Computer Algebra Independent Integration Tests

Summer 2024

4-Trig-functions/4-Miscellaneous/259-4.5

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**330**]. This is test number [259].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (330)	0.00 (0)
Mathematica	92.42 (305)	7.58 (25)
Fricas	55.45 (183)	44.55 (147)
Maple	45.45 (150)	54.55 (180)
Mupad	45.15 (149)	54.85 (181)
Maxima	42.73 (141)	57.27 (189)
Reduce	33.03 (109)	66.97 (221)
Giac	27.27 (90)	72.73 (240)
Sympy	20.30 (67)	79.70 (263)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

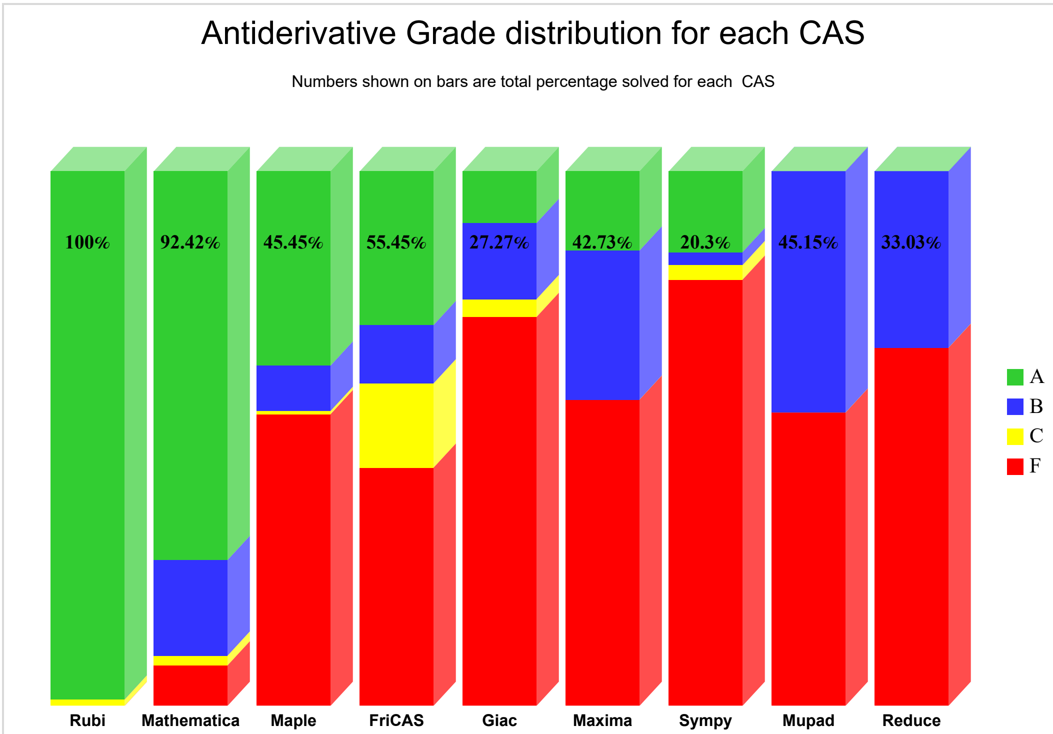
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

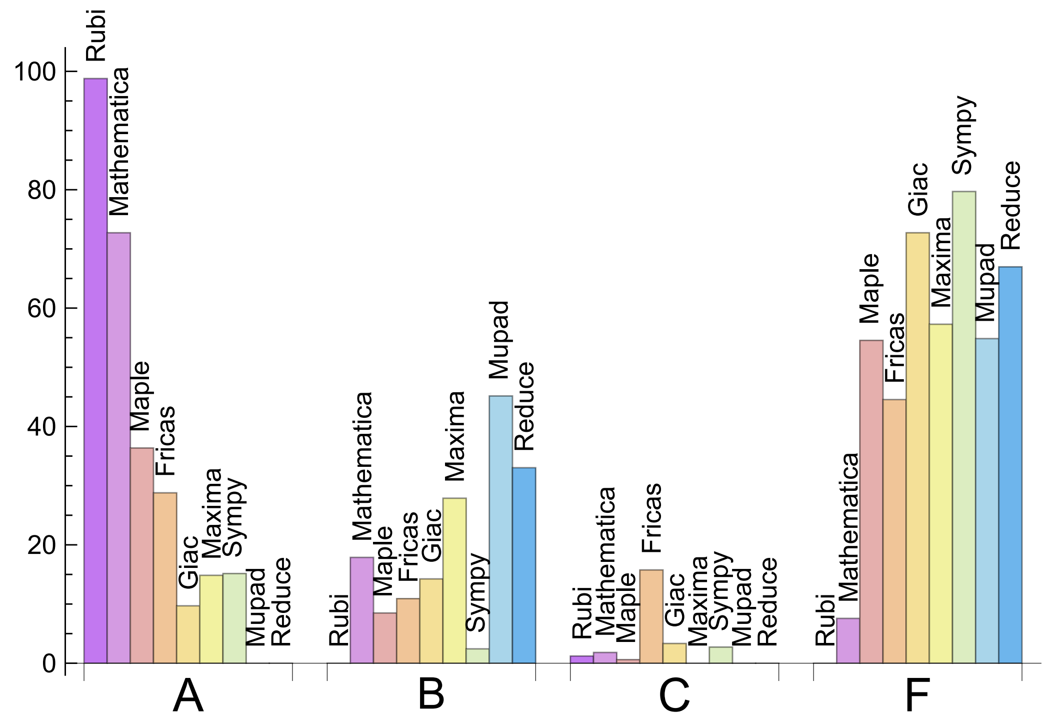
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.788	0.000	1.212	0.000
Mathematica	72.727	17.879	1.818	7.576
Maple	36.364	8.485	0.606	54.545
Fricas	28.788	10.909	15.758	44.545
Sympy	15.152	2.424	2.727	79.697
Maxima	14.848	27.879	0.000	57.273
Giac	9.697	14.242	3.333	72.727
Mupad	0.000	45.152	0.000	54.848
Reduce	0.000	33.030	0.000	66.970

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	25	100.00	0.00	0.00
Fricas	147	66.67	0.00	33.33
Maple	180	100.00	0.00	0.00
Mupad	181	0.00	100.00	0.00
Maxima	189	98.94	0.00	1.06
Reduce	221	100.00	0.00	0.00
Giac	240	72.92	25.42	1.67
Sympy	263	81.37	18.63	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.08
Maxima	0.09
Reduce	0.17
Rubi	0.31
Giac	1.28
Mathematica	1.79
Sympy	5.36
Maple	10.22
Mupad	20.92

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	77.91	1.11	59.00	0.93
Rubi	107.51	1.09	105.00	1.00
Maple	110.02	1.53	59.00	0.97
Fricas	111.07	1.37	77.00	1.06
Sympy	112.36	1.68	54.00	1.22
Reduce	113.19	1.23	67.00	1.00
Mathematica	148.96	1.54	127.00	1.23
Maxima	690.54	7.47	195.00	3.38
Giac	17082.89	69.27	159.50	2.56

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

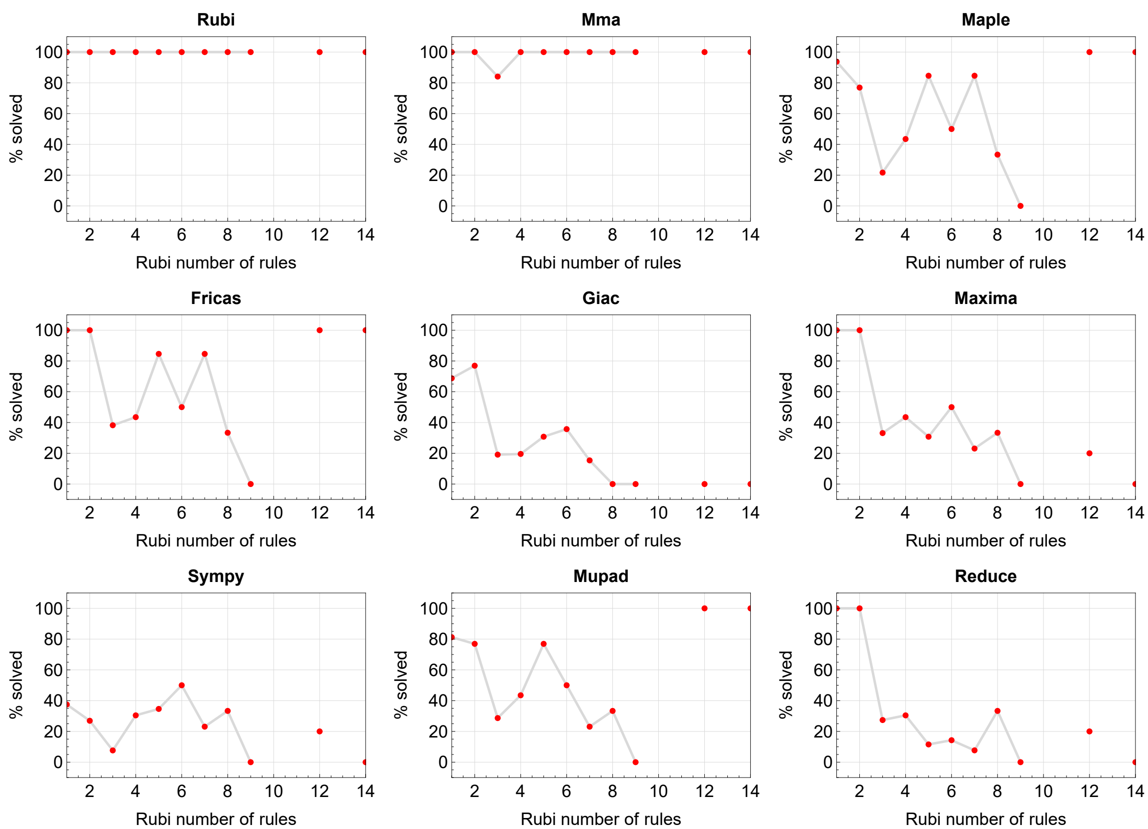


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

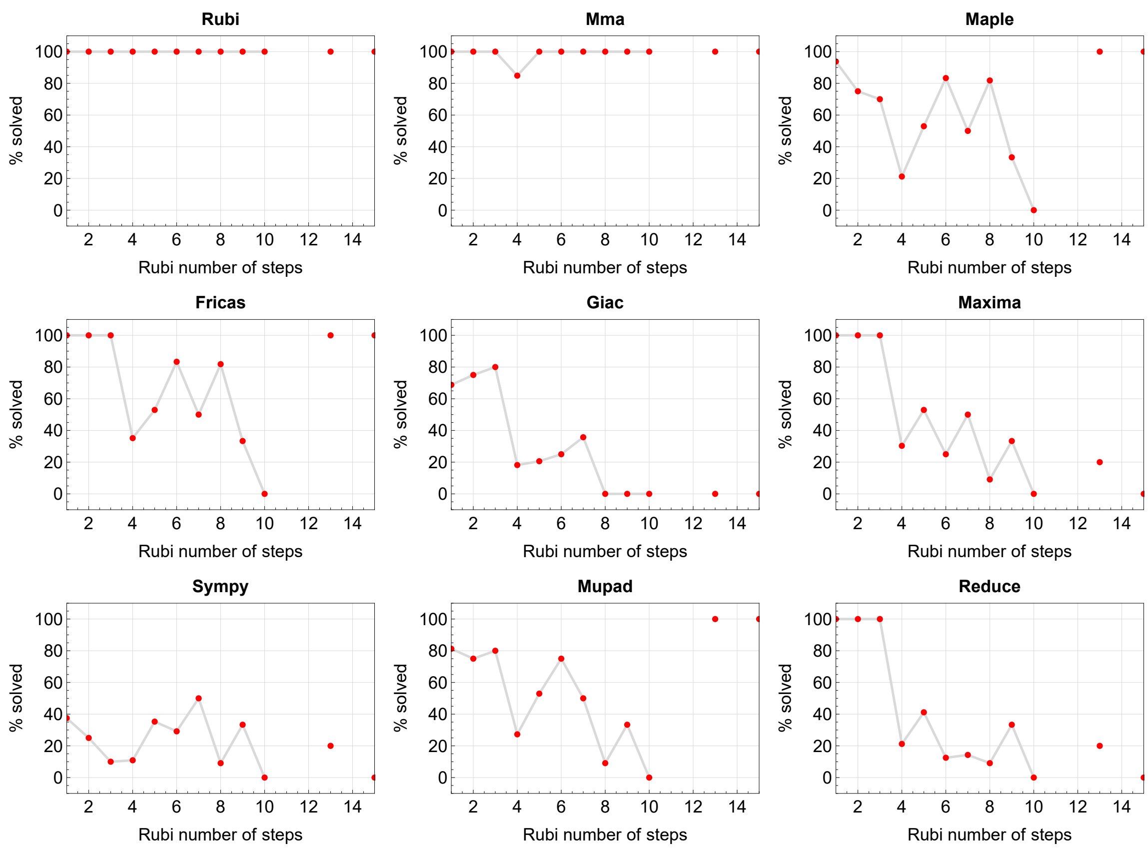


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

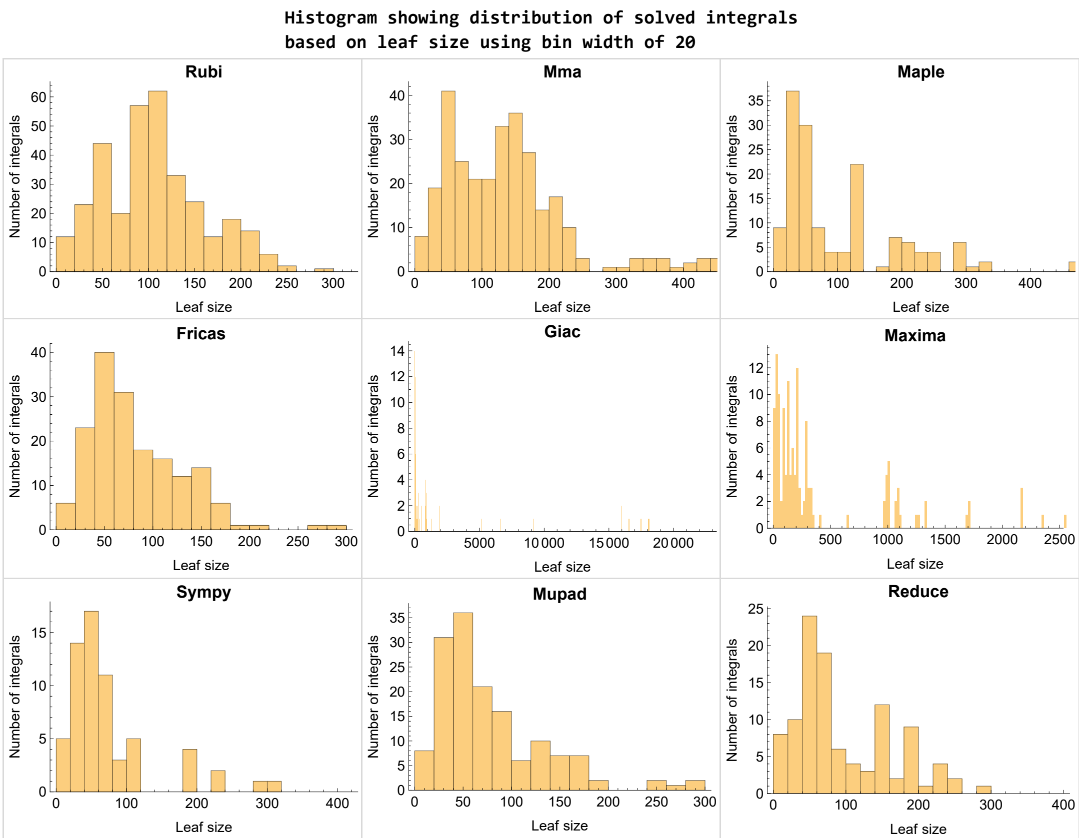


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

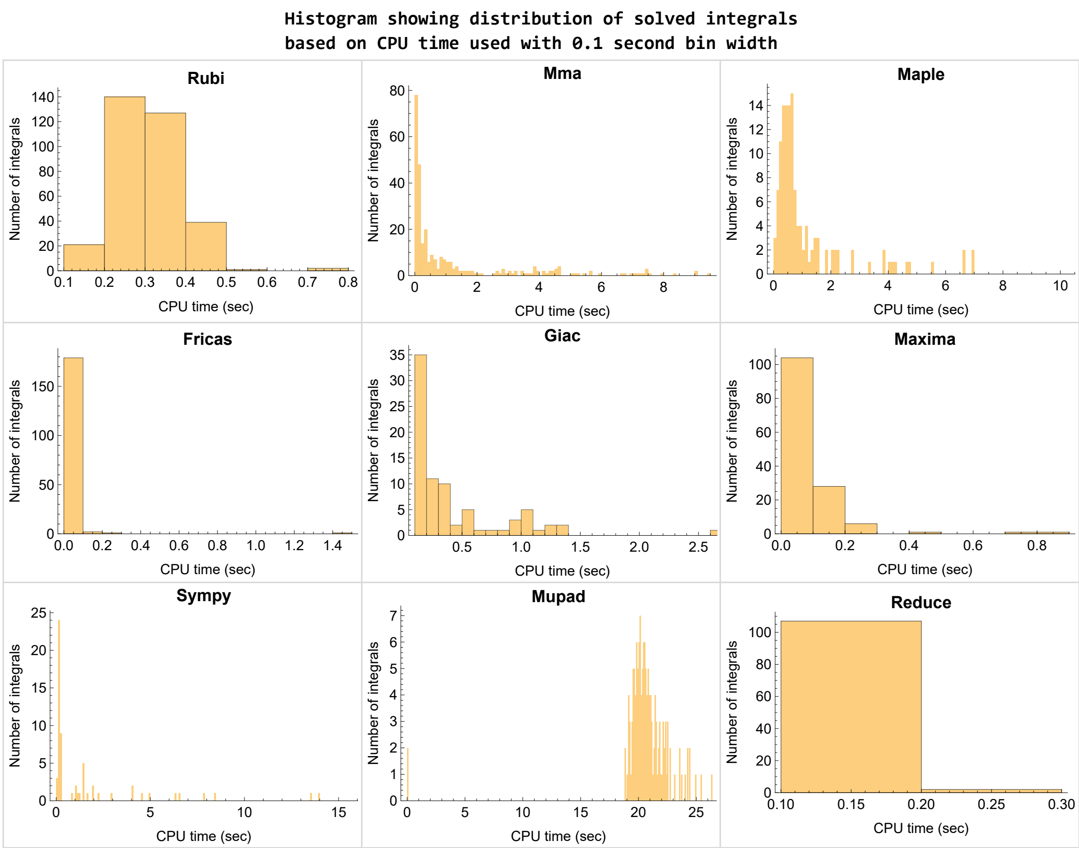


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

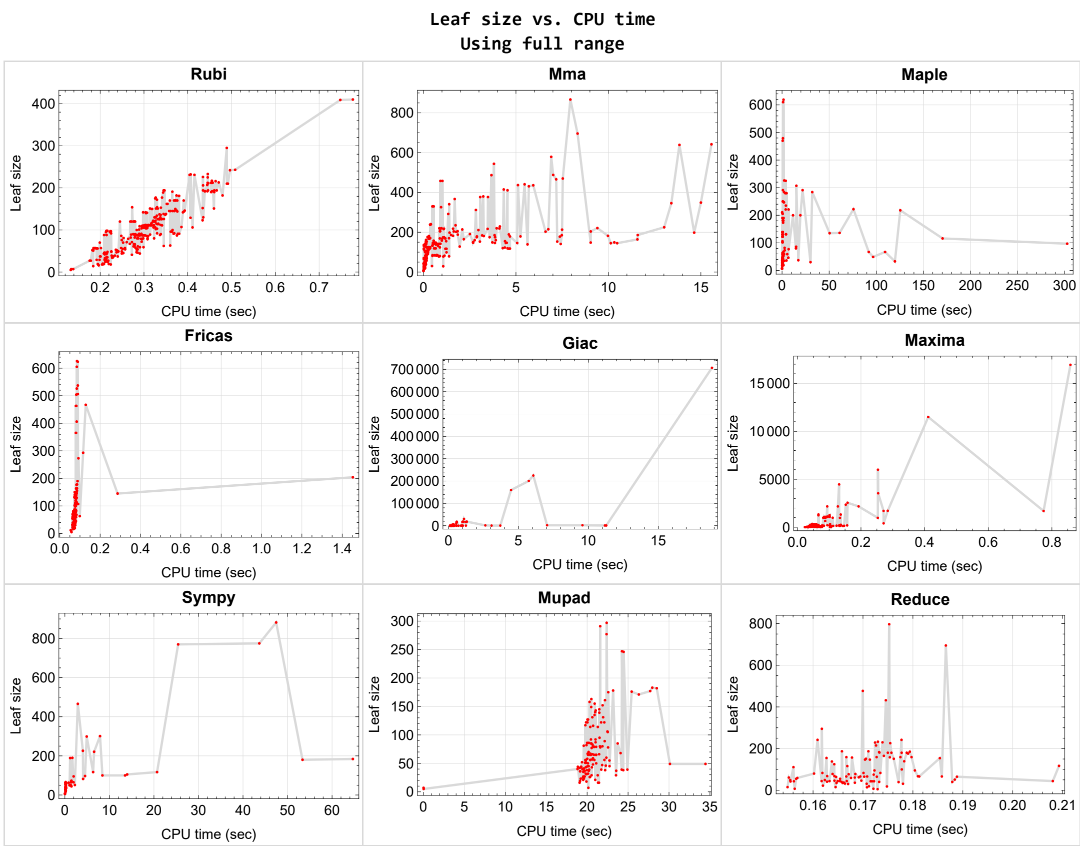


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 38, 39, 41, 42, 43, 45, 46, 47, 49, 51, 104, 105, 106, 107, 109}

Mathematica {78, 127, 129, 131, 132, 153, 155, 156, 157, 178, 204, 206, 207, 208, 229, 264, 265, 268, 276, 281, 282, 285, 306, 310, 312, 316, 323, 324, 325, 327}

Maple $\{261, 303\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

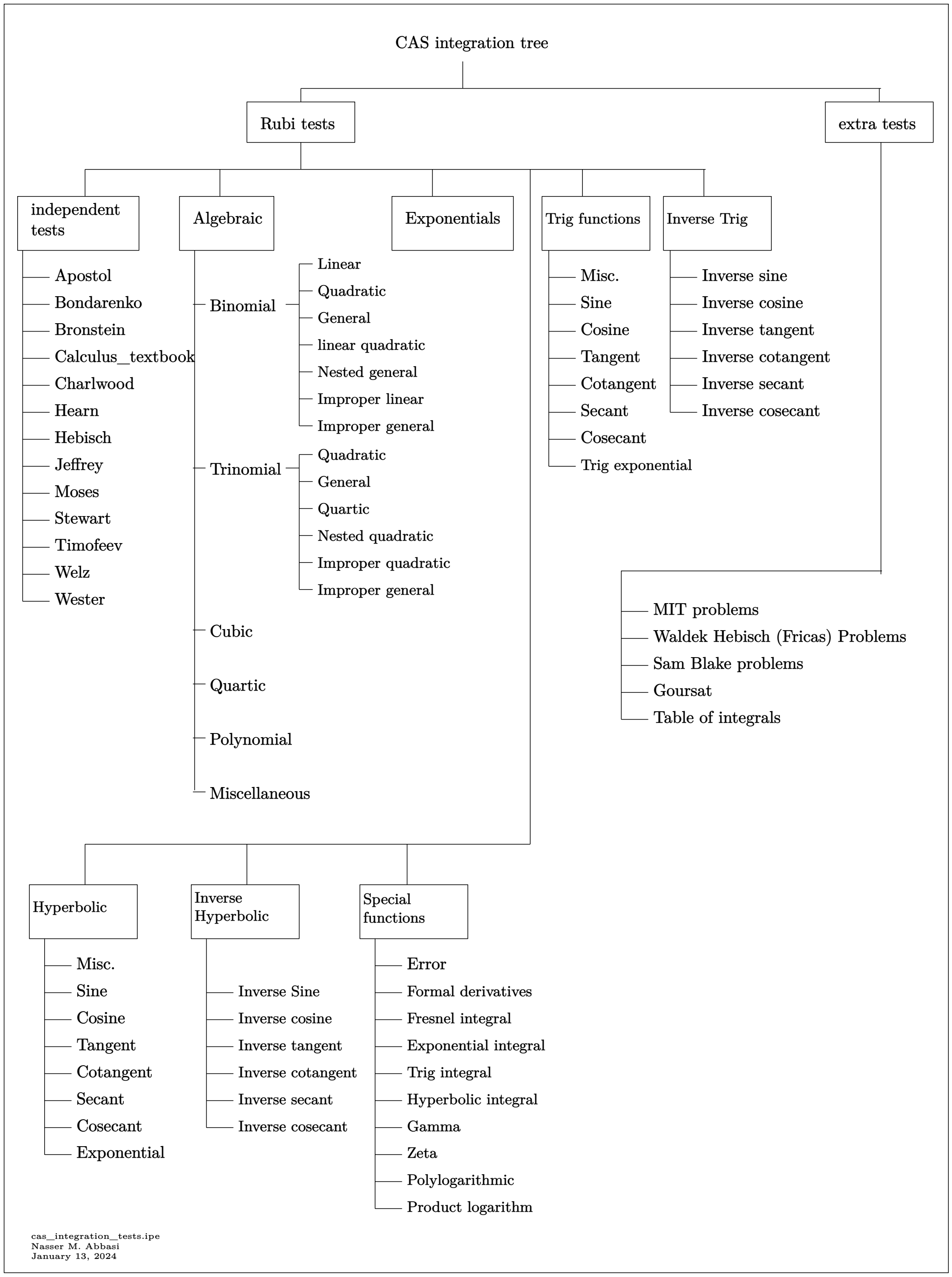
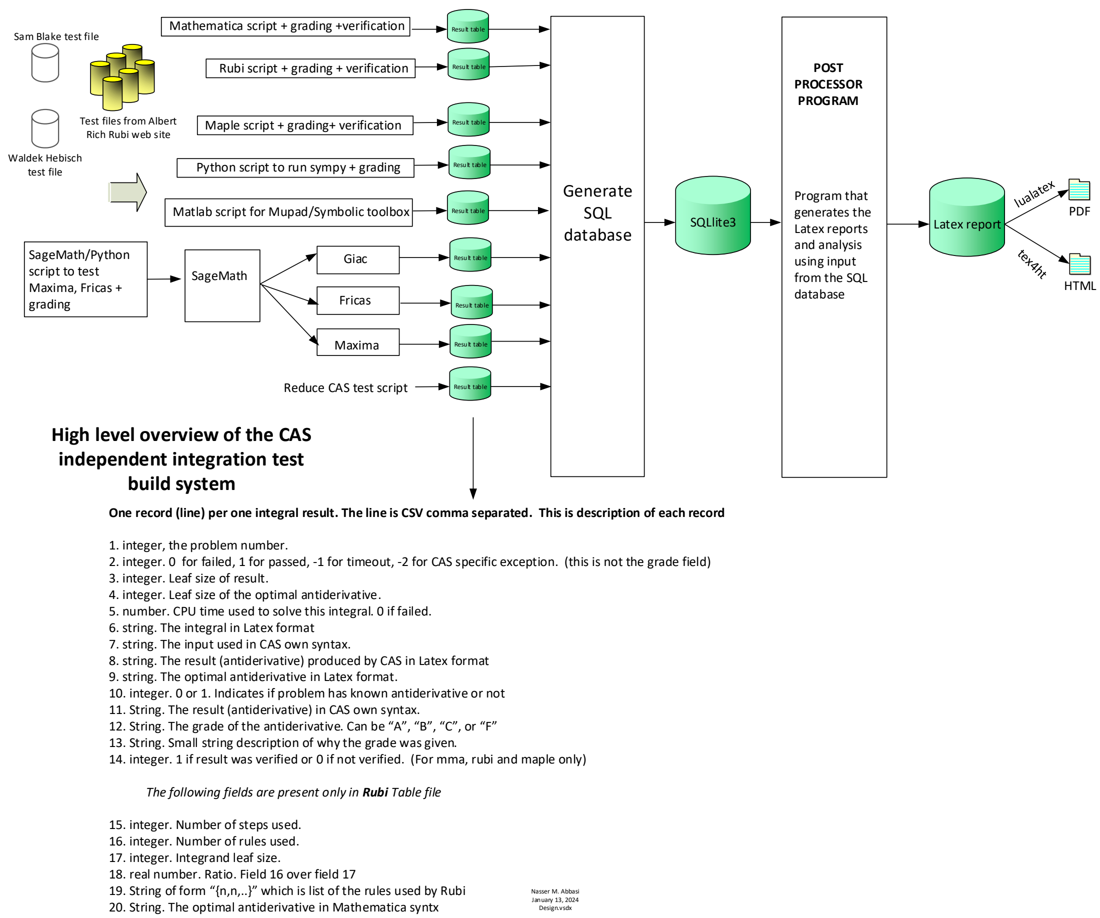


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330 }

B grade { }

C grade { 259, 260, 301, 302 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 30, 37, 40, 44, 48, 50, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 76, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 108, 111, 113, 115, 116, 117, 119, 120, 121, 122, 123, 124, 126, 127, 129, 131, 132, 133, 134, 136, 138, 139, 140, 142, 144, 146, 147, 148, 150, 151, 152, 154, 155, 156, 157, 162, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 184, 185, 187, 189, 190, 191, 193, 195, 197, 199, 201, 202, 203, 205, 206, 207, 208, 213, 216, 217, 218, 219, 221, 222, 223, 225, 226, 227, 228, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 256, 259, 260, 264, 265, 266, 267, 269, 270, 271, 273, 274, 275, 277, 278, 279, 280, 281, 283, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 300, 301, 302, 306, 307, 308, 309, 311, 312, 313, 315, 316, 317, 319, 321, 322, 323, 325, 327, 328, 329, 330 }

B grade { 75, 77, 89, 110, 112, 114, 118, 128, 130, 135, 137, 141, 143, 145, 149, 153, 158, 159, 160, 161, 163, 164, 178, 186, 188, 192, 194, 196, 200, 204, 209, 210, 211, 212, 214, 215, 229, 254, 255, 257, 258, 261, 262, 263, 268, 272, 276, 282, 284, 299, 303, 304, 305, 310, 314, 318, 320, 324, 326 }

C grade { 55, 72, 125, 198, 220, 224 }

F normal fail { 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 38, 39, 41, 42, 43, 45, 46, 47, 49, 51, 104, 105, 106, 107, 109 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 3, 4, 5, 6, 9, 10, 11, 12, 15, 16, 17, 18, 21, 22, 23, 24, 25, 30, 31, 32, 37, 38, 39, 40, 44, 45, 46, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 98, 99, 100, 101, 102, 103, 108, 117, 123, 124, 125, 126, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 162, 169, 172, 173, 174, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 213, 220, 223, 224, 225, 231, 232, 233, 234, 235, 236, 240, 246, 251, 256, 259, 260, 262, 263, 292, 296, 298, 300, 301, 302, 304, 305, 309, 313, 317 }

B grade { 1, 2, 27, 28, 48, 50, 52, 55, 60, 64, 66, 68, 96, 97, 111, 113, 115, 119, 121, 267, 269, 271, 273, 275, 277, 311, 315, 319 }

C grade { 261, 303 }

F normal fail { 7, 8, 13, 14, 19, 20, 26, 29, 33, 34, 35, 36, 41, 42, 43, 47, 49, 51, 53, 54, 56, 57, 58, 59, 61, 62, 63, 65, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 104, 105, 106, 107, 109, 110, 112, 114, 116, 118, 120, 122, 127, 128, 129, 130, 131, 132, 133, 134, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 170, 171, 175, 176, 177, 178, 179, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 221, 222, 226, 227, 228, 229, 230, 237, 238, 239, 241, 242, 243, 244, 245, 247, 248, 249, 250, 252, 253, 254, 255, 257, 258, 264, 265, 266, 268, 270, 272, 274, 276, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 297, 299, 306, 307, 308, 310, 312, 314, 316, 318, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 30, 37, 44, 48, 69, 70, 71, 72, 73, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 122, 123, 124, 125, 126, 135, 136, 137, 138, 139, 141, 142, 143, 144, 145, 149, 162, 172, 186, 188, 190, 191, 192, 193, 194, 196, 197, 200, 213, 223, 246, 251, 256, 259, 261, 262, 264, 296, 300, 301, 303, 304, 306 }

B grade { 50, 52, 140, 146, 147, 148, 169, 173, 174, 180, 181, 182, 183, 184, 185, 187, 189, 195, 198, 199, 220, 224, 225, 231, 232, 233, 234, 235, 236, 240, 263, 265, 292, 298, 305, 307 }

C grade { 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 45, 46, 47, 49, 51, 55, 60, 64, 66, 68, 104, 105, 106, 107, 108, 109, 111, 113, 115, 117, 119, 121, 260, 267, 269, 271, 273, 275, 277, 302, 309, 311, 313, 315, 317, 319 }

F normal fail { 79, 80, 81, 82, 83, 84, 85, 132, 133, 134, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 170, 171, 175, 176, 177, 178, 179, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 221, 222, 226, 227, 228, 229, 230, 237, 238, 239, 241, 242, 243, 244, 245, 247, 248, 249, 250, 252, 253, 254, 255, 257, 258, 278, 279, 280, 286, 287, 288, 289, 290, 291, 293, 294, 295, 297, 299, 320, 321, 322, 328, 329, 330 }

F(-1) timedout fail { }

F(-2) exception fail { 53, 54, 56, 57, 58, 59, 61, 62, 63, 65, 67, 74, 75, 76, 77, 78, 110, 112, 114, 116, 118, 120, 127, 128, 129, 130, 131, 266, 268, 270, 272, 274, 276, 281, 282, 283, 284, 285, 308, 310, 312, 314, 316, 318, 323, 324, 325, 326, 327 }

Maxima

A grade { 4, 10, 22, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 89, 94, 102, 103, 104, 105, 106, 107, 108, 109, 139, 147, 162, 190, 198, 213, 240, 292 }

B grade { 1, 2, 3, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 69, 70, 71, 72, 73, 86, 87, 88, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 122, 123, 124, 125, 126, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 148, 169, 172, 173, 174, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 199, 220, 223, 224, 225, 246, 256, 259, 260, 261, 262, 263, 296, 298, 300, 301, 302, 303, 304, 305 }

C grade { }

F normal fail { 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 127, 128, 129, 130, 131, 132, 133, 134, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 170, 171, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 221, 222, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 297, 299, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330 }

F(-1) timedout fail { }

F(-2) exception fail { 149, 200 }

Giac

A grade { 25, 27, 28, 29, 30, 34, 35, 36, 37, 44, 48, 50, 103, 105, 107, 135, 136, 137, 138, 140, 141, 142, 147, 186, 187, 188, 189, 191, 193, 195, 262, 304 }

B grade { 1, 2, 3, 7, 8, 9, 13, 14, 15, 19, 20, 21, 70, 71, 72, 73, 86, 87, 88, 91, 92, 93, 96, 97, 98, 101, 123, 124, 125, 126, 139, 143, 144, 145, 146, 148, 149, 190, 192, 194, 196, 197, 198, 199, 200, 263, 305 }

C grade { 26, 33, 40, 47, 49, 51, 104, 106, 108, 260, 302 }

F normal fail { 4, 5, 6, 10, 11, 12, 16, 17, 18, 22, 23, 24, 31, 32, 38, 39, 45, 46, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 69, 74, 75, 76, 79, 80, 81, 82, 83, 84, 85, 89, 90, 94, 95, 99, 100, 102, 110, 111, 112, 113, 114, 115, 116, 117, 122, 127, 128, 129, 132, 133, 134, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 164, 165, 166, 167, 168, 171, 175, 176, 177, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 215, 222, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 261, 264, 265, 266, 267, 272, 273, 274, 275, 276, 277, 278, 279, 280, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 306, 307, 308, 309, 314, 315, 316, 317, 318, 319, 320, 321, 322, 325, 326, 327, 328, 329, 330 }

F(-1) timedout fail { 65, 66, 67, 68, 77, 78, 118, 119, 120, 121, 130, 131, 162, 163, 169, 170, 172, 173, 174, 178, 179, 180, 181, 182, 183, 184, 185, 213, 214, 216, 217, 218, 219, 220, 221, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 268, 269, 270, 271, 281, 282, 310, 311, 312, 313, 323, 324 }

F(-2) exception fail { 41, 42, 43, 109 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 7, 8, 9, 10, 13, 14, 15, 16, 19, 20, 21, 22, 25, 26, 27, 28, 29, 30, 33, 34, 35, 36, 37, 40, 42, 43, 44, 47, 49, 51, 55, 60, 64, 66, 68, 69, 70, 71, 72, 73, 83, 86, 87, 88, 89, 91, 92, 93, 94, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 113, 115, 117, 119, 121, 122, 123, 124, 125, 126, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 162, 169, 172, 173, 174, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 213, 220, 223, 224, 225, 231, 232, 233, 234, 235, 236, 240, 246, 251, 256, 259, 260, 261, 262, 263, 267, 292, 296, 298, 300, 301, 302, 303, 304, 305, 309 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 6, 11, 12, 17, 18, 23, 24, 31, 32, 38, 39, 41, 45, 46, 48, 50, 52, 53, 54, 56, 57, 58, 59, 61, 62, 63, 65, 67, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 90, 95, 100, 110, 112, 114, 116, 118, 120, 127, 128, 129, 130, 131, 132, 133, 134, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 170, 171, 175, 176, 177, 178, 179, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 221, 222, 226, 227, 228, 229, 230, 237, 238, 239, 241, 242, 243, 244, 245, 247, 248, 249, 250, 252, 253, 254, 255, 257, 258, 264, 265, 266, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 297, 299, 306, 307, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330 }

F(-2) exception fail { }

Sympy

A grade { 10, 22, 25, 30, 31, 32, 37, 38, 44, 45, 46, 94, 102, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 162, 172, 173, 174, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 224, 240, 292 }

B grade { 4, 16, 39, 89, 99, 213, 223, 225 }

C grade { 5, 6, 11, 12, 17, 18, 90, 95, 100 }

F normal fail { 1, 2, 3, 7, 8, 9, 15, 26, 27, 28, 29, 33, 35, 36, 40, 43, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64, 65, 66, 69, 70, 71, 72, 73, 75, 76, 80, 81, 82, 83, 84, 85, 86, 87, 88, 91, 92, 93, 97, 98, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 116, 117, 118, 119, 122, 125, 126, 128, 129, 130, 133, 134, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 166, 167, 168, 169, 170, 171, 175, 176, 177, 178, 181, 182, 183, 184, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 220, 221, 222, 226, 227, 229, 230, 232, 233, 234, 235, 237, 238, 239, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 272, 273, 274, 275, 278, 279, 280, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 314, 315, 316, 317, 321, 322, 325, 326, 327, 328, 329, 330 }

F(-1) timedout fail { 13, 14, 19, 20, 21, 23, 24, 34, 41, 42, 58, 67, 68, 74, 77, 78, 79, 96, 114, 115, 120, 121, 123, 124, 127, 131, 132, 165, 179, 180, 185, 228, 231, 236, 260, 270, 271, 276, 277, 281, 282, 302, 312, 313, 318, 319, 320, 323, 324 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 70, 71, 72, 73, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 123, 124, 125, 126, 139, 147, 162, 169, 172, 173, 174, 190, 198, 213, 220, 223, 224, 225, 240, 246, 251, 256, 259, 261, 292, 296, 298, 300, 301, 303 }

C grade { }

F normal fail { 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 170, 171, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 221, 222, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 247, 248, 249, 250, 252, 253, 254, 255, 257, 258, 260, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 297, 299, 302, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	44	479	219	49	0	923	44	44
N.S.	1	1.00	0.77	8.40	3.84	0.86	0.00	16.19	0.77	0.77
time (sec)	N/A	0.185	0.058	0.922	0.051	0.076	0.000	0.184	0.165	19.259

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	44	470	219	49	0	923	44	44
N.S.	1	1.00	0.77	8.25	3.84	0.86	0.00	16.19	0.77	0.77
time (sec)	N/A	0.202	0.045	0.637	0.054	0.073	0.000	0.188	0.162	18.867

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	40	43	206	45	0	882	40	40
N.S.	1	1.00	0.77	0.83	3.96	0.87	0.00	16.96	0.77	0.77
time (sec)	N/A	0.182	0.039	0.290	0.094	0.074	0.000	0.147	0.166	18.828

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	38	20	19	20	36	0	19	19
N.S.	1	1.00	2.00	1.05	1.00	1.05	1.89	0.00	1.00	1.00
time (sec)	N/A	0.194	0.023	0.224	0.029	0.074	0.279	0.000	0.171	19.136

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	40	45	209	44	190	0	44	0
N.S.	1	1.00	0.70	0.79	3.67	0.77	3.33	0.00	0.77	0.00
time (sec)	N/A	0.189	0.045	0.319	0.059	0.073	1.686	0.000	0.163	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	44	45	216	46	226	0	44	0
N.S.	1	1.00	0.77	0.79	3.79	0.81	3.96	0.00	0.77	0.00
time (sec)	N/A	0.187	0.047	0.532	0.059	0.073	4.044	0.000	0.167	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	61	0	301	80	0	830	66	67
N.S.	1	1.00	0.63	0.00	3.10	0.82	0.00	8.56	0.68	0.69
time (sec)	N/A	0.224	0.110	0.000	0.060	0.075	0.000	0.335	0.167	20.061

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	57	0	282	78	0	817	65	67
N.S.	1	1.00	0.58	0.00	2.88	0.80	0.00	8.34	0.66	0.68
time (sec)	N/A	0.214	0.081	0.000	0.053	0.073	0.000	0.310	0.170	20.405

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	56	59	280	73	0	783	62	56
N.S.	1	1.00	0.64	0.67	3.18	0.83	0.00	8.90	0.70	0.64
time (sec)	N/A	0.209	0.070	0.631	0.055	0.076	0.000	0.261	0.169	19.786

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	44	36	32	55	40	51	0	41	32
N.S.	1	1.13	0.92	0.82	1.41	1.03	1.31	0.00	1.05	0.82
time (sec)	N/A	0.208	0.060	0.656	0.043	0.075	2.240	0.000	0.163	20.116

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	57	59	283	71	301	0	66	0
N.S.	1	1.00	0.60	0.62	2.98	0.75	3.17	0.00	0.69	0.00
time (sec)	N/A	0.213	0.082	1.238	0.056	0.075	7.895	0.000	0.171	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	58	59	280	69	466	0	66	0
N.S.	1	1.00	0.59	0.60	2.86	0.70	4.76	0.00	0.67	0.00
time (sec)	N/A	0.215	0.078	2.134	0.054	0.075	2.909	0.000	0.169	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	154	122	0	1008	138	0	18085	158	122
N.S.	1	0.96	0.76	0.00	6.30	0.86	0.00	113.03	0.99	0.76
time (sec)	N/A	0.330	0.358	0.000	0.084	0.081	0.000	1.317	0.175	20.041

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	151	125	0	1016	140	0	18117	159	122
N.S.	1	0.96	0.79	0.00	6.43	0.89	0.00	114.66	1.01	0.77
time (sec)	N/A	0.314	0.360	0.000	0.083	0.081	0.000	1.071	0.180	19.877

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	141	121	190	990	130	0	17522	154	114
N.S.	1	0.95	0.81	1.28	6.64	0.87	0.00	117.60	1.03	0.77
time (sec)	N/A	0.307	0.319	1.457	0.111	0.079	0.000	0.598	0.185	20.384

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	37	45	35	233	37	73	0	49	37
N.S.	1	0.86	1.05	0.81	5.42	0.86	1.70	0.00	1.14	0.86
time (sec)	N/A	0.229	0.047	2.196	0.055	0.073	1.480	0.000	0.162	20.213

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	151	125	221	995	127	775	0	159	0
N.S.	1	0.96	0.79	1.40	6.30	0.80	4.91	0.00	1.01	0.00
time (sec)	N/A	0.327	0.238	3.839	0.088	0.079	43.667	0.000	0.173	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	151	125	221	1007	129	882	0	159	0
N.S.	1	0.96	0.79	1.40	6.37	0.82	5.58	0.00	1.01	0.00
time (sec)	N/A	0.331	0.279	6.913	0.086	0.081	47.448	0.000	0.177	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	202	191	171	0	1107	178	0	16604	180	127
N.S.	1	0.95	0.85	0.00	5.48	0.88	0.00	82.20	0.89	0.63
time (sec)	N/A	0.365	0.356	0.000	0.095	0.086	0.000	1.223	0.178	20.120

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	210	194	169	0	1085	177	0	16554	180	127
N.S.	1	0.92	0.80	0.00	5.17	0.84	0.00	78.83	0.86	0.60
time (sec)	N/A	0.345	0.323	0.000	0.090	0.083	0.000	0.998	0.179	20.043

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	177	168	131	1078	165	0	15992	176	117
N.S.	1	0.93	0.88	0.69	5.64	0.86	0.00	83.73	0.92	0.61
time (sec)	N/A	0.341	0.280	4.103	0.092	0.082	0.000	0.572	0.179	19.786

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	80	51	46	93	59	100	0	68	51
N.S.	1	1.10	0.70	0.63	1.27	0.81	1.37	0.00	0.93	0.70
time (sec)	N/A	0.284	0.074	6.654	0.053	0.081	13.520	0.000	0.164	19.893

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	189	170	200	1085	162	0	0	180	0
N.S.	1	0.94	0.84	0.99	5.37	0.80	0.00	0.00	0.89	0.00
time (sec)	N/A	0.344	0.355	11.631	0.095	0.083	0.000	0.000	0.176	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	194	169	200	1082	163	0	0	181	0
N.S.	1	0.92	0.80	0.95	5.15	0.78	0.00	0.00	0.86	0.00
time (sec)	N/A	0.347	0.329	19.460	0.102	0.081	0.000	0.000	0.173	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	37	29	34	27	33	56	35	46	36
N.S.	1	0.95	0.74	0.87	0.69	0.85	1.44	0.90	1.18	0.92
time (sec)	N/A	0.201	0.015	0.238	0.036	0.073	0.199	0.126	0.164	19.136

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	A	C	F	C	B	B
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	123	0	0	82	63	0	272	61	135
N.S.	1	0.92	0.00	0.00	0.62	0.47	0.00	2.05	0.46	1.02
time (sec)	N/A	0.434	0.000	0.000	0.107	0.099	0.000	0.898	0.168	20.906

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	A	C	F	A	B	B
verified	N/A	No	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	80	0	619	31	42	0	1	83	85
N.S.	1	0.91	0.00	7.03	0.35	0.48	0.00	0.01	0.94	0.97
time (sec)	N/A	0.323	0.000	1.559	0.065	0.072	0.000	0.277	0.162	20.527

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	A	C	F	A	B	B
verified	N/A	No	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	80	0	610	31	42	0	1	83	85
N.S.	1	0.91	0.00	6.93	0.35	0.48	0.00	0.01	0.94	0.97
time (sec)	N/A	0.298	0.000	1.053	0.076	0.071	0.000	0.279	0.173	20.456

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	A	C	F	A	B	B
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	78	0	0	29	42	0	1	77	81
N.S.	1	0.95	0.00	0.00	0.35	0.51	0.00	0.01	0.94	0.99
time (sec)	N/A	0.295	0.000	0.000	0.064	0.070	0.000	0.210	0.167	19.583

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	6	5	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	1.20	1.00	1.00
time (sec)	N/A	0.133	0.001	0.026	0.033	0.058	0.022	0.123	0.173	0.034

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	A	C	A	F	B	F(-1)
verified	N/A	No	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	78	0	68	33	45	95	0	78	0
N.S.	1	0.91	0.00	0.79	0.38	0.52	1.10	0.00	0.91	0.00
time (sec)	N/A	0.308	0.000	1.454	0.068	0.072	1.965	0.000	0.168	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	A	C	A	F	B	F(-1)
verified	N/A	No	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	80	0	73	35	45	117	0	83	0
N.S.	1	0.91	0.00	0.83	0.40	0.51	1.33	0.00	0.94	0.00
time (sec)	N/A	0.308	0.000	2.793	0.060	0.071	6.343	0.000	0.170	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	A	C	F	C	B	B
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	130	0	0	173	107	0	498	95	145
N.S.	1	1.11	0.00	0.00	1.48	0.91	0.00	4.26	0.81	1.24
time (sec)	N/A	0.436	0.000	0.000	0.102	0.091	0.000	3.085	0.172	21.489

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	A	C	F(-1)	A	B	B
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	92	0	0	47	59	0	1	165	92
N.S.	1	1.21	0.00	0.00	0.62	0.78	0.00	0.01	2.17	1.21
time (sec)	N/A	0.330	0.000	0.000	0.071	0.073	0.000	0.720	0.174	21.023

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	A	C	F	A	B	B
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	89	0	0	47	60	0	1	147	92
N.S.	1	1.17	0.00	0.00	0.62	0.79	0.00	0.01	1.93	1.21
time (sec)	N/A	0.314	0.000	0.000	0.055	0.071	0.000	0.582	0.171	20.466

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	A	C	F	A	B	B
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	86	0	0	41	57	0	1	157	86
N.S.	1	1.26	0.00	0.00	0.60	0.84	0.00	0.01	2.31	1.26
time (sec)	N/A	0.306	0.000	0.000	0.078	0.070	0.000	0.397	0.168	20.005

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	10	7	8	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.43	1.00	1.14	1.00	1.00
time (sec)	N/A	0.134	0.001	0.030	0.035	0.056	0.019	0.128	0.172	0.022

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	A	C	A	F	B	F(-1)
verified	N/A	No	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	88	0	77	48	62	105	0	159	0
N.S.	1	1.19	0.00	1.04	0.65	0.84	1.42	0.00	2.15	0.00
time (sec)	N/A	0.337	0.000	6.983	0.066	0.071	13.990	0.000	0.167	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	A	C	B	F	B	F(-1)
verified	N/A	No	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	90	0	86	54	65	221	0	147	0
N.S.	1	1.18	0.00	1.13	0.71	0.86	2.91	0.00	1.93	0.00
time (sec)	N/A	0.338	0.000	14.925	0.065	0.071	6.566	0.000	0.170	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	231	169	135	195	128	0	1870	151	297
N.S.	1	1.02	0.75	0.60	0.86	0.57	0.00	8.27	0.67	1.31
time (sec)	N/A	0.415	0.882	50.477	0.132	0.078	0.000	7.053	0.176	22.385

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	A	C	F(-1)	F(-2)	B	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	172	142	0	0	90	82	0	0	215	0
N.S.	1	0.83	0.00	0.00	0.52	0.48	0.00	0.00	1.25	0.00
time (sec)	N/A	0.389	0.000	0.000	0.063	0.074	0.000	0.000	0.173	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	A	C	F(-1)	F(-2)	B	B
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	178	146	0	0	112	84	0	0	242	163
N.S.	1	0.82	0.00	0.00	0.63	0.47	0.00	0.00	1.36	0.92
time (sec)	N/A	0.375	0.000	0.000	0.071	0.073	0.000	0.000	0.178	20.519

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	A	C	F	F(-2)	B	B
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	144	0	0	106	84	0	0	230	155
N.S.	1	0.86	0.00	0.00	0.63	0.50	0.00	0.00	1.37	0.92
time (sec)	N/A	0.376	0.000	0.000	0.067	0.074	0.000	0.000	0.174	20.915

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	12	7	8	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.71	1.00	1.14	1.00	1.00
time (sec)	N/A	0.138	0.001	0.033	0.027	0.054	0.019	0.129	0.156	20.153

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	A	C	A	F	B	F(-1)
verified	N/A	No	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	144	0	284	122	87	180	0	233	0
N.S.	1	0.82	0.00	1.61	0.69	0.49	1.02	0.00	1.32	0.00
time (sec)	N/A	0.392	0.000	32.050	0.066	0.073	53.366	0.000	0.173	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	A	C	A	F	B	F(-1)
verified	N/A	No	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	146	0	136	128	87	184	0	242	0
N.S.	1	0.82	0.00	0.76	0.72	0.49	1.03	0.00	1.36	0.00
time (sec)	N/A	0.393	0.000	61.093	0.064	0.075	64.667	0.000	0.161	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	A	C	F	C	B	B
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	100	0	0	48	51	0	189	55	139
N.S.	1	0.89	0.00	0.00	0.43	0.46	0.00	1.69	0.49	1.24
time (sec)	N/A	0.365	0.000	0.000	0.043	0.075	0.000	0.445	0.157	21.494

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	44	44	106	31	24	0	24	59	0
N.S.	1	0.85	0.85	2.04	0.60	0.46	0.00	0.46	1.13	0.00
time (sec)	N/A	0.249	0.044	0.441	0.041	0.065	0.000	0.145	0.157	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	A	C	F	C	B	B
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	107	0	0	134	75	0	350	80	149
N.S.	1	1.01	0.00	0.00	1.26	0.71	0.00	3.30	0.75	1.41
time (sec)	N/A	0.386	0.000	0.000	0.046	0.075	0.000	1.251	0.160	20.503

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	54	60	173	47	145	0	27	111	0
N.S.	1	1.02	1.13	3.26	0.89	2.74	0.00	0.51	2.09	0.00
time (sec)	N/A	0.257	0.073	1.135	0.038	0.286	0.000	0.202	0.156	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	A	C	F	C	B	B
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	218	182	0	0	206	98	0	1297	125	291
N.S.	1	0.83	0.00	0.00	0.94	0.45	0.00	5.95	0.57	1.33
time (sec)	N/A	0.479	0.000	0.000	0.064	0.076	0.000	2.640	0.164	21.598

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	82	103	284	75	204	0	0	16	0
N.S.	1	0.84	1.05	2.90	0.77	2.08	0.00	0.00	0.16	0.00
time (sec)	N/A	0.279	0.088	2.061	0.056	1.453	0.000	0.000	0.160	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	145	0	0	0	0	0	62	0
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.322	10.128	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	148	0	0	0	0	0	58	0
N.S.	1	1.00	1.35	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.305	9.041	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	29	82	129	0	85	0	0	18	26
N.S.	1	0.97	2.73	4.30	0.00	2.83	0.00	0.00	0.60	0.87
time (sec)	N/A	0.210	0.168	0.428	0.000	0.078	0.000	0.000	0.159	20.362

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	149	0	0	0	0	0	66	0
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.316	10.319	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	145	0	0	0	0	0	68	0
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.323	10.473	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	218	0	0	0	0	0	60	0
N.S.	1	1.00	1.96	0.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.306	1.107	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	218	0	0	0	0	0	56	0
N.S.	1	1.00	2.00	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.296	0.905	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	66	58	131	0	106	0	0	29	65
N.S.	1	0.96	0.84	1.90	0.00	1.54	0.00	0.00	0.42	0.94
time (sec)	N/A	0.279	0.114	0.402	0.000	0.083	0.000	0.000	0.160	20.110

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	220	0	0	0	0	0	65	0
N.S.	1	1.00	1.98	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.313	0.977	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	216	0	0	0	0	0	67	0
N.S.	1	1.00	1.95	0.00	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.318	1.090	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	132	0	0	0	0	0	28	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.296	0.487	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	29	32	102	0	77	0	0	31	26
N.S.	1	0.97	1.07	3.40	0.00	2.57	0.00	0.00	1.03	0.87
time (sec)	N/A	0.214	0.084	0.367	0.000	0.078	0.000	0.000	0.165	20.319

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	164	0	0	0	0	0	28	0
N.S.	1	1.00	1.50	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.297	11.570	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	62	57	190	0	150	0	0	31	65
N.S.	1	0.95	0.88	2.92	0.00	2.31	0.00	0.00	0.48	1.00
time (sec)	N/A	0.279	0.164	0.396	0.000	0.083	0.000	0.000	0.167	21.071

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	191	0	0	0	0	0	28	0
N.S.	1	1.00	1.75	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.302	2.511	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	66	61	131	0	164	0	0	31	65
N.S.	1	0.96	0.88	1.90	0.00	2.38	0.00	0.00	0.45	0.94
time (sec)	N/A	0.277	0.160	0.406	0.000	0.085	0.000	0.000	0.165	20.841

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	81	0	402	43	0	0	32	50
N.S.	1	1.00	1.65	0.00	8.20	0.88	0.00	0.00	0.65	1.02
time (sec)	N/A	0.226	0.124	0.000	0.271	0.069	0.000	0.000	0.167	21.110

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	337	295	341	0	16932	467	0	706991	695	175
N.S.	1	0.88	1.01	0.00	50.24	1.39	0.00	2097.90	2.06	0.52
time (sec)	N/A	0.489	1.391	0.000	0.858	0.129	0.000	18.842	0.187	22.584

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	256	233	326	0	11491	293	0	200416	432	161
N.S.	1	0.91	1.27	0.00	44.89	1.14	0.00	782.88	1.69	0.63
time (sec)	N/A	0.446	0.950	0.000	0.411	0.116	0.000	5.744	0.175	22.182

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	B	A	F	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	102	0	2551	155	0	30585	180	95
N.S.	1	1.00	0.66	0.00	16.56	1.01	0.00	198.60	1.17	0.62
time (sec)	N/A	0.273	0.245	0.000	0.158	0.085	0.000	1.113	0.175	21.515

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	63	0	1263	86	0	6580	78	80
N.S.	1	1.00	0.68	0.00	13.73	0.93	0.00	71.52	0.85	0.87
time (sec)	N/A	0.225	0.116	0.000	0.101	0.078	0.000	0.367	0.164	21.563

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	145	235	0	0	0	0	0	87	0
N.S.	1	0.97	1.57	0.00	0.00	0.00	0.00	0.00	0.58	0.00
time (sec)	N/A	0.370	1.734	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	145	488	0	0	0	0	0	89	0
N.S.	1	0.97	3.28	0.00	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.366	7.013	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	131	0	0	0	0	0	41	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.372	0.789	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	145	544	0	0	0	0	0	41	0
N.S.	1	0.97	3.63	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.357	3.811	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	145	214	0	0	0	0	0	41	0
N.S.	1	0.97	1.43	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.361	2.098	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	181	174	0	0	0	0	0	89	0
N.S.	1	1.26	1.21	0.00	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	0.380	1.054	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	148	0	0	0	0	0	67	0
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.333	0.663	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	144	0	0	0	0	0	65	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.317	0.553	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	146	0	0	0	0	0	61	0
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.303	0.508	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	0	0	0	0	0	19	77
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.22	0.90
time (sec)	N/A	0.248	0.128	0.000	0.000	0.000	0.000	0.000	0.173	20.549

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	146	0	0	0	0	0	68	0
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.333	0.549	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	142	0	0	0	0	0	71	0
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	0.62	0.00
time (sec)	N/A	0.329	0.574	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	43	44	218	48	0	923	43	43
N.S.	1	1.00	0.77	0.79	3.89	0.86	0.00	16.48	0.77	0.77
time (sec)	N/A	0.194	0.056	0.904	0.053	0.075	0.000	0.201	0.172	20.579

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	43	44	218	48	0	915	43	43
N.S.	1	1.00	0.77	0.79	3.89	0.86	0.00	16.34	0.77	0.77
time (sec)	N/A	0.191	0.049	0.673	0.048	0.073	0.000	0.192	0.172	19.828

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	39	40	205	43	0	878	39	39
N.S.	1	1.00	0.76	0.78	4.02	0.84	0.00	17.22	0.76	0.76
time (sec)	N/A	0.184	0.037	0.303	0.059	0.073	0.000	0.147	0.177	20.253

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	37	19	18	19	34	0	18	18
N.S.	1	1.00	2.06	1.06	1.00	1.06	1.89	0.00	1.00	1.00
time (sec)	N/A	0.195	0.023	0.221	0.025	0.074	0.258	0.000	0.175	19.917

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	41	44	208	45	189	0	43	0
N.S.	1	1.00	0.73	0.79	3.71	0.80	3.38	0.00	0.77	0.00
time (sec)	N/A	0.185	0.046	0.319	0.055	0.078	1.181	0.000	0.172	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	61	61	301	76	0	830	67	66
N.S.	1	1.00	0.63	0.63	3.10	0.78	0.00	8.56	0.69	0.68
time (sec)	N/A	0.223	0.114	1.175	0.057	0.077	0.000	0.315	0.181	20.461

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	54	57	282	74	0	817	66	66
N.S.	1	1.00	0.55	0.58	2.88	0.76	0.00	8.34	0.67	0.67
time (sec)	N/A	0.216	0.073	0.688	0.059	0.076	0.000	0.320	0.186	20.674

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	54	57	280	68	0	783	65	56
N.S.	1	1.00	0.61	0.65	3.18	0.77	0.00	8.90	0.74	0.64
time (sec)	N/A	0.215	0.063	0.615	0.054	0.077	0.000	0.244	0.189	20.378

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	44	36	30	53	39	51	0	40	32
N.S.	1	1.13	0.92	0.77	1.36	1.00	1.31	0.00	1.03	0.82
time (sec)	N/A	0.215	0.048	0.645	0.047	0.078	1.298	0.000	0.173	19.818

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	57	61	285	68	299	0	65	0
N.S.	1	1.00	0.60	0.64	3.00	0.72	3.15	0.00	0.68	0.00
time (sec)	N/A	0.222	0.096	1.194	0.055	0.083	4.901	0.000	0.173	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	153	120	325	1007	127	0	18053	187	122
N.S.	1	0.96	0.75	2.03	6.29	0.79	0.00	112.83	1.17	0.76
time (sec)	N/A	0.339	0.358	3.882	0.088	0.082	0.000	1.310	0.171	21.712

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	150	123	326	1015	129	0	18069	187	122
N.S.	1	0.95	0.78	2.06	6.42	0.82	0.00	114.36	1.18	0.77
time (sec)	N/A	0.319	0.339	2.295	0.086	0.079	0.000	1.012	0.166	22.100

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	140	117	190	989	119	0	17458	183	114
N.S.	1	0.94	0.79	1.28	6.64	0.80	0.00	117.17	1.23	0.77
time (sec)	N/A	0.305	0.314	1.550	0.081	0.078	0.000	0.593	0.173	21.306

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	39	42	35	232	36	71	0	36	37
N.S.	1	0.93	1.00	0.83	5.52	0.86	1.69	0.00	0.86	0.88
time (sec)	N/A	0.226	0.042	2.249	0.066	0.075	1.431	0.000	0.168	19.627

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	150	122	235	994	119	770	0	185	0
N.S.	1	0.95	0.77	1.49	6.29	0.75	4.87	0.00	1.17	0.00
time (sec)	N/A	0.318	0.347	4.062	0.089	0.079	25.439	0.000	0.179	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	177	167	131	1078	144	0	15993	226	116
N.S.	1	0.93	0.87	0.69	5.64	0.75	0.00	83.73	1.18	0.61
time (sec)	N/A	0.327	0.302	4.299	0.085	0.081	0.000	0.608	0.175	20.730

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	80	51	46	93	59	100	0	68	50
N.S.	1	1.10	0.70	0.63	1.27	0.81	1.37	0.00	0.93	0.68
time (sec)	N/A	0.287	0.084	6.663	0.057	0.078	8.433	0.000	0.163	20.100

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	37	22	27	20	25	0	25	35	21
N.S.	1	1.28	0.76	0.93	0.69	0.86	0.00	0.86	1.21	0.72
time (sec)	N/A	0.200	0.017	0.219	0.033	0.072	0.000	0.118	0.166	20.994

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	A	C	F	C	B	B
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	106	0	0	82	60	0	267	59	131
N.S.	1	1.05	0.00	0.00	0.81	0.59	0.00	2.64	0.58	1.30
time (sec)	N/A	0.411	0.000	0.000	0.073	0.077	0.000	0.960	0.168	21.954

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	A	C	F	A	B	B
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	72	0	0	29	40	0	1	75	83
N.S.	1	1.16	0.00	0.00	0.47	0.65	0.00	0.02	1.21	1.34
time (sec)	N/A	0.295	0.000	0.000	0.057	0.068	0.000	0.188	0.172	20.659

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	A	C	F	C	B	B
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	129	0	0	172	107	0	498	95	143
N.S.	1	1.10	0.00	0.00	1.47	0.91	0.00	4.26	0.81	1.22
time (sec)	N/A	0.406	0.000	0.000	0.077	0.081	0.000	3.708	0.180	21.043

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	A	C	F	A	B	B
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	85	0	0	41	57	0	1	116	86
N.S.	1	1.25	0.00	0.00	0.60	0.84	0.00	0.01	1.71	1.26
time (sec)	N/A	0.309	0.000	0.000	0.062	0.072	0.000	0.435	0.167	20.209

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	231	158	218	195	128	0	1870	151	277
N.S.	1	1.02	0.70	0.96	0.86	0.57	0.00	8.27	0.67	1.23
time (sec)	N/A	0.404	0.991	125.530	0.102	0.078	0.000	9.583	0.171	22.371

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	A	C	F	F(-2)	B	B
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	138	0	0	106	84	0	0	229	158
N.S.	1	1.08	0.00	0.00	0.83	0.66	0.00	0.00	1.79	1.23
time (sec)	N/A	0.350	0.000	0.000	0.076	0.073	0.000	0.000	0.172	20.295

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	377	0	0	0	0	0	58	0
N.S.	1	1.00	3.43	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.308	3.487	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	181	0	84	0	0	18	23
N.S.	1	1.00	1.00	7.54	0.00	3.50	0.00	0.00	0.75	0.96
time (sec)	N/A	0.217	0.080	1.157	0.000	0.078	0.000	0.000	0.170	19.210

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	220	0	0	0	0	0	26	0
N.S.	1	1.00	2.02	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.298	0.858	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	61	54	247	0	107	0	0	29	56
N.S.	1	0.97	0.86	3.92	0.00	1.70	0.00	0.00	0.46	0.89
time (sec)	N/A	0.286	0.095	1.851	0.000	0.082	0.000	0.000	0.165	20.884

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	696	0	0	0	0	0	28	0
N.S.	1	1.00	6.33	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.299	8.332	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	61	58	280	0	113	0	0	31	65
N.S.	1	0.97	0.92	4.44	0.00	1.79	0.00	0.00	0.49	1.03
time (sec)	N/A	0.278	0.113	4.786	0.000	0.082	0.000	0.000	0.174	21.121

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	134	0	0	0	0	0	28	0
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.298	0.483	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	26	0	78	0	0	31	23
N.S.	1	1.00	1.00	1.08	0.00	3.25	0.00	0.00	1.29	0.96
time (sec)	N/A	0.212	0.075	0.322	0.000	0.077	0.000	0.000	0.171	19.604

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	431	0	0	0	0	0	28	0
N.S.	1	1.00	3.95	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.297	5.698	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	57	54	250	0	150	0	0	31	65
N.S.	1	0.97	0.92	4.24	0.00	2.54	0.00	0.00	0.53	1.10
time (sec)	N/A	0.281	0.129	0.700	0.000	0.076	0.000	0.000	0.167	20.478

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	188	0	0	0	0	0	28	0
N.S.	1	1.00	1.72	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.302	1.854	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	61	54	291	0	149	0	0	31	65
N.S.	1	0.97	0.86	4.62	0.00	2.37	0.00	0.00	0.49	1.03
time (sec)	N/A	0.276	0.120	0.897	0.000	0.082	0.000	0.000	0.160	20.359

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	82	0	187	39	0	0	30	48
N.S.	1	1.00	1.71	0.00	3.90	0.81	0.00	0.00	0.62	1.00
time (sec)	N/A	0.226	0.112	0.000	0.157	0.064	0.000	0.000	0.158	20.714

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	232	312	222	3537	273	0	224645	797	152
N.S.	1	0.87	1.17	0.83	13.30	1.03	0.00	844.53	3.00	0.57
time (sec)	N/A	0.407	2.834	75.805	0.254	0.092	0.000	6.070	0.175	21.893

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	182	292	307	2352	190	0	159584	477	140
N.S.	1	0.91	1.45	1.53	11.70	0.95	0.00	793.95	2.37	0.70
time (sec)	N/A	0.374	1.338	15.208	0.152	0.088	0.000	4.478	0.170	20.984

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	91	82	646	105	0	9145	155	82
N.S.	1	1.00	0.76	0.68	5.38	0.88	0.00	76.21	1.29	0.68
time (sec)	N/A	0.245	0.240	2.706	0.088	0.077	0.000	0.563	0.163	19.921

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	53	63	313	58	0	5162	61	70
N.S.	1	1.00	0.76	0.90	4.47	0.83	0.00	73.74	0.87	1.00
time (sec)	N/A	0.199	0.098	0.414	0.059	0.075	0.000	0.294	0.155	20.177

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	126	204	0	0	0	0	0	29	0
N.S.	1	0.97	1.57	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.345	1.641	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	126	436	0	0	0	0	0	72	0
N.S.	1	0.98	3.38	0.00	0.00	0.00	0.00	0.00	0.56	0.00
time (sec)	N/A	0.335	5.936	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	119	0	0	0	0	0	31	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.327	0.659	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	126	487	0	0	0	0	0	31	0
N.S.	1	0.97	3.75	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.331	3.674	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	126	205	0	0	0	0	0	31	0
N.S.	1	0.97	1.58	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.335	1.845	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	181	170	0	0	0	0	0	88	0
N.S.	1	1.26	1.18	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.380	1.380	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	141	0	0	0	0	0	65	0
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.330	0.674	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	143	0	0	0	0	0	60	0
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	0.54	0.00
time (sec)	N/A	0.310	0.516	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	132	37	88	30	37	37	13	36
N.S.	1	1.00	2.81	0.79	1.87	0.64	0.79	0.79	0.28	0.77
time (sec)	N/A	0.237	0.028	0.876	0.050	0.063	0.115	0.178	0.156	19.160

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	42	66	33	149	42	61	26	13	36
N.S.	1	0.98	1.53	0.77	3.47	0.98	1.42	0.60	0.30	0.84
time (sec)	N/A	0.210	0.015	0.659	0.147	0.074	0.113	0.185	0.158	19.636

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	114	26	70	21	26	28	11	25
N.S.	1	1.00	3.45	0.79	2.12	0.64	0.79	0.85	0.33	0.76
time (sec)	N/A	0.211	0.019	0.504	0.037	0.065	0.113	0.187	0.151	19.608

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	42	22	122	33	27	17	9	25
N.S.	1	1.00	1.56	0.81	4.52	1.22	1.00	0.63	0.33	0.93
time (sec)	N/A	0.176	0.007	0.344	0.143	0.071	0.114	0.147	0.158	19.478

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	17	10	16	17	73	15	16
N.S.	1	1.00	1.00	1.21	0.71	1.14	1.21	5.21	1.07	1.14
time (sec)	N/A	0.184	0.018	0.146	0.032	0.068	0.156	0.124	0.155	21.337

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	44	24	127	39	27	19	13	27
N.S.	1	1.00	1.52	0.83	4.38	1.34	0.93	0.66	0.45	0.93
time (sec)	N/A	0.192	0.020	0.359	0.132	0.073	0.138	0.252	0.155	19.700

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	132	36	94	37	39	36	13	35
N.S.	1	1.00	3.77	1.03	2.69	1.06	1.11	1.03	0.37	1.00
time (sec)	N/A	0.214	0.026	0.514	0.035	0.067	0.206	0.180	0.156	19.532

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	46	70	35	156	53	53	28	13	40
N.S.	1	1.02	1.56	0.78	3.47	1.18	1.18	0.62	0.29	0.89
time (sec)	N/A	0.209	0.019	0.752	0.136	0.071	0.184	0.178	0.160	19.445

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	67	155	52	217	64	54	261	35	51
N.S.	1	1.06	2.46	0.83	3.44	1.02	0.86	4.14	0.56	0.81
time (sec)	N/A	0.268	0.133	1.083	0.040	0.065	0.181	0.346	0.162	19.756

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	79	100	48	254	86	66	141	35	52
N.S.	1	1.32	1.67	0.80	4.23	1.43	1.10	2.35	0.58	0.87
time (sec)	N/A	0.264	0.087	0.748	0.137	0.079	0.175	0.388	0.168	19.375

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	53	135	42	185	54	42	221	33	41
N.S.	1	1.04	2.65	0.82	3.63	1.06	0.82	4.33	0.65	0.80
time (sec)	N/A	0.236	0.083	0.519	0.040	0.063	0.161	0.302	0.161	19.591

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	70	36	218	77	51	114	26	42
N.S.	1	1.00	1.52	0.78	4.74	1.67	1.11	2.48	0.57	0.91
time (sec)	N/A	0.211	0.061	0.482	0.134	0.071	0.159	0.213	0.168	19.101

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	28	17	17	30	22	17	15	16
N.S.	1	1.00	1.56	0.94	0.94	1.67	1.22	0.94	0.83	0.89
time (sec)	N/A	0.206	0.030	0.214	0.102	0.077	0.205	0.144	0.165	19.080

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	76	72	38	223	78	54	73	33	45
N.S.	1	1.95	1.85	0.97	5.72	2.00	1.38	1.87	0.85	1.15
time (sec)	N/A	0.249	0.080	0.378	0.134	0.075	0.205	0.309	0.172	20.189

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	150	51	0	74	61	178	36	56
N.S.	1	1.00	2.73	0.93	0.00	1.35	1.11	3.24	0.65	1.02
time (sec)	N/A	0.251	0.130	0.542	0.000	0.067	0.278	0.285	0.163	20.400

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	124	0	0	0	0	0	17	0
N.S.	1	1.00	1.75	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.261	0.167	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	122	86	0	0	0	0	0	95	0
N.S.	1	1.58	1.12	0.00	0.00	0.00	0.00	0.00	1.23	0.00
time (sec)	N/A	0.326	0.188	0.000	0.000	0.000	0.000	0.000	0.162	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	225	125	0	0	0	0	0	103	0
N.S.	1	1.79	0.99	0.00	0.00	0.00	0.00	0.00	0.82	0.00
time (sec)	N/A	0.446	0.205	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	330	0	0	0	0	0	11	0
N.S.	1	1.00	2.32	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.304	0.552	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	157	0	0	0	0	0	19	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.352	0.548	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	240	0	0	0	0	0	9	0
N.S.	1	1.00	2.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.276	0.388	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	240	0	0	0	0	0	11	0
N.S.	1	1.00	2.00	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.279	0.391	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	240	0	0	0	0	0	11	0
N.S.	1	1.00	2.00	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.278	0.397	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	107	146	0	0	0	0	0	20	0
N.S.	1	1.51	2.06	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.297	5.068	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	111	155	0	0	0	0	0	20	0
N.S.	1	1.48	2.07	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.307	4.409	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	105	146	0	0	0	0	0	18	0
N.S.	1	1.52	2.12	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.305	4.453	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	101	153	0	0	0	0	0	16	0
N.S.	1	1.51	2.28	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.289	7.234	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	25	30	24	35	44	0	30	38
N.S.	1	1.00	0.96	1.15	0.92	1.35	1.69	0.00	1.15	1.46
time (sec)	N/A	0.213	0.042	0.171	0.029	0.072	1.475	0.000	0.167	22.202

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	105	153	0	0	0	0	0	20	0
N.S.	1	1.48	2.15	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.308	3.050	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	105	147	0	0	0	0	0	20	0
N.S.	1	1.52	2.13	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.300	2.684	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	213	179	0	0	0	0	0	61	0
N.S.	1	1.34	1.13	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.471	5.243	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	217	189	0	0	0	0	0	61	0
N.S.	1	1.33	1.16	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.455	4.616	0.000	0.000	0.000	0.000	0.000	0.177	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	213	179	0	0	0	0	0	59	0
N.S.	1	1.34	1.13	0.00	0.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.456	4.675	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	207	185	0	0	0	0	0	51	0
N.S.	1	1.34	1.20	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.444	7.479	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	34	51	35	320	85	0	0	35	39
N.S.	1	1.17	1.76	1.21	11.03	2.93	0.00	0.00	1.21	1.34
time (sec)	N/A	0.227	0.077	0.160	0.053	0.074	0.000	0.000	0.165	21.730

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	212	184	0	0	0	0	0	54	0
N.S.	1	1.35	1.17	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.454	3.173	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	156	213	179	0	0	0	0	0	60	0
N.S.	1	1.37	1.15	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.456	2.888	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	41	38	41	1242	69	63	0	40	105
N.S.	1	0.95	0.88	0.95	28.88	1.60	1.47	0.00	0.93	2.44
time (sec)	N/A	0.272	0.130	0.250	0.066	0.074	0.852	0.000	0.188	22.515

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	47	62	44	2171	140	65	0	44	183
N.S.	1	1.04	1.38	0.98	48.24	3.11	1.44	0.00	0.98	4.07
time (sec)	N/A	0.283	0.077	0.357	0.094	0.074	1.943	0.000	0.165	27.957

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	62	65	57	4466	129	82	0	55	247
N.S.	1	0.93	0.97	0.85	66.66	1.93	1.22	0.00	0.82	3.69
time (sec)	N/A	0.345	0.102	0.645	0.131	0.076	4.095	0.000	0.165	24.253

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	137	186	0	0	0	0	0	24	0
N.S.	1	1.36	1.84	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.335	11.585	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	196	243	349	0	0	0	0	0	137	0
N.S.	1	1.24	1.78	0.00	0.00	0.00	0.00	0.00	0.70	0.00
time (sec)	N/A	0.508	15.001	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	351	410	642	0	0	0	0	0	186	0
N.S.	1	1.17	1.83	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.777	15.574	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	458	0	0	0	0	0	18	0
N.S.	1	1.00	2.41	0.00	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.461	1.025	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	205	0	0	0	0	0	26	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.492	0.903	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	194	121	139	0	526	0	0	31	79
N.S.	1	1.30	0.81	0.93	0.00	3.53	0.00	0.00	0.21	0.53
time (sec)	N/A	0.461	0.259	0.563	0.000	0.084	0.000	0.000	0.161	21.102

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	192	176	139	0	406	0	0	57	78
N.S.	1	1.30	1.19	0.94	0.00	2.74	0.00	0.00	0.39	0.53
time (sec)	N/A	0.439	0.221	0.466	0.000	0.084	0.000	0.000	0.179	21.607

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	170	82	122	0	365	0	0	18	131
N.S.	1	1.36	0.66	0.98	0.00	2.92	0.00	0.00	0.14	1.05
time (sec)	N/A	0.359	0.102	0.476	0.000	0.080	0.000	0.000	0.166	20.638

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	170	142	122	0	365	0	0	31	59
N.S.	1	1.37	1.15	0.98	0.00	2.94	0.00	0.00	0.25	0.48
time (sec)	N/A	0.376	0.137	0.467	0.000	0.080	0.000	0.000	0.171	20.713

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	192	105	139	0	506	0	0	31	79
N.S.	1	1.31	0.71	0.95	0.00	3.44	0.00	0.00	0.21	0.54
time (sec)	N/A	0.439	0.182	0.459	0.000	0.089	0.000	0.000	0.162	20.849

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	194	109	139	0	537	0	0	31	78
N.S.	1	1.29	0.73	0.93	0.00	3.58	0.00	0.00	0.21	0.52
time (sec)	N/A	0.446	0.209	0.459	0.000	0.089	0.000	0.000	0.188	22.559

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	137	39	131	32	39	50	13	38
N.S.	1	1.00	2.80	0.80	2.67	0.65	0.80	1.02	0.27	0.78
time (sec)	N/A	0.242	0.029	0.402	0.045	0.062	0.172	0.132	0.179	19.987

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	66	33	126	78	63	41	13	40
N.S.	1	1.00	1.53	0.77	2.93	1.81	1.47	0.95	0.30	0.93
time (sec)	N/A	0.214	0.016	0.378	0.038	0.068	0.118	0.152	0.173	19.422

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	118	28	109	23	27	41	11	27
N.S.	1	1.00	3.37	0.80	3.11	0.66	0.77	1.17	0.31	0.77
time (sec)	N/A	0.216	0.018	0.358	0.037	0.063	0.120	0.128	0.174	19.537

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	42	22	94	49	29	32	9	29
N.S.	1	1.00	1.56	0.81	3.48	1.81	1.07	1.19	0.33	1.07
time (sec)	N/A	0.179	0.007	0.263	0.035	0.071	0.105	0.133	0.200	20.544

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	16	14	17	10	18	17	75	31	21
N.S.	1	1.14	1.00	1.21	0.71	1.29	1.21	5.36	2.21	1.50
time (sec)	N/A	0.197	0.015	0.349	0.024	0.065	0.162	0.142	0.171	19.520

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	44	24	99	36	29	34	13	31
N.S.	1	1.00	1.52	0.83	3.41	1.24	1.00	1.17	0.45	1.07
time (sec)	N/A	0.199	0.018	0.194	0.041	0.067	0.138	0.131	0.189	19.935

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	136	38	135	39	39	49	13	37
N.S.	1	1.00	3.78	1.06	3.75	1.08	1.08	1.36	0.36	1.03
time (sec)	N/A	0.215	0.025	0.198	0.042	0.067	0.200	0.142	0.192	19.713

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	70	35	139	55	54	43	13	44
N.S.	1	1.00	1.56	0.78	3.09	1.22	1.20	0.96	0.29	0.98
time (sec)	N/A	0.213	0.020	0.211	0.043	0.072	0.174	0.138	0.184	19.933

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	71	162	54	345	70	54	139	15	55
N.S.	1	1.06	2.42	0.81	5.15	1.04	0.81	2.07	0.22	0.82
time (sec)	N/A	0.273	0.141	0.754	0.046	0.064	0.191	0.180	0.184	19.823

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	82	100	48	335	102	60	83	15	57
N.S.	1	1.32	1.61	0.77	5.40	1.65	0.97	1.34	0.24	0.92
time (sec)	N/A	0.270	0.101	0.501	0.048	0.074	0.198	0.175	0.167	20.655

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	57	142	44	290	61	42	118	13	45
N.S.	1	1.04	2.58	0.80	5.27	1.11	0.76	2.15	0.24	0.82
time (sec)	N/A	0.257	0.097	0.520	0.050	0.064	0.191	0.181	0.169	20.086

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	70	36	270	72	42	79	11	44
N.S.	1	1.00	1.46	0.75	5.62	1.50	0.88	1.65	0.23	0.92
time (sec)	N/A	0.217	0.059	0.634	0.044	0.065	0.154	0.145	0.194	19.283

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	34	17	19	34	20	32	14	16
N.S.	1	1.00	1.89	0.94	1.06	1.89	1.11	1.78	0.78	0.89
time (sec)	N/A	0.204	0.051	0.431	0.106	0.066	0.184	0.122	0.171	21.446

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	81	72	38	276	74	46	87	15	47
N.S.	1	1.98	1.76	0.93	6.73	1.80	1.12	2.12	0.37	1.15
time (sec)	N/A	0.253	0.088	0.930	0.051	0.069	0.227	0.187	0.190	21.841

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F(-2)	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	153	53	0	81	60	190	15	60
N.S.	1	1.00	2.68	0.93	0.00	1.42	1.05	3.33	0.26	1.05
time (sec)	N/A	0.248	0.176	1.490	0.000	0.067	0.284	0.165	0.194	22.355

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	103	0	0	0	0	0	17	0
N.S.	1	1.00	1.47	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.252	0.214	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	122	84	0	0	0	0	0	19	0
N.S.	1	1.58	1.09	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.321	0.160	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	226	122	0	0	0	0	0	19	0
N.S.	1	1.78	0.96	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.435	0.198	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	330	0	0	0	0	0	58	0
N.S.	1	1.00	2.32	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.298	0.482	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	157	0	0	0	0	0	76	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.47	0.00
time (sec)	N/A	0.354	0.554	0.000	0.000	0.000	0.000	0.000	0.211	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	238	0	0	0	0	0	46	0
N.S.	1	1.00	1.98	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.280	0.362	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	238	0	0	0	0	0	58	0
N.S.	1	1.00	1.98	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.271	0.366	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	238	0	0	0	0	0	58	0
N.S.	1	1.00	1.98	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.269	0.364	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	106	220	0	0	0	0	0	20	0
N.S.	1	1.51	3.14	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.298	3.956	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	110	229	0	0	0	0	0	20	0
N.S.	1	1.49	3.09	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.296	4.130	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	104	219	0	0	0	0	0	18	0
N.S.	1	1.53	3.22	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.295	4.105	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	100	141	0	0	0	0	0	16	0
N.S.	1	1.52	2.14	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.283	7.419	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	27	25	30	24	35	46	0	51	37
N.S.	1	1.08	1.00	1.20	0.96	1.40	1.84	0.00	2.04	1.48
time (sec)	N/A	0.219	0.037	0.234	0.053	0.075	1.427	0.000	0.188	23.555

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	104	217	0	0	0	0	0	20	0
N.S.	1	1.49	3.10	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.306	3.469	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	104	211	0	0	0	0	0	20	0
N.S.	1	1.53	3.10	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.304	2.970	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	212	175	0	0	0	0	0	22	0
N.S.	1	1.34	1.11	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.453	3.575	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	216	185	0	0	0	0	0	22	0
N.S.	1	1.33	1.14	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.464	3.844	0.000	0.000	0.000	0.000	0.000	0.187	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	212	175	0	0	0	0	0	20	0
N.S.	1	1.34	1.11	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.445	3.870	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	206	178	0	0	0	0	0	18	0
N.S.	1	1.35	1.16	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.435	7.373	0.000	0.000	0.000	0.000	0.000	0.168	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	35	51	34	322	78	0	0	37	39
N.S.	1	1.17	1.70	1.13	10.73	2.60	0.00	0.00	1.23	1.30
time (sec)	N/A	0.229	0.113	0.264	0.103	0.077	0.000	0.000	0.166	24.921

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	156	211	181	0	0	0	0	0	22	0
N.S.	1	1.35	1.16	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.447	3.261	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	212	175	0	0	0	0	0	22	0
N.S.	1	1.37	1.13	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.457	2.881	0.000	0.000	0.000	0.000	0.000	0.169	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	42	1713	70	97	0	100	106
N.S.	1	1.00	1.00	0.95	38.93	1.59	2.20	0.00	2.27	2.41
time (sec)	N/A	0.291	0.120	0.508	0.284	0.075	4.528	0.000	0.178	22.482

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	46	46	44	2172	132	65	0	46	182
N.S.	1	1.05	1.05	1.00	49.36	3.00	1.48	0.00	1.05	4.14
time (sec)	N/A	0.285	0.096	0.746	0.192	0.072	1.426	0.000	0.170	28.503

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	63	63	57	5998	129	117	0	117	246
N.S.	1	0.95	0.95	0.86	90.88	1.95	1.77	0.00	1.77	3.73
time (sec)	N/A	0.360	0.098	1.380	0.253	0.077	20.703	0.000	0.209	24.457

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	136	182	0	0	0	0	0	24	0
N.S.	1	1.36	1.82	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.349	9.977	0.000	0.000	0.000	0.000	0.000	0.190	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	195	242	346	0	0	0	0	0	26	0
N.S.	1	1.24	1.77	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.496	13.404	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	350	409	639	0	0	0	0	0	26	0
N.S.	1	1.17	1.83	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.748	13.841	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	458	0	0	0	0	0	97	0
N.S.	1	1.00	2.41	0.00	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	0.453	0.915	0.000	0.000	0.000	0.000	0.000	0.217	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	205	0	0	0	0	0	115	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.55	0.00
time (sec)	N/A	0.489	0.887	0.000	0.000	0.000	0.000	0.000	0.226	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	194	121	139	0	605	0	0	31	79
N.S.	1	1.29	0.81	0.93	0.00	4.03	0.00	0.00	0.21	0.53
time (sec)	N/A	0.466	0.317	0.764	0.000	0.084	0.000	0.000	0.178	20.844

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	192	177	139	0	504	0	0	57	80
N.S.	1	1.31	1.20	0.95	0.00	3.43	0.00	0.00	0.39	0.54
time (sec)	N/A	0.445	0.252	0.562	0.000	0.082	0.000	0.000	0.173	20.717

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	170	82	122	0	463	0	0	18	58
N.S.	1	1.37	0.66	0.98	0.00	3.73	0.00	0.00	0.15	0.47
time (sec)	N/A	0.369	0.112	0.564	0.000	0.080	0.000	0.000	0.191	20.019

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	170	142	122	0	463	0	0	31	57
N.S.	1	1.36	1.14	0.98	0.00	3.70	0.00	0.00	0.25	0.46
time (sec)	N/A	0.373	0.151	0.590	0.000	0.082	0.000	0.000	0.172	20.631

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	192	105	139	0	623	0	0	31	79
N.S.	1	1.30	0.71	0.94	0.00	4.21	0.00	0.00	0.21	0.53
time (sec)	N/A	0.456	0.162	0.563	0.000	0.089	0.000	0.000	0.170	20.821

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	194	109	139	0	626	0	0	31	80
N.S.	1	1.30	0.73	0.93	0.00	4.20	0.00	0.00	0.21	0.54
time (sec)	N/A	0.435	0.263	0.567	0.000	0.085	0.000	0.000	0.165	21.872

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	0	0	0	0	0	647	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	7.44	0.00
time (sec)	N/A	0.280	0.721	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	82	0	0	0	0	0	630	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	7.24	0.00
time (sec)	N/A	0.280	0.697	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	89	84	0	0	0	0	0	622	0
N.S.	1	1.05	0.99	0.00	0.00	0.00	0.00	0.00	7.32	0.00
time (sec)	N/A	0.271	0.604	0.000	0.000	0.000	0.000	0.000	0.234	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	32	31	43	51	0	44	66
N.S.	1	1.00	1.00	1.68	1.63	2.26	2.68	0.00	2.32	3.47
time (sec)	N/A	0.209	0.035	0.438	0.072	0.077	1.055	0.000	0.208	21.466

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	85	0	0	0	0	0	698	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	8.02	0.00
time (sec)	N/A	0.289	0.490	0.000	0.000	0.000	0.000	0.000	0.205	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	81	0	0	0	0	0	699	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	8.03	0.00
time (sec)	N/A	0.291	0.487	0.000	0.000	0.000	0.000	0.000	0.195	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	160	0	0	0	0	0	0	0
N.S.	1	1.00	1.84	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.300	3.910	0.000	0.000	0.000	0.000	0.000	0.260	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	91	149	0	0	0	0	0	0	0
N.S.	1	1.15	1.89	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.294	3.724	0.000	0.000	0.000	0.000	0.000	0.278	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	89	147	0	0	0	0	0	0	0
N.S.	1	1.05	1.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.281	4.501	0.000	0.000	0.000	0.000	0.000	0.261	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	165	33	0	0	31	29
N.S.	1	1.00	1.00	1.06	9.17	1.83	0.00	0.00	1.72	1.61
time (sec)	N/A	0.224	0.077	1.519	0.080	0.066	0.000	0.000	0.178	21.294

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	160	0	0	0	0	0	0	0
N.S.	1	1.00	1.84	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.303	2.767	0.000	0.000	0.000	0.000	0.000	0.326	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	150	0	0	0	0	0	0	0
N.S.	1	1.00	1.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.293	2.680	0.000	0.000	0.000	0.000	0.000	0.274	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	118	0	0	0	0	0	0	0
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.286	4.674	0.000	0.000	0.000	0.000	0.000	0.490	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	89	120	0	0	0	0	0	0	0
N.S.	1	1.05	1.41	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.281	4.285	0.000	0.000	0.000	0.000	0.000	0.483	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	53	55	59	0	100	0	0	138	178
N.S.	1	0.96	1.00	1.07	0.00	1.82	0.00	0.00	2.51	3.24
time (sec)	N/A	0.284	0.064	5.544	0.000	0.080	0.000	0.000	0.164	23.174

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	123	0	0	0	0	0	0	0
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.298	4.399	0.000	0.000	0.000	0.000	0.000	0.524	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	119	0	0	0	0	0	0	0
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.296	4.545	0.000	0.000	0.000	0.000	0.000	0.528	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	91	204	0	0	0	0	0	0	0
N.S.	1	1.15	2.58	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.289	9.044	0.000	0.000	0.000	0.000	0.000	0.688	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	89	213	0	0	0	0	0	0	0
N.S.	1	1.05	2.51	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.278	7.495	0.000	0.000	0.000	0.000	0.000	0.677	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	39	36	37	1323	52	0	0	66	49
N.S.	1	0.93	0.86	0.88	31.50	1.24	0.00	0.00	1.57	1.17
time (sec)	N/A	0.234	0.096	17.289	0.136	0.071	0.000	0.000	0.181	30.112

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	215	0	0	0	0	0	0	0
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.289	6.751	0.000	0.000	0.000	0.000	0.000	0.793	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	203	0	0	0	0	0	0	0
N.S.	1	1.00	2.57	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.294	6.601	0.000	0.000	0.000	0.000	0.000	0.771	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	175	29	30	1696	47	0	0	295	87
N.S.	1	4.27	0.71	0.73	41.37	1.15	0.00	0.00	7.20	2.12
time (sec)	N/A	0.326	1.068	30.211	0.774	0.071	0.000	0.000	0.162	21.030

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	C	F(-1)	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	151	198	97	976	81	0	834	23	176
N.S.	1	1.37	1.80	0.88	8.87	0.74	0.00	7.58	0.21	1.60
time (sec)	N/A	0.460	1.407	302.691	0.253	0.073	0.000	11.285	0.175	25.439

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	B	A	F	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	127	209	139	55	0	0	139	46
N.S.	1	1.00	2.82	4.64	3.09	1.22	0.00	0.00	3.09	1.02
time (sec)	N/A	0.253	0.113	0.156	0.112	0.065	0.000	0.000	0.178	22.281

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	48	137	33	151	55	0	74	52	56
N.S.	1	0.83	2.36	0.57	2.60	0.95	0.00	1.28	0.90	0.97
time (sec)	N/A	0.240	0.086	119.793	0.104	0.064	0.000	1.022	0.177	22.109

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	139	49	162	57	0	83	58	39
N.S.	1	1.00	2.90	1.02	3.38	1.19	0.00	1.73	1.21	0.81
time (sec)	N/A	0.240	0.104	96.670	0.123	0.066	0.000	0.985	0.188	24.216

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	102	117	0	0	149	0	0	138	0
N.S.	1	1.07	1.23	0.00	0.00	1.57	0.00	0.00	1.45	0.00
time (sec)	N/A	0.327	1.185	0.000	0.000	0.075	0.000	0.000	0.185	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	111	117	0	0	149	0	0	142	0
N.S.	1	1.59	1.67	0.00	0.00	2.13	0.00	0.00	2.03	0.00
time (sec)	N/A	0.341	1.244	0.000	0.000	0.077	0.000	0.000	0.225	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	99	0	0	0	0	0	45	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.303	0.244	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	181	0	78	0	0	18	51
N.S.	1	1.00	1.00	3.35	0.00	1.44	0.00	0.00	0.33	0.94
time (sec)	N/A	0.286	0.117	0.805	0.000	0.079	0.000	0.000	0.154	19.808

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	415	0	0	0	0	0	67	0
N.S.	1	1.00	3.81	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.307	4.340	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	87	68	250	0	112	0	0	29	0
N.S.	1	0.98	0.76	2.81	0.00	1.26	0.00	0.00	0.33	0.00
time (sec)	N/A	0.375	0.139	0.977	0.000	0.078	0.000	0.000	0.154	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	124	0	0	0	0	0	71	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	0.65	0.00
time (sec)	N/A	0.301	0.884	0.000	0.000	0.000	0.000	0.000	0.247	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	91	69	291	0	145	0	0	31	0
N.S.	1	0.98	0.74	3.13	0.00	1.56	0.00	0.00	0.33	0.00
time (sec)	N/A	0.377	0.174	21.796	0.000	0.083	0.000	0.000	0.162	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	380	0	0	0	0	0	28	0
N.S.	1	1.00	3.45	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.303	3.233	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	181	0	84	0	0	31	0
N.S.	1	1.00	1.00	3.35	0.00	1.56	0.00	0.00	0.57	0.00
time (sec)	N/A	0.284	0.105	1.371	0.000	0.076	0.000	0.000	0.160	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	168	0	0	0	0	0	28	0
N.S.	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.301	1.017	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	91	72	247	0	107	0	0	31	0
N.S.	1	0.98	0.77	2.66	0.00	1.15	0.00	0.00	0.33	0.00
time (sec)	N/A	0.368	0.142	2.007	0.000	0.084	0.000	0.000	0.160	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	867	0	0	0	0	0	28	0
N.S.	1	1.00	7.88	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.300	7.943	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	91	83	280	0	113	0	0	31	0
N.S.	1	0.98	0.89	3.01	0.00	1.22	0.00	0.00	0.33	0.00
time (sec)	N/A	0.380	0.157	3.372	0.000	0.084	0.000	0.000	0.152	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	119	139	0	0	0	0	0	0	0
N.S.	1	1.17	1.36	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.325	5.626	0.000	0.000	0.000	0.000	0.000	3.626	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	119	198	0	0	0	0	0	0	0
N.S.	1	1.17	1.94	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.320	14.639	0.000	0.000	0.000	0.000	0.000	11.421	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	116	99	0	0	0	0	0	0	0
N.S.	1	1.13	0.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	1.245	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	126	182	0	0	0	0	0	86	0
N.S.	1	0.97	1.40	0.00	0.00	0.00	0.00	0.00	0.66	0.00
time (sec)	N/A	0.340	1.597	0.000	0.000	0.000	0.000	0.000	0.295	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	126	470	0	0	0	0	0	82	0
N.S.	1	0.97	3.62	0.00	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.340	7.532	0.000	0.000	0.000	0.000	0.000	0.250	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	119	0	0	0	0	0	60	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.337	0.576	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	126	437	0	0	0	0	0	31	0
N.S.	1	0.98	3.39	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.337	5.108	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	126	202	0	0	0	0	0	31	0
N.S.	1	0.97	1.55	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.336	1.767	0.000	0.000	0.000	0.000	0.000	0.179	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	170	169	0	0	0	0	0	73	0
N.S.	1	1.22	1.22	0.00	0.00	0.00	0.00	0.00	0.53	0.00
time (sec)	N/A	0.391	1.175	0.000	0.000	0.000	0.000	0.000	0.214	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	142	0	0	0	0	0	52	0
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.322	0.842	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	111	142	0	0	0	0	0	48	0
N.S.	1	1.04	1.33	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.301	0.687	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	88	82	0	0	0	0	0	17	0
N.S.	1	1.02	0.95	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.286	1.133	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	88	78	0	0	0	0	0	15	0
N.S.	1	1.02	0.91	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.284	1.075	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	90	80	0	0	0	0	0	13	0
N.S.	1	1.07	0.95	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.275	0.887	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	24	32	45	49	0	22	68
N.S.	1	1.00	1.00	1.20	1.60	2.25	2.45	0.00	1.10	3.40
time (sec)	N/A	0.206	0.039	0.387	0.029	0.080	1.057	0.000	0.162	24.084

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	88	82	0	0	0	0	0	17	0
N.S.	1	1.04	0.96	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.283	0.809	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	88	78	0	0	0	0	0	17	0
N.S.	1	1.04	0.92	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.283	0.833	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	88	146	0	0	0	0	0	15	0
N.S.	1	1.05	1.74	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.286	3.861	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	168	34	0	0	32	29
N.S.	1	1.00	1.00	1.05	8.84	1.79	0.00	0.00	1.68	1.53
time (sec)	N/A	0.220	0.082	0.812	0.045	0.071	0.000	0.000	0.166	23.528

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	90	117	0	0	0	0	0	15	0
N.S.	1	1.07	1.39	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.280	4.623	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	53	107	61	2168	110	0	0	64	177
N.S.	1	0.96	1.95	1.11	39.42	2.00	0.00	0.00	1.16	3.22
time (sec)	N/A	0.284	0.076	1.819	0.127	0.077	0.000	0.000	0.165	27.701

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	88	221	0	0	0	0	0	15	0
N.S.	1	1.05	2.63	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.289	9.404	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	37	56	36	1332	71	0	0	49	49
N.S.	1	0.86	1.30	0.84	30.98	1.65	0.00	0.00	1.14	1.14
time (sec)	N/A	0.240	0.079	4.661	0.066	0.068	0.000	0.000	0.155	34.456

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	172	30	80	1701	50	0	0	42	85
N.S.	1	4.10	0.71	1.90	40.50	1.19	0.00	0.00	1.00	2.02
time (sec)	N/A	0.330	0.460	14.750	0.271	0.072	0.000	0.000	0.155	23.727

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	C	F(-1)	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	152	79	116	974	83	0	839	23	171
N.S.	1	1.38	0.72	1.05	8.85	0.75	0.00	7.63	0.21	1.55
time (sec)	N/A	0.434	1.385	170.404	0.134	0.080	0.000	11.187	0.156	26.315

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	C	B	A	F	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	127	211	139	56	0	0	43	45
N.S.	1	1.00	2.59	4.31	2.84	1.14	0.00	0.00	0.88	0.92
time (sec)	N/A	0.249	0.137	0.164	0.048	0.063	0.000	0.000	0.156	22.736

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	51	137	67	153	57	0	74	17	55
N.S.	1	0.88	2.36	1.16	2.64	0.98	0.00	1.28	0.29	0.95
time (sec)	N/A	0.238	0.108	92.029	0.084	0.064	0.000	1.024	0.150	22.724

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	137	67	162	57	0	83	19	38
N.S.	1	1.00	2.69	1.31	3.18	1.12	0.00	1.63	0.37	0.75
time (sec)	N/A	0.239	0.108	109.371	0.079	0.065	0.000	1.004	0.160	24.416

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	103	155	0	0	150	0	0	32	0
N.S.	1	1.07	1.61	0.00	0.00	1.56	0.00	0.00	0.33	0.00
time (sec)	N/A	0.330	1.226	0.000	0.000	0.074	0.000	0.000	0.176	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	112	128	0	0	150	0	0	37	0
N.S.	1	1.58	1.80	0.00	0.00	2.11	0.00	0.00	0.52	0.00
time (sec)	N/A	0.338	1.957	0.000	0.000	0.074	0.000	0.000	0.188	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	115	0	0	0	0	0	14	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.304	0.313	0.000	0.000	0.000	0.000	0.000	0.153	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	59	58	102	0	78	0	0	18	89
N.S.	1	0.98	0.97	1.70	0.00	1.30	0.00	0.00	0.30	1.48
time (sec)	N/A	0.299	0.101	0.658	0.000	0.079	0.000	0.000	0.151	22.146

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	411	0	0	0	0	0	26	0
N.S.	1	1.00	3.77	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.308	4.544	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	92	72	190	0	111	0	0	29	0
N.S.	1	0.97	0.76	2.00	0.00	1.17	0.00	0.00	0.31	0.00
time (sec)	N/A	0.369	0.120	0.677	0.000	0.083	0.000	0.000	0.160	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	174	0	0	0	0	0	28	0
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.299	1.124	0.000	0.000	0.000	0.000	0.000	0.150	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	96	73	131	0	145	0	0	31	0
N.S.	1	0.97	0.74	1.32	0.00	1.46	0.00	0.00	0.31	0.00
time (sec)	N/A	0.364	0.160	0.779	0.000	0.082	0.000	0.000	0.162	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	377	0	0	0	0	0	28	0
N.S.	1	1.00	3.43	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.303	3.068	0.000	0.000	0.000	0.000	0.000	0.158	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	59	58	129	0	82	0	0	31	0
N.S.	1	0.98	0.97	2.15	0.00	1.37	0.00	0.00	0.52	0.00
time (sec)	N/A	0.291	0.095	0.687	0.000	0.081	0.000	0.000	0.161	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	186	0	0	0	0	0	28	0
N.S.	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	0.26	0.00
time (sec)	N/A	0.290	1.494	0.000	0.000	0.000	0.000	0.000	0.150	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	96	76	131	0	107	0	0	31	0
N.S.	1	0.97	0.77	1.32	0.00	1.08	0.00	0.00	0.31	0.00
time (sec)	N/A	0.364	0.145	0.686	0.000	0.083	0.000	0.000	0.162	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	579	0	0	0	0	0	28	0
N.S.	1	1.00	5.26	0.00	0.00	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.297	6.907	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	96	88	205	0	130	0	0	31	0
N.S.	1	0.97	0.89	2.07	0.00	1.31	0.00	0.00	0.31	0.00
time (sec)	N/A	0.361	0.155	0.681	0.000	0.084	0.000	0.000	0.164	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	138	367	0	0	0	0	0	26	0
N.S.	1	1.13	3.01	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.345	1.687	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	136	225	0	0	0	0	0	26	0
N.S.	1	1.14	1.89	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.338	13.017	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	134	181	0	0	0	0	0	24	0
N.S.	1	1.09	1.47	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.329	0.295	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	126	165	0	0	0	0	0	31	0
N.S.	1	0.97	1.27	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.341	2.179	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	126	466	0	0	0	0	0	29	0
N.S.	1	0.97	3.58	0.00	0.00	0.00	0.00	0.00	0.22	0.00
time (sec)	N/A	0.332	7.177	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	138	0	0	0	0	0	18	0
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.324	0.675	0.000	0.000	0.000	0.000	0.000	0.151	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	126	441	0	0	0	0	0	31	0
N.S.	1	0.98	3.42	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.335	5.467	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	126	218	0	0	0	0	0	31	0
N.S.	1	0.97	1.68	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.333	1.598	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	170	169	0	0	0	0	0	26	0
N.S.	1	1.22	1.22	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.371	1.180	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	142	0	0	0	0	0	17	0
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	0.326	0.918	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	111	142	0	0	0	0	0	15	0
N.S.	1	1.04	1.33	0.00	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	0.306	0.741	0.000	0.000	0.000	0.000	0.000	0.160	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [180] had the largest ratio of [.7368419999999999997]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	15	0.067
2	A	1	1	1.00	13	0.077
3	A	1	1	1.00	11	0.091
4	A	4	3	1.00	15	0.200
5	A	1	1	1.00	15	0.067
6	A	1	1	1.00	15	0.067
7	A	2	2	1.00	17	0.118
8	A	2	2	1.00	15	0.133
9	A	2	2	1.00	13	0.154
10	A	5	4	1.13	17	0.235
11	A	2	2	1.00	17	0.118
12	A	2	2	1.00	17	0.118
13	A	2	2	0.96	17	0.118
14	A	2	2	0.96	15	0.133
15	A	2	2	0.95	13	0.154
16	A	5	4	0.86	17	0.235
17	A	2	2	0.96	17	0.118
18	A	2	2	0.96	17	0.118
19	A	3	3	0.95	17	0.176
20	A	3	3	0.92	15	0.200
21	A	3	3	0.93	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	7	6	1.10	17	0.353
23	A	3	3	0.94	17	0.176
24	A	3	3	0.92	17	0.176
25	A	3	2	0.95	7	0.286
26	A	4	3	0.92	28	0.107
27	A	4	3	0.91	24	0.125
28	A	4	3	0.91	22	0.136
29	A	4	3	0.95	19	0.158
30	A	1	1	1.00	6	0.167
31	A	4	3	0.91	23	0.130
32	A	4	3	0.91	24	0.125
33	A	4	3	1.11	33	0.091
34	A	4	3	1.21	28	0.107
35	A	4	3	1.17	23	0.130
36	A	4	3	1.26	24	0.125
37	A	1	1	1.00	8	0.125
38	A	4	3	1.19	28	0.107
39	A	4	3	1.18	25	0.120
40	A	2	2	1.02	33	0.061
41	A	4	3	0.83	25	0.120
42	A	4	3	0.82	26	0.115
43	A	4	3	0.86	24	0.125
44	A	1	1	1.00	8	0.125
45	A	4	3	0.82	28	0.107
46	A	4	3	0.82	28	0.107
47	A	4	3	0.89	28	0.107
48	A	4	3	0.85	15	0.200
49	A	4	3	1.01	30	0.100
50	A	4	3	1.02	17	0.176
51	A	4	3	0.83	30	0.100
52	A	4	3	0.84	17	0.176
53	A	4	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	4	3	1.00	15	0.200
55	A	4	3	0.97	19	0.158
56	A	4	3	1.00	19	0.158
57	A	4	3	1.00	19	0.158
58	A	4	3	1.00	17	0.176
59	A	4	3	1.00	15	0.200
60	A	6	5	0.96	19	0.263
61	A	4	3	1.00	19	0.158
62	A	4	3	1.00	19	0.158
63	A	4	3	1.00	15	0.200
64	A	4	3	0.97	19	0.158
65	A	4	3	1.00	15	0.200
66	A	6	5	0.95	19	0.263
67	A	4	3	1.00	15	0.200
68	A	6	5	0.96	19	0.263
69	A	4	3	1.00	15	0.200
70	A	3	3	0.88	21	0.143
71	A	2	2	0.91	21	0.095
72	A	2	2	1.00	21	0.095
73	A	1	1	1.00	19	0.053
74	A	4	3	0.97	23	0.130
75	A	4	3	0.97	23	0.130
76	A	4	3	1.00	23	0.130
77	A	4	3	0.97	23	0.130
78	A	4	3	0.97	23	0.130
79	A	4	3	1.26	21	0.143
80	A	4	3	1.00	17	0.176
81	A	4	3	1.00	15	0.200
82	A	4	3	1.00	13	0.231
83	A	4	3	1.00	17	0.176
84	A	4	3	1.00	17	0.176
85	A	4	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	1	1	1.00	15	0.067
87	A	1	1	1.00	13	0.077
88	A	1	1	1.00	11	0.091
89	A	4	3	1.00	15	0.200
90	A	1	1	1.00	15	0.067
91	A	2	2	1.00	17	0.118
92	A	2	2	1.00	15	0.133
93	A	2	2	1.00	13	0.154
94	A	5	4	1.13	17	0.235
95	A	2	2	1.00	17	0.118
96	A	2	2	0.96	17	0.118
97	A	2	2	0.95	15	0.133
98	A	2	2	0.94	13	0.154
99	A	5	4	0.93	17	0.235
100	A	2	2	0.95	17	0.118
101	A	3	3	0.93	13	0.231
102	A	7	6	1.10	17	0.353
103	A	3	2	1.28	7	0.286
104	A	4	3	1.05	28	0.107
105	A	4	3	1.16	19	0.158
106	A	4	3	1.10	33	0.091
107	A	4	3	1.25	24	0.125
108	A	2	2	1.02	33	0.061
109	A	4	3	1.08	24	0.125
110	A	4	3	1.00	15	0.200
111	A	4	3	1.00	19	0.158
112	A	4	3	1.00	15	0.200
113	A	6	5	0.97	19	0.263
114	A	4	3	1.00	15	0.200
115	A	6	5	0.97	19	0.263
116	A	4	3	1.00	15	0.200
117	A	4	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	4	3	1.00	15	0.200
119	A	6	5	0.97	19	0.263
120	A	4	3	1.00	15	0.200
121	A	6	5	0.97	19	0.263
122	A	4	3	1.00	15	0.200
123	A	3	3	0.87	17	0.176
124	A	2	2	0.91	17	0.118
125	A	2	2	1.00	17	0.118
126	A	1	1	1.00	15	0.067
127	A	4	3	0.97	19	0.158
128	A	4	3	0.98	19	0.158
129	A	4	3	1.00	19	0.158
130	A	4	3	0.97	19	0.158
131	A	4	3	0.97	19	0.158
132	A	4	3	1.26	21	0.143
133	A	4	3	1.00	15	0.200
134	A	4	3	1.00	13	0.231
135	A	7	6	1.00	13	0.462
136	A	5	5	0.98	13	0.385
137	A	7	6	1.00	11	0.545
138	A	4	4	1.00	9	0.444
139	A	4	3	1.00	13	0.231
140	A	4	4	1.00	13	0.308
141	A	6	5	1.00	13	0.385
142	A	5	5	1.02	13	0.385
143	A	7	6	1.06	15	0.400
144	A	7	7	1.32	15	0.467
145	A	7	6	1.04	13	0.462
146	A	4	4	1.00	11	0.364
147	A	5	4	1.00	15	0.267
148	A	5	5	1.95	15	0.333
149	A	6	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	5	4	1.00	15	0.267
151	A	8	7	1.58	17	0.412
152	A	10	9	1.79	17	0.529
153	A	4	4	1.00	9	0.444
154	A	4	4	1.00	15	0.267
155	A	4	4	1.00	7	0.571
156	A	4	4	1.00	9	0.444
157	A	4	4	1.00	9	0.444
158	A	5	4	1.51	17	0.235
159	A	5	4	1.48	17	0.235
160	A	5	4	1.52	15	0.267
161	A	5	4	1.51	13	0.308
162	A	4	3	1.00	17	0.176
163	A	5	4	1.48	17	0.235
164	A	5	4	1.52	17	0.235
165	A	7	6	1.34	19	0.316
166	A	7	6	1.33	19	0.316
167	A	7	6	1.34	17	0.353
168	A	7	6	1.34	15	0.400
169	A	5	4	1.17	19	0.211
170	A	7	6	1.35	19	0.316
171	A	7	6	1.37	19	0.316
172	A	6	5	0.95	17	0.294
173	A	7	6	1.04	17	0.353
174	A	8	7	0.93	17	0.412
175	A	5	4	1.36	19	0.211
176	A	7	6	1.24	21	0.286
177	A	9	8	1.17	21	0.381
178	A	6	5	1.00	15	0.333
179	A	6	5	1.00	21	0.238
180	A	15	14	1.30	19	0.737
181	A	15	14	1.30	19	0.737

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	13	12	1.36	19	0.632
183	A	13	12	1.37	19	0.632
184	A	15	14	1.31	19	0.737
185	A	15	14	1.29	19	0.737
186	A	7	6	1.00	13	0.462
187	A	6	6	1.00	13	0.462
188	A	7	6	1.00	11	0.545
189	A	4	4	1.00	9	0.444
190	A	5	4	1.14	13	0.308
191	A	4	4	1.00	13	0.308
192	A	6	5	1.00	13	0.385
193	A	6	6	1.00	13	0.462
194	A	7	6	1.06	15	0.400
195	A	7	7	1.32	15	0.467
196	A	7	6	1.04	13	0.462
197	A	4	4	1.00	11	0.364
198	A	5	4	1.00	15	0.267
199	A	5	5	1.98	15	0.333
200	A	6	5	1.00	15	0.333
201	A	5	4	1.00	15	0.267
202	A	8	7	1.58	17	0.412
203	A	10	9	1.78	17	0.529
204	A	4	4	1.00	9	0.444
205	A	4	4	1.00	15	0.267
206	A	4	4	1.00	7	0.571
207	A	4	4	1.00	9	0.444
208	A	4	4	1.00	9	0.444
209	A	5	4	1.51	17	0.235
210	A	5	4	1.49	17	0.235
211	A	5	4	1.53	15	0.267
212	A	5	4	1.52	13	0.308
213	A	5	4	1.08	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	5	4	1.49	17	0.235
215	A	5	4	1.53	17	0.235
216	A	7	6	1.34	19	0.316
217	A	7	6	1.33	19	0.316
218	A	7	6	1.34	17	0.353
219	A	7	6	1.35	15	0.400
220	A	5	4	1.17	19	0.211
221	A	7	6	1.35	19	0.316
222	A	7	6	1.37	19	0.316
223	A	9	8	1.00	17	0.471
224	A	7	6	1.05	17	0.353
225	A	13	12	0.95	17	0.706
226	A	5	4	1.36	19	0.211
227	A	7	6	1.24	21	0.286
228	A	9	8	1.17	21	0.381
229	A	6	5	1.00	15	0.333
230	A	6	5	1.00	21	0.238
231	A	15	14	1.29	19	0.737
232	A	15	14	1.31	19	0.737
233	A	13	12	1.37	19	0.632
234	A	13	12	1.36	19	0.632
235	A	15	14	1.30	19	0.737
236	A	15	14	1.30	19	0.737
237	A	4	3	1.00	15	0.200
238	A	4	3	1.00	13	0.231
239	A	4	3	1.05	11	0.273
240	A	4	3	1.00	15	0.200
241	A	4	3	1.00	15	0.200
242	A	4	3	1.00	15	0.200
243	A	4	3	1.00	17	0.176
244	A	4	3	1.15	15	0.200
245	A	4	3	1.05	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	5	4	1.00	17	0.235
247	A	4	3	1.00	17	0.176
248	A	4	3	1.00	17	0.176
249	A	4	3	1.00	15	0.200
250	A	4	3	1.05	13	0.231
251	A	6	5	0.96	17	0.294
252	A	4	3	1.00	17	0.176
253	A	4	3	1.00	17	0.176
254	A	4	3	1.15	15	0.200
255	A	4	3	1.05	13	0.231
256	A	5	4	0.93	17	0.235
257	A	4	3	1.00	17	0.176
258	A	4	3	1.00	17	0.176
259	C	1	1	4.27	44	0.023
260	C	4	3	1.37	31	0.097
261	A	4	3	1.00	17	0.176
262	A	4	3	0.83	17	0.176
263	A	4	3	1.00	17	0.176
264	A	4	3	1.07	23	0.130
265	A	4	3	1.59	23	0.130
266	A	4	3	1.00	15	0.200
267	A	6	5	1.00	19	0.263
268	A	4	3	1.00	15	0.200
269	A	8	7	0.98	19	0.368
270	A	4	3	1.00	15	0.200
271	A	8	7	0.98	19	0.368
272	A	4	3	1.00	15	0.200
273	A	6	5	1.00	19	0.263
274	A	4	3	1.00	15	0.200
275	A	8	7	0.98	19	0.368
276	A	4	3	1.00	15	0.200
277	A	8	7	0.98	19	0.368

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	4	3	1.17	17	0.176
279	A	4	3	1.17	17	0.176
280	A	4	3	1.13	15	0.200
281	A	4	3	0.97	19	0.158
282	A	4	3	0.97	19	0.158
283	A	4	3	1.00	19	0.158
284	A	4	3	0.98	19	0.158
285	A	4	3	0.97	19	0.158
286	A	4	3	1.22	21	0.143
287	A	4	3	1.00	15	0.200
288	A	4	3	1.04	13	0.231
289	A	4	3	1.02	15	0.200
290	A	4	3	1.02	13	0.231
291	A	4	3	1.07	11	0.273
292	A	4	3	1.00	15	0.200
293	A	4	3	1.04	15	0.200
294	A	4	3	1.04	15	0.200
295	A	4	3	1.05	13	0.231
296	A	5	4	1.00	17	0.235
297	A	4	3	1.07	13	0.231
298	A	6	5	0.96	17	0.294
299	A	4	3	1.05	13	0.231
300	A	5	4	0.86	17	0.235
301	C	1	1	4.10	44	0.023
302	C	4	3	1.38	31	0.097
303	A	4	3	1.00	17	0.176
304	A	4	3	0.88	17	0.176
305	A	4	3	1.00	17	0.176
306	A	4	3	1.07	23	0.130
307	A	4	3	1.58	23	0.130
308	A	4	3	1.00	15	0.200
309	A	6	5	0.98	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	4	3	1.00	15	0.200
311	A	8	7	0.97	19	0.368
312	A	4	3	1.00	15	0.200
313	A	8	7	0.97	19	0.368
314	A	4	3	1.00	15	0.200
315	A	6	5	0.98	19	0.263
316	A	4	3	1.00	15	0.200
317	A	8	7	0.97	19	0.368
318	A	4	3	1.00	15	0.200
319	A	8	7	0.97	19	0.368
320	A	4	3	1.13	21	0.143
321	A	4	3	1.14	21	0.143
322	A	4	3	1.09	19	0.158
323	A	4	3	0.97	19	0.158
324	A	4	3	0.97	19	0.158
325	A	4	3	1.00	19	0.158
326	A	4	3	0.98	19	0.158
327	A	4	3	0.97	19	0.158
328	A	4	3	1.22	21	0.143
329	A	4	3	1.00	15	0.200
330	A	4	3	1.04	13	0.231

CHAPTER 3

Listing of integrals

3.1 $\int x^2 \sin(a + b\log(cx^n))\, dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int x^2 \sin(a + b\log(cx^n))\, dx = -\frac{bnx^3 \cos(a + b\log(cx^n))}{9 + b^2n^2} + \frac{3x^3 \sin(a + b\log(cx^n))}{9 + b^2n^2}$$

output
```
-b*n*x^3*cos(a+b*ln(c*x^n))/(b^2*n^2+9)+3*x^3*sin(a+b*ln(c*x^n))/(b^2*n^2+
9)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int x^2 \sin(a + b\log(cx^n))\, dx = -\frac{x^3(bn\cos(a + b\log(cx^n)) - 3\sin(a + b\log(cx^n)))}{9 + b^2n^2}$$

input
```
Integrate[x^2*Sin[a + b*Log[c*x^n]],x]
```

output
```
-((x^3*(b*n*Cos[a + b*Log[c*x^n]] - 3*Sin[a + b*Log[c*x^n]]))/(9 + b^2*n^2
))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {4988}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin(a + b\log(cx^n))\,dx$$

$$\Big\downarrow \; 4988$$

$$\frac{3x^3 \sin(a + b\log(cx^n))}{b^2n^2 + 9} - \frac{bnx^3 \cos(a + b\log(cx^n))}{b^2n^2 + 9}$$

input
```
Int[x^2*Sin[a + b*Log[c*x^n]],x]
```

output
```
-((b*n*x^3*Cos[a + b*Log[c*x^n]])/(9 + b^2*n^2)) + (3*x^3*Sin[a + b*Log[c*
x^n]])/(9 + b^2*n^2)
```

Defintions of rubi rules used

rule 4988
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(57) = 114$.

Time = 0.92 (sec) , antiderivative size = 479, normalized size of antiderivative = 8.40

method	result
parts	$-\frac{x^2 b\, \mathrm{e}^{\frac{\ln(c\,x^n)}{n}-\frac{\ln(c)}{n}} \cos(a+b\ln(c\,x^n))}{n\left(\frac{1}{n^2}+b^2\right)}+\frac{x^2 \mathrm{e}^{\frac{\ln(c\,x^n)}{n}-\frac{\ln(c)}{n}} \sin(a+b\ln(c\,x^n))}{n^2\left(\frac{1}{n^2}+b^2\right)}-\frac{2\left(\frac{n\left(\frac{bn\,c^{-\frac{1}{n}}\mathrm{e}^{\frac{\ln(c\,x^n)-n\ln(x)}{n}}x^3\tan\left(\frac{a}{2}+\frac{b\ln(\ldots}{\ldots}\right.}{b^2n^2+9}\ldots\right.}{\ldots}\ldots\right.}{\ldots}$

input
```
int(x^2*sin(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

output
```
-1/n*x^2*b/(1/n^2+b^2)*exp(1/n*ln(c*x^n)-1/n*ln(c))*cos(a+b*ln(c*x^n))+1/n
^2*x^2/(1/n^2+b^2)*exp(1/n*ln(c*x^n)-1/n*ln(c))*sin(a+b*ln(c*x^n))-2/n*(n/
(b^2*n^2+1)*(b*n/(c^(1/n))/(b^2*n^2+9)*exp(1/n*(ln(c*x^n)-n*ln(x)))*x^3*ta
n(1/2*a+1/2*b*ln(c*x^n))^2+6/(c^(1/n))/(b^2*n^2+9)*exp(1/n*(ln(c*x^n)-n*ln
(x)))*x^3*tan(1/2*a+1/2*b*ln(c*x^n))-b*n/(c^(1/n))/(b^2*n^2+9)*exp(1/n*(ln
(c*x^n)-n*ln(x)))*x^3)/(1+tan(1/2*a+1/2*b*ln(c*x^n))^2)-b*n^2/(b^2*n^2+1)*
(3/(c^(1/n))/(b^2*n^2+9)*exp(1/n*(ln(c*x^n)-n*ln(x)))*x^3-3/(c^(1/n))/(b^2
*n^2+9)*exp(1/n*(ln(c*x^n)-n*ln(x)))*x^3*tan(1/2*a+1/2*b*ln(c*x^n))^2+2*b*
n/(c^(1/n))/(b^2*n^2+9)*exp(1/n*(ln(c*x^n)-n*ln(x)))*x^3*tan(1/2*a+1/2*b*l
n(c*x^n)))/(1+tan(1/2*a+1/2*b*ln(c*x^n))^2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int x^2 \sin\left(a + b\log\left(cx^n\right)\right)\, dx$$
$$= -\frac{bnx^3 \cos\left(bn\log\left(x\right) + b\log\left(c\right) + a\right) - 3\,x^3 \sin\left(bn\log\left(x\right) + b\log\left(c\right) + a\right)}{b^2n^2 + 9}$$

input
```
integrate(x^2*sin(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
-(b*n*x^3*cos(b*n*log(x) + b*log(c) + a) - 3*x^3*sin(b*n*log(x) + b*log(c)
 + a))/(b^2*n^2 + 9)
```

Sympy [F]

$$\int x^2 \sin(a + b\log(cx^n))\, dx = \begin{cases} \int x^2 \sin\left(a - \frac{3i\log(cx^n)}{n}\right) dx & \text{for } b = -\frac{3i}{n} \\ \int x^2 \sin\left(a + \frac{3i\log(cx^n)}{n}\right) dx & \text{for } b = \frac{3i}{n} \\ -\frac{bnx^3\cos(a+b\log(cx^n))}{b^2n^2+9} + \frac{3x^3\sin(a+b\log(cx^n))}{b^2n^2+9} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*sin(a+b*ln(c*x**n)),x)
```

output
```
Piecewise((Integral(x**2*sin(a - 3*I*log(c*x**n)/n), x), Eq(b, -3*I/n)), (
Integral(x**2*sin(a + 3*I*log(c*x**n)/n), x), Eq(b, 3*I/n)), (-b*n*x**3*co
s(a + b*log(c*x**n))/(b**2*n**2 + 9) + 3*x**3*sin(a + b*log(c*x**n))/(b**2
*n**2 + 9), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(57) = 114.

Time = 0.05 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.84

$$\int x^2 \sin(a + b\log(cx^n))\, dx = -\frac{((b\cos(2b\log(c))\cos(b\log(c)) + b\sin(2b\log(c))\sin(b\log(c)) + b\cos(b\log(c)))n - 3\cos(b\log(c))}{}$$

input
```
integrate(x^2*sin(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
-1/2*(((b*cos(2*b*log(c))*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c))
+ b*cos(b*log(c)))*n - 3*cos(b*log(c))*sin(2*b*log(c)) + 3*cos(2*b*log(c))
*sin(b*log(c)) - 3*sin(b*log(c)))*x^3*cos(b*log(x^n) + a) - ((b*cos(b*log(
c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)) + b*sin(b*log(c)))*n
 + 3*cos(2*b*log(c))*cos(b*log(c)) + 3*sin(2*b*log(c))*sin(b*log(c)) + 3*c
os(b*log(c)))*x^3*sin(b*log(x^n) + a))/((b^2*cos(b*log(c))^2 + b^2*sin(b*l
og(c))^2)*n^2 + 9*cos(b*log(c))^2 + 9*sin(b*log(c))^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 923 vs. $2(57) = 114$.

Time = 0.18 (sec) , antiderivative size = 923, normalized size of antiderivative = 16.19

$$\int x^2 \sin(a + b\log(cx^n))\, dx = \text{Too large to display}$$

input
```
integrate(x^2*sin(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
-1/2*(b*n*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi
*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 + b*n*x^3*
e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b
*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - b*n*x^3*e^(1/2*pi*b*n
*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x))
 + 1/2*b*log(abs(c)))^2 - b*n*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2
*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 -
4*b*n*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*
tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a) - 4*b*n*x^3*e^(-1/
2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log
(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a) - b*n*x^3*e^(1/2*pi*b*n*sgn(x) -
1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*a)^2 - b*n*x^3*e^(-1/2*pi
*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*a)^2 + 6*x^
3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*
b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a) + 6*x^3*e^(-1/2*pi*b*n*s
gn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) +
 1/2*b*log(abs(c)))^2*tan(1/2*a) + 6*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n
 + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c))
)*tan(1/2*a)^2 + 6*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c
) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a)^2...
```

Mupad [B] (verification not implemented)

Time = 19.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int x^2 \sin(a+b\log(cx^n))\,dx = \frac{x^3\,(3\,\sin(a+b\,\ln(c\,x^n)) - b\,n\,\cos(a+b\,\ln(c\,x^n)))}{b^2\,n^2+9}$$

input
```
int(x^2*sin(a + b*log(c*x^n)),x)
```

output
```
(x^3*(3*sin(a + b*log(c*x^n)) - b*n*cos(a + b*log(c*x^n))))/(b^2*n^2 + 9)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int x^2 \sin(a + b \log(cx^n))\, dx = \frac{x^3(-\cos(\log(x^n c)\, b + a)\, bn + 3 \sin(\log(x^n c)\, b + a))}{b^2 n^2 + 9}$$

input
```
int(x^2*sin(a+b*log(c*x^n)),x)
```

output
```
(x**3*( - cos(log(x**n*c)*b + a)*b*n + 3*sin(log(x**n*c)*b + a)))/(b**2*n*
*2 + 9)
```

3.2 $\int x \sin(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 57

$$\int x\sin(a+b\log(cx^n))\,dx=-\frac{bnx^2\cos(a+b\log(cx^n))}{4+b^2n^2}+\frac{2x^2\sin(a+b\log(cx^n))}{4+b^2n^2}$$

output
```
-b*n*x^2*cos(a+b*ln(c*x^n))/(b^2*n^2+4)+2*x^2*sin(a+b*ln(c*x^n))/(b^2*n^2+
4)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int x\sin(a+b\log(cx^n))\,dx=-\frac{x^2(bn\cos(a+b\log(cx^n))-2\sin(a+b\log(cx^n)))}{4+b^2n^2}$$

input
```
Integrate[x*Sin[a + b*Log[c*x^n]],x]
```

output
```
-((x^2*(b*n*Cos[a + b*Log[c*x^n]] - 2*Sin[a + b*Log[c*x^n]]))/(4 + b^2*n^2
))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4988}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin(a + b\log(cx^n))\,dx$$

$$\Big\downarrow \ 4988$$

$$\frac{2x^2\sin(a + b\log(cx^n))}{b^2n^2 + 4} - \frac{bnx^2\cos(a + b\log(cx^n))}{b^2n^2 + 4}$$

input
```
Int[x*Sin[a + b*Log[c*x^n]],x]
```

output
```
-((b*n*x^2*Cos[a + b*Log[c*x^n]])/(4 + b^2*n^2)) + (2*x^2*Sin[a + b*Log[c*
x^n]])/(4 + b^2*n^2)
```

Defintions of rubi rules used

rule 4988
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(57) = 114$.

Time = 0.64 (sec) , antiderivative size = 470, normalized size of antiderivative = 8.25

method	result
parts	$-\frac{xb\,\mathrm{e}^{\frac{\ln(c\,x^n)}{n}-\frac{\ln(c)}{n}}\cos(a+b\ln(c\,x^n))}{n\left(\frac{1}{n^2}+b^2\right)}+\frac{x\,\mathrm{e}^{\frac{\ln(c\,x^n)}{n}-\frac{\ln(c)}{n}}\sin(a+b\ln(c\,x^n))}{n^2\left(\frac{1}{n^2}+b^2\right)}-\frac{\frac{bn\,c^{-\frac{1}{n}}\,\mathrm{e}^{\frac{\ln(c\,x^n)-n\ln(x)}{n}}x^2\tan\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}{b^2n^2+4}}{n\left(\right.}$

input
```
int(x*sin(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

output
```
-1/n*x*b/(1/n^2+b^2)*exp(1/n*ln(c*x^n)-1/n*ln(c))*cos(a+b*ln(c*x^n))+1/n^2
*x/(1/n^2+b^2)*exp(1/n*ln(c*x^n)-1/n*ln(c))*sin(a+b*ln(c*x^n))-1/n*(1/n/(1
/n^2+b^2)*(b*n/(b^2*n^2+4)/(c^(1/n))*exp(1/n*(ln(c*x^n)-n*ln(x)))*x^2*tan(
1/2*a+1/2*b*ln(c*x^n))^2+4/(b^2*n^2+4)/(c^(1/n))*exp(1/n*(ln(c*x^n)-n*ln(x
)))*x^2*tan(1/2*a+1/2*b*ln(c*x^n))-b*n/(b^2*n^2+4)/(c^(1/n))*exp(1/n*(ln(c
*x^n)-n*ln(x)))*x^2)/(1+tan(1/2*a+1/2*b*ln(c*x^n))^2)-b/(1/n^2+b^2)*(2/(b^
2*n^2+4)/(c^(1/n))*exp(1/n*(ln(c*x^n)-n*ln(x)))*x^2-2/(b^2*n^2+4)/(c^(1/n)
)*exp(1/n*(ln(c*x^n)-n*ln(x)))*x^2*tan(1/2*a+1/2*b*ln(c*x^n))^2+2*b*n/(b^2
*n^2+4)/(c^(1/n))*exp(1/n*(ln(c*x^n)-n*ln(x)))*x^2*tan(1/2*a+1/2*b*ln(c*x^
n)))/(1+tan(1/2*a+1/2*b*ln(c*x^n))^2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int x\sin\left(a+b\log\left(cx^n\right)\right)\,dx = -\frac{bnx^2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-2\,x^2\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{b^2n^2+4}$$

input
```
integrate(x*sin(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
-(b*n*x^2*cos(b*n*log(x) + b*log(c) + a) - 2*x^2*sin(b*n*log(x) + b*log(c)
 + a))/(b^2*n^2 + 4)
```

Sympy [F]

$$\int x\sin\left(a+b\log\left(cx^n\right)\right)\,dx=\begin{cases}\int x\sin\left(a-\frac{2i\log\left(cx^n\right)}{n}\right)dx & \text{for } b=-\frac{2i}{n}\\ \int x\sin\left(a+\frac{2i\log\left(cx^n\right)}{n}\right)dx & \text{for } b=\frac{2i}{n}\\ -\frac{bnx^2\cos\left(a+b\log\left(cx^n\right)\right)}{b^2n^2+4}+\frac{2x^2\sin\left(a+b\log\left(cx^n\right)\right)}{b^2n^2+4} & \text{otherwise}\end{cases}$$

input
```
integrate(x*sin(a+b*ln(c*x**n)),x)
```

output
```
Piecewise((Integral(x*sin(a - 2*I*log(c*x**n)/n), x), Eq(b, -2*I/n)), (Int
egral(x*sin(a + 2*I*log(c*x**n)/n), x), Eq(b, 2*I/n)), (-b*n*x**2*cos(a +
b*log(c*x**n))/(b**2*n**2 + 4) + 2*x**2*sin(a + b*log(c*x**n))/(b**2*n**2
+ 4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(57) = 114$.

Time = 0.05 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.84

$$\int x\sin\left(a+b\log\left(cx^n\right)\right)\,dx=$$
$$-\frac{\left(\left(b\cos\left(2\,b\log\left(c\right)\right)\cos\left(b\log\left(c\right)\right)+b\sin\left(2\,b\log\left(c\right)\right)\sin\left(b\log\left(c\right)\right)+b\cos\left(b\log\left(c\right)\right)\right)n-2\,\cos\left(b\log\left(c\right)\right)}{}$$

input
```
integrate(x*sin(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
-1/2*(((b*cos(2*b*log(c))*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c))
+ b*cos(b*log(c)))*n - 2*cos(b*log(c))*sin(2*b*log(c)) + 2*cos(2*b*log(c))
*sin(b*log(c)) - 2*sin(b*log(c)))*x^2*cos(b*log(x^n) + a) - ((b*cos(b*log(
c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)) + b*sin(b*log(c)))*n
 + 2*cos(2*b*log(c))*cos(b*log(c)) + 2*sin(2*b*log(c))*sin(b*log(c)) + 2*c
os(b*log(c)))*x^2*sin(b*log(x^n) + a))/((b^2*cos(b*log(c))^2 + b^2*sin(b*l
og(c))^2)*n^2 + 4*cos(b*log(c))^2 + 4*sin(b*log(c))^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 923 vs. $2(57) = 114$.

Time = 0.19 (sec) , antiderivative size = 923, normalized size of antiderivative = 16.19

$$\int x \sin \left(a + b \log \left(c x^n\right)\right) \, dx = \text{Too large to display}$$

input
```
integrate(x*sin(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
-1/2*(b*n*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi
*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 + b*n*x^2*
e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b
*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - b*n*x^2*e^(1/2*pi*b*n
*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x))
 + 1/2*b*log(abs(c)))^2 - b*n*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2
*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 -
4*b*n*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*
tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a) - 4*b*n*x^2*e^(-1/
2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log
(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a) - b*n*x^2*e^(1/2*pi*b*n*sgn(x) -
1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*a)^2 - b*n*x^2*e^(-1/2*pi
*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*a)^2 + 4*x^
2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*
b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a) + 4*x^2*e^(-1/2*pi*b*n*s
gn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) +
 1/2*b*log(abs(c)))^2*tan(1/2*a) + 4*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n
 + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c))
)*tan(1/2*a)^2 + 4*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c
) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a)^2...
```

Mupad [B] (verification not implemented)

Time = 18.87 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int x\sin\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{x^2\left(2\,\sin\left(a+b\,\ln\left(c\,x^n\right)\right)-b\,n\,\cos\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}{b^2\,n^2+4}$$

input
```
int(x*sin(a + b*log(c*x^n)),x)
```

output
```
(x^2*(2*sin(a + b*log(c*x^n)) - b*n*cos(a + b*log(c*x^n))))/(b^2*n^2 + 4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int x \sin(a + b\log(cx^n))\, dx = \frac{x^2(-\cos(\log(x^n c)\, b + a)\, bn + 2\sin(\log(x^n c)\, b + a))}{b^2n^2 + 4}$$

input
```
int(x*sin(a+b*log(c*x^n)),x)
```

output
```
(x**2*( - cos(log(x**n*c)*b + a)*b*n + 2*sin(log(x**n*c)*b + a)))/(b**2*n*
*2 + 4)
```

3.3 $\int \sin(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 52

$$\int \sin(a+b\log(cx^n))\,dx = -\frac{bnx\cos(a+b\log(cx^n))}{1+b^2n^2}+\frac{x\sin(a+b\log(cx^n))}{1+b^2n^2}$$

output
```
-b*n*x*cos(a+b*ln(c*x^n))/(b^2*n^2+1)+x*sin(a+b*ln(c*x^n))/(b^2*n^2+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \sin(a+b\log(cx^n))\,dx = \frac{x(-bn\cos(a+b\log(cx^n))+\sin(a+b\log(cx^n)))}{1+b^2n^2}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]],x]
```

output
```
(x*(-(b*n*Cos[a + b*Log[c*x^n]]) + Sin[a + b*Log[c*x^n]]))/(1 + b^2*n^2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4978}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin \left(a+b \log \left(c x^n\right)\right) \, dx$$

$$\Big\downarrow \ 4978$$

$$\frac{x \sin \left(a+b \log \left(c x^n\right)\right)}{b^2 n^2+1}-\frac{b n x \cos \left(a+b \log \left(c x^n\right)\right)}{b^2 n^2+1}$$

input
```
Int[Sin[a + b*Log[c*x^n]],x]
```

output
```
-((b*n*x*Cos[a + b*Log[c*x^n]])/(1 + b^2*n^2)) + (x*Sin[a + b*Log[c*x^n]])
/(1 + b^2*n^2)
```

Defintions of rubi rules used

rule 4978
```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] - Simp[b*d*n*x*(Cos[d*(a +
 b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

method	result	size
parallelrisch	$-\frac{x(\cos(a+b\ln(c\,x^n))bn-\sin(a+b\ln(c\,x^n)))}{b^2n^2+1}$	43
default	$\frac{-\frac{b\,e^{\frac{\ln(c\,x^n)}{n}-\frac{\ln(c)}{n}}\cos(a+b\ln(c\,x^n))}{\frac{1}{n^2}+b^2}+\frac{e^{\frac{\ln(c\,x^n)}{n}-\frac{\ln(c)}{n}}\sin(a+b\ln(c\,x^n))}{n\left(\frac{1}{n^2}+b^2\right)}}{n}$	91

input
```
int(sin(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

output
```
-x*(cos(a+b*ln(c*x^n))*b*n-sin(a+b*ln(c*x^n)))/(b^2*n^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \sin\left(a+b\log\left(cx^n\right)\right)\,dx = -\frac{bnx\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-x\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{b^2n^2+1}$$

input
```
integrate(sin(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
-(b*n*x*cos(b*n*log(x) + b*log(c) + a) - x*sin(b*n*log(x) + b*log(c) + a))
/(b^2*n^2 + 1)
```

Sympy [F]

$$\int \sin\left(a+b\log\left(cx^n\right)\right)\,dx = \begin{cases} \int \sin\left(a-\frac{i\log\left(cx^n\right)}{n}\right)\,dx & \text{for}\ b=-\frac{i}{n} \\ \int \sin\left(a+\frac{i\log\left(cx^n\right)}{n}\right)\,dx & \text{for}\ b=\frac{i}{n} \\ -\frac{bnx\cos\left(a+b\log\left(cx^n\right)\right)}{b^2n^2+1}+\frac{x\sin\left(a+b\log\left(cx^n\right)\right)}{b^2n^2+1} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(a+b*ln(c*x**n)),x)
```

output
```
Piecewise((Integral(sin(a - I*log(c*x**n)/n), x), Eq(b, -I/n)), (Integral(
sin(a + I*log(c*x**n)/n), x), Eq(b, I/n)), (-b*n*x*cos(a + b*log(c*x**n))/
(b**2*n**2 + 1) + x*sin(a + b*log(c*x**n))/(b**2*n**2 + 1), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(52) = 104$.

Time = 0.09 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.96

$$\int \sin (a + b \log (cx^n)) \, dx = -\frac{((b \cos (2 b \log (c)) \cos (b \log (c)) + b \sin (2 b \log (c)) \sin (b \log (c)) + b \cos (b \log (c)))n - \cos (b \log (c)) \sin}{}$$

input
```
integrate(sin(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
-1/2*(((b*cos(2*b*log(c))*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c))
+ b*cos(b*log(c)))*n - cos(b*log(c))*sin(2*b*log(c)) + cos(2*b*log(c))*sin
(b*log(c)) - sin(b*log(c)))*x*cos(b*log(x^n) + a) - ((b*cos(b*log(c))*sin(
2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)) + b*sin(b*log(c)))*n + cos(2
*b*log(c))*cos(b*log(c)) + sin(2*b*log(c))*sin(b*log(c)) + cos(b*log(c)))*
x*sin(b*log(x^n) + a))/((b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))^2)*n^2 +
cos(b*log(c))^2 + sin(b*log(c))^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(52) = 104$.

Time = 0.15 (sec) , antiderivative size = 882, normalized size of antiderivative = 16.96

$$\int \sin (a + b \log (cx^n)) \, dx = \text{Too large to display}$$

input
```
integrate(sin(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
-1/2*(b*n*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b
)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 + b*n*x*e^(-
1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*l
og(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - b*n*x*e^(1/2*pi*b*n*sgn(x
) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2
*b*log(abs(c)))^2 - b*n*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sg
n(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 - 4*b*n*x*
e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*
n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a) - 4*b*n*x*e^(-1/2*pi*b*n*sgn
(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1
/2*b*log(abs(c)))*tan(1/2*a) - b*n*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1
/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*a)^2 - b*n*x*e^(-1/2*pi*b*n*sgn(x) + 1/
2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*a)^2 + 2*x*e^(1/2*pi*b*n*sg
n(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) +
1/2*b*log(abs(c)))^2*tan(1/2*a) + 2*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n -
 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^
2*tan(1/2*a) + 2*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1
/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a)^2 + 2*x*e
^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*
n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/2*a)^2 + b*n*x*e^(1/2*pi*b*n*s...
```

Mupad [B] (verification not implemented)

Time = 18.83 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \sin \left(a+b\log \left(cx^n\right)\right)\,dx=\frac{x\,\left(\sin \left(a+b\,\ln \left(c\,x^n\right)\right)-b\,n\,\cos \left(a+b\,\ln \left(c\,x^n\right)\right)\right)}{b^2\,n^2+1}$$

input
```
int(sin(a + b*log(c*x^n)),x)
```

output
```
(x*(sin(a + b*log(c*x^n)) - b*n*cos(a + b*log(c*x^n))))/(b^2*n^2 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \sin\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{x(-\cos\left(\log(x^n c)\,b+a\right)bn+\sin\left(\log(x^n c)\,b+a\right))}{b^2n^2+1}$$

input
```
int(sin(a+b*log(c*x^n)),x)
```

output
```
(x*( - cos(log(x**n*c)*b + a)*b*n + sin(log(x**n*c)*b + a)))/(b**2*n**2 +
1)
```

3.4 $\int \frac{\sin(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{\sin(a+b\log(cx^n))}{x}\,dx = -\frac{\cos(a+b\log(cx^n))}{bn}$$

output
```
-cos(a+b*ln(c*x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.00

$$\int \frac{\sin(a+b\log(cx^n))}{x}\,dx = -\frac{\cos(a)\cos(b\log(cx^n))}{bn} + \frac{\sin(a)\sin(b\log(cx^n))}{bn}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]/x,x]
```

output
```
-((Cos[a]*Cos[b*Log[c*x^n]])/(b*n)) + (Sin[a]*Sin[b*Log[c*x^n]])/(b*n)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3039, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \sin(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3118}$$

$$-\frac{\cos(a+b\log(cx^n))}{bn}$$

input
```
Int[Sin[a + b*Log[c*x^n]]/x,x]
```

output
```
-(Cos[a + b*Log[c*x^n]]/(b*n))
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$-\frac{\cos(a+b\ln(c\,x^{n}))}{bn}$	20
default	$-\frac{\cos(a+b\ln(c\,x^{n}))}{bn}$	20
parallelrisch	$\frac{-1-\cos\left(a+2b\ln\left(\sqrt{c\,x^{n}}\right)\right)}{bn}$	26

input
```
int(sin(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
```

output
```
-cos(a+b*ln(c*x^n))/b/n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\sin\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx = -\frac{\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{bn}$$

input
```
integrate(sin(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

output
```
-cos(b*n*log(x) + b*log(c) + a)/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{\sin(a+b\log(cx^n))}{x}\,dx = \begin{cases} \log(x)\sin(a) & \text{for } b=0 \wedge (b=0 \vee n=0) \\ \log(x)\sin(a+b\log(c)) & \text{for } n=0 \\ -\frac{\cos(a+b\log(cx^n))}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(a+b*ln(c*x**n))/x,x)
```

output
```
Piecewise((log(x)*sin(a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*sin(a
 + b*log(c)), Eq(n, 0)), (-cos(a + b*log(c*x**n))/(b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a+b\log(cx^n))}{x}\,dx = -\frac{\cos(b\log(cx^n)+a)}{bn}$$

input
```
integrate(sin(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

output
```
-cos(b*log(c*x^n) + a)/(b*n)
```

Giac [F]

$$\int \frac{\sin(a+b\log(cx^n))}{x}\,dx = \int \frac{\sin(b\log(cx^n)+a)}{x}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))/x,x, algorithm="giac")
```

output
```
integrate(sin(b*log(c*x^n) + a)/x, x)
```

Mupad [B] (verification not implemented)

Time = 19.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\sin (a+b\log (cx^n))}{x}\,dx = -\frac{\cos \left(a+b\,\ln \left(c\,x^n\right)\right)}{b\,n}$$

input
```
int(sin(a + b*log(c*x^n))/x,x)
```

output
```
-cos(a + b*log(c*x^n))/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\sin (a+b\log (cx^n))}{x}\,dx = -\frac{\cos \left(\log(x^n c)\,b+a\right)}{bn}$$

input
```
int(sin(a+b*log(c*x^n))/x,x)
```

output
```
( - cos(log(x**n*c)*b + a))/(b*n)
```

3.5 $\int \frac{\sin(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{\sin(a+b\log(cx^n))}{x^2}\,dx = -\frac{bn\cos(a+b\log(cx^n))}{(1+b^2n^2)\,x} - \frac{\sin(a+b\log(cx^n))}{(1+b^2n^2)\,x}$$

output
```
-b*n*cos(a+b*ln(c*x^n))/(b^2*n^2+1)/x-sin(a+b*ln(c*x^n))/(b^2*n^2+1)/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.70

$$\int \frac{\sin(a+b\log(cx^n))}{x^2}\,dx = -\frac{bn\cos(a+b\log(cx^n))+\sin(a+b\log(cx^n))}{x+b^2n^2x}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]/x^2,x]
```

output
```
-((b*n*Cos[a + b*Log[c*x^n]] + Sin[a + b*Log[c*x^n]])/(x + b^2*n^2*x))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {4988}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+b\log(cx^n))}{x^2}\,dx$$

$$\Big\downarrow \; 4988$$

$$-\frac{\sin(a+b\log(cx^n))}{x(b^2n^2+1)}-\frac{bn\cos(a+b\log(cx^n))}{x(b^2n^2+1)}$$

input
```
Int[Sin[a + b*Log[c*x^n]]/x^2,x]
```

output
```
-((b*n*Cos[a + b*Log[c*x^n]])/((1 + b^2*n^2)*x)) - Sin[a + b*Log[c*x^n]]/(
(1 + b^2*n^2)*x)
```

Defintions of rubi rules used

rule 4988
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

method	result	size
parallelrisch	$\frac{-\cos(a+b\ln(c\,x^{n}))bn-\sin(a+b\ln(c\,x^{n}))}{x(b^{2}n^{2}+1)}$	45

input
```
int(sin(a+b*ln(c*x^n))/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x/(b^2*n^2+1)*(-cos(a+b*ln(c*x^n))*b*n-sin(a+b*ln(c*x^n)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int \frac{\sin(a+b\log(cx^n))}{x^2}\,dx = -\frac{bn\cos(bn\log(x)+b\log(c)+a)+\sin(bn\log(x)+b\log(c)+a)}{(b^2n^2+1)x}$$

input
```
integrate(sin(a+b*log(c*x^n))/x^2,x, algorithm="fricas")
```

output
```
-(b*n*cos(b*n*log(x) + b*log(c) + a) + sin(b*n*log(x) + b*log(c) + a))/((b
^2*n^2 + 1)*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.69 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.33

$$\int \frac{\sin (a + b\log (cx^n))}{x^2}\,dx = \begin{cases} -\frac{\sin\left(a-\frac{i\log(cx^n)}{n}\right)}{2x} + \frac{\log(cx^n)\sin\left(a-\frac{i\log(cx^n)}{n}\right)}{2nx} - \frac{i\log(cx^n)\cos\left(a-\frac{i\log(cx^n)}{n}\right)}{2nx} & \text{for } b = -\frac{i}{n} \\ -\frac{\sin\left(a+\frac{i\log(cx^n)}{n}\right)}{2x} + \frac{\log(cx^n)\sin\left(a+\frac{i\log(cx^n)}{n}\right)}{2nx} + \frac{i\log(cx^n)\cos\left(a+\frac{i\log(cx^n)}{n}\right)}{2nx} & \text{for } b = \frac{i}{n} \\ -\frac{bn\cos(a+b\log(cx^n))}{b^2n^2x+x} - \frac{\sin(a+b\log(cx^n))}{b^2n^2x+x} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(a+b*ln(c*x**n))/x**2,x)
```

output
```
Piecewise((-sin(a - I*log(c*x**n)/n)/(2*x) + log(c*x**n)*sin(a - I*log(c*x
**n)/n)/(2*n*x) - I*log(c*x**n)*cos(a - I*log(c*x**n)/n)/(2*n*x), Eq(b, -I
/n)), (-sin(a + I*log(c*x**n)/n)/(2*x) + log(c*x**n)*sin(a + I*log(c*x**n)
/n)/(2*n*x) + I*log(c*x**n)*cos(a + I*log(c*x**n)/n)/(2*n*x), Eq(b, I/n)),
 (-b*n*cos(a + b*log(c*x**n))/(b**2*n**2*x + x) - sin(a + b*log(c*x**n))/(
b**2*n**2*x + x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(57) = 114.

