + 3*b*x)*b*c*d - 6*
sin(3*a + 3*b*x)*b*d**2*x + 18*a*b*c*d - 9*b**2*c**2 + 2*d**2)/(27*b**3)
```

3.385 $\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx$

Optimal result . . . 3130
Mathematica [B] (warning: unable to verify) . . . 3131
Rubi [A] (verified) . . . 3131
Maple [A] (verified) . . . 3133
Fricas [B] (verification not implemented) . . . 3133
Sympy [F(-1)] . . . 3134
Maxima [F] . . . 3134
Giac [F] . . . 3135
Mupad [F(-1)] . . . 3135
Reduce [F] . . . 3135

Optimal result

Integrand size = 21, antiderivative size = 107

$$\begin{aligned}\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx = &-\frac{dx}{b}-\frac{i(c+dx)^2}{2d}\\ &+\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}\\ &-\frac{id\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}\\ &+\frac{d\cos(a+bx)\sin(a+bx)}{b^2}\\ &+\frac{2(c+dx)\sin^2(a+bx)}{b}\end{aligned}$$

output

```
-d*x/b-1/2*I*(d*x+c)^2/d+(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b-1/2*I*d*polylog(
2,-exp(2*I*(b*x+a)))/b^2+d*cos(b*x+a)*sin(b*x+a)/b^2+2*(d*x+c)*sin(b*x+a)^
2/b
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 257 vs. $2(107) = 214$.

Time = 3.50 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.40

$$\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx = \frac{c\log(\cos(a+bx))}{b}$$
$$+\frac{d\csc(a)\left(b^2e^{-i\arctan(\cot(a))}x^2 - \frac{\cot(a)(ibx(-\pi-2\arctan(\cot(a)))-\pi\log(1+e^{-2ibx})-2(bx-\arctan(\cot(a)))\log(1-e^{2i(bx-\arctan(}}{}\right.}{2b^2\sqrt{\csc^2(a)\left(\cos^2(a)+\right.}$$
$$-\frac{d\cos(2bx)(2bx\cos(2a)-\sin(2a))}{2b^2}$$
$$+\frac{d(\cos(2a)+2bx\sin(2a))\sin(2bx)}{2b^2}+\frac{2c\sin^2(a+bx)}{b}-\frac{1}{2}dx^2\tan(a)$$

input
```
Integrate[(c + d*x)*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
(c*Log[Cos[a + b*x]])/b + (d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot
[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x
- ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b
*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((
2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Cs
c[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (d*Cos[2*b*x]*(2*b*x*Cos[2*a] - Sin[2*a])
)/(2*b^2) + (d*(Cos[2*a] + 2*b*x*Sin[2*a])*Sin[2*b*x])/(2*b^2) + (2*c*Sin[
a + b*x]^2)/b - (d*x^2*Tan[a])/2
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin(3a+3bx)\sec(a+bx)\,dx$$

$$\downarrow 4931$$

$$\int \left(3(c+dx)\sin(a+bx)\cos(a+bx) - (c+dx)\sin^2(a+bx)\tan(a+bx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{id\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} + \frac{d\sin(a+bx)\cos(a+bx)}{b^2} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2(c+dx)\sin^2(a+bx)}{b} - \frac{dx}{b} - \frac{i(c+dx)^2}{2d}$$

input
```
Int[(c + d*x)*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
-((d*x)/b) - ((I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 + E^((2*I)*(a + b*x)
)])/b - ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + (d*Cos[a + b*x]*S
in[a + b*x])/b^2 + (2*(c + d*x)*Sin[a + b*x]^2)/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.65

method	result
risch	$-\frac{id\,x^2}{2}+ixc-\frac{(2bdx+2cb+id)e^{2i(bx+a)}}{4b^2}-\frac{(2bdx+2cb-id)e^{-2i(bx+a)}}{4b^2}+\frac{c\ln\left(e^{2i(bx+a)}+1\right)}{b}-\frac{2c\ln\left(e^{i(bx+a)}\right)}{b}-\frac{2idax}{b}$

input
```
int((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*d*x^2+I*x*c-1/4*(2*b*d*x+I*d+2*c*b)/b^2*exp(2*I*(b*x+a))-1/4*(2*b*d
*x-I*d+2*c*b)/b^2*exp(-2*I*(b*x+a))+1/b*c*ln(exp(2*I*(b*x+a))+1)-2/b*c*ln(
exp(I*(b*x+a)))-2*I/b*d*a*x-I/b^2*d*a^2+1/b*d*ln(exp(2*I*(b*x+a))+1)*x-1/2
*I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+2/b^2*d*a*ln(exp(I*(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(94) = 188$.

Time = 0.11 (sec) , antiderivative size = 340, normalized size of antiderivative = 3.18

$$\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{2\,bdx - 4\,(bdx+bc)\cos(bx+a)^2 + 2\,d\cos(bx+a)\sin(bx+a) + i\,d\mathrm{Li}_2(i\,\cos(bx+a)+\sin(bx+a)) -$$

input
```
integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
1/2*(2*b*d*x - 4*(b*d*x + b*c)*cos(b*x + a)^2 + 2*d*cos(b*x + a)*sin(b*x +
 a) + I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) - I*d*dilog(I*cos(b*x + a)
- sin(b*x + a)) - I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(-I
*cos(b*x + a) - sin(b*x + a)) + (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x +
 a) + I) + (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) + (b*d*x + a
*d)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x + a*d)*log(I*cos(b*x +
 a) - sin(b*x + a) + 1) + (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a)
 + 1) + (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d
)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)*log(-cos(b*x + a)
- I*sin(b*x + a) + I))/b^2
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx = \int (dx+c)\sec\left(bx+a\right)\sin\left(3\,bx+3\,a\right)\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
-1/2*c*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + co
s(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b - 1/2*(2*
b*x*cos(2*b*x + 2*a) + 4*b^2*integrate(x*sin(2*b*x + 2*a)/(cos(2*b*x + 2*a
)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1), x) - sin(2*b*x + 2*a))
*d/b^2
```

Giac [F]

$$\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx = \int (dx+c)\sec(bx+a)\sin(3\,bx+3\,a)\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx = \int \frac{\sin(3\,a+3\,b\,x)\,(c+d\,x)}{\cos(a+b\,x)}\,dx$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x))/cos(a + b*x),x)
```

output
```
int((sin(3*a + 3*b*x)*(c + d*x))/cos(a + b*x), x)
```

Reduce [F]

$$\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{3\left(\int \frac{\sin(3bx+3a)}{\cos(bx+a)}dx\right)b^2c + 3\left(\int \frac{\sin(3bx+3a)x}{\cos(bx+a)}dx\right)b^2d + ad - bc}{3b^2}$$

input
```
int((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x)
```

output
```
(3*int(sin(3*a + 3*b*x)/cos(a + b*x),x)*b**2*c + 3*int((sin(3*a + 3*b*x)*x
)/cos(a + b*x),x)*b**2*d + a*d - b*c)/(3*b**2)
```

3.386 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

Optimal result . . . 3136
Mathematica [N/A] . . . 3136
Rubi [N/A] . . . 3137
Maple [N/A] . . . 3138
Fricas [N/A] . . . 3138
Sympy [F(-2)] . . . 3139
Maxima [N/A] . . . 3139
Giac [N/A] . . . 3140
Mupad [N/A] . . . 3140
Reduce [N/A] . . . 3140

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \frac{2\,\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d} + \frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d} - \text{Int}\left(\frac{\tan(a+bx)}{c+dx},x\right)$$

output
```
2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d+2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x
)/d-Defer(Int)(tan(b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

input
```
Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

output
```
Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\sec(a+bx)}{c+dx}\,dx$$

$$\downarrow 4931$$

$$\int \left(\frac{3\sin(a+bx)\cos(a+bx)}{c+dx} - \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\right)dx$$

$$\downarrow 2009$$

$$-\int \frac{\tan(a+bx)}{c+dx}dx + \frac{2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d} + \frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}$$

input
```
Int[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sec(bx+a)\sin(3bx+3a)}{dx+c}dx$$

input

```
int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

output

```
int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)\sin(3\,bx+3\,a)}{dx+c}\,dx$$

input

```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
```

output

```
integral(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 182, normalized size of antiderivative = 7.91

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)\sin(3\,bx+3\,a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

output
```
-((-I*exp_integral_e(1, 2*(-I*b*d*x - I*b*c)/d) + I*exp_integral_e(1, -2*(
-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 2*d*integrate(sin(2*b*x + 2*
a)/((d*x + c)*cos(2*b*x + 2*a)^2 + (d*x + c)*sin(2*b*x + 2*a)^2 + d*x + 2*
(d*x + c)*cos(2*b*x + 2*a) + c), x) + (exp_integral_e(1, 2*(-I*b*d*x - I*b
*c)/d) + exp_integral_e(1, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b*c - a*d)/d)
)/d
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)\sin(3\,bx+3\,a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 20.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sin(3\,a+3\,b\,x)}{\cos(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)),x)
```

output
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)\sin(3bx+3a)}{dx+c}dx$$

input
```
int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

output
```
int((sec(a + b*x)*sin(3*a + 3*b*x))/(c + d*x),x)
```

3.387 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$

Optimal result . . . 3142
Mathematica [N/A] . . . 3142
Rubi [N/A] . . . 3143
Maple [N/A] . . . 3144
Fricas [N/A] . . . 3144
Sympy [F(-2)] . . . 3145
Maxima [N/A] . . . 3145
Giac [N/A] . . . 3146
Mupad [N/A] . . . 3146
Reduce [N/A] . . . 3146

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \frac{4b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2} - \frac{2\sin(2a+2bx)}{d(c+dx)} - \frac{4b\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2} - \operatorname{Int}\left(\frac{\tan(a+bx)}{(c+dx)^2},x\right)$$

output
```
4*b*cos(2*a-2*b*c/d)*Ci(2*b*c/d+2*b*x)/d^2-2*sin(2*b*x+2*a)/d/(d*x+c)-4*b*
sin(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^2-Defer(Int)(tan(b*x+a)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 3.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

$$\downarrow 4931$$

$$\int \left(\frac{3\sin(a+bx)\cos(a+bx)}{(c+dx)^2} - \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\right)dx$$

$$\downarrow 2009$$

$$-\int \frac{\tan(a+bx)}{(c+dx)^2}dx + \frac{4b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2} - \frac{4b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2} - \frac{2\sin(2a+2bx)}{d(c+dx)}$$

input
```
Int[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sec(bx+a)\sin(3bx+3a)}{(dx+c)^2}dx$$

input
```
int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

output
```
int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\sec(bx+a)\sin(3\,bx+3\,a)}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)*sin(3*b*x + 3*a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 242, normalized size of antiderivative = 10.52

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\sec(bx+a)\sin(3bx+3a)}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-((-I*exp_integral_e(2, 2*(-I*b*d*x - I*b*c)/d) + I*exp_integral_e(2, -2*(
-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 2*(d^2*x + c*d)*integrate(si
n(2*b*x + 2*a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x +
2*a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(2*b*x + 2*a)^2 + c^2 + 2*(d^2*x^2 +
 2*c*d*x + c^2)*cos(2*b*x + 2*a)), x) + (exp_integral_e(2, 2*(-I*b*d*x - I
*b*c)/d) + exp_integral_e(2, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b*c - a*d)/
d))/(d^2*x + c*d)
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\sec(bx+a)\sin(3\,bx+3\,a)}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 21.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\sin(3\,a+3\,b\,x)}{\cos(a+b\,x)\,(c+d\,x)^2}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)^2),x)
```

output
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\sec(bx+a)\sin(3bx+3a)}{d^2x^2+2cdx+c^2}dx$$

input
```
int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

output
```
int((sec(a + b*x)*sin(3*a + 3*b*x))/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.388 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

Optimal result . . . 3148
Mathematica [N/A] . . . 3149
Rubi [N/A] . . . 3149
Maple [N/A] . . . 3150
Fricas [N/A] . . . 3150
Sympy [F(-1)] . . . 3151
Maxima [N/A] . . . 3151
Giac [N/A] . . . 3152
Mupad [N/A] . . . 3152
Reduce [N/A] . . . 3152

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = -\frac{2b\cos(2a+2bx)}{d^2(c+dx)} - \frac{4b^2\,\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^3} - \frac{\sin(2a+2bx)}{d(c+dx)^2} - \frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3} - \text{Int}\left(\frac{\tan(a+bx)}{(c+dx)^3},x\right)$$

output