Time = 0.06 (sec) , antiderivative size = 209, normalized size of antiderivative = 3.67

$$\int \frac{\sin (a + b\log (cx^n))}{x^2}\,dx = -\frac{((b\cos(2\,b\log(c))\cos(b\log(c)) + b\sin(2\,b\log(c))\sin(b\log(c)) + b\cos(b\log(c)))n + \cos(b\log(c))\,\mathrm{si}}{}$$

input
```
integrate(sin(a+b*log(c*x^n))/x^2,x, algorithm="maxima")
```

output
```
-1/2*(((b*cos(2*b*log(c))*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c))
+ b*cos(b*log(c)))*n + cos(b*log(c))*sin(2*b*log(c)) - cos(2*b*log(c))*sin
(b*log(c)) + sin(b*log(c)))*cos(b*log(x^n) + a) - ((b*cos(b*log(c))*sin(2*
b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)) + b*sin(b*log(c)))*n - cos(2*b
*log(c))*cos(b*log(c)) - sin(2*b*log(c))*sin(b*log(c)) - cos(b*log(c)))*si
n(b*log(x^n) + a))/(((b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))^2)*n^2 + cos
(b*log(c))^2 + sin(b*log(c))^2)*x)
```

Giac [F]

$$\int \frac{\sin(a + b\log(cx^n))}{x^2}\,dx = \int \frac{\sin(b\log(cx^n) + a)}{x^2}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))/x^2,x, algorithm="giac")
```

output
```
integrate(sin(b*log(c*x^n) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a + b\log(cx^n))}{x^2}\,dx = \int \frac{\sin(a + b\ln(c\,x^n))}{x^2}\,dx$$

input
```
int(sin(a + b*log(c*x^n))/x^2,x)
```

output
```
int(sin(a + b*log(c*x^n))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int \frac{\sin\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx = \frac{-\cos\left(\log(x^nc)\,b+a\right)bn-\sin\left(\log(x^nc)\,b+a\right)}{x\left(b^2n^2+1\right)}$$

input
```
int(sin(a+b*log(c*x^n))/x^2,x)
```

output
```
( - (cos(log(x**n*c)*b + a)*b*n + sin(log(x**n*c)*b + a)))/(x*(b**2*n**2 +
 1))
```

3.6 $\int \frac{\sin(a+b\log(cx^n))}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{\sin(a+b\log(cx^n))}{x^3}\,dx = -\frac{bn\cos(a+b\log(cx^n))}{(4+b^2n^2)\,x^2} - \frac{2\sin(a+b\log(cx^n))}{(4+b^2n^2)\,x^2}$$

output
```
-b*n*cos(a+b*ln(c*x^n))/(b^2*n^2+4)/x^2-2*sin(a+b*ln(c*x^n))/(b^2*n^2+4)/x
^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int \frac{\sin(a+b\log(cx^n))}{x^3}\,dx = -\frac{bn\cos(a+b\log(cx^n))+2\sin(a+b\log(cx^n))}{(4+b^2n^2)\,x^2}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]/x^3,x]
```

output
```
-((b*n*Cos[a + b*Log[c*x^n]] + 2*Sin[a + b*Log[c*x^n]])/((4 + b^2*n^2)*x^2
))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {4988}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+b\log(cx^n))}{x^3}\,dx$$

$$\Big\downarrow \text{4988}$$

$$-\frac{2\sin(a+b\log(cx^n))}{x^2(b^2n^2+4)}-\frac{bn\cos(a+b\log(cx^n))}{x^2(b^2n^2+4)}$$

input
```
Int[Sin[a + b*Log[c*x^n]]/x^3,x]
```

output
```
-((b*n*Cos[a + b*Log[c*x^n]])/((4 + b^2*n^2)*x^2)) - (2*Sin[a + b*Log[c*x^
n]])/((4 + b^2*n^2)*x^2)
```

Defintions of rubi rules used

rule 4988
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

method	result	size
parallelrisch	$\frac{-\cos(a+b\ln(c\,x^{n}))bn-2\sin(a+b\ln(c\,x^{n}))}{x^{2}(b^{2}n^{2}+4)}$	45

input
```
int(sin(a+b*ln(c*x^n))/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/x^2/(b^2*n^2+4)*(-cos(a+b*ln(c*x^n))*b*n-2*sin(a+b*ln(c*x^n)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{\sin(a+b\log(cx^n))}{x^3}\,dx = -\frac{bn\cos(bn\log(x)+b\log(c)+a)+2\sin(bn\log(x)+b\log(c)+a)}{(b^2n^2+4)x^2}$$

input
```
integrate(sin(a+b*log(c*x^n))/x^3,x, algorithm="fricas")
```

output
```
-(b*n*cos(b*n*log(x) + b*log(c) + a) + 2*sin(b*n*log(x) + b*log(c) + a))/(
(b^2*n^2 + 4)*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.04 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.96

$$\int \frac{\sin(a+b\log(cx^n))}{x^3}\,dx = \begin{cases} -\frac{\sin\left(a-\frac{2i\log(cx^n)}{n}\right)}{4x^2} + \frac{\log(cx^n)\sin\left(a-\frac{2i\log(cx^n)}{n}\right)}{2nx^2} - \frac{i\log(cx^n)\cos\left(a-\frac{2i\log(cx^n)}{n}\right)}{2nx^2} & \text{for } b=-\frac{2i}{n} \\ -\frac{\sin\left(a+\frac{2i\log(cx^n)}{n}\right)}{4x^2} + \frac{\log(cx^n)\sin\left(a+\frac{2i\log(cx^n)}{n}\right)}{2nx^2} + \frac{i\log(cx^n)\cos\left(a+\frac{2i\log(cx^n)}{n}\right)}{2nx^2} & \text{for } b=\frac{2i}{n} \\ -\frac{bn\cos(a+b\log(cx^n))}{b^2n^2x^2+4x^2} - \frac{2\sin(a+b\log(cx^n))}{b^2n^2x^2+4x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(a+b*ln(c*x**n))/x**3,x)
```

output
```
Piecewise((-sin(a - 2*I*log(c*x**n)/n)/(4*x**2) + log(c*x**n)*sin(a - 2*I*
log(c*x**n)/n)/(2*n*x**2) - I*log(c*x**n)*cos(a - 2*I*log(c*x**n)/n)/(2*n*
x**2), Eq(b, -2*I/n)), (-sin(a + 2*I*log(c*x**n)/n)/(4*x**2) + log(c*x**n)
*sin(a + 2*I*log(c*x**n)/n)/(2*n*x**2) + I*log(c*x**n)*cos(a + 2*I*log(c*x
**n)/n)/(2*n*x**2), Eq(b, 2*I/n)), (-b*n*cos(a + b*log(c*x**n))/(b**2*n**2
*x**2 + 4*x**2) - 2*sin(a + b*log(c*x**n))/(b**2*n**2*x**2 + 4*x**2), True
))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(57) = 114$.

Time = 0.06 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.79

$$\int \frac{\sin(a+b\log(cx^n))}{x^3}\,dx = -\frac{((b\cos(2b\log(c))\cos(b\log(c)) + b\sin(2b\log(c))\sin(b\log(c)) + b\cos(b\log(c)))n + 2\cos(b\log(c))}{}$$

input
```
integrate(sin(a+b*log(c*x^n))/x^3,x, algorithm="maxima")
```

output
```
-1/2*(((b*cos(2*b*log(c))*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c))
+ b*cos(b*log(c)))*n + 2*cos(b*log(c))*sin(2*b*log(c)) - 2*cos(2*b*log(c))
*sin(b*log(c)) + 2*sin(b*log(c)))*cos(b*log(x^n) + a) - ((b*cos(b*log(c))*
sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)) + b*sin(b*log(c)))*n - 2
*cos(2*b*log(c))*cos(b*log(c)) - 2*sin(2*b*log(c))*sin(b*log(c)) - 2*cos(b
*log(c)))*sin(b*log(x^n) + a))/(((b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))^
2)*n^2 + 4*cos(b*log(c))^2 + 4*sin(b*log(c))^2)*x^2)
```

Giac [F]

$$\int \frac{\sin(a+b\log(cx^n))}{x^3}\,dx = \int \frac{\sin(b\log(cx^n)+a)}{x^3}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))/x^3,x, algorithm="giac")
```

output
```
integrate(sin(b*log(c*x^n) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+b\log(cx^n))}{x^3}\,dx = \int \frac{\sin(a+b\,\ln(c\,x^n))}{x^3}\,dx$$

input
```
int(sin(a + b*log(c*x^n))/x^3,x)
```

output
```
int(sin(a + b*log(c*x^n))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int \frac{\sin(a+b\log(cx^n))}{x^3}\,dx = \frac{-\cos(\log(x^nc)\,b+a)\,bn-2\sin(\log(x^nc)\,b+a)}{x^2\,(b^2n^2+4)}$$

input
```
int(sin(a+b*log(c*x^n))/x^3,x)
```

output
```
( - cos(log(x**n*c)*b + a)*b*n - 2*sin(log(x**n*c)*b + a))/(x**2*(b**2*n**
2 + 4))
```

3.7 $\int x^2 \sin^2(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 97

$$\int x^2 \sin^2(a+b\log(cx^n))\,dx = \frac{2b^2n^2x^3}{3(9+4b^2n^2)} - \frac{2bnx^3\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{9+4b^2n^2} + \frac{3x^3\sin^2(a+b\log(cx^n))}{9+4b^2n^2}$$

output
```
2*b^2*n^2*x^3/(12*b^2*n^2+27)-2*b*n*x^3*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^
n))/(4*b^2*n^2+9)+3*x^3*sin(a+b*ln(c*x^n))^2/(4*b^2*n^2+9)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.63

$$\int x^2 \sin^2(a+b\log(cx^n))\,dx = \frac{x^3(9+4b^2n^2-9\cos(2(a+b\log(cx^n)))-6bn\sin(2(a+b\log(cx^n))))}{6(9+4b^2n^2)}$$

input
```
Integrate[x^2*Sin[a + b*Log[c*x^n]]^2,x]
```

output
```
(x^3*(9 + 4*b^2*n^2 - 9*Cos[2*(a + b*Log[c*x^n])] - 6*b*n*Sin[2*(a + b*Log
[c*x^n])]))/(6*(9 + 4*b^2*n^2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4990, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin^2(a + b\log(cx^n)) \, dx$$

$$\Big\downarrow \text{4990}$$

$$\frac{2b^2n^2\int x^2dx}{4b^2n^2+9} + \frac{3x^3\sin^2(a+b\log(cx^n))}{4b^2n^2+9} - \frac{2bnx^3\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+9}$$

$$\Big\downarrow \text{15}$$

$$\frac{3x^3\sin^2(a+b\log(cx^n))}{4b^2n^2+9} - \frac{2bnx^3\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+9} + \frac{2b^2n^2x^3}{3(4b^2n^2+9)}$$

input
```
Int[x^2*Sin[a + b*Log[c*x^n]]^2,x]
```

output
```
(2*b^2*n^2*x^3)/(3*(9 + 4*b^2*n^2)) - (2*b*n*x^3*Cos[a + b*Log[c*x^n]]*Sin
[a + b*Log[c*x^n]])/(9 + 4*b^2*n^2) + (3*x^3*Sin[a + b*Log[c*x^n]]^2)/(9 +
 4*b^2*n^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 4990
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (-Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a
 + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e
*(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2
))   Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
 c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [F]

$$\int x^2 \sin\left(a+b\ln\left(c\,x^n\right)\right)^2 dx$$

input
```
int(x^2*sin(a+b*ln(c*x^n))^2,x)
```

output
```
int(x^2*sin(a+b*ln(c*x^n))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int x^2 \sin^2\left(a+b\log\left(cx^n\right)\right)\,dx = -\frac{6\,bnx^3\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+9\,x^3\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right.}{3\left(4\,b^2n^2+9\right)}$$

input
```
integrate(x^2*sin(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
-1/3*(6*b*n*x^3*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x) + b*log(c) +
 a) + 9*x^3*cos(b*n*log(x) + b*log(c) + a)^2 - (2*b^2*n^2 + 9)*x^3)/(4*b^2
*n^2 + 9)
```

Sympy [F]

$$\int x^2\sin^2\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\begin{cases}\int x^2\sin^2\left(a-\frac{3i\log\left(cx^n\right)}{2n}\right)dx\\ \int x^2\sin^2\left(a+\frac{3i\log\left(cx^n\right)}{2n}\right)dx\\ \frac{2b^2n^2x^3\sin^2\left(a+b\log\left(cx^n\right)\right)}{12b^2n^2+27}+\frac{2b^2n^2x^3\cos^2\left(a+b\log\left(cx^n\right)\right)}{12b^2n^2+27}-\frac{6bnx^3\sin\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{12b^2n^2+27}+\frac{9x^3\sin^2\left(a+b\log\left(cx^n\right)\right)}{12b^2n^2+27}\end{cases}$$

input
```
integrate(x**2*sin(a+b*ln(c*x**n))**2,x)
```

output
```
Piecewise((Integral(x**2*sin(a - 3*I*log(c*x**n)/(2*n))**2, x), Eq(b, -3*I
/(2*n))), (Integral(x**2*sin(a + 3*I*log(c*x**n)/(2*n))**2, x), Eq(b, 3*I/
(2*n))), (2*b**2*n**2*x**3*sin(a + b*log(c*x**n))**2/(12*b**2*n**2 + 27) +
 2*b**2*n**2*x**3*cos(a + b*log(c*x**n))**2/(12*b**2*n**2 + 27) - 6*b*n*x*
*3*sin(a + b*log(c*x**n))*cos(a + b*log(c*x**n))/(12*b**2*n**2 + 27) + 9*x
**3*sin(a + b*log(c*x**n))**2/(12*b**2*n**2 + 27), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(95) = 190.

Time = 0.06 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.10

$$\int x^2\sin^2\left(a+b\log\left(cx^n\right)\right)\,dx=$$
$$-\frac{3\left(2\left(b\cos\left(2\,b\log\left(c\right)\right)\sin\left(4\,b\log\left(c\right)\right)-b\cos\left(4\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)+b\sin\left(2\,b\log\left(c\right)\right)\right)n+3\,\cos\left(4\right.}{}$$

input
```
integrate(x^2*sin(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
-1/12*(3*(2*(b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b
*log(c)) + b*sin(2*b*log(c)))*n + 3*cos(4*b*log(c))*cos(2*b*log(c)) + 3*si
n(4*b*log(c))*sin(2*b*log(c)) + 3*cos(2*b*log(c)))*x^3*cos(2*b*log(x^n) +
2*a) + 3*(2*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b
*log(c)) + b*cos(2*b*log(c)))*n - 3*cos(2*b*log(c))*sin(4*b*log(c)) + 3*co
s(4*b*log(c))*sin(2*b*log(c)) - 3*sin(2*b*log(c)))*x^3*sin(2*b*log(x^n) +
2*a) - 2*(4*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + 9*cos(2*
b*log(c))^2 + 9*sin(2*b*log(c))^2)*x^3)/(4*(b^2*cos(2*b*log(c))^2 + b^2*si
n(2*b*log(c))^2)*n^2 + 9*cos(2*b*log(c))^2 + 9*sin(2*b*log(c))^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(95) = 190$.

Time = 0.33 (sec) , antiderivative size = 830, normalized size of antiderivative = 8.56

$$\int x^2 \sin^2(a + b\log(cx^n))\, dx = \text{Too large to display}$$

input
```
integrate(x^2*sin(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
1/6*x^3 + 1/4*(4*b*n*x^3*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2
*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 4*b*n*x^3*e^(2*pi*b
*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*log
(abs(c)))*tan(a)^2 + 4*b*n*x^3*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(
a) + 4*b*n*x^3*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a)^2 - 3*x^3*e^(2*
pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b
*log(abs(c)))^2*tan(a)^2 - 4*b*n*x^3*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*
b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c))) - 4*b*n*x^3*e^(2*p
i*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(a) - 3*x^3*tan(b*n*l
og(abs(x)) + b*log(abs(c)))^2*tan(a)^2 - 4*b*n*x^3*tan(b*n*log(abs(x)) + b
*log(abs(c))) + 3*x^3*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi
*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 - 4*b*n*x^3*tan(a) + 12*x^3*e^(
2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) +
 b*log(abs(c)))*tan(a) + 3*x^3*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(
c) - 2*pi*b)*tan(a)^2 + 3*x^3*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + 12*
x^3*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a) + 3*x^3*tan(a)^2 - 3*x^3*e
^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b) - 3*x^3)/(4*b^2*n^2
*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*l
og(abs(c)))^2*tan(a)^2 + 4*b^2*n^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c)
 - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + 4*b^2*n^2*e^(pi*b*n*s...
```

Mupad [B] (verification not implemented)

Time = 20.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.69

$$\int x^2 \sin^2(a + b\log(cx^n))\,dx = \frac{x^3}{6} - \frac{x^3\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{{(c\,x^n)}^{b\,2\mathrm{i}}}\,1\mathrm{i}}{8\,b\,n + 12\mathrm{i}} - \frac{x^3\,\mathrm{e}^{a\,2\mathrm{i}}\,{(c\,x^n)}^{b\,2\mathrm{i}}}{12 + b\,n\,8\mathrm{i}}$$

input
```
int(x^2*sin(a + b*log(c*x^n))^2,x)
```

output
```
x^3/6 - (x^3*exp(-a*2i)/(c*x^n)^(b*2i)*1i)/(8*b*n + 12i) - (x^3*exp(a*2i)*
(c*x^n)^(b*2i))/(b*n*8i + 12)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.68

$$\int x^2 \sin^2(a + b\log(cx^n)) \, dx$$
$$= \frac{x^3(-6\cos(\log(x^n c) b + a) \sin(\log(x^n c) b + a) bn + 9\sin(\log(x^n c) b + a)^2 + 2b^2n^2)}{12b^2n^2 + 27}$$

input
```
int(x^2*sin(a+b*log(c*x^n))^2,x)
```

output
```
(x**3*( - 6*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*b*n + 9*sin(log(
x**n*c)*b + a)**2 + 2*b**2*n**2))/(3*(4*b**2*n**2 + 9))
```

3.8 $\int x\sin^2(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 98

$$\int x\sin^2(a+b\log(cx^n))\,dx=\frac{b^2n^2x^2}{4(1+b^2n^2)}-\frac{bnx^2\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{2(1+b^2n^2)}+\frac{x^2\sin^2(a+b\log(cx^n))}{2(1+b^2n^2)}$$

output
```
b^2*n^2*x^2/(4*b^2*n^2+4)-b*n*x^2*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/(2
*b^2*n^2+2)+x^2*sin(a+b*ln(c*x^n))^2/(2*b^2*n^2+2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int x\sin^2(a+b\log(cx^n))\,dx=\frac{x^2(1+b^2n^2-\cos(2(a+b\log(cx^n)))-bn\sin(2(a+b\log(cx^n))))}{4+4b^2n^2}$$

input
```
Integrate[x*Sin[a + b*Log[c*x^n]]^2,x]
```

output
```
(x^2*(1 + b^2*n^2 - Cos[2*(a + b*Log[c*x^n])] - b*n*Sin[2*(a + b*Log[c*x^n
])]))/(4 + 4*b^2*n^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4990, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin^2(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{4990}$$

$$\frac{b^2n^2\int xdx}{2(b^2n^2+1)}+\frac{x^2\sin^2(a+b\log(cx^n))}{2(b^2n^2+1)}-\frac{bnx^2\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{2(b^2n^2+1)}$$

$$\Big\downarrow \text{15}$$

$$\frac{x^2\sin^2(a+b\log(cx^n))}{2(b^2n^2+1)}-\frac{bnx^2\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{2(b^2n^2+1)}+\frac{b^2n^2x^2}{4(b^2n^2+1)}$$

input
```
Int[x*Sin[a + b*Log[c*x^n]]^2,x]
```

output
```
(b^2*n^2*x^2)/(4*(1 + b^2*n^2)) - (b*n*x^2*Cos[a + b*Log[c*x^n]]*Sin[a + b
*Log[c*x^n]])/(2*(1 + b^2*n^2)) + (x^2*Sin[a + b*Log[c*x^n]]^2)/(2*(1 + b^
2*n^2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 4990
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (-Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a
 + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e
*(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2
))   Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
 c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [F]

$$\int x\sin\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}dx$$

input
```
int(x*sin(a+b*ln(c*x^n))^2,x)
```

output
```
int(x*sin(a+b*ln(c*x^n))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int x\sin^2\left(a+b\log\left(cx^n\right)\right)\,dx =$$
$$-\frac{2\,bnx^2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+2\,x^2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right.}{4\left(b^2n^2+1\right)}$$

input
```
integrate(x*sin(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
-1/4*(2*b*n*x^2*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x) + b*log(c) +
 a) + 2*x^2*cos(b*n*log(x) + b*log(c) + a)^2 - (b^2*n^2 + 2)*x^2)/(b^2*n^2
 + 1)
```

Sympy [F]

$$\int x\sin^2\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\begin{cases}\int x\sin^2\left(a-\frac{i\log\left(cx^n\right)}{n}\right)dx\\ \int x\sin^2\left(a+\frac{i\log\left(cx^n\right)}{n}\right)dx\\ \frac{b^2n^2x^2\sin^2\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2+4}+\frac{b^2n^2x^2\cos^2\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2+4}-\frac{2bnx^2\sin\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2+4}+\frac{2x^2\sin^2\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2+4}\end{cases}$$

input
```
integrate(x*sin(a+b*ln(c*x**n))**2,x)
```

output
```
Piecewise((Integral(x*sin(a - I*log(c*x**n)/n)**2, x), Eq(b, -I/n)), (Inte
gral(x*sin(a + I*log(c*x**n)/n)**2, x), Eq(b, I/n)), (b**2*n**2*x**2*sin(a
 + b*log(c*x**n))**2/(4*b**2*n**2 + 4) + b**2*n**2*x**2*cos(a + b*log(c*x*
*n))**2/(4*b**2*n**2 + 4) - 2*b*n*x**2*sin(a + b*log(c*x**n))*cos(a + b*lo
g(c*x**n))/(4*b**2*n**2 + 4) + 2*x**2*sin(a + b*log(c*x**n))**2/(4*b**2*n*
*2 + 4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(92) = 184$.

Time = 0.05 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.88

$$\int x\sin^2\left(a+b\log\left(cx^n\right)\right)\,dx =$$
$$-\frac{((b\cos\left(2\,b\log\left(c\right)\right)\sin\left(4\,b\log\left(c\right)\right)-b\cos\left(4\,b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right)+b\sin\left(2\,b\log\left(c\right)\right))n+\cos\left(4\,b\log\right.}{}$$

input
```
integrate(x*sin(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
-1/8*(((b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(
c)) + b*sin(2*b*log(c)))*n + cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log
(c))*sin(2*b*log(c)) + cos(2*b*log(c)))*x^2*cos(2*b*log(x^n) + 2*a) + ((b*
cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)) + b*co
s(2*b*log(c)))*n - cos(2*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin(2
*b*log(c)) - sin(2*b*log(c)))*x^2*sin(2*b*log(x^n) + 2*a) - 2*((b^2*cos(2*
b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*b*log(c))^2 + sin(2*b*log
(c))^2)*x^2)/((b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*
b*log(c))^2 + sin(2*b*log(c))^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 817 vs. $2(92) = 184$.

Time = 0.31 (sec) , antiderivative size = 817, normalized size of antiderivative = 8.34

$$\int x\sin^2\left(a+b\log\left(cx^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x*sin(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
1/4*x^2 + 1/8*(2*b*n*x^2*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2
*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 2*b*n*x^2*e^(2*pi*b
*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*log
(abs(c)))*tan(a)^2 + 2*b*n*x^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(
a) + 2*b*n*x^2*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a)^2 - x^2*e^(2*pi
*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*l
og(abs(c)))^2*tan(a)^2 - 2*b*n*x^2*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*
sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c))) - 2*b*n*x^2*e^(2*pi*
b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(a) - x^2*tan(b*n*log(a
bs(x)) + b*log(abs(c)))^2*tan(a)^2 - 2*b*n*x^2*tan(b*n*log(abs(x)) + b*log
(abs(c))) + x^2*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*ta
n(b*n*log(abs(x)) + b*log(abs(c)))^2 - 2*b*n*x^2*tan(a) + 4*x^2*e^(2*pi*b*
n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*log(
abs(c)))*tan(a) + x^2*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi
*b)*tan(a)^2 + x^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + 4*x^2*tan(b*n*
log(abs(x)) + b*log(abs(c)))*tan(a) + x^2*tan(a)^2 - x^2*e^(2*pi*b*n*sgn(x
) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b) - x^2)/(b^2*n^2*e^(pi*b*n*sgn(x) -
pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)
^2 + b^2*n^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(a
bs(x)) + b*log(abs(c)))^2 + b^2*n^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sg...
```

Mupad [B] (verification not implemented)

Time = 20.40 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.68

$$\int x\sin^2(a+b\log(cx^n))\,dx=\frac{x^2}{4}-\frac{x^2\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,2\mathrm{i}}}\,1\mathrm{i}}{8\,b\,n+8\mathrm{i}}-\frac{x^2\,\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}}}{8+b\,n\,8\mathrm{i}}$$

input
```
int(x*sin(a + b*log(c*x^n))^2,x)
```

output
```
x^2/4 - (x^2*exp(-a*2i)/(c*x^n)^(b*2i)*1i)/(8*b*n + 8i) - (x^2*exp(a*2i)*(
c*x^n)^(b*2i))/(b*n*8i + 8)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.66

$$\int x\sin^2\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\frac{x^2\left(-2\cos\left(\log(x^nc)\,b+a\right)\sin\left(\log(x^nc)\,b+a\right)bn+2\sin\left(\log(x^nc)\,b+a\right)^2+b^2n^2\right)}{4b^2n^2+4}$$

input
```
int(x*sin(a+b*log(c*x^n))^2,x)
```

output
```
(x**2*( - 2*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*b*n + 2*sin(log(
x**n*c)*b + a)**2 + b**2*n**2))/(4*(b**2*n**2 + 1))
```

3.9 $\int \sin^2(a + b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 88

$$\int \sin^2(a + b\log(cx^n))\,dx = \frac{2b^2n^2x}{1+4b^2n^2} - \frac{2bnx\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{1+4b^2n^2} + \frac{x\sin^2(a+b\log(cx^n))}{1+4b^2n^2}$$

output
```
2*b^2*n^2*x/(4*b^2*n^2+1)-2*b*n*x*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/(4
*b^2*n^2+1)+x*sin(a+b*ln(c*x^n))^2/(4*b^2*n^2+1)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \sin^2(a + b\log(cx^n))\,dx = \frac{x(1+4b^2n^2-\cos(2(a+b\log(cx^n)))-2bn\sin(2(a+b\log(cx^n))))}{2+8b^2n^2}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]^2,x]
```

output
```
(x*(1 + 4*b^2*n^2 - Cos[2*(a + b*Log[c*x^n])] - 2*b*n*Sin[2*(a + b*Log[c*x
^n])]))/(2 + 8*b^2*n^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4980, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \; 4980$$

$$\frac{2b^2n^2\int 1dx}{4b^2n^2+1}+\frac{x\sin^2(a+b\log(cx^n))}{4b^2n^2+1}-\frac{2bnx\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+1}$$

$$\Big\downarrow \; 24$$

$$\frac{x\sin^2(a+b\log(cx^n))}{4b^2n^2+1}-\frac{2bnx\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+1}+\frac{2b^2n^2x}{4b^2n^2+1}$$

input
```
Int[Sin[a + b*Log[c*x^n]]^2,x]
```

output
```
(2*b^2*n^2*x)/(1 + 4*b^2*n^2) - (2*b*n*x*Cos[a + b*Log[c*x^n]]*Sin[a + b*L
og[c*x^n]])/(1 + 4*b^2*n^2) + (x*Sin[a + b*Log[c*x^n]]^2)/(1 + 4*b^2*n^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 4980
```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Sim
p[x*(Sin[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 + 1)), x] + (-Simp[b*d*n*
p*x*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*n
^2*p^2 + 1)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + 1))   Int
[Sin[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b, c, d, n}, x] &&
 IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + 1, 0]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{x\left(4b^{2}n^{2}-2bn\sin\left(2b\ln\left(c\,x^{n}\right)+2a\right)-\cos\left(2b\ln\left(c\,x^{n}\right)+2a\right)+1\right)}{8b^{2}n^{2}+2}$	59
default	$\frac{x}{2}-\frac{{\mathrm e}^{\frac{\ln\left(c\,x^{n}\right)}{n}-\frac{\ln\left(c\right)}{n}}\cos\left(2b\ln\left(c\,x^{n}\right)+2a\right)}{2n^{2}\left(\frac{1}{n^{2}}+4b^{2}\right)}-\frac{b\,{\mathrm e}^{\frac{\ln\left(c\,x^{n}\right)}{n}-\frac{\ln\left(c\right)}{n}}\sin\left(2b\ln\left(c\,x^{n}\right)+2a\right)}{n\left(\frac{1}{n^{2}}+4b^{2}\right)}$	104

input
```
int(sin(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

output
```
x*(4*b^2*n^2-2*b*n*sin(2*b*ln(c*x^n)+2*a)-cos(2*b*ln(c*x^n)+2*a)+1)/(8*b^2
*n^2+2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

$$\int \sin^2\left(a+b\log\left(cx^n\right)\right)\,dx =$$
$$-\frac{2\,bnx\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2}{4\,b^2n^2+1}$$

input
```
integrate(sin(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
-(2*b*n*x*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x) + b*log(c) + a) +
x*cos(b*n*log(x) + b*log(c) + a)^2 - (2*b^2*n^2 + 1)*x)/(4*b^2*n^2 + 1)
```

Sympy [F]

$$\int \sin^2(a+b\log(cx^n))\,dx$$
$$= \begin{cases} \int \sin^2\left(a - \frac{i\log(cx^n)}{2n}\right)dx & \text{fo} \\ \int \sin^2\left(a + \frac{i\log(cx^n)}{2n}\right)dx & \text{fo} \\ \frac{2b^2n^2x\sin^2(a+b\log(cx^n))}{4b^2n^2+1} + \frac{2b^2n^2x\cos^2(a+b\log(cx^n))}{4b^2n^2+1} - \frac{2bnx\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+1} + \frac{x\sin^2(a+b\log(cx^n))}{4b^2n^2+1} & \text{ot} \end{cases}$$

input
```
integrate(sin(a+b*ln(c*x**n))**2,x)
```

output
```
Piecewise((Integral(sin(a - I*log(c*x**n)/(2*n))**2, x), Eq(b, -I/(2*n))),
 (Integral(sin(a + I*log(c*x**n)/(2*n))**2, x), Eq(b, I/(2*n))), (2*b**2*n
**2*x*sin(a + b*log(c*x**n))**2/(4*b**2*n**2 + 1) + 2*b**2*n**2*x*cos(a +
b*log(c*x**n))**2/(4*b**2*n**2 + 1) - 2*b*n*x*sin(a + b*log(c*x**n))*cos(a
 + b*log(c*x**n))/(4*b**2*n**2 + 1) + x*sin(a + b*log(c*x**n))**2/(4*b**2*
n**2 + 1), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(88) = 176.

Time = 0.06 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.18

$$\int \sin^2(a+b\log(cx^n))\,dx =$$
$$-\frac{(2(b\cos(2b\log(c))\sin(4b\log(c)) - b\cos(4b\log(c))\sin(2b\log(c)) + b\sin(2b\log(c)))n + \cos(4b\log\ldots}{\ldots}$$

input
```
integrate(sin(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
-1/4*((2*(b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*lo
g(c)) + b*sin(2*b*log(c)))*n + cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*l
og(c))*sin(2*b*log(c)) + cos(2*b*log(c)))*x*cos(2*b*log(x^n) + 2*a) + (2*(
b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)) + b*
cos(2*b*log(c)))*n - cos(2*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin
(2*b*log(c)) - sin(2*b*log(c)))*x*sin(2*b*log(x^n) + 2*a) - 2*(4*(b^2*cos(
2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*b*log(c))^2 + sin(2*b*l
og(c))^2)*x)/(4*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(
2*b*log(c))^2 + sin(2*b*log(c))^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 783 vs. $2(88) = 176$.

Time = 0.26 (sec) , antiderivative size = 783, normalized size of antiderivative = 8.90

$$\int \sin^2(a + b\log(cx^n))\, dx = \text{Too large to display}$$

input
```
integrate(sin(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
1/2*x + 1/4*(4*b*n*x*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*
b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 4*b*n*x*e^(2*pi*b*n*sgn
(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c
)))*tan(a)^2 + 4*b*n*x*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 4*b
*n*x*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a)^2 - x*e^(2*pi*b*n*sgn(x)
- 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^
2*tan(a)^2 - 4*b*n*x*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*
b)*tan(b*n*log(abs(x)) + b*log(abs(c))) - 4*b*n*x*e^(2*pi*b*n*sgn(x) - 2*p
i*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(a) - x*tan(b*n*log(abs(x)) + b*log(abs
(c)))^2*tan(a)^2 - 4*b*n*x*tan(b*n*log(abs(x)) + b*log(abs(c))) + x*e^(2*p
i*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*
log(abs(c)))^2 - 4*b*n*x*tan(a) + 4*x*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi
*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a) + x*e^(2*p
i*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(a)^2 + x*tan(b*n*log
(abs(x)) + b*log(abs(c)))^2 + 4*x*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan
(a) + x*tan(a)^2 - x*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*
b) - x)/(4*b^2*n^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n
*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 + 4*b^2*n^2*e^(pi*b*n*sgn(x) - pi
*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + 4*b^2*
n^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(a)^2 + 4*b^2*n^...
```

Mupad [B] (verification not implemented)

Time = 19.79 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \sin^2(a+b\log(cx^n))\,dx = \frac{x\left(2\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^2+4\,b^2\,n^2-2\,b\,n\,\sin\left(2\,a+2\,b\,\ln\left(c\,x^n\right)\right)\right)}{8\,b^2\,n^2+2}$$

input
```
int(sin(a + b*log(c*x^n))^2,x)
```

output
```
(x*(2*sin(a + b*log(c*x^n))^2 + 4*b^2*n^2 - 2*b*n*sin(2*a + 2*b*log(c*x^n)
)))/(8*b^2*n^2 + 2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

$$\int \sin^2(a+b\log(cx^n))\,dx = \frac{x\left(-2\cos(\log(x^n c)\,b+a)\sin(\log(x^n c)\,b+a)\,bn+\sin(\log(x^n c)\,b+a)^2+2b^2n^2\right)}{4b^2n^2+1}$$

input
```
int(sin(a+b*log(c*x^n))^2,x)
```

output
```
(x*( - 2*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*b*n + sin(log(x**n*
c)*b + a)**2 + 2*b**2*n**2))/(4*b**2*n**2 + 1)
```

3.10 $\int \frac{\sin^2(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \frac{\sin^2(a+b\log(cx^n))}{x}\,dx=\frac{\log(x)}{2}-\frac{\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{2bn}$$

output
```
1/2*ln(x)-1/2*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{\sin^2(a+b\log(cx^n))}{x}\,dx=-\frac{-2(a+b\log(cx^n))+\sin(2(a+b\log(cx^n)))}{4bn}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]^2/x,x]
```

output
```
-1/4*(-2*(a + b*Log[c*x^n]) + Sin[2*(a + b*Log[c*x^n])])/(b*n)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3039, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{ 3039}$$

$$\frac{\int \sin^2(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sin(a+b\log(cx^n))^2d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{\frac{1}{2}\int 1d\log(cx^n)-\frac{\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{2b}}{n}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\frac{1}{2}\log(cx^n)-\frac{\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{2b}}{n}$$

input
```
Int[Sin[a + b*Log[c*x^n]]^2/x,x]
```

output
```
(Log[c*x^n]/2 - (Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/(2*b))/n
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

method	result	size
parallelrisch	$\frac{2\ln(x)bn-\sin(2b\ln(c\,x^{n})+2a)}{4bn}$	32
derivativedivides	$\frac{-\frac{\cos(a+b\ln(c\,x^{n}))\sin(a+b\ln(c\,x^{n}))}{2}+\frac{b\ln(c\,x^{n})}{2}+\frac{a}{2}}{nb}$	45
default	$\frac{-\frac{\cos(a+b\ln(c\,x^{n}))\sin(a+b\ln(c\,x^{n}))}{2}+\frac{b\ln(c\,x^{n})}{2}+\frac{a}{2}}{nb}$	45

input
```
int(sin(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*ln(x)*b*n-sin(2*b*ln(c*x^n)+2*a))/b/n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{\sin^2(a+b\log(cx^n))}{x}\,dx = \frac{bn\log(x) - \cos(bn\log(x) + b\log(c) + a)\sin(bn\log(x) + b\log(c) + a)}{2\,bn}$$

input
```
integrate(sin(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

output
```
1/2*(b*n*log(x) - cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x) + b*log(c)
 + a))/(b*n)
```

Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31

$$\int \frac{\sin^2(a+b\log(cx^n))}{x}\,dx = -\frac{\begin{cases} \log(x)\cos(2a) & \text{for } b=0 \wedge (b=0 \vee n=0) \\ \log(x)\cos(2a+2b\log(c)) & \text{for } n=0 \\ \frac{\sin(2a+2b\log(cx^n))}{2bn} & \text{otherwise} \end{cases}}{2} + \frac{\log(x)}{2}$$

input
```
integrate(sin(a+b*ln(c*x**n))**2/x,x)
```

output
```
-Piecewise((log(x)*cos(2*a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*co
s(2*a + 2*b*log(c)), Eq(n, 0)), (sin(2*a + 2*b*log(c*x**n))/(2*b*n), True)
)/2 + log(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.41

$$\int \frac{\sin^2(a+b\log(cx^n))}{x}\,dx$$
$$=\frac{2\,bn\log(x)-\cos(2\,b\log(x^n)+2\,a)\sin(2\,b\log(c))-\cos(2\,b\log(c))\sin(2\,b\log(x^n)+2\,a)}{4\,bn}$$

input
```
integrate(sin(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

output
```
1/4*(2*b*n*log(x) - cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) - cos(2*b*log(
c))*sin(2*b*log(x^n) + 2*a))/(b*n)
```

Giac [F]

$$\int \frac{\sin^2(a+b\log(cx^n))}{x}\,dx=\int \frac{\sin(b\log(cx^n)+a)^2}{x}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

output
```
integrate(sin(b*log(c*x^n) + a)^2/x, x)
```

Mupad [B] (verification not implemented)

Time = 20.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{\sin^2(a+b\log(cx^n))}{x}\,dx=\frac{\ln(x^n)}{2\,n}-\frac{\sin(2\,a+2\,b\,\ln(c\,x^n))}{4\,b\,n}$$

input
```
int(sin(a + b*log(c*x^n))^2/x,x)
```

output
```
log(x^n)/(2*n) - sin(2*a + 2*b*log(c*x^n))/(4*b*n)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{\sin^2(a+b\log(cx^n))}{x}\,dx = \frac{-\cos(\log(x^nc)\,b+a)\sin(\log(x^nc)\,b+a)+\log(x^nc)\,b}{2bn}$$

input
```
int(sin(a+b*log(c*x^n))^2/x,x)
```

output
```
( - cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a) + log(x**n*c)*b)/(2*b*n)
```

3.11 $\int \frac{\sin^2(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 95

$$\int \frac{\sin^2(a+b\log(cx^n))}{x^2}\,dx = -\frac{2b^2n^2}{(1+4b^2n^2)\,x} - \frac{2bn\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{(1+4b^2n^2)\,x} - \frac{\sin^2(a+b\log(cx^n))}{(1+4b^2n^2)\,x}$$

output
```
-2*b^2*n^2/(4*b^2*n^2+1)/x-2*b*n*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/(4*
b^2*n^2+1)/x-sin(a+b*ln(c*x^n))^2/(4*b^2*n^2+1)/x
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.60

$$\int \frac{\sin^2(a+b\log(cx^n))}{x^2}\,dx = \frac{-1-4b^2n^2+\cos(2(a+b\log(cx^n)))-2bn\sin(2(a+b\log(cx^n)))}{2(x+4b^2n^2x)}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]^2/x^2,x]
```

output
```
(-1 - 4*b^2*n^2 + Cos[2*(a + b*Log[c*x^n])] - 2*b*n*Sin[2*(a + b*Log[c*x^n
])])/(2*(x + 4*b^2*n^2*x))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{4990, 15\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+b\log(cx^n))}{x^2}dx$$

$$\Big\downarrow \text{ 4990}$$

$$\frac{2b^2n^2\int \frac{1}{x^2}dx}{4b^2n^2+1}-\frac{\sin^2(a+b\log(cx^n))}{x(4b^2n^2+1)}-\frac{2bn\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{x(4b^2n^2+1)}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\sin^2(a+b\log(cx^n))}{x(4b^2n^2+1)}-\frac{2bn\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{x(4b^2n^2+1)}-\frac{2b^2n^2}{x(4b^2n^2+1)}$$

input
```
Int[Sin[a + b*Log[c*x^n]]^2/x^2,x]
```

output
```
(-2*b^2*n^2)/((1 + 4*b^2*n^2)*x) - (2*b*n*Cos[a + b*Log[c*x^n]]*Sin[a + b*
Log[c*x^n]])/((1 + 4*b^2*n^2)*x) - Sin[a + b*Log[c*x^n]]^2/((1 + 4*b^2*n^2
)*x)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 4990
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (-Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a
 + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e
*(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2
))   Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
 c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.62

method	result	size
parallelrisch	$\frac{-4b^2n^2-2bn\sin(2b\ln(c\,x^n)+2a)+\cos(2b\ln(c\,x^n)+2a)-1}{8b^2n^2x+2x}$	59

input
```
int(sin(a+b*ln(c*x^n))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
(-4*b^2*n^2-2*b*n*sin(2*b*ln(c*x^n)+2*a)+cos(2*b*ln(c*x^n)+2*a)-1)/(8*b^2*
n^2*x+2*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.75

$$\int \frac{\sin^2\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx =$$
$$-\frac{2\,b^2n^2+2\,bn\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-\cos\left(bn\log\left(x\right)+b\log\left(c\right)+\right.}{(4\,b^2n^2+1)x}$$

input
```
integrate(sin(a+b*log(c*x^n))^2/x^2,x, algorithm="fricas")
```

output
```
-(2*b^2*n^2 + 2*b*n*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x) + b*log(
c) + a) - cos(b*n*log(x) + b*log(c) + a)^2 + 1)/((4*b^2*n^2 + 1)*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.89 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.17

$$\int \frac{\sin^2(a+b\log(cx^n))}{x^2}\,dx = \begin{cases} \frac{\cos\left(2a-\frac{i\log(cx^n)}{n}\right)}{4x} - \frac{1}{2x} - \frac{i\log(cx^n)\sin\left(2a-\frac{i\log(cx^n)}{n}\right)}{4nx} - \frac{\log(cx^n)\cos\left(2a-\frac{i\log(cx^n)}{n}\right)}{4nx} & \text{for } b \\ \frac{\cos\left(2a+\frac{i\log(cx^n)}{n}\right)}{4x} - \frac{1}{2x} + \frac{i\log(cx^n)\sin\left(2a+\frac{i\log(cx^n)}{n}\right)}{4nx} - \frac{\log(cx^n)\cos\left(2a+\frac{i\log(cx^n)}{n}\right)}{4nx} & \text{for } b \\ -\frac{2b^2n^2\sin^2(a+b\log(cx^n))}{4b^2n^2x+x} - \frac{2b^2n^2\cos^2(a+b\log(cx^n))}{4b^2n^2x+x} - \frac{2bn\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2x+x} - \frac{\sin^2(a+b\log(cx^n))}{4b^2n^2x+x} & \text{othe} \end{cases}$$

input
```
integrate(sin(a+b*ln(c*x**n))**2/x**2,x)
```

output
```
Piecewise((cos(2*a - I*log(c*x**n)/n)/(4*x) - 1/(2*x) - I*log(c*x**n)*sin(
2*a - I*log(c*x**n)/n)/(4*n*x) - log(c*x**n)*cos(2*a - I*log(c*x**n)/n)/(4
*n*x), Eq(b, -I/(2*n))), (cos(2*a + I*log(c*x**n)/n)/(4*x) - 1/(2*x) + I*l
og(c*x**n)*sin(2*a + I*log(c*x**n)/n)/(4*n*x) - log(c*x**n)*cos(2*a + I*lo
g(c*x**n)/n)/(4*n*x), Eq(b, I/(2*n))), (-2*b**2*n**2*sin(a + b*log(c*x**n)
)**2/(4*b**2*n**2*x + x) - 2*b**2*n**2*cos(a + b*log(c*x**n))**2/(4*b**2*n
**2*x + x) - 2*b*n*sin(a + b*log(c*x**n))*cos(a + b*log(c*x**n))/(4*b**2*n
**2*x + x) - sin(a + b*log(c*x**n))**2/(4*b**2*n**2*x + x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(95) = 190.

Time = 0.06 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.98

$$\int \frac{\sin^2(a+b\log(cx^n))}{x^2}\,dx = -\frac{8\left(b^2\cos(2b\log(c))^2 + b^2\sin(2b\log(c))^2\right)n^2 + 2\cos(2b\log(c))^2 + (2(b\cos(2b\log(c))\sin(4b\log(c)}{}$$

input
```
integrate(sin(a+b*log(c*x^n))^2/x^2,x, algorithm="maxima")
```

output
```
-1/4*(8*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + 2*cos(2*b*lo
g(c))^2 + (2*(b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*
b*log(c)) + b*sin(2*b*log(c)))*n - cos(4*b*log(c))*cos(2*b*log(c)) - sin(4
*b*log(c))*sin(2*b*log(c)) - cos(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 2*
sin(2*b*log(c))^2 + (2*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(
c))*sin(2*b*log(c)) + b*cos(2*b*log(c)))*n + cos(2*b*log(c))*sin(4*b*log(c
)) - cos(4*b*log(c))*sin(2*b*log(c)) + sin(2*b*log(c)))*sin(2*b*log(x^n) +
 2*a))/((4*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*b*l
og(c))^2 + sin(2*b*log(c))^2)*x)
```

Giac [F]

$$\int \frac{\sin^2(a+b\log(cx^n))}{x^2}\,dx = \int \frac{\sin(b\log(cx^n)+a)^2}{x^2}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^2/x^2,x, algorithm="giac")
```

output
```
integrate(sin(b*log(c*x^n) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+b\log(cx^n))}{x^2}\,dx = \int \frac{\sin(a+b\,\ln(c\,x^n))^2}{x^2}\,dx$$

input
```
int(sin(a + b*log(c*x^n))^2/x^2,x)
```

output
```
int(sin(a + b*log(c*x^n))^2/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

$$\int \frac{\sin^2(a+b\log(cx^n))}{x^2}\,dx = \frac{-2\cos(\log(x^n c)\,b+a)\sin(\log(x^n c)\,b+a)\,bn-\sin(\log(x^n c)\,b+a)^2-2b^2n^2}{x(4b^2n^2+1)}$$

input
```
int(sin(a+b*log(c*x^n))^2/x^2,x)
```

output
```
( - 2*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*b*n - sin(log(x**n*c)*
b + a)**2 - 2*b**2*n**2)/(x*(4*b**2*n**2 + 1))
```

3.12 $\int \frac{\sin^2(a+b\log(cx^n))}{x^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 98

$$\int \frac{\sin^2(a+b\log(cx^n))}{x^3} dx = -\frac{b^2n^2}{4(1+b^2n^2)x^2} - \frac{bn\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{2(1+b^2n^2)x^2} - \frac{\sin^2(a+b\log(cx^n))}{2(1+b^2n^2)x^2}$$

output
```
-1/4*b^2*n^2/(b^2*n^2+1)/x^2-1/2*b*n*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))
/(b^2*n^2+1)/x^2-1/2*sin(a+b*ln(c*x^n))^2/(b^2*n^2+1)/x^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.59

$$\int \frac{\sin^2(a+b\log(cx^n))}{x^3} dx = -\frac{1+b^2n^2-\cos(2(a+b\log(cx^n)))+bn\sin(2(a+b\log(cx^n)))}{4(1+b^2n^2)x^2}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]^2/x^3,x]
```

output
```
-1/4*(1 + b^2*n^2 - Cos[2*(a + b*Log[c*x^n])] + b*n*Sin[2*(a + b*Log[c*x^n
])])/((1 + b^2*n^2)*x^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4990, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+b\log(cx^n))}{x^3}dx$$

$$\Big\downarrow \text{4990}$$

$$\frac{b^2n^2\int \frac{1}{x^3}dx}{2(b^2n^2+1)}-\frac{\sin^2(a+b\log(cx^n))}{2x^2(b^2n^2+1)}-\frac{bn\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{2x^2(b^2n^2+1)}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\sin^2(a+b\log(cx^n))}{2x^2(b^2n^2+1)}-\frac{bn\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{2x^2(b^2n^2+1)}-\frac{b^2n^2}{4x^2(b^2n^2+1)}$$

input
```
Int[Sin[a + b*Log[c*x^n]]^2/x^3,x]
```

output
```
-1/4*(b^2*n^2)/((1 + b^2*n^2)*x^2) - (b*n*Cos[a + b*Log[c*x^n]]*Sin[a + b*
Log[c*x^n]])/(2*(1 + b^2*n^2)*x^2) - Sin[a + b*Log[c*x^n]]^2/(2*(1 + b^2*n
^2)*x^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 4990
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (-Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a
 + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e
*(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2
))   Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
 c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.60

method	result	size
parallelrisch	$\frac{-b^{2} n^{2}-b n \sin \left(2 b \ln \left(c \,x^{n}\right)+2 a \right)+\cos \left(2 b \ln \left(c \,x^{n}\right)+2 a \right)-1}{4 x^{2} \left(b^{2} n^{2}+1\right)}$	59

input
```
int(sin(a+b*ln(c*x^n))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-b^2*n^2-b*n*sin(2*b*ln(c*x^n)+2*a)+cos(2*b*ln(c*x^n)+2*a)-1)/x^2/(b^
2*n^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int \frac{\sin^2\left(a+b\log\left(cx^n\right)\right)}{x^3}\,dx =$$
$$-\frac{b^2n^2 + 2\,bn\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right) - 2\,\cos\left(bn\log\left(x\right)+b\log\left(c\right)\right.}{4\,(b^2n^2+1)x^2}$$

input
```
integrate(sin(a+b*log(c*x^n))^2/x^3,x, algorithm="fricas")
```

output
```
-1/4*(b^2*n^2 + 2*b*n*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x) + b*lo
g(c) + a) - 2*cos(b*n*log(x) + b*log(c) + a)^2 + 2)/((b^2*n^2 + 1)*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.91 (sec) , antiderivative size = 466, normalized size of antiderivative = 4.76

$$\int \frac{\sin^2(a + b\log(cx^n))}{x^3}\,dx$$
$$= \begin{cases} -\frac{\sin^2\left(a - \frac{i\log(cx^n)}{n}\right)}{2x^2} + \frac{i\sin\left(a - \frac{i\log(cx^n)}{n}\right)\cos\left(a - \frac{i\log(cx^n)}{n}\right)}{4x^2} + \frac{\log(cx^n)\sin^2\left(a - \frac{i\log(cx^n)}{n}\right)}{4nx^2} - \frac{i\log(cx^n)\sin\left(a - \frac{i\log(cx^n)}{n}\right)\cos}{2nx^2} \\ -\frac{\sin^2\left(a + \frac{i\log(cx^n)}{n}\right)}{2x^2} - \frac{i\sin\left(a + \frac{i\log(cx^n)}{n}\right)\cos\left(a + \frac{i\log(cx^n)}{n}\right)}{4x^2} + \frac{\log(cx^n)\sin^2\left(a + \frac{i\log(cx^n)}{n}\right)}{4nx^2} + \frac{i\log(cx^n)\sin\left(a + \frac{i\log(cx^n)}{n}\right)\cos}{2nx^2} \\ -\frac{b^2n^2\sin^2(a+b\log(cx^n))}{4b^2n^2x^2+4x^2} - \frac{b^2n^2\cos^2(a+b\log(cx^n))}{4b^2n^2x^2+4x^2} - \frac{2bn\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2x^2+4x^2} - \frac{2\sin^2(a+b\log(cx^n))}{4b^2n^2x^2+4x^2} \end{cases}$$

input
```
integrate(sin(a+b*ln(c*x**n))**2/x**3,x)
```

output
```
Piecewise((-sin(a - I*log(c*x**n)/n)**2/(2*x**2) + I*sin(a - I*log(c*x**n)
/n)*cos(a - I*log(c*x**n)/n)/(4*x**2) + log(c*x**n)*sin(a - I*log(c*x**n)/
n)**2/(4*n*x**2) - I*log(c*x**n)*sin(a - I*log(c*x**n)/n)*cos(a - I*log(c*
x**n)/n)/(2*n*x**2) - log(c*x**n)*cos(a - I*log(c*x**n)/n)**2/(4*n*x**2),
Eq(b, -I/n)), (-sin(a + I*log(c*x**n)/n)**2/(2*x**2) - I*sin(a + I*log(c*x
**n)/n)*cos(a + I*log(c*x**n)/n)/(4*x**2) + log(c*x**n)*sin(a + I*log(c*x*
*n)/n)**2/(4*n*x**2) + I*log(c*x**n)*sin(a + I*log(c*x**n)/n)*cos(a + I*lo
g(c*x**n)/n)/(2*n*x**2) - log(c*x**n)*cos(a + I*log(c*x**n)/n)**2/(4*n*x**
2), Eq(b, I/n)), (-b**2*n**2*sin(a + b*log(c*x**n))**2/(4*b**2*n**2*x**2 +
 4*x**2) - b**2*n**2*cos(a + b*log(c*x**n))**2/(4*b**2*n**2*x**2 + 4*x**2)
 - 2*b*n*sin(a + b*log(c*x**n))*cos(a + b*log(c*x**n))/(4*b**2*n**2*x**2 +
 4*x**2) - 2*sin(a + b*log(c*x**n))**2/(4*b**2*n**2*x**2 + 4*x**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(92) = 184$.

Time = 0.05 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.86

$$\int \frac{\sin^2(a + b\log(cx^n))}{x^3}\,dx = -\frac{2\left(b^2\cos(2\,b\log(c))^2 + b^2\sin(2\,b\log(c))^2\right)n^2 + 2\,\cos(2\,b\log(c))^2 + ((b\cos(2\,b\log(c))\sin(4\,b\log(c))}{}$$

input
```
integrate(sin(a+b*log(c*x^n))^2/x^3,x, algorithm="maxima")
```

output
```
-1/8*(2*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + 2*cos(2*b*lo
g(c))^2 + ((b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*
log(c)) + b*sin(2*b*log(c)))*n - cos(4*b*log(c))*cos(2*b*log(c)) - sin(4*b
*log(c))*sin(2*b*log(c)) - cos(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 2*si
n(2*b*log(c))^2 + ((b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*
sin(2*b*log(c)) + b*cos(2*b*log(c)))*n + cos(2*b*log(c))*sin(4*b*log(c)) -
 cos(4*b*log(c))*sin(2*b*log(c)) + sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a
))/(((b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*b*log(c))
^2 + sin(2*b*log(c))^2)*x^2)
```

Giac [F]

$$\int \frac{\sin^2(a + b\log(cx^n))}{x^3}\,dx = \int \frac{\sin(b\log(cx^n) + a)^2}{x^3}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^2/x^3,x, algorithm="giac")
```

output
```
integrate(sin(b*log(c*x^n) + a)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2(a+b\log(cx^n))}{x^3}\,dx = \int \frac{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^2}{x^3}\,dx$$

input
```
int(sin(a + b*log(c*x^n))^2/x^3,x)
```

output
```
int(sin(a + b*log(c*x^n))^2/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.67

$$\int \frac{\sin^2(a+b\log(cx^n))}{x^3}\,dx = \frac{-2\cos(\log(x^nc)\,b+a)\sin(\log(x^nc)\,b+a)\,bn-2\sin(\log(x^nc)\,b+a)^2-b^2n^2}{4x^2\,(b^2n^2+1)}$$

input
```
int(sin(a+b*log(c*x^n))^2/x^3,x)
```

output
```
( - 2*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*b*n - 2*sin(log(x**n*c
)*b + a)**2 - b**2*n**2)/(4*x**2*(b**2*n**2 + 1))
```

3.13 $\int x^2 \sin^3(a + b\log(cx^n))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 160

$$\int x^2 \sin^3(a + b\log(cx^n))\,dx = -\frac{2b^3n^3x^3\cos(a + b\log(cx^n))}{3(9 + 10b^2n^2 + b^4n^4)} + \frac{2b^2n^2x^3\sin(a + b\log(cx^n))}{9 + 10b^2n^2 + b^4n^4} - \frac{bnx^3\cos(a + b\log(cx^n))\sin^2(a + b\log(cx^n))}{3(1 + b^2n^2)} + \frac{x^3\sin^3(a + b\log(cx^n))}{3(1 + b^2n^2)}$$

output
```
-2*b^3*n^3*x^3*cos(a+b*ln(c*x^n))/(3*b^4*n^4+30*b^2*n^2+27)+2*b^2*n^2*x^3*
sin(a+b*ln(c*x^n))/(b^4*n^4+10*b^2*n^2+9)-b*n*x^3*cos(a+b*ln(c*x^n))*sin(a
+b*ln(c*x^n))^2/(3*b^2*n^2+3)+x^3*sin(a+b*ln(c*x^n))^3/(3*b^2*n^2+3)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.76

$$\int x^2 \sin^3(a+b\log(cx^n))\,dx$$
$$= \frac{x^3(-9bn(1+b^2n^2)\cos(a+b\log(cx^n))+bn(9+b^2n^2)\cos(3(a+b\log(cx^n)))-2(-9-13b^2n^2+(9+\ldots}{12(9+10b^2n^2+b^4n^4)}$$

input
```
Integrate[x^2*Sin[a + b*Log[c*x^n]]^3,x]
```

output
```
(x^3*(-9*b*n*(1 + b^2*n^2)*Cos[a + b*Log[c*x^n]] + b*n*(9 + b^2*n^2)*Cos[3
*(a + b*Log[c*x^n])] - 2*(-9 - 13*b^2*n^2 + (9 + b^2*n^2)*Cos[2*(a + b*Log
[c*x^n])])*Sin[a + b*Log[c*x^n]]))/(12*(9 + 10*b^2*n^2 + b^4*n^4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4990, 4988}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin^3(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{4990}$$

$$\frac{2b^2n^2\int x^2\sin(a+b\log(cx^n))dx}{3(b^2n^2+1)}+\frac{x^3\sin^3(a+b\log(cx^n))}{3(b^2n^2+1)}-\frac{bnx^3\sin^2(a+b\log(cx^n))\cos(a+b\log(cx^n))}{3(b^2n^2+1)}$$

$$\Big\downarrow \text{4988}$$

$$\frac{x^3\sin^3(a+b\log(cx^n))}{3(b^2n^2+1)}-\frac{bnx^3\sin^2(a+b\log(cx^n))\cos(a+b\log(cx^n))}{3(b^2n^2+1)}+\frac{2b^2n^2\left(\frac{3x^3\sin(a+b\log(cx^n))}{b^2n^2+9}-\frac{bnx^3\cos(a+b\log(cx^n))}{b^2n^2+9}\right)}{3(b^2n^2+1)}$$

input
```
Int[x^2*Sin[a + b*Log[c*x^n]]^3,x]
```

output
```
-1/3*(b*n*x^3*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^2)/(1 + b^2*n^2)
 + (x^3*Sin[a + b*Log[c*x^n]]^3)/(3*(1 + b^2*n^2)) + (2*b^2*n^2*(-((b*n*x^
3*Cos[a + b*Log[c*x^n]])/(9 + b^2*n^2)) + (3*x^3*Sin[a + b*Log[c*x^n]])/(9
 + b^2*n^2)))/(3*(1 + b^2*n^2))
```

Defintions of rubi rules used

rule 4988
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

rule 4990
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (-Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a
 + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e
*(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2
))   Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
 c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [F]

$$\int x^2 \sin\left(a+b\ln\left(c\,x^n\right)\right)^3 dx$$

input
```
int(x^2*sin(a+b*ln(c*x^n))^3,x)
```

output
```
int(x^2*sin(a+b*ln(c*x^n))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.86

$$\int x^2 \sin^3(a + b\log(cx^n))\,dx = \frac{(b^3n^3 + 9\,bn)x^3\cos(bn\log(x) + b\log(c) + a)^3 - 3\,(b^3n^3 + 3\,bn)x^3\cos(bn\log(x) + b\log(c) + a) - ((b^2n^2 + 9)x^3\cos(bn\log(x) + b\log(c) + a)^2 - (7\,b^2n^2 + 9)x^3)\sin(bn\log(x) + b\log(c) + a)}{3\,(b^4n^4 + 10\,b^2n^2 + 9)}$$

input
```
integrate(x^2*sin(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
1/3*((b^3*n^3 + 9*b*n)*x^3*cos(b*n*log(x) + b*log(c) + a)^3 - 3*(b^3*n^3 +
 3*b*n)*x^3*cos(b*n*log(x) + b*log(c) + a) - ((b^2*n^2 + 9)*x^3*cos(b*n*lo
g(x) + b*log(c) + a)^2 - (7*b^2*n^2 + 9)*x^3)*sin(b*n*log(x) + b*log(c) +
a))/(b^4*n^4 + 10*b^2*n^2 + 9)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \sin^3(a + b\log(cx^n))\,dx = \text{Timed out}$$

input
```
integrate(x**2*sin(a+b*ln(c*x**n))**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1008 vs. 2(154) = 308.

Time = 0.08 (sec) , antiderivative size = 1008, normalized size of antiderivative = 6.30

$$\int x^2 \sin^3(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(x^2*sin(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
1/24*(((b^3*cos(6*b*log(c))*cos(3*b*log(c)) + b^3*sin(6*b*log(c))*sin(3*b*
log(c)) + b^3*cos(3*b*log(c)))*n^3 - (b^2*cos(3*b*log(c))*sin(6*b*log(c))
- b^2*cos(6*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c)))*n^2 + 9*(b*co
s(6*b*log(c))*cos(3*b*log(c)) + b*sin(6*b*log(c))*sin(3*b*log(c)) + b*cos(
3*b*log(c)))*n - 9*cos(3*b*log(c))*sin(6*b*log(c)) + 9*cos(6*b*log(c))*sin
(3*b*log(c)) - 9*sin(3*b*log(c)))*x^3*cos(3*b*log(x^n) + 3*a) - 9*((b^3*co
s(4*b*log(c))*cos(3*b*log(c)) + b^3*cos(3*b*log(c))*cos(2*b*log(c)) + b^3*
sin(4*b*log(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c))*sin(2*b*log(c)))*n^3
 - 3*(b^2*cos(3*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(3*b*lo
g(c)) + b^2*cos(2*b*log(c))*sin(3*b*log(c)) - b^2*cos(3*b*log(c))*sin(2*b*
log(c)))*n^2 + (b*cos(4*b*log(c))*cos(3*b*log(c)) + b*cos(3*b*log(c))*cos(
2*b*log(c)) + b*sin(4*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c))*sin(2*
b*log(c)))*n - 3*cos(3*b*log(c))*sin(4*b*log(c)) + 3*cos(4*b*log(c))*sin(3
*b*log(c)) - 3*cos(2*b*log(c))*sin(3*b*log(c)) + 3*cos(3*b*log(c))*sin(2*b
*log(c)))*x^3*cos(b*log(x^n) + a) - ((b^3*cos(3*b*log(c))*sin(6*b*log(c))
- b^3*cos(6*b*log(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c)))*n^3 + (b^2*co
s(6*b*log(c))*cos(3*b*log(c)) + b^2*sin(6*b*log(c))*sin(3*b*log(c)) + b^2*
cos(3*b*log(c)))*n^2 + 9*(b*cos(3*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*lo
g(c))*sin(3*b*log(c)) + b*sin(3*b*log(c)))*n + 9*cos(6*b*log(c))*cos(3*b*l
og(c)) + 9*sin(6*b*log(c))*sin(3*b*log(c)) + 9*cos(3*b*log(c)))*x^3*sin...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18085 vs. $2(154) = 308$.

Time = 1.32 (sec) , antiderivative size = 18085, normalized size of antiderivative = 113.03

$$\int x^2 \sin^3(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(x^2*sin(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output

```
1/24*(b^3*n^3*x^3*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/
2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs
(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 - 9*b^3*n^3*x^3*e^(1
/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*lo
g(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c
)))^2*tan(3/2*a)^2*tan(1/2*a)^2 - 9*b^3*n^3*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/
2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log
(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*ta
n(1/2*a)^2 + b^3*n^3*x^3*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn
(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n
*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 + b^3*n^3*x^
3*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*
b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log
(abs(c)))^2*tan(3/2*a)^2 + 9*b^3*n^3*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n
 + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c))
)^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2 + 9*b^3*n^
3*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan
(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*
b*log(abs(c)))^2*tan(3/2*a)^2 + b^3*n^3*x^3*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi
*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(...
```

Mupad [B] (verification not implemented)

Time = 20.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.76

$$\int x^2 \sin^3(a + b\log(cx^n))\,dx = -\frac{x^3\,e^{-a\,1i}\,\frac{1}{(c\,x^n)^{b\,1i}}\,3i}{-24 + b\,n\,8i} - \frac{3\,x^3\,e^{a\,1i}\,(c\,x^n)^{b\,1i}}{8\,b\,n - 24i} + \frac{x^3\,e^{-a\,3i}\,\frac{1}{(c\,x^n)^{b\,3i}}\,1i}{-24 + b\,n\,24i} + \frac{x^3\,e^{a\,3i}\,(c\,x^n)^{b\,3i}}{24\,b\,n - 24i}$$

input

```
int(x^2*sin(a + b*log(c*x^n))^3,x)
```

output

```
(x^3*exp(-a*3i)/(c*x^n)^(b*3i)*1i)/(b*n*24i - 24) - (3*x^3*exp(a*1i)*(c*x^
n)^(b*1i))/(8*b*n - 24i) - (x^3*exp(-a*1i)/(c*x^n)^(b*1i)*3i)/(b*n*8i - 24
) + (x^3*exp(a*3i)*(c*x^n)^(b*3i))/(24*b*n - 24i)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.99

$$\int x^2 \sin^3(a + b\log(cx^n))\,dx$$
$$= \frac{x^3\left(-\cos(\log(x^n c)\,b + a)\sin(\log(x^n c)\,b + a)^2 b^3 n^3 - 9\cos(\log(x^n c)\,b + a)\sin(\log(x^n c)\,b + a)^2 bn - 2\cos\right.}{3b^4n^4 + 30}$$

input
```
int(x^2*sin(a+b*log(c*x^n))^3,x)
```

output
```
(x**3*( - cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**2*b**3*n**3 - 9*c
os(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**2*b*n - 2*cos(log(x**n*c)*b
+ a)*b**3*n**3 + sin(log(x**n*c)*b + a)**3*b**2*n**2 + 9*sin(log(x**n*c)*b
 + a)**3 + 6*sin(log(x**n*c)*b + a)*b**2*n**2))/(3*(b**4*n**4 + 10*b**2*n*
*2 + 9))
```

3.14 $\int x \sin^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 158

$$\int x\sin^3(a+b\log(cx^n))\,dx=-\frac{6b^3n^3x^2\cos(a+b\log(cx^n))}{16+40b^2n^2+9b^4n^4}+\frac{12b^2n^2x^2\sin(a+b\log(cx^n))}{16+40b^2n^2+9b^4n^4}-\frac{3bnx^2\cos(a+b\log(cx^n))\sin^2(a+b\log(cx^n))}{4+9b^2n^2}+\frac{2x^2\sin^3(a+b\log(cx^n))}{4+9b^2n^2}$$

output
```
-6*b^3*n^3*x^2*cos(a+b*ln(c*x^n))/(9*b^4*n^4+40*b^2*n^2+16)+12*b^2*n^2*x^2
*sin(a+b*ln(c*x^n))/(9*b^4*n^4+40*b^2*n^2+16)-3*b*n*x^2*cos(a+b*ln(c*x^n))
*sin(a+b*ln(c*x^n))^2/(9*b^2*n^2+4)+2*x^2*sin(a+b*ln(c*x^n))^3/(9*b^2*n^2+
4)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.79

$$\int x\sin^3(a+b\log(cx^n))\,dx = \frac{x^2(-3bn(4+9b^2n^2)\cos(a+b\log(cx^n))+3bn(4+b^2n^2)\cos(3(a+b\log(cx^n)))-4(-4-13b^2n^2+(4}{4(16+40b^2n^2+9b^4n^4)}$$

input
```
Integrate[x*Sin[a + b*Log[c*x^n]]^3,x]
```

output
```
(x^2*(-3*b*n*(4 + 9*b^2*n^2)*Cos[a + b*Log[c*x^n]] + 3*b*n*(4 + b^2*n^2)*C
os[3*(a + b*Log[c*x^n])] - 4*(-4 - 13*b^2*n^2 + (4 + b^2*n^2)*Cos[2*(a + b
*Log[c*x^n])])*Sin[a + b*Log[c*x^n]]))/(4*(16 + 40*b^2*n^2 + 9*b^4*n^4))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4990, 4988}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sin^3(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{4990}$$

$$\frac{6b^2n^2\int x\sin(a+b\log(cx^n))dx}{9b^2n^2+4}+\frac{2x^2\sin^3(a+b\log(cx^n))}{9b^2n^2+4}-\frac{3bnx^2\sin^2(a+b\log(cx^n))\cos(a+b\log(cx^n))}{9b^2n^2+4}$$

$$\Big\downarrow \text{4988}$$

$$\frac{2x^2\sin^3(a+b\log(cx^n))}{9b^2n^2+4}-\frac{3bnx^2\sin^2(a+b\log(cx^n))\cos(a+b\log(cx^n))}{9b^2n^2+4}+\frac{6b^2n^2\left(\frac{2x^2\sin(a+b\log(cx^n))}{b^2n^2+4}-\frac{bnx^2\cos(a+b\log(cx^n))}{b^2n^2+4}\right)}{9b^2n^2+4}$$

input
```
Int[x*Sin[a + b*Log[c*x^n]]^3,x]
```

output
```
(-3*b*n*x^2*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^2)/(4 + 9*b^2*n^2)
 + (2*x^2*Sin[a + b*Log[c*x^n]]^3)/(4 + 9*b^2*n^2) + (6*b^2*n^2*(-((b*n*x^
2*Cos[a + b*Log[c*x^n]])/(4 + b^2*n^2)) + (2*x^2*Sin[a + b*Log[c*x^n]])/(4
 + b^2*n^2)))/(4 + 9*b^2*n^2)
```

Defintions of rubi rules used

rule 4988
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

rule 4990
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (-Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a
 + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e
*(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2
))   Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
 c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [F]

$$\int x\sin\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}dx$$

input
```
int(x*sin(a+b*ln(c*x^n))^3,x)
```

output
```
int(x*sin(a+b*ln(c*x^n))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.89

$$\int x\sin^3\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\frac{3\left(b^3n^3+4\,bn\right)x^2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3-3\left(3\,b^3n^3+4\,bn\right)x^2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-2}{9\,b^4n^4+40\,b^2n^2}$$

input
```
integrate(x*sin(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
(3*(b^3*n^3 + 4*b*n)*x^2*cos(b*n*log(x) + b*log(c) + a)^3 - 3*(3*b^3*n^3 +
 4*b*n)*x^2*cos(b*n*log(x) + b*log(c) + a) - 2*((b^2*n^2 + 4)*x^2*cos(b*n*
log(x) + b*log(c) + a)^2 - (7*b^2*n^2 + 4)*x^2)*sin(b*n*log(x) + b*log(c)
+ a))/(9*b^4*n^4 + 40*b^2*n^2 + 16)
```

Sympy [F(-1)]

Timed out.

$$\int x\sin^3\left(a+b\log\left(cx^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x*sin(a+b*ln(c*x**n))**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1016 vs. 2(158) = 316.

Time = 0.08 (sec) , antiderivative size = 1016, normalized size of antiderivative = 6.43

$$\int x\sin^3\left(a+b\log\left(cx^n\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x*sin(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
1/8*((3*(b^3*cos(6*b*log(c))*cos(3*b*log(c)) + b^3*sin(6*b*log(c))*sin(3*b
*log(c)) + b^3*cos(3*b*log(c)))*n^3 - 2*(b^2*cos(3*b*log(c))*sin(6*b*log(c
)) - b^2*cos(6*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c)))*n^2 + 12*(
b*cos(6*b*log(c))*cos(3*b*log(c)) + b*sin(6*b*log(c))*sin(3*b*log(c)) + b*
cos(3*b*log(c)))*n - 8*cos(3*b*log(c))*sin(6*b*log(c)) + 8*cos(6*b*log(c))
*sin(3*b*log(c)) - 8*sin(3*b*log(c)))*x^2*cos(3*b*log(x^n) + 3*a) - 3*(9*(
b^3*cos(4*b*log(c))*cos(3*b*log(c)) + b^3*cos(3*b*log(c))*cos(2*b*log(c))
+ b^3*sin(4*b*log(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c))*sin(2*b*log(c)
))*n^3 - 18*(b^2*cos(3*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin
(3*b*log(c)) + b^2*cos(2*b*log(c))*sin(3*b*log(c)) - b^2*cos(3*b*log(c))*s
in(2*b*log(c)))*n^2 + 4*(b*cos(4*b*log(c))*cos(3*b*log(c)) + b*cos(3*b*log
(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c
))*sin(2*b*log(c)))*n - 8*cos(3*b*log(c))*sin(4*b*log(c)) + 8*cos(4*b*log(
c))*sin(3*b*log(c)) - 8*cos(2*b*log(c))*sin(3*b*log(c)) + 8*cos(3*b*log(c)
)*sin(2*b*log(c)))*x^2*cos(b*log(x^n) + a) - (3*(b^3*cos(3*b*log(c))*sin(6
*b*log(c)) - b^3*cos(6*b*log(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c)))*n^
3 + 2*(b^2*cos(6*b*log(c))*cos(3*b*log(c)) + b^2*sin(6*b*log(c))*sin(3*b*l
og(c)) + b^2*cos(3*b*log(c)))*n^2 + 12*(b*cos(3*b*log(c))*sin(6*b*log(c))
- b*cos(6*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c)))*n + 8*cos(6*b*log
(c))*cos(3*b*log(c)) + 8*sin(6*b*log(c))*sin(3*b*log(c)) + 8*cos(3*b*lo...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18117 vs. $2(158) = 316$.

Time = 1.07 (sec) , antiderivative size = 18117, normalized size of antiderivative = 114.66

$$\int x \sin^3(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(x*sin(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
1/8*(3*b^3*n^3*x^2*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3
/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(ab
s(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 - 27*b^3*n^3*x^2*e^
(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*
log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs
(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 - 27*b^3*n^3*x^2*e^(-1/2*pi*b*n*sgn(x) +
 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*
log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2
*tan(1/2*a)^2 + 3*b^3*n^3*x^2*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*
b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/
2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 + 3*b^3
*n^3*x^2*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*t
an(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/
2*b*log(abs(c)))^2*tan(3/2*a)^2 + 27*b^3*n^3*x^2*e^(1/2*pi*b*n*sgn(x) - 1/
2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log
(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2 +
27*b^3*n^3*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*
pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x
)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2 + 3*b^3*n^3*x^2*e^(-3/2*pi*b*n*sgn(
x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) +...
```

Mupad [B] (verification not implemented)

Time = 19.88 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.77

$$\int x\sin^3(a+b\log(cx^n))\,dx = -\frac{x^2\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,1\mathrm{i}}}\,3\mathrm{i}}{-16+b\,n\,8\mathrm{i}}-\frac{3\,x^2\,\mathrm{e}^{a\,1\mathrm{i}}\,(c\,x^n)^{b\,1\mathrm{i}}}{8\,b\,n-16\mathrm{i}}+\frac{x^2\,\mathrm{e}^{-a\,3\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,3\mathrm{i}}}\,1\mathrm{i}}{-16+b\,n\,24\mathrm{i}}+\frac{x^2\,\mathrm{e}^{a\,3\mathrm{i}}\,(c\,x^n)^{b\,3\mathrm{i}}}{24\,b\,n-16\mathrm{i}}$$

input
```
int(x*sin(a + b*log(c*x^n))^3,x)
```

output
```
(x^2*exp(-a*3i)/(c*x^n)^(b*3i)*1i)/(b*n*24i - 16) - (3*x^2*exp(a*1i)*(c*x^
n)^(b*1i))/(8*b*n - 16i) - (x^2*exp(-a*1i)/(c*x^n)^(b*1i)*3i)/(b*n*8i - 16
) + (x^2*exp(a*3i)*(c*x^n)^(b*3i))/(24*b*n - 16i)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01

$$\int x\sin^3(a+b\log(cx^n))\,dx$$
$$=\frac{x^2\left(-3\cos(\log(x^nc)\,b+a)\sin(\log(x^nc)\,b+a)^2b^3n^3-12\cos(\log(x^nc)\,b+a)\sin(\log(x^nc)\,b+a)^2bn-6\right.}{9b^4n^4+}$$

input
```
int(x*sin(a+b*log(c*x^n))^3,x)
```

output
```
(x**2*( - 3*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**2*b**3*n**3 - 1
2*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**2*b*n - 6*cos(log(x**n*c)
*b + a)*b**3*n**3 + 2*sin(log(x**n*c)*b + a)**3*b**2*n**2 + 8*sin(log(x**n
*c)*b + a)**3 + 12*sin(log(x**n*c)*b + a)*b**2*n**2))/(9*b**4*n**4 + 40*b*
*2*n**2 + 16)
```

3.15 $\int \sin^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 149

$$\int \sin^3(a+b\log(cx^n))\,dx = -\frac{6b^3n^3x\cos(a+b\log(cx^n))}{1+10b^2n^2+9b^4n^4}+\frac{6b^2n^2x\sin(a+b\log(cx^n))}{1+10b^2n^2+9b^4n^4} - \frac{3bnx\cos(a+b\log(cx^n))\sin^2(a+b\log(cx^n))}{1+9b^2n^2} + \frac{x\sin^3(a+b\log(cx^n))}{1+9b^2n^2}$$

output
```
-6*b^3*n^3*x*cos(a+b*ln(c*x^n))/(9*b^4*n^4+10*b^2*n^2+1)+6*b^2*n^2*x*sin(a
+b*ln(c*x^n))/(9*b^4*n^4+10*b^2*n^2+1)-3*b*n*x*cos(a+b*ln(c*x^n))*sin(a+b*
ln(c*x^n))^2/(9*b^2*n^2+1)+x*sin(a+b*ln(c*x^n))^3/(9*b^2*n^2+1)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.81

$$\int \sin^3(a+b\log(cx^n))\,dx = -\frac{x(3bn(1+9b^2n^2)\cos(a+b\log(cx^n))-3(bn+b^3n^3)\cos(3(a+b\log(cx^n)))+2(-1-13b^2n^2+(1+}{4+40b^2n^2+36b^4n^4}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]^3,x]
```

output
```
-((x*(3*b*n*(1 + 9*b^2*n^2)*Cos[a + b*Log[c*x^n]] - 3*(b*n + b^3*n^3)*Cos[
3*(a + b*Log[c*x^n])] + 2*(-1 - 13*b^2*n^2 + (1 + b^2*n^2)*Cos[2*(a + b*Lo
g[c*x^n])])*Sin[a + b*Log[c*x^n]]))/(4 + 40*b^2*n^2 + 36*b^4*n^4))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4980, 4978}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \; 4980$$

$$\frac{6b^2n^2\int \sin(a+b\log(cx^n))\,dx}{9b^2n^2+1}+\frac{x\sin^3(a+b\log(cx^n))}{9b^2n^2+1}-\frac{3bnx\sin^2(a+b\log(cx^n))\cos(a+b\log(cx^n))}{9b^2n^2+1}$$

$$\Big\downarrow \; 4978$$

$$\frac{x\sin^3(a+b\log(cx^n))}{9b^2n^2+1}-\frac{3bnx\sin^2(a+b\log(cx^n))\cos(a+b\log(cx^n))}{9b^2n^2+1}+\frac{6b^2n^2\left(\frac{x\sin(a+b\log(cx^n))}{b^2n^2+1}-\frac{bnx\cos(a+b\log(cx^n))}{b^2n^2+1}\right)}{9b^2n^2+1}$$

input
```
Int[Sin[a + b*Log[c*x^n]]^3,x]
```

output
```
(-3*b*n*x*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^2)/(1 + 9*b^2*n^2) +
 (x*Sin[a + b*Log[c*x^n]]^3)/(1 + 9*b^2*n^2) + (6*b^2*n^2*(-((b*n*x*Cos[a
+ b*Log[c*x^n]])/(1 + b^2*n^2)) + (x*Sin[a + b*Log[c*x^n]])/(1 + b^2*n^2))
)/(1 + 9*b^2*n^2)
```

Defintions of rubi rules used

rule 4978
```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] - Simp[b*d*n*x*(Cos[d*(a +
 b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

rule 4980
```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Sim
p[x*(Sin[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 + 1)), x] + (-Simp[b*d*n*
p*x*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*n
^2*p^2 + 1)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + 1))   Int
[Sin[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b, c, d, n}, x] &&
 IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + 1, 0]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.28

method	result
default	$-\frac{3 b \,{\mathrm e}^{\frac{\ln \left(c \,x^{n}\right)}{n}-\frac{\ln \left(c \right)}{n}} \cos \left(a +b \ln \left(c \,x^{n}\right)\right)}{4 n \left(\frac{1}{n^{2}}+b^{2}\right)}+\frac{3 \,{\mathrm e}^{\frac{\ln \left(c \,x^{n}\right)}{n}-\frac{\ln \left(c \right)}{n}} \sin \left(a +b \ln \left(c \,x^{n}\right)\right)}{4 n^{2} \left(\frac{1}{n^{2}}+b^{2}\right)}+\frac{3 b \,{\mathrm e}^{\frac{\ln \left(c \,x^{n}\right)}{n}-\frac{\ln \left(c \right)}{n}} \cos \left(3 b \ln \left(c \,x^{n}\right)+3 a \right)}{4 n \left(\frac{1}{n^{2}}+9 b^{2}\right)}-$
parallelrisch	$\frac{2 \left(b^{3} n^{3} \tan \left(\frac{a}{2}+b \ln \left(\sqrt{c \,x^{n}}\right)\right)^{6}+2 b^{2} n^{2} \tan \left(\frac{a}{2}+b \ln \left(\sqrt{c \,x^{n}}\right)\right)^{5}+\left(3 b^{3} n^{3}+2 b n \right) \tan \left(\frac{a}{2}+b \ln \left(\sqrt{c \,x^{n}}\right)\right)^{4}+\frac{4 \left(4 b^{2} n^{2}+1\right) \tan \left(\frac{a}{2}+b \ln \left(\sqrt{}\right.\right.}{3}\right.}{3 \left(b^{2} n^{2}+1\right) \left(1+\tan \left(\frac{a}{2}+b \ln \left(\sqrt{c \,x^{n}}\right)\right)^{2}\right)^{3} \left(\frac{1}{9}+\right.}$

input
```
int(sin(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
```

output
```
-3/4/n*b/(1/n^2+b^2)*exp(1/n*ln(c*x^n)-1/n*ln(c))*cos(a+b*ln(c*x^n))+3/4/n
^2/(1/n^2+b^2)*exp(1/n*ln(c*x^n)-1/n*ln(c))*sin(a+b*ln(c*x^n))+3/4/n*b/(1/
n^2+9*b^2)*exp(1/n*ln(c*x^n)-1/n*ln(c))*cos(3*b*ln(c*x^n)+3*a)-1/4/n^2/(1/
n^2+9*b^2)*exp(1/n*ln(c*x^n)-1/n*ln(c))*sin(3*b*ln(c*x^n)+3*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.87

$$\int \sin^3(a + b\log(cx^n)) \, dx$$
$$= \frac{3(b^3n^3 + bn)x\cos(bn\log(x) + b\log(c) + a)^3 - 3(3b^3n^3 + bn)x\cos(bn\log(x) + b\log(c) + a) - ((b^2n^2}{9b^4n^4 + 10b^2n^2 + 1}$$

input
```
integrate(sin(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
(3*(b^3*n^3 + b*n)*x*cos(b*n*log(x) + b*log(c) + a)^3 - 3*(3*b^3*n^3 + b*n
)*x*cos(b*n*log(x) + b*log(c) + a) - ((b^2*n^2 + 1)*x*cos(b*n*log(x) + b*l
og(c) + a)^2 - (7*b^2*n^2 + 1)*x)*sin(b*n*log(x) + b*log(c) + a))/(9*b^4*n
^4 + 10*b^2*n^2 + 1)
```

Sympy [F]

$$\int \sin^3(a + b\log(cx^n)) \, dx$$
$$= \begin{cases} \int \sin^3\left(a - \frac{i\log(cx^n)}{n}\right) dx \\ \int \sin^3\left(a - \frac{i\log(cx^n)}{3n}\right) dx \\ \int \sin^3\left(a + \frac{i\log(cx^n)}{3n}\right) dx \\ \int \sin^3\left(a + \frac{i\log(cx^n)}{n}\right) dx \\ -\frac{9b^3n^3x\sin^2(a+b\log(cx^n))\cos(a+b\log(cx^n))}{9b^4n^4+10b^2n^2+1} - \frac{6b^3n^3x\cos^3(a+b\log(cx^n))}{9b^4n^4+10b^2n^2+1} + \frac{7b^2n^2x\sin^3(a+b\log(cx^n))}{9b^4n^4+10b^2n^2+1} + \frac{6b^2n^2x\sin(a+b\log(c}{9b^4n^4+} \end{cases}$$

input
```
integrate(sin(a+b*ln(c*x**n))**3,x)
```

output
```
Piecewise((Integral(sin(a - I*log(c*x**n)/n)**3, x), Eq(b, -I/n)), (Integr
al(sin(a - I*log(c*x**n)/(3*n))**3, x), Eq(b, -I/(3*n))), (Integral(sin(a
+ I*log(c*x**n)/(3*n))**3, x), Eq(b, I/(3*n))), (Integral(sin(a + I*log(c*
x**n)/n)**3, x), Eq(b, I/n)), (-9*b**3*n**3*x*sin(a + b*log(c*x**n))**2*co
s(a + b*log(c*x**n))/(9*b**4*n**4 + 10*b**2*n**2 + 1) - 6*b**3*n**3*x*cos(
a + b*log(c*x**n))**3/(9*b**4*n**4 + 10*b**2*n**2 + 1) + 7*b**2*n**2*x*sin
(a + b*log(c*x**n))**3/(9*b**4*n**4 + 10*b**2*n**2 + 1) + 6*b**2*n**2*x*si
n(a + b*log(c*x**n))*cos(a + b*log(c*x**n))**2/(9*b**4*n**4 + 10*b**2*n**2
 + 1) - 3*b*n*x*sin(a + b*log(c*x**n))**2*cos(a + b*log(c*x**n))/(9*b**4*n
**4 + 10*b**2*n**2 + 1) + x*sin(a + b*log(c*x**n))**3/(9*b**4*n**4 + 10*b*
*2*n**2 + 1), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 990 vs. $2(149) = 298$.

Time = 0.11 (sec) , antiderivative size = 990, normalized size of antiderivative = 6.64

$$\int \sin^3(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(sin(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
1/8*((3*(b^3*cos(6*b*log(c))*cos(3*b*log(c)) + b^3*sin(6*b*log(c))*sin(3*b
*log(c)) + b^3*cos(3*b*log(c)))*n^3 - (b^2*cos(3*b*log(c))*sin(6*b*log(c))
 - b^2*cos(6*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c)))*n^2 + 3*(b*c
os(6*b*log(c))*cos(3*b*log(c)) + b*sin(6*b*log(c))*sin(3*b*log(c)) + b*cos
(3*b*log(c)))*n - cos(3*b*log(c))*sin(6*b*log(c)) + cos(6*b*log(c))*sin(3*
b*log(c)) - sin(3*b*log(c)))*x*cos(3*b*log(x^n) + 3*a) - 3*(9*(b^3*cos(4*b
*log(c))*cos(3*b*log(c)) + b^3*cos(3*b*log(c))*cos(2*b*log(c)) + b^3*sin(4
*b*log(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c))*sin(2*b*log(c)))*n^3 - 9*
(b^2*cos(3*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(3*b*log(c))
 + b^2*cos(2*b*log(c))*sin(3*b*log(c)) - b^2*cos(3*b*log(c))*sin(2*b*log(c
)))*n^2 + (b*cos(4*b*log(c))*cos(3*b*log(c)) + b*cos(3*b*log(c))*cos(2*b*l
og(c)) + b*sin(4*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log
(c)))*n - cos(3*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin(3*b*log(c)
) - cos(2*b*log(c))*sin(3*b*log(c)) + cos(3*b*log(c))*sin(2*b*log(c)))*x*c
os(b*log(x^n) + a) - (3*(b^3*cos(3*b*log(c))*sin(6*b*log(c)) - b^3*cos(6*b
*log(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c)))*n^3 + (b^2*cos(6*b*log(c))
*cos(3*b*log(c)) + b^2*sin(6*b*log(c))*sin(3*b*log(c)) + b^2*cos(3*b*log(c
)))*n^2 + 3*(b*cos(3*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(3*b
*log(c)) + b*sin(3*b*log(c)))*n + cos(6*b*log(c))*cos(3*b*log(c)) + sin(6*
b*log(c))*sin(3*b*log(c)) + cos(3*b*log(c)))*x*sin(3*b*log(x^n) + 3*a) ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17522 vs. $2(149) = 298$.

Time = 0.60 (sec) , antiderivative size = 17522, normalized size of antiderivative = 117.60

$$\int \sin^3\left(a+b\log\left(cx^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate(sin(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
1/8*(3*b^3*n^3*x*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2
*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(
x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 - 27*b^3*n^3*x*e^(1/2
*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(
abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c))
)^2*tan(3/2*a)^2*tan(1/2*a)^2 - 27*b^3*n^3*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*p
i*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(ab
s(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1
/2*a)^2 + 3*b^3*n^3*x*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c)
 + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*lo
g(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 + 3*b^3*n^3*x*e
^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n
*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(ab
s(c)))^2*tan(3/2*a)^2 + 27*b^3*n^3*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1
/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*
tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2 + 27*b^3*n^3*x
*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*
b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log
(abs(c)))^2*tan(3/2*a)^2 + 3*b^3*n^3*x*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n
- 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c...
```

Mupad [B] (verification not implemented)

Time = 20.38 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.77

$$\int \sin^3(a+b\log(cx^n))\,dx = -\frac{x\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{(c\,x^n)}^{b\,1\mathrm{i}}}\,3\mathrm{i}}{-8+b\,n\,8\mathrm{i}}-\frac{3\,x\,\mathrm{e}^{a\,1\mathrm{i}}\,{(c\,x^n)}^{b\,1\mathrm{i}}}{8\,b\,n-8\mathrm{i}}+\frac{x\,\mathrm{e}^{-a\,3\mathrm{i}}\,\frac{1}{{(c\,x^n)}^{b\,3\mathrm{i}}}\,1\mathrm{i}}{-8+b\,n\,24\mathrm{i}}+\frac{x\,\mathrm{e}^{a\,3\mathrm{i}}\,{(c\,x^n)}^{b\,3\mathrm{i}}}{24\,b\,n-8\mathrm{i}}$$

input
```
int(sin(a + b*log(c*x^n))^3,x)
```

output
```
(x*exp(-a*3i)/(c*x^n)^(b*3i)*1i)/(b*n*24i - 8) - (3*x*exp(a*1i)*(c*x^n)^(b
*1i))/(8*b*n - 8i) - (x*exp(-a*1i)/(c*x^n)^(b*1i)*3i)/(b*n*8i - 8) + (x*ex
p(a*3i)*(c*x^n)^(b*3i))/(24*b*n - 8i)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.03

$$\int \sin^3(a+b\log(cx^n))\,dx$$
$$= \frac{x\left(-3\cos(\log(x^nc)\,b+a)\sin(\log(x^nc)\,b+a)^2b^3n^3-3\cos(\log(x^nc)\,b+a)\sin(\log(x^nc)\,b+a)^2bn-6\,\mathrm{co}\right.}{9b^4n^4+10}$$

input
```
int(sin(a+b*log(c*x^n))^3,x)
```

output
```
(x*( - 3*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**2*b**3*n**3 - 3*co
s(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**2*b*n - 6*cos(log(x**n*c)*b +
 a)*b**3*n**3 + sin(log(x**n*c)*b + a)**3*b**2*n**2 + sin(log(x**n*c)*b +
a)**3 + 6*sin(log(x**n*c)*b + a)*b**2*n**2))/(9*b**4*n**4 + 10*b**2*n**2 +
 1)
```

3.16 $\int \frac{\sin^3(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{\sin^3(a+b\log(cx^n))}{x}\,dx = -\frac{\cos(a+b\log(cx^n))}{bn}+\frac{\cos^3(a+b\log(cx^n))}{3bn}$$

output
```
-cos(a+b*ln(c*x^n))/b/n+1/3*cos(a+b*ln(c*x^n))^3/b/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{\sin^3(a+b\log(cx^n))}{x}\,dx = -\frac{3\cos(a+b\log(cx^n))}{4bn}+\frac{\cos(3(a+b\log(cx^n)))}{12bn}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]^3/x,x]
```

output
```
(-3*Cos[a + b*Log[c*x^n]])/(4*b*n) + Cos[3*(a + b*Log[c*x^n])]/(12*b*n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3039, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \sin^3(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin(a+b\log(cx^n))^3d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{\int (1-\cos^2(a+b\log(cx^n)))d\cos(a+b\log(cx^n))}{bn}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cos(a+b\log(cx^n))-\frac{1}{3}\cos^3(a+b\log(cx^n))}{bn}$$

input
```
Int[Sin[a + b*Log[c*x^n]]^3/x,x]
```

output
```
-((Cos[a + b*Log[c*x^n]] - Cos[a + b*Log[c*x^n]]^3/3)/(b*n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{\left(2+\sin(a+b\ln(c\,x^{n}))^{2}\right)\cos(a+b\ln(c\,x^{n}))}{3nb}$	35
default	$-\frac{\left(2+\sin(a+b\ln(c\,x^{n}))^{2}\right)\cos(a+b\ln(c\,x^{n}))}{3nb}$	35
parallelrisch	$\frac{-8-9\cos(a+b\ln(c\,x^{n}))+\cos(3b\ln(c\,x^{n})+3a)}{12bn}$	38

input
```
int(sin(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

output
```
-1/3/n/b*(2+sin(a+b*ln(c*x^n))^2)*cos(a+b*ln(c*x^n))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\sin^3(a+b\log(cx^n))}{x}\,dx = \frac{\cos(bn\log(x)+b\log(c)+a)^3 - 3\cos(bn\log(x)+b\log(c)+a)}{3\,bn}$$

input
```
integrate(sin(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

output
```
1/3*(cos(b*n*log(x) + b*log(c) + a)^3 - 3*cos(b*n*log(x) + b*log(c) + a))/
(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(32) = 64.

Time = 1.48 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int \frac{\sin^3(a+b\log(cx^n))}{x}\,dx = \begin{cases} \log(x)\sin^3(a) & \text{for } b=0 \wedge (b=0 \vee n=0) \\ \log(x)\sin^3(a+b\log(c)) & \text{for } n=0 \\ -\frac{\sin^2(a+b\log(cx^n))\cos(a+b\log(cx^n))}{bn} - \frac{2\cos^3(a+b\log(cx^n))}{3bn} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(a+b*ln(c*x**n))**3/x,x)
```

output
```
Piecewise((log(x)*sin(a)**3, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*si
n(a + b*log(c))**3, Eq(n, 0)), (-sin(a + b*log(c*x**n))**2*cos(a + b*log(c
*x**n))/(b*n) - 2*cos(a + b*log(c*x**n))**3/(3*b*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(41) = 82$.

Time = 0.06 (sec) , antiderivative size = 233, normalized size of antiderivative = 5.42

$$\int \frac{\sin^3\left(a + b\log\left(cx^n\right)\right)}{x}\,dx$$
$$= \frac{\left(\cos\left(6\,b\log\left(c\right)\right)\cos\left(3\,b\log\left(c\right)\right) + \sin\left(6\,b\log\left(c\right)\right)\sin\left(3\,b\log\left(c\right)\right) + \cos\left(3\,b\log\left(c\right)\right)\right)\cos\left(3\,b\log\left(x^n\right) + 3\,a\right)}{}$$

input
```
integrate(sin(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

output
```
1/24*((cos(6*b*log(c))*cos(3*b*log(c)) + sin(6*b*log(c))*sin(3*b*log(c)) +
 cos(3*b*log(c)))*cos(3*b*log(x^n) + 3*a) - 9*(cos(4*b*log(c))*cos(3*b*log
(c)) + cos(3*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(3*b*log(c)) +
 sin(3*b*log(c))*sin(2*b*log(c)))*cos(b*log(x^n) + a) - (cos(3*b*log(c))*s
in(6*b*log(c)) - cos(6*b*log(c))*sin(3*b*log(c)) + sin(3*b*log(c)))*sin(3*
b*log(x^n) + 3*a) + 9*(cos(3*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*s
in(3*b*log(c)) + cos(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b
*log(c)))*sin(b*log(x^n) + a))/(b*n)
```

Giac [F]

$$\int \frac{\sin^3\left(a + b\log\left(cx^n\right)\right)}{x}\,dx = \int \frac{\sin\left(b\log\left(cx^n\right) + a\right)^3}{x}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

output
```
integrate(sin(b*log(c*x^n) + a)^3/x, x)
```

Mupad [B] (verification not implemented)

Time = 20.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\sin^3(a+b\log(cx^n))}{x}\,dx = -\frac{3\,\cos\left(a+b\,\ln\left(c\,x^n\right)\right)-{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}^3}{3\,b\,n}$$

input
```
int(sin(a + b*log(c*x^n))^3/x,x)
```

output
```
-(3*cos(a + b*log(c*x^n)) - cos(a + b*log(c*x^n))^3)/(3*b*n)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{\sin^3(a+b\log(cx^n))}{x}\,dx = \frac{-\cos(\log(x^n c)\,b+a)\sin(\log(x^n c)\,b+a)^2-2\cos(\log(x^n c)\,b+a)+2}{3bn}$$

input
```
int(sin(a+b*log(c*x^n))^3/x,x)
```

output
```
( - cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**2 - 2*cos(log(x**n*c)*b
 + a) + 2)/(3*b*n)
```

3.17 $\int \frac{\sin^3(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 158

$$\int \frac{\sin^3(a+b\log(cx^n))}{x^2}\,dx = -\frac{6b^3n^3\cos(a+b\log(cx^n))}{(1+10b^2n^2+9b^4n^4)\,x} - \frac{6b^2n^2\sin(a+b\log(cx^n))}{(1+10b^2n^2+9b^4n^4)\,x} - \frac{3bn\cos(a+b\log(cx^n))\sin^2(a+b\log(cx^n))}{(1+9b^2n^2)\,x} - \frac{\sin^3(a+b\log(cx^n))}{(1+9b^2n^2)\,x}$$

output
```
-6*b^3*n^3*cos(a+b*ln(c*x^n))/(9*b^4*n^4+10*b^2*n^2+1)/x-6*b^2*n^2*sin(a+b
*ln(c*x^n))/(9*b^4*n^4+10*b^2*n^2+1)/x-3*b*n*cos(a+b*ln(c*x^n))*sin(a+b*ln
(c*x^n))^2/(9*b^2*n^2+1)/x-sin(a+b*ln(c*x^n))^3/(9*b^2*n^2+1)/x
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.79

$$\int \frac{\sin^3(a+b\log(cx^n))}{x^2}\,dx = \frac{-3bn(1+9b^2n^2)\cos(a+b\log(cx^n)) + 3(bn+b^3n^3)\cos(3(a+b\log(cx^n))) + 2(-1-13b^2n^2+(1+b^2 \cdots}{4\,(1+10b^2n^2+9b^4n^4)\,x}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]^3/x^2,x]
```

output
```
(-3*b*n*(1 + 9*b^2*n^2)*Cos[a + b*Log[c*x^n]] + 3*(b*n + b^3*n^3)*Cos[3*(a
 + b*Log[c*x^n])] + 2*(-1 - 13*b^2*n^2 + (1 + b^2*n^2)*Cos[2*(a + b*Log[c*
x^n])])*Sin[a + b*Log[c*x^n]])/(4*(1 + 10*b^2*n^2 + 9*b^4*n^4)*x)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4990, 4988}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+b\log(cx^n))}{x^2}dx$$

$$\Big\downarrow \ 4990$$

$$\frac{6b^2n^2\int \frac{\sin(a+b\log(cx^n))}{x^2}dx}{9b^2n^2+1}-\frac{\sin^3(a+b\log(cx^n))}{x(9b^2n^2+1)}-\frac{3bn\sin^2(a+b\log(cx^n))\cos(a+b\log(cx^n))}{x(9b^2n^2+1)}$$

$$\Big\downarrow \ 4988$$

$$-\frac{\sin^3(a+b\log(cx^n))}{x(9b^2n^2+1)}-\frac{3bn\sin^2(a+b\log(cx^n))\cos(a+b\log(cx^n))}{x(9b^2n^2+1)}+\frac{6b^2n^2\left(-\frac{\sin(a+b\log(cx^n))}{x(b^2n^2+1)}-\frac{bn\cos(a+b\log(cx^n))}{x(b^2n^2+1)}\right)}{9b^2n^2+1}$$

input
```
Int[Sin[a + b*Log[c*x^n]]^3/x^2,x]
```

output
```
(-3*b*n*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^2)/((1 + 9*b^2*n^2)*x)
 - Sin[a + b*Log[c*x^n]]^3/((1 + 9*b^2*n^2)*x) + (6*b^2*n^2*(-((b*n*Cos[a
+ b*Log[c*x^n]])/((1 + b^2*n^2)*x)) - Sin[a + b*Log[c*x^n]]/((1 + b^2*n^2)
*x)))/(1 + 9*b^2*n^2)
```

Defintions of rubi rules used

rule 4988
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

rule 4990
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (-Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a
 + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e
*(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2
))   Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
 c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [A] (verified)

Time = 3.84 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.40

method	result
parallelrisch	$\frac{6b^3n^3\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^6-12b^2n^2\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^5+\left(18b^3n^3+12bn\right)\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^4+\left(-32b^2n^2-8\right)\tan\left(\frac{a}{2}+b}{9\left(b^2n^2+1\right)x\left(1+\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^2\right)^3}$

input
```
int(sin(a+b*ln(c*x^n))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*(6*b^3*n^3*tan(1/2*a+b*ln((c*x^n)^(1/2)))^6-12*b^2*n^2*tan(1/2*a+b*ln(
(c*x^n)^(1/2)))^5+(18*b^3*n^3+12*b*n)*tan(1/2*a+b*ln((c*x^n)^(1/2)))^4+(-3
2*b^2*n^2-8)*tan(1/2*a+b*ln((c*x^n)^(1/2)))^3+(-18*b^3*n^3-12*b*n)*tan(1/2
*a+b*ln((c*x^n)^(1/2)))^2-12*b^2*n^2*tan(1/2*a+b*ln((c*x^n)^(1/2)))-6*b^3*
n^3)/(b^2*n^2+1)/x/(1+tan(1/2*a+b*ln((c*x^n)^(1/2)))^2)^3/(1/9+b^2*n^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.80

$$\int \frac{\sin^3(a+b\log(cx^n))}{x^2}\,dx = \frac{3\left(b^3n^3+bn\right)\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3-3\left(3\,b^3n^3+bn\right)\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-\left(7\,b^2n^2-\right.}{(9\,b^4n^4+10\,b^2n^2+1)x}$$

input
```
integrate(sin(a+b*log(c*x^n))^3/x^2,x, algorithm="fricas")
```

output
```
(3*(b^3*n^3 + b*n)*cos(b*n*log(x) + b*log(c) + a)^3 - 3*(3*b^3*n^3 + b*n)*
cos(b*n*log(x) + b*log(c) + a) - (7*b^2*n^2 - (b^2*n^2 + 1)*cos(b*n*log(x)
 + b*log(c) + a)^2 + 1)*sin(b*n*log(x) + b*log(c) + a))/((9*b^4*n^4 + 10*b
^2*n^2 + 1)*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 43.67 (sec) , antiderivative size = 775, normalized size of antiderivative = 4.91

$$\int \frac{\sin^3(a+b\log(cx^n))}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(a+b*ln(c*x**n))**3/x**2,x)
```

output
```
Piecewise((-3*sin(a - I*log(c*x**n)/n)/(8*x) - sin(3*a - 3*I*log(c*x**n)/n
)/(32*x) + 3*I*cos(3*a - 3*I*log(c*x**n)/n)/(32*x) + 3*log(c*x**n)*sin(a -
 I*log(c*x**n)/n)/(8*n*x) - 3*I*log(c*x**n)*cos(a - I*log(c*x**n)/n)/(8*n*
x), Eq(b, -I/n)), (-27*sin(a - I*log(c*x**n)/(3*n))/(32*x) + 9*I*cos(a - I
*log(c*x**n)/(3*n))/(32*x) + I*cos(3*a - I*log(c*x**n)/n)/(8*x) - log(c*x*
*n)*sin(3*a - I*log(c*x**n)/n)/(8*n*x) + I*log(c*x**n)*cos(3*a - I*log(c*x
**n)/n)/(8*n*x), Eq(b, -I/(3*n))), (-27*sin(a + I*log(c*x**n)/(3*n))/(32*x
) - 9*I*cos(a + I*log(c*x**n)/(3*n))/(32*x) - I*cos(3*a + I*log(c*x**n)/n)
/(8*x) - log(c*x**n)*sin(3*a + I*log(c*x**n)/n)/(8*n*x) - I*log(c*x**n)*co
s(3*a + I*log(c*x**n)/n)/(8*n*x), Eq(b, I/(3*n))), (-3*sin(a + I*log(c*x**
n)/n)/(8*x) - sin(3*a + 3*I*log(c*x**n)/n)/(32*x) - 3*I*cos(3*a + 3*I*log(
c*x**n)/n)/(32*x) + 3*log(c*x**n)*sin(a + I*log(c*x**n)/n)/(8*n*x) + 3*I*l
og(c*x**n)*cos(a + I*log(c*x**n)/n)/(8*n*x), Eq(b, I/n)), (-9*b**3*n**3*si
n(a + b*log(c*x**n))**2*cos(a + b*log(c*x**n))/(9*b**4*n**4*x + 10*b**2*n*
*2*x + x) - 6*b**3*n**3*cos(a + b*log(c*x**n))**3/(9*b**4*n**4*x + 10*b**2
*n**2*x + x) - 7*b**2*n**2*sin(a + b*log(c*x**n))**3/(9*b**4*n**4*x + 10*b
**2*n**2*x + x) - 6*b**2*n**2*sin(a + b*log(c*x**n))*cos(a + b*log(c*x**n)
)**2/(9*b**4*n**4*x + 10*b**2*n**2*x + x) - 3*b*n*sin(a + b*log(c*x**n))**
2*cos(a + b*log(c*x**n))/(9*b**4*n**4*x + 10*b**2*n**2*x + x) - sin(a + b*
log(c*x**n))**3/(9*b**4*n**4*x + 10*b**2*n**2*x + x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 995 vs. $2(158) = 316$.

Time = 0.09 (sec) , antiderivative size = 995, normalized size of antiderivative = 6.30

$$\int \frac{\sin^3(a + b\log(cx^n))}{x^2}\, dx = \text{Too large to display}$$

input
```
integrate(sin(a+b*log(c*x^n))^3/x^2,x, algorithm="maxima")
```

output
```
1/8*((3*(b^3*cos(6*b*log(c))*cos(3*b*log(c)) + b^3*sin(6*b*log(c))*sin(3*b
*log(c)) + b^3*cos(3*b*log(c)))*n^3 + (b^2*cos(3*b*log(c))*sin(6*b*log(c))
 - b^2*cos(6*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c)))*n^2 + 3*(b*c
os(6*b*log(c))*cos(3*b*log(c)) + b*sin(6*b*log(c))*sin(3*b*log(c)) + b*cos
(3*b*log(c)))*n + cos(3*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(3*
b*log(c)) + sin(3*b*log(c)))*cos(3*b*log(x^n) + 3*a) - 3*(9*(b^3*cos(4*b*l
og(c))*cos(3*b*log(c)) + b^3*cos(3*b*log(c))*cos(2*b*log(c)) + b^3*sin(4*b
*log(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c))*sin(2*b*log(c)))*n^3 + 9*(b
^2*cos(3*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(3*b*log(c)) +
 b^2*cos(2*b*log(c))*sin(3*b*log(c)) - b^2*cos(3*b*log(c))*sin(2*b*log(c))
)*n^2 + (b*cos(4*b*log(c))*cos(3*b*log(c)) + b*cos(3*b*log(c))*cos(2*b*log
(c)) + b*sin(4*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c
)))*n + cos(3*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(3*b*log(c))
+ cos(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*log(c)))*cos(b
*log(x^n) + a) - (3*(b^3*cos(3*b*log(c))*sin(6*b*log(c)) - b^3*cos(6*b*log
(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c)))*n^3 - (b^2*cos(6*b*log(c))*cos
(3*b*log(c)) + b^2*sin(6*b*log(c))*sin(3*b*log(c)) + b^2*cos(3*b*log(c)))*
n^2 + 3*(b*cos(3*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(3*b*log
(c)) + b*sin(3*b*log(c)))*n - cos(6*b*log(c))*cos(3*b*log(c)) - sin(6*b*lo
g(c))*sin(3*b*log(c)) - cos(3*b*log(c)))*sin(3*b*log(x^n) + 3*a) + 3*(9...
```

Giac [F]

$$\int \frac{\sin^3(a+b\log(cx^n))}{x^2}\,dx = \int \frac{\sin(b\log(cx^n)+a)^3}{x^2}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^3/x^2,x, algorithm="giac")
```

output
```
integrate(sin(b*log(c*x^n) + a)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+b\log(cx^n))}{x^2}\,dx = \int \frac{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x^2}\,dx$$

input
```
int(sin(a + b*log(c*x^n))^3/x^2,x)
```

output
```
int(sin(a + b*log(c*x^n))^3/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01

$$\int \frac{\sin^3(a+b\log(cx^n))}{x^2}\,dx = \frac{-3\cos(\log(x^nc)\,b+a)\sin(\log(x^nc)\,b+a)^2b^3n^3 - 3\cos(\log(x^nc)\,b+a)\sin(\log(x^nc)\,b+a)^2bn - 6\cos(\mathrm{l}}{x\,(9b^4n^4+10b}$$

input
```
int(sin(a+b*log(c*x^n))^3/x^2,x)
```

output
```
( - 3*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**2*b**3*n**3 - 3*cos(l
og(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**2*b*n - 6*cos(log(x**n*c)*b + a)
*b**3*n**3 - sin(log(x**n*c)*b + a)**3*b**2*n**2 - sin(log(x**n*c)*b + a)*
*3 - 6*sin(log(x**n*c)*b + a)*b**2*n**2)/(x*(9*b**4*n**4 + 10*b**2*n**2 +
1))
```

3.18 $\int \frac{\sin^3(a+b\log(cx^n))}{x^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 158

$$\int \frac{\sin^3(a+b\log(cx^n))}{x^3} dx = -\frac{6b^3n^3\cos(a+b\log(cx^n))}{(16+40b^2n^2+9b^4n^4)x^2} - \frac{12b^2n^2\sin(a+b\log(cx^n))}{(16+40b^2n^2+9b^4n^4)x^2} - \frac{3bn\cos(a+b\log(cx^n))\sin^2(a+b\log(cx^n))}{(4+9b^2n^2)x^2} - \frac{2\sin^3(a+b\log(cx^n))}{(4+9b^2n^2)x^2}$$

output
```
-6*b^3*n^3*cos(a+b*ln(c*x^n))/(9*b^4*n^4+40*b^2*n^2+16)/x^2-12*b^2*n^2*sin
(a+b*ln(c*x^n))/(9*b^4*n^4+40*b^2*n^2+16)/x^2-3*b*n*cos(a+b*ln(c*x^n))*sin
(a+b*ln(c*x^n))^2/(9*b^2*n^2+4)/x^2-2*sin(a+b*ln(c*x^n))^3/(9*b^2*n^2+4)/x
^2
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.79

$$\int \frac{\sin^3(a+b\log(cx^n))}{x^3}\,dx = \frac{-3bn(4+9b^2n^2)\cos(a+b\log(cx^n))+3bn(4+b^2n^2)\cos(3(a+b\log(cx^n)))+4(-4-13b^2n^2+(4+b}{4(16+40b^2n^2+9b^4n^4)x^2}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]^3/x^3,x]
```

output
```
(-3*b*n*(4 + 9*b^2*n^2)*Cos[a + b*Log[c*x^n]] + 3*b*n*(4 + b^2*n^2)*Cos[3*
(a + b*Log[c*x^n])] + 4*(-4 - 13*b^2*n^2 + (4 + b^2*n^2)*Cos[2*(a + b*Log[
c*x^n])])*Sin[a + b*Log[c*x^n]])/(4*(16 + 40*b^2*n^2 + 9*b^4*n^4)*x^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4990, 4988}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+b\log(cx^n))}{x^3}\,dx$$

$$\Big\downarrow \text{4990}$$

$$\frac{6b^2n^2\int \frac{\sin(a+b\log(cx^n))}{x^3}dx}{9b^2n^2+4}-\frac{2\sin^3(a+b\log(cx^n))}{x^2(9b^2n^2+4)}-\frac{3bn\sin^2(a+b\log(cx^n))\cos(a+b\log(cx^n))}{x^2(9b^2n^2+4)}$$

$$\Big\downarrow \text{4988}$$

$$-\frac{2\sin^3(a+b\log(cx^n))}{x^2(9b^2n^2+4)}-\frac{3bn\sin^2(a+b\log(cx^n))\cos(a+b\log(cx^n))}{x^2(9b^2n^2+4)}+\frac{6b^2n^2\left(-\frac{2\sin(a+b\log(cx^n))}{x^2(b^2n^2+4)}-\frac{bn\cos(a+b\log(cx^n))}{x^2(b^2n^2+4)}\right)}{9b^2n^2+4}$$

input
```
Int[Sin[a + b*Log[c*x^n]]^3/x^3,x]
```

output
```
(-3*b*n*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^2)/((4 + 9*b^2*n^2)*x^
2) - (2*Sin[a + b*Log[c*x^n]]^3)/((4 + 9*b^2*n^2)*x^2) + (6*b^2*n^2*(-((b*
n*Cos[a + b*Log[c*x^n]])/((4 + b^2*n^2)*x^2)) - (2*Sin[a + b*Log[c*x^n]])/
((4 + b^2*n^2)*x^2)))/(4 + 9*b^2*n^2)
```

Defintions of rubi rules used

rule 4988
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

rule 4990
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (-Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a
 + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e
*(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2
))   Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
 c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [A] (verified)

Time = 6.91 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.40

method	result
parallelrisch	$\frac{6b^3n^3\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^6-24b^2n^2\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^5+\left(18b^3n^3+48bn\right)\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^4+\left(-64b^2n^2-64\right)\tan\left(\frac{a}{2}+\right.}{9\left(b^2n^2+\frac{4}{9}\right)x^2\left(1+\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^2\right)}$

input
```
int(sin(a+b*ln(c*x^n))^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/9*(6*b^3*n^3*tan(1/2*a+b*ln((c*x^n)^(1/2)))^6-24*b^2*n^2*tan(1/2*a+b*ln(
(c*x^n)^(1/2)))^5+(18*b^3*n^3+48*b*n)*tan(1/2*a+b*ln((c*x^n)^(1/2)))^4+(-6
4*b^2*n^2-64)*tan(1/2*a+b*ln((c*x^n)^(1/2)))^3+(-18*b^3*n^3-48*b*n)*tan(1/
2*a+b*ln((c*x^n)^(1/2)))^2-24*b^2*n^2*tan(1/2*a+b*ln((c*x^n)^(1/2)))-6*b^3
*n^3)/(b^2*n^2+4/9)/x^2/(1+tan(1/2*a+b*ln((c*x^n)^(1/2)))^2)^3/(b^2*n^2+4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.82

$$\int \frac{\sin^3(a+b\log(cx^n))}{x^3}\,dx = \frac{3\left(b^3n^3+4\,bn\right)\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3-3\left(3\,b^3n^3+4\,bn\right)\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-2\left(7\,b^2\right.}{\left(9\,b^4n^4+40\,b^2n^2+16\right)x}$$

input
```
integrate(sin(a+b*log(c*x^n))^3/x^3,x, algorithm="fricas")
```

output
```
(3*(b^3*n^3 + 4*b*n)*cos(b*n*log(x) + b*log(c) + a)^3 - 3*(3*b^3*n^3 + 4*b
*n)*cos(b*n*log(x) + b*log(c) + a) - 2*(7*b^2*n^2 - (b^2*n^2 + 4)*cos(b*n*
log(x) + b*log(c) + a)^2 + 4)*sin(b*n*log(x) + b*log(c) + a))/((9*b^4*n^4
+ 40*b^2*n^2 + 16)*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 47.45 (sec) , antiderivative size = 882, normalized size of antiderivative = 5.58

$$\int \frac{\sin^3(a+b\log(cx^n))}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(a+b*ln(c*x**n))**3/x**3,x)
```

output
```
Piecewise((-3*sin(a - 2*I*log(c*x**n)/n)/(16*x**2) - sin(3*a - 6*I*log(c*x
**n)/n)/(64*x**2) + 3*I*cos(3*a - 6*I*log(c*x**n)/n)/(64*x**2) + 3*log(c*x
**n)*sin(a - 2*I*log(c*x**n)/n)/(8*n*x**2) - 3*I*log(c*x**n)*cos(a - 2*I*l
og(c*x**n)/n)/(8*n*x**2), Eq(b, -2*I/n)), (-27*sin(a - 2*I*log(c*x**n)/(3*
n))/(64*x**2) + 9*I*cos(a - 2*I*log(c*x**n)/(3*n))/(64*x**2) + I*cos(3*a -
 2*I*log(c*x**n)/n)/(16*x**2) - log(c*x**n)*sin(3*a - 2*I*log(c*x**n)/n)/(
8*n*x**2) + I*log(c*x**n)*cos(3*a - 2*I*log(c*x**n)/n)/(8*n*x**2), Eq(b, -
2*I/(3*n))), (-27*sin(a + 2*I*log(c*x**n)/(3*n))/(64*x**2) + sin(3*a + 2*I
*log(c*x**n)/n)/(16*x**2) - 9*I*cos(a + 2*I*log(c*x**n)/(3*n))/(64*x**2) -
 log(c*x**n)*sin(3*a + 2*I*log(c*x**n)/n)/(8*n*x**2) - I*log(c*x**n)*cos(3
*a + 2*I*log(c*x**n)/n)/(8*n*x**2), Eq(b, 2*I/(3*n))), (-3*sin(a + 2*I*log
(c*x**n)/n)/(16*x**2) - sin(3*a + 6*I*log(c*x**n)/n)/(64*x**2) - 3*I*cos(3
*a + 6*I*log(c*x**n)/n)/(64*x**2) + 3*log(c*x**n)*sin(a + 2*I*log(c*x**n)/
n)/(8*n*x**2) + 3*I*log(c*x**n)*cos(a + 2*I*log(c*x**n)/n)/(8*n*x**2), Eq(
b, 2*I/n)), (-9*b**3*n**3*sin(a + b*log(c*x**n))**2*cos(a + b*log(c*x**n))
/(9*b**4*n**4*x**2 + 40*b**2*n**2*x**2 + 16*x**2) - 6*b**3*n**3*cos(a + b*
log(c*x**n))**3/(9*b**4*n**4*x**2 + 40*b**2*n**2*x**2 + 16*x**2) - 14*b**2
*n**2*sin(a + b*log(c*x**n))**3/(9*b**4*n**4*x**2 + 40*b**2*n**2*x**2 + 16
*x**2) - 12*b**2*n**2*sin(a + b*log(c*x**n))*cos(a + b*log(c*x**n))**2/(9*
b**4*n**4*x**2 + 40*b**2*n**2*x**2 + 16*x**2) - 12*b*n*sin(a + b*log(c*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1007 vs. $2(158) = 316$.

Time = 0.09 (sec) , antiderivative size = 1007, normalized size of antiderivative = 6.37

$$\int \frac{\sin^3(a + b\log(cx^n))}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(a+b*log(c*x^n))^3/x^3,x, algorithm="maxima")
```

output
```
1/8*((3*(b^3*cos(6*b*log(c))*cos(3*b*log(c)) + b^3*sin(6*b*log(c))*sin(3*b
*log(c)) + b^3*cos(3*b*log(c)))*n^3 + 2*(b^2*cos(3*b*log(c))*sin(6*b*log(c
)) - b^2*cos(6*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c)))*n^2 + 12*(
b*cos(6*b*log(c))*cos(3*b*log(c)) + b*sin(6*b*log(c))*sin(3*b*log(c)) + b*
cos(3*b*log(c)))*n + 8*cos(3*b*log(c))*sin(6*b*log(c)) - 8*cos(6*b*log(c))
*sin(3*b*log(c)) + 8*sin(3*b*log(c)))*cos(3*b*log(x^n) + 3*a) - 3*(9*(b^3*
cos(4*b*log(c))*cos(3*b*log(c)) + b^3*cos(3*b*log(c))*cos(2*b*log(c)) + b^
3*sin(4*b*log(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c))*sin(2*b*log(c)))*n
^3 + 18*(b^2*cos(3*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(3*b
*log(c)) + b^2*cos(2*b*log(c))*sin(3*b*log(c)) - b^2*cos(3*b*log(c))*sin(2
*b*log(c)))*n^2 + 4*(b*cos(4*b*log(c))*cos(3*b*log(c)) + b*cos(3*b*log(c))
*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c))*s
in(2*b*log(c)))*n + 8*cos(3*b*log(c))*sin(4*b*log(c)) - 8*cos(4*b*log(c))*
sin(3*b*log(c)) + 8*cos(2*b*log(c))*sin(3*b*log(c)) - 8*cos(3*b*log(c))*si
n(2*b*log(c)))*cos(b*log(x^n) + a) - (3*(b^3*cos(3*b*log(c))*sin(6*b*log(c
)) - b^3*cos(6*b*log(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c)))*n^3 - 2*(b
^2*cos(6*b*log(c))*cos(3*b*log(c)) + b^2*sin(6*b*log(c))*sin(3*b*log(c)) +
 b^2*cos(3*b*log(c)))*n^2 + 12*(b*cos(3*b*log(c))*sin(6*b*log(c)) - b*cos(
6*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c)))*n - 8*cos(6*b*log(c))*cos
(3*b*log(c)) - 8*sin(6*b*log(c))*sin(3*b*log(c)) - 8*cos(3*b*log(c)))*s...
```

Giac [F]

$$\int \frac{\sin^3(a+b\log(cx^n))}{x^3}\,dx = \int \frac{\sin(b\log(cx^n)+a)^3}{x^3}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^3/x^3,x, algorithm="giac")
```

output
```
integrate(sin(b*log(c*x^n) + a)^3/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(a+b\log(cx^n))}{x^3}\,dx = \int \frac{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{x^3}\,dx$$

input
```
int(sin(a + b*log(c*x^n))^3/x^3,x)
```

output
```
int(sin(a + b*log(c*x^n))^3/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01

$$\int \frac{\sin^3(a+b\log(cx^n))}{x^3}\,dx$$
$$= \frac{-3\cos(\log(x^nc)\,b+a)\sin(\log(x^nc)\,b+a)^2b^3n^3 - 12\cos(\log(x^nc)\,b+a)\sin(\log(x^nc)\,b+a)^2bn - 6\cos}{x^2\left(9b^4n^4+4\right.}$$

input
```
int(sin(a+b*log(c*x^n))^3/x^3,x)
```

output
```
( - 3*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**2*b**3*n**3 - 12*cos(
log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**2*b*n - 6*cos(log(x**n*c)*b + a
)*b**3*n**3 - 2*sin(log(x**n*c)*b + a)**3*b**2*n**2 - 8*sin(log(x**n*c)*b
+ a)**3 - 12*sin(log(x**n*c)*b + a)*b**2*n**2)/(x**2*(9*b**4*n**4 + 40*b**
2*n**2 + 16))
```

3.19 $\int x^2 \sin^4(a + b\log(cx^n))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 202

$$\int x^2 \sin^4(a+b\log(cx^n))\,dx = \frac{8b^4n^4x^3}{81+180b^2n^2+64b^4n^4} - \frac{24b^3n^3x^3\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{81+180b^2n^2+64b^4n^4} + \frac{36b^2n^2x^3\sin^2(a+b\log(cx^n))}{81+180b^2n^2+64b^4n^4} - \frac{4bnx^3\cos(a+b\log(cx^n))\sin^3(a+b\log(cx^n))}{9+16b^2n^2} + \frac{3x^3\sin^4(a+b\log(cx^n))}{9+16b^2n^2}$$

output
```
8*b^4*n^4*x^3/(64*b^4*n^4+180*b^2*n^2+81)-24*b^3*n^3*x^3*cos(a+b*ln(c*x^n)
)*sin(a+b*ln(c*x^n))/(64*b^4*n^4+180*b^2*n^2+81)+36*b^2*n^2*x^3*sin(a+b*ln
(c*x^n))^2/(64*b^4*n^4+180*b^2*n^2+81)-4*b*n*x^3*cos(a+b*ln(c*x^n))*sin(a+
b*ln(c*x^n))^3/(16*b^2*n^2+9)+3*x^3*sin(a+b*ln(c*x^n))^4/(16*b^2*n^2+9)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.85

$$\int x^2 \sin^4(a + b\log(cx^n))\,dx$$
$$= \frac{x^3(81 + 180b^2n^2 + 64b^4n^4 - 12(9 + 16b^2n^2)\cos(2(a + b\log(cx^n))) + 3(9 + 4b^2n^2)\cos(4(a + b\log(cx^n)}{}$$

input
```
Integrate[x^2*Sin[a + b*Log[c*x^n]]^4,x]
```

output
```
(x^3*(81 + 180*b^2*n^2 + 64*b^4*n^4 - 12*(9 + 16*b^2*n^2)*Cos[2*(a + b*Log
[c*x^n])] + 3*(9 + 4*b^2*n^2)*Cos[4*(a + b*Log[c*x^n])] - 72*b*n*Sin[2*(a
+ b*Log[c*x^n])] - 128*b^3*n^3*Sin[2*(a + b*Log[c*x^n])] + 36*b*n*Sin[4*(a
 + b*Log[c*x^n])] + 16*b^3*n^3*Sin[4*(a + b*Log[c*x^n])]))/(8*(81 + 180*b^
2*n^2 + 64*b^4*n^4))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4990, 4990, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin^4(a + b\log(cx^n))\,dx$$
$$\Big\downarrow \; 4990$$
$$\frac{12b^2n^2 \int x^2 \sin^2(a + b\log(cx^n))\,dx}{16b^2n^2 + 9} + \frac{3x^3 \sin^4(a + b\log(cx^n))}{16b^2n^2 + 9} - \frac{4bnx^3 \sin^3(a + b\log(cx^n))\cos(a + b\log(cx^n))}{16b^2n^2 + 9}$$
$$\Big\downarrow \; 4990$$

$$\frac{12b^2n^2\left(\frac{2b^2n^2\int x^2dx}{4b^2n^2+9}+\frac{3x^3\sin^2(a+b\log(cx^n))}{4b^2n^2+9}-\frac{2bnx^3\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+9}\right)}{16b^2n^2+9}+\frac{3x^3\sin^4(a+b\log(cx^n))}{16b^2n^2+9}-\frac{4bnx^3\sin^3(a+b\log(cx^n))\cos(a+b\log(cx^n))}{16b^2n^2+9}$$

$$\Big\downarrow \text{15}$$

$$\frac{3x^3\sin^4(a+b\log(cx^n))}{16b^2n^2+9}-\frac{4bnx^3\sin^3(a+b\log(cx^n))\cos(a+b\log(cx^n))}{16b^2n^2+9}+\frac{12b^2n^2\left(\frac{3x^3\sin^2(a+b\log(cx^n))}{4b^2n^2+9}-\frac{2bnx^3\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+9}+\frac{2b^2n^2x^3}{3(4b^2n^2+9)}\right)}{16b^2n^2+9}$$

input
```
Int[x^2*Sin[a + b*Log[c*x^n]]^4,x]
```

output
```
(-4*b*n*x^3*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^3)/(9 + 16*b^2*n^2
) + (3*x^3*Sin[a + b*Log[c*x^n]]^4)/(9 + 16*b^2*n^2) + (12*b^2*n^2*((2*b^2
*n^2*x^3)/(3*(9 + 4*b^2*n^2)) - (2*b*n*x^3*Cos[a + b*Log[c*x^n]]*Sin[a + b
*Log[c*x^n]])/(9 + 4*b^2*n^2) + (3*x^3*Sin[a + b*Log[c*x^n]]^2)/(9 + 4*b^2
*n^2)))/(9 + 16*b^2*n^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 4990
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (-Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a
 + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e
*(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2
))   Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
 c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [F]

$$\int x^2 \sin\left(a+b\ln\left(c\,x^n\right)\right)^4 dx$$

input
```
int(x^2*sin(a+b*ln(c*x^n))^4,x)
```

output
```
int(x^2*sin(a+b*ln(c*x^n))^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.88

$$\int x^2 \sin^4\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$= \frac{3\left(4\,b^2n^2+9\right)x^3\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^4-6\left(10\,b^2n^2+9\right)x^3\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+(8\,b}{}$$

input
```
integrate(x^2*sin(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

output
```
(3*(4*b^2*n^2 + 9)*x^3*cos(b*n*log(x) + b*log(c) + a)^4 - 6*(10*b^2*n^2 +
9)*x^3*cos(b*n*log(x) + b*log(c) + a)^2 + (8*b^4*n^4 + 48*b^2*n^2 + 27)*x^
3 + 4*((4*b^3*n^3 + 9*b*n)*x^3*cos(b*n*log(x) + b*log(c) + a)^3 - (10*b^3*
n^3 + 9*b*n)*x^3*cos(b*n*log(x) + b*log(c) + a))*sin(b*n*log(x) + b*log(c)
 + a))/(64*b^4*n^4 + 180*b^2*n^2 + 81)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \sin^4\left(a+b\log\left(cx^n\right)\right)\,dx = \text{Timed out}$$

input
```
integrate(x**2*sin(a+b*ln(c*x**n))**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1107 vs. $2(202) = 404$.

Time = 0.10 (sec) , antiderivative size = 1107, normalized size of antiderivative = 5.48

$$\int x^2 \sin^4(a + b\log(cx^n)) \, dx = \text{Too large to display}$$

input
```
integrate(x^2*sin(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

output
```
1/16*((16*(b^3*cos(4*b*log(c))*sin(8*b*log(c)) - b^3*cos(8*b*log(c))*sin(4
*b*log(c)) + b^3*sin(4*b*log(c)))*n^3 + 12*(b^2*cos(8*b*log(c))*cos(4*b*lo
g(c)) + b^2*sin(8*b*log(c))*sin(4*b*log(c)) + b^2*cos(4*b*log(c)))*n^2 + 3
6*(b*cos(4*b*log(c))*sin(8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c)) +
 b*sin(4*b*log(c)))*n + 27*cos(8*b*log(c))*cos(4*b*log(c)) + 27*sin(8*b*lo
g(c))*sin(4*b*log(c)) + 27*cos(4*b*log(c)))*x^3*cos(4*b*log(x^n) + 4*a) -
4*(32*(b^3*cos(4*b*log(c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(4*b*l
og(c)) + b^3*cos(2*b*log(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(2*b
*log(c)))*n^3 + 48*(b^2*cos(6*b*log(c))*cos(4*b*log(c)) + b^2*cos(4*b*log(
c))*cos(2*b*log(c)) + b^2*sin(6*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*lo
g(c))*sin(2*b*log(c)))*n^2 + 18*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos
(6*b*log(c))*sin(4*b*log(c)) + b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4
*b*log(c))*sin(2*b*log(c)))*n + 27*cos(6*b*log(c))*cos(4*b*log(c)) + 27*co
s(4*b*log(c))*cos(2*b*log(c)) + 27*sin(6*b*log(c))*sin(4*b*log(c)) + 27*si
n(4*b*log(c))*sin(2*b*log(c)))*x^3*cos(2*b*log(x^n) + 2*a) + (16*(b^3*cos(
8*b*log(c))*cos(4*b*log(c)) + b^3*sin(8*b*log(c))*sin(4*b*log(c)) + b^3*co
s(4*b*log(c)))*n^3 - 12*(b^2*cos(4*b*log(c))*sin(8*b*log(c)) - b^2*cos(8*b
*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c)))*n^2 + 36*(b*cos(8*b*log(c)
)*cos(4*b*log(c)) + b*sin(8*b*log(c))*sin(4*b*log(c)) + b*cos(4*b*log(c)))
*n - 27*cos(4*b*log(c))*sin(8*b*log(c)) + 27*cos(8*b*log(c))*sin(4*b*lo...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16604 vs. $2(202) = 404$.

Time = 1.22 (sec) , antiderivative size = 16604, normalized size of antiderivative = 82.20

$$\int x^2 \sin^4(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(x^2*sin(a+b*log(c*x^n))^4,x, algorithm="giac")
```

output
```
1/8*x^3 + 1/16*(256*b^3*n^3*x^3*e^(3*pi*b*n*sgn(x) - 3*pi*b*n + 3*pi*b*sgn
(c) - 3*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x
)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a) + 256*b^3*n^3*x^3*e^(pi*b*n*sgn(x)
 - pi*b*n + pi*b*sgn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2
*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a) - 32*b^3*n^3*x^3
*e^(4*pi*b*n*sgn(x) - 4*pi*b*n + 4*pi*b*sgn(c) - 4*pi*b)*tan(2*b*n*log(abs
(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)*
tan(a)^2 + 256*b^3*n^3*x^3*e^(3*pi*b*n*sgn(x) - 3*pi*b*n + 3*pi*b*sgn(c) -
 3*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) +
b*log(abs(c)))*tan(2*a)^2*tan(a)^2 + 256*b^3*n^3*x^3*e^(pi*b*n*sgn(x) - pi
*b*n + pi*b*sgn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(
b*n*log(abs(x)) + b*log(abs(c)))*tan(2*a)^2*tan(a)^2 - 32*b^3*n^3*x^3*e^(4
*pi*b*n*sgn(x) - 4*pi*b*n + 4*pi*b*sgn(c) - 4*pi*b)*tan(2*b*n*log(abs(x))
+ 2*b*log(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a
)^2 - 32*b^3*n^3*x^3*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*lo
g(abs(x)) + b*log(abs(c)))^2*tan(2*a)*tan(a)^2 - 32*b^3*n^3*x^3*tan(2*b*n*
log(abs(x)) + 2*b*log(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(
2*a)^2*tan(a)^2 + 12*b^2*n^2*x^3*e^(4*pi*b*n*sgn(x) - 4*pi*b*n + 4*pi*b*sg
n(c) - 4*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(
x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a)^2 - 192*b^2*n^2*x^3*e^(3*pi*b*...
```

Mupad [B] (verification not implemented)

Time = 20.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.63

$$\int x^2 \sin^4(a+b\log(cx^n))\,dx = \frac{x^3}{8} - \frac{x^3\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,2\mathrm{i}}}\,1\mathrm{i}}{8\,b\,n+12\mathrm{i}} - \frac{x^3\,\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}}}{12+b\,n\,8\mathrm{i}} + \frac{x^3\,\mathrm{e}^{-a\,4\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,4\mathrm{i}}}\,1\mathrm{i}}{64\,b\,n+48\mathrm{i}} + \frac{x^3\,\mathrm{e}^{a\,4\mathrm{i}}\,(c\,x^n)^{b\,4\mathrm{i}}}{48+b\,n\,64\mathrm{i}}$$

input
```
int(x^2*sin(a + b*log(c*x^n))^4,x)
```

output
```
x^3/8 - (x^3*exp(-a*2i)/(c*x^n)^(b*2i)*1i)/(8*b*n + 12i) - (x^3*exp(a*2i)*
(c*x^n)^(b*2i))/(b*n*8i + 12) + (x^3*exp(-a*4i)/(c*x^n)^(b*4i)*1i)/(64*b*n
 + 48i) + (x^3*exp(a*4i)*(c*x^n)^(b*4i))/(b*n*64i + 48)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.89

$$\int x^2 \sin^4(a+b\log(cx^n))\,dx$$
$$= \frac{x^3\left(-16\cos(\log(x^nc)\,b+a)\sin(\log(x^nc)\,b+a)^3b^3n^3 - 36\cos(\log(x^nc)\,b+a)\sin(\log(x^nc)\,b+a)^3bn - \cdots\right.}{\cdots}$$

input
```
int(x^2*sin(a+b*log(c*x^n))^4,x)
```

output
```
(x**3*( - 16*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*b**3*n**3 -
36*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*b*n - 24*cos(log(x**n*
c)*b + a)*sin(log(x**n*c)*b + a)*b**3*n**3 + 12*sin(log(x**n*c)*b + a)**4*
b**2*n**2 + 27*sin(log(x**n*c)*b + a)**4 + 36*sin(log(x**n*c)*b + a)**2*b*
*2*n**2 + 8*b**4*n**4))/(64*b**4*n**4 + 180*b**2*n**2 + 81)
```

3.20 $\int x\sin^4(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 210

$$\int x\sin^4(a+b\log(cx^n))\,dx=\frac{3b^4n^4x^2}{4(1+5b^2n^2+4b^4n^4)}-\frac{3b^3n^3x^2\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{2(1+5b^2n^2+4b^4n^4)}+\frac{3b^2n^2x^2\sin^2(a+b\log(cx^n))}{2(1+5b^2n^2+4b^4n^4)}-\frac{bnx^2\cos(a+b\log(cx^n))\sin^3(a+b\log(cx^n))}{1+4b^2n^2}+\frac{x^2\sin^4(a+b\log(cx^n))}{2(1+4b^2n^2)}$$

output
```
3*b^4*n^4*x^2/(16*b^4*n^4+20*b^2*n^2+4)-3*b^3*n^3*x^2*cos(a+b*ln(c*x^n))*s
in(a+b*ln(c*x^n))/(8*b^4*n^4+10*b^2*n^2+2)+3*b^2*n^2*x^2*sin(a+b*ln(c*x^n)
)^2/(8*b^4*n^4+10*b^2*n^2+2)-b*n*x^2*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))
^3/(4*b^2*n^2+1)+x^2*sin(a+b*ln(c*x^n))^4/(8*b^2*n^2+2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.80

$$\int x\sin^4(a+b\log(cx^n))\,dx$$
$$=\frac{x^2(3+15b^2n^2+12b^4n^4-4(1+4b^2n^2)\cos(2(a+b\log(cx^n)))+(1+b^2n^2)\cos(4(a+b\log(cx^n)))-4b}{16(}$$

input
```
Integrate[x*Sin[a + b*Log[c*x^n]]^4,x]
```

output
```
(x^2*(3 + 15*b^2*n^2 + 12*b^4*n^4 - 4*(1 + 4*b^2*n^2)*Cos[2*(a + b*Log[c*x
^n])] + (1 + b^2*n^2)*Cos[4*(a + b*Log[c*x^n])] - 4*b*n*Sin[2*(a + b*Log[c
*x^n])] - 16*b^3*n^3*Sin[2*(a + b*Log[c*x^n])] + 2*b*n*Sin[4*(a + b*Log[c*
x^n])] + 2*b^3*n^3*Sin[4*(a + b*Log[c*x^n])]))/(16*(1 + 5*b^2*n^2 + 4*b^4*
n^4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4990, 4990, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sin^4(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{4990}$$

$$\frac{3b^2n^2\int x\sin^2(a+b\log(cx^n))\,dx}{4b^2n^2+1}+\frac{x^2\sin^4(a+b\log(cx^n))}{2(4b^2n^2+1)}-\frac{bnx^2\sin^3(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+1}$$

$$\Big\downarrow \text{4990}$$

$$\frac{3b^2n^2\left(\frac{b^2n^2\int xdx}{2(b^2n^2+1)}+\frac{x^2\sin^2(a+b\log(cx^n))}{2(b^2n^2+1)}-\frac{bnx^2\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{2(b^2n^2+1)}\right)}{4b^2n^2+1}+\frac{x^2\sin^4\left(a+b\log\left(cx^n\right)\right)}{2\left(4b^2n^2+1\right)}-\frac{bnx^2\sin^3\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2+1}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x^2\sin^4\left(a+b\log\left(cx^n\right)\right)}{2\left(4b^2n^2+1\right)}-\frac{bnx^2\sin^3\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{4b^2n^2+1}+\frac{3b^2n^2\left(\frac{x^2\sin^2(a+b\log(cx^n))}{2(b^2n^2+1)}-\frac{bnx^2\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{2(b^2n^2+1)}+\frac{b^2n^2x^2}{4(b^2n^2+1)}\right)}{4b^2n^2+1}$$

input
```
Int[x*Sin[a + b*Log[c*x^n]]^4,x]
```

output
```
-((b*n*x^2*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^3)/(1 + 4*b^2*n^2))
 + (x^2*Sin[a + b*Log[c*x^n]]^4)/(2*(1 + 4*b^2*n^2)) + (3*b^2*n^2*((b^2*n^
2*x^2)/(4*(1 + b^2*n^2)) - (b*n*x^2*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*
x^n]])/(2*(1 + b^2*n^2)) + (x^2*Sin[a + b*Log[c*x^n]]^2)/(2*(1 + b^2*n^2))
))/(1 + 4*b^2*n^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 4990
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (-Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a
 + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e
*(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2
))   Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
 c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [F]

$$\int x\sin\left(a+b\ln\left(c\,x^{n}\right)\right)^{4}dx$$

input
```
int(x*sin(a+b*ln(c*x^n))^4,x)
```

output
```
int(x*sin(a+b*ln(c*x^n))^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.84

$$\int x\sin^{4}\left(a+b\log\left(cx^{n}\right)\right)\,dx$$
$$= \frac{2\,(b^{2}n^{2}+1)x^{2}\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}-2\,(5\,b^{2}n^{2}+2)x^{2}\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+(3\,b^{4}n}{}$$

input
```
integrate(x*sin(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

output
```
1/4*(2*(b^2*n^2 + 1)*x^2*cos(b*n*log(x) + b*log(c) + a)^4 - 2*(5*b^2*n^2 +
 2)*x^2*cos(b*n*log(x) + b*log(c) + a)^2 + (3*b^4*n^4 + 8*b^2*n^2 + 2)*x^2
 + 2*(2*(b^3*n^3 + b*n)*x^2*cos(b*n*log(x) + b*log(c) + a)^3 - (5*b^3*n^3
+ 2*b*n)*x^2*cos(b*n*log(x) + b*log(c) + a))*sin(b*n*log(x) + b*log(c) + a
))/(4*b^4*n^4 + 5*b^2*n^2 + 1)
```

Sympy [F(-1)]

Timed out.

$$\int x\sin^{4}\left(a+b\log\left(cx^{n}\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x*sin(a+b*ln(c*x**n))**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. $2(202) = 404$.

Time = 0.09 (sec) , antiderivative size = 1085, normalized size of antiderivative = 5.17

$$\int x \sin^4(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(x*sin(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

output
```
1/32*((2*(b^3*cos(4*b*log(c))*sin(8*b*log(c)) - b^3*cos(8*b*log(c))*sin(4*
b*log(c)) + b^3*sin(4*b*log(c)))*n^3 + (b^2*cos(8*b*log(c))*cos(4*b*log(c)
) + b^2*sin(8*b*log(c))*sin(4*b*log(c)) + b^2*cos(4*b*log(c)))*n^2 + 2*(b*
cos(4*b*log(c))*sin(8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c)) + b*si
n(4*b*log(c)))*n + cos(8*b*log(c))*cos(4*b*log(c)) + sin(8*b*log(c))*sin(4
*b*log(c)) + cos(4*b*log(c)))*x^2*cos(4*b*log(x^n) + 4*a) - 4*(4*(b^3*cos(
4*b*log(c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(4*b*log(c)) + b^3*co
s(2*b*log(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(2*b*log(c)))*n^3 +
 4*(b^2*cos(6*b*log(c))*cos(4*b*log(c)) + b^2*cos(4*b*log(c))*cos(2*b*log(
c)) + b^2*sin(6*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c))*sin(2*b*lo
g(c)))*n^2 + (b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*
b*log(c)) + b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*
log(c)))*n + cos(6*b*log(c))*cos(4*b*log(c)) + cos(4*b*log(c))*cos(2*b*log
(c)) + sin(6*b*log(c))*sin(4*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*
x^2*cos(2*b*log(x^n) + 2*a) + (2*(b^3*cos(8*b*log(c))*cos(4*b*log(c)) + b^
3*sin(8*b*log(c))*sin(4*b*log(c)) + b^3*cos(4*b*log(c)))*n^3 - (b^2*cos(4*
b*log(c))*sin(8*b*log(c)) - b^2*cos(8*b*log(c))*sin(4*b*log(c)) + b^2*sin(
4*b*log(c)))*n^2 + 2*(b*cos(8*b*log(c))*cos(4*b*log(c)) + b*sin(8*b*log(c)
)*sin(4*b*log(c)) + b*cos(4*b*log(c)))*n - cos(4*b*log(c))*sin(8*b*log(c))
 + cos(8*b*log(c))*sin(4*b*log(c)) - sin(4*b*log(c)))*x^2*sin(4*b*log(x...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16554 vs. $2(202) = 404$.

Time = 1.00 (sec) , antiderivative size = 16554, normalized size of antiderivative = 78.83

$$\int x \sin^4(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(x*sin(a+b*log(c*x^n))^4,x, algorithm="giac")
```

output
```
3/16*x^2 + 1/32*(32*b^3*n^3*x^2*e^(3*pi*b*n*sgn(x) - 3*pi*b*n + 3*pi*b*sgn
(c) - 3*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x
)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a) + 32*b^3*n^3*x^2*e^(pi*b*n*sgn(x)
- pi*b*n + pi*b*sgn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*
tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a) - 4*b^3*n^3*x^2*e
^(4*pi*b*n*sgn(x) - 4*pi*b*n + 4*pi*b*sgn(c) - 4*pi*b)*tan(2*b*n*log(abs(x
)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)*ta
n(a)^2 + 32*b^3*n^3*x^2*e^(3*pi*b*n*sgn(x) - 3*pi*b*n + 3*pi*b*sgn(c) - 3*
pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*l
og(abs(c)))*tan(2*a)^2*tan(a)^2 + 32*b^3*n^3*x^2*e^(pi*b*n*sgn(x) - pi*b*n
 + pi*b*sgn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*
log(abs(x)) + b*log(abs(c)))*tan(2*a)^2*tan(a)^2 - 4*b^3*n^3*x^2*e^(4*pi*b
*n*sgn(x) - 4*pi*b*n + 4*pi*b*sgn(c) - 4*pi*b)*tan(2*b*n*log(abs(x)) + 2*b
*log(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a)^2 -
 4*b^3*n^3*x^2*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(
x)) + b*log(abs(c)))^2*tan(2*a)*tan(a)^2 - 4*b^3*n^3*x^2*tan(2*b*n*log(abs
(x)) + 2*b*log(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*
tan(a)^2 + b^2*n^2*x^2*e^(4*pi*b*n*sgn(x) - 4*pi*b*n + 4*pi*b*sgn(c) - 4*p
i*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*lo
g(abs(c)))^2*tan(2*a)^2*tan(a)^2 - 16*b^2*n^2*x^2*e^(3*pi*b*n*sgn(x) - ...
```

Mupad [B] (verification not implemented)

Time = 20.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.60

$$\int x\sin^4\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{3\,x^2}{16}-\frac{x^2\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,2\mathrm{i}}}\,1\mathrm{i}}{8\,b\,n+8\mathrm{i}}-\frac{x^2\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}}{8+b\,n\,8\mathrm{i}}+\frac{x^2\,\mathrm{e}^{-a\,4\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,4\mathrm{i}}}\,1\mathrm{i}}{64\,b\,n+32\mathrm{i}}+\frac{x^2\,\mathrm{e}^{a\,4\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,4\mathrm{i}}}{32+b\,n\,64\mathrm{i}}$$

input
```
int(x*sin(a + b*log(c*x^n))^4,x)
```

output
```
(3*x^2)/16 - (x^2*exp(-a*2i)/(c*x^n)^(b*2i)*1i)/(8*b*n + 8i) - (x^2*exp(a*
2i)*(c*x^n)^(b*2i))/(b*n*8i + 8) + (x^2*exp(-a*4i)/(c*x^n)^(b*4i)*1i)/(64*
b*n + 32i) + (x^2*exp(a*4i)*(c*x^n)^(b*4i))/(b*n*64i + 32)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.86

$$\int x\sin^4\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{x^2\left(-4\cos\left(\log(x^nc)\,b+a\right)\sin\left(\log(x^nc)\,b+a\right)^3b^3n^3-4\cos\left(\log(x^nc)\,b+a\right)\sin\left(\log(x^nc)\,b+a\right)^3bn-6\,\mathrm{co}\right.}{}$$

input
```
int(x*sin(a+b*log(c*x^n))^4,x)
```

output
```
(x**2*( - 4*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*b**3*n**3 - 4
*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*b*n - 6*cos(log(x**n*c)*
b + a)*sin(log(x**n*c)*b + a)*b**3*n**3 + 2*sin(log(x**n*c)*b + a)**4*b**2
*n**2 + 2*sin(log(x**n*c)*b + a)**4 + 6*sin(log(x**n*c)*b + a)**2*b**2*n**
2 + 3*b**4*n**4))/(4*(4*b**4*n**4 + 5*b**2*n**2 + 1))
```

3.21 $\int \sin^4(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 191

$$\int \sin^4(a+b\log(cx^n))\,dx = \frac{24b^4n^4x}{1+20b^2n^2+64b^4n^4} - \frac{24b^3n^3x\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{1+20b^2n^2+64b^4n^4} + \frac{12b^2n^2x\sin^2(a+b\log(cx^n))}{1+20b^2n^2+64b^4n^4} - \frac{4bnx\cos(a+b\log(cx^n))\sin^3(a+b\log(cx^n))}{1+16b^2n^2} + \frac{x\sin^4(a+b\log(cx^n))}{1+16b^2n^2}$$

output
```
24*b^4*n^4*x/(64*b^4*n^4+20*b^2*n^2+1)-24*b^3*n^3*x*cos(a+b*ln(c*x^n))*sin
(a+b*ln(c*x^n))/(64*b^4*n^4+20*b^2*n^2+1)+12*b^2*n^2*x*sin(a+b*ln(c*x^n))^
2/(64*b^4*n^4+20*b^2*n^2+1)-4*b*n*x*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))^
3/(16*b^2*n^2+1)+x*sin(a+b*ln(c*x^n))^4/(16*b^2*n^2+1)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.88

$$\int \sin^4\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\frac{x(3+60b^2n^2+192b^4n^4-4(1+16b^2n^2)\cos\left(2(a+b\log\left(cx^n\right))\right)+(1+4b^2n^2)\cos\left(4(a+b\log\left(cx^n\right))\right)-}{8(}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]^4,x]
```

output
```
(x*(3 + 60*b^2*n^2 + 192*b^4*n^4 - 4*(1 + 16*b^2*n^2)*Cos[2*(a + b*Log[c*x
^n])] + (1 + 4*b^2*n^2)*Cos[4*(a + b*Log[c*x^n])] - 8*b*n*Sin[2*(a + b*Log
[c*x^n])] - 128*b^3*n^3*Sin[2*(a + b*Log[c*x^n])] + 4*b*n*Sin[4*(a + b*Log
[c*x^n])] + 16*b^3*n^3*Sin[4*(a + b*Log[c*x^n])]))/(8*(1 + 20*b^2*n^2 + 64
*b^4*n^4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4980, 4980, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^4\left(a+b\log\left(cx^n\right)\right)\,dx$$

$$\Big\downarrow \; 4980$$

$$\frac{12b^2n^2\int \sin^2\left(a+b\log\left(cx^n\right)\right)dx}{16b^2n^2+1}+\frac{x\sin^4\left(a+b\log\left(cx^n\right)\right)}{16b^2n^2+1}-\frac{4bnx\sin^3\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{16b^2n^2+1}$$

$$\Big\downarrow \; 4980$$

$$\frac{12b^2n^2\left(\frac{2b^2n^2\int 1dx}{4b^2n^2+1}+\frac{x\sin^2(a+b\log(cx^n))}{4b^2n^2+1}-\frac{2bnx\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+1}\right)}{16b^2n^2+1}+\frac{x\sin^4\left(a+b\log\left(cx^n\right)\right)}{16b^2n^2+1}-\frac{4bnx\sin^3\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{16b^2n^2+1}$$

$$\Big\downarrow \text{24}$$

$$\frac{x\sin^4\left(a+b\log\left(cx^n\right)\right)}{16b^2n^2+1}-\frac{4bnx\sin^3\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{16b^2n^2+1}+\frac{12b^2n^2\left(\frac{x\sin^2(a+b\log(cx^n))}{4b^2n^2+1}-\frac{2bnx\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+1}+\frac{2b^2n^2x}{4b^2n^2+1}\right)}{16b^2n^2+1}$$

input

```
Int[Sin[a + b*Log[c*x^n]]^4,x]
```

output

```
(-4*b*n*x*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^3)/(1 + 16*b^2*n^2)
+ (x*Sin[a + b*Log[c*x^n]]^4)/(1 + 16*b^2*n^2) + (12*b^2*n^2*((2*b^2*n^2*x
)/(1 + 4*b^2*n^2) - (2*b*n*x*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/
(1 + 4*b^2*n^2) + (x*Sin[a + b*Log[c*x^n]]^2)/(1 + 4*b^2*n^2)))/(1 + 16*b^
2*n^2)
```

Defintions of rubi rules used

rule 24

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 4980

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Sim
p[x*(Sin[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 + 1)), x] + (-Simp[b*d*n*
p*x*Cos[d*(a + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*n
^2*p^2 + 1)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + 1))   Int
[Sin[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b, c, d, n}, x] &&
 IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + 1, 0]
```

Maple [A] (verified)

Time = 4.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.69

method	result
parallelrisch	$-\frac{128\left(\left(-\frac{1}{8}b^{3}n^{3}-\frac{1}{32}bn\right)\sin(4b\ln(c\,x^{n})+4a)+\left(-\frac{b^{2}n^{2}}{32}-\frac{1}{128}\right)\cos(4b\ln(c\,x^{n})+4a)+\left(\frac{1}{16}+b^{2}n^{2}\right)\left(-\frac{3b^{2}n^{2}}{2}+bn\sin(2b\ln(c\,x^{n})\right.\right.}{512b^{4}n^{4}+160b^{2}n^{2}+8}$
default	$\frac{3x}{8}-\frac{\mathrm{e}^{\frac{\ln(c\,x^{n})}{n}-\frac{\ln(c)}{n}}\cos(2b\ln(c\,x^{n})+2a)}{2n^{2}\left(\frac{1}{n^{2}}+4b^{2}\right)}-\frac{b\,\mathrm{e}^{\frac{\ln(c\,x^{n})}{n}-\frac{\ln(c)}{n}}\sin(2b\ln(c\,x^{n})+2a)}{n\left(\frac{1}{n^{2}}+4b^{2}\right)}+\frac{\mathrm{e}^{\frac{\ln(c\,x^{n})}{n}-\frac{\ln(c)}{n}}\cos(4b\ln(c\,x^{n})+4}{8n^{2}\left(\frac{1}{n^{2}}+16b^{2}\right)}$

input
```
int(sin(a+b*ln(c*x^n))^4,x,method=_RETURNVERBOSE)
```

output
```
-128*((-1/8*b^3*n^3-1/32*b*n)*sin(4*b*ln(c*x^n)+4*a)+(-1/32*b^2*n^2-1/128)
*cos(4*b*ln(c*x^n)+4*a)+(1/16+b^2*n^2)*(-3/2*b^2*n^2+b*n*sin(2*b*ln(c*x^n)
+2*a)+1/2*cos(2*b*ln(c*x^n)+2*a)-3/8))*x/(512*b^4*n^4+160*b^2*n^2+8)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.86

$$\int \sin^{4}\left(a+b\log\left(cx^{n}\right)\right)\,dx$$
$$=\frac{(4\,b^{2}n^{2}+1)x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{4}-2\left(10\,b^{2}n^{2}+1\right)x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+(24\,b^{4}n}{}$$

input
```
integrate(sin(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

output
```
((4*b^2*n^2 + 1)*x*cos(b*n*log(x) + b*log(c) + a)^4 - 2*(10*b^2*n^2 + 1)*x
*cos(b*n*log(x) + b*log(c) + a)^2 + (24*b^4*n^4 + 16*b^2*n^2 + 1)*x + 4*((
4*b^3*n^3 + b*n)*x*cos(b*n*log(x) + b*log(c) + a)^3 - (10*b^3*n^3 + b*n)*x
*cos(b*n*log(x) + b*log(c) + a))*sin(b*n*log(x) + b*log(c) + a))/(64*b^4*n
^4 + 20*b^2*n^2 + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \sin^4\left(a+b\log\left(cx^n\right)\right)\,dx = \text{Timed out}$$

input
```
integrate(sin(a+b*ln(c*x**n))**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1078 vs. $2(191) = 382$.

Time = 0.09 (sec) , antiderivative size = 1078, normalized size of antiderivative = 5.64

$$\int \sin^4\left(a+b\log\left(cx^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate(sin(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

output
```
1/16*((16*(b^3*cos(4*b*log(c))*sin(8*b*log(c)) - b^3*cos(8*b*log(c))*sin(4
*b*log(c)) + b^3*sin(4*b*log(c)))*n^3 + 4*(b^2*cos(8*b*log(c))*cos(4*b*log
(c)) + b^2*sin(8*b*log(c))*sin(4*b*log(c)) + b^2*cos(4*b*log(c)))*n^2 + 4*
(b*cos(4*b*log(c))*sin(8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c)) + b
*sin(4*b*log(c)))*n + cos(8*b*log(c))*cos(4*b*log(c)) + sin(8*b*log(c))*si
n(4*b*log(c)) + cos(4*b*log(c)))*x*cos(4*b*log(x^n) + 4*a) - 4*(32*(b^3*co
s(4*b*log(c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(4*b*log(c)) + b^3*
cos(2*b*log(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(2*b*log(c)))*n^3
 + 16*(b^2*cos(6*b*log(c))*cos(4*b*log(c)) + b^2*cos(4*b*log(c))*cos(2*b*l
og(c)) + b^2*sin(6*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c))*sin(2*b
*log(c)))*n^2 + 2*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*s
in(4*b*log(c)) + b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin
(2*b*log(c)))*n + cos(6*b*log(c))*cos(4*b*log(c)) + cos(4*b*log(c))*cos(2*
b*log(c)) + sin(6*b*log(c))*sin(4*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(
c)))*x*cos(2*b*log(x^n) + 2*a) + (16*(b^3*cos(8*b*log(c))*cos(4*b*log(c))
+ b^3*sin(8*b*log(c))*sin(4*b*log(c)) + b^3*cos(4*b*log(c)))*n^3 - 4*(b^2*
cos(4*b*log(c))*sin(8*b*log(c)) - b^2*cos(8*b*log(c))*sin(4*b*log(c)) + b^
2*sin(4*b*log(c)))*n^2 + 4*(b*cos(8*b*log(c))*cos(4*b*log(c)) + b*sin(8*b*
log(c))*sin(4*b*log(c)) + b*cos(4*b*log(c)))*n - cos(4*b*log(c))*sin(8*b*l
og(c)) + cos(8*b*log(c))*sin(4*b*log(c)) - sin(4*b*log(c)))*x*sin(4*b*l...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15992 vs. $2(191) = 382$.

Time = 0.57 (sec) , antiderivative size = 15992, normalized size of antiderivative = 83.73

$$\int \sin^4(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(sin(a+b*log(c*x^n))^4,x, algorithm="giac")
```

output
```
3/8*x + 1/16*(256*b^3*n^3*x*e^(3*pi*b*n*sgn(x) - 3*pi*b*n + 3*pi*b*sgn(c)
- 3*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) +
 b*log(abs(c)))^2*tan(2*a)^2*tan(a) + 256*b^3*n^3*x*e^(pi*b*n*sgn(x) - pi*
b*n + pi*b*sgn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b
*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a) - 32*b^3*n^3*x*e^(4*pi
*b*n*sgn(x) - 4*pi*b*n + 4*pi*b*sgn(c) - 4*pi*b)*tan(2*b*n*log(abs(x)) + 2
*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)*tan(a)^2
 + 256*b^3*n^3*x*e^(3*pi*b*n*sgn(x) - 3*pi*b*n + 3*pi*b*sgn(c) - 3*pi*b)*t
an(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(
c)))*tan(2*a)^2*tan(a)^2 + 256*b^3*n^3*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*
sgn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(
x)) + b*log(abs(c)))*tan(2*a)^2*tan(a)^2 - 32*b^3*n^3*x*e^(4*pi*b*n*sgn(x)
 - 4*pi*b*n + 4*pi*b*sgn(c) - 4*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(
c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a)^2 - 32*b^3*n
^3*x*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*lo
g(abs(c)))^2*tan(2*a)*tan(a)^2 - 32*b^3*n^3*x*tan(2*b*n*log(abs(x)) + 2*b*
log(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a)^2 +
4*b^2*n^2*x*e^(4*pi*b*n*sgn(x) - 4*pi*b*n + 4*pi*b*sgn(c) - 4*pi*b)*tan(2*
b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^
2*tan(2*a)^2*tan(a)^2 - 64*b^2*n^2*x*e^(3*pi*b*n*sgn(x) - 3*pi*b*n + 3*...
```

Mupad [B] (verification not implemented)

Time = 19.79 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.61

$$\int \sin^4(a+b\log(cx^n))\,dx = \frac{3x}{8} - \frac{x\,e^{-a\,2i}\,\frac{1}{(c\,x^n)^{b\,2i}}\,1i}{8\,b\,n+4i} - \frac{x\,e^{a\,2i}\,(c\,x^n)^{b\,2i}}{4+b\,n\,8i} + \frac{x\,e^{-a\,4i}\,\frac{1}{(c\,x^n)^{b\,4i}}\,1i}{64\,b\,n+16i} + \frac{x\,e^{a\,4i}\,(c\,x^n)^{b\,4i}}{16+b\,n\,64i}$$

input
```
int(sin(a + b*log(c*x^n))^4,x)
```

output
```
(3*x)/8 - (x*exp(-a*2i)/(c*x^n)^(b*2i)*1i)/(8*b*n + 4i) - (x*exp(a*2i)*(c*
x^n)^(b*2i))/(b*n*8i + 4) + (x*exp(-a*4i)/(c*x^n)^(b*4i)*1i)/(64*b*n + 16i
) + (x*exp(a*4i)*(c*x^n)^(b*4i))/(b*n*64i + 16)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.92

$$\int \sin^4\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\frac{x\left(-16\cos\left(\log(x^nc)\,b+a\right)\sin\left(\log(x^nc)\,b+a\right)^3b^3n^3-4\cos\left(\log(x^nc)\,b+a\right)\sin\left(\log(x^nc)\,b+a\right)^3bn-24\right.}{}$$

input
```
int(sin(a+b*log(c*x^n))^4,x)
```

output
```
(x*( - 16*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*b**3*n**3 - 4*c
os(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*b*n - 24*cos(log(x**n*c)*b
 + a)*sin(log(x**n*c)*b + a)*b**3*n**3 + 4*sin(log(x**n*c)*b + a)**4*b**2*
n**2 + sin(log(x**n*c)*b + a)**4 + 12*sin(log(x**n*c)*b + a)**2*b**2*n**2
+ 24*b**4*n**4))/(64*b**4*n**4 + 20*b**2*n**2 + 1)
```

3.22 $\int \frac{\sin^4(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 73

$$\int \frac{\sin^4(a+b\log(cx^n))}{x}\,dx = \frac{3\log(x)}{8} - \frac{3\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{8bn} - \frac{\cos(a+b\log(cx^n))\sin^3(a+b\log(cx^n))}{4bn}$$

output
```
3/8*ln(x)-3/8*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/b/n-1/4*cos(a+b*ln(c*x
^n))*sin(a+b*ln(c*x^n))^3/b/n
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \frac{\sin^4(a+b\log(cx^n))}{x}\,dx = \frac{12(a+b\log(cx^n)) - 8\sin(2(a+b\log(cx^n))) + \sin(4(a+b\log(cx^n)))}{32bn}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]^4/x,x]
```

output
```
(12*(a + b*Log[c*x^n]) - 8*Sin[2*(a + b*Log[c*x^n])] + Sin[4*(a + b*Log[c*
x^n])])/(32*b*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3039, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \ 3039$$

$$\frac{\int \sin^4(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \sin(a+b\log(cx^n))^4d\log(cx^n)}{n}$$

$$\Big\downarrow \ 3115$$

$$\frac{\frac{3}{4}\int \sin^2(a+b\log(cx^n))d\log(cx^n)-\frac{\sin^3(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b}}{n}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{3}{4}\int \sin(a+b\log(cx^n))^2d\log(cx^n)-\frac{\sin^3(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b}}{n}$$

$$\Big\downarrow \ 3115$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int 1d\log(cx^n)-\frac{\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{2b}\right)-\frac{\sin^3(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b}}{n}$$

$$\Big\downarrow \ 24$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\log(cx^n)-\frac{\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{2b}\right)-\frac{\sin^3(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b}}{n}$$

input
```
Int[Sin[a + b*Log[c*x^n]]^4/x,x]
```

output
```
(-1/4*(Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^3)/b + (3*(Log[c*x^n]/2
 - (Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/(2*b)))/4)/n
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 6.65 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$\frac{12\ln(x) b n+\sin(4b\ln(c\,x^{n})+4a)-8\sin(2b\ln(c\,x^{n})+2a)}{32bn}$	46
derivativedivides	$\frac{-\frac{\left(\sin(a+b\ln(c\,x^{n}))^{3}+\frac{3\sin(a+b\ln(c\,x^{n}))}{2}\right)\cos(a+b\ln(c\,x^{n}))}{4}+\frac{3b\ln(c\,x^{n})}{8}+\frac{3a}{8}}{nb}$	61
default	$\frac{-\frac{\left(\sin(a+b\ln(c\,x^{n}))^{3}+\frac{3\sin(a+b\ln(c\,x^{n}))}{2}\right)\cos(a+b\ln(c\,x^{n}))}{4}+\frac{3b\ln(c\,x^{n})}{8}+\frac{3a}{8}}{nb}$	61

input
```
int(sin(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
```

output
```
1/32*(12*ln(x)*b*n+sin(4*b*ln(c*x^n)+4*a)-8*sin(2*b*ln(c*x^n)+2*a))/b/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{\sin^4(a+b\log(cx^n))}{x}\,dx = \frac{3\,bn\log(x) + \left(2\,\cos\left(bn\log(x)+b\log(c)+a\right)^3 - 5\,\cos\left(bn\log(x)+b\log(c)+a\right)\right)\sin\left(bn\log(x)+b\log\right.}{8\,bn}$$

input
```
integrate(sin(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

output
```
1/8*(3*b*n*log(x) + (2*cos(b*n*log(x) + b*log(c) + a)^3 - 5*cos(b*n*log(x)
 + b*log(c) + a))*sin(b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [A] (verification not implemented)

Time = 13.52 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.37

$$\int \frac{\sin^4(a+b\log(cx^n))}{x}\,dx = -\frac{\begin{cases} \log(x)\cos(2a) & \text{for } b=0 \wedge (b=0 \vee n=0) \\ \log(x)\cos(2a+2b\log(c)) & \text{for } n=0 \\ \frac{\sin(2a+2b\log(cx^n))}{2bn} & \text{otherwise} \end{cases}}{2} + \frac{\begin{cases} \log(x)\cos(4a) & \text{for } b=0 \wedge (b=0 \vee n=0) \\ \log(x)\cos(4a+4b\log(c)) & \text{for } n=0 \\ \frac{\sin(4a+4b\log(cx^n))}{4bn} & \text{otherwise} \end{cases}}{8} + \frac{3\log(x)}{8}$$

input
```
integrate(sin(a+b*ln(c*x**n))**4/x,x)
```

output
```
-Piecewise((log(x)*cos(2*a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*co
s(2*a + 2*b*log(c)), Eq(n, 0)), (sin(2*a + 2*b*log(c*x**n))/(2*b*n), True)
)/2 + Piecewise((log(x)*cos(4*a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(
x)*cos(4*a + 4*b*log(c)), Eq(n, 0)), (sin(4*a + 4*b*log(c*x**n))/(4*b*n),
True))/8 + 3*log(x)/8
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

$$\int \frac{\sin^4(a+b\log(cx^n))}{x}\,dx = \frac{12\,bn\log(x)+\cos(4\,b\log(x^n)+4\,a)\sin(4\,b\log(c))-8\,\cos(2\,b\log(x^n)+2\,a)\sin(2\,b\log(c))+\cos(4\,b}{32\,bn}$$

input
```
integrate(sin(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

output
```
1/32*(12*b*n*log(x) + cos(4*b*log(x^n) + 4*a)*sin(4*b*log(c)) - 8*cos(2*b*
log(x^n) + 2*a)*sin(2*b*log(c)) + cos(4*b*log(c))*sin(4*b*log(x^n) + 4*a)
- 8*cos(2*b*log(c))*sin(2*b*log(x^n) + 2*a))/(b*n)
```

Giac [F]

$$\int \frac{\sin^4(a+b\log(cx^n))}{x}\,dx = \int \frac{\sin(b\log(cx^n)+a)^4}{x}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

output
```
integrate(sin(b*log(c*x^n) + a)^4/x, x)
```

Mupad [B] (verification not implemented)

Time = 19.89 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \frac{\sin^4(a+b\log(cx^n))}{x}\,dx = \frac{3\ \ln(x^n)}{8\,n} - \frac{\frac{\sin(2\,a+2\,b\ \ln(c\,x^n))}{4} - \frac{\sin(4\,a+4\,b\ \ln(c\,x^n))}{32}}{b\,n}$$

input
```
int(sin(a + b*log(c*x^n))^4/x,x)
```

output
```
(3*log(x^n))/(8*n) - (sin(2*a + 2*b*log(c*x^n))/4 - sin(4*a + 4*b*log(c*x^
n))/32)/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{\sin^4(a+b\log(cx^n))}{x}\,dx = \frac{-2\cos(\log(x^nc)\,b+a)\sin(\log(x^nc)\,b+a)^3 - 3\cos(\log(x^nc)\,b+a)\sin(\log(x^nc)\,b+a) + 3\log(x^nc)\,b}{8bn}$$

input
```
int(sin(a+b*log(c*x^n))^4/x,x)
```

output
```
( - 2*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3 - 3*cos(log(x**n*c)
*b + a)*sin(log(x**n*c)*b + a) + 3*log(x**n*c)*b)/(8*b*n)
```

3.23 $\int \frac{\sin^4(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 202

$$\begin{aligned}\int \frac{\sin^4(a+b\log(cx^n))}{x^2}\,dx = &-\frac{24b^4n^4}{(1+20b^2n^2+64b^4n^4)\,x}\\ &-\frac{24b^3n^3\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{(1+20b^2n^2+64b^4n^4)\,x}\\ &-\frac{12b^2n^2\sin^2(a+b\log(cx^n))}{(1+20b^2n^2+64b^4n^4)\,x}\\ &-\frac{4bn\cos(a+b\log(cx^n))\sin^3(a+b\log(cx^n))}{(1+16b^2n^2)\,x}\\ &-\frac{\sin^4(a+b\log(cx^n))}{(1+16b^2n^2)\,x}\end{aligned}$$

output
```
-24*b^4*n^4/(64*b^4*n^4+20*b^2*n^2+1)/x-24*b^3*n^3*cos(a+b*ln(c*x^n))*sin(
a+b*ln(c*x^n))/(64*b^4*n^4+20*b^2*n^2+1)/x-12*b^2*n^2*sin(a+b*ln(c*x^n))^2
/(64*b^4*n^4+20*b^2*n^2+1)/x-4*b*n*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))^3
/(16*b^2*n^2+1)/x-sin(a+b*ln(c*x^n))^4/(16*b^2*n^2+1)/x
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.84

$$\int \frac{\sin^4(a+b\log(cx^n))}{x^2}\,dx = -\frac{3+60b^2n^2+192b^4n^4-4(1+16b^2n^2)\cos(2(a+b\log(cx^n)))+(1+4b^2n^2)\cos(4(a+b\log(cx^n)))+}{8(}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]^4/x^2,x]
```

output
```
-1/8*(3 + 60*b^2*n^2 + 192*b^4*n^4 - 4*(1 + 16*b^2*n^2)*Cos[2*(a + b*Log[c
*x^n])] + (1 + 4*b^2*n^2)*Cos[4*(a + b*Log[c*x^n])] + 8*b*n*Sin[2*(a + b*L
og[c*x^n])] + 128*b^3*n^3*Sin[2*(a + b*Log[c*x^n])] - 4*b*n*Sin[4*(a + b*L
og[c*x^n])] - 16*b^3*n^3*Sin[4*(a + b*Log[c*x^n])])/((1 + 20*b^2*n^2 + 64*
b^4*n^4)*x)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4990, 4990, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(a+b\log(cx^n))}{x^2}\,dx$$

$$\Big\downarrow \text{4990}$$

$$\frac{12b^2n^2\int \frac{\sin^2(a+b\log(cx^n))}{x^2}dx}{16b^2n^2+1}-\frac{\sin^4(a+b\log(cx^n))}{x(16b^2n^2+1)}-\frac{4bn\sin^3(a+b\log(cx^n))\cos(a+b\log(cx^n))}{x(16b^2n^2+1)}$$

$$\Big\downarrow \text{4990}$$

$$\frac{12b^2n^2\left(\frac{2b^2n^2\int\frac{1}{x^2}dx}{4b^2n^2+1}-\frac{\sin^2(a+b\log(cx^n))}{x(4b^2n^2+1)}-\frac{2bn\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{x(4b^2n^2+1)}\right)}{16b^2n^2+1}-\frac{\sin^4\left(a+b\log\left(cx^n\right)\right)}{x\left(16b^2n^2+1\right)}-\frac{4bn\sin^3\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{x\left(16b^2n^2+1\right)}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\sin^4\left(a+b\log\left(cx^n\right)\right)}{x\left(16b^2n^2+1\right)}-\frac{4bn\sin^3\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{x\left(16b^2n^2+1\right)}+\frac{12b^2n^2\left(-\frac{\sin^2(a+b\log(cx^n))}{x(4b^2n^2+1)}-\frac{2bn\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{x(4b^2n^2+1)}-\frac{2b^2n^2}{x(4b^2n^2+1)}\right)}{16b^2n^2+1}$$

input
```
Int[Sin[a + b*Log[c*x^n]]^4/x^2,x]
```

output
```
(-4*b*n*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^3)/((1 + 16*b^2*n^2)*x
) - Sin[a + b*Log[c*x^n]]^4/((1 + 16*b^2*n^2)*x) + (12*b^2*n^2*((-2*b^2*n^
2)/((1 + 4*b^2*n^2)*x) - (2*b*n*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]
])/((1 + 4*b^2*n^2)*x) - Sin[a + b*Log[c*x^n]]^2/((1 + 4*b^2*n^2)*x)))/(1
+ 16*b^2*n^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 4990
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (-Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a
 + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e
*(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2
))   Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
 c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [A] (verified)

Time = 11.63 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.99

method	result
parallelrisch	$\frac{-3-192b^4n^4-60b^2n^2-\cos(4b\ln(c\,x^n)+4a)+4\cos(2b\ln(c\,x^n)+2a)-4b^2n^2\cos(4b\ln(c\,x^n)+4a)+64b^2n^2\cos(2b\ln(c\,x^n)+2a)}{8x(64b^4n^4+20b^2n}$

input
```
int(sin(a+b*ln(c*x^n))^4/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(-3-192*b^4*n^4-60*b^2*n^2-cos(4*b*ln(c*x^n)+4*a)+4*cos(2*b*ln(c*x^n)+
2*a)-4*b^2*n^2*cos(4*b*ln(c*x^n)+4*a)+64*b^2*n^2*cos(2*b*ln(c*x^n)+2*a)+4*
b*n*sin(4*b*ln(c*x^n)+4*a)-8*b*n*sin(2*b*ln(c*x^n)+2*a)+16*b^3*n^3*sin(4*b
*ln(c*x^n)+4*a)-128*b^3*n^3*sin(2*b*ln(c*x^n)+2*a))/x/(64*b^4*n^4+20*b^2*n
^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.80

$$\int \frac{\sin^4(a+b\log(cx^n))}{x^2}\,dx = -\frac{24\,b^4n^4+(4\,b^2n^2+1)\cos(bn\log(x)+b\log(c)+a)^4+16\,b^2n^2-2\,(10\,b^2n^2+1)\cos(bn\log(x)+b\log}{}$$

input
```
integrate(sin(a+b*log(c*x^n))^4/x^2,x, algorithm="fricas")
```

output
```
-(24*b^4*n^4 + (4*b^2*n^2 + 1)*cos(b*n*log(x) + b*log(c) + a)^4 + 16*b^2*n
^2 - 2*(10*b^2*n^2 + 1)*cos(b*n*log(x) + b*log(c) + a)^2 - 4*((4*b^3*n^3 +
 b*n)*cos(b*n*log(x) + b*log(c) + a)^3 - (10*b^3*n^3 + b*n)*cos(b*n*log(x)
 + b*log(c) + a))*sin(b*n*log(x) + b*log(c) + a) + 1)/((64*b^4*n^4 + 20*b^
2*n^2 + 1)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+b\log(cx^n))}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(a+b*ln(c*x**n))**4/x**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. $2(202) = 404$.

Time = 0.10 (sec) , antiderivative size = 1085, normalized size of antiderivative = 5.37

$$\int \frac{\sin^4(a+b\log(cx^n))}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(a+b*log(c*x^n))^4/x^2,x, algorithm="maxima")
```

output
```
-1/16*(384*(b^4*cos(4*b*log(c))^2 + b^4*sin(4*b*log(c))^2)*n^4 + 120*(b^2*
cos(4*b*log(c))^2 + b^2*sin(4*b*log(c))^2)*n^2 + 6*cos(4*b*log(c))^2 - (16
*(b^3*cos(4*b*log(c))*sin(8*b*log(c)) - b^3*cos(8*b*log(c))*sin(4*b*log(c)
) + b^3*sin(4*b*log(c)))*n^3 - 4*(b^2*cos(8*b*log(c))*cos(4*b*log(c)) + b^
2*sin(8*b*log(c))*sin(4*b*log(c)) + b^2*cos(4*b*log(c)))*n^2 + 4*(b*cos(4*
b*log(c))*sin(8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c)) + b*sin(4*b*
log(c)))*n - cos(8*b*log(c))*cos(4*b*log(c)) - sin(8*b*log(c))*sin(4*b*log
(c)) - cos(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) + 4*(32*(b^3*cos(4*b*log(c
))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(4*b*log(c)) + b^3*cos(2*b*log
(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(2*b*log(c)))*n^3 - 16*(b^2*
cos(6*b*log(c))*cos(4*b*log(c)) + b^2*cos(4*b*log(c))*cos(2*b*log(c)) + b^
2*sin(6*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c))*sin(2*b*log(c)))*n
^2 + 2*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*log(
c)) + b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)
))*n - cos(6*b*log(c))*cos(4*b*log(c)) - cos(4*b*log(c))*cos(2*b*log(c)) -
 sin(6*b*log(c))*sin(4*b*log(c)) - sin(4*b*log(c))*sin(2*b*log(c)))*cos(2*
b*log(x^n) + 2*a) + 6*sin(4*b*log(c))^2 - (16*(b^3*cos(8*b*log(c))*cos(4*b
*log(c)) + b^3*sin(8*b*log(c))*sin(4*b*log(c)) + b^3*cos(4*b*log(c)))*n^3
+ 4*(b^2*cos(4*b*log(c))*sin(8*b*log(c)) - b^2*cos(8*b*log(c))*sin(4*b*log
(c)) + b^2*sin(4*b*log(c)))*n^2 + 4*(b*cos(8*b*log(c))*cos(4*b*log(c)) ...
```

Giac [F]

$$\int \frac{\sin^4\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx=\int \frac{\sin\left(b\log\left(cx^n\right)+a\right)^4}{x^2}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^4/x^2,x, algorithm="giac")
```

output
```
integrate(sin(b*log(c*x^n) + a)^4/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+b\log(cx^n))}{x^2}\,dx=\int \frac{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^4}{x^2}\,dx$$

input
```
int(sin(a + b*log(c*x^n))^4/x^2,x)
```

output
```
int(sin(a + b*log(c*x^n))^4/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.89

$$\int \frac{\sin^4(a+b\log(cx^n))}{x^2}\,dx$$
$$= \frac{-16\cos\left(\log(x^n c)\,b+a\right)\sin\left(\log(x^n c)\,b+a\right)^3 b^3n^3-4\cos\left(\log(x^n c)\,b+a\right)\sin\left(\log(x^n c)\,b+a\right)^3 bn-24\cos}{}$$

input
```
int(sin(a+b*log(c*x^n))^4/x^2,x)
```

output
```
( - 16*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*b**3*n**3 - 4*cos(
log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*b*n - 24*cos(log(x**n*c)*b +
a)*sin(log(x**n*c)*b + a)*b**3*n**3 - 4*sin(log(x**n*c)*b + a)**4*b**2*n**
2 - sin(log(x**n*c)*b + a)**4 - 12*sin(log(x**n*c)*b + a)**2*b**2*n**2 - 2
4*b**4*n**4)/(x*(64*b**4*n**4 + 20*b**2*n**2 + 1))
```

3.24 $\int \frac{\sin^4(a+b\log(cx^n))}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 210

$$\int \frac{\sin^4(a+b\log(cx^n))}{x^3}\,dx = -\frac{3b^4n^4}{4(1+5b^2n^2+4b^4n^4)x^2} - \frac{3b^3n^3\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{2(1+5b^2n^2+4b^4n^4)x^2} - \frac{3b^2n^2\sin^2(a+b\log(cx^n))}{2(1+5b^2n^2+4b^4n^4)x^2} - \frac{bn\cos(a+b\log(cx^n))\sin^3(a+b\log(cx^n))}{(1+4b^2n^2)x^2} - \frac{\sin^4(a+b\log(cx^n))}{2(1+4b^2n^2)x^2}$$

output

```
-3/4*b^4*n^4/(4*b^4*n^4+5*b^2*n^2+1)/x^2-3/2*b^3*n^3*cos(a+b*ln(c*x^n))*si
n(a+b*ln(c*x^n))/(4*b^4*n^4+5*b^2*n^2+1)/x^2-3/2*b^2*n^2*sin(a+b*ln(c*x^n)
)^2/(4*b^4*n^4+5*b^2*n^2+1)/x^2-b*n*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))^
3/(4*b^2*n^2+1)/x^2-1/2*sin(a+b*ln(c*x^n))^4/(4*b^2*n^2+1)/x^2
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.80

$$\int \frac{\sin^4(a+b\log(cx^n))}{x^3}\,dx =$$
$$-\frac{3+15b^2n^2+12b^4n^4-4(1+4b^2n^2)\cos(2(a+b\log(cx^n)))+(1+b^2n^2)\cos(4(a+b\log(cx^n)))+4bn}{16(1}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]^4/x^3,x]
```

output
```
-1/16*(3 + 15*b^2*n^2 + 12*b^4*n^4 - 4*(1 + 4*b^2*n^2)*Cos[2*(a + b*Log[c*
x^n])] + (1 + b^2*n^2)*Cos[4*(a + b*Log[c*x^n])] + 4*b*n*Sin[2*(a + b*Log[
c*x^n])] + 16*b^3*n^3*Sin[2*(a + b*Log[c*x^n])] - 2*b*n*Sin[4*(a + b*Log[c
*x^n])] - 2*b^3*n^3*Sin[4*(a + b*Log[c*x^n])])/((1 + 5*b^2*n^2 + 4*b^4*n^4
)*x^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4990, 4990, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(a+b\log(cx^n))}{x^3}\,dx$$

$$\Big\downarrow \; 4990$$

$$\frac{3b^2n^2\int \frac{\sin^2(a+b\log(cx^n))}{x^3}dx}{4b^2n^2+1}-\frac{\sin^4(a+b\log(cx^n))}{2x^2(4b^2n^2+1)}-\frac{bn\sin^3(a+b\log(cx^n))\cos(a+b\log(cx^n))}{x^2(4b^2n^2+1)}$$

$$\Big\downarrow \; 4990$$

$$\frac{3b^2n^2\left(\frac{b^2n^2\int\frac{1}{x^3}dx}{2(b^2n^2+1)}-\frac{\sin^2(a+b\log(cx^n))}{2x^2(b^2n^2+1)}-\frac{bn\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{2x^2(b^2n^2+1)}\right)}{4b^2n^2+1}-\frac{\sin^4(a+b\log(cx^n))}{2x^2(4b^2n^2+1)}-\frac{bn\sin^3(a+b\log(cx^n))\cos(a+b\log(cx^n))}{x^2(4b^2n^2+1)}$$

$$\Big\downarrow \; 15$$

$$-\frac{\sin^4(a+b\log(cx^n))}{2x^2(4b^2n^2+1)}-\frac{bn\sin^3(a+b\log(cx^n))\cos(a+b\log(cx^n))}{x^2(4b^2n^2+1)}+\frac{3b^2n^2\left(-\frac{\sin^2(a+b\log(cx^n))}{2x^2(b^2n^2+1)}-\frac{bn\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{2x^2(b^2n^2+1)}-\frac{b^2n^2}{4x^2(b^2n^2+1)}\right)}{4b^2n^2+1}$$

input
```
Int[Sin[a + b*Log[c*x^n]]^4/x^3,x]
```

output
```
-((b*n*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^3)/((1 + 4*b^2*n^2)*x^2
)) - Sin[a + b*Log[c*x^n]]^4/(2*(1 + 4*b^2*n^2)*x^2) + (3*b^2*n^2*(-1/4*(b
^2*n^2)/((1 + b^2*n^2)*x^2) - (b*n*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x
^n]])/(2*(1 + b^2*n^2)*x^2) - Sin[a + b*Log[c*x^n]]^2/(2*(1 + b^2*n^2)*x^2
)))/(1 + 4*b^2*n^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 4990
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (-Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a
 + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e
*(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2
))   Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
 c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [A] (verified)

Time = 19.46 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.95

method	result
parallelrisch	$\frac{-3-12b^4n^4-15b^2n^2+2bn\sin(4b\ln(c\,x^n)+4a)-4bn\sin(2b\ln(c\,x^n)+2a)+2b^3n^3\sin(4b\ln(c\,x^n)+4a)-16b^3n^3\sin(2b\ln(c\,x^n)-}{16x^2(4b^4n^4+5b^2n^2+}$

input
```
int(sin(a+b*ln(c*x^n))^4/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*(-3-12*b^4*n^4-15*b^2*n^2+2*b*n*sin(4*b*ln(c*x^n)+4*a)-4*b*n*sin(2*b*
ln(c*x^n)+2*a)+2*b^3*n^3*sin(4*b*ln(c*x^n)+4*a)-16*b^3*n^3*sin(2*b*ln(c*x^
n)+2*a)+16*b^2*n^2*cos(2*b*ln(c*x^n)+2*a)-b^2*n^2*cos(4*b*ln(c*x^n)+4*a)-c
os(4*b*ln(c*x^n)+4*a)+4*cos(2*b*ln(c*x^n)+2*a))/x^2/(4*b^4*n^4+5*b^2*n^2+1
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.78

$$\int \frac{\sin^4\left(a+b\log\left(cx^n\right)\right)}{x^3}\,dx =$$

$$-\frac{3\,b^4n^4+2\left(b^2n^2+1\right)\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^4+8\,b^2n^2-2\left(5\,b^2n^2+2\right)\cos\left(bn\log\left(x\right)+b\log\left(c\right)\right.}{}$$

input
```
integrate(sin(a+b*log(c*x^n))^4/x^3,x, algorithm="fricas")
```

output
```
-1/4*(3*b^4*n^4 + 2*(b^2*n^2 + 1)*cos(b*n*log(x) + b*log(c) + a)^4 + 8*b^2
*n^2 - 2*(5*b^2*n^2 + 2)*cos(b*n*log(x) + b*log(c) + a)^2 - 2*(2*(b^3*n^3
+ b*n)*cos(b*n*log(x) + b*log(c) + a)^3 - (5*b^3*n^3 + 2*b*n)*cos(b*n*log(
x) + b*log(c) + a))*sin(b*n*log(x) + b*log(c) + a) + 2)/((4*b^4*n^4 + 5*b^
2*n^2 + 1)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+b\log(cx^n))}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(a+b*ln(c*x**n))**4/x**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1082 vs. $2(202) = 404$.

Time = 0.10 (sec) , antiderivative size = 1082, normalized size of antiderivative = 5.15

$$\int \frac{\sin^4(a+b\log(cx^n))}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate(sin(a+b*log(c*x^n))^4/x^3,x, algorithm="maxima")
```

output
```
-1/32*(24*(b^4*cos(4*b*log(c))^2 + b^4*sin(4*b*log(c))^2)*n^4 + 30*(b^2*co
s(4*b*log(c))^2 + b^2*sin(4*b*log(c))^2)*n^2 + 6*cos(4*b*log(c))^2 - (2*(b
^3*cos(4*b*log(c))*sin(8*b*log(c)) - b^3*cos(8*b*log(c))*sin(4*b*log(c)) +
 b^3*sin(4*b*log(c)))*n^3 - (b^2*cos(8*b*log(c))*cos(4*b*log(c)) + b^2*sin
(8*b*log(c))*sin(4*b*log(c)) + b^2*cos(4*b*log(c)))*n^2 + 2*(b*cos(4*b*log
(c))*sin(8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c)) + b*sin(4*b*log(c
)))*n - cos(8*b*log(c))*cos(4*b*log(c)) - sin(8*b*log(c))*sin(4*b*log(c))
- cos(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) + 4*(4*(b^3*cos(4*b*log(c))*sin
(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(4*b*log(c)) + b^3*cos(2*b*log(c))*s
in(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(2*b*log(c)))*n^3 - 4*(b^2*cos(6*b
*log(c))*cos(4*b*log(c)) + b^2*cos(4*b*log(c))*cos(2*b*log(c)) + b^2*sin(6
*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c))*sin(2*b*log(c)))*n^2 + (b
*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*log(c)) + b*c
os(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)))*n - co
s(6*b*log(c))*cos(4*b*log(c)) - cos(4*b*log(c))*cos(2*b*log(c)) - sin(6*b*
log(c))*sin(4*b*log(c)) - sin(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n
) + 2*a) + 6*sin(4*b*log(c))^2 - (2*(b^3*cos(8*b*log(c))*cos(4*b*log(c)) +
 b^3*sin(8*b*log(c))*sin(4*b*log(c)) + b^3*cos(4*b*log(c)))*n^3 + (b^2*cos
(4*b*log(c))*sin(8*b*log(c)) - b^2*cos(8*b*log(c))*sin(4*b*log(c)) + b^2*s
in(4*b*log(c)))*n^2 + 2*(b*cos(8*b*log(c))*cos(4*b*log(c)) + b*sin(8*b*...
```

Giac [F]

$$\int \frac{\sin^4\left(a+b\log\left(cx^n\right)\right)}{x^3}\,dx=\int \frac{\sin\left(b\log\left(cx^n\right)+a\right)^4}{x^3}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^4/x^3,x, algorithm="giac")
```

output
```
integrate(sin(b*log(c*x^n) + a)^4/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(a+b\log(cx^n))}{x^3}\,dx=\int \frac{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^4}{x^3}\,dx$$

input
```
int(sin(a + b*log(c*x^n))^4/x^3,x)
```

output
```
int(sin(a + b*log(c*x^n))^4/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.86

$$\int \frac{\sin^4(a+b\log(cx^n))}{x^3}\,dx$$
$$=\frac{-4\cos\left(\log(x^n c)\,b+a\right)\sin\left(\log(x^n c)\,b+a\right)^3 b^3 n^3-4\cos\left(\log(x^n c)\,b+a\right)\sin\left(\log(x^n c)\,b+a\right)^3 bn-6\cos\left(\mathrm{l}\right.}{}$$

input
```
int(sin(a+b*log(c*x^n))^4/x^3,x)
```

output
```
( - 4*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*b**3*n**3 - 4*cos(l
og(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*b*n - 6*cos(log(x**n*c)*b + a)
*sin(log(x**n*c)*b + a)*b**3*n**3 - 2*sin(log(x**n*c)*b + a)**4*b**2*n**2
- 2*sin(log(x**n*c)*b + a)**4 - 6*sin(log(x**n*c)*b + a)**2*b**2*n**2 - 3*
b**4*n**4)/(4*x**2*(4*b**4*n**4 + 5*b**2*n**2 + 1))
```

3.25 $\int \sin(\log(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 39

$$\int \sin(\log(a+bx))\,dx = -\frac{(a+bx)\cos(\log(a+bx))}{2b} + \frac{(a+bx)\sin(\log(a+bx))}{2b}$$

output
```
-1/2*(b*x+a)*cos(ln(b*x+a))/b+1/2*(b*x+a)*sin(ln(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \sin(\log(a+bx))\,dx = -\frac{(a+bx)(\cos(\log(a+bx))-\sin(\log(a+bx)))}{2b}$$

input
```
Integrate[Sin[Log[a + b*x]],x]
```

output
```
-1/2*((a + b*x)*(Cos[Log[a + b*x]] - Sin[Log[a + b*x]]))/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {7281, 4978}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(\log(a+bx))\,dx$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{\int \sin(\log(a+bx))d(a+bx)}{b}$$

$$\Big\downarrow \text{ 4978}$$

$$\frac{\frac{1}{2}(a+bx)\sin(\log(a+bx))-\frac{1}{2}(a+bx)\cos(\log(a+bx))}{b}$$

input
```
Int[Sin[Log[a + b*x]],x]
```

output
```
(-1/2*((a + b*x)*Cos[Log[a + b*x]]) + ((a + b*x)*Sin[Log[a + b*x]])/2)/b
```

Defintions of rubi rules used

rule 4978
```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] - Simp[b*d*n*x*(Cos[d*(a +
 b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{-\frac{(bx+a)\cos(\ln(bx+a))}{2}+\frac{(bx+a)\sin(\ln(bx+a))}{2}}{b}$	34
parallelrisch	$\frac{(-bx-a)\cos(\ln(bx+a))+(bx+a)\sin(\ln(bx+a))-2a}{2b}$	39
risch	$\frac{\left(-\frac{1}{4}-\frac{i}{4}\right)(bx+a)(bx+a)^{i}}{b}+\frac{\left(-\frac{1}{4}+\frac{i}{4}\right)(bx+a)(bx+a)^{-i}}{b}$	44
norman	$\frac{x\tan\left(\frac{\ln(bx+a)}{2}\right)+\frac{a\tan\left(\frac{\ln(bx+a)}{2}\right)}{b}+\frac{a\tan\left(\frac{\ln(bx+a)}{2}\right)^{2}}{b}-\frac{x}{2}+\frac{x\tan\left(\frac{\ln(bx+a)}{2}\right)^{2}}{2}}{1+\tan\left(\frac{\ln(bx+a)}{2}\right)^{2}}$	76

input
```
int(sin(ln(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2*(b*x+a)*cos(ln(b*x+a))+1/2*(b*x+a)*sin(ln(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \sin(\log(a+bx))\,dx = -\frac{(bx+a)\cos\left(\log\left(bx+a\right)\right)-(bx+a)\sin\left(\log\left(bx+a\right)\right)}{2\,b}$$

input
```
integrate(sin(log(b*x+a)),x, algorithm="fricas")
```

output
```
-1/2*((b*x + a)*cos(log(b*x + a)) - (b*x + a)*sin(log(b*x + a)))/b
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int \sin(\log(a+bx))\,dx = \begin{cases} \frac{a\sin(\log(a+bx))}{2b} - \frac{a\cos(\log(a+bx))}{2b} + \frac{x\sin(\log(a+bx))}{2} - \frac{x\cos(\log(a+bx))}{2} & \text{for } b\neq 0 \\ x\sin(\log(a)) & \text{otherwise} \end{cases}$$

input
```
integrate(sin(ln(b*x+a)),x)
```

output
```
Piecewise((a*sin(log(a + b*x))/(2*b) - a*cos(log(a + b*x))/(2*b) + x*sin(l
og(a + b*x))/2 - x*cos(log(a + b*x))/2, Ne(b, 0)), (x*sin(log(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \sin(\log(a+bx))\,dx = -\frac{(bx+a)(\cos(\log(bx+a)) - \sin(\log(bx+a)))}{2\,b}$$

input
```
integrate(sin(log(b*x+a)),x, algorithm="maxima")
```

output
```
-1/2*(b*x + a)*(cos(log(b*x + a)) - sin(log(b*x + a)))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \sin(\log(a+bx))\,dx = -\frac{(bx+a)\cos(\log(bx+a))}{2\,b} + \frac{(bx+a)\sin(\log(bx+a))}{2\,b}$$

input
```
integrate(sin(log(b*x+a)),x, algorithm="giac")
```

output
```
-1/2*(b*x + a)*cos(log(b*x + a))/b + 1/2*(b*x + a)*sin(log(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 19.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \sin(\log(a+bx))\,dx = \begin{cases} x\sin\left(\ln\left(a\right)\right) & \text{if } b=0 \\ -\frac{\sqrt{2}\cos\left(\frac{\pi}{4}+\ln(a+b\,x)\right)\,(a+b\,x)}{2\,b} & \text{if } b\neq 0 \end{cases}$$

input
```
int(sin(log(a + b*x)),x)
```

output
```
piecewise(b == 0, x*sin(log(a)), b ~= 0, -(2^(1/2)*cos(pi/4 + log(a + b*x)
)*(a + b*x))/(2*b))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \sin(\log(a+bx))\,dx = \frac{-\cos\left(\log(bx+a)\right)a-\cos\left(\log(bx+a)\right)bx+\sin\left(\log(bx+a)\right)a+\sin\left(\log(bx+a)\right)bx}{2b}$$

input
```
int(sin(log(b*x+a)),x)
```

output
```
( - cos(log(a + b*x))*a - cos(log(a + b*x))*b*x + sin(log(a + b*x))*a + si
n(log(a + b*x))*b*x)/(2*b)
```

3.26 $\int x^m \sin\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 133

$$\int x^m \sin\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)\,dx = -\frac{e^{\frac{a(1+m)}{\sqrt{-\frac{(1+m)^2}{n^2}}\,n}}x^{1+m}(cx^n)^{\frac{1+m}{n}}}{4\sqrt{-\frac{(1+m)^2}{n^2}}\,n}+\frac{e^{\frac{a\sqrt{-\frac{(1+m)^2}{n^2}}\,n}{1+m}}(1+m)x^{1+m}(cx^n)^{-\frac{1+m}{n}}\log(x)}{2\sqrt{-\frac{(1+m)^2}{n^2}}\,n}$$

output

```
-1/4*exp(a*(1+m)/(-(1+m)^2/n^2)^(1/2)/n)*x^(1+m)*(c*x^n)^((1+m)/n)/(-(1+m)
^2/n^2)^(1/2)/n+1/2*exp(a*(-(1+m)^2/n^2)^(1/2)*n/(1+m))*(1+m)*x^(1+m)*ln(x
)/(-(1+m)^2/n^2)^(1/2)/n/((c*x^n)^((1+m)/n))
```

Mathematica [F]

$$\int x^m \sin\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)dx = \int x^m \sin\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)dx$$

input
```
Integrate[x^m*Sin[a + Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n]],x]
```

output
```
Integrate[x^m*Sin[a + Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n]], x]
```

Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {4996, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sin\left(a+\sqrt{-\frac{(m+1)^2}{n^2}}\log(cx^n)\right)dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\sin\left(a+\sqrt{-\frac{(m+1)^2}{n^2}}\log(cx^n)\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{4992}$$

$$\frac{(m+1)x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int \left(\frac{e^{\frac{a\sqrt{-\frac{(m+1)^2}{n^2}}n}{m+1}}x^{-n}}{c}-e^{\frac{a(m+1)}{\sqrt{-\frac{(m+1)^2}{n^2}}n}}(cx^n)^{\frac{2(m+1)}{n}-1}\right)d(cx^n)}{2n^2\sqrt{-\frac{(m+1)^2}{n^2}}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(m+1)x^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(e^{\frac{an\sqrt{-\frac{(m+1)^2}{n^2}}}{m+1}}\log(cx^n)-\frac{ne^{\frac{a(m+1)}{n\sqrt{-\frac{(m+1)^2}{n^2}}}}(cx^n)^{\frac{2(m+1)}{n}}}{2(m+1)}\right)}{2n^2\sqrt{-\frac{(m+1)^2}{n^2}}}$$

input
```
Int[x^m*Sin[a + Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n]],x]
```

output
```
((1 + m)*x^(1 + m)*(-1/2*(E^((a*(1 + m))/(Sqrt[-((1 + m)^2/n^2)]*n))*n*(c*
x^n)^((2*(1 + m))/n))/(1 + m) + E^((a*Sqrt[-((1 + m)^2/n^2)]*n)/(1 + m))*L
og[c*x^n]))/(2*Sqrt[-((1 + m)^2/n^2)]*n^2*(c*x^n)^((1 + m)/n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^m \sin\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\,\ln\left(c\,x^n\right)\right)dx$$

input
```
int(x^m*sin(a+(-(1+m)^2/n^2)^(1/2)*ln(c*x^n)),x)
```

output
```
int(x^m*sin(a+(-(1+m)^2/n^2)^(1/2)*ln(c*x^n)),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.47

$$\int x^m \sin\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)dx = \frac{\left(i\,x^2x^{2\,m}-2\,(i\,m+i)e^{\left(\frac{2\,(i\,an-(m+1)\log(c))}{n}\right)}\log\left(x\right)\right)e^{\left(-\frac{i\,an-(m+1)\log(c)}{n}\right)}}{4\,(m+1)}$$

input
```
integrate(x^m*sin(a+(-(1+m)^2/n^2)^(1/2)*log(c*x^n)),x, algorithm="fricas"
)
```

output
```
1/4*(I*x^2*x^(2*m) - 2*(I*m + I)*e^(2*(I*a*n - (m + 1)*log(c))/n)*log(x))*
e^(-(I*a*n - (m + 1)*log(c))/n)/(m + 1)
```

Sympy [F]

$$\int x^m \sin\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)dx$$
$$=\int x^m \sin\left(a+\sqrt{-\frac{m^2}{n^2}-\frac{2m}{n^2}-\frac{1}{n^2}}\log(cx^n)\right)dx$$

input
```
integrate(x**m*sin(a+(-(1+m)**2/n**2)**(1/2)*ln(c*x**n)),x)
```

output
```
Integral(x**m*sin(a + sqrt(-m**2/n**2 - 2*m/n**2 - 1/n**2)*log(c*x**n)), x
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.62

$$\int x^m \sin\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)dx$$
$$=\frac{c^{\frac{2m}{n}+\frac{2}{n}}xe^{\left(m\log(x)+\frac{m\log(x^n)}{n}+\frac{\log(x^n)}{n}\right)}\sin(a)+2(m\sin(a)+\sin(a))\log(x)}{4\left(c^{\frac{m}{n}+\frac{1}{n}}m+c^{\frac{m}{n}+\frac{1}{n}}\right)}$$

input
```
integrate(x^m*sin(a+(-(1+m)^2/n^2)^(1/2)*log(c*x^n)),x, algorithm="maxima"
)
```

output
```
1/4*(c^(2*m/n + 2/n)*x*e^(m*log(x) + m*log(x^n)/n + log(x^n)/n)*sin(a) + 2
*(m*sin(a) + sin(a))*log(x))/(c^(m/n + 1/n)*m + c^(m/n + 1/n))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.05

$$\int x^m \sin\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)dx$$
$$= \frac{-i\,mn^2xx^me^{\left(i\,a-\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+i\,mn^2xx^me^{\left(-i\,a+\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}-i\,n^2xx^me^{\left(i\,a-\frac{n|mn+n|}{}\right.}}{}$$

input
```
integrate(x^m*sin(a+(-(1+m)^2/n^2)^(1/2)*log(c*x^n)),x, algorithm="giac")
```

output
```
1/2*(-I*m*n^2*x*x^m*e^(I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))
/n^2) + I*m*n^2*x*x^m*e^(-I*a + (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(
c))/n^2) - I*n^2*x*x^m*e^(I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(
c))/n^2) - I*n*x*x^m*abs(m*n + n)*e^(I*a - (n*abs(m*n + n)*log(x) + abs(m*
n + n)*log(c))/n^2) + I*n^2*x*x^m*e^(-I*a + (n*abs(m*n + n)*log(x) + abs(m
*n + n)*log(c))/n^2) - I*n*x*x^m*abs(m*n + n)*e^(-I*a + (n*abs(m*n + n)*lo
g(x) + abs(m*n + n)*log(c))/n^2))/(m^2*n^2 + 2*m*n^2 - (m*n + n)^2 + n^2)
```

Mupad [B] (verification not implemented)

Time = 20.91 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02

$$\int x^m \sin\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)dx$$
$$= \frac{x\,x^m\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{(c\,x^n)^{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}}\,1\mathrm{i}}{2\,m+2-n\,\sqrt{-\frac{(m+1)^2}{n^2}}\,2\mathrm{i}}-\frac{x\,x^m\,\mathrm{e}^{a\,1\mathrm{i}}\,(c\,x^n)^{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}\,1\mathrm{i}}{2\,m+2+n\,\sqrt{-\frac{(m+1)^2}{n^2}}\,2\mathrm{i}}$$

input
```
int(x^m*sin(a + log(c*x^n)*(-(m + 1)^2/n^2)^(1/2)),x)
```

output
```
(x*x^m*exp(-a*1i)/(c*x^n)^((- (2*m)/n^2 - 1/n^2 - m^2/n^2)^(1/2)*1i)*1i)/(
2*m - n*(-(m + 1)^2/n^2)^(1/2)*2i + 2) - (x*x^m*exp(a*1i)*(c*x^n)^((- (2*m
)/n^2 - 1/n^2 - m^2/n^2)^(1/2)*1i)*1i)/(2*m + n*(-(m + 1)^2/n^2)^(1/2)*2i
+ 2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.46

$$\int x^m \sin\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx = \frac{x^m x\left(-\cos\left(\frac{\log(x^n c)m+\log(x^n c)+an}{n}\right)+\sin\left(\frac{\log(x^n c)m+\log(x^n c)+an}{n}\right)\right)}{2m+2}$$

input
```
int(x^m*sin(a+(-(1+m)^2/n^2)^(1/2)*log(c*x^n)),x)
```

output
```
(x**m*x*( - cos((log(x**n*c)*m + log(x**n*c) + a*n)/n) + sin((log(x**n*c)*
m + log(x**n*c) + a*n)/n)))/(2*(m + 1))
```

3.27 $\int x^2 \sin\left(a+3\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int x^2 \sin\left(a+3\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx = \frac{1}{12}e^{-a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx^3(cx^n)^{3/n} - \frac{1}{2}e^{a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx^3(cx^n)^{-3/n}\log(x)$$

output
```
1/12*(-1/n^2)^(1/2)*n*x^3*(c*x^n)^(3/n)/exp(a*(-1/n^2)^(1/2)*n)-1/2*exp(a*
(-1/n^2)^(1/2)*n)*(-1/n^2)^(1/2)*n*x^3*ln(x)/((c*x^n)^(3/n))
```

Mathematica [F]

$$\int x^2 \sin\left(a+3\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx = \int x^2 \sin\left(a+3\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$

input
```
Integrate[x^2*Sin[a + 3*Sqrt[-n^(-2)]*Log[c*x^n]],x]
```

output
```
Integrate[x^2*Sin[a + 3*Sqrt[-n^(-2)]*Log[c*x^n]], x]
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4996, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin \left(a+3 \sqrt{-\frac{1}{n^2}} \log \left(c x^n\right)\right) \, dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{x^3 \left(c x^n\right)^{-3/n} \int \left(c x^n\right)^{\frac{3}{n}-1} \sin \left(a+3 \sqrt{-\frac{1}{n^2}} \log \left(c x^n\right)\right) d\left(c x^n\right)}{n}$$

$$\Big\downarrow \text{4992}$$

$$-\frac{1}{2} \sqrt{-\frac{1}{n^2}} x^3 \left(c x^n\right)^{-3/n} \int \left(\frac{e^{a \sqrt{-\frac{1}{n^2}} n} x^{-n}}{c}-e^{-a \sqrt{-\frac{1}{n^2}} n} \left(c x^n\right)^{\frac{6}{n}-1}\right) d\left(c x^n\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2} \sqrt{-\frac{1}{n^2}} x^3 \left(c x^n\right)^{-3/n} \left(e^{a \sqrt{-\frac{1}{n^2}} n} \log \left(c x^n\right)-\frac{1}{6} n e^{-a \sqrt{-\frac{1}{n^2}} n} \left(c x^n\right)^{6/n}\right)$$

input
```
Int[x^2*Sin[a + 3*Sqrt[-n^(-2)]*Log[c*x^n]],x]
```

output
```
-1/2*(Sqrt[-n^(-2)]*x^3*(-1/6*(n*(c*x^n)^(6/n))/E^(a*Sqrt[-n^(-2)]*n) + E^
(a*Sqrt[-n^(-2)]*n)*Log[c*x^n]))/(c*x^n)^(3/n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(77) = 154$.

Time = 1.56 (sec) , antiderivative size = 619, normalized size of antiderivative = 7.03

method	result
parts	$\frac{3n\,x^{2}\sqrt{-\frac{1}{n^{2}}}\,\mathrm{e}^{\frac{\ln\left(c\,x^{n}\right)}{n}-\frac{\ln\left(c\right)}{n}}\cos\left(a+3\sqrt{-\frac{1}{n^{2}}}\,\ln\left(c\,x^{n}\right)\right)}{8}-\frac{x^{2}\mathrm{e}^{\frac{\ln\left(c\,x^{n}\right)}{n}-\frac{\ln\left(c\right)}{n}}\sin\left(a+3\sqrt{-\frac{1}{n^{2}}}\,\ln\left(c\,x^{n}\right)\right)}{8}-\frac{n\left(-\frac{\sqrt{-\frac{1}{n^{2}}}\,n\,c^{-\frac{1}{n}}\cdots}{\cdots}\right.}{\cdots}$

input
```
int(x^2*sin(a+3*(-1/n^2)^(1/2)*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

output
```
3/8*n*x^2*(-1/n^2)^(1/2)*exp(1/n*ln(c*x^n)-1/n*ln(c))*cos(a+3*(-1/n^2)^(1/
2)*ln(c*x^n))-1/8*x^2*exp(1/n*ln(c*x^n)-1/n*ln(c))*sin(a+3*(-1/n^2)^(1/2)*
ln(c*x^n))-1/4/n*(-n*(-1/2*(-1/n^2)^(1/2)*n/(c^(1/n))*exp(1/n*(ln(c*x^n)-n
*ln(x)))*x^3*ln(x)+1/(c^(1/n))*exp(1/n*(ln(c*x^n)-n*ln(x)))*x^3*ln(x)*tan(
1/2*a+3/2*(-1/n^2)^(1/2)*ln(c*x^n))+1/6*(-1/n^2)^(1/2)*n/(c^(1/n))*exp(1/n
*(ln(c*x^n)-n*ln(x)))*x^3-1/6*(-1/n^2)^(1/2)*n*exp(1/n*(ln(c*x^n)-n*ln(x))
)/(c^(1/n))*x^3*tan(1/2*a+3/2*(-1/n^2)^(1/2)*ln(c*x^n))^2+1/2*(-1/n^2)^(1/
2)*n/(c^(1/n))*exp(1/n*(ln(c*x^n)-n*ln(x)))*x^3*ln(x)*tan(1/2*a+3/2*(-1/n^
2)^(1/2)*ln(c*x^n))^2)/(1+tan(1/2*a+3/2*(-1/n^2)^(1/2)*ln(c*x^n))^2)+3*(-1
/n^2)^(1/2)*n^2*(1/2/(c^(1/n))*exp(1/n*(ln(c*x^n)-n*ln(x)))*x^3*ln(x)+1/3/
(-1/n^2)^(1/2)/(c^(1/n))/n*exp(1/n*(ln(c*x^n)-n*ln(x)))*x^3*tan(1/2*a+3/2*
(-1/n^2)^(1/2)*ln(c*x^n))-1/2/(c^(1/n))*exp(1/n*(ln(c*x^n)-n*ln(x)))*x^3*l
n(x)*tan(1/2*a+3/2*(-1/n^2)^(1/2)*ln(c*x^n))^2-1/(-1/n^2)^(1/2)/(c^(1/n))/
n*exp(1/n*(ln(c*x^n)-n*ln(x)))*x^3*ln(x)*tan(1/2*a+3/2*(-1/n^2)^(1/2)*ln(c
*x^n)))/(1+tan(1/2*a+3/2*(-1/n^2)^(1/2)*ln(c*x^n))^2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.48

$$\int x^2 \sin\left(a+3\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx = \frac{1}{12}\left(i\,x^6-6i\,e^{\left(\frac{2\,(i\,an-3\,\log(c))}{n}\right)}\log\left(x\right)\right)e^{\left(-\frac{i\,an-3\,\log(c)}{n}\right)}$$

input
```
integrate(x^2*sin(a+3*(-1/n^2)^(1/2)*log(c*x^n)),x, algorithm="fricas")
```

output
```
1/12*(I*x^6 - 6*I*e^(2*(I*a*n - 3*log(c))/n)*log(x))*e^(-(I*a*n - 3*log(c)
)/n)
```

Sympy [F]

$$\int x^2\sin\left(a+3\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)dx=\int x^2\sin\left(a+3\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)dx$$

input
```
integrate(x**2*sin(a+3*(-1/n**2)**(1/2)*ln(c*x**n)),x)
```

output
```
Integral(x**2*sin(a + 3*sqrt(-1/n**2)*log(c*x**n)), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.35

$$\int x^2\sin\left(a+3\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)dx=\frac{c^{\frac{6}{n}}x^6\sin(a)+6\log(x)\sin(a)}{12\,c^{\frac{3}{n}}}$$

input
```
integrate(x^2*sin(a+3*(-1/n^2)^(1/2)*log(c*x^n)),x, algorithm="maxima")
```

output
```
1/12*(c^(6/n)*x^6*sin(a) + 6*log(x)*sin(a))/c^(3/n)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int x^2\sin\left(a+3\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)dx=+\infty$$

input
```
integrate(x^2*sin(a+3*(-1/n^2)^(1/2)*log(c*x^n)),x, algorithm="giac")
```

output
```
+Infinity
```

Mupad [B] (verification not implemented)

Time = 20.53 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int x^2 \sin\left(a+3\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=-\frac{x^3\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,3\mathrm{i}}}}{6\,n\,\sqrt{-\frac{1}{n^2}}+6\mathrm{i}}-\frac{x^3\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,3\mathrm{i}}}{6\,n\,\sqrt{-\frac{1}{n^2}}-6\mathrm{i}}$$

input
```
int(x^2*sin(a + 3*log(c*x^n)*(-1/n^2)^(1/2)),x)
```

output
```
- (x^3*exp(-a*1i)/(c*x^n)^((-1/n^2)^(1/2)*3i))/(6*n*(-1/n^2)^(1/2) + 6i) -
 (x^3*exp(a*1i)*(c*x^n)^((-1/n^2)^(1/2)*3i))/(6*n*(-1/n^2)^(1/2) - 6i)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int x^2 \sin\left(a+3\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx$$
$$=\frac{x^3\left(-3\cos\left(\frac{3\log(x^n c)i+an}{n}\right)\log(x^n c)\,i+\cos\left(\frac{3\log(x^n c)i+an}{n}\right)in+3\log(x^n c)\sin\left(\frac{3\log(x^n c)i+an}{n}\right)\right)}{6n}$$

input
```
int(x^2*sin(a+3*(-1/n^2)^(1/2)*log(c*x^n)),x)
```

output
```
(x**3*( - 3*cos((3*log(x**n*c)*i + a*n)/n)*log(x**n*c)*i + cos((3*log(x**n
*c)*i + a*n)/n)*i*n + 3*log(x**n*c)*sin((3*log(x**n*c)*i + a*n)/n)))/(6*n)
```

3.28 $\int x \sin \left(a+2\sqrt{-\frac{1}{n^2}} \log \left(c x^n\right)\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 88

$$\int x \sin \left(a+2\sqrt{-\frac{1}{n^2}} \log \left(c x^n\right)\right) \, dx = \frac{1}{8} e^{-a\sqrt{-\frac{1}{n^2}}n} \sqrt{-\frac{1}{n^2}} n x^2 (c x^n)^{2/n} - \frac{1}{2} e^{a\sqrt{-\frac{1}{n^2}}n} \sqrt{-\frac{1}{n^2}} n x^2 (c x^n)^{-2/n} \log(x)$$

output
```
1/8*(-1/n^2)^(1/2)*n*x^2*(c*x^n)^(2/n)/exp(a*(-1/n^2)^(1/2)*n)-1/2*exp(a*(
-1/n^2)^(1/2)*n)*(-1/n^2)^(1/2)*n*x^2*ln(x)/((c*x^n)^(2/n))
```

Mathematica [F]

$$\int x \sin \left(a+2\sqrt{-\frac{1}{n^2}} \log \left(c x^n\right)\right) \, dx = \int x \sin \left(a+2\sqrt{-\frac{1}{n^2}} \log \left(c x^n\right)\right) \, dx$$

input
```
Integrate[x*Sin[a + 2*Sqrt[-n^(-2)]*Log[c*x^n]],x]
```

output
```
Integrate[x*Sin[a + 2*Sqrt[-n^(-2)]*Log[c*x^n]], x]
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4996, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin \left(a+2\sqrt{-\frac{1}{n^2}}\log \left(cx^n\right)\right) dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{x^2(cx^n)^{-2/n} \int (cx^n)^{\frac{2}{n}-1} \sin \left(a+2\sqrt{-\frac{1}{n^2}}\log \left(cx^n\right)\right) d(cx^n)}{n}$$

$$\Big\downarrow \text{4992}$$

$$-\frac{1}{2}\sqrt{-\frac{1}{n^2}}x^2(cx^n)^{-2/n} \int \left(\frac{e^{a\sqrt{-\frac{1}{n^2}}n}x^{-n}}{c}-e^{-a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{4}{n}-1}\right) d(cx^n)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}\sqrt{-\frac{1}{n^2}}x^2(cx^n)^{-2/n} \left(e^{a\sqrt{-\frac{1}{n^2}}n}\log \left(cx^n\right)-\frac{1}{4}ne^{-a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{4/n}\right)$$

input
```
Int[x*Sin[a + 2*Sqrt[-n^(-2)]*Log[c*x^n]],x]
```

output
```
-1/2*(Sqrt[-n^(-2)]*x^2*(-1/4*(n*(c*x^n)^(4/n))/E^(a*Sqrt[-n^(-2)]*n) + E^
(a*Sqrt[-n^(-2)]*n)*Log[c*x^n]))/(c*x^n)^(2/n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(77) = 154$.

Time = 1.05 (sec) , antiderivative size = 610, normalized size of antiderivative = 6.93

method	result
parts	$\frac{2nx\sqrt{-\frac{1}{n^2}}\,\mathrm{e}^{\frac{\ln(c\,x^n)}{n}-\frac{\ln(c)}{n}}\cos\left(a+2\sqrt{-\frac{1}{n^2}}\,\ln(c\,x^n)\right)}{3}-\frac{x\,\mathrm{e}^{\frac{\ln(c\,x^n)}{n}-\frac{\ln(c)}{n}}\sin\left(a+2\sqrt{-\frac{1}{n^2}}\,\ln(c\,x^n)\right)}{3}-\frac{n\left(-\frac{c^{-\frac{1}{n}}\mathrm{e}^{\frac{\ln(c\,x^n)-\dots}{n}}}{4\sqrt{-\frac{1}{n^2}}\dots}\dots\right.}{\dots}$

input
```
int(x*sin(a+2*(-1/n^2)^(1/2)*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

output
```
2/3*n*x*(-1/n^2)^(1/2)*exp(1/n*ln(c*x^n)-1/n*ln(c))*cos(a+2*(-1/n^2)^(1/2)
*ln(c*x^n))-1/3*x*exp(1/n*ln(c*x^n)-1/n*ln(c))*sin(a+2*(-1/n^2)^(1/2)*ln(c
*x^n))-1/3/n*(-n*(-1/4/(-1/n^2)^(1/2)/(c^(1/n))/n*exp(1/n*(ln(c*x^n)-n*ln(
x)))*x^2+1/2/(-1/n^2)^(1/2)/(c^(1/n))/n*exp(1/n*(ln(c*x^n)-n*ln(x)))*x^2*l
n(x)+1/4/(-1/n^2)^(1/2)/(c^(1/n))/n*exp(1/n*(ln(c*x^n)-n*ln(x)))*x^2*tan(1
/2*a+(-1/n^2)^(1/2)*ln(c*x^n))^2+1/(c^(1/n))*exp(1/n*(ln(c*x^n)-n*ln(x)))*
x^2*ln(x)*tan(1/2*a+(-1/n^2)^(1/2)*ln(c*x^n))-1/2/(-1/n^2)^(1/2)/(c^(1/n))
/n*exp(1/n*(ln(c*x^n)-n*ln(x)))*x^2*ln(x)*tan(1/2*a+(-1/n^2)^(1/2)*ln(c*x^
n))^2)/(1+tan(1/2*a+(-1/n^2)^(1/2)*ln(c*x^n))^2)+2*(-1/n^2)^(1/2)*n^2*(-1/
2/(c^(1/n))*exp(1/n*(ln(c*x^n)-n*ln(x)))*x^2*ln(x)*tan(1/2*a+(-1/n^2)^(1/2
)*ln(c*x^n))^2+1/2/(c^(1/n))*exp(1/n*(ln(c*x^n)-n*ln(x)))*x^2*ln(x)-1/2*(-
1/n^2)^(1/2)*n*exp(1/n*(ln(c*x^n)-n*ln(x)))/(c^(1/n))*x^2*tan(1/2*a+(-1/n^
2)^(1/2)*ln(c*x^n))+1/(c^(1/n))*exp(1/n*(ln(c*x^n)-n*ln(x)))*(-1/n^2)^(1/2
)*n*x^2*ln(x)*tan(1/2*a+(-1/n^2)^(1/2)*ln(c*x^n)))/(1+tan(1/2*a+(-1/n^2)^(
1/2)*ln(c*x^n))^2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.48

$$\int x\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\frac{1}{8}\left(i\,x^4-4i\,e^{\left(\frac{2\,(i\,an-2\,\log(c))}{n}\right)}\log\left(x\right)\right)e^{\left(-\frac{i\,an-2\,\log(c)}{n}\right)}$$

input
```
integrate(x*sin(a+2*(-1/n^2)^(1/2)*log(c*x^n)),x, algorithm="fricas")
```

output
```
1/8*(I*x^4 - 4*I*e^(2*(I*a*n - 2*log(c))/n)*log(x))*e^(-(I*a*n - 2*log(c))
/n)
```

Sympy [F]

$$\int x\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\int x\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$

input
```
integrate(x*sin(a+2*(-1/n**2)**(1/2)*ln(c*x**n)),x)
```

output
```
Integral(x*sin(a + 2*sqrt(-1/n**2)*log(c*x**n)), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.35

$$\int x\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\frac{c^{\frac{4}{n}}x^4\sin\left(a\right)+4\,\log\left(x\right)\sin\left(a\right)}{8\,c^{\frac{2}{n}}}$$

input
```
integrate(x*sin(a+2*(-1/n^2)^(1/2)*log(c*x^n)),x, algorithm="maxima")
```

output
```
1/8*(c^(4/n)*x^4*sin(a) + 4*log(x)*sin(a))/c^(2/n)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int x\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=+\infty$$

input
```
integrate(x*sin(a+2*(-1/n^2)^(1/2)*log(c*x^n)),x, algorithm="giac")
```

output
```
+Infinity
```

Mupad [B] (verification not implemented)

Time = 20.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int x\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=-\frac{x^2\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{\left(c\,x^n\right)^{\sqrt{-\frac{1}{n^2}}\,2\mathrm{i}}}}{4\,n\,\sqrt{-\frac{1}{n^2}}+4\mathrm{i}}-\frac{x^2\,\mathrm{e}^{a\,1\mathrm{i}}\,\left(c\,x^n\right)^{\sqrt{-\frac{1}{n^2}}\,2\mathrm{i}}}{4\,n\,\sqrt{-\frac{1}{n^2}}-4\mathrm{i}}$$

input
```
int(x*sin(a + 2*log(c*x^n)*(-1/n^2)^(1/2)),x)
```

output
```
- (x^2*exp(-a*1i)/(c*x^n)^((-1/n^2)^(1/2)*2i))/(4*n*(-1/n^2)^(1/2) + 4i) -
 (x^2*exp(a*1i)*(c*x^n)^((-1/n^2)^(1/2)*2i))/(4*n*(-1/n^2)^(1/2) - 4i)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int x\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\frac{x^2\left(-2\cos\left(\frac{2\log(x^nc)i+an}{n}\right)\log(x^nc)\,i+\cos\left(\frac{2\log(x^nc)i+an}{n}\right)in+2\log(x^nc)\sin\left(\frac{2\log(x^nc)i+an}{n}\right)\right)}{4n}$$

input
```
int(x*sin(a+2*(-1/n^2)^(1/2)*log(c*x^n)),x)
```

output
```
(x**2*( - 2*cos((2*log(x**n*c)*i + a*n)/n)*log(x**n*c)*i + cos((2*log(x**n
*c)*i + a*n)/n)*i*n + 2*log(x**n*c)*sin((2*log(x**n*c)*i + a*n)/n)))/(4*n)
```

3.29 $\int \sin \left(a+\sqrt{-\frac{1}{n^2}} \log \left(c x^n\right)\right) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \sin \left(a+\sqrt{-\frac{1}{n^2}} \log \left(c x^n\right)\right) \, dx=\frac{1}{4} e^{-a \sqrt{-\frac{1}{n^2}} n} \sqrt{-\frac{1}{n^2}} n x \left(c x^n\right)^{\frac{1}{n}}-\frac{1}{2} e^{a \sqrt{-\frac{1}{n^2}} n} \sqrt{-\frac{1}{n^2}} n x \left(c x^n\right)^{-1/n} \log (x)$$

output
```
1/4*(-1/n^2)^(1/2)*n*x*(c*x^n)^(1/n)/exp(a*(-1/n^2)^(1/2)*n)-1/2*exp(a*(-1
/n^2)^(1/2)*n)*(-1/n^2)^(1/2)*n*x*ln(x)/((c*x^n)^(1/n))
```

Mathematica [F]

$$\int \sin \left(a+\sqrt{-\frac{1}{n^2}} \log \left(c x^n\right)\right) \, dx=\int \sin \left(a+\sqrt{-\frac{1}{n^2}} \log \left(c x^n\right)\right) \, dx$$

input
```
Integrate[Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]],x]
```

output
```
Integrate[Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]], x]
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4986, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx$$

$$\Big\downarrow \text{4986}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{4992}$$

$$-\frac{1}{2}\sqrt{-\frac{1}{n^2}}x(cx^n)^{-1/n}\int\left(\frac{e^{a\sqrt{-\frac{1}{n^2}}n}x^{-n}}{c}-e^{-a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{2}{n}-1}\right)d(cx^n)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}\sqrt{-\frac{1}{n^2}}x(cx^n)^{-1/n}\left(e^{a\sqrt{-\frac{1}{n^2}}n}\log\left(cx^n\right)-\frac{1}{2}ne^{-a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{2/n}\right)$$

input
```
Int[Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]],x]
```

output
```
-1/2*(Sqrt[-n^(-2)]*x*(-1/2*(n*(c*x^n)^(2/n))/E^(a*Sqrt[-n^(-2)]*n) + E^(a
*Sqrt[-n^(-2)]*n)*Log[c*x^n]))/(c*x^n)^n^(-1)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4986
```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

Maple [F]

$$\int \sin\left(a+\sqrt{-\frac{1}{n^2}}\,\ln\left(c\,x^n\right)\right)dx$$

input
```
int(sin(a+(-1/n^2)^(1/2)*ln(c*x^n)),x)
```

output
```
int(sin(a+(-1/n^2)^(1/2)*ln(c*x^n)),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.51

$$\int \sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\frac{1}{4}\left(i\,x^2-2i\,e^{\left(\frac{2\,(i\,an-\log(c))}{n}\right)}\log\left(x\right)\right)e^{\left(-\frac{i\,an-\log(c)}{n}\right)}$$

input
```
integrate(sin(a+(-1/n^2)^(1/2)*log(c*x^n)),x, algorithm="fricas")
```

output
```
1/4*(I*x^2 - 2*I*e^(2*(I*a*n - log(c))/n)*log(x))*e^(-(I*a*n - log(c))/n)
```

Sympy [F]

$$\int \sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\int \sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$

input
```
integrate(sin(a+(-1/n**2)**(1/2)*ln(c*x**n)),x)
```

output
```
Integral(sin(a + sqrt(-1/n**2)*log(c*x**n)), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.35

$$\int \sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\frac{c^{\frac{2}{n}}x^2\sin\left(a\right)+2\,\log\left(x\right)\sin\left(a\right)}{4\,c^{\left(\frac{1}{n}\right)}}$$

input
```
integrate(sin(a+(-1/n^2)^(1/2)*log(c*x^n)),x, algorithm="maxima")
```

output
```
1/4*(c^(2/n)*x^2*sin(a) + 2*log(x)*sin(a))/c^(1/n)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int \sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=+\infty$$

input
```
integrate(sin(a+(-1/n^2)^(1/2)*log(c*x^n)),x, algorithm="giac")
```

output
```
+Infinity
```

Mupad [B] (verification not implemented)

Time = 19.58 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99

$$\int \sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx = -\frac{x\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}}}{2\,n\,\sqrt{-\frac{1}{n^2}}+2\mathrm{i}}-\frac{x\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}}{2\,n\,\sqrt{-\frac{1}{n^2}}-2\mathrm{i}}$$

input
```
int(sin(a + log(c*x^n)*(-1/n^2)^(1/2)),x)
```

output
```
- (x*exp(-a*1i)/(c*x^n)^((-1/n^2)^(1/2)*1i))/(2*n*(-1/n^2)^(1/2) + 2i) - (
x*exp(a*1i)*(c*x^n)^((-1/n^2)^(1/2)*1i))/(2*n*(-1/n^2)^(1/2) - 2i)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94

$$\int \sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx = \frac{x\left(-\cos\left(\frac{\log(x^nc)i+an}{n}\right)\log(x^nc)\,i+\cos\left(\frac{\log(x^nc)i+an}{n}\right)in+\log(x^nc)\sin\left(\frac{\log(x^nc)i+an}{n}\right)\right)}{2n}$$

input
```
int(sin(a+(-1/n^2)^(1/2)*log(c*x^n)),x)
```

output
```
(x*( - cos((log(x**n*c)*i + a*n)/n)*log(x**n*c)*i + cos((log(x**n*c)*i + a
*n)/n)*i*n + log(x**n*c)*sin((log(x**n*c)*i + a*n)/n)))/(2*n)
```

3.30 $\int \frac{\sin(a)}{x} \, dx$

Optimal result

Integrand size = 6, antiderivative size = 5

$$\int \frac{\sin(a)}{x} \, dx = \log(x)\sin(a)$$

output
```
ln(x)*sin(a)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a)}{x} \, dx = \log(x)\sin(a)$$

input
```
Integrate[Sin[a]/x,x]
```

output
```
Log[x]*Sin[a]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a)}{x}\,dx$$

$$\Big\downarrow \; 14$$

$$\sin(a)\log(x)$$

input
```
Int[Sin[a]/x,x]
```

output
```
Log[x]*Sin[a]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
default	$\ln(x)\sin(a)$	6
norman	$\ln(x)\sin(a)$	6
risch	$\ln(x)\sin(a)$	6
parallelrisch	$\ln(x)\sin(a)$	6

input
```
int(sin(a)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*sin(a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a)}{x} \, dx = \log{(x)} \sin{(a)}$$

input
```
integrate(sin(a)/x,x, algorithm="fricas")
```

output
```
log(x)*sin(a)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a)}{x} \, dx = \log{(x)} \sin{(a)}$$

input
```
integrate(sin(a)/x,x)
```

output
```
log(x)*sin(a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a)}{x} \, dx = \log{(x)} \sin{(a)}$$

input
```
integrate(sin(a)/x,x, algorithm="maxima")
```

output
```
log(x)*sin(a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

$$\int \frac{\sin(a)}{x}\,dx = \log(|x|)\sin(a)$$

input
```
integrate(sin(a)/x,x, algorithm="giac")
```

output
```
log(abs(x))*sin(a)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a)}{x}\,dx = \sin(a)\ \ln(x)$$

input
```
int(sin(a)/x,x)
```

output
```
sin(a)*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a)}{x}\,dx = \log(x)\sin(a)$$

input
```
int(sin(a)/x,x)
```

output
```
log(x)*sin(a)
```

3.31 $\int \frac{\sin\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{\sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx = \frac{e^{a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}n(cx^n)^{-1/n}}{4x} + \frac{e^{-a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}n(cx^n)^{\frac{1}{n}}\log(x)}{2x}$$

output
```
1/4*exp(a*(-1/n^2)^(1/2)*n)*(-1/n^2)^(1/2)*n/x/((c*x^n)^(1/n))+1/2*(-1/n^2
)^(1/2)*n*(c*x^n)^(1/n)*ln(x)/exp(a*(-1/n^2)^(1/2)*n)/x
```

Mathematica [F]

$$\int \frac{\sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx = \int \frac{\sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx$$

input
```
Integrate[Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]/x^2,x]
```

output
```
Integrate[Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]/x^2, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4996, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}dx$$

$$\Big\downarrow \text{ 4996}$$

$$\frac{(cx^n)^{\frac{1}{n}}\int (cx^n)^{-1-\frac{1}{n}}\sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)d(cx^n)}{nx}$$

$$\Big\downarrow \text{ 4992}$$

$$\frac{\sqrt{-\frac{1}{n^2}}(cx^n)^{\frac{1}{n}}\int \left(\frac{e^{-a\sqrt{-\frac{1}{n^2}}n}x^{-n}}{c}-e^{a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{-\frac{n+2}{n}}\right)d(cx^n)}{2x}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{-\frac{1}{n^2}}(cx^n)^{\frac{1}{n}}\left(\frac{1}{2}ne^{a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{-2/n}+e^{-a\sqrt{-\frac{1}{n^2}}n}\log\left(cx^n\right)\right)}{2x}$$

input
```
Int[Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]/x^2,x]
```

output
```
(Sqrt[-n^(-2)]*(c*x^n)^n^(-1)*((E^(a*Sqrt[-n^(-2)]*n)*n)/(2*(c*x^n)^(2/n))
 + Log[c*x^n]/E^(a*Sqrt[-n^(-2)]*n)))/(2*x)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.79

method	result	size
parallelrisch	$\frac{n\sqrt{-\frac{1}{n^{2}}}\,(n+\ln(c\,x^{n}))\cos\left(a+\sqrt{-\frac{1}{n^{2}}}\,\ln(c\,x^{n})\right)+\ln(c\,x^{n})\sin\left(a+\sqrt{-\frac{1}{n^{2}}}\,\ln(c\,x^{n})\right)}{2xn}$	68

input
```
int(sin(a+(-1/n^2)^(1/2)*ln(c*x^n))/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(n*(-1/n^2)^(1/2)*(n+ln(c*x^n))*cos(a+(-1/n^2)^(1/2)*ln(c*x^n))+ln(c*x
^n)*sin(a+(-1/n^2)^(1/2)*ln(c*x^n)))/x/n
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.52

$$\int \frac{\sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx = \frac{\left(2i\,x^2\log\left(x\right)+i\,e^{\left(\frac{2\,(i\,an-\log(c))}{n}\right)}\right)e^{\left(-\frac{i\,an-\log(c)}{n}\right)}}{4\,x^2}$$

input
```
integrate(sin(a+(-1/n^2)^(1/2)*log(c*x^n))/x^2,x, algorithm="fricas")
```

output
```
1/4*(2*I*x^2*log(x) + I*e^(2*(I*a*n - log(c))/n))*e^(-(I*a*n - log(c))/n)/
x^2
```

Sympy [A] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.10

$$\int \frac{\sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx = \frac{\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\cos\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{2x} - \frac{\sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{2x} + \frac{\log\left(cx^n\right)\sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{2nx}$$

input
```
integrate(sin(a+(-1/n**2)**(1/2)*ln(c*x**n))/x**2,x)
```

output
```
sqrt(-1/n**2)*log(c*x**n)*cos(a + sqrt(-1/n**2)*log(c*x**n))/(2*x) - sin(a
 + sqrt(-1/n**2)*log(c*x**n))/(2*x) + log(c*x**n)*sin(a + sqrt(-1/n**2)*lo
g(c*x**n))/(2*n*x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.38

$$\int \frac{\sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx = \frac{2\,c^{\frac{2}{n}}x^2\log\left(x\right)\sin\left(a\right)-\sin\left(a\right)}{4\,c^{\left(\frac{1}{n}\right)}x^2}$$

input
```
integrate(sin(a+(-1/n^2)^(1/2)*log(c*x^n))/x^2,x, algorithm="maxima")
```

output
```
1/4*(2*c^(2/n)*x^2*log(x)*sin(a) - sin(a))/(c^(1/n)*x^2)
```

Giac [F]

$$\int \frac{\sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx = \int \frac{\sin\left(\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)+a\right)}{x^2}\,dx$$

input
```
integrate(sin(a+(-1/n^2)^(1/2)*log(c*x^n))/x^2,x, algorithm="giac")
```

output
```
integrate(sin(sqrt(-1/n^2)*log(c*x^n) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx = \int \frac{\sin\left(a+\ln\left(c\,x^n\right)\,\sqrt{-\frac{1}{n^2}}\right)}{x^2}\,dx$$

input
```
int(sin(a + log(c*x^n)*(-1/n^2)^(1/2))/x^2,x)
```

output
```
int(sin(a + log(c*x^n)*(-1/n^2)^(1/2))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int \frac{\sin\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx$$
$$= \frac{\cos\left(\frac{\log(x^n c)i+an}{n}\right)\log(x^n c)\,i+\cos\left(\frac{\log(x^n c)i+an}{n}\right)in+\log(x^n c)\sin\left(\frac{\log(x^n c)i+an}{n}\right)}{2nx}$$

input
```
int(sin(a+(-1/n^2)^(1/2)*log(c*x^n))/x^2,x)
```

output
```
(cos((log(x**n*c)*i + a*n)/n)*log(x**n*c)*i + cos((log(x**n*c)*i + a*n)/n)
*i*n + log(x**n*c)*sin((log(x**n*c)*i + a*n)/n))/(2*n*x)
```

3.32 $\int \frac{\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \frac{\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx = \frac{e^{a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}n(cx^n)^{-2/n}}{8x^2} + \frac{e^{-a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}n(cx^n)^{2/n}\log(x)}{2x^2}$$

output
```
1/8*exp(a*(-1/n^2)^(1/2)*n)*(-1/n^2)^(1/2)*n/x^2/((c*x^n)^(2/n))+1/2*(-1/n
^2)^(1/2)*n*(c*x^n)^(2/n)*ln(x)/exp(a*(-1/n^2)^(1/2)*n)/x^2
```

Mathematica [F]

$$\int \frac{\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx = \int \frac{\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx$$

input
```
Integrate[Sin[a + 2*Sqrt[-n^(-2)]*Log[c*x^n]]/x^3,x]
```

output
```
Integrate[Sin[a + 2*Sqrt[-n^(-2)]*Log[c*x^n]]/x^3, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4996, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin \left(a+2\sqrt{-\frac{1}{n^2}}\log \left(cx^n\right)\right)}{x^3}dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{\left(cx^n\right)^{2/n}\int \left(cx^n\right)^{-1-\frac{2}{n}}\sin \left(a+2\sqrt{-\frac{1}{n^2}}\log \left(cx^n\right)\right)d\left(cx^n\right)}{nx^2}$$

$$\Big\downarrow \text{4992}$$

$$\frac{\sqrt{-\frac{1}{n^2}}\left(cx^n\right)^{2/n}\int \left(\frac{e^{-a\sqrt{-\frac{1}{n^2}}n}x^{-n}}{c}-e^{a\sqrt{-\frac{1}{n^2}}n}\left(cx^n\right)^{-\frac{n+4}{n}}\right)d\left(cx^n\right)}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{-\frac{1}{n^2}}\left(cx^n\right)^{2/n}\left(\frac{1}{4}ne^{a\sqrt{-\frac{1}{n^2}}n}\left(cx^n\right)^{-4/n}+e^{-a\sqrt{-\frac{1}{n^2}}n}\log \left(cx^n\right)\right)}{2x^2}$$

input
```
Int[Sin[a + 2*Sqrt[-n^(-2)]*Log[c*x^n]]/x^3,x]
```

output
```
(Sqrt[-n^(-2)]*(c*x^n)^(2/n)*((E^(a*Sqrt[-n^(-2)]*n)*n)/(4*(c*x^n)^(4/n))
+ Log[c*x^n]/E^(a*Sqrt[-n^(-2)]*n)))/(2*x^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 2.79 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

method	result	size
parallelrisch	$\frac{n\sqrt{-\frac{1}{n^2}}\,(n+2\ln(c\,x^n))\cos\left(a+2\sqrt{-\frac{1}{n^2}}\,\ln(c\,x^n)\right)+2\ln(c\,x^n)\sin\left(a+2\sqrt{-\frac{1}{n^2}}\,\ln(c\,x^n)\right)}{4x^2n}$	73

input
```
int(sin(a+2*(-1/n^2)^(1/2)*ln(c*x^n))/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(n*(-1/n^2)^(1/2)*(n+2*ln(c*x^n))*cos(a+2*(-1/n^2)^(1/2)*ln(c*x^n))+2*
ln(c*x^n)*sin(a+2*(-1/n^2)^(1/2)*ln(c*x^n)))/x^2/n
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.51

$$\int \frac{\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx=\frac{\left(4i\,x^4\log\left(x\right)+i\,e^{\left(\frac{2\,(i\,an-2\,\log(c))}{n}\right)}\right)e^{\left(-\frac{i\,an-2\,\log(c)}{n}\right)}}{8\,x^4}$$

input
```
integrate(sin(a+2*(-1/n^2)^(1/2)*log(c*x^n))/x^3,x, algorithm="fricas")
```

output
```
1/8*(4*I*x^4*log(x) + I*e^(2*(I*a*n - 2*log(c))/n))*e^(-(I*a*n - 2*log(c))
/n)/x^4
```

Sympy [A] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.33

$$\int \frac{\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx=\frac{n\sqrt{-\frac{1}{n^2}}\cos\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{4x^2}+\frac{\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\cos\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{2x^2}+\frac{\log\left(cx^n\right)\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{2nx^2}$$

input
```
integrate(sin(a+2*(-1/n**2)**(1/2)*ln(c*x**n))/x**3,x)
```

output
```
n*sqrt(-1/n**2)*cos(a + 2*sqrt(-1/n**2)*log(c*x**n))/(4*x**2) + sqrt(-1/n*
*2)*log(c*x**n)*cos(a + 2*sqrt(-1/n**2)*log(c*x**n))/(2*x**2) + log(c*x**n
)*sin(a + 2*sqrt(-1/n**2)*log(c*x**n))/(2*n*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.40

$$\int \frac{\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx = \frac{4\,c^{\frac{4}{n}}x^4\log\left(x\right)\sin\left(a\right)-\sin\left(a\right)}{8\,c^{\frac{2}{n}}x^4}$$

input
```
integrate(sin(a+2*(-1/n^2)^(1/2)*log(c*x^n))/x^3,x, algorithm="maxima")
```

output
```
1/8*(4*c^(4/n)*x^4*log(x)*sin(a) - sin(a))/(c^(2/n)*x^4)
```

Giac [F]

$$\int \frac{\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx = \int \frac{\sin\left(2\,\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)+a\right)}{x^3}\,dx$$

input
```
integrate(sin(a+2*(-1/n^2)^(1/2)*log(c*x^n))/x^3,x, algorithm="giac")
```

output
```
integrate(sin(2*sqrt(-1/n^2)*log(c*x^n) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx = \int \frac{\sin\left(a+2\,\ln\left(c\,x^n\right)\,\sqrt{-\frac{1}{n^2}}\right)}{x^3}\,dx$$

input
```
int(sin(a + 2*log(c*x^n)*(-1/n^2)^(1/2))/x^3,x)
```

output
```
int(sin(a + 2*log(c*x^n)*(-1/n^2)^(1/2))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int \frac{\sin \left(a+2 \sqrt{-\frac{1}{n^2}} \log \left(c x^n\right)\right)}{x^3} \, dx = \frac{2 \cos \left(\frac{2 \log(x^n c) i+a n}{n}\right) \log(x^n c)\, i+\cos \left(\frac{2 \log(x^n c) i+a n}{n}\right) i n+2 \log(x^n c) \sin \left(\frac{2 \log(x^n c) i+a n}{n}\right)}{4 n\, x^2}$$

input
```
int(sin(a+2*(-1/n^2)^(1/2)*log(c*x^n))/x^3,x)
```

output
```
(2*cos((2*log(x**n*c)*i + a*n)/n)*log(x**n*c)*i + cos((2*log(x**n*c)*i + a
*n)/n)*i*n + 2*log(x**n*c)*sin((2*log(x**n*c)*i + a*n)/n))/(4*n*x**2)
```

3.33 $\int x^m \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 117

$$\int x^m \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)\,dx$$
$$=\frac{x^{1+m}}{2(1+m)}-\frac{e^{-\frac{2a\sqrt{-\frac{(1+m)^2}{n^2}}n}{1+m}}x^{1+m}(cx^n)^{\frac{1+m}{n}}}{8(1+m)}-\frac{1}{4}e^{\frac{2a\sqrt{-\frac{(1+m)^2}{n^2}}n}{1+m}}x^{1+m}(cx^n)^{-\frac{1+m}{n}}\log(x)$$

output
```
x^(1+m)/(2+2*m)-1/8*x^(1+m)*(c*x^n)^((1+m)/n)/exp(2*a*(-(1+m)^2/n^2)^(1/2)
*n/(1+m))/(1+m)-1/4*exp(2*a*(-(1+m)^2/n^2)^(1/2)*n/(1+m))*x^(1+m)*ln(x)/((
c*x^n)^((1+m)/n))
```

Mathematica [F]

$$\int x^m \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)\,dx$$
$$=\int x^m \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)\,dx$$

input
```
Integrate[x^m*Sin[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^2,x]
```

output
```
Integrate[x^m*Sin[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^2, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4996, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{4992}$$

$$-\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int \left(-2(cx^n)^{\frac{m+1}{n}-1}+e^{-\frac{2a\sqrt{-\frac{(m+1)^2}{n^2}}n}{m+1}}(cx^n)^{\frac{2(m+1)}{n}-1}+\frac{e^{\frac{2a\sqrt{-\frac{(m+1)^2}{n^2}}n}{m+1}}x^{-n}}{c}\right)d(cx^n)}{4n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(-\frac{ne^{-\frac{2an\sqrt{-\frac{(m+1)^2}{n^2}}}{m+1}}(cx^n)^{\frac{2(m+1)}{n}}}{2(m+1)}-e^{\frac{2an\sqrt{-\frac{(m+1)^2}{n^2}}}{m+1}}\log\left(cx^n\right)+\frac{2n(cx^n)^{\frac{m+1}{n}}}{m+1}\right)}{4n}$$

input
```
Int[x^m*Sin[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^2,x]
```

output
```
(x^(1 + m)*((2*n*(c*x^n)^((1 + m)/n))/(1 + m) - (n*(c*x^n)^((2*(1 + m))/n)
)/(2*E^((2*a*Sqrt[-((1 + m)^2/n^2)]*n)/(1 + m))*(1 + m)) - E^((2*a*Sqrt[-(
(1 + m)^2/n^2)]*n)/(1 + m))*Log[c*x^n]))/(4*n*(c*x^n)^((1 + m)/n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^{m} \sin\left(a +\frac{\sqrt{-\frac{(1+m)^{2}}{n^{2}}}\, \ln\left(c\,x^{n}\right)}{2}\right)^{2}d x$$

input
```
int(x^m*sin(a+1/2*(-(1+m)^2/n^2)^(1/2)*ln(c*x^n))^2,x)
```

output
```
int(x^m*sin(a+1/2*(-(1+m)^2/n^2)^(1/2)*ln(c*x^n))^2,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^m \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx =$$

$$-\frac{\left(2\,(m+1)e^{\left(-\frac{2\,((m+1)n\log(x)-2i\,an+(m+1)\log(c))}{n}\right)}\log\left(x\right)-4\,e^{\left(-\frac{(m+1)n\log(x)-2i\,an+(m+1)\log(c)}{n}\right)}+1\right)e^{\left(\frac{2\,((m+1)n\log(x)\cdots}{\cdots}\right)}}{8\,(m+1)}$$

input
```
integrate(x^m*sin(a+1/2*(-(1+m)^2/n^2)^(1/2)*log(c*x^n))^2,x, algorithm="f
ricas")
```

output
```
-1/8*(2*(m + 1)*e^(-2*((m + 1)*n*log(x) - 2*I*a*n + (m + 1)*log(c))/n)*log
(x) - 4*e^(-((m + 1)*n*log(x) - 2*I*a*n + (m + 1)*log(c))/n) + 1)*e^(2*((m
 + 1)*n*log(x) - 2*I*a*n + (m + 1)*log(c))/n + (2*I*a*n - (m + 1)*log(c))/
n)/(m + 1)
```

Sympy [F]

$$\int x^m \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\int x^m \sin^2\left(a+\frac{\sqrt{-\frac{m^2}{n^2}-\frac{2m}{n^2}-\frac{1}{n^2}}\log\left(cx^n\right)}{2}\right)\,dx$$

input
```
integrate(x**m*sin(a+1/2*(-(1+m)**2/n**2)**(1/2)*ln(c*x**n))**2,x)
```

output
```
Integral(x**m*sin(a + sqrt(-m**2/n**2 - 2*m/n**2 - 1/n**2)*log(c*x**n)/2)*
*2, x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.48

$$\int x^m \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\frac{4\left(\cos\left(2\,a\right)^2+\sin\left(2\,a\right)^2\right)c^{\frac{m}{n}+\frac{1}{n}}xx^m-c^{\frac{2\,m}{n}+\frac{2}{n}}x\cos\left(2\,a\right)e^{\left(m\log(x)+\frac{m\log(x^n)}{n}+\frac{\log(x^n)}{n}\right)}-2\left(\cos\left(2\,a\right)^3+\cos\left(2\,a\right.\right.}{8\left(\left(\cos\left(2\,a\right)^2+\sin\left(2\,a\right)^2\right)c^{\frac{m}{n}+\frac{1}{n}}m+\left(\cos\left(2\,a\right)^2+\sin\left(2\,a\right.\right.\right.}$$

input
```
integrate(x^m*sin(a+1/2*(-(1+m)^2/n^2)^(1/2)*log(c*x^n))^2,x, algorithm="m
axima")
```

output
```
1/8*(4*(cos(2*a)^2 + sin(2*a)^2)*c^(m/n + 1/n)*x*x^m - c^(2*m/n + 2/n)*x*c
os(2*a)*e^(m*log(x) + m*log(x^n)/n + log(x^n)/n) - 2*(cos(2*a)^3 + cos(2*a
)*sin(2*a)^2 + (cos(2*a)^3 + cos(2*a)*sin(2*a)^2)*m)*log(x))/((cos(2*a)^2
+ sin(2*a)^2)*c^(m/n + 1/n)*m + (cos(2*a)^2 + sin(2*a)^2)*c^(m/n + 1/n))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.08 (sec) , antiderivative size = 498, normalized size of antiderivative = 4.26

$$\int x^m \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx=$$
$$-\frac{m^2n^2xx^me^{\left(2i\,a-\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+m^2n^2xx^me^{\left(-2i\,a+\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}-2\,m^2n^2xx^m+2\,m}{}$$

input
```
integrate(x^m*sin(a+1/2*(-(1+m)^2/n^2)^(1/2)*log(c*x^n))^2,x, algorithm="g
iac")
```

output
```
-1/4*(m^2*n^2*x*x^m*e^(2*I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c
))/n^2) + m^2*n^2*x*x^m*e^(-2*I*a + (n*abs(m*n + n)*log(x) + abs(m*n + n)*
log(c))/n^2) - 2*m^2*n^2*x*x^m + 2*m*n^2*x*x^m*e^(2*I*a - (n*abs(m*n + n)*
log(x) + abs(m*n + n)*log(c))/n^2) + m*n*x*x^m*abs(m*n + n)*e^(2*I*a - (n*
abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 2*m*n^2*x*x^m*e^(-2*I*a
+ (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - m*n*x*x^m*abs(m*n +
 n)*e^(-2*I*a + (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 4*m*n
^2*x*x^m + n^2*x*x^m*e^(2*I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(
c))/n^2) + n*x*x^m*abs(m*n + n)*e^(2*I*a - (n*abs(m*n + n)*log(x) + abs(m*
n + n)*log(c))/n^2) + n^2*x*x^m*e^(-2*I*a + (n*abs(m*n + n)*log(x) + abs(m
*n + n)*log(c))/n^2) - n*x*x^m*abs(m*n + n)*e^(-2*I*a + (n*abs(m*n + n)*lo
g(x) + abs(m*n + n)*log(c))/n^2) + 2*(m*n + n)^2*x*x^m - 2*n^2*x*x^m)/(m^3
*n^2 + 3*m^2*n^2 - (m*n + n)^2*m + 3*m*n^2 - (m*n + n)^2 + n^2)
```

Mupad [B] (verification not implemented)

Time = 21.49 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.24

$$\int x^m \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\frac{x\,x^m}{2\,m+2}-\frac{x\,x^m\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}}}{4\,m+4-n\,\sqrt{-\frac{{(m+1)}^2}{n^2}}\,4\mathrm{i}}-\frac{x\,x^m\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}}{4\,m+4+n\,\sqrt{-\frac{{(m+1)}^2}{n^2}}\,4\mathrm{i}}$$

input
```
int(x^m*sin(a + (log(c*x^n)*(-(m + 1)^2/n^2)^(1/2))/2)^2,x)
```

output
```
(x*x^m)/(2*m + 2) - (x*x^m*exp(-a*2i)/(c*x^n)^((- (2*m)/n^2 - 1/n^2 - m^2/
n^2)^(1/2)*1i))/(4*m - n*(-(m + 1)^2/n^2)^(1/2)*4i + 4) - (x*x^m*exp(a*2i)
*(c*x^n)^((- (2*m)/n^2 - 1/n^2 - m^2/n^2)^(1/2)*1i))/(4*m + n*(-(m + 1)^2/
n^2)^(1/2)*4i + 4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81

$$\int x^m \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\frac{x^m x\left(-2\cos\left(\frac{\log(x^n c)m+\log(x^n c)+2an}{2n}\right)\sin\left(\frac{\log(x^n c)m+\log(x^n c)+2an}{2n}\right)+2\sin\left(\frac{\log(x^n c)m+\log(x^n c)+2an}{2n}\right)^2+1\right)}{4m+4}$$

input
```
int(x^m*sin(a+1/2*(-(1+m)^2/n^2)^(1/2)*log(c*x^n))^2,x)
```

output
```
(x**m*x*( - 2*cos((log(x**n*c)*m + log(x**n*c) + 2*a*n)/(2*n))*sin((log(x*
*n*c)*m + log(x**n*c) + 2*a*n)/(2*n)) + 2*sin((log(x**n*c)*m + log(x**n*c)
 + 2*a*n)/(2*n))**2 + 1))/(4*(m + 1))
```

3.34 $\int x^2 \sin^2\left(a+\frac{3}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 76

$$\int x^2 \sin^2\left(a+\frac{3}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx = \frac{x^3}{6}-\frac{1}{24}e^{-2a\sqrt{-\frac{1}{n^2}}n}x^3\left(cx^n\right)^{3/n} -\frac{1}{4}e^{2a\sqrt{-\frac{1}{n^2}}n}x^3\left(cx^n\right)^{-3/n}\log(x)$$

output
```
1/6*x^3-1/24*x^3*(c*x^n)^(3/n)/exp(2*a*(-1/n^2)^(1/2)*n)-1/4*exp(2*a*(-1/n
^2)^(1/2)*n)*x^3*ln(x)/((c*x^n)^(3/n))
```

Mathematica [F]

$$\int x^2 \sin^2\left(a+\frac{3}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx = \int x^2 \sin^2\left(a+\frac{3}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$

input
```
Integrate[x^2*Sin[a + (3*Sqrt[-n^(-2)]*Log[c*x^n])/2]^2,x]
```

output
```
Integrate[x^2*Sin[a + (3*Sqrt[-n^(-2)]*Log[c*x^n])/2]^2, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {4996, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin^2\left(a+\frac{3}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{x^3(cx^n)^{-3/n}\int (cx^n)^{\frac{3}{n}-1}\sin^2\left(a+\frac{3}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{4992}$$

$$-\frac{x^3(cx^n)^{-3/n}\int \left(-2(cx^n)^{\frac{3}{n}-1}+e^{-2a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{6}{n}-1}+\frac{e^{2a\sqrt{-\frac{1}{n^2}}n}x^{-n}}{c}\right)d(cx^n)}{4n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3(cx^n)^{-3/n}\left(-\frac{1}{6}ne^{-2a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{6/n}-e^{2a\sqrt{-\frac{1}{n^2}}n}\log\left(cx^n\right)+\frac{2}{3}n(cx^n)^{3/n}\right)}{4n}$$

input
```
Int[x^2*Sin[a + (3*Sqrt[-n^(-2)]*Log[c*x^n])/2]^2,x]
```

output
```
(x^3*((2*n*(c*x^n)^(3/n))/3 - (n*(c*x^n)^(6/n))/(6*E^(2*a*Sqrt[-n^(-2)]*n)
) - E^(2*a*Sqrt[-n^(-2)]*n)*Log[c*x^n]))/(4*n*(c*x^n)^(3/n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^2\sin\left(a+\frac{3\sqrt{-\frac{1}{n^2}}\,\ln\left(c\,x^n\right)}{2}\right)^2dx$$

input
```
int(x^2*sin(a+3/2*(-1/n^2)^(1/2)*ln(c*x^n))^2,x)
```

output
```
int(x^2*sin(a+3/2*(-1/n^2)^(1/2)*ln(c*x^n))^2,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.78

$$\int x^2\sin^2\left(a+\frac{3}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=-\frac{1}{24}\left(x^6-4\,x^3e^{\left(\frac{2i\,an-3\,\log(c)}{n}\right)}+6\,e^{\left(\frac{2\,(2i\,an-3\,\log(c))}{n}\right)}\log\left(x\right)\right)e^{\left(-\frac{2i\,an-3\,\log(c)}{n}\right)}$$

input
```
integrate(x^2*sin(a+3/2*(-1/n^2)^(1/2)*log(c*x^n))^2,x, algorithm="fricas"
)
```

output
```
-1/24*(x^6 - 4*x^3*e^((2*I*a*n - 3*log(c))/n) + 6*e^(2*(2*I*a*n - 3*log(c)
)/n)*log(x))*e^(-(2*I*a*n - 3*log(c))/n)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \sin^2\left(a+\frac{3}{2}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)\,dx=\text{Timed out}$$

input
```
integrate(x**2*sin(a+3/2*(-1/n**2)**(1/2)*ln(c*x**n))**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.62

$$\int x^2 \sin^2\left(a+\frac{3}{2}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)\,dx=-\frac{c^{\frac{6}{n}}x^6\cos(2a)-4\,c^{\frac{3}{n}}x^3+6\,\cos(2a)\log(x)}{24\,c^{\frac{3}{n}}}$$

input
```
integrate(x^2*sin(a+3/2*(-1/n^2)^(1/2)*log(c*x^n))^2,x, algorithm="maxima"
)
```

output
```
-1/24*(c^(6/n)*x^6*cos(2*a) - 4*c^(3/n)*x^3 + 6*cos(2*a)*log(x))/c^(3/n)
```

Giac [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int x^2 \sin^2\left(a+\frac{3}{2}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)\,dx=+\infty$$

input
```
integrate(x^2*sin(a+3/2*(-1/n^2)^(1/2)*log(c*x^n))^2,x, algorithm="giac")
```

output
```
+Infinity
```

Mupad [B] (verification not implemented)

Time = 21.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

$$\int x^2 \sin^2\left(a+\frac{3}{2}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)\,dx=\frac{x^3}{6}-\frac{x^3\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,3\mathrm{i}}}\,1\mathrm{i}}{12\,n\,\sqrt{-\frac{1}{n^2}}+12\mathrm{i}}+\frac{x^3\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,3\mathrm{i}}\,1\mathrm{i}}{12\,n\,\sqrt{-\frac{1}{n^2}}-12\mathrm{i}}$$

input
```
int(x^2*sin(a + (3*log(c*x^n)*(-1/n^2)^(1/2))/2)^2,x)
```

output
```
x^3/6 - (x^3*exp(-a*2i)/(c*x^n)^((-1/n^2)^(1/2)*3i)*1i)/(12*n*(-1/n^2)^(1/
2) + 12i) + (x^3*exp(a*2i)*(c*x^n)^((-1/n^2)^(1/2)*3i)*1i)/(12*n*(-1/n^2)^
(1/2) - 12i)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.17

$$\int x^2 \sin^2\left(a + \frac{3}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right) dx$$
$$= \frac{x^3\left(-6\cos\left(\frac{3\log(x^n c)i+2an}{2n}\right)\log(x^n c)\sin\left(\frac{3\log(x^n c)i+2an}{2n}\right)i - 2\cos\left(\frac{3\log(x^n c)i+2an}{2n}\right)\sin\left(\frac{3\log(x^n c)i+2an}{2n}\right)in + \right.}{12n}$$

input
```
int(x^2*sin(a+3/2*(-1/n^2)^(1/2)*log(c*x^n))^2,x)
```

output
```
(x**3*( - 6*cos((3*log(x**n*c)*i + 2*a*n)/(2*n))*log(x**n*c)*sin((3*log(x*
*n*c)*i + 2*a*n)/(2*n))*i - 2*cos((3*log(x**n*c)*i + 2*a*n)/(2*n))*sin((3*
log(x**n*c)*i + 2*a*n)/(2*n))*i*n + 6*log(x**n*c)*sin((3*log(x**n*c)*i + 2
*a*n)/(2*n))**2 - 3*log(x**n*c) + 4*sin((3*log(x**n*c)*i + 2*a*n)/(2*n))**
2*n))/(12*n)
```

3.35 $\int x \sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right) dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int x\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)dx=\frac{x^2}{4}-\frac{1}{16}e^{-2a\sqrt{-\frac{1}{n^2}}n}x^2(cx^n)^{2/n}-\frac{1}{4}e^{2a\sqrt{-\frac{1}{n^2}}n}x^2(cx^n)^{-2/n}\log(x)$$

output
```
1/4*x^2-1/16*x^2*(c*x^n)^(2/n)/exp(2*a*(-1/n^2)^(1/2)*n)-1/4*exp(2*a*(-1/n
^2)^(1/2)*n)*x^2*ln(x)/((c*x^n)^(2/n))
```

Mathematica [F]

$$\int x\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)dx=\int x\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)dx$$

input
```
Integrate[x*Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]^2,x]
```

output
```
Integrate[x*Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]^2, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {4996, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{x^2(cx^n)^{-2/n}\int (cx^n)^{\frac{2}{n}-1}\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{4992}$$

$$-\frac{x^2(cx^n)^{-2/n}\int \left(-2(cx^n)^{\frac{2}{n}-1}+e^{-2a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{4}{n}-1}+\frac{e^{2a\sqrt{-\frac{1}{n^2}}n}x^{-n}}{c}\right)d(cx^n)}{4n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2(cx^n)^{-2/n}\left(-\frac{1}{4}ne^{-2a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{4/n}-e^{2a\sqrt{-\frac{1}{n^2}}n}\log\left(cx^n\right)+n(cx^n)^{2/n}\right)}{4n}$$

input
```
Int[x*Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]^2,x]
```

output
```
(x^2*(n*(c*x^n)^(2/n) - (n*(c*x^n)^(4/n))/(4*E^(2*a*Sqrt[-n^(-2)]*n)) - E^
(2*a*Sqrt[-n^(-2)]*n)*Log[c*x^n]))/(4*n*(c*x^n)^(2/n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x\sin \left(a+\sqrt{-\frac{1}{n^{2}}}\, \ln \left(c\,x^{n}\right)\right)^{2}dx$$

input
```
int(x*sin(a+(-1/n^2)^(1/2)*ln(c*x^n))^2,x)
```

output
```
int(x*sin(a+(-1/n^2)^(1/2)*ln(c*x^n))^2,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int x\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=-\frac{1}{16}\left(x^4-4\,x^2e^{\left(\frac{2\,(i\,an-\log(c))}{n}\right)}+4\,e^{\left(\frac{4\,(i\,an-\log(c))}{n}\right)}\log\left(x\right)\right)e^{\left(-\frac{2\,(i\,an-\log(c))}{n}\right)}$$

input
```
integrate(x*sin(a+(-1/n^2)^(1/2)*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
-1/16*(x^4 - 4*x^2*e^(2*(I*a*n - log(c))/n) + 4*e^(4*(I*a*n - log(c))/n)*l
og(x))*e^(-2*(I*a*n - log(c))/n)
```

Sympy [F]

$$\int x\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\int x\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$

input
```
integrate(x*sin(a+(-1/n**2)**(1/2)*ln(c*x**n))**2,x)
```

output
```
Integral(x*sin(a + sqrt(-1/n**2)*log(c*x**n))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.62

$$\int x\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=-\frac{c^{\frac{4}{n}}x^4\cos\left(2\,a\right)-4\,c^{\frac{2}{n}}x^2+4\,\cos\left(2\,a\right)\log\left(x\right)}{16\,c^{\frac{2}{n}}}$$

input
```
integrate(x*sin(a+(-1/n^2)^(1/2)*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
-1/16*(c^(4/n)*x^4*cos(2*a) - 4*c^(2/n)*x^2 + 4*cos(2*a)*log(x))/c^(2/n)
```

Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int x\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=+\infty$$

input
```
integrate(x*sin(a+(-1/n^2)^(1/2)*log(c*x^n))^2,x, algorithm="giac")
```

output
```
+Infinity
```

Mupad [B] (verification not implemented)

Time = 20.47 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

$$\int x\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\frac{x^2}{4}-\frac{x^2\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,2\mathrm{i}}}\,1\mathrm{i}}{8\,n\,\sqrt{-\frac{1}{n^2}}+8\mathrm{i}}+\frac{x^2\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,2\mathrm{i}}\,1\mathrm{i}}{8\,n\,\sqrt{-\frac{1}{n^2}}-8\mathrm{i}}$$

input
```
int(x*sin(a + log(c*x^n)*(-1/n^2)^(1/2))^2,x)
```

output
```
x^2/4 - (x^2*exp(-a*2i)/(c*x^n)^((-1/n^2)^(1/2)*2i)*1i)/(8*n*(-1/n^2)^(1/2
) + 8i) + (x^2*exp(a*2i)*(c*x^n)^((-1/n^2)^(1/2)*2i)*1i)/(8*n*(-1/n^2)^(1/
2) - 8i)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.93

$$\int x\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$

$$=\frac{x^2\left(-2\cos\left(\frac{\log(x^nc)i+an}{n}\right)\log(x^nc)\sin\left(\frac{\log(x^nc)i+an}{n}\right)i-\cos\left(\frac{\log(x^nc)i+an}{n}\right)\sin\left(\frac{\log(x^nc)i+an}{n}\right)in+2\log(x^nc\right.}{4n}$$

input
```
int(x*sin(a+(-1/n^2)^(1/2)*log(c*x^n))^2,x)
```

output
```
(x**2*( - 2*cos((log(x**n*c)*i + a*n)/n)*log(x**n*c)*sin((log(x**n*c)*i +
a*n)/n)*i - cos((log(x**n*c)*i + a*n)/n)*sin((log(x**n*c)*i + a*n)/n)*i*n
+ 2*log(x**n*c)*sin((log(x**n*c)*i + a*n)/n)**2 - log(x**n*c) + 2*sin((log
(x**n*c)*i + a*n)/n)**2*n))/(4*n)
```

3.36 $\int \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\frac{x}{2}-\frac{1}{8}e^{-2a\sqrt{-\frac{1}{n^2}}n}x(cx^n)^{\frac{1}{n}}-\frac{1}{4}e^{2a\sqrt{-\frac{1}{n^2}}n}x(cx^n)^{-1/n}\log(x)$$

output
```
1/2*x-1/8*x*(c*x^n)^(1/n)/exp(2*a*(-1/n^2)^(1/2)*n)-1/4*exp(2*a*(-1/n^2)^(
1/2)*n)*x*ln(x)/((c*x^n)^(1/n))
```

Mathematica [F]

$$\int \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\int \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$

input
```
Integrate[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/2]^2,x]
```

output
```
Integrate[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/2]^2, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4986, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx$$

$$\Big\downarrow \text{4986}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{4992}$$

$$-\frac{x(cx^n)^{-1/n}\int \left(-2(cx^n)^{\frac{1}{n}-1}+e^{-2a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{2}{n}-1}+\frac{e^{2a\sqrt{-\frac{1}{n^2}}n}x^{-n}}{c}\right)d(cx^n)}{4n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x(cx^n)^{-1/n}\left(-\frac{1}{2}ne^{-2a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{2/n}-e^{2a\sqrt{-\frac{1}{n^2}}n}\log\left(cx^n\right)+2n(cx^n)^{\frac{1}{n}}\right)}{4n}$$

input
```
Int[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/2]^2,x]
```

output
```
(x*(2*n*(c*x^n)^n^(-1) - (n*(c*x^n)^(2/n))/(2*E^(2*a*Sqrt[-n^(-2)]*n)) - E
^(2*a*Sqrt[-n^(-2)]*n)*Log[c*x^n]))/(4*n*(c*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4986
```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

Maple [F]

$$\int \sin \left(a+\frac{\sqrt{-\frac{1}{n^{2}}}\, \ln \left(c\, x^{n}\right)}{2}\right)^{2}d x$$

input
```
int(sin(a+1/2*(-1/n^2)^(1/2)*ln(c*x^n))^2,x)
```

output
```
int(sin(a+1/2*(-1/n^2)^(1/2)*ln(c*x^n))^2,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

$$\int \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log \left(cx^n\right)\right)\,dx = -\frac{1}{8}\,\left(x^{2} - 4\,x e^{\left(\frac{2i\,an - \log\left(c\right)}{n}\right)} + 2\,e^{\left(\frac{2\,\left(2i\,an - \log\left(c\right)\right)}{n}\right)} \log\left(x\right)\right) e^{\left(-\frac{2i\,an - \log\left(c\right)}{n}\right)}$$

input
```
integrate(sin(a+1/2*(-1/n^2)^(1/2)*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
-1/8*(x^2 - 4*x*e^((2*I*a*n - log(c))/n) + 2*e^(2*(2*I*a*n - log(c))/n)*lo
g(x))*e^(-(2*I*a*n - log(c))/n)
```

Sympy [F]

$$\int \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\int \sin^2\left(a+\frac{\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)}{2}\right)\,dx$$

input
```
integrate(sin(a+1/2*(-1/n**2)**(1/2)*ln(c*x**n))**2,x)
```

output
```
Integral(sin(a + sqrt(-1/n**2)*log(c*x**n)/2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.60

$$\int \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=-\frac{c^{\frac{2}{n}}x^2\cos\left(2\,a\right)-4\,c^{\left(\frac{1}{n}\right)}x+2\,\cos\left(2\,a\right)\log\left(x\right)}{8\,c^{\left(\frac{1}{n}\right)}}$$

input
```
integrate(sin(a+1/2*(-1/n^2)^(1/2)*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
-1/8*(c^(2/n)*x^2*cos(2*a) - 4*c^(1/n)*x + 2*cos(2*a)*log(x))/c^(1/n)
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=+\infty$$

input
```
integrate(sin(a+1/2*(-1/n^2)^(1/2)*log(c*x^n))^2,x, algorithm="giac")
```

output
```
+Infinity
```

Mupad [B] (verification not implemented)

Time = 20.00 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.26

$$\int \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\frac{x}{2}-\frac{x\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}}\,1\mathrm{i}}{4\,n\,\sqrt{-\frac{1}{n^2}}+4\mathrm{i}}+\frac{x\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}\,1\mathrm{i}}{4\,n\,\sqrt{-\frac{1}{n^2}}-4\mathrm{i}}$$

input
```
int(sin(a + (log(c*x^n)*(-1/n^2)^(1/2))/2)^2,x)
```

output
```
x/2 - (x*exp(-a*2i)/(c*x^n)^((-1/n^2)^(1/2)*1i)*1i)/(4*n*(-1/n^2)^(1/2) +
4i) + (x*exp(a*2i)*(c*x^n)^((-1/n^2)^(1/2)*1i)*1i)/(4*n*(-1/n^2)^(1/2) - 4
i)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.31

$$\int \sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$

$$=\frac{x\left(-2\cos\left(\frac{\log(x^nc)i+2an}{2n}\right)\log(x^nc)\sin\left(\frac{\log(x^nc)i+2an}{2n}\right)i-2\cos\left(\frac{\log(x^nc)i+2an}{2n}\right)\sin\left(\frac{\log(x^nc)i+2an}{2n}\right)in+2\log(\right.}{4n}$$

input
```
int(sin(a+1/2*(-1/n^2)^(1/2)*log(c*x^n))^2,x)
```

output
```
(x*( - 2*cos((log(x**n*c)*i + 2*a*n)/(2*n))*log(x**n*c)*sin((log(x**n*c)*i
 + 2*a*n)/(2*n))*i - 2*cos((log(x**n*c)*i + 2*a*n)/(2*n))*sin((log(x**n*c)
*i + 2*a*n)/(2*n))*i*n + 2*log(x**n*c)*sin((log(x**n*c)*i + 2*a*n)/(2*n))*
*2 - log(x**n*c) + 4*sin((log(x**n*c)*i + 2*a*n)/(2*n))**2*n))/(4*n)
```

3.37 $\int \frac{\sin^2(a)}{x} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 7

$$\int \frac{\sin^2(a)}{x} \, dx = \log(x) \sin^2(a)$$

output
```
ln(x)*sin(a)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(a)}{x} \, dx = \log(x) \sin^2(a)$$

input
```
Integrate[Sin[a]^2/x,x]
```

output
```
Log[x]*Sin[a]^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a)}{x} \, dx$$
$$\Big\downarrow \; 14$$
$$\sin^2(a)\log(x)$$

input
```
Int[Sin[a]^2/x,x]
```

output
```
Log[x]*Sin[a]^2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
default	$\ln\left(x\right)\sin\left(a\right)^{2}$	8
norman	$\ln\left(x\right)\sin\left(a\right)^{2}$	8
risch	$\ln\left(x\right)\sin\left(a\right)^{2}$	8
parallelrisch	$\ln\left(x\right)\sin\left(a\right)^{2}$	8

input
```
int(sin(a)^2/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*sin(a)^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.43

$$\int \frac{\sin^2(a)}{x}\,dx = -\left(\cos{(a)}^2 - 1\right)\log{(x)}$$

input
```
integrate(sin(a)^2/x,x, algorithm="fricas")
```

output
```
-(cos(a)^2 - 1)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(a)}{x}\,dx = \log{(x)}\sin^2{(a)}$$

input
```
integrate(sin(a)**2/x,x)
```

output
```
log(x)*sin(a)**2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(a)}{x}\,dx = \log{(x)}\sin{(a)}^2$$

input
```
integrate(sin(a)^2/x,x, algorithm="maxima")
```

output
```
log(x)*sin(a)^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int \frac{\sin^2(a)}{x} \, dx = \log(|x|) \sin(a)^2$$

input
```
integrate(sin(a)^2/x,x, algorithm="giac")
```

output
```
log(abs(x))*sin(a)^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(a)}{x} \, dx = \sin(a)^2 \ln(x)$$

input
```
int(sin(a)^2/x,x)
```

output
```
sin(a)^2*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(a)}{x} \, dx = \log(x) \sin(a)^2$$

input
```
int(sin(a)^2/x,x)
```

output
```
log(x)*sin(a)**2
```

3.38 $\int \frac{\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 74

$$\int \frac{\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx = -\frac{1}{2x}+\frac{e^{2a\sqrt{-\frac{1}{n^2}}n}\left(cx^n\right)^{-1/n}}{8x}-\frac{e^{-2a\sqrt{-\frac{1}{n^2}}n}\left(cx^n\right)^{\frac{1}{n}}\log(x)}{4x}$$

output
```
-1/2/x+1/8*exp(2*a*(-1/n^2)^(1/2)*n)/x/((c*x^n)^(1/n))-1/4*(c*x^n)^(1/n)*l
n(x)/exp(2*a*(-1/n^2)^(1/2)*n)/x
```

Mathematica [F]

$$\int \frac{\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx = \int \frac{\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx$$

input
```
Integrate[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/2]^2/x^2,x]
```

output
```
Integrate[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/2]^2/x^2, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {4996, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{\left(cx^n\right)^{\frac{1}{n}}\int \left(cx^n\right)^{-1-\frac{1}{n}}\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)d(cx^n)}{nx}$$

$$\Big\downarrow \text{4992}$$

$$-\frac{\left(cx^n\right)^{\frac{1}{n}}\int \left(\frac{e^{-2a\sqrt{-\frac{1}{n^2}}n}x^{-n}}{c}-2\left(cx^n\right)^{-\frac{n+1}{n}}+e^{2a\sqrt{-\frac{1}{n^2}}n}\left(cx^n\right)^{-\frac{n+2}{n}}\right)d(cx^n)}{4nx}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(cx^n\right)^{\frac{1}{n}}\left(\frac{1}{2}ne^{2a\sqrt{-\frac{1}{n^2}}n}\left(cx^n\right)^{-2/n}-e^{-2a\sqrt{-\frac{1}{n^2}}n}\log\left(cx^n\right)-2n\left(cx^n\right)^{-1/n}\right)}{4nx}$$

input
```
Int[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/2]^2/x^2,x]
```

output
```
((c*x^n)^n^(-1)*((E^(2*a*Sqrt[-n^(-2)]*n)*n)/(2*(c*x^n)^(2/n)) - (2*n)/(c*
x^n)^n^(-1) - Log[c*x^n]/E^(2*a*Sqrt[-n^(-2)]*n)))/(4*n*x)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 6.98 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

method	result	size
parallelrisch	$\frac{(n-\ln(c\,x^n))\cos\left(\sqrt{-\frac{1}{n^2}}\,\ln(c\,x^n)+2a\right)+n\left(\sqrt{-\frac{1}{n^2}}\,\ln(c\,x^n)\sin\left(\sqrt{-\frac{1}{n^2}}\,\ln(c\,x^n)+2a\right)-2\right)}{4xn}$	77

input
```
int(sin(a+1/2*(-1/n^2)^(1/2)*ln(c*x^n))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*((n-ln(c*x^n))*cos((-1/n^2)^(1/2)*ln(c*x^n)+2*a)+n*((-1/n^2)^(1/2)*ln(
c*x^n)*sin((-1/n^2)^(1/2)*ln(c*x^n)+2*a)-2))/x/n
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx = -\frac{\left(2\,x^2\log\left(x\right)+4\,xe^{\left(\frac{2i\,an-\log\left(c\right)}{n}\right)}-e^{\left(\frac{2\,(2i\,an-\log\left(c\right))}{n}\right)}\right)e^{\left(-\frac{2i\,an-\log\left(c\right)}{n}\right)}}{8\,x^2}$$

input
```
integrate(sin(a+1/2*(-1/n^2)^(1/2)*log(c*x^n))^2/x^2,x, algorithm="fricas"
)
```

output
```
-1/8*(2*x^2*log(x) + 4*x*e^((2*I*a*n - log(c))/n) - e^(2*(2*I*a*n - log(c)
)/n))*e^(-(2*I*a*n - log(c))/n)/x^2
```

Sympy [A] (verification not implemented)

Time = 13.99 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.42

$$\int \frac{\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx = \frac{\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\sin\left(2a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{4x} + \frac{\cos\left(2a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{4x} - \frac{1}{2x} - \frac{\log\left(cx^n\right)\cos\left(2a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{4nx}$$

input
```
integrate(sin(a+1/2*(-1/n**2)**(1/2)*ln(c*x**n))**2/x**2,x)
```

output
```
sqrt(-1/n**2)*log(c*x**n)*sin(2*a + sqrt(-1/n**2)*log(c*x**n))/(4*x) + cos
(2*a + sqrt(-1/n**2)*log(c*x**n))/(4*x) - 1/(2*x) - log(c*x**n)*cos(2*a +
sqrt(-1/n**2)*log(c*x**n))/(4*n*x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

$$\int \frac{\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx = -\frac{2\,c^{\frac{2}{n}}x^3\cos\left(2\,a\right)\log\left(x\right)+4\,c^{\left(\frac{1}{n}\right)}x^2-x\cos\left(2\,a\right)}{8\,c^{\left(\frac{1}{n}\right)}x^3}$$

input
```
integrate(sin(a+1/2*(-1/n^2)^(1/2)*log(c*x^n))^2/x^2,x, algorithm="maxima"
)
```

output
```
-1/8*(2*c^(2/n)*x^3*cos(2*a)*log(x) + 4*c^(1/n)*x^2 - x*cos(2*a))/(c^(1/n)
*x^3)
```

Giac [F]

$$\int \frac{\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx = \int \frac{\sin\left(\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate(sin(a+1/2*(-1/n^2)^(1/2)*log(c*x^n))^2/x^2,x, algorithm="giac")
```

output
```
integrate(sin(1/2*sqrt(-1/n^2)*log(c*x^n) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx = \int \frac{\sin\left(a+\frac{\ln\left(c\,x^n\right)\sqrt{-\frac{1}{n^2}}}{2}\right)^2}{x^2}\,dx$$

input
```
int(sin(a + (log(c*x^n)*(-1/n^2)^(1/2))/2)^2/x^2,x)
```

output
```
int(sin(a + (log(c*x^n)*(-1/n^2)^(1/2))/2)^2/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.15

$$\int \frac{\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx$$
$$= \frac{2\cos\left(\frac{\log(x^nc)i+2an}{2n}\right)\log(x^nc)\sin\left(\frac{\log(x^nc)i+2an}{2n}\right)i-2\cos\left(\frac{\log(x^nc)i+2an}{2n}\right)\sin\left(\frac{\log(x^nc)i+2an}{2n}\right)in+2\log(x^nc)}{4nx}$$

input
```
int(sin(a+1/2*(-1/n^2)^(1/2)*log(c*x^n))^2/x^2,x)
```

output
```
(2*cos((log(x**n*c)*i + 2*a*n)/(2*n))*log(x**n*c)*sin((log(x**n*c)*i + 2*a
*n)/(2*n))*i - 2*cos((log(x**n*c)*i + 2*a*n)/(2*n))*sin((log(x**n*c)*i + 2
*a*n)/(2*n))*i*n + 2*log(x**n*c)*sin((log(x**n*c)*i + 2*a*n)/(2*n))**2 - l
og(x**n*c) - 4*sin((log(x**n*c)*i + 2*a*n)/(2*n))**2*n)/(4*n*x)
```

3.39 $\int \frac{\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 76

$$\int \frac{\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^3}\,dx = -\frac{1}{4x^2}+\frac{e^{2a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{-2/n}}{16x^2} - \frac{e^{-2a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{2/n}\log(x)}{4x^2}$$

output
```
-1/4/x^2+1/16*exp(2*a*(-1/n^2)^(1/2)*n)/x^2/((c*x^n)^(2/n))-1/4*(c*x^n)^(2
/n)*ln(x)/exp(2*a*(-1/n^2)^(1/2)*n)/x^2
```

Mathematica [F]

$$\int \frac{\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^3}\,dx = \int \frac{\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^3}\,dx$$

input
```
Integrate[Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]^2/x^3,x]
```

output
```
Integrate[Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]^2/x^3, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4996, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}dx$$

$$\Big\downarrow \text{ 4996}$$

$$\frac{\left(cx^n\right)^{2/n}\int \left(cx^n\right)^{-1-\frac{2}{n}}\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)d\left(cx^n\right)}{nx^2}$$

$$\Big\downarrow \text{ 4992}$$

$$-\frac{\left(cx^n\right)^{2/n}\int \left(\frac{e^{-2a\sqrt{-\frac{1}{n^2}}n}x^{-n}}{c}-2\left(cx^n\right)^{-\frac{n+2}{n}}+e^{2a\sqrt{-\frac{1}{n^2}}n}\left(cx^n\right)^{-\frac{n+4}{n}}\right)d\left(cx^n\right)}{4nx^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(cx^n\right)^{2/n}\left(\frac{1}{4}ne^{2a\sqrt{-\frac{1}{n^2}}n}\left(cx^n\right)^{-4/n}-e^{-2a\sqrt{-\frac{1}{n^2}}n}\log\left(cx^n\right)-n\left(cx^n\right)^{-2/n}\right)}{4nx^2}$$

input
```
Int[Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]^2/x^3,x]
```

output
```
((c*x^n)^(2/n)*((E^(2*a*Sqrt[-n^(-2)]*n)*n)/(4*(c*x^n)^(4/n)) - n/(c*x^n)^
(2/n) - Log[c*x^n]/E^(2*a*Sqrt[-n^(-2)]*n)))/(4*n*x^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 14.92 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13

method	result	size
parallelrisch	$\frac{(-2n-6\ln(c\,x^{n}))\cos\left(2\sqrt{-\frac{1}{n^{2}}}\,\ln(c\,x^{n})+2a\right)+5n\left(-\frac{6}{5}+\sqrt{-\frac{1}{n^{2}}}\,\left(n+\frac{6\ln(c\,x^{n})}{5}\right)\sin\left(2\sqrt{-\frac{1}{n^{2}}}\,\ln(c\,x^{n})+2a\right)\right)}{24x^{2}n}$	86

input
```
int(sin(a+(-1/n^2)^(1/2)*ln(c*x^n))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/24*((-2*n-6*ln(c*x^n))*cos(2*(-1/n^2)^(1/2)*ln(c*x^n)+2*a)+5*n*(-6/5+(-1
/n^2)^(1/2)*(n+6/5*ln(c*x^n))*sin(2*(-1/n^2)^(1/2)*ln(c*x^n)+2*a)))/x^2/n
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.86

$$\int \frac{\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^3}\,dx = -\frac{\left(4\,x^4\log(x)+4\,x^2e^{\left(\frac{2\,(i\,an-\log(c))}{n}\right)}-e^{\left(\frac{4\,(i\,an-\log(c))}{n}\right)}\right)e^{\left(-\frac{2\,(i\,an-\log(c))}{n}\right)}}{16\,x^4}$$

input
```
integrate(sin(a+(-1/n^2)^(1/2)*log(c*x^n))^2/x^3,x, algorithm="fricas")
```

output
```
-1/16*(4*x^4*log(x) + 4*x^2*e^(2*(I*a*n - log(c))/n) - e^(4*(I*a*n - log(c
))/n))*e^(-2*(I*a*n - log(c))/n)/x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(70) = 140.