```
-2*b*cos(2*b*x+2*a)/d^2/(d*x+c)-4*b^2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d
^3-sin(2*b*x+2*a)/d/(d*x+c)^2-4*b^2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^3
-Defer(Int)(tan(b*x+a)/(d*x+c)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 4.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

input
```
Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

output
```
Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\sec(a+bx)}{(c+dx)^3}\,dx$$

$$\downarrow 4931$$

$$\int \left(\frac{3\sin(a+bx)\cos(a+bx)}{(c+dx)^3} - \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^3}\right)dx$$

$$\downarrow 2009$$

$$-\int \frac{\tan(a+bx)}{(c+dx)^3}dx - \frac{4b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3} - \frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3} - \frac{2b\cos(2a+2bx)}{d^2(c+dx)} - \frac{\sin(2a+2bx)}{d(c+dx)^2}$$

input
```
Int[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sec(bx+a)\sin(3bx+3a)}{(dx+c)^3}dx$$

input
```
int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

output
```
int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\sec(bx+a)\sin(3\,bx+3\,a)}{(dx+c)^3}\,dx$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)*sin(3*b*x + 3*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
+ c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 308, normalized size of antiderivative = 13.39

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\sec(bx+a)\sin(3\,bx+3\,a)}{(dx+c)^3}\,dx$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-((-I*exp_integral_e(3, 2*(-I*b*d*x - I*b*c)/d) + I*exp_integral_e(3, -2*(
-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 2*(d^3*x^2 + 2*c*d^2*x + c^2
*d)*integrate(sin(2*b*x + 2*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3 +
(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*cos(2*b*x + 2*a)^2 + (d^3*x^3 +
3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sin(2*b*x + 2*a)^2 + 2*(d^3*x^3 + 3*c*d^2*x
^2 + 3*c^2*d*x + c^3)*cos(2*b*x + 2*a)), x) + (exp_integral_e(3, 2*(-I*b*d
*x - I*b*c)/d) + exp_integral_e(3, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b*c -
 a*d)/d))/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\sec(bx+a)\sin(3\,bx+3\,a)}{(dx+c)^3}\,dx$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 22.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\sin(3\,a+3\,b\,x)}{\cos(a+b\,x)\,(c+d\,x)^3}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)^3),x)
```

output
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\sec(bx+a)\sin(3bx+3a)}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}\,dx$$

input
```
int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

output
```
int((sec(a + b*x)*sin(3*a + 3*b*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d
**3*x**3),x)
```

3.389 $\int (c+dx)^3 \sec^2(a+bx)\sin(3a+3bx)\,dx$

Optimal result . . . 3154
Mathematica [B] (verified) . . . 3155
Rubi [A] (verified) . . . 3155
Maple [B] (verified) . . . 3157
Fricas [B] (verification not implemented) . . . 3157
Sympy [F(-1)] . . . 3158
Maxima [F] . . . 3159
Giac [F] . . . 3159
Mupad [F(-1)] . . . 3160
Reduce [F] . . . 3160

Optimal result

Integrand size = 25, antiderivative size = 230

$$\begin{aligned}
&\int (c+dx)^3 \sec^2(a+bx)\sin(3a+3bx)\,dx \\
&= -\frac{6id(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^2(c+dx)\cos(a+bx)}{b^3} - \frac{4(c+dx)^3\cos(a+bx)}{b} \\
&\quad + \frac{6id^2(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} - \frac{6id^2(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} \\
&\quad - \frac{6d^3\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4} + \frac{6d^3\,\mathrm{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4} \\
&\quad - \frac{(c+dx)^3\sec(a+bx)}{b} - \frac{24d^3\sin(a+bx)}{b^4} + \frac{12d(c+dx)^2\sin(a+bx)}{b^2}
\end{aligned}$$

output

```
-6*I*d*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b^2+24*d^2*(d*x+c)*cos(b*x+a)/b^3-
4*(d*x+c)^3*cos(b*x+a)/b+6*I*d^2*(d*x+c)*polylog(2,-I*exp(I*(b*x+a)))/b^3-
6*I*d^2*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^3-6*d^3*polylog(3,-I*exp(I*(
b*x+a)))/b^4+6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4-(d*x+c)^3*sec(b*x+a)/b-
24*d^3*sin(b*x+a)/b^4+12*d*(d*x+c)^2*sin(b*x+a)/b^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 532 vs. $2(230) = 460$.

Time = 1.57 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.31

$$\int (c+dx)^3 \sec^2(a+bx)\sin(3a+3bx)\,dx =$$

$$-\frac{\sec(a+bx)\left(3b^3c^3-12bcd^2+9b^3c^2dx-12bd^3x+9b^3cd^2x^2+3b^3d^3x^3+6ib^2c^2d\arctan\left(e^{i(a+bx)}\right)\cos(\right.}{}$$

input
```
Integrate[(c + d*x)^3*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output
```
-((Sec[a + b*x]*(3*b^3*c^3 - 12*b*c*d^2 + 9*b^3*c^2*d*x - 12*b*d^3*x + 9*b
^3*c*d^2*x^2 + 3*b^3*d^3*x^3 + (6*I)*b^2*c^2*d*ArcTan[E^(I*(a + b*x))]*Cos
[a + b*x] + 2*b^3*c^3*Cos[2*(a + b*x)] - 12*b*c*d^2*Cos[2*(a + b*x)] + 6*b
^3*c^2*d*x*Cos[2*(a + b*x)] - 12*b*d^3*x*Cos[2*(a + b*x)] + 6*b^3*c*d^2*x^
2*Cos[2*(a + b*x)] + 2*b^3*d^3*x^3*Cos[2*(a + b*x)] - 6*b^2*c*d^2*x*Cos[a
+ b*x]*Log[1 - I*E^(I*(a + b*x))] - 3*b^2*d^3*x^2*Cos[a + b*x]*Log[1 - I*E
^(I*(a + b*x))] + 6*b^2*c*d^2*x*Cos[a + b*x]*Log[1 + I*E^(I*(a + b*x))] +
3*b^2*d^3*x^2*Cos[a + b*x]*Log[1 + I*E^(I*(a + b*x))] - (6*I)*b*d^2*(c + d
*x)*Cos[a + b*x]*PolyLog[2, (-I)*E^(I*(a + b*x))] + (6*I)*b*d^2*(c + d*x)*
Cos[a + b*x]*PolyLog[2, I*E^(I*(a + b*x))] + 6*d^3*Cos[a + b*x]*PolyLog[3,
 (-I)*E^(I*(a + b*x))] - 6*d^3*Cos[a + b*x]*PolyLog[3, I*E^(I*(a + b*x))]
- 6*b^2*c^2*d*Sin[2*(a + b*x)] + 12*d^3*Sin[2*(a + b*x)] - 12*b^2*c*d^2*x*
Sin[2*(a + b*x)] - 6*b^2*d^3*x^2*Sin[2*(a + b*x)]))/b^4)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin(3a+3bx) \sec^2(a+bx)\, dx$$

$$\downarrow 4931$$

$$\int \left(3(c+dx)^3 \sin(a+bx) - (c+dx)^3 \sin(a+bx) \tan^2(a+bx)\right) dx$$

$$\downarrow 2009$$

$$-\frac{6id(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^3 \operatorname{PolyLog}\left(3, -ie^{i(a+bx)}\right)}{b^4} + \frac{6d^3 \operatorname{PolyLog}\left(3, ie^{i(a+bx)}\right)}{b^4} - \frac{24d^3 \sin(a+bx)}{b^4} + \frac{6id^2(c+dx) \operatorname{PolyLog}\left(2, -ie^{i(a+bx)}\right)}{b^3} - \frac{6id^2(c+dx) \operatorname{PolyLog}\left(2, ie^{i(a+bx)}\right)}{b^3} + \frac{24d^2(c+dx)\cos(a+bx)}{b^3} + \frac{12d(c+dx)^2 \sin(a+bx)}{b^2} - \frac{4(c+dx)^3 \cos(a+bx)}{b} - \frac{(c+dx)^3 \sec(a+bx)}{b}$$

input

```
Int[(c + d*x)^3*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output

```
((-6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 + (24*d^2*(c + d*x)*Cos
[a + b*x])/b^3 - (4*(c + d*x)^3*Cos[a + b*x])/b + ((6*I)*d^2*(c + d*x)*Pol
yLog[2, (-I)*E^(I*(a + b*x))])/b^3 - ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(
I*(a + b*x))])/b^3 - (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 + (6*d^3
*PolyLog[3, I*E^(I*(a + b*x))])/b^4 - ((c + d*x)^3*Sec[a + b*x])/b - (24*d
^3*Sin[a + b*x])/b^4 + (12*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(213) = 426$.

Time = 8.58 (sec) , antiderivative size = 677, normalized size of antiderivative = 2.94

method	result
risch	$-\frac{2(d^3x^3b^3+3b^3cd^2x^2+3ib^2d^3x^2+3b^3c^2dx+6ib^2cd^2x+b^3c^3+3ib^2c^2d-6bd^3x-6cd^2b-6id^3)e^{i(bx+a)}}{b^4}$ $-$ $\frac{2(d^3x^3b^3+3b^3cd^2x^2-3}{}$

input
```
int((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
-2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b*
d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))-
2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d
^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))-
2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))+1
)+3/b^2*d^3*ln(1-I*exp(I*(b*x+a)))*x^2-6*d^3*polylog(3,-I*exp(I*(b*x+a)))/
b^4+6/b^2*c*d^2*ln(1-I*exp(I*(b*x+a)))*x-6/b^2*c*d^2*ln(I*exp(I*(b*x+a))+1
)*x+12*I/b^3*d^2*c*a*arctan(exp(I*(b*x+a)))-6/b^3*c*d^2*ln(I*exp(I*(b*x+a)
)+1)*a+3/b^4*a^2*d^3*ln(I*exp(I*(b*x+a))+1)-6*I/b^3*d^3*polylog(2,I*exp(I*
(b*x+a)))*x-6*I/b^3*d^2*c*polylog(2,I*exp(I*(b*x+a)))-6*I/b^2*d*c^2*arctan
(exp(I*(b*x+a)))+6/b^3*c*d^2*ln(1-I*exp(I*(b*x+a)))*a-6*I/b^4*d^3*a^2*arct
an(exp(I*(b*x+a)))-3/b^2*d^3*ln(I*exp(I*(b*x+a))+1)*x^2+6*I/b^3*d^2*c*poly
log(2,-I*exp(I*(b*x+a)))-3/b^4*a^2*d^3*ln(1-I*exp(I*(b*x+a)))+6*d^3*polylo
g(3,I*exp(I*(b*x+a)))/b^4+6*I/b^3*d^3*polylog(2,-I*exp(I*(b*x+a)))*x
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(204) = 408$.

Time = 0.13 (sec) , antiderivative size = 896, normalized size of antiderivative = 3.90

$$\int (c+dx)^3 \sec^2(a+bx)\sin(3a+3bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

output