Time = 6.57 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.91

$$\int \frac{\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^3}\,dx = -\frac{\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{2x^2}+\frac{\log(cx^n)\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{4nx^2} - \frac{\log(cx^n)\cos^2\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{4nx^2} + \frac{\sin\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)\cos\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{4nx^2\sqrt{-\frac{1}{n^2}}} - \frac{\log(cx^n)\sin\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)\cos\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{2n^2x^2\sqrt{-\frac{1}{n^2}}}$$

input
```
integrate(sin(a+(-1/n**2)**(1/2)*ln(c*x**n))**2/x**3,x)
```

output
```
-sin(a + sqrt(-1/n**2)*log(c*x**n))**2/(2*x**2) + log(c*x**n)*sin(a + sqrt
(-1/n**2)*log(c*x**n))**2/(4*n*x**2) - log(c*x**n)*cos(a + sqrt(-1/n**2)*l
og(c*x**n))**2/(4*n*x**2) + sin(a + sqrt(-1/n**2)*log(c*x**n))*cos(a + sqr
t(-1/n**2)*log(c*x**n))/(4*n*x**2*sqrt(-1/n**2)) - log(c*x**n)*sin(a + sqr
t(-1/n**2)*log(c*x**n))*cos(a + sqrt(-1/n**2)*log(c*x**n))/(2*n**2*x**2*sq
rt(-1/n**2))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.71

$$\int \frac{\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx = -\frac{4\,c^{\frac{4}{n}}x^6\cos\left(2\,a\right)\log\left(x\right)+4\,c^{\frac{2}{n}}x^4-x^2\cos\left(2\,a\right)}{16\,c^{\frac{2}{n}}x^6}$$

input
```
integrate(sin(a+(-1/n^2)^(1/2)*log(c*x^n))^2/x^3,x, algorithm="maxima")
```

output
```
-1/16*(4*c^(4/n)*x^6*cos(2*a)*log(x) + 4*c^(2/n)*x^4 - x^2*cos(2*a))/(c^(2
/n)*x^6)
```

Giac [F]

$$\int \frac{\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx = \int \frac{\sin\left(\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)+a\right)^2}{x^3}\,dx$$

input
```
integrate(sin(a+(-1/n^2)^(1/2)*log(c*x^n))^2/x^3,x, algorithm="giac")
```

output
```
integrate(sin(sqrt(-1/n^2)*log(c*x^n) + a)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx = \int \frac{\sin\left(a+\ln\left(c\,x^n\right)\,\sqrt{-\frac{1}{n^2}}\right)^2}{x^3}\,dx$$

input
```
int(sin(a + log(c*x^n)*(-1/n^2)^(1/2))^2/x^3,x)
```

output
```
int(sin(a + log(c*x^n)*(-1/n^2)^(1/2))^2/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.93

$$\int \frac{\sin^2\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx$$
$$= \frac{2\cos\left(\frac{\log(x^nc)i+an}{n}\right)\log(x^nc)\sin\left(\frac{\log(x^nc)i+an}{n}\right)i-\cos\left(\frac{\log(x^nc)i+an}{n}\right)\sin\left(\frac{\log(x^nc)i+an}{n}\right)in+2\log(x^nc)\sin\left(\right.}{4n\,x^2}$$

input
```
int(sin(a+(-1/n^2)^(1/2)*log(c*x^n))^2/x^3,x)
```

output
```
(2*cos((log(x**n*c)*i + a*n)/n)*log(x**n*c)*sin((log(x**n*c)*i + a*n)/n)*i
 - cos((log(x**n*c)*i + a*n)/n)*sin((log(x**n*c)*i + a*n)/n)*i*n + 2*log(x
**n*c)*sin((log(x**n*c)*i + a*n)/n)**2 - log(x**n*c) - 2*sin((log(x**n*c)*
i + a*n)/n)**2*n)/(4*n*x**2)
```

3.40 $\int x^m \sin^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 226

$$\begin{aligned}
&\int x^m \sin^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)\,dx \\
&= -\frac{4\sqrt{-\frac{(1+m)^2}{n^2}}\,nx^{1+m}\cos\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)}{5(1+m)^2} \\
&\quad+\frac{8x^{1+m}\sin\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)}{5(1+m)} \\
&\quad+\frac{6\sqrt{-\frac{(1+m)^2}{n^2}}\,nx^{1+m}\cos\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)}{5(1+m)^2} \\
&\quad-\frac{4x^{1+m}\sin^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)}{5(1+m)}
\end{aligned}$$

output
```
-4/5*(-(1+m)^2/n^2)^(1/2)*n*x^(1+m)*cos(a+1/2*(-(1+m)^2/n^2)^(1/2)*ln(c*x^
n))/(1+m)^2+8*x^(1+m)*sin(a+1/2*(-(1+m)^2/n^2)^(1/2)*ln(c*x^n))/(5+5*m)+6/
5*(-(1+m)^2/n^2)^(1/2)*n*x^(1+m)*cos(a+1/2*(-(1+m)^2/n^2)^(1/2)*ln(c*x^n))
*sin(a+1/2*(-(1+m)^2/n^2)^(1/2)*ln(c*x^n))^2/(1+m)^2-4*x^(1+m)*sin(a+1/2*(
-(1+m)^2/n^2)^(1/2)*ln(c*x^n))^3/(5+5*m)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.75

$$\int x^m \sin^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\frac{x^{1+m}\left(-5\sqrt{-\frac{(1+m)^2}{n^2}}n\cos\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)-3\sqrt{-\frac{(1+m)^2}{n^2}}n\cos\left(3a+\frac{3}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right.\right.}{10(1+m)^2}$$

input
```
Integrate[x^m*Sin[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^3,x]
```

output
```
(x^(1 + m)*(-5*Sqrt[-((1 + m)^2/n^2)]*n*Cos[a + (Sqrt[-((1 + m)^2/n^2)]*Lo
g[c*x^n])/2] - 3*Sqrt[-((1 + m)^2/n^2)]*n*Cos[3*a + (3*Sqrt[-((1 + m)^2/n^
2)]*Log[c*x^n])/2] + 2*(1 + m)*(5*Sin[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^
n])/2] + Sin[3*a + (3*Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2])))/(10*(1 + m)
^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {4990, 4988}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sin^3\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$$

$$\Big\downarrow \text{4990}$$

$$\frac{6}{5}\int x^m \sin\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)dx-\frac{4x^{m+1}\sin^3\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)}{5(m+1)}+\frac{6n\sqrt{-\frac{(m+1)^2}{n^2}}x^{m+1}\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)\cos\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)}{5(m+1)^2}$$

$$\Big\downarrow \text{4988}$$

$$-\frac{4x^{m+1}\sin^3\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)}{5(m+1)}+\frac{6n\sqrt{-\frac{(m+1)^2}{n^2}}x^{m+1}\sin^2\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)\cos\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)}{5(m+1)^2}+\frac{6}{5}\left(\frac{4x^{m+1}\sin\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)}{3(m+1)}-\frac{2n\sqrt{-\frac{(m+1)^2}{n^2}}x^{m+1}\cos\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)}{3(m+1)^2}\right)$$

input

```
Int[x^m*Sin[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^3,x]
```

output

```
(6*Sqrt[-((1 + m)^2/n^2)]*n*x^(1 + m)*Cos[a + (Sqrt[-((1 + m)^2/n^2)]*Log[
c*x^n])/2]*Sin[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^2)/(5*(1 + m)^2)
 - (4*x^(1 + m)*Sin[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^3)/(5*(1 +
m)) + (6*((-2*Sqrt[-((1 + m)^2/n^2)]*n*x^(1 + m)*Cos[a + (Sqrt[-((1 + m)^2
/n^2)]*Log[c*x^n])/2])/(3*(1 + m)^2) + (4*x^(1 + m)*Sin[a + (Sqrt[-((1 + m
)^2/n^2)]*Log[c*x^n])/2])/(3*(1 + m))))/5
```

Defintions of rubi rules used

rule 4988

```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

rule 4990
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (-Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a
 + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e
*(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2
))   Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
 c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [A] (verified)

Time = 50.48 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.60

method	result
parallelrisch	$\frac{8x^{1+m}\left(5\sqrt{-\frac{(1+m)^2}{n^2}}\,n\tan\left(\frac{a}{2}+\sqrt{-\frac{(1+m)^2}{n^2}}\,\ln\left(\left(c\,x^n\right)^{\frac{1}{4}}\right)\right)^2+4(1+m)\tan\left(\frac{a}{2}+\sqrt{-\frac{(1+m)^2}{n^2}}\,\ln\left(\left(c\,x^n\right)^{\frac{1}{4}}\right)\right)-\sqrt{-\frac{(1+m)^2}{n^2}}\,n\right.}{5(1+m)^2\left(1+\tan\left(\frac{a}{2}+\sqrt{-\frac{(1+m)^2}{n^2}}\,\ln\left(\left(c\,x^n\right)^{\frac{1}{4}}\right)\right)^2\right)^3}$

input
```
int(x^m*sin(a+1/2*(-(1+m)^2/n^2)^(1/2)*ln(c*x^n))^3,x,method=_RETURNVERBOS
E)
```

output
```
8/5*x^(1+m)*(5*(-(1+m)^2/n^2)^(1/2)*n*tan(1/2*a+(-(1+m)^2/n^2)^(1/2)*ln((c
*x^n)^(1/4)))^2+4*(1+m)*tan(1/2*a+(-(1+m)^2/n^2)^(1/2)*ln((c*x^n)^(1/4)))-
(-(1+m)^2/n^2)^(1/2)*n)/(1+m)^2/(1+tan(1/2*a+(-(1+m)^2/n^2)^(1/2)*ln((c*x^
n)^(1/4)))^2)^3
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.57

$$\int x^m\sin^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)dx$$
$$=\frac{\left(5i\,e^{\left(-\frac{(m+1)n\log(x)-2i\,an+(m+1)\log(c)}{n}\right)}-15i\,e^{\left(-\frac{2((m+1)n\log(x)-2i\,an+(m+1)\log(c))}{n}\right)}-5i\,e^{\left(-\frac{3((m+1)n\log(x)-2i\,an+(m+1)\log(c)}{n}\right.}\right.}{20\,(m+1)}$$

input
```
integrate(x^m*sin(a+1/2*(-(1+m)^2/n^2)^(1/2)*log(c*x^n))^3,x, algorithm="f
ricas")
```

output
```
1/20*(5*I*e^(-((m + 1)*n*log(x) - 2*I*a*n + (m + 1)*log(c))/n) - 15*I*e^(-
2*((m + 1)*n*log(x) - 2*I*a*n + (m + 1)*log(c))/n) - 5*I*e^(-3*((m + 1)*n*
log(x) - 2*I*a*n + (m + 1)*log(c))/n) - I)*e^(5/2*((m + 1)*n*log(x) - 2*I*
a*n + (m + 1)*log(c))/n + (2*I*a*n - (m + 1)*log(c))/n)/(m + 1)
```

Sympy [F]

$$\int x^m \sin^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\int x^m \sin^3\left(a+\frac{\sqrt{-\frac{m^2}{n^2}-\frac{2m}{n^2}-\frac{1}{n^2}}\log\left(cx^n\right)}{2}\right)dx$$

input
```
integrate(x**m*sin(a+1/2*(-(1+m)**2/n**2)**(1/2)*ln(c*x**n))**3,x)
```

output
```
Integral(x**m*sin(a + sqrt(-m**2/n**2 - 2*m/n**2 - 1/n**2)*log(c*x**n)/2)*
*3, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.86

$$\int x^m \sin^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx =$$
$$-\frac{\left(c^{\frac{3m}{n}+\frac{3}{n}}xe^{\left(m\log(x)+\frac{3m\log(x^n)}{n}+\frac{3\log(x^n)}{n}\right)}\sin\left(3a\right)-5c^{\frac{2m}{n}+\frac{2}{n}}xe^{\left(m\log(x)+\frac{2m\log(x^n)}{n}+\frac{2\log(x^n)}{n}\right)}\sin\left(a\right)-15c^{\frac{m}{n}+}\right.}{20\left(c^{\frac{3m}{2n}+\frac{3}{2n}}m+c^{\frac{3m}{2n}+\frac{3}{2n}}\right)}$$

input
```
integrate(x^m*sin(a+1/2*(-(1+m)^2/n^2)^(1/2)*log(c*x^n))^3,x, algorithm="m
axima")
```

output
```
-1/20*(c^(3*m/n + 3/n)*x*e^(m*log(x) + 3*m*log(x^n)/n + 3*log(x^n)/n)*sin(
3*a) - 5*c^(2*m/n + 2/n)*x*e^(m*log(x) + 2*m*log(x^n)/n + 2*log(x^n)/n)*si
n(a) - 15*c^(m/n + 1/n)*x*e^(m*log(x) + m*log(x^n)/n + log(x^n)/n)*sin(a)
- 5*x*x^m*sin(3*a))*e^(-3/2*m*log(x^n)/n - 3/2*log(x^n)/n)/(c^(3/2*m/n + 3
/2/n)*m + c^(3/2*m/n + 3/2/n))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.05 (sec) , antiderivative size = 1870, normalized size of antiderivative = 8.27

$$\int x^m \sin^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^m*sin(a+1/2*(-(1+m)^2/n^2)^(1/2)*log(c*x^n))^3,x, algorithm="g
iac")
```

output
```
1/4*(8*I*m^3*n^4*x*x^m*e^(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n
)*log(c))/n^2) - 24*I*m^3*n^4*x*x^m*e^(I*a - 1/2*(n*abs(m*n + n)*log(x) +
abs(m*n + n)*log(c))/n^2) + 24*I*m^3*n^4*x*x^m*e^(-I*a + 1/2*(n*abs(m*n +
n)*log(x) + abs(m*n + n)*log(c))/n^2) - 8*I*m^3*n^4*x*x^m*e^(-3*I*a + 3/2*
(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 24*I*m^2*n^4*x*x^m*e^
(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 12*I*m^2
*n^3*x*x^m*abs(m*n + n)*e^(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n +
n)*log(c))/n^2) - 72*I*m^2*n^4*x*x^m*e^(I*a - 1/2*(n*abs(m*n + n)*log(x) +
 abs(m*n + n)*log(c))/n^2) - 12*I*m^2*n^3*x*x^m*abs(m*n + n)*e^(I*a - 1/2*
(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 72*I*m^2*n^4*x*x^m*e^
(-I*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 12*I*m^2*
n^3*x*x^m*abs(m*n + n)*e^(-I*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)
*log(c))/n^2) - 24*I*m^2*n^4*x*x^m*e^(-3*I*a + 3/2*(n*abs(m*n + n)*log(x)
+ abs(m*n + n)*log(c))/n^2) + 12*I*m^2*n^3*x*x^m*abs(m*n + n)*e^(-3*I*a +
3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 2*I*(m*n + n)^2*m
*n^2*x*x^m*e^(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^
2) + 24*I*m*n^4*x*x^m*e^(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)
*log(c))/n^2) + 24*I*m*n^3*x*x^m*abs(m*n + n)*e^(3*I*a - 3/2*(n*abs(m*n +
n)*log(x) + abs(m*n + n)*log(c))/n^2) + 54*I*(m*n + n)^2*m*n^2*x*x^m*e^(I*
a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 72*I*m*n^4...
```

Mupad [B] (verification not implemented)

Time = 22.38 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.31

$$\int x^m \sin^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$= -\frac{x\,x^m\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{(c\,x^n)^{\frac{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}{2}}}\left(2\,m+2+n\,\sqrt{-\frac{(m+1)^2}{n^2}}\,1\mathrm{i}\right)\,1\mathrm{i}}{4\,(m\,1\mathrm{i}+1\mathrm{i})^2}$$
$$+\frac{x\,x^m\,\mathrm{e}^{a\,1\mathrm{i}}\,(c\,x^n)^{\frac{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}{2}}\left(2\,m+2-n\,\sqrt{-\frac{(m+1)^2}{n^2}}\,1\mathrm{i}\right)\,1\mathrm{i}}{4\,(m\,1\mathrm{i}+1\mathrm{i})^2}$$
$$-\frac{x\,x^m\,\mathrm{e}^{-a\,3\mathrm{i}}\,\frac{1}{(c\,x^n)^{\frac{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,3\mathrm{i}}{2}}}\left(2\,m+2+n\,\sqrt{-\frac{(m+1)^2}{n^2}}\,3\mathrm{i}\right)\,1\mathrm{i}}{20\,(m\,1\mathrm{i}+1\mathrm{i})^2}$$
$$+\frac{x\,x^m\,\mathrm{e}^{a\,3\mathrm{i}}\,(c\,x^n)^{\frac{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,3\mathrm{i}}{2}}\left(2\,m+2-n\,\sqrt{-\frac{(m+1)^2}{n^2}}\,3\mathrm{i}\right)\,1\mathrm{i}}{20\,(m\,1\mathrm{i}+1\mathrm{i})^2}$$

input
```
int(x^m*sin(a + (log(c*x^n)*(-(m + 1)^2/n^2)^(1/2))/2)^3,x)
```

output
```
(x*x^m*exp(a*1i)*(c*x^n)^(((- (2*m)/n^2 - 1/n^2 - m^2/n^2)^(1/2)*1i)/2)*(2
*m - n*(-(m + 1)^2/n^2)^(1/2)*1i + 2)*1i)/(4*(m*1i + 1i)^2) - (x*x^m*exp(-
a*1i)/(c*x^n)^(((- (2*m)/n^2 - 1/n^2 - m^2/n^2)^(1/2)*1i)/2)*(2*m + n*(-(m
 + 1)^2/n^2)^(1/2)*1i + 2)*1i)/(4*(m*1i + 1i)^2) - (x*x^m*exp(-a*3i)/(c*x^
n)^(((- (2*m)/n^2 - 1/n^2 - m^2/n^2)^(1/2)*3i)/2)*(2*m + n*(-(m + 1)^2/n^2
)^(1/2)*3i + 2)*1i)/(20*(m*1i + 1i)^2) + (x*x^m*exp(a*3i)*(c*x^n)^(((- (2*
m)/n^2 - 1/n^2 - m^2/n^2)^(1/2)*3i)/2)*(2*m - n*(-(m + 1)^2/n^2)^(1/2)*3i
+ 2)*1i)/(20*(m*1i + 1i)^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.67

$$\int x^m \sin^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\frac{2x^m x\left(-15\cos\left(\frac{\log(x^n c)m+\log(x^n c)+2an}{2n}\right)\sin\left(\frac{\log(x^n c)m+\log(x^n c)+2an}{2n}\right)^2-6\cos\left(\frac{\log(x^n c)m+\log(x^n c)+2an}{2n}\right)+10\mathrm{si}\right.}{65m+65}$$

input
```
int(x^m*sin(a+1/2*(-(1+m)^2/n^2)^(1/2)*log(c*x^n))^3,x)
```

output
```
(2*x**m*x*( - 15*cos((log(x**n*c)*m + log(x**n*c) + 2*a*n)/(2*n))*sin((log
(x**n*c)*m + log(x**n*c) + 2*a*n)/(2*n))**2 - 6*cos((log(x**n*c)*m + log(x
**n*c) + 2*a*n)/(2*n)) + 10*sin((log(x**n*c)*m + log(x**n*c) + 2*a*n)/(2*n
))**3 + 12*sin((log(x**n*c)*m + log(x**n*c) + 2*a*n)/(2*n))))/(65*(m + 1))
```

3.41 $\int x^2 \sin^3\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 172

$$\int x^2\sin^3\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx = -\frac{3}{16}e^{a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx^3\left(cx^n\right)^{-1/n} + \frac{3}{32}e^{-a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx^3\left(cx^n\right)^{\frac{1}{n}} - \frac{1}{48}e^{-3a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx^3\left(cx^n\right)^{3/n} + \frac{1}{8}e^{3a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx^3\left(cx^n\right)^{-3/n}\log(x)$$

output
```
-3/16*exp(a*(-1/n^2)^(1/2)*n)*(-1/n^2)^(1/2)*n*x^3/((c*x^n)^(1/n))+3/32*(-
1/n^2)^(1/2)*n*x^3*(c*x^n)^(1/n)/exp(a*(-1/n^2)^(1/2)*n)-1/48*(-1/n^2)^(1/
2)*n*x^3*(c*x^n)^(3/n)/exp(3*a*(-1/n^2)^(1/2)*n)+1/8*exp(3*a*(-1/n^2)^(1/2
)*n)*(-1/n^2)^(1/2)*n*x^3*ln(x)/((c*x^n)^(3/n))
```

Mathematica [F]

$$\int x^2 \sin^3\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\int x^2 \sin^3\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$

input
```
Integrate[x^2*Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]^3,x]
```

output
```
Integrate[x^2*Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]^3, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {4996, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin^3\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{x^3(cx^n)^{-3/n}\int (cx^n)^{\frac{3}{n}-1}\sin^3\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{4992}$$

$$\frac{1}{8}\sqrt{-\frac{1}{n^2}}x^3(cx^n)^{-3/n}\int \left(-3e^{a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{2}{n}-1}+3e^{-a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{4}{n}-1}-e^{-3a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{6}{n}-1}+\frac{e^{3a\sqrt{-\frac{1}{n^2}}n}x^{-n}}{c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\sqrt{-\frac{1}{n^2}}x^3(cx^n)^{-3/n}\left(-\frac{3}{2}ne^{a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{2/n}+\frac{3}{4}ne^{-a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{4/n}-\frac{1}{6}ne^{-3a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{6/n}+e^{3a\sqrt{-\frac{1}{n^2}}n}\log\right.$$

input
```
Int[x^2*Sin[a + Sqrt[-n^(-2)]*Log[c*x^n]]^3,x]
```

output
```
(Sqrt[-n^(-2)]*x^3*((-3*E^(a*Sqrt[-n^(-2)]*n)*n*(c*x^n)^(2/n))/2 + (3*n*(c
*x^n)^(4/n))/(4*E^(a*Sqrt[-n^(-2)]*n)) - (n*(c*x^n)^(6/n))/(6*E^(3*a*Sqrt[
-n^(-2)]*n)) + E^(3*a*Sqrt[-n^(-2)]*n)*Log[c*x^n]))/(8*(c*x^n)^(3/n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^2\sin\left(a+\sqrt{-\frac{1}{n^2}}\,\ln\left(c\,x^n\right)\right)^3dx$$

input
```
int(x^2*sin(a+(-1/n^2)^(1/2)*ln(c*x^n))^3,x)
```

output
```
int(x^2*sin(a+(-1/n^2)^(1/2)*ln(c*x^n))^3,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.48

$$\int x^2 \sin^3\left(a + \sqrt{-\frac{1}{n^2}} \log\left(cx^n\right)\right) dx$$
$$= \frac{1}{96}\left(-2i\,x^6 + 9i\,x^4 e^{\left(\frac{2\,(i\,an - \log(c))}{n}\right)} - 18i\,x^2 e^{\left(\frac{4\,(i\,an - \log(c))}{n}\right)} + 12i\,e^{\left(\frac{6\,(i\,an - \log(c))}{n}\right)} \log\left(x\right)\right) e^{\left(-\frac{3\,(i\,an - \log(c))}{n}\right)}$$

input
```
integrate(x^2*sin(a+(-1/n^2)^(1/2)*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
1/96*(-2*I*x^6 + 9*I*x^4*e^(2*(I*a*n - log(c))/n) - 18*I*x^2*e^(4*(I*a*n -
 log(c))/n) + 12*I*e^(6*(I*a*n - log(c))/n)*log(x))*e^(-3*(I*a*n - log(c))
/n)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \sin^3\left(a + \sqrt{-\frac{1}{n^2}} \log\left(cx^n\right)\right) dx = \text{Timed out}$$

input
```
integrate(x**2*sin(a+(-1/n**2)**(1/2)*ln(c*x**n))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.52

$$\int x^2 \sin^3\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\frac{18\,c^{\frac{2}{n}}x^3\sin\left(a\right)-12\,(x^n)^{\left(\frac{1}{n}\right)}\log\left(x\right)\sin\left(3\,a\right)-\left(2\,c^{\frac{6}{n}}x^6\sin\left(3\,a\right)-9\,c^{\frac{4}{n}}x^4\sin\left(a\right)\right)(x^n)^{\left(\frac{1}{n}\right)}}{96\,c^{\frac{3}{n}}\,(x^n)^{\left(\frac{1}{n}\right)}}$$

input
```
integrate(x^2*sin(a+(-1/n^2)^(1/2)*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
1/96*(18*c^(2/n)*x^3*sin(a) - 12*(x^n)^(1/n)*log(x)*sin(3*a) - (2*c^(6/n)*
x^6*sin(3*a) - 9*c^(4/n)*x^4*sin(a))*(x^n)^(1/n))/(c^(3/n)*(x^n)^(1/n))
```

Giac [F(-2)]

Exception generated.

$$\int x^2 \sin^3\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\text{Exception raised: NotImplementedError}$$

input
```
integrate(x^2*sin(a+(-1/n^2)^(1/2)*log(c*x^n))^3,x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: ((-9
*i)*sageVARn^4*sageVARx^3*exp((-3*i)*sageVARa)*exp((3*sageVARn*abs(sageVAR
n)*ln(sageVARx)+3*abs(sageVARn)*ln(sageVARc))/sageVARn^2)+27*i*sageVARn^4*
sageVARx^3*exp((-i)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sin^3\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\int x^2 \sin\left(a+\ln\left(c\,x^n\right)\,\sqrt{-\frac{1}{n^2}}\right)^3\,dx$$

input
```
int(x^2*sin(a + log(c*x^n)*(-1/n^2)^(1/2))^3,x)
```

output
```
int(x^2*sin(a + log(c*x^n)*(-1/n^2)^(1/2))^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.25

$$\int x^2 \sin^3\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$

$$=\frac{x^3\left(-12\cos\left(\frac{\log(x^n c)i+an}{n}\right)\log(x^n c)\sin\left(\frac{\log(x^n c)i+an}{n}\right)^2 i+3\cos\left(\frac{\log(x^n c)i+an}{n}\right)\log(x^n c)\,i-5\cos\left(\frac{\log(x^n c)i+a}{n}\right)\right.}{}$$

input
```
int(x^2*sin(a+(-1/n^2)^(1/2)*log(c*x^n))^3,x)
```

output
```
(x**3*( - 12*cos((log(x**n*c)*i + a*n)/n)*log(x**n*c)*sin((log(x**n*c)*i +
 a*n)/n)**2*i + 3*cos((log(x**n*c)*i + a*n)/n)*log(x**n*c)*i - 5*cos((log(
x**n*c)*i + a*n)/n)*sin((log(x**n*c)*i + a*n)/n)**2*i*n - cos((log(x**n*c)
*i + a*n)/n)*i*n + 12*log(x**n*c)*sin((log(x**n*c)*i + a*n)/n)**3 - 9*log(
x**n*c)*sin((log(x**n*c)*i + a*n)/n) + 9*sin((log(x**n*c)*i + a*n)/n)**3*n
))/(24*n)
```

3.42 $\int x \sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 178

$$\int x\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)\,dx=-\frac{9}{32}e^{a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx^2(cx^n)^{-\frac{2}{3}/n}+\frac{9}{64}e^{-a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx^2(cx^n)^{\frac{2}{3}/n}-\frac{1}{32}e^{-3a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx^2(cx^n)^{2/n}+\frac{1}{8}e^{3a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx^2(cx^n)^{-2/n}\log(x)$$

output
```
-9/32*exp(a*(-1/n^2)^(1/2)*n)*(-1/n^2)^(1/2)*n*x^2/((c*x^n)^(2/3/n))+9/64*
(-1/n^2)^(1/2)*n*x^2*(c*x^n)^(2/3/n)/exp(a*(-1/n^2)^(1/2)*n)-1/32*(-1/n^2)
^(1/2)*n*x^2*(c*x^n)^(2/n)/exp(3*a*(-1/n^2)^(1/2)*n)+1/8*exp(3*a*(-1/n^2)^
(1/2)*n)*(-1/n^2)^(1/2)*n*x^2*ln(x)/((c*x^n)^(2/n))
```

Mathematica [F]

$$\int x\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\int x\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$

input
```
Integrate[x*Sin[a + (2*Sqrt[-n^(-2)]*Log[c*x^n])/3]^3,x]
```

output
```
Integrate[x*Sin[a + (2*Sqrt[-n^(-2)]*Log[c*x^n])/3]^3, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {4996, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{x^2(cx^n)^{-2/n}\int(cx^n)^{\frac{2}{n}-1}\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{4992}$$

$$\frac{1}{8}\sqrt{-\frac{1}{n^2}}x^2(cx^n)^{-2/n}\int\left(-3e^{a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{4}{3n}-1}+3e^{-a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{8}{3n}-1}-e^{-3a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{4}{n}-1}+\frac{e^{3a\sqrt{-\frac{1}{n^2}}n}x^-}{c}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\sqrt{-\frac{1}{n^2}}x^2(cx^n)^{-2/n}\left(-\frac{9}{4}ne^{a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{4}{3}/n}+\frac{9}{8}ne^{-a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{8}{3}/n}-\frac{1}{4}ne^{-3a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{4/n}+e^{3a\sqrt{-\frac{1}{n^2}}n}\text{lo}\right.$$

input
```
Int[x*Sin[a + (2*Sqrt[-n^(-2)]*Log[c*x^n])/3]^3,x]
```

output
```
(Sqrt[-n^(-2)]*x^2*((-9*E^(a*Sqrt[-n^(-2)]*n)*n*(c*x^n)^(4/(3*n)))/4 + (9*
n*(c*x^n)^(8/(3*n)))/(8*E^(a*Sqrt[-n^(-2)]*n)) - (n*(c*x^n)^(4/n))/(4*E^(3
*a*Sqrt[-n^(-2)]*n)) + E^(3*a*Sqrt[-n^(-2)]*n)*Log[c*x^n]))/(8*(c*x^n)^(2/
n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x \sin\left(a + \frac{2\sqrt{-\frac{1}{n^2}}\,\ln\left(c\,x^n\right)}{3}\right)^3 dx$$

input
```
int(x*sin(a+2/3*(-1/n^2)^(1/2)*ln(c*x^n))^3,x)
```

output
```
int(x*sin(a+2/3*(-1/n^2)^(1/2)*ln(c*x^n))^3,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.47

$$\int x\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\frac{1}{64}\left(-2i\,x^4+9i\,x^{\frac{8}{3}}e^{\left(\frac{2\,(3i\,an-2\,\log(c))}{3\,n}\right)}-18i\,x^{\frac{4}{3}}e^{\left(\frac{4\,(3i\,an-2\,\log(c))}{3\,n}\right)}+24i\,e^{\left(\frac{2\,(3i\,an-2\,\log(c))}{n}\right)}\log\left(x^{\frac{1}{3}}\right)\right)e^{\left(-\frac{3i\,an-2\,\log(}{n}\right.}$$

input
```
integrate(x*sin(a+2/3*(-1/n^2)^(1/2)*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
1/64*(-2*I*x^4 + 9*I*x^(8/3)*e^(2/3*(3*I*a*n - 2*log(c))/n) - 18*I*x^(4/3)
*e^(4/3*(3*I*a*n - 2*log(c))/n) + 24*I*e^(2*(3*I*a*n - 2*log(c))/n)*log(x^
(1/3)))*e^(-(3*I*a*n - 2*log(c))/n)
```

Sympy [F(-1)]

Timed out.

$$\int x\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x*sin(a+2/3*(-1/n**2)**(1/2)*ln(c*x**n))**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.63

$$\int x\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx = \frac{9\,c^{\frac{10}{3\,n}}x^2(x^n)^{\frac{4}{3\,n}}\sin\left(a\right)-8\,c^{\frac{2}{3\,n}}(x^n)^{\frac{2}{3\,n}}\log\left(x\right)\sin\left(3\,a\right)+18\,c^{\frac{2}{n}}x^2\sin\left(a\right)-2\,c^{\frac{14}{3\,n}}e^{\left(\frac{2\,\log(x^n)}{3\,n}+4\,\log(x)\right)}\sin\left(3\,a\right)}{64\,c^{\frac{8}{3\,n}}(x^n)^{\frac{2}{3\,n}}}$$

input
```
integrate(x*sin(a+2/3*(-1/n^2)^(1/2)*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
1/64*(9*c^(10/3/n)*x^2*(x^n)^(4/3/n)*sin(a) - 8*c^(2/3/n)*(x^n)^(2/3/n)*lo
g(x)*sin(3*a) + 18*c^(2/n)*x^2*sin(a) - 2*c^(14/3/n)*e^(2/3*log(x^n)/n + 4
*log(x))*sin(3*a))/(c^(8/3/n)*(x^n)^(2/3/n))
```

Giac [F(-2)]

Exception generated.

$$\int x\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(x*sin(a+2/3*(-1/n^2)^(1/2)*log(c*x^n))^3,x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: ((-9
*i)*sageVARn^4*sageVARx^2*exp((-3*i)*sageVARa)*exp((2*sageVARn*abs(sageVAR
n)*ln(sageVARx)+2*abs(sageVARn)*ln(sageVARc))/sageVARn^2)+27*i*sageVARn^4*
sageVARx^2*exp((-i)
```

Mupad [B] (verification not implemented)

Time = 20.52 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.92

$$\int x\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx = -x^2\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{\left(c\,x^n\right)^{\frac{\sqrt{-\frac{1}{n^2}}\,2\mathrm{i}}{3}}}\left(\frac{9\,n\,\sqrt{-\frac{1}{n^2}}}{128}-\frac{27}{128}\mathrm{i}\right) - x^2\,\mathrm{e}^{a\,1\mathrm{i}}\,\left(c\,x^n\right)^{\frac{\sqrt{-\frac{1}{n^2}}\,2\mathrm{i}}{3}}\left(\frac{9\,n\,\sqrt{-\frac{1}{n^2}}}{128}+\frac{27}{128}\mathrm{i}\right) + \frac{x^2\,\mathrm{e}^{-a\,3\mathrm{i}}\,\frac{1}{\left(c\,x^n\right)^{\sqrt{-\frac{1}{n^2}}\,2\mathrm{i}}}}{16\,n\,\sqrt{-\frac{1}{n^2}}+16\mathrm{i}} + \frac{x^2\,\mathrm{e}^{a\,3\mathrm{i}}\,\left(c\,x^n\right)^{\sqrt{-\frac{1}{n^2}}\,2\mathrm{i}}}{16\,n\,\sqrt{-\frac{1}{n^2}}-16\mathrm{i}}$$

input
```
int(x*sin(a + (2*log(c*x^n)*(-1/n^2)^(1/2))/3)^3,x)
```

output
```
(x^2*exp(-a*3i)/(c*x^n)^((-1/n^2)^(1/2)*2i))/(16*n*(-1/n^2)^(1/2) + 16i) -
 x^2*exp(a*1i)*(c*x^n)^(((-1/n^2)^(1/2)*2i)/3)*((9*n*(-1/n^2)^(1/2))/128 +
 27i/128) - x^2*exp(-a*1i)/(c*x^n)^(((-1/n^2)^(1/2)*2i)/3)*((9*n*(-1/n^2)^
(1/2))/128 - 27i/128) + (x^2*exp(a*3i)*(c*x^n)^((-1/n^2)^(1/2)*2i))/(16*n*
(-1/n^2)^(1/2) - 16i)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.36

$$\int x\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$

$$= \frac{x^2\left(-8\cos\left(\frac{2\log(x^nc)i+3an}{3n}\right)\log(x^nc)\sin\left(\frac{2\log(x^nc)i+3an}{3n}\right)^2 i+2\cos\left(\frac{2\log(x^nc)i+3an}{3n}\right)\log(x^nc)\,i-5\cos\left(\frac{2\log(\right.\right.}{}$$

input
```
int(x*sin(a+2/3*(-1/n^2)^(1/2)*log(c*x^n))^3,x)
```

output

```
(x**2*( - 8*cos((2*log(x***n*c)*i + 3*a*n)/(3*n))*log(x***n*c)*sin((2*log(x*
*n*c)*i + 3*a*n)/(3*n))**2*i + 2*cos((2*log(x***n*c)*i + 3*a*n)/(3*n))*log(
x***n*c)*i - 5*cos((2*log(x***n*c)*i + 3*a*n)/(3*n))*sin((2*log(x***n*c)*i +
3*a*n)/(3*n))**2*i*n - cos((2*log(x***n*c)*i + 3*a*n)/(3*n))*i*n + 8*log(x*
*n*c)*sin((2*log(x***n*c)*i + 3*a*n)/(3*n))**3 - 6*log(x***n*c)*sin((2*log(x
**n*c)*i + 3*a*n)/(3*n)) + 9*sin((2*log(x***n*c)*i + 3*a*n)/(3*n))**3*n))/(
16*n)
```

3.43 $\int \sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx = -\frac{9}{16}e^{a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx(cx^n)^{-\frac{1}{3}/n} + \frac{9}{32}e^{-a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx(cx^n)^{\frac{1}{3}/n} - \frac{1}{16}e^{-3a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx(cx^n)^{\frac{1}{n}} + \frac{1}{8}e^{3a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}nx(cx^n)^{-1/n}\log(x)$$

output
```
-9/16*exp(a*(-1/n^2)^(1/2)*n)*(-1/n^2)^(1/2)*n*x/((c*x^n)^(1/3/n))+9/32*(-
1/n^2)^(1/2)*n*x*(c*x^n)^(1/3/n)/exp(a*(-1/n^2)^(1/2)*n)-1/16*(-1/n^2)^(1/
2)*n*x*(c*x^n)^(1/n)/exp(3*a*(-1/n^2)^(1/2)*n)+1/8*exp(3*a*(-1/n^2)^(1/2)*
n)*(-1/n^2)^(1/2)*n*x*ln(x)/((c*x^n)^(1/n))
```

Mathematica [F]

$$\int \sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)\,dx=\int \sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)\,dx$$

input
```
Integrate[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/3]^3,x]
```

output
```
Integrate[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/3]^3, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4986, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)\,dx$$

$$\Big\downarrow \text{4986}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{4992}$$

$$\frac{1}{8}\sqrt{-\frac{1}{n^2}}x(cx^n)^{-1/n}\int\left(-3e^{a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{2}{3n}-1}+3e^{-a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{4}{3n}-1}-e^{-3a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{2}{n}-1}+\frac{e^{3a\sqrt{-\frac{1}{n^2}}n}x^{-n}}{c}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\sqrt{-\frac{1}{n^2}}x(cx^n)^{-1/n}\left(-\frac{9}{2}ne^{a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{2}{3}/n}+\frac{9}{4}ne^{-a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{4}{3}/n}-\frac{1}{2}ne^{-3a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{2/n}+e^{3a\sqrt{-\frac{1}{n^2}}n}\log\right.$$

input
```
Int[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/3]^3,x]
```

output
```
(Sqrt[-n^(-2)]*x*((-9*E^(a*Sqrt[-n^(-2)]*n)*n*(c*x^n)^(2/(3*n)))/2 + (9*n*
(c*x^n)^(4/(3*n)))/(4*E^(a*Sqrt[-n^(-2)]*n)) - (n*(c*x^n)^(2/n))/(2*E^(3*a
*Sqrt[-n^(-2)]*n)) + E^(3*a*Sqrt[-n^(-2)]*n)*Log[c*x^n]))/(8*(c*x^n)^n^(-1
))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4986
```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

Maple [F]

$$\int \sin\left(a+\frac{\sqrt{-\frac{1}{n^2}}\,\ln\left(c\,x^n\right)}{3}\right)^3 dx$$

input
```
int(sin(a+1/3*(-1/n^2)^(1/2)*ln(c*x^n))^3,x)
```

output
```
int(sin(a+1/3*(-1/n^2)^(1/2)*ln(c*x^n))^3,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.50

$$\int \sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\frac{1}{32}\left(9i\,x^{\frac{4}{3}}e^{\left(\frac{2\,(3i\,an-\log(c))}{3\,n}\right)}-2i\,x^2+12i\,e^{\left(\frac{2\,(3i\,an-\log(c))}{n}\right)}\log\left(x^{\frac{1}{3}}\right)-18i\,x^{\frac{2}{3}}e^{\left(\frac{4\,(3i\,an-\log(c))}{3\,n}\right)}\right)e^{\left(-\frac{3i\,an-\log(c)}{n}\right)}$$

input
```
integrate(sin(a+1/3*(-1/n^2)^(1/2)*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
1/32*(9*I*x^(4/3)*e^(2/3*(3*I*a*n - log(c))/n) - 2*I*x^2 + 12*I*e^(2*(3*I*
a*n - log(c))/n)*log(x^(1/3)) - 18*I*x^(2/3)*e^(4/3*(3*I*a*n - log(c))/n))
*e^(-(3*I*a*n - log(c))/n)
```

Sympy [F]

$$\int \sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\int \sin^3\left(a+\frac{\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)}{3}\right)\,dx$$

input
```
integrate(sin(a+1/3*(-1/n**2)**(1/2)*ln(c*x**n))**3,x)
```

output
```
Integral(sin(a + sqrt(-1/n**2)*log(c*x**n)/3)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.63

$$\int \sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)dx =$$

$$-\frac{4\,c^{\frac{1}{3\,n}}(x^n)^{\frac{1}{3\,n}}\log(x)\sin(3\,a)-9\,c^{\frac{5}{3\,n}}x(x^n)^{\frac{2}{3\,n}}\sin(a)+2\,c^{\frac{7}{3\,n}}e^{\left(\frac{\log(x^n)}{3\,n}+2\,\log(x)\right)}\sin(3\,a)-18\,c^{\left(\frac{1}{n}\right)}x\sin(a)}{32\,c^{\frac{4}{3\,n}}(x^n)^{\frac{1}{3\,n}}}$$

input
```
integrate(sin(a+1/3*(-1/n^2)^(1/2)*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
-1/32*(4*c^(1/3/n)*(x^n)^(1/3/n)*log(x)*sin(3*a) - 9*c^(5/3/n)*x*(x^n)^(2/
3/n)*sin(a) + 2*c^(7/3/n)*e^(1/3*log(x^n)/n + 2*log(x))*sin(3*a) - 18*c^(1
/n)*x*sin(a))/(c^(4/3/n)*(x^n)^(1/3/n))
```

Giac [F(-2)]

Exception generated.

$$\int \sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(sin(a+1/3*(-1/n^2)^(1/2)*log(c*x^n))^3,x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: ((-9
*i)*sageVARn^4*sageVARx*exp((-3*i)*sageVARa)*exp((sageVARn*abs(sageVARn)*l
n(sageVARx)+abs(sageVARn)*ln(sageVARc))/sageVARn^2)+27*i*sageVARn^4*sageVA
Rx*exp((-i)*sageVAR
```

Mupad [B] (verification not implemented)

Time = 20.92 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.92

$$\int \sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx = -x\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{\left(c\,x^n\right)^{\frac{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}{3}}}\,\left(\frac{9\,n\,\sqrt{-\frac{1}{n^2}}}{64}-\frac{27}{64}\mathrm{i}\right) - x\,\mathrm{e}^{a\,1\mathrm{i}}\,\left(c\,x^n\right)^{\frac{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}{3}}\,\left(\frac{9\,n\,\sqrt{-\frac{1}{n^2}}}{64}+\frac{27}{64}\mathrm{i}\right) + \frac{x\,\mathrm{e}^{-a\,3\mathrm{i}}\,\frac{1}{\left(c\,x^n\right)^{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}}}{8\,n\,\sqrt{-\frac{1}{n^2}}+8\mathrm{i}} + \frac{x\,\mathrm{e}^{a\,3\mathrm{i}}\,\left(c\,x^n\right)^{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}}{8\,n\,\sqrt{-\frac{1}{n^2}}-8\mathrm{i}}$$

input
```
int(sin(a + (log(c*x^n)*(-1/n^2)^(1/2))/3)^3,x)
```

output
```
(x*exp(-a*3i)/(c*x^n)^((-1/n^2)^(1/2)*1i))/(8*n*(-1/n^2)^(1/2) + 8i) - x*e
xp(a*1i)*(c*x^n)^(((-1/n^2)^(1/2)*1i)/3)*((9*n*(-1/n^2)^(1/2))/64 + 27i/64
) - x*exp(-a*1i)/(c*x^n)^(((-1/n^2)^(1/2)*1i)/3)*((9*n*(-1/n^2)^(1/2))/64
- 27i/64) + (x*exp(a*3i)*(c*x^n)^((-1/n^2)^(1/2)*1i))/(8*n*(-1/n^2)^(1/2)
- 8i)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.37

$$\int \sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx = \frac{x\left(-4\cos\left(\frac{\log(x^nc)i+3an}{3n}\right)\log(x^nc)\sin\left(\frac{\log(x^nc)i+3an}{3n}\right)^2 i+\cos\left(\frac{\log(x^nc)i+3an}{3n}\right)\log(x^nc)\,i-5\cos\left(\frac{\log(x^nc)i+3a}{3n}\right.\right.}{}$$

input
```
int(sin(a+1/3*(-1/n^2)^(1/2)*log(c*x^n))^3,x)
```

output

```
(x*( - 4*cos((log(x**n*c)*i + 3*a*n)/(3*n))*log(x**n*c)*sin((log(x**n*c)*i
 + 3*a*n)/(3*n))**2*i + cos((log(x**n*c)*i + 3*a*n)/(3*n))*log(x**n*c)*i -
 5*cos((log(x**n*c)*i + 3*a*n)/(3*n))*sin((log(x**n*c)*i + 3*a*n)/(3*n))**
2*i*n - cos((log(x**n*c)*i + 3*a*n)/(3*n))*i*n + 4*log(x**n*c)*sin((log(x*
*n*c)*i + 3*a*n)/(3*n))**3 - 3*log(x**n*c)*sin((log(x**n*c)*i + 3*a*n)/(3*
n)) + 9*sin((log(x**n*c)*i + 3*a*n)/(3*n))**3*n))/(8*n)
```

3.44 $\int \frac{\sin^3(a)}{x}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 7

$$\int \frac{\sin^3(a)}{x}\,dx = \log(x)\sin^3(a)$$

output
```
ln(x)*sin(a)^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(a)}{x}\,dx = \log(x)\sin^3(a)$$

input
```
Integrate[Sin[a]^3/x,x]
```

output
```
Log[x]*Sin[a]^3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{14\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a)}{x} \, dx$$
$$\Big\downarrow \text{14}$$
$$\sin^3(a) \log(x)$$

input
```
Int[Sin[a]^3/x,x]
```

output
```
Log[x]*Sin[a]^3
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
default	$\ln(x)\sin(a)^3$	8
norman	$\ln(x)\sin(a)^3$	8
risch	$\ln(x)\sin(a)^3$	8
parallelrisch	$\ln(x)\sin(a)^3$	8

input
```
int(sin(a)^3/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*sin(a)^3
```

Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int \frac{\sin^3(a)}{x}\,dx = -\left(\cos\left(a\right)^2 - 1\right)\log\left(x\right)\sin\left(a\right)$$

input
```
integrate(sin(a)^3/x,x, algorithm="fricas")
```

output
```
-(cos(a)^2 - 1)*log(x)*sin(a)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(a)}{x}\,dx = \log\left(x\right)\sin^3\left(a\right)$$

input
```
integrate(sin(a)**3/x,x)
```

output
```
log(x)*sin(a)**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(a)}{x}\,dx = \log\left(x\right)\sin\left(a\right)^3$$

input
```
integrate(sin(a)^3/x,x, algorithm="maxima")
```

output
```
log(x)*sin(a)^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int \frac{\sin^3(a)}{x}\,dx = \log(|x|)\sin(a)^3$$

input
```
integrate(sin(a)^3/x,x, algorithm="giac")
```

output
```
log(abs(x))*sin(a)^3
```

Mupad [B] (verification not implemented)

Time = 20.15 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(a)}{x}\,dx = \sin(a)^3\,\ln(x)$$

input
```
int(sin(a)^3/x,x)
```

output
```
sin(a)^3*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(a)}{x}\,dx = \log(x)\sin(a)^3$$

input
```
int(sin(a)^3/x,x)
```

output
```
log(x)*sin(a)**3
```

3.45 $\int \frac{\sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 176

$$\int \frac{\sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^2}\,dx = -\frac{e^{3a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}n(cx^n)^{-1/n}}{16x} + \frac{9e^{a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}n(cx^n)^{-\frac{1}{3}/n}}{32x} - \frac{9e^{-a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}n(cx^n)^{\frac{1}{3}/n}}{16x} - \frac{e^{-3a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}n(cx^n)^{\frac{1}{n}}\log(x)}{8x}$$

output
```
-1/16*exp(3*a*(-1/n^2)^(1/2)*n)*(-1/n^2)^(1/2)*n/x/((c*x^n)^(1/n))+9/32*ex
p(a*(-1/n^2)^(1/2)*n)*(-1/n^2)^(1/2)*n/x/((c*x^n)^(1/3/n))-9/16*(-1/n^2)^(
1/2)*n*(c*x^n)^(1/3/n)/exp(a*(-1/n^2)^(1/2)*n)/x-1/8*(-1/n^2)^(1/2)*n*(c*x
^n)^(1/n)*ln(x)/exp(3*a*(-1/n^2)^(1/2)*n)/x
```

Mathematica [F]

$$\int \frac{\sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx=\int \frac{\sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx$$

input
```
Integrate[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/3]^3/x^2,x]
```

output
```
Integrate[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/3]^3/x^2, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {4996, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{\left(cx^n\right)^{\frac{1}{n}}\int \left(cx^n\right)^{-1-\frac{1}{n}}\sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)d\left(cx^n\right)}{nx}$$

$$\Big\downarrow \text{4992}$$

$$-\frac{\sqrt{-\frac{1}{n^2}}\left(cx^n\right)^{\frac{1}{n}}\int \left(3e^{a\sqrt{-\frac{1}{n^2}}n}\left(cx^n\right)^{-1-\frac{4}{3n}}-3e^{-a\sqrt{-\frac{1}{n^2}}n}\left(cx^n\right)^{-1-\frac{2}{3n}}-e^{3a\sqrt{-\frac{1}{n^2}}n}\left(cx^n\right)^{-\frac{n+2}{n}}+\frac{e^{-3a\sqrt{-\frac{1}{n^2}}n}x^{-n}}{c}\right)d(}{8x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{-\frac{1}{n^2}}\left(cx^n\right)^{\frac{1}{n}}\left(\frac{1}{2}ne^{3a\sqrt{-\frac{1}{n^2}}n}\left(cx^n\right)^{-2/n}-\frac{9}{4}ne^{a\sqrt{-\frac{1}{n^2}}n}\left(cx^n\right)^{-\frac{4}{3}/n}+\frac{9}{2}ne^{-a\sqrt{-\frac{1}{n^2}}n}\left(cx^n\right)^{-\frac{2}{3}/n}+e^{-3a\sqrt{-\frac{1}{n^2}}n}\log(}{8x}$$

input
```
Int[Sin[a + (Sqrt[-n^(-2)]*Log[c*x^n])/3]^3/x^2,x]
```

output
```
-1/8*(Sqrt[-n^(-2)]*(c*x^n)^n^(-1)*((E^(3*a*Sqrt[-n^(-2)]*n)*n)/(2*(c*x^n)
^(2/n)) - (9*E^(a*Sqrt[-n^(-2)]*n)*n)/(4*(c*x^n)^(4/(3*n))) + (9*n)/(2*E^(
a*Sqrt[-n^(-2)]*n)*(c*x^n)^(2/(3*n))) + Log[c*x^n]/E^(3*a*Sqrt[-n^(-2)]*n)
))/x
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 32.05 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.61

method	result
parallelrisch	$\frac{12\left(n+\frac{5\ln\left(c\,x^{n}\right)}{12}\right)n\sqrt{-\frac{1}{n^{2}}}\,\tan\left(\frac{a}{2}+\sqrt{-\frac{1}{n^{2}}}\,\ln\left(\left(c\,x^{n}\right)^{\frac{1}{6}}\right)\right)^{6}+\left(-30\ln\left(c\,x^{n}\right)-42n\right)\tan\left(\frac{a}{2}+\sqrt{-\frac{1}{n^{2}}}\,\ln\left(\left(c\,x^{n}\right)^{\frac{1}{6}}\right)\right)^{5}-75\sqrt{-\frac{1}{n^{2}}}}{}$

input
```
int(sin(a+1/3*(-1/n^2)^(1/2)*ln(c*x^n))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/40*(12*(n+5/12*ln(c*x^n))*n*(-1/n^2)^(1/2)*tan(1/2*a+(-1/n^2)^(1/2)*ln((
c*x^n)^(1/6)))^6+(-30*ln(c*x^n)-42*n)*tan(1/2*a+(-1/n^2)^(1/2)*ln((c*x^n)^
(1/6)))^5-75*(-1/n^2)^(1/2)*tan(1/2*a+(-1/n^2)^(1/2)*ln((c*x^n)^(1/6)))^4*
ln(c*x^n)*n+(100*ln(c*x^n)-220*n)*tan(1/2*a+(-1/n^2)^(1/2)*ln((c*x^n)^(1/6
)))^3+75*(-1/n^2)^(1/2)*tan(1/2*a+(-1/n^2)^(1/2)*ln((c*x^n)^(1/6)))^2*ln(c
*x^n)*n+(-30*ln(c*x^n)-42*n)*tan(1/2*a+(-1/n^2)^(1/2)*ln((c*x^n)^(1/6)))-1
2*(n+5/12*ln(c*x^n))*n*(-1/n^2)^(1/2))/x/n/(1+tan(1/2*a+(-1/n^2)^(1/2)*ln(
(c*x^n)^(1/6)))^2)^3
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.49

$$\int \frac{\sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx$$
$$=\frac{\left(-12i\,x^2\log\left(x^{\frac{1}{3}}\right)-18i\,x^{\frac{4}{3}}e^{\left(\frac{2\,(3i\,an-\log(c))}{3\,n}\right)}+9i\,x^{\frac{2}{3}}e^{\left(\frac{4\,(3i\,an-\log(c))}{3\,n}\right)}-2i\,e^{\left(\frac{2\,(3i\,an-\log(c))}{n}\right)}\right)e^{\left(-\frac{3i\,an-\log(c)}{n}\right)}}{32\,x^2}$$

input
```
integrate(sin(a+1/3*(-1/n^2)^(1/2)*log(c*x^n))^3/x^2,x, algorithm="fricas"
)
```

output
```
1/32*(-12*I*x^2*log(x^(1/3)) - 18*I*x^(4/3)*e^(2/3*(3*I*a*n - log(c))/n) +
 9*I*x^(2/3)*e^(4/3*(3*I*a*n - log(c))/n) - 2*I*e^(2*(3*I*a*n - log(c))/n)
)*e^(-(3*I*a*n - log(c))/n)/x^2
```

Sympy [A] (verification not implemented)

Time = 53.37 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.02

$$\int \frac{\sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx = -\frac{9n\sqrt{-\frac{1}{n^2}}\cos\left(a+\frac{\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)}{3}\right)}{32x} - \frac{n\sqrt{-\frac{1}{n^2}}\cos\left(3a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{8x} - \frac{\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\cos\left(3a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{8x} - \frac{27\sin\left(a+\frac{\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)}{3}\right)}{32x} - \frac{\log\left(cx^n\right)\sin\left(3a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{8nx}$$

input
```
integrate(sin(a+1/3*(-1/n**2)**(1/2)*ln(c*x**n))**3/x**2,x)
```

output
```
-9*n*sqrt(-1/n**2)*cos(a + sqrt(-1/n**2)*log(c*x**n)/3)/(32*x) - n*sqrt(-1
/n**2)*cos(3*a + sqrt(-1/n**2)*log(c*x**n))/(8*x) - sqrt(-1/n**2)*log(c*x*
*n)*cos(3*a + sqrt(-1/n**2)*log(c*x**n))/(8*x) - 27*sin(a + sqrt(-1/n**2)*
log(c*x**n)/3)/(32*x) - log(c*x**n)*sin(3*a + sqrt(-1/n**2)*log(c*x**n))/(
8*n*x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.69

$$\int \frac{\sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx = -\frac{\left(4\,c^{\frac{7}{3\,n}}xe^{\left(\frac{\log(x^n)}{3\,n}+2\,\log(x)\right)}\log\left(x\right)\sin\left(3\,a\right) - 2\,c^{\frac{1}{3\,n}}x\left(x^n\right)^{\frac{1}{3\,n}}\sin\left(3\,a\right) + 9\,c^{\left(\frac{1}{n}\right)}x^2\sin\left(a\right) + 18\,c^{\frac{5}{3\,n}}e^{\left(\frac{2\,\log(x^n)}{3\,n}+\right.}}{32\,c^{\frac{4}{3\,n}}x}$$

input
```
integrate(sin(a+1/3*(-1/n^2)^(1/2)*log(c*x^n))^3/x^2,x, algorithm="maxima"
)
```

output
```
-1/32*(4*c^(7/3/n)*x*e^(1/3*log(x^n)/n + 2*log(x))*log(x)*sin(3*a) - 2*c^(
1/3/n)*x*(x^n)^(1/3/n)*sin(3*a) + 9*c^(1/n)*x^2*sin(a) + 18*c^(5/3/n)*e^(2
/3*log(x^n)/n + 2*log(x))*sin(a))*e^(-1/3*log(x^n)/n - 2*log(x))/(c^(4/3/n
)*x)
```

Giac [F]

$$\int \frac{\sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx=\int \frac{\sin\left(\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)+a\right)^3}{x^2}\,dx$$

input
```
integrate(sin(a+1/3*(-1/n^2)^(1/2)*log(c*x^n))^3/x^2,x, algorithm="giac")
```

output
```
integrate(sin(1/3*sqrt(-1/n^2)*log(c*x^n) + a)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^2}\,dx=\int \frac{\sin\left(a+\frac{\ln\left(c\,x^n\right)\sqrt{-\frac{1}{n^2}}}{3}\right)^3}{x^2}\,dx$$

input
```
int(sin(a + (log(c*x^n)*(-1/n^2)^(1/2))/3)^3/x^2,x)
```

output
```
int(sin(a + (log(c*x^n)*(-1/n^2)^(1/2))/3)^3/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.32

$$\int \frac{\sin^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^2}\,dx$$
$$= \frac{4\cos\left(\frac{\log(x^n c)i+3an}{3n}\right)\log(x^n c)\sin\left(\frac{\log(x^n c)i+3an}{3n}\right)^2 i-\cos\left(\frac{\log(x^n c)i+3an}{3n}\right)\log(x^n c)\,i-5\cos\left(\frac{\log(x^n c)i+3an}{3n}\right)\sin\cdots}{\cdots}$$

input
```
int(sin(a+1/3*(-1/n^2)^(1/2)*log(c*x^n))^3/x^2,x)
```

output
```
(4*cos((log(x**n*c)*i + 3*a*n)/(3*n))*log(x**n*c)*sin((log(x**n*c)*i + 3*a
*n)/(3*n))**2*i - cos((log(x**n*c)*i + 3*a*n)/(3*n))*log(x**n*c)*i - 5*cos
((log(x**n*c)*i + 3*a*n)/(3*n))*sin((log(x**n*c)*i + 3*a*n)/(3*n))**2*i*n
- cos((log(x**n*c)*i + 3*a*n)/(3*n))*i*n + 4*log(x**n*c)*sin((log(x**n*c)*
i + 3*a*n)/(3*n))**3 - 3*log(x**n*c)*sin((log(x**n*c)*i + 3*a*n)/(3*n)) -
9*sin((log(x**n*c)*i + 3*a*n)/(3*n))**3*n)/(8*n*x)
```

3.46 $\int \frac{\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)}{x^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 178

$$\int \frac{\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx = -\frac{e^{3a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}n(cx^n)^{-2/n}}{32x^2} + \frac{9e^{a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}n(cx^n)^{-\frac{2}{3}/n}}{64x^2} - \frac{9e^{-a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}n(cx^n)^{\frac{2}{3}/n}}{32x^2} - \frac{e^{-3a\sqrt{-\frac{1}{n^2}}n}\sqrt{-\frac{1}{n^2}}n(cx^n)^{2/n}\log(x)}{8x^2}$$

output
```
-1/32*exp(3*a*(-1/n^2)^(1/2)*n)*(-1/n^2)^(1/2)*n/x^2/((c*x^n)^(2/n))+9/64*
exp(a*(-1/n^2)^(1/2)*n)*(-1/n^2)^(1/2)*n/x^2/((c*x^n)^(2/3/n))-9/32*(-1/n^
2)^(1/2)*n*(c*x^n)^(2/3/n)/exp(a*(-1/n^2)^(1/2)*n)/x^2-1/8*(-1/n^2)^(1/2)*
n*(c*x^n)^(2/n)*ln(x)/exp(3*a*(-1/n^2)^(1/2)*n)/x^2
```

Mathematica [F]

$$\int \frac{\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx=\int \frac{\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx$$

input
```
Integrate[Sin[a + (2*Sqrt[-n^(-2)]*Log[c*x^n])/3]^3/x^3,x]
```

output
```
Integrate[Sin[a + (2*Sqrt[-n^(-2)]*Log[c*x^n])/3]^3/x^3, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.107$, Rules used = {4996, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{\left(cx^n\right)^{2/n}\int \left(cx^n\right)^{-1-\frac{2}{n}}\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)d\left(cx^n\right)}{nx^2}$$

$$\Big\downarrow \text{4992}$$

$$-\frac{\sqrt{-\frac{1}{n^2}}\left(cx^n\right)^{2/n}\int \left(3e^{a\sqrt{-\frac{1}{n^2}}n}\left(cx^n\right)^{-1-\frac{8}{3n}}-3e^{-a\sqrt{-\frac{1}{n^2}}n}\left(cx^n\right)^{-1-\frac{4}{3n}}-e^{3a\sqrt{-\frac{1}{n^2}}n}\left(cx^n\right)^{-\frac{n+4}{n}}+\frac{e^{-3a\sqrt{-\frac{1}{n^2}}n}x^{-n}}{c}\right)d}{8x^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{-\frac{1}{n^2}}\left(cx^n\right)^{2/n}\left(\frac{1}{4}ne^{3a\sqrt{-\frac{1}{n^2}}n}\left(cx^n\right)^{-4/n}-\frac{9}{8}ne^{a\sqrt{-\frac{1}{n^2}}n}\left(cx^n\right)^{-\frac{8}{3}/n}+\frac{9}{4}ne^{-a\sqrt{-\frac{1}{n^2}}n}\left(cx^n\right)^{-\frac{4}{3}/n}+e^{-3a\sqrt{-\frac{1}{n^2}}n}\log\right.}{8x^2}$$

input
```
Int[Sin[a + (2*Sqrt[-n^(-2)]*Log[c*x^n])/3]^3/x^3,x]
```

output
```
-1/8*(Sqrt[-n^(-2)]*(c*x^n)^(2/n)*((E^(3*a*Sqrt[-n^(-2)]*n)*n)/(4*(c*x^n)^
(4/n)) - (9*E^(a*Sqrt[-n^(-2)]*n)*n)/(8*(c*x^n)^(8/(3*n))) + (9*n)/(4*E^(a
*Sqrt[-n^(-2)]*n)*(c*x^n)^(4/(3*n))) + Log[c*x^n]/E^(3*a*Sqrt[-n^(-2)]*n))
)/x^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 61.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.76

method	result
parallelrisch	$\frac{-47n\left(n+\frac{40\ln\left(c\,x^{n}\right)}{47}\right)\sqrt{-\frac{1}{n^{2}}}\,\cos\left(2\sqrt{-\frac{1}{n^{2}}}\,\ln(c\,x^{n})+3a\right)+\left(-27n-40\ln(c\,x^{n})\right)\sin\left(2\sqrt{-\frac{1}{n^{2}}}\,\ln(c\,x^{n})+3a\right)-45n\left(\cos\left(a+\sqrt{\right.\right.}{320x^{2}n}$

input
```
int(sin(a+2/3*(-1/n^2)^(1/2)*ln(c*x^n))^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/320*(-47*n*(n+40/47*ln(c*x^n))*(-1/n^2)^(1/2)*cos(2*(-1/n^2)^(1/2)*ln(c*
x^n)+3*a)+(-27*n-40*ln(c*x^n))*sin(2*(-1/n^2)^(1/2)*ln(c*x^n)+3*a)-45*n*(c
os(a+(-1/n^2)^(1/2)*ln((c*x^n)^(2/3)))*n*(-1/n^2)^(1/2)+3*sin(a+(-1/n^2)^(
1/2)*ln((c*x^n)^(2/3)))))/x^2/n
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.49

$$\int \frac{\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx = \frac{\left(-24i\,x^4\log\left(x^{\frac{1}{3}}\right)-18i\,x^{\frac{8}{3}}e^{\left(\frac{2\,(3i\,an-2\,\log(c))}{3\,n}\right)}+9i\,x^{\frac{4}{3}}e^{\left(\frac{4\,(3i\,an-2\,\log(c))}{3\,n}\right)}-2i\,e^{\left(\frac{2\,(3i\,an-2\,\log(c))}{n}\right)}\right)e^{\left(-\frac{3i\,an-2\,\log(c)}{n}\right)}}{64\,x^4}$$

input
```
integrate(sin(a+2/3*(-1/n^2)^(1/2)*log(c*x^n))^3/x^3,x, algorithm="fricas"
)
```

output
```
1/64*(-24*I*x^4*log(x^(1/3)) - 18*I*x^(8/3)*e^(2/3*(3*I*a*n - 2*log(c))/n)
 + 9*I*x^(4/3)*e^(4/3*(3*I*a*n - 2*log(c))/n) - 2*I*e^(2*(3*I*a*n - 2*log(
c))/n))*e^(-(3*I*a*n - 2*log(c))/n)/x^4
```

Sympy [A] (verification not implemented)

Time = 64.67 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.03

$$\int \frac{\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx = -\frac{9n\sqrt{-\frac{1}{n^2}}\cos\left(a+\frac{2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)}{3}\right)}{64x^2} - \frac{\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\cos\left(3a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{8x^2} - \frac{27\sin\left(a+\frac{2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)}{3}\right)}{64x^2} + \frac{\sin\left(3a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{16x^2} - \frac{\log\left(cx^n\right)\sin\left(3a+2\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{8nx^2}$$

input
```
integrate(sin(a+2/3*(-1/n**2)**(1/2)*ln(c*x**n))**3/x**3,x)
```

output
```
-9*n*sqrt(-1/n**2)*cos(a + 2*sqrt(-1/n**2)*log(c*x**n)/3)/(64*x**2) - sqrt
(-1/n**2)*log(c*x**n)*cos(3*a + 2*sqrt(-1/n**2)*log(c*x**n))/(8*x**2) - 27
*sin(a + 2*sqrt(-1/n**2)*log(c*x**n)/3)/(64*x**2) + sin(3*a + 2*sqrt(-1/n*
*2)*log(c*x**n))/(16*x**2) - log(c*x**n)*sin(3*a + 2*sqrt(-1/n**2)*log(c*x
**n))/(8*n*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.72

$$\int \frac{\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx = -\frac{\left(8\,c^{\frac{14}{3\,n}}x^2e^{\left(\frac{2\,\log\left(x^n\right)}{3\,n}+4\,\log\left(x\right)\right)}\log\left(x\right)\sin\left(3\,a\right)+9\,c^{\frac{2}{n}}x^4\sin\left(a\right)-2\,c^{\frac{2}{3\,n}}x^2\left(x^n\right)^{\frac{2}{3\,n}}\sin\left(3\,a\right)+18\,c^{\frac{10}{3\,n}}e^{\left(\frac{4\,\log\left(x^n\right)}{3\,n}\right.}}{64\,c^{\frac{8}{3\,n}}x^2}$$

input
```
integrate(sin(a+2/3*(-1/n^2)^(1/2)*log(c*x^n))^3/x^3,x, algorithm="maxima"
)
```

output
```
-1/64*(8*c^(14/3/n)*x^2*e^(2/3*log(x^n)/n + 4*log(x))*log(x)*sin(3*a) + 9*
c^(2/n)*x^4*sin(a) - 2*c^(2/3/n)*x^2*(x^n)^(2/3/n)*sin(3*a) + 18*c^(10/3/n
)*e^(4/3*log(x^n)/n + 4*log(x))*sin(a))*e^(-2/3*log(x^n)/n - 4*log(x))/(c^
(8/3/n)*x^2)
```

Giac [F]

$$\int \frac{\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx=\int \frac{\sin\left(\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate(sin(a+2/3*(-1/n^2)^(1/2)*log(c*x^n))^3/x^3,x, algorithm="giac")
```

output
```
integrate(sin(2/3*sqrt(-1/n^2)*log(c*x^n) + a)^3/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx=\int \frac{\sin\left(a+\frac{2\,\ln\left(c\,x^n\right)\,\sqrt{-\frac{1}{n^2}}}{3}\right)^3}{x^3}\,dx$$

input
```
int(sin(a + (2*log(c*x^n)*(-1/n^2)^(1/2))/3)^3/x^3,x)
```

output
```
int(sin(a + (2*log(c*x^n)*(-1/n^2)^(1/2))/3)^3/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.36

$$\int \frac{\sin^3\left(a+\frac{2}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)}{x^3}\,dx$$
$$= \frac{8\cos\left(\frac{2\log(x^nc)i+3an}{3n}\right)\log(x^nc)\sin\left(\frac{2\log(x^nc)i+3an}{3n}\right)^2 i - 2\cos\left(\frac{2\log(x^nc)i+3an}{3n}\right)\log(x^nc)\,i - 5\cos\left(\frac{2\log(x^nc)i+3}{3n}\right.}{}$$

input
```
int(sin(a+2/3*(-1/n^2)^(1/2)*log(c*x^n))^3/x^3,x)
```

output
```
(8*cos((2*log(x**n*c)*i + 3*a*n)/(3*n))*log(x**n*c)*sin((2*log(x**n*c)*i +
 3*a*n)/(3*n))**2*i - 2*cos((2*log(x**n*c)*i + 3*a*n)/(3*n))*log(x**n*c)*i
 - 5*cos((2*log(x**n*c)*i + 3*a*n)/(3*n))*sin((2*log(x**n*c)*i + 3*a*n)/(3
*n))**2*i*n - cos((2*log(x**n*c)*i + 3*a*n)/(3*n))*i*n + 8*log(x**n*c)*sin
((2*log(x**n*c)*i + 3*a*n)/(3*n))**3 - 6*log(x**n*c)*sin((2*log(x**n*c)*i
+ 3*a*n)/(3*n)) - 9*sin((2*log(x**n*c)*i + 3*a*n)/(3*n))**3*n)/(16*n*x**2)
```

3.47 $\int x^m \sin\left(a+\frac{1}{2}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 112

$$\int x^m \sin\left(a+\frac{1}{2}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx = -\frac{e^{\frac{a(1+m)}{\sqrt{-(1+m)^2}}}x^{1+m}\left(cx^2\right)^{\frac{1+m}{2}}}{4\sqrt{-(1+m)^2}}+\frac{e^{\frac{a\sqrt{-(1+m)^2}}{1+m}}(1+m)x^{1+m}\left(cx^2\right)^{\frac{1}{2}(-1-m)}\log(x)}{2\sqrt{-(1+m)^2}}$$

output
```
-1/4*exp(a*(1+m)/(-(1+m)^2)^(1/2))*x^(1+m)*(c*x^2)^(1/2+1/2*m)/(-(1+m)^2)^
(1/2)+1/2*exp(a*(-(1+m)^2)^(1/2)/(1+m))*(1+m)*x^(1+m)*(c*x^2)^(-1/2-1/2*m)
*ln(x)/(-(1+m)^2)^(1/2)
```

Mathematica [F]

$$\int x^m \sin\left(a+\frac{1}{2}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx = \int x^m \sin\left(a+\frac{1}{2}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx$$

input
```
Integrate[x^m*Sin[a + (Sqrt[-(1 + m)^2]*Log[c*x^2])/2],x]
```

output
```
Integrate[x^m*Sin[a + (Sqrt[-(1 + m)^2]*Log[c*x^2])/2], x]
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {4996, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sin \left(a+\frac{1}{2}\sqrt{-(m+1)^2}\log \left(cx^2\right)\right) \, dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{1}{2}x^{m+1}\left(cx^2\right)^{\frac{1}{2}(-m-1)}\int \left(cx^2\right)^{\frac{m-1}{2}}\sin \left(a+\frac{1}{2}\sqrt{-(m+1)^2}\log \left(cx^2\right)\right)d\left(cx^2\right)$$

$$\Big\downarrow \text{4992}$$

$$\frac{(m+1)x^{m+1}\left(cx^2\right)^{\frac{1}{2}(-m-1)}\int \left(\frac{e^{\frac{a\sqrt{-(m+1)^2}}{m+1}}}{cx^2}-e^{\frac{a(m+1)}{\sqrt{-(m+1)^2}}}\left(cx^2\right)^m\right)d\left(cx^2\right)}{4\sqrt{-(m+1)^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(m+1)x^{m+1}\left(cx^2\right)^{\frac{1}{2}(-m-1)}\left(e^{\frac{a\sqrt{-(m+1)^2}}{m+1}}\log \left(cx^2\right)-\frac{e^{\frac{a(m+1)}{\sqrt{-(m+1)^2}}}\left(cx^2\right)^{m+1}}{m+1}\right)}{4\sqrt{-(m+1)^2}}$$

input
```
Int[x^m*Sin[a + (Sqrt[-(1 + m)^2]*Log[c*x^2])/2],x]
```

output
```
((1 + m)*x^(1 + m)*(c*x^2)^((-1 - m)/2)*(-((E^((a*(1 + m))/Sqrt[-(1 + m)^2
])*(c*x^2)^(1 + m))/(1 + m)) + E^((a*Sqrt[-(1 + m)^2])/(1 + m))*Log[c*x^2]
))/(4*Sqrt[-(1 + m)^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^m \sin\left(a+\frac{\sqrt{-\left(1+m\right)^2}\,\ln\left(c\,x^2\right)}{2}\right)dx$$

input
```
int(x^m*sin(a+1/2*(-(1+m)^2)^(1/2)*ln(c*x^2)),x)
```

output
```
int(x^m*sin(a+1/2*(-(1+m)^2)^(1/2)*ln(c*x^2)),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.46

$$\int x^m \sin\left(a+\frac{1}{2}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx = \frac{\left(i\,x^2x^{2\,m}-2\,(i\,m+i)e^{\left(-(m+1)\log(c)+2\,i\,a\right)}\log\left(x\right)\right)e^{\left(\frac{1}{2}\,(m+1)\log(c)-i\,a\right)}}{4\,(m+1)}$$

input
```
integrate(x^m*sin(a+1/2*(-(1+m)^2)^(1/2)*log(c*x^2)),x, algorithm="fricas"
)
```

output
```
1/4*(I*x^2*x^(2*m) - 2*(I*m + I)*e^(-(m + 1)*log(c) + 2*I*a)*log(x))*e^(1/
2*(m + 1)*log(c) - I*a)/(m + 1)
```

Sympy [F]

$$\int x^m \sin\left(a + \frac{1}{2}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)dx$$
$$= \int x^m \sin\left(a + \frac{\sqrt{-m^2-2m-1}\log\left(cx^2\right)}{2}\right)dx$$

input
```
integrate(x**m*sin(a+1/2*(-(1+m)**2)**(1/2)*ln(c*x**2)),x)
```

output
```
Integral(x**m*sin(a + sqrt(-m**2 - 2*m - 1)*log(c*x**2)/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.43

$$\int x^m \sin\left(a + \frac{1}{2}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)dx$$
$$= \frac{c^{m+1}x^2x^{2m}\sin(a) + 2(m\sin(a) + \sin(a))\log(x)}{4\left(c^{\frac{1}{2}m}m + c^{\frac{1}{2}m}\right)\sqrt{c}}$$

input
```
integrate(x^m*sin(a+1/2*(-(1+m)^2)^(1/2)*log(c*x^2)),x, algorithm="maxima"
)
```

output
```
1/4*(c^(m + 1)*x^2*x^(2*m)*sin(a) + 2*(m*sin(a) + sin(a))*log(x))/((c^(1/2
*m)*m + c^(1/2*m))*sqrt(c))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.69

$$\int x^m \sin\left(a+\frac{1}{2}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx =$$
$$-\frac{i\,mxx^m e^{\left(\frac{1}{2}\,|m+1|\log(c)+|m+1|\log(x)-i\,a\right)} - i\,xx^m|m+1|e^{\left(\frac{1}{2}\,|m+1|\log(c)+|m+1|\log(x)-i\,a\right)} - i\,mxx^m e^{\left(-\frac{1}{2}\,|m+1|\log(\right.}}{}$$

input
```
integrate(x^m*sin(a+1/2*(-(1+m)^2)^(1/2)*log(c*x^2)),x, algorithm="giac")
```

output
```
-1/2*(I*m*x*x^m*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*log(x) - I*a) - I*x*
x^m*abs(m + 1)*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*log(x) - I*a) - I*m*x
*x^m*e^(-1/2*abs(m + 1)*log(c) - abs(m + 1)*log(x) + I*a) - I*x*x^m*abs(m
+ 1)*e^(-1/2*abs(m + 1)*log(c) - abs(m + 1)*log(x) + I*a) + I*x*x^m*e^(1/2
*abs(m + 1)*log(c) + abs(m + 1)*log(x) - I*a) - I*x*x^m*e^(-1/2*abs(m + 1)
*log(c) - abs(m + 1)*log(x) + I*a))/((m + 1)^2 - m^2 - 2*m - 1)
```

Mupad [B] (verification not implemented)

Time = 21.49 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.24

$$\int x^m \sin\left(a+\frac{1}{2}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx$$
$$=\frac{\frac{1}{c^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{2}}}\,x\,x^m\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(x^2\right)}^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{2}}}\,1\mathrm{i}}{2\,m+2-\sqrt{-{\left(m+1\right)}^2}\,2\mathrm{i}}-\frac{c^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{2}}\,x\,x^m\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(x^2\right)}^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{2}}\,1\mathrm{i}}{2\,m+2+\sqrt{-{\left(m+1\right)}^2}\,2\mathrm{i}}$$

input
```
int(x^m*sin(a + (log(c*x^2)*(-(m + 1)^2)^(1/2))/2),x)
```

output
```
(1/c^(((- 2*m - m^2 - 1)^(1/2)*1i)/2)*x*x^m*exp(-a*1i)/(x^2)^(((- 2*m - m^
2 - 1)^(1/2)*1i)/2)*1i)/(2*m - (-(m + 1)^2)^(1/2)*2i + 2) - (c^(((- 2*m -
m^2 - 1)^(1/2)*1i)/2)*x*x^m*exp(a*1i)*(x^2)^(((- 2*m - m^2 - 1)^(1/2)*1i)/
2)*1i)/(2*m + (-(m + 1)^2)^(1/2)*2i + 2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.49

$$\int x^m \sin\left(a + \frac{1}{2}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right) dx = \frac{x^m x\left(-\cos\left(\frac{\log(c x^2)m}{2} + \frac{\log(c x^2)}{2} + a\right) + \sin\left(\frac{\log(c x^2)m}{2} + \frac{\log(c x^2)}{2} + a\right)\right)}{2m+2}$$

input
```
int(x^m*sin(a+1/2*(-(1+m)^2)^(1/2)*log(c*x^2)),x)
```

output
```
(x**m*x*( - cos((log(c*x**2)*m + log(c*x**2) + 2*a)/2) + sin((log(c*x**2)*
m + log(c*x**2) + 2*a)/2)))/(2*(m + 1))
```

3.48 $\int \sin \left(a+\frac{1}{2}i \log \left(cx^2\right)\right) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \sin \left(a+\frac{1}{2}i \log \left(cx^2\right)\right) \, dx=\frac{ice^{-ia}x^3}{4\sqrt{cx^2}}-\frac{ie^{ia}x\log(x)}{2\sqrt{cx^2}}$$

output
```
1/4*I*c*x^3/exp(I*a)/(c*x^2)^(1/2)-1/2*I*exp(I*a)*x*ln(x)/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \sin \left(a+\frac{1}{2}i \log \left(cx^2\right)\right) \, dx=\frac{x\left(i\cos(a)\left(cx^2-2\log(x)\right)+\left(cx^2+2\log(x)\right)\sin(a)\right)}{4\sqrt{cx^2}}$$

input
```
Integrate[Sin[a + (I/2)*Log[c*x^2]],x]
```

output
```
(x*(I*Cos[a]*(c*x^2 - 2*Log[x]) + (c*x^2 + 2*Log[x])*Sin[a]))/(4*Sqrt[c*x^
2])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4986, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(a+\frac{1}{2}i\log\left(cx^2\right)\right)dx$$
$$\Big\downarrow \text{4986}$$
$$\frac{x\int \frac{\sin\left(a+\frac{1}{2}i\log\left(cx^2\right)\right)}{\sqrt{cx^2}}d\left(cx^2\right)}{2\sqrt{cx^2}}$$
$$\Big\downarrow \text{4992}$$
$$-\frac{ix\int \left(\frac{e^{ia}}{cx^2}-e^{-ia}\right)d\left(cx^2\right)}{4\sqrt{cx^2}}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{ix\left(e^{ia}\log\left(cx^2\right)-e^{-ia}cx^2\right)}{4\sqrt{cx^2}}$$

input
```
Int[Sin[a + (I/2)*Log[c*x^2]],x]
```

output
```
((-1/4*I)*x*(-((c*x^2)/E^(I*a)) + E^(I*a)*Log[c*x^2]))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4986
```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(40) = 80$.

Time = 0.44 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.04

method	result	size
norman	$\frac{\frac{ix}{2}-\frac{ix\tan\left(\frac{a}{2}+\frac{i\ln\left(c\,x^{2}\right)}{4}\right)^{2}}{2}+\frac{x\ln\left(c\,x^{2}\right)\tan\left(\frac{a}{2}+\frac{i\ln\left(c\,x^{2}\right)}{4}\right)}{2}-\frac{ix\ln\left(c\,x^{2}\right)}{4}+\frac{ix\ln\left(c\,x^{2}\right)\tan\left(\frac{a}{2}+\frac{i\ln\left(c\,x^{2}\right)}{4}\right)^{2}}{4}}{1+\tan\left(\frac{a}{2}+\frac{i\ln\left(c\,x^{2}\right)}{4}\right)^{2}}$	106

input
```
int(sin(a+1/2*I*ln(c*x^2)),x,method=_RETURNVERBOSE)
```

output
```
(1/2*I*x-1/2*I*x*tan(1/2*a+1/4*I*ln(c*x^2))^2+1/2*x*ln(c*x^2)*tan(1/2*a+1/
4*I*ln(c*x^2))-1/4*I*x*ln(c*x^2)+1/4*I*x*ln(c*x^2)*tan(1/2*a+1/4*I*ln(c*x^
2))^2)/(1+tan(1/2*a+1/4*I*ln(c*x^2))^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.46

$$\int \sin\left(a+\frac{1}{2}i\log\left(cx^2\right)\right)\,dx=\frac{\left(i\,cx^2-2i\,e^{(2i\,a)}\log\left(x\right)\right)e^{(-i\,a)}}{4\sqrt{c}}$$

input
```
integrate(sin(a+1/2*I*log(c*x^2)),x, algorithm="fricas")
```

output
```
1/4*(I*c*x^2 - 2*I*e^(2*I*a)*log(x))*e^(-I*a)/sqrt(c)
```

Sympy [F]

$$\int \sin\left(a + \frac{1}{2}i\log\left(cx^2\right)\right)\,dx = \int \sin\left(a + \frac{i\log\left(cx^2\right)}{2}\right)dx$$

input
```
integrate(sin(a+1/2*I*ln(c*x**2)),x)
```

output
```
Integral(sin(a + I*log(c*x**2)/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.60

$$\int \sin\left(a + \frac{1}{2}i\log\left(cx^2\right)\right)\,dx = \frac{cx^2(i\,\cos\left(a\right) + \sin\left(a\right)) - 2\left(i\,\cos\left(a\right) - \sin\left(a\right)\right)\log\left(x\right)}{4\sqrt{c}}$$

input
```
integrate(sin(a+1/2*I*log(c*x^2)),x, algorithm="maxima")
```

output
```
1/4*(c*x^2*(I*cos(a) + sin(a)) - 2*(I*cos(a) - sin(a))*log(x))/sqrt(c)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.46

$$\int \sin\left(a + \frac{1}{2}i\log\left(cx^2\right)\right)\,dx = -\frac{-i\,cx^2e^{(-i\,a)} + 2i\,e^{(i\,a)}\log\left(x\right)}{4\sqrt{c}}$$

input
```
integrate(sin(a+1/2*I*log(c*x^2)),x, algorithm="giac")
```

output
```
-1/4*(-I*c*x^2*e^(-I*a) + 2*I*e^(I*a)*log(x))/sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int \sin \left(a+\frac{1}{2}i\log \left(cx^2\right)\right) dx = \int \sin \left(a+\frac{\ln \left(c\,x^2\right)\,1\mathrm{i}}{2}\right) dx$$

input
```
int(sin(a + (log(c*x^2)*1i)/2),x)
```

output
```
int(sin(a + (log(c*x^2)*1i)/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \sin \left(a+\frac{1}{2}i\log \left(cx^2\right)\right) dx = \frac{x\left(-\cos\left(\frac{\log(c\,x^2)\,i}{2}+a\right)\log(c\,x^2)\,i+2\cos\left(\frac{\log(c\,x^2)\,i}{2}+a\right)i+\log(c\,x^2)\sin\left(\frac{\log(c\,x^2)\,i}{2}+a\right)\right)}{4}$$

input
```
int(sin(a+1/2*I*log(c*x^2)),x)
```

output
```
(x*( - cos((log(c*x**2)*i + 2*a)/2)*log(c*x**2)*i + 2*cos((log(c*x**2)*i +
 2*a)/2)*i + log(c*x**2)*sin((log(c*x**2)*i + 2*a)/2)))/4
```

3.49 $\int x^m \sin^2\left(a+\frac{1}{4}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 106

$$\int x^m \sin^2\left(a+\frac{1}{4}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx = \frac{x^{1+m}}{2(1+m)}-\frac{e^{\frac{2a(1+m)}{\sqrt{-(1+m)^2}}}x^{1+m}\left(cx^2\right)^{\frac{1+m}{2}}}{8(1+m)}-\frac{1}{4}e^{-\frac{2a(1+m)}{\sqrt{-(1+m)^2}}}x^{1+m}\left(cx^2\right)^{\frac{1}{2}(-1-m)}\log(x)$$

output
```
x^(1+m)/(2+2*m)-exp(2*a*(1+m)/(-(1+m)^2)^(1/2))*x^(1+m)*(c*x^2)^(1/2+1/2*m
)/(8+8*m)-1/4*x^(1+m)*(c*x^2)^(-1/2-1/2*m)*ln(x)/exp(2*a*(1+m)/(-(1+m)^2)^
(1/2))
```

Mathematica [F]

$$\int x^m \sin^2\left(a+\frac{1}{4}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx = \int x^m \sin^2\left(a+\frac{1}{4}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx$$

input
```
Integrate[x^m*Sin[a + (Sqrt[-(1 + m)^2]*Log[c*x^2])/4]^2,x]
```

output
```
Integrate[x^m*Sin[a + (Sqrt[-(1 + m)^2]*Log[c*x^2])/4]^2, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4996, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sin^2\left(a+\frac{1}{4}\sqrt{-(m+1)^2}\log\left(cx^2\right)\right)dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{1}{2}x^{m+1}\left(cx^2\right)^{\frac{1}{2}(-m-1)}\int \left(cx^2\right)^{\frac{m-1}{2}}\sin^2\left(a+\frac{1}{4}\sqrt{-(m+1)^2}\log\left(cx^2\right)\right)d\left(cx^2\right)$$

$$\Big\downarrow \text{4992}$$

$$-\frac{1}{8}x^{m+1}\left(cx^2\right)^{\frac{1}{2}(-m-1)}\int \left(-2\left(cx^2\right)^{\frac{m-1}{2}}+e^{\frac{2a(m+1)}{\sqrt{-(m+1)^2}}}\left(cx^2\right)^m+\frac{e^{-\frac{2a(m+1)}{\sqrt{-(m+1)^2}}}}{cx^2}\right)d\left(cx^2\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}x^{m+1}\left(cx^2\right)^{\frac{1}{2}(-m-1)}\left(-\frac{e^{\frac{2a(m+1)}{\sqrt{-(m+1)^2}}}\left(cx^2\right)^{m+1}}{m+1}-e^{-\frac{2a(m+1)}{\sqrt{-(m+1)^2}}}\log\left(cx^2\right)+\frac{4\left(cx^2\right)^{\frac{m+1}{2}}}{m+1}\right)$$

input
```
Int[x^m*Sin[a + (Sqrt[-(1 + m)^2]*Log[c*x^2])/4]^2,x]
```

output
```
(x^(1 + m)*(c*x^2)^((-1 - m)/2)*((4*(c*x^2)^((1 + m)/2))/(1 + m) - (E^((2*
a*(1 + m))/Sqrt[-(1 + m)^2])*(c*x^2)^(1 + m))/(1 + m) - Log[c*x^2]/E^((2*a
*(1 + m))/Sqrt[-(1 + m)^2])))/8
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^{m} \sin\left(a+\frac{\sqrt{-\left(1+m\right)^{2}}\, \ln\left(c\,x^{2}\right)}{4}\right)^{2} dx$$

input
```
int(x^m*sin(a+1/4*(-(1+m)^2)^(1/2)*ln(c*x^2))^2,x)
```

output
```
int(x^m*sin(a+1/4*(-(1+m)^2)^(1/2)*ln(c*x^2))^2,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.71

$$\int x^m \sin^2\left(a+\frac{1}{4}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx =$$
$$-\frac{\left(2\,(m+1)e^{\left(-(m+1)\log(c)-2\,(m+1)\log(x)+4i\,a\right)}\log\left(x\right)-4\,e^{\left(-\frac{1}{2}\,(m+1)\log(c)-(m+1)\log(x)+2i\,a\right)}+1\right)e^{\left(\frac{1}{2}\,(m+1)\log(c)\right.}}{8\,(m+1)}$$

input
```
integrate(x^m*sin(a+1/4*(-(1+m)^2)^(1/2)*log(c*x^2))^2,x, algorithm="frica
s")
```

output
```
-1/8*(2*(m + 1)*e^(-(m + 1)*log(c) - 2*(m + 1)*log(x) + 4*I*a)*log(x) - 4*
e^(-1/2*(m + 1)*log(c) - (m + 1)*log(x) + 2*I*a) + 1)*e^(1/2*(m + 1)*log(c
) + 2*(m + 1)*log(x) - 2*I*a)/(m + 1)
```

Sympy [F]

$$\int x^m \sin^2\left(a+\frac{1}{4}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx$$
$$=\int x^m \sin^2\left(a+\frac{\sqrt{-m^2-2m-1}\log\left(cx^2\right)}{4}\right)dx$$

input
```
integrate(x**m*sin(a+1/4*(-(1+m)**2)**(1/2)*ln(c*x**2))**2,x)
```

output
```
Integral(x**m*sin(a + sqrt(-m**2 - 2*m - 1)*log(c*x**2)/4)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.26

$$\int x^m \sin^2\left(a+\frac{1}{4}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx =$$
$$-\frac{c^{m+1}x^2x^{2m}\cos(2a)-4\left(\cos(2a)^2+\sin(2a)^2\right)c^{\frac{1}{2}m+\frac{1}{2}}xx^m+2\left(\cos(2a)^3+\cos(2a)\sin(2a)^2+(\cos\right.}{8\left(\left(\cos(2a)^2+\sin(2a)^2\right)c^{\frac{1}{2}m}m+\left(\cos(2a)^2+\sin(2a)^2\right)c^{\frac{1}{2}m}\right)\sqrt{}}$$

input
```
integrate(x^m*sin(a+1/4*(-(1+m)^2)^(1/2)*log(c*x^2))^2,x, algorithm="maxim
a")
```

output
```
-1/8*(c^(m + 1)*x^2*x^(2*m)*cos(2*a) - 4*(cos(2*a)^2 + sin(2*a)^2)*c^(1/2*
m + 1/2)*x*x^m + 2*(cos(2*a)^3 + cos(2*a)*sin(2*a)^2 + (cos(2*a)^3 + cos(2
*a)*sin(2*a)^2)*m)*log(x))/(((cos(2*a)^2 + sin(2*a)^2)*c^(1/2*m)*m + (cos(
2*a)^2 + sin(2*a)^2)*c^(1/2*m))*sqrt(c))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.25 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.30

$$\int x^m \sin^2\left(a+\frac{1}{4}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx = \frac{m^2xx^m e^{\left(\frac{1}{2}|m+1|\log(c)+|m+1|\log(x)-2ia\right)} - mxx^m|m+1|e^{\left(\frac{1}{2}|m+1|\log(c)+|m+1|\log(x)-2ia\right)} + m^2xx^m e^{\left(-\frac{1}{2}|m+1|\log(c\right.}}{}$$

input
```
integrate(x^m*sin(a+1/4*(-(1+m)^2)^(1/2)*log(c*x^2))^2,x, algorithm="giac"
)
```

output
```
1/4*(m^2*x*x^m*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*log(x) - 2*I*a) - m*x
*x^m*abs(m + 1)*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*log(x) - 2*I*a) + m^
2*x*x^m*e^(-1/2*abs(m + 1)*log(c) - abs(m + 1)*log(x) + 2*I*a) + m*x*x^m*a
bs(m + 1)*e^(-1/2*abs(m + 1)*log(c) - abs(m + 1)*log(x) + 2*I*a) + 2*(m +
1)^2*x*x^m - 2*m^2*x*x^m + 2*m*x*x^m*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)
*log(x) - 2*I*a) - x*x^m*abs(m + 1)*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*
log(x) - 2*I*a) + 2*m*x*x^m*e^(-1/2*abs(m + 1)*log(c) - abs(m + 1)*log(x)
+ 2*I*a) + x*x^m*abs(m + 1)*e^(-1/2*abs(m + 1)*log(c) - abs(m + 1)*log(x)
+ 2*I*a) - 4*m*x*x^m + x*x^m*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*log(x)
- 2*I*a) + x*x^m*e^(-1/2*abs(m + 1)*log(c) - abs(m + 1)*log(x) + 2*I*a) -
2*x*x^m)/((m + 1)^2*m - m^3 + (m + 1)^2 - 3*m^2 - 3*m - 1)
```

Mupad [B] (verification not implemented)

Time = 20.50 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.41

$$\int x^m \sin^2\left(a+\frac{1}{4}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx = \frac{x\,x^m}{2\,m+2} - \frac{\frac{1}{c^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{2}}}\,x\,x^m\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{\left(x^2\right)^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{2}}}}{4\,m+4-\sqrt{-\left(m+1\right)^2}\,4\mathrm{i}} - \frac{c^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{2}}\,x\,x^m\,\mathrm{e}^{a\,2\mathrm{i}}\,\left(x^2\right)^{\frac{\sqrt{-m^2-2\,m-1}\,1\mathrm{i}}{2}}}{4\,m+4+\sqrt{-\left(m+1\right)^2}\,4\mathrm{i}}$$

input
```
int(x^m*sin(a + (log(c*x^2)*(-(m + 1)^2)^(1/2))/4)^2,x)
```

output
```
(x*x^m)/(2*m + 2) - (1/c^(((- 2*m - m^2 - 1)^(1/2)*1i)/2)*x*x^m*exp(-a*2i)
/(x^2)^(((- 2*m - m^2 - 1)^(1/2)*1i)/2))/(4*m - (-(m + 1)^2)^(1/2)*4i + 4)
 - (c^(((- 2*m - m^2 - 1)^(1/2)*1i)/2)*x*x^m*exp(a*2i)*(x^2)^(((- 2*m - m^
2 - 1)^(1/2)*1i)/2))/(4*m + (-(m + 1)^2)^(1/2)*4i + 4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75

$$\int x^m \sin^2\left(a+\frac{1}{4}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx = \frac{x^m x\left(-2\cos\left(\frac{\log(c\,x^2)m}{4}+\frac{\log(c\,x^2)}{4}+a\right)\sin\left(\frac{\log(c\,x^2)m}{4}+\frac{\log(c\,x^2)}{4}+a\right)+2\sin\left(\frac{\log(c\,x^2)m}{4}+\frac{\log(c\,x^2)}{4}+a\right)^2+\right.}{4m+4}$$

input
```
int(x^m*sin(a+1/4*(-(1+m)^2)^(1/2)*log(c*x^2))^2,x)
```

output
```
(x**m*x*( - 2*cos((log(c*x**2)*m + log(c*x**2) + 4*a)/4)*sin((log(c*x**2)*
m + log(c*x**2) + 4*a)/4) + 2*sin((log(c*x**2)*m + log(c*x**2) + 4*a)/4)**
2 + 1))/(4*(m + 1))
```

3.50 $\int \sin^2\left(a+\frac{1}{4}i\log\left(cx^2\right)\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \sin^2\left(a+\frac{1}{4}i\log\left(cx^2\right)\right)\,dx=\frac{x}{2}-\frac{ce^{-2ia}x^3}{8\sqrt{cx^2}}-\frac{e^{2ia}x\log(x)}{4\sqrt{cx^2}}$$

output
```
1/2*x-1/8*c*x^3/exp(2*I*a)/(c*x^2)^(1/2)-1/4*exp(2*I*a)*x*ln(x)/(c*x^2)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

$$\int \sin^2\left(a+\frac{1}{4}i\log\left(cx^2\right)\right)\,dx
=\frac{x\left(4\sqrt{cx^2}-\cos(2a)\left(cx^2+2\log(x)\right)+i(cx^2-2\log(x))\sin(2a)\right)}{8\sqrt{cx^2}}$$

input
```
Integrate[Sin[a + (I/4)*Log[c*x^2]]^2,x]
```

output
```
(x*(4*Sqrt[c*x^2] - Cos[2*a]*(c*x^2 + 2*Log[x]) + I*(c*x^2 - 2*Log[x])*Sin
[2*a]))/(8*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4986, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2\left(a+\frac{1}{4}i\log\left(cx^2\right)\right)dx$$

$$\Big\downarrow \; 4986$$

$$\frac{x\int \frac{\sin^2\left(a+\frac{1}{4}i\log\left(cx^2\right)\right)}{\sqrt{cx^2}}d\left(cx^2\right)}{2\sqrt{cx^2}}$$

$$\Big\downarrow \; 4992$$

$$-\frac{x\int \left(e^{-2ia}+\frac{e^{2ia}}{cx^2}-\frac{2}{\sqrt{cx^2}}\right)d\left(cx^2\right)}{8\sqrt{cx^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{x\left(e^{-2ia}(-c)x^2-e^{2ia}\log\left(cx^2\right)+4\sqrt{cx^2}\right)}{8\sqrt{cx^2}}$$

input
```
Int[Sin[a + (I/4)*Log[c*x^2]]^2,x]
```

output
```
(x*(-((c*x^2)/E^((2*I)*a)) + 4*Sqrt[c*x^2] - E^((2*I)*a)*Log[c*x^2]))/(8*S
qrt[c*x^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4986
```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(41) = 82$.

Time = 1.14 (sec) , antiderivative size = 173, normalized size of antiderivative = 3.26

method	result
norman	$\frac{\frac{x}{4}+\frac{5x\tan\left(\frac{a}{2}+\frac{i\ln\left(c\,x^{2}\right)}{8}\right)^{2}}{2}+\frac{x\tan\left(\frac{a}{2}+\frac{i\ln\left(c\,x^{2}\right)}{8}\right)^{4}}{4}-\frac{x\ln\left(c\,x^{2}\right)}{8}+\frac{3x\ln\left(c\,x^{2}\right)\tan\left(\frac{a}{2}+\frac{i\ln\left(c\,x^{2}\right)}{8}\right)^{2}}{4}-\frac{x\ln\left(c\,x^{2}\right)\tan\left(\frac{a}{2}+\frac{i\ln\left(c\,x^{2}\right)}{8}\right)^{4}}{8}-\cdots}{\left(1+\tan\left(\frac{a}{2}+\frac{i\ln\left(c\,x^{2}\right)}{8}\right)^{2}\right)^{2}}$

input
```
int(sin(a+1/4*I*ln(c*x^2))^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4*x+5/2*x*tan(1/2*a+1/8*I*ln(c*x^2))^2+1/4*x*tan(1/2*a+1/8*I*ln(c*x^2))
^4-1/8*x*ln(c*x^2)+3/4*x*ln(c*x^2)*tan(1/2*a+1/8*I*ln(c*x^2))^2-1/8*x*ln(c
*x^2)*tan(1/2*a+1/8*I*ln(c*x^2))^4-1/2*I*x*ln(c*x^2)*tan(1/2*a+1/8*I*ln(c*
x^2))+1/2*I*x*ln(c*x^2)*tan(1/2*a+1/8*I*ln(c*x^2))^3)/(1+tan(1/2*a+1/8*I*l
n(c*x^2))^2)^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(37) = 74$.

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.74

$$\int \sin^2\left(a+\frac{1}{4}i\log\left(cx^2\right)\right)\,dx$$
$$= \frac{\left(4\,x^2e^{(2i\,a)} - \frac{xe^{(4i\,a)}\log\left(\frac{\left(\sqrt{cx^2}\left(x^2+1\right)e^{(2i\,a)}+\frac{\left(cx^3-cx\right)e^{(2i\,a)}}{\sqrt{c}}\right)e^{(-2i\,a)}}{8\,x^2}\right)}{\sqrt{c}} + \frac{xe^{(4i\,a)}\log\left(\frac{\left(\sqrt{cx^2}\left(x^2+1\right)e^{(2i\,a)}-\frac{\left(cx^3-cx\right)e^{(2i\,a)}}{\sqrt{c}}\right)e^{(-2}}{8\,x^2}\right.}{\sqrt{c}}\right.}{8\,x}$$

input
```
integrate(sin(a+1/4*I*log(c*x^2))^2,x, algorithm="fricas")
```

output
```
1/8*(4*x^2*e^(2*I*a) - x*e^(4*I*a)*log(1/8*(sqrt(c*x^2)*(x^2 + 1)*e^(2*I*a
) + (c*x^3 - c*x)*e^(2*I*a)/sqrt(c))*e^(-2*I*a)/x^2)/sqrt(c) + x*e^(4*I*a)
*log(1/8*(sqrt(c*x^2)*(x^2 + 1)*e^(2*I*a) - (c*x^3 - c*x)*e^(2*I*a)/sqrt(c
))*e^(-2*I*a)/x^2)/sqrt(c) - sqrt(c*x^2)*(x^2 - 1))*e^(-2*I*a)/x
```

Sympy [F]

$$\int \sin^2\left(a+\frac{1}{4}i\log\left(cx^2\right)\right)\,dx = \int \sin^2\left(a+\frac{i\log\left(cx^2\right)}{4}\right)\,dx$$

input
```
integrate(sin(a+1/4*I*ln(c*x**2))**2,x)
```

output
```
Integral(sin(a + I*log(c*x**2)/4)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \sin^2\left(a+\frac{1}{4}i\log\left(cx^2\right)\right)\,dx = \frac{4\,cx-\left(cx^2(\cos\left(2\,a\right)-i\,\sin\left(2\,a\right))+2\left(\cos\left(2\,a\right)+i\,\sin\left(2\,a\right)\right)\log\left(x\right)\right)\sqrt{c}}{8\,c}$$

input
```
integrate(sin(a+1/4*I*log(c*x^2))^2,x, algorithm="maxima")
```

output
```
1/8*(4*c*x - (c*x^2*(cos(2*a) - I*sin(2*a)) + 2*(cos(2*a) + I*sin(2*a))*lo
g(x))*sqrt(c))/c
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.51

$$\int \sin^2\left(a+\frac{1}{4}i\log\left(cx^2\right)\right)\,dx = \frac{1}{2}\,x-\frac{cx^2e^{(-2i\,a)}+2\,e^{(2i\,a)}\log\left(x\right)}{8\,\sqrt{c}}$$

input
```
integrate(sin(a+1/4*I*log(c*x^2))^2,x, algorithm="giac")
```

output
```
1/2*x - 1/8*(c*x^2*e^(-2*I*a) + 2*e^(2*I*a)*log(x))/sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2\left(a+\frac{1}{4}i\log\left(cx^2\right)\right)\,dx = \int \sin\left(a+\frac{\ln\left(c\,x^2\right)\,1\mathrm{i}}{4}\right)^2\,dx$$

input
```
int(sin(a + (log(c*x^2)*1i)/4)^2,x)
```

output
```
int(sin(a + (log(c*x^2)*1i)/4)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.09

$$\int \sin^2\left(a+\frac{1}{4}i\log\left(cx^2\right)\right)\,dx$$
$$= \frac{x\left(-6\cos\left(\frac{\log(c\,x^2)i}{4}+a\right)\log(c\,x^2)\sin\left(\frac{\log(c\,x^2)i}{4}+a\right)i-76\cos\left(\frac{\log(c\,x^2)i}{4}+a\right)\sin\left(\frac{\log(c\,x^2)i}{4}+a\right)i+6\log\right.}{24}$$

input
```
int(sin(a+1/4*I*log(c*x^2))^2,x)
```

output
```
(x*( - 6*cos((log(c*x**2)*i + 4*a)/4)*log(c*x**2)*sin((log(c*x**2)*i + 4*a
)/4)*i - 76*cos((log(c*x**2)*i + 4*a)/4)*sin((log(c*x**2)*i + 4*a)/4)*i +
6*log(c*x**2)*sin((log(c*x**2)*i + 4*a)/4)**2 - 3*log(c*x**2) + 88*sin((lo
g(c*x**2)*i + 4*a)/4)**2 - 32))/24
```

3.51 $\int x^m \sin^3\left(a+\frac{1}{6}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 218

$$\int x^m \sin^3\left(a+\frac{1}{6}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx = \frac{9e^{\frac{a\sqrt{-(1+m)^2}}{1+m}}x^{1+m}\left(cx^2\right)^{\frac{1}{6}(-1-m)}}{16\sqrt{-(1+m)^2}} - \frac{9e^{\frac{a(1+m)}{\sqrt{-(1+m)^2}}}x^{1+m}\left(cx^2\right)^{\frac{1+m}{6}}}{32\sqrt{-(1+m)^2}} + \frac{e^{\frac{3a(1+m)}{\sqrt{-(1+m)^2}}}x^{1+m}\left(cx^2\right)^{\frac{1+m}{2}}}{16\sqrt{-(1+m)^2}} - \frac{e^{-\frac{3a(1+m)}{\sqrt{-(1+m)^2}}}(1+m)x^{1+m}\left(cx^2\right)^{\frac{1}{2}(-1-m)}\log(x)}{8\sqrt{-(1+m)^2}}$$

output
```
9/16*exp(a*(-(1+m)^2)^(1/2)/(1+m))*x^(1+m)*(c*x^2)^(-1/6-1/6*m)/(-(1+m)^2)
^(1/2)-9/32*exp(a*(1+m)/(-(1+m)^2)^(1/2))*x^(1+m)*(c*x^2)^(1/6+1/6*m)/(-(1
+m)^2)^(1/2)+1/16*exp(3*a*(1+m)/(-(1+m)^2)^(1/2))*x^(1+m)*(c*x^2)^(1/2+1/2
*m)/(-(1+m)^2)^(1/2)-1/8*(1+m)*x^(1+m)*(c*x^2)^(-1/2-1/2*m)*ln(x)/exp(3*a*
(1+m)/(-(1+m)^2)^(1/2))/(-(1+m)^2)^(1/2)
```

Mathematica [F]

$$\int x^m \sin^3\left(a+\frac{1}{6}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx$$
$$=\int x^m \sin^3\left(a+\frac{1}{6}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx$$

input
```
Integrate[x^m*Sin[a + (Sqrt[-(1 + m)^2]*Log[c*x^2])/6]^3,x]
```

output
```
Integrate[x^m*Sin[a + (Sqrt[-(1 + m)^2]*Log[c*x^2])/6]^3, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4996, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sin^3\left(a+\frac{1}{6}\sqrt{-(m+1)^2}\log\left(cx^2\right)\right)\,dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{1}{2}x^{m+1}\left(cx^2\right)^{\frac{1}{2}(-m-1)}\int \left(cx^2\right)^{\frac{m-1}{2}}\sin^3\left(a+\frac{1}{6}\sqrt{-(m+1)^2}\log\left(cx^2\right)\right)d\left(cx^2\right)$$

$$\Big\downarrow \text{4992}$$

$$\frac{\sqrt{-(m+1)^2}x^{m+1}\left(cx^2\right)^{\frac{1}{2}(-m-1)}\int\left(-3e^{\frac{a\sqrt{-(m+1)^2}}{m+1}}\left(cx^2\right)^{\frac{m-2}{3}}-e^{\frac{3a(m+1)}{\sqrt{-(m+1)^2}}}\left(cx^2\right)^m+3e^{\frac{a(m+1)}{\sqrt{-(m+1)^2}}}\left(cx^2\right)^{\frac{1}{3}(2m-1)}+\frac{e}{}\right.}{16(m+1)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{-(m+1)^2}x^{m+1}\left(cx^2\right)^{\frac{1}{2}(-m-1)}\left(-\frac{9e^{\frac{a\sqrt{-(m+1)^2}}{m+1}}\left(cx^2\right)^{\frac{m+1}{3}}}{m+1}+\frac{9e^{\frac{a(m+1)}{\sqrt{-(m+1)^2}}}\left(cx^2\right)^{\frac{2(m+1)}{3}}}{2(m+1)}-\frac{e^{\frac{3a(m+1)}{\sqrt{-(m+1)^2}}}\left(cx^2\right)^{m+1}}{m+1}+e^{-\frac{3a(m+1)}{\sqrt{-(m+1)^2}}}\cdots\right.}{16(m+1)}$$

input
```
Int[x^m*Sin[a + (Sqrt[-(1 + m)^2]*Log[c*x^2])/6]^3,x]
```

output
```
(Sqrt[-(1 + m)^2]*x^(1 + m)*(c*x^2)^((-1 - m)/2)*((-9*E^((a*Sqrt[-(1 + m)^
2])/(1 + m))*(c*x^2)^((1 + m)/3))/(1 + m) + (9*E^((a*(1 + m))/Sqrt[-(1 + m
)^2])*(c*x^2)^((2*(1 + m))/3))/(2*(1 + m)) - (E^((3*a*(1 + m))/Sqrt[-(1 +
m)^2])*(c*x^2)^(1 + m))/(1 + m) + Log[c*x^2]/E^((3*a*(1 + m))/Sqrt[-(1 + m
)^2])))/(16*(1 + m))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^{m}\sin\left(a+\frac{\sqrt{-\left(1+m\right)^{2}}\,\ln\left(c\,x^{2}\right)}{6}\right)^{3}dx$$

input
```
int(x^m*sin(a+1/6*(-(1+m)^2)^(1/2)*ln(c*x^2))^3,x)
```

output
```
int(x^m*sin(a+1/6*(-(1+m)^2)^(1/2)*ln(c*x^2))^3,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.45

$$\int x^m \sin^3\left(a+\frac{1}{6}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx = -\frac{\left(4\left(-i\,m-i\right)e^{\left(-(m+1)\log(c)-2\,(m+1)\log(x)+6i\,a\right)}\log\left(x\right)-9i\,e^{\left(-\frac{1}{3}\,(m+1)\log(c)-\frac{2}{3}\,(m+1)\log(x)+2i\,a\right)}+18i\,e^{\left(-\frac{2}{3}\,(m\right.}}{32\,(m+1)}$$

input
```
integrate(x^m*sin(a+1/6*(-(1+m)^2)^(1/2)*log(c*x^2))^3,x, algorithm="frica
s")
```

output
```
-1/32*(4*(-I*m - I)*e^(-(m + 1)*log(c) - 2*(m + 1)*log(x) + 6*I*a)*log(x)
- 9*I*e^(-1/3*(m + 1)*log(c) - 2/3*(m + 1)*log(x) + 2*I*a) + 18*I*e^(-2/3*
(m + 1)*log(c) - 4/3*(m + 1)*log(x) + 4*I*a) + 2*I)*e^(1/2*(m + 1)*log(c)
+ 2*(m + 1)*log(x) - 3*I*a)/(m + 1)
```

Sympy [F]

$$\int x^m \sin^3\left(a+\frac{1}{6}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx$$
$$=\int x^m \sin^3\left(a+\frac{\sqrt{-m^2-2m-1}\log\left(cx^2\right)}{6}\right)\,dx$$

input
```
integrate(x**m*sin(a+1/6*(-(1+m)**2)**(1/2)*ln(c*x**2))**3,x)
```

output
```
Integral(x**m*sin(a + sqrt(-m**2 - 2*m - 1)*log(c*x**2)/6)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.94

$$\int x^m \sin^3\left(a+\frac{1}{6}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx$$
$$=\frac{9\,(\cos(2\,a)\sin(3\,a)-\cos(3\,a)\sin(2\,a))c^{\frac{5}{6}\,m+\frac{5}{6}}x^{\frac{5}{3}}x^{\frac{4}{3}\,m}+18\,(\cos(3\,a)\sin(4\,a)-\cos(4\,a)\sin(3\,a))c^{\frac{1}{2}\,m+\frac{1}{2}}}{32\,\left(\left(\cos(3\,a)^2+\sin(\right.\right.}$$

input
```
integrate(x^m*sin(a+1/6*(-(1+m)^2)^(1/2)*log(c*x^2))^3,x, algorithm="maxim
a")
```

output
```
1/32*(9*(cos(2*a)*sin(3*a) - cos(3*a)*sin(2*a))*c^(5/6*m + 5/6)*x^(5/3)*x^
(4/3*m) + 18*(cos(3*a)*sin(4*a) - cos(4*a)*sin(3*a))*c^(1/2*m + 1/2)*x*x^(
2/3*m) - 2*(c^(7/6*m + 1)*x^2*x^(2*m)*sin(3*a) + 2*((cos(3*a)^2*sin(3*a) +
 sin(3*a)^3)*c^(1/6*m)*m + (cos(3*a)^2*sin(3*a) + sin(3*a)^3)*c^(1/6*m))*l
og(x))*c^(1/6)*x^(1/3))/(((cos(3*a)^2 + sin(3*a)^2)*c^(2/3*m)*m + (cos(3*a
)^2 + sin(3*a)^2)*c^(2/3*m))*c^(2/3)*x^(1/3))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.64 (sec) , antiderivative size = 1297, normalized size of antiderivative = 5.95

$$\int x^m \sin^3\left(a+\frac{1}{6}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^m*sin(a+1/6*(-(1+m)^2)^(1/2)*log(c*x^2))^3,x, algorithm="giac"
)
```

output
```
1/8*(I*(m + 1)^2*m*x*x^m*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*log(x) - 3*
I*a) - 9*I*m^3*x*x^m*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*log(x) - 3*I*a)
 - I*(m + 1)^2*x*x^m*abs(m + 1)*e^(1/2*abs(m + 1)*log(c) + abs(m + 1)*log(
x) - 3*I*a) + 9*I*m^2*x*x^m*abs(m + 1)*e^(1/2*abs(m + 1)*log(c) + abs(m +
1)*log(x) - 3*I*a) - 27*I*(m + 1)^2*m*x*x^m*e^(1/6*abs(m + 1)*log(c) + 1/3
*abs(m + 1)*log(x) - I*a) + 27*I*m^3*x*x^m*e^(1/6*abs(m + 1)*log(c) + 1/3*
abs(m + 1)*log(x) - I*a) + 9*I*(m + 1)^2*x*x^m*abs(m + 1)*e^(1/6*abs(m + 1
)*log(c) + 1/3*abs(m + 1)*log(x) - I*a) - 9*I*m^2*x*x^m*abs(m + 1)*e^(1/6*
abs(m + 1)*log(c) + 1/3*abs(m + 1)*log(x) - I*a) + 27*I*(m + 1)^2*m*x*x^m*
e^(-1/6*abs(m + 1)*log(c) - 1/3*abs(m + 1)*log(x) + I*a) - 27*I*m^3*x*x^m*
e^(-1/6*abs(m + 1)*log(c) - 1/3*abs(m + 1)*log(x) + I*a) + 9*I*(m + 1)^2*x
*x^m*abs(m + 1)*e^(-1/6*abs(m + 1)*log(c) - 1/3*abs(m + 1)*log(x) + I*a) -
 9*I*m^2*x*x^m*abs(m + 1)*e^(-1/6*abs(m + 1)*log(c) - 1/3*abs(m + 1)*log(x
) + I*a) - I*(m + 1)^2*m*x*x^m*e^(-1/2*abs(m + 1)*log(c) - abs(m + 1)*log(
x) + 3*I*a) + 9*I*m^3*x*x^m*e^(-1/2*abs(m + 1)*log(c) - abs(m + 1)*log(x)
+ 3*I*a) - I*(m + 1)^2*x*x^m*abs(m + 1)*e^(-1/2*abs(m + 1)*log(c) - abs(m
+ 1)*log(x) + 3*I*a) + 9*I*m^2*x*x^m*abs(m + 1)*e^(-1/2*abs(m + 1)*log(c)
- abs(m + 1)*log(x) + 3*I*a) + I*(m + 1)^2*x*x^m*e^(1/2*abs(m + 1)*log(c)
+ abs(m + 1)*log(x) - 3*I*a) - 27*I*m^2*x*x^m*e^(1/2*abs(m + 1)*log(c) + a
bs(m + 1)*log(x) - 3*I*a) + 18*I*m*x*x^m*abs(m + 1)*e^(1/2*abs(m + 1)*l...
```

Mupad [B] (verification not implemented)

Time = 21.60 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.33

$$\int x^m \sin^3\left(a+\frac{1}{6}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)\,dx = -\frac{\frac{1}{c^{\frac{\sqrt{-m^2-2m-1}\,1\mathrm{i}}{2}}}\,x\,x^m\,\mathrm{e}^{-a\,3\mathrm{i}}\,\frac{1}{(x^2)^{\frac{\sqrt{-m^2-2m-1}\,1\mathrm{i}}{2}}}\,1\mathrm{i}}{8\,m+8-\sqrt{-(m+1)^2}\,8\mathrm{i}} + \frac{c^{\frac{\sqrt{-m^2-2m-1}\,1\mathrm{i}}{2}}\,x\,x^m\,\mathrm{e}^{a\,3\mathrm{i}}\,(x^2)^{\frac{\sqrt{-m^2-2m-1}\,1\mathrm{i}}{2}}\,1\mathrm{i}}{8\,m+8+\sqrt{-(m+1)^2}\,8\mathrm{i}} - \frac{\frac{1}{c^{\frac{\sqrt{-m^2-2m-1}\,1\mathrm{i}}{6}}}\,x\,x^m\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{(x^2)^{\frac{\sqrt{-m^2-2m-1}\,1\mathrm{i}}{6}}}\left(27\,m+27+\sqrt{-(m+1)^2}\,9\mathrm{i}\right)\,1\mathrm{i}}{64\,(m\,1\mathrm{i}+1\mathrm{i})^2} + \frac{c^{\frac{\sqrt{-m^2-2m-1}\,1\mathrm{i}}{6}}\,x\,x^m\,\mathrm{e}^{a\,1\mathrm{i}}\,(x^2)^{\frac{\sqrt{-m^2-2m-1}\,1\mathrm{i}}{6}}\left(27\,m+27-\sqrt{-(m+1)^2}\,9\mathrm{i}\right)\,1\mathrm{i}}{64\,(m\,1\mathrm{i}+1\mathrm{i})^2}$$

input
```
int(x^m*sin(a + (log(c*x^2)*(-(m + 1)^2)^(1/2))/6)^3,x)
```

output
```
(c^(((- 2*m - m^2 - 1)^(1/2)*1i)/2)*x*x^m*exp(a*3i)*(x^2)^(((- 2*m - m^2 -
 1)^(1/2)*1i)/2)*1i)/(8*m + (-(m + 1)^2)^(1/2)*8i + 8) - (1/c^(((- 2*m - m
^2 - 1)^(1/2)*1i)/2)*x*x^m*exp(-a*3i)/(x^2)^(((- 2*m - m^2 - 1)^(1/2)*1i)/
2)*1i)/(8*m - (-(m + 1)^2)^(1/2)*8i + 8) - (1/c^(((- 2*m - m^2 - 1)^(1/2)*
1i)/6)*x*x^m*exp(-a*1i)/(x^2)^(((- 2*m - m^2 - 1)^(1/2)*1i)/6)*(27*m + (-(
m + 1)^2)^(1/2)*9i + 27)*1i)/(64*(m*1i + 1i)^2) + (c^(((- 2*m - m^2 - 1)^(
1/2)*1i)/6)*x*x^m*exp(a*1i)*(x^2)^(((- 2*m - m^2 - 1)^(1/2)*1i)/6)*(27*m -
 (-(m + 1)^2)^(1/2)*9i + 27)*1i)/(64*(m*1i + 1i)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.57

$$\int x^m \sin^3\left(a+\frac{1}{6}\sqrt{-(1+m)^2}\log\left(cx^2\right)\right)dx$$
$$=\frac{x^m x\left(-5\cos\left(\frac{\log(cx^2)m}{6}+\frac{\log(cx^2)}{6}+a\right)\sin\left(\frac{\log(cx^2)m}{6}+\frac{\log(cx^2)}{6}+a\right)^2-\cos\left(\frac{\log(cx^2)m}{6}+\frac{\log(cx^2)}{6}+a\right)+\right.}{10m+10}$$

input
```
int(x^m*sin(a+1/6*(-(1+m)^2)^(1/2)*log(c*x^2))^3,x)
```

output
```
(x**m*x*( - 5*cos((log(c*x**2)*m + log(c*x**2) + 6*a)/6)*sin((log(c*x**2)*
m + log(c*x**2) + 6*a)/6)**2 - cos((log(c*x**2)*m + log(c*x**2) + 6*a)/6)
+ 5*sin((log(c*x**2)*m + log(c*x**2) + 6*a)/6)**3 + 3*sin((log(c*x**2)*m +
 log(c*x**2) + 6*a)/6)))/(10*(m + 1))
```

3.52 $\int \sin^3\left(a+\frac{1}{6}i\log\left(cx^2\right)\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 98

$$\int \sin^3\left(a+\frac{1}{6}i\log\left(cx^2\right)\right)\,dx=-\frac{ice^{-3ia}x^3}{16\sqrt{cx^2}}-\frac{9ie^{ia}x}{16\sqrt[6]{cx^2}}+\frac{9}{32}ie^{-ia}x\sqrt[6]{cx^2}+\frac{ie^{3ia}x\log(x)}{8\sqrt{cx^2}}$$

output
```
-1/16*I*c*x^3/exp(3*I*a)/(c*x^2)^(1/2)-9/16*I*exp(I*a)*x/(c*x^2)^(1/6)+9/3
2*I*x*(c*x^2)^(1/6)/exp(I*a)+1/8*I*exp(3*I*a)*x*ln(x)/(c*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05

$$\int \sin^3\left(a+\frac{1}{6}i\log\left(cx^2\right)\right)\,dx$$
$$=\frac{x\left(9i\sqrt[3]{cx^2}\left(-2+\sqrt[3]{cx^2}\right)\cos(a)-2i\cos(3a)\left(cx^2-2\log(x)\right)+18\sqrt[3]{cx^2}\sin(a)+9(cx^2)^{2/3}\sin(a)-2cx^2\,\mathrm{s}}{32\sqrt{cx^2}}$$

input
```
Integrate[Sin[a + (I/6)*Log[c*x^2]]^3,x]
```

output
```
(x*((9*I)*(c*x^2)^(1/3)*(-2 + (c*x^2)^(1/3))*Cos[a] - (2*I)*Cos[3*a]*(c*x^
2 - 2*Log[x]) + 18*(c*x^2)^(1/3)*Sin[a] + 9*(c*x^2)^(2/3)*Sin[a] - 2*c*x^2
*Sin[3*a] - 4*Log[x]*Sin[3*a]))/(32*Sqrt[c*x^2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4986, 4992, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3\left(a+\frac{1}{6}i\log\left(cx^2\right)\right)dx$$

$$\Big\downarrow \ 4986$$

$$\frac{x\int \frac{\sin^3\left(a+\frac{1}{6}i\log\left(cx^2\right)\right)}{\sqrt{cx^2}}d\left(cx^2\right)}{2\sqrt{cx^2}}$$

$$\Big\downarrow \ 4992$$

$$\frac{ix\int \left(-e^{-3ia}+\frac{e^{3ia}}{cx^2}+\frac{3e^{-ia}}{\sqrt[3]{cx^2}}-\frac{3e^{ia}}{\left(cx^2\right)^{2/3}}\right)d\left(cx^2\right)}{16\sqrt{cx^2}}$$

$$\Big\downarrow \ 2009$$

$$\frac{ix\left(e^{-3ia}(-c)x^2+\frac{9}{2}e^{-ia}\left(cx^2\right)^{2/3}-9e^{ia}\sqrt[3]{cx^2}+e^{3ia}\log\left(cx^2\right)\right)}{16\sqrt{cx^2}}$$

input
```
Int[Sin[a + (I/6)*Log[c*x^2]]^3,x]
```

output
```
((I/16)*x*(-((c*x^2)/E^((3*I)*a)) - 9*E^(I*a)*(c*x^2)^(1/3) + (9*(c*x^2)^(
2/3))/(2*E^(I*a)) + E^((3*I)*a)*Log[c*x^2]))/Sqrt[c*x^2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4986
```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 4992
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[(m + 1)^p/(2^p*b^p*d^p*p^p)   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d
^2*(p/(m + 1)))/x^((m + 1)/p) - x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x
], x], x] /; FreeQ[{a, b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (
m + 1)^2, 0]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(74) = 148$.

Time = 2.06 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.90

method	result
norman	$-\frac{23ix}{40}+\frac{27x\tan\left(\frac{a}{2}+\frac{i\ln\left(c\,x^2\right)}{12}\right)}{10}+\frac{27x\tan\left(\frac{a}{2}+\frac{i\ln\left(c\,x^2\right)}{12}\right)^5}{10}+\frac{33ix\tan\left(\frac{a}{2}+\frac{i\ln\left(c\,x^2\right)}{12}\right)^2}{8}+\frac{23ix\tan\left(\frac{a}{2}+\frac{i\ln\left(c\,x^2\right)}{12}\right)^6}{40}-\frac{33ix\tan\left(\frac{a}{2}+\frac{i\ln\left(c\right.}{12}\right.}{8}$

input
```
int(sin(a+1/6*I*ln(c*x^2))^3,x,method=_RETURNVERBOSE)
```

output
```
(-23/40*I*x+27/10*x*tan(1/2*a+1/12*I*ln(c*x^2))+27/10*x*tan(1/2*a+1/12*I*l
n(c*x^2))^5+33/8*I*x*tan(1/2*a+1/12*I*ln(c*x^2))^2+23/40*I*x*tan(1/2*a+1/1
2*I*ln(c*x^2))^6-33/8*I*x*tan(1/2*a+1/12*I*ln(c*x^2))^4-3/8*x*ln(c*x^2)*ta
n(1/2*a+1/12*I*ln(c*x^2))+5/4*x*ln(c*x^2)*tan(1/2*a+1/12*I*ln(c*x^2))^3-3/
8*x*ln(c*x^2)*tan(1/2*a+1/12*I*ln(c*x^2))^5+1/16*I*x*ln(c*x^2)-15/16*I*x*l
n(c*x^2)*tan(1/2*a+1/12*I*ln(c*x^2))^2+15/16*I*x*ln(c*x^2)*tan(1/2*a+1/12*
I*ln(c*x^2))^4-1/16*I*x*ln(c*x^2)*tan(1/2*a+1/12*I*ln(c*x^2))^6)/(1+tan(1/
2*a+1/12*I*ln(c*x^2))^2)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(62) = 124$.

Time = 1.45 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.08

$$\int \sin^3\left(a+\frac{1}{6}i\log\left(cx^2\right)\right)\,dx$$

$$= \frac{\left(2\,cx\sqrt{-\frac{e^{(6i\,a)}}{c}}e^{(3i\,a)}\log\left(\frac{\left(\sqrt{cx^2}\,(x^2+1)e^{(3i\,a)}-(i\,cx^3-i\,cx)\sqrt{-\frac{e^{(6i\,a)}}{c}}\right)e^{(-3i\,a)}}{8\,x^2}\right)-2\,cx\sqrt{-\frac{e^{(6i\,a)}}{c}}e^{(3i\,a)}\log\left(\frac{\left(\sqrt{cx^2}\,(x^2\cdots\right.}{}\right.}{32\cdots}$$

input
```
integrate(sin(a+1/6*I*log(c*x^2))^3,x, algorithm="fricas")
```

output
```
1/32*(2*c*x*sqrt(-e^(6*I*a)/c)*e^(3*I*a)*log(1/8*(sqrt(c*x^2)*(x^2 + 1)*e^
(3*I*a) - (I*c*x^3 - I*c*x)*sqrt(-e^(6*I*a)/c))*e^(-3*I*a)/x^2) - 2*c*x*sq
rt(-e^(6*I*a)/c)*e^(3*I*a)*log(1/8*(sqrt(c*x^2)*(x^2 + 1)*e^(3*I*a) - (-I*
c*x^3 + I*c*x)*sqrt(-e^(6*I*a)/c))*e^(-3*I*a)/x^2) + 9*I*(c*x^2)^(1/6)*c*x
^2*e^(2*I*a) - 18*I*(c*x^2)^(5/6)*e^(4*I*a) - 2*sqrt(c*x^2)*(I*c*x^2 - I*c
))*e^(-3*I*a)/(c*x)
```

Sympy [F]

$$\int \sin^3\left(a+\frac{1}{6}i\log\left(cx^2\right)\right)\,dx=\int \sin^3\left(a+\frac{i\log\left(cx^2\right)}{6}\right)dx$$

input
```
integrate(sin(a+1/6*I*ln(c*x**2))**3,x)
```

output
```
Integral(sin(a + I*log(c*x**2)/6)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \sin^3\left(a+\frac{1}{6}i\log\left(cx^2\right)\right)\,dx = -\frac{9\,c^{\frac{4}{3}}x^{\frac{4}{3}}(-i\,\cos(a)-\sin(a))+18\,cx^{\frac{2}{3}}(i\,\cos(a)-\sin(a))+2\,(cx^2(i\,\cos(3\,a)+\sin(3\,a))+2\,(-i\,\cos(}{32\,c^{\frac{7}{6}}}$$

input
```
integrate(sin(a+1/6*I*log(c*x^2))^3,x, algorithm="maxima")
```

output
```
-1/32*(9*c^(4/3)*x^(4/3)*(-I*cos(a) - sin(a)) + 18*c*x^(2/3)*(I*cos(a) - s
in(a)) + 2*(c*x^2*(I*cos(3*a) + sin(3*a)) + 2*(-I*cos(3*a) + sin(3*a))*log
(x))*c^(2/3))/c^(7/6)
```

Giac [F]

$$\int \sin^3\left(a+\frac{1}{6}i\log\left(cx^2\right)\right)\,dx=\int \sin\left(a+\frac{1}{6}i\,\log\left(cx^2\right)\right)^3\,dx$$

input
```
integrate(sin(a+1/6*I*log(c*x^2))^3,x, algorithm="giac")
```

output
```
integrate(sin(a + 1/6*I*log(c*x^2))^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^3\left(a+\frac{1}{6}i\log\left(cx^2\right)\right)\,dx=\int \sin\left(a+\frac{\ln\left(c\,x^2\right)\,1\mathrm{i}}{6}\right)^3\,dx$$

input
```
int(sin(a + (log(c*x^2)*1i)/6)^3,x)
```

output
```
int(sin(a + (log(c*x^2)*1i)/6)^3, x)
```

Reduce [F]

$$\int \sin^3\left(a+\frac{1}{6}i\log\left(cx^2\right)\right)\,dx=\int \sin\left(\frac{\log(c\,x^2)\,i}{6}+a\right)^3dx$$

input
```
int(sin(a+1/6*I*log(c*x^2))^3,x)
```

output
```
int(sin((log(c*x**2)*i + 6*a)/6)**3,x)
```

3.53 $\int x\sqrt{\sin(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 111

$$\int x\sqrt{\sin(a+b\log(cx^n))}\,dx = \frac{2x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-1-\frac{4i}{bn}\right),\frac{1}{4}\left(3-\frac{4i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sin(a+b\log(cx^n))}}{(4-ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

output
```
2*x^2*hypergeom([-1/2, -1/4-I/b/n],[3/4-I/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))
*sin(a+b*ln(c*x^n))^(1/2)/(4-I*b*n)/(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)
```

Mathematica [A] (verified)

Time = 10.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.31

$$\int x\sqrt{\sin(a+b\log(cx^n))}\,dx = \frac{i\sqrt{2}x^2\sqrt{-ie^{-ia}(cx^n)^{-ib}\left(-1+e^{2ia}(cx^n)^{2ib}\right)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4}-\frac{i}{bn},\frac{3}{4}-\frac{i}{bn},e^{2ia}(cx^n)^{2ib}\right)}{(4i+bn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

input
```
Integrate[x*Sqrt[Sin[a + b*Log[c*x^n]]],x]
```

output
```
(I*Sqrt[2]*x^2*Sqrt[((-I)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))/(E^(I*a)*(
c*x^n)^(I*b))]*Hypergeometric2F1[-1/2, -1/4 - I/(b*n), 3/4 - I/(b*n), E^((
2*I)*a)*(c*x^n)^((2*I)*b)])/((4*I + b*n)*Sqrt[1 - E^((2*I)*a)*(c*x^n)^((2*
I)*b)])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4996, 4994, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\sin(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{x^2(cx^n)^{-2/n}\int (cx^n)^{\frac{2}{n}-1}\sqrt{\sin(a+b\log(cx^n))}d(cx^n)}{n}$$

$$\Big\downarrow \text{4994}$$

$$\frac{x^2(cx^n)^{-\frac{2}{n}+\frac{ib}{2}}\sqrt{\sin(a+b\log(cx^n))}\int (cx^n)^{-\frac{ib}{2}+\frac{2}{n}-1}\sqrt{1-e^{2ia}(cx^n)^{2ib}}d(cx^n)}{n\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-1-\frac{4i}{bn}\right),\frac{1}{4}\left(3-\frac{4i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sin(a+b\log(cx^n))}}{(4-ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

input
```
Int[x*Sqrt[Sin[a + b*Log[c*x^n]]],x]
```

output
```
(2*x^2*Hypergeometric2F1[-1/2, (-1 - (4*I)/(b*n))/4, (3 - (4*I)/(b*n))/4,
E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[Sin[a + b*Log[c*x^n]]])/((4 - I*b*n)*S
qrt[1 - E^((2*I)*a)*(c*x^n)^((2*I)*b)])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4994
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Simp[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x\sqrt{\sin \left(a+b\ln \left(c\,x^{n}\right)\right)}dx$$

input
```
int(x*sin(a+b*ln(c*x^n))^(1/2),x)
```

output
```
int(x*sin(a+b*ln(c*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{\sin\left(a+b\log\left(cx^{n}\right)\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sin(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x\sqrt{\sin\left(a+b\log\left(cx^{n}\right)\right)}\,dx = \int x\sqrt{\sin\left(a+b\log\left(cx^{n}\right)\right)}\,dx$$

input
```
integrate(x*sin(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(x*sqrt(sin(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int x\sqrt{\sin\left(a+b\log\left(cx^{n}\right)\right)}\,dx = \int x\sqrt{\sin\left(b\log\left(cx^{n}\right)+a\right)}\,dx$$

input
```
integrate(x*sin(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*sqrt(sin(b*log(c*x^n) + a)), x)
```

Giac [F]

$$\int x\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}\,dx=\int x\sqrt{\sin\left(b\log\left(cx^n\right)+a\right)}\,dx$$

input
```
integrate(x*sin(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x*sqrt(sin(b*log(c*x^n) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}\,dx=\int x\,\sqrt{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}\,dx$$

input
```
int(x*sin(a + b*log(c*x^n))^(1/2),x)
```

output
```
int(x*sin(a + b*log(c*x^n))^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}\,dx=\frac{\sqrt{\sin\left(\log\left(x^nc\right)b+a\right)}\,x^2}{2}-\frac{\left(\int\frac{\sqrt{\sin(\log(x^nc)b+a)}\,\cos(\log(x^nc)b+a)x}{\sin(\log(x^nc)b+a)}dx\right)bn}{4}$$

input
```
int(x*sin(a+b*log(c*x^n))^(1/2),x)
```

output
```
(2*sqrt(sin(log(x**n*c)*b + a))*x**2 - int((sqrt(sin(log(x**n*c)*b + a))*c
os(log(x**n*c)*b + a)*x)/sin(log(x**n*c)*b + a),x)*b*n)/4
```

3.54 $\int \sqrt{\sin(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \sqrt{\sin(a+b\log(cx^n))}\,dx = \frac{2x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2i+bn}{4bn},\frac{1}{4}\left(3-\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sin(a+b\log(cx^n))}}{(2-ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

output
```
2*x*hypergeom([-1/2, -1/4*(2*I+b*n)/b/n],[3/4-1/2*I/b/n],exp(2*I*a)*(c*x^n
)^(2*I*b))*sin(a+b*ln(c*x^n))^(1/2)/(2-I*b*n)/(1-exp(2*I*a)*(c*x^n)^(2*I*b
))^(1/2)
```

Mathematica [A] (verified)

Time = 9.04 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.35

$$\int \sqrt{\sin(a+b\log(cx^n))}\,dx = \frac{i\sqrt{2}x\sqrt{-ie^{-ia}(cx^n)^{-ib}\left(-1+e^{2ia}(cx^n)^{2ib}\right)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2i+bn}{4bn},\frac{3}{4}-\frac{i}{2bn},e^{2ia}(cx^n)^{2ib}\right)}{(2i+bn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

input
```
Integrate[Sqrt[Sin[a + b*Log[c*x^n]]],x]
```

output
```
(I*Sqrt[2]*x*Sqrt[((-I)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))/(E^(I*a)*(c*
x^n)^(I*b))]*Hypergeometric2F1[-1/2, -1/4*(2*I + b*n)/(b*n), 3/4 - (I/2)/(
b*n), E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((2*I + b*n)*Sqrt[1 - E^((2*I)*a)*(c
*x^n)^((2*I)*b)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4986, 4994, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin (a+b\log (cx^n))}dx$$

$$\Big\downarrow \text{ 4986}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\sqrt{\sin (a+b\log (cx^n))}d(cx^n)}{n}$$

$$\Big\downarrow \text{ 4994}$$

$$\frac{x(cx^n)^{-\frac{1}{n}+\frac{ib}{2}}\sqrt{\sin (a+b\log (cx^n))}\int (cx^n)^{-\frac{ib}{2}+\frac{1}{n}-1}\sqrt{1-e^{2ia}(cx^n)^{2ib}}d(cx^n)}{n\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{2x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{bn+2i}{4bn},\frac{1}{4}\left(3-\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sin (a+b\log (cx^n))}}{(2-ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

input
```
Int[Sqrt[Sin[a + b*Log[c*x^n]]],x]
```

output
```
(2*x*Hypergeometric2F1[-1/2, -1/4*(2*I + b*n)/(b*n), (3 - (2*I)/(b*n))/4,
E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[Sin[a + b*Log[c*x^n]]])/((2 - I*b*n)*S
qrt[1 - E^((2*I)*a)*(c*x^n)^((2*I)*b)])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4986
```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 4994
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Simp[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \sqrt{\sin \left(a+b\ln \left(c\,x^{n}\right)\right)}dx$$

input
```
int(sin(a+b*ln(c*x^n))^(1/2),x)
```

output
```
int(sin(a+b*ln(c*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\sin (a+b\log (cx^n))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sin(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{\sin (a+b\log (cx^n))}\,dx = \int \sqrt{\sin (a+b\log (cx^n))}\,dx$$

input
```
integrate(sin(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(sqrt(sin(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \sqrt{\sin (a+b\log (cx^n))}\,dx = \int \sqrt{\sin (b\log (cx^n)+a)}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sin(b*log(c*x^n) + a)), x)
```

Giac [F]

$$\int \sqrt{\sin (a+b\log (cx^n))}\,dx=\int \sqrt{\sin (b\log (cx^n)+a)}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sin(b*log(c*x^n) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sin (a+b\log (cx^n))}\,dx=\int \sqrt{\sin \left(a+b\,\ln \left(c\,x^n\right)\right)}\,dx$$

input
```
int(sin(a + b*log(c*x^n))^(1/2),x)
```

output
```
int(sin(a + b*log(c*x^n))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\sin (a+b\log (cx^n))}\,dx=\sqrt{\sin (\log (x^n c)\,b+a)}\,x-\frac{\left(\int \frac{\sqrt{\sin(\log(x^n c)b+a)}\,\cos(\log(x^n c)b+a)}{\sin(\log(x^n c)b+a)}dx\right)bn}{2}$$

input
```
int(sin(a+b*log(c*x^n))^(1/2),x)
```

output
```
(2*sqrt(sin(log(x**n*c)*b + a))*x - int((sqrt(sin(log(x**n*c)*b + a))*cos(
log(x**n*c)*b + a))/sin(log(x**n*c)*b + a),x)*b*n)/2
```

3.55 $\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 30

$$\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x}\,dx = \frac{2E\left(\frac{1}{4}(2a-\pi+2b\log(cx^n))\middle|2\right)}{bn}$$

output
```
-2*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*ln(c*x^n)),2^(1/2))/b/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x}\,dx = -\frac{2\sec(a+b\log(cx^n))\left(-3+2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},-\tan^2(a+b\log(cx^n))\right)\sqrt[4]{\sec^2(a+b\log}\right.}{3bn}$$

input
```
Integrate[Sqrt[Sin[a + b*Log[c*x^n]]]/x,x]
```

output
```
(-2*Sec[a + b*Log[c*x^n]]*(-3 + 2*Hypergeometric2F1[1/4, 3/4, 7/4, -Tan[a
+ b*Log[c*x^n]]^2]*(Sec[a + b*Log[c*x^n]]^2)^(1/4))*Sin[a + b*Log[c*x^n]]^
(3/2))/(3*b*n)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3039, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x}\,dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \sqrt{\sin(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\sin(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2E\left(\frac{1}{2}\left(a+b\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{bn}$$

input
```
Int[Sqrt[Sin[a + b*Log[c*x^n]]]/x,x]
```

output
```
(2*EllipticE[(a - Pi/2 + b*Log[c*x^n])/2, 2])/(b*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(29) = 58$.