```
-1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*
cos(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a
)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3
, -I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b
*x + a) - sin(b*x + a)) + 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b
*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 6*(I*b*d^3*x + I*b*c*
d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) + 6*(I*b*d^3*x + I*
b*c*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a)) + 6*(-I*b*d^3*x
 - I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 6*(-I*b
*d^3*x - I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3
*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin
(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(co
s(b*x + a) - I*sin(b*x + a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*
c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(
b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*co
s(b*x + a) - sin(b*x + a) + 1) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*
d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b
^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*co
s(b*x + a) - sin(b*x + a) + 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos
(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*...
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^3 \sec^2(a+bx) \sin(3a+3bx)\,dx = \text{Timed out}$$

input

```
integrate((d*x+c)**3*sec(b*x+a)**2*sin(3*b*x+3*a),x)
```

output

```
Timed out
```

Maxima [F]

$$\int (c+dx)^3 \sec^2(a+bx) \sin(3a+3bx)\,dx = \int (dx+c)^3 \sec(bx+a)^2 \sin(3\,bx+3\,a)\,dx$$

input
```
integrate((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
-2*((cos(3*b*x + 3*a) + cos(b*x + a))*cos(4*b*x + 4*a) + (3*cos(2*b*x + 2*
a) + 1)*cos(3*b*x + 3*a) + 3*cos(2*b*x + 2*a)*cos(b*x + a) + (sin(3*b*x +
3*a) + sin(b*x + a))*sin(4*b*x + 4*a) + 3*sin(3*b*x + 3*a)*sin(2*b*x + 2*a
) + 3*sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))*c^3/(b*cos(3*b*x + 3*a
)^2 + 2*b*cos(3*b*x + 3*a)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x +
 3*a)^2 + 2*b*sin(3*b*x + 3*a)*sin(b*x + a) + b*sin(b*x + a)^2) - 3/2*(4*(
cos(a)^2 + sin(a)^2)*b*x*cos(b*x + a) + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x +
 2*a) + b*x*cos(b*x + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) +
b*x*sin(b*x + 2*a)*sin(a))*cos(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(
b*x + a))*cos(2*b*x + 3*a)^2 + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) + b
*x*cos(b*x + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(b
*x + 2*a)*sin(a))*sin(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))
*sin(2*b*x + 3*a)^2 + 4*((b*x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x +
3*a) + sin(a))*cos(3*b*x + 3*a)^2 + (b*x*cos(a) + sin(a))*cos(b*x + a)^2 +
 (b*x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*sin(3*b*x
 + 3*a)^2 + (b*x*cos(a) + sin(a))*sin(b*x + a)^2 + 2*(b*x*cos(2*b*x + 3*a)
*cos(b*x + a) + (b*x*cos(a) + sin(a))*cos(b*x + a) + cos(b*x + a)*sin(2*b*
x + 3*a))*cos(3*b*x + 3*a) + (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos
(2*b*x + 3*a) + 2*(b*x*cos(2*b*x + 3*a)*sin(b*x + a) + (b*x*cos(a) + sin(a
))*sin(b*x + a) + sin(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a) + (co...
```

Giac [F]

$$\int (c+dx)^3 \sec^2(a+bx) \sin(3a+3bx)\,dx = \int (dx+c)^3 \sec(bx+a)^2 \sin(3\,bx+3\,a)\,dx$$

input
```
integrate((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*sec(b*x + a)^2*sin(3*b*x + 3*a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \sec^2(a+bx) \sin(3a+3bx)\,dx = \text{Hanged}$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x)^3)/cos(a + b*x)^2,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^3 \sec^2(a+bx) \sin(3a+3bx)\,dx = \text{too large to display}$$

input
```
int((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x)
```

output

```
(35937*cos(3*a + 3*b*x)*cos(a + b*x)*tan((3*a + 3*b*x)/2)**2*tan((a + b*x)
/2)**2*b**3*c**3 + 107811*cos(3*a + 3*b*x)*cos(a + b*x)*tan((3*a + 3*b*x)/
2)**2*tan((a + b*x)/2)**2*b**3*c**2*d*x + 107811*cos(3*a + 3*b*x)*cos(a +
b*x)*tan((3*a + 3*b*x)/2)**2*tan((a + b*x)/2)**2*b**3*c*d**2*x**2 + 35937*
cos(3*a + 3*b*x)*cos(a + b*x)*tan((3*a + 3*b*x)/2)**2*tan((a + b*x)/2)**2*
b**3*d**3*x**3 - 5346*cos(3*a + 3*b*x)*cos(a + b*x)*tan((3*a + 3*b*x)/2)**
2*tan((a + b*x)/2)**2*b*c*d**2 - 5346*cos(3*a + 3*b*x)*cos(a + b*x)*tan((3
*a + 3*b*x)/2)**2*tan((a + b*x)/2)**2*b*d**3*x - 35937*cos(3*a + 3*b*x)*co
s(a + b*x)*tan((3*a + 3*b*x)/2)**2*b**3*c**3 - 107811*cos(3*a + 3*b*x)*cos
(a + b*x)*tan((3*a + 3*b*x)/2)**2*b**3*c**2*d*x - 107811*cos(3*a + 3*b*x)*
cos(a + b*x)*tan((3*a + 3*b*x)/2)**2*b**3*c*d**2*x**2 - 35937*cos(3*a + 3*
b*x)*cos(a + b*x)*tan((3*a + 3*b*x)/2)**2*b**3*d**3*x**3 + 5346*cos(3*a +
3*b*x)*cos(a + b*x)*tan((3*a + 3*b*x)/2)**2*b*c*d**2 + 5346*cos(3*a + 3*b*
x)*cos(a + b*x)*tan((3*a + 3*b*x)/2)**2*b*d**3*x + 35937*cos(3*a + 3*b*x)*
cos(a + b*x)*tan((a + b*x)/2)**2*b**3*c**3 + 107811*cos(3*a + 3*b*x)*cos(a
 + b*x)*tan((a + b*x)/2)**2*b**3*c**2*d*x + 107811*cos(3*a + 3*b*x)*cos(a
+ b*x)*tan((a + b*x)/2)**2*b**3*c*d**2*x**2 + 35937*cos(3*a + 3*b*x)*cos(a
 + b*x)*tan((a + b*x)/2)**2*b**3*d**3*x**3 - 5346*cos(3*a + 3*b*x)*cos(a +
 b*x)*tan((a + b*x)/2)**2*b*c*d**2 - 5346*cos(3*a + 3*b*x)*cos(a + b*x)*ta
n((a + b*x)/2)**2*b*d**3*x - 35937*cos(3*a + 3*b*x)*cos(a + b*x)*b**3*c...
```

3.390 $\int (c+dx)^2 \sec^2(a+bx)\sin(3a+3bx)\,dx$

Optimal result . . . 3162
Mathematica [B] (warning: unable to verify) . . . 3163
Rubi [A] (verified) . . . 3163
Maple [B] (verified) . . . 3164
Fricas [B] (verification not implemented) . . . 3165
Sympy [F(-1)] . . . 3166
Maxima [F] . . . 3166
Giac [F] . . . 3167
Mupad [F(-1)] . . . 3168
Reduce [F] . . . 3168

Optimal result

Integrand size = 25, antiderivative size = 147

$$\begin{aligned}
&\int (c+dx)^2 \sec^2(a+bx)\sin(3a+3bx)\,dx \\
&= -\frac{4id(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^2} + \frac{8d^2\cos(a+bx)}{b^3} - \frac{4(c+dx)^2\cos(a+bx)}{b} \\
&\quad + \frac{2id^2\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} - \frac{2id^2\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} \\
&\quad - \frac{(c+dx)^2\sec(a+bx)}{b} + \frac{8d(c+dx)\sin(a+bx)}{b^2}
\end{aligned}$$

output

```
-4*I*d*(d*x+c)*arctan(exp(I*(b*x+a)))/b^2+8*d^2*cos(b*x+a)/b^3-4*(d*x+c)^2
*cos(b*x+a)/b+2*I*d^2*polylog(2,-I*exp(I*(b*x+a)))/b^3-2*I*d^2*polylog(2,I
*exp(I*(b*x+a)))/b^3-(d*x+c)^2*sec(b*x+a)/b+8*d*(d*x+c)*sin(b*x+a)/b^2
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 364 vs. $2(147) = 294$.

Time $= 2.55$ (sec) , antiderivative size $= 364$, normalized size of antiderivative $= 2.48$

$$\int (c+dx)^2 \sec^2(a+bx)\sin(3a+3bx)\,dx$$

$$= \frac{4bcd\operatorname{arctanh}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right)+2d^2\left(2\arctan(\cot(a))\operatorname{arctanh}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right)-\frac{\csc(a)}{}\right.}{}$$

input
```
Integrate[(c + d*x)^2*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output
```
(4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + 2*d^2*(2*ArcTan[Cot[a]]*A
rcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - (Csc[a]*((b*x - ArcTan[Cot[a]])*(Lo
g[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan[Cot[a]]))
]) + I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyLog[2, E^(I*(b*x
 - ArcTan[Cot[a]]))]))/Sqrt[Csc[a]^2]) - b^2*(c + d*x)^2*Sec[a] - 4*Cos[b*x
]*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] - 2*b*d*(c + d*x)*Sin[a]) + 4*(2*b*d*
(c + d*x)*Cos[a] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x] - (b^2*(c +
 d*x)^2*Sin[(b*x)/2])/((Cos[a/2] - Sin[a/2])*(Cos[(a + b*x)/2] - Sin[(a +
b*x)/2])) + (b^2*(c + d*x)^2*Sin[(b*x)/2])/((Cos[a/2] + Sin[a/2])*(Cos[(a
+ b*x)/2] + Sin[(a + b*x)/2])))/b^3
```

Rubi [A] (verified)

Time $= 0.42$ (sec) , antiderivative size $= 147$, normalized size of antiderivative $= 1.00$, number of steps used $= 2$, number of rules used $= 2$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used $= \{4931, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin(3a+3bx)\sec^2(a+bx)\,dx$$

$$\downarrow 4931$$

$$\int \left(3(c+dx)^2\sin(a+bx) - (c+dx)^2\sin(a+bx)\tan^2(a+bx)\right)dx$$

$$\downarrow 2009$$

$$-\frac{4id(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^2} + \frac{2id^2\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} - \frac{2id^2\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} + \frac{8d^2\cos(a+bx)}{b^3} + \frac{8d(c+dx)\sin(a+bx)}{b^2} - \frac{4(c+dx)^2\cos(a+bx)}{b} - \frac{(c+dx)^2\sec(a+bx)}{b}$$

input
```
Int[(c + d*x)^2*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output
```
((-4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 + (8*d^2*Cos[a + b*x])/b^
3 - (4*(c + d*x)^2*Cos[a + b*x])/b + ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a +
b*x))])/b^3 - ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((c + d*x)^2
*Sec[a + b*x])/b + (8*d*(c + d*x)*Sin[a + b*x])/b^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(136) = 272$.

Time = 3.87 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.27

method	result
default	$-\frac{4c^2\cos(bx+a)}{b}-\frac{c^2}{b\cos(bx+a)}+\frac{4d^2\left(-(bx+a)^2\cos(bx+a)+2\cos(bx+a)+2(bx+a)\sin(bx+a)-2a(\sin(bx+a)-(bx+a)\cos(bx$
risch	$-\frac{2\left(x^2d^2b^2+2b^2cdx+2ib\,d^2x+b^2c^2+2ibcd-2d^2\right)e^{i(bx+a)}}{b^3}-\frac{2\left(x^2d^2b^2+2b^2cdx-2ib\,d^2x+b^2c^2-2ibcd-2d^2\right)e^{-i(bx+a)}}{b^3}-\frac{2\,e^{i(bx$

input
```
int((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
-4*c^2/b*cos(b*x+a)-c^2/b/cos(b*x+a)+4*d^2/b^3*(-(b*x+a)^2*cos(b*x+a)+2*co
s(b*x+a)+2*(b*x+a)*sin(b*x+a)-2*a*(sin(b*x+a)-(b*x+a)*cos(b*x+a))-a^2*cos(
b*x+a))-d^2/b^3*((b*x+a)^2/cos(b*x+a)+2*(b*x+a)*ln(I*exp(I*(b*x+a))+1)-2*(
b*x+a)*ln(1-I*exp(I*(b*x+a)))-2*I*dilog(I*exp(I*(b*x+a))+1)+2*I*dilog(1-I*
exp(I*(b*x+a)))-2*a*((b*x+a)/cos(b*x+a)-ln(sec(b*x+a)+tan(b*x+a)))+a^2/cos
(b*x+a))+8*c*d/b^2*(sin(b*x+a)-(b*x+a)*cos(b*x+a)+a*cos(b*x+a))-2*c*d/b/co
s(b*x+a)*x+2*c*d/b^2*ln(sec(b*x+a)+tan(b*x+a))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(129)=258$.