Time = 0.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 4.30

method	result
derivativedivides	$-\frac{\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\left(2\,\mathrm{EllipticE}\left(\sqrt{\sin(a+b\ln(c\,x^n))+1},\frac{\sqrt{2}}{2}\right)-\right.}{n\cos(a+b\ln(c\,x^n))\sqrt{\sin(a+b\ln(c\,x^n))}\,b}$
default	$-\frac{\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\left(2\,\mathrm{EllipticE}\left(\sqrt{\sin(a+b\ln(c\,x^n))+1},\frac{\sqrt{2}}{2}\right)-\right.}{n\cos(a+b\ln(c\,x^n))\sqrt{\sin(a+b\ln(c\,x^n))}\,b}$

input
```
int(sin(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/n*(sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin(a+
b*ln(c*x^n)))^(1/2)*(2*EllipticE((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2))
-EllipticF((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2)))/cos(a+b*ln(c*x^n))/s
in(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.83

$$\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x}\,dx =$$
$$-\frac{2\left(-i\sqrt{-\frac{1}{2}i}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(bn\log(x)+b\log(c)+a)+i\sin(bn\log\right.}{}$$

input
```
integrate(sin(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

output
```
-2*(-I*sqrt(-1/2*I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*
n*log(x) + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a))) + I*sqrt(1/2
*I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*n*log(x) + b*log
(c) + a) - I*sin(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F]

$$\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x}\,dx = \int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x}\,dx$$

input
```
integrate(sin(a+b*ln(c*x**n))**(1/2)/x,x)
```

output
```
Integral(sqrt(sin(a + b*log(c*x**n)))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x}\,dx = \int \frac{\sqrt{\sin(b\log(cx^n)+a)}}{x}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(sin(b*log(c*x^n) + a))/x, x)
```

Giac [F]

$$\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x}\,dx = \int \frac{\sqrt{\sin(b\log(cx^n)+a)}}{x}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(sin(b*log(c*x^n) + a))/x, x)
```

Mupad [B] (verification not implemented)

Time = 20.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x}\,dx = \frac{2\,\mathrm{E}\left(\frac{a}{2}-\frac{\pi}{4}+\frac{b\ln(c\,x^n)}{2}\,\middle|\,2\right)}{b\,n}$$

input
```
int(sin(a + b*log(c*x^n))^(1/2)/x,x)
```

output
```
(2*ellipticE(a/2 - pi/4 + (b*log(c*x^n))/2, 2))/(b*n)
```

Reduce [F]

$$\int \frac{\sqrt{\sin \left(a+b \log \left(c x^n\right)\right)}}{x} \, dx=\int \frac{\sqrt{\sin \left(\log \left(x^n c\right) b+a\right)}}{x}dx$$

input
```
int(sin(a+b*log(c*x^n))^(1/2)/x,x)
```

output
```
int(sqrt(sin(log(x**n*c)*b + a))/x,x)
```

3.56 $\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 111

$$\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x^2}\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-1+\frac{2i}{bn}\right),\frac{1}{4}\left(3+\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sin(a+b\log(cx^n))}}{(2+ibn)x\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

output
```
-2*hypergeom([-1/2, -1/4+1/2*I/b/n],[3/4+1/2*I/b/n],exp(2*I*a)*(c*x^n)^(2*
I*b))*sin(a+b*ln(c*x^n))^(1/2)/(2+I*b*n)/x/(1-exp(2*I*a)*(c*x^n)^(2*I*b))^
(1/2)
```

Mathematica [A] (verified)

Time = 10.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x^2}\,dx = \frac{i\sqrt{2}\sqrt{-ie^{-ia}(cx^n)^{-ib}\left(-1+e^{2ia}(cx^n)^{2ib}\right)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4}+\frac{i}{2bn},\frac{3}{4}+\frac{i}{2bn},e^{2ia}(cx^n)^{2ib}\right)}{(-2i+bn)x\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

input
```
Integrate[Sqrt[Sin[a + b*Log[c*x^n]]]/x^2,x]
```

output
```
(I*Sqrt[2]*Sqrt[((-I)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))/(E^(I*a)*(c*x^
n)^(I*b))]*Hypergeometric2F1[-1/2, -1/4 + (I/2)/(b*n), 3/4 + (I/2)/(b*n),
E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((-2*I + b*n)*x*Sqrt[1 - E^((2*I)*a)*(c*x^
n)^((2*I)*b)])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4996, 4994, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sin (a+b\log (cx^n))}}{x^2}dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{(cx^n)^{\frac{1}{n}}\int (cx^n)^{-1-\frac{1}{n}}\sqrt{\sin (a+b\log (cx^n))}d(cx^n)}{nx}$$

$$\Big\downarrow \text{4994}$$

$$\frac{(cx^n)^{\frac{1}{n}+\frac{ib}{2}}\sqrt{\sin (a+b\log (cx^n))}\int (cx^n)^{-\frac{ib}{2}-\frac{1}{n}-1}\sqrt{1-e^{2ia}(cx^n)^{2ib}}d(cx^n)}{nx\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(\frac{2i}{bn}-1\right),\frac{1}{4}\left(3+\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sin (a+b\log (cx^n))}}{x(2+ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

input
```
Int[Sqrt[Sin[a + b*Log[c*x^n]]]/x^2,x]
```

output
```
(-2*Hypergeometric2F1[-1/2, (-1 + (2*I)/(b*n))/4, (3 + (2*I)/(b*n))/4, E^(
(2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[Sin[a + b*Log[c*x^n]]])/((2 + I*b*n)*x*Sq
rt[1 - E^((2*I)*a)*(c*x^n)^((2*I)*b)])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4994
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Simp[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\sqrt{\sin \left(a+b\ln \left(c\,x^{n}\right)\right)}}{x^{2}}dx$$

input
```
int(sin(a+b*ln(c*x^n))^(1/2)/x^2,x)
```

output
```
int(sin(a+b*ln(c*x^n))^(1/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sin(a+b*log(c*x^n))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x^2}\,dx = \int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x^2}\,dx$$

input
```
integrate(sin(a+b*ln(c*x**n))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(sin(a + b*log(c*x**n)))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x^2}\,dx = \int \frac{\sqrt{\sin(b\log(cx^n)+a)}}{x^2}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(sin(b*log(c*x^n) + a))/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}{x^2}\,dx=\int \frac{\sqrt{\sin\left(b\log\left(cx^n\right)+a\right)}}{x^2}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(sin(b*log(c*x^n) + a))/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}{x^2}\,dx=\int \frac{\sqrt{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}}{x^2}\,dx$$

input
```
int(sin(a + b*log(c*x^n))^(1/2)/x^2,x)
```

output
```
int(sin(a + b*log(c*x^n))^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}{x^2}\,dx = \frac{-2\sqrt{\sin\left(\log\left(x^n c\right)b+a\right)}+\left(\int \frac{\sqrt{\sin(\log(x^n c)b+a)}\,\cos(\log(x^n c)b+a)}{\sin(\log(x^n c)b+a)x^2}dx\right)bnx}{2x}$$

input
```
int(sin(a+b*log(c*x^n))^(1/2)/x^2,x)
```

output
```
( - 2*sqrt(sin(log(x**n*c)*b + a)) + int((sqrt(sin(log(x**n*c)*b + a))*cos
(log(x**n*c)*b + a))/(sin(log(x**n*c)*b + a)*x**2),x)*b*n*x)/(2*x)
```

3.57 $\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 111

$$\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x^3}\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-1+\frac{4i}{bn}\right),\frac{1}{4}\left(3+\frac{4i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sin(a+b\log(cx^n))}}{(4+ibn)x^2\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

output
```
-2*hypergeom([-1/2, -1/4+I/b/n],[3/4+I/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))*si
n(a+b*ln(c*x^n))^(1/2)/(4+I*b*n)/x^2/(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)
```

Mathematica [A] (verified)

Time = 10.47 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x^3}\,dx = \frac{i\sqrt{2}\sqrt{-ie^{-ia}(cx^n)^{-ib}\left(-1+e^{2ia}(cx^n)^{2ib}\right)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4}+\frac{i}{bn},\frac{3}{4}+\frac{i}{bn},e^{2ia}(cx^n)^{2ib}\right)}{(-4i+bn)x^2\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

input
```
Integrate[Sqrt[Sin[a + b*Log[c*x^n]]]/x^3,x]
```

output
```
(I*Sqrt[2]*Sqrt[((-I)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))/(E^(I*a)*(c*x^
n)^(I*b))]*Hypergeometric2F1[-1/2, -1/4 + I/(b*n), 3/4 + I/(b*n), E^((2*I)
*a)*(c*x^n)^((2*I)*b)])/((-4*I + b*n)*x^2*Sqrt[1 - E^((2*I)*a)*(c*x^n)^((2
*I)*b)])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4996, 4994, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sin (a+b\log (cx^n))}}{x^3}dx$$

$$\Big\downarrow \ 4996$$

$$\frac{(cx^n)^{2/n}\int (cx^n)^{-1-\frac{2}{n}}\sqrt{\sin (a+b\log (cx^n))}d(cx^n)}{nx^2}$$

$$\Big\downarrow \ 4994$$

$$\frac{(cx^n)^{\frac{2}{n}+\frac{ib}{2}}\sqrt{\sin (a+b\log (cx^n))}\int (cx^n)^{-\frac{ib}{2}-\frac{2}{n}-1}\sqrt{1-e^{2ia}(cx^n)^{2ib}}d(cx^n)}{nx^2\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

$$\Big\downarrow \ 888$$

$$-\frac{2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(\frac{4i}{bn}-1\right),\frac{1}{4}\left(3+\frac{4i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sin (a+b\log (cx^n))}}{x^2(4+ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

input
```
Int[Sqrt[Sin[a + b*Log[c*x^n]]]/x^3,x]
```

output
```
(-2*Hypergeometric2F1[-1/2, (-1 + (4*I)/(b*n))/4, (3 + (4*I)/(b*n))/4, E^(
(2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[Sin[a + b*Log[c*x^n]]])/((4 + I*b*n)*x^2*
Sqrt[1 - E^((2*I)*a)*(c*x^n)^((2*I)*b)])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4994
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Simp[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\sqrt{\sin\left(a+b\ln\left(c\,x^{n}\right)\right)}}{x^{3}}dx$$

input
```
int(sin(a+b*ln(c*x^n))^(1/2)/x^3,x)
```

output
```
int(sin(a+b*ln(c*x^n))^(1/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\sin(a + b\log(cx^n))}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sin(a+b*log(c*x^n))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{\sin(a + b\log(cx^n))}}{x^3}\,dx = \int \frac{\sqrt{\sin(a + b\log(cx^n))}}{x^3}\,dx$$

input
```
integrate(sin(a+b*ln(c*x**n))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(sin(a + b*log(c*x**n)))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{\sin(a + b\log(cx^n))}}{x^3}\,dx = \int \frac{\sqrt{\sin(b\log(cx^n) + a)}}{x^3}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(sin(b*log(c*x^n) + a))/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x^3}\,dx = \int \frac{\sqrt{\sin(b\log(cx^n)+a)}}{x^3}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(sin(b*log(c*x^n) + a))/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x^3}\,dx = \int \frac{\sqrt{\sin(a+b\,\ln(c\,x^n))}}{x^3}\,dx$$

input
```
int(sin(a + b*log(c*x^n))^(1/2)/x^3,x)
```

output
```
int(sin(a + b*log(c*x^n))^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{\sin(a+b\log(cx^n))}}{x^3}\,dx = \frac{-2\sqrt{\sin(\log(x^n c)\,b+a)}+\left(\int \frac{\sqrt{\sin(\log(x^n c)b+a)}\,\cos(\log(x^n c)b+a)}{\sin(\log(x^n c)b+a)x^3}dx\right)bn\,x^2}{4x^2}$$

input
```
int(sin(a+b*log(c*x^n))^(1/2)/x^3,x)
```

output
```
( - 2*sqrt(sin(log(x**n*c)*b + a)) + int((sqrt(sin(log(x**n*c)*b + a))*cos
(log(x**n*c)*b + a))/(sin(log(x**n*c)*b + a)*x**3),x)*b*n*x**2)/(4*x**2)
```

3.58 $\int x \sin^{\frac{3}{2}}(a + b\log(cx^n)) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 111

$$\int x \sin^{\frac{3}{2}}(a + b\log(cx^n)) \, dx = \frac{2x^2 \text{Hypergeometric2F1}\left(-\frac{3}{2}, \frac{1}{4}\left(-3 - \frac{4i}{bn}\right), \frac{1}{4}\left(1 - \frac{4i}{bn}\right), e^{2ia}(cx^n)^{2ib}\right) \sin^{\frac{3}{2}}(a + b\log(cx^n))}{(4 - 3ibn)\left(1 - e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

output
```
2*x^2*hypergeom([-3/2, -3/4-I/b/n],[1/4-I/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))
*sin(a+b*ln(c*x^n))^(3/2)/(4-3*I*b*n)/(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.96

$$\int x \sin^{\frac{3}{2}}(a + b\log(cx^n)) \, dx = -\frac{6ib^2\sqrt{2 - 2e^{2i(a+b\log(cx^n))}}n^2x^2 \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{4} - \frac{i}{bn}, \frac{5}{4} - \frac{i}{bn}, e^{2i(a+b\log(cx^n))}\right)}{\sqrt{-ie^{-i(a+b\log(cx^n))}}\left(-1 + e^{2i(a+b\log(cx^n))}\right)(-4i + bn)(-4i + 3bn)(4i + 3bn)} + \frac{2x^2\sqrt{\sin(a + b\log(cx^n))}(-3bn\cos(a + b\log(cx^n)) + 4\sin(a + b\log(cx^n)))}{16 + 9b^2n^2}$$

input
```
Integrate[x*Sin[a + b*Log[c*x^n]]^(3/2),x]
```

output
```
((-6*I)*b^2*Sqrt[2 - 2*E^((2*I)*(a + b*Log[c*x^n]))]*n^2*x^2*Hypergeometri
c2F1[1/2, 1/4 - I/(b*n), 5/4 - I/(b*n), E^((2*I)*(a + b*Log[c*x^n]))])/(Sq
rt[((-I)*(-1 + E^((2*I)*(a + b*Log[c*x^n]))))/E^(I*(a + b*Log[c*x^n]))]*(-
4*I + b*n)*(-4*I + 3*b*n)*(4*I + 3*b*n)) + (2*x^2*Sqrt[Sin[a + b*Log[c*x^n
]]]*(-3*b*n*Cos[a + b*Log[c*x^n]] + 4*Sin[a + b*Log[c*x^n]]))/(16 + 9*b^2*
n^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4996, 4994, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin^{\frac{3}{2}}(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{x^2(cx^n)^{-2/n}\int (cx^n)^{\frac{2}{n}-1}\sin^{\frac{3}{2}}(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{4994}$$

$$\frac{x^2(cx^n)^{-\frac{2}{n}+\frac{3ib}{2}}\sin^{\frac{3}{2}}(a+b\log(cx^n))\int (cx^n)^{-\frac{3ib}{2}+\frac{2}{n}-1}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}d(cx^n)}{n\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{4i}{bn}\right),\frac{1}{4}\left(1-\frac{4i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)\sin^{\frac{3}{2}}(a+b\log(cx^n))}{(4-3ibn)\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

input
```
Int[x*Sin[a + b*Log[c*x^n]]^(3/2),x]
```

output
```
(2*x^2*Hypergeometric2F1[-3/2, (-3 - (4*I)/(b*n))/4, (1 - (4*I)/(b*n))/4,
E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sin[a + b*Log[c*x^n]]^(3/2))/((4 - (3*I)*b*
n)*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4994
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Simp[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x \sin\left(a+b\ln\left(c\,x^{n}\right)\right)^{\frac{3}{2}}dx$$

input
```
int(x*sin(a+b*ln(c*x^n))^(3/2),x)
```

output
```
int(x*sin(a+b*ln(c*x^n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\sin^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*sin(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int x\sin^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x*sin(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x\sin^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\,dx=\int x\sin\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*sin(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x*sin(b*log(c*x^n) + a)^(3/2), x)
```

Giac [F]

$$\int x \sin^{\frac{3}{2}}(a + b\log(cx^n)) \, dx = \int x \sin(b\log(cx^n) + a)^{\frac{3}{2}} \, dx$$

input
```
integrate(x*sin(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate(x*sin(b*log(c*x^n) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \sin^{\frac{3}{2}}(a + b\log(cx^n)) \, dx = \int x \sin(a + b \ln(c x^n))^{3/2} \, dx$$

input
```
int(x*sin(a + b*log(c*x^n))^(3/2),x)
```

output
```
int(x*sin(a + b*log(c*x^n))^(3/2), x)
```

Reduce [F]

$$\int x \sin^{\frac{3}{2}}(a + b\log(cx^n)) \, dx = \frac{\sqrt{\sin(\log(x^n c) b + a)} \sin(\log(x^n c) b + a) x^2}{2} - \frac{3\left(\int \sqrt{\sin(\log(x^n c) b + a)} \cos(\log(x^n c) b + a) x dx\right) bn}{4}$$

input
```
int(x*sin(a+b*log(c*x^n))^(3/2),x)
```

output
```
(2*sqrt(sin(log(x**n*c)*b + a))*sin(log(x**n*c)*b + a)*x**2 - 3*int(sqrt(s
in(log(x**n*c)*b + a))*cos(log(x**n*c)*b + a)*x,x)*b*n)/4
```

3.59 $\int \sin^{\frac{3}{2}}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \sin^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \frac{2x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i}{bn}\right),\frac{1}{4}\left(1-\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)\sin^{\frac{3}{2}}(a+b\log(cx^n))}{(2-3ibn)\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

output
```
2*x*hypergeom([-3/2, -3/4-1/2*I/b/n],[1/4-1/2*I/b/n],exp(2*I*a)*(c*x^n)^(2
*I*b))*sin(a+b*ln(c*x^n))^(3/2)/(2-3*I*b*n)/(1-exp(2*I*a)*(c*x^n)^(2*I*b))
^(3/2)
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.00

$$\int \sin^{\frac{3}{2}}(a+b\log(cx^n))\,dx = -\frac{6ib^2\sqrt{2-2e^{2i(a+b\log(cx^n))}}n^2x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}-\frac{i}{2bn},\frac{5}{4}-\frac{i}{2bn},e^{2i(a+b\log(cx^n))}\right)}{\sqrt{-ie^{-i(a+b\log(cx^n))}\left(-1+e^{2i(a+b\log(cx^n))}\right)}(-2i+bn)(-2i+3bn)(2i+3bn)} + \frac{2x\sqrt{\sin(a+b\log(cx^n))}(-3bn\cos(a+b\log(cx^n))+2\sin(a+b\log(cx^n)))}{4+9b^2n^2}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]^(3/2),x]
```

output
```
((-6*I)*b^2*Sqrt[2 - 2*E^((2*I)*(a + b*Log[c*x^n]))]*n^2*x*Hypergeometric2
F1[1/2, 1/4 - (I/2)/(b*n), 5/4 - (I/2)/(b*n), E^((2*I)*(a + b*Log[c*x^n]))
])/(Sqrt[((-I)*(-1 + E^((2*I)*(a + b*Log[c*x^n]))))/E^(I*(a + b*Log[c*x^n]
))]*(-2*I + b*n)*(-2*I + 3*b*n)*(2*I + 3*b*n)) + (2*x*Sqrt[Sin[a + b*Log[c
*x^n]]]*(-3*b*n*Cos[a + b*Log[c*x^n]] + 2*Sin[a + b*Log[c*x^n]]))/(4 + 9*b
^2*n^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4986, 4994, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{3}{2}}(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{4986}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\sin^{\frac{3}{2}}(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{4994}$$

$$\frac{x(cx^n)^{-\frac{1}{n}+\frac{3ib}{2}}\sin^{\frac{3}{2}}(a+b\log(cx^n))\int (cx^n)^{-\frac{3ib}{2}+\frac{1}{n}-1}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}d(cx^n)}{n\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i}{bn}\right),\frac{1}{4}\left(1-\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)\sin^{\frac{3}{2}}(a+b\log(cx^n))}{(2-3ibn)\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

input
```
Int[Sin[a + b*Log[c*x^n]]^(3/2),x]
```

output
```
(2*x*Hypergeometric2F1[-3/2, (-3 - (2*I)/(b*n))/4, (1 - (2*I)/(b*n))/4, E^
((2*I)*a)*(c*x^n)^((2*I)*b)]*Sin[a + b*Log[c*x^n]]^(3/2))/((2 - (3*I)*b*n)
*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4986
```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 4994
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Simp[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Maple [F]

$$\int \sin \left(a+b\ln \left(c\,x^{n}\right)\right)^{\frac{3}{2}}dx$$

input
```
int(sin(a+b*ln(c*x^n))^(3/2),x)
```

output
```
int(sin(a+b*ln(c*x^n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sin^{\frac{3}{2}}(a + b\log(cx^n)) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(sin(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sin^{\frac{3}{2}}(a + b\log(cx^n)) \, dx = \int \sin^{\frac{3}{2}}(a + b\log(cx^n)) \, dx$$

input
```
integrate(sin(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Integral(sin(a + b*log(c*x**n))**(3/2), x)
```

Maxima [F]

$$\int \sin^{\frac{3}{2}}(a + b\log(cx^n)) \, dx = \int \sin(b\log(cx^n) + a)^{\frac{3}{2}} \, dx$$

input
```
integrate(sin(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*log(c*x^n) + a)^(3/2), x)
```

Giac [F]

$$\int \sin^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \int \sin(b\log(cx^n)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(b*log(c*x^n) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \int \sin(a+b\,\ln(c\,x^n))^{3/2}\,dx$$

input
```
int(sin(a + b*log(c*x^n))^(3/2),x)
```

output
```
int(sin(a + b*log(c*x^n))^(3/2), x)
```

Reduce [F]

$$\int \sin^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \sqrt{\sin(\log(x^n c)\,b+a)}\,\sin(\log(x^n c)\,b+a)\,x - \frac{3\left(\int \sqrt{\sin(\log(x^n c)\,b+a)}\,\cos(\log(x^n c)\,b+a)\,dx\right)bn}{2}$$

input
```
int(sin(a+b*log(c*x^n))^(3/2),x)
```

output
```
(2*sqrt(sin(log(x**n*c)*b + a))*sin(log(x**n*c)*b + a)*x - 3*int(sqrt(sin(
log(x**n*c)*b + a))*cos(log(x**n*c)*b + a),x)*b*n)/2
```

3.60 $\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x} \, dx = \frac{2\,\text{EllipticF}\left(\frac{1}{4}(2a-\pi+2b\log(cx^n)),2\right)}{3bn} - \frac{2\cos(a+b\log(cx^n))\sqrt{\sin(a+b\log(cx^n))}}{3bn}$$

output
```
2/3*InverseJacobiAM(1/2*a-1/4*Pi+1/2*b*ln(c*x^n),2^(1/2))/b/n-2/3*cos(a+b*
ln(c*x^n))*sin(a+b*ln(c*x^n))^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x} \, dx = -\frac{2\left(\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2b\log(cx^n)),2\right)+\cos(a+b\log(cx^n))\sqrt{\sin(a+b\log(cx^n))}\right)}{3bn}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
(-2*(EllipticF[(-2*a + Pi - 2*b*Log[c*x^n])/4, 2] + Cos[a + b*Log[c*x^n]]*
Sqrt[Sin[a + b*Log[c*x^n]]]))/(3*b*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3039, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \sin^{\frac{3}{2}}(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin(a+b\log(cx^n))^{3/2}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+b\log(cx^n))}}d\log(cx^n)-\frac{2\sqrt{\sin(a+b\log(cx^n))}\cos(a+b\log(cx^n))}{3b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+b\log(cx^n))}}d\log(cx^n)-\frac{2\sqrt{\sin(a+b\log(cx^n))}\cos(a+b\log(cx^n))}{3b}}{n}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2\text{EllipticF}\left(\frac{1}{2}\left(a+b\log(cx^n)-\frac{\pi}{2}\right),2\right)}{3b}-\frac{2\sqrt{\sin(a+b\log(cx^n))}\cos(a+b\log(cx^n))}{3b}}{n}$$

input
```
Int[Sin[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
((2*EllipticF[(a - Pi/2 + b*Log[c*x^n])/2, 2])/(3*b) - (2*Cos[a + b*Log[c*
x^n]]*Sqrt[Sin[a + b*Log[c*x^n]]])/(3*b))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(61) = 122$.

Time = 0.40 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.90

method	result
derivativedivides	$\frac{\frac{\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\;\text{EllipticF}\left(\sqrt{\sin(a+b\ln(c\,x^n))+1},\frac{\sqrt{2}}{2}\right)}{3}-\frac{2\cos(a+b\ln(c\,x^n))}{3}}{n\cos(a+b\ln(c\,x^n))\sqrt{\sin(a+b\ln(c\,x^n))}\,b}$
default	$\frac{\frac{\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\;\text{EllipticF}\left(\sqrt{\sin(a+b\ln(c\,x^n))+1},\frac{\sqrt{2}}{2}\right)}{3}-\frac{2\cos(a+b\ln(c\,x^n))}{3}}{n\cos(a+b\ln(c\,x^n))\sqrt{\sin(a+b\ln(c\,x^n))}\,b}$

input
```
int(sin(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(1/3*(sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-si
n(a+b*ln(c*x^n)))^(1/2)*EllipticF((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2)
)-2/3*cos(a+b*ln(c*x^n))^2*sin(a+b*ln(c*x^n)))/cos(a+b*ln(c*x^n))/sin(a+b*
ln(c*x^n))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.54

$$\int \frac{\sin^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = -\frac{2\left(\cos(bn\log(x) + b\log(c) + a)\sqrt{\sin(bn\log(x) + b\log(c) + a)} - \sqrt{-\frac{1}{2}i}\text{weierstrassPInverse}(4, 0, \text{co}$$

input
```
integrate(sin(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

output
```
-2/3*(cos(b*n*log(x) + b*log(c) + a)*sqrt(sin(b*n*log(x) + b*log(c) + a))
- sqrt(-1/2*I)*weierstrassPInverse(4, 0, cos(b*n*log(x) + b*log(c) + a) +
I*sin(b*n*log(x) + b*log(c) + a)) - sqrt(1/2*I)*weierstrassPInverse(4, 0,
cos(b*n*log(x) + b*log(c) + a) - I*sin(b*n*log(x) + b*log(c) + a)))/(b*n)
```

Sympy [F]

$$\int \frac{\sin^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \int \frac{\sin^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx$$

input
```
integrate(sin(a+b*ln(c*x**n))**(3/2)/x,x)
```

output
```
Integral(sin(a + b*log(c*x**n))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\sin^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \int \frac{\sin(b\log(cx^n) + a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(sin(b*log(c*x^n) + a)^(3/2)/x, x)
```

Giac [F]

$$\int \frac{\sin^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \int \frac{\sin(b\log(cx^n) + a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

output
```
integrate(sin(b*log(c*x^n) + a)^(3/2)/x, x)
```

Mupad [B] (verification not implemented)

Time = 20.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{\sin^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = -\frac{\cos(a + b\ln(c\,x^n))\,\sin(a + b\ln(c\,x^n))^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{1}{4}, \frac{1}{2};\ \frac{3}{2};\ \cos(a + b\ln(c\,x^n))^2\right)}{b\,n\left(\sin(a + b\ln(c\,x^n))^2\right)^{5/4}}$$

input
```
int(sin(a + b*log(c*x^n))^(3/2)/x,x)
```

output
```
-(cos(a + b*log(c*x^n))*sin(a + b*log(c*x^n))^(5/2)*hypergeom([-1/4, 1/2],
 3/2, cos(a + b*log(c*x^n))^2))/(b*n*(sin(a + b*log(c*x^n))^2)^(5/4))
```

Reduce [F]

$$\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sqrt{\sin(\log(x^n c)\,b+a)}\,\sin(\log(x^n c)\,b+a)}{x}dx$$

input
```
int(sin(a+b*log(c*x^n))^(3/2)/x,x)
```

output
```
int((sqrt(sin(log(x**n*c)*b + a))*sin(log(x**n*c)*b + a))/x,x)
```

3.61 $\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 111

$$\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^2}\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4}\left(-3+\frac{2i}{bn}\right),\frac{1}{4}\left(1+\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)\sin^{\frac{3}{2}}(a+b\log(cx^n))}{(2+3ibn)x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

output
```
-2*hypergeom([-3/2, -3/4+1/2*I/b/n],[1/4+1/2*I/b/n],exp(2*I*a)*(c*x^n)^(2*
I*b))*sin(a+b*ln(c*x^n))^(3/2)/(2+3*I*b*n)/x/(1-exp(2*I*a)*(c*x^n)^(2*I*b)
)^(3/2)
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.98

$$\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^2}\,dx = \frac{6b^2\sqrt{2-2e^{2i(a+b\log(cx^n))}}n^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}+\frac{i}{2bn},\frac{5}{4}+\frac{i}{2bn},e^{2i(a+b\log(cx^n))}\right)}{\sqrt{-ie^{-i(a+b\log(cx^n))}\left(-1+e^{2i(a+b\log(cx^n))}\right)}(2+3ibn)(2i+bn)(2i+3bn)x} - \frac{2\sqrt{\sin(a+b\log(cx^n))}(3bn\cos(a+b\log(cx^n))+2\sin(a+b\log(cx^n)))}{(4+9b^2n^2)\,x}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]^(3/2)/x^2,x]
```

output
```
(6*b^2*Sqrt[2 - 2*E^((2*I)*(a + b*Log[c*x^n]))]*n^2*Hypergeometric2F1[1/2,
 1/4 + (I/2)/(b*n), 5/4 + (I/2)/(b*n), E^((2*I)*(a + b*Log[c*x^n]))])/(Sqr
t[((-I)*(-1 + E^((2*I)*(a + b*Log[c*x^n]))))/E^(I*(a + b*Log[c*x^n]))]*(2
+ (3*I)*b*n)*(2*I + b*n)*(2*I + 3*b*n)*x) - (2*Sqrt[Sin[a + b*Log[c*x^n]]]
*(3*b*n*Cos[a + b*Log[c*x^n]] + 2*Sin[a + b*Log[c*x^n]]))/((4 + 9*b^2*n^2)
*x)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4996, 4994, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^2}dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{(cx^n)^{\frac{1}{n}}\int (cx^n)^{-1-\frac{1}{n}}\sin^{\frac{3}{2}}(a+b\log(cx^n))d(cx^n)}{nx}$$

$$\Big\downarrow \text{4994}$$

$$\frac{(cx^n)^{\frac{1}{n}+\frac{3ib}{2}}\sin^{\frac{3}{2}}(a+b\log(cx^n))\int (cx^n)^{-\frac{3ib}{2}-\frac{1}{n}-1}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}d(cx^n)}{nx\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4}\left(\frac{2i}{bn}-3\right),\frac{1}{4}\left(1+\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x(2+3ibn)\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

input
```
Int[Sin[a + b*Log[c*x^n]]^(3/2)/x^2,x]
```

output
```
(-2*Hypergeometric2F1[-3/2, (-3 + (2*I)/(b*n))/4, (1 + (2*I)/(b*n))/4, E^(
(2*I)*a)*(c*x^n)^((2*I)*b)]*Sin[a + b*Log[c*x^n]]^(3/2))/((2 + (3*I)*b*n)*
x*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4994
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Simp[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\sin \left(a+b \ln \left(c\,x^{n}\right)\right)^{\frac{3}{2}}}{x^{2}}dx$$

input
```
int(sin(a+b*ln(c*x^n))^(3/2)/x^2,x)
```

output
```
int(sin(a+b*ln(c*x^n))^(3/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sin(a+b*log(c*x^n))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^2}\,dx = \int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^2}\,dx$$

input
```
integrate(sin(a+b*ln(c*x**n))**(3/2)/x**2,x)
```

output
```
Integral(sin(a + b*log(c*x**n))**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^2}\,dx = \int \frac{\sin(b\log(cx^n)+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sin(b*log(c*x^n) + a)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{\sin^{\frac{3}{2}}(a + b\log(cx^n))}{x^2}\,dx = \int \frac{\sin(b\log(cx^n) + a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sin(b*log(c*x^n) + a)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{3}{2}}(a + b\log(cx^n))}{x^2}\,dx = \int \frac{\sin(a + b\ln(c\,x^n))^{3/2}}{x^2}\,dx$$

input
```
int(sin(a + b*log(c*x^n))^(3/2)/x^2,x)
```

output
```
int(sin(a + b*log(c*x^n))^(3/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sin^{\frac{3}{2}}(a + b\log(cx^n))}{x^2}\,dx = \frac{-2\sqrt{\sin(\log(x^n c)\,b + a)}\,\sin(\log(x^n c)\,b + a) + 3\left(\int \frac{\sqrt{\sin(\log(x^n c)b+a)}\,\cos(\log(x^n c)b+a)}{x^2}dx\right)bnx}{2x}$$

input
```
int(sin(a+b*log(c*x^n))^(3/2)/x^2,x)
```

output
```
( - 2*sqrt(sin(log(x**n*c)*b + a))*sin(log(x**n*c)*b + a) + 3*int((sqrt(si
n(log(x**n*c)*b + a))*cos(log(x**n*c)*b + a))/x**2,x)*b*n*x)/(2*x)
```

3.62 $\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 111

$$\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^3}\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4}\left(-3+\frac{4i}{bn}\right),\frac{1}{4}\left(1+\frac{4i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)\sin^{\frac{3}{2}}(a+b\log(cx^n))}{(4+3ibn)x^2\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

output
```
-2*hypergeom([-3/2, -3/4+I/b/n],[1/4+I/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))*si
n(a+b*ln(c*x^n))^(3/2)/(4+3*I*b*n)/x^2/(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2
)
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.95

$$\begin{aligned}\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^3}\,dx &= \frac{6b^2\sqrt{2-2e^{2i(a+b\log(cx^n))}}\,n^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}+\frac{i}{bn},\frac{5}{4}+\frac{i}{bn},e^{2i(a+b\log(cx^n))}\right)}{\sqrt{-ie^{-i(a+b\log(cx^n))}\left(-1+e^{2i(a+b\log(cx^n))}\right)}(4+3ibn)(4i+bn)(4i+3bn)x^2} \\ &\quad - \frac{2\sqrt{\sin(a+b\log(cx^n))}(3bn\cos(a+b\log(cx^n))+4\sin(a+b\log(cx^n)))}{(16+9b^2n^2)\,x^2}\end{aligned}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]^(3/2)/x^3,x]
```

output
```
(6*b^2*Sqrt[2 - 2*E^((2*I)*(a + b*Log[c*x^n]))]*n^2*Hypergeometric2F1[1/2,
 1/4 + I/(b*n), 5/4 + I/(b*n), E^((2*I)*(a + b*Log[c*x^n]))])/(Sqrt[((-I)*
(-1 + E^((2*I)*(a + b*Log[c*x^n]))))/E^(I*(a + b*Log[c*x^n]))]*(4 + (3*I)*
b*n)*(4*I + b*n)*(4*I + 3*b*n)*x^2) - (2*Sqrt[Sin[a + b*Log[c*x^n]]]*(3*b*
n*Cos[a + b*Log[c*x^n]] + 4*Sin[a + b*Log[c*x^n]]))/((16 + 9*b^2*n^2)*x^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4996, 4994, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^3}dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{(cx^n)^{2/n}\int (cx^n)^{-1-\frac{2}{n}}\sin^{\frac{3}{2}}(a+b\log(cx^n))\,d(cx^n)}{nx^2}$$

$$\Big\downarrow \text{4994}$$

$$\frac{(cx^n)^{\frac{2}{n}+\frac{3ib}{2}}\sin^{\frac{3}{2}}(a+b\log(cx^n))\int (cx^n)^{-\frac{3ib}{2}-\frac{2}{n}-1}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}d(cx^n)}{nx^2\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4}\left(\frac{4i}{bn}-3\right),\frac{1}{4}\left(1+\frac{4i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^2(4+3ibn)\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

input
```
Int[Sin[a + b*Log[c*x^n]]^(3/2)/x^3,x]
```

output
```
(-2*Hypergeometric2F1[-3/2, (-3 + (4*I)/(b*n))/4, (1 + (4*I)/(b*n))/4, E^(
(2*I)*a)*(c*x^n)^((2*I)*b)]*Sin[a + b*Log[c*x^n]]^(3/2))/((4 + (3*I)*b*n)*
x^2*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4994
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Simp[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\sin \left(a+b\ln \left(c\,x^{n}\right)\right)^{\frac{3}{2}}}{x^{3}}dx$$

input
```
int(sin(a+b*ln(c*x^n))^(3/2)/x^3,x)
```

output
```
int(sin(a+b*ln(c*x^n))^(3/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sin(a+b*log(c*x^n))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^3}\,dx = \int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^3}\,dx$$

input
```
integrate(sin(a+b*ln(c*x**n))**(3/2)/x**3,x)
```

output
```
Integral(sin(a + b*log(c*x**n))**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^3}\,dx = \int \frac{\sin(b\log(cx^n)+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sin(b*log(c*x^n) + a)^(3/2)/x^3, x)
```

Giac [F]

$$\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^3}\,dx=\int \frac{\sin(b\log(cx^n)+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sin(b*log(c*x^n) + a)^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^3}\,dx=\int \frac{\sin(a+b\ln(c\,x^n))^{3/2}}{x^3}\,dx$$

input
```
int(sin(a + b*log(c*x^n))^(3/2)/x^3,x)
```

output
```
int(sin(a + b*log(c*x^n))^(3/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sin^{\frac{3}{2}}(a+b\log(cx^n))}{x^3}\,dx=\frac{-2\sqrt{\sin(\log(x^nc)\,b+a)}\,\sin(\log(x^nc)\,b+a)+3\left(\int \frac{\sqrt{\sin(\log(x^nc)b+a)}\,\cos(\log(x^nc)b+a)}{x^3}dx\right)bn\,x^2}{4x^2}$$

input
```
int(sin(a+b*log(c*x^n))^(3/2)/x^3,x)
```

output
```
( - 2*sqrt(sin(log(x**n*c)*b + a))*sin(log(x**n*c)*b + a) + 3*int((sqrt(si
n(log(x**n*c)*b + a))*cos(log(x**n*c)*b + a))/x**3,x)*b*n*x**2)/(4*x**2)
```

3.63 $\int \frac{1}{\sqrt{\sin(a+b\log(cx^n))}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \frac{1}{\sqrt{\sin(a+b\log(cx^n))}}\,dx = \frac{2x\sqrt{1-e^{2ia}(cx^n)^{2ib}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}\left(1-\frac{2i}{bn}\right),\frac{1}{4}\left(5-\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{(2+ibn)\sqrt{\sin(a+b\log(cx^n))}}$$

output
```
2*x*(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)*hypergeom([1/2, 1/4-1/2*I/b/n],[5
/4-1/2*I/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))/(2+I*b*n)/sin(a+b*ln(c*x^n))^(1/
2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{\sin(a+b\log(cx^n))}}\,dx = -\frac{2i\sqrt{2-2e^{2i(a+b\log(cx^n))}}\,x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}-\frac{i}{2bn},\frac{5}{4}-\frac{i}{2bn},e^{2i(a+b\log(cx^n))}\right)}{\sqrt{-ie^{-i(a+b\log(cx^n))}\left(-1+e^{2i(a+b\log(cx^n))}\right)}(-2i+bn)}$$

input
```
Integrate[1/Sqrt[Sin[a + b*Log[c*x^n]]],x]
```

output
```
((-2*I)*Sqrt[2 - 2*E^((2*I)*(a + b*Log[c*x^n]))]*x*Hypergeometric2F1[1/2,
1/4 - (I/2)/(b*n), 5/4 - (I/2)/(b*n), E^((2*I)*(a + b*Log[c*x^n]))])/(Sqrt
[((-I)*(-1 + E^((2*I)*(a + b*Log[c*x^n]))))/E^(I*(a + b*Log[c*x^n]))]*(-2*
I + b*n))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4986, 4994, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sin (a+b\log (cx^n))}}dx$$

$$\Big\downarrow \text{ 4986}$$

$$\frac{x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}-1}}{\sqrt{\sin(a+b\log(cx^n))}}d(cx^n)}{n}$$

$$\Big\downarrow \text{ 4994}$$

$$\frac{x(cx^n)^{-\frac{1}{n}-\frac{ib}{2}}\sqrt{1-e^{2ia}(cx^n)^{2ib}}\int \frac{(cx^n)^{\frac{ib}{2}+\frac{1}{n}-1}}{\sqrt{1-e^{2ia}(cx^n)^{2ib}}}d(cx^n)}{n\sqrt{\sin (a+b\log (cx^n))}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{2x\sqrt{1-e^{2ia}(cx^n)^{2ib}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}\left(1-\frac{2i}{bn}\right),\frac{1}{4}\left(5-\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{(2+ibn)\sqrt{\sin (a+b\log (cx^n))}}$$

input
```
Int[1/Sqrt[Sin[a + b*Log[c*x^n]]],x]
```

output
```
(2*x*Sqrt[1 - E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Hypergeometric2F1[1/2, (1 - (
2*I)/(b*n))/4, (5 - (2*I)/(b*n))/4, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((2 +
I*b*n)*Sqrt[Sin[a + b*Log[c*x^n]]])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4986
```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 4994
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Simp[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\sqrt{\sin \left(a+b\ln \left(c\,x^{n}\right)\right)}}dx$$

input
```
int(1/sin(a+b*ln(c*x^n))^(1/2),x)
```

output
```
int(1/sin(a+b*ln(c*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/sin(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}\,dx$$

input
```
integrate(1/sin(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(1/sqrt(sin(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{\sqrt{\sin\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

input
```
integrate(1/sin(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(sin(b*log(c*x^n) + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\sin(a+b\log(cx^n))}}\,dx = \int \frac{1}{\sqrt{\sin(b\log(cx^n)+a)}}\,dx$$

input
```
integrate(1/sin(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(sin(b*log(c*x^n) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sin(a+b\log(cx^n))}}\,dx = \int \frac{1}{\sqrt{\sin(a+b\,\ln(c\,x^n))}}\,dx$$

input
```
int(1/sin(a + b*log(c*x^n))^(1/2),x)
```

output
```
int(1/sin(a + b*log(c*x^n))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sin(a+b\log(cx^n))}}\,dx = \int \frac{\sqrt{\sin(\log(x^n c)\,b+a)}}{\sin(\log(x^n c)\,b+a)}\,dx$$

input
```
int(1/sin(a+b*log(c*x^n))^(1/2),x)
```

output
```
int(sqrt(sin(log(x**n*c)*b + a))/sin(log(x**n*c)*b + a),x)
```

3.64 $\int \frac{1}{x\sqrt{\sin(a+b\log(cx^n))}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 30

$$\int \frac{1}{x\sqrt{\sin(a+b\log(cx^n))}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{4}(2a-\pi+2b\log(cx^n)),2\right)}{bn}$$

output
```
2*InverseJacobiAM(1/2*a-1/4*Pi+1/2*b*ln(c*x^n),2^(1/2))/b/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\sqrt{\sin(a+b\log(cx^n))}}\,dx = -\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(-a+\frac{\pi}{2}-b\log(cx^n)\right),2\right)}{bn}$$

input
```
Integrate[1/(x*Sqrt[Sin[a + b*Log[c*x^n]]]),x]
```

output
```
(-2*EllipticF[(-a + Pi/2 - b*Log[c*x^n])/2, 2])/(b*n)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3039, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\sin(a+b\log(cx^n))}}dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \frac{1}{\sqrt{\sin(a+b\log(cx^n))}}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \frac{1}{\sqrt{\sin(a+b\log(cx^n))}}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3120}$$
$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a+b\log(cx^n)-\frac{\pi}{2}\right),2\right)}{bn}$$

input
```
Int[1/(x*Sqrt[Sin[a + b*Log[c*x^n]]]),x]
```

output
```
(2*EllipticF[(a - Pi/2 + b*Log[c*x^n])/2, 2])/(b*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(28) = 56.

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.40

method	result	siz
derivativedivides	$\frac{\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\;\mathrm{EllipticF}\left(\sqrt{\sin(a+b\ln(c\,x^n))+1},\frac{\sqrt{2}}{2}\right)}{n\cos(a+b\ln(c\,x^n))\sqrt{\sin(a+b\ln(c\,x^n))}\,b}$	10
default	$\frac{\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\;\mathrm{EllipticF}\left(\sqrt{\sin(a+b\ln(c\,x^n))+1},\frac{\sqrt{2}}{2}\right)}{n\cos(a+b\ln(c\,x^n))\sqrt{\sin(a+b\ln(c\,x^n))}\,b}$	10

input
```
int(1/x/sin(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/n*(sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin(a+b
*ln(c*x^n)))^(1/2)*EllipticF((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2))/cos
(a+b*ln(c*x^n))/sin(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.57

$$\int \frac{1}{x\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}\,dx$$
$$= \frac{2\left(\sqrt{-\frac{1}{2}i}\mathrm{weierstrassPInverse}(4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+i\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right))+\sqrt{\vphantom{x}}\right.}{bn}$$

input
```
integrate(1/x/sin(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(-1/2*I)*weierstrassPInverse(4, 0, cos(b*n*log(x) + b*log(c) + a) +
 I*sin(b*n*log(x) + b*log(c) + a)) + sqrt(1/2*I)*weierstrassPInverse(4, 0,
 cos(b*n*log(x) + b*log(c) + a) - I*sin(b*n*log(x) + b*log(c) + a)))/(b*n)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{x\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}\,dx$$

input
```
integrate(1/x/sin(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(sin(a + b*log(c*x**n)))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{x\sqrt{\sin\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

input
```
integrate(1/x/sin(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sqrt(sin(b*log(c*x^n) + a))), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{x\sqrt{\sin\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

input
```
integrate(1/x/sin(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(x*sqrt(sin(b*log(c*x^n) + a))), x)
```

Mupad [B] (verification not implemented)

Time = 20.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\sqrt{\sin(a+b\log(cx^n))}}\,dx = -\frac{2\,\mathrm{F}\left(\frac{\pi}{4}-\frac{a}{2}-\frac{b\,\ln(c\,x^n)}{2}\middle|2\right)}{b\,n}$$

input
```
int(1/(x*sin(a + b*log(c*x^n))^(1/2)),x)
```

output
```
-(2*ellipticF(pi/4 - a/2 - (b*log(c*x^n))/2, 2))/(b*n)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{\sin(a+b\log(cx^n))}}\,dx = \int \frac{\sqrt{\sin(\log(x^nc)\,b+a)}}{\sin(\log(x^nc)\,b+a)\,x}dx$$

input
```
int(1/x/sin(a+b*log(c*x^n))^(1/2),x)
```

output
```
int(sqrt(sin(log(x**n*c)*b + a))/(sin(log(x**n*c)*b + a)*x),x)
```

3.65 $\int \frac{1}{\sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{2x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i}{bn}\right),\frac{1}{4}\left(7-\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{(2+3ibn)\sin^{\frac{3}{2}}(a+b\log(cx^n))}$$

output
```
2*x*(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)*hypergeom([3/2, 3/4-1/2*I/b/n],[7
/4-1/2*I/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))/(2+3*I*b*n)/sin(a+b*ln(c*x^n))^(
3/2)
```

Mathematica [A] (verified)

Time = 11.57 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{4\sqrt{2}e^{2ia}x(cx^n)^{2ib}\sqrt{-ie^{-ia}(cx^n)^{-ib}\left(-1+e^{2ia}(cx^n)^{2ib}\right)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{3}{4}-\frac{i}{2bn},\frac{7}{4}-\frac{i}{2bn},e^{2ia}(cx^n)^{2ib}\right)}{(-2-3ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]^(-3/2),x]
```

output
```
(4*Sqrt[2]*E^((2*I)*a)*x*(c*x^n)^((2*I)*b)*Sqrt[((-I)*(-1 + E^((2*I)*a)*(c
*x^n)^((2*I)*b)))/(E^(I*a)*(c*x^n)^(I*b))]*Hypergeometric2F1[3/2, 3/4 - (I
/2)/(b*n), 7/4 - (I/2)/(b*n), E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((-2 - (3*I)
*b*n)*Sqrt[1 - E^((2*I)*a)*(c*x^n)^((2*I)*b)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4986, 4994, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{4986}$$

$$\frac{x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}-1}}{\sin^{\frac{3}{2}}(a+b\log(cx^n))}d(cx^n)}{n}$$

$$\Big\downarrow \text{4994}$$

$$\frac{x(cx^n)^{-\frac{1}{n}-\frac{3ib}{2}}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\int \frac{(cx^n)^{\frac{3ib}{2}+\frac{1}{n}-1}}{\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}}d(cx^n)}{n\sin^{\frac{3}{2}}(a+b\log(cx^n))}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i}{bn}\right),\frac{1}{4}\left(7-\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{(2+3ibn)\sin^{\frac{3}{2}}(a+b\log(cx^n))}$$

input
```
Int[Sin[a + b*Log[c*x^n]]^(-3/2),x]
```

output
```
(2*x*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2)*Hypergeometric2F1[3/2, (3 -
 (2*I)/(b*n))/4, (7 - (2*I)/(b*n))/4, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((2
+ (3*I)*b*n)*Sin[a + b*Log[c*x^n]]^(3/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4986
```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 4994
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Simp[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\sin \left(a + b \ln \left(c\, x^{n}\right)\right)^{\frac{3}{2}}} dx$$

input
```
int(1/sin(a+b*ln(c*x^n))^(3/2),x)
```

output
```
int(1/sin(a+b*ln(c*x^n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/sin(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{\sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$

input
```
integrate(1/sin(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Integral(sin(a + b*log(c*x**n))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{\sin(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sin(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*log(c*x^n) + a)^(-3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/sin(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^{3/2}}\,dx$$

input
```
int(1/sin(a + b*log(c*x^n))^(3/2),x)
```

output
```
int(1/sin(a + b*log(c*x^n))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{\sqrt{\sin(\log(x^n c)\,b+a)}}{\sin(\log(x^n c)\,b+a)^2}\,dx$$

input
```
int(1/sin(a+b*log(c*x^n))^(3/2),x)
```

output
```
int(sqrt(sin(log(x**n*c)*b + a))/sin(log(x**n*c)*b + a)**2,x)
```

3.66 $\int \frac{1}{x \sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 65

$$\int \frac{1}{x \sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = -\frac{2E\left(\frac{1}{4}(2a-\pi+2b\log(cx^n))\middle|2\right)}{bn} - \frac{2\cos(a+b\log(cx^n))}{bn\sqrt{\sin(a+b\log(cx^n))}}$$

output
```
2*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*ln(c*x^n)),2^(1/2))/b/n-2*cos(a+b*ln(c*
x^n))/b/n/sin(a+b*ln(c*x^n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{1}{x \sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{2\left(E\left(\frac{1}{4}(-2a+\pi-2b\log(cx^n))\middle|2\right) - \frac{\cos(a+b\log(cx^n))}{\sqrt{\sin(a+b\log(cx^n))}}\right)}{bn}$$

input
```
Integrate[1/(x*Sin[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
(2*(EllipticE[(-2*a + Pi - 2*b*Log[c*x^n])/4, 2] - Cos[a + b*Log[c*x^n]]/S
qrt[Sin[a + b*Log[c*x^n]]]))/(b*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3039, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sin^{\frac{3}{2}}(a+b\log(cx^n))}dx$$
$$\Big\downarrow \; 3039$$
$$\frac{\int \frac{1}{\sin^{\frac{3}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$
$$\Big\downarrow \; 3042$$
$$\frac{\int \frac{1}{\sin(a+b\log(cx^n))^{3/2}}d\log(cx^n)}{n}$$
$$\Big\downarrow \; 3116$$
$$\frac{-\int \sqrt{\sin(a+b\log(cx^n))}d\log(cx^n)-\frac{2\cos(a+b\log(cx^n))}{b\sqrt{\sin(a+b\log(cx^n))}}}{n}$$
$$\Big\downarrow \; 3042$$
$$\frac{-\int \sqrt{\sin(a+b\log(cx^n))}d\log(cx^n)-\frac{2\cos(a+b\log(cx^n))}{b\sqrt{\sin(a+b\log(cx^n))}}}{n}$$
$$\Big\downarrow \; 3119$$
$$\frac{-\frac{2E\left(\frac{1}{2}\left(a+b\log(cx^n)-\frac{\pi}{2}\right)|2\right)}{b}-\frac{2\cos(a+b\log(cx^n))}{b\sqrt{\sin(a+b\log(cx^n))}}}{n}$$

input
```
Int[1/(x*Sin[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
((-2*EllipticE[(a - Pi/2 + b*Log[c*x^n])/2, 2])/b - (2*Cos[a + b*Log[c*x^n
]])/(b*Sqrt[Sin[a + b*Log[c*x^n]]]))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(62) = 124$.

Time = 0.40 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.92

method	result
derivativedivides	$\frac{2\sqrt{\sin(a+b\ln(c\,x^{n}))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^{n}))+2}\,\sqrt{-\sin(a+b\ln(c\,x^{n}))}\,\operatorname{EllipticE}\left(\sqrt{\sin(a+b\ln(c\,x^{n}))+1},\frac{\sqrt{2}}{2}\right)-\sqrt{\mathrm{si}}}{n\cos(a+b\ln(c}$
default	$\frac{2\sqrt{\sin(a+b\ln(c\,x^{n}))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^{n}))+2}\,\sqrt{-\sin(a+b\ln(c\,x^{n}))}\,\operatorname{EllipticE}\left(\sqrt{\sin(a+b\ln(c\,x^{n}))+1},\frac{\sqrt{2}}{2}\right)-\sqrt{\mathrm{si}}}{n\cos(a+b\ln(c}$

input
```
int(1/x/sin(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/n*(2*(sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin(
a+b*ln(c*x^n)))^(1/2)*EllipticE((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2))-
(sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin(a+b*ln(
c*x^n)))^(1/2)*EllipticF((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2))-2*cos(a
+b*ln(c*x^n))^2)/cos(a+b*ln(c*x^n))/sin(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.31

$$\int \frac{1}{x\sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = -\frac{2\left(i\sqrt{-\frac{1}{2}i}\sin(bn\log(x)+b\log(c)+a)\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(bn\log(x)\right.}{}$$

input
```
integrate(1/x/sin(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
-2*(I*sqrt(-1/2*I)*sin(b*n*log(x) + b*log(c) + a)*weierstrassZeta(4, 0, we
ierstrassPInverse(4, 0, cos(b*n*log(x) + b*log(c) + a) + I*sin(b*n*log(x)
+ b*log(c) + a))) - I*sqrt(1/2*I)*sin(b*n*log(x) + b*log(c) + a)*weierstra
ssZeta(4, 0, weierstrassPInverse(4, 0, cos(b*n*log(x) + b*log(c) + a) - I*
sin(b*n*log(x) + b*log(c) + a))) + cos(b*n*log(x) + b*log(c) + a)*sqrt(sin
(b*n*log(x) + b*log(c) + a)))/(b*n*sin(b*n*log(x) + b*log(c) + a))
```

Sympy [F]

$$\int \frac{1}{x\sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$

input
```
integrate(1/x/sin(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Integral(1/(x*sin(a + b*log(c*x**n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\sin(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/sin(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sin(b*log(c*x^n) + a)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/x/sin(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 21.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = -\frac{\cos(a+b\ln(c\,x^n))\,\left(\sin(a+b\ln(c\,x^n))^2\right)^{1/4}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\ \frac{3}{2};\ \cos(a+b\ln(c\,x^n))^2\right)}{b\,n\,\sqrt{\sin(a+b\ln(c\,x^n))}}$$

input
```
int(1/(x*sin(a + b*log(c*x^n))^(3/2)),x)
```

output
```
-(cos(a + b*log(c*x^n))*(sin(a + b*log(c*x^n))^2)^(1/4)*hypergeom([1/2, 5/
4], 3/2, cos(a + b*log(c*x^n))^2))/(b*n*sin(a + b*log(c*x^n))^(1/2))
```

Reduce [F]

$$\int \frac{1}{x\sin^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{\sqrt{\sin(\log(x^nc)\,b+a)}}{\sin(\log(x^nc)\,b+a)^2\,x}dx$$

input
```
int(1/x/sin(a+b*log(c*x^n))^(3/2),x)
```

output
```
int(sqrt(sin(log(x**n*c)*b + a))/(sin(log(x**n*c)*b + a)**2*x),x)
```

3.67 $\int \frac{1}{\sin^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{2x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i}{bn}\right),\frac{1}{4}\left(9-\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{(2+5ibn)\sin^{\frac{5}{2}}(a+b\log(cx^n))}$$

output
```
2*x*(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(5/2)*hypergeom([5/2, 5/4-1/2*I/b/n],[9
/4-1/2*I/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))/(2+5*I*b*n)/sin(a+b*ln(c*x^n))^(
5/2)
```

Mathematica [A] (verified)

Time = 2.51 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{2x\left(\frac{(2-ibn)\sqrt{2-2e^{2ia}(cx^n)^{2ib}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}-\frac{i}{2bn},\frac{5}{4}-\frac{i}{2bn},e^{2i(a+b\log(cx^n))}\right)}{\sqrt{-ie^{-ia}(cx^n)^{-ib}\left(-1+e^{2ia}(cx^n)^{2ib}\right)}}-\frac{bn\cos(a+b\log(cx^n))+2\sin(a+b\log(cx^n))}{\sin^{\frac{3}{2}}(a+b\log(cx^n))}\right)}{3b^2n^2}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]^(-5/2),x]
```

output
```
(2*x*(((2 - I*b*n)*Sqrt[2 - 2*E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Hypergeometri
c2F1[1/2, 1/4 - (I/2)/(b*n), 5/4 - (I/2)/(b*n), E^((2*I)*(a + b*Log[c*x^n]
))])/Sqrt[((-I)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))/(E^(I*a)*(c*x^n)^(I*
b))] - (b*n*Cos[a + b*Log[c*x^n]] + 2*Sin[a + b*Log[c*x^n]])/Sin[a + b*Log
[c*x^n]]^(3/2)))/(3*b^2*n^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4986, 4994, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{4986}$$

$$\frac{x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}-1}}{\sin^{\frac{5}{2}}(a+b\log(cx^n))}d(cx^n)}{n}$$

$$\Big\downarrow \text{4994}$$

$$\frac{x(cx^n)^{-\frac{1}{n}-\frac{5ib}{2}}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}\int \frac{(cx^n)^{\frac{5ib}{2}+\frac{1}{n}-1}}{\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}}d(cx^n)}{n\sin^{\frac{5}{2}}(a+b\log(cx^n))}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i}{bn}\right),\frac{1}{4}\left(9-\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{(2+5ibn)\sin^{\frac{5}{2}}(a+b\log(cx^n))}$$

input
```
Int[Sin[a + b*Log[c*x^n]]^(-5/2),x]
```

output
```
(2*x*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^(5/2)*Hypergeometric2F1[5/2, (5 -
 (2*I)/(b*n))/4, (9 - (2*I)/(b*n))/4, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((2
+ (5*I)*b*n)*Sin[a + b*Log[c*x^n]]^(5/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4986
```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 4994
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Simp[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\sin\left(a+b\ln\left(c\,x^{n}\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/sin(a+b*ln(c*x^n))^(5/2),x)
```

output
```
int(1/sin(a+b*ln(c*x^n))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sin^{\frac{5}{2}}(a + b\log(cx^n))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/sin(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sin^{\frac{5}{2}}(a + b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/sin(a+b*ln(c*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sin^{\frac{5}{2}}(a + b\log(cx^n))}\,dx = \int \frac{1}{\sin(b\log(cx^n) + a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sin(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sin(b*log(c*x^n) + a)^(-5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/sin(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^{5/2}}\,dx$$

input
```
int(1/sin(a + b*log(c*x^n))^(5/2),x)
```

output
```
int(1/sin(a + b*log(c*x^n))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{\sqrt{\sin(\log(x^n c)\,b+a)}}{\sin(\log(x^n c)\,b+a)^3}dx$$

input
```
int(1/sin(a+b*log(c*x^n))^(5/2),x)
```

output
```
int(sqrt(sin(log(x**n*c)*b + a))/sin(log(x**n*c)*b + a)**3,x)
```

3.68 $\int \frac{1}{x \sin^{\frac{5}{2}}(a+b \log(cx^n))} dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \frac{1}{x \sin^{\frac{5}{2}}(a+b \log(cx^n))} dx = \frac{2 \operatorname{EllipticF}\left(\frac{1}{4}(2a-\pi+2b \log(cx^n)),2\right)}{3bn} - \frac{2 \cos(a+b \log(cx^n))}{3bn \sin^{\frac{3}{2}}(a+b \log(cx^n))}$$

output
```
2/3*InverseJacobiAM(1/2*a-1/4*Pi+1/2*b*ln(c*x^n),2^(1/2))/b/n-2/3*cos(a+b*
ln(c*x^n))/b/n/sin(a+b*ln(c*x^n))^(3/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \frac{1}{x \sin^{\frac{5}{2}}(a+b \log(cx^n))} dx = \frac{2\left(\operatorname{EllipticF}\left(\frac{1}{4}(2a-\pi+2b \log(cx^n)),2\right) - \frac{\cos(a+b \log(cx^n))}{\sin^{\frac{3}{2}}(a+b \log(cx^n))}\right)}{3bn}$$

input
```
Integrate[1/(x*Sin[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
(2*(EllipticF[(2*a - Pi + 2*b*Log[c*x^n])/4, 2] - Cos[a + b*Log[c*x^n]]/Si
n[a + b*Log[c*x^n]]^(3/2)))/(3*b*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3039, 3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sin^{\frac{5}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\sin^{\frac{5}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sin(a+b\log(cx^n))^{5/2}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+b\log(cx^n))}}d\log(cx^n)-\frac{2\cos(a+b\log(cx^n))}{3b\sin^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+b\log(cx^n))}}d\log(cx^n)-\frac{2\cos(a+b\log(cx^n))}{3b\sin^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a+b\log(cx^n)-\frac{\pi}{2}\right),2\right)}{3b}-\frac{2\cos(a+b\log(cx^n))}{3b\sin^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

input
```
Int[1/(x*Sin[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
((2*EllipticF[(a - Pi/2 + b*Log[c*x^n])/2, 2])/(3*b) - (2*Cos[a + b*Log[c*
x^n]])/(3*b*Sin[a + b*Log[c*x^n]]^(3/2)))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(61) = 122$.

Time = 0.41 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.90

method	result
derivativedivides	$\frac{\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\ \mathrm{EllipticF}\left(\sqrt{\sin(a+b\ln(c\,x^n))+1},\frac{\sqrt{2}}{2}\right)\sin(a+}{3n\sin(a+b\ln(c\,x^n))^{\frac{3}{2}}\cos(a+b\ln(c\,x^n))b}$
default	$\frac{\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\ \mathrm{EllipticF}\left(\sqrt{\sin(a+b\ln(c\,x^n))+1},\frac{\sqrt{2}}{2}\right)\sin(a+}{3n\sin(a+b\ln(c\,x^n))^{\frac{3}{2}}\cos(a+b\ln(c\,x^n))b}$

input
```
int(1/x/sin(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/n/sin(a+b*ln(c*x^n))^(3/2)*((sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*l
n(c*x^n))+2)^(1/2)*(-sin(a+b*ln(c*x^n)))^(1/2)*EllipticF((sin(a+b*ln(c*x^n
))+1)^(1/2),1/2*2^(1/2))*sin(a+b*ln(c*x^n))-2*cos(a+b*ln(c*x^n))^2)/cos(a+
b*ln(c*x^n))/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.38

$$\int \frac{1}{x\sin^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{2\left(\left(\sqrt{-\frac{1}{2}i}\cos(bn\log(x)+b\log(c)+a)^2-\sqrt{-\frac{1}{2}i}\right)\text{weierstrassPInverse}(4,0,\cos(bn\log(x)+b\log(c)+\cdots$$

input
```
integrate(1/x/sin(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

output
```
2/3*((sqrt(-1/2*I)*cos(b*n*log(x) + b*log(c) + a)^2 - sqrt(-1/2*I))*weiers
trassPInverse(4, 0, cos(b*n*log(x) + b*log(c) + a) + I*sin(b*n*log(x) + b*
log(c) + a)) + (sqrt(1/2*I)*cos(b*n*log(x) + b*log(c) + a)^2 - sqrt(1/2*I)
)*weierstrassPInverse(4, 0, cos(b*n*log(x) + b*log(c) + a) - I*sin(b*n*log
(x) + b*log(c) + a)) + cos(b*n*log(x) + b*log(c) + a)*sqrt(sin(b*n*log(x)
+ b*log(c) + a)))/(b*n*cos(b*n*log(x) + b*log(c) + a)^2 - b*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\sin^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/x/sin(a+b*ln(c*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x \sin^{\frac{5}{2}}(a + b\log(cx^n))}\, dx = \int \frac{1}{x \sin(b\log(cx^n) + a)^{\frac{5}{2}}}\, dx$$

input
```
integrate(1/x/sin(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sin(b*log(c*x^n) + a)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x \sin^{\frac{5}{2}}(a + b\log(cx^n))}\, dx = \text{Timed out}$$

input
```
integrate(1/x/sin(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 20.84 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{1}{x \sin^{\frac{5}{2}}(a + b\log(cx^n))}\, dx$$
$$= -\frac{\cos(a + b\,\ln(c\,x^n))\,\left(\sin(a + b\,\ln(c\,x^n))^2\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{7}{4};\ \frac{3}{2};\ \cos(a + b\,\ln(c\,x^n))^2\right)}{b\,n\sin(a + b\,\ln(c\,x^n))^{3/2}}$$

input
```
int(1/(x*sin(a + b*log(c*x^n))^(5/2)),x)
```

output
```
-(cos(a + b*log(c*x^n))*(sin(a + b*log(c*x^n))^2)^(3/4)*hypergeom([1/2, 7/
4], 3/2, cos(a + b*log(c*x^n))^2))/(b*n*sin(a + b*log(c*x^n))^(3/2))
```

Reduce [F]

$$\int \frac{1}{x \sin^{\frac{5}{2}}(a + b\log(cx^n))} \, dx = \int \frac{\sqrt{\sin(\log(x^n c) b + a)}}{\sin(\log(x^n c) b + a)^3 x} dx$$

input
```
int(1/x/sin(a+b*log(c*x^n))^(5/2),x)
```

output
```
int(sqrt(sin(log(x**n*c)*b + a))/(sin(log(x**n*c)*b + a)**3*x),x)
```

3.69 $\int \frac{1}{\sin^{\frac{3}{2}}(a-2i\log(cx))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{1}{\sin^{\frac{3}{2}}(a-2i\log(cx))}\,dx = \frac{e^{-2ia}(1-c^4e^{2ia}x^4)}{2c^4x^3\sin^{\frac{3}{2}}(a-2i\log(cx))}$$

output
```
1/2*(1-c^4*exp(2*I*a)*x^4)/c^4/exp(2*I*a)/x^3/sin(a-2*I*ln(c*x))^(3/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.65

$$\int \frac{1}{\sin^{\frac{3}{2}}(a-2i\log(cx))}\,dx = \frac{x(\cos(a)-i\sin(a))\sqrt{\frac{-2i(-1+c^4x^4)\cos(a)+2(1+c^4x^4)\sin(a)}{c^2x^2}}}{(-1+c^4x^4)\cos(a)+i(1+c^4x^4)\sin(a)}$$

input
```
Integrate[Sin[a - (2*I)*Log[c*x]]^(-3/2),x]
```

output
```
(x*(Cos[a] - I*Sin[a])*Sqrt[((-2*I)*(-1 + c^4*x^4)*Cos[a] + 2*(1 + c^4*x^4
)*Sin[a])/(c^2*x^2)])/((-1 + c^4*x^4)*Cos[a] + I*(1 + c^4*x^4)*Sin[a])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4986, 4984, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^{\frac{3}{2}}(a-2i\log(cx))}dx$$

$$\Big\downarrow \text{4986}$$

$$\frac{\int \frac{1}{\sin^{\frac{3}{2}}(a-2i\log(cx))}d(cx)}{c}$$

$$\Big\downarrow \text{4984}$$

$$\frac{\left(1-e^{2ia}c^4x^4\right)^{3/2}\int \frac{c^3x^3}{\left(1-c^4e^{2ia}x^4\right)^{3/2}}d(cx)}{c^4x^3\sin^{\frac{3}{2}}(a-2i\log(cx))}$$

$$\Big\downarrow \text{793}$$

$$\frac{e^{-2ia}\left(1-e^{2ia}c^4x^4\right)}{2c^4x^3\sin^{\frac{3}{2}}(a-2i\log(cx))}$$

input
```
Int[Sin[a - (2*I)*Log[c*x]]^(-3/2),x]
```

output
```
(1 - c^4*E^((2*I)*a)*x^4)/(2*c^4*E^((2*I)*a)*x^3*Sin[a - (2*I)*Log[c*x]]^(
3/2))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 4984
```
Int[Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :> Simp[Sin[d*(a + b
*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p)   Int[(1 - E^(2*I
*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p), x], x] /; FreeQ[{a, b, d, p}, x] &&  !Int
egerQ[p]
```

rule 4986
```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{1}{\sin\left(a-2i\ln\left(xc\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/sin(a-2*I*ln(x*c))^(3/2),x)
```

output
```
int(1/sin(a-2*I*ln(x*c))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sin^{\frac{3}{2}}(a-2i\log(cx))}\,dx=\frac{2\sqrt{\frac{1}{2}}\sqrt{-i\,c^4x^4+i\,e^{(-2i\,a)}}e^{\left(-\frac{3}{2}i\,a\right)}}{c^5x^4-ce^{(-2i\,a)}}$$

input
```
integrate(1/sin(a-2*I*log(c*x))^(3/2),x, algorithm="fricas")
```

output
```
2*sqrt(1/2)*sqrt(-I*c^4*x^4 + I*e^(-2*I*a))*e^(-3/2*I*a)/(c^5*x^4 - c*e^(-
2*I*a))
```

Sympy [F]

$$\int \frac{1}{\sin^{\frac{3}{2}}(a-2i\log(cx))}\,dx = \int \frac{1}{\sin^{\frac{3}{2}}\left(a-2i\log\left(cx\right)\right)}\,dx$$

input
```
integrate(1/sin(a-2*I*ln(c*x))**(3/2),x)
```

output
```
Integral(sin(a - 2*I*log(c*x))**(-3/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(36) = 72$.

Time = 0.27 (sec) , antiderivative size = 402, normalized size of antiderivative = 8.20

$$\int \frac{1}{\sin^{\frac{3}{2}}(a-2i\log(cx))}\,dx = \text{Too large to display}$$

input
```
integrate(1/sin(a-2*I*log(c*x))^(3/2),x, algorithm="maxima")
```

output
```
((cos(a)^2 + sin(a)^2)*c^4*x^4 + 2*c^2*x^2*cos(a) + 1)^(1/4)*((cos(a)^2 +
sin(a)^2)*c^4*x^4 - 2*c^2*x^2*cos(a) + 1)^(1/4)*(((c^4*x^4*((I + 1)*cos(3/
2*a) + (I - 1)*sin(3/2*a)) - (I + 1)*cos(1/2*a) + (I - 1)*sin(1/2*a))*cos(
3/2*arctan2(c^2*x^2*sin(a), -c^2*x^2*cos(a) + 1)) + (c^4*x^4*((I - 1)*cos(
3/2*a) - (I + 1)*sin(3/2*a)) - (I - 1)*cos(1/2*a) - (I + 1)*sin(1/2*a))*si
n(3/2*arctan2(c^2*x^2*sin(a), -c^2*x^2*cos(a) + 1)))*cos(3/2*arctan2(c^2*x
^2*sin(a), c^2*x^2*cos(a) + 1)) + ((c^4*x^4*(-(I - 1)*cos(3/2*a) + (I + 1)
*sin(3/2*a)) + (I - 1)*cos(1/2*a) + (I + 1)*sin(1/2*a))*cos(3/2*arctan2(c^
2*x^2*sin(a), -c^2*x^2*cos(a) + 1)) + (c^4*x^4*((I + 1)*cos(3/2*a) + (I -
1)*sin(3/2*a)) - (I + 1)*cos(1/2*a) + (I - 1)*sin(1/2*a))*sin(3/2*arctan2(
c^2*x^2*sin(a), -c^2*x^2*cos(a) + 1)))*sin(3/2*arctan2(c^2*x^2*sin(a), c^2
*x^2*cos(a) + 1)))/(((cos(a)^4 + 2*cos(a)^2*sin(a)^2 + sin(a)^4)*c^8*x^8 -
 2*(cos(a)^2 - sin(a)^2)*c^4*x^4 + 1)*c)
```

Giac [F]

$$\int \frac{1}{\sin^{\frac{3}{2}}(a-2i\log(cx))}\,dx=\int \frac{1}{\sin\left(a-2i\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sin(a-2*I*log(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate(sin(a - 2*I*log(c*x))^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 21.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sin^{\frac{3}{2}}(a-2i\log(cx))}\,dx=\frac{2\,x\,\sqrt{\frac{{\mathrm{e}}^{-a\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2\,c^2\,x^2}-\frac{c^2\,x^2\,{\mathrm{e}}^{a\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}}}{c^4\,x^4\,{\mathrm{e}}^{a\,2{}\mathrm{i}}-1}$$

input
```
int(1/sin(a - log(c*x)*2i)^(3/2),x)
```

output
```
(2*x*((exp(-a*1i)*1i)/(2*c^2*x^2) - (c^2*x^2*exp(a*1i)*1i)/2)^(1/2))/(c^4*
x^4*exp(a*2i) - 1)
```

Reduce [F]

$$\int \frac{1}{\sin^{\frac{3}{2}}(a-2i\log(cx))}\,dx = \left(\int \frac{\sqrt{\sin(2\log(cx)\,i-a)}}{\sin(2\log(cx)\,i-a)^2}dx\right)i$$

input
```
int(1/sin(a-2*I*log(c*x))^(3/2),x)
```

output
```
int(sqrt(sin(2*log(c*x)*i - a))/sin(2*log(c*x)*i - a)**2,x)*i
```

3.70 $\int (ex)^m \sin^4(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 337

$$\begin{aligned}\int (ex)^m \sin^4(d(a+b\log(cx^n)))\,dx &= \frac{24b^4d^4n^4(ex)^{1+m}}{e(1+m)\left((1+m)^2+4b^2d^2n^2\right)\left((1+m)^2+16b^2d^2n^2\right)} \\ &\quad - \frac{24b^3d^3n^3(ex)^{1+m}\cos(d(a+b\log(cx^n)))\sin(d(a+b\log(cx^n)))}{e\left((1+m)^2+4b^2d^2n^2\right)\left((1+m)^2+16b^2d^2n^2\right)} \\ &\quad + \frac{12b^2d^2(1+m)n^2(ex)^{1+m}\sin^2(d(a+b\log(cx^n)))}{e\left((1+m)^2+4b^2d^2n^2\right)\left((1+m)^2+16b^2d^2n^2\right)} \\ &\quad - \frac{4bdn(ex)^{1+m}\cos(d(a+b\log(cx^n)))\sin^3(d(a+b\log(cx^n)))}{e\left((1+m)^2+16b^2d^2n^2\right)} \\ &\quad + \frac{(1+m)(ex)^{1+m}\sin^4(d(a+b\log(cx^n)))}{e\left((1+m)^2+16b^2d^2n^2\right)}\end{aligned}$$

output
```
24*b^4*d^4*n^4*(e*x)^(1+m)/e/(1+m)/((1+m)^2+4*b^2*d^2*n^2)/((1+m)^2+16*b^2
*d^2*n^2)-24*b^3*d^3*n^3*(e*x)^(1+m)*cos(d*(a+b*ln(c*x^n)))*sin(d*(a+b*ln(
c*x^n)))/e/((1+m)^2+4*b^2*d^2*n^2)/((1+m)^2+16*b^2*d^2*n^2)+12*b^2*d^2*(1+
m)*n^2*(e*x)^(1+m)*sin(d*(a+b*ln(c*x^n)))^2/e/((1+m)^2+4*b^2*d^2*n^2)/((1+
m)^2+16*b^2*d^2*n^2)-4*b*d*n*(e*x)^(1+m)*cos(d*(a+b*ln(c*x^n)))*sin(d*(a+b
*ln(c*x^n)))^3/e/((1+m)^2+16*b^2*d^2*n^2)+(1+m)*(e*x)^(1+m)*sin(d*(a+b*ln(
c*x^n)))^4/e/((1+m)^2+16*b^2*d^2*n^2)
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.01

$$\int (ex)^m \sin^4(d(a+b\log(cx^n)))\,dx = \frac{1}{8}x(ex)^m\left(\frac{3}{1+m} + \frac{4\sin(2bdn\log(x))\left(-2bdn\cos(2d(a-bn\log(x)+b\log(cx^n)))+(1+m)\sin(2d(a-bn\log(x)+b\log(cx^n)))\right)}{1+2m+m^2+4b^2d^2n^2} - \frac{4\cos(2bdn\log(x))\left((1+m)\cos(2d(a-bn\log(x)+b\log(cx^n)))+2bdn\sin(2d(a-bn\log(x)+b\log(cx^n)))\right)}{1+2m+m^2+4b^2d^2n^2} - \frac{\sin(4bdn\log(x))\left(-4bdn\cos(4d(a-bn\log(x)+b\log(cx^n)))+(1+m)\sin(4d(a-bn\log(x)+b\log(cx^n)))\right)}{1+2m+m^2+16b^2d^2n^2} + \frac{\cos(4bdn\log(x))\left((1+m)\cos(4d(a-bn\log(x)+b\log(cx^n)))+4bdn\sin(4d(a-bn\log(x)+b\log(cx^n)))\right)}{1+2m+m^2+16b^2d^2n^2}\right)$$

input
```
Integrate[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^4,x]
```

output
```
(x*(e*x)^m*(3/(1 + m) + (4*Sin[2*b*d*n*Log[x]]*(-2*b*d*n*Cos[2*d*(a - b*n*
Log[x] + b*Log[c*x^n])] + (1 + m)*Sin[2*d*(a - b*n*Log[x] + b*Log[c*x^n])]
))/(1 + 2*m + m^2 + 4*b^2*d^2*n^2) - (4*Cos[2*b*d*n*Log[x]]*((1 + m)*Cos[2
*d*(a - b*n*Log[x] + b*Log[c*x^n])] + 2*b*d*n*Sin[2*d*(a - b*n*Log[x] + b*
Log[c*x^n])]))/(1 + 2*m + m^2 + 4*b^2*d^2*n^2) - (Sin[4*b*d*n*Log[x]]*(-4*
b*d*n*Cos[4*d*(a - b*n*Log[x] + b*Log[c*x^n])] + (1 + m)*Sin[4*d*(a - b*n*
Log[x] + b*Log[c*x^n])]))/(1 + 2*m + m^2 + 16*b^2*d^2*n^2) + (Cos[4*b*d*n*
Log[x]]*((1 + m)*Cos[4*d*(a - b*n*Log[x] + b*Log[c*x^n])] + 4*b*d*n*Sin[4*
d*(a - b*n*Log[x] + b*Log[c*x^n])]))/(1 + 2*m + m^2 + 16*b^2*d^2*n^2)))/8
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.88, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4990, 4990, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \sin^4(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \; 4990$$

$$\frac{12b^2d^2n^2\int(ex)^m\sin^2(d(a+b\log(cx^n)))dx}{16b^2d^2n^2+(m+1)^2}+\frac{(m+1)(ex)^{m+1}\sin^4(d(a+b\log(cx^n)))}{e(16b^2d^2n^2+(m+1)^2)}-\frac{4bdn(ex)^{m+1}\sin^3(d(a+b\log(cx^n)))\cos(d(a+b\log(cx^n)))}{e(16b^2d^2n^2+(m+1)^2)}$$

$$\Big\downarrow \; 4990$$

$$\frac{12b^2d^2n^2\left(\frac{2b^2d^2n^2\int(ex)^mdx}{4b^2d^2n^2+(m+1)^2}+\frac{(m+1)(ex)^{m+1}\sin^2(d(a+b\log(cx^n)))}{e(4b^2d^2n^2+(m+1)^2)}-\frac{2bdn(ex)^{m+1}\sin(d(a+b\log(cx^n)))\cos(d(a+b\log(cx^n)))}{e(4b^2d^2n^2+(m+1)^2)}\right)}{16b^2d^2n^2+(m+1)^2}+\frac{(m+1)(ex)^{m+1}\sin^4(d(a+b\log(cx^n)))}{e(16b^2d^2n^2+(m+1)^2)}-\frac{4bdn(ex)^{m+1}\sin^3(d(a+b\log(cx^n)))\cos(d(a+b\log(cx^n)))}{e(16b^2d^2n^2+(m+1)^2)}$$

$$\Big\downarrow \; 17$$

$$\frac{(m+1)(ex)^{m+1}\sin^4(d(a+b\log(cx^n)))}{e(16b^2d^2n^2+(m+1)^2)}-\frac{4bdn(ex)^{m+1}\sin^3(d(a+b\log(cx^n)))\cos(d(a+b\log(cx^n)))}{e(16b^2d^2n^2+(m+1)^2)}+\frac{12b^2d^2n^2\left(\frac{(m+1)(ex)^{m+1}\sin^2(d(a+b\log(cx^n)))}{e(4b^2d^2n^2+(m+1)^2)}-\frac{2bdn(ex)^{m+1}\sin(d(a+b\log(cx^n)))\cos(d(a+b\log(cx^n)))}{e(4b^2d^2n^2+(m+1)^2)}+\frac{2b^2d^2n^2(ex)^{m+1}}{e(m+1)(4b^2d^2n^2+(m+1)^2}\right.}{16b^2d^2n^2+(m+1)^2}$$

input
```
Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^4,x]
```

output
```
(-4*b*d*n*(e*x)^(1 + m)*Cos[d*(a + b*Log[c*x^n])]*Sin[d*(a + b*Log[c*x^n])
]^3)/(e*((1 + m)^2 + 16*b^2*d^2*n^2)) + ((1 + m)*(e*x)^(1 + m)*Sin[d*(a +
b*Log[c*x^n])]^4)/(e*((1 + m)^2 + 16*b^2*d^2*n^2)) + (12*b^2*d^2*n^2*((2*b
^2*d^2*n^2*(e*x)^(1 + m))/(e*(1 + m)*((1 + m)^2 + 4*b^2*d^2*n^2)) - (2*b*d
*n*(e*x)^(1 + m)*Cos[d*(a + b*Log[c*x^n])]*Sin[d*(a + b*Log[c*x^n])])/(e*(
(1 + m)^2 + 4*b^2*d^2*n^2)) + ((1 + m)*(e*x)^(1 + m)*Sin[d*(a + b*Log[c*x^
n])]^2)/(e*((1 + m)^2 + 4*b^2*d^2*n^2))))/((1 + m)^2 + 16*b^2*d^2*n^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 4990
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (-Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a
 + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e
*(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2
))   Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
 c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [F]

$$\int (ex)^m \sin\left(d(a+b\ln\left(c\,x^n\right))\right)^4 dx$$

input
```
int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^4,x)
```

output
```
int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.39

$$\int (ex)^m \sin^4\left(d(a+b\log\left(cx^n\right))\right)\,dx = \frac{4\left((4\left(b^3d^3m+b^3d^3\right)n^3+\left(bdm^3+3\,bdm^2+3\,bdm+bd\right)n\right)x\cos\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)^3-(10\,(b^3\cdots}{\cdots}$$

input
```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^4,x, algorithm="fricas")
```

output
```
(4*((4*(b^3*d^3*m + b^3*d^3)*n^3 + (b*d*m^3 + 3*b*d*m^2 + 3*b*d*m + b*d)*n
)*x*cos(b*d*n*log(x) + b*d*log(c) + a*d)^3 - (10*(b^3*d^3*m + b^3*d^3)*n^3
 + (b*d*m^3 + 3*b*d*m^2 + 3*b*d*m + b*d)*n)*x*cos(b*d*n*log(x) + b*d*log(c
) + a*d))*e^(m*log(e) + m*log(x))*sin(b*d*n*log(x) + b*d*log(c) + a*d) + (
(m^4 + 4*m^3 + 4*(b^2*d^2*m^2 + 2*b^2*d^2*m + b^2*d^2)*n^2 + 6*m^2 + 4*m +
 1)*x*cos(b*d*n*log(x) + b*d*log(c) + a*d)^4 - 2*(m^4 + 4*m^3 + 10*(b^2*d^
2*m^2 + 2*b^2*d^2*m + b^2*d^2)*n^2 + 6*m^2 + 4*m + 1)*x*cos(b*d*n*log(x) +
 b*d*log(c) + a*d)^2 + (24*b^4*d^4*n^4 + m^4 + 4*m^3 + 16*(b^2*d^2*m^2 + 2
*b^2*d^2*m + b^2*d^2)*n^2 + 6*m^2 + 4*m + 1)*x)*e^(m*log(e) + m*log(x)))/(
m^5 + 64*(b^4*d^4*m + b^4*d^4)*n^4 + 5*m^4 + 10*m^3 + 20*(b^2*d^2*m^3 + 3*
b^2*d^2*m^2 + 3*b^2*d^2*m + b^2*d^2)*n^2 + 10*m^2 + 5*m + 1)
```

Sympy [F]

$$\int (ex)^m \sin^4(d(a+b\log(cx^n)))\,dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*sin(d*(a+b*ln(c*x**n)))**4,x)
```

output
```
-Piecewise((log(x)*cos(2*a*d)/e, Eq(b, 0) & Eq(m, -1)), (Integral((e*x)**m
*cos(-2*a*d + I*m*log(c*x**n)/n + I*log(c*x**n)/n), x), Eq(b, -I*(m + 1)/(
2*d*n))), (Integral((e*x)**m*cos(2*a*d + I*m*log(c*x**n)/n + I*log(c*x**n)
/n), x), Eq(b, I*(m + 1)/(2*d*n))), (2*b*d*n*x*(e*x)**m*sin(2*a*d + 2*b*d*
log(c*x**n))/(4*b**2*d**2*n**2 + m**2 + 2*m + 1) + m*x*(e*x)**m*cos(2*a*d
+ 2*b*d*log(c*x**n))/(4*b**2*d**2*n**2 + m**2 + 2*m + 1) + x*(e*x)**m*cos(
2*a*d + 2*b*d*log(c*x**n))/(4*b**2*d**2*n**2 + m**2 + 2*m + 1), True))/2 +
 Piecewise((log(x)*cos(4*a*d)/e, Eq(b, 0) & Eq(m, -1)), (Integral((e*x)**m
*cos(-4*a*d + I*m*log(c*x**n)/n + I*log(c*x**n)/n), x), Eq(b, -I*(m + 1)/(
4*d*n))), (Integral((e*x)**m*cos(4*a*d + I*m*log(c*x**n)/n + I*log(c*x**n)
/n), x), Eq(b, I*(m + 1)/(4*d*n))), (4*b*d*n*x*(e*x)**m*sin(4*a*d + 4*b*d*
log(c*x**n))/(16*b**2*d**2*n**2 + m**2 + 2*m + 1) + m*x*(e*x)**m*cos(4*a*d
 + 4*b*d*log(c*x**n))/(16*b**2*d**2*n**2 + m**2 + 2*m + 1) + x*(e*x)**m*co
s(4*a*d + 4*b*d*log(c*x**n))/(16*b**2*d**2*n**2 + m**2 + 2*m + 1), True))/
8 + 3*Piecewise(((e*x)**(m + 1)/(m + 1), Ne(m, -1)), (log(e*x), True))/(8*
e)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16932 vs. $2(337) = 674$.

Time = 0.86 (sec) , antiderivative size = 16932, normalized size of antiderivative = 50.24

$$\int (ex)^m \sin^4(d(a + b\log(cx^n)))\,dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^4,x, algorithm="maxima")
```

output
```
1/16*(((((cos(8*a*d)*cos(4*a*d) + sin(8*a*d)*sin(4*a*d))*cos(4*b*d*log(c))
 + (cos(4*a*d)*sin(8*a*d) - cos(8*a*d)*sin(4*a*d))*sin(4*b*d*log(c)))*cos(
8*b*d*log(c)) + cos(4*b*d*log(c))*cos(4*a*d) - ((cos(4*a*d)*sin(8*a*d) - c
os(8*a*d)*sin(4*a*d))*cos(4*b*d*log(c)) - (cos(8*a*d)*cos(4*a*d) + sin(8*a
*d)*sin(4*a*d))*sin(4*b*d*log(c)))*sin(8*b*d*log(c)) - sin(4*b*d*log(c))*s
in(4*a*d))*e^m*m^4 + 4*(((cos(8*a*d)*cos(4*a*d) + sin(8*a*d)*sin(4*a*d))*c
os(4*b*d*log(c)) + (cos(4*a*d)*sin(8*a*d) - cos(8*a*d)*sin(4*a*d))*sin(4*b
*d*log(c)))*cos(8*b*d*log(c)) + cos(4*b*d*log(c))*cos(4*a*d) - ((cos(4*a*d
)*sin(8*a*d) - cos(8*a*d)*sin(4*a*d))*cos(4*b*d*log(c)) - (cos(8*a*d)*cos(
4*a*d) + sin(8*a*d)*sin(4*a*d))*sin(4*b*d*log(c)))*sin(8*b*d*log(c)) - sin
(4*b*d*log(c))*sin(4*a*d))*e^m*m^3 + 6*(((cos(8*a*d)*cos(4*a*d) + sin(8*a*
d)*sin(4*a*d))*cos(4*b*d*log(c)) + (cos(4*a*d)*sin(8*a*d) - cos(8*a*d)*sin
(4*a*d))*sin(4*b*d*log(c)))*cos(8*b*d*log(c)) + cos(4*b*d*log(c))*cos(4*a*
d) - ((cos(4*a*d)*sin(8*a*d) - cos(8*a*d)*sin(4*a*d))*cos(4*b*d*log(c)) -
(cos(8*a*d)*cos(4*a*d) + sin(8*a*d)*sin(4*a*d))*sin(4*b*d*log(c)))*sin(8*b
*d*log(c)) - sin(4*b*d*log(c))*sin(4*a*d))*e^m*m^2 + 16*((b^3*d^3*cos(4*a*
d)*sin(4*b*d*log(c)) + b^3*d^3*cos(4*b*d*log(c))*sin(4*a*d) + ((b^3*d^3*co
s(4*a*d)*sin(8*a*d) - b^3*d^3*cos(8*a*d)*sin(4*a*d))*cos(4*b*d*log(c)) - (
b^3*d^3*cos(8*a*d)*cos(4*a*d) + b^3*d^3*sin(8*a*d)*sin(4*a*d))*sin(4*b*d*l
og(c)))*cos(8*b*d*log(c)) + ((b^3*d^3*cos(8*a*d)*cos(4*a*d) + b^3*d^3*s...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706991 vs. $2(337) = 674$.

Time = 18.84 (sec) , antiderivative size = 706991, normalized size of antiderivative = 2097.90

$$\int (ex)^m \sin^4(d(a+b\log(cx^n)))\,dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^4,x, algorithm="giac")
```

output
```
-1/16*(384*(abs(e)*abs(x))^m*b^4*d^4*n^4*x*tan(2*b*d*n*log(abs(x)) + 2*b*d
*log(abs(c)))^2*tan(b*d*n*log(abs(x)) + b*d*log(abs(c)))^2*tan(pi*m*floor(
-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi
*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(2*a*d)^2*tan
(a*d)^2 + 384*(abs(e)*abs(x))^m*b^4*d^4*n^4*x*tan(2*b*d*n*log(abs(x)) + 2*
b*d*log(abs(c)))^2*tan(b*d*n*log(abs(x)) + b*d*log(abs(c)))^2*tan(pi*m*flo
or(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2
*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(2*a*d)^2
+ 384*(abs(e)*abs(x))^m*b^4*d^4*n^4*x*tan(2*b*d*n*log(abs(x)) + 2*b*d*log(
abs(c)))^2*tan(b*d*n*log(abs(x)) + b*d*log(abs(c)))^2*tan(pi*m*floor(-1/4*
sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2
*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(a*d)^2 + 384*(abs
(e)*abs(x))^m*b^4*d^4*n^4*x*tan(2*b*d*n*log(abs(x)) + 2*b*d*log(abs(c)))^2
*tan(b*d*n*log(abs(x)) + b*d*log(abs(c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1
/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(2*a*d
)^2*tan(a*d)^2 - 384*(abs(e)*abs(x))^m*b^4*d^4*n^4*x*tan(2*b*d*n*log(abs(x
)) + 2*b*d*log(abs(c)))^2*tan(b*d*n*log(abs(x)) + b*d*log(abs(c)))^2*tan(1
/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(2*a*d)^2*tan(a*d)^2 + 3
84*(abs(e)*abs(x))^m*b^4*d^4*n^4*x*tan(2*b*d*n*log(abs(x)) + 2*b*d*log(abs
(c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) ...
```

Mupad [B] (verification not implemented)

Time = 22.58 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.52

$$\int (ex)^m \sin^4(d(a+b\log(cx^n)))\,dx = \frac{3\,x\,(e\,x)^m}{8\,m+8} - \frac{x\,\mathrm{e}^{a\,d\,2\mathrm{i}}\,(c\,x^n)^{b\,d\,2\mathrm{i}}\,(e\,x)^m}{4\,m+4+b\,d\,n\,8\mathrm{i}} - \frac{x\,\mathrm{e}^{-a\,d\,2\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,d\,2\mathrm{i}}}\,(e\,x)^m\,1\mathrm{i}}{m\,4\mathrm{i}+8\,b\,d\,n+4\mathrm{i}} + \frac{x\,\mathrm{e}^{a\,d\,4\mathrm{i}}\,(c\,x^n)^{b\,d\,4\mathrm{i}}\,(e\,x)^m}{16\,m+16+b\,d\,n\,64\mathrm{i}} + \frac{x\,\mathrm{e}^{-a\,d\,4\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,d\,4\mathrm{i}}}\,(e\,x)^m\,1\mathrm{i}}{m\,16\mathrm{i}+64\,b\,d\,n+16\mathrm{i}}$$

input
```
int(sin(d*(a + b*log(c*x^n)))^4*(e*x)^m,x)
```

output
```
(3*x*(e*x)^m)/(8*m + 8) - (x*exp(a*d*2i)*(c*x^n)^(b*d*2i)*(e*x)^m)/(4*m +
b*d*n*8i + 4) - (x*exp(-a*d*2i)/(c*x^n)^(b*d*2i)*(e*x)^m*1i)/(m*4i + 8*b*d
*n + 4i) + (x*exp(a*d*4i)*(c*x^n)^(b*d*4i)*(e*x)^m)/(16*m + b*d*n*64i + 16
) + (x*exp(-a*d*4i)/(c*x^n)^(b*d*4i)*(e*x)^m*1i)/(m*16i + 64*b*d*n + 16i)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 695, normalized size of antiderivative = 2.06

$$\int (ex)^m \sin^4(d(a+b\log(cx^n)))\,dx = \text{Too large to display}$$

input
```
int((e*x)^m*sin(d*(a+b*log(c*x^n)))^4,x)
```

output
```
(x**m*e**m*x*( - 16*cos(log(x**n*c)*b*d + a*d)*sin(log(x**n*c)*b*d + a*d)*
*3*b**3*d**3*m*n**3 - 16*cos(log(x**n*c)*b*d + a*d)*sin(log(x**n*c)*b*d +
a*d)**3*b**3*d**3*n**3 - 4*cos(log(x**n*c)*b*d + a*d)*sin(log(x**n*c)*b*d
+ a*d)**3*b*d*m**3*n - 12*cos(log(x**n*c)*b*d + a*d)*sin(log(x**n*c)*b*d +
 a*d)**3*b*d*m**2*n - 12*cos(log(x**n*c)*b*d + a*d)*sin(log(x**n*c)*b*d +
a*d)**3*b*d*m*n - 4*cos(log(x**n*c)*b*d + a*d)*sin(log(x**n*c)*b*d + a*d)*
*3*b*d*n - 24*cos(log(x**n*c)*b*d + a*d)*sin(log(x**n*c)*b*d + a*d)*b**3*d
**3*m*n**3 - 24*cos(log(x**n*c)*b*d + a*d)*sin(log(x**n*c)*b*d + a*d)*b**3
*d**3*n**3 + 4*sin(log(x**n*c)*b*d + a*d)**4*b**2*d**2*m**2*n**2 + 8*sin(l
og(x**n*c)*b*d + a*d)**4*b**2*d**2*m*n**2 + 4*sin(log(x**n*c)*b*d + a*d)**
4*b**2*d**2*n**2 + sin(log(x**n*c)*b*d + a*d)**4*m**4 + 4*sin(log(x**n*c)*
b*d + a*d)**4*m**3 + 6*sin(log(x**n*c)*b*d + a*d)**4*m**2 + 4*sin(log(x**n
*c)*b*d + a*d)**4*m + sin(log(x**n*c)*b*d + a*d)**4 + 12*sin(log(x**n*c)*b
*d + a*d)**2*b**2*d**2*m**2*n**2 + 24*sin(log(x**n*c)*b*d + a*d)**2*b**2*d
**2*m*n**2 + 12*sin(log(x**n*c)*b*d + a*d)**2*b**2*d**2*n**2 + 24*b**4*d**
4*n**4))/(64*b**4*d**4*m*n**4 + 64*b**4*d**4*n**4 + 20*b**2*d**2*m**3*n**2
 + 60*b**2*d**2*m**2*n**2 + 60*b**2*d**2*m*n**2 + 20*b**2*d**2*n**2 + m**5
 + 5*m**4 + 10*m**3 + 10*m**2 + 5*m + 1)
```

3.71 $\int (ex)^m \sin^3(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 256

$$\begin{aligned}\int (ex)^m \sin^3(d(a+b\log(cx^n)))\,dx &= -\frac{6b^3d^3n^3(ex)^{1+m}\cos(d(a+b\log(cx^n)))}{e((1+m)^2+b^2d^2n^2)((1+m)^2+9b^2d^2n^2)} \\ &\quad+\frac{6b^2d^2(1+m)n^2(ex)^{1+m}\sin(d(a+b\log(cx^n)))}{e((1+m)^2+b^2d^2n^2)((1+m)^2+9b^2d^2n^2)} \\ &\quad-\frac{3bdn(ex)^{1+m}\cos(d(a+b\log(cx^n)))\sin^2(d(a+b\log(cx^n)))}{e((1+m)^2+9b^2d^2n^2)} \\ &\quad+\frac{(1+m)(ex)^{1+m}\sin^3(d(a+b\log(cx^n)))}{e((1+m)^2+9b^2d^2n^2)}\end{aligned}$$

output
```
-6*b^3*d^3*n^3*(e*x)^(1+m)*cos(d*(a+b*ln(c*x^n)))/e/((1+m)^2+b^2*d^2*n^2)/
((1+m)^2+9*b^2*d^2*n^2)+6*b^2*d^2*(1+m)*n^2*(e*x)^(1+m)*sin(d*(a+b*ln(c*x^
n)))/e/((1+m)^2+b^2*d^2*n^2)/((1+m)^2+9*b^2*d^2*n^2)-3*b*d*n*(e*x)^(1+m)*c
os(d*(a+b*ln(c*x^n)))*sin(d*(a+b*ln(c*x^n)))^2/e/((1+m)^2+9*b^2*d^2*n^2)+(
1+m)*(e*x)^(1+m)*sin(d*(a+b*ln(c*x^n)))^3/e/((1+m)^2+9*b^2*d^2*n^2)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.27

$$\int (ex)^m \sin^3(d(a+b\log(cx^n)))\,dx$$

$$= \frac{1}{4}x(ex)^m \left(\frac{3\cos(bdn\log(x))\left(-bdn\cos\left(d(a-bn\log(x)+b\log(cx^n))\right)+(1+m)\sin\left(d(a-bn\log(x)+\right.\right.}{1+2m+m^2+b^2d^2n^2} \right.$$

$$+ \frac{3\sin(bdn\log(x))\left((1+m)\cos\left(d(a-bn\log(x)+b\log(cx^n))\right)+bdn\sin\left(d(a-bn\log(x)+b\log(cx^n)\right)\right.}{1+2m+m^2+b^2d^2n^2}$$

$$- \frac{\cos(3bdn\log(x))\left(-3bdn\cos\left(3d(a-bn\log(x)+b\log(cx^n))\right)+(1+m)\sin\left(3d(a-bn\log(x)+b\log\right.\right.}{1+2m+m^2+9b^2d^2n^2}$$

$$- \frac{\sin(3bdn\log(x))\left((1+m)\cos\left(3d(a-bn\log(x)+b\log(cx^n))\right)+3bdn\sin\left(3d(a-bn\log(x)+b\log(c\right.\right.}{1+2m+m^2+9b^2d^2n^2}$$

input
```
Integrate[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^3,x]
```

output
```
(x*(e*x)^m*((3*Cos[b*d*n*Log[x]]*(-(b*d*n*Cos[d*(a - b*n*Log[x] + b*Log[c*
x^n])]) + (1 + m)*Sin[d*(a - b*n*Log[x] + b*Log[c*x^n])]))/(1 + 2*m + m^2
+ b^2*d^2*n^2) + (3*Sin[b*d*n*Log[x]]*((1 + m)*Cos[d*(a - b*n*Log[x] + b*L
og[c*x^n])] + b*d*n*Sin[d*(a - b*n*Log[x] + b*Log[c*x^n])]))/(1 + 2*m + m^
2 + b^2*d^2*n^2) - (Cos[3*b*d*n*Log[x]]*(-3*b*d*n*Cos[3*d*(a - b*n*Log[x]
+ b*Log[c*x^n])] + (1 + m)*Sin[3*d*(a - b*n*Log[x] + b*Log[c*x^n])]))/(1 +
 2*m + m^2 + 9*b^2*d^2*n^2) - (Sin[3*b*d*n*Log[x]]*((1 + m)*Cos[3*d*(a - b
*n*Log[x] + b*Log[c*x^n])] + 3*b*d*n*Sin[3*d*(a - b*n*Log[x] + b*Log[c*x^n
])]))/(1 + 2*m + m^2 + 9*b^2*d^2*n^2)))/4
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4990, 4988}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \sin^3(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{4990}$$

$$\frac{6b^2d^2n^2\int (ex)^m \sin (d(a+b\log (cx^n)))dx}{9b^2d^2n^2+(m+1)^2}+\frac{(m+1)(ex)^{m+1}\sin ^3(d(a+b\log (cx^n)))}{e(9b^2d^2n^2+(m+1)^2)}-\frac{3bdn(ex)^{m+1}\sin ^2(d(a+b\log (cx^n)))\cos (d(a+b\log (cx^n)))}{e(9b^2d^2n^2+(m+1)^2)}$$

$$\Big\downarrow \text{4988}$$

$$\frac{(m+1)(ex)^{m+1}\sin ^3(d(a+b\log (cx^n)))}{e(9b^2d^2n^2+(m+1)^2)}-\frac{3bdn(ex)^{m+1}\sin ^2(d(a+b\log (cx^n)))\cos (d(a+b\log (cx^n)))}{e(9b^2d^2n^2+(m+1)^2)}+\frac{6b^2d^2n^2\left(\frac{(m+1)(ex)^{m+1}\sin (d(a+b\log (cx^n)))}{e(b^2d^2n^2+(m+1)^2)}-\frac{bdn(ex)^{m+1}\cos (d(a+b\log (cx^n)))}{e(b^2d^2n^2+(m+1)^2)}\right)}{9b^2d^2n^2+(m+1)^2}$$

input
```
Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^3,x]
```

output
```
(-3*b*d*n*(e*x)^(1 + m)*Cos[d*(a + b*Log[c*x^n])]*Sin[d*(a + b*Log[c*x^n])
]^2)/(e*((1 + m)^2 + 9*b^2*d^2*n^2)) + ((1 + m)*(e*x)^(1 + m)*Sin[d*(a + b
*Log[c*x^n])]^3)/(e*((1 + m)^2 + 9*b^2*d^2*n^2)) + (6*b^2*d^2*n^2*(-((b*d*
n*(e*x)^(1 + m)*Cos[d*(a + b*Log[c*x^n])])/(e*((1 + m)^2 + b^2*d^2*n^2)))
+ ((1 + m)*(e*x)^(1 + m)*Sin[d*(a + b*Log[c*x^n])])/(e*((1 + m)^2 + b^2*d^
2*n^2))))/((1 + m)^2 + 9*b^2*d^2*n^2)
```

Defintions of rubi rules used

rule 4988
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

rule 4990
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (-Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a
 + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e
*(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2
))   Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
 c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [F]

$$\int (ex)^m \sin (d(a+b\ln (c\,x^n)))^3 dx$$

input
```
int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^3,x)
```

output
```
int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.14

$$\int (ex)^m \sin^3 (d(a+b\log (cx^n)))\, dx =$$
$$-\frac{((m^3+(b^2d^2m+b^2d^2)n^2+3\,m^2+3\,m+1)x\cos (bdn\log (x)+bd\log (c)+ad)^2-(m^3+7\,(b^2d^2m+$$

input
```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^3,x, algorithm="fricas")
```

output
```
-(((m^3 + (b^2*d^2*m + b^2*d^2)*n^2 + 3*m^2 + 3*m + 1)*x*cos(b*d*n*log(x)
+ b*d*log(c) + a*d)^2 - (m^3 + 7*(b^2*d^2*m + b^2*d^2)*n^2 + 3*m^2 + 3*m +
 1)*x)*e^(m*log(e) + m*log(x))*sin(b*d*n*log(x) + b*d*log(c) + a*d) - 3*((
b^3*d^3*n^3 + (b*d*m^2 + 2*b*d*m + b*d)*n)*x*cos(b*d*n*log(x) + b*d*log(c)
 + a*d)^3 - (3*b^3*d^3*n^3 + (b*d*m^2 + 2*b*d*m + b*d)*n)*x*cos(b*d*n*log(
x) + b*d*log(c) + a*d))*e^(m*log(e) + m*log(x)))/(9*b^4*d^4*n^4 + m^4 + 4*
m^3 + 10*(b^2*d^2*m^2 + 2*b^2*d^2*m + b^2*d^2)*n^2 + 6*m^2 + 4*m + 1)
```

Sympy [F]

$$\int (ex)^m \sin^3 (d(a+b\log (cx^n)))\,dx$$

$$= \frac{3\begin{cases} \frac{\log(x)\sin(ad)}{e} & \text{for } b=0 \wedge m=-1 \\ -\int (ex)^m \sin\left(-ad+\frac{im\log(cx^n)}{n}+\frac{i\log(cx^n)}{n}\right)dx & \text{for } b=-\frac{i(m+1)}{dn} \\ \int (ex)^m \sin\left(ad+\frac{im\log(cx^n)}{n}+\frac{i\log(cx^n)}{n}\right)dx & \text{for } b=\frac{i(m+1)}{dn} \\ -\frac{bdnx(ex)^m\cos(ad+bd\log(cx^n))}{b^2d^2n^2+m^2+2m+1}+\frac{mx(ex)^m\sin(ad+bd\log(cx^n))}{b^2d^2n^2+m^2+2m+1}+\frac{x(ex)^m\sin(ad+bd\log(cx^n))}{b^2d^2n^2+m^2+2m+1} & \text{otherwise} \end{cases}}{4}$$

$$- \frac{\begin{cases} \frac{\log(x)\sin(3ad)}{e} & \text{for } b=0 \wedge m \\ -\int (ex)^m \sin\left(-3ad+\frac{im\log(cx^n)}{n}+\frac{i\log(cx^n)}{n}\right)dx & \text{for } b=-\frac{i(m+1}{3dn} \\ \int (ex)^m \sin\left(3ad+\frac{im\log(cx^n)}{n}+\frac{i\log(cx^n)}{n}\right)dx & \text{for } b=\frac{i(m+1)}{3dn} \\ -\frac{3bdnx(ex)^m\cos(3ad+3bd\log(cx^n))}{9b^2d^2n^2+m^2+2m+1}+\frac{mx(ex)^m\sin(3ad+3bd\log(cx^n))}{9b^2d^2n^2+m^2+2m+1}+\frac{x(ex)^m\sin(3ad+3bd\log(cx^n))}{9b^2d^2n^2+m^2+2m+1} & \text{otherwise} \end{cases}}{4}$$

input
```
integrate((e*x)**m*sin(d*(a+b*ln(c*x**n)))**3,x)
```

output
```
3*Piecewise((log(x)*sin(a*d)/e, Eq(b, 0) & Eq(m, -1)), (-Integral((e*x)**m
*sin(-a*d + I*m*log(c*x**n)/n + I*log(c*x**n)/n), x), Eq(b, -I*(m + 1)/(d*
n))), (Integral((e*x)**m*sin(a*d + I*m*log(c*x**n)/n + I*log(c*x**n)/n), x
), Eq(b, I*(m + 1)/(d*n))), (-b*d*n*x*(e*x)**m*cos(a*d + b*d*log(c*x**n))/
(b**2*d**2*n**2 + m**2 + 2*m + 1) + m*x*(e*x)**m*sin(a*d + b*d*log(c*x**n)
)/(b**2*d**2*n**2 + m**2 + 2*m + 1) + x*(e*x)**m*sin(a*d + b*d*log(c*x**n)
)/(b**2*d**2*n**2 + m**2 + 2*m + 1), True))/4 - Piecewise((log(x)*sin(3*a*
d)/e, Eq(b, 0) & Eq(m, -1)), (-Integral((e*x)**m*sin(-3*a*d + I*m*log(c*x*
*n)/n + I*log(c*x**n)/n), x), Eq(b, -I*(m + 1)/(3*d*n))), (Integral((e*x)*
*m*sin(3*a*d + I*m*log(c*x**n)/n + I*log(c*x**n)/n), x), Eq(b, I*(m + 1)/(
3*d*n))), (-3*b*d*n*x*(e*x)**m*cos(3*a*d + 3*b*d*log(c*x**n))/(9*b**2*d**2
*n**2 + m**2 + 2*m + 1) + m*x*(e*x)**m*sin(3*a*d + 3*b*d*log(c*x**n))/(9*b
**2*d**2*n**2 + m**2 + 2*m + 1) + x*(e*x)**m*sin(3*a*d + 3*b*d*log(c*x**n)
)/(9*b**2*d**2*n**2 + m**2 + 2*m + 1), True))/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11491 vs. $2(256) = 512$.

Time = 0.41 (sec) , antiderivative size = 11491, normalized size of antiderivative = 44.89

$$\int (ex)^m \sin^3(d(a + b\log(cx^n)))\,dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^3,x, algorithm="maxima")
```

output
```
-1/8*(((((cos(3*a*d)*sin(6*a*d) - cos(6*a*d)*sin(3*a*d))*cos(3*b*d*log(c))
 - (cos(6*a*d)*cos(3*a*d) + sin(6*a*d)*sin(3*a*d))*sin(3*b*d*log(c)))*cos(
6*b*d*log(c)) + ((cos(6*a*d)*cos(3*a*d) + sin(6*a*d)*sin(3*a*d))*cos(3*b*d
*log(c)) + (cos(3*a*d)*sin(6*a*d) - cos(6*a*d)*sin(3*a*d))*sin(3*b*d*log(c
)))*sin(6*b*d*log(c)) + cos(3*a*d)*sin(3*b*d*log(c)) + cos(3*b*d*log(c))*s
in(3*a*d))*e^m*m^3 - 3*(b^3*d^3*cos(3*b*d*log(c))*cos(3*a*d) - b^3*d^3*sin
(3*b*d*log(c))*sin(3*a*d) + ((b^3*d^3*cos(6*a*d)*cos(3*a*d) + b^3*d^3*sin(
6*a*d)*sin(3*a*d))*cos(3*b*d*log(c)) + (b^3*d^3*cos(3*a*d)*sin(6*a*d) - b^
3*d^3*cos(6*a*d)*sin(3*a*d))*sin(3*b*d*log(c)))*cos(6*b*d*log(c)) - ((b^3*
d^3*cos(3*a*d)*sin(6*a*d) - b^3*d^3*cos(6*a*d)*sin(3*a*d))*cos(3*b*d*log(c
)) - (b^3*d^3*cos(6*a*d)*cos(3*a*d) + b^3*d^3*sin(6*a*d)*sin(3*a*d))*sin(3
*b*d*log(c)))*sin(6*b*d*log(c)))*e^m*n^3 + 3*(((cos(3*a*d)*sin(6*a*d) - co
s(6*a*d)*sin(3*a*d))*cos(3*b*d*log(c)) - (cos(6*a*d)*cos(3*a*d) + sin(6*a*
d)*sin(3*a*d))*sin(3*b*d*log(c)))*cos(6*b*d*log(c)) + ((cos(6*a*d)*cos(3*a
*d) + sin(6*a*d)*sin(3*a*d))*cos(3*b*d*log(c)) + (cos(3*a*d)*sin(6*a*d) -
cos(6*a*d)*sin(3*a*d))*sin(3*b*d*log(c)))*sin(6*b*d*log(c)) + cos(3*a*d)*s
in(3*b*d*log(c)) + cos(3*b*d*log(c))*sin(3*a*d))*e^m*m^2 + 3*(((cos(3*a*d)
*sin(6*a*d) - cos(6*a*d)*sin(3*a*d))*cos(3*b*d*log(c)) - (cos(6*a*d)*cos(3
*a*d) + sin(6*a*d)*sin(3*a*d))*sin(3*b*d*log(c)))*cos(6*b*d*log(c)) + ((co
s(6*a*d)*cos(3*a*d) + sin(6*a*d)*sin(3*a*d))*cos(3*b*d*log(c)) + (cos(3...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200416 vs. $2(256) = 512$.

Time = 5.74 (sec) , antiderivative size = 200416, normalized size of antiderivative = 782.88

$$\int (ex)^m \sin^3(d(a + b\log(cx^n)))\,dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^3,x, algorithm="giac")
```

output
```
-1/8*(3*b^3*d^3*n^3*x*e^(3/2*pi*b*d*n*sgn(x) - 3/2*pi*b*d*n + 3/2*pi*b*d*s
gn(c) - 3/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(3/2*b*d*n*log(abs(
x)) + 3/2*b*d*log(abs(c)))^2*tan(1/2*b*d*n*log(abs(x)) + 1/2*b*d*log(abs(c
)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(3/2*a*d)^2*t
an(1/2*a*d)^2 - 27*b^3*d^3*n^3*x*e^(1/2*pi*b*d*n*sgn(x) - 1/2*pi*b*d*n + 1
/2*pi*b*d*sgn(c) - 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(3/2*b*d
*n*log(abs(x)) + 3/2*b*d*log(abs(c)))^2*tan(1/2*b*d*n*log(abs(x)) + 1/2*b*
d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(3
/2*a*d)^2*tan(1/2*a*d)^2 - 27*b^3*d^3*n^3*x*e^(-1/2*pi*b*d*n*sgn(x) + 1/2*
pi*b*d*n - 1/2*pi*b*d*sgn(c) + 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))
*tan(3/2*b*d*n*log(abs(x)) + 3/2*b*d*log(abs(c)))^2*tan(1/2*b*d*n*log(abs(
x)) + 1/2*b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*p
i*m)^2*tan(3/2*a*d)^2*tan(1/2*a*d)^2 + 3*b^3*d^3*n^3*x*e^(-3/2*pi*b*d*n*sg
n(x) + 3/2*pi*b*d*n - 3/2*pi*b*d*sgn(c) + 3/2*pi*b*d + m*log(abs(e)) + m*l
og(abs(x)))*tan(3/2*b*d*n*log(abs(x)) + 3/2*b*d*log(abs(c)))^2*tan(1/2*b*d
*n*log(abs(x)) + 1/2*b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn
(x) - 1/2*pi*m)^2*tan(3/2*a*d)^2*tan(1/2*a*d)^2 + 3*b^3*d^3*n^3*x*e^(3/2*p
i*b*d*n*sgn(x) - 3/2*pi*b*d*n + 3/2*pi*b*d*sgn(c) - 3/2*pi*b*d + m*log(abs
(e)) + m*log(abs(x)))*tan(3/2*b*d*n*log(abs(x)) + 3/2*b*d*log(abs(c)))^2*t
an(1/2*b*d*n*log(abs(x)) + 1/2*b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) +...
```

Mupad [B] (verification not implemented)

Time = 22.18 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.63

$$\int (ex)^m \sin^3(d(a+b\log(cx^n)))\,dx = \frac{x\,\mathrm{e}^{-a\,d\,1\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,d\,1\mathrm{i}}}\,(e\,x)^m\,3\mathrm{i}}{8\,m+8-b\,d\,n\,8\mathrm{i}} + \frac{3\,x\,\mathrm{e}^{a\,d\,1\mathrm{i}}\,(c\,x^n)^{b\,d\,1\mathrm{i}}\,(e\,x)^m}{m\,8\mathrm{i}-8\,b\,d\,n+8\mathrm{i}} - \frac{x\,\mathrm{e}^{-a\,d\,3\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,d\,3\mathrm{i}}}\,(e\,x)^m\,1\mathrm{i}}{8\,m+8-b\,d\,n\,24\mathrm{i}} - \frac{x\,\mathrm{e}^{a\,d\,3\mathrm{i}}\,(c\,x^n)^{b\,d\,3\mathrm{i}}\,(e\,x)^m}{m\,8\mathrm{i}-24\,b\,d\,n+8\mathrm{i}}$$

input
```
int(sin(d*(a + b*log(c*x^n)))^3*(e*x)^m,x)
```

output
```
(x*exp(-a*d*1i)/(c*x^n)^(b*d*1i)*(e*x)^m*3i)/(8*m - b*d*n*8i + 8) + (3*x*e
xp(a*d*1i)*(c*x^n)^(b*d*1i)*(e*x)^m)/(m*8i - 8*b*d*n + 8i) - (x*exp(-a*d*3
i)/(c*x^n)^(b*d*3i)*(e*x)^m*1i)/(8*m - b*d*n*24i + 8) - (x*exp(a*d*3i)*(c*
x^n)^(b*d*3i)*(e*x)^m)/(m*8i - 24*b*d*n + 8i)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.69

$$\int (ex)^m \sin^3(d(a+b\log(cx^n)))\,dx$$
$$= \frac{x^m e^m x\left(-3\cos(\log(x^n c)\,bd+ad)\sin(\log(x^n c)\,bd+ad)^2 b^3d^3n^3 - 3\cos(\log(x^n c)\,bd+ad)\sin(\log(x^n c)\,b\right.}{}$$

input
```
int((e*x)^m*sin(d*(a+b*log(c*x^n)))^3,x)
```

output

```
(x**m*e**m*x*( - 3*cos(log(x**n*c)*b*d + a*d)*sin(log(x**n*c)*b*d + a*d)**
2*b**3*d**3*n**3 - 3*cos(log(x**n*c)*b*d + a*d)*sin(log(x**n*c)*b*d + a*d)
**2*b*d*m**2*n - 6*cos(log(x**n*c)*b*d + a*d)*sin(log(x**n*c)*b*d + a*d)**
2*b*d*m*n - 3*cos(log(x**n*c)*b*d + a*d)*sin(log(x**n*c)*b*d + a*d)**2*b*d
*n - 6*cos(log(x**n*c)*b*d + a*d)*b**3*d**3*n**3 + sin(log(x**n*c)*b*d + a
*d)**3*b**2*d**2*m*n**2 + sin(log(x**n*c)*b*d + a*d)**3*b**2*d**2*n**2 + s
in(log(x**n*c)*b*d + a*d)**3*m**3 + 3*sin(log(x**n*c)*b*d + a*d)**3*m**2 +
 3*sin(log(x**n*c)*b*d + a*d)**3*m + sin(log(x**n*c)*b*d + a*d)**3 + 6*sin
(log(x**n*c)*b*d + a*d)*b**2*d**2*m*n**2 + 6*sin(log(x**n*c)*b*d + a*d)*b*
*2*d**2*n**2))/(9*b**4*d**4*n**4 + 10*b**2*d**2*m**2*n**2 + 20*b**2*d**2*m
*n**2 + 10*b**2*d**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1)
```

3.72 $\int (ex)^m \sin^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 154

$$\int (ex)^m \sin^2(d(a+b\log(cx^n)))\,dx = \frac{2b^2d^2n^2(ex)^{1+m}}{e(1+m)\left((1+m)^2+4b^2d^2n^2\right)} - \frac{2bdn(ex)^{1+m}\cos(d(a+b\log(cx^n)))\sin(d(a+b\log(cx^n)))}{e\left((1+m)^2+4b^2d^2n^2\right)} + \frac{(1+m)(ex)^{1+m}\sin^2(d(a+b\log(cx^n)))}{e\left((1+m)^2+4b^2d^2n^2\right)}$$

output
```
2*b^2*d^2*n^2*(e*x)^(1+m)/e/(1+m)/((1+m)^2+4*b^2*d^2*n^2)-2*b*d*n*(e*x)^(1
+m)*cos(d*(a+b*ln(c*x^n)))*sin(d*(a+b*ln(c*x^n)))/e/((1+m)^2+4*b^2*d^2*n^2
)+(1+m)*(e*x)^(1+m)*sin(d*(a+b*ln(c*x^n)))^2/e/((1+m)^2+4*b^2*d^2*n^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.66

$$\int (ex)^m \sin^2(d(a+b\log(cx^n)))\,dx =$$
$$-\frac{x(ex)^m\left(-1-2m-m^2-4b^2d^2n^2+(1+m)^2\cos(2d(a+b\log(cx^n)))+2bd(1+m)n\sin(2d(a+b\,\text{lo}\right.}{2(1+m)(1+m-2ibdn)(1+m+2ibdn)}$$

input
```
Integrate[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
-1/2*(x*(e*x)^m*(-1 - 2*m - m^2 - 4*b^2*d^2*n^2 + (1 + m)^2*Cos[2*d*(a + b
*Log[c*x^n])] + 2*b*d*(1 + m)*n*Sin[2*d*(a + b*Log[c*x^n])]))/((1 + m)*(1
+ m - (2*I)*b*d*n)*(1 + m + (2*I)*b*d*n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4990, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \sin^2(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{4990}$$

$$\frac{2b^2d^2n^2\int (ex)^m dx}{4b^2d^2n^2+(m+1)^2}+\frac{(m+1)(ex)^{m+1}\sin^2(d(a+b\log(cx^n)))}{e(4b^2d^2n^2+(m+1)^2)}-\frac{2bdn(ex)^{m+1}\sin(d(a+b\log(cx^n)))\cos(d(a+b\log(cx^n)))}{e(4b^2d^2n^2+(m+1)^2)}$$

$$\Big\downarrow \text{17}$$

$$\frac{(m+1)(ex)^{m+1}\sin^2\left(d(a+b\log\left(cx^n\right))\right)}{e\left(4b^2d^2n^2+(m+1)^2\right)} - \frac{2bdn(ex)^{m+1}\sin\left(d(a+b\log\left(cx^n\right))\right)\cos\left(d(a+b\log\left(cx^n\right))\right)}{e\left(4b^2d^2n^2+(m+1)^2\right)} + \frac{2b^2d^2n^2(ex)^{m+1}}{e(m+1)\left(4b^2d^2n^2+(m+1)^2\right)}$$

input
```
Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
(2*b^2*d^2*n^2*(e*x)^(1 + m))/(e*(1 + m)*((1 + m)^2 + 4*b^2*d^2*n^2)) - (2
*b*d*n*(e*x)^(1 + m)*Cos[d*(a + b*Log[c*x^n])]*Sin[d*(a + b*Log[c*x^n])])/
(e*((1 + m)^2 + 4*b^2*d^2*n^2)) + ((1 + m)*(e*x)^(1 + m)*Sin[d*(a + b*Log[
c*x^n])]^2)/(e*((1 + m)^2 + 4*b^2*d^2*n^2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 4990
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (-Simp[b*d*n*p*(e*x)^(m + 1)*Cos[d*(a
 + b*Log[c*x^n])]*(Sin[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e
*(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2
))   Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
 c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [F]

$$\int (ex)^m \sin\left(d(a+b\ln\left(c\,x^n\right))\right)^2 dx$$

input
```
int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^2,x)
```

output
```
int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.01

$$\int (ex)^m \sin^2(d(a+b\log(cx^n)))\,dx = -\frac{2\,(bdm+bd)nx\cos(bdn\log(x)+bd\log(c)+ad)\,e^{(m\log(e)+m\log(x))}\sin(bdn\log(x)+bd\log(c)+ad)+}{m^3+4\,(b^2d^2m+b^2d^2}$$

input
```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

output
```
-(2*(b*d*m + b*d)*n*x*cos(b*d*n*log(x) + b*d*log(c) + a*d)*e^(m*log(e) + m
*log(x))*sin(b*d*n*log(x) + b*d*log(c) + a*d) + ((m^2 + 2*m + 1)*x*cos(b*d
*n*log(x) + b*d*log(c) + a*d)^2 - (2*b^2*d^2*n^2 + m^2 + 2*m + 1)*x)*e^(m*
log(e) + m*log(x)))/(m^3 + 4*(b^2*d^2*m + b^2*d^2)*n^2 + 3*m^2 + 3*m + 1)
```

Sympy [F]

$$\int (ex)^m \sin^2(d(a+b\log(cx^n)))\,dx = -\frac{\begin{cases} \frac{\log(x)\cos(2ad)}{e} & \text{for } b=0 \wedge m= \\ \int (ex)^m\cos\left(-2ad+\frac{im\log(cx^n)}{n}+\frac{i\log(cx^n)}{n}\right)dx & \text{for } b=-\frac{i(m+1)}{2dn} \\ \int (ex)^m\cos\left(2ad+\frac{im\log(cx^n)}{n}+\frac{i\log(cx^n)}{n}\right)dx & \text{for } b=\frac{i(m+1)}{2dn} \\ \frac{2bdnx(ex)^m\sin(2ad+2bd\log(cx^n))}{4b^2d^2n^2+m^2+2m+1}+\frac{mx(ex)^m\cos(2ad+2bd\log(cx^n))}{4b^2d^2n^2+m^2+2m+1}+\frac{x(ex)^m\cos(2ad+2bd\log(cx^n))}{4b^2d^2n^2+m^2+2m+1} & \text{otherwise} \end{cases}}{2} + \frac{\begin{cases} \frac{(ex)^{m+1}}{m+1} & \text{for } m\neq -1 \\ \log(ex) & \text{otherwise} \end{cases}}{2e}$$

input
```
integrate((e*x)**m*sin(d*(a+b*ln(c*x**n)))**2,x)
```

output
```
-Piecewise((log(x)*cos(2*a*d)/e, Eq(b, 0) & Eq(m, -1)), (Integral((e*x)**m
*cos(-2*a*d + I*m*log(c*x**n)/n + I*log(c*x**n)/n), x), Eq(b, -I*(m + 1)/(
2*d*n))), (Integral((e*x)**m*cos(2*a*d + I*m*log(c*x**n)/n + I*log(c*x**n)
/n), x), Eq(b, I*(m + 1)/(2*d*n))), (2*b*d*n*x*(e*x)**m*sin(2*a*d + 2*b*d*
log(c*x**n))/(4*b**2*d**2*n**2 + m**2 + 2*m + 1) + m*x*(e*x)**m*cos(2*a*d
+ 2*b*d*log(c*x**n))/(4*b**2*d**2*n**2 + m**2 + 2*m + 1) + x*(e*x)**m*cos(
2*a*d + 2*b*d*log(c*x**n))/(4*b**2*d**2*n**2 + m**2 + 2*m + 1), True))/2 +
 Piecewise(((e*x)**(m + 1)/(m + 1), Ne(m, -1)), (log(e*x), True))/(2*e)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2551 vs. $2(154) = 308$.

Time = 0.16 (sec) , antiderivative size = 2551, normalized size of antiderivative = 16.56

$$\int (ex)^m \sin^2(d(a + b\log(cx^n)))\, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

output
```
-1/4*(((((cos(4*a*d)*cos(2*a*d) + sin(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c))
 + (cos(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c)))*cos(
4*b*d*log(c)) + cos(2*b*d*log(c))*cos(2*a*d) - ((cos(2*a*d)*sin(4*a*d) - c
os(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c)) - (cos(4*a*d)*cos(2*a*d) + sin(4*a
*d)*sin(2*a*d))*sin(2*b*d*log(c)))*sin(4*b*d*log(c)) - sin(2*b*d*log(c))*s
in(2*a*d))*e^m*m^2 + 2*(((cos(4*a*d)*cos(2*a*d) + sin(4*a*d)*sin(2*a*d))*c
os(2*b*d*log(c)) + (cos(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*sin(2*b
*d*log(c)))*cos(4*b*d*log(c)) + cos(2*b*d*log(c))*cos(2*a*d) - ((cos(2*a*d
)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c)) - (cos(4*a*d)*cos(
2*a*d) + sin(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c)))*sin(4*b*d*log(c)) - sin
(2*b*d*log(c))*sin(2*a*d))*e^m*m + (((cos(4*a*d)*cos(2*a*d) + sin(4*a*d)*s
in(2*a*d))*cos(2*b*d*log(c)) + (cos(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a
*d))*sin(2*b*d*log(c)))*cos(4*b*d*log(c)) + cos(2*b*d*log(c))*cos(2*a*d) -
 ((cos(2*a*d)*sin(4*a*d) - cos(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c)) - (cos
(4*a*d)*cos(2*a*d) + sin(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c)))*sin(4*b*d*l
og(c)) - sin(2*b*d*log(c))*sin(2*a*d))*e^m + 2*((b*d*cos(2*a*d)*sin(2*b*d*
log(c)) + b*d*cos(2*b*d*log(c))*sin(2*a*d) + ((b*d*cos(2*a*d)*sin(4*a*d) -
 b*d*cos(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c)) - (b*d*cos(4*a*d)*cos(2*a*d)
 + b*d*sin(4*a*d)*sin(2*a*d))*sin(2*b*d*log(c)))*cos(4*b*d*log(c)) + ((b*d
*cos(4*a*d)*cos(2*a*d) + b*d*sin(4*a*d)*sin(2*a*d))*cos(2*b*d*log(c)) +...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30585 vs. $2(154) = 308$.

Time = 1.11 (sec) , antiderivative size = 30585, normalized size of antiderivative = 198.60

$$\int (ex)^m \sin^2(d(a + b\log(cx^n))) \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

output
```
-1/4*(8*(abs(e)*abs(x))^m*b^2*d^2*n^2*x*tan(b*d*n*log(abs(x)) + b*d*log(ab
s(c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sgn(e) +
1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*
pi*m)^2*tan(a*d)^2 + 8*(abs(e)*abs(x))^m*b^2*d^2*n^2*x*tan(b*d*n*log(abs(x
)) + b*d*log(abs(c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4
*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*
m*sgn(x) - 1/2*pi*m)^2 + 8*(abs(e)*abs(x))^m*b^2*d^2*n^2*x*tan(b*d*n*log(a
bs(x)) + b*d*log(abs(c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) +
 1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(a*d)^2 - 8*(abs(e)*ab
s(x))^m*b^2*d^2*n^2*x*tan(b*d*n*log(abs(x)) + b*d*log(abs(c)))^2*tan(1/4*p
i*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(a*d)^2 + 8*(abs(e)*abs(x))^
m*b^2*d^2*n^2*x*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*pi*m*sg
n(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x)
 - 1/2*pi*m)^2*tan(a*d)^2 + 4*b*d*m*n*x*e^(pi*b*d*n*sgn(x) - pi*b*d*n + pi
*b*d*sgn(c) - pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(b*d*n*log(abs(x)
) + b*d*log(abs(c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + 1) + 1/4*
pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m
*sgn(x) - 1/2*pi*m)^2*tan(a*d) + 4*b*d*m*n*x*e^(-pi*b*d*n*sgn(x) + pi*b*d*
n - pi*b*d*sgn(c) + pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(b*d*n*log(
abs(x)) + b*d*log(abs(c)))^2*tan(pi*m*floor(-1/4*sgn(e) - 1/4*sgn(x) + ...
```

Mupad [B] (verification not implemented)

Time = 21.52 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.62

$$\int (ex)^m \sin^2(d(a + b\log(cx^n))) \, dx = \frac{x\,(e\,x)^m}{2\,m+2} - \frac{x\,\mathrm{e}^{a\,d\,2\mathrm{i}}\,(c\,x^n)^{b\,d\,2\mathrm{i}}\,(e\,x)^m}{4\,m+4+b\,d\,n\,8\mathrm{i}} - \frac{x\,\mathrm{e}^{-a\,d\,2\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,d\,2\mathrm{i}}}\,(e\,x)^m\,1\mathrm{i}}{m\,4\mathrm{i}+8\,b\,d\,n+4\mathrm{i}}$$

input
```
int(sin(d*(a + b*log(c*x^n)))^2*(e*x)^m,x)
```

output
```
(x*(e*x)^m)/(2*m + 2) - (x*exp(a*d*2i)*(c*x^n)^(b*d*2i)*(e*x)^m)/(4*m + b*
d*n*8i + 4) - (x*exp(-a*d*2i)/(c*x^n)^(b*d*2i)*(e*x)^m*1i)/(m*4i + 8*b*d*n
 + 4i)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.17

$$\int (ex)^m \sin^2(d(a+b\log(cx^n)))\,dx$$
$$= \frac{x^m e^m x\left(-2\cos(\log(x^n c)\,bd+ad)\sin(\log(x^n c)\,bd+ad)\,bdmn-2\cos(\log(x^n c)\,bd+ad)\sin(\log(x^n c)\,bd}{4b^2d^2m\,n^2+4b^2d^2n^2}$$

input
```
int((e*x)^m*sin(d*(a+b*log(c*x^n)))^2,x)
```

output
```
(x**m*e**m*x*( - 2*cos(log(x**n*c)*b*d + a*d)*sin(log(x**n*c)*b*d + a*d)*b
*d*m*n - 2*cos(log(x**n*c)*b*d + a*d)*sin(log(x**n*c)*b*d + a*d)*b*d*n + s
in(log(x**n*c)*b*d + a*d)**2*m**2 + 2*sin(log(x**n*c)*b*d + a*d)**2*m + si
n(log(x**n*c)*b*d + a*d)**2 + 2*b**2*d**2*n**2))/(4*b**2*d**2*m*n**2 + 4*b
**2*d**2*n**2 + m**3 + 3*m**2 + 3*m + 1)
```

3.73 $\int (ex)^m \sin(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 92

$$\int (ex)^m \sin(d(a+b\log(cx^n)))\,dx = -\frac{bdn(ex)^{1+m}\cos(d(a+b\log(cx^n)))}{e((1+m)^2+b^2d^2n^2)} + \frac{(1+m)(ex)^{1+m}\sin(d(a+b\log(cx^n)))}{e((1+m)^2+b^2d^2n^2)}$$

output
```
-b*d*n*(e*x)^(1+m)*cos(d*(a+b*ln(c*x^n)))/e/((1+m)^2+b^2*d^2*n^2)+(1+m)*(e
*x)^(1+m)*sin(d*(a+b*ln(c*x^n)))/e/((1+m)^2+b^2*d^2*n^2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.68

$$\int (ex)^m \sin(d(a+b\log(cx^n)))\,dx = \frac{x(ex)^m(-bdn\cos(d(a+b\log(cx^n)))+(1+m)\sin(d(a+b\log(cx^n))))}{1+2m+m^2+b^2d^2n^2}$$

input
```
Integrate[(e*x)^m*Sin[d*(a + b*Log[c*x^n])],x]
```

output
```
(x*(e*x)^m*(-(b*d*n*Cos[d*(a + b*Log[c*x^n])]) + (1 + m)*Sin[d*(a + b*Log[
c*x^n])]))/(1 + 2*m + m^2 + b^2*d^2*n^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {4988}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \sin (d(a+b\log (cx^n)))\, dx$$

$$\Big\downarrow \; 4988$$

$$\frac{(m+1)(ex)^{m+1}\sin (d(a+b\log (cx^n)))}{e\left(b^2d^2n^2+(m+1)^2\right)}-\frac{bdn(ex)^{m+1}\cos (d(a+b\log (cx^n)))}{e\left(b^2d^2n^2+(m+1)^2\right)}$$

input
```
Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])],x]
```

output
```
-((b*d*n*(e*x)^(1 + m)*Cos[d*(a + b*Log[c*x^n])])/(e*((1 + m)^2 + b^2*d^2*
n^2))) + ((1 + m)*(e*x)^(1 + m)*Sin[d*(a + b*Log[c*x^n])])/(e*((1 + m)^2 +
 b^2*d^2*n^2))
```

Defintions of rubi rules used

rule 4988
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] - Simp[b*d*n*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Maple [F]

$$\int (ex)^m \sin\left(d(a+b\ln\left(c\,x^n\right))\right)dx$$

input
```
int((e*x)^m*sin(d*(a+b*ln(c*x^n))),x)
```

output
```
int((e*x)^m*sin(d*(a+b*ln(c*x^n))),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.93

$$\int (ex)^m \sin\left(d(a+b\log\left(cx^n\right))\right)\,dx = -\frac{bdnx\cos\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)e^{(m\log(e)+m\log(x))}-(m+1)xe^{(m\log(e)+m\log(x))}\sin\left(bdn\log\left(x\right)+\right.}{b^2d^2n^2+m^2+2\,m+1}$$

input
```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
-(b*d*n*x*cos(b*d*n*log(x) + b*d*log(c) + a*d)*e^(m*log(e) + m*log(x)) - (
m + 1)*x*e^(m*log(e) + m*log(x))*sin(b*d*n*log(x) + b*d*log(c) + a*d))/(b^
2*d^2*n^2 + m^2 + 2*m + 1)
```

Sympy [F]

$$\int (ex)^m \sin\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int (ex)^m \sin\left(ad+bd\log\left(cx^n\right)\right)dx$$

input
```
integrate((e*x)**m*sin(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral((e*x)**m*sin(a*d + b*d*log(c*x**n)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1263 vs. $2(92) = 184$.

Time = 0.10 (sec) , antiderivative size = 1263, normalized size of antiderivative = 13.73

$$\int (ex)^m \sin(d(a + b\log(cx^n)))\, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
1/2*(((((cos(a*d)*sin(2*a*d) - cos(2*a*d)*sin(a*d))*cos(b*d*log(c)) - (cos
(2*a*d)*cos(a*d) + sin(2*a*d)*sin(a*d))*sin(b*d*log(c)))*cos(2*b*d*log(c))
 + ((cos(2*a*d)*cos(a*d) + sin(2*a*d)*sin(a*d))*cos(b*d*log(c)) + (cos(a*d
)*sin(2*a*d) - cos(2*a*d)*sin(a*d))*sin(b*d*log(c)))*sin(2*b*d*log(c)) + c
os(a*d)*sin(b*d*log(c)) + cos(b*d*log(c))*sin(a*d))*e^m*m - (b*d*cos(b*d*l
og(c))*cos(a*d) - b*d*sin(b*d*log(c))*sin(a*d) + ((b*d*cos(2*a*d)*cos(a*d)
 + b*d*sin(2*a*d)*sin(a*d))*cos(b*d*log(c)) + (b*d*cos(a*d)*sin(2*a*d) - b
*d*cos(2*a*d)*sin(a*d))*sin(b*d*log(c)))*cos(2*b*d*log(c)) - ((b*d*cos(a*d
)*sin(2*a*d) - b*d*cos(2*a*d)*sin(a*d))*cos(b*d*log(c)) - (b*d*cos(2*a*d)*
cos(a*d) + b*d*sin(2*a*d)*sin(a*d))*sin(b*d*log(c)))*sin(2*b*d*log(c)))*e^
m*n + (((cos(a*d)*sin(2*a*d) - cos(2*a*d)*sin(a*d))*cos(b*d*log(c)) - (cos
(2*a*d)*cos(a*d) + sin(2*a*d)*sin(a*d))*sin(b*d*log(c)))*cos(2*b*d*log(c))
 + ((cos(2*a*d)*cos(a*d) + sin(2*a*d)*sin(a*d))*cos(b*d*log(c)) + (cos(a*d
)*sin(2*a*d) - cos(2*a*d)*sin(a*d))*sin(b*d*log(c)))*sin(2*b*d*log(c)) + c
os(a*d)*sin(b*d*log(c)) + cos(b*d*log(c))*sin(a*d))*e^m)*x*x^m*cos(b*d*log
(x^n)) + ((((cos(2*a*d)*cos(a*d) + sin(2*a*d)*sin(a*d))*cos(b*d*log(c)) +
(cos(a*d)*sin(2*a*d) - cos(2*a*d)*sin(a*d))*sin(b*d*log(c)))*cos(2*b*d*log
(c)) + cos(b*d*log(c))*cos(a*d) - ((cos(a*d)*sin(2*a*d) - cos(2*a*d)*sin(a
*d))*cos(b*d*log(c)) - (cos(2*a*d)*cos(a*d) + sin(2*a*d)*sin(a*d))*sin(b*d
*log(c)))*sin(2*b*d*log(c)) - sin(b*d*log(c))*sin(a*d))*e^m*m + (b*d*co...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6580 vs. $2(92) = 184$.

Time = 0.37 (sec) , antiderivative size = 6580, normalized size of antiderivative = 71.52

$$\int (ex)^m \sin \left(d(a + b\log \left(cx^n\right))\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
1/2*(b*d*n*x*e^(1/2*pi*b*d*n*sgn(x) - 1/2*pi*b*d*n + 1/2*pi*b*d*sgn(c) - 1
/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*b*d*n*log(abs(x)) + 1/2
*b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2*pi*m)^2*ta
n(1/2*a*d)^2 + b*d*n*x*e^(-1/2*pi*b*d*n*sgn(x) + 1/2*pi*b*d*n - 1/2*pi*b*d
*sgn(c) + 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*b*d*n*log(ab
s(x)) + 1/2*b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) - 1/2
*pi*m)^2*tan(1/2*a*d)^2 - b*d*n*x*e^(1/2*pi*b*d*n*sgn(x) - 1/2*pi*b*d*n +
1/2*pi*b*d*sgn(c) - 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*b*
d*n*log(abs(x)) + 1/2*b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sg
n(x) - 1/2*pi*m)^2 - b*d*n*x*e^(-1/2*pi*b*d*n*sgn(x) + 1/2*pi*b*d*n - 1/2*
pi*b*d*sgn(c) + 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*b*d*n*
log(abs(x)) + 1/2*b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x)
 - 1/2*pi*m)^2 + 4*b*d*n*x*e^(1/2*pi*b*d*n*sgn(x) - 1/2*pi*b*d*n + 1/2*pi*
b*d*sgn(c) - 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*b*d*n*log
(abs(x)) + 1/2*b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*sgn(x) -
1/2*pi*m)*tan(1/2*a*d) - 4*b*d*n*x*e^(-1/2*pi*b*d*n*sgn(x) + 1/2*pi*b*d*n
- 1/2*pi*b*d*sgn(c) + 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)))*tan(1/2*
b*d*n*log(abs(x)) + 1/2*b*d*log(abs(c)))^2*tan(1/4*pi*m*sgn(e) + 1/4*pi*m*
sgn(x) - 1/2*pi*m)*tan(1/2*a*d) - 4*b*d*n*x*e^(1/2*pi*b*d*n*sgn(x) - 1/2*p
i*b*d*n + 1/2*pi*b*d*sgn(c) - 1/2*pi*b*d + m*log(abs(e)) + m*log(abs(x)...
```

Mupad [B] (verification not implemented)

Time = 21.56 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int (ex)^m \sin (d(a+b\log (cx^n)))\, dx = \frac{x\,\mathrm{e}^{-a\,d\,1\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,d\,1\mathrm{i}}}\,(e\,x)^m\,1\mathrm{i}}{2\,m+2-b\,d\,n\,2\mathrm{i}}+\frac{x\,\mathrm{e}^{a\,d\,1\mathrm{i}}\,(c\,x^n)^{b\,d\,1\mathrm{i}}\,(e\,x)^m}{m\,2\mathrm{i}-2\,b\,d\,n+2\mathrm{i}}$$

input
```
int(sin(d*(a + b*log(c*x^n)))*(e*x)^m,x)
```

output
```
(x*exp(-a*d*1i)/(c*x^n)^(b*d*1i)*(e*x)^m*1i)/(2*m - b*d*n*2i + 2) + (x*exp
(a*d*1i)*(c*x^n)^(b*d*1i)*(e*x)^m)/(m*2i - 2*b*d*n + 2i)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.85

$$\int (ex)^m \sin (d(a+b\log (cx^n)))\, dx$$
$$= \frac{x^m e^m x(-\cos (\log(x^n c)\, bd+ad)\, bdn+\sin (\log(x^n c)\, bd+ad)\, m+\sin (\log(x^n c)\, bd+ad))}{b^2d^2n^2+m^2+2m+1}$$

input
```
int((e*x)^m*sin(d*(a+b*log(c*x^n))),x)
```

output
```
(x**m*e**m*x*( - cos(log(x**n*c)*b*d + a*d)*b*d*n + sin(log(x**n*c)*b*d +
a*d)*m + sin(log(x**n*c)*b*d + a*d)))/(b**2*d**2*n**2 + m**2 + 2*m + 1)
```

3.74 $\int (ex)^m \sin^{\frac{3}{2}}(d(a+b\log(cx^n)))\, dx$

Optimal result

Integrand size = 23, antiderivative size = 150

$$\int (ex)^m \sin^{\frac{3}{2}}(d(a+b\log(cx^n)))\, dx = \frac{2(ex)^{1+m}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{2i+2im+3bdn}{4bdn},-\frac{2i+2im-bdn}{4bdn},e^{2iad}(cx^n)^{2ibd}\right)\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))}{e(2+2m-3ibdn)\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{3/2}}$$

output
```
2*(e*x)^(1+m)*hypergeom([-3/2, -1/4*(2*I+2*I*m+3*b*d*n)/b/d/n],[-1/4*(2*I+
2*I*m-b*d*n)/b/d/n],exp(2*I*a*d)*(c*x^n)^(2*I*b*d))*sin(d*(a+b*ln(c*x^n)))
^(3/2)/e/(2+2*m-3*I*b*d*n)/(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^(3/2)
```

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.57

$$\int (ex)^m \sin^{\frac{3}{2}}(d(a+b\log(cx^n)))\, dx = \frac{2(ex)^m\left(-3b^2d^2\left(-1+e^{2id(a+b\log(cx^n))}\right)n^2x\,\text{Hypergeometric2F1}\left(1,-\frac{2i+2im-3bdn}{4bdn},-\frac{2i+2im-5bdn}{4bdn},e^{2id(a+b\log\cdots}\right.\right.}{(2+2m+ibdn)(2+2m-3ib\cdots}$$

input
```
Integrate[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(3/2),x]
```

output
```
(2*(e*x)^m*(-3*b^2*d^2*(-1 + E^((2*I)*d*(a + b*Log[c*x^n])))*n^2*x*Hyperge
ometric2F1[1, -1/4*(2*I + (2*I)*m - 3*b*d*n)/(b*d*n), -1/4*(2*I + (2*I)*m
- 5*b*d*n)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + (2 + 2*m + I*b*d*n)*
x*Sin[d*(a + b*Log[c*x^n])]*(-3*b*d*n*Cos[d*(a + b*Log[c*x^n])] + 2*(1 + m
)*Sin[d*(a + b*Log[c*x^n])])))/((2 + 2*m + I*b*d*n)*(2 + 2*m - (3*I)*b*d*n
)*(2 + 2*m + (3*I)*b*d*n)*Sqrt[Sin[d*(a + b*Log[c*x^n])]])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4996, 4994, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \sin^{\frac{3}{2}}(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))\,d(cx^n)}{en}$$

$$\Big\downarrow \text{4994}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}+\frac{3ibd}{2}}\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))\int (cx^n)^{-\frac{3}{2}ibd+\frac{m+1}{n}-1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{3/2}d(cx^n)}{en\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2(ex)^{m+1}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4}\left(-\frac{2i(m+1)}{bdn}-3\right),-\frac{2im-bdn+2i}{4bdn},e^{2iad}(cx^n)^{2ibd}\right)\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))}{e(-3ibdn+2m+2)\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{3/2}}$$

input
```
Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^(3/2),x]
```

output
```
(2*(e*x)^(1 + m)*Hypergeometric2F1[-3/2, (-3 - ((2*I)*(1 + m))/(b*d*n))/4,
 -1/4*(2*I + (2*I)*m - b*d*n)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)]*
Sin[d*(a + b*Log[c*x^n])]^(3/2))/(e*(2 + 2*m - (3*I)*b*d*n)*(1 - E^((2*I)*
a*d)*(c*x^n)^((2*I)*b*d))^(3/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4994
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Simp[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int (ex)^m \sin (d(a + b \ln (c x^n)))^{\frac{3}{2}} dx$$

input
```
int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^(3/2),x)
```

output
```
int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ex)^m \sin^{\frac{3}{2}}(d(a + b\log(cx^n)))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \sin^{\frac{3}{2}}(d(a + b\log(cx^n)))\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m*sin(d*(a+b*ln(c*x**n)))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m \sin^{\frac{3}{2}}(d(a + b\log(cx^n)))\,dx = \int (ex)^m \sin((b\log(cx^n) + a)d)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^m*sin((b*log(c*x^n) + a)*d)^(3/2), x)
```

Giac [F]

$$\int (ex)^m \sin^{\frac{3}{2}}(d(a+b\log(cx^n)))\,dx = \int (ex)^m \sin((b\log(cx^n)+a)d)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x)^m*sin((b*log(c*x^n) + a)*d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \sin^{\frac{3}{2}}(d(a+b\log(cx^n)))\,dx = \int \sin(d(a+b\ln(c\,x^n)))^{3/2}\,(e\,x)^m\,dx$$

input
```
int(sin(d*(a + b*log(c*x^n)))^(3/2)*(e*x)^m,x)
```

output
```
int(sin(d*(a + b*log(c*x^n)))^(3/2)*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \sin^{\frac{3}{2}}(d(a+b\log(cx^n)))\,dx$$
$$= \frac{e^m\left(2x^m\sqrt{\sin(\log(x^nc)\,bd+ad)}\,\sin(\log(x^nc)\,bd+ad)\,x-3\left(\int x^m\sqrt{\sin(\log(x^nc)\,bd+ad)}\,\cos(\log(x^n\right.\right.}{2m+2}$$

input
```
int((e*x)^m*sin(d*(a+b*log(c*x^n)))^(3/2),x)
```

output
```
(e**m*(2*x**m*sqrt(sin(log(x**n*c)*b*d + a*d))*sin(log(x**n*c)*b*d + a*d)*
x - 3*int(x**m*sqrt(sin(log(x**n*c)*b*d + a*d))*cos(log(x**n*c)*b*d + a*d)
,x)*b*d*n))/(2*(m + 1))
```

3.75 $\int (ex)^m \sqrt{\sin (d (a+b \log (c x^n)))} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 149

$$\int (ex)^m \sqrt{\sin (d (a+b \log (c x^n)))} \, dx = \frac{2(ex)^{1+m} \text{Hypergeometric2F1}\left(-\frac{1}{2}, -\frac{2i+2im+bdn}{4bdn}, -\frac{2i+2im-3bdn}{4bdn}, e^{2iad}(cx^n)^{2ibd}\right) \sqrt{\sin (d (a+b \log (c x^n)))}}{e(2+2m-ibdn)\sqrt{1-e^{2iad}(cx^n)^{2ibd}}}$$

output
```
2*(e*x)^(1+m)*hypergeom([-1/2, -1/4*(2*I+2*I*m+b*d*n)/b/d/n],[-1/4*(2*I+2*
I*m-3*b*d*n)/b/d/n],exp(2*I*a*d)*(c*x^n)^(2*I*b*d))*sin(d*(a+b*ln(c*x^n)))
^(1/2)/e/(2+2*m-I*b*d*n)/(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^(1/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 488 vs. 2(149) = 298.

Time = 7.01 (sec) , antiderivative size = 488, normalized size of antiderivative = 3.28

$$\int (ex)^m \sqrt{\sin \left(d \left(a+b \log \left(c x^n\right)\right)\right)} \, dx$$
$$= 2x(ex)^m \left(-\frac{bde^{id(a-bn\log(x)+b\log(cx^n))}nx^{-ibdn}\sqrt{2-2e^{2iad}\left(cx^n\right)^{2ibd}}\left((2i+2im+bdn)x^{2ibdn}\,\text{Hypergeometr}\right.}{(2+2m-ibdn)(2+2m+3ibdn)\left(2i+2i\right.} \right.$$
$$\left. + \frac{\sqrt{\sin \left(d \left(a+b \log \left(c x^n\right)\right)\right)} \sin \left(d(a-bn\log(x)+b\log \left(cx^n\right)\right))}{bdn\cos \left(d \left(a-bn\log(x)+b\log \left(cx^n\right)\right)\right)+2(1+m)\sin \left(d \left(a-bn\log(x)+b\log \left(cx^n\right)\right)\right)} \right)$$

input
```
Integrate[(e*x)^m*Sqrt[Sin[d*(a + b*Log[c*x^n])]],x]
```

output
```
2*x*(e*x)^m*(-((b*d*E^(I*d*(a - b*n*Log[x] + b*Log[c*x^n]))*n*Sqrt[2 - 2*E
^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)]*((2*I + (2*I)*m + b*d*n)*x^((2*I)*b*d*n)
*Hypergeometric2F1[1/2, ((-1/2*I)*(1 + m + ((3*I)/2)*b*d*n))/(b*d*n), -1/4
*(2*I + (2*I)*m - 7*b*d*n)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)] + (
-2*I - (2*I)*m + 3*b*d*n)*Hypergeometric2F1[1/2, -1/4*(2*I + (2*I)*m + b*d
*n)/(b*d*n), -1/4*(2*I + (2*I)*m - 3*b*d*n)/(b*d*n), E^((2*I)*a*d)*(c*x^n)
^((2*I)*b*d)]))/((2 + 2*m - I*b*d*n)*(2 + 2*m + (3*I)*b*d*n)*(2*I + (2*I)*
m + b*d*n + E^((2*I)*d*(a - b*n*Log[x] + b*Log[c*x^n]))*(-2*I - (2*I)*m +
b*d*n))*x^(I*b*d*n)*Sqrt[((-I)*(-1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(
E^(I*a*d)*(c*x^n)^(I*b*d))])) + (Sqrt[Sin[d*(a + b*Log[c*x^n])]]*Sin[d*(a
- b*n*Log[x] + b*Log[c*x^n])])/(b*d*n*Cos[d*(a - b*n*Log[x] + b*Log[c*x^n]
)] + 2*(1 + m)*Sin[d*(a - b*n*Log[x] + b*Log[c*x^n])]))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4996, 4994, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \sqrt{\sin (d (a+b \log (cx^n)))} \, dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{(ex)^{m+1} (cx^n)^{-\frac{m+1}{n}} \int (cx^n)^{\frac{m+1}{n}-1} \sqrt{\sin (d (a+b \log (cx^n)))} d(cx^n)}{en}$$

$$\Big\downarrow \text{4994}$$

$$\frac{(ex)^{m+1} (cx^n)^{-\frac{m+1}{n}+\frac{ibd}{2}} \sqrt{\sin (d (a+b \log (cx^n)))} \int (cx^n)^{-\frac{1}{2}ibd+\frac{m+1}{n}-1} \sqrt{1-e^{2iad} (cx^n)^{2ibd}} d(cx^n)}{en\sqrt{1-e^{2iad} (cx^n)^{2ibd}}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2(ex)^{m+1} \text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{1}{4}\left(-\frac{2i(m+1)}{bdn}-1\right), -\frac{2im-3bdn+2i}{4bdn}, e^{2iad}(cx^n)^{2ibd}\right) \sqrt{\sin (d (a+b \log (cx^n)))}}{e(-ibdn+2m+2)\sqrt{1-e^{2iad} (cx^n)^{2ibd}}}$$

input
```
Int[(e*x)^m*Sqrt[Sin[d*(a + b*Log[c*x^n])]],x]
```

output
```
(2*(e*x)^(1 + m)*Hypergeometric2F1[-1/2, (-1 - ((2*I)*(1 + m))/(b*d*n))/4,
 -1/4*(2*I + (2*I)*m - 3*b*d*n)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)
]*Sqrt[Sin[d*(a + b*Log[c*x^n])]])/(e*(2 + 2*m - I*b*d*n)*Sqrt[1 - E^((2*I
)*a*d)*(c*x^n)^((2*I)*b*d)])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4994
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Simp[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int (ex)^m \sqrt{\sin (d (a+b\ln (c\,x^n)))}dx$$

input
```
int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^(1/2),x)
```

output
```
int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ex)^m \sqrt{\sin (d (a+b\log (cx^n)))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (ex)^m \sqrt{\sin (d (a+b\log (cx^n)))}\,dx = \int (ex)^m \sqrt{\sin (ad+bd\log (cx^n))}\,dx$$

input
```
integrate((e*x)**m*sin(d*(a+b*ln(c*x**n)))**(1/2),x)
```

output
```
Integral((e*x)**m*sqrt(sin(a*d + b*d*log(c*x**n))), x)
```

Maxima [F]

$$\int (ex)^m \sqrt{\sin \left(d \left(a+b \log \left(cx^n\right)\right)\right)}\,dx=\int (ex)^m \sqrt{\sin \left(\left(b \log \left(cx^n\right)+a\right)d\right)}\,dx$$

input
```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^m*sqrt(sin((b*log(c*x^n) + a)*d)), x)
```

Giac [F]

$$\int (ex)^m \sqrt{\sin \left(d \left(a+b \log \left(cx^n\right)\right)\right)}\,dx=\int (ex)^m \sqrt{\sin \left(\left(b \log \left(cx^n\right)+a\right)d\right)}\,dx$$

input
```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x)^m*sqrt(sin((b*log(c*x^n) + a)*d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \sqrt{\sin \left(d \left(a+b \log \left(cx^n\right)\right)\right)}\,dx=\int \sqrt{\sin \left(d\,\left(a+b\,\ln \left(c\,x^n\right)\right)\right)}\,{\left(e\,x\right)}^m\,dx$$

input
```
int(sin(d*(a + b*log(c*x^n)))^(1/2)*(e*x)^m,x)
```

output
```
int(sin(d*(a + b*log(c*x^n)))^(1/2)*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \sqrt{\sin (d (a + b \log (cx^n)))}\, dx$$
$$= \frac{e^m \left(2x^m \sqrt{\sin (\log (x^n c)\, bd + ad)}\, x - \left(\int \frac{x^m \sqrt{\sin(\log(x^n c)bd+ad)}\, \cos(\log(x^n c)bd+ad)}{\sin(\log(x^n c)bd+ad)} dx\right) bdn\right)}{2m+2}$$

input
```
int((e*x)^m*sin(d*(a+b*log(c*x^n)))^(1/2),x)
```

output
```
(e**m*(2*x**m*sqrt(sin(log(x**n*c)*b*d + a*d))*x - int((x**m*sqrt(sin(log(
x**n*c)*b*d + a*d))*cos(log(x**n*c)*b*d + a*d))/sin(log(x**n*c)*b*d + a*d)
,x)*b*d*n))/(2*(m + 1))
```

3.76 $\int \frac{(ex)^m}{\sqrt{\sin(d(a+b\log(cx^n)))}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 150

$$\int \frac{(ex)^m}{\sqrt{\sin\left(d\left(a+b\log\left(cx^n\right)\right)\right)}}\,dx = \frac{2(ex)^{1+m}\sqrt{1-e^{2iad}\left(cx^n\right)^{2ibd}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{2i+2im-bdn}{4bdn},-\frac{2i+2im-5bdn}{4bdn},e^{2iad}\left(cx^n\right)^{2ibd}\right)}{e(2+2m+ibdn)\sqrt{\sin\left(d\left(a+b\log\left(cx^n\right)\right)\right)}}$$

output
```
2*(e*x)^(1+m)*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^(1/2)*hypergeom([1/2, -1/
4*(2*I+2*I*m-b*d*n)/b/d/n],[-1/4*(2*I+2*I*m-5*b*d*n)/b/d/n],exp(2*I*a*d)*(
c*x^n)^(2*I*b*d))/e/(2+2*m+I*b*d*n)/sin(d*(a+b*ln(c*x^n)))^(1/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.87

$$\int \frac{(ex)^m}{\sqrt{\sin\left(d\left(a+b\log\left(cx^n\right)\right)\right)}}\,dx = -\frac{2\left(-1+e^{2id(a+b\log(cx^n))}\right)x(ex)^m\,\text{Hypergeometric2F1}\left(1,-\frac{2i+2im-3bdn}{4bdn},-\frac{2i+2im-5bdn}{4bdn},e^{2id(a+b\log(cx^n))}\right)}{(2+2m+ibdn)\sqrt{\sin\left(d\left(a+b\log\left(cx^n\right)\right)\right)}}$$

input
```
Integrate[(e*x)^m/Sqrt[Sin[d*(a + b*Log[c*x^n])]],x]
```

output
```
(-2*(-1 + E^((2*I)*d*(a + b*Log[c*x^n])))*x*(e*x)^m*Hypergeometric2F1[1, -
1/4*(2*I + (2*I)*m - 3*b*d*n)/(b*d*n), -1/4*(2*I + (2*I)*m - 5*b*d*n)/(b*d
*n), E^((2*I)*d*(a + b*Log[c*x^n]))])/((2 + 2*m + I*b*d*n)*Sqrt[Sin[d*(a +
 b*Log[c*x^n])]])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4996, 4994, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{\sqrt{\sin(d(a+b\log(cx^n)))}}dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int \frac{(cx^n)^{\frac{m+1}{n}-1}}{\sqrt{\sin(d(a+b\log(cx^n)))}}d(cx^n)}{en}$$

$$\Big\downarrow \text{4994}$$

$$\frac{(ex)^{m+1}\sqrt{1-e^{2iad}(cx^n)^{2ibd}}(cx^n)^{-\frac{m+1}{n}-\frac{1}{2}ibd}\int \frac{(cx^n)^{\frac{ibd}{2}+\frac{m+1}{n}-1}}{\sqrt{1-e^{2iad}(cx^n)^{2ibd}}}d(cx^n)}{en\sqrt{\sin(d(a+b\log(cx^n)))}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2(ex)^{m+1}\sqrt{1-e^{2iad}(cx^n)^{2ibd}}\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{2im-bdn+2i}{4bdn},-\frac{2im-5bdn+2i}{4bdn},e^{2iad}(cx^n)^{2ibd}\right)}{e(ibdn+2m+2)\sqrt{\sin(d(a+b\log(cx^n)))}}$$

input
```
Int[(e*x)^m/Sqrt[Sin[d*(a + b*Log[c*x^n])]],x]
```

output
```
(2*(e*x)^(1 + m)*Sqrt[1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)]*Hypergeometri
c2F1[1/2, -1/4*(2*I + (2*I)*m - b*d*n)/(b*d*n), -1/4*(2*I + (2*I)*m - 5*b*
d*n)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(e*(2 + 2*m + I*b*d*n)*S
qrt[Sin[d*(a + b*Log[c*x^n])]])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4994
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Simp[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{(ex)^m}{\sqrt{\sin\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)}}dx$$

input
```
int((e*x)^m/sin(d*(a+b*ln(c*x^n)))^(1/2),x)
```

output
```
int((e*x)^m/sin(d*(a+b*ln(c*x^n)))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ex)^m}{\sqrt{\sin\left(d\left(a+b\log\left(cx^n\right)\right)\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^m/sin(d*(a+b*log(c*x^n)))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{(ex)^m}{\sqrt{\sin\left(d\left(a+b\log\left(cx^n\right)\right)\right)}}\,dx = \int \frac{(ex)^m}{\sqrt{\sin\left(ad+bd\log\left(cx^n\right)\right)}}\,dx$$

input
```
integrate((e*x)**m/sin(d*(a+b*ln(c*x**n)))**(1/2),x)
```

output
```
Integral((e*x)**m/sqrt(sin(a*d + b*d*log(c*x**n))), x)
```

Maxima [F]

$$\int \frac{(ex)^m}{\sqrt{\sin\left(d\left(a+b\log\left(cx^n\right)\right)\right)}}\,dx = \int \frac{(ex)^m}{\sqrt{\sin\left(\left(b\log\left(cx^n\right)+a\right)d\right)}}\,dx$$

input
```
integrate((e*x)^m/sin(d*(a+b*log(c*x^n)))^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/sqrt(sin((b*log(c*x^n) + a)*d)), x)
```

Giac [F]

$$\int \frac{(ex)^m}{\sqrt{\sin (d (a+b \log (c x^n)))}} \, dx = \int \frac{(ex)^m}{\sqrt{\sin ((b \log (c x^n) + a)d)}} \, dx$$

input
```
integrate((e*x)^m/sin(d*(a+b*log(c*x^n)))^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x)^m/sqrt(sin((b*log(c*x^n) + a)*d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{\sqrt{\sin (d (a+b \log (c x^n)))}} \, dx = \int \frac{(e\,x)^m}{\sqrt{\sin \left(d\,\left(a+b\,\ln \left(c\,x^n\right)\right)\right)}} \, dx$$

input
```
int((e*x)^m/sin(d*(a + b*log(c*x^n)))^(1/2),x)
```

output
```
int((e*x)^m/sin(d*(a + b*log(c*x^n)))^(1/2), x)
```

Reduce [F]

$$\int \frac{(ex)^m}{\sqrt{\sin (d (a+b \log (c x^n)))}} \, dx = e^m \left(\int \frac{x^m \sqrt{\sin (\log (x^n c) bd + ad)}}{\sin (\log (x^n c) bd + ad)} dx\right)$$

input
```
int((e*x)^m/sin(d*(a+b*log(c*x^n)))^(1/2),x)
```

output
```
e**m*int((x**m*sqrt(sin(log(x**n*c)*b*d + a*d)))/sin(log(x**n*c)*b*d + a*d
),x)
```

3.77 $\int \frac{(ex)^m}{\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 150

$$\int \frac{(ex)^m}{\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))}\,dx = \frac{2(ex)^{1+m}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{2i+2im-3bdn}{4bdn},-\frac{2i+2im-7bdn}{4bdn},e^{2iad}(cx^n)^{2ibd}\right)}{e(2+2m+3ibdn)\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))}$$

output
```
2*(e*x)^(1+m)*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^(3/2)*hypergeom([3/2, -1/
4*(2*I+2*I*m-3*b*d*n)/b/d/n],[-1/4*(2*I+2*I*m-7*b*d*n)/b/d/n],exp(2*I*a*d)
*(c*x^n)^(2*I*b*d))/e/(2+2*m+3*I*b*d*n)/sin(d*(a+b*ln(c*x^n)))^(3/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 544 vs. 2(150) = 300.

Time = 3.81 (sec) , antiderivative size = 544, normalized size of antiderivative = 3.63

$$\int \frac{(ex)^m}{\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))}\,dx$$

$$= \frac{(4+8m+4m^2+b^2d^2n^2)\,x^{1+ibdn}(ex)^m\sqrt{2-2e^{2iad}(cx^n)^{2ibd}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{i(1+m+\frac{3}{2}ibdn)}{2bdn},-\frac{2i}{\ldots}\right.}{bdn(-2i-2im+\ldots}$$

input
```
Integrate[(e*x)^m/Sin[d*(a + b*Log[c*x^n])]^(3/2),x]
```

output
```
((4 + 8*m + 4*m^2 + b^2*d^2*n^2)*x^(1 + I*b*d*n)*(e*x)^m*Sqrt[2 - 2*E^((2*
I)*a*d)*(c*x^n)^((2*I)*b*d)]*Hypergeometric2F1[1/2, ((-1/2*I)*(1 + m + ((3
*I)/2)*b*d*n))/(b*d*n), -1/4*(2*I + (2*I)*m - 7*b*d*n)/(b*d*n), E^((2*I)*a
*d)*(c*x^n)^((2*I)*b*d)] + ((-2*I - (2*I)*m + 3*b*d*n)*x^(1 - I*b*d*n)*(e*
x)^m*(-2*x^(I*b*d*n)*Sqrt[((-I)*(-1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/
(E^(I*a*d)*(c*x^n)^(I*b*d))]*(b*d*n*Cos[b*d*n*Log[x]] - 2*(1 + m)*Sin[b*d*
n*Log[x]]) + (-2*I - (2*I)*m + b*d*n)*Sqrt[2 - 2*E^((2*I)*a*d)*(c*x^n)^((2
*I)*b*d)]*Hypergeometric2F1[1/2, -1/4*(2*I + (2*I)*m + b*d*n)/(b*d*n), -1/
4*(2*I + (2*I)*m - 3*b*d*n)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)]*Sq
rt[Sin[d*(a + b*Log[c*x^n])]]))/Sqrt[Sin[d*(a + b*Log[c*x^n])]])/(b*d*n*(-
2*I - (2*I)*m + 3*b*d*n)*Sqrt[((-I)*(-1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d
)))/(E^(I*a*d)*(c*x^n)^(I*b*d))]*(b*d*n*Cos[d*(a - b*n*Log[x] + b*Log[c*x^
n])] + 2*(1 + m)*Sin[d*(a - b*n*Log[x] + b*Log[c*x^n])]))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4996, 4994, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))}\,dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int\frac{(cx^n)^{\frac{m+1}{n}-1}}{\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))}d(cx^n)}{en}$$

$$\Big\downarrow \text{4994}$$

$$\frac{(ex)^{m+1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{3/2}(cx^n)^{-\frac{m+1}{n}-\frac{3}{2}ibd}\int\frac{(cx^n)^{\frac{3ibd}{2}+\frac{m+1}{n}-1}}{\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{3/2}}d(cx^n)}{en\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))}$$

$$\Big\downarrow \text{888}$$

$$\frac{2(ex)^{m+1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i(m+1)}{bdn}\right),-\frac{2im-7bdn+2i}{4bdn},e^{2iad}(cx^n)^{2ibd}\right)}{e(3ibdn+2m+2)\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))}$$

input
```
Int[(e*x)^m/Sin[d*(a + b*Log[c*x^n])]^(3/2),x]
```

output
```
(2*(e*x)^(1 + m)*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^(3/2)*Hypergeomet
ric2F1[3/2, (3 - ((2*I)*(1 + m))/(b*d*n))/4, -1/4*(2*I + (2*I)*m - 7*b*d*n
)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(e*(2 + 2*m + (3*I)*b*d*n)*
Sin[d*(a + b*Log[c*x^n])]^(3/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4994
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Simp[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{(ex)^m}{\sin\left(d\left(a+b\ln\left(c\,x^n\right)\right)\right)^{\frac{3}{2}}}dx$$

input
```
int((e*x)^m/sin(d*(a+b*ln(c*x^n)))^(3/2),x)
```

output
```
int((e*x)^m/sin(d*(a+b*ln(c*x^n)))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ex)^m}{\sin^{\frac{3}{2}}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^m/sin(d*(a+b*log(c*x^n)))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m/sin(d*(a+b*ln(c*x**n)))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m}{\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))}\,dx = \int \frac{(ex)^m}{\sin((b\log(cx^n)+a)d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m/sin(d*(a+b*log(c*x^n)))^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/sin((b*log(c*x^n) + a)*d)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^m/sin(d*(a+b*log(c*x^n)))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))}\,dx = \int \frac{(e\,x)^m}{\sin\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)^{3/2}}\,dx$$

input
```
int((e*x)^m/sin(d*(a + b*log(c*x^n)))^(3/2),x)
```

output
```
int((e*x)^m/sin(d*(a + b*log(c*x^n)))^(3/2), x)
```

Reduce [F]

$$\int \frac{(ex)^m}{\sin^{\frac{3}{2}}(d(a+b\log(cx^n)))}\,dx = e^m\left(\int \frac{x^m\sqrt{\sin(\log(x^n c)\,bd+ad)}}{\sin(\log(x^n c)\,bd+ad)^2}dx\right)$$

input
```
int((e*x)^m/sin(d*(a+b*log(c*x^n)))^(3/2),x)
```

output
```
e**m*int((x**m*sqrt(sin(log(x**n*c)*b*d + a*d)))/sin(log(x**n*c)*b*d + a*d
)**2,x)
```

3.78 $\int \frac{(ex)^m}{\sin^{\frac{5}{2}}(d(a+b\log(cx^n)))}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 150

$$\int \frac{(ex)^m}{\sin^{\frac{5}{2}}(d(a+b\log(cx^n)))}\,dx = \frac{2(ex)^{1+m}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{2i+2im-5bdn}{4bdn},-\frac{2i+2im-9bdn}{4bdn},e^{2iad}(cx^n)^{2ibd}\right)}{e(2+2m+5ibdn)\sin^{\frac{5}{2}}(d(a+b\log(cx^n)))}$$

output
```
2*(e*x)^(1+m)*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^(5/2)*hypergeom([5/2, -1/
4*(2*I+2*I*m-5*b*d*n)/b/d/n],[-1/4*(2*I+2*I*m-9*b*d*n)/b/d/n],exp(2*I*a*d)
*(c*x^n)^(2*I*b*d))/e/(2+2*m+5*I*b*d*n)/sin(d*(a+b*ln(c*x^n)))^(5/2)
```

Mathematica [A] (warning: unable to verify)

Time = 2.10 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.43

$$\int \frac{(ex)^m}{\sin^{\frac{5}{2}}(d(a+b\log(cx^n)))}\,dx = \frac{2x(ex)^m\left(-2-2m-bdn\cot(d(a-bn\log(x)+b\log(cx^n)))-\left(-1+e^{2id(a+b\log(cx^n))}\right)(2+2m-ibdn)\right.}{}$$

input
```
Integrate[(e*x)^m/Sin[d*(a + b*Log[c*x^n])]^(5/2),x]
```

output
```
(2*x*(e*x)^m*(-2 - 2*m - b*d*n*Cot[d*(a - b*n*Log[x] + b*Log[c*x^n])] - (-
1 + E^((2*I)*d*(a + b*Log[c*x^n])))*(2 + 2*m - I*b*d*n)*Hypergeometric2F1[
1, -1/4*(2*I + (2*I)*m - 3*b*d*n)/(b*d*n), -1/4*(2*I + (2*I)*m - 5*b*d*n)/
(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + b*d*n*Csc[d*(a + b*Log[c*x^n])]
*Csc[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sin[b*d*n*Log[x]]))/(3*b^2*d^2*n^2
*Sqrt[Sin[d*(a + b*Log[c*x^n])]])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4996, 4994, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^m}{\sin^{\frac{5}{2}}(d(a+b\log(cx^n)))}dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int \frac{(cx^n)^{\frac{m+1}{n}-1}}{\sin^{\frac{5}{2}}(d(a+b\log(cx^n)))}d(cx^n)}{en}$$

$$\Big\downarrow \text{4994}$$

$$\frac{(ex)^{m+1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{5/2}(cx^n)^{-\frac{m+1}{n}-\frac{5}{2}ibd}\int \frac{(cx^n)^{\frac{5ibd}{2}+\frac{m+1}{n}-1}}{\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{5/2}}d(cx^n)}{en\sin^{\frac{5}{2}}(d(a+b\log(cx^n)))}$$

$$\Big\downarrow \text{888}$$

$$\frac{2(ex)^{m+1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i(m+1)}{bdn}\right),-\frac{2im-9bdn+2i}{4bdn},e^{2iad}(cx^n)^{2ibd}\right)}{e(5ibdn+2m+2)\sin^{\frac{5}{2}}(d(a+b\log(cx^n)))}$$

input
```
Int[(e*x)^m/Sin[d*(a + b*Log[c*x^n])]^(5/2),x]
```

output
```
(2*(e*x)^(1 + m)*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^(5/2)*Hypergeomet
ric2F1[5/2, (5 - ((2*I)*(1 + m))/(b*d*n))/4, -1/4*(2*I + (2*I)*m - 9*b*d*n
)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(e*(2 + 2*m + (5*I)*b*d*n)*
Sin[d*(a + b*Log[c*x^n])]^(5/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4994
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Simp[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{(ex)^m}{\sin \left(d \left(a+b \ln \left(c\, x^n\right)\right)\right)^{\frac{5}{2}}}dx$$

input
```
int((e*x)^m/sin(d*(a+b*ln(c*x^n)))^(5/2),x)
```

output
```
int((e*x)^m/sin(d*(a+b*ln(c*x^n)))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ex)^m}{\sin^{\frac{5}{2}}(d(a+b\log(cx^n)))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^m/sin(d*(a+b*log(c*x^n)))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{\sin^{\frac{5}{2}}(d(a+b\log(cx^n)))}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m/sin(d*(a+b*ln(c*x**n)))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m}{\sin^{\frac{5}{2}}(d(a+b\log(cx^n)))}\,dx = \int \frac{(ex)^m}{\sin\left((b\log(cx^n)+a)d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^m/sin(d*(a+b*log(c*x^n)))^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x)^m/sin((b*log(c*x^n) + a)*d)^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{\sin^{\frac{5}{2}}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}\,dx=\text{Timed out}$$

input
```
integrate((e*x)^m/sin(d*(a+b*log(c*x^n)))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m}{\sin^{\frac{5}{2}}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}\,dx=\int \frac{(e\,x)^m}{\sin\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)^{5/2}}\,dx$$

input
```
int((e*x)^m/sin(d*(a + b*log(c*x^n)))^(5/2),x)
```

output
```
int((e*x)^m/sin(d*(a + b*log(c*x^n)))^(5/2), x)
```

Reduce [F]

$$\int \frac{(ex)^m}{\sin^{\frac{5}{2}}\left(d\left(a+b\log\left(cx^n\right)\right)\right)}\,dx=e^m\left(\int \frac{x^m\sqrt{\sin\left(\log\left(x^n c\right)bd+ad\right)}}{\sin\left(\log\left(x^n c\right)bd+ad\right)^3}dx\right)$$

input
```
int((e*x)^m/sin(d*(a+b*log(c*x^n)))^(5/2),x)
```

output
```
e**m*int((x**m*sqrt(sin(log(x**n*c)*b*d + a*d)))/sin(log(x**n*c)*b*d + a*d
)**3,x)
```

3.79 $\int (ex)^m \sin^p(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 144

$$\int (ex)^m \sin^p(d(a+b\log(cx^n)))\,dx = \frac{(ex)^{1+m}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-\frac{i+im+bdnp}{2bdn},\frac{1}{2}\left(2-\frac{i(1+m)}{bdn}-p\right),e^{2iad}(cx^n)^{2ibd}\right)}{e(1+m-ibdnp)}$$

output
```
(e*x)^(1+m)*hypergeom([-p, -1/2*(I+I*m+b*d*n*p)/b/d/n],[1-1/2*I*(1+m)/b/d/
n-1/2*p],exp(2*I*a*d)*(c*x^n)^(2*I*b*d))*sin(d*(a+b*ln(c*x^n)))^p/e/(1+m-I
*b*d*n*p)/((1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.21

$$\int (ex)^m \sin^p(d(a+b\log(cx^n)))\,dx = \frac{x(ex)^m\left(2-2e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(-ie^{-iad}(cx^n)^{-ibd}\left(-1+e^{2iad}(cx^n)^{2ibd}\right)\right)^p\text{Hypergeometric2F1}\left(-p,-\frac{i\cdots}{}\right.}{1+m-ibdnp}$$

input
```
Integrate[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^p,x]
```

output
```
(x*(e*x)^m*(((-I)*(-1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(E^(I*a*d)*(c*
x^n)^(I*b*d)))^p*Hypergeometric2F1[-p, -1/2*(I + I*m + b*d*n*p)/(b*d*n), 1
 - ((I/2)*(1 + m))/(b*d*n) - p/2, E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/((1
+ m - I*b*d*n*p)*(2 - 2*E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4996, 4994, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \sin^p(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\sin^p(d(a+b\log(cx^n)))\,d(cx^n)}{en}$$

$$\Big\downarrow \text{4994}$$

$$\frac{(ex)^{m+1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{-p}(cx^n)^{-\frac{m+1}{n}+ibdp}\sin^p(d(a+b\log(cx^n)))\int (cx^n)^{\frac{m+1}{n}-ibdp-1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{p}}{en}$$

$$\Big\downarrow \text{888}$$

$$\frac{(ex)^{m+1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{-p}(cx^n)^{\frac{-ibdnp+m+1}{n}+ibdp-\frac{m+1}{n}}\text{Hypergeometric2F1}\left(-p,-\frac{im+bdnp+i}{2bdn},\frac{1}{2}\left(-\frac{i(m+1)}{bdn}-p\right.\right.}{e(-ibdnp+m+1)}$$

input
```
Int[(e*x)^m*Sin[d*(a + b*Log[c*x^n])]^p,x]
```

output
```
((e*x)^(1 + m)*(c*x^n)^(-((1 + m)/n) + I*b*d*p + (1 + m - I*b*d*n*p)/n)*Hy
pergeometric2F1[-p, -1/2*(I + I*m + b*d*n*p)/(b*d*n), (2 - (I*(1 + m))/(b*
d*n) - p)/2, E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)]*Sin[d*(a + b*Log[c*x^n])]^
p)/(e*(1 + m - I*b*d*n*p)*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4994
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Simp[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int (ex)^m \sin (d(a+b\ln (c\,x^n)))^p dx$$

input
```
int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^p,x)
```

output
```
int((e*x)^m*sin(d*(a+b*ln(c*x^n)))^p,x)
```

Fricas [F]

$$\int (ex)^m \sin^p(d(a+b\log(cx^n)))\,dx=\int (ex)^m \sin((b\log(cx^n)+a)d)^p\,dx$$

input
```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^m*sin(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \sin^p(d(a+b\log(cx^n)))\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m*sin(d*(a+b*ln(c*x**n)))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m \sin^p(d(a+b\log(cx^n)))\,dx = \int (ex)^m \sin((b\log(cx^n)+a)d)^p\,dx$$

input
```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*sin((b*log(c*x^n) + a)*d)^p, x)
```

Giac [F]

$$\int (ex)^m \sin^p(d(a+b\log(cx^n)))\,dx = \int (ex)^m \sin((b\log(cx^n)+a)d)^p\,dx$$

input
```
integrate((e*x)^m*sin(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^m*sin((b*log(c*x^n) + a)*d)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \sin^p(d(a+b\log(cx^n)))\, dx = \int \sin(d(a+b\ln(c x^n)))^p (e x)^m\, dx$$

input
```
int(sin(d*(a + b*log(c*x^n)))^p*(e*x)^m,x)
```

output
```
int(sin(d*(a + b*log(c*x^n)))^p*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \sin^p(d(a+b\log(cx^n)))\, dx$$
$$= \frac{e^m \left(x^m \sin(\log(x^n c)\, bd + ad)^p x - \left(\int \frac{x^m \sin(\log(x^n c)bd+ad)^p \cos(\log(x^n c)bd+ad)}{\sin(\log(x^n c)bd+ad)} dx\right) bdnp\right)}{m+1}$$

input
```
int((e*x)^m*sin(d*(a+b*log(c*x^n)))^p,x)
```

output
```
(e**m*(x**m*sin(log(x**n*c)*b*d + a*d)**p*x - int((x**m*sin(log(x**n*c)*b*
d + a*d)**p*cos(log(x**n*c)*b*d + a*d))/sin(log(x**n*c)*b*d + a*d),x)*b*d*
n*p))/(m + 1)
```

3.80 $\int x^2 \sin^p(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 114

$$\int x^2 \sin^p(a+b\log(cx^n))\,dx$$
$$= \frac{x^3\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-\frac{3i+bnp}{2bn},\frac{1}{2}\left(2-\frac{3i}{bn}-p\right),e^{2ia}(cx^n)^{2ib}\right)\sin^p(a+b\log(cx}{3-ibnp}$$

output
```
x^3*hypergeom([-p, -1/2*(3*I+b*n*p)/b/n],[1-3/2*I/b/n-1/2*p],exp(2*I*a)*(c
*x^n)^(2*I*b))*sin(a+b*ln(c*x^n))^p/(3-I*b*n*p)/((1-exp(2*I*a)*(c*x^n)^(2*
I*b))^p)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.30

$$\int x^2 \sin^p(a+b\log(cx^n))\,dx$$
$$= \frac{ix^3\left(2-2e^{2ia}(cx^n)^{2ib}\right)^{-p}\left(-ie^{-ia}(cx^n)^{-ib}\left(-1+e^{2ia}(cx^n)^{2ib}\right)\right)^p\text{Hypergeometric2F1}\left(-p,-\frac{3i+bnp}{2bn},1-\right.}{3i+bnp}$$

input
```
Integrate[x^2*Sin[a + b*Log[c*x^n]]^p,x]
```

output
```
(I*x^3*(((-I)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))/(E^(I*a)*(c*x^n)^(I*b)
))^p*Hypergeometric2F1[-p, -1/2*(3*I + b*n*p)/(b*n), 1 - ((3*I)/2)/(b*n) -
 p/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((3*I + b*n*p)*(2 - 2*E^((2*I)*a)*(c
*x^n)^((2*I)*b))^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4996, 4994, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin^p(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{x^3(cx^n)^{-3/n}\int(cx^n)^{\frac{3}{n}-1}\sin^p(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{4994}$$

$$\frac{x^3(cx^n)^{-\frac{3}{n}+ibp}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\sin^p(a+b\log(cx^n))\int(cx^n)^{-ibp+\frac{3}{n}-1}\left(1-e^{2ia}(cx^n)^{2ib}\right)^p d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^3\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-\frac{bnp+3i}{2bn},\frac{1}{2}\left(-p-\frac{3i}{bn}+2\right),e^{2ia}(cx^n)^{2ib}\right)\sin^p(a+b\log(cx^n))}{3-ibnp}$$

input
```
Int[x^2*Sin[a + b*Log[c*x^n]]^p,x]
```

output
```
(x^3*Hypergeometric2F1[-p, -1/2*(3*I + b*n*p)/(b*n), (2 - (3*I)/(b*n) - p)
/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sin[a + b*Log[c*x^n]]^p)/((3 - I*b*n*p)
*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4994
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Simp[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^2 \sin\left(a + b\ln\left(c\,x^n\right)\right)^p dx$$

input
```
int(x^2*sin(a+b*ln(c*x^n))^p,x)
```

output
```
int(x^2*sin(a+b*ln(c*x^n))^p,x)
```

Fricas [F]

$$\int x^2 \sin^p\left(a + b\log\left(cx^n\right)\right)\,dx = \int x^2 \sin\left(b\log\left(cx^n\right) + a\right)^p\,dx$$

input
```
integrate(x^2*sin(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

output
```
integral(x^2*sin(b*log(c*x^n) + a)^p, x)
```

Sympy [F]

$$\int x^2 \sin^p\left(a + b\log\left(cx^n\right)\right)\, dx = \int x^2 \sin^p\left(a + b\log\left(cx^n\right)\right) dx$$

input
```
integrate(x**2*sin(a+b*ln(c*x**n))**p,x)
```

output
```
Integral(x**2*sin(a + b*log(c*x**n))**p, x)
```

Maxima [F]

$$\int x^2 \sin^p\left(a + b\log\left(cx^n\right)\right)\, dx = \int x^2 \sin\left(b\log\left(cx^n\right) + a\right)^p\, dx$$

input
```
integrate(x^2*sin(a+b*log(c*x^n))^p,x, algorithm="maxima")
```

output
```
integrate(x^2*sin(b*log(c*x^n) + a)^p, x)
```

Giac [F]

$$\int x^2 \sin^p\left(a + b\log\left(cx^n\right)\right)\, dx = \int x^2 \sin\left(b\log\left(cx^n\right) + a\right)^p\, dx$$

input
```
integrate(x^2*sin(a+b*log(c*x^n))^p,x, algorithm="giac")
```

output
```
integrate(x^2*sin(b*log(c*x^n) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sin^p(a+b\log(cx^n))\,dx = \int x^2 \sin(a+b\ln(c\,x^n))^p\,dx$$

input
```
int(x^2*sin(a + b*log(c*x^n))^p,x)
```

output
```
int(x^2*sin(a + b*log(c*x^n))^p, x)
```

Reduce [F]

$$\int x^2 \sin^p(a+b\log(cx^n))\,dx = \frac{\sin(\log(x^n c)\,b+a)^p x^3}{3} - \frac{\left(\int \frac{\sin(\log(x^n c)b+a)^p \cos(\log(x^n c)b+a)x^2}{\sin(\log(x^n c)b+a)}dx\right) bnp}{3}$$

input
```
int(x^2*sin(a+b*log(c*x^n))^p,x)
```

output
```
(sin(log(x**n*c)*b + a)**p*x**3 - int((sin(log(x**n*c)*b + a)**p*cos(log(x
**n*c)*b + a)*x**2)/sin(log(x**n*c)*b + a),x)*b*n*p)/3
```

3.81 $\int x \sin^p(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 114

$$\int x \sin^p(a+b\log(cx^n))\,dx = \frac{x^2\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-\frac{2i}{bn}-p\right),-p,\frac{1}{2}\left(2-\frac{2i}{bn}-p\right),e^{2ia}(cx^n)^{2ib}\right)\sin^p(a+b\log(cx^n))}{2-ibnp}$$

output
```
x^2*hypergeom([-p, -I/b/n-1/2*p],[1-I/b/n-1/2*p],exp(2*I*a)*(c*x^n)^(2*I*b
))*sin(a+b*ln(c*x^n))^p/(2-I*b*n*p)/((1-exp(2*I*a)*(c*x^n)^(2*I*b))^p)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.26

$$\int x \sin^p(a+b\log(cx^n))\,dx = \frac{ix^2\left(2-2e^{2ia}(cx^n)^{2ib}\right)^{-p}\left(-ie^{-ia}(cx^n)^{-ib}\left(-1+e^{2ia}(cx^n)^{2ib}\right)\right)^p\text{Hypergeometric2F1}\left(-\frac{i}{bn}-\frac{p}{2},-p,1-\right.}{2i+bnp}$$

input
```
Integrate[x*Sin[a + b*Log[c*x^n]]^p,x]
```

output
```
(I*x^2*(((-I)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))/(E^(I*a)*(c*x^n)^(I*b)
))^p*Hypergeometric2F1[(-I)/(b*n) - p/2, -p, 1 - I/(b*n) - p/2, E^((2*I)*a
)*(c*x^n)^((2*I)*b)])/((2*I + b*n*p)*(2 - 2*E^((2*I)*a)*(c*x^n)^((2*I)*b))
^p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4996, 4994, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sin^p(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{x^2(cx^n)^{-2/n}\int (cx^n)^{\frac{2}{n}-1}\sin^p(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{4994}$$

$$\frac{x^2(cx^n)^{-\frac{2}{n}+ibp}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\sin^p(a+b\log(cx^n))\int (cx^n)^{-ibp+\frac{2}{n}-1}\left(1-e^{2ia}(cx^n)^{2ib}\right)^p d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-p-\frac{2i}{bn}\right),-p,\frac{1}{2}\left(-p-\frac{2i}{bn}+2\right),e^{2ia}(cx^n)^{2ib}\right)\sin^p(a+b\log(cx^n))}{2-ibnp}$$

input
```
Int[x*Sin[a + b*Log[c*x^n]]^p,x]
```

output
```
(x^2*Hypergeometric2F1[((-2*I)/(b*n) - p)/2, -p, (2 - (2*I)/(b*n) - p)/2,
E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sin[a + b*Log[c*x^n]]^p)/((2 - I*b*n*p)*(1
- E^((2*I)*a)*(c*x^n)^((2*I)*b))^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4994
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Simp[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x\sin\left(a+b\ln\left(c\,x^{n}\right)\right)^{p}dx$$

input
```
int(x*sin(a+b*ln(c*x^n))^p,x)
```

output
```
int(x*sin(a+b*ln(c*x^n))^p,x)
```

Fricas [F]

$$\int x\sin^p\left(a+b\log\left(cx^n\right)\right)\,dx=\int x\sin\left(b\log\left(cx^n\right)+a\right)^p\,dx$$

input
```
integrate(x*sin(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

output
```
integral(x*sin(b*log(c*x^n) + a)^p, x)
```

Sympy [F]

$$\int x\sin^p\left(a+b\log\left(cx^n\right)\right)\,dx=\int x\sin^p\left(a+b\log\left(cx^n\right)\right)dx$$

input
```
integrate(x*sin(a+b*ln(c*x**n))**p,x)
```

output
```
Integral(x*sin(a + b*log(c*x**n))**p, x)
```

Maxima [F]

$$\int x\sin^p\left(a+b\log\left(cx^n\right)\right)\,dx=\int x\sin\left(b\log\left(cx^n\right)+a\right)^p\,dx$$

input
```
integrate(x*sin(a+b*log(c*x^n))^p,x, algorithm="maxima")
```

output
```
integrate(x*sin(b*log(c*x^n) + a)^p, x)
```

Giac [F]

$$\int x\sin^p\left(a+b\log\left(cx^n\right)\right)\,dx=\int x\sin\left(b\log\left(cx^n\right)+a\right)^p\,dx$$

input
```
integrate(x*sin(a+b*log(c*x^n))^p,x, algorithm="giac")
```

output
```
integrate(x*sin(b*log(c*x^n) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int x \sin^p(a + b\log(cx^n)) \, dx = \int x \sin(a + b \ln(c x^n))^p \, dx$$

input
```
int(x*sin(a + b*log(c*x^n))^p,x)
```

output
```
int(x*sin(a + b*log(c*x^n))^p, x)
```

Reduce [F]

$$\int x \sin^p(a + b\log(cx^n)) \, dx = \frac{\sin(\log(x^n c) b + a)^p x^2}{2} - \frac{\left(\int \frac{\sin(\log(x^n c)b+a)^p \cos(\log(x^n c)b+a)x}{\sin(\log(x^n c)b+a)} dx\right) bnp}{2}$$

input
```
int(x*sin(a+b*log(c*x^n))^p,x)
```

output
```
(sin(log(x**n*c)*b + a)**p*x**2 - int((sin(log(x**n*c)*b + a)**p*cos(log(x
**n*c)*b + a)*x)/sin(log(x**n*c)*b + a),x)*b*n*p)/2
```

3.82 $\int \sin^p(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 112

$$\int \sin^p(a+b\log(cx^n))\,dx = \frac{x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-\frac{i+bnp}{2bn},\frac{1}{2}\left(2-\frac{i}{bn}-p\right),e^{2ia}(cx^n)^{2ib}\right)\sin^p(a+b\log(cx^n}{1-ibnp}$$

output
```
x*hypergeom([-p, -1/2*(I+b*n*p)/b/n],[1-1/2*I/b/n-1/2*p],exp(2*I*a)*(c*x^n
)^(2*I*b))*sin(a+b*ln(c*x^n))^p/(1-I*b*n*p)/((1-exp(2*I*a)*(c*x^n)^(2*I*b)
)^p)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.30

$$\int \sin^p(a+b\log(cx^n))\,dx = \frac{ix\left(2-2e^{2ia}(cx^n)^{2ib}\right)^{-p}\left(-ie^{-ia}(cx^n)^{-ib}\left(-1+e^{2ia}(cx^n)^{2ib}\right)\right)^p\text{Hypergeometric2F1}\left(-p,-\frac{i+bnp}{2bn},1-\frac{i}{2b}\right.}{i+bnp}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]^p,x]
```

output
```
(I*x*(((-I)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))/(E^(I*a)*(c*x^n)^(I*b)))
^p*Hypergeometric2F1[-p, -1/2*(I + b*n*p)/(b*n), 1 - (I/2)/(b*n) - p/2, E^
((2*I)*a)*(c*x^n)^((2*I)*b)])/((I + b*n*p)*(2 - 2*E^((2*I)*a)*(c*x^n)^((2*
I)*b))^p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4986, 4994, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^p(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{4986}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\sin^p(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{4994}$$

$$\frac{x(cx^n)^{-\frac{1}{n}+ibp}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\sin^p(a+b\log(cx^n))\int (cx^n)^{-ibp+\frac{1}{n}-1}\left(1-e^{2ia}(cx^n)^{2ib}\right)^p d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-\frac{bnp+i}{2bn},\frac{1}{2}\left(-p-\frac{i}{bn}+2\right),e^{2ia}(cx^n)^{2ib}\right)\sin^p(a+b\log(cx^n))}{1-ibnp}$$

input
```
Int[Sin[a + b*Log[c*x^n]]^p,x]
```

output
```
(x*Hypergeometric2F1[-p, -1/2*(I + b*n*p)/(b*n), (2 - I/(b*n) - p)/2, E^((
2*I)*a)*(c*x^n)^((2*I)*b)]*Sin[a + b*Log[c*x^n]]^p)/((1 - I*b*n*p)*(1 - E^
((2*I)*a)*(c*x^n)^((2*I)*b))^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4986
```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sin[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 4994
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Simp[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Maple [F]

$$\int \sin \left(a +b \ln \left(c \,x^{n}\right)\right)^{p}d x$$

input
```
int(sin(a+b*ln(c*x^n))^p,x)
```

output
```
int(sin(a+b*ln(c*x^n))^p,x)
```

Fricas [F]

$$\int \sin^p\left(a+b\log\left(cx^n\right)\right)\,dx=\int \sin\left(b\log\left(cx^n\right)+a\right)^p\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

output
```
integral(sin(b*log(c*x^n) + a)^p, x)
```

Sympy [F]

$$\int \sin^p(a+b\log(cx^n))\,dx = \int \sin^p(a+b\log(cx^n))\,dx$$

input
```
integrate(sin(a+b*ln(c*x**n))**p,x)
```

output
```
Integral(sin(a + b*log(c*x**n))**p, x)
```

Maxima [F]

$$\int \sin^p(a+b\log(cx^n))\,dx = \int \sin(b\log(cx^n)+a)^p\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^p,x, algorithm="maxima")
```

output
```
integrate(sin(b*log(c*x^n) + a)^p, x)
```

Giac [F]

$$\int \sin^p(a+b\log(cx^n))\,dx = \int \sin(b\log(cx^n)+a)^p\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^p,x, algorithm="giac")
```

output
```
integrate(sin(b*log(c*x^n) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^p\left(a+b\log\left(cx^n\right)\right)\,dx=\int \sin\left(a+b\,\ln\left(c\,x^n\right)\right)^p\,dx$$

input
```
int(sin(a + b*log(c*x^n))^p,x)
```

output
```
int(sin(a + b*log(c*x^n))^p, x)
```

Reduce [F]

$$\int \sin^p\left(a+b\log\left(cx^n\right)\right)\,dx=\sin\left(\log(x^nc)\,b+a\right)^p x-\left(\int \frac{\sin\left(\log(x^nc)\,b+a\right)^p\cos\left(\log(x^nc)\,b+a\right)}{\sin\left(\log\left(x^nc\right)b+a\right)}dx\right)bnp$$

input
```
int(sin(a+b*log(c*x^n))^p,x)
```

output
```
sin(log(x**n*c)*b + a)**p*x - int((sin(log(x**n*c)*b + a)**p*cos(log(x**n*
c)*b + a))/sin(log(x**n*c)*b + a),x)*b*n*p
```

3.83 $\int \frac{\sin^p(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 86

$$\int \frac{\sin^p(a+b\log(cx^n))}{x}\,dx = \frac{\cos(a+b\log(cx^n))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+p}{2},\frac{3+p}{2},\sin^2(a+b\log(cx^n))\right)\sin^{1+p}(a+b\log(cx^n))}{bn(1+p)\sqrt{\cos^2(a+b\log(cx^n))}}$$

output
```
cos(a+b*ln(c*x^n))*hypergeom([1/2, 1/2*p+1/2],[3/2+1/2*p],sin(a+b*ln(c*x^n
))^2)*sin(a+b*ln(c*x^n))^(p+1)/b/n/(p+1)/(cos(a+b*ln(c*x^n))^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int \frac{\sin^p(a+b\log(cx^n))}{x}\,dx = \frac{\sqrt{\cos^2(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+p}{2},\frac{3+p}{2},\sin^2(a+b\log(cx^n))\right)\sec(a+b\log(cx^n))\sin^{1+}}{bn(1+p)}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]^p/x,x]
```

output
```
(Sqrt[Cos[a + b*Log[c*x^n]]^2]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2
, Sin[a + b*Log[c*x^n]]^2]*Sec[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]]^(1
+ p))/(b*n*(1 + p))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3039, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^p(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \ 3039$$

$$\frac{\int \sin^p(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \sin(a+b\log(cx^n))^p d\log(cx^n)}{n}$$

$$\Big\downarrow \ 3122$$

$$\frac{\cos(a+b\log(cx^n))\sin^{p+1}(a+b\log(cx^n))\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{p+1}{2},\frac{p+3}{2},\sin^2(a+b\log(cx^n))\right)}{bn(p+1)\sqrt{\cos^2(a+b\log(cx^n))}}$$

input
```
Int[Sin[a + b*Log[c*x^n]]^p/x,x]
```

output
```
(Cos[a + b*Log[c*x^n]]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Sin[a
+ b*Log[c*x^n]]^2]*Sin[a + b*Log[c*x^n]]^(1 + p))/(b*n*(1 + p)*Sqrt[Cos[a
+ b*Log[c*x^n]]^2])
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Maple [F]

$$\int \frac{\sin\left(a+b\ln\left(c\,x^{n}\right)\right)^{p}}{x}dx$$

input
```
int(sin(a+b*ln(c*x^n))^p/x,x)
```

output
```
int(sin(a+b*ln(c*x^n))^p/x,x)
```

Fricas [F]

$$\int \frac{\sin^{p}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx=\int \frac{\sin\left(b\log\left(cx^{n}\right)+a\right)^{p}}{x}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^p/x,x, algorithm="fricas")
```

output
```
integral(sin(b*log(c*x^n) + a)^p/x, x)
```

Sympy [F]

$$\int \frac{\sin^p(a+b\log(cx^n))}{x}\,dx = \int \frac{\sin^p(a+b\log(cx^n))}{x}\,dx$$

input
```
integrate(sin(a+b*ln(c*x**n))**p/x,x)
```

output
```
Integral(sin(a + b*log(c*x**n))**p/x, x)
```

Maxima [F]

$$\int \frac{\sin^p(a+b\log(cx^n))}{x}\,dx = \int \frac{\sin(b\log(cx^n)+a)^p}{x}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^p/x,x, algorithm="maxima")
```

output
```
integrate(sin(b*log(c*x^n) + a)^p/x, x)
```

Giac [F]

$$\int \frac{\sin^p(a+b\log(cx^n))}{x}\,dx = \int \frac{\sin(b\log(cx^n)+a)^p}{x}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^p/x,x, algorithm="giac")
```

output
```
integrate(sin(b*log(c*x^n) + a)^p/x, x)
```

Mupad [B] (verification not implemented)

Time = 20.55 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \frac{\sin^p(a+b\log(cx^n))}{x}\,dx = -\frac{\cos(a+b\ln(c\,x^n))\,\sin(a+b\ln(c\,x^n))^{p+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-\frac{p}{2};\ \frac{3}{2};\ \cos(a+b\ln(c\,x^n))^2\right)}{b\,n\,\left(\sin(a+b\ln(c\,x^n))^2\right)^{\frac{p}{2}+\frac{1}{2}}}$$

input
```
int(sin(a + b*log(c*x^n))^p/x,x)
```

output
```
-(cos(a + b*log(c*x^n))*sin(a + b*log(c*x^n))^(p + 1)*hypergeom([1/2, 1/2
- p/2], 3/2, cos(a + b*log(c*x^n))^2))/(b*n*(sin(a + b*log(c*x^n))^2)^(p/2
 + 1/2))
```

Reduce [F]

$$\int \frac{\sin^p(a+b\log(cx^n))}{x}\,dx = \int \frac{\sin(\log(x^n c)\,b+a)^p}{x}dx$$

input
```
int(sin(a+b*log(c*x^n))^p/x,x)
```

output
```
int(sin(log(x**n*c)*b + a)**p/x,x)
```

3.84 $\int \frac{\sin^p(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 115

$$\int \frac{\sin^p(a+b\log(cx^n))}{x^2}\,dx = -\frac{\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(\frac{i}{bn}-p\right),-p,\frac{1}{2}\left(2+\frac{i}{bn}-p\right),e^{2ia}(cx^n)^{2ib}\right)\sin^p(a+b\log}{(1+ibnp)x}$$

output
```
-hypergeom([-p, 1/2*I/b/n-1/2*p],[1+1/2*I/b/n-1/2*p],exp(2*I*a)*(c*x^n)^(2
*I*b))*sin(a+b*ln(c*x^n))^p/(1+I*b*n*p)/x/((1-exp(2*I*a)*(c*x^n)^(2*I*b))^
p)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.27

$$\int \frac{\sin^p(a+b\log(cx^n))}{x^2}\,dx = \frac{\left(2-2e^{2ia}(cx^n)^{2ib}\right)^{-p}\left(-ie^{-ia}(cx^n)^{-ib}\left(-1+e^{2ia}(cx^n)^{2ib}\right)\right)^p\text{Hypergeometric2F1}\left(\frac{i}{2bn}-\frac{p}{2},-p,1+\frac{i}{2bn}\right.}{(-1-ibnp)x}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]^p/x^2,x]
```

output
```
((((-I)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))/(E^(I*a)*(c*x^n)^(I*b)))^p*H
ypergeometric2F1[(I/2)/(b*n) - p/2, -p, 1 + (I/2)/(b*n) - p/2, E^((2*I)*a)
*(c*x^n)^((2*I)*b)])/((-1 - I*b*n*p)*x*(2 - 2*E^((2*I)*a)*(c*x^n)^((2*I)*b
))^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4996, 4994, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^p(a+b\log(cx^n))}{x^2}dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{(cx^n)^{\frac{1}{n}}\int (cx^n)^{-1-\frac{1}{n}}\sin^p(a+b\log(cx^n))d(cx^n)}{nx}$$

$$\Big\downarrow \text{4994}$$

$$\frac{(cx^n)^{\frac{1}{n}+ibp}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\sin^p(a+b\log(cx^n))\int (cx^n)^{-ibp-\frac{1}{n}-1}\left(1-e^{2ia}(cx^n)^{2ib}\right)^p d(cx^n)}{nx}$$

$$\Big\downarrow \text{888}$$

$$-\frac{\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(\frac{i}{bn}-p\right),-p,\frac{1}{2}\left(-p+\frac{i}{bn}+2\right),e^{2ia}(cx^n)^{2ib}\right)\sin^p(a+b\log(cx^n))}{x(1+ibnp)}$$

input
```
Int[Sin[a + b*Log[c*x^n]]^p/x^2,x]
```

output
```
-((Hypergeometric2F1[(I/(b*n) - p)/2, -p, (2 + I/(b*n) - p)/2, E^((2*I)*a)
*(c*x^n)^((2*I)*b)]*Sin[a + b*Log[c*x^n]]^p)/((1 + I*b*n*p)*x*(1 - E^((2*I
)*a)*(c*x^n)^((2*I)*b))^p))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4994
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Simp[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\sin\left(a+b\ln\left(c\,x^{n}\right)\right)^{p}}{x^{2}}dx$$

input
```
int(sin(a+b*ln(c*x^n))^p/x^2,x)
```

output
```
int(sin(a+b*ln(c*x^n))^p/x^2,x)
```

Fricas [F]

$$\int \frac{\sin^p\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx=\int \frac{\sin\left(b\log\left(cx^n\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^p/x^2,x, algorithm="fricas")
```

output
```
integral(sin(b*log(c*x^n) + a)^p/x^2, x)
```

Sympy [F]

$$\int \frac{\sin^p\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx=\int \frac{\sin^p\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx$$

input
```
integrate(sin(a+b*ln(c*x**n))**p/x**2,x)
```

output
```
Integral(sin(a + b*log(c*x**n))**p/x**2, x)
```

Maxima [F]

$$\int \frac{\sin^p\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx=\int \frac{\sin\left(b\log\left(cx^n\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^p/x^2,x, algorithm="maxima")
```

output
```
integrate(sin(b*log(c*x^n) + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{\sin^p\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx=\int \frac{\sin\left(b\log\left(cx^n\right)+a\right)^p}{x^2}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^p/x^2,x, algorithm="giac")
```

output
```
integrate(sin(b*log(c*x^n) + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^p(a+b\log(cx^n))}{x^2}\,dx=\int \frac{\sin(a+b\,\ln(c\,x^n))^p}{x^2}\,dx$$

input
```
int(sin(a + b*log(c*x^n))^p/x^2,x)
```

output
```
int(sin(a + b*log(c*x^n))^p/x^2, x)
```

Reduce [F]

$$\int \frac{\sin^p(a+b\log(cx^n))}{x^2}\,dx = \frac{-\sin(\log(x^nc)\,b+a)^p+\left(\int \frac{\sin(\log(x^nc)b+a)^p\cos(\log(x^nc)b+a)}{\sin(\log(x^nc)b+a)x^2}dx\right)bnpx}{x}$$

input
```
int(sin(a+b*log(c*x^n))^p/x^2,x)
```

output
```
( - sin(log(x**n*c)*b + a)**p + int((sin(log(x**n*c)*b + a)**p*cos(log(x**
n*c)*b + a))/(sin(log(x**n*c)*b + a)*x**2),x)*b*n*p*x)/x
```

3.85 $\int \frac{\sin^p(a+b\log(cx^n))}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 115

$$\int \frac{\sin^p(a+b\log(cx^n))}{x^3}\,dx = -\frac{\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(\frac{2i}{bn}-p\right),-p,\frac{1}{2}\left(2+\frac{2i}{bn}-p\right),e^{2ia}(cx^n)^{2ib}\right)\sin^p(a+b\log}{(2+ibnp)x^2}$$

output
```
-hypergeom([-p, I/b/n-1/2*p],[1+I/b/n-1/2*p],exp(2*I*a)*(c*x^n)^(2*I*b))*s
in(a+b*ln(c*x^n))^p/(2+I*b*n*p)/x^2/((1-exp(2*I*a)*(c*x^n)^(2*I*b))^p)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.23

$$\int \frac{\sin^p(a+b\log(cx^n))}{x^3}\,dx = \frac{\left(2-2e^{2ia}(cx^n)^{2ib}\right)^{-p}\left(-ie^{-ia}(cx^n)^{-ib}\left(-1+e^{2ia}(cx^n)^{2ib}\right)\right)^p\text{Hypergeometric2F1}\left(\frac{i}{bn}-\frac{p}{2},-p,1+\frac{i}{bn}-\right.}{(-2-ibnp)x^2}$$

input
```
Integrate[Sin[a + b*Log[c*x^n]]^p/x^3,x]
```

output
```
((((-I)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))/(E^(I*a)*(c*x^n)^(I*b)))^p*H
ypergeometric2F1[I/(b*n) - p/2, -p, 1 + I/(b*n) - p/2, E^((2*I)*a)*(c*x^n)
^((2*I)*b)])/((-2 - I*b*n*p)*x^2*(2 - 2*E^((2*I)*a)*(c*x^n)^((2*I)*b))^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4996, 4994, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^p(a+b\log(cx^n))}{x^3}dx$$

$$\Big\downarrow \text{4996}$$

$$\frac{(cx^n)^{2/n}\int (cx^n)^{-1-\frac{2}{n}}\sin^p(a+b\log(cx^n))d(cx^n)}{nx^2}$$

$$\Big\downarrow \text{4994}$$

$$\frac{(cx^n)^{\frac{2}{n}+ibp}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\sin^p(a+b\log(cx^n))\int (cx^n)^{-ibp-\frac{2}{n}-1}\left(1-e^{2ia}(cx^n)^{2ib}\right)^p d(cx^n)}{nx^2}$$

$$\Big\downarrow \text{888}$$

$$-\frac{\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(\frac{2i}{bn}-p\right),-p,\frac{1}{2}\left(-p+\frac{2i}{bn}+2\right),e^{2ia}(cx^n)^{2ib}\right)\sin^p(a+b\log(cx^n))}{x^2(2+ibnp)}$$

input
```
Int[Sin[a + b*Log[c*x^n]]^p/x^3,x]
```

output
```
-((Hypergeometric2F1[((2*I)/(b*n) - p)/2, -p, (2 + (2*I)/(b*n) - p)/2, E^(
(2*I)*a)*(c*x^n)^((2*I)*b)]*Sin[a + b*Log[c*x^n]]^p)/((2 + I*b*n*p)*x^2*(1
 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^p))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4994
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :
> Simp[Sin[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 4996
```
Int[((e_.)*(x_))^(m_.)*Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sin[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\sin \left(a+b\ln \left(c\,x^{n}\right)\right)^{p}}{x^{3}}dx$$

input
```
int(sin(a+b*ln(c*x^n))^p/x^3,x)
```

output
```
int(sin(a+b*ln(c*x^n))^p/x^3,x)
```

Fricas [F]

$$\int \frac{\sin^p\left(a+b\log\left(cx^n\right)\right)}{x^3}\,dx=\int \frac{\sin\left(b\log\left(cx^n\right)+a\right)^p}{x^3}\,dx$$

input
```
integrate(sin(a+b*log(c*x^n))^p/x^3,x, algorithm="fricas")
```

output
```
integral(sin(b*log(c*x^n) + a)^p/x^3, x)
```

Sympy [F]

$$\int \frac{\sin^p(a + b\log(cx^n))}{x^3} dx = \int \frac{\sin^p(a + b\log(cx^n))}{x^3} dx$$

input
```
integrate(sin(a+b*ln(c*x**n))**p/x**3,x)
```

output
```
Integral(sin(a + b*log(c*x**n))**p/x**3, x)
```

Maxima [F]

$$\int \frac{\sin^p(a + b\log(cx^n))}{x^3} dx = \int \frac{\sin(b\log(cx^n) + a)^p}{x^3} dx$$

input
```
integrate(sin(a+b*log(c*x^n))^p/x^3,x, algorithm="maxima")
```

output
```
integrate(sin(b*log(c*x^n) + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{\sin^p(a + b\log(cx^n))}{x^3} dx = \int \frac{\sin(b\log(cx^n) + a)^p}{x^3} dx$$

input
```
integrate(sin(a+b*log(c*x^n))^p/x^3,x, algorithm="giac")
```

output
```
integrate(sin(b*log(c*x^n) + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^p(a+b\log(cx^n))}{x^3}\,dx = \int \frac{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^p}{x^3}\,dx$$

input
```
int(sin(a + b*log(c*x^n))^p/x^3,x)
```

output
```
int(sin(a + b*log(c*x^n))^p/x^3, x)
```

Reduce [F]

$$\int \frac{\sin^p(a+b\log(cx^n))}{x^3}\,dx = \frac{-\sin\left(\log(x^n c)\,b+a\right)^p + \left(\int \frac{\sin(\log(x^n c)b+a)^p\cos(\log(x^n c)b+a)}{\sin(\log(x^n c)b+a)x^3}dx\right)bnp\,x^2}{2x^2}$$

input
```
int(sin(a+b*log(c*x^n))^p/x^3,x)
```

output
```
( - sin(log(x**n*c)*b + a)**p + int((sin(log(x**n*c)*b + a)**p*cos(log(x**
n*c)*b + a))/(sin(log(x**n*c)*b + a)*x**3),x)*b*n*p*x**2)/(2*x**2)
```

3.86 $\int x^2 \cos(a + b\log(cx^n))\, dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int x^2 \cos(a + b\log(cx^n))\, dx = \frac{3x^3 \cos(a + b\log(cx^n))}{9 + b^2n^2} + \frac{bnx^3 \sin(a + b\log(cx^n))}{9 + b^2n^2}$$

output
```
3*x^3*cos(a+b*ln(c*x^n))/(b^2*n^2+9)+b*n*x^3*sin(a+b*ln(c*x^n))/(b^2*n^2+9
)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int x^2 \cos(a + b\log(cx^n))\, dx = \frac{x^3(3\cos(a + b\log(cx^n)) + bn\sin(a + b\log(cx^n)))}{9 + b^2n^2}$$

input
```
Integrate[x^2*Cos[a + b*Log[c*x^n]],x]
```

output
```
(x^3*(3*Cos[a + b*Log[c*x^n]] + b*n*Sin[a + b*Log[c*x^n]]))/(9 + b^2*n^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {4989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cos (a+b \log (c x^n)) \, dx$$

$$\Big\downarrow \text{4989}$$

$$\frac{b n x^3 \sin (a+b \log (c x^n))}{b^2 n^2+9}+\frac{3 x^3 \cos (a+b \log (c x^n))}{b^2 n^2+9}$$

input
```
Int[x^2*Cos[a + b*Log[c*x^n]],x]
```

output
```
(3*x^3*Cos[a + b*Log[c*x^n]])/(9 + b^2*n^2) + (b*n*x^3*Sin[a + b*Log[c*x^n
]])/(9 + b^2*n^2)
```

Defintions of rubi rules used

rule 4989
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

method	result
parallelrisch	$\frac{(\sin(a+b\ln(c\,x^{n}))bn+3\cos(a+b\ln(c\,x^{n})))x^{3}}{b^{2}n^{2}+9}$
parts	$\frac{x^{2}\mathrm{e}^{\frac{\ln(c\,x^{n})}{n}-\frac{\ln(c)}{n}}\cos(a+b\ln(c\,x^{n}))}{n^{2}\left(\frac{1}{n^{2}}+b^{2}\right)}+\frac{x^{2}b\,\mathrm{e}^{\frac{\ln(c\,x^{n})}{n}-\frac{\ln(c)}{n}}\sin(a+b\ln(c\,x^{n}))}{n\left(\frac{1}{n^{2}}+b^{2}\right)}-\frac{2\left(\frac{n\left(\frac{3c^{-\frac{1}{n}}\mathrm{e}^{\frac{\ln(c\,x^{n})-n\ln(x)}{n}}}{b^{2}n^{2}+9}x^{3}-\frac{3c^{-\frac{1}{n}}\mathrm{e}^{\cdots}}{\cdots}\right.}{\cdots}\right.}{\cdots}$

input
```
int(x^2*cos(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

output
```
(sin(a+b*ln(c*x^n))*b*n+3*cos(a+b*ln(c*x^n)))*x^3/(b^2*n^2+9)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int x^2\cos\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{bnx^3\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+3\,x^3\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{b^2n^2+9}$$

input
```
integrate(x^2*cos(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
(b*n*x^3*sin(b*n*log(x) + b*log(c) + a) + 3*x^3*cos(b*n*log(x) + b*log(c)
+ a))/(b^2*n^2 + 9)
```

Sympy [F]

$$\int x^2 \cos(a + b\log(cx^n))\, dx = \begin{cases} \int x^2 \cos\left(a - \frac{3i\log(cx^n)}{n}\right) dx & \text{for } b = -\frac{3i}{n} \\ \int x^2 \cos\left(a + \frac{3i\log(cx^n)}{n}\right) dx & \text{for } b = \frac{3i}{n} \\ \frac{bnx^3 \sin(a+b\log(cx^n))}{b^2n^2+9} + \frac{3x^3\cos(a+b\log(cx^n))}{b^2n^2+9} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*cos(a+b*ln(c*x**n)),x)
```

output
```
Piecewise((Integral(x**2*cos(a - 3*I*log(c*x**n)/n), x), Eq(b, -3*I/n)), (
Integral(x**2*cos(a + 3*I*log(c*x**n)/n), x), Eq(b, 3*I/n)), (b*n*x**3*sin
(a + b*log(c*x**n))/(b**2*n**2 + 9) + 3*x**3*cos(a + b*log(c*x**n))/(b**2*
n**2 + 9), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(56) = 112.

Time = 0.05 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.89

$$\int x^2 \cos(a + b\log(cx^n))\, dx = \frac{((b\cos(b\log(c))\sin(2b\log(c)) - b\cos(2b\log(c))\sin(b\log(c)) + b\sin(b\log(c)))n + 3\cos(2b\log(c))\,c}{}$$

input
```
integrate(x^2*cos(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
1/2*(((b*cos(b*log(c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)) +
 b*sin(b*log(c)))*n + 3*cos(2*b*log(c))*cos(b*log(c)) + 3*sin(2*b*log(c))*
sin(b*log(c)) + 3*cos(b*log(c)))*x^3*cos(b*log(x^n) + a) + ((b*cos(2*b*log
(c))*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c)) + b*cos(b*log(c)))*n
- 3*cos(b*log(c))*sin(2*b*log(c)) + 3*cos(2*b*log(c))*sin(b*log(c)) - 3*si
n(b*log(c)))*x^3*sin(b*log(x^n) + a))/((b^2*cos(b*log(c))^2 + b^2*sin(b*lo
g(c))^2)*n^2 + 9*cos(b*log(c))^2 + 9*sin(b*log(c))^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 923 vs. $2(56) = 112$.

Time = 0.20 (sec) , antiderivative size = 923, normalized size of antiderivative = 16.48

$$\int x^2 \cos(a + b\log(cx^n))\, dx = \text{Too large to display}$$

input
```
integrate(x^2*cos(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
-1/2*(2*b*n*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*
pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a) + 2*b*n*x^
3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2
*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a) + 2*b*n*x^3*e^(1/2*pi*b
*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x
)) + 1/2*b*log(abs(c)))*tan(1/2*a)^2 + 2*b*n*x^3*e^(-1/2*pi*b*n*sgn(x) + 1
/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*lo
g(abs(c)))*tan(1/2*a)^2 - 3*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi
*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1
/2*a)^2 - 3*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2
*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - 2*b*n
*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1
/2*b*n*log(abs(x)) + 1/2*b*log(abs(c))) - 2*b*n*x^3*e^(-1/2*pi*b*n*sgn(x)
+ 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b
*log(abs(c))) - 2*b*n*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn
(c) - 1/2*pi*b)*tan(1/2*a) - 2*b*n*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n
- 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*a) + 3*x^3*e^(1/2*pi*b*n*sgn(x) - 1/
2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log
(abs(c)))^2 + 3*x^3*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) +
 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 + 12*x^3*e^(1...
```

Mupad [B] (verification not implemented)

Time = 20.58 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int x^2 \cos(a + b\log(cx^n))\, dx = \frac{x^3\,(3\,\cos(a + b\,\ln(c\,x^n)) + b\,n\,\sin(a + b\,\ln(c\,x^n)))}{b^2\,n^2 + 9}$$

input
```
int(x^2*cos(a + b*log(c*x^n)),x)
```

output
```
(x^3*(3*cos(a + b*log(c*x^n)) + b*n*sin(a + b*log(c*x^n))))/(b^2*n^2 + 9)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int x^2 \cos(a + b\log(cx^n))\, dx = \frac{x^3(3\cos(\log(x^n c)\,b + a) + \sin(\log(x^n c)\,b + a)\,bn)}{b^2n^2 + 9}$$

input
```
int(x^2*cos(a+b*log(c*x^n)),x)
```

output
```
(x**3*(3*cos(log(x**n*c)*b + a) + sin(log(x**n*c)*b + a)*b*n))/(b**2*n**2
+ 9)
```

3.87 $\int x \cos (a+b \log (c x^n)) \, dx$

Optimal result

Integrand size = 13, antiderivative size = 56

$$\int x \cos (a+b \log (c x^n)) \, dx = \frac{2x^2 \cos (a+b \log (c x^n))}{4+b^2n^2} + \frac{bnx^2 \sin (a+b \log (c x^n))}{4+b^2n^2}$$

output
```
2*x^2*cos(a+b*ln(c*x^n))/(b^2*n^2+4)+b*n*x^2*sin(a+b*ln(c*x^n))/(b^2*n^2+4
)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int x \cos (a+b \log (c x^n)) \, dx = \frac{x^2(2 \cos (a+b \log (c x^n)) + bn \sin (a+b \log (c x^n)))}{4+b^2n^2}$$

input
```
Integrate[x*Cos[a + b*Log[c*x^n]],x]
```

output
```
(x^2*(2*Cos[a + b*Log[c*x^n]] + b*n*Sin[a + b*Log[c*x^n]]))/(4 + b^2*n^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cos(a + b \log(cx^n)) \, dx$$

$$\Big\downarrow \text{ 4989}$$

$$\frac{bnx^2 \sin(a + b \log(cx^n))}{b^2n^2 + 4} + \frac{2x^2 \cos(a + b \log(cx^n))}{b^2n^2 + 4}$$

input
```
Int[x*Cos[a + b*Log[c*x^n]],x]
```

output
```
(2*x^2*Cos[a + b*Log[c*x^n]])/(4 + b^2*n^2) + (b*n*x^2*Sin[a + b*Log[c*x^n
]])/(4 + b^2*n^2)
```

Defintions of rubi rules used

rule 4989
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

method	result
parallelrisch	$\frac{(\sin(a+b\ln(c\,x^n))bn+2\cos(a+b\ln(c\,x^n)))x^2}{b^2n^2+4}$
parts	$\frac{x\,\mathrm{e}^{\frac{\ln(c\,x^n)}{n}-\frac{\ln(c)}{n}}\cos(a+b\ln(c\,x^n))}{n^2\left(\frac{1}{n^2}+b^2\right)}+\frac{xb\,\mathrm{e}^{\frac{\ln(c\,x^n)}{n}-\frac{\ln(c)}{n}}\sin(a+b\ln(c\,x^n))}{n\left(\frac{1}{n^2}+b^2\right)}-\frac{b\left(\frac{bn\,c^{-\frac{1}{n}}\mathrm{e}^{\frac{\ln(c\,x^n)-n\ln(x)}{n}}x^2\tan\left(\frac{a}{2}+\frac{b\ln(c}{2}\right.}{b^2n^2+4}\right.}{}$

input
```
int(x*cos(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

output
```
(sin(a+b*ln(c*x^n))*b*n+2*cos(a+b*ln(c*x^n)))*x^2/(b^2*n^2+4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int x\cos\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{bnx^2\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+2\,x^2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{b^2n^2+4}$$

input
```
integrate(x*cos(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
(b*n*x^2*sin(b*n*log(x) + b*log(c) + a) + 2*x^2*cos(b*n*log(x) + b*log(c)
+ a))/(b^2*n^2 + 4)
```

Sympy [F]

$$\int x\cos\left(a+b\log\left(cx^n\right)\right)\,dx=\begin{cases}\int x\cos\left(a-\frac{2i\log\left(cx^n\right)}{n}\right)dx & \text{for } b=-\frac{2i}{n}\\ \int x\cos\left(a+\frac{2i\log\left(cx^n\right)}{n}\right)dx & \text{for } b=\frac{2i}{n}\\ \frac{bnx^2\sin\left(a+b\log\left(cx^n\right)\right)}{b^2n^2+4}+\frac{2x^2\cos\left(a+b\log\left(cx^n\right)\right)}{b^2n^2+4} & \text{otherwise}\end{cases}$$

input
```
integrate(x*cos(a+b*ln(c*x**n)),x)
```

output
```
Piecewise((Integral(x*cos(a - 2*I*log(c*x**n)/n), x), Eq(b, -2*I/n)), (Int
egral(x*cos(a + 2*I*log(c*x**n)/n), x), Eq(b, 2*I/n)), (b*n*x**2*sin(a + b
*log(c*x**n))/(b**2*n**2 + 4) + 2*x**2*cos(a + b*log(c*x**n))/(b**2*n**2 +
 4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(56) = 112.

Time = 0.05 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.89

$$\int x\cos\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\frac{((b\cos(b\log(c))\sin(2b\log(c))-b\cos(2b\log(c))\sin(b\log(c))+b\sin(b\log(c)))n+2\cos(2b\log(c))\,c}{}$$

input
```
integrate(x*cos(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
1/2*(((b*cos(b*log(c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)) +
 b*sin(b*log(c)))*n + 2*cos(2*b*log(c))*cos(b*log(c)) + 2*sin(2*b*log(c))*
sin(b*log(c)) + 2*cos(b*log(c)))*x^2*cos(b*log(x^n) + a) + ((b*cos(2*b*log
(c))*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c)) + b*cos(b*log(c)))*n
- 2*cos(b*log(c))*sin(2*b*log(c)) + 2*cos(2*b*log(c))*sin(b*log(c)) - 2*si
n(b*log(c)))*x^2*sin(b*log(x^n) + a))/((b^2*cos(b*log(c))^2 + b^2*sin(b*lo
g(c))^2)*n^2 + 4*cos(b*log(c))^2 + 4*sin(b*log(c))^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(56) = 112$.

Time = 0.19 (sec) , antiderivative size = 915, normalized size of antiderivative = 16.34

$$\int x \cos(a + b \log(cx^n)) \, dx = \text{Too large to display}$$

input
```
integrate(x*cos(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
-(b*n*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*
tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a) + b*n*x^2*e^(-1/
2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log
(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a) + b*n*x^2*e^(1/2*pi*b*n*sgn(x)
- 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b
*log(abs(c)))*tan(1/2*a)^2 + b*n*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n -
1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*t
an(1/2*a)^2 - x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/
2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - x^2*
e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b
*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - b*n*x^2*e^(1/2*pi*b*n
*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x))
 + 1/2*b*log(abs(c))) - b*n*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*p
i*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c))) - b*n*
x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/
2*a) - b*n*x^2*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*
pi*b)*tan(1/2*a) + x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c)
 - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 + x^2*e^(-1/2*
pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(a
bs(x)) + 1/2*b*log(abs(c)))^2 + 4*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n...
```

Mupad [B] (verification not implemented)

Time = 19.83 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int x \cos (a + b \log (cx^n)) \, dx = \frac{x^2 \left(2 \cos \left(a + b \ln \left(c\, x^n\right)\right) + b\, n \sin \left(a + b \ln \left(c\, x^n\right)\right)\right)}{b^2\, n^2 + 4}$$

input
```
int(x*cos(a + b*log(c*x^n)),x)
```

output
```
(x^2*(2*cos(a + b*log(c*x^n)) + b*n*sin(a + b*log(c*x^n))))/(b^2*n^2 + 4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int x \cos (a + b \log (cx^n)) \, dx = \frac{x^2 (2 \cos (\log(x^n c)\, b + a) + \sin (\log(x^n c)\, b + a)\, bn)}{b^2 n^2 + 4}$$

input
```
int(x*cos(a+b*log(c*x^n)),x)
```

output
```
(x**2*(2*cos(log(x**n*c)*b + a) + sin(log(x**n*c)*b + a)*b*n))/(b**2*n**2
+ 4)
```

3.88 $\int \cos(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 51

$$\int \cos(a+b\log(cx^n))\,dx = \frac{x\cos(a+b\log(cx^n))}{1+b^2n^2}+\frac{bnx\sin(a+b\log(cx^n))}{1+b^2n^2}$$

output
```
x*cos(a+b*ln(c*x^n))/(b^2*n^2+1)+b*n*x*sin(a+b*ln(c*x^n))/(b^2*n^2+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \cos(a+b\log(cx^n))\,dx = \frac{x(\cos(a+b\log(cx^n))+bn\sin(a+b\log(cx^n)))}{1+b^2n^2}$$

input
```
Integrate[Cos[a + b*Log[c*x^n]],x]
```

output
```
(x*(Cos[a + b*Log[c*x^n]] + b*n*Sin[a + b*Log[c*x^n]]))/(1 + b^2*n^2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4979}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a + b\log(cx^n))\,dx$$

$$\Big\downarrow \text{4979}$$

$$\frac{bnx\sin(a + b\log(cx^n))}{b^2n^2 + 1} + \frac{x\cos(a + b\log(cx^n))}{b^2n^2 + 1}$$

input
```
Int[Cos[a + b*Log[c*x^n]],x]
```

output
```
(x*Cos[a + b*Log[c*x^n]])/(1 + b^2*n^2) + (b*n*x*Sin[a + b*Log[c*x^n]])/(1
 + b^2*n^2)
```

Defintions of rubi rules used

rule 4979
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] + Simp[b*d*n*x*(Sin[d*(a +
 b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

method	result	size
parallelrisch	$\frac{(\sin(a+b\ln(c\,x^n))bn+\cos(a+b\ln(c\,x^n)))x}{b^2n^2+1}$	40
default	$\frac{\frac{e^{\frac{\ln(c\,x^n)}{n}-\frac{\ln(c)}{n}}\cos(a+b\ln(c\,x^n))}{n\left(\frac{1}{n^2}+b^2\right)}+\frac{b\,e^{\frac{\ln(c\,x^n)}{n}-\frac{\ln(c)}{n}}\sin(a+b\ln(c\,x^n))}{\frac{1}{n^2}+b^2}}{n}$	90

input
```
int(cos(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

output
```
(sin(a+b*ln(c*x^n))*b*n+cos(a+b*ln(c*x^n)))*x/(b^2*n^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \cos(a+b\log(cx^n))\,dx = \frac{bnx\sin(bn\log(x)+b\log(c)+a)+x\cos(bn\log(x)+b\log(c)+a)}{b^2n^2+1}$$

input
```
integrate(cos(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
(b*n*x*sin(b*n*log(x) + b*log(c) + a) + x*cos(b*n*log(x) + b*log(c) + a))/
(b^2*n^2 + 1)
```

Sympy [F]

$$\int \cos(a+b\log(cx^n))\,dx = \begin{cases} \int \cos\left(a-\frac{i\log(cx^n)}{n}\right)dx & \text{for}\ b=-\frac{i}{n} \\ \int \cos\left(a+\frac{i\log(cx^n)}{n}\right)dx & \text{for}\ b=\frac{i}{n} \\ \frac{bnx\sin(a+b\log(cx^n))}{b^2n^2+1}+\frac{x\cos(a+b\log(cx^n))}{b^2n^2+1} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(a+b*ln(c*x**n)),x)
```

output
```
Piecewise((Integral(cos(a - I*log(c*x**n)/n), x), Eq(b, -I/n)), (Integral(
cos(a + I*log(c*x**n)/n), x), Eq(b, I/n)), (b*n*x*sin(a + b*log(c*x**n))/(
b**2*n**2 + 1) + x*cos(a + b*log(c*x**n))/(b**2*n**2 + 1), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(51) = 102$.

Time = 0.06 (sec) , antiderivative size = 205, normalized size of antiderivative = 4.02

$$\int \cos\left(a + b\log\left(cx^n\right)\right)\,dx = \frac{\left(\left(b\cos\left(b\log\left(c\right)\right)\sin\left(2\,b\log\left(c\right)\right) - b\cos\left(2\,b\log\left(c\right)\right)\sin\left(b\log\left(c\right)\right) + b\sin\left(b\log\left(c\right)\right)\right)n + \cos\left(2\,b\log\left(c\right)\right)\cos}{}$$

input
```
integrate(cos(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
1/2*(((b*cos(b*log(c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)) +
 b*sin(b*log(c)))*n + cos(2*b*log(c))*cos(b*log(c)) + sin(2*b*log(c))*sin(
b*log(c)) + cos(b*log(c)))*x*cos(b*log(x^n) + a) + ((b*cos(2*b*log(c))*cos
(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c)) + b*cos(b*log(c)))*n - cos(b*
log(c))*sin(2*b*log(c)) + cos(2*b*log(c))*sin(b*log(c)) - sin(b*log(c)))*x
*sin(b*log(x^n) + a))/((b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))^2)*n^2 + c
os(b*log(c))^2 + sin(b*log(c))^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 878 vs. $2(51) = 102$.

Time = 0.15 (sec) , antiderivative size = 878, normalized size of antiderivative = 17.22

$$\int \cos\left(a + b\log\left(cx^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate(cos(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
-1/2*(2*b*n*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi
*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a) + 2*b*n*x*e^
(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n
*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a) + 2*b*n*x*e^(1/2*pi*b*n*sgn
(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1
/2*b*log(abs(c)))*tan(1/2*a)^2 + 2*b*n*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*
n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)
))*tan(1/2*a)^2 - x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) -
1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - x*
e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b
*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/2*a)^2 - 2*b*n*x*e^(1/2*pi*b*n
*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x))
 + 1/2*b*log(abs(c))) - 2*b*n*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*p
i*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c))) - 2*b*
n*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/
2*a) - 2*b*n*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*
pi*b)*tan(1/2*a) + x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) -
 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2 + x*e^(-1/2*pi*b
*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x
)) + 1/2*b*log(abs(c)))^2 + 4*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2...
```

Mupad [B] (verification not implemented)

Time = 20.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \cos(a+b\log(cx^n))\,dx=\frac{x\left(\cos\left(a+b\,\ln\left(c\,x^n\right)\right)+b\,n\,\sin\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}{b^2\,n^2+1}$$

input
```
int(cos(a + b*log(c*x^n)),x)
```

output
```
(x*(cos(a + b*log(c*x^n)) + b*n*sin(a + b*log(c*x^n))))/(b^2*n^2 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \cos \left(a+b\log \left(cx^n\right)\right)\,dx=\frac{x(\cos \left(\log(x^n c)\,b+a\right)+\sin \left(\log(x^n c)\,b+a\right)bn)}{b^2n^2+1}$$

input
```
int(cos(a+b*log(c*x^n)),x)
```

output
```
(x*(cos(log(x**n*c)*b + a) + sin(log(x**n*c)*b + a)*b*n))/(b**2*n**2 + 1)
```

3.89 $\int \frac{\cos(a+b\log(cx^n))}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\cos(a+b\log(cx^n))}{x} dx = \frac{\sin(a+b\log(cx^n))}{bn}$$

output
```
sin(a+b*ln(c*x^n))/b/n
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(18) = 36.

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \frac{\cos(a+b\log(cx^n))}{x} dx = \frac{\cos(b\log(cx^n))\sin(a)}{bn} + \frac{\cos(a)\sin(b\log(cx^n))}{bn}$$

input
```
Integrate[Cos[a + b*Log[c*x^n]]/x,x]
```

output
```
(Cos[b*Log[c*x^n]]*Sin[a])/(b*n) + (Cos[a]*Sin[b*Log[c*x^n]])/(b*n)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3039, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos (a+b\log (cx^n))}{x}dx$$
$$\Big\downarrow \; 3039$$
$$\frac{\int \cos (a+b\log (cx^n))d\log (cx^n)}{n}$$
$$\Big\downarrow \; 3042$$
$$\frac{\int \sin \left(a+b\log (cx^n)+\frac{\pi}{2}\right)d\log (cx^n)}{n}$$
$$\Big\downarrow \; 3117$$
$$\frac{\sin (a+b\log (cx^n))}{bn}$$

input
```
Int[Cos[a + b*Log[c*x^n]]/x,x]
```

output
```
Sin[a + b*Log[c*x^n]]/(b*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\sin(a+b\ln(c\,x^{n}))}{bn}$	19
default	$\frac{\sin(a+b\ln(c\,x^{n}))}{bn}$	19
parallelrisch	$\frac{\sin(a+b\ln(c\,x^{n}))}{bn}$	19

input
```
int(cos(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
```

output
```
sin(a+b*ln(c*x^n))/b/n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\cos(a+b\log(cx^n))}{x}\,dx = \frac{\sin(bn\log(x)+b\log(c)+a)}{bn}$$

input
```
integrate(cos(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

output
```
sin(b*n*log(x) + b*log(c) + a)/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(14) = 28$.

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{\cos(a + b\log(cx^n))}{x}\,dx = \begin{cases} \log(x)\cos(a) & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \log(x)\cos(a + b\log(c)) & \text{for } n = 0 \\ \frac{\sin(a+b\log(cx^n))}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(a+b*ln(c*x**n))/x,x)
```

output
```
Piecewise((log(x)*cos(a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*cos(a
 + b*log(c)), Eq(n, 0)), (sin(a + b*log(c*x**n))/(b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cos(a + b\log(cx^n))}{x}\,dx = \frac{\sin(b\log(cx^n) + a)}{bn}$$

input
```
integrate(cos(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

output
```
sin(b*log(c*x^n) + a)/(b*n)
```

Giac [F]

$$\int \frac{\cos(a + b\log(cx^n))}{x}\,dx = \int \frac{\cos(b\log(cx^n) + a)}{x}\,dx$$

input
```
integrate(cos(a+b*log(c*x^n))/x,x, algorithm="giac")
```

output
```
integrate(cos(b*log(c*x^n) + a)/x, x)
```

Mupad [B] (verification not implemented)

Time = 19.92 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cos(a+b\log(cx^n))}{x}\,dx = \frac{\sin(a+b\,\ln(c\,x^n))}{b\,n}$$

input
```
int(cos(a + b*log(c*x^n))/x,x)
```

output
```
sin(a + b*log(c*x^n))/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cos(a+b\log(cx^n))}{x}\,dx = \frac{\sin(\log(x^n c)\,b+a)}{bn}$$

input
```
int(cos(a+b*log(c*x^n))/x,x)
```

output
```
sin(log(x**n*c)*b + a)/(b*n)
```

3.90 $\int \frac{\cos(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{\cos(a+b\log(cx^n))}{x^2}\,dx = -\frac{\cos(a+b\log(cx^n))}{(1+b^2n^2)\,x}+\frac{bn\sin(a+b\log(cx^n))}{(1+b^2n^2)\,x}$$

output
```
-cos(a+b*ln(c*x^n))/(b^2*n^2+1)/x+b*n*sin(a+b*ln(c*x^n))/(b^2*n^2+1)/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.73

$$\int \frac{\cos(a+b\log(cx^n))}{x^2}\,dx = \frac{-\cos(a+b\log(cx^n))+bn\sin(a+b\log(cx^n))}{x+b^2n^2x}$$

input
```
Integrate[Cos[a + b*Log[c*x^n]]/x^2,x]
```

output
```
(-Cos[a + b*Log[c*x^n]] + b*n*Sin[a + b*Log[c*x^n]])/(x + b^2*n^2*x)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{4989\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+b\log(cx^n))}{x^2}\,dx$$

$$\Big\downarrow \text{4989}$$

$$\frac{bn\sin(a+b\log(cx^n))}{x(b^2n^2+1)} - \frac{\cos(a+b\log(cx^n))}{x(b^2n^2+1)}$$

input
```
Int[Cos[a + b*Log[c*x^n]]/x^2,x]
```

output
```
-(Cos[a + b*Log[c*x^n]]/((1 + b^2*n^2)*x)) + (b*n*Sin[a + b*Log[c*x^n]])/(
(1 + b^2*n^2)*x)
```

Defintions of rubi rules used

rule 4989
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

method	result	size
parallelrisch	$\frac{-\cos(a+b\ln(c\,x^{n}))+\sin(a+b\ln(c\,x^{n}))bn}{x(b^{2}n^{2}+1)}$	44

input
```
int(cos(a+b*ln(c*x^n))/x^2,x,method=_RETURNVERBOSE)
```

output
```
(-cos(a+b*ln(c*x^n))+sin(a+b*ln(c*x^n))*b*n)/x/(b^2*n^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \frac{\cos(a+b\log(cx^n))}{x^2}\,dx = \frac{bn\sin(bn\log(x)+b\log(c)+a)-\cos(bn\log(x)+b\log(c)+a)}{(b^2n^2+1)x}$$

input
```
integrate(cos(a+b*log(c*x^n))/x^2,x, algorithm="fricas")
```

output
```
(b*n*sin(b*n*log(x) + b*log(c) + a) - cos(b*n*log(x) + b*log(c) + a))/((b^
2*n^2 + 1)*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 189, normalized size of antiderivative = 3.38

$$\int \frac{\cos(a+b\log(cx^n))}{x^2}\,dx = \begin{cases} -\frac{\cos\left(a-\frac{i\log(cx^n)}{n}\right)}{2x} + \frac{i\log(cx^n)\sin\left(a-\frac{i\log(cx^n)}{n}\right)}{2nx} + \frac{\log(cx^n)\cos\left(a-\frac{i\log(cx^n)}{n}\right)}{2nx} & \text{for } b=-\frac{i}{n} \\ -\frac{\cos\left(a+\frac{i\log(cx^n)}{n}\right)}{2x} - \frac{i\log(cx^n)\sin\left(a+\frac{i\log(cx^n)}{n}\right)}{2nx} + \frac{\log(cx^n)\cos\left(a+\frac{i\log(cx^n)}{n}\right)}{2nx} & \text{for } b=\frac{i}{n} \\ \frac{bn\sin(a+b\log(cx^n))}{b^2n^2x+x} - \frac{\cos(a+b\log(cx^n))}{b^2n^2x+x} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(a+b*ln(c*x**n))/x**2,x)
```

output
```
Piecewise((-cos(a - I*log(c*x**n)/n)/(2*x) + I*log(c*x**n)*sin(a - I*log(c
*x**n)/n)/(2*n*x) + log(c*x**n)*cos(a - I*log(c*x**n)/n)/(2*n*x), Eq(b, -I
/n)), (-cos(a + I*log(c*x**n)/n)/(2*x) - I*log(c*x**n)*sin(a + I*log(c*x**
n)/n)/(2*n*x) + log(c*x**n)*cos(a + I*log(c*x**n)/n)/(2*n*x), Eq(b, I/n)),
 (b*n*sin(a + b*log(c*x**n))/(b**2*n**2*x + x) - cos(a + b*log(c*x**n))/(b
**2*n**2*x + x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(56) = 112.

Time = 0.05 (sec) , antiderivative size = 208, normalized size of antiderivative = 3.71

$$\int \frac{\cos(a+b\log(cx^n))}{x^2}\,dx = \frac{((b\cos(b\log(c))\sin(2b\log(c)) - b\cos(2b\log(c))\sin(b\log(c)) + b\sin(b\log(c)))n - \cos(2b\log(c))\cos}{}$$

input
```
integrate(cos(a+b*log(c*x^n))/x^2,x, algorithm="maxima")
```

output
```
1/2*(((b*cos(b*log(c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)) +
 b*sin(b*log(c)))*n - cos(2*b*log(c))*cos(b*log(c)) - sin(2*b*log(c))*sin(
b*log(c)) - cos(b*log(c)))*cos(b*log(x^n) + a) + ((b*cos(2*b*log(c))*cos(b
*log(c)) + b*sin(2*b*log(c))*sin(b*log(c)) + b*cos(b*log(c)))*n + cos(b*lo
g(c))*sin(2*b*log(c)) - cos(2*b*log(c))*sin(b*log(c)) + sin(b*log(c)))*sin
(b*log(x^n) + a))/(((b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))^2)*n^2 + cos(
b*log(c))^2 + sin(b*log(c))^2)*x)
```

Giac [F]

$$\int \frac{\cos(a + b\log(cx^n))}{x^2} dx = \int \frac{\cos(b\log(cx^n) + a)}{x^2} dx$$

input
```
integrate(cos(a+b*log(c*x^n))/x^2,x, algorithm="giac")
```

output
```
integrate(cos(b*log(c*x^n) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a + b\log(cx^n))}{x^2} dx = \int \frac{\cos(a + b\ln(c\,x^n))}{x^2} dx$$

input
```
int(cos(a + b*log(c*x^n))/x^2,x)
```

output
```
int(cos(a + b*log(c*x^n))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{\cos\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx = \frac{-\cos\left(\log(x^n c)\,b+a\right)+\sin\left(\log(x^n c)\,b+a\right)bn}{x\,(b^2n^2+1)}$$

input
```
int(cos(a+b*log(c*x^n))/x^2,x)
```

output
```
( - cos(log(x**n*c)*b + a) + sin(log(x**n*c)*b + a)*b*n)/(x*(b**2*n**2 + 1
))
```

3.91 $\int x^2 \cos^2(a + b\log(cx^n))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 97

$$\int x^2\cos^2(a+b\log(cx^n))\,dx=\frac{2b^2n^2x^3}{3(9+4b^2n^2)}+\frac{3x^3\cos^2(a+b\log(cx^n))}{9+4b^2n^2}+\frac{2bnx^3\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{9+4b^2n^2}$$

output
```
2*b^2*n^2*x^3/(12*b^2*n^2+27)+3*x^3*cos(a+b*ln(c*x^n))^2/(4*b^2*n^2+9)+2*b
*n*x^3*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/(4*b^2*n^2+9)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.63

$$\int x^2\cos^2(a+b\log(cx^n))\,dx=\frac{x^3(9+4b^2n^2+9\cos(2(a+b\log(cx^n)))+6bn\sin(2(a+b\log(cx^n))))}{6(9+4b^2n^2)}$$

input
```
Integrate[x^2*Cos[a + b*Log[c*x^n]]^2,x]
```

output
```
(x^3*(9 + 4*b^2*n^2 + 9*Cos[2*(a + b*Log[c*x^n])] + 6*b*n*Sin[2*(a + b*Log
[c*x^n])]))/(6*(9 + 4*b^2*n^2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4991, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cos^2(a + b\log(cx^n))\,dx$$

$$\Big\downarrow \text{ 4991}$$

$$\frac{2b^2n^2\int x^2dx}{4b^2n^2+9} + \frac{3x^3\cos^2(a+b\log(cx^n))}{4b^2n^2+9} + \frac{2bnx^3\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+9}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{3x^3\cos^2(a+b\log(cx^n))}{4b^2n^2+9} + \frac{2bnx^3\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+9} + \frac{2b^2n^2x^3}{3(4b^2n^2+9)}$$

input
```
Int[x^2*Cos[a + b*Log[c*x^n]]^2,x]
```

output
```
(2*b^2*n^2*x^3)/(3*(9 + 4*b^2*n^2)) + (3*x^3*Cos[a + b*Log[c*x^n]]^2)/(9 +
 4*b^2*n^2) + (2*b*n*x^3*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/(9 +
 4*b^2*n^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 4991
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Simp[b*d*n*p*(e*x)^(m + 1)*Sin[d*(a
+ b*Log[c*x^n])]*(Cos[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*
(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2)
)   Int[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$\frac{x^{3}\left(4 b^{2} n^{2}+6 b n \sin\left(2 b \ln\left(c\,x^{n}\right)+2 a\right)+9+9 \cos\left(2 b \ln\left(c\,x^{n}\right)+2 a\right)\right)}{24 b^{2} n^{2}+54}$	61

input
```
int(x^2*cos(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

output
```
x^3*(4*b^2*n^2+6*b*n*sin(2*b*ln(c*x^n)+2*a)+9+9*cos(2*b*ln(c*x^n)+2*a))/(2
4*b^2*n^2+54)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.78

$$\int x^2\cos^2\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\frac{2\,b^2n^2x^3+6\,bnx^3\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+9\,x^3\cos\left(bn\log\left(x\right)+b\,\mathrm{l}\right.}{3\left(4\,b^2n^2+9\right)}$$

input
```
integrate(x^2*cos(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
1/3*(2*b^2*n^2*x^3 + 6*b*n*x^3*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(
x) + b*log(c) + a) + 9*x^3*cos(b*n*log(x) + b*log(c) + a)^2)/(4*b^2*n^2 +
9)
```

Sympy [F]

$$\int x^2 \cos^2(a + b\log(cx^n))\,dx$$
$$= \begin{cases} \int x^2 \cos^2\left(a - \frac{3i\log(cx^n)}{2n}\right) dx \\ \int x^2 \cos^2\left(a + \frac{3i\log(cx^n)}{2n}\right) dx \\ \frac{2b^2n^2x^3\sin^2(a+b\log(cx^n))}{12b^2n^2+27} + \frac{2b^2n^2x^3\cos^2(a+b\log(cx^n))}{12b^2n^2+27} + \frac{6bnx^3\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{12b^2n^2+27} + \frac{9x^3\cos^2(a+b\log(cx^n))}{12b^2n^2+27} \end{cases}$$

input
```
integrate(x**2*cos(a+b*ln(c*x**n))**2,x)
```

output
```
Piecewise((Integral(x**2*cos(a - 3*I*log(c*x**n)/(2*n))**2, x), Eq(b, -3*I
/(2*n))), (Integral(x**2*cos(a + 3*I*log(c*x**n)/(2*n))**2, x), Eq(b, 3*I/
(2*n))), (2*b**2*n**2*x**3*sin(a + b*log(c*x**n))**2/(12*b**2*n**2 + 27) +
 2*b**2*n**2*x**3*cos(a + b*log(c*x**n))**2/(12*b**2*n**2 + 27) + 6*b*n*x*
*3*sin(a + b*log(c*x**n))*cos(a + b*log(c*x**n))/(12*b**2*n**2 + 27) + 9*x
**3*cos(a + b*log(c*x**n))**2/(12*b**2*n**2 + 27), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(95) = 190$.

Time = 0.06 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.10

$$\int x^2 \cos^2(a + b\log(cx^n))\,dx$$
$$= \frac{3\,(2\,(b\cos(2\,b\log(c))\sin(4\,b\log(c)) - b\cos(4\,b\log(c))\sin(2\,b\log(c)) + b\sin(2\,b\log(c)))n + 3\,\cos(4\,b}{}$$

input
```
integrate(x^2*cos(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
1/12*(3*(2*(b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*
log(c)) + b*sin(2*b*log(c)))*n + 3*cos(4*b*log(c))*cos(2*b*log(c)) + 3*sin
(4*b*log(c))*sin(2*b*log(c)) + 3*cos(2*b*log(c)))*x^3*cos(2*b*log(x^n) + 2
*a) + 3*(2*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*
log(c)) + b*cos(2*b*log(c)))*n - 3*cos(2*b*log(c))*sin(4*b*log(c)) + 3*cos
(4*b*log(c))*sin(2*b*log(c)) - 3*sin(2*b*log(c)))*x^3*sin(2*b*log(x^n) + 2
*a) + 2*(4*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + 9*cos(2*b
*log(c))^2 + 9*sin(2*b*log(c))^2)*x^3)/(4*(b^2*cos(2*b*log(c))^2 + b^2*sin
(2*b*log(c))^2)*n^2 + 9*cos(2*b*log(c))^2 + 9*sin(2*b*log(c))^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 830 vs. 2(95) = 190.

Time = 0.31 (sec) , antiderivative size = 830, normalized size of antiderivative = 8.56

$$\int x^2 \cos^2(a + b\log(cx^n)) \, dx = \text{Too large to display}$$

input
```
integrate(x^2*cos(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
1/6*x^3 - 1/4*(4*b*n*x^3*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2
*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 4*b*n*x^3*e^(2*pi*b
*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*log
(abs(c)))*tan(a)^2 + 4*b*n*x^3*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(
a) + 4*b*n*x^3*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a)^2 - 3*x^3*e^(2*
pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b
*log(abs(c)))^2*tan(a)^2 - 4*b*n*x^3*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*
b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c))) - 4*b*n*x^3*e^(2*p
i*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(a) - 3*x^3*tan(b*n*l
og(abs(x)) + b*log(abs(c)))^2*tan(a)^2 - 4*b*n*x^3*tan(b*n*log(abs(x)) + b
*log(abs(c))) + 3*x^3*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi
*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 - 4*b*n*x^3*tan(a) + 12*x^3*e^(
2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) +
 b*log(abs(c)))*tan(a) + 3*x^3*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(
c) - 2*pi*b)*tan(a)^2 + 3*x^3*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + 12*
x^3*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a) + 3*x^3*tan(a)^2 - 3*x^3*e
^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b) - 3*x^3)/(4*b^2*n^2
*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*l
og(abs(c)))^2*tan(a)^2 + 4*b^2*n^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c)
 - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + 4*b^2*n^2*e^(pi*b*n*s...
```

Mupad [B] (verification not implemented)

Time = 20.46 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.68

$$\int x^2 \cos^2(a + b\log(cx^n)) \, dx = \frac{x^3}{6} + \frac{x^3 \, e^{-a\,2i} \, \frac{1}{(c\,x^n)^{b\,2i}} \, 1i}{8\,b\,n + 12i} + \frac{x^3 \, e^{a\,2i} \, (c\,x^n)^{b\,2i}}{12 + b\,n\,8i}$$

input
```
int(x^2*cos(a + b*log(c*x^n))^2,x)
```

output
```
x^3/6 + (x^3*exp(-a*2i)/(c*x^n)^(b*2i)*1i)/(8*b*n + 12i) + (x^3*exp(a*2i)*
(c*x^n)^(b*2i))/(b*n*8i + 12)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.69

$$\int x^2 \cos^2(a + b\log(cx^n))\, dx$$
$$= \frac{x^3 \left(6\cos(\log(x^n c)\, b + a)\sin(\log(x^n c)\, b + a)\, bn - 9\sin(\log(x^n c)\, b + a)^2 + 2b^2n^2 + 9\right)}{12b^2n^2 + 27}$$

input
```
int(x^2*cos(a+b*log(c*x^n))^2,x)
```

output
```
(x**3*(6*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*b*n - 9*sin(log(x**
n*c)*b + a)**2 + 2*b**2*n**2 + 9))/(3*(4*b**2*n**2 + 9))
```

3.92 $\int x \cos^2(a + b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 98

$$\int x \cos^2(a + b\log(cx^n))\,dx = \frac{b^2n^2x^2}{4(1 + b^2n^2)} + \frac{x^2\cos^2(a + b\log(cx^n))}{2(1 + b^2n^2)} + \frac{bnx^2\cos(a + b\log(cx^n))\sin(a + b\log(cx^n))}{2(1 + b^2n^2)}$$

output
```
b^2*n^2*x^2/(4*b^2*n^2+4)+x^2*cos(a+b*ln(c*x^n))^2/(2*b^2*n^2+2)+b*n*x^2*c
os(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/(2*b^2*n^2+2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.55

$$\int x \cos^2(a + b\log(cx^n))\,dx = \frac{x^2(1 + b^2n^2 + \cos(2(a + b\log(cx^n))) + bn\sin(2(a + b\log(cx^n))))}{4 + 4b^2n^2}$$

input
```
Integrate[x*Cos[a + b*Log[c*x^n]]^2,x]
```

output
```
(x^2*(1 + b^2*n^2 + Cos[2*(a + b*Log[c*x^n])] + b*n*Sin[2*(a + b*Log[c*x^n
])]))/(4 + 4*b^2*n^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4991, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos^2(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \; 4991$$

$$\frac{b^2n^2\int xdx}{2(b^2n^2+1)}+\frac{x^2\cos^2(a+b\log(cx^n))}{2(b^2n^2+1)}+\frac{bnx^2\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{2(b^2n^2+1)}$$

$$\Big\downarrow \; 15$$

$$\frac{x^2\cos^2(a+b\log(cx^n))}{2(b^2n^2+1)}+\frac{bnx^2\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{2(b^2n^2+1)}+\frac{b^2n^2x^2}{4(b^2n^2+1)}$$

input
```
Int[x*Cos[a + b*Log[c*x^n]]^2,x]
```

output
```
(b^2*n^2*x^2)/(4*(1 + b^2*n^2)) + (x^2*Cos[a + b*Log[c*x^n]]^2)/(2*(1 + b^
2*n^2)) + (b*n*x^2*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/(2*(1 + b^
2*n^2))
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 4991
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Simp[b*d*n*p*(e*x)^(m + 1)*Sin[d*(a
+ b*Log[c*x^n])]*(Cos[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*
(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2)
)   Int[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

method	result	size
parallelrisch	$\frac{x^{2}\left(b^{2} n^{2}+b n \sin\left(2 b \ln\left(c \,x^{n}\right)+2 a\right)+\cos\left(2 b \ln\left(c \,x^{n}\right)+2 a\right)+1\right)}{4 b^{2} n^{2}+4}$	57

input
```
int(x*cos(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

output
```
x^2*(b^2*n^2+b*n*sin(2*b*ln(c*x^n)+2*a)+cos(2*b*ln(c*x^n)+2*a)+1)/(4*b^2*n
^2+4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int x\cos^2\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\frac{b^2n^2x^2+2\,bnx^2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+2\,x^2\cos\left(bn\log\left(x\right)+b\log\right.}{4\left(b^2n^2+1\right)}$$

input
```
integrate(x*cos(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
1/4*(b^2*n^2*x^2 + 2*b*n*x^2*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x)
 + b*log(c) + a) + 2*x^2*cos(b*n*log(x) + b*log(c) + a)^2)/(b^2*n^2 + 1)
```

Sympy [F]

$$\int x\cos^2(a+b\log(cx^n))\,dx$$
$$=\begin{cases}\int x\cos^2\left(a-\frac{i\log(cx^n)}{n}\right)dx\\ \int x\cos^2\left(a+\frac{i\log(cx^n)}{n}\right)dx\\ \frac{b^2n^2x^2\sin^2(a+b\log(cx^n))}{4b^2n^2+4}+\frac{b^2n^2x^2\cos^2(a+b\log(cx^n))}{4b^2n^2+4}+\frac{2bnx^2\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+4}+\frac{2x^2\cos^2(a+b\log(cx^n))}{4b^2n^2+4}\end{cases}$$

input
```
integrate(x*cos(a+b*ln(c*x**n))**2,x)
```

output
```
Piecewise((Integral(x*cos(a - I*log(c*x**n)/n)**2, x), Eq(b, -I/n)), (Inte
gral(x*cos(a + I*log(c*x**n)/n)**2, x), Eq(b, I/n)), (b**2*n**2*x**2*sin(a
 + b*log(c*x**n))**2/(4*b**2*n**2 + 4) + b**2*n**2*x**2*cos(a + b*log(c*x*
*n))**2/(4*b**2*n**2 + 4) + 2*b*n*x**2*sin(a + b*log(c*x**n))*cos(a + b*lo
g(c*x**n))/(4*b**2*n**2 + 4) + 2*x**2*cos(a + b*log(c*x**n))**2/(4*b**2*n*
*2 + 4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(92) = 184.

Time = 0.06 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.88

$$\int x\cos^2(a+b\log(cx^n))\,dx$$
$$=\frac{((b\cos(2\,b\log(c))\sin(4\,b\log(c))-b\cos(4\,b\log(c))\sin(2\,b\log(c))+b\sin(2\,b\log(c)))n+\cos(4\,b\log(c$$

input
```
integrate(x*cos(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
1/8*(((b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log(c
)) + b*sin(2*b*log(c)))*n + cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(
c))*sin(2*b*log(c)) + cos(2*b*log(c)))*x^2*cos(2*b*log(x^n) + 2*a) + ((b*c
os(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)) + b*cos
(2*b*log(c)))*n - cos(2*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin(2*
b*log(c)) - sin(2*b*log(c)))*x^2*sin(2*b*log(x^n) + 2*a) + 2*((b^2*cos(2*b
*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*b*log(c))^2 + sin(2*b*log(
c))^2)*x^2)/((b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*b
*log(c))^2 + sin(2*b*log(c))^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 817 vs. $2(92) = 184$.

Time = 0.32 (sec) , antiderivative size = 817, normalized size of antiderivative = 8.34

$$\int x\cos^2\left(a+b\log\left(cx^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x*cos(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
1/4*x^2 - 1/8*(2*b*n*x^2*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2
*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 2*b*n*x^2*e^(2*pi*b
*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*log
(abs(c)))*tan(a)^2 + 2*b*n*x^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(
a) + 2*b*n*x^2*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a)^2 - x^2*e^(2*pi
*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*l
og(abs(c)))^2*tan(a)^2 - 2*b*n*x^2*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*
sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c))) - 2*b*n*x^2*e^(2*pi*
b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(a) - x^2*tan(b*n*log(a
bs(x)) + b*log(abs(c)))^2*tan(a)^2 - 2*b*n*x^2*tan(b*n*log(abs(x)) + b*log
(abs(c))) + x^2*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*ta
n(b*n*log(abs(x)) + b*log(abs(c)))^2 - 2*b*n*x^2*tan(a) + 4*x^2*e^(2*pi*b*
n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*log(
abs(c)))*tan(a) + x^2*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi
*b)*tan(a)^2 + x^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + 4*x^2*tan(b*n*
log(abs(x)) + b*log(abs(c)))*tan(a) + x^2*tan(a)^2 - x^2*e^(2*pi*b*n*sgn(x
) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b) - x^2)/(b^2*n^2*e^(pi*b*n*sgn(x) -
pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)
^2 + b^2*n^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(a
bs(x)) + b*log(abs(c)))^2 + b^2*n^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sg...
```

Mupad [B] (verification not implemented)

Time = 20.67 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.67

$$\int x\cos^2(a+b\log(cx^n))\,dx=\frac{x^2}{4}+\frac{x^2\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,2\mathrm{i}}}\,1\mathrm{i}}{8\,b\,n+8\mathrm{i}}+\frac{x^2\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}}{8+b\,n\,8\mathrm{i}}$$

input
```
int(x*cos(a + b*log(c*x^n))^2,x)
```

output
```
x^2/4 + (x^2*exp(-a*2i)/(c*x^n)^(b*2i)*1i)/(8*b*n + 8i) + (x^2*exp(a*2i)*(
c*x^n)^(b*2i))/(b*n*8i + 8)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.67

$$\int x\cos^2\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{x^2\left(2\cos\left(\log(x^nc)\,b+a\right)\sin\left(\log(x^nc)\,b+a\right)bn-2\sin\left(\log(x^nc)\,b+a\right)^2+b^2n^2+2\right)}{4b^2n^2+4}$$

input
```
int(x*cos(a+b*log(c*x^n))^2,x)
```

output
```
(x**2*(2*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*b*n - 2*sin(log(x**
n*c)*b + a)**2 + b**2*n**2 + 2))/(4*(b**2*n**2 + 1))
```

3.93 $\int \cos^2(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 88

$$\int \cos^2(a+b\log(cx^n))\,dx=\frac{2b^2n^2x}{1+4b^2n^2}+\frac{x\cos^2(a+b\log(cx^n))}{1+4b^2n^2}+\frac{2bnx\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{1+4b^2n^2}$$

output
```
2*b^2*n^2*x/(4*b^2*n^2+1)+x*cos(a+b*ln(c*x^n))^2/(4*b^2*n^2+1)+2*b*n*x*cos
(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/(4*b^2*n^2+1)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int \cos^2(a+b\log(cx^n))\,dx=\frac{x(1+4b^2n^2+\cos(2(a+b\log(cx^n)))+2bn\sin(2(a+b\log(cx^n))))}{2+8b^2n^2}$$

input
```
Integrate[Cos[a + b*Log[c*x^n]]^2,x]
```

output
```
(x*(1 + 4*b^2*n^2 + Cos[2*(a + b*Log[c*x^n])] + 2*b*n*Sin[2*(a + b*Log[c*x
^n])]))/(2 + 8*b^2*n^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4981, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{ 4981}$$

$$\frac{2b^2n^2\int 1dx}{4b^2n^2+1}+\frac{x\cos^2(a+b\log(cx^n))}{4b^2n^2+1}+\frac{2bnx\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+1}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{x\cos^2(a+b\log(cx^n))}{4b^2n^2+1}+\frac{2bnx\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+1}+\frac{2b^2n^2x}{4b^2n^2+1}$$

input
```
Int[Cos[a + b*Log[c*x^n]]^2,x]
```

output
```
(2*b^2*n^2*x)/(1 + 4*b^2*n^2) + (x*Cos[a + b*Log[c*x^n]]^2)/(1 + 4*b^2*n^2
) + (2*b*n*x*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/(1 + 4*b^2*n^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 4981
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Sim
p[x*(Cos[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 + 1)), x] + (Simp[b*d*n*p
*x*Cos[d*(a + b*Log[c*x^n])]^(p - 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^
2*p^2 + 1)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + 1))   Int[
Cos[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b, c, d, n}, x] &&
IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + 1, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$\frac{x\left(\cos(2b\ln(c\,x^{n})+2a)+1+2bn\sin(2b\ln(c\,x^{n})+2a)+4b^{2}n^{2}\right)}{8b^{2}n^{2}+2}$	57
default	$\frac{x}{2}+\frac{\mathrm{e}^{\frac{\ln(c\,x^{n})}{n}-\frac{\ln(c)}{n}}\cos(2b\ln(c\,x^{n})+2a)}{2n^{2}\left(\frac{1}{n^{2}}+4b^{2}\right)}+\frac{b\,\mathrm{e}^{\frac{\ln(c\,x^{n})}{n}-\frac{\ln(c)}{n}}\sin(2b\ln(c\,x^{n})+2a)}{n\left(\frac{1}{n^{2}}+4b^{2}\right)}$	103

input
```
int(cos(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

output
```
x*(cos(2*b*ln(c*x^n)+2*a)+1+2*b*n*sin(2*b*ln(c*x^n)+2*a)+4*b^2*n^2)/(8*b^2
*n^2+2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int \cos^{2}\left(a+b\log\left(cx^{n}\right)\right)\,dx$$
$$=\frac{2\,b^{2}n^{2}x+2\,bnx\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+x\cos\left(bn\log\left(x\right)+b\log\left(c\right.\right.}{4\,b^{2}n^{2}+1}$$

input
```
integrate(cos(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
(2*b^2*n^2*x + 2*b*n*x*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x) + b*l
og(c) + a) + x*cos(b*n*log(x) + b*log(c) + a)^2)/(4*b^2*n^2 + 1)
```

Sympy [F]

$$\int \cos^2(a + b\log(cx^n))\,dx$$
$$= \begin{cases} \int \cos^2\left(a - \frac{i\log(cx^n)}{2n}\right)dx & \text{fo} \\ \int \cos^2\left(a + \frac{i\log(cx^n)}{2n}\right)dx & \text{fo} \\ \frac{2b^2n^2x\sin^2(a+b\log(cx^n))}{4b^2n^2+1} + \frac{2b^2n^2x\cos^2(a+b\log(cx^n))}{4b^2n^2+1} + \frac{2bnx\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+1} + \frac{x\cos^2(a+b\log(cx^n))}{4b^2n^2+1} & \text{ot} \end{cases}$$

input
```
integrate(cos(a+b*ln(c*x**n))**2,x)
```

output
```
Piecewise((Integral(cos(a - I*log(c*x**n)/(2*n))**2, x), Eq(b, -I/(2*n))),
 (Integral(cos(a + I*log(c*x**n)/(2*n))**2, x), Eq(b, I/(2*n))), (2*b**2*n
**2*x*sin(a + b*log(c*x**n))**2/(4*b**2*n**2 + 1) + 2*b**2*n**2*x*cos(a +
b*log(c*x**n))**2/(4*b**2*n**2 + 1) + 2*b*n*x*sin(a + b*log(c*x**n))*cos(a
 + b*log(c*x**n))/(4*b**2*n**2 + 1) + x*cos(a + b*log(c*x**n))**2/(4*b**2*
n**2 + 1), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(88) = 176$.

Time = 0.05 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.18

$$\int \cos^2(a + b\log(cx^n))\,dx$$
$$= \frac{(2(b\cos(2b\log(c))\sin(4b\log(c)) - b\cos(4b\log(c))\sin(2b\log(c)) + b\sin(2b\log(c)))n + \cos(4b\log(}{}$$

input
```
integrate(cos(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
1/4*((2*(b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*b*log
(c)) + b*sin(2*b*log(c)))*n + cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*lo
g(c))*sin(2*b*log(c)) + cos(2*b*log(c)))*x*cos(2*b*log(x^n) + 2*a) + (2*(b
*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)) + b*c
os(2*b*log(c)))*n - cos(2*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin(
2*b*log(c)) - sin(2*b*log(c)))*x*sin(2*b*log(x^n) + 2*a) + 2*(4*(b^2*cos(2
*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*b*log(c))^2 + sin(2*b*lo
g(c))^2)*x)/(4*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2
*b*log(c))^2 + sin(2*b*log(c))^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 783 vs. $2(88) = 176$.

Time = 0.24 (sec) , antiderivative size = 783, normalized size of antiderivative = 8.90

$$\int \cos^2(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(cos(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
1/2*x - 1/4*(4*b*n*x*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*
b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 4*b*n*x*e^(2*pi*b*n*sgn
(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c
)))*tan(a)^2 + 4*b*n*x*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a) + 4*b
*n*x*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a)^2 - x*e^(2*pi*b*n*sgn(x)
- 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^
2*tan(a)^2 - 4*b*n*x*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*
b)*tan(b*n*log(abs(x)) + b*log(abs(c))) - 4*b*n*x*e^(2*pi*b*n*sgn(x) - 2*p
i*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(a) - x*tan(b*n*log(abs(x)) + b*log(abs
(c)))^2*tan(a)^2 - 4*b*n*x*tan(b*n*log(abs(x)) + b*log(abs(c))) + x*e^(2*p
i*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*
log(abs(c)))^2 - 4*b*n*x*tan(a) + 4*x*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi
*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan(a) + x*e^(2*p
i*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(a)^2 + x*tan(b*n*log
(abs(x)) + b*log(abs(c)))^2 + 4*x*tan(b*n*log(abs(x)) + b*log(abs(c)))*tan
(a) + x*tan(a)^2 - x*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*
b) - x)/(4*b^2*n^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n
*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 + 4*b^2*n^2*e^(pi*b*n*sgn(x) - pi
*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2 + 4*b^2*
n^2*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(a)^2 + 4*b^2*n^...
```

Mupad [B] (verification not implemented)

Time = 20.38 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \cos^2(a+b\log(cx^n))\,dx = \frac{x\left(2\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^2+4\,b^2\,n^2+2\,b\,n\,\sin\left(2\,a+2\,b\,\ln\left(c\,x^n\right)\right)\right)}{8\,b^2\,n^2+2}$$

input
```
int(cos(a + b*log(c*x^n))^2,x)
```

output
```
(x*(2*cos(a + b*log(c*x^n))^2 + 4*b^2*n^2 + 2*b*n*sin(2*a + 2*b*log(c*x^n)
)))/(8*b^2*n^2 + 2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

$$\int \cos^2(a + b\log(cx^n))\,dx$$
$$= \frac{x\left(2\cos(\log(x^n c)\,b + a)\sin(\log(x^n c)\,b + a)\,bn - \sin(\log(x^n c)\,b + a)^2 + 2b^2n^2 + 1\right)}{4b^2n^2 + 1}$$

input
```
int(cos(a+b*log(c*x^n))^2,x)
```

output
```
(x*(2*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*b*n - sin(log(x**n*c)*
b + a)**2 + 2*b**2*n**2 + 1))/(4*b**2*n**2 + 1)
```

3.94 $\int \frac{\cos^2(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \frac{\cos^2(a+b\log(cx^n))}{x}\,dx=\frac{\log(x)}{2}+\frac{\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{2bn}$$

output
```
1/2*ln(x)+1/2*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{\cos^2(a+b\log(cx^n))}{x}\,dx=\frac{2(a+b\log(cx^n))+\sin(2(a+b\log(cx^n)))}{4bn}$$

input
```
Integrate[Cos[a + b*Log[c*x^n]]^2/x,x]
```

output
```
(2*(a + b*Log[c*x^n]) + Sin[2*(a + b*Log[c*x^n])])/(4*b*n)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3039, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \cos^2(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(a+b\log(cx^n)+\frac{\pi}{2}\right)^2d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{1}{2}\int 1d\log(cx^n)+\frac{\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{2b}}{n}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{2b}+\frac{1}{2}\log(cx^n)}{n}$$

input
```
Int[Cos[a + b*Log[c*x^n]]^2/x,x]
```

output
```
(Log[c*x^n]/2 + (Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/(2*b))/n
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

method	result	size
parallelrisch	$\frac{2\ln(x)bn+\sin(2b\ln(c\,x^n)+2a)}{4bn}$	30
derivativedivides	$\frac{\frac{\cos(a+b\ln(c\,x^n))\sin(a+b\ln(c\,x^n))}{2}+\frac{b\ln(c\,x^n)}{2}+\frac{a}{2}}{nb}$	45
default	$\frac{\frac{\cos(a+b\ln(c\,x^n))\sin(a+b\ln(c\,x^n))}{2}+\frac{b\ln(c\,x^n)}{2}+\frac{a}{2}}{nb}$	45

input
```
int(cos(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*ln(x)*b*n+sin(2*b*ln(c*x^n)+2*a))/b/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\cos^2(a+b\log(cx^n))}{x}\,dx = \frac{bn\log(x)+\cos(bn\log(x)+b\log(c)+a)\sin(bn\log(x)+b\log(c)+a)}{2\,bn}$$

input
```
integrate(cos(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

output
```
1/2*(b*n*log(x) + cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x) + b*log(c)
 + a))/(b*n)
```

Sympy [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31

$$\int \frac{\cos^2(a+b\log(cx^n))}{x}\,dx = \frac{\begin{cases}\log(x)\cos(2a) & \text{for } b=0\wedge(b=0\vee n=0)\\ \log(x)\cos(2a+2b\log(c)) & \text{for } n=0\\ \frac{\sin(2a+2b\log(cx^n))}{2bn} & \text{otherwise}\end{cases}}{2} + \frac{\log(x)}{2}$$

input
```
integrate(cos(a+b*ln(c*x**n))**2/x,x)
```

output
```
Piecewise((log(x)*cos(2*a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*cos
(2*a + 2*b*log(c)), Eq(n, 0)), (sin(2*a + 2*b*log(c*x**n))/(2*b*n), True))
/2 + log(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.36

$$\int \frac{\cos^2(a + b\log(cx^n))}{x}\,dx = \frac{2\,bn\log(x) + \cos(2\,b\log(x^n) + 2\,a)\sin(2\,b\log(c)) + \cos(2\,b\log(c))\sin(2\,b\log(x^n) + 2\,a)}{4\,bn}$$

input
```
integrate(cos(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

output
```
1/4*(2*b*n*log(x) + cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + cos(2*b*log(
c))*sin(2*b*log(x^n) + 2*a))/(b*n)
```

Giac [F]

$$\int \frac{\cos^2(a + b\log(cx^n))}{x}\,dx = \int \frac{\cos(b\log(cx^n) + a)^2}{x}\,dx$$

input
```
integrate(cos(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

output
```
integrate(cos(b*log(c*x^n) + a)^2/x, x)
```

Mupad [B] (verification not implemented)

Time = 19.82 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{\cos^2(a + b\log(cx^n))}{x}\,dx = \frac{\ln(x^n)}{2\,n} + \frac{\sin(2\,a + 2\,b\,\ln(c\,x^n))}{4\,b\,n}$$

input
```
int(cos(a + b*log(c*x^n))^2/x,x)
```

output
```
log(x^n)/(2*n) + sin(2*a + 2*b*log(c*x^n))/(4*b*n)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{\cos^2(a+b\log(cx^n))}{x}\,dx = \frac{\cos(\log(x^nc)\,b+a)\sin(\log(x^nc)\,b+a)+\log(x^nc)\,b}{2bn}$$

input
```
int(cos(a+b*log(c*x^n))^2/x,x)
```

output
```
(cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a) + log(x**n*c)*b)/(2*b*n)
```

3.95 $\int \frac{\cos^2(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 95

$$\int \frac{\cos^2(a+b\log(cx^n))}{x^2}\,dx = -\frac{2b^2n^2}{(1+4b^2n^2)\,x}-\frac{\cos^2(a+b\log(cx^n))}{(1+4b^2n^2)\,x}+\frac{2bn\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{(1+4b^2n^2)\,x}$$

output
```
-2*b^2*n^2/(4*b^2*n^2+1)/x-cos(a+b*ln(c*x^n))^2/(4*b^2*n^2+1)/x+2*b*n*cos(
a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/(4*b^2*n^2+1)/x
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.60

$$\int \frac{\cos^2(a+b\log(cx^n))}{x^2}\,dx = -\frac{1+4b^2n^2+\cos(2(a+b\log(cx^n)))-2bn\sin(2(a+b\log(cx^n)))}{2\,(x+4b^2n^2x)}$$

input
```
Integrate[Cos[a + b*Log[c*x^n]]^2/x^2,x]
```

output
```
-1/2*(1 + 4*b^2*n^2 + Cos[2*(a + b*Log[c*x^n])] - 2*b*n*Sin[2*(a + b*Log[c
*x^n])])/(x + 4*b^2*n^2*x)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4991, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+b\log(cx^n))}{x^2}dx$$

$$\Big\downarrow \textbf{4991}$$

$$\frac{2b^2n^2\int \frac{1}{x^2}dx}{4b^2n^2+1}-\frac{\cos^2(a+b\log(cx^n))}{x(4b^2n^2+1)}+\frac{2bn\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{x(4b^2n^2+1)}$$

$$\Big\downarrow \textbf{15}$$

$$-\frac{\cos^2(a+b\log(cx^n))}{x(4b^2n^2+1)}+\frac{2bn\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{x(4b^2n^2+1)}-\frac{2b^2n^2}{x(4b^2n^2+1)}$$

input
```
Int[Cos[a + b*Log[c*x^n]]^2/x^2,x]
```

output
```
(-2*b^2*n^2)/((1 + 4*b^2*n^2)*x) - Cos[a + b*Log[c*x^n]]^2/((1 + 4*b^2*n^2
)*x) + (2*b*n*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/((1 + 4*b^2*n^2
)*x)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 4991
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Simp[b*d*n*p*(e*x)^(m + 1)*Sin[d*(a
+ b*Log[c*x^n])]*(Cos[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*
(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2)
)   Int[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.64

method	result	size
parallelrisch	$\frac{-\cos(2b\ln(c\,x^{n})+2a)-1-4b^{2}n^{2}+2bn\sin(2b\ln(c\,x^{n})+2a)}{8x\,b^{2}n^{2}+2x}$	61

input
```
int(cos(a+b*ln(c*x^n))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
(-cos(2*b*ln(c*x^n)+2*a)-1-4*b^2*n^2+2*b*n*sin(2*b*ln(c*x^n)+2*a))/(8*b^2*
n^2*x+2*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72

$$\int \frac{\cos^2(a+b\log(cx^n))}{x^2}\,dx = -\frac{2\,b^2n^2 - 2\,bn\cos(bn\log(x)+b\log(c)+a)\sin(bn\log(x)+b\log(c)+a) + \cos(bn\log(x)+b\log(c)}{(4\,b^2n^2+1)x}$$

input
```
integrate(cos(a+b*log(c*x^n))^2/x^2,x, algorithm="fricas")
```

output
```
-(2*b^2*n^2 - 2*b*n*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x) + b*log(
c) + a) + cos(b*n*log(x) + b*log(c) + a)^2)/((4*b^2*n^2 + 1)*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.90 (sec) , antiderivative size = 299, normalized size of antiderivative = 3.15

$$\int \frac{\cos^2(a+b\log(cx^n))}{x^2}\,dx = \begin{cases} -\frac{\cos\left(2a-\frac{i\log(cx^n)}{n}\right)}{4x} - \frac{1}{2x} + \frac{i\log(cx^n)\sin\left(2a-\frac{i\log(cx^n)}{n}\right)}{4nx} + \frac{\log(cx^n)\cos\left(2a-\frac{i\log(cx^n)}{n}\right)}{4nx} & \text{for } b \\ -\frac{\cos\left(2a+\frac{i\log(cx^n)}{n}\right)}{4x} - \frac{1}{2x} - \frac{i\log(cx^n)\sin\left(2a+\frac{i\log(cx^n)}{n}\right)}{4nx} + \frac{\log(cx^n)\cos\left(2a+\frac{i\log(cx^n)}{n}\right)}{4nx} & \text{for } b \\ -\frac{2b^2n^2\sin^2(a+b\log(cx^n))}{4b^2n^2x+x} - \frac{2b^2n^2\cos^2(a+b\log(cx^n))}{4b^2n^2x+x} + \frac{2bn\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2x+x} - \frac{\cos^2(a+b\log(cx^n))}{4b^2n^2x+x} & \text{othe} \end{cases}$$

input
```
integrate(cos(a+b*ln(c*x**n))**2/x**2,x)
```

output
```
Piecewise((-cos(2*a - I*log(c*x**n)/n)/(4*x) - 1/(2*x) + I*log(c*x**n)*sin
(2*a - I*log(c*x**n)/n)/(4*n*x) + log(c*x**n)*cos(2*a - I*log(c*x**n)/n)/(
4*n*x), Eq(b, -I/(2*n))), (-cos(2*a + I*log(c*x**n)/n)/(4*x) - 1/(2*x) - I
*log(c*x**n)*sin(2*a + I*log(c*x**n)/n)/(4*n*x) + log(c*x**n)*cos(2*a + I*
log(c*x**n)/n)/(4*n*x), Eq(b, I/(2*n))), (-2*b**2*n**2*sin(a + b*log(c*x**
n))**2/(4*b**2*n**2*x + x) - 2*b**2*n**2*cos(a + b*log(c*x**n))**2/(4*b**2
*n**2*x + x) + 2*b*n*sin(a + b*log(c*x**n))*cos(a + b*log(c*x**n))/(4*b**2
*n**2*x + x) - cos(a + b*log(c*x**n))**2/(4*b**2*n**2*x + x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(95) = 190.

Time = 0.05 (sec) , antiderivative size = 285, normalized size of antiderivative = 3.00

$$\int \frac{\cos^2(a+b\log(cx^n))}{x^2}\,dx = -\frac{8\left(b^2\cos(2b\log(c))^2 + b^2\sin(2b\log(c))^2\right)n^2 + 2\cos(2b\log(c))^2 - (2(b\cos(2b\log(c))\sin(4b\log(c

input
```
integrate(cos(a+b*log(c*x^n))^2/x^2,x, algorithm="maxima")
```

output
```
-1/4*(8*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + 2*cos(2*b*lo
g(c))^2 - (2*(b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(2*
b*log(c)) + b*sin(2*b*log(c)))*n - cos(4*b*log(c))*cos(2*b*log(c)) - sin(4
*b*log(c))*sin(2*b*log(c)) - cos(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 2*
sin(2*b*log(c))^2 - (2*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(
c))*sin(2*b*log(c)) + b*cos(2*b*log(c)))*n + cos(2*b*log(c))*sin(4*b*log(c
)) - cos(4*b*log(c))*sin(2*b*log(c)) + sin(2*b*log(c)))*sin(2*b*log(x^n) +
 2*a))/((4*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + cos(2*b*l
og(c))^2 + sin(2*b*log(c))^2)*x)
```

Giac [F]

$$\int \frac{\cos^2(a+b\log(cx^n))}{x^2}\,dx=\int \frac{\cos(b\log(cx^n)+a)^2}{x^2}\,dx$$

input
```
integrate(cos(a+b*log(c*x^n))^2/x^2,x, algorithm="giac")
```

output
```
integrate(cos(b*log(c*x^n) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+b\log(cx^n))}{x^2}\,dx=\int \frac{\cos(a+b\,\ln(c\,x^n))^2}{x^2}\,dx$$

input
```
int(cos(a + b*log(c*x^n))^2/x^2,x)
```

output
```
int(cos(a + b*log(c*x^n))^2/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

$$\int \frac{\cos^2(a+b\log(cx^n))}{x^2}\,dx$$
$$=\frac{2\cos(\log(x^n c)\,b+a)\sin(\log(x^n c)\,b+a)\,bn+\sin(\log(x^n c)\,b+a)^2-2b^2n^2-1}{x\,(4b^2n^2+1)}$$

input
```
int(cos(a+b*log(c*x^n))^2/x^2,x)
```

output
```
(2*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*b*n + sin(log(x**n*c)*b +
 a)**2 - 2*b**2*n**2 - 1)/(x*(4*b**2*n**2 + 1))
```

3.96 $\int x^2 \cos^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 160

$$\int x^2\cos^3(a+b\log(cx^n))\,dx = \frac{2b^2n^2x^3\cos(a+b\log(cx^n))}{9+10b^2n^2+b^4n^4}+\frac{x^3\cos^3(a+b\log(cx^n))}{3(1+b^2n^2)}+\frac{2b^3n^3x^3\sin(a+b\log(cx^n))}{3(9+10b^2n^2+b^4n^4)}+\frac{bnx^3\cos^2(a+b\log(cx^n))\sin(a+b\log(cx^n))}{3(1+b^2n^2)}$$

output
```
2*b^2*n^2*x^3*cos(a+b*ln(c*x^n))/(b^4*n^4+10*b^2*n^2+9)+x^3*cos(a+b*ln(c*x
^n))^3/(3*b^2*n^2+3)+2*b^3*n^3*x^3*sin(a+b*ln(c*x^n))/(3*b^4*n^4+30*b^2*n^
2+27)+b*n*x^3*cos(a+b*ln(c*x^n))^2*sin(a+b*ln(c*x^n))/(3*b^2*n^2+3)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.75

$$\int x^2\cos^3(a+b\log(cx^n))\,dx$$
$$= \frac{x^3(27(1+b^2n^2)\cos(a+b\log(cx^n))+(9+b^2n^2)\cos(3(a+b\log(cx^n)))+2bn(9+5b^2n^2+(9+b^2n^2)\,c}{12(9+10b^2n^2+b^4n^4)}$$

input
```
Integrate[x^2*Cos[a + b*Log[c*x^n]]^3,x]
```

output
```
(x^3*(27*(1 + b^2*n^2)*Cos[a + b*Log[c*x^n]] + (9 + b^2*n^2)*Cos[3*(a + b*
Log[c*x^n])] + 2*b*n*(9 + 5*b^2*n^2 + (9 + b^2*n^2)*Cos[2*(a + b*Log[c*x^n
])])*Sin[a + b*Log[c*x^n]]))/(12*(9 + 10*b^2*n^2 + b^4*n^4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4991, 4989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cos^3(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{4991}$$

$$\frac{2b^2n^2\int x^2\cos(a+b\log(cx^n))\,dx}{3(b^2n^2+1)}+\frac{x^3\cos^3(a+b\log(cx^n))}{3(b^2n^2+1)}+\frac{bnx^3\sin(a+b\log(cx^n))\cos^2(a+b\log(cx^n))}{3(b^2n^2+1)}$$

$$\Big\downarrow \text{4989}$$

$$\frac{x^3\cos^3(a+b\log(cx^n))}{3(b^2n^2+1)}+\frac{bnx^3\sin(a+b\log(cx^n))\cos^2(a+b\log(cx^n))}{3(b^2n^2+1)}+\frac{2b^2n^2\left(\frac{bnx^3\sin(a+b\log(cx^n))}{b^2n^2+9}+\frac{3x^3\cos(a+b\log(cx^n))}{b^2n^2+9}\right)}{3(b^2n^2+1)}$$

input
```
Int[x^2*Cos[a + b*Log[c*x^n]]^3,x]
```

output
```
(x^3*Cos[a + b*Log[c*x^n]]^3)/(3*(1 + b^2*n^2)) + (b*n*x^3*Cos[a + b*Log[c
*x^n]]^2*Sin[a + b*Log[c*x^n]])/(3*(1 + b^2*n^2)) + (2*b^2*n^2*((3*x^3*Cos
[a + b*Log[c*x^n]])/(9 + b^2*n^2) + (b*n*x^3*Sin[a + b*Log[c*x^n]])/(9 + b
^2*n^2)))/(3*(1 + b^2*n^2))
```

Defintions of rubi rules used

rule 4989
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

rule 4991
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Simp[b*d*n*p*(e*x)^(m + 1)*Sin[d*(a
+ b*Log[c*x^n])]*(Cos[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*
(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2)
)   Int[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(155) = 310$.

Time = 3.88 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.03

method	result
parallelrisch	$\frac{x^3\left(6\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^5 b^3n^3-7b^2n^2\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^6+4\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^3 b^3n^3-3b^2n^2\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^4+1}{}$

input
```
int(x^2*cos(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*(6*tan(1/2*a+b*ln((c*x^n)^(1/2)))^5*b^3*n^3-7*b^2*n^2*tan(1/2*a+b*
ln((c*x^n)^(1/2)))^6+4*tan(1/2*a+b*ln((c*x^n)^(1/2)))^3*b^3*n^3-3*b^2*n^2*
tan(1/2*a+b*ln((c*x^n)^(1/2)))^4+18*b*n*tan(1/2*a+b*ln((c*x^n)^(1/2)))^5+6
*b^3*n^3*tan(1/2*a+b*ln((c*x^n)^(1/2)))-9*tan(1/2*a+b*ln((c*x^n)^(1/2)))^6
+3*tan(1/2*a+b*ln((c*x^n)^(1/2)))^2*b^2*n^2-36*b*n*tan(1/2*a+b*ln((c*x^n)^
(1/2)))^3+27*tan(1/2*a+b*ln((c*x^n)^(1/2)))^4+7*b^2*n^2+18*b*n*tan(1/2*a+b
*ln((c*x^n)^(1/2)))-27*tan(1/2*a+b*ln((c*x^n)^(1/2)))^2+9)/(b^2*n^2+9)/(b^
2*n^2+1)/(1+tan(1/2*a+b*ln((c*x^n)^(1/2)))^2)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.79

$$\int x^2\cos^3\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\frac{6\,b^2n^2x^3\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+(b^2n^2+9)x^3\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3+\left(2\,b^3n^3x^3+(b^3n\right)}{3\left(b^4n^4+10\,b^2n^2+9\right)}$$

input
```
integrate(x^2*cos(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
1/3*(6*b^2*n^2*x^3*cos(b*n*log(x) + b*log(c) + a) + (b^2*n^2 + 9)*x^3*cos(
b*n*log(x) + b*log(c) + a)^3 + (2*b^3*n^3*x^3 + (b^3*n^3 + 9*b*n)*x^3*cos(
b*n*log(x) + b*log(c) + a)^2)*sin(b*n*log(x) + b*log(c) + a))/(b^4*n^4 + 1
0*b^2*n^2 + 9)
```

Sympy [F(-1)]

Timed out.

$$\int x^2\cos^3\left(a+b\log\left(cx^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x**2*cos(a+b*ln(c*x**n))**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1007 vs. 2(154) = 308.

Time = 0.09 (sec) , antiderivative size = 1007, normalized size of antiderivative = 6.29

$$\int x^2\cos^3\left(a+b\log\left(cx^n\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x^2*cos(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
1/24*(((b^3*cos(3*b*log(c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(3*b*
log(c)) + b^3*sin(3*b*log(c)))*n^3 + (b^2*cos(6*b*log(c))*cos(3*b*log(c))
+ b^2*sin(6*b*log(c))*sin(3*b*log(c)) + b^2*cos(3*b*log(c)))*n^2 + 9*(b*co
s(3*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(3*b*log(c)) + b*sin(
3*b*log(c)))*n + 9*cos(6*b*log(c))*cos(3*b*log(c)) + 9*sin(6*b*log(c))*sin
(3*b*log(c)) + 9*cos(3*b*log(c)))*x^3*cos(3*b*log(x^n) + 3*a) + 9*((b^3*co
s(3*b*log(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(3*b*log(c)) + b^3*
cos(2*b*log(c))*sin(3*b*log(c)) - b^3*cos(3*b*log(c))*sin(2*b*log(c)))*n^3
 + 3*(b^2*cos(4*b*log(c))*cos(3*b*log(c)) + b^2*cos(3*b*log(c))*cos(2*b*lo
g(c)) + b^2*sin(4*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c))*sin(2*b*
log(c)))*n^2 + (b*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(
3*b*log(c)) + b*cos(2*b*log(c))*sin(3*b*log(c)) - b*cos(3*b*log(c))*sin(2*
b*log(c)))*n + 3*cos(4*b*log(c))*cos(3*b*log(c)) + 3*cos(3*b*log(c))*cos(2
*b*log(c)) + 3*sin(4*b*log(c))*sin(3*b*log(c)) + 3*sin(3*b*log(c))*sin(2*b
*log(c)))*x^3*cos(b*log(x^n) + a) + ((b^3*cos(6*b*log(c))*cos(3*b*log(c))
+ b^3*sin(6*b*log(c))*sin(3*b*log(c)) + b^3*cos(3*b*log(c)))*n^3 - (b^2*co
s(3*b*log(c))*sin(6*b*log(c)) - b^2*cos(6*b*log(c))*sin(3*b*log(c)) + b^2*
sin(3*b*log(c)))*n^2 + 9*(b*cos(6*b*log(c))*cos(3*b*log(c)) + b*sin(6*b*lo
g(c))*sin(3*b*log(c)) + b*cos(3*b*log(c)))*n - 9*cos(3*b*log(c))*sin(6*b*l
og(c)) + 9*cos(6*b*log(c))*sin(3*b*log(c)) - 9*sin(3*b*log(c)))*x^3*sin...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18053 vs. $2(154) = 308$.

Time = 1.31 (sec) , antiderivative size = 18053, normalized size of antiderivative = 112.83

$$\int x^2 \cos^3(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(x^2*cos(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
-1/24*(18*b^3*n^3*x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c)
- 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log
(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a) + 18*b^3*n^3*x^3*e
^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*
n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(a
bs(c)))^2*tan(3/2*a)^2*tan(1/2*a) + 2*b^3*n^3*x^3*e^(3/2*pi*b*n*sgn(x) - 3
/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*lo
g(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)*tan
(1/2*a)^2 + 2*b^3*n^3*x^3*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sg
n(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*
n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)*tan(1/2*a)^2 + 18*b^3*n^3*
x^3*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/
2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*l
og(abs(c)))*tan(3/2*a)^2*tan(1/2*a)^2 + 18*b^3*n^3*x^3*e^(-1/2*pi*b*n*sgn(
x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/
2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(3/2*a)
^2*tan(1/2*a)^2 + 2*b^3*n^3*x^3*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi
*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))*tan(1/2
*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 + 2*b^3*
n^3*x^3*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b...
```

Mupad [B] (verification not implemented)

Time = 21.71 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.76

$$\int x^2 \cos^3(a + b\log(cx^n))\,dx = \frac{x^3\,e^{-a\,1i}\,\frac{1}{(c\,x^n)^{b\,1i}}\,3i}{8\,b\,n+24i} + \frac{3\,x^3\,e^{a\,1i}\,(c\,x^n)^{b\,1i}}{24+b\,n\,8i} + \frac{x^3\,e^{-a\,3i}\,\frac{1}{(c\,x^n)^{b\,3i}}\,1i}{24\,b\,n+24i} + \frac{x^3\,e^{a\,3i}\,(c\,x^n)^{b\,3i}}{24+b\,n\,24i}$$

input
```
int(x^2*cos(a + b*log(c*x^n))^3,x)
```

output
```
(x^3*exp(-a*1i)/(c*x^n)^(b*1i)*3i)/(8*b*n + 24i) + (3*x^3*exp(a*1i)*(c*x^n
)^(b*1i))/(b*n*8i + 24) + (x^3*exp(-a*3i)/(c*x^n)^(b*3i)*1i)/(24*b*n + 24i
) + (x^3*exp(a*3i)*(c*x^n)^(b*3i))/(b*n*24i + 24)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.17

$$\int x^2 \cos^3(a+b\log(cx^n))\,dx$$
$$= \frac{x^3\left(-\cos(\log(x^n c)\,b+a)\sin(\log(x^n c)\,b+a)^2 b^2 n^2 - 9\cos(\log(x^n c)\,b+a)\sin(\log(x^n c)\,b+a)^2 + 7\cos(\mathrm{l}\right.}{}$$

input
```
int(x^2*cos(a+b*log(c*x^n))^3,x)
```

output
```
(x**3*( - cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**2*b**2*n**2 - 9*c
os(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**2 + 7*cos(log(x**n*c)*b + a)
*b**2*n**2 + 9*cos(log(x**n*c)*b + a) - sin(log(x**n*c)*b + a)**3*b**3*n**
3 - 9*sin(log(x**n*c)*b + a)**3*b*n + 3*sin(log(x**n*c)*b + a)*b**3*n**3 +
 9*sin(log(x**n*c)*b + a)*b*n))/(3*(b**4*n**4 + 10*b**2*n**2 + 9))
```

3.97 $\int x \cos^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 158

$$\int x\cos^3(a+b\log(cx^n))\,dx=\frac{12b^2n^2x^2\cos(a+b\log(cx^n))}{16+40b^2n^2+9b^4n^4}+\frac{2x^2\cos^3(a+b\log(cx^n))}{4+9b^2n^2}+\frac{6b^3n^3x^2\sin(a+b\log(cx^n))}{16+40b^2n^2+9b^4n^4}+\frac{3bnx^2\cos^2(a+b\log(cx^n))\sin(a+b\log(cx^n))}{4+9b^2n^2}$$

output
```
12*b^2*n^2*x^2*cos(a+b*ln(c*x^n))/(9*b^4*n^4+40*b^2*n^2+16)+2*x^2*cos(a+b*
ln(c*x^n))^3/(9*b^2*n^2+4)+6*b^3*n^3*x^2*sin(a+b*ln(c*x^n))/(9*b^4*n^4+40*
b^2*n^2+16)+3*b*n*x^2*cos(a+b*ln(c*x^n))^2*sin(a+b*ln(c*x^n))/(9*b^2*n^2+4
)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.78

$$\int x\cos^3(a+b\log(cx^n))\,dx=\frac{x^2(6(4+9b^2n^2)\cos(a+b\log(cx^n))+2(4+b^2n^2)\cos(3(a+b\log(cx^n)))+6bn(4+5b^2n^2+(4+b^2n^2)}{4(16+40b^2n^2+9b^4n^4)}$$

input
```
Integrate[x*Cos[a + b*Log[c*x^n]]^3,x]
```

output
```
(x^2*(6*(4 + 9*b^2*n^2)*Cos[a + b*Log[c*x^n]] + 2*(4 + b^2*n^2)*Cos[3*(a +
 b*Log[c*x^n])] + 6*b*n*(4 + 5*b^2*n^2 + (4 + b^2*n^2)*Cos[2*(a + b*Log[c*
x^n])])*Sin[a + b*Log[c*x^n]]))/(4*(16 + 40*b^2*n^2 + 9*b^4*n^4))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4991, 4989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cos^3(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{4991}$$

$$\frac{6b^2n^2\int x\cos(a+b\log(cx^n))\,dx}{9b^2n^2+4}+\frac{2x^2\cos^3(a+b\log(cx^n))}{9b^2n^2+4}+\frac{3bnx^2\sin(a+b\log(cx^n))\cos^2(a+b\log(cx^n))}{9b^2n^2+4}$$

$$\Big\downarrow \text{4989}$$

$$\frac{2x^2\cos^3(a+b\log(cx^n))}{9b^2n^2+4}+\frac{3bnx^2\sin(a+b\log(cx^n))\cos^2(a+b\log(cx^n))}{9b^2n^2+4}+\frac{6b^2n^2\left(\frac{bnx^2\sin(a+b\log(cx^n))}{b^2n^2+4}+\frac{2x^2\cos(a+b\log(cx^n))}{b^2n^2+4}\right)}{9b^2n^2+4}$$

input
```
Int[x*Cos[a + b*Log[c*x^n]]^3,x]
```

output
```
(2*x^2*Cos[a + b*Log[c*x^n]]^3)/(4 + 9*b^2*n^2) + (3*b*n*x^2*Cos[a + b*Log
[c*x^n]]^2*Sin[a + b*Log[c*x^n]])/(4 + 9*b^2*n^2) + (6*b^2*n^2*((2*x^2*Cos
[a + b*Log[c*x^n]])/(4 + b^2*n^2) + (b*n*x^2*Sin[a + b*Log[c*x^n]])/(4 + b
^2*n^2)))/(4 + 9*b^2*n^2)
```

Defintions of rubi rules used

rule 4989
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

rule 4991
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Simp[b*d*n*p*(e*x)^(m + 1)*Sin[d*(a
+ b*Log[c*x^n])]*(Cos[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*
(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2)
)   Int[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(158) = 316$.