Time $=0.11$ (sec) , antiderivative size $=513$, normalized size of antiderivative $=3.49$

$$\int (c+dx)^2\sec^2(a+bx)\sin(3a+3bx)\,dx=$$

$$-\frac{b^2d^2x^2+2\,b^2cdx+b^2c^2+i\,d^2\cos(bx+a)\,\mathrm{Li}_2(i\,\cos(bx+a)+\sin(bx+a))+i\,d^2\cos(bx+a)\,\mathrm{Li}_2(i\,c$$

input
```
integrate((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

output

```
-(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*cos(b*x + a)*dilog(I*cos(b*x
 + a) + sin(b*x + a)) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x
+ a)) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d^2*c
os(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 4*(b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - (b*c*d - a*d^2)*cos(b*x + a)*lo
g(cos(b*x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(co
s(b*x + a) - I*sin(b*x + a) + I) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*co
s(b*x + a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(
b*x + a) - sin(b*x + a) + 1) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b
*x + a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*
x + a) - sin(b*x + a) + 1) - (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a
) + I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) -
 I*sin(b*x + a) + I) - 8*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3
*cos(b*x + a))
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^2 \sec^2(a+bx) \sin(3a+3bx)\,dx = \text{Timed out}$$

input

```
integrate((d*x+c)**2*sec(b*x+a)**2*sin(3*b*x+3*a),x)
```

output

```
Timed out
```

Maxima [F]

$$\int (c+dx)^2 \sec^2(a+bx) \sin(3a+3bx)\,dx = \int (dx+c)^2 \sec(bx+a)^2 \sin(3\,bx+3\,a)\,dx$$

input

```
integrate((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

output

```
-2*((cos(3*b*x + 3*a) + cos(b*x + a))*cos(4*b*x + 4*a) + (3*cos(2*b*x + 2*
a) + 1)*cos(3*b*x + 3*a) + 3*cos(2*b*x + 2*a)*cos(b*x + a) + (sin(3*b*x +
3*a) + sin(b*x + a))*sin(4*b*x + 4*a) + 3*sin(3*b*x + 3*a)*sin(2*b*x + 2*a
) + 3*sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))*c^2/(b*cos(3*b*x + 3*a
)^2 + 2*b*cos(3*b*x + 3*a)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x +
 3*a)^2 + 2*b*sin(3*b*x + 3*a)*sin(b*x + a) + b*sin(b*x + a)^2) - (4*(cos(
a)^2 + sin(a)^2)*b*x*cos(b*x + a) + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a
) + b*x*cos(b*x + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*
sin(b*x + 2*a)*sin(a))*cos(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x
+ a))*cos(2*b*x + 3*a)^2 + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) + b*x*c
os(b*x + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(b*x +
 2*a)*sin(a))*sin(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))*sin
(2*b*x + 3*a)^2 + 4*((b*x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a)
 + sin(a))*cos(3*b*x + 3*a)^2 + (b*x*cos(a) + sin(a))*cos(b*x + a)^2 + (b*
x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*sin(3*b*x + 3
*a)^2 + (b*x*cos(a) + sin(a))*sin(b*x + a)^2 + 2*(b*x*cos(2*b*x + 3*a)*cos
(b*x + a) + (b*x*cos(a) + sin(a))*cos(b*x + a) + cos(b*x + a)*sin(2*b*x +
3*a))*cos(3*b*x + 3*a) + (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(2*b
*x + 3*a) + 2*(b*x*cos(2*b*x + 3*a)*sin(b*x + a) + (b*x*cos(a) + sin(a))*s
in(b*x + a) + sin(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a) + (cos(b*...
```

Giac [F]

$$\int (c+dx)^2 \sec^2(a+bx)\sin(3a+3bx)\,dx = \int (dx+c)^2 \sec(bx+a)^2 \sin(3\,bx+3\,a)\,dx$$

input

```
integrate((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

output

```
integrate((d*x + c)^2*sec(b*x + a)^2*sin(3*b*x + 3*a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \sec^2(a+bx) \sin(3a+3bx)\,dx = \text{Hanged}$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x)^2)/cos(a + b*x)^2,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (c+dx)^2 \sec^2(a+bx) \sin(3a+3bx)\,dx = \text{too large to display}$$

input
```
int((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x)
```

output

```
(9801*cos(3*a + 3*b*x)*cos(a + b*x)*tan((3*a + 3*b*x)/2)**2*tan((a + b*x)/
2)**2*b**2*c**2 + 19602*cos(3*a + 3*b*x)*cos(a + b*x)*tan((3*a + 3*b*x)/2)
**2*tan((a + b*x)/2)**2*b**2*c*d*x + 9801*cos(3*a + 3*b*x)*cos(a + b*x)*ta
n((3*a + 3*b*x)/2)**2*tan((a + b*x)/2)**2*b**2*d**2*x**2 - 486*cos(3*a + 3
*b*x)*cos(a + b*x)*tan((3*a + 3*b*x)/2)**2*tan((a + b*x)/2)**2*d**2 - 9801
*cos(3*a + 3*b*x)*cos(a + b*x)*tan((3*a + 3*b*x)/2)**2*b**2*c**2 - 19602*c
os(3*a + 3*b*x)*cos(a + b*x)*tan((3*a + 3*b*x)/2)**2*b**2*c*d*x - 9801*cos
(3*a + 3*b*x)*cos(a + b*x)*tan((3*a + 3*b*x)/2)**2*b**2*d**2*x**2 + 486*co
s(3*a + 3*b*x)*cos(a + b*x)*tan((3*a + 3*b*x)/2)**2*d**2 + 9801*cos(3*a +
3*b*x)*cos(a + b*x)*tan((a + b*x)/2)**2*b**2*c**2 + 19602*cos(3*a + 3*b*x)
*cos(a + b*x)*tan((a + b*x)/2)**2*b**2*c*d*x + 9801*cos(3*a + 3*b*x)*cos(a
 + b*x)*tan((a + b*x)/2)**2*b**2*d**2*x**2 - 486*cos(3*a + 3*b*x)*cos(a +
b*x)*tan((a + b*x)/2)**2*d**2 - 9801*cos(3*a + 3*b*x)*cos(a + b*x)*b**2*c*
*2 - 19602*cos(3*a + 3*b*x)*cos(a + b*x)*b**2*c*d*x - 9801*cos(3*a + 3*b*x
)*cos(a + b*x)*b**2*d**2*x**2 + 486*cos(3*a + 3*b*x)*cos(a + b*x)*d**2 + 7
128*cos(3*a + 3*b*x)*sin(a + b*x)*tan((3*a + 3*b*x)/2)**2*tan((a + b*x)/2)
**2*b*c*d + 7128*cos(3*a + 3*b*x)*sin(a + b*x)*tan((3*a + 3*b*x)/2)**2*tan
((a + b*x)/2)**2*b*d**2*x - 7128*cos(3*a + 3*b*x)*sin(a + b*x)*tan((3*a +
3*b*x)/2)**2*b*c*d - 7128*cos(3*a + 3*b*x)*sin(a + b*x)*tan((3*a + 3*b*x)/
2)**2*b*d**2*x + 7128*cos(3*a + 3*b*x)*sin(a + b*x)*tan((a + b*x)/2)**2...
```

3.391 $\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx$

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Optimal result

Integrand size = 23, antiderivative size = 57

$$\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx = \frac{d\operatorname{arctanh}(\sin(a+bx))}{b^2} - \frac{4(c+dx)\cos(a+bx)}{b} - \frac{(c+dx)\sec(a+bx)}{b} + \frac{4d\sin(a+bx)}{b^2}$$

output
```
d*arctanh(sin(b*x+a))/b^2-4*(d*x+c)*cos(b*x+a)/b-(d*x+c)*sec(b*x+a)/b+4*d*
sin(b*x+a)/b^2
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.84

$$\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx = -\frac{\sec(a+bx)\left(3bc+3bdx+2b(c+dx)\cos(2(a+bx))+d\cos(a+bx)\left(\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right.\right.\right.}{b^2}$$

input
```
Integrate[(c + d*x)*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output
```
-((Sec[a + b*x]*(3*b*c + 3*b*d*x + 2*b*(c + d*x)*Cos[2*(a + b*x)] + d*Cos[
a + b*x]*(Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]] - Log[Cos[(a + b*x)/2]
+ Sin[(a + b*x)/2]]) - 2*d*Sin[2*(a + b*x)]))/b^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin(3a+3bx)\sec^2(a+bx)\,dx$$

$$\downarrow 4931$$

$$\int \left(3(c+dx)\sin(a+bx) - (c+dx)\sin(a+bx)\tan^2(a+bx)\right)dx$$

$$\downarrow 2009$$

$$\frac{d\operatorname{arctanh}(\sin(a+bx))}{b^2} + \frac{4d\sin(a+bx)}{b^2} - \frac{4(c+dx)\cos(a+bx)}{b} - \frac{(c+dx)\sec(a+bx)}{b}$$

input
```
Int[(c + d*x)*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output
```
(d*ArcTanh[Sin[a + b*x]])/b^2 - (4*(c + d*x)*Cos[a + b*x])/b - ((c + d*x)*
Sec[a + b*x])/b + (4*d*Sin[a + b*x])/b^2
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.68

method	result
default	$-\frac{4c\cos(bx+a)}{b}-\frac{c}{b\cos(bx+a)}+\frac{4d(\sin(bx+a)-(bx+a)\cos(bx+a)+a\cos(bx+a))}{b^2}-\frac{dx}{b\cos(bx+a)}+\frac{d\ln(\sec(bx+a)+\tan(bx-}{b^2}$
risch	$-\frac{2(bdx+cb+id)e^{i(bx+a)}}{b^2}-\frac{2(bdx+cb-id)e^{-i(bx+a)}}{b^2}-\frac{2e^{i(bx+a)}(dx+c)}{b(e^{2i(bx+a)}+1)}-\frac{d\ln(e^{i(bx+a)}-i)}{b^2}+\frac{d\ln(e^{i(bx+a)}+i)}{b^2}$

input

```
int((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output

```
-4*c/b*cos(b*x+a)-c/b/cos(b*x+a)+4*d/b^2*(sin(b*x+a)-(b*x+a)*cos(b*x+a)+a*
cos(b*x+a))-d/b/cos(b*x+a)*x+d/b^2*ln(sec(b*x+a)+tan(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.63

$$\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx =$$

$$-\frac{2\,bdx+8\,(bdx+bc)\cos(bx+a)^2-d\cos(bx+a)\log(\sin(bx+a)+1)+d\cos(bx+a)\log(-\sin(bx}{2\,b^2\cos(bx+a)}$$

input

```
integrate((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
-1/2*(2*b*d*x + 8*(b*d*x + b*c)*cos(b*x + a)^2 - d*cos(b*x + a)*log(sin(b*
x + a) + 1) + d*cos(b*x + a)*log(-sin(b*x + a) + 1) - 8*d*cos(b*x + a)*sin
(b*x + a) + 2*b*c)/(b^2*cos(b*x + a))
```

Sympy [F(-2)]

Exception generated.

$$\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d*x+c)*sec(b*x+a)**2*sin(3*b*x+3*a),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3330 vs. $2(57) = 114$.

Time $= 0.31$ (sec) , antiderivative size $= 3330$, normalized size of antiderivative $= 58.42$

$$\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

output

```
-2*((cos(3*b*x + 3*a) + cos(b*x + a))*cos(4*b*x + 4*a) + (3*cos(2*b*x + 2*
a) + 1)*cos(3*b*x + 3*a) + 3*cos(2*b*x + 2*a)*cos(b*x + a) + (sin(3*b*x +
3*a) + sin(b*x + a))*sin(4*b*x + 4*a) + 3*sin(3*b*x + 3*a)*sin(2*b*x + 2*a
) + 3*sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))*c/(b*cos(3*b*x + 3*a)^
2 + 2*b*cos(3*b*x + 3*a)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x + 3
*a)^2 + 2*b*sin(3*b*x + 3*a)*sin(b*x + a) + b*sin(b*x + a)^2) - 1/2*(4*(co
s(a)^2 + sin(a)^2)*b*x*cos(b*x + a) + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2
*a) + b*x*cos(b*x + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*
x*sin(b*x + 2*a)*sin(a))*cos(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*
x + a))*cos(2*b*x + 3*a)^2 + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) + b*x
*cos(b*x + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(b*x
 + 2*a)*sin(a))*sin(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))*s
in(2*b*x + 3*a)^2 + 4*((b*x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*
a) + sin(a))*cos(3*b*x + 3*a)^2 + (b*x*cos(a) + sin(a))*cos(b*x + a)^2 + (
b*x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*sin(3*b*x +
 3*a)^2 + (b*x*cos(a) + sin(a))*sin(b*x + a)^2 + 2*(b*x*cos(2*b*x + 3*a)*c
os(b*x + a) + (b*x*cos(a) + sin(a))*cos(b*x + a) + cos(b*x + a)*sin(2*b*x
+ 3*a))*cos(3*b*x + 3*a) + (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(2
*b*x + 3*a) + 2*(b*x*cos(2*b*x + 3*a)*sin(b*x + a) + (b*x*cos(a) + sin(a))
*sin(b*x + a) + sin(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a) + (cos(...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(57) = 114$.

Time $= 0.21$ (sec) , antiderivative size $= 365$, normalized size of antiderivative $= 6.40$

$$\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx$$

$$= \frac{10\,bc\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^4 + 10\,(bx+a)d\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^4 - 10\,ad\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^4 + d\log\left(\frac{2\left(\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2\right.}{\tan\left(\frac{1}{2}\right.}\right.}$$

input

```
integrate((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
1/2*(10*b*c*tan(1/2*b*x + 1/2*a)^4 + 10*(b*x + a)*d*tan(1/2*b*x + 1/2*a)^4
 - 10*a*d*tan(1/2*b*x + 1/2*a)^4 + d*log(2*(tan(1/2*b*x + 1/2*a)^2 + 2*tan
(1/2*b*x + 1/2*a) + 1)/(tan(1/2*b*x + 1/2*a)^2 + 1))*tan(1/2*b*x + 1/2*a)^
4 - d*log(2*(tan(1/2*b*x + 1/2*a)^2 - 2*tan(1/2*b*x + 1/2*a) + 1)/(tan(1/2
*b*x + 1/2*a)^2 + 1))*tan(1/2*b*x + 1/2*a)^4 - 12*b*c*tan(1/2*b*x + 1/2*a)
^2 - 12*(b*x + a)*d*tan(1/2*b*x + 1/2*a)^2 + 12*a*d*tan(1/2*b*x + 1/2*a)^2
 + 16*d*tan(1/2*b*x + 1/2*a)^3 + 10*b*c + 10*(b*x + a)*d - 10*a*d - d*log(
2*(tan(1/2*b*x + 1/2*a)^2 + 2*tan(1/2*b*x + 1/2*a) + 1)/(tan(1/2*b*x + 1/2
*a)^2 + 1)) + d*log(2*(tan(1/2*b*x + 1/2*a)^2 - 2*tan(1/2*b*x + 1/2*a) + 1
)/(tan(1/2*b*x + 1/2*a)^2 + 1)) - 16*d*tan(1/2*b*x + 1/2*a))/((b*tan(1/2*b
*x + 1/2*a)^4 - b)*b)
```

Mupad [B] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.63

$$\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx = \mathrm{e}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}\left(\frac{-2\,b\,c+d\,2\mathrm{i}}{b^2}-\frac{2\,d\,x}{b}\right) - \mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\left(\frac{2\,b\,c+d\,2\mathrm{i}}{b^2}+\frac{2\,d\,x}{b}\right) - \frac{d\,\ln\left(\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}-\mathrm{i}\right)}{b^2}+\frac{d\,\ln\left(\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}+1\mathrm{i}\right)}{b^2} - \frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,(c+d\,x)\,2\mathrm{i}}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x))/cos(a + b*x)^2,x)
```

output
```
exp(- a*1i - b*x*1i)*((d*2i - 2*b*c)/b^2 - (2*d*x)/b) - exp(a*1i + b*x*1i)
*((d*2i + 2*b*c)/b^2 + (2*d*x)/b) - (d*log(exp(a*1i + b*x*1i) - 1i))/b^2 +
 (d*log(exp(a*1i + b*x*1i) + 1i))/b^2 - (exp(a*1i + b*x*1i)*(c + d*x)*2i)/
(b*(exp(a*2i + b*x*2i)*1i + 1i))
```

Reduce [F]

$$\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx = \text{Too large to display}$$

input
```
int((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x)
```

output
```
(33*cos(3*a + 3*b*x)*cos(a + b*x)*b*c + 33*cos(3*a + 3*b*x)*cos(a + b*x)*b
*d*x + 12*cos(3*a + 3*b*x)*sin(a + b*x)*d + 66*cos(3*a + 3*b*x)*b*c + 66*c
os(3*a + 3*b*x)*b*d*x + 792*cos(a + b*x)*int(tan((3*a + 3*b*x)/2)/(tan((3*
a + 3*b*x)/2)**2*tan((a + b*x)/2)**4 - 2*tan((3*a + 3*b*x)/2)**2*tan((a +
b*x)/2)**2 + tan((3*a + 3*b*x)/2)**2 + tan((a + b*x)/2)**4 - 2*tan((a + b*
x)/2)**2 + 1),x)*b**2*c - 528*cos(a + b*x)*int(tan((a + b*x)/2)/(tan((3*a
+ 3*b*x)/2)**2*tan((a + b*x)/2)**4 - 2*tan((3*a + 3*b*x)/2)**2*tan((a + b*
x)/2)**2 + tan((3*a + 3*b*x)/2)**2 + tan((a + b*x)/2)**4 - 2*tan((a + b*x)
/2)**2 + 1),x)*b**2*c + 264*cos(a + b*x)*int((tan((3*a + 3*b*x)/2)**2*tan(
(a + b*x)/2)*x)/(tan((3*a + 3*b*x)/2)**2*tan((a + b*x)/2)**4 - 2*tan((3*a
+ 3*b*x)/2)**2*tan((a + b*x)/2)**2 + tan((3*a + 3*b*x)/2)**2 + tan((a + b*
x)/2)**4 - 2*tan((a + b*x)/2)**2 + 1),x)*b**2*d + 112*cos(a + b*x)*int((ta
n((3*a + 3*b*x)/2)*tan((a + b*x)/2))/(tan((3*a + 3*b*x)/2)**2*tan((a + b*x
)/2)**4 - 2*tan((3*a + 3*b*x)/2)**2*tan((a + b*x)/2)**2 + tan((3*a + 3*b*x
)/2)**2 + tan((a + b*x)/2)**4 - 2*tan((a + b*x)/2)**2 + 1),x)*b*d + 792*co
s(a + b*x)*int((tan((3*a + 3*b*x)/2)*x)/(tan((3*a + 3*b*x)/2)**2*tan((a +
b*x)/2)**4 - 2*tan((3*a + 3*b*x)/2)**2*tan((a + b*x)/2)**2 + tan((3*a + 3*
b*x)/2)**2 + tan((a + b*x)/2)**4 - 2*tan((a + b*x)/2)**2 + 1),x)*b**2*d -
264*cos(a + b*x)*int((tan((a + b*x)/2)*x)/(tan((3*a + 3*b*x)/2)**2*tan((a
+ b*x)/2)**4 - 2*tan((3*a + 3*b*x)/2)**2*tan((a + b*x)/2)**2 + tan((3*a...
```

3.392 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

Optimal result . . . 3177
Mathematica [N/A] . . . 3177
Rubi [N/A] . . . 3178
Maple [N/A] . . . 3179
Fricas [N/A] . . . 3179
Sympy [F(-2)] . . . 3180
Maxima [N/A] . . . 3180
Giac [N/A] . . . 3181
Mupad [N/A] . . . 3182
Reduce [N/A] . . . 3182

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \frac{4\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d} + \frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d} - \text{Int}\left(\frac{\sec(a+bx)\tan(a+bx)}{c+dx},x\right)$$

output
```
4*Ci(b*c/d+b*x)*sin(a-b*c/d)/d+4*cos(a-b*c/d)*Si(b*c/d+b*x)/d-Defer(Int)(s
ec(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 9.64 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

input
```
Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
```

output
```
Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\sec^2(a+bx)}{c+dx}\,dx$$

$$\downarrow 4931$$

$$\int \left(\frac{3\sin(a+bx)}{c+dx} - \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\right)dx$$

$$\downarrow 2009$$

$$-\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}dx + \frac{4\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d} + \frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

input
```
Int[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sec(bx+a)^2 \sin(3bx+3a)}{dx+c} dx$$

input
```
int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

output
```
int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)^2 \sin(3\,bx+3\,a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(sec(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 1275, normalized size of antiderivative = 51.00

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sec\left(bx+a\right)^2 \sin\left(3\,bx+3\,a\right)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

output

```
2*(b*c*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(1, -(
I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_integral_e(1, (I*b*d*x
 + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d
) + (b*c*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(1,
-(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_integral_e(1, (I*b*d
*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)
/d) + (b*d*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(1
, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integral_e(1, (I*b
*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*
d)/d))*x)*cos(2*b*x + 2*a)^2 + (b*c*(-I*exp_integral_e(1, (I*b*d*x + I*b*c
)/d) + I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*
c*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x +
I*b*c)/d))*sin(-(b*c - a*d)/d) + (b*d*(-I*exp_integral_e(1, (I*b*d*x + I*b
*c)/d) + I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) -
b*d*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x
+ I*b*c)/d))*sin(-(b*c - a*d)/d))*x)*sin(2*b*x + 2*a)^2 - d*sin(2*b*x + 2*
a)*sin(b*x + a) + (b*d*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*exp_
integral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integr
al_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*si
n(-(b*c - a*d)/d))*x + (2*b*c*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d...
```

Giac [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sec\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)}{dx+c}\,dx$$

input