Time = 2.30 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.06

method	result
parallelrisch	$-\frac{2x^2\left(-4+24bn\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^3-3\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^2b^2n^2+3b^2n^2\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^4-12bn\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right.\right.\right.}{}$

input
```
int(x*cos(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
```

output
```
-2*x^2*(-4+24*b*n*tan(1/2*a+b*ln((c*x^n)^(1/2)))^3-3*tan(1/2*a+b*ln((c*x^n
)^(1/2)))^2*b^2*n^2+3*b^2*n^2*tan(1/2*a+b*ln((c*x^n)^(1/2)))^4-12*b*n*tan(
1/2*a+b*ln((c*x^n)^(1/2)))+12*tan(1/2*a+b*ln((c*x^n)^(1/2)))^2-12*tan(1/2*
a+b*ln((c*x^n)^(1/2)))^4-9*tan(1/2*a+b*ln((c*x^n)^(1/2)))^5*b^3*n^3+7*b^2*
n^2*tan(1/2*a+b*ln((c*x^n)^(1/2)))^6-6*tan(1/2*a+b*ln((c*x^n)^(1/2)))^3*b^
3*n^3-9*b^3*n^3*tan(1/2*a+b*ln((c*x^n)^(1/2)))-12*b*n*tan(1/2*a+b*ln((c*x^
n)^(1/2)))^5+4*tan(1/2*a+b*ln((c*x^n)^(1/2)))^6-7*b^2*n^2)/(9*b^2*n^2+4)/(
1+tan(1/2*a+b*ln((c*x^n)^(1/2)))^2)^3/(b^2*n^2+4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.82

$$\int x\cos^3\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{12\,b^2n^2x^2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+2\,(b^2n^2+4)x^2\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3+3\,\left(2\,b^3n^3x^2+\right.}{9\,b^4n^4+40\,b^2n^2+16}$$

input
```
integrate(x*cos(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
(12*b^2*n^2*x^2*cos(b*n*log(x) + b*log(c) + a) + 2*(b^2*n^2 + 4)*x^2*cos(b
*n*log(x) + b*log(c) + a)^3 + 3*(2*b^3*n^3*x^2 + (b^3*n^3 + 4*b*n)*x^2*cos
(b*n*log(x) + b*log(c) + a)^2)*sin(b*n*log(x) + b*log(c) + a))/(9*b^4*n^4
+ 40*b^2*n^2 + 16)
```

Sympy [F]

$$\int x\cos^3\left(a+b\log\left(cx^n\right)\right)\,dx = \begin{cases} \int x\cos^3\left(a-\frac{2i\log\left(cx^n\right)}{n}\right)dx \\ \int x\cos^3\left(a-\frac{2i\log\left(cx^n\right)}{3n}\right)dx \\ \int x\cos^3\left(a+\frac{2i\log\left(cx^n\right)}{3n}\right)dx \\ \int x\cos^3\left(a+\frac{2i\log\left(cx^n\right)}{n}\right)dx \\ \frac{6b^3n^3x^2\sin^3\left(a+b\log\left(cx^n\right)\right)}{9b^4n^4+40b^2n^2+16}+\frac{9b^3n^3x^2\sin\left(a+b\log\left(cx^n\right)\right)\cos^2\left(a+b\log\left(cx^n\right)\right)}{9b^4n^4+40b^2n^2+16}+\frac{12b^2n^2x^2\sin^2\left(a+b\log\left(cx^n\right)\right)\cos\left(a+b\log\left(cx^n\right)\right)}{9b^4n^4+40b^2n^2+16}+ \end{cases}$$

input
```
integrate(x*cos(a+b*ln(c*x**n))**3,x)
```

output
```
Piecewise((Integral(x*cos(a - 2*I*log(c*x**n)/n)**3, x), Eq(b, -2*I/n)), (
Integral(x*cos(a - 2*I*log(c*x**n)/(3*n))**3, x), Eq(b, -2*I/(3*n))), (Int
egral(x*cos(a + 2*I*log(c*x**n)/(3*n))**3, x), Eq(b, 2*I/(3*n))), (Integra
l(x*cos(a + 2*I*log(c*x**n)/n)**3, x), Eq(b, 2*I/n)), (6*b**3*n**3*x**2*si
n(a + b*log(c*x**n))**3/(9*b**4*n**4 + 40*b**2*n**2 + 16) + 9*b**3*n**3*x*
*2*sin(a + b*log(c*x**n))*cos(a + b*log(c*x**n))**2/(9*b**4*n**4 + 40*b**2
*n**2 + 16) + 12*b**2*n**2*x**2*sin(a + b*log(c*x**n))**2*cos(a + b*log(c*
x**n))/(9*b**4*n**4 + 40*b**2*n**2 + 16) + 14*b**2*n**2*x**2*cos(a + b*log
(c*x**n))**3/(9*b**4*n**4 + 40*b**2*n**2 + 16) + 12*b*n*x**2*sin(a + b*log
(c*x**n))*cos(a + b*log(c*x**n))**2/(9*b**4*n**4 + 40*b**2*n**2 + 16) + 8*
x**2*cos(a + b*log(c*x**n))**3/(9*b**4*n**4 + 40*b**2*n**2 + 16), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. 2(158) = 316.

Time = 0.09 (sec) , antiderivative size = 1015, normalized size of antiderivative = 6.42

$$\int x\cos^3(a+b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(x*cos(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
1/8*((3*(b^3*cos(3*b*log(c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(3*b
*log(c)) + b^3*sin(3*b*log(c)))*n^3 + 2*(b^2*cos(6*b*log(c))*cos(3*b*log(c
)) + b^2*sin(6*b*log(c))*sin(3*b*log(c)) + b^2*cos(3*b*log(c)))*n^2 + 12*(
b*cos(3*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(3*b*log(c)) + b*
sin(3*b*log(c)))*n + 8*cos(6*b*log(c))*cos(3*b*log(c)) + 8*sin(6*b*log(c))
*sin(3*b*log(c)) + 8*cos(3*b*log(c)))*x^2*cos(3*b*log(x^n) + 3*a) + 3*(9*(
b^3*cos(3*b*log(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(3*b*log(c))
+ b^3*cos(2*b*log(c))*sin(3*b*log(c)) - b^3*cos(3*b*log(c))*sin(2*b*log(c)
))*n^3 + 18*(b^2*cos(4*b*log(c))*cos(3*b*log(c)) + b^2*cos(3*b*log(c))*cos
(2*b*log(c)) + b^2*sin(4*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c))*s
in(2*b*log(c)))*n^2 + 4*(b*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log
(c))*sin(3*b*log(c)) + b*cos(2*b*log(c))*sin(3*b*log(c)) - b*cos(3*b*log(c
))*sin(2*b*log(c)))*n + 8*cos(4*b*log(c))*cos(3*b*log(c)) + 8*cos(3*b*log(
c))*cos(2*b*log(c)) + 8*sin(4*b*log(c))*sin(3*b*log(c)) + 8*sin(3*b*log(c)
)*sin(2*b*log(c)))*x^2*cos(b*log(x^n) + a) + (3*(b^3*cos(6*b*log(c))*cos(3
*b*log(c)) + b^3*sin(6*b*log(c))*sin(3*b*log(c)) + b^3*cos(3*b*log(c)))*n^
3 - 2*(b^2*cos(3*b*log(c))*sin(6*b*log(c)) - b^2*cos(6*b*log(c))*sin(3*b*l
og(c)) + b^2*sin(3*b*log(c)))*n^2 + 12*(b*cos(6*b*log(c))*cos(3*b*log(c))
+ b*sin(6*b*log(c))*sin(3*b*log(c)) + b*cos(3*b*log(c)))*n - 8*cos(3*b*log
(c))*sin(6*b*log(c)) + 8*cos(6*b*log(c))*sin(3*b*log(c)) - 8*sin(3*b*lo...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18069 vs. $2(158) = 316$.

Time = 1.01 (sec) , antiderivative size = 18069, normalized size of antiderivative = 114.36

$$\int x\cos^3\left(a+b\log\left(cx^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x*cos(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
-1/4*(27*b^3*n^3*x^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) -
 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(
abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a) + 27*b^3*n^3*x^2*e^
(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n
*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(ab
s(c)))^2*tan(3/2*a)^2*tan(1/2*a) + 3*b^3*n^3*x^2*e^(3/2*pi*b*n*sgn(x) - 3/
2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log
(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)*tan(
1/2*a)^2 + 3*b^3*n^3*x^2*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn
(c) + 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n
*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)*tan(1/2*a)^2 + 27*b^3*n^3*x
^2*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2
*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*lo
g(abs(c)))*tan(3/2*a)^2*tan(1/2*a)^2 + 27*b^3*n^3*x^2*e^(-1/2*pi*b*n*sgn(x
) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2
*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(3/2*a)^
2*tan(1/2*a)^2 + 3*b^3*n^3*x^2*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*
b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))*tan(1/2*
b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 + 3*b^3*n
^3*x^2*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b)...
```

Mupad [B] (verification not implemented)

Time = 22.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.77

$$\int x\cos^3(a+b\log(cx^n))\,dx = \frac{x^2\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,1\mathrm{i}}}\,3\mathrm{i}}{8\,b\,n+16\mathrm{i}}+\frac{3\,x^2\,\mathrm{e}^{a\,1\mathrm{i}}\,(c\,x^n)^{b\,1\mathrm{i}}}{16+b\,n\,8\mathrm{i}}+\frac{x^2\,\mathrm{e}^{-a\,3\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,3\mathrm{i}}}\,1\mathrm{i}}{24\,b\,n+16\mathrm{i}}+\frac{x^2\,\mathrm{e}^{a\,3\mathrm{i}}\,(c\,x^n)^{b\,3\mathrm{i}}}{16+b\,n\,24\mathrm{i}}$$

input
```
int(x*cos(a + b*log(c*x^n))^3,x)
```

output
```
(x^2*exp(-a*1i)/(c*x^n)^(b*1i)*3i)/(8*b*n + 16i) + (3*x^2*exp(a*1i)*(c*x^n
)^(b*1i))/(b*n*8i + 16) + (x^2*exp(-a*3i)/(c*x^n)^(b*3i)*1i)/(24*b*n + 16i
) + (x^2*exp(a*3i)*(c*x^n)^(b*3i))/(b*n*24i + 16)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.18

$$\int x\cos^3\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$= \frac{x^2\left(-2\cos\left(\log(x^nc)\,b+a\right)\sin\left(\log(x^nc)\,b+a\right)^2b^2n^2-8\cos\left(\log(x^nc)\,b+a\right)\sin\left(\log(x^nc)\,b+a\right)^2+14\cos\right.}{}$$

input
```
int(x*cos(a+b*log(c*x^n))^3,x)
```

output
```
(x**2*( - 2*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**2*b**2*n**2 - 8
*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**2 + 14*cos(log(x**n*c)*b +
 a)*b**2*n**2 + 8*cos(log(x**n*c)*b + a) - 3*sin(log(x**n*c)*b + a)**3*b**
3*n**3 - 12*sin(log(x**n*c)*b + a)**3*b*n + 9*sin(log(x**n*c)*b + a)*b**3*
n**3 + 12*sin(log(x**n*c)*b + a)*b*n))/(9*b**4*n**4 + 40*b**2*n**2 + 16)
```

3.98 $\int \cos^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 149

$$\int \cos^3(a+b\log(cx^n))\,dx = \frac{6b^2n^2x\cos(a+b\log(cx^n))}{1+10b^2n^2+9b^4n^4} + \frac{x\cos^3(a+b\log(cx^n))}{1+9b^2n^2} + \frac{6b^3n^3x\sin(a+b\log(cx^n))}{1+10b^2n^2+9b^4n^4} + \frac{3bnx\cos^2(a+b\log(cx^n))\sin(a+b\log(cx^n))}{1+9b^2n^2}$$

output
```
6*b^2*n^2*x*cos(a+b*ln(c*x^n))/(9*b^4*n^4+10*b^2*n^2+1)+x*cos(a+b*ln(c*x^n
))^3/(9*b^2*n^2+1)+6*b^3*n^3*x*sin(a+b*ln(c*x^n))/(9*b^4*n^4+10*b^2*n^2+1)
+3*b*n*x*cos(a+b*ln(c*x^n))^2*sin(a+b*ln(c*x^n))/(9*b^2*n^2+1)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.79

$$\int \cos^3(a+b\log(cx^n))\,dx = \frac{x(3(1+9b^2n^2)\cos(a+b\log(cx^n))+(1+b^2n^2)\cos(3(a+b\log(cx^n)))+6bn(1+5b^2n^2+(1+b^2n^2)\,\mathrm{co}}{4+40b^2n^2+36b^4n^4}$$

input
```
Integrate[Cos[a + b*Log[c*x^n]]^3,x]
```

output
```
(x*(3*(1 + 9*b^2*n^2)*Cos[a + b*Log[c*x^n]] + (1 + b^2*n^2)*Cos[3*(a + b*L
og[c*x^n])] + 6*b*n*(1 + 5*b^2*n^2 + (1 + b^2*n^2)*Cos[2*(a + b*Log[c*x^n]
)])*Sin[a + b*Log[c*x^n]]))/(4 + 40*b^2*n^2 + 36*b^4*n^4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4981, 4979}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{4981}$$

$$\frac{6b^2n^2\int\cos(a+b\log(cx^n))\,dx}{9b^2n^2+1}+\frac{x\cos^3(a+b\log(cx^n))}{9b^2n^2+1}+\frac{3bnx\sin(a+b\log(cx^n))\cos^2(a+b\log(cx^n))}{9b^2n^2+1}$$

$$\Big\downarrow \text{4979}$$

$$\frac{x\cos^3(a+b\log(cx^n))}{9b^2n^2+1}+\frac{3bnx\sin(a+b\log(cx^n))\cos^2(a+b\log(cx^n))}{9b^2n^2+1}+\frac{6b^2n^2\left(\frac{bnx\sin(a+b\log(cx^n))}{b^2n^2+1}+\frac{x\cos(a+b\log(cx^n))}{b^2n^2+1}\right)}{9b^2n^2+1}$$

input
```
Int[Cos[a + b*Log[c*x^n]]^3,x]
```

output
```
(x*Cos[a + b*Log[c*x^n]]^3)/(1 + 9*b^2*n^2) + (3*b*n*x*Cos[a + b*Log[c*x^n
]]^2*Sin[a + b*Log[c*x^n]])/(1 + 9*b^2*n^2) + (6*b^2*n^2*((x*Cos[a + b*Log
[c*x^n]])/(1 + b^2*n^2) + (b*n*x*Sin[a + b*Log[c*x^n]])/(1 + b^2*n^2)))/(1
 + 9*b^2*n^2)
```

Defintions of rubi rules used

rule 4979
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] + Simp[b*d*n*x*(Sin[d*(a +
 b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

rule 4981
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Sim
p[x*(Cos[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 + 1)), x] + (Simp[b*d*n*p
*x*Cos[d*(a + b*Log[c*x^n])]^(p - 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^
2*p^2 + 1)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + 1))   Int[
Cos[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b, c, d, n}, x] &&
IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + 1, 0]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.28

method	result
default	$\frac{3\,\mathrm{e}^{\frac{\ln(c\,x^{n})}{n}-\frac{\ln(c)}{n}}\cos(a+b\ln(c\,x^{n}))}{4n^{2}\left(\frac{1}{n^{2}}+b^{2}\right)}+\frac{3b\,\mathrm{e}^{\frac{\ln(c\,x^{n})}{n}-\frac{\ln(c)}{n}}\sin(a+b\ln(c\,x^{n}))}{4n\left(\frac{1}{n^{2}}+b^{2}\right)}+\frac{\mathrm{e}^{\frac{\ln(c\,x^{n})}{n}-\frac{\ln(c)}{n}}\cos(3b\ln(c\,x^{n})+3a)}{4n^{2}\left(\frac{1}{n^{2}}+9b^{2}\right)}+3b$
parallelrisch	$\frac{x\left(1+12\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)^{3}b^{3}n^{3}+18b^{3}n^{3}\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)+6bn\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)^{5}+18\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)^{5}b^{3}n^{3}\right.}{}$

input
```
int(cos(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
```

output
```
3/4/n^2/(1/n^2+b^2)*exp(1/n*ln(c*x^n)-1/n*ln(c))*cos(a+b*ln(c*x^n))+3/4/n*
b/(1/n^2+b^2)*exp(1/n*ln(c*x^n)-1/n*ln(c))*sin(a+b*ln(c*x^n))+1/4/n^2/(1/n
^2+9*b^2)*exp(1/n*ln(c*x^n)-1/n*ln(c))*cos(3*b*ln(c*x^n)+3*a)+3/4/n*b/(1/n
^2+9*b^2)*exp(1/n*ln(c*x^n)-1/n*ln(c))*sin(3*b*ln(c*x^n)+3*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.80

$$\int \cos^3(a + b\log(cx^n))\,dx = \frac{6\,b^2n^2x\cos(bn\log(x) + b\log(c) + a) + (b^2n^2 + 1)x\cos(bn\log(x) + b\log(c) + a)^3 + 3\left(2\,b^3n^3x + (b^3n^3\right.}{9\,b^4n^4 + 10\,b^2n^2 + 1}$$

input
```
integrate(cos(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
(6*b^2*n^2*x*cos(b*n*log(x) + b*log(c) + a) + (b^2*n^2 + 1)*x*cos(b*n*log(
x) + b*log(c) + a)^3 + 3*(2*b^3*n^3*x + (b^3*n^3 + b*n)*x*cos(b*n*log(x) +
 b*log(c) + a)^2)*sin(b*n*log(x) + b*log(c) + a))/(9*b^4*n^4 + 10*b^2*n^2
+ 1)
```

Sympy [F]

$$\int \cos^3(a + b\log(cx^n))\,dx = \begin{cases} \int \cos^3\left(a - \frac{i\log(cx^n)}{n}\right)dx \\ \int \cos^3\left(a - \frac{i\log(cx^n)}{3n}\right)dx \\ \int \cos^3\left(a + \frac{i\log(cx^n)}{3n}\right)dx \\ \int \cos^3\left(a + \frac{i\log(cx^n)}{n}\right)dx \\ \frac{6b^3n^3x\sin^3(a+b\log(cx^n))}{9b^4n^4+10b^2n^2+1} + \frac{9b^3n^3x\sin(a+b\log(cx^n))\cos^2(a+b\log(cx^n))}{9b^4n^4+10b^2n^2+1} + \frac{6b^2n^2x\sin^2(a+b\log(cx^n))\cos(a+b\log(cx^n))}{9b^4n^4+10b^2n^2+1} + \frac{7b^2n}{} \end{cases}$$

input
```
integrate(cos(a+b*ln(c*x**n))**3,x)
```

output
```
Piecewise((Integral(cos(a - I*log(c*x**n)/n)**3, x), Eq(b, -I/n)), (Integr
al(cos(a - I*log(c*x**n)/(3*n))**3, x), Eq(b, -I/(3*n))), (Integral(cos(a
+ I*log(c*x**n)/(3*n))**3, x), Eq(b, I/(3*n))), (Integral(cos(a + I*log(c*
x**n)/n)**3, x), Eq(b, I/n)), (6*b**3*n**3*x*sin(a + b*log(c*x**n))**3/(9*
b**4*n**4 + 10*b**2*n**2 + 1) + 9*b**3*n**3*x*sin(a + b*log(c*x**n))*cos(a
 + b*log(c*x**n))**2/(9*b**4*n**4 + 10*b**2*n**2 + 1) + 6*b**2*n**2*x*sin(
a + b*log(c*x**n))**2*cos(a + b*log(c*x**n))/(9*b**4*n**4 + 10*b**2*n**2 +
 1) + 7*b**2*n**2*x*cos(a + b*log(c*x**n))**3/(9*b**4*n**4 + 10*b**2*n**2
+ 1) + 3*b*n*x*sin(a + b*log(c*x**n))*cos(a + b*log(c*x**n))**2/(9*b**4*n*
*4 + 10*b**2*n**2 + 1) + x*cos(a + b*log(c*x**n))**3/(9*b**4*n**4 + 10*b**
2*n**2 + 1), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 989 vs. $2(149) = 298$.

Time = 0.08 (sec) , antiderivative size = 989, normalized size of antiderivative = 6.64

$$\int \cos^3(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(cos(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
1/8*((3*(b^3*cos(3*b*log(c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(3*b
*log(c)) + b^3*sin(3*b*log(c)))*n^3 + (b^2*cos(6*b*log(c))*cos(3*b*log(c))
 + b^2*sin(6*b*log(c))*sin(3*b*log(c)) + b^2*cos(3*b*log(c)))*n^2 + 3*(b*c
os(3*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(3*b*log(c)) + b*sin
(3*b*log(c)))*n + cos(6*b*log(c))*cos(3*b*log(c)) + sin(6*b*log(c))*sin(3*
b*log(c)) + cos(3*b*log(c)))*x*cos(3*b*log(x^n) + 3*a) + 3*(9*(b^3*cos(3*b
*log(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(3*b*log(c)) + b^3*cos(2
*b*log(c))*sin(3*b*log(c)) - b^3*cos(3*b*log(c))*sin(2*b*log(c)))*n^3 + 9*
(b^2*cos(4*b*log(c))*cos(3*b*log(c)) + b^2*cos(3*b*log(c))*cos(2*b*log(c))
 + b^2*sin(4*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c))*sin(2*b*log(c
)))*n^2 + (b*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(3*b*l
og(c)) + b*cos(2*b*log(c))*sin(3*b*log(c)) - b*cos(3*b*log(c))*sin(2*b*log
(c)))*n + cos(4*b*log(c))*cos(3*b*log(c)) + cos(3*b*log(c))*cos(2*b*log(c)
) + sin(4*b*log(c))*sin(3*b*log(c)) + sin(3*b*log(c))*sin(2*b*log(c)))*x*c
os(b*log(x^n) + a) + (3*(b^3*cos(6*b*log(c))*cos(3*b*log(c)) + b^3*sin(6*b
*log(c))*sin(3*b*log(c)) + b^3*cos(3*b*log(c)))*n^3 - (b^2*cos(3*b*log(c))
*sin(6*b*log(c)) - b^2*cos(6*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c
)))*n^2 + 3*(b*cos(6*b*log(c))*cos(3*b*log(c)) + b*sin(6*b*log(c))*sin(3*b
*log(c)) + b*cos(3*b*log(c)))*n - cos(3*b*log(c))*sin(6*b*log(c)) + cos(6*
b*log(c))*sin(3*b*log(c)) - sin(3*b*log(c)))*x*sin(3*b*log(x^n) + 3*a) ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17458 vs. $2(149) = 298$.

Time = 0.59 (sec) , antiderivative size = 17458, normalized size of antiderivative = 117.17

$$\int \cos^3(a + b\log(cx^n))\, dx = \text{Too large to display}$$

input
```
integrate(cos(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
-1/8*(54*b^3*n^3*x*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1
/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(ab
s(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a) + 54*b^3*n^3*x*e^(-1/
2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log
(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)
))^2*tan(3/2*a)^2*tan(1/2*a) + 6*b^3*n^3*x*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b
*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c
)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)*tan(1/2*a)
^2 + 6*b^3*n^3*x*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/
2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs
(x)) + 1/2*b*log(abs(c)))^2*tan(3/2*a)*tan(1/2*a)^2 + 54*b^3*n^3*x*e^(1/2*
pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(a
bs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))
*tan(3/2*a)^2*tan(1/2*a)^2 + 54*b^3*n^3*x*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b
*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c
)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(3/2*a)^2*tan(1/2*a)
^2 + 6*b^3*n^3*x*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2
*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))*tan(1/2*b*n*log(abs(x)
) + 1/2*b*log(abs(c)))^2*tan(3/2*a)^2*tan(1/2*a)^2 + 6*b^3*n^3*x*e^(-3/2*p
i*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan(3/2*b*n*log...
```

Mupad [B] (verification not implemented)

Time = 21.31 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.77

$$\int \cos^3(a+b\log(cx^n))\,dx = \frac{x\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,1\mathrm{i}}}\,3\mathrm{i}}{8\,b\,n+8\mathrm{i}}+\frac{3\,x\,\mathrm{e}^{a\,1\mathrm{i}}\,(c\,x^n)^{b\,1\mathrm{i}}}{8+b\,n\,8\mathrm{i}}+\frac{x\,\mathrm{e}^{-a\,3\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,3\mathrm{i}}}\,1\mathrm{i}}{24\,b\,n+8\mathrm{i}}+\frac{x\,\mathrm{e}^{a\,3\mathrm{i}}\,(c\,x^n)^{b\,3\mathrm{i}}}{8+b\,n\,24\mathrm{i}}$$

input
```
int(cos(a + b*log(c*x^n))^3,x)
```

output
```
(x*exp(-a*1i)/(c*x^n)^(b*1i)*3i)/(8*b*n + 8i) + (3*x*exp(a*1i)*(c*x^n)^(b*
1i))/(b*n*8i + 8) + (x*exp(-a*3i)/(c*x^n)^(b*3i)*1i)/(24*b*n + 8i) + (x*ex
p(a*3i)*(c*x^n)^(b*3i))/(b*n*24i + 8)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.23

$$\int \cos^3(a + b\log(cx^n)) \, dx$$
$$= \frac{x\left(-\cos(\log(x^n c)\,b + a)\sin(\log(x^n c)\,b + a)^2 b^2 n^2 - \cos(\log(x^n c)\,b + a)\sin(\log(x^n c)\,b + a)^2 + 7\cos(\log$$

input
```
int(cos(a+b*log(c*x^n))^3,x)
```

output
```
(x*( - cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**2*b**2*n**2 - cos(lo
g(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**2 + 7*cos(log(x**n*c)*b + a)*b**2
*n**2 + cos(log(x**n*c)*b + a) - 3*sin(log(x**n*c)*b + a)**3*b**3*n**3 - 3
*sin(log(x**n*c)*b + a)**3*b*n + 9*sin(log(x**n*c)*b + a)*b**3*n**3 + 3*si
n(log(x**n*c)*b + a)*b*n))/(9*b**4*n**4 + 10*b**2*n**2 + 1)
```

3.99 $\int \frac{\cos^3(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{\cos^3(a+b\log(cx^n))}{x}\,dx=\frac{\sin(a+b\log(cx^n))}{bn}-\frac{\sin^3(a+b\log(cx^n))}{3bn}$$

output
```
sin(a+b*ln(c*x^n))/b/n-1/3*sin(a+b*ln(c*x^n))^3/b/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\cos^3(a+b\log(cx^n))}{x}\,dx=\frac{\sin(a+b\log(cx^n))}{bn}-\frac{\sin^3(a+b\log(cx^n))}{3bn}$$

input
```
Integrate[Cos[a + b*Log[c*x^n]]^3/x,x]
```

output
```
Sin[a + b*Log[c*x^n]]/(b*n) - Sin[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3039, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \cos^3(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(a+b\log(cx^n)+\frac{\pi}{2}\right)^3d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{\int \left(1-\sin^2(a+b\log(cx^n))\right)d(-\sin(a+b\log(cx^n)))}{bn}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{3}\sin^3(a+b\log(cx^n))-\sin(a+b\log(cx^n))}{bn}$$

input
```
Int[Cos[a + b*Log[c*x^n]]^3/x,x]
```

output
```
-((-Sin[a + b*Log[c*x^n]] + Sin[a + b*Log[c*x^n]]^3/3)/(b*n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\left(2+\cos(a+b\ln(c\,x^n))^2\right)\sin(a+b\ln(c\,x^n))}{3nb}$	35
default	$\frac{\left(2+\cos(a+b\ln(c\,x^n))^2\right)\sin(a+b\ln(c\,x^n))}{3nb}$	35
parallelrisch	$\frac{\sin(3b\ln(c\,x^n)+3a)+9\sin(a+b\ln(c\,x^n))}{12bn}$	37

input
```
int(cos(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/3/n/b*(2+cos(a+b*ln(c*x^n))^2)*sin(a+b*ln(c*x^n))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{\cos^3(a+b\log(cx^n))}{x}\,dx = \frac{\left(\cos(bn\log(x)+b\log(c)+a)^2+2\right)\sin(bn\log(x)+b\log(c)+a)}{3\,bn}$$

input
```
integrate(cos(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

output
```
1/3*(cos(b*n*log(x) + b*log(c) + a)^2 + 2)*sin(b*n*log(x) + b*log(c) + a)/
(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(32) = 64.

Time = 1.43 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.69

$$\int \frac{\cos^3(a+b\log(cx^n))}{x}\,dx = \begin{cases} \log(x)\cos^3(a) & \text{for } b=0 \wedge (b=0 \vee n=0) \\ \log(x)\cos^3(a+b\log(c)) & \text{for } n=0 \\ \frac{2\sin^3(a+b\log(cx^n))}{3bn} + \frac{\sin(a+b\log(cx^n))\cos^2(a+b\log(cx^n))}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(a+b*ln(c*x**n))**3/x,x)
```

output
```
Piecewise((log(x)*cos(a)**3, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*co
s(a + b*log(c))**3, Eq(n, 0)), (2*sin(a + b*log(c*x**n))**3/(3*b*n) + sin(
a + b*log(c*x**n))*cos(a + b*log(c*x**n))**2/(b*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(40) = 80$.

Time = 0.07 (sec) , antiderivative size = 232, normalized size of antiderivative = 5.52

$$\int \frac{\cos^3(a + b\log(cx^n))}{x}\,dx$$
$$= \frac{(\cos(3\,b\log(c))\sin(6\,b\log(c)) - \cos(6\,b\log(c))\sin(3\,b\log(c)) + \sin(3\,b\log(c)))\cos(3\,b\log(x^n) + 3\,a)}{}$$

input
```
integrate(cos(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

output
```
1/24*((cos(3*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(3*b*log(c)) +
 sin(3*b*log(c)))*cos(3*b*log(x^n) + 3*a) + 9*(cos(3*b*log(c))*sin(4*b*log
(c)) - cos(4*b*log(c))*sin(3*b*log(c)) + cos(2*b*log(c))*sin(3*b*log(c)) -
 cos(3*b*log(c))*sin(2*b*log(c)))*cos(b*log(x^n) + a) + (cos(6*b*log(c))*c
os(3*b*log(c)) + sin(6*b*log(c))*sin(3*b*log(c)) + cos(3*b*log(c)))*sin(3*
b*log(x^n) + 3*a) + 9*(cos(4*b*log(c))*cos(3*b*log(c)) + cos(3*b*log(c))*c
os(2*b*log(c)) + sin(4*b*log(c))*sin(3*b*log(c)) + sin(3*b*log(c))*sin(2*b
*log(c)))*sin(b*log(x^n) + a))/(b*n)
```

Giac [F]

$$\int \frac{\cos^3(a + b\log(cx^n))}{x}\,dx = \int \frac{\cos(b\log(cx^n) + a)^3}{x}\,dx$$

input
```
integrate(cos(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

output
```
integrate(cos(b*log(c*x^n) + a)^3/x, x)
```

Mupad [B] (verification not implemented)

Time = 19.63 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{\cos^3(a+b\log(cx^n))}{x}\,dx = \frac{3\,\sin(a+b\,\ln(c\,x^n))-\sin(a+b\,\ln(c\,x^n))^3}{3\,b\,n}$$

input
```
int(cos(a + b*log(c*x^n))^3/x,x)
```

output
```
(3*sin(a + b*log(c*x^n)) - sin(a + b*log(c*x^n))^3)/(3*b*n)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{\cos^3(a+b\log(cx^n))}{x}\,dx = \frac{\sin(\log(x^nc)\,b+a)\left(-\sin(\log(x^nc)\,b+a)^2+3\right)}{3bn}$$

input
```
int(cos(a+b*log(c*x^n))^3/x,x)
```

output
```
(sin(log(x**n*c)*b + a)*( - sin(log(x**n*c)*b + a)**2 + 3))/(3*b*n)
```

3.100 $\int \frac{\cos^3(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 158

$$\int \frac{\cos^3(a+b\log(cx^n))}{x^2}\,dx = -\frac{6b^2n^2\cos(a+b\log(cx^n))}{(1+10b^2n^2+9b^4n^4)\,x} - \frac{\cos^3(a+b\log(cx^n))}{(1+9b^2n^2)\,x} + \frac{6b^3n^3\sin(a+b\log(cx^n))}{(1+10b^2n^2+9b^4n^4)\,x} + \frac{3bn\cos^2(a+b\log(cx^n))\sin(a+b\log(cx^n))}{(1+9b^2n^2)\,x}$$

output
```
-6*b^2*n^2*cos(a+b*ln(c*x^n))/(9*b^4*n^4+10*b^2*n^2+1)/x-cos(a+b*ln(c*x^n)
)^3/(9*b^2*n^2+1)/x+6*b^3*n^3*sin(a+b*ln(c*x^n))/(9*b^4*n^4+10*b^2*n^2+1)/
x+3*b*n*cos(a+b*ln(c*x^n))^2*sin(a+b*ln(c*x^n))/(9*b^2*n^2+1)/x
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.77

$$\int \frac{\cos^3(a+b\log(cx^n))}{x^2}\,dx = -\frac{3(1+9b^2n^2)\cos(a+b\log(cx^n)) + (1+b^2n^2)\cos(3(a+b\log(cx^n))) - 6bn(1+5b^2n^2+(1+b^2n^2)\,\mathrm{co}}{4(1+10b^2n^2+9b^4n^4)\,x}$$

input
```
Integrate[Cos[a + b*Log[c*x^n]]^3/x^2,x]
```

output
```
-1/4*(3*(1 + 9*b^2*n^2)*Cos[a + b*Log[c*x^n]] + (1 + b^2*n^2)*Cos[3*(a + b
*Log[c*x^n])] - 6*b*n*(1 + 5*b^2*n^2 + (1 + b^2*n^2)*Cos[2*(a + b*Log[c*x^
n])])*Sin[a + b*Log[c*x^n]])/((1 + 10*b^2*n^2 + 9*b^4*n^4)*x)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4991, 4989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+b\log(cx^n))}{x^2}\,dx$$

$$\Big\downarrow \text{4991}$$

$$\frac{6b^2n^2\int \frac{\cos(a+b\log(cx^n))}{x^2}dx}{9b^2n^2+1}-\frac{\cos^3(a+b\log(cx^n))}{x(9b^2n^2+1)}+\frac{3bn\sin(a+b\log(cx^n))\cos^2(a+b\log(cx^n))}{x(9b^2n^2+1)}$$

$$\Big\downarrow \text{4989}$$

$$-\frac{\cos^3(a+b\log(cx^n))}{x(9b^2n^2+1)}+\frac{3bn\sin(a+b\log(cx^n))\cos^2(a+b\log(cx^n))}{x(9b^2n^2+1)}+\frac{6b^2n^2\left(\frac{bn\sin(a+b\log(cx^n))}{x(b^2n^2+1)}-\frac{\cos(a+b\log(cx^n))}{x(b^2n^2+1)}\right)}{9b^2n^2+1}$$

input
```
Int[Cos[a + b*Log[c*x^n]]^3/x^2,x]
```

output
```
-(Cos[a + b*Log[c*x^n]]^3/((1 + 9*b^2*n^2)*x)) + (3*b*n*Cos[a + b*Log[c*x^
n]]^2*Sin[a + b*Log[c*x^n]])/((1 + 9*b^2*n^2)*x) + (6*b^2*n^2*(-(Cos[a + b
*Log[c*x^n]]/((1 + b^2*n^2)*x)) + (b*n*Sin[a + b*Log[c*x^n]])/((1 + b^2*n^
2)*x)))/(1 + 9*b^2*n^2)
```

Defintions of rubi rules used

rule 4989
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

rule 4991
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Simp[b*d*n*p*(e*x)^(m + 1)*Sin[d*(a
+ b*Log[c*x^n])]*(Cos[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*
(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2)
)   Int[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [A] (verified)

Time = 4.06 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.49

method	result
parallelrisch	$\frac{-1+(7b^2n^2+1)\tan(\frac{a}{2}+b\ln(\sqrt{c\,x^n}))^6+6(3b^3n^3+bn)\tan(\frac{a}{2}+b\ln(\sqrt{c\,x^n}))^5+3(b^2n^2-1)\tan(\frac{a}{2}+b\ln(\sqrt{c\,x^n}))^4+12(b^3n^3-bn}{9(b^2n^2+1)x\left(1+\tan(\frac{a}{2}+b\ln(\sqrt{c\,x^n}\right.}$

input
```
int(cos(a+b*ln(c*x^n))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*(-1+(7*b^2*n^2+1)*tan(1/2*a+b*ln((c*x^n)^(1/2)))^6+6*(3*b^3*n^3+b*n)*t
an(1/2*a+b*ln((c*x^n)^(1/2)))^5+3*(b^2*n^2-1)*tan(1/2*a+b*ln((c*x^n)^(1/2)
))^4+12*(b^3*n^3-b*n)*tan(1/2*a+b*ln((c*x^n)^(1/2)))^3+3*(-b^2*n^2+1)*tan(
1/2*a+b*ln((c*x^n)^(1/2)))^2+6*(3*b^3*n^3+b*n)*tan(1/2*a+b*ln((c*x^n)^(1/2
)))-7*b^2*n^2)/(b^2*n^2+1)/x/(1+tan(1/2*a+b*ln((c*x^n)^(1/2)))^2)^3/(1/9+b
^2*n^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.75

$$\int \frac{\cos^3(a+b\log(cx^n))}{x^2}\,dx = -\frac{6\,b^2n^2\cos(bn\log(x)+b\log(c)+a)+(b^2n^2+1)\cos(bn\log(x)+b\log(c)+a)^3-3\left(2\,b^3n^3+(b^3n^3+\ldots\right.}{(9\,b^4n^4+10\,b^2n^2+1)x}$$

input
```
integrate(cos(a+b*log(c*x^n))^3/x^2,x, algorithm="fricas")
```

output
```
-(6*b^2*n^2*cos(b*n*log(x) + b*log(c) + a) + (b^2*n^2 + 1)*cos(b*n*log(x)
+ b*log(c) + a)^3 - 3*(2*b^3*n^3 + (b^3*n^3 + b*n)*cos(b*n*log(x) + b*log(
c) + a)^2)*sin(b*n*log(x) + b*log(c) + a))/((9*b^4*n^4 + 10*b^2*n^2 + 1)*x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 25.44 (sec) , antiderivative size = 770, normalized size of antiderivative = 4.87

$$\int \frac{\cos^3(a+b\log(cx^n))}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(a+b*ln(c*x**n))**3/x**2,x)
```

output
```
Piecewise((3*I*sin(3*a - 3*I*log(c*x**n)/n)/(32*x) - 3*cos(a - I*log(c*x**
n)/n)/(8*x) + cos(3*a - 3*I*log(c*x**n)/n)/(32*x) + 3*I*log(c*x**n)*sin(a
- I*log(c*x**n)/n)/(8*n*x) + 3*log(c*x**n)*cos(a - I*log(c*x**n)/n)/(8*n*x
), Eq(b, -I/n)), (-9*I*sin(a - I*log(c*x**n)/(3*n))/(32*x) + I*sin(3*a - I
*log(c*x**n)/n)/(8*x) - 27*cos(a - I*log(c*x**n)/(3*n))/(32*x) + I*log(c*x
**n)*sin(3*a - I*log(c*x**n)/n)/(8*n*x) + log(c*x**n)*cos(3*a - I*log(c*x*
*n)/n)/(8*n*x), Eq(b, -I/(3*n))), (9*I*sin(a + I*log(c*x**n)/(3*n))/(32*x)
 - 27*cos(a + I*log(c*x**n)/(3*n))/(32*x) - cos(3*a + I*log(c*x**n)/n)/(8*
x) - I*log(c*x**n)*sin(3*a + I*log(c*x**n)/n)/(8*n*x) + log(c*x**n)*cos(3*
a + I*log(c*x**n)/n)/(8*n*x), Eq(b, I/(3*n))), (-3*I*sin(3*a + 3*I*log(c*x
**n)/n)/(32*x) - 3*cos(a + I*log(c*x**n)/n)/(8*x) + cos(3*a + 3*I*log(c*x*
*n)/n)/(32*x) - 3*I*log(c*x**n)*sin(a + I*log(c*x**n)/n)/(8*n*x) + 3*log(c
*x**n)*cos(a + I*log(c*x**n)/n)/(8*n*x), Eq(b, I/n)), (6*b**3*n**3*sin(a +
 b*log(c*x**n))**3/(9*b**4*n**4*x + 10*b**2*n**2*x + x) + 9*b**3*n**3*sin(
a + b*log(c*x**n))*cos(a + b*log(c*x**n))**2/(9*b**4*n**4*x + 10*b**2*n**2
*x + x) - 6*b**2*n**2*sin(a + b*log(c*x**n))**2*cos(a + b*log(c*x**n))/(9*
b**4*n**4*x + 10*b**2*n**2*x + x) - 7*b**2*n**2*cos(a + b*log(c*x**n))**3/
(9*b**4*n**4*x + 10*b**2*n**2*x + x) + 3*b*n*sin(a + b*log(c*x**n))*cos(a
+ b*log(c*x**n))**2/(9*b**4*n**4*x + 10*b**2*n**2*x + x) - cos(a + b*log(c
*x**n))**3/(9*b**4*n**4*x + 10*b**2*n**2*x + x), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 994 vs. $2(158) = 316$.

Time = 0.09 (sec) , antiderivative size = 994, normalized size of antiderivative = 6.29

$$\int \frac{\cos^3(a + b\log(cx^n))}{x^2} \, dx = \text{Too large to display}$$

input
```
integrate(cos(a+b*log(c*x^n))^3/x^2,x, algorithm="maxima")
```

output
```
1/8*((3*(b^3*cos(3*b*log(c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(3*b
*log(c)) + b^3*sin(3*b*log(c)))*n^3 - (b^2*cos(6*b*log(c))*cos(3*b*log(c))
 + b^2*sin(6*b*log(c))*sin(3*b*log(c)) + b^2*cos(3*b*log(c)))*n^2 + 3*(b*c
os(3*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(3*b*log(c)) + b*sin
(3*b*log(c)))*n - cos(6*b*log(c))*cos(3*b*log(c)) - sin(6*b*log(c))*sin(3*
b*log(c)) - cos(3*b*log(c)))*cos(3*b*log(x^n) + 3*a) + 3*(9*(b^3*cos(3*b*l
og(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(3*b*log(c)) + b^3*cos(2*b
*log(c))*sin(3*b*log(c)) - b^3*cos(3*b*log(c))*sin(2*b*log(c)))*n^3 - 9*(b
^2*cos(4*b*log(c))*cos(3*b*log(c)) + b^2*cos(3*b*log(c))*cos(2*b*log(c)) +
 b^2*sin(4*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c))*sin(2*b*log(c))
)*n^2 + (b*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(3*b*log
(c)) + b*cos(2*b*log(c))*sin(3*b*log(c)) - b*cos(3*b*log(c))*sin(2*b*log(c
)))*n - cos(4*b*log(c))*cos(3*b*log(c)) - cos(3*b*log(c))*cos(2*b*log(c))
- sin(4*b*log(c))*sin(3*b*log(c)) - sin(3*b*log(c))*sin(2*b*log(c)))*cos(b
*log(x^n) + a) + (3*(b^3*cos(6*b*log(c))*cos(3*b*log(c)) + b^3*sin(6*b*log
(c))*sin(3*b*log(c)) + b^3*cos(3*b*log(c)))*n^3 + (b^2*cos(3*b*log(c))*sin
(6*b*log(c)) - b^2*cos(6*b*log(c))*sin(3*b*log(c)) + b^2*sin(3*b*log(c)))*
n^2 + 3*(b*cos(6*b*log(c))*cos(3*b*log(c)) + b*sin(6*b*log(c))*sin(3*b*log
(c)) + b*cos(3*b*log(c)))*n + cos(3*b*log(c))*sin(6*b*log(c)) - cos(6*b*lo
g(c))*sin(3*b*log(c)) + sin(3*b*log(c)))*sin(3*b*log(x^n) + 3*a) + 3*(9...
```

Giac [F]

$$\int \frac{\cos^3(a+b\log(cx^n))}{x^2}\,dx=\int \frac{\cos(b\log(cx^n)+a)^3}{x^2}\,dx$$

input
```
integrate(cos(a+b*log(c*x^n))^3/x^2,x, algorithm="giac")
```

output
```
integrate(cos(b*log(c*x^n) + a)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+b\log(cx^n))}{x^2}\,dx=\int \frac{\cos\left(a+b\ln\left(c\,x^n\right)\right)^3}{x^2}\,dx$$

input
```
int(cos(a + b*log(c*x^n))^3/x^2,x)
```

output
```
int(cos(a + b*log(c*x^n))^3/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.17

$$\int \frac{\cos^3(a+b\log(cx^n))}{x^2}\,dx$$
$$=\frac{\cos\left(\log(x^nc)\,b+a\right)\sin\left(\log(x^nc)\,b+a\right)^2b^2n^2+\cos\left(\log(x^nc)\,b+a\right)\sin\left(\log(x^nc)\,b+a\right)^2-7\cos\left(\log(x^nc\right)$$

input
```
int(cos(a+b*log(c*x^n))^3/x^2,x)
```

output
```
(cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**2*b**2*n**2 + cos(log(x**n
*c)*b + a)*sin(log(x**n*c)*b + a)**2 - 7*cos(log(x**n*c)*b + a)*b**2*n**2
- cos(log(x**n*c)*b + a) - 3*sin(log(x**n*c)*b + a)**3*b**3*n**3 - 3*sin(l
og(x**n*c)*b + a)**3*b*n + 9*sin(log(x**n*c)*b + a)*b**3*n**3 + 3*sin(log(
x**n*c)*b + a)*b*n)/(x*(9*b**4*n**4 + 10*b**2*n**2 + 1))
```

3.101 $\int \cos^4(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 191

$$\int \cos^4(a+b\log(cx^n))\,dx = \frac{24b^4n^4x}{1+20b^2n^2+64b^4n^4}+\frac{12b^2n^2x\cos^2(a+b\log(cx^n))}{1+20b^2n^2+64b^4n^4}+\frac{x\cos^4(a+b\log(cx^n))}{1+16b^2n^2}+\frac{24b^3n^3x\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{1+20b^2n^2+64b^4n^4}+\frac{4bnx\cos^3(a+b\log(cx^n))\sin(a+b\log(cx^n))}{1+16b^2n^2}$$

output
```
24*b^4*n^4*x/(64*b^4*n^4+20*b^2*n^2+1)+12*b^2*n^2*x*cos(a+b*ln(c*x^n))^2/(
64*b^4*n^4+20*b^2*n^2+1)+x*cos(a+b*ln(c*x^n))^4/(16*b^2*n^2+1)+24*b^3*n^3*
x*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/(64*b^4*n^4+20*b^2*n^2+1)+4*b*n*x*
cos(a+b*ln(c*x^n))^3*sin(a+b*ln(c*x^n))/(16*b^2*n^2+1)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

$$\int \cos^4(a+b\log(cx^n))\,dx = \frac{x(3+60b^2n^2+192b^4n^4+(4+64b^2n^2)\cos(2(a+b\log(cx^n)))+(1+4b^2n^2)\cos(4(a+b\log(cx^n)))+8}{8(}$$

input
```
Integrate[Cos[a + b*Log[c*x^n]]^4,x]
```

output
```
(x*(3 + 60*b^2*n^2 + 192*b^4*n^4 + (4 + 64*b^2*n^2)*Cos[2*(a + b*Log[c*x^n
])] + (1 + 4*b^2*n^2)*Cos[4*(a + b*Log[c*x^n])] + 8*b*n*Sin[2*(a + b*Log[c
*x^n])] + 128*b^3*n^3*Sin[2*(a + b*Log[c*x^n])] + 4*b*n*Sin[4*(a + b*Log[c
*x^n])] + 16*b^3*n^3*Sin[4*(a + b*Log[c*x^n])]))/(8*(1 + 20*b^2*n^2 + 64*b
^4*n^4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4981, 4981, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{4981}$$

$$\frac{12b^2n^2\int \cos^2(a+b\log(cx^n))\,dx}{16b^2n^2+1}+\frac{x\cos^4(a+b\log(cx^n))}{16b^2n^2+1}+\frac{4bnx\sin(a+b\log(cx^n))\cos^3(a+b\log(cx^n))}{16b^2n^2+1}$$

$$\Big\downarrow \text{4981}$$

$$\frac{12b^2n^2\left(\frac{2b^2n^2\int 1dx}{4b^2n^2+1}+\frac{x\cos^2(a+b\log(cx^n))}{4b^2n^2+1}+\frac{2bnx\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+1}\right)}{16b^2n^2+1}+\frac{x\cos^4(a+b\log(cx^n))}{16b^2n^2+1}+\frac{4bnx\sin(a+b\log(cx^n))\cos^3(a+b\log(cx^n))}{16b^2n^2+1}$$

$$\Big\downarrow \text{24}$$

$$\frac{x\cos^4(a+b\log(cx^n))}{16b^2n^2+1}+\frac{4bnx\sin(a+b\log(cx^n))\cos^3(a+b\log(cx^n))}{16b^2n^2+1}+\frac{12b^2n^2\left(\frac{x\cos^2(a+b\log(cx^n))}{4b^2n^2+1}+\frac{2bnx\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+1}+\frac{2b^2n^2x}{4b^2n^2+1}\right)}{16b^2n^2+1}$$

input
```
Int[Cos[a + b*Log[c*x^n]]^4,x]
```

output
```
(x*Cos[a + b*Log[c*x^n]]^4)/(1 + 16*b^2*n^2) + (4*b*n*x*Cos[a + b*Log[c*x^
n]]^3*Sin[a + b*Log[c*x^n]])/(1 + 16*b^2*n^2) + (12*b^2*n^2*((2*b^2*n^2*x)
/(1 + 4*b^2*n^2) + (x*Cos[a + b*Log[c*x^n]]^2)/(1 + 4*b^2*n^2) + (2*b*n*x*
Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/(1 + 4*b^2*n^2)))/(1 + 16*b^2
*n^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 4981
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Sim
p[x*(Cos[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 + 1)), x] + (Simp[b*d*n*p
*x*Cos[d*(a + b*Log[c*x^n])]^(p - 1)*(Sin[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^
2*p^2 + 1)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + 1))   Int[
Cos[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b, c, d, n}, x] &&
IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + 1, 0]
```

Maple [A] (verified)

Time = 4.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.69

method	result
parallelrisch	$\frac{128x\left(\left(\frac{1}{8}b^3n^3+\frac{1}{32}bn\right)\sin(4b\ln(c\,x^n)+4a)+\left(\frac{b^2n^2}{32}+\frac{1}{128}\right)\cos(4b\ln(c\,x^n)+4a)+\left(\frac{3b^2n^2}{2}+bn\sin(2b\ln(c\,x^n)+2a)+\frac{\cos(2b\ln(c\,x}{2}\right.\right.}{512b^4n^4+160b^2n^2+8}$
default	$\frac{3x}{8}+\frac{\mathrm{e}^{\frac{\ln(c\,x^n)}{n}-\frac{\ln(c)}{n}}\cos(2b\ln(c\,x^n)+2a)}{2n^2\left(\frac{1}{n^2}+4b^2\right)}+\frac{b\,\mathrm{e}^{\frac{\ln(c\,x^n)}{n}-\frac{\ln(c)}{n}}\sin(2b\ln(c\,x^n)+2a)}{n\left(\frac{1}{n^2}+4b^2\right)}+\frac{\mathrm{e}^{\frac{\ln(c\,x^n)}{n}-\frac{\ln(c)}{n}}\cos(4b\ln(c\,x^n)+4a}{8n^2\left(\frac{1}{n^2}+16b^2\right)}$

input
```
int(cos(a+b*ln(c*x^n))^4,x,method=_RETURNVERBOSE)
```

output
```
128*x*((1/8*b^3*n^3+1/32*b*n)*sin(4*b*ln(c*x^n)+4*a)+(1/32*b^2*n^2+1/128)*
cos(4*b*ln(c*x^n)+4*a)+(3/2*b^2*n^2+b*n*sin(2*b*ln(c*x^n)+2*a)+1/2*cos(2*b
*ln(c*x^n)+2*a)+3/8)*(1/16+b^2*n^2))/(512*b^4*n^4+160*b^2*n^2+8)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.75

$$\int \cos^4\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\frac{24\,b^4n^4x+12\,b^2n^2x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+(4\,b^2n^2+1)x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^4+4\,(}{64\,b^4}$$

input
```
integrate(cos(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

output
```
(24*b^4*n^4*x + 12*b^2*n^2*x*cos(b*n*log(x) + b*log(c) + a)^2 + (4*b^2*n^2
 + 1)*x*cos(b*n*log(x) + b*log(c) + a)^4 + 4*(6*b^3*n^3*x*cos(b*n*log(x) +
 b*log(c) + a) + (4*b^3*n^3 + b*n)*x*cos(b*n*log(x) + b*log(c) + a)^3)*sin
(b*n*log(x) + b*log(c) + a))/(64*b^4*n^4 + 20*b^2*n^2 + 1)
```

Sympy [F]

$$\int \cos^4(a+b\log(cx^n))\,dx$$

$$= \begin{cases} \int \cos^4\left(a-\frac{i\log(cx^n)}{2n}\right)dx \\ \int \cos^4\left(a-\frac{i\log(cx^n)}{4n}\right)dx \\ \int \cos^4\left(a+\frac{i\log(cx^n)}{4n}\right)dx \\ \int \cos^4\left(a+\frac{i\log(cx^n)}{2n}\right)dx \\ \frac{24b^4n^4x\sin^4(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1}+\frac{48b^4n^4x\sin^2(a+b\log(cx^n))\cos^2(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1}+\frac{24b^4n^4x\cos^4(a+b\log(cx^n))}{64b^4n^4+20b^2n^2+1}+\frac{24b^3n^3x\sin^3(a+b\log}{64b^4n} \end{cases}$$

input

```
integrate(cos(a+b*ln(c*x**n))**4,x)
```

output

```
Piecewise((Integral(cos(a - I*log(c*x**n)/(2*n))**4, x), Eq(b, -I/(2*n))),
 (Integral(cos(a - I*log(c*x**n)/(4*n))**4, x), Eq(b, -I/(4*n))), (Integra
l(cos(a + I*log(c*x**n)/(4*n))**4, x), Eq(b, I/(4*n))), (Integral(cos(a +
I*log(c*x**n)/(2*n))**4, x), Eq(b, I/(2*n))), (24*b**4*n**4*x*sin(a + b*lo
g(c*x**n))**4/(64*b**4*n**4 + 20*b**2*n**2 + 1) + 48*b**4*n**4*x*sin(a + b
*log(c*x**n))**2*cos(a + b*log(c*x**n))**2/(64*b**4*n**4 + 20*b**2*n**2 +
1) + 24*b**4*n**4*x*cos(a + b*log(c*x**n))**4/(64*b**4*n**4 + 20*b**2*n**2
 + 1) + 24*b**3*n**3*x*sin(a + b*log(c*x**n))**3*cos(a + b*log(c*x**n))/(6
4*b**4*n**4 + 20*b**2*n**2 + 1) + 40*b**3*n**3*x*sin(a + b*log(c*x**n))*co
s(a + b*log(c*x**n))**3/(64*b**4*n**4 + 20*b**2*n**2 + 1) + 12*b**2*n**2*x
*sin(a + b*log(c*x**n))**2*cos(a + b*log(c*x**n))**2/(64*b**4*n**4 + 20*b*
*2*n**2 + 1) + 16*b**2*n**2*x*cos(a + b*log(c*x**n))**4/(64*b**4*n**4 + 20
*b**2*n**2 + 1) + 4*b*n*x*sin(a + b*log(c*x**n))*cos(a + b*log(c*x**n))**3
/(64*b**4*n**4 + 20*b**2*n**2 + 1) + x*cos(a + b*log(c*x**n))**4/(64*b**4*
n**4 + 20*b**2*n**2 + 1), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1078 vs. 2(191) = 382.

Time = 0.08 (sec) , antiderivative size = 1078, normalized size of antiderivative = 5.64

$$\int \cos^4(a+b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(cos(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

output
```
1/16*((16*(b^3*cos(4*b*log(c))*sin(8*b*log(c)) - b^3*cos(8*b*log(c))*sin(4
*b*log(c)) + b^3*sin(4*b*log(c)))*n^3 + 4*(b^2*cos(8*b*log(c))*cos(4*b*log
(c)) + b^2*sin(8*b*log(c))*sin(4*b*log(c)) + b^2*cos(4*b*log(c)))*n^2 + 4*
(b*cos(4*b*log(c))*sin(8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c)) + b
*sin(4*b*log(c)))*n + cos(8*b*log(c))*cos(4*b*log(c)) + sin(8*b*log(c))*si
n(4*b*log(c)) + cos(4*b*log(c)))*x*cos(4*b*log(x^n) + 4*a) + 4*(32*(b^3*co
s(4*b*log(c))*sin(6*b*log(c)) - b^3*cos(6*b*log(c))*sin(4*b*log(c)) + b^3*
cos(2*b*log(c))*sin(4*b*log(c)) - b^3*cos(4*b*log(c))*sin(2*b*log(c)))*n^3
 + 16*(b^2*cos(6*b*log(c))*cos(4*b*log(c)) + b^2*cos(4*b*log(c))*cos(2*b*l
og(c)) + b^2*sin(6*b*log(c))*sin(4*b*log(c)) + b^2*sin(4*b*log(c))*sin(2*b
*log(c)))*n^2 + 2*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*s
in(4*b*log(c)) + b*cos(2*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin
(2*b*log(c)))*n + cos(6*b*log(c))*cos(4*b*log(c)) + cos(4*b*log(c))*cos(2*
b*log(c)) + sin(6*b*log(c))*sin(4*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(
c)))*x*cos(2*b*log(x^n) + 2*a) + (16*(b^3*cos(8*b*log(c))*cos(4*b*log(c))
+ b^3*sin(8*b*log(c))*sin(4*b*log(c)) + b^3*cos(4*b*log(c)))*n^3 - 4*(b^2*
cos(4*b*log(c))*sin(8*b*log(c)) - b^2*cos(8*b*log(c))*sin(4*b*log(c)) + b^
2*sin(4*b*log(c)))*n^2 + 4*(b*cos(8*b*log(c))*cos(4*b*log(c)) + b*sin(8*b*
log(c))*sin(4*b*log(c)) + b*cos(4*b*log(c)))*n - cos(4*b*log(c))*sin(8*b*l
og(c)) + cos(8*b*log(c))*sin(4*b*log(c)) - sin(4*b*log(c)))*x*sin(4*b*l...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15993 vs. $2(191) = 382$.

Time = 0.61 (sec) , antiderivative size = 15993, normalized size of antiderivative = 83.73

$$\int \cos^4(a + b\log(cx^n))\, dx = \text{Too large to display}$$

input
```
integrate(cos(a+b*log(c*x^n))^4,x, algorithm="giac")
```

output
```
3/8*x - 1/16*(256*b^3*n^3*x*e^(3*pi*b*n*sgn(x) - 3*pi*b*n + 3*pi*b*sgn(c)
- 3*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) +
 b*log(abs(c)))^2*tan(2*a)^2*tan(a) + 256*b^3*n^3*x*e^(pi*b*n*sgn(x) - pi*
b*n + pi*b*sgn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b
*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a) + 32*b^3*n^3*x*e^(4*pi
*b*n*sgn(x) - 4*pi*b*n + 4*pi*b*sgn(c) - 4*pi*b)*tan(2*b*n*log(abs(x)) + 2
*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)*tan(a)^2
 + 256*b^3*n^3*x*e^(3*pi*b*n*sgn(x) - 3*pi*b*n + 3*pi*b*sgn(c) - 3*pi*b)*t
an(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(
c)))*tan(2*a)^2*tan(a)^2 + 256*b^3*n^3*x*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*
sgn(c) - pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(
x)) + b*log(abs(c)))*tan(2*a)^2*tan(a)^2 + 32*b^3*n^3*x*e^(4*pi*b*n*sgn(x)
 - 4*pi*b*n + 4*pi*b*sgn(c) - 4*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(
c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a)^2 + 32*b^3*n
^3*x*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*lo
g(abs(c)))^2*tan(2*a)*tan(a)^2 + 32*b^3*n^3*x*tan(2*b*n*log(abs(x)) + 2*b*
log(abs(c)))*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a)^2 -
4*b^2*n^2*x*e^(4*pi*b*n*sgn(x) - 4*pi*b*n + 4*pi*b*sgn(c) - 4*pi*b)*tan(2*
b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^
2*tan(2*a)^2*tan(a)^2 - 64*b^2*n^2*x*e^(3*pi*b*n*sgn(x) - 3*pi*b*n + 3*...
```

Mupad [B] (verification not implemented)

Time = 20.73 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.61

$$\int \cos^4(a+b\log(cx^n))\,dx = \frac{3\,x}{8}+\frac{x\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,2\mathrm{i}}}\,1\mathrm{i}}{8\,b\,n+4\mathrm{i}}+\frac{x\,\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}}}{4+b\,n\,8\mathrm{i}} + \frac{x\,\mathrm{e}^{-a\,4\mathrm{i}}\,\frac{1}{(c\,x^n)^{b\,4\mathrm{i}}}\,1\mathrm{i}}{64\,b\,n+16\mathrm{i}}+\frac{x\,\mathrm{e}^{a\,4\mathrm{i}}\,(c\,x^n)^{b\,4\mathrm{i}}}{16+b\,n\,64\mathrm{i}}$$

input
```
int(cos(a + b*log(c*x^n))^4,x)
```

output
```
(3*x)/8 + (x*exp(-a*2i)/(c*x^n)^(b*2i)*1i)/(8*b*n + 4i) + (x*exp(a*2i)*(c*
x^n)^(b*2i))/(b*n*8i + 4) + (x*exp(-a*4i)/(c*x^n)^(b*4i)*1i)/(64*b*n + 16i
) + (x*exp(a*4i)*(c*x^n)^(b*4i))/(b*n*64i + 16)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.18

$$\int \cos^4(a+b\log(cx^n))\,dx = \frac{x\left(-16\cos(\log(x^n c)\,b+a)\sin(\log(x^n c)\,b+a)^3 b^3 n^3-4\cos(\log(x^n c)\,b+a)\sin(\log(x^n c)\,b+a)^3 bn+40\right.}{}$$

input
```
int(cos(a+b*log(c*x^n))^4,x)
```

output
```
(x*( - 16*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*b**3*n**3 - 4*c
os(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*b*n + 40*cos(log(x**n*c)*b
 + a)*sin(log(x**n*c)*b + a)*b**3*n**3 + 4*cos(log(x**n*c)*b + a)*sin(log(
x**n*c)*b + a)*b*n + 4*sin(log(x**n*c)*b + a)**4*b**2*n**2 + sin(log(x**n*
c)*b + a)**4 - 20*sin(log(x**n*c)*b + a)**2*b**2*n**2 - 2*sin(log(x**n*c)*
b + a)**2 + 24*b**4*n**4 + 16*b**2*n**2 + 1))/(64*b**4*n**4 + 20*b**2*n**2
 + 1)
```

3.102 $\int \frac{\cos^4(a+b\log(cx^n))}{x} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 73

$$\int \frac{\cos^4(a+b\log(cx^n))}{x} \, dx = \frac{3\log(x)}{8} + \frac{3\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{8bn} + \frac{\cos^3(a+b\log(cx^n))\sin(a+b\log(cx^n))}{4bn}$$

output
```
3/8*ln(x)+3/8*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/b/n+1/4*cos(a+b*ln(c*x
^n))^3*sin(a+b*ln(c*x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \frac{\cos^4(a+b\log(cx^n))}{x} \, dx = \frac{12(a+b\log(cx^n)) + 8\sin(2(a+b\log(cx^n))) + \sin(4(a+b\log(cx^n)))}{32bn}$$

input
```
Integrate[Cos[a + b*Log[c*x^n]]^4/x,x]
```

output
```
(12*(a + b*Log[c*x^n]) + 8*Sin[2*(a + b*Log[c*x^n])] + Sin[4*(a + b*Log[c*
x^n])])/(32*b*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3039, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \cos^4(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(a+b\log(cx^n)+\frac{\pi}{2}\right)^4d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{3}{4}\int \cos^2(a+b\log(cx^n))d\log(cx^n)+\frac{\sin(a+b\log(cx^n))\cos^3(a+b\log(cx^n))}{4b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{4}\int \sin\left(a+b\log(cx^n)+\frac{\pi}{2}\right)^2d\log(cx^n)+\frac{\sin(a+b\log(cx^n))\cos^3(a+b\log(cx^n))}{4b}}{n}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int 1d\log(cx^n)+\frac{\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{2b}\right)+\frac{\sin(a+b\log(cx^n))\cos^3(a+b\log(cx^n))}{4b}}{n}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{\sin(a+b\log(cx^n))\cos^3(a+b\log(cx^n))}{4b}+\frac{3}{4}\left(\frac{\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{2b}+\frac{1}{2}\log(cx^n)\right)}{n}$$

input
```
Int[Cos[a + b*Log[c*x^n]]^4/x,x]
```

output
```
((Cos[a + b*Log[c*x^n]]^3*Sin[a + b*Log[c*x^n]])/(4*b) + (3*(Log[c*x^n]/2
+ (Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/(2*b)))/4)/n
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 6.66 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$\frac{\sin(4b\ln(c\,x^n)+4a)+8\sin(2b\ln(c\,x^n)+2a)+12\ln(x)bn}{32bn}$	46
derivativedivides	$\frac{\frac{\left(\cos(a+b\ln(c\,x^n))^3+\frac{3\cos(a+b\ln(c\,x^n))}{2}\right)\sin(a+b\ln(c\,x^n))}{4}+\frac{3b\ln(c\,x^n)}{8}+\frac{3a}{8}}{nb}$	61
default	$\frac{\frac{\left(\cos(a+b\ln(c\,x^n))^3+\frac{3\cos(a+b\ln(c\,x^n))}{2}\right)\sin(a+b\ln(c\,x^n))}{4}+\frac{3b\ln(c\,x^n)}{8}+\frac{3a}{8}}{nb}$	61

input
```
int(cos(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
```

output
```
1/32*(sin(4*b*ln(c*x^n)+4*a)+8*sin(2*b*ln(c*x^n)+2*a)+12*ln(x)*b*n)/b/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \frac{\cos^4(a+b\log(cx^n))}{x}\,dx = \frac{3\,bn\log(x) + \left(2\,\cos(bn\log(x)+b\log(c)+a)^3 + 3\,\cos(bn\log(x)+b\log(c)+a)\right)\sin(bn\log(x)+b\log}{8\,bn}$$

input
```
integrate(cos(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

output
```
1/8*(3*b*n*log(x) + (2*cos(b*n*log(x) + b*log(c) + a)^3 + 3*cos(b*n*log(x)
 + b*log(c) + a))*sin(b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [A] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.37

$$\int \frac{\cos^4(a+b\log(cx^n))}{x}\,dx = \frac{\begin{cases} \log(x)\cos(2a) & \text{for } b=0 \wedge (b=0 \vee n=0) \\ \log(x)\cos(2a+2b\log(c)) & \text{for } n=0 \\ \frac{\sin(2a+2b\log(cx^n))}{2bn} & \text{otherwise} \end{cases}}{2} + \frac{\begin{cases} \log(x)\cos(4a) & \text{for } b=0 \wedge (b=0 \vee n=0) \\ \log(x)\cos(4a+4b\log(c)) & \text{for } n=0 \\ \frac{\sin(4a+4b\log(cx^n))}{4bn} & \text{otherwise} \end{cases}}{8} + \frac{3\log(x)}{8}$$

input
```
integrate(cos(a+b*ln(c*x**n))**4/x,x)
```

output
```
Piecewise((log(x)*cos(2*a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*cos
(2*a + 2*b*log(c)), Eq(n, 0)), (sin(2*a + 2*b*log(c*x**n))/(2*b*n), True))
/2 + Piecewise((log(x)*cos(4*a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x
)*cos(4*a + 4*b*log(c)), Eq(n, 0)), (sin(4*a + 4*b*log(c*x**n))/(4*b*n), T
rue))/8 + 3*log(x)/8
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

$$\int \frac{\cos^4(a+b\log(cx^n))}{x}\,dx = \frac{12\,bn\log(x)+\cos(4\,b\log(x^n)+4\,a)\sin(4\,b\log(c))+8\,\cos(2\,b\log(x^n)+2\,a)\sin(2\,b\log(c))+\cos(4\,b}{32\,bn}$$

input
```
integrate(cos(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

output
```
1/32*(12*b*n*log(x) + cos(4*b*log(x^n) + 4*a)*sin(4*b*log(c)) + 8*cos(2*b*
log(x^n) + 2*a)*sin(2*b*log(c)) + cos(4*b*log(c))*sin(4*b*log(x^n) + 4*a)
+ 8*cos(2*b*log(c))*sin(2*b*log(x^n) + 2*a))/(b*n)
```

Giac [F]

$$\int \frac{\cos^4(a+b\log(cx^n))}{x}\,dx = \int \frac{\cos(b\log(cx^n)+a)^4}{x}\,dx$$

input
```
integrate(cos(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

output
```
integrate(cos(b*log(c*x^n) + a)^4/x, x)
```

Mupad [B] (verification not implemented)

Time = 20.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int \frac{\cos^4(a+b\log(cx^n))}{x}\,dx = \frac{3\,\ln(x^n)}{8\,n} + \frac{\frac{\sin(2\,a+2\,b\,\ln(c\,x^n))}{4} + \frac{\sin(4\,a+4\,b\,\ln(c\,x^n))}{32}}{b\,n}$$

input
```
int(cos(a + b*log(c*x^n))^4/x,x)
```

output
```
(3*log(x^n))/(8*n) + (sin(2*a + 2*b*log(c*x^n))/4 + sin(4*a + 4*b*log(c*x^
n))/32)/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{\cos^4(a+b\log(cx^n))}{x}\,dx = \frac{-2\cos(\log(x^n c)\,b+a)\sin(\log(x^n c)\,b+a)^3 + 5\cos(\log(x^n c)\,b+a)\sin(\log(x^n c)\,b+a) + 3\log(x^n c)\,b}{8bn}$$

input
```
int(cos(a+b*log(c*x^n))^4/x,x)
```

output
```
( - 2*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3 + 5*cos(log(x**n*c)
*b + a)*sin(log(x**n*c)*b + a) + 3*log(x**n*c)*b)/(8*b*n)
```

3.103 $\int \cos(\log(6+3x))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 29

$$\int \cos(\log(6+3x))\,dx = \frac{1}{2}(2+x)\cos(\log(3(2+x))) + \frac{1}{2}(2+x)\sin(\log(3(2+x)))$$

output
```
1/2*(2+x)*cos(ln(6+3*x))+1/2*(2+x)*sin(ln(6+3*x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

$$\int \cos(\log(6+3x))\,dx = \frac{1}{2}(2+x)(\cos(\log(3(2+x))) + \sin(\log(3(2+x))))$$

input
```
Integrate[Cos[Log[6 + 3*x]],x]
```

output
```
((2 + x)*(Cos[Log[3*(2 + x)]] + Sin[Log[3*(2 + x)]]))/2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {7281, 4979}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(\log(3x+6))\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{1}{3}\int \cos(\log(3x+6))d(3x+6)$$

$$\Big\downarrow \text{4979}$$

$$\frac{1}{3}\left(\frac{1}{2}(3x+6)\sin(\log(3x+6))+\frac{1}{2}(3x+6)\cos(\log(3x+6))\right)$$

input
```
Int[Cos[Log[6 + 3*x]],x]
```

output
```
(((6 + 3*x)*Cos[Log[6 + 3*x]])/2 + ((6 + 3*x)*Sin[Log[6 + 3*x]])/2)/3
```

Defintions of rubi rules used

rule 4979
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[x*(
Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] + Simp[b*d*n*x*(Sin[d*(a +
 b*Log[c*x^n])]/(b^2*d^2*n^2 + 1)), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$\frac{(2+x)\cos(\ln(6+3x))}{2}+2+\frac{(2+x)\sin(\ln(6+3x))}{2}$	27
default	$\frac{\cos(\ln(6+3x))(6+3x)}{6}+\frac{(6+3x)\sin(\ln(6+3x))}{6}$	30
risch	$\left(\frac{1}{4}-\frac{i}{4}\right)(2+x)(6+3x)^{i}+\left(\frac{1}{4}+\frac{i}{4}\right)(2+x)(6+3x)^{-i}$	34

input
```
int(cos(ln(6+3*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2+x)*cos(ln(6+3*x))+2+1/2*(2+x)*sin(ln(6+3*x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \cos(\log(6+3x))\,dx=\frac{1}{2}\,(x+2)\cos\left(\log\left(3\,x+6\right)\right)+\frac{1}{2}\,(x+2)\sin\left(\log\left(3\,x+6\right)\right)$$

input
```
integrate(cos(log(6+3*x)),x, algorithm="fricas")
```

output
```
1/2*(x + 2)*cos(log(3*x + 6)) + 1/2*(x + 2)*sin(log(3*x + 6))
```

Sympy [F]

$$\int \cos(\log(6+3x))\,dx=\int \cos\left(\log\left(3x+6\right)\right)\,dx$$

input
```
integrate(cos(ln(6+3*x)),x)
```

output
```
Integral(cos(log(3*x + 6)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

$$\int \cos(\log(6+3x))\,dx = \frac{1}{2}\,(x+2)(\cos\left(\log\left(3\,x+6\right)\right)+\sin\left(\log\left(3\,x+6\right)\right))$$

input
```
integrate(cos(log(6+3*x)),x, algorithm="maxima")
```

output
```
1/2*(x + 2)*(cos(log(3*x + 6)) + sin(log(3*x + 6)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \cos(\log(6+3x))\,dx = \frac{1}{2}\,(x+2)\cos\left(\log\left(3\,x+6\right)\right)+\frac{1}{2}\,(x+2)\sin\left(\log\left(3\,x+6\right)\right)$$

input
```
integrate(cos(log(6+3*x)),x, algorithm="giac")
```

output
```
1/2*(x + 2)*cos(log(3*x + 6)) + 1/2*(x + 2)*sin(log(3*x + 6))
```

Mupad [B] (verification not implemented)

Time = 20.99 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \cos(\log(6+3x))\,dx = \frac{\sqrt{2}\,\sin\left(\frac{\pi}{4}+\ln\left(3\,x+6\right)\right)\,\left(3\,x+6\right)}{6}$$

input
```
int(cos(log(3*x + 6)),x)
```

output
```
(2^(1/2)*sin(pi/4 + log(3*x + 6))*(3*x + 6))/6
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \cos(\log(6+3x))\,dx = \frac{\cos(\log(3x+6))\,x}{2} + \cos(\log(3x+6)) + \frac{\sin(\log(3x+6))\,x}{2} + \sin(\log(3x+6))$$

input
```
int(cos(log(6+3*x)),x)
```

output
```
(cos(log(3*x + 6))*x + 2*cos(log(3*x + 6)) + sin(log(3*x + 6))*x + 2*sin(l
og(3*x + 6)))/2
```

3.104 $\int x^m \cos\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 101

$$\int x^m \cos\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)\,dx = \frac{e^{\frac{a(1+m)}{\sqrt{-\frac{(1+m)^2}{n^2}}\,n}}x^{1+m}(cx^n)^{\frac{1+m}{n}}}{4(1+m)}+\frac{1}{2}e^{\frac{a\sqrt{-\frac{(1+m)^2}{n^2}}\,n}{1+m}}x^{1+m}(cx^n)^{-\frac{1+m}{n}}\log(x)$$

output
```
exp(a*(1+m)/(-(1+m)^2/n^2)^(1/2)/n)*x^(1+m)*(c*x^n)^((1+m)/n)/(4+4*m)+1/2*
exp(a*(-(1+m)^2/n^2)^(1/2)*n/(1+m))*x^(1+m)*ln(x)/((c*x^n)^((1+m)/n))
```

Mathematica [F]

$$\int x^m \cos\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)\,dx = \int x^m \cos\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)\,dx$$

input
```
Integrate[x^m*Cos[a + Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n]],x]
```

output
```
Integrate[x^m*Cos[a + Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n]], x]
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {4997, 4993, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cos\left(a+\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)dx$$

$$\Big\downarrow \text{4997}$$

$$\frac{x^{m+1}\left(cx^n\right)^{-\frac{m+1}{n}}\int \left(cx^n\right)^{\frac{m+1}{n}-1}\cos\left(a+\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)d\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{4993}$$

$$\frac{x^{m+1}\left(cx^n\right)^{-\frac{m+1}{n}}\int \left(e^{\frac{a(m+1)}{\sqrt{-\frac{(m+1)^2}{n^2}}n}}\left(cx^n\right)^{\frac{2(m+1)}{n}-1}+\frac{e^{\frac{a\sqrt{-\frac{(m+1)^2}{n^2}}n}{m+1}}x^{-n}}{c}\right)d\left(cx^n\right)}{2n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{m+1}\left(cx^n\right)^{-\frac{m+1}{n}}\left(\frac{ne^{\frac{a(m+1)}{n\sqrt{-\frac{(m+1)^2}{n^2}}}}\left(cx^n\right)^{\frac{2(m+1)}{n}}}{2(m+1)}+e^{\frac{an\sqrt{-\frac{(m+1)^2}{n^2}}}{m+1}}\log\left(cx^n\right)\right)}{2n}$$

input
```
Int[x^m*Cos[a + Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n]],x]
```

output
```
(x^(1 + m)*((E^((a*(1 + m))/(Sqrt[-((1 + m)^2/n^2)]*n))*n*(c*x^n)^((2*(1 +
 m))/n))/(2*(1 + m)) + E^((a*Sqrt[-((1 + m)^2/n^2)]*n)/(1 + m))*Log[c*x^n]
))/(2*n*(c*x^n)^((1 + m)/n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4993
```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[1/2^p   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^2*(p/(m + 1)))/x^((m
+ 1)/p) + x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x], x], x] /; FreeQ[{a,
 b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m + 1)^2, 0]
```

rule 4997
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cos[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^{m} \cos \left(a+\sqrt{-\frac{(1+m)^{2}}{n^{2}}}\, \ln \left(c\, x^{n}\right)\right) dx$$

input
```
int(x^m*cos(a+(-(1+m)^2/n^2)^(1/2)*ln(c*x^n)),x)
```

output
```
int(x^m*cos(a+(-(1+m)^2/n^2)^(1/2)*ln(c*x^n)),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.59

$$\int x^m \cos \left(a+\sqrt{-\frac{(1+m)^2}{n^2}} \log \left(cx^n\right)\right) dx = \frac{\left(x^2 x^{2\,m} + 2\,(m+1)e^{\left(\frac{2\,(i\,an-(m+1)\log(c))}{n}\right)}\log\left(x\right)\right)e^{\left(-\frac{i\,an-(m+1)\log(c)}{n}\right)}}{4\,(m+1)}$$

input
```
integrate(x^m*cos(a+(-(1+m)^2/n^2)^(1/2)*log(c*x^n)),x, algorithm="fricas"
)
```

output
```
1/4*(x^2*x^(2*m) + 2*(m + 1)*e^(2*(I*a*n - (m + 1)*log(c))/n)*log(x))*e^(-
(I*a*n - (m + 1)*log(c))/n)/(m + 1)
```

Sympy [F]

$$\int x^m \cos\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\int x^m \cos\left(a+\sqrt{-\frac{m^2}{n^2}-\frac{2m}{n^2}-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$

input
```
integrate(x**m*cos(a+(-(1+m)**2/n**2)**(1/2)*ln(c*x**n)),x)
```

output
```
Integral(x**m*cos(a + sqrt(-m**2/n**2 - 2*m/n**2 - 1/n**2)*log(c*x**n)), x
)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.81

$$\int x^m \cos\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\frac{c^{\frac{2m}{n}+\frac{2}{n}}x\cos(a)\,e^{\left(m\log(x)+\frac{m\log(x^n)}{n}+\frac{\log(x^n)}{n}\right)}+2\left(m\cos(a)+\cos(a)\right)\log(x)}{4\left(c^{\frac{m}{n}+\frac{1}{n}}m+c^{\frac{m}{n}+\frac{1}{n}}\right)}$$

input
```
integrate(x^m*cos(a+(-(1+m)^2/n^2)^(1/2)*log(c*x^n)),x, algorithm="maxima"
)
```

output
```
1/4*(c^(2*m/n + 2/n)*x*cos(a)*e^(m*log(x) + m*log(x^n)/n + log(x^n)/n) + 2
*(m*cos(a) + cos(a))*log(x))/(c^(m/n + 1/n)*m + c^(m/n + 1/n))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.64

$$\int x^m \cos\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\frac{mn^2xx^m e^{\left(ia-\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+mn^2xx^m e^{\left(-ia+\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+n^2xx^m e^{\left(ia-\frac{n|mn+n|\log(x)+|}{n^2}\right.}}{}$$

input
```
integrate(x^m*cos(a+(-(1+m)^2/n^2)^(1/2)*log(c*x^n)),x, algorithm="giac")
```

output
```
1/2*(m*n^2*x*x^m*e^(I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^
2) + m*n^2*x*x^m*e^(-I*a + (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n
^2) + n^2*x*x^m*e^(I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2
) + n*x*x^m*abs(m*n + n)*e^(I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*lo
g(c))/n^2) + n^2*x*x^m*e^(-I*a + (n*abs(m*n + n)*log(x) + abs(m*n + n)*log
(c))/n^2) - n*x*x^m*abs(m*n + n)*e^(-I*a + (n*abs(m*n + n)*log(x) + abs(m*
n + n)*log(c))/n^2))/(m^2*n^2 + 2*m*n^2 - (m*n + n)^2 + n^2)
```

Mupad [B] (verification not implemented)

Time = 21.95 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.30

$$\int x^m \cos\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\frac{x\,x^m\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}}}{2\,m+2-n\,\sqrt{-\frac{{\left(m+1\right)}^2}{n^2}}\,2\mathrm{i}}+\frac{x\,x^m\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}}{2\,m+2+n\,\sqrt{-\frac{{\left(m+1\right)}^2}{n^2}}\,2\mathrm{i}}$$

input
```
int(x^m*cos(a + log(c*x^n)*(-(m + 1)^2/n^2)^(1/2)),x)
```

output
```
(x*x^m*exp(-a*1i)/(c*x^n)^((- (2*m)/n^2 - 1/n^2 - m^2/n^2)^(1/2)*1i))/(2*m
 - n*(-(m + 1)^2/n^2)^(1/2)*2i + 2) + (x*x^m*exp(a*1i)*(c*x^n)^((- (2*m)/n
^2 - 1/n^2 - m^2/n^2)^(1/2)*1i))/(2*m + n*(-(m + 1)^2/n^2)^(1/2)*2i + 2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.58

$$\int x^m \cos\left(a+\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx = \frac{x^m x\left(\cos\left(\frac{\log(x^n c)m+\log(x^n c)+an}{n}\right)+\sin\left(\frac{\log(x^n c)m+\log(x^n c)+an}{n}\right)\right)}{2m+2}$$

input
```
int(x^m*cos(a+(-(1+m)^2/n^2)^(1/2)*log(c*x^n)),x)
```

output
```
(x**m*x*(cos((log(x**n*c)*m + log(x**n*c) + a*n)/n) + sin((log(x**n*c)*m +
 log(x**n*c) + a*n)/n)))/(2*(m + 1))
```

3.105 $\int \cos\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 62

$$\int \cos\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\frac{1}{4}e^{-a\sqrt{-\frac{1}{n^2}}n}x(cx^n)^{\frac{1}{n}}+\frac{1}{2}e^{a\sqrt{-\frac{1}{n^2}}n}x(cx^n)^{-1/n}\log(x)$$

output
```
1/4*x*(c*x^n)^(1/n)/exp(a*(-1/n^2)^(1/2)*n)+1/2*exp(a*(-1/n^2)^(1/2)*n)*x*
ln(x)/((c*x^n)^(1/n))
```

Mathematica [F]

$$\int \cos\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\int \cos\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$

input
```
Integrate[Cos[a + Sqrt[-n^(-2)]*Log[c*x^n]],x]
```

output
```
Integrate[Cos[a + Sqrt[-n^(-2)]*Log[c*x^n]], x]
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4987, 4993, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)dx$$

$$\big\downarrow \text{4987}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\cos\left(a+\sqrt{-\frac{1}{n^2}}\log(cx^n)\right)d(cx^n)}{n}$$

$$\big\downarrow \text{4993}$$

$$\frac{x(cx^n)^{-1/n}\int \left(e^{-a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{2}{n}-1}+\frac{e^{a\sqrt{-\frac{1}{n^2}}n}x^{-n}}{c}\right)d(cx^n)}{2n}$$

$$\big\downarrow \text{2009}$$

$$\frac{x(cx^n)^{-1/n}\left(\frac{1}{2}ne^{-a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{2/n}+e^{a\sqrt{-\frac{1}{n^2}}n}\log(cx^n)\right)}{2n}$$

input
```
Int[Cos[a + Sqrt[-n^(-2)]*Log[c*x^n]],x]
```

output
```
(x*((n*(c*x^n)^(2/n))/(2*E^(a*Sqrt[-n^(-2)]*n)) + E^(a*Sqrt[-n^(-2)]*n)*Lo
g[c*x^n]))/(2*n*(c*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4987
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cos[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 4993
```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[1/2^p   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^2*(p/(m + 1)))/x^((m
+ 1)/p) + x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x], x], x] /; FreeQ[{a,
 b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m + 1)^2, 0]
```

Maple [F]

$$\int \cos\left(a+\sqrt{-\frac{1}{n^2}}\,\ln\left(c\,x^n\right)\right)dx$$

input
```
int(cos(a+(-1/n^2)^(1/2)*ln(c*x^n)),x)
```

output
```
int(cos(a+(-1/n^2)^(1/2)*ln(c*x^n)),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

$$\int \cos\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\frac{1}{4}\left(x^2+2\,e^{\left(\frac{2\,(i\,an-\log(c))}{n}\right)}\log\left(x\right)\right)e^{\left(-\frac{i\,an-\log(c)}{n}\right)}$$

input
```
integrate(cos(a+(-1/n^2)^(1/2)*log(c*x^n)),x, algorithm="fricas")
```

output
```
1/4*(x^2 + 2*e^(2*(I*a*n - log(c))/n)*log(x))*e^(-(I*a*n - log(c))/n)
```

Sympy [F]

$$\int \cos \left(a + \sqrt{-\frac{1}{n^2}} \log \left(cx^n\right)\right) dx = \int \cos \left(a + \sqrt{-\frac{1}{n^2}} \log \left(cx^n\right)\right) dx$$

input
```
integrate(cos(a+(-1/n**2)**(1/2)*ln(c*x**n)),x)
```

output
```
Integral(cos(a + sqrt(-1/n**2)*log(c*x**n)), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.47

$$\int \cos \left(a + \sqrt{-\frac{1}{n^2}} \log \left(cx^n\right)\right) dx = \frac{c^{\frac{2}{n}} x^2 \cos \left(a\right) + 2 \cos \left(a\right) \log \left(x\right)}{4 c^{\left(\frac{1}{n}\right)}}$$

input
```
integrate(cos(a+(-1/n^2)^(1/2)*log(c*x^n)),x, algorithm="maxima")
```

output
```
1/4*(c^(2/n)*x^2*cos(a) + 2*cos(a)*log(x))/c^(1/n)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.02

$$\int \cos \left(a + \sqrt{-\frac{1}{n^2}} \log \left(cx^n\right)\right) dx = +\infty$$

input
```
integrate(cos(a+(-1/n^2)^(1/2)*log(c*x^n)),x, algorithm="giac")
```

output
```
+Infinity
```

Mupad [B] (verification not implemented)

Time = 20.66 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.34

$$\int \cos\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\frac{x\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}}\,1\mathrm{i}}{2\,n\,\sqrt{-\frac{1}{n^2}}+2\mathrm{i}}-\frac{x\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}\,1\mathrm{i}}{2\,n\,\sqrt{-\frac{1}{n^2}}-2\mathrm{i}}$$

input
```
int(cos(a + log(c*x^n)*(-1/n^2)^(1/2)),x)
```

output
```
(x*exp(-a*1i)/(c*x^n)^((-1/n^2)^(1/2)*1i)*1i)/(2*n*(-1/n^2)^(1/2) + 2i) -
(x*exp(a*1i)*(c*x^n)^((-1/n^2)^(1/2)*1i)*1i)/(2*n*(-1/n^2)^(1/2) - 2i)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.21

$$\int \cos\left(a+\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\frac{x\left(\cos\left(\frac{\log(x^nc)i+an}{n}\right)\log(x^nc)+\cos\left(\frac{\log(x^nc)i+an}{n}\right)n+\log(x^nc)\sin\left(\frac{\log(x^nc)i+an}{n}\right)i\right)}{2n}$$

input
```
int(cos(a+(-1/n^2)^(1/2)*log(c*x^n)),x)
```

output
```
(x*(cos((log(x**n*c)*i + a*n)/n)*log(x**n*c) + cos((log(x**n*c)*i + a*n)/n
)*n + log(x**n*c)*sin((log(x**n*c)*i + a*n)/n)*i))/(2*n)
```

3.106 $\int x^m \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 117

$$\int x^m \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)\,dx = \frac{x^{1+m}}{2(1+m)}+\frac{e^{-\frac{2a\sqrt{-\frac{(1+m)^2}{n^2}}\,n}{1+m}}x^{1+m}(cx^n)^{\frac{1+m}{n}}}{8(1+m)}+\frac{1}{4}e^{\frac{2a\sqrt{-\frac{(1+m)^2}{n^2}}\,n}{1+m}}x^{1+m}(cx^n)^{-\frac{1+m}{n}}\log(x)$$

output
```
x^(1+m)/(2+2*m)+1/8*x^(1+m)*(c*x^n)^((1+m)/n)/exp(2*a*(-(1+m)^2/n^2)^(1/2)
*n/(1+m))/(1+m)+1/4*exp(2*a*(-(1+m)^2/n^2)^(1/2)*n/(1+m))*x^(1+m)*ln(x)/((
c*x^n)^((1+m)/n))
```

Mathematica [F]

$$\int x^m \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)\,dx = \int x^m \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)\,dx$$

input
```
Integrate[x^m*Cos[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^2,x]
```

output
```
Integrate[x^m*Cos[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^2, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4997, 4993, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)dx$$

$$\Big\downarrow \text{ 4997}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\cos^2\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{ 4993}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int \left(2(cx^n)^{\frac{m+1}{n}-1}+e^{-\frac{2a\sqrt{-\frac{(m+1)^2}{n^2}}n}{m+1}}(cx^n)^{\frac{2(m+1)}{n}-1}+\frac{e^{\frac{2a\sqrt{-\frac{(m+1)^2}{n^2}}n}{m+1}}x^{-n}}{c}\right)d(cx^n)}{4n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{ne^{-\frac{2an\sqrt{-\frac{(m+1)^2}{n^2}}}{m+1}}(cx^n)^{\frac{2(m+1)}{n}}}{2(m+1)}+e^{\frac{2an\sqrt{-\frac{(m+1)^2}{n^2}}}{m+1}}\log\left(cx^n\right)+\frac{2n(cx^n)^{\frac{m+1}{n}}}{m+1}\right)}{4n}$$

input
```
Int[x^m*Cos[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^2,x]
```

output
```
(x^(1 + m)*((2*n*(c*x^n)^((1 + m)/n))/(1 + m) + (n*(c*x^n)^((2*(1 + m))/n)
)/(2*E^((2*a*Sqrt[-((1 + m)^2/n^2)]*n)/(1 + m))*(1 + m)) + E^((2*a*Sqrt[-(
(1 + m)^2/n^2)]*n)/(1 + m))*Log[c*x^n]))/(4*n*(c*x^n)^((1 + m)/n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4993
```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[1/2^p   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^2*(p/(m + 1)))/x^((m
+ 1)/p) + x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x], x], x] /; FreeQ[{a,
 b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m + 1)^2, 0]
```

rule 4997
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cos[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^{m} \cos\left(a+\frac{\sqrt{-\frac{(1+m)^{2}}{n^{2}}}\, \ln\left(c\,x^{n}\right)}{2}\right)^{2} dx$$

input
```
int(x^m*cos(a+1/2*(-(1+m)^2/n^2)^(1/2)*ln(c*x^n))^2,x)
```

output
```
int(x^m*cos(a+1/2*(-(1+m)^2/n^2)^(1/2)*ln(c*x^n))^2,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int x^m \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\frac{\left(2\,(m+1)e^{\left(-\frac{2\,((m+1)n\log(x)-2i\,an+(m+1)\log(c))}{n}\right)}\log\left(x\right)+4\,e^{\left(-\frac{(m+1)n\log(x)-2i\,an+(m+1)\log(c)}{n}\right)}+1\right)e^{\left(\frac{2\,((m+1)n\log(x)-2i}{n}\right.}}{8\,(m+1)}$$

input
```
integrate(x^m*cos(a+1/2*(-(1+m)^2/n^2)^(1/2)*log(c*x^n))^2,x, algorithm="f
ricas")
```

output
```
1/8*(2*(m + 1)*e^(-2*((m + 1)*n*log(x) - 2*I*a*n + (m + 1)*log(c))/n)*log(
x) + 4*e^(-((m + 1)*n*log(x) - 2*I*a*n + (m + 1)*log(c))/n) + 1)*e^(2*((m
+ 1)*n*log(x) - 2*I*a*n + (m + 1)*log(c))/n + (2*I*a*n - (m + 1)*log(c))/n
)/(m + 1)
```

Sympy [F]

$$\int x^m \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)dx$$
$$=\int x^m \cos^2\left(a+\frac{\sqrt{-\frac{m^2}{n^2}-\frac{2m}{n^2}-\frac{1}{n^2}}\log\left(cx^n\right)}{2}\right)dx$$

input
```
integrate(x**m*cos(a+1/2*(-(1+m)**2/n**2)**(1/2)*ln(c*x**n))**2,x)
```

output
```
Integral(x**m*cos(a + sqrt(-m**2/n**2 - 2*m/n**2 - 1/n**2)*log(c*x**n)/2)*
*2, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.47

$$\int x^m \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\frac{4\left(\cos\left(2a\right)^2+\sin\left(2a\right)^2\right)c^{\frac{m}{n}+\frac{1}{n}}xx^m+c^{\frac{2m}{n}+\frac{2}{n}}x\cos\left(2a\right)e^{\left(m\log(x)+\frac{m\log(x^n)}{n}+\frac{\log(x^n)}{n}\right)}+2\left(\cos\left(2a\right)^3+\cos\left(2\right.\right.}{8\left(\left(\cos\left(2a\right)^2+\sin\left(2a\right)^2\right)c^{\frac{m}{n}+\frac{1}{n}}m+\left(\cos\left(2a\right)^2+\sin\left(2a\right)\right.\right.}$$

input
```
integrate(x^m*cos(a+1/2*(-(1+m)^2/n^2)^(1/2)*log(c*x^n))^2,x, algorithm="m
axima")
```

output
```
1/8*(4*(cos(2*a)^2 + sin(2*a)^2)*c^(m/n + 1/n)*x*x^m + c^(2*m/n + 2/n)*x*c
os(2*a)*e^(m*log(x) + m*log(x^n)/n + log(x^n)/n) + 2*(cos(2*a)^3 + cos(2*a
)*sin(2*a)^2 + (cos(2*a)^3 + cos(2*a)*sin(2*a)^2)*m)*log(x))/((cos(2*a)^2
+ sin(2*a)^2)*c^(m/n + 1/n)*m + (cos(2*a)^2 + sin(2*a)^2)*c^(m/n + 1/n))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.71 (sec) , antiderivative size = 498, normalized size of antiderivative = 4.26

$$\int x^m \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\frac{m^2n^2xx^me^{\left(2ia-\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+m^2n^2xx^me^{\left(-2ia+\frac{n|mn+n|\log(x)+|mn+n|\log(c)}{n^2}\right)}+2m^2n^2xx^m+2mn^2\cdots}{}$$

input
```
integrate(x^m*cos(a+1/2*(-(1+m)^2/n^2)^(1/2)*log(c*x^n))^2,x, algorithm="g
iac")
```

output
```
1/4*(m^2*n^2*x*x^m*e^(2*I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c)
)/n^2) + m^2*n^2*x*x^m*e^(-2*I*a + (n*abs(m*n + n)*log(x) + abs(m*n + n)*l
og(c))/n^2) + 2*m^2*n^2*x*x^m + 2*m*n^2*x*x^m*e^(2*I*a - (n*abs(m*n + n)*l
og(x) + abs(m*n + n)*log(c))/n^2) + m*n*x*x^m*abs(m*n + n)*e^(2*I*a - (n*a
bs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 2*m*n^2*x*x^m*e^(-2*I*a +
 (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - m*n*x*x^m*abs(m*n +
n)*e^(-2*I*a + (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 4*m*n^
2*x*x^m + n^2*x*x^m*e^(2*I*a - (n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c
))/n^2) + n*x*x^m*abs(m*n + n)*e^(2*I*a - (n*abs(m*n + n)*log(x) + abs(m*n
 + n)*log(c))/n^2) + n^2*x*x^m*e^(-2*I*a + (n*abs(m*n + n)*log(x) + abs(m*
n + n)*log(c))/n^2) - n*x*x^m*abs(m*n + n)*e^(-2*I*a + (n*abs(m*n + n)*log
(x) + abs(m*n + n)*log(c))/n^2) - 2*(m*n + n)^2*x*x^m + 2*n^2*x*x^m)/(m^3*
n^2 + 3*m^2*n^2 - (m*n + n)^2*m + 3*m*n^2 - (m*n + n)^2 + n^2)
```

Mupad [B] (verification not implemented)

Time = 21.04 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.22

$$\int x^m \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx = \frac{x\,x^m}{2\,m+2}+\frac{x\,x^m\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}}}{4\,m+4-n\,\sqrt{-\frac{{\left(m+1\right)}^2}{n^2}}\,4\mathrm{i}}+\frac{x\,x^m\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}}{4\,m+4+n\,\sqrt{-\frac{{\left(m+1\right)}^2}{n^2}}\,4\mathrm{i}}$$

input
```
int(x^m*cos(a + (log(c*x^n)*(-(m + 1)^2/n^2)^(1/2))/2)^2,x)
```

output
```
(x*x^m)/(2*m + 2) + (x*x^m*exp(-a*2i)/(c*x^n)^((- (2*m)/n^2 - 1/n^2 - m^2/
n^2)^(1/2)*1i))/(4*m - n*(-(m + 1)^2/n^2)^(1/2)*4i + 4) + (x*x^m*exp(a*2i)
*(c*x^n)^((- (2*m)/n^2 - 1/n^2 - m^2/n^2)^(1/2)*1i))/(4*m + n*(-(m + 1)^2/
n^2)^(1/2)*4i + 4)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81

$$\int x^m \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\frac{x^m x\left(2\cos\left(\frac{\log(x^n c)m+\log(x^n c)+2an}{2n}\right)\sin\left(\frac{\log(x^n c)m+\log(x^n c)+2an}{2n}\right)-2\sin\left(\frac{\log(x^n c)m+\log(x^n c)+2an}{2n}\right)^2+3\right)}{4m+4}$$

input
```
int(x^m*cos(a+1/2*(-(1+m)^2/n^2)^(1/2)*log(c*x^n))^2,x)
```

output
```
(x**m*x*(2*cos((log(x**n*c)*m + log(x**n*c) + 2*a*n)/(2*n))*sin((log(x**n*
c)*m + log(x**n*c) + 2*a*n)/(2*n)) - 2*sin((log(x**n*c)*m + log(x**n*c) +
2*a*n)/(2*n))**2 + 3))/(4*(m + 1))
```

3.107 $\int \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\frac{x}{2}+\frac{1}{8}e^{-2a\sqrt{-\frac{1}{n^2}}n}x(cx^n)^{\frac{1}{n}}+\frac{1}{4}e^{2a\sqrt{-\frac{1}{n^2}}n}x(cx^n)^{-1/n}\log(x)$$

output
```
1/2*x+1/8*x*(c*x^n)^(1/n)/exp(2*a*(-1/n^2)^(1/2)*n)+1/4*exp(2*a*(-1/n^2)^(
1/2)*n)*x*ln(x)/((c*x^n)^(1/n))
```

Mathematica [F]

$$\int \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\int \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$

input
```
Integrate[Cos[a + (Sqrt[-n^(-2)]*Log[c*x^n])/2]^2,x]
```

output
```
Integrate[Cos[a + (Sqrt[-n^(-2)]*Log[c*x^n])/2]^2, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4987, 4993, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx$$
$$\Big\downarrow \text{4987}$$
$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\cos^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)d(cx^n)}{n}$$
$$\Big\downarrow \text{4993}$$
$$\frac{x(cx^n)^{-1/n}\int \left(2(cx^n)^{\frac{1}{n}-1}+e^{-2a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{2}{n}-1}+\frac{e^{2a\sqrt{-\frac{1}{n^2}}n}x^{-n}}{c}\right)d(cx^n)}{4n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{x(cx^n)^{-1/n}\left(\frac{1}{2}ne^{-2a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{2/n}+e^{2a\sqrt{-\frac{1}{n^2}}n}\log\left(cx^n\right)+2n(cx^n)^{\frac{1}{n}}\right)}{4n}$$

input
```
Int[Cos[a + (Sqrt[-n^(-2)]*Log[c*x^n])/2]^2,x]
```

output
```
(x*(2*n*(c*x^n)^n^(-1) + (n*(c*x^n)^(2/n))/(2*E^(2*a*Sqrt[-n^(-2)]*n)) + E
^(2*a*Sqrt[-n^(-2)]*n)*Log[c*x^n]))/(4*n*(c*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4987
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cos[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 4993
```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[1/2^p   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^2*(p/(m + 1)))/x^((m
+ 1)/p) + x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x], x], x] /; FreeQ[{a,
 b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m + 1)^2, 0]
```

Maple [F]

$$\int \cos\left(a+\frac{\sqrt{-\frac{1}{n^2}}\,\ln\left(c\,x^n\right)}{2}\right)^2 dx$$

input
```
int(cos(a+1/2*(-1/n^2)^(1/2)*ln(c*x^n))^2,x)
```

output
```
int(cos(a+1/2*(-1/n^2)^(1/2)*ln(c*x^n))^2,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

$$\int \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx = \frac{1}{8}\left(x^2+4\,xe^{\left(\frac{2i\,an-\log(c)}{n}\right)}+2\,e^{\left(\frac{2\,(2i\,an-\log(c))}{n}\right)}\log\left(x\right)\right)e^{\left(-\frac{2i\,an-\log(c)}{n}\right)}$$

input
```
integrate(cos(a+1/2*(-1/n^2)^(1/2)*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
1/8*(x^2 + 4*x*e^((2*I*a*n - log(c))/n) + 2*e^(2*(2*I*a*n - log(c))/n)*log
(x))*e^(-(2*I*a*n - log(c))/n)
```

Sympy [F]

$$\int \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\int \cos^2\left(a+\frac{\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)}{2}\right)\,dx$$

input
```
integrate(cos(a+1/2*(-1/n**2)**(1/2)*ln(c*x**n))**2,x)
```

output
```
Integral(cos(a + sqrt(-1/n**2)*log(c*x**n)/2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.60

$$\int \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\frac{c^{\frac{2}{n}}x^2\cos\left(2\,a\right)+4\,c^{\left(\frac{1}{n}\right)}x+2\,\cos\left(2\,a\right)\log\left(x\right)}{8\,c^{\left(\frac{1}{n}\right)}}$$

input
```
integrate(cos(a+1/2*(-1/n^2)^(1/2)*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
1/8*(c^(2/n)*x^2*cos(2*a) + 4*c^(1/n)*x + 2*cos(2*a)*log(x))/c^(1/n)
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.01

$$\int \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=+\infty$$

input
```
integrate(cos(a+1/2*(-1/n^2)^(1/2)*log(c*x^n))^2,x, algorithm="giac")
```

output
```
+Infinity
```

Mupad [B] (verification not implemented)

Time = 20.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.26

$$\int \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\frac{x}{2}+\frac{x\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}}\,1\mathrm{i}}{4\,n\,\sqrt{-\frac{1}{n^2}}+4\mathrm{i}}-\frac{x\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{\sqrt{-\frac{1}{n^2}}\,1\mathrm{i}}\,1\mathrm{i}}{4\,n\,\sqrt{-\frac{1}{n^2}}-4\mathrm{i}}$$

input
```
int(cos(a + (log(c*x^n)*(-1/n^2)^(1/2))/2)^2,x)
```

output
```
x/2 + (x*exp(-a*2i)/(c*x^n)^((-1/n^2)^(1/2)*1i)*1i)/(4*n*(-1/n^2)^(1/2) +
4i) - (x*exp(a*2i)*(c*x^n)^((-1/n^2)^(1/2)*1i)*1i)/(4*n*(-1/n^2)^(1/2) - 4
i)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.71

$$\int \cos^2\left(a+\frac{1}{2}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$

$$=\frac{x\left(2\cos\left(\frac{\log(x^nc)i+2an}{2n}\right)\log(x^nc)\sin\left(\frac{\log(x^nc)i+2an}{2n}\right)i-2\log(x^nc)\sin\left(\frac{\log(x^nc)i+2an}{2n}\right)^2+\log(x^nc)-2\sin\left(\frac{\log\right.\right.}{4n}$$

input
```
int(cos(a+1/2*(-1/n^2)^(1/2)*log(c*x^n))^2,x)
```

output
```
(x*(2*cos((log(x**n*c)*i + 2*a*n)/(2*n))*log(x**n*c)*sin((log(x**n*c)*i +
2*a*n)/(2*n))*i - 2*log(x**n*c)*sin((log(x**n*c)*i + 2*a*n)/(2*n))**2 + lo
g(x**n*c) - 2*sin((log(x**n*c)*i + 2*a*n)/(2*n))**2*n + 3*n))/(4*n)
```

3.108 $\int x^m \cos^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 226

$$\int x^m \cos^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx = \frac{8x^{1+m}\cos\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)}{5(1+m)} - \frac{4x^{1+m}\cos^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)}{5(1+m)} + \frac{4\sqrt{-\frac{(1+m)^2}{n^2}}nx^{1+m}\sin\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)}{5(1+m)^2} - \frac{6\sqrt{-\frac{(1+m)^2}{n^2}}nx^{1+m}\cos^2\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\sin\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)}{5(1+m)^2}$$

output

```
8*x^(1+m)*cos(a+1/2*(-(1+m)^2/n^2)^(1/2)*ln(c*x^n))/(5+5*m)-4*x^(1+m)*cos(
a+1/2*(-(1+m)^2/n^2)^(1/2)*ln(c*x^n))^3/(5+5*m)+4/5*(-(1+m)^2/n^2)^(1/2)*n
*x^(1+m)*sin(a+1/2*(-(1+m)^2/n^2)^(1/2)*ln(c*x^n))/(1+m)^2-6/5*(-(1+m)^2/n
^2)^(1/2)*n*x^(1+m)*cos(a+1/2*(-(1+m)^2/n^2)^(1/2)*ln(c*x^n))^2*sin(a+1/2*
(-(1+m)^2/n^2)^(1/2)*ln(c*x^n))/(1+m)^2
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.70

$$\int x^m \cos^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\frac{x^{1+m}\left(10(1+m)\cos\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)-2(1+m)\cos\left(3a+\frac{3}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)+\sqrt{-\cdots}\right.}{10(1+m)^2}$$

input
```
Integrate[x^m*Cos[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^3,x]
```

output
```
(x^(1 + m)*(10*(1 + m)*Cos[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2] - 2*
(1 + m)*Cos[3*a + (3*Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2] + Sqrt[-((1 + m
)^2/n^2)]*n*(5*Sin[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2] - 3*Sin[3*a
+ (3*Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2])))/(10*(1 + m)^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {4991, 4989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cos^3\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$\Big\downarrow \ 4991$$
$$\frac{6}{5}\int x^m \cos\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)dx-\frac{4x^{m+1}\cos^3\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)}{5(m+1)}-\frac{6n\sqrt{-\frac{(m+1)^2}{n^2}}x^{m+1}\sin\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)\cos^2\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)}{5(m+1)^2}$$

$$\Big\downarrow \text{4989}$$

$$-\frac{4x^{m+1}\cos^3\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)}{5(m+1)}-\frac{6n\sqrt{-\frac{(m+1)^2}{n^2}}x^{m+1}\sin\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)\cos^2\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)}{5(m+1)^2}+\frac{6}{5}\left(\frac{2n\sqrt{-\frac{(m+1)^2}{n^2}}x^{m+1}\sin\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)}{3(m+1)^2}+\frac{4x^{m+1}\cos\left(a+\frac{1}{2}\sqrt{-\frac{(m+1)^2}{n^2}}\log\left(cx^n\right)\right)}{3(m+1)}\right)$$

input
```
Int[x^m*Cos[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^3,x]
```

output
```
(-4*x^(1 + m)*Cos[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2]^3)/(5*(1 + m)
) - (6*Sqrt[-((1 + m)^2/n^2)]*n*x^(1 + m)*Cos[a + (Sqrt[-((1 + m)^2/n^2)]*
Log[c*x^n])/2]^2*Sin[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2])/(5*(1 + m
)^2) + (6*((4*x^(1 + m)*Cos[a + (Sqrt[-((1 + m)^2/n^2)]*Log[c*x^n])/2])/(3
*(1 + m)) + (2*Sqrt[-((1 + m)^2/n^2)]*n*x^(1 + m)*Sin[a + (Sqrt[-((1 + m)^
2/n^2)]*Log[c*x^n])/2])/(3*(1 + m)^2)))/5
```

Defintions of rubi rules used

rule 4989
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

rule 4991
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Simp[b*d*n*p*(e*x)^(m + 1)*Sin[d*(a
+ b*Log[c*x^n])]*(Cos[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*
(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2)
)   Int[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [A] (verified)

Time = 125.53 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.96

method	result
parallelrisch	$-\frac{x^{1+m}\left((1+m)\cos\left(3a+\frac{3\sqrt{-\frac{(1+m)^2}{n^2}}\,\ln(c\,x^n)}{2}\right)+\frac{3\sqrt{-\frac{(1+m)^2}{n^2}}\,n\sin\left(3a+\frac{3\sqrt{-\frac{(1+m)^2}{n^2}}\,\ln(c\,x^n)}{2}\right)}{2}+(1+m)\cos\left(\sqrt{-\frac{(1+m)^2}{n^2}}\right.\right.}{}$

input
```
int(x^m*cos(a+1/2*(-(1+m)^2/n^2)^(1/2)*ln(c*x^n))^3,x,method=_RETURNVERBOS
E)
```

output
```
-1/5*x^(1+m)*((1+m)*cos(3*a+3/2*(-(1+m)^2/n^2)^(1/2)*ln(c*x^n))+3/2*(-(1+m
)^2/n^2)^(1/2)*n*sin(3*a+3/2*(-(1+m)^2/n^2)^(1/2)*ln(c*x^n))+(1+m)*cos((-(
1+m)^2/n^2)^(1/2)*ln(c*x^n)+2*a)+(-(1+m)^2/n^2)^(1/2)*n*sin((-(1+m)^2/n^2)
^(1/2)*ln(c*x^n)+2*a)+(-5*m-5)*cos(a+(-(1+m)^2/n^2)^(1/2)*ln((c*x^n)^(1/2)
))-5/2*(-(1+m)^2/n^2)^(1/2)*sin(a+(-(1+m)^2/n^2)^(1/2)*ln((c*x^n)^(1/2)))*
n)/(1+m)^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.57

$$\int x^m \cos^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)dx$$
$$=\frac{\left(5\,e^{\left(-\frac{(m+1)n\log(x)-2i\,an+(m+1)\log(c)}{n}\right)}+15\,e^{\left(-\frac{2\,((m+1)n\log(x)-2i\,an+(m+1)\log(c))}{n}\right)}-5\,e^{\left(-\frac{3\,((m+1)n\log(x)-2i\,an+(m+1)\log(c))}{n}\right)}\right.}{20\,(m+1)}$$

input
```
integrate(x^m*cos(a+1/2*(-(1+m)^2/n^2)^(1/2)*log(c*x^n))^3,x, algorithm="f
ricas")
```

output
```
1/20*(5*e^(-((m + 1)*n*log(x) - 2*I*a*n + (m + 1)*log(c))/n) + 15*e^(-2*((
m + 1)*n*log(x) - 2*I*a*n + (m + 1)*log(c))/n) - 5*e^(-3*((m + 1)*n*log(x)
 - 2*I*a*n + (m + 1)*log(c))/n) + 1)*e^(5/2*((m + 1)*n*log(x) - 2*I*a*n +
(m + 1)*log(c))/n + (2*I*a*n - (m + 1)*log(c))/n)/(m + 1)
```

Sympy [F]

$$\int x^m \cos^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)\,dx$$
$$=\int x^m \cos^3\left(a+\frac{\sqrt{-\frac{m^2}{n^2}-\frac{2m}{n^2}-\frac{1}{n^2}}\log(cx^n)}{2}\right)\,dx$$

input
```
integrate(x**m*cos(a+1/2*(-(1+m)**2/n**2)**(1/2)*ln(c*x**n))**3,x)
```

output
```
Integral(x**m*cos(a + sqrt(-m**2/n**2 - 2*m/n**2 - 1/n**2)*log(c*x**n)/2)*
*3, x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.86

$$\int x^m \cos^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log(cx^n)\right)\,dx$$
$$=\frac{\left(c^{\frac{3m}{n}+\frac{3}{n}}x\cos(3a)e^{\left(m\log(x)+\frac{3m\log(x^n)}{n}+\frac{3\log(x^n)}{n}\right)}+5c^{\frac{2m}{n}+\frac{2}{n}}x\cos(a)e^{\left(m\log(x)+\frac{2m\log(x^n)}{n}+\frac{2\log(x^n)}{n}\right)}+15c^{\frac{m}{n}+\frac{1}{n}}\cdots\right.}{20\left(c^{\frac{3m}{2n}+\frac{3}{2n}}m+c^{\frac{3m}{2n}+\frac{3}{2n}}\right)}$$

input
```
integrate(x^m*cos(a+1/2*(-(1+m)^2/n^2)^(1/2)*log(c*x^n))^3,x, algorithm="m
axima")
```

output
```
1/20*(c^(3*m/n + 3/n)*x*cos(3*a)*e^(m*log(x) + 3*m*log(x^n)/n + 3*log(x^n)
/n) + 5*c^(2*m/n + 2/n)*x*cos(a)*e^(m*log(x) + 2*m*log(x^n)/n + 2*log(x^n)
/n) + 15*c^(m/n + 1/n)*x*cos(a)*e^(m*log(x) + m*log(x^n)/n + log(x^n)/n) -
 5*x*x^m*cos(3*a))*e^(-3/2*m*log(x^n)/n - 3/2*log(x^n)/n)/(c^(3/2*m/n + 3/
2/n)*m + c^(3/2*m/n + 3/2/n))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.58 (sec) , antiderivative size = 1870, normalized size of antiderivative = 8.27

$$\int x^m \cos^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right) dx = \text{Too large to display}$$

input
```
integrate(x^m*cos(a+1/2*(-(1+m)^2/n^2)^(1/2)*log(c*x^n))^3,x, algorithm="g
iac")
```

output
```
1/4*(8*m^3*n^4*x*x^m*e^(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*
log(c))/n^2) + 24*m^3*n^4*x*x^m*e^(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(
m*n + n)*log(c))/n^2) + 24*m^3*n^4*x*x^m*e^(-I*a + 1/2*(n*abs(m*n + n)*log
(x) + abs(m*n + n)*log(c))/n^2) + 8*m^3*n^4*x*x^m*e^(-3*I*a + 3/2*(n*abs(m
*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 24*m^2*n^4*x*x^m*e^(3*I*a - 3
/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 12*m^2*n^3*x*x^m*a
bs(m*n + n)*e^(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n
^2) + 72*m^2*n^4*x*x^m*e^(I*a - 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*
log(c))/n^2) + 12*m^2*n^3*x*x^m*abs(m*n + n)*e^(I*a - 1/2*(n*abs(m*n + n)*
log(x) + abs(m*n + n)*log(c))/n^2) + 72*m^2*n^4*x*x^m*e^(-I*a + 1/2*(n*abs
(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) - 12*m^2*n^3*x*x^m*abs(m*n +
n)*e^(-I*a + 1/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 24*m
^2*n^4*x*x^m*e^(-3*I*a + 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))
/n^2) - 12*m^2*n^3*x*x^m*abs(m*n + n)*e^(-3*I*a + 3/2*(n*abs(m*n + n)*log(
x) + abs(m*n + n)*log(c))/n^2) - 2*(m*n + n)^2*m*n^2*x*x^m*e^(3*I*a - 3/2*
(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 24*m*n^4*x*x^m*e^(3*I
*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c))/n^2) + 24*m*n^3*x*x
^m*abs(m*n + n)*e^(3*I*a - 3/2*(n*abs(m*n + n)*log(x) + abs(m*n + n)*log(c
))/n^2) - 54*(m*n + n)^2*m*n^2*x*x^m*e^(I*a - 1/2*(n*abs(m*n + n)*log(x) +
 abs(m*n + n)*log(c))/n^2) + 72*m*n^4*x*x^m*e^(I*a - 1/2*(n*abs(m*n + n...
```

Mupad [B] (verification not implemented)

Time = 22.37 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.23

$$\int x^m \cos^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx = \frac{x\,x^m\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{(c\,x^n)^{\frac{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}{2}}}\left(2\,m+2+n\,\sqrt{-\frac{(m+1)^2}{n^2}}\,1\mathrm{i}\right)}{4\,(m+1)^2} + \frac{x\,x^m\,\mathrm{e}^{a\,1\mathrm{i}}\,(c\,x^n)^{\frac{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,1\mathrm{i}}{2}}\left(2\,m+2-n\,\sqrt{-\frac{(m+1)^2}{n^2}}\,1\mathrm{i}\right)}{4\,(m+1)^2} - \frac{x\,x^m\,\mathrm{e}^{-a\,3\mathrm{i}}\,\frac{1}{(c\,x^n)^{\frac{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,3\mathrm{i}}{2}}}\left(2\,m+2+n\,\sqrt{-\frac{(m+1)^2}{n^2}}\,3\mathrm{i}\right)}{20\,(m+1)^2} - \frac{x\,x^m\,\mathrm{e}^{a\,3\mathrm{i}}\,(c\,x^n)^{\frac{\sqrt{-\frac{2\,m}{n^2}-\frac{1}{n^2}-\frac{m^2}{n^2}}\,3\mathrm{i}}{2}}\left(2\,m+2-n\,\sqrt{-\frac{(m+1)^2}{n^2}}\,3\mathrm{i}\right)}{20\,(m+1)^2}$$

input
```
int(x^m*cos(a + (log(c*x^n)*(-(m + 1)^2/n^2)^(1/2))/2)^3,x)
```

output
```
(x*x^m*exp(-a*1i)/(c*x^n)^(((- (2*m)/n^2 - 1/n^2 - m^2/n^2)^(1/2)*1i)/2)*(
2*m + n*(-(m + 1)^2/n^2)^(1/2)*1i + 2))/(4*(m + 1)^2) + (x*x^m*exp(a*1i)*(
c*x^n)^(((- (2*m)/n^2 - 1/n^2 - m^2/n^2)^(1/2)*1i)/2)*(2*m - n*(-(m + 1)^2
/n^2)^(1/2)*1i + 2))/(4*(m + 1)^2) - (x*x^m*exp(-a*3i)/(c*x^n)^(((- (2*m)/
n^2 - 1/n^2 - m^2/n^2)^(1/2)*3i)/2)*(2*m + n*(-(m + 1)^2/n^2)^(1/2)*3i + 2
))/(20*(m + 1)^2) - (x*x^m*exp(a*3i)*(c*x^n)^(((- (2*m)/n^2 - 1/n^2 - m^2/
n^2)^(1/2)*3i)/2)*(2*m - n*(-(m + 1)^2/n^2)^(1/2)*3i + 2))/(20*(m + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.67

$$\int x^m \cos^3\left(a+\frac{1}{2}\sqrt{-\frac{(1+m)^2}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\frac{2x^m x\left(-10\cos\left(\frac{\log(x^n c)m+\log(x^n c)+2an}{2n}\right)\sin\left(\frac{\log(x^n c)m+\log(x^n c)+2an}{2n}\right)^2+22\cos\left(\frac{\log(x^n c)m+\log(x^n c)+2an}{2n}\right)-15s\right.}{65m+65}$$

input
```
int(x^m*cos(a+1/2*(-(1+m)^2/n^2)^(1/2)*log(c*x^n))^3,x)
```

output
```
(2*x**m*x*( - 10*cos((log(x**n*c)*m + log(x**n*c) + 2*a*n)/(2*n))*sin((log
(x**n*c)*m + log(x**n*c) + 2*a*n)/(2*n))**2 + 22*cos((log(x**n*c)*m + log(
x**n*c) + 2*a*n)/(2*n)) - 15*sin((log(x**n*c)*m + log(x**n*c) + 2*a*n)/(2*
n))**3 + 21*sin((log(x**n*c)*m + log(x**n*c) + 2*a*n)/(2*n))))/(65*(m + 1)
)
```

3.109 $\int \cos^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 128

$$\int \cos^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx = \frac{9}{16}e^{a\sqrt{-\frac{1}{n^2}}n}x(cx^n)^{-\frac{1}{3}/n}+\frac{9}{32}e^{-a\sqrt{-\frac{1}{n^2}}n}x(cx^n)^{\frac{1}{3}/n} + \frac{1}{16}e^{-3a\sqrt{-\frac{1}{n^2}}n}x(cx^n)^{\frac{1}{n}} + \frac{1}{8}e^{3a\sqrt{-\frac{1}{n^2}}n}x(cx^n)^{-1/n}\log(x)$$

output
```
9/16*exp(a*(-1/n^2)^(1/2)*n)*x/((c*x^n)^(1/3/n))+9/32*x*(c*x^n)^(1/3/n)/ex
p(a*(-1/n^2)^(1/2)*n)+1/16*x*(c*x^n)^(1/n)/exp(3*a*(-1/n^2)^(1/2)*n)+1/8*e
xp(3*a*(-1/n^2)^(1/2)*n)*x*ln(x)/((c*x^n)^(1/n))
```

Mathematica [F]

$$\int \cos^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx = \int \cos^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$

input
```
Integrate[Cos[a + (Sqrt[-n^(-2)]*Log[c*x^n])/3]^3,x]
```

output
```
Integrate[Cos[a + (Sqrt[-n^(-2)]*Log[c*x^n])/3]^3, x]
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4987, 4993, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)dx$$

$$\Big\downarrow \text{ 4987}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\cos^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{ 4993}$$

$$\frac{x(cx^n)^{-1/n}\int \left(3e^{a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{2}{3n}-1}+3e^{-a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{4}{3n}-1}+e^{-3a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{2}{n}-1}+\frac{e^{3a\sqrt{-\frac{1}{n^2}}n}x^{-n}}{c}\right)d(cx^n)}{8n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x(cx^n)^{-1/n}\left(\frac{9}{2}ne^{a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{2}{3}/n}+\frac{9}{4}ne^{-a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{\frac{4}{3}/n}+\frac{1}{2}ne^{-3a\sqrt{-\frac{1}{n^2}}n}(cx^n)^{2/n}+e^{3a\sqrt{-\frac{1}{n^2}}n}\log\left(cx^n\right)\right)}{8n}$$

input
```
Int[Cos[a + (Sqrt[-n^(-2)]*Log[c*x^n])/3]^3,x]
```

output
```
(x*((9*E^(a*Sqrt[-n^(-2)]*n)*n*(c*x^n)^(2/(3*n)))/2 + (9*n*(c*x^n)^(4/(3*n
)))/(4*E^(a*Sqrt[-n^(-2)]*n)) + (n*(c*x^n)^(2/n))/(2*E^(3*a*Sqrt[-n^(-2)]*
n)) + E^(3*a*Sqrt[-n^(-2)]*n)*Log[c*x^n]))/(8*n*(c*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4987
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cos[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 4993
```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[1/2^p   Int[ExpandIntegrand[(e*x)^m*(E^(a*b*d^2*(p/(m + 1)))/x^((m
+ 1)/p) + x^((m + 1)/p)/E^(a*b*d^2*(p/(m + 1))))^p, x], x], x] /; FreeQ[{a,
 b, d, e, m}, x] && IGtQ[p, 0] && EqQ[b^2*d^2*p^2 + (m + 1)^2, 0]
```

Maple [F]

$$\int \cos\left(a+\frac{\sqrt{-\frac{1}{n^2}}\,\ln\left(c\,x^n\right)}{3}\right)^3 dx$$

input
```
int(cos(a+1/3*(-1/n^2)^(1/2)*ln(c*x^n))^3,x)
```

output
```
int(cos(a+1/3*(-1/n^2)^(1/2)*ln(c*x^n))^3,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.66

$$\int \cos^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\frac{1}{32}\left(9\,x^{\frac{4}{3}}e^{\left(\frac{2\,(3i\,an-\log(c))}{3\,n}\right)}+2\,x^2+12\,e^{\left(\frac{2\,(3i\,an-\log(c))}{n}\right)}\log\left(x^{\frac{1}{3}}\right)+18\,x^{\frac{2}{3}}e^{\left(\frac{4\,(3i\,an-\log(c))}{3\,n}\right)}\right)e^{\left(-\frac{3i\,an-\log(c)}{n}\right)}$$

input
```
integrate(cos(a+1/3*(-1/n^2)^(1/2)*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
1/32*(9*x^(4/3)*e^(2/3*(3*I*a*n - log(c))/n) + 2*x^2 + 12*e^(2*(3*I*a*n -
log(c))/n)*log(x^(1/3)) + 18*x^(2/3)*e^(4/3*(3*I*a*n - log(c))/n))*e^(-(3*
I*a*n - log(c))/n)
```

Sympy [F]

$$\int \cos^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx=\int \cos^3\left(a+\frac{\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)}{3}\right)dx$$

input
```
integrate(cos(a+1/3*(-1/n**2)**(1/2)*ln(c*x**n))**3,x)
```

output
```
Integral(cos(a + sqrt(-1/n**2)*log(c*x**n)/3)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.83

$$\int \cos^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$
$$=\frac{9\,c^{\frac{5}{3\,n}}x(x^n)^{\frac{2}{3\,n}}\cos\left(a\right)+4\,c^{\frac{1}{3\,n}}(x^n)^{\frac{1}{3\,n}}\cos\left(3\,a\right)\log\left(x\right)+18\,c^{\left(\frac{1}{n}\right)}x\cos\left(a\right)+2\,c^{\frac{7}{3\,n}}\cos\left(3\,a\right)e^{\left(\frac{\log(x^n)}{3\,n}+2\,\log(x)\right)}}{32\,c^{\frac{4}{3\,n}}(x^n)^{\frac{1}{3\,n}}}$$

input
```
integrate(cos(a+1/3*(-1/n^2)^(1/2)*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
1/32*(9*c^(5/3/n)*x*(x^n)^(2/3/n)*cos(a) + 4*c^(1/3/n)*(x^n)^(1/3/n)*cos(3
*a)*log(x) + 18*c^(1/n)*x*cos(a) + 2*c^(7/3/n)*cos(3*a)*e^(1/3*log(x^n)/n
 + 2*log(x)))/(c^(4/3/n)*(x^n)^(1/3/n))
```

Giac [F(-2)]

Exception generated.

$$\int \cos^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(cos(a+1/3*(-1/n^2)^(1/2)*log(c*x^n))^3,x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: (9*s
ageVARn^4*sageVARx*exp((-3*i)*sageVARa)*exp((sageVARn*abs(sageVARn)*ln(sag
eVARx)+abs(sageVARn)*ln(sageVARc))/sageVARn^2)+27*sageVARn^4*sageVARx*exp(
(-i)*sageVARa)*exp(
```

Mupad [B] (verification not implemented)

Time = 20.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.23

$$\int \cos^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx = x\,e^{-a\,1i}\,\frac{1}{\left(c\,x^n\right)^{\frac{\sqrt{-\frac{1}{n^2}}\,1i}{3}}}\left(\frac{27}{64}+\frac{n\,\sqrt{-\frac{1}{n^2}}\,9i}{64}\right) - x\,e^{a\,1i}\left(c\,x^n\right)^{\frac{\sqrt{-\frac{1}{n^2}}\,1i}{3}}\left(-\frac{27}{64}+\frac{n\,\sqrt{-\frac{1}{n^2}}\,9i}{64}\right) + \frac{x\,e^{-a\,3i}\,\frac{1}{\left(c\,x^n\right)^{\sqrt{-\frac{1}{n^2}}\,1i}}\,1i}{8\,n\,\sqrt{-\frac{1}{n^2}}+8i} - \frac{x\,e^{a\,3i}\left(c\,x^n\right)^{\sqrt{-\frac{1}{n^2}}\,1i}\,1i}{8\,n\,\sqrt{-\frac{1}{n^2}}-8i}$$

input
```
int(cos(a + (log(c*x^n)*(-1/n^2)^(1/2))/3)^3,x)
```

output
```
x*exp(-a*1i)/(c*x^n)^(((-1/n^2)^(1/2)*1i)/3)*((n*(-1/n^2)^(1/2)*9i)/64 + 2
7/64) - x*exp(a*1i)*(c*x^n)^(((-1/n^2)^(1/2)*1i)/3)*((n*(-1/n^2)^(1/2)*9i)
/64 - 27/64) + (x*exp(-a*3i)/(c*x^n)^((-1/n^2)^(1/2)*1i)*1i)/(8*n*(-1/n^2)
^(1/2) + 8i) - (x*exp(a*3i)*(c*x^n)^((-1/n^2)^(1/2)*1i)*1i)/(8*n*(-1/n^2)^
(1/2) - 8i)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.79

$$\int \cos^3\left(a+\frac{1}{3}\sqrt{-\frac{1}{n^2}}\log\left(cx^n\right)\right)\,dx$$

$$= \frac{x\left(-4\cos\left(\frac{\log(x^nc)i+3an}{3n}\right)\log(x^nc)\sin\left(\frac{\log(x^nc)i+3an}{3n}\right)^2+\cos\left(\frac{\log(x^nc)i+3an}{3n}\right)\log(x^nc)-\cos\left(\frac{\log(x^nc)i+3an}{3n}\right)\text{s}\right.}{}$$

input
```
int(cos(a+1/3*(-1/n^2)^(1/2)*log(c*x^n))^3,x)
```

output
```
(x*( - 4*cos((log(x**n*c)*i + 3*a*n)/(3*n))*log(x**n*c)*sin((log(x**n*c)*i
 + 3*a*n)/(3*n))**2 + cos((log(x**n*c)*i + 3*a*n)/(3*n))*log(x**n*c) - cos
((log(x**n*c)*i + 3*a*n)/(3*n))*sin((log(x**n*c)*i + 3*a*n)/(3*n))**2*n +
7*cos((log(x**n*c)*i + 3*a*n)/(3*n))*n - 4*log(x**n*c)*sin((log(x**n*c)*i
+ 3*a*n)/(3*n))**3*i + 3*log(x**n*c)*sin((log(x**n*c)*i + 3*a*n)/(3*n))*i
+ 3*sin((log(x**n*c)*i + 3*a*n)/(3*n))**3*i*n))/(8*n)
```

3.110 $\int \sqrt{\cos(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \sqrt{\cos(a+b\log(cx^n))}\,dx = \frac{2x\sqrt{\cos(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2i+bn}{4bn},\frac{1}{4}\left(3-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{(2-ibn)\sqrt{1+e^{2ia}(cx^n)^{2ib}}}$$

output
```
2*x*cos(a+b*ln(c*x^n))^(1/2)*hypergeom([-1/2, -1/4*(2*I+b*n)/b/n],[3/4-1/2
*I/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(2-I*b*n)/(1+exp(2*I*a)*(c*x^n)^(2*I*
b))^(1/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 377 vs. 2(110) = 220.

Time = 3.49 (sec) , antiderivative size = 377, normalized size of antiderivative = 3.43

$$\int \sqrt{\cos \left(a+b \log \left(c x^{n}\right)\right)}\, dx = \frac{2 b e^{i a} n x (c x^{n})^{i b} \sqrt{2+2 e^{2 i a} \left(c x^{n}\right)^{2 i b}} \left((2 i+b n) x^{2 i b n} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}-\frac{i}{2 b n}, \frac{7}{4}-\frac{i}{2 b n},-e^{2 i a}(c x^{n})^{2 i b}\right.\right.}{(2 i+b n)(-2 i+3 b n) \sqrt{e^{-i a} \left(c x^{n}\right)^{-i b}+e^{i a} \left(c x^{n}\right)^{i b}} \left((-2+i b n)\right.} - \frac{2 x \sqrt{\cos \left(a+b \log \left(c x^{n}\right)\right)} \cos \left(a-b n \log (x)+b \log \left(c x^{n}\right)\right)}{-2 \cos \left(a-b n \log (x)+b \log \left(c x^{n}\right)\right)+b n \sin \left(a-b n \log (x)+b \log \left(c x^{n}\right)\right)}$$

input
```
Integrate[Sqrt[Cos[a + b*Log[c*x^n]]],x]
```

output
```
(2*b*E^(I*a)*n*x*(c*x^n)^(I*b)*Sqrt[2 + 2*E^((2*I)*a)*(c*x^n)^((2*I)*b)]*(
(2*I + b*n)*x^((2*I)*b*n)*Hypergeometric2F1[1/2, 3/4 - (I/2)/(b*n), 7/4 -
(I/2)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))] + (-2*I + 3*b*n)*Hypergeome
tric2F1[1/2, -1/4*(2*I + b*n)/(b*n), 3/4 - (I/2)/(b*n), -(E^((2*I)*a)*(c*x
^n)^((2*I)*b))]))/((2*I + b*n)*(-2*I + 3*b*n)*Sqrt[1/(E^(I*a)*(c*x^n)^(I*b
)) + E^(I*a)*(c*x^n)^(I*b)]*((-2 + I*b*n)*x^((2*I)*b*n) - I*E^((2*I)*a)*(-
2*I + b*n)*(c*x^n)^((2*I)*b))) - (2*x*Sqrt[Cos[a + b*Log[c*x^n]]]*Cos[a -
b*n*Log[x] + b*Log[c*x^n]])/(-2*Cos[a - b*n*Log[x] + b*Log[c*x^n]] + b*n*S
in[a - b*n*Log[x] + b*Log[c*x^n]])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4987, 4995, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos \left(a+b \log \left(c x^{n}\right)\right)}\, dx$$

$$\Big\downarrow \text{ 4987}$$

$$\frac{x (c x^{n})^{-1/n} \int \left(c x^{n}\right)^{\frac{1}{n}-1} \sqrt{\cos \left(a+b \log \left(c x^{n}\right)\right)} d(c x^{n})}{n}$$

$$\Big\downarrow \text{4995}$$

$$\frac{x(cx^n)^{-\frac{1}{n}+\frac{ib}{2}}\sqrt{\cos(a+b\log(cx^n))}\int(cx^n)^{-\frac{ib}{2}+\frac{1}{n}-1}\sqrt{e^{2ia}(cx^n)^{2ib}+1}d(cx^n)}{n\sqrt{1+e^{2ia}(cx^n)^{2ib}}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{bn+2i}{4bn},\frac{1}{4}\left(3-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)\sqrt{\cos(a+b\log(cx^n))}}{(2-ibn)\sqrt{1+e^{2ia}(cx^n)^{2ib}}}$$

input
```
Int[Sqrt[Cos[a + b*Log[c*x^n]]],x]
```

output
```
(2*x*Sqrt[Cos[a + b*Log[c*x^n]]]*Hypergeometric2F1[-1/2, -1/4*(2*I + b*n)/
(b*n), (3 - (2*I)/(b*n))/4, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 - I*b*n
)*Sqrt[1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4987
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cos[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 4995
```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Simp[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Maple [F]

$$\int \sqrt{\cos(a+b\ln(c\,x^n))}dx$$

input
```
int(cos(a+b*ln(c*x^n))^(1/2),x)
```

output
```
int(cos(a+b*ln(c*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\cos(a+b\log(cx^n))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{\cos(a+b\log(cx^n))}\,dx = \int \sqrt{\cos(a+b\log(cx^n))}\,dx$$

input
```
integrate(cos(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(sqrt(cos(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \sqrt{\cos(a + b\log(cx^n))}\,dx = \int \sqrt{\cos(b\log(cx^n) + a)}\,dx$$

input
```
integrate(cos(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(b*log(c*x^n) + a)), x)
```

Giac [F]

$$\int \sqrt{\cos(a + b\log(cx^n))}\,dx = \int \sqrt{\cos(b\log(cx^n) + a)}\,dx$$

input
```
integrate(cos(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(b*log(c*x^n) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(a + b\log(cx^n))}\,dx = \int \sqrt{\cos(a + b\ln(c\,x^n))}\,dx$$

input
```
int(cos(a + b*log(c*x^n))^(1/2),x)
```

output
```
int(cos(a + b*log(c*x^n))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\cos \left(a+b \log \left(c x^{n}\right)\right)} \, dx=\sqrt{\cos \left(\log \left(x^{n} c\right) b+a\right)}\, x+\frac{\left(\int \frac{\sqrt{\cos \left(\log \left(x^{n} c\right) b+a\right)}\, \sin \left(\log \left(x^{n} c\right) b+a\right)}{\cos \left(\log \left(x^{n} c\right) b+a\right)} d x\right) b n}{2}$$

input
```
int(cos(a+b*log(c*x^n))^(1/2),x)
```

output
```
(2*sqrt(cos(log(x**n*c)*b + a))*x + int((sqrt(cos(log(x**n*c)*b + a))*sin(
log(x**n*c)*b + a))/cos(log(x**n*c)*b + a),x)*b*n)/2
```

3.111 $\int \frac{\sqrt{\cos(a+b\log(cx^n))}}{x} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \frac{\sqrt{\cos(a+b\log(cx^n))}}{x} \, dx = \frac{2E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{bn}$$

output
```
2*EllipticE(sin(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\cos(a+b\log(cx^n))}}{x} \, dx = \frac{2E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{bn}$$

input
```
Integrate[Sqrt[Cos[a + b*Log[c*x^n]]]/x,x]
```

output
```
(2*EllipticE[(a + b*Log[c*x^n])/2, 2])/(b*n)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3039, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(a+b\log(cx^n))}}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \sqrt{\cos(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\sin\left(a+b\log(cx^n)+\frac{\pi}{2}\right)}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{bn}$$

input
```
Int[Sqrt[Cos[a + b*Log[c*x^n]]]/x,x]
```

output
```
(2*EllipticE[(a + b*Log[c*x^n])/2, 2])/(b*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(26) = 52$.