```
integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
```

output

```
integrate(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 22.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sin(3\,a+3\,b\,x)}{\cos(a+b\,x)^2\,(c+d\,x)}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)),x)
```

output
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)^2\sin(3bx+3a)}{dx+c}dx$$

input
```
int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

output
```
int((sec(a + b*x)**2*sin(3*a + 3*b*x))/(c + d*x),x)
```

3.393 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$

Optimal result . . . 3183
Mathematica [N/A] . . . 3183
Rubi [N/A] . . . 3184
Maple [N/A] . . . 3185
Fricas [N/A] . . . 3185
Sympy [F(-2)] . . . 3186
Maxima [N/A] . . . 3186
Giac [N/A] . . . 3187
Mupad [N/A] . . . 3188
Reduce [N/A] . . . 3188

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \frac{4b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2} - \frac{4\sin(a+bx)}{d(c+dx)} - \frac{4b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2} - \text{Int}\left(\frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2},x\right)$$

output

```
4*b*cos(a-b*c/d)*Ci(b*c/d+b*x)/d^2-4*sin(b*x+a)/d/(d*x+c)-4*b*sin(a-b*c/d)
*Si(b*c/d+b*x)/d^2-Defer(Int)(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 15.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

input

```
Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

$$\downarrow 4931$$

$$\int \left(\frac{3\sin(a+bx)}{(c+dx)^2} - \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\right)dx$$

$$\downarrow 2009$$

$$-\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}dx + \frac{4b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2} - \frac{4b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2} - \frac{4\sin(a+bx)}{d(c+dx)}$$

input
```
Int[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sec(bx+a)^2 \sin(3bx+3a)}{(dx+c)^2} dx$$

input
```
int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

output
```
int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2} dx = \int \frac{\sec(bx+a)^2 \sin(3\,bx+3\,a)}{(dx+c)^2} dx$$

input
```
integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(sec(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 1419, normalized size of antiderivative = 56.76

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\sec\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
```

output

```
2*(b*c*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(2, -(
I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_integral_e(2, (I*b*d*x
 + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d
) + (b*c*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(2,
-(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_integral_e(2, (I*b*d
*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)
/d) + (b*d*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(2
, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integral_e(2, (I*b
*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*
d)/d))*x)*cos(2*b*x + 2*a)^2 + (b*c*(-I*exp_integral_e(2, (I*b*d*x + I*b*c
)/d) + I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*
c*(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x +
I*b*c)/d))*sin(-(b*c - a*d)/d) + (b*d*(-I*exp_integral_e(2, (I*b*d*x + I*b
*c)/d) + I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) -
b*d*(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x
+ I*b*c)/d))*sin(-(b*c - a*d)/d))*x)*sin(2*b*x + 2*a)^2 - d*sin(2*b*x + 2*
a)*sin(b*x + a) + (b*d*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*exp_
integral_e(2, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integr
al_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*si
n(-(b*c - a*d)/d))*x + (2*b*c*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d...
```

Giac [N/A]

Not integrable

Time = 3.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\sec\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)}{\left(dx+c\right)^2}\,dx$$

input

```
integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

output

```
integrate(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 24.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\sin(3a+3bx)}{\cos(a+bx)^2(c+dx)^2}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)^2),x)
```

output
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\sec(bx+a)^2\sin(3bx+3a)}{d^2x^2+2cdx+c^2}dx$$

input
```
int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

output
```
int((sec(a + b*x)**2*sin(3*a + 3*b*x))/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.394 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

Optimal result . . . 3189
Mathematica [N/A] . . . 3190
Rubi [N/A] . . . 3190
Maple [N/A] . . . 3191
Fricas [N/A] . . . 3191
Sympy [F(-1)] . . . 3192
Maxima [N/A] . . . 3192
Giac [N/A] . . . 3193
Mupad [N/A] . . . 3194
Reduce [N/A] . . . 3194

Optimal result

Integrand size = 25, antiderivative size = 25

$$\begin{aligned}\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = &-\frac{2b\cos(a+bx)}{d^2(c+dx)}\\ &-\frac{2b^2\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^3}\\ &-\frac{2\sin(a+bx)}{d(c+dx)^2}-\frac{2b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^3}\\ &-\text{Int}\left(\frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^3},x\right)\end{aligned}$$

output

```
-2*b*cos(b*x+a)/d^2/(d*x+c)-2*b^2*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^3-2*sin(b*x
+a)/d/(d*x+c)^2-2*b^2*cos(a-b*c/d)*Si(b*c/d+b*x)/d^3-Defer(Int)(sec(b*x+a)
*tan(b*x+a)/(d*x+c)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 16.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

input
```
Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

output
```
Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\sec^2(a+bx)}{(c+dx)^3}\,dx$$

$$\downarrow 4931$$

$$\int \left(\frac{3\sin(a+bx)}{(c+dx)^3} - \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^3}\right)dx$$

$$\downarrow 2009$$

$$-\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^3}dx - \frac{2b^2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^3} - \frac{2b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^3} - \frac{2b\cos(a+bx)}{d^2(c+dx)} - \frac{2\sin(a+bx)}{d(c+dx)^2}$$

input
```
Int[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sec(bx+a)^2 \sin(3bx+3a)}{(dx+c)^3} dx$$

input
```
int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

output
```
int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3} dx = \int \frac{\sec(bx+a)^2 \sin(3\,bx+3\,a)}{(dx+c)^3} dx$$

input
```
integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*
x + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 1563, normalized size of antiderivative = 62.52

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\sec\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)}{\left(dx+c\right)^3}\,dx$$

input
```
integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
```

output

```
2*(b*c*(-I*exp_integral_e(3, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(3, -(
I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_integral_e(3, (I*b*d*x
 + I*b*c)/d) + exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d
) + (b*c*(-I*exp_integral_e(3, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(3,
-(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_integral_e(3, (I*b*d
*x + I*b*c)/d) + exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)
/d) + (b*d*(-I*exp_integral_e(3, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(3
, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integral_e(3, (I*b
*d*x + I*b*c)/d) + exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*
d)/d))*x)*cos(2*b*x + 2*a)^2 + (b*c*(-I*exp_integral_e(3, (I*b*d*x + I*b*c
)/d) + I*exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*
c*(exp_integral_e(3, (I*b*d*x + I*b*c)/d) + exp_integral_e(3, -(I*b*d*x +
I*b*c)/d))*sin(-(b*c - a*d)/d) + (b*d*(-I*exp_integral_e(3, (I*b*d*x + I*b
*c)/d) + I*exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) -
b*d*(exp_integral_e(3, (I*b*d*x + I*b*c)/d) + exp_integral_e(3, -(I*b*d*x
+ I*b*c)/d))*sin(-(b*c - a*d)/d))*x)*sin(2*b*x + 2*a)^2 - d*sin(2*b*x + 2*
a)*sin(b*x + a) + (b*d*(-I*exp_integral_e(3, (I*b*d*x + I*b*c)/d) + I*exp_
integral_e(3, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integr
al_e(3, (I*b*d*x + I*b*c)/d) + exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*si
n(-(b*c - a*d)/d))*x + (2*b*c*(-I*exp_integral_e(3, (I*b*d*x + I*b*c)/d...
```

Giac [N/A]

Not integrable

Time = 9.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\sec\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)}{\left(dx+c\right)^3}\,dx$$

input

```
integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

output

```
integrate(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^3, x)
```

Mupad [N/A]

Not integrable

Time = 25.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\sin(3a+3bx)}{\cos(a+bx)^2(c+dx)^3}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)^3),x)
```

output
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\sec(bx+a)^2\sin(3bx+3a)}{d^3x^3+3cd^2x^2+3c^2dx+c^3}dx$$

input
```
int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

output
```
int((sec(a + b*x)**2*sin(3*a + 3*b*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
+ d**3*x**3),x)
```

3.395 $\int x\cos(2x)\sec(x)\,dx$

Optimal result . . . 3195
Mathematica [A] (verified) . . . 3195
Rubi [A] (verified) . . . 3196
Maple [A] (verified) . . . 3197
Fricas [B] (verification not implemented) . . . 3197
Sympy [F] . . . 3198
Maxima [F] . . . 3198
Giac [F] . . . 3198
Mupad [B] (verification not implemented) . . . 3199
Reduce [F] . . . 3199

Optimal result

Integrand size = 8, antiderivative size = 57

$$\int x\cos(2x)\sec(x)\,dx = 2ix\arctan\left(e^{ix}\right) + 2\cos(x) - i\,\mathrm{PolyLog}\left(2,-ie^{ix}\right) + i\,\mathrm{PolyLog}\left(2,ie^{ix}\right) + 2x\sin(x)$$

output
```
2*I*x*arctan(exp(I*x))+2*cos(x)-I*polylog(2,-I*exp(I*x))+I*polylog(2,I*exp
(I*x))+2*x*sin(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.35

$$\int x\cos(2x)\sec(x)\,dx = 2\cos(x) - x\left(\log\left(1-ie^{ix}\right) - \log\left(1+ie^{ix}\right)\right) - i\left(\mathrm{PolyLog}\left(2,-ie^{ix}\right) - \mathrm{PolyLog}\left(2,ie^{ix}\right)\right) + 2x\sin(x)$$

input
```
Integrate[x*Cos[2*x]*Sec[x],x]
```

output
```
2*Cos[x] - x*(Log[1 - I*E^(I*x)] - Log[1 + I*E^(I*x)]) - I*(PolyLog[2, (-I
)*E^(I*x)] - PolyLog[2, I*E^(I*x)]) + 2*x*Sin[x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos(2x)\sec(x)\,dx$$

$$\downarrow 4931$$

$$\int (x\cos(x) - x\sin(x)\tan(x))dx$$

$$\downarrow 2009$$

$$2ix\arctan\left(e^{ix}\right) - i\,\text{PolyLog}\left(2, -ie^{ix}\right) + i\,\text{PolyLog}\left(2, ie^{ix}\right) + 2x\sin(x) + 2\cos(x)$$

input
```
Int[x*Cos[2*x]*Sec[x],x]
```

output
```
(2*I)*x*ArcTan[E^(I*x)] + 2*Cos[x] - I*PolyLog[2, (-I)*E^(I*x)] + I*PolyLo
g[2, I*E^(I*x)] + 2*x*Sin[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16

method	result
default	$x \ln(1+ie^{ix}) - x \ln(1-ie^{ix}) - i\,\mathrm{dilog}(1+ie^{ix}) + i\,\mathrm{dilog}(1-ie^{ix}) + 2\cos(x) + 2x\sin(x)$
risch	$-i(x+i)\,e^{ix} + i(x-i)\,e^{-ix} + x \ln(1+ie^{ix}) - x \ln(1-ie^{ix}) - i\,\mathrm{dilog}(1+ie^{ix}) + i\,\mathrm{dilog}(1-$

input
```
int(x*cos(2*x)*sec(x),x,method=_RETURNVERBOSE)
```

output
```
x*ln(1+I*exp(I*x))-x*ln(1-I*exp(I*x))-I*dilog(1+I*exp(I*x))+I*dilog(1-I*ex
p(I*x))+2*cos(x)+2*x*sin(x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(36) = 72$.

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.86

$$\begin{aligned}\int x\cos(2x)\sec(x)\,dx = &-\frac{1}{2}x\log(i\,\cos(x)+\sin(x)+1)+\frac{1}{2}x\log(i\,\cos(x)-\sin(x)+1)\\ &-\frac{1}{2}x\log(-i\,\cos(x)+\sin(x)+1)\\ &+\frac{1}{2}x\log(-i\,\cos(x)-\sin(x)+1)+2\,x\sin(x)+2\,\cos(x)\\ &+\frac{1}{2}i\,\mathrm{Li}_2(i\,\cos(x)+\sin(x))+\frac{1}{2}i\,\mathrm{Li}_2(i\,\cos(x)-\sin(x))\\ &-\frac{1}{2}i\,\mathrm{Li}_2(-i\,\cos(x)+\sin(x))-\frac{1}{2}i\,\mathrm{Li}_2(-i\,\cos(x)-\sin(x))\end{aligned}$$

input
```
integrate(x*cos(2*x)*sec(x),x, algorithm="fricas")
```

output
```
-1/2*x*log(I*cos(x) + sin(x) + 1) + 1/2*x*log(I*cos(x) - sin(x) + 1) - 1/2
*x*log(-I*cos(x) + sin(x) + 1) + 1/2*x*log(-I*cos(x) - sin(x) + 1) + 2*x*s
in(x) + 2*cos(x) + 1/2*I*dilog(I*cos(x) + sin(x)) + 1/2*I*dilog(I*cos(x) -
 sin(x)) - 1/2*I*dilog(-I*cos(x) + sin(x)) - 1/2*I*dilog(-I*cos(x) - sin(x
))
```

Sympy [F]

$$\int x \cos(2x) \sec(x)\, dx = \int x \cos(2x) \sec(x)\, dx$$

input
```
integrate(x*cos(2*x)*sec(x),x)
```

output
```
Integral(x*cos(2*x)*sec(x), x)
```

Maxima [F]

$$\int x \cos(2x) \sec(x)\, dx = \int x \cos(2\,x) \sec(x)\ dx$$

input
```
integrate(x*cos(2*x)*sec(x),x, algorithm="maxima")
```

output
```
2*x*sin(x) + 2*cos(x) - 2*integrate((x*cos(2*x)*cos(x) + x*sin(2*x)*sin(x)
 + x*cos(x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1), x)
```

Giac [F]

$$\int x \cos(2x) \sec(x)\, dx = \int x \cos(2\,x) \sec(x)\ dx$$

input
```
integrate(x*cos(2*x)*sec(x),x, algorithm="giac")
```

output
```
integrate(x*cos(2*x)*sec(x), x)
```

Mupad [B] (verification not implemented)

Time = 18.95 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int x\cos(2x)\sec(x)\,dx = 2\cos(x) + 2x\sin(x) - \text{polylog}\left(2, -e^{x\,1i}\,1i\right)\,1i + \text{polylog}\left(2, e^{x\,1i}\,1i\right)\,1i + x\,\text{atan}\left(e^{x\,1i}\right)\,2i$$

input
```
int((x*cos(2*x))/cos(x),x)
```

output
```
2*cos(x) - polylog(2, -exp(x*1i)*1i)*1i + polylog(2, exp(x*1i)*1i)*1i + x*
atan(exp(x*1i))*2i + 2*x*sin(x)
```

Reduce [F]

$$\int x\cos(2x)\sec(x)\,dx$$
$$= \frac{\cos(2x)\tan(x)^2 + \cos(2x) + 4\left(\int \frac{\cos(2x)x}{\cos(x)}dx\right)\tan(x)^2 + 4\left(\int \frac{\cos(2x)x}{\cos(x)}dx\right) + 2\sin(2x)\tan(x)^2 x + 2\sin(}{4\tan(x)^2 + 4}$$

input
```
int(x*cos(2*x)*sec(x),x)
```

output
```
(cos(2*x)*tan(x)**2 + cos(2*x) + 4*int((cos(2*x)*x)/cos(x),x)*tan(x)**2 +
4*int((cos(2*x)*x)/cos(x),x) + 2*sin(2*x)*tan(x)**2*x + 2*sin(2*x)*x + 2*t
an(x)**2 - 4*tan(x)*x)/(4*(tan(x)**2 + 1))
```

3.396 $\int x \cos(2x) \sec^2(x)\, dx$

Optimal result . . . 3200
Mathematica [A] (verified) . . . 3200
Rubi [A] (verified) . . . 3201
Maple [A] (verified) . . . 3202
Fricas [A] (verification not implemented) . . . 3202
Sympy [B] (verification not implemented) . . . 3203
Maxima [B] (verification not implemented) . . . 3203
Giac [B] (verification not implemented) . . . 3204
Mupad [B] (verification not implemented) . . . 3204
Reduce [F] . . . 3205

Optimal result

Integrand size = 10, antiderivative size = 14

$$\int x \cos(2x) \sec^2(x)\, dx = x^2 - \log(\cos(x)) - x \tan(x)$$

output
```
x^2-ln(cos(x))-x*tan(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int x \cos(2x) \sec^2(x)\, dx = x^2 - \log(\cos(x)) - x \tan(x)$$

input
```
Integrate[x*Cos[2*x]*Sec[x]^2,x]
```

output
```
x^2 - Log[Cos[x]] - x*Tan[x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos(2x)\sec^2(x)\,dx$$

$$\downarrow 4931$$

$$\int (x - x\tan^2(x))\,dx$$

$$\downarrow 2009$$

$$x^2 - x\tan(x) - \log(\cos(x))$$

input
```
Int[x*Cos[2*x]*Sec[x]^2,x]
```

output
```
x^2 - Log[Cos[x]] - x*Tan[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$x^2 - \ln(\cos(x)) - x\tan(x)$	15
risch	$x^2 + 2ix - \frac{2ix}{e^{2ix}+1} - \ln(e^{2ix}+1)$	32

input
```
int(x*cos(2*x)*sec(x)^2,x,method=_RETURNVERBOSE)
```

output
```
x^2-ln(cos(x))-x*tan(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int x\cos(2x)\sec^2(x)\,dx = \frac{x^2\cos(x) - \cos(x)\log(-\cos(x)) - x\sin(x)}{\cos(x)}$$

input
```
integrate(x*cos(2*x)*sec(x)^2,x, algorithm="fricas")
```

output
```
(x^2*cos(x) - cos(x)*log(-cos(x)) - x*sin(x))/cos(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(12) = 24$.

Time = 3.37 (sec) , antiderivative size = 144, normalized size of antiderivative = 10.29

$$\begin{aligned}\int x\cos(2x)\sec^2(x)\,dx = x^2 &+ \frac{2x\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1} - \frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1} \\ &+ \frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{\tan^2\left(\frac{x}{2}\right)-1} - \frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1} \\ &+ \frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{\tan^2\left(\frac{x}{2}\right)-1} + \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1} \\ &- \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{\tan^2\left(\frac{x}{2}\right)-1}\end{aligned}$$

input
```
integrate(x*cos(2*x)*sec(x)**2,x)
```

output
```
x**2 + 2*x*tan(x/2)/(tan(x/2)**2 - 1) - log(tan(x/2) - 1)*tan(x/2)**2/(tan
(x/2)**2 - 1) + log(tan(x/2) - 1)/(tan(x/2)**2 - 1) - log(tan(x/2) + 1)*ta
n(x/2)**2/(tan(x/2)**2 - 1) + log(tan(x/2) + 1)/(tan(x/2)**2 - 1) + log(ta
n(x/2)**2 + 1)*tan(x/2)**2/(tan(x/2)**2 - 1) - log(tan(x/2)**2 + 1)/(tan(x
/2)**2 - 1)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(14) = 28$.

Time = 0.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 7.93

$$\int x\cos(2x)\sec^2(x)\,dx$$
$$= \frac{2\,x^2\cos\left(2\,x\right)^2 + 2\,x^2\sin\left(2\,x\right)^2 + 4\,x^2\cos\left(2\,x\right) + 2\,x^2 - \left(\cos\left(2\,x\right)^2 + \sin\left(2\,x\right)^2 + 2\,\cos\left(2\,x\right) + 1\right)\log\left(\cos\right.}{2\left(\cos\left(2\,x\right)^2 + \sin\left(2\,x\right)^2 + 2\,\cos\left(2\,x\right) + 1\right)}$$

input
```
integrate(x*cos(2*x)*sec(x)^2,x, algorithm="maxima")
```

output
```
1/2*(2*x^2*cos(2*x)^2 + 2*x^2*sin(2*x)^2 + 4*x^2*cos(2*x) + 2*x^2 - (cos(2
*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2
*x) + 1) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(14) = 28$.

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 8.43

$$\int x\cos(2x)\sec^2(x)\,dx$$
$$= \frac{2\,x^2\tan\left(\frac{1}{2}\,x\right)^2 - \log\left(\frac{4\left(\tan\left(\frac{1}{2}\,x\right)^4 - 2\tan\left(\frac{1}{2}\,x\right)^2 + 1\right)}{\tan\left(\frac{1}{2}\,x\right)^4 + 2\tan\left(\frac{1}{2}\,x\right)^2 + 1}\right)\tan\left(\frac{1}{2}\,x\right)^2 - 2\,x^2 + 4\,x\tan\left(\frac{1}{2}\,x\right) + \log\left(\frac{4\left(\tan\left(\frac{1}{2}\,x\right)^4 - 2\tan\right.\right.}{2\left(\tan\left(\frac{1}{2}\,x\right)^2 - 1\right)}$$

input
```
integrate(x*cos(2*x)*sec(x)^2,x, algorithm="giac")
```

output
```
1/2*(2*x^2*tan(1/2*x)^2 - log(4*(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)/(tan(1
/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - 2*x^2 + 4*x*tan(1/2*x) + log
(4*(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)
))/(tan(1/2*x)^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int x\cos(2x)\sec^2(x)\,dx = x^2 - \ln\left(\mathrm{e}^{x\,2\mathrm{i}} + 1\right) + x\,2\mathrm{i} - \frac{x\,2\mathrm{i}}{\mathrm{e}^{x\,2\mathrm{i}} + 1}$$

input
```
int((x*cos(2*x))/cos(x)^2,x)
```

output
```
x*2i - log(exp(x*2i) + 1) - (x*2i)/(exp(x*2i) + 1) + x^2
```

Reduce [F]

$$\int x\cos(2x)\sec^2(x)\,dx$$

$$= \frac{\cos(2x)\cos(x) - 4\cos(x)\left(\int \frac{\cos(2x)\sin(x)^2 x}{\sin(x)^2-1}dx\right) - 4\cos(x)\left(\int \frac{\sin(x)^2 x}{\sin(x)^2-1}dx\right) + 4\cos(x)\log\left(\tan\left(\frac{x}{2}\right)^2+1\right)}$$

input
```
int(x*cos(2*x)*sec(x)^2,x)
```

output
```
(cos(2*x)*cos(x) - 4*cos(x)*int((cos(2*x)*sin(x)**2*x)/(sin(x)**2 - 1),x)
- 4*cos(x)*int((sin(x)**2*x)/(sin(x)**2 - 1),x) + 4*cos(x)*log(tan(x/2)**2
 + 1) - 4*cos(x)*log(tan(x/2) - 1) - 4*cos(x)*log(tan(x/2) + 1) + 2*cos(x)
*sin(2*x)*x + 2*cos(x)*x**2 + 2*cos(x) - 4*sin(x)*x)/(4*cos(x))
```

3.397 $\int x\cos(2x)\sec^3(x)\,dx$

Optimal result . . . 3206
Mathematica [B] (verified) . . . 3206
Rubi [A] (verified) . . . 3207
Maple [B] (verified) . . . 3208
Fricas [B] (verification not implemented) . . . 3208
Sympy [F] . . . 3209
Maxima [F] . . . 3209
Giac [F] . . . 3210
Mupad [B] (verification not implemented) . . . 3210
Reduce [F] . . . 3210

Optimal result

Integrand size = 10, antiderivative size = 67

$$\int x\cos(2x)\sec^3(x)\,dx = -3ix\arctan\left(e^{ix}\right) + \frac{3}{2}i\,\mathrm{PolyLog}\left(2,-ie^{ix}\right) - \frac{3}{2}i\,\mathrm{PolyLog}\left(2,ie^{ix}\right) + \frac{\sec(x)}{2} - \frac{1}{2}x\sec(x)\tan(x)$$

output

```
-3*I*x*arctan(exp(I*x))+3/2*I*polylog(2,-I*exp(I*x))-3/2*I*polylog(2,I*exp
(I*x))+1/2*sec(x)-1/2*x*sec(x)*tan(x)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 146 vs. 2(67) = 134.

Time = 0.21 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.18

$$\int x\cos(2x)\sec^3(x)\,dx = \frac{1}{4}\left(6x\log\left(1-ie^{ix}\right) - 6x\log\left(1+ie^{ix}\right) + 6i\,\mathrm{PolyLog}\left(2,-ie^{ix}\right) - 6i\,\mathrm{PolyLog}\left(2,ie^{ix}\right) + \frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)} + \frac{x}{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2} - \frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)} + \frac{x}{-1+\sin(x)}\right)$$

input
```
Integrate[x*Cos[2*x]*Sec[x]^3,x]
```

output
```
(6*x*Log[1 - I*E^(I*x)] - 6*x*Log[1 + I*E^(I*x)] + (6*I)*PolyLog[2, (-I)*E
^(I*x)] - (6*I)*PolyLog[2, I*E^(I*x)] + (2*Sin[x/2])/(Cos[x/2] - Sin[x/2])
 + x/(Cos[x/2] + Sin[x/2])^2 - (2*Sin[x/2])/(Cos[x/2] + Sin[x/2]) + x/(-1
+ Sin[x]))/4
```

Rubi [A] (verified)

Time $= 0.31$ (sec) , antiderivative size $= 67$, normalized size of antiderivative $= 1.00$, number of steps used $= 2$, number of rules used $= 2$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used $= \{4931, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cos(2x) \sec^3(x)\, dx$$

$$\downarrow 4931$$

$$\int (x \sec(x) - x \tan^2(x) \sec(x))\, dx$$

$$\downarrow 2009$$

$$-3ix \arctan\left(e^{ix}\right) + \frac{3}{2} i \operatorname{PolyLog}\left(2, -ie^{ix}\right) - \frac{3}{2} i \operatorname{PolyLog}\left(2, ie^{ix}\right) + \frac{\sec(x)}{2} - \frac{1}{2} x \tan(x) \sec(x)$$

input
```
Int[x*Cos[2*x]*Sec[x]^3,x]
```

output
```
(-3*I)*x*ArcTan[E^(I*x)] + ((3*I)/2)*PolyLog[2, (-I)*E^(I*x)] - ((3*I)/2)*
PolyLog[2, I*E^(I*x)] + Sec[x]/2 - (x*Sec[x]*Tan[x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(48) = 96$.

Time = 5.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.52

method	result	size
risch	$\frac{ie^{ix}\left(e^{2ix}x-x-ie^{2ix}-i\right)}{\left(e^{2ix}+1\right)^2}-\frac{3x\ln\left(1+ie^{ix}\right)}{2}+\frac{3x\ln\left(1-ie^{ix}\right)}{2}+\frac{3i\,\mathrm{dilog}\left(1+ie^{ix}\right)}{2}-\frac{3i\,\mathrm{dilog}\left(1-ie^{ix}\right)}{2}$	102

input
```
int(x*cos(2*x)*sec(x)^3,x,method=_RETURNVERBOSE)
```

output
```
I*exp(I*x)*(exp(I*x)^2*x-x-I*exp(I*x)^2-I)/(exp(I*x)^2+1)^2-3/2*x*ln(1+I*e
xp(I*x))+3/2*x*ln(1-I*exp(I*x))+3/2*I*dilog(1+I*exp(I*x))-3/2*I*dilog(1-I*
exp(I*x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(38) = 76$.

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.15

$$\int x\cos(2x)\sec^3(x)\,dx$$

$$= \frac{3\,x\cos(x)^2\log(i\,\cos(x)+\sin(x)+1)-3\,x\cos(x)^2\log(i\,\cos(x)-\sin(x)+1)+3\,x\cos(x)^2\log(-i\,c}{}$$

input
```
integrate(x*cos(2*x)*sec(x)^3,x, algorithm="fricas")
```

output
```
1/4*(3*x*cos(x)^2*log(I*cos(x) + sin(x) + 1) - 3*x*cos(x)^2*log(I*cos(x) -
 sin(x) + 1) + 3*x*cos(x)^2*log(-I*cos(x) + sin(x) + 1) - 3*x*cos(x)^2*log
(-I*cos(x) - sin(x) + 1) - 3*I*cos(x)^2*dilog(I*cos(x) + sin(x)) - 3*I*cos
(x)^2*dilog(I*cos(x) - sin(x)) + 3*I*cos(x)^2*dilog(-I*cos(x) + sin(x)) +
3*I*cos(x)^2*dilog(-I*cos(x) - sin(x)) - 2*x*sin(x) + 2*cos(x))/cos(x)^2
```

Sympy [F]

$$\int x \cos(2x) \sec^3(x)\, dx = \int x \cos\left(2x\right) \sec^{3}\left(x\right)\, dx$$

input
```
integrate(x*cos(2*x)*sec(x)**3,x)
```

output
```
Integral(x*cos(2*x)*sec(x)**3, x)
```

Maxima [F]

$$\int x \cos(2x) \sec^3(x)\, dx = \int x \cos\left(2\,x\right) \sec\left(x\right)^{3}\, dx$$

input
```
integrate(x*cos(2*x)*sec(x)^3,x, algorithm="maxima")
```

output
```
-((x*sin(3*x) - x*sin(x) - cos(3*x) - cos(x))*cos(4*x) - (2*x*sin(2*x) + 2
*cos(2*x) + 1)*cos(3*x) - 2*(x*sin(x) + cos(x))*cos(2*x) - 3*(2*(2*cos(2*x
) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(
2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)*integrate((x*cos(2*x)*cos(x) + x*sin
(2*x)*sin(x) + x*cos(x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1), x) -
(x*cos(3*x) - x*cos(x) + sin(3*x) + sin(x))*sin(4*x) + (2*x*cos(2*x) + x -
 2*sin(2*x))*sin(3*x) + 2*(x*cos(x) - sin(x))*sin(2*x) - x*sin(x) - cos(x)
)/(2*(2*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 +
4*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)
```

Giac [F]

$$\int x\cos(2x)\sec^3(x)\,dx = \int x\cos\left(2\,x\right)\sec\left(x\right)^3\,dx$$

input
```
integrate(x*cos(2*x)*sec(x)^3,x, algorithm="giac")
```

output
```
integrate(x*cos(2*x)*sec(x)^3, x)
```

Mupad [B] (verification not implemented)

Time = 19.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\begin{aligned}\int x\cos(2x)\sec^3(x)\,dx = {} & \frac{1}{2\cos\left(x\right)} + x\,\mathrm{atanh}\left(\mathrm{e}^{x\,1\mathrm{i}}\,1\mathrm{i}\right) \\ & - \frac{x\,\sin\left(x\right)}{2\cos\left(x\right)^2} + \frac{\mathrm{polylog}\left(2,-\mathrm{e}^{x\,1\mathrm{i}}\,1\mathrm{i}\right)\,3\mathrm{i}}{2} \\ & - \frac{\mathrm{polylog}\left(2,\mathrm{e}^{x\,1\mathrm{i}}\,1\mathrm{i}\right)\,3\mathrm{i}}{2} - x\,\mathrm{atan}\left(\mathrm{e}^{x\,1\mathrm{i}}\right)\,4\mathrm{i}\end{aligned}$$

input
```
int((x*cos(2*x))/cos(x)^3,x)
```

output
```
(polylog(2, -exp(x*1i)*1i)*3i)/2 - (polylog(2, exp(x*1i)*1i)*3i)/2 + 1/(2*
cos(x)) - x*atan(exp(x*1i))*4i + x*atanh(exp(x*1i)*1i) - (x*sin(x))/(2*cos
(x)^2)
```

Reduce [F]

$$\int x\cos(2x)\sec^3(x)\,dx = \text{Too large to display}$$

input
```
int(x*cos(2*x)*sec(x)^3,x)
```

output

```
( - 12*cos(2*x)*cos(x) - 9*cos(2*x)*sin(x)**2 - 12*cos(2*x)*sin(x)*x + 5*c
os(2*x) - 8*cos(x)*sin(2*x)*sin(x) - 24*cos(x)*sin(2*x)*x - 60*cos(x) - 64
*int(tan(x/2)**3/(tan(x/2)**6*tan(x)**2 + tan(x/2)**6 - 3*tan(x/2)**4*tan(
x)**2 - 3*tan(x/2)**4 + 3*tan(x/2)**2*tan(x)**2 + 3*tan(x/2)**2 - tan(x)**
2 - 1),x)*sin(x)**2 + 64*int(tan(x/2)**3/(tan(x/2)**6*tan(x)**2 + tan(x/2)
**6 - 3*tan(x/2)**4*tan(x)**2 - 3*tan(x/2)**4 + 3*tan(x/2)**2*tan(x)**2 +
3*tan(x/2)**2 - tan(x)**2 - 1),x) + 192*int(tan(x/2)/(tan(x/2)**6*tan(x)**
2 + tan(x/2)**6 - 3*tan(x/2)**4*tan(x)**2 - 3*tan(x/2)**4 + 3*tan(x/2)**2*
tan(x)**2 + 3*tan(x/2)**2 - tan(x)**2 - 1),x)*sin(x)**2 - 192*int(tan(x/2)
/(tan(x/2)**6*tan(x)**2 + tan(x/2)**6 - 3*tan(x/2)**4*tan(x)**2 - 3*tan(x/
2)**4 + 3*tan(x/2)**2*tan(x)**2 + 3*tan(x/2)**2 - tan(x)**2 - 1),x) - 720*
int((tan(x/2)**4*x)/(tan(x/2)**6*tan(x)**2 + tan(x/2)**6 - 3*tan(x/2)**4*t
an(x)**2 - 3*tan(x/2)**4 + 3*tan(x/2)**2*tan(x)**2 + 3*tan(x/2)**2 - tan(x
)**2 - 1),x)*sin(x)**2 + 720*int((tan(x/2)**4*x)/(tan(x/2)**6*tan(x)**2 +
tan(x/2)**6 - 3*tan(x/2)**4*tan(x)**2 - 3*tan(x/2)**4 + 3*tan(x/2)**2*tan(
x)**2 + 3*tan(x/2)**2 - tan(x)**2 - 1),x) - 288*int((tan(x/2)**2*x)/(tan(x
/2)**6*tan(x)**2 + tan(x/2)**6 - 3*tan(x/2)**4*tan(x)**2 - 3*tan(x/2)**4 +
 3*tan(x/2)**2*tan(x)**2 + 3*tan(x/2)**2 - tan(x)**2 - 1),x)*sin(x)**2 + 2
88*int((tan(x/2)**2*x)/(tan(x/2)**6*tan(x)**2 + tan(x/2)**6 - 3*tan(x/2)**
4*tan(x)**2 - 3*tan(x/2)**4 + 3*tan(x/2)**2*tan(x)**2 + 3*tan(x/2)**2 -...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions 3212
4.2 Links to plain text integration problems used in this report for each CAS . 3230

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                    convert(ExpnType_result,string)," vs. order ",
                    convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
         'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
         'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
         'elliptic_pi','exp_integral_e','log_integral',
         'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
         'weierstrassPPrime','weierstrassSigma']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if  not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
          return 1
        else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
      return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
   if result.has(I):
      if optimal.has(I): #both result and optimal complex
         if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
         else:
            grade = "B"
            grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
      else: #result contains complex but optimal is not
         grade = "C"
         grade_annotation ="Result contains complex when optimal does not."
   else: # result do not contain complex, this assumes optimal do not as well
      if leaf_count_result <= 2*leaf_count_optimal:
         grade = "A"
         grade_annotation ="none"
      else:
         grade = "B"
         grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
   grade = "C"
   grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file