Time = 1.16 (sec) , antiderivative size = 181, normalized size of antiderivative = 7.54

method	result
derivativedivides	$\frac{2\sqrt{\left(2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}-1\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos\left(a+2b\ln\left(\sqrt{c\,x^{n}}\right)\right)}{2}}\,\sqrt{-2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}+1}\ \mathrm{EllipticE}\left(\right.}{n\sqrt{-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{4}+\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sqrt{2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}-1}}$
default	$\frac{2\sqrt{\left(2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}-1\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\sqrt{\frac{1}{2}-\frac{\cos\left(a+2b\ln\left(\sqrt{c\,x^{n}}\right)\right)}{2}}\,\sqrt{-2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}+1}\ \mathrm{EllipticE}\left(\right.}{n\sqrt{-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{4}+\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sqrt{2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}-1}}$

input
```
int(cos(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/n*((2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2
)*(sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cos(1/2*a+1/2*b*ln(c*x^n))^2+1)
^(1/2)*EllipticE(cos(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/(-2*sin(1/2*a+1/2*b*l
n(c*x^n))^4+sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sin(1/2*a+1/2*b*ln(c*x^n))
/(2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.50

$$\int \frac{\sqrt{\cos\left(a+b\log\left(cx^{n}\right)\right)}}{x}\,dx = \frac{i\sqrt{2}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+i\,\sin\left(bn\log\left(x\right)\right.\right.\right.}{}$$

input
```
integrate(cos(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

output
```
(I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*n*log(x
) + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a))) - I*sqrt(2)*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*n*log(x) + b*log(c) + a)
- I*sin(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F]

$$\int \frac{\sqrt{\cos(a+b\log(cx^n))}}{x}\,dx = \int \frac{\sqrt{\cos(a+b\log(cx^n))}}{x}\,dx$$

input
```
integrate(cos(a+b*ln(c*x**n))**(1/2)/x,x)
```

output
```
Integral(sqrt(cos(a + b*log(c*x**n)))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{\cos(a+b\log(cx^n))}}{x}\,dx = \int \frac{\sqrt{\cos(b\log(cx^n)+a)}}{x}\,dx$$

input
```
integrate(cos(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(b*log(c*x^n) + a))/x, x)
```

Giac [F]

$$\int \frac{\sqrt{\cos(a+b\log(cx^n))}}{x}\,dx = \int \frac{\sqrt{\cos(b\log(cx^n)+a)}}{x}\,dx$$

input
```
integrate(cos(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(cos(b*log(c*x^n) + a))/x, x)
```

Mupad [B] (verification not implemented)

Time = 19.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{\cos(a+b\log(cx^n))}}{x}\,dx = \frac{2\,\mathrm{E}\left(\frac{a}{2}+\frac{b\,\ln(c\,x^n)}{2}\,\middle|\,2\right)}{b\,n}$$

input
```
int(cos(a + b*log(c*x^n))^(1/2)/x,x)
```

output
```
(2*ellipticE(a/2 + (b*log(c*x^n))/2, 2))/(b*n)
```

Reduce [F]

$$\int \frac{\sqrt{\cos(a+b\log(cx^n))}}{x}\,dx = \int \frac{\sqrt{\cos(\log(x^n c)\,b+a)}}{x}\,dx$$

input
```
int(cos(a+b*log(c*x^n))^(1/2)/x,x)
```

output
```
int(sqrt(cos(log(x**n*c)*b + a))/x,x)
```

3.112 $\int \cos^{\frac{3}{2}}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \cos^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \frac{2x\cos^{\frac{3}{2}}(a+b\log(cx^n))\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i}{bn}\right),\frac{1}{4}\left(1-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{(2-3ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

output
```
2*x*cos(a+b*ln(c*x^n))^(3/2)*hypergeom([-3/2, -3/4-1/2*I/b/n],[1/4-1/2*I/b
/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(2-3*I*b*n)/(1+exp(2*I*a)*(c*x^n)^(2*I*b)
)^(3/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 220 vs. 2(109) = 218.

Time = 0.86 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.02

$$\int \cos^{\frac{3}{2}}(a+b\log(cx^n))\,dx = -\frac{6i\sqrt{2}b^2\sqrt{1+e^{2i(a+b\log(cx^n))}}n^2x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}-\frac{i}{2bn},\frac{5}{4}-\frac{i}{2bn},-e^{2i(a+b\log(cx^n))}\right)}{\sqrt{e^{-i(a+b\log(cx^n))}\left(1+e^{2i(a+b\log(cx^n))}\right)}(-2i+bn)(-2i+3bn)(2i+3bn)} + \frac{2x\sqrt{\cos(a+b\log(cx^n))}(2\cos(a+b\log(cx^n))+3bn\sin(a+b\log(cx^n)))}{4+9b^2n^2}$$

input
```
Integrate[Cos[a + b*Log[c*x^n]]^(3/2),x]
```

output
```
((-6*I)*Sqrt[2]*b^2*Sqrt[1 + E^((2*I)*(a + b*Log[c*x^n]))]*n^2*x*Hypergeom
etric2F1[1/2, 1/4 - (I/2)/(b*n), 5/4 - (I/2)/(b*n), -E^((2*I)*(a + b*Log[c
*x^n]))])/(Sqrt[(1 + E^((2*I)*(a + b*Log[c*x^n])))/E^(I*(a + b*Log[c*x^n])
)]*(-2*I + b*n)*(-2*I + 3*b*n)*(2*I + 3*b*n)) + (2*x*Sqrt[Cos[a + b*Log[c*
x^n]]]*(2*Cos[a + b*Log[c*x^n]] + 3*b*n*Sin[a + b*Log[c*x^n]]))/(4 + 9*b^2
*n^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4987, 4995, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{4987}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\cos^{\frac{3}{2}}(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{4995}$$

$$\frac{x(cx^n)^{-\frac{1}{n}+\frac{3ib}{2}} \cos^{\frac{3}{2}}(a+b\log(cx^n)) \int (cx^n)^{-\frac{3ib}{2}+\frac{1}{n}-1} \left(e^{2ia}(cx^n)^{2ib}+1\right)^{3/2} d(cx^n)}{n\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i}{bn}\right),\frac{1}{4}\left(1-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)\cos^{\frac{3}{2}}(a+b\log(cx^n))}{(2-3ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

input
```
Int[Cos[a + b*Log[c*x^n]]^(3/2),x]
```

output
```
(2*x*Cos[a + b*Log[c*x^n]]^(3/2)*Hypergeometric2F1[-3/2, (-3 - (2*I)/(b*n)
)/4, (1 - (2*I)/(b*n))/4, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 - (3*I)*b
*n)*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4987
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cos[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 4995
```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Simp[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Maple [F]

$$\int \cos\left(a+b\ln\left(c\,x^{n}\right)\right)^{\frac{3}{2}}dx$$

input
```
int(cos(a+b*ln(c*x^n))^(3/2),x)
```

output
```
int(cos(a+b*ln(c*x^n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \cos^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \cos^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)\,dx = \int \cos^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)dx$$

input
```
integrate(cos(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Integral(cos(a + b*log(c*x**n))**(3/2), x)
```

Maxima [F]

$$\int \cos^{\frac{3}{2}}(a + b\log(cx^n))\,dx = \int \cos(b\log(cx^n) + a)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*log(c*x^n) + a)^(3/2), x)
```

Giac [F]

$$\int \cos^{\frac{3}{2}}(a + b\log(cx^n))\,dx = \int \cos(b\log(cx^n) + a)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*log(c*x^n) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(a + b\log(cx^n))\,dx = \int \cos(a + b\ln(c\,x^n))^{3/2}\,dx$$

input
```
int(cos(a + b*log(c*x^n))^(3/2),x)
```

output
```
int(cos(a + b*log(c*x^n))^(3/2), x)
```

Reduce [F]

$$\int \cos^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\,dx=\int \sqrt{\cos\left(\log\left(x^n c\right)b+a\right)}\,\cos\left(\log(x^n c)\,b+a\right)dx$$

input
```
int(cos(a+b*log(c*x^n))^(3/2),x)
```

output
```
int(sqrt(cos(log(x**n*c)*b + a))*cos(log(x**n*c)*b + a),x)
```

3.113 $\int \frac{\cos^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \frac{\cos^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+b\log(cx^n)),2\right)}{3bn} + \frac{2\sqrt{\cos(a+b\log(cx^n))}\sin(a+b\log(cx^n))}{3bn}$$

output
```
2/3*InverseJacobiAM(1/2*a+1/2*b*ln(c*x^n),2^(1/2))/b/n+2/3*cos(a+b*ln(c*x^
n))^(1/2)*sin(a+b*ln(c*x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \frac{\cos^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2\left(\text{EllipticF}\left(\frac{1}{2}(a+b\log(cx^n)),2\right)+\sqrt{\cos(a+b\log(cx^n))}\sin(a+b\log(cx^n))\right)}{3bn}$$

input
```
Integrate[Cos[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
(2*(EllipticF[(a + b*Log[c*x^n])/2, 2] + Sqrt[Cos[a + b*Log[c*x^n]]]*Sin[a
 + b*Log[c*x^n]]))/(3*b*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3039, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \cos^{\frac{3}{2}}(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(a+b\log(cx^n)+\frac{\pi}{2}\right)^{3/2}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{1}{3}\int \frac{1}{\sqrt{\cos(a+b\log(cx^n))}}d\log(cx^n)+\frac{2\sin(a+b\log(cx^n))\sqrt{\cos(a+b\log(cx^n))}}{3b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(a+b\log(cx^n)+\frac{\pi}{2}\right)}}d\log(cx^n)+\frac{2\sin(a+b\log(cx^n))\sqrt{\cos(a+b\log(cx^n))}}{3b}}{n}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+b\log(cx^n)),2\right)}{3b}+\frac{2\sin(a+b\log(cx^n))\sqrt{\cos(a+b\log(cx^n))}}{3b}}{n}$$

input
```
Int[Cos[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
((2*EllipticF[(a + b*Log[c*x^n])/2, 2])/(3*b) + (2*Sqrt[Cos[a + b*Log[c*x^
n]]]*Sin[a + b*Log[c*x^n]])/(3*b))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(58) = 116$.

Time = 1.85 (sec) , antiderivative size = 247, normalized size of antiderivative = 3.92

method	result
derivativedivides	$-\frac{2\sqrt{\left(2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}-1\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\left(4\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{4}-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\cdots\right.}{3n\sqrt{-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{4}+\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\cdots}}$
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}-1\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\left(4\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{4}-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\cdots\right.}{3n\sqrt{-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{4}+\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\cdots}}$

input
```
int(cos(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2/3/n*((2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*(4*cos(1/2*a+1/2*b*ln(c*x^n))*sin(1/2*a+1/2*b*ln(c*x^n))^4-2*sin(1/2*
a+1/2*b*ln(c*x^n))^2*cos(1/2*a+1/2*b*ln(c*x^n))+(sin(1/2*a+1/2*b*ln(c*x^n)
)^2)^(1/2)*(-1+2*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*EllipticF(cos(1/2*a+1
/2*b*ln(c*x^n)),2^(1/2)))/(-2*sin(1/2*a+1/2*b*ln(c*x^n))^4+sin(1/2*a+1/2*b
*ln(c*x^n))^2)^(1/2)/sin(1/2*a+1/2*b*ln(c*x^n))/(2*cos(1/2*a+1/2*b*ln(c*x^
n))^2-1)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.70

$$\int \frac{\cos^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2\sqrt{\cos(bn\log(x)+b\log(c)+a)}\sin(bn\log(x)+b\log(c)+a)-i\sqrt{2}\text{weierstrassPInverse}(-4,0,\cos(b$$

input
```
integrate(cos(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(cos(b*n*log(x) + b*log(c) + a))*sin(b*n*log(x) + b*log(c) + a)
 - I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b*n*log(x) + b*log(c) + a) + I
*sin(b*n*log(x) + b*log(c) + a)) + I*sqrt(2)*weierstrassPInverse(-4, 0, co
s(b*n*log(x) + b*log(c) + a) - I*sin(b*n*log(x) + b*log(c) + a)))/(b*n)
```

Sympy [F]

$$\int \frac{\cos^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx=\int \frac{\cos^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx$$

input
```
integrate(cos(a+b*ln(c*x**n))**(3/2)/x,x)
```

output
```
Integral(cos(a + b*log(c*x**n))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\cos^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx=\int \frac{\cos\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(cos(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(cos(b*log(c*x^n) + a)^(3/2)/x, x)
```

Giac [F]

$$\int \frac{\cos^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx=\int \frac{\cos\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(cos(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

output
```
integrate(cos(b*log(c*x^n) + a)^(3/2)/x, x)
```

Mupad [B] (verification not implemented)

Time = 20.88 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{\cos^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2\,\mathrm{F}\left(\frac{a}{2}+\frac{b\,\ln(c\,x^n)}{2}\middle|2\right)}{3\,b\,n} + \frac{2\,\sqrt{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}\,\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}{3\,b\,n}$$

input
```
int(cos(a + b*log(c*x^n))^(3/2)/x,x)
```

output
```
(2*ellipticF(a/2 + (b*log(c*x^n))/2, 2))/(3*b*n) + (2*cos(a + b*log(c*x^n)
)^(1/2)*sin(a + b*log(c*x^n)))/(3*b*n)
```

Reduce [F]

$$\int \frac{\cos^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sqrt{\cos\left(\log\left(x^n c\right)b+a\right)}\,\cos\left(\log(x^n c)\,b+a\right)}{x}dx$$

input
```
int(cos(a+b*log(c*x^n))^(3/2)/x,x)
```

output
```
int((sqrt(cos(log(x**n*c)*b + a))*cos(log(x**n*c)*b + a))/x,x)
```

3.114 $\int \cos^{\frac{5}{2}}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \cos^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \frac{2x\cos^{\frac{5}{2}}(a+b\log(cx^n))\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4}\left(-5-\frac{2i}{bn}\right),-\frac{2i+bn}{4bn},-e^{2ia}(cx^n)^{2ib}\right)}{(2-5ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}}$$

output
```
2*x*cos(a+b*ln(c*x^n))^(5/2)*hypergeom([-5/2, -5/4-1/2*I/b/n],[-1/4*(2*I+b
*n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(2-5*I*b*n)/(1+exp(2*I*a)*(c*x^n)^(2
*I*b))^(5/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 696 vs. 2(110) = 220.

Time = 8.33 (sec) , antiderivative size = 696, normalized size of antiderivative = 6.33

$$\int \cos^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
Integrate[Cos[a + b*Log[c*x^n]]^(5/2),x]
```

output
```
(30*b^3*E^(I*(a + b*(-(n*Log[x]) + Log[c*x^n])))*n^3*x^(1 - I*b*n)*Sqrt[2
+ 2*E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*x^((2*I)*b*n)]*((2*I + b*
n)*x^((2*I)*b*n)*Hypergeometric2F1[1/2, 3/4 - (I/2)/(b*n), 7/4 - (I/2)/(b*
n), -(E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*x^((2*I)*b*n))] + (-2*I
 + 3*b*n)*Hypergeometric2F1[1/2, -1/4*(2*I + b*n)/(b*n), 3/4 - (I/2)/(b*n)
, -(E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*x^((2*I)*b*n))]))/((2 - (
5*I)*b*n)*(2*I + b*n)*(-2*I + 3*b*n)*(-2*I + 5*b*n)*(-2*I - b*n + E^((2*I)
*(a + b*(-(n*Log[x]) + Log[c*x^n])))*(-2*I + b*n))*Sqrt[(1 + E^((2*I)*(a +
 b*(-(n*Log[x]) + Log[c*x^n])))*x^((2*I)*b*n))/(E^(I*(a + b*(-(n*Log[x]) +
 Log[c*x^n])))*x^(I*b*n))]) + Sqrt[Cos[a + b*n*Log[x] + b*(-(n*Log[x]) + L
og[c*x^n])]]*((-2*x*(2*Cos[a + b*(-(n*Log[x]) + Log[c*x^n])] + 15*b^2*n^2*
Cos[a + b*(-(n*Log[x]) + Log[c*x^n])] - b*n*Sin[a + b*(-(n*Log[x]) + Log[c
*x^n])]))/((-2*I + 5*b*n)*(2*I + 5*b*n)*(-2*Cos[a + b*(-(n*Log[x]) + Log[c
*x^n])] + b*n*Sin[a + b*(-(n*Log[x]) + Log[c*x^n])])) + (x*Sin[2*b*n*Log[x
]]*(5*b*n*Cos[2*(a + b*(-(n*Log[x]) + Log[c*x^n]))] - 2*Sin[2*(a + b*(-(n*
Log[x]) + Log[c*x^n]))]))/((-2*I + 5*b*n)*(2*I + 5*b*n)) + (x*Cos[2*b*n*Lo
g[x]]*(2*Cos[2*(a + b*(-(n*Log[x]) + Log[c*x^n]))] + 5*b*n*Sin[2*(a + b*(-
(n*Log[x]) + Log[c*x^n]))]))/((-2*I + 5*b*n)*(2*I + 5*b*n)))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4987, 4995, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{5}{2}}(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{4987}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\cos^{\frac{5}{2}}(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{4995}$$

$$\frac{x(cx^n)^{-\frac{1}{n}+\frac{5ib}{2}}\cos^{\frac{5}{2}}(a+b\log(cx^n))\int(cx^n)^{-\frac{5ib}{2}+\frac{1}{n}-1}\left(e^{2ia}(cx^n)^{2ib}+1\right)^{5/2}d(cx^n)}{n\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{2x\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4}\left(-5-\frac{2i}{bn}\right),-\frac{bn+2i}{4bn},-e^{2ia}(cx^n)^{2ib}\right)\cos^{\frac{5}{2}}(a+b\log(cx^n))}{(2-5ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}}$$

input
```
Int[Cos[a + b*Log[c*x^n]]^(5/2),x]
```

output
```
(2*x*Cos[a + b*Log[c*x^n]]^(5/2)*Hypergeometric2F1[-5/2, (-5 - (2*I)/(b*n)
)/4, -1/4*(2*I + b*n)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 - (5*I
)*b*n)*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^(5/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4987
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cos[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 4995
```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Simp[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Maple [F]

$$\int \cos\left(a+b\ln\left(c\,x^{n}\right)\right)^{\frac{5}{2}}dx$$

input
```
int(cos(a+b*ln(c*x^n))^(5/2),x)
```

output
```
int(cos(a+b*ln(c*x^n))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \cos^{\frac{5}{2}}\left(a+b\log\left(cx^{n}\right)\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(cos(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}\left(a+b\log\left(cx^{n}\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(cos(a+b*ln(c*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cos^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \int \cos(b\log(cx^n)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*log(c*x^n) + a)^(5/2), x)
```

Giac [F]

$$\int \cos^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \int \cos(b\log(cx^n)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate(cos(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(b*log(c*x^n) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \int \cos\left(a+b\,\ln\left(c\,x^n\right)\right)^{5/2}\,dx$$

input
```
int(cos(a + b*log(c*x^n))^(5/2),x)
```

output
```
int(cos(a + b*log(c*x^n))^(5/2), x)
```

Reduce [F]

$$\int \cos^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)\,dx=\int \sqrt{\cos\left(\log\left(x^nc\right)b+a\right)}\cos\left(\log(x^nc)\,b+a\right)^2dx$$

input
```
int(cos(a+b*log(c*x^n))^(5/2),x)
```

output
```
int(sqrt(cos(log(x**n*c)*b + a))*cos(log(x**n*c)*b + a)**2,x)
```

3.115 $\int \frac{\cos^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \frac{\cos^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{6E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{5bn} + \frac{2\cos^{\frac{3}{2}}(a+b\log(cx^n))\sin(a+b\log(cx^n))}{5bn}$$

output
```
6/5*EllipticE(sin(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n+2/5*cos(a+b*ln(c*x^n
))^(3/2)*sin(a+b*ln(c*x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{\cos^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{6E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)+\sqrt{\cos(a+b\log(cx^n))}\sin(2(a+b\log(cx^n)))}{5bn}$$

input
```
Integrate[Cos[a + b*Log[c*x^n]]^(5/2)/x,x]
```

output
```
(6*EllipticE[(a + b*Log[c*x^n])/2, 2] + Sqrt[Cos[a + b*Log[c*x^n]]]*Sin[2*
(a + b*Log[c*x^n])])/(5*b*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3039, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{5}{2}}(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \cos^{\frac{5}{2}}(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(a+b\log(cx^n)+\frac{\pi}{2}\right)^{5/2}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{3}{5}\int \sqrt{\cos(a+b\log(cx^n))}d\log(cx^n)+\frac{2\sin(a+b\log(cx^n))\cos^{\frac{3}{2}}(a+b\log(cx^n))}{5b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}\int \sqrt{\sin\left(a+b\log(cx^n)+\frac{\pi}{2}\right)}d\log(cx^n)+\frac{2\sin(a+b\log(cx^n))\cos^{\frac{3}{2}}(a+b\log(cx^n))}{5b}}{n}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{6E\left(\frac{1}{2}(a+b\log(cx^n))|2\right)}{5b}+\frac{2\sin(a+b\log(cx^n))\cos^{\frac{3}{2}}(a+b\log(cx^n))}{5b}}{n}$$

input
```
Int[Cos[a + b*Log[c*x^n]]^(5/2)/x,x]
```

output
```
((6*EllipticE[(a + b*Log[c*x^n])/2, 2])/(5*b) + (2*Cos[a + b*Log[c*x^n]]^(
3/2)*Sin[a + b*Log[c*x^n]])/(5*b))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(59) = 118$.

Time = 4.79 (sec) , antiderivative size = 280, normalized size of antiderivative = 4.44

method	result
derivativedivides	$-\frac{2\sqrt{\left(2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\left(-8\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^6+8\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n}{2}\right.\right.}{5n\sqrt{-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right.}}$
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\left(-8\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^6+8\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n}{2}\right.\right.}{5n\sqrt{-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right.}}$

input
```
int(cos(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2/5/n*((2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*(-8*cos(1/2*a+1/2*b*ln(c*x^n))*sin(1/2*a+1/2*b*ln(c*x^n))^6+8*cos(1/2
*a+1/2*b*ln(c*x^n))*sin(1/2*a+1/2*b*ln(c*x^n))^4-2*sin(1/2*a+1/2*b*ln(c*x^
n))^2*cos(1/2*a+1/2*b*ln(c*x^n))-3*(sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-
1+2*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*EllipticE(cos(1/2*a+1/2*b*ln(c*x^n
)),2^(1/2)))/(-2*sin(1/2*a+1/2*b*ln(c*x^n))^4+sin(1/2*a+1/2*b*ln(c*x^n))^2
)^(1/2)/sin(1/2*a+1/2*b*ln(c*x^n))/(2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2
)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.79

$$\int \frac{\cos^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2\cos(bn\log(x)+b\log(c)+a)^{\frac{3}{2}}\sin(bn\log(x)+b\log(c)+a)+3i\sqrt{2}\text{weierstrassZeta}(-4,0,\text{weierstra}}{}$$

input
```
integrate(cos(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

output
```
1/5*(2*cos(b*n*log(x) + b*log(c) + a)^(3/2)*sin(b*n*log(x) + b*log(c) + a)
 + 3*I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*n*l
og(x) + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a))) - 3*I*sqrt(2)*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*n*log(x) + b*log(c)
 + a) - I*sin(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \text{Timed out}$$

input
```
integrate(cos(a+b*ln(c*x**n))**(5/2)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cos^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \int \frac{\cos\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate(cos(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate(cos(b*log(c*x^n) + a)^(5/2)/x, x)
```

Giac [F]

$$\int \frac{\cos^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \int \frac{\cos\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate(cos(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

output
```
integrate(cos(b*log(c*x^n) + a)^(5/2)/x, x)
```

Mupad [B] (verification not implemented)

Time = 21.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{\cos^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$$
$$= -\frac{2\cos(a+b\ln(c\,x^n))^{7/2}\,\sin(a+b\ln(c\,x^n))\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\;\frac{11}{4};\;\cos(a+b\ln(c\,x^n))^2\right)}{7\,b\,n\,\sqrt{\sin(a+b\ln(c\,x^n))^2}}$$

input
```
int(cos(a + b*log(c*x^n))^(5/2)/x,x)
```

output
```
-(2*cos(a + b*log(c*x^n))^(7/2)*sin(a + b*log(c*x^n))*hypergeom([1/2, 7/4]
, 11/4, cos(a + b*log(c*x^n))^2))/(7*b*n*(sin(a + b*log(c*x^n))^2)^(1/2))
```

Reduce [F]

$$\int \frac{\cos^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sqrt{\cos(\log(x^n c)\,b+a)}\,\cos(\log(x^n c)\,b+a)^2}{x}dx$$

input
```
int(cos(a+b*log(c*x^n))^(5/2)/x,x)
```

output
```
int((sqrt(cos(log(x**n*c)*b + a))*cos(log(x**n*c)*b + a)**2)/x,x)
```

3.116 $\int \frac{1}{\sqrt{\cos(a+b\log(cx^n))}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \frac{1}{\sqrt{\cos(a+b\log(cx^n))}}\,dx = \frac{2x\sqrt{1+e^{2ia}(cx^n)^{2ib}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}\left(1-\frac{2i}{bn}\right),\frac{1}{4}\left(5-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{(2+ibn)\sqrt{\cos(a+b\log(cx^n))}}$$

output
```
2*x*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)*hypergeom([1/2, 1/4-1/2*I/b/n],[5
/4-1/2*I/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(2+I*b*n)/cos(a+b*ln(c*x^n))^(1
/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{\cos(a+b\log(cx^n))}}\,dx = -\frac{2i\sqrt{2}\sqrt{1+e^{2i(a+b\log(cx^n))}}\,x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}-\frac{i}{2bn},\frac{5}{4}-\frac{i}{2bn},-e^{2i(a+b\log(cx^n))}\right)}{\sqrt{e^{-i(a+b\log(cx^n))}\left(1+e^{2i(a+b\log(cx^n))}\right)}(-2i+bn)}$$

input
```
Integrate[1/Sqrt[Cos[a + b*Log[c*x^n]]],x]
```

output
```
((-2*I)*Sqrt[2]*Sqrt[1 + E^((2*I)*(a + b*Log[c*x^n]))]*x*Hypergeometric2F1
[1/2, 1/4 - (I/2)/(b*n), 5/4 - (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))]
)/(Sqrt[(1 + E^((2*I)*(a + b*Log[c*x^n])))/E^(I*(a + b*Log[c*x^n]))]*(-2*I
 + b*n))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4987, 4995, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos (a+b\log (cx^n))}}dx$$

$$\Big\downarrow \text{4987}$$

$$\frac{x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}-1}}{\sqrt{\cos (a+b\log (cx^n))}}d(cx^n)}{n}$$

$$\Big\downarrow \text{4995}$$

$$\frac{x(cx^n)^{-\frac{1}{n}-\frac{ib}{2}}\sqrt{1+e^{2ia}(cx^n)^{2ib}}\int \frac{(cx^n)^{\frac{ib}{2}+\frac{1}{n}-1}}{\sqrt{e^{2ia}(cx^n)^{2ib}+1}}d(cx^n)}{n\sqrt{\cos (a+b\log (cx^n))}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x\sqrt{1+e^{2ia}(cx^n)^{2ib}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}\left(1-\frac{2i}{bn}\right),\frac{1}{4}\left(5-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{(2+ibn)\sqrt{\cos (a+b\log (cx^n))}}$$

input
```
Int[1/Sqrt[Cos[a + b*Log[c*x^n]]],x]
```

output
```
(2*x*Sqrt[1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Hypergeometric2F1[1/2, (1 - (
2*I)/(b*n))/4, (5 - (2*I)/(b*n))/4, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2
 + I*b*n)*Sqrt[Cos[a + b*Log[c*x^n]]])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4987
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cos[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 4995
```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Simp[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\sqrt{\cos \left(a+b\ln \left(c\,x^{n}\right)\right)}}dx$$

input
```
int(1/cos(a+b*ln(c*x^n))^(1/2),x)
```

output
```
int(1/cos(a+b*ln(c*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\cos(a+b\log(cx^n))}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/cos(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\cos(a+b\log(cx^n))}}\,dx = \int \frac{1}{\sqrt{\cos(a+b\log(cx^n))}}\,dx$$

input
```
integrate(1/cos(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(1/sqrt(cos(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\cos(a+b\log(cx^n))}}\,dx = \int \frac{1}{\sqrt{\cos(b\log(cx^n)+a)}}\,dx$$

input
```
integrate(1/cos(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(cos(b*log(c*x^n) + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\cos\left(a+b\log\left(cx^{n}\right)\right)}}\,dx=\int \frac{1}{\sqrt{\cos\left(b\log\left(cx^{n}\right)+a\right)}}\,dx$$

input
```
integrate(1/cos(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(cos(b*log(c*x^n) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos\left(a+b\log\left(cx^{n}\right)\right)}}\,dx=\int \frac{1}{\sqrt{\cos\left(a+b\,\ln\left(c\,x^{n}\right)\right)}}\,dx$$

input
```
int(1/cos(a + b*log(c*x^n))^(1/2),x)
```

output
```
int(1/cos(a + b*log(c*x^n))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\cos\left(a+b\log\left(cx^{n}\right)\right)}}\,dx=\int \frac{\sqrt{\cos\left(\log\left(x^{n}c\right)b+a\right)}}{\cos\left(\log\left(x^{n}c\right)b+a\right)}\,dx$$

input
```
int(1/cos(a+b*log(c*x^n))^(1/2),x)
```

output
```
int(sqrt(cos(log(x**n*c)*b + a))/cos(log(x**n*c)*b + a),x)
```

3.117 $\int \frac{1}{x\sqrt{\cos(a+b\log(cx^n))}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \frac{1}{x\sqrt{\cos(a+b\log(cx^n))}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+b\log(cx^n)),2\right)}{bn}$$

output
```
2*InverseJacobiAM(1/2*a+1/2*b*ln(c*x^n),2^(1/2))/b/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\cos(a+b\log(cx^n))}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+b\log(cx^n)),2\right)}{bn}$$

input
```
Integrate[1/(x*Sqrt[Cos[a + b*Log[c*x^n]]]),x]
```

output
```
(2*EllipticF[(a + b*Log[c*x^n])/2, 2])/(b*n)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3039, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\cos(a+b\log(cx^n))}}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\sqrt{\cos(a+b\log(cx^n))}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{\sin\left(a+b\log(cx^n)+\frac{\pi}{2}\right)}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+b\log(cx^n)),2\right)}{bn}$$

input
```
Int[1/(x*Sqrt[Cos[a + b*Log[c*x^n]]]),x]
```

output
```
(2*EllipticF[(a + b*Log[c*x^n])/2, 2])/(b*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{2\ \mathrm{InverseJacobiAM}\left(\frac{a}{2}+\frac{b\ln\left(c\,x^{n}\right)}{2},\sqrt{2}\right)}{bn}$	26
default	$\frac{2\ \mathrm{InverseJacobiAM}\left(\frac{a}{2}+\frac{b\ln\left(c\,x^{n}\right)}{2},\sqrt{2}\right)}{bn}$	26

input
```
int(1/x/cos(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*InverseJacobiAM(1/2*a+1/2*b*ln(c*x^n),2^(1/2))/b/n
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.25

$$\int \frac{1}{x\sqrt{\cos\left(a+b\log\left(cx^{n}\right)\right)}}\,dx$$
$$= \frac{-i\sqrt{2}\mathrm{weierstrassPInverse}(-4,0,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+i\,\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right))+i\,\sqrt{}}{bn}$$

input
```
integrate(1/x/cos(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b*n*log(x) + b*log(c) + a) + I*
sin(b*n*log(x) + b*log(c) + a)) + I*sqrt(2)*weierstrassPInverse(-4, 0, cos
(b*n*log(x) + b*log(c) + a) - I*sin(b*n*log(x) + b*log(c) + a)))/(b*n)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{\cos(a+b\log(cx^n))}}\,dx = \int \frac{1}{x\sqrt{\cos(a+b\log(cx^n))}}\,dx$$

input
```
integrate(1/x/cos(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(cos(a + b*log(c*x**n)))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{\cos(a+b\log(cx^n))}}\,dx = \int \frac{1}{x\sqrt{\cos(b\log(cx^n)+a)}}\,dx$$

input
```
integrate(1/x/cos(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sqrt(cos(b*log(c*x^n) + a))), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{\cos(a+b\log(cx^n))}}\,dx = \int \frac{1}{x\sqrt{\cos(b\log(cx^n)+a)}}\,dx$$

input
```
integrate(1/x/cos(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(x*sqrt(cos(b*log(c*x^n) + a))), x)
```

Mupad [B] (verification not implemented)

Time = 19.60 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\sqrt{\cos(a+b\log(cx^n))}}\,dx = \frac{2\,\mathrm{F}\left(\frac{a}{2}+\frac{b\,\ln(c\,x^n)}{2}\middle|2\right)}{b\,n}$$

input
```
int(1/(x*cos(a + b*log(c*x^n))^(1/2)),x)
```

output
```
(2*ellipticF(a/2 + (b*log(c*x^n))/2, 2))/(b*n)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{\cos(a+b\log(cx^n))}}\,dx = \int \frac{\sqrt{\cos(\log(x^n c)\,b+a)}}{\cos(\log(x^n c)\,b+a)\,x}dx$$

input
```
int(1/x/cos(a+b*log(c*x^n))^(1/2),x)
```

output
```
int(sqrt(cos(log(x**n*c)*b + a))/(cos(log(x**n*c)*b + a)*x),x)
```

3.118 $\int \frac{1}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{2x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i}{bn}\right),\frac{1}{4}\left(7-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{(2+3ibn)\cos^{\frac{3}{2}}(a+b\log(cx^n))}$$

output
```
2*x*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)*hypergeom([3/2, 3/4-1/2*I/b/n],[7
/4-1/2*I/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(2+3*I*b*n)/cos(a+b*ln(c*x^n))^
(3/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 431 vs. 2(109) = 218.

Time = 5.70 (sec) , antiderivative size = 431, normalized size of antiderivative = 3.95

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$
$$= \frac{x\left(-\left((4+b^2n^2)\,x^{ibn}\sqrt{2+2e^{2ia}\,(cx^n)^{2ib}}\sqrt{\cos(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4}-\frac{i}{2bn},\frac{7}{4}-\frac{i}{2bn}\right.\right.\right.}{bn(-2i}$$

input
```
Integrate[Cos[a + b*Log[c*x^n]]^(-3/2),x]
```

output
```
(x*(-((4 + b^2*n^2)*x^(I*b*n)*Sqrt[2 + 2*E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sq
rt[Cos[a + b*Log[c*x^n]]]*Hypergeometric2F1[1/2, 3/4 - (I/2)/(b*n), 7/4 -
(I/2)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))]) + ((-2*I + 3*b*n)*(-((-2*I
 + b*n)*Sqrt[2 + 2*E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[Cos[a + b*Log[c*x^n
]]]*Hypergeometric2F1[1/2, -1/4*(2*I + b*n)/(b*n), 3/4 - (I/2)/(b*n), -(E^
((2*I)*a)*(c*x^n)^((2*I)*b))]) + 2*x^(I*b*n)*Sqrt[1/(E^(I*a)*(c*x^n)^(I*b)
) + E^(I*a)*(c*x^n)^(I*b)]*(b*n*Cos[b*n*Log[x]] - 2*Sin[b*n*Log[x]])))/x^(
I*b*n)))/(b*n*(-2*I + 3*b*n)*Sqrt[1/(E^(I*a)*(c*x^n)^(I*b)) + E^(I*a)*(c*x
^n)^(I*b)]*Sqrt[Cos[a + b*Log[c*x^n]]]*(-2*Cos[a - b*n*Log[x] + b*Log[c*x^
n]] + b*n*Sin[a - b*n*Log[x] + b*Log[c*x^n]]))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4987, 4995, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$
$$\Big\downarrow \; 4987$$
$$\frac{x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}-1}}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}d(cx^n)}{n}$$

$$\Big\downarrow \text{4995}$$

$$\frac{x(cx^n)^{-\frac{1}{n}-\frac{3ib}{2}}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\int\frac{(cx^n)^{\frac{3ib}{2}+\frac{1}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^{3/2}}d(cx^n)}{n\cos^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i}{bn}\right),\frac{1}{4}\left(7-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{(2+3ibn)\cos^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

input

```
Int[Cos[a + b*Log[c*x^n]]^(-3/2),x]
```

output

```
(2*x*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2)*Hypergeometric2F1[3/2, (3 -
 (2*I)/(b*n))/4, (7 - (2*I)/(b*n))/4, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(
(2 + (3*I)*b*n)*Cos[a + b*Log[c*x^n]]^(3/2))
```

Defintions of rubi rules used

rule 888

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4987

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cos[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 4995

```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Simp[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\cos \left(a+b\ln \left(c\,x^{n}\right)\right)^{\frac{3}{2}}}d x$$

input
```
int(1/cos(a+b*ln(c*x^n))^(3/2),x)
```

output
```
int(1/cos(a+b*ln(c*x^n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\cos^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/cos(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\,dx = \int \frac{1}{\cos^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\,dx$$

input
```
integrate(1/cos(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Integral(cos(a + b*log(c*x**n))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{\cos(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*log(c*x^n) + a)^(-3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{\cos(a+b\,\ln(c\,x^n))^{3/2}}\,dx$$

input
```
int(1/cos(a + b*log(c*x^n))^(3/2),x)
```

output
```
int(1/cos(a + b*log(c*x^n))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \int \frac{\sqrt{\cos\left(\log\left(x^n c\right)b+a\right)}}{\cos\left(\log\left(x^n c\right)b+a\right)^2}dx$$

input
```
int(1/cos(a+b*log(c*x^n))^(3/2),x)
```

output
```
int(sqrt(cos(log(x**n*c)*b + a))/cos(log(x**n*c)*b + a)**2,x)
```

3.119 $\int \frac{1}{x\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{1}{x\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx=-\frac{2E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{bn}+\frac{2\sin(a+b\log(cx^n))}{bn\sqrt{\cos(a+b\log(cx^n))}}$$

output
```
-2*EllipticE(sin(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n+2*sin(a+b*ln(c*x^n))/
b/n/cos(a+b*ln(c*x^n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx=\frac{2\left(-E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)+\frac{\sin(a+b\log(cx^n))}{\sqrt{\cos(a+b\log(cx^n))}}\right)}{bn}$$

input
```
Integrate[1/(x*Cos[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
(2*(-EllipticE[(a + b*Log[c*x^n])/2, 2] + Sin[a + b*Log[c*x^n]]/Sqrt[Cos[a
 + b*Log[c*x^n]]]))/(b*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3039, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\cos^{\frac{3}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sin\left(a+b\log(cx^n)+\frac{\pi}{2}\right)^{3/2}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\frac{2\sin(a+b\log(cx^n))}{b\sqrt{\cos(a+b\log(cx^n))}}-\int \sqrt{\cos(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sin(a+b\log(cx^n))}{b\sqrt{\cos(a+b\log(cx^n))}}-\int \sqrt{\sin\left(a+b\log(cx^n)+\frac{\pi}{2}\right)}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{2\sin(a+b\log(cx^n))}{b\sqrt{\cos(a+b\log(cx^n))}}-\frac{2E\left(\frac{1}{2}(a+b\log(cx^n))|2\right)}{b}}{n}$$

input
```
Int[1/(x*Cos[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
((-2*EllipticE[(a + b*Log[c*x^n])/2, 2])/b + (2*Sin[a + b*Log[c*x^n]])/(b*
Sqrt[Cos[a + b*Log[c*x^n]]]))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(59) = 118$.

Time = 0.70 (sec) , antiderivative size = 250, normalized size of antiderivative = 4.24

method	result
derivativedivides	$-\frac{2\left(-2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sqrt{-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{4}+\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos\left(a+2b\ln\left(\sqrt{c\cdots}\right.\right.}{2}}\cdots\right.}{n\sqrt{-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{4}+\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\cdots}}$
default	$-\frac{2\left(-2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sqrt{-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{4}+\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}+\sqrt{\frac{1}{2}-\frac{\cos\left(a+2b\ln\left(\sqrt{c\cdots}\right.\right.}{2}}\cdots\right.}{n\sqrt{-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{4}+\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\cdots}}$

input
```
int(1/x/cos(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/n*(-2*cos(1/2*a+1/2*b*ln(c*x^n))*(-2*sin(1/2*a+1/2*b*ln(c*x^n))^4+sin(1
/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*sin(1/2*a+1/2*b*ln(c*x^n))^2+(sin(1/2*a+1/2
*b*ln(c*x^n))^2)^(1/2)*(-1+2*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*sin(1
/2*a+1/2*b*ln(c*x^n))^4+sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*EllipticE(cos(
1/2*a+1/2*b*ln(c*x^n)),2^(1/2)))/(-2*sin(1/2*a+1/2*b*ln(c*x^n))^4+sin(1/2*
a+1/2*b*ln(c*x^n))^2)^(1/2)/sin(1/2*a+1/2*b*ln(c*x^n))/(2*cos(1/2*a+1/2*b*
ln(c*x^n))^2-1)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.54

$$\int \frac{1}{x\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$
$$= \frac{-i\sqrt{2}\cos(bn\log(x)+b\log(c)+a)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bn\log(x)+}{}$$

input
```
integrate(1/x/cos(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*cos(b*n*log(x) + b*log(c) + a)*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(b*n*log(x) + b*log(c) + a) + I*sin(b*n*log(x) + b*l
og(c) + a))) + I*sqrt(2)*cos(b*n*log(x) + b*log(c) + a)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(b*n*log(x) + b*log(c) + a) - I*sin(b*n
*log(x) + b*log(c) + a))) + 2*sqrt(cos(b*n*log(x) + b*log(c) + a))*sin(b*n
*log(x) + b*log(c) + a))/(b*n*cos(b*n*log(x) + b*log(c) + a))
```

Sympy [F]

$$\int \frac{1}{x\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$

input
```
integrate(1/x/cos(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Integral(1/(x*cos(a + b*log(c*x**n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\cos(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/cos(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*cos(b*log(c*x^n) + a)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/x/cos(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 20.48 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{2\sin(a+b\ln(c\,x^n))\;{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(a+b\ln(c\,x^n))^2\right)}{b\,n\,\sqrt{\cos(a+b\ln(c\,x^n))}\,\sqrt{\sin(a+b\ln(c\,x^n))^2}}$$

input
```
int(1/(x*cos(a + b*log(c*x^n))^(3/2)),x)
```

output
```
(2*sin(a + b*log(c*x^n))*hypergeom([-1/4, 1/2], 3/4, cos(a + b*log(c*x^n))
^2))/(b*n*cos(a + b*log(c*x^n))^(1/2)*(sin(a + b*log(c*x^n))^2)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x\cos^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \int \frac{\sqrt{\cos\left(\log\left(x^n c\right)b+a\right)}}{\cos\left(\log\left(x^n c\right)b+a\right)^2 x}dx$$

input
```
int(1/x/cos(a+b*log(c*x^n))^(3/2),x)
```

output
```
int(sqrt(cos(log(x**n*c)*b + a))/(cos(log(x**n*c)*b + a)**2*x),x)
```

3.120 $\int \frac{1}{\cos^{\frac{5}{2}}(a+b\log(cx^n))} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \frac{1}{\cos^{\frac{5}{2}}(a+b\log(cx^n))} \, dx = \frac{2x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2} \text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i}{bn}\right),\frac{1}{4}\left(9-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{(2+5ibn)\cos^{\frac{5}{2}}(a+b\log(cx^n))}$$

output
```
2*x*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(5/2)*hypergeom([5/2, 5/4-1/2*I/b/n],[9
/4-1/2*I/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(2+5*I*b*n)/cos(a+b*ln(c*x^n))^
(5/2)
```

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.72

$$\int \frac{1}{\cos^{\frac{5}{2}}(a+b\log(cx^n))} \, dx = \frac{2x\left(\frac{(2-ibn)\sqrt{2+2e^{2ia}(cx^n)^{2ib}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}-\frac{i}{2bn},\frac{5}{4}-\frac{i}{2bn},-e^{2i(a+b\log(cx^n))}\right)}{\sqrt{e^{-ia}(cx^n)^{-ib}+e^{ia}(cx^n)^{ib}}}+\frac{-2\cos(a+b\log(cx^n))+bn\sin(a+b\log(cx^n))}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}\right.}{3b^2n^2}$$

input
```
Integrate[Cos[a + b*Log[c*x^n]]^(-5/2),x]
```

output
```
(2*x*(((2 - I*b*n)*Sqrt[2 + 2*E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Hypergeometri
c2F1[1/2, 1/4 - (I/2)/(b*n), 5/4 - (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n
]))])/Sqrt[1/(E^(I*a)*(c*x^n)^(I*b)) + E^(I*a)*(c*x^n)^(I*b)] + (-2*Cos[a
+ b*Log[c*x^n]] + b*n*Sin[a + b*Log[c*x^n]])/Cos[a + b*Log[c*x^n]]^(3/2)))
/(3*b^2*n^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4987, 4995, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{4987}$$

$$\frac{x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}-1}}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}d(cx^n)}{n}$$

$$\Big\downarrow \text{4995}$$

$$\frac{x(cx^n)^{-\frac{1}{n}-\frac{5ib}{2}}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\int \frac{(cx^n)^{\frac{5ib}{2}+\frac{1}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^{5/2}}d(cx^n)}{n\cos^{\frac{5}{2}}(a+b\log(cx^n))}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i}{bn}\right),\frac{1}{4}\left(9-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{(2+5ibn)\cos^{\frac{5}{2}}(a+b\log(cx^n))}$$

input
```
Int[Cos[a + b*Log[c*x^n]]^(-5/2),x]
```

output
```
(2*x*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^(5/2)*Hypergeometric2F1[5/2, (5 -
 (2*I)/(b*n))/4, (9 - (2*I)/(b*n))/4, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(
(2 + (5*I)*b*n)*Cos[a + b*Log[c*x^n]]^(5/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4987
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cos[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 4995
```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Simp[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\cos \left(a+b\ln \left(c\,x^{n}\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/cos(a+b*ln(c*x^n))^(5/2),x)
```

output
```
int(1/cos(a+b*ln(c*x^n))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/cos(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(a+b*ln(c*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{\cos(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cos(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*log(c*x^n) + a)^(-5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^{5/2}}\,dx$$

input
```
int(1/cos(a + b*log(c*x^n))^(5/2),x)
```

output
```
int(1/cos(a + b*log(c*x^n))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{\sqrt{\cos(\log(x^n c)\,b+a)}}{\cos(\log(x^n c)\,b+a)^3}\,dx$$

input
```
int(1/cos(a+b*log(c*x^n))^(5/2),x)
```

output
```
int(sqrt(cos(log(x**n*c)*b + a))/cos(log(x**n*c)*b + a)**3,x)
```

3.121 $\int \frac{1}{x \cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \frac{1}{x\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+b\log(cx^n)),2\right)}{3bn} + \frac{2\sin(a+b\log(cx^n))}{3bn\cos^{\frac{3}{2}}(a+b\log(cx^n))}$$

output
```
2/3*InverseJacobiAM(1/2*a+1/2*b*ln(c*x^n),2^(1/2))/b/n+2/3*sin(a+b*ln(c*x^
n))/b/n/cos(a+b*ln(c*x^n))^(3/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{2\left(\text{EllipticF}\left(\frac{1}{2}(a+b\log(cx^n)),2\right)+\frac{\sin(a+b\log(cx^n))}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}\right)}{3bn}$$

input
```
Integrate[1/(x*Cos[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
(2*(EllipticF[(a + b*Log[c*x^n])/2, 2] + Sin[a + b*Log[c*x^n]]/Cos[a + b*L
og[c*x^n]]^(3/2)))/(3*b*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3039, 3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \cos^{\frac{5}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \ 3039$$

$$\frac{\int \frac{1}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{1}{\sin\left(a+b\log(cx^n)+\frac{\pi}{2}\right)^{5/2}}d\log(cx^n)}{n}$$

$$\Big\downarrow \ 3116$$

$$\frac{\frac{1}{3}\int \frac{1}{\sqrt{\cos(a+b\log(cx^n))}}d\log(cx^n)+\frac{2\sin(a+b\log(cx^n))}{3b\cos^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(a+b\log(cx^n)+\frac{\pi}{2}\right)}}d\log(cx^n)+\frac{2\sin(a+b\log(cx^n))}{3b\cos^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$$\Big\downarrow \ 3120$$

$$\frac{\frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+b\log(cx^n)),2\right)}{3b}+\frac{2\sin(a+b\log(cx^n))}{3b\cos^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

input
```
Int[1/(x*Cos[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
((2*EllipticF[(a + b*Log[c*x^n])/2, 2])/(3*b) + (2*Sin[a + b*Log[c*x^n]])/
(3*b*Cos[a + b*Log[c*x^n]]^(3/2)))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(58) = 116$.

Time = 0.90 (sec) , antiderivative size = 291, normalized size of antiderivative = 4.62

method	result
derivativedivides	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos\left(a+2b\ln\left(\sqrt{c\,x^{n}}\right)\right)}{2}}\,\sqrt{-1+2\sin\left(\frac{a}{2}+\frac{b\ln\left(c\,x^{n}\right)}{2}\right)^{2}}\,\operatorname{EllipticF}\left(\cos\left(\frac{a}{2}+\frac{b\ln\left(c\,x^{n}\right)}{2}\right),\sqrt{2}\right)\sin\left(\frac{a}{2}+\frac{b\ln\left(c\,x^{n}\right)}{2}\right)}{3n\sqrt{-2\sin\left(\frac{a}{2}}$
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos\left(a+2b\ln\left(\sqrt{c\,x^{n}}\right)\right)}{2}}\,\sqrt{-1+2\sin\left(\frac{a}{2}+\frac{b\ln\left(c\,x^{n}\right)}{2}\right)^{2}}\,\operatorname{EllipticF}\left(\cos\left(\frac{a}{2}+\frac{b\ln\left(c\,x^{n}\right)}{2}\right),\sqrt{2}\right)\sin\left(\frac{a}{2}+\frac{b\ln\left(c\,x^{n}\right)}{2}\right)}{3n\sqrt{-2\sin\left(\frac{a}{2}}$

input
```
int(1/x/cos(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/n*(-2*(sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-1+2*sin(1/2*a+1/2*b*ln(c
*x^n))^2)^(1/2)*EllipticF(cos(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*sin(1/2*a+1/
2*b*ln(c*x^n))^2-2*sin(1/2*a+1/2*b*ln(c*x^n))^2*cos(1/2*a+1/2*b*ln(c*x^n))
+(sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-1+2*sin(1/2*a+1/2*b*ln(c*x^n))^2)^
(1/2)*EllipticF(cos(1/2*a+1/2*b*ln(c*x^n)),2^(1/2)))*((2*cos(1/2*a+1/2*b*l
n(c*x^n))^2-1)*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(-2*sin(1/2*a+1/2*b*ln(
c*x^n))^4+sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(2*cos(1/2*a+1/2*b*ln(c*x^n)
)^2-1)^(3/2)/sin(1/2*a+1/2*b*ln(c*x^n))/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.37

$$\int \frac{1}{x\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{-i\sqrt{2}\cos(bn\log(x)+b\log(c)+a)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(bn\log(x)+b\log(c)+a)+i\,\sin(\ldots}{}$$

input
```
integrate(1/x/cos(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*cos(b*n*log(x) + b*log(c) + a)^2*weierstrassPInverse(-4, 0
, cos(b*n*log(x) + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a)) + I*s
qrt(2)*cos(b*n*log(x) + b*log(c) + a)^2*weierstrassPInverse(-4, 0, cos(b*n
*log(x) + b*log(c) + a) - I*sin(b*n*log(x) + b*log(c) + a)) + 2*sqrt(cos(b
*n*log(x) + b*log(c) + a))*sin(b*n*log(x) + b*log(c) + a))/(b*n*cos(b*n*lo
g(x) + b*log(c) + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/x/cos(a+b*ln(c*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\cos(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/cos(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*cos(b*log(c*x^n) + a)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/x/cos(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 20.36 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{1}{x\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{2\sin(a+b\ln(c\,x^n))\;{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(a+b\ln(c\,x^n))^2\right)}{3\,b\,n\cos(a+b\ln(c\,x^n))^{3/2}\sqrt{\sin(a+b\ln(c\,x^n))^2}}$$

input
```
int(1/(x*cos(a + b*log(c*x^n))^(5/2)),x)
```

output
```
(2*sin(a + b*log(c*x^n))*hypergeom([-3/4, 1/2], 1/4, cos(a + b*log(c*x^n))
^2))/(3*b*n*cos(a + b*log(c*x^n))^(3/2)*(sin(a + b*log(c*x^n))^2)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{\sqrt{\cos(\log(x^n c)\,b+a)}}{\cos(\log(x^n c)\,b+a)^3 x}dx$$

input
```
int(1/x/cos(a+b*log(c*x^n))^(5/2),x)
```

output
```
int(sqrt(cos(log(x**n*c)*b + a))/(cos(log(x**n*c)*b + a)**3*x),x)
```

3.122 $\int \frac{1}{\cos^{\frac{3}{2}}(a-2i\log(cx))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 48

$$\int \frac{1}{\cos^{\frac{3}{2}}(a-2i\log(cx))}\,dx = -\frac{e^{-2ia}(1+c^4e^{2ia}x^4)}{2c^4x^3\cos^{\frac{3}{2}}(a-2i\log(cx))}$$

output
```
-1/2*(1+c^4*exp(2*I*a)*x^4)/c^4/exp(2*I*a)/x^3/cos(a-2*I*ln(c*x))^(3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.71

$$\int \frac{1}{\cos^{\frac{3}{2}}(a-2i\log(cx))}\,dx = -\frac{x(\cos(a)-i\sin(a))\sqrt{\frac{2(1+c^4x^4)\cos(a)+2i(-1+c^4x^4)\sin(a)}{c^2x^2}}}{(1+c^4x^4)\cos(a)+i\,(-1+c^4x^4)\sin(a)}$$

input
```
Integrate[Cos[a - (2*I)*Log[c*x]]^(-3/2),x]
```

output
```
-((x*(Cos[a] - I*Sin[a])*Sqrt[(2*(1 + c^4*x^4)*Cos[a] + (2*I)*(-1 + c^4*x^
4)*Sin[a])/(c^2*x^2)])/((1 + c^4*x^4)*Cos[a] + I*(-1 + c^4*x^4)*Sin[a]))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4987, 4985, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(a-2i\log(cx))}dx$$

$$\Big\downarrow \text{4987}$$

$$\frac{\int \frac{1}{\cos^{\frac{3}{2}}(a-2i\log(cx))}d(cx)}{c}$$

$$\Big\downarrow \text{4985}$$

$$\frac{\left(1+e^{2ia}c^4x^4\right)^{3/2}\int \frac{c^3x^3}{\left(c^4e^{2ia}x^4+1\right)^{3/2}}d(cx)}{c^4x^3\cos^{\frac{3}{2}}(a-2i\log(cx))}$$

$$\Big\downarrow \text{793}$$

$$-\frac{e^{-2ia}\left(1+e^{2ia}c^4x^4\right)}{2c^4x^3\cos^{\frac{3}{2}}(a-2i\log(cx))}$$

input
```
Int[Cos[a - (2*I)*Log[c*x]]^(-3/2),x]
```

output
```
-1/2*(1 + c^4*E^((2*I)*a)*x^4)/(c^4*E^((2*I)*a)*x^3*Cos[a - (2*I)*Log[c*x]
]^(3/2))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 4985
```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol] :> Simp[Cos[d*(a + b
*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p)   Int[(1 + E^(2*I
*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p), x], x] /; FreeQ[{a, b, d, p}, x] &&  !Int
egerQ[p]
```

rule 4987
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cos[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{1}{\cos\left(a-2 i \ln\left(x c\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/cos(a-2*I*ln(x*c))^(3/2),x)
```

output
```
int(1/cos(a-2*I*ln(x*c))^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{1}{\cos^{\frac{3}{2}}(a-2 i \log(c x))}\,dx = -\frac{2\,\sqrt{\frac{1}{2}}\sqrt{c^4x^4+e^{(-2i\,a)}}e^{\left(-\frac{3}{2}i\,a\right)}}{c^5x^4+ce^{(-2i\,a)}}$$

input
```
integrate(1/cos(a-2*I*log(c*x))^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(1/2)*sqrt(c^4*x^4 + e^(-2*I*a))*e^(-3/2*I*a)/(c^5*x^4 + c*e^(-2*I*
a))
```

Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(a-2i\log(cx))}\,dx = \int \frac{1}{\cos^{\frac{3}{2}}\left(a-2i\log\left(cx\right)\right)}\,dx$$

input
```
integrate(1/cos(a-2*I*ln(c*x))**(3/2),x)
```

output
```
Integral(cos(a - 2*I*log(c*x))**(-3/2), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(36) = 72$.

Time = 0.16 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.90

$$\int \frac{1}{\cos^{\frac{3}{2}}(a-2i\log(cx))}\,dx = -\frac{\left(\left(\sqrt{2}\cos\left(\frac{3}{2}a\right)+i\sqrt{2}\sin\left(\frac{3}{2}a\right)\right)c^4x^4+\sqrt{2}\cos\left(\frac{1}{2}a\right)-i\sqrt{2}\sin\left(\frac{1}{2}a\right)\right)\cos\left(\frac{3}{2}\arctan\left(c^4x^4\sin\left(2a\right),c^4x\right.\right.}{\left(\left(\cos\left(2\right.\right.\right.}$$

input
```
integrate(1/cos(a-2*I*log(c*x))^(3/2),x, algorithm="maxima")
```

output
```
-(((sqrt(2)*cos(3/2*a) + I*sqrt(2)*sin(3/2*a))*c^4*x^4 + sqrt(2)*cos(1/2*a
) - I*sqrt(2)*sin(1/2*a))*cos(3/2*arctan2(c^4*x^4*sin(2*a), c^4*x^4*cos(2*
a) + 1)) + ((-I*sqrt(2)*cos(3/2*a) + sqrt(2)*sin(3/2*a))*c^4*x^4 - I*sqrt(
2)*cos(1/2*a) - sqrt(2)*sin(1/2*a))*sin(3/2*arctan2(c^4*x^4*sin(2*a), c^4*
x^4*cos(2*a) + 1)))/(((cos(2*a)^2 + sin(2*a)^2)*c^8*x^8 + 2*c^4*x^4*cos(2*
a) + 1)^(3/4)*c)
```

Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(a-2i\log(cx))}\,dx = \int \frac{1}{\cos\left(a-2i\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(a-2*I*log(c*x))^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(a - 2*I*log(c*x))^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 20.71 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cos^{\frac{3}{2}}(a-2i\log(cx))}\,dx = -\frac{2\,x\,\sqrt{\frac{\mathrm{e}^{-a\,1\mathrm{i}}}{2\,c^2\,x^2}+\frac{c^2\,x^2\,\mathrm{e}^{a\,1\mathrm{i}}}{2}}}{\mathrm{e}^{a\,2\mathrm{i}}\,c^4\,x^4+1}$$

input
```
int(1/cos(a - log(c*x)*2i)^(3/2),x)
```

output
```
-(2*x*(exp(-a*1i)/(2*c^2*x^2) + (c^2*x^2*exp(a*1i))/2)^(1/2))/(c^4*x^4*exp
(a*2i) + 1)
```

Reduce [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(a-2i\log(cx))}\,dx = \int \frac{\sqrt{\cos\left(2\log\left(cx\right)i-a\right)}}{\cos\left(2\log\left(cx\right)i-a\right)^{2}}dx$$

input
```
int(1/cos(a-2*I*log(c*x))^(3/2),x)
```

output
```
int(sqrt(cos(2*log(c*x)*i - a))/cos(2*log(c*x)*i - a)**2,x)
```

3.123 $\int x^m \cos^4(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 266

$$\begin{aligned}\int x^m\cos^4(a+b\log(cx^n))\,dx &= \frac{24b^4n^4x^{1+m}}{(1+m)\left((1+m)^2+4b^2n^2\right)\left((1+m)^2+16b^2n^2\right)}\\ &\quad+\frac{12b^2(1+m)n^2x^{1+m}\cos^2(a+b\log(cx^n))}{\left((1+m)^2+4b^2n^2\right)\left((1+m)^2+16b^2n^2\right)}\\ &\quad+\frac{(1+m)x^{1+m}\cos^4(a+b\log(cx^n))}{(1+m)^2+16b^2n^2}\\ &\quad+\frac{24b^3n^3x^{1+m}\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{\left((1+m)^2+4b^2n^2\right)\left((1+m)^2+16b^2n^2\right)}\\ &\quad+\frac{4bnx^{1+m}\cos^3(a+b\log(cx^n))\sin(a+b\log(cx^n))}{(1+m)^2+16b^2n^2}\end{aligned}$$

output
```
24*b^4*n^4*x^(1+m)/(1+m)/((1+m)^2+4*b^2*n^2)/((1+m)^2+16*b^2*n^2)+12*b^2*(
1+m)*n^2*x^(1+m)*cos(a+b*ln(c*x^n))^2/((1+m)^2+4*b^2*n^2)/((1+m)^2+16*b^2*
n^2)+(1+m)*x^(1+m)*cos(a+b*ln(c*x^n))^4/((1+m)^2+16*b^2*n^2)+24*b^3*n^3*x^
(1+m)*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n))/((1+m)^2+4*b^2*n^2)/((1+m)^2+1
6*b^2*n^2)+4*b*n*x^(1+m)*cos(a+b*ln(c*x^n))^3*sin(a+b*ln(c*x^n))/((1+m)^2+
16*b^2*n^2)
```

Mathematica [A] (verified)

Time = 2.83 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.17

$$\int x^m \cos^4(a+b\log(cx^n))\,dx = \frac{1}{8}x^{1+m}\left(\frac{3}{1+m} - \frac{4\sin(2bn\log(x))(-2bn\cos(2(a-bn\log(x)+b\log(cx^n)))+(1+m)\sin(2(a-bn\log(x)+b\log(cx^n))))}{1+2m+m^2+4b^2n^2} + \frac{4\cos(2bn\log(x))((1+m)\cos(2(a-bn\log(x)+b\log(cx^n)))+2bn\sin(2(a-bn\log(x)+b\log(cx^n))))}{1+2m+m^2+4b^2n^2} - \frac{\sin(4bn\log(x))(-4bn\cos(4(a-bn\log(x)+b\log(cx^n)))+(1+m)\sin(4(a-bn\log(x)+b\log(cx^n))))}{1+2m+m^2+16b^2n^2} + \frac{\cos(4bn\log(x))((1+m)\cos(4(a-bn\log(x)+b\log(cx^n)))+4bn\sin(4(a-bn\log(x)+b\log(cx^n))))}{1+2m+m^2+16b^2n^2}\right)$$

input
```
Integrate[x^m*Cos[a + b*Log[c*x^n]]^4,x]
```

output
```
(x^(1 + m)*(3/(1 + m) - (4*Sin[2*b*n*Log[x]]*(-2*b*n*Cos[2*(a - b*n*Log[x]
 + b*Log[c*x^n])] + (1 + m)*Sin[2*(a - b*n*Log[x] + b*Log[c*x^n])]))/(1 +
2*m + m^2 + 4*b^2*n^2) + (4*Cos[2*b*n*Log[x]]*((1 + m)*Cos[2*(a - b*n*Log[
x] + b*Log[c*x^n])] + 2*b*n*Sin[2*(a - b*n*Log[x] + b*Log[c*x^n])]))/(1 +
2*m + m^2 + 4*b^2*n^2) - (Sin[4*b*n*Log[x]]*(-4*b*n*Cos[4*(a - b*n*Log[x]
+ b*Log[c*x^n])] + (1 + m)*Sin[4*(a - b*n*Log[x] + b*Log[c*x^n])]))/(1 + 2
*m + m^2 + 16*b^2*n^2) + (Cos[4*b*n*Log[x]]*((1 + m)*Cos[4*(a - b*n*Log[x]
 + b*Log[c*x^n])] + 4*b*n*Sin[4*(a - b*n*Log[x] + b*Log[c*x^n])]))/(1 + 2*
m + m^2 + 16*b^2*n^2)))/8
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.87, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4991, 4991, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cos^4(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \; 4991$$

$$\frac{12b^2n^2\int x^m\cos^2(a+b\log(cx^n))\,dx}{16b^2n^2+(m+1)^2}+\frac{(m+1)x^{m+1}\cos^4(a+b\log(cx^n))}{16b^2n^2+(m+1)^2}+\frac{4bnx^{m+1}\sin(a+b\log(cx^n))\cos^3(a+b\log(cx^n))}{16b^2n^2+(m+1)^2}$$

$$\Big\downarrow \; 4991$$

$$\frac{12b^2n^2\left(\frac{2b^2n^2\int x^m dx}{4b^2n^2+(m+1)^2}+\frac{(m+1)x^{m+1}\cos^2(a+b\log(cx^n))}{4b^2n^2+(m+1)^2}+\frac{2bnx^{m+1}\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+(m+1)^2}\right)}{16b^2n^2+(m+1)^2}+\frac{(m+1)x^{m+1}\cos^4(a+b\log(cx^n))}{16b^2n^2+(m+1)^2}+\frac{4bnx^{m+1}\sin(a+b\log(cx^n))\cos^3(a+b\log(cx^n))}{16b^2n^2+(m+1)^2}$$

$$\Big\downarrow \; 15$$

$$\frac{(m+1)x^{m+1}\cos^4(a+b\log(cx^n))}{16b^2n^2+(m+1)^2}+\frac{4bnx^{m+1}\sin(a+b\log(cx^n))\cos^3(a+b\log(cx^n))}{16b^2n^2+(m+1)^2}+\frac{12b^2n^2\left(\frac{(m+1)x^{m+1}\cos^2(a+b\log(cx^n))}{4b^2n^2+(m+1)^2}+\frac{2bnx^{m+1}\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+(m+1)^2}+\frac{2b^2n^2x^{m+1}}{(m+1)(4b^2n^2+(m+1)^2)}\right)}{16b^2n^2+(m+1)^2}$$

input
```
Int[x^m*Cos[a + b*Log[c*x^n]]^4,x]
```

output
```
((1 + m)*x^(1 + m)*Cos[a + b*Log[c*x^n]]^4)/((1 + m)^2 + 16*b^2*n^2) + (4*
b*n*x^(1 + m)*Cos[a + b*Log[c*x^n]]^3*Sin[a + b*Log[c*x^n]])/((1 + m)^2 +
16*b^2*n^2) + (12*b^2*n^2*((2*b^2*n^2*x^(1 + m))/((1 + m)*((1 + m)^2 + 4*b
^2*n^2)) + ((1 + m)*x^(1 + m)*Cos[a + b*Log[c*x^n]]^2)/((1 + m)^2 + 4*b^2*
n^2) + (2*b*n*x^(1 + m)*Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/((1 +
 m)^2 + 4*b^2*n^2)))/((1 + m)^2 + 16*b^2*n^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 4991
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Simp[b*d*n*p*(e*x)^(m + 1)*Sin[d*(a
+ b*Log[c*x^n])]*(Cos[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*
(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2)
)   Int[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [A] (verified)

Time = 75.80 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{\left((1+m)^2\left(b^2n^2+\frac{1}{16}m^2+\frac{1}{8}m+\frac{1}{16}\right)\cos(2b\ln(c\,x^n)+2a)+\frac{(1+m)^2\left(b^2n^2+\frac{1}{4}m^2+\frac{1}{2}m+\frac{1}{4}\right)\cos(4b\ln(c\,x^n)+4a)}{16}+2n(1+m)\left(b^2n^2+\right.\right.}{8(1+m)\left(b^2n^2+\frac{1}{4}m^2+\frac{1}{2}m+\frac{1}{4}\right.}$

input
```
int(x^m*cos(a+b*ln(c*x^n))^4,x,method=_RETURNVERBOSE)
```

output
```
1/8*((1+m)^2*(b^2*n^2+1/16*m^2+1/8*m+1/16)*cos(2*b*ln(c*x^n)+2*a)+1/16*(1+
m)^2*(b^2*n^2+1/4*m^2+1/2*m+1/4)*cos(4*b*ln(c*x^n)+4*a)+2*n*(1+m)*(b^2*n^2
+1/16*m^2+1/8*m+1/16)*b*sin(2*b*ln(c*x^n)+2*a)+1/4*(b^2*n^2+1/4*m^2+1/2*m+
1/4)*((1+m)*b*n*sin(4*b*ln(c*x^n)+4*a)+12*b^2*n^2+3/4*m^2+3/2*m+3/4))*x^(1
+m)/(1+m)/(b^2*n^2+1/4*m^2+1/2*m+1/4)/(b^2*n^2+1/16*m^2+1/8*m+1/16)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.03

$$\int x^m \cos^4(a + b\log(cx^n))\,dx$$
$$= \frac{4\left(6\left(b^3m + b^3\right)n^3x\cos(bn\log(x) + b\log(c) + a) + \left(4\left(b^3m + b^3\right)n^3 + (bm^3 + 3\,bm^2 + 3\,bm + b)n\right)x\cos\right.}{}$$

input
```
integrate(x^m*cos(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

output
```
(4*(6*(b^3*m + b^3)*n^3*x*cos(b*n*log(x) + b*log(c) + a) + (4*(b^3*m + b^3
)*n^3 + (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cos(b*n*log(x) + b*log(c) + a)^
3)*x^m*sin(b*n*log(x) + b*log(c) + a) + (24*b^4*n^4*x + 12*(b^2*m^2 + 2*b^
2*m + b^2)*n^2*x*cos(b*n*log(x) + b*log(c) + a)^2 + (m^4 + 4*m^3 + 4*(b^2*
m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cos(b*n*log(x) + b*log(c) +
a)^4)*x^m)/(m^5 + 64*(b^4*m + b^4)*n^4 + 5*m^4 + 10*m^3 + 20*(b^2*m^3 + 3*
b^2*m^2 + 3*b^2*m + b^2)*n^2 + 10*m^2 + 5*m + 1)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \cos^4(a + b\log(cx^n))\,dx = \text{Timed out}$$

input
```
integrate(x**m*cos(a+b*ln(c*x**n))**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3537 vs. $2(266) = 532$.

Time = 0.25 (sec) , antiderivative size = 3537, normalized size of antiderivative = 13.30

$$\int x^m \cos^4\left(a + b\log\left(cx^n\right)\right) dx = \text{Too large to display}$$

input
```
integrate(x^m*cos(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

output
```
1/16*(((cos(8*b*log(c))*cos(4*b*log(c)) + sin(8*b*log(c))*sin(4*b*log(c))
+ cos(4*b*log(c)))*m^4 + 4*(cos(8*b*log(c))*cos(4*b*log(c)) + sin(8*b*log(
c))*sin(4*b*log(c)) + cos(4*b*log(c)))*m^3 + 16*(b^3*cos(4*b*log(c))*sin(8
*b*log(c)) - b^3*cos(8*b*log(c))*sin(4*b*log(c)) + b^3*sin(4*b*log(c)) + (
b^3*cos(4*b*log(c))*sin(8*b*log(c)) - b^3*cos(8*b*log(c))*sin(4*b*log(c))
+ b^3*sin(4*b*log(c)))*m)*n^3 + 6*(cos(8*b*log(c))*cos(4*b*log(c)) + sin(8
*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c)))*m^2 + 4*(b^2*cos(8*b*log(c))
*cos(4*b*log(c)) + b^2*sin(8*b*log(c))*sin(4*b*log(c)) + (b^2*cos(8*b*log(
c))*cos(4*b*log(c)) + b^2*sin(8*b*log(c))*sin(4*b*log(c)) + b^2*cos(4*b*lo
g(c)))*m^2 + b^2*cos(4*b*log(c)) + 2*(b^2*cos(8*b*log(c))*cos(4*b*log(c))
+ b^2*sin(8*b*log(c))*sin(4*b*log(c)) + b^2*cos(4*b*log(c)))*m)*n^2 + 4*(c
os(8*b*log(c))*cos(4*b*log(c)) + sin(8*b*log(c))*sin(4*b*log(c)) + cos(4*b
*log(c)))*m + 4*((b*cos(4*b*log(c))*sin(8*b*log(c)) - b*cos(8*b*log(c))*si
n(4*b*log(c)) + b*sin(4*b*log(c)))*m^3 + 3*(b*cos(4*b*log(c))*sin(8*b*log(
c)) - b*cos(8*b*log(c))*sin(4*b*log(c)) + b*sin(4*b*log(c)))*m^2 + b*cos(4
*b*log(c))*sin(8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c)) + 3*(b*cos(
4*b*log(c))*sin(8*b*log(c)) - b*cos(8*b*log(c))*sin(4*b*log(c)) + b*sin(4*
b*log(c)))*m + b*sin(4*b*log(c)))*n + cos(8*b*log(c))*cos(4*b*log(c)) + si
n(8*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c)))*x*x^m*cos(4*b*log(x^n) +
4*a) + 4*((cos(6*b*log(c))*cos(4*b*log(c)) + cos(4*b*log(c))*cos(2*b*lo...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224645 vs. $2(266) = 532$.

Time = 6.07 (sec) , antiderivative size = 224645, normalized size of antiderivative = 844.53

$$\int x^m \cos^4(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(x^m*cos(a+b*log(c*x^n))^4,x, algorithm="giac")
```

output
```
-1/16*(384*b^4*n^4*x*abs(x)^m*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c
) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x))
 + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(2*a)^2*tan(a)^2
+ 384*b^4*n^4*x*abs(x)^m*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2
*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*
log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(2*a)^2 - 256*b^3*m*n^
3*x*abs(x)^m*e^(3*pi*b*n*sgn(x) - 3*pi*b*n + 3*pi*b*sgn(c) - 3*pi*b)*tan(2
*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))
^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(2*a)^2*tan(a) - 256*b^3*m*n^3*x*a
bs(x)^m*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(2*b*n*log(abs(
x)) + 2*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*pi
*m*sgn(x) - 1/4*pi*m)^2*tan(2*a)^2*tan(a) + 384*b^4*n^4*x*abs(x)^m*e^(2*pi
*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2
*b*log(abs(c)))^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(
x) - 1/4*pi*m)^2*tan(a)^2 - 32*b^3*m*n^3*x*abs(x)^m*e^(4*pi*b*n*sgn(x) - 4
*pi*b*n + 4*pi*b*sgn(c) - 4*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))
^2*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^
2*tan(2*a)*tan(a)^2 - 384*b^4*n^4*x*abs(x)^m*e^(2*pi*b*n*sgn(x) - 2*pi*b*n
 + 2*pi*b*sgn(c) - 2*pi*b)*tan(2*b*n*log(abs(x)) + 2*b*log(abs(c)))^2*tan(
b*n*log(abs(x)) + b*log(abs(c)))^2*tan(2*a)^2*tan(a)^2 + 32*b^3*m*n^3*x...
```

Mupad [B] (verification not implemented)

Time = 21.89 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.57

$$\int x^m \cos^4(a+b\log(cx^n))\,dx = \frac{3\,x\,x^m}{8\,m+8} + \frac{x\,x^m\,\mathrm{e}^{a\,2\mathrm{i}}\,\left(c\,x^n\right)^{b\,2\mathrm{i}}}{4\,m+4+b\,n\,8\mathrm{i}} + \frac{x\,x^m\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{\left(c\,x^n\right)^{b\,2\mathrm{i}}}\,1\mathrm{i}}{m\,4\mathrm{i}+8\,b\,n+4\mathrm{i}} + \frac{x\,x^m\,\mathrm{e}^{a\,4\mathrm{i}}\,\left(c\,x^n\right)^{b\,4\mathrm{i}}}{16\,m+16+b\,n\,64\mathrm{i}} + \frac{x\,x^m\,\mathrm{e}^{-a\,4\mathrm{i}}\,\frac{1}{\left(c\,x^n\right)^{b\,4\mathrm{i}}}\,1\mathrm{i}}{m\,16\mathrm{i}+64\,b\,n+16\mathrm{i}}$$

input
```
int(x^m*cos(a + b*log(c*x^n))^4,x)
```

output
```
(3*x*x^m)/(8*m + 8) + (x*x^m*exp(a*2i)*(c*x^n)^(b*2i))/(4*m + b*n*8i + 4)
+ (x*x^m*exp(-a*2i)/(c*x^n)^(b*2i)*1i)/(m*4i + 8*b*n + 4i) + (x*x^m*exp(a*
4i)*(c*x^n)^(b*4i))/(16*m + b*n*64i + 16) + (x*x^m*exp(-a*4i)/(c*x^n)^(b*4
i)*1i)/(m*16i + 64*b*n + 16i)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 797, normalized size of antiderivative = 3.00

$$\int x^m \cos^4(a+b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
int(x^m*cos(a+b*log(c*x^n))^4,x)
```

output

```
(x**m*x*( - 16*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*b**3*m*n**
3 - 16*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*b**3*n**3 - 4*cos(
log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*b*m**3*n - 12*cos(log(x**n*c)
*b + a)*sin(log(x**n*c)*b + a)**3*b*m**2*n - 12*cos(log(x**n*c)*b + a)*sin
(log(x**n*c)*b + a)**3*b*m*n - 4*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b
+ a)**3*b*n + 40*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*b**3*m*n**3
 + 40*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*b**3*n**3 + 4*cos(log(
x**n*c)*b + a)*sin(log(x**n*c)*b + a)*b*m**3*n + 12*cos(log(x**n*c)*b + a)
*sin(log(x**n*c)*b + a)*b*m**2*n + 12*cos(log(x**n*c)*b + a)*sin(log(x**n*
c)*b + a)*b*m*n + 4*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*b*n + 4*
sin(log(x**n*c)*b + a)**4*b**2*m**2*n**2 + 8*sin(log(x**n*c)*b + a)**4*b**
2*m*n**2 + 4*sin(log(x**n*c)*b + a)**4*b**2*n**2 + sin(log(x**n*c)*b + a)*
*4*m**4 + 4*sin(log(x**n*c)*b + a)**4*m**3 + 6*sin(log(x**n*c)*b + a)**4*m
**2 + 4*sin(log(x**n*c)*b + a)**4*m + sin(log(x**n*c)*b + a)**4 - 20*sin(l
og(x**n*c)*b + a)**2*b**2*m**2*n**2 - 40*sin(log(x**n*c)*b + a)**2*b**2*m*
n**2 - 20*sin(log(x**n*c)*b + a)**2*b**2*n**2 - 2*sin(log(x**n*c)*b + a)**
2*m**4 - 8*sin(log(x**n*c)*b + a)**2*m**3 - 12*sin(log(x**n*c)*b + a)**2*m
**2 - 8*sin(log(x**n*c)*b + a)**2*m - 2*sin(log(x**n*c)*b + a)**2 + 24*b**
4*n**4 + 16*b**2*m**2*n**2 + 32*b**2*m*n**2 + 16*b**2*n**2 + m**4 + 4*m**3
 + 6*m**2 + 4*m + 1))/(64*b**4*m*n**4 + 64*b**4*n**4 + 20*b**2*m**3*n**...
```

3.124 $\int x^m \cos^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 201

$$\int x^m \cos^3(a+b\log(cx^n))\,dx = \frac{6b^2(1+m)n^2x^{1+m}\cos(a+b\log(cx^n))}{((1+m)^2+b^2n^2)((1+m)^2+9b^2n^2)} + \frac{(1+m)x^{1+m}\cos^3(a+b\log(cx^n))}{(1+m)^2+9b^2n^2} + \frac{6b^3n^3x^{1+m}\sin(a+b\log(cx^n))}{((1+m)^2+b^2n^2)((1+m)^2+9b^2n^2)} + \frac{3bnx^{1+m}\cos^2(a+b\log(cx^n))\sin(a+b\log(cx^n))}{(1+m)^2+9b^2n^2}$$

output
```
6*b^2*(1+m)*n^2*x^(1+m)*cos(a+b*ln(c*x^n))/((1+m)^2+b^2*n^2)/((1+m)^2+9*b^
2*n^2)+(1+m)*x^(1+m)*cos(a+b*ln(c*x^n))^3/((1+m)^2+9*b^2*n^2)+6*b^3*n^3*x^
(1+m)*sin(a+b*ln(c*x^n))/((1+m)^2+b^2*n^2)/((1+m)^2+9*b^2*n^2)+3*b*n*x^(1+
m)*cos(a+b*ln(c*x^n))^2*sin(a+b*ln(c*x^n))/((1+m)^2+9*b^2*n^2)
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.45

$$\int x^m \cos^3(a+b\log(cx^n))\,dx$$

$$=\frac{1}{4}x^{1+m}\Bigg(-\frac{3\sin(bn\log(x))(-bn\cos(a-bn\log(x)+b\log(cx^n))+(1+m)\sin(a-bn\log(x)+b\log(cx}{1+2m+m^2+b^2n^2}$$

$$+\frac{3\cos(bn\log(x))((1+m)\cos(a-bn\log(x)+b\log(cx^n))+bn\sin(a-bn\log(x)+b\log(cx^n)))}{1+2m+m^2+b^2n^2}$$

$$-\frac{\sin(3bn\log(x))(-3bn\cos(3(a-bn\log(x)+b\log(cx^n)))+(1+m)\sin(3(a-bn\log(x)+b\log(cx^n)}{1+2m+m^2+9b^2n^2}$$

$$+\frac{\cos(3bn\log(x))((1+m)\cos(3(a-bn\log(x)+b\log(cx^n)))+3bn\sin(3(a-bn\log(x)+b\log(cx^n)))}{1+2m+m^2+9b^2n^2}$$

input
```
Integrate[x^m*Cos[a + b*Log[c*x^n]]^3,x]
```

output
```
(x^(1 + m)*((-3*Sin[b*n*Log[x]]*(-(b*n*Cos[a - b*n*Log[x] + b*Log[c*x^n]])
 + (1 + m)*Sin[a - b*n*Log[x] + b*Log[c*x^n]]))/(1 + 2*m + m^2 + b^2*n^2)
+ (3*Cos[b*n*Log[x]]*((1 + m)*Cos[a - b*n*Log[x] + b*Log[c*x^n]] + b*n*Sin
[a - b*n*Log[x] + b*Log[c*x^n]]))/(1 + 2*m + m^2 + b^2*n^2) - (Sin[3*b*n*L
og[x]]*(-3*b*n*Cos[3*(a - b*n*Log[x] + b*Log[c*x^n])] + (1 + m)*Sin[3*(a -
 b*n*Log[x] + b*Log[c*x^n])]))/(1 + 2*m + m^2 + 9*b^2*n^2) + (Cos[3*b*n*Lo
g[x]]*((1 + m)*Cos[3*(a - b*n*Log[x] + b*Log[c*x^n])] + 3*b*n*Sin[3*(a - b
*n*Log[x] + b*Log[c*x^n])]))/(1 + 2*m + m^2 + 9*b^2*n^2)))/4
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4991, 4989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cos^3(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{ 4991}$$

$$\frac{6b^2n^2\int x^m\cos(a+b\log(cx^n))\,dx}{9b^2n^2+(m+1)^2}+\frac{(m+1)x^{m+1}\cos^3(a+b\log(cx^n))}{9b^2n^2+(m+1)^2}+\frac{3bnx^{m+1}\sin(a+b\log(cx^n))\cos^2(a+b\log(cx^n))}{9b^2n^2+(m+1)^2}$$

$$\Big\downarrow \text{ 4989}$$

$$\frac{(m+1)x^{m+1}\cos^3(a+b\log(cx^n))}{9b^2n^2+(m+1)^2}+\frac{3bnx^{m+1}\sin(a+b\log(cx^n))\cos^2(a+b\log(cx^n))}{9b^2n^2+(m+1)^2}+\frac{6b^2n^2\left(\frac{bnx^{m+1}\sin(a+b\log(cx^n))}{b^2n^2+(m+1)^2}+\frac{(m+1)x^{m+1}\cos(a+b\log(cx^n))}{b^2n^2+(m+1)^2}\right)}{9b^2n^2+(m+1)^2}$$

input
```
Int[x^m*Cos[a + b*Log[c*x^n]]^3,x]
```

output
```
((1 + m)*x^(1 + m)*Cos[a + b*Log[c*x^n]]^3)/((1 + m)^2 + 9*b^2*n^2) + (3*b
*n*x^(1 + m)*Cos[a + b*Log[c*x^n]]^2*Sin[a + b*Log[c*x^n]])/((1 + m)^2 + 9
*b^2*n^2) + (6*b^2*n^2*(((1 + m)*x^(1 + m)*Cos[a + b*Log[c*x^n]])/((1 + m)
^2 + b^2*n^2) + (b*n*x^(1 + m)*Sin[a + b*Log[c*x^n]])/((1 + m)^2 + b^2*n^2
)))/((1 + m)^2 + 9*b^2*n^2)
```

Defintions of rubi rules used

rule 4989
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

rule 4991
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Simp[b*d*n*p*(e*x)^(m + 1)*Sin[d*(a
+ b*Log[c*x^n])]*(Cos[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*
(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2)
)   Int[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [A] (verified)

Time = 15.21 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.53

method	result
parallelrisch	$-\frac{7x^{1+m}\left(\left(b^2n^2+\frac{1}{7}m^2+\frac{2}{7}m+\frac{1}{7}\right)(1+m)\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^6-\frac{18n\left(b^2n^2+\frac{1}{3}m^2+\frac{2}{3}m+\frac{1}{3}\right)b\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^5}{7}+\frac{3(1+m)(bn \cdots}{\cdots}}{\cdots}$

input
```
int(x^m*cos(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
```

output
```
-7/9*x^(1+m)*((b^2*n^2+1/7*m^2+2/7*m+1/7)*(1+m)*tan(1/2*a+b*ln((c*x^n)^(1/
2)))^6-18/7*n*(b^2*n^2+1/3*m^2+2/3*m+1/3)*b*tan(1/2*a+b*ln((c*x^n)^(1/2)))
^5+3/7*(1+m)*(b*n+m+1)*(b*n-m-1)*tan(1/2*a+b*ln((c*x^n)^(1/2)))^4-12/7*b*n
*(b*n+m+1)*(b*n-m-1)*tan(1/2*a+b*ln((c*x^n)^(1/2)))^3-3/7*(1+m)*(b*n+m+1)*
(b*n-m-1)*tan(1/2*a+b*ln((c*x^n)^(1/2)))^2-18/7*n*(b^2*n^2+1/3*m^2+2/3*m+1
/3)*b*tan(1/2*a+b*ln((c*x^n)^(1/2)))-(b^2*n^2+1/7*m^2+2/7*m+1/7)*(1+m))/(b
^2*n^2+m^2+2*m+1)/(b^2*n^2+1/9*m^2+2/9*m+1/9)/(1+tan(1/2*a+b*ln((c*x^n)^(1
/2)))^2)^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.95

$$\int x^m\cos^3\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{3\left(2\,b^3n^3x+(b^3n^3+(bm^2+2\,bm+b)n)x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2\right)x^m\sin\left(bn\log\left(x\right)+b\log\left(c\right)+\cdots\right.}{9\,b^4n^4+m^4+4\,m^3\cdots}$$

input
```
integrate(x^m*cos(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
(3*(2*b^3*n^3*x + (b^3*n^3 + (b*m^2 + 2*b*m + b)*n)*x*cos(b*n*log(x) + b*l
og(c) + a)^2)*x^m*sin(b*n*log(x) + b*log(c) + a) + (6*(b^2*m + b^2)*n^2*x*
cos(b*n*log(x) + b*log(c) + a) + (m^3 + (b^2*m + b^2)*n^2 + 3*m^2 + 3*m +
1)*x*cos(b*n*log(x) + b*log(c) + a)^3)*x^m)/(9*b^4*n^4 + m^4 + 4*m^3 + 10*
(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \cos^3(a + b\log(cx^n)) \, dx = \text{Timed out}$$

input
```
integrate(x**m*cos(a+b*ln(c*x**n))**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2352 vs. $2(201) = 402$.

Time = 0.15 (sec) , antiderivative size = 2352, normalized size of antiderivative = 11.70

$$\int x^m \cos^3(a + b\log(cx^n)) \, dx = \text{Too large to display}$$

input
```
integrate(x^m*cos(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
1/8*(((cos(6*b*log(c))*cos(3*b*log(c)) + sin(6*b*log(c))*sin(3*b*log(c)) +
 cos(3*b*log(c)))*m^3 + 3*(b^3*cos(3*b*log(c))*sin(6*b*log(c)) - b^3*cos(6
*b*log(c))*sin(3*b*log(c)) + b^3*sin(3*b*log(c)))*n^3 + 3*(cos(6*b*log(c))
*cos(3*b*log(c)) + sin(6*b*log(c))*sin(3*b*log(c)) + cos(3*b*log(c)))*m^2
+ (b^2*cos(6*b*log(c))*cos(3*b*log(c)) + b^2*sin(6*b*log(c))*sin(3*b*log(c
)) + b^2*cos(3*b*log(c)) + (b^2*cos(6*b*log(c))*cos(3*b*log(c)) + b^2*sin(
6*b*log(c))*sin(3*b*log(c)) + b^2*cos(3*b*log(c)))*m)*n^2 + 3*(cos(6*b*log
(c))*cos(3*b*log(c)) + sin(6*b*log(c))*sin(3*b*log(c)) + cos(3*b*log(c)))*
m + 3*((b*cos(3*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(3*b*log(
c)) + b*sin(3*b*log(c)))*m^2 + b*cos(3*b*log(c))*sin(6*b*log(c)) - b*cos(6
*b*log(c))*sin(3*b*log(c)) + 2*(b*cos(3*b*log(c))*sin(6*b*log(c)) - b*cos(
6*b*log(c))*sin(3*b*log(c)) + b*sin(3*b*log(c)))*m + b*sin(3*b*log(c)))*n
+ cos(6*b*log(c))*cos(3*b*log(c)) + sin(6*b*log(c))*sin(3*b*log(c)) + cos(
3*b*log(c)))*x*x^m*cos(3*b*log(x^n) + 3*a) + 3*((cos(4*b*log(c))*cos(3*b*l
og(c)) + cos(3*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(3*b*log(c))
 + sin(3*b*log(c))*sin(2*b*log(c)))*m^3 + 9*(b^3*cos(3*b*log(c))*sin(4*b*l
og(c)) - b^3*cos(4*b*log(c))*sin(3*b*log(c)) + b^3*cos(2*b*log(c))*sin(3*b
*log(c)) - b^3*cos(3*b*log(c))*sin(2*b*log(c)))*n^3 + 3*(cos(4*b*log(c))*c
os(3*b*log(c)) + cos(3*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(3*b
*log(c)) + sin(3*b*log(c))*sin(2*b*log(c)))*m^2 + 9*(b^2*cos(4*b*log(c)...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159584 vs. 2(201) = 402.

Time = 4.48 (sec) , antiderivative size = 159584, normalized size of antiderivative = 793.95

$$\int x^m \cos^3(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(x^m*cos(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
1/8*(54*b^3*n^3*x*abs(x)^m*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sg
n(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*
n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan
(3/2*a)^2*tan(1/2*a) + 54*b^3*n^3*x*abs(x)^m*e^(-1/2*pi*b*n*sgn(x) + 1/2*p
i*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(ab
s(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x
) - 1/4*pi*m)^2*tan(3/2*a)^2*tan(1/2*a) + 6*b^3*n^3*x*abs(x)^m*e^(3/2*pi*b
*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x
)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*t
an(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(3/2*a)*tan(1/2*a)^2 + 6*b^3*n^3*x*abs
(x)^m*e^(-3/2*pi*b*n*sgn(x) + 3/2*pi*b*n - 3/2*pi*b*sgn(c) + 3/2*pi*b)*tan
(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*log(abs(x)) + 1/2*
b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(3/2*a)*tan(1/2*a)^2
 - 6*b^3*n^3*x*abs(x)^m*e^(3/2*pi*b*n*sgn(x) - 3/2*pi*b*n + 3/2*pi*b*sgn(c
) - 3/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)))^2*tan(1/2*b*n*l
og(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)*tan(3/2*
a)^2*tan(1/2*a)^2 - 54*b^3*n^3*x*abs(x)^m*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*
n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(3/2*b*n*log(abs(x)) + 3/2*b*log(abs(c)
))^2*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) -
1/4*pi*m)*tan(3/2*a)^2*tan(1/2*a)^2 + 54*b^3*n^3*x*abs(x)^m*e^(-1/2*pi*...
```

Mupad [B] (verification not implemented)

Time = 20.98 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

$$\int x^m \cos^3(a+b\log(cx^n))\,dx = \frac{3\,x\,x^m\,\mathrm{e}^{a\,1\mathrm{i}}\,{(c\,x^n)}^{b\,1\mathrm{i}}}{8\,m+8+b\,n\,8\mathrm{i}}+\frac{x\,x^m\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{(c\,x^n)}^{b\,1\mathrm{i}}}\,3\mathrm{i}}{m\,8\mathrm{i}+8\,b\,n+8\mathrm{i}}+\frac{x\,x^m\,\mathrm{e}^{a\,3\mathrm{i}}\,{(c\,x^n)}^{b\,3\mathrm{i}}}{8\,m+8+b\,n\,24\mathrm{i}}+\frac{x\,x^m\,\mathrm{e}^{-a\,3\mathrm{i}}\,\frac{1}{{(c\,x^n)}^{b\,3\mathrm{i}}}\,1\mathrm{i}}{m\,8\mathrm{i}+24\,b\,n+8\mathrm{i}}$$

input
```
int(x^m*cos(a + b*log(c*x^n))^3,x)
```

output
```
(3*x*x^m*exp(a*1i)*(c*x^n)^(b*1i))/(8*m + b*n*8i + 8) + (x*x^m*exp(-a*1i)/
(c*x^n)^(b*1i)*3i)/(m*8i + 8*b*n + 8i) + (x*x^m*exp(a*3i)*(c*x^n)^(b*3i))/
(8*m + b*n*24i + 8) + (x*x^m*exp(-a*3i)/(c*x^n)^(b*3i)*1i)/(m*8i + 24*b*n
+ 8i)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.37

$$\int x^m \cos^3(a + b\log(cx^n)) \, dx$$
$$= \frac{x^m x(-\cos(\log(x^n c) b + a) \sin(\log(x^n c) b + a)^2 b^2 m n^2 - \cos(\log(x^n c) b + a) \sin(\log(x^n c) b + a)^2 b^2 n^2 -}{}$$

input
```
int(x^m*cos(a+b*log(c*x^n))^3,x)
```

output
```
(x**m*x*( - cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**2*b**2*m*n**2 -
 cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**2*b**2*n**2 - cos(log(x**n
*c)*b + a)*sin(log(x**n*c)*b + a)**2*m**3 - 3*cos(log(x**n*c)*b + a)*sin(l
og(x**n*c)*b + a)**2*m**2 - 3*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a
)**2*m - cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**2 + 7*cos(log(x**n
*c)*b + a)*b**2*m*n**2 + 7*cos(log(x**n*c)*b + a)*b**2*n**2 + cos(log(x**n
*c)*b + a)*m**3 + 3*cos(log(x**n*c)*b + a)*m**2 + 3*cos(log(x**n*c)*b + a)
*m + cos(log(x**n*c)*b + a) - 3*sin(log(x**n*c)*b + a)**3*b**3*n**3 - 3*si
n(log(x**n*c)*b + a)**3*b*m**2*n - 6*sin(log(x**n*c)*b + a)**3*b*m*n - 3*s
in(log(x**n*c)*b + a)**3*b*n + 9*sin(log(x**n*c)*b + a)*b**3*n**3 + 3*sin(
log(x**n*c)*b + a)*b*m**2*n + 6*sin(log(x**n*c)*b + a)*b*m*n + 3*sin(log(x
**n*c)*b + a)*b*n))/(9*b**4*n**4 + 10*b**2*m**2*n**2 + 20*b**2*m*n**2 + 10
*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1)
```

3.125 $\int x^m \cos^2(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 120

$$\int x^m \cos^2(a+b\log(cx^n))\,dx = \frac{2b^2n^2x^{1+m}}{(1+m)\left((1+m)^2+4b^2n^2\right)} + \frac{(1+m)x^{1+m}\cos^2(a+b\log(cx^n))}{(1+m)^2+4b^2n^2} + \frac{2bnx^{1+m}\cos(a+b\log(cx^n))\sin(a+b\log(cx^n))}{(1+m)^2+4b^2n^2}$$

output
```
2*b^2*n^2*x^(1+m)/(1+m)/((1+m)^2+4*b^2*n^2)+(1+m)*x^(1+m)*cos(a+b*ln(c*x^n
))^2/((1+m)^2+4*b^2*n^2)+2*b*n*x^(1+m)*cos(a+b*ln(c*x^n))*sin(a+b*ln(c*x^n
))/((1+m)^2+4*b^2*n^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int x^m \cos^2(a+b\log(cx^n))\,dx = \frac{x^{1+m}\left(1+2m+m^2+4b^2n^2+(1+m)^2\cos(2(a+b\log(cx^n)))+2b(1+m)n\sin(2(a+b\log(cx^n)))\right)}{2(1+m)(1+m-2ibn)(1+m+2ibn)}$$

input
```
Integrate[x^m*Cos[a + b*Log[c*x^n]]^2,x]
```

output
```
(x^(1 + m)*(1 + 2*m + m^2 + 4*b^2*n^2 + (1 + m)^2*Cos[2*(a + b*Log[c*x^n])
] + 2*b*(1 + m)*n*Sin[2*(a + b*Log[c*x^n])]))/(2*(1 + m)*(1 + m - (2*I)*b*
n)*(1 + m + (2*I)*b*n))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4991, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cos^2(a + b\log(cx^n))\,dx$$

$$\Big\downarrow \text{4991}$$

$$\frac{2b^2n^2\int x^m dx}{4b^2n^2+(m+1)^2} + \frac{(m+1)x^{m+1}\cos^2(a+b\log(cx^n))}{4b^2n^2+(m+1)^2} + \frac{2bnx^{m+1}\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+(m+1)^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{(m+1)x^{m+1}\cos^2(a+b\log(cx^n))}{4b^2n^2+(m+1)^2} + \frac{2bnx^{m+1}\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2n^2+(m+1)^2} + \frac{2b^2n^2x^{m+1}}{(m+1)(4b^2n^2+(m+1)^2)}$$

input
```
Int[x^m*Cos[a + b*Log[c*x^n]]^2,x]
```

output
```
(2*b^2*n^2*x^(1 + m))/((1 + m)*((1 + m)^2 + 4*b^2*n^2)) + ((1 + m)*x^(1 +
m)*Cos[a + b*Log[c*x^n]]^2)/((1 + m)^2 + 4*b^2*n^2) + (2*b*n*x^(1 + m)*Cos
[a + b*Log[c*x^n]]*Sin[a + b*Log[c*x^n]])/((1 + m)^2 + 4*b^2*n^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 4991
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.
), x_Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]^p/(b^
2*d^2*e*n^2*p^2 + e*(m + 1)^2)), x] + (Simp[b*d*n*p*(e*x)^(m + 1)*Sin[d*(a
+ b*Log[c*x^n])]*(Cos[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 + e*
(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 + (m + 1)^2)
)   Int[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b,
c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 + (m + 1)^2, 0]
```

Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{x^{1+m}\left(2b^2n^2+(1+m)bn\sin(2b\ln(c\,x^n)+2a)+\frac{(\cos(2b\ln(c\,x^n)+2a)+1)(1+m)^2}{2}\right)}{(4b^2n^2+m^2+2m+1)(1+m)}$	82

input
```
int(x^m*cos(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

output
```
x^(1+m)*(2*b^2*n^2+(1+m)*b*n*sin(2*b*ln(c*x^n)+2*a)+1/2*(cos(2*b*ln(c*x^n)
+2*a)+1)*(1+m)^2)/(4*b^2*n^2+m^2+2*m+1)/(1+m)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88

$$\int x^m\cos^2(a+b\log(cx^n))\,dx$$
$$=\frac{2\,(bm+b)nxx^m\cos(bn\log(x)+b\log(c)+a)\sin(bn\log(x)+b\log(c)+a)+\left(2\,b^2n^2x+(m^2+2\,m+\right.}{m^3+4\,(b^2m+b^2)n^2+3\,m^2+3\,m+1}$$

input
```
integrate(x^m*cos(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
(2*(b*m + b)*n*x*x^m*cos(b*n*log(x) + b*log(c) + a)*sin(b*n*log(x) + b*log
(c) + a) + (2*b^2*n^2*x + (m^2 + 2*m + 1)*x*cos(b*n*log(x) + b*log(c) + a)
^2)*x^m)/(m^3 + 4*(b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)
```

Sympy [F]

$$\int x^m \cos^2(a + b\log(cx^n))\, dx$$
$$= \begin{cases} \log(x)\cos^2(a) \\ \int x^m \cos^2\left(-a + \frac{im\log(cx^n)}{2n} + \frac{i\log(cx^n)}{2n}\right) dx \\ \int x^m \cos^2\left(a + \frac{im\log(cx^n)}{2n} + \frac{i\log(cx^n)}{2n}\right) dx \\ \frac{\begin{cases} \log(x)\cos(2a) & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \log(x)\cos(2a + 2b\log(c)) & \text{for } n = 0 \\ \frac{\sin(2a+2b\log(cx^n))}{2bn} & \text{otherwise} \end{cases}}{2} + \frac{\log(x)}{2} \\ \frac{2b^2n^2xx^m\sin^2(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2+m^3+3m^2+3m+1} + \frac{2b^2n^2xx^m\cos^2(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2+m^3+3m^2+3m+1} + \frac{2bmnxx^m\sin(a+b\log(cx^n))\cos(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2+m^3+3m^2+3m+1} + \frac{2bnxx^m}{4} \end{cases}$$

input
```
integrate(x**m*cos(a+b*ln(c*x**n))**2,x)
```

output
```
Piecewise((log(x)*cos(a)**2, Eq(b, 0) & Eq(m, -1)), (Integral(x**m*cos(-a
+ I*m*log(c*x**n)/(2*n) + I*log(c*x**n)/(2*n))**2, x), Eq(b, -I*(m + 1)/(2
*n))), (Integral(x**m*cos(a + I*m*log(c*x**n)/(2*n) + I*log(c*x**n)/(2*n))
**2, x), Eq(b, I*(m + 1)/(2*n))), (Piecewise((log(x)*cos(2*a), Eq(b, 0) &
(Eq(b, 0) | Eq(n, 0))), (log(x)*cos(2*a + 2*b*log(c)), Eq(n, 0)), (sin(2*a
 + 2*b*log(c*x**n))/(2*b*n), True))/2 + log(x)/2, Eq(m, -1)), (2*b**2*n**2
*x*x**m*sin(a + b*log(c*x**n))**2/(4*b**2*m*n**2 + 4*b**2*n**2 + m**3 + 3*
m**2 + 3*m + 1) + 2*b**2*n**2*x*x**m*cos(a + b*log(c*x**n))**2/(4*b**2*m*n
**2 + 4*b**2*n**2 + m**3 + 3*m**2 + 3*m + 1) + 2*b*m*n*x*x**m*sin(a + b*lo
g(c*x**n))*cos(a + b*log(c*x**n))/(4*b**2*m*n**2 + 4*b**2*n**2 + m**3 + 3*
m**2 + 3*m + 1) + 2*b*n*x*x**m*sin(a + b*log(c*x**n))*cos(a + b*log(c*x**n
))/(4*b**2*m*n**2 + 4*b**2*n**2 + m**3 + 3*m**2 + 3*m + 1) + m**2*x*x**m*c
os(a + b*log(c*x**n))**2/(4*b**2*m*n**2 + 4*b**2*n**2 + m**3 + 3*m**2 + 3*
m + 1) + 2*m*x*x**m*cos(a + b*log(c*x**n))**2/(4*b**2*m*n**2 + 4*b**2*n**2
 + m**3 + 3*m**2 + 3*m + 1) + x*x**m*cos(a + b*log(c*x**n))**2/(4*b**2*m*n
**2 + 4*b**2*n**2 + m**3 + 3*m**2 + 3*m + 1), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. 2(120) = 240.

Time = 0.09 (sec) , antiderivative size = 646, normalized size of antiderivative = 5.38

$$\int x^m \cos^2(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(x^m*cos(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
1/4*(((cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)) +
 cos(2*b*log(c)))*m^2 + 2*(cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c
))*sin(2*b*log(c)) + cos(2*b*log(c)))*m + 2*(b*cos(2*b*log(c))*sin(4*b*log
(c)) - b*cos(4*b*log(c))*sin(2*b*log(c)) + (b*cos(2*b*log(c))*sin(4*b*log(
c)) - b*cos(4*b*log(c))*sin(2*b*log(c)) + b*sin(2*b*log(c)))*m + b*sin(2*b
*log(c)))*n + cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*lo
g(c)) + cos(2*b*log(c)))*x*x^m*cos(2*b*log(x^n) + 2*a) - ((cos(2*b*log(c))
*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)) + sin(2*b*log(c)))*m^2
+ 2*(cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)) + s
in(2*b*log(c)))*m - 2*(b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c
))*sin(2*b*log(c)) + (b*cos(4*b*log(c))*cos(2*b*log(c)) + b*sin(4*b*log(c)
)*sin(2*b*log(c)) + b*cos(2*b*log(c)))*m + b*cos(2*b*log(c)))*n + cos(2*b*
log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)) + sin(2*b*log(c)
))*x*x^m*sin(2*b*log(x^n) + 2*a) + 2*((cos(2*b*log(c))^2 + sin(2*b*log(c))
^2)*m^2 + 4*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2 + 2*(cos(2
*b*log(c))^2 + sin(2*b*log(c))^2)*m + cos(2*b*log(c))^2 + sin(2*b*log(c))^
2)*x*x^m)/((cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*m^3 + 3*(cos(2*b*log(c)
)^2 + sin(2*b*log(c))^2)*m^2 + 4*(b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(
c))^2 + (b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*m)*n^2 + 3*(cos(2*
b*log(c))^2 + sin(2*b*log(c))^2)*m + cos(2*b*log(c))^2 + sin(2*b*log(c)...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9145 vs. $2(120) = 240$.

Time = 0.56 (sec) , antiderivative size = 9145, normalized size of antiderivative = 76.21

$$\int x^m \cos^2(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(x^m*cos(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
-1/4*(8*b^2*n^2*x*abs(x)^m*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)
*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*
tan(a)^2 + 8*b^2*n^2*x*abs(x)^m*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) -
pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*
m)^2 - 4*b*m*n*x*abs(x)^m*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) -
2*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*p
i*m)^2*tan(a) - 8*b^2*n^2*x*abs(x)^m*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(
c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(a)^2 + 4*b*m*n*x*abs
(x)^m*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(
abs(x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)*tan(a)^2 + 8*b^
2*n^2*x*abs(x)^m*e^(pi*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(1/4*p
i*m*sgn(x) - 1/4*pi*m)^2*tan(a)^2 - 4*b*m*n*x*abs(x)^m*e^(2*pi*b*n*sgn(x)
- 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(c)))*
tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(a)^2 + m^2*x*abs(x)^m*e^(2*pi*b*n*sg
n(x) - 2*pi*b*n + 2*pi*b*sgn(c) - 2*pi*b)*tan(b*n*log(abs(x)) + b*log(abs(
c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(a)^2 + 2*m^2*x*abs(x)^m*e^(pi
*b*n*sgn(x) - pi*b*n + pi*b*sgn(c) - pi*b)*tan(b*n*log(abs(x)) + b*log(abs
(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(a)^2 - 4*b*m*n*x*abs(x)^m*ta
n(b*n*log(abs(x)) + b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan
(a) - 4*b*n*x*abs(x)^m*e^(2*pi*b*n*sgn(x) - 2*pi*b*n + 2*pi*b*sgn(c) - ...
```

Mupad [B] (verification not implemented)

Time = 19.92 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.68

$$\int x^m \cos^2(a + b\log(cx^n))\, dx = \frac{x\,x^m}{2\,m+2} + \frac{x\,x^m\,\mathrm{e}^{a\,2\mathrm{i}}\,\left(c\,x^n\right)^{b\,2\mathrm{i}}}{4\,m+4+b\,n\,8\mathrm{i}} + \frac{x\,x^m\,\mathrm{e}^{-a\,2\mathrm{i}}\,\frac{1}{\left(c\,x^n\right)^{b\,2\mathrm{i}}}\,1\mathrm{i}}{m\,4\mathrm{i}+8\,b\,n+4\mathrm{i}}$$

input
```
int(x^m*cos(a + b*log(c*x^n))^2,x)
```

output
```
(x*x^m)/(2*m + 2) + (x*x^m*exp(a*2i)*(c*x^n)^(b*2i))/(4*m + b*n*8i + 4) +
(x*x^m*exp(-a*2i)/(c*x^n)^(b*2i)*1i)/(m*4i + 8*b*n + 4i)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.29

$$\int x^m \cos^2(a + b\log(cx^n))\,dx$$
$$= \frac{x^m x\left(2\cos(\log(x^n c)\,b + a)\sin(\log(x^n c)\,b + a)\,bmn + 2\cos(\log(x^n c)\,b + a)\sin(\log(x^n c)\,b + a)\,bn - \sin(\right.}{4b^2 m\,n^2 + 4b^2 n^2 + m^3 +}$$

input
```
int(x^m*cos(a+b*log(c*x^n))^2,x)
```

output
```
(x**m*x*(2*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*b*m*n + 2*cos(log
(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*b*n - sin(log(x**n*c)*b + a)**2*m**
2 - 2*sin(log(x**n*c)*b + a)**2*m - sin(log(x**n*c)*b + a)**2 + 2*b**2*n**
2 + m**2 + 2*m + 1))/(4*b**2*m*n**2 + 4*b**2*n**2 + m**3 + 3*m**2 + 3*m +
1)
```

3.126 $\int x^m \cos(a + b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int x^m \cos(a + b\log(cx^n))\,dx = \frac{(1+m)x^{1+m}\cos(a+b\log(cx^n))}{(1+m)^2+b^2n^2} + \frac{bnx^{1+m}\sin(a+b\log(cx^n))}{(1+m)^2+b^2n^2}$$

output
```
(1+m)*x^(1+m)*cos(a+b*ln(c*x^n))/((1+m)^2+b^2*n^2)+b*n*x^(1+m)*sin(a+b*ln(
c*x^n))/((1+m)^2+b^2*n^2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.76

$$\int x^m \cos(a+b\log(cx^n))\,dx = \frac{x^{1+m}((1+m)\cos(a+b\log(cx^n))+bn\sin(a+b\log(cx^n)))}{1+2m+m^2+b^2n^2}$$

input
```
Integrate[x^m*Cos[a + b*Log[c*x^n]],x]
```

output
```
(x^(1 + m)*((1 + m)*Cos[a + b*Log[c*x^n]] + b*n*Sin[a + b*Log[c*x^n]]))/(1
 + 2*m + m^2 + b^2*n^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {4989}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cos(a + b\log(cx^n)) \, dx$$

$$\Big\downarrow \text{4989}$$

$$\frac{bnx^{m+1}\sin(a + b\log(cx^n))}{b^2n^2 + (m+1)^2} + \frac{(m+1)x^{m+1}\cos(a + b\log(cx^n))}{b^2n^2 + (m+1)^2}$$

input
```
Int[x^m*Cos[a + b*Log[c*x^n]],x]
```

output
```
((1 + m)*x^(1 + m)*Cos[a + b*Log[c*x^n]])/((1 + m)^2 + b^2*n^2) + (b*n*x^(
1 + m)*Sin[a + b*Log[c*x^n]])/((1 + m)^2 + b^2*n^2)
```

Defintions of rubi rules used

rule 4989
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x_
Symbol] :> Simp[(m + 1)*(e*x)^(m + 1)*(Cos[d*(a + b*Log[c*x^n])]/(b^2*d^2*e
*n^2 + e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sin[d*(a + b*Log[c*x^n
])]/(b^2*d^2*e*n^2 + e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}, x] &
& NeQ[b^2*d^2*n^2 + (m + 1)^2, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

method	result	size
parallelrisch	$\frac{x^{1+m}\left(\sin\left(a+b\ln\left(c\,x^{n}\right)\right) b n+\cos\left(a+b\ln\left(c\,x^{n}\right)\right) m+\cos\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{b^{2} n^{2}+m^{2}+2 m+1}$	63

input
```
int(x^m*cos(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

output
```
x^(1+m)/(b^2*n^2+m^2+2*m+1)*(sin(a+b*ln(c*x^n))*b*n+cos(a+b*ln(c*x^n))*m+c
os(a+b*ln(c*x^n)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int x^m \cos(a+b\log(cx^n))\,dx = \frac{bnxx^m \sin(bn\log(x)+b\log(c)+a)+(m+1)xx^m \cos(bn\log(x)+b\log(c)+a)}{b^2n^2+m^2+2\,m+1}$$

input
```
integrate(x^m*cos(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
(b*n*x*x^m*sin(b*n*log(x) + b*log(c) + a) + (m + 1)*x*x^m*cos(b*n*log(x) +
 b*log(c) + a))/(b^2*n^2 + m^2 + 2*m + 1)
```

Sympy [F]

$$\int x^m \cos(a + b\log(cx^n))\,dx$$
$$= \begin{cases} \log(x)\cos(a) & \text{for } b = 0 \wedge m = -1 \\ \int x^m \cos\left(-a + \frac{im\log(cx^n)}{n} + \frac{i\log(cx^n)}{n}\right) dx & \text{for } b = -\frac{i(m+1)}{n} \\ \int x^m \cos\left(a + \frac{im\log(cx^n)}{n} + \frac{i\log(cx^n)}{n}\right) dx & \text{for } b = \frac{i(m+1)}{n} \\ \frac{bnxx^m \sin(a+b\log(cx^n))}{b^2n^2+m^2+2m+1} + \frac{mxx^m \cos(a+b\log(cx^n))}{b^2n^2+m^2+2m+1} + \frac{xx^m \cos(a+b\log(cx^n))}{b^2n^2+m^2+2m+1} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*cos(a+b*ln(c*x**n)),x)
```

output
```
Piecewise((log(x)*cos(a), Eq(b, 0) & Eq(m, -1)), (Integral(x**m*cos(-a + I
*m*log(c*x**n)/n + I*log(c*x**n)/n), x), Eq(b, -I*(m + 1)/n)), (Integral(x
**m*cos(a + I*m*log(c*x**n)/n + I*log(c*x**n)/n), x), Eq(b, I*(m + 1)/n)),
 (b*n*x*x**m*sin(a + b*log(c*x**n))/(b**2*n**2 + m**2 + 2*m + 1) + m*x*x**
m*cos(a + b*log(c*x**n))/(b**2*n**2 + m**2 + 2*m + 1) + x*x**m*cos(a + b*l
og(c*x**n))/(b**2*n**2 + m**2 + 2*m + 1), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(70) = 140.

Time = 0.06 (sec) , antiderivative size = 313, normalized size of antiderivative = 4.47

$$\int x^m \cos(a + b\log(cx^n))\,dx$$
$$= \frac{((\cos(2\,b\log(c))\cos(b\log(c)) + \sin(2\,b\log(c))\sin(b\log(c)) + \cos(b\log(c)))m + (b\cos(b\log(c))\sin(2$$

input
```
integrate(x^m*cos(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
1/2*(((cos(2*b*log(c))*cos(b*log(c)) + sin(2*b*log(c))*sin(b*log(c)) + cos
(b*log(c)))*m + (b*cos(b*log(c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b
*log(c)) + b*sin(b*log(c)))*n + cos(2*b*log(c))*cos(b*log(c)) + sin(2*b*lo
g(c))*sin(b*log(c)) + cos(b*log(c)))*x*x^m*cos(b*log(x^n) + a) - ((cos(b*l
og(c))*sin(2*b*log(c)) - cos(2*b*log(c))*sin(b*log(c)) + sin(b*log(c)))*m
- (b*cos(2*b*log(c))*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c)) + b*c
os(b*log(c)))*n + cos(b*log(c))*sin(2*b*log(c)) - cos(2*b*log(c))*sin(b*lo
g(c)) + sin(b*log(c)))*x*x^m*sin(b*log(x^n) + a))/((cos(b*log(c))^2 + sin(
b*log(c))^2)*m^2 + (b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))^2)*n^2 + 2*(co
s(b*log(c))^2 + sin(b*log(c))^2)*m + cos(b*log(c))^2 + sin(b*log(c))^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5162 vs. $2(70) = 140$.

Time = 0.29 (sec) , antiderivative size = 5162, normalized size of antiderivative = 73.74

$$\int x^m \cos (a + b \log (cx^n)) \, dx = \text{Too large to display}$$

input
```
integrate(x^m*cos(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
1/2*(2*b*n*x*abs(x)^m*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c)
- 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sg
n(x) - 1/4*pi*m)^2*tan(1/2*a) + 2*b*n*x*abs(x)^m*e^(-1/2*pi*b*n*sgn(x) + 1
/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*lo
g(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(1/2*a) - 2*b*n*x*abs(x)
^m*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2
*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)*ta
n(1/2*a)^2 + 2*b*n*x*abs(x)^m*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*
b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*tan(1/
4*pi*m*sgn(x) - 1/4*pi*m)*tan(1/2*a)^2 + 2*b*n*x*abs(x)^m*e^(1/2*pi*b*n*sg
n(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) +
1/2*b*log(abs(c)))*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(1/2*a)^2 + 2*b*n*
x*abs(x)^m*e^(-1/2*pi*b*n*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b
)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))*tan(1/4*pi*m*sgn(x) - 1/4*p
i*m)^2*tan(1/2*a)^2 - m*x*abs(x)^m*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2
*pi*b*sgn(c) - 1/2*pi*b)*tan(1/2*b*n*log(abs(x)) + 1/2*b*log(abs(c)))^2*ta
n(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(1/2*a)^2 - m*x*abs(x)^m*e^(-1/2*pi*b*n
*sgn(x) + 1/2*pi*b*n - 1/2*pi*b*sgn(c) + 1/2*pi*b)*tan(1/2*b*n*log(abs(x))
 + 1/2*b*log(abs(c)))^2*tan(1/4*pi*m*sgn(x) - 1/4*pi*m)^2*tan(1/2*a)^2 - x
*abs(x)^m*e^(1/2*pi*b*n*sgn(x) - 1/2*pi*b*n + 1/2*pi*b*sgn(c) - 1/2*pi*...
```

Mupad [B] (verification not implemented)

Time = 20.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int x^m \cos\left(a + b\log\left(cx^n\right)\right)\,dx = \frac{x\,x^m\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}}{2\,m+2+b\,n\,2\mathrm{i}} + \frac{x\,x^m\,\mathrm{e}^{-a\,1\mathrm{i}}\,\frac{1}{{\left(c\,x^n\right)}^{b\,1\mathrm{i}}}\,1\mathrm{i}}{m\,2\mathrm{i}+2\,b\,n+2\mathrm{i}}$$

input
```
int(x^m*cos(a + b*log(c*x^n)),x)
```

output
```
(x*x^m*exp(a*1i)*(c*x^n)^(b*1i))/(2*m + b*n*2i + 2) + (x*x^m*exp(-a*1i)/(c
*x^n)^(b*1i)*1i)/(m*2i + 2*b*n + 2i)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int x^m \cos(a + b\log(cx^n))\,dx$$
$$= \frac{x^m x(\cos(\log(x^n c)\,b + a)\,m + \cos(\log(x^n c)\,b + a) + \sin(\log(x^n c)\,b + a)\,bn)}{b^2n^2 + m^2 + 2m + 1}$$

input
```
int(x^m*cos(a+b*log(c*x^n)),x)
```

output
```
(x**m*x*(cos(log(x**n*c)*b + a)*m + cos(log(x**n*c)*b + a) + sin(log(x**n*
c)*b + a)*b*n))/(b**2*n**2 + m**2 + 2*m + 1)
```

3.127 $\int x^m \cos^{\frac{3}{2}}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int x^m \cos^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \frac{2x^{1+m}\cos^{\frac{3}{2}}(a+b\log(cx^n))\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{2i+2im+3bn}{4bn},-\frac{2i+2im-bn}{4bn},-e^{2ia}(cx^n)^{2ib}\right)}{(2+2m-3ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

output
```
2*x^(1+m)*cos(a+b*ln(c*x^n))^(3/2)*hypergeom([-3/2, -1/4*(2*I+2*I*m+3*b*n)
/b/n],[-1/4*(2*I+2*I*m-b*n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(2+2*m-3*I*b
*n)/(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.64 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.57

$$\int x^m \cos^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \frac{x^{1+m}\left(6b^2n^2\left(1+e^{2ia}(cx^n)^{2ib}\right)\text{Hypergeometric2F1}\left(1,-\frac{2i+2im-3bn}{4bn},-\frac{2i+2im-5bn}{4bn},-e^{2i(a+b\log(cx^n))}\right)+(2-\cdots\right.}{(2+2m+ibn)(2+2m-3ibn)(2+2m+3ibn)\sqrt{\cdots}}$$

input
```
Integrate[x^m*Cos[a + b*Log[c*x^n]]^(3/2),x]
```

output
```
(x^(1 + m)*(6*b^2*n^2*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))*Hypergeometric2F
1[1, -1/4*(2*I + (2*I)*m - 3*b*n)/(b*n), -1/4*(2*I + (2*I)*m - 5*b*n)/(b*n
), -E^((2*I)*(a + b*Log[c*x^n]))] + (2 + 2*m + I*b*n)*(4*(1 + m)*Cos[a + b
*Log[c*x^n]]^2 + 3*b*n*Sin[2*(a + b*Log[c*x^n])])))/((2 + 2*m + I*b*n)*(2
+ 2*m - (3*I)*b*n)*(2 + 2*m + (3*I)*b*n)*Sqrt[Cos[a + b*Log[c*x^n]]])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4997, 4995, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cos^{\frac{3}{2}}(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{4997}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\cos^{\frac{3}{2}}(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{4995}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}+\frac{3ib}{2}}\cos^{\frac{3}{2}}(a+b\log(cx^n))\int (cx^n)^{-\frac{3ib}{2}+\frac{m+1}{n}-1}\left(e^{2ia}(cx^n)^{2ib}+1\right)^{3/2}d(cx^n)}{n\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4}\left(-\frac{2i(m+1)}{bn}-3\right),-\frac{2im-bn+2i}{4bn},-e^{2ia}(cx^n)^{2ib}\right)\cos^{\frac{3}{2}}(a+b\log(cx^n))}{(-3ibn+2m+2)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}}$$

input
```
Int[x^m*Cos[a + b*Log[c*x^n]]^(3/2),x]
```

output
```
(2*x^(1 + m)*Cos[a + b*Log[c*x^n]]^(3/2)*Hypergeometric2F1[-3/2, (-3 - ((2
*I)*(1 + m))/(b*n))/4, -1/4*(2*I + (2*I)*m - b*n)/(b*n), -(E^((2*I)*a)*(c*
x^n)^((2*I)*b))])/((2 + 2*m - (3*I)*b*n)*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b
))^(3/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4995
```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Simp[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

rule 4997
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cos[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^m \cos\left(a + b\ln\left(c\,x^n\right)\right)^{\frac{3}{2}} dx$$

input
```
int(x^m*cos(a+b*ln(c*x^n))^(3/2),x)
```

output
```
int(x^m*cos(a+b*ln(c*x^n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^m \cos^{\frac{3}{2}}(a + b\log(cx^n))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*cos(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \cos^{\frac{3}{2}}(a + b\log(cx^n))\,dx = \text{Timed out}$$

input
```
integrate(x**m*cos(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^m \cos^{\frac{3}{2}}(a + b\log(cx^n))\,dx = \int x^m \cos(b\log(cx^n) + a)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^m*cos(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^m*cos(b*log(c*x^n) + a)^(3/2), x)
```

Giac [F]

$$\int x^m \cos^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \int x^m \cos(b\log(cx^n)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^m*cos(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m*cos(b*log(c*x^n) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \cos^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \int x^m \cos(a+b\,\ln(c\,x^n))^{3/2}\,dx$$

input
```
int(x^m*cos(a + b*log(c*x^n))^(3/2),x)
```

output
```
int(x^m*cos(a + b*log(c*x^n))^(3/2), x)
```

Reduce [F]

$$\int x^m \cos^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \int x^m \sqrt{\cos(\log(x^n c)\,b+a)}\,\cos(\log(x^n c)\,b+a)\,dx$$

input
```
int(x^m*cos(a+b*log(c*x^n))^(3/2),x)
```

output
```
int(x**m*sqrt(cos(log(x**n*c)*b + a))*cos(log(x**n*c)*b + a),x)
```

3.128 $\int x^m \sqrt{\cos(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 129

$$\int x^m \sqrt{\cos(a+b\log(cx^n))}\,dx = \frac{2x^{1+m}\sqrt{\cos(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2i+2im+bn}{4bn},-\frac{2i+2im-3bn}{4bn},-e^{2ia}(cx^n)^{2ib}\right)}{(2+2m-ibn)\sqrt{1+e^{2ia}(cx^n)^{2ib}}}$$

output
```
2*x^(1+m)*cos(a+b*ln(c*x^n))^(1/2)*hypergeom([-1/2, -1/4*(2*I+2*I*m+b*n)/b
/n],[-1/4*(2*I+2*I*m-3*b*n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(2+2*m-I*b*n
)/(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 436 vs. 2(129) = 258.

Time = 5.94 (sec) , antiderivative size = 436, normalized size of antiderivative = 3.38

$$\int x^m \sqrt{\cos(a+b\log(cx^n))}\,dx =$$
$$-\frac{2be^{ia}nx^{1+m}(cx^n)^{ib}\sqrt{2+2e^{2ia}(cx^n)^{2ib}}\Big((2i+2im+bn)x^{2ibn}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{i\left(1+m+\frac{3ibn}{2}\right)}{2bn},-\right.}{(2+2m-ibn)(2+2m+3ibn)\sqrt{e^{-ia}(cx^n)^{-ib}+}}$$
$$+\frac{2x^{1+m}\sqrt{\cos(a+b\log(cx^n))}\cos(a-bn\log(x)+b\log(cx^n))}{2(1+m)\cos(a-bn\log(x)+b\log(cx^n))-bn\sin(a-bn\log(x)+b\log(cx^n))}$$

input
```
Integrate[x^m*Sqrt[Cos[a + b*Log[c*x^n]]],x]
```

output
```
(-2*b*E^(I*a)*n*x^(1 + m)*(c*x^n)^(I*b)*Sqrt[2 + 2*E^((2*I)*a)*(c*x^n)^((2
*I)*b)]*((2*I + (2*I)*m + b*n)*x^((2*I)*b*n)*Hypergeometric2F1[1/2, ((-1/2
*I)*(1 + m + ((3*I)/2)*b*n))/(b*n), -1/4*(2*I + (2*I)*m - 7*b*n)/(b*n), -(
E^((2*I)*a)*(c*x^n)^((2*I)*b))] + (-2*I - (2*I)*m + 3*b*n)*Hypergeometric2
F1[1/2, -1/4*(2*I + (2*I)*m + b*n)/(b*n), -1/4*(2*I + (2*I)*m - 3*b*n)/(b*
n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))]))/((2 + 2*m - I*b*n)*(2 + 2*m + (3*I
)*b*n)*Sqrt[1/(E^(I*a)*(c*x^n)^(I*b)) + E^(I*a)*(c*x^n)^(I*b)]*((2 + 2*m -
 I*b*n)*x^((2*I)*b*n) + E^((2*I)*a)*(2 + 2*m + I*b*n)*(c*x^n)^((2*I)*b)))
+ (2*x^(1 + m)*Sqrt[Cos[a + b*Log[c*x^n]]]*Cos[a - b*n*Log[x] + b*Log[c*x^
n]])/(2*(1 + m)*Cos[a - b*n*Log[x] + b*Log[c*x^n]] - b*n*Sin[a - b*n*Log[x
] + b*Log[c*x^n]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4997, 4995, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{\cos(a+b\log(cx^n))}\,dx$$

$$\Big\downarrow \; 4997$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\sqrt{\cos (a+b\log (cx^n))}d(cx^n)}{n}$$

$$\Big\downarrow \text{4995}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}+\frac{ib}{2}}\sqrt{\cos (a+b\log (cx^n))}\int (cx^n)^{-\frac{ib}{2}+\frac{m+1}{n}-1}\sqrt{e^{2ia}(cx^n)^{2ib}+1}d(cx^n)}{n\sqrt{1+e^{2ia}(cx^n)^{2ib}}}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^{m+1}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-\frac{2i(m+1)}{bn}-1\right),-\frac{2im-3bn+2i}{4bn},-e^{2ia}(cx^n)^{2ib}\right)\sqrt{\cos (a+b\log (cx^n))}}{(-ibn+2m+2)\sqrt{1+e^{2ia}(cx^n)^{2ib}}}$$

input

```
Int[x^m*Sqrt[Cos[a + b*Log[c*x^n]]],x]
```

output

```
(2*x^(1 + m)*Sqrt[Cos[a + b*Log[c*x^n]]]*Hypergeometric2F1[-1/2, (-1 - ((2
*I)*(1 + m))/(b*n))/4, -1/4*(2*I + (2*I)*m - 3*b*n)/(b*n), -(E^((2*I)*a)*(
c*x^n)^((2*I)*b))])/((2 + 2*m - I*b*n)*Sqrt[1 + E^((2*I)*a)*(c*x^n)^((2*I)
*b)])
```

Defintions of rubi rules used

rule 888

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4995

```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Simp[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 4997
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cos[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^m \sqrt{\cos(a + b \ln(c x^n))} dx$$

input
```
int(x^m*cos(a+b*ln(c*x^n))^(1/2),x)
```

output
```
int(x^m*cos(a+b*ln(c*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^m \sqrt{\cos(a + b \log(cx^n))}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*cos(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^m \sqrt{\cos(a + b \log(cx^n))}\, dx = \int x^m \sqrt{\cos(a + b \log(cx^n))}\, dx$$

input
```
integrate(x**m*cos(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(x**m*sqrt(cos(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int x^m \sqrt{\cos(a+b\log(cx^n))}\,dx = \int x^m \sqrt{\cos(b\log(cx^n)+a)}\,dx$$

input
```
integrate(x^m*cos(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m*sqrt(cos(b*log(c*x^n) + a)), x)
```

Giac [F]

$$\int x^m \sqrt{\cos(a+b\log(cx^n))}\,dx = \int x^m \sqrt{\cos(b\log(cx^n)+a)}\,dx$$

input
```
integrate(x^m*cos(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m*sqrt(cos(b*log(c*x^n) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sqrt{\cos(a+b\log(cx^n))}\,dx = \int x^m \sqrt{\cos(a+b\ln(c\,x^n))}\,dx$$

input
```
int(x^m*cos(a + b*log(c*x^n))^(1/2),x)
```

output
```
int(x^m*cos(a + b*log(c*x^n))^(1/2), x)
```

Reduce [F]

$$\int x^m \sqrt{\cos(a+b\log(cx^n))}\,dx$$
$$= \frac{2x^m\sqrt{\cos(\log(x^n c)\,b+a)}\,x + \left(\int \frac{x^m\sqrt{\cos(\log(x^n c)b+a)}\,\sin(\log(x^n c)b+a)}{\cos(\log(x^n c)b+a)}dx\right)bn}{2m+2}$$

input
```
int(x^m*cos(a+b*log(c*x^n))^(1/2),x)
```

output
```
(2*x**m*sqrt(cos(log(x**n*c)*b + a))*x + int((x**m*sqrt(cos(log(x**n*c)*b
+ a))*sin(log(x**n*c)*b + a))/cos(log(x**n*c)*b + a),x)*b*n)/(2*(m + 1))
```

3.129 $\int \frac{x^m}{\sqrt{\cos(a+b\log(cx^n))}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{x^m}{\sqrt{\cos(a+b\log(cx^n))}}\,dx = \frac{2x^{1+m}\sqrt{1+e^{2ia}(cx^n)^{2ib}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{2i+2im-bn}{4bn},-\frac{2i+2im-5bn}{4bn},-e^{2ia}(cx^n)^{2ib}\right)}{(2+2m+ibn)\sqrt{\cos(a+b\log(cx^n))}}$$

output
```
2*x^(1+m)*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)*hypergeom([1/2, -1/4*(2*I+2
*I*m-b*n)/b/n],[-1/4*(2*I+2*I*m-5*b*n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(
2+2*m+I*b*n)/cos(a+b*ln(c*x^n))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{\cos(a+b\log(cx^n))}}\,dx = \frac{2\left(1+e^{2i(a+b\log(cx^n))}\right)x^{1+m}\,\text{Hypergeometric2F1}\left(1,-\frac{2i+2im-3bn}{4bn},-\frac{2i+2im-5bn}{4bn},-e^{2i(a+b\log(cx^n))}\right)}{(2+2m+ibn)\sqrt{\cos(a+b\log(cx^n))}}$$

input
```
Integrate[x^m/Sqrt[Cos[a + b*Log[c*x^n]]],x]
```

output
```
(2*(1 + E^((2*I)*(a + b*Log[c*x^n])))*x^(1 + m)*Hypergeometric2F1[1, -1/4*
(2*I + (2*I)*m - 3*b*n)/(b*n), -1/4*(2*I + (2*I)*m - 5*b*n)/(b*n), -E^((2*
I)*(a + b*Log[c*x^n]))])/((2 + 2*m + I*b*n)*Sqrt[Cos[a + b*Log[c*x^n]]])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4997, 4995, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{\cos(a+b\log(cx^n))}}dx$$

$$\Big\downarrow \text{ 4997}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int \frac{(cx^n)^{\frac{m+1}{n}-1}}{\sqrt{\cos(a+b\log(cx^n))}}d(cx^n)}{n}$$

$$\Big\downarrow \text{ 4995}$$

$$\frac{x^{m+1}\sqrt{1+e^{2ia}(cx^n)^{2ib}}(cx^n)^{-\frac{m+1}{n}-\frac{ib}{2}}\int \frac{(cx^n)^{\frac{ib}{2}+\frac{m+1}{n}-1}}{\sqrt{e^{2ia}(cx^n)^{2ib}+1}}d(cx^n)}{n\sqrt{\cos(a+b\log(cx^n))}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{2x^{m+1}\sqrt{1+e^{2ia}(cx^n)^{2ib}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{2im-bn+2i}{4bn},-\frac{2im-5bn+2i}{4bn},-e^{2ia}(cx^n)^{2ib}\right)}{(ibn+2m+2)\sqrt{\cos(a+b\log(cx^n))}}$$

input
```
Int[x^m/Sqrt[Cos[a + b*Log[c*x^n]]],x]
```

output
```
(2*x^(1 + m)*Sqrt[1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Hypergeometric2F1[1/2
, -1/4*(2*I + (2*I)*m - b*n)/(b*n), -1/4*(2*I + (2*I)*m - 5*b*n)/(b*n), -(
E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 + 2*m + I*b*n)*Sqrt[Cos[a + b*Log[c*x
^n]]])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4995
```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Simp[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 4997
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cos[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{x^m}{\sqrt{\cos \left(a+b\ln \left(c\,x^n\right)\right)}}dx$$

input
```
int(x^m/cos(a+b*ln(c*x^n))^(1/2),x)
```

output
```
int(x^m/cos(a+b*ln(c*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^m}{\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/cos(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^m}{\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{x^m}{\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}}\,dx$$

input
```
integrate(x**m/cos(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(x**m/sqrt(cos(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \frac{x^m}{\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{x^m}{\sqrt{\cos\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

input
```
integrate(x^m/cos(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/sqrt(cos(b*log(c*x^n) + a)), x)
```

Giac [F]

$$\int \frac{x^m}{\sqrt{\cos (a+b\log (cx^n))}}\, dx = \int \frac{x^m}{\sqrt{\cos (b\log (cx^n)+a)}}\, dx$$

input
```
integrate(x^m/cos(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/sqrt(cos(b*log(c*x^n) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{\cos (a+b\log (cx^n))}}\, dx = \int \frac{x^m}{\sqrt{\cos \left(a+b\,\ln \left(c\,x^n\right)\right)}}\, dx$$

input
```
int(x^m/cos(a + b*log(c*x^n))^(1/2),x)
```

output
```
int(x^m/cos(a + b*log(c*x^n))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^m}{\sqrt{\cos (a+b\log (cx^n))}}\, dx = \int \frac{x^m \sqrt{\cos (\log (x^n c)\, b+a)}}{\cos (\log (x^n c)\, b+a)}\, dx$$

input
```
int(x^m/cos(a+b*log(c*x^n))^(1/2),x)
```

output
```
int((x**m*sqrt(cos(log(x**n*c)*b + a)))/cos(log(x**n*c)*b + a),x)
```

3.130 $\int \frac{x^m}{\cos^{\frac{3}{2}}(a+b\log(cx^n))} dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{x^m}{\cos^{\frac{3}{2}}(a+b\log(cx^n))} dx = \frac{2x^{1+m}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{2i+2im-3bn}{4bn},-\frac{2i+2im-7bn}{4bn},-e^{2ia}(cx^n)^{2ib}\right)}{(2+2m+3ibn)\cos^{\frac{3}{2}}(a+b\log(cx^n))}$$

output
```
2*x^(1+m)*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)*hypergeom([3/2, -1/4*(2*I+2
*I*m-3*b*n)/b/n],[-1/4*(2*I+2*I*m-7*b*n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))
/(2+2*m+3*I*b*n)/cos(a+b*ln(c*x^n))^(3/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 487 vs. 2(130) = 260.

Time = 3.67 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.75

$$\int \frac{x^m}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = -\frac{x^{1+m-ibn}\left((4+8m+4m^2+b^2n^2)\,x^{2ibn}\sqrt{2+2e^{2ia}(cx^n)^{2ib}}\sqrt{\cos(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\right.}{}$$

input
```
Integrate[x^m/Cos[a + b*Log[c*x^n]]^(3/2),x]
```

output
```
-((x^(1 + m - I*b*n)*((4 + 8*m + 4*m^2 + b^2*n^2)*x^((2*I)*b*n)*Sqrt[2 + 2
*E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[Cos[a + b*Log[c*x^n]]]*Hypergeometric
2F1[1/2, ((-1/2*I)*(1 + m + ((3*I)/2)*b*n))/(b*n), -1/4*(2*I + (2*I)*m - 7
*b*n)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))] + (-2*I - (2*I)*m + 3*b*n)*
((-2*I - (2*I)*m + b*n)*Sqrt[2 + 2*E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[Cos
[a + b*Log[c*x^n]]]*Hypergeometric2F1[1/2, -1/4*(2*I + (2*I)*m + b*n)/(b*n
), -1/4*(2*I + (2*I)*m - 3*b*n)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))] -
 2*x^(I*b*n)*Sqrt[1/(E^(I*a)*(c*x^n)^(I*b)) + E^(I*a)*(c*x^n)^(I*b)]*(b*n*
Cos[b*n*Log[x]] - 2*(1 + m)*Sin[b*n*Log[x]]))))/(b*n*(-2*I - (2*I)*m + 3*b
*n)*Sqrt[1/(E^(I*a)*(c*x^n)^(I*b)) + E^(I*a)*(c*x^n)^(I*b)]*Sqrt[Cos[a + b
*Log[c*x^n]]]*(-2*(1 + m)*Cos[a - b*n*Log[x] + b*Log[c*x^n]] + b*n*Sin[a -
 b*n*Log[x] + b*Log[c*x^n]])))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4997, 4995, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{4997}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int\frac{(cx^n)^{\frac{m+1}{n}-1}}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}d(cx^n)}{n}$$

$$\Big\downarrow \text{4995}$$

$$\frac{x^{m+1}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}(cx^n)^{-\frac{m+1}{n}-\frac{3ib}{2}}\int\frac{(cx^n)^{\frac{3ib}{2}+\frac{m+1}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^{3/2}}d(cx^n)}{n\cos^{\frac{3}{2}}(a+b\log(cx^n))}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^{m+1}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i(m+1)}{bn}\right),-\frac{2im-7bn+2i}{4bn},-e^{2ia}(cx^n)^{2ib}\right)}{(3ibn+2m+2)\cos^{\frac{3}{2}}(a+b\log(cx^n))}$$

input
```
Int[x^m/Cos[a + b*Log[c*x^n]]^(3/2),x]
```

output
```
(2*x^(1 + m)*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2)*Hypergeometric2F1[3
/2, (3 - ((2*I)*(1 + m))/(b*n))/4, -1/4*(2*I + (2*I)*m - 7*b*n)/(b*n), -(E
^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 + 2*m + (3*I)*b*n)*Cos[a + b*Log[c*x^n
]]^(3/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4995
```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Simp[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 4997
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cos[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{x^m}{\cos\left(a+b\ln\left(c\,x^n\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m/cos(a+b*ln(c*x^n))^(3/2),x)
```

output
```
int(x^m/cos(a+b*ln(c*x^n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^m}{\cos^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/cos(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^m}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{x^m}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$

input
```
integrate(x**m/cos(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Integral(x**m/cos(a + b*log(c*x**n))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^m}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{x^m}{\cos(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/cos(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^m/cos(b*log(c*x^n) + a)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^m}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(x^m/cos(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{x^m}{\cos(a+b\,\ln(c\,x^n))^{3/2}}\,dx$$

input
```
int(x^m/cos(a + b*log(c*x^n))^(3/2),x)
```

output
```
int(x^m/cos(a + b*log(c*x^n))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^m}{\cos^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{x^m\sqrt{\cos(\log(x^nc)\,b+a)}}{\cos(\log(x^nc)\,b+a)^2}dx$$

input
```
int(x^m/cos(a+b*log(c*x^n))^(3/2),x)
```

output
```
int((x**m*sqrt(cos(log(x**n*c)*b + a)))/cos(log(x**n*c)*b + a)**2,x)
```

3.131 $\int \frac{x^m}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{x^m}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{2x^{1+m}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{2i+2im-5bn}{4bn},-\frac{2i+2im-9bn}{4bn},-e^{2ia}(cx^n)^{2ib}\right)}{(2+2m+5ibn)\cos^{\frac{5}{2}}(a+b\log(cx^n))}$$

output
```
2*x^(1+m)*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(5/2)*hypergeom([5/2, -1/4*(2*I+2
*I*m-5*b*n)/b/n],[-1/4*(2*I+2*I*m-9*b*n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))
/(2+2*m+5*I*b*n)/cos(a+b*ln(c*x^n))^(5/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.84 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.58

$$\int \frac{x^m}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{2x^{1+m}\left((2+2m-ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right)\cos(a+b\log(cx^n))\,\text{Hypergeometric2F1}\left(1,-\frac{2i+2im-3bn}{4bn},-\frac{2i+\cdots}{}\right.}{}$$

input
```
Integrate[x^m/Cos[a + b*Log[c*x^n]]^(5/2),x]
```

output
```
(2*x^(1 + m)*((2 + 2*m - I*b*n)*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))*Cos[a
+ b*Log[c*x^n]]*Hypergeometric2F1[1, -1/4*(2*I + (2*I)*m - 3*b*n)/(b*n), -
1/4*(2*I + (2*I)*m - 5*b*n)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] + b*n*Se
c[a - b*n*Log[x] + b*Log[c*x^n]]*Sin[b*n*Log[x]] + Cos[a + b*Log[c*x^n]]*(
-2*(1 + m) + b*n*Tan[a - b*n*Log[x] + b*Log[c*x^n]])))/(3*b^2*n^2*Cos[a +
b*Log[c*x^n]]^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4997, 4995, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{4997}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int \frac{(cx^n)^{\frac{m+1}{n}-1}}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}d(cx^n)}{n}$$

$$\Big\downarrow \text{4995}$$

$$\frac{x^{m+1}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}(cx^n)^{-\frac{m+1}{n}-\frac{5ib}{2}}\int \frac{(cx^n)^{\frac{5ib}{2}+\frac{m+1}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^{5/2}}d(cx^n)}{n\cos^{\frac{5}{2}}(a+b\log(cx^n))}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^{m+1}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i(m+1)}{bn}\right),-\frac{2im-9bn+2i}{4bn},-e^{2ia}(cx^n)^{2ib}\right)}{(5ibn+2m+2)\cos^{\frac{5}{2}}(a+b\log(cx^n))}$$

input
```
Int[x^m/Cos[a + b*Log[c*x^n]]^(5/2),x]
```

output
```
(2*x^(1 + m)*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^(5/2)*Hypergeometric2F1[5
/2, (5 - ((2*I)*(1 + m))/(b*n))/4, -1/4*(2*I + (2*I)*m - 9*b*n)/(b*n), -(E
^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 + 2*m + (5*I)*b*n)*Cos[a + b*Log[c*x^n
]]^(5/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4995
```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Simp[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

rule 4997
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cos[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{x^m}{\cos \left(a+b\ln \left(c\,x^n\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(x^m/cos(a+b*ln(c*x^n))^(5/2),x)
```

output
```
int(x^m/cos(a+b*ln(c*x^n))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^m}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/cos(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(x**m/cos(a+b*ln(c*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^m}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{x^m}{\cos(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/cos(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^m/cos(b*log(c*x^n) + a)^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^m}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(x^m/cos(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{x^m}{\cos(a+b\,\ln(c\,x^n))^{5/2}}\,dx$$

input
```
int(x^m/cos(a + b*log(c*x^n))^(5/2),x)
```

output
```
int(x^m/cos(a + b*log(c*x^n))^(5/2), x)
```

Reduce [F]

$$\int \frac{x^m}{\cos^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{x^m\sqrt{\cos(\log(x^nc)\,b+a)}}{\cos(\log(x^nc)\,b+a)^3}dx$$

input
```
int(x^m/cos(a+b*log(c*x^n))^(5/2),x)
```

output
```
int((x**m*sqrt(cos(log(x**n*c)*b + a)))/cos(log(x**n*c)*b + a)**3,x)
```

3.132 $\int (ex)^m \cos^p(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 144

$$\int (ex)^m \cos^p(d(a+b\log(cx^n)))\,dx$$
$$= \frac{(ex)^{1+m}\left(1+e^{2iad}(cx^n)^{2ibd}\right)^{-p}\cos^p(d(a+b\log(cx^n)))\,\text{Hypergeometric2F1}\left(-p,-\frac{i+im+bdnp}{2bdn},\frac{1}{2}\left(2-\frac{i(1+\cdots}{bd\cdots}\right.\right.}{e(1+m-ibdnp)}$$

output
```
(e*x)^(1+m)*cos(d*(a+b*ln(c*x^n)))^p*hypergeom([-p, -1/2*(I+I*m+b*d*n*p)/b
/d/n],[1-1/2*I*(1+m)/b/d/n-1/2*p],-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/e/(1+m-
I*b*d*n*p)/((1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 1.38 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.18

$$\int (ex)^m \cos^p(d(a+b\log(cx^n)))\,dx$$
$$= \frac{x(ex)^m\left(e^{-iad}(cx^n)^{-ibd}+e^{iad}(cx^n)^{ibd}\right)^p\left(2+2e^{-2iad}(cx^n)^{-2ibd}\right)^{-p}\text{Hypergeometric2F1}\left(-p,\frac{i(1+m+ibdnp)}{2bdn}\cdots\right.}{1+m+ibdnp}$$

input
```
Integrate[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^p,x]
```

output
```
(x*(e*x)^m*(1/(E^(I*a*d)*(c*x^n)^(I*b*d)) + E^(I*a*d)*(c*x^n)^(I*b*d))^p*H
ypergeometric2F1[-p, ((I/2)*(1 + m + I*b*d*n*p))/(b*d*n), 1 + ((I/2)*(1 +
m))/(b*d*n) - p/2, -(1/(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))])/((1 + m + I*
b*d*n*p)*(2 + 2/(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))^p)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.26, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4997, 4995, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \cos^p(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{4997}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\cos^p(d(a+b\log(cx^n)))\,d(cx^n)}{en}$$

$$\Big\downarrow \text{4995}$$

$$\frac{(ex)^{m+1}\left(1+e^{2iad}(cx^n)^{2ibd}\right)^{-p}(cx^n)^{-\frac{m+1}{n}+ibdp}\cos^p(d(a+b\log(cx^n)))\int (cx^n)^{\frac{m+1}{n}-ibdp-1}\left(e^{2iad}(cx^n)^{2ibd}+1\right)^{p}}{en}$$

$$\Big\downarrow \text{888}$$

$$\frac{(ex)^{m+1}\left(1+e^{2iad}(cx^n)^{2ibd}\right)^{-p}(cx^n)^{\frac{-ibdnp+m+1}{n}+ibdp-\frac{m+1}{n}}\text{Hypergeometric2F1}\left(-p,-\frac{im+bdnp+i}{2bdn},\frac{1}{2}\left(-\frac{i(m+1)}{bdn}-p\right.\right.}{e(-ibdnp+m+1)}$$

input
```
Int[(e*x)^m*Cos[d*(a + b*Log[c*x^n])]^p,x]
```

output
```
((e*x)^(1 + m)*(c*x^n)^(-((1 + m)/n) + I*b*d*p + (1 + m - I*b*d*n*p)/n)*Co
s[d*(a + b*Log[c*x^n])]^p*Hypergeometric2F1[-p, -1/2*(I + I*m + b*d*n*p)/(
b*d*n), (2 - (I*(1 + m))/(b*d*n) - p)/2, -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*
d))])/(e*(1 + m - I*b*d*n*p)*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4995
```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Simp[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

rule 4997
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cos[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int (ex)^m \cos \left(d(a+b\ln \left(c\,x^n\right))\right)^p dx$$

input
```
int((e*x)^m*cos(d*(a+b*ln(c*x^n)))^p,x)
```

output
```
int((e*x)^m*cos(d*(a+b*ln(c*x^n)))^p,x)
```

Fricas [F]

$$\int (ex)^m \cos^p\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int (ex)^m \cos\left((b\log\left(cx^n\right)+a)d\right)^p\,dx$$

input
```
integrate((e*x)^m*cos(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^m*cos(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \cos^p\left(d(a+b\log\left(cx^n\right))\right)\,dx=\text{Timed out}$$

input
```
integrate((e*x)**m*cos(d*(a+b*ln(c*x**n)))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m \cos^p\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int (ex)^m \cos\left((b\log\left(cx^n\right)+a)d\right)^p\,dx$$

input
```
integrate((e*x)^m*cos(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*cos((b*log(c*x^n) + a)*d)^p, x)
```

Giac [F]

$$\int (ex)^m \cos^p\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int (ex)^m \cos\left((b\log\left(cx^n\right)+a)d\right)^p\,dx$$

input
```
integrate((e*x)^m*cos(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^m*cos((b*log(c*x^n) + a)*d)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \cos^p(d(a+b\log(cx^n)))\,dx=\int \cos\left(d\left(a+b\,\ln\left(c\,x^n\right)\right)\right)^p\,(e\,x)^m\,dx$$

input
```
int(cos(d*(a + b*log(c*x^n)))^p*(e*x)^m,x)
```

output
```
int(cos(d*(a + b*log(c*x^n)))^p*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \cos^p(d(a+b\log(cx^n)))\,dx$$
$$=\frac{e^m\left(x^m\cos(\log(x^nc)\,bd+ad)^p x+\left(\int\frac{x^m\cos(\log(x^nc)bd+ad)^p\sin(\log(x^nc)bd+ad)}{\cos(\log(x^nc)bd+ad)}dx\right)bdnp\right)}{m+1}$$

input
```
int((e*x)^m*cos(d*(a+b*log(c*x^n)))^p,x)
```

output
```
(e**m*(x**m*cos(log(x**n*c)*b*d + a*d)**p*x + int((x**m*cos(log(x**n*c)*b*
d + a*d)**p*sin(log(x**n*c)*b*d + a*d))/cos(log(x**n*c)*b*d + a*d),x)*b*d*
n*p))/(m + 1)
```

3.133 $\int x \cos^p(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 114

$$\int x \cos^p(a+b\log(cx^n))\,dx = \frac{x^2\left(1+e^{2ia}(cx^n)^{2ib}\right)^{-p}\cos^p(a+b\log(cx^n))\,\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-\frac{2i}{bn}-p\right),-p,\frac{1}{2}\left(2-\frac{2i}{bn}-p\right),-e^{2i}\right.}{2-ibnp}$$

output
```
x^2*cos(a+b*ln(c*x^n))^p*hypergeom([-p, -I/b/n-1/2*p],[1-I/b/n-1/2*p],-exp
(2*I*a)*(c*x^n)^(2*I*b))/(2-I*b*n*p)/((1+exp(2*I*a)*(c*x^n)^(2*I*b))^p)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.24

$$\int x \cos^p(a+b\log(cx^n))\,dx = \frac{ix^2\left(e^{-ia}(cx^n)^{-ib}+e^{ia}(cx^n)^{ib}\right)^p\left(2+2e^{2ia}(cx^n)^{2ib}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{i}{bn}-\frac{p}{2},-p,1-\frac{i}{bn}-\frac{p}{2},-e\right.}{2i+bnp}$$

input
```
Integrate[x*Cos[a + b*Log[c*x^n]]^p,x]
```

output
```
(I*x^2*(1/(E^(I*a)*(c*x^n)^(I*b)) + E^(I*a)*(c*x^n)^(I*b))^p*Hypergeometri
c2F1[(-I)/(b*n) - p/2, -p, 1 - I/(b*n) - p/2, -(E^((2*I)*a)*(c*x^n)^((2*I)
*b))])/((2*I + b*n*p)*(2 + 2*E^((2*I)*a)*(c*x^n)^((2*I)*b))^p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4997, 4995, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos^p(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{4997}$$

$$\frac{x^2(cx^n)^{-2/n}\int (cx^n)^{\frac{2}{n}-1}\cos^p(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{4995}$$

$$\frac{x^2(cx^n)^{-\frac{2}{n}+ibp}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{-p}\cos^p(a+b\log(cx^n))\int (cx^n)^{-ibp+\frac{2}{n}-1}\left(e^{2ia}(cx^n)^{2ib}+1\right)^p d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2\left(1+e^{2ia}(cx^n)^{2ib}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}\left(-p-\frac{2i}{bn}\right),-p,\frac{1}{2}\left(-p-\frac{2i}{bn}+2\right),-e^{2ia}(cx^n)^{2ib}\right)\cos^p(a+b\log(c}{2-ibnp}$$

input
```
Int[x*Cos[a + b*Log[c*x^n]]^p,x]
```

output
```
(x^2*Cos[a + b*Log[c*x^n]]^p*Hypergeometric2F1[((-2*I)/(b*n) - p)/2, -p, (
2 - (2*I)/(b*n) - p)/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 - I*b*n*p)*
(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4995
```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Simp[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 4997
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cos[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x\cos\left(a+b\ln\left(c\,x^{n}\right)\right)^{p}dx$$

input
```
int(x*cos(a+b*ln(c*x^n))^p,x)
```

output
```
int(x*cos(a+b*ln(c*x^n))^p,x)
```

Fricas [F]

$$\int x\cos^{p}\left(a+b\log\left(cx^{n}\right)\right)\,dx=\int x\cos\left(b\log\left(cx^{n}\right)+a\right)^{p}\,dx$$

input
```
integrate(x*cos(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

output
```
integral(x*cos(b*log(c*x^n) + a)^p, x)
```

Sympy [F]

$$\int x\cos^p(a+b\log(cx^n))\,dx=\int x\cos^p(a+b\log(cx^n))\,dx$$

input
```
integrate(x*cos(a+b*ln(c*x**n))**p,x)
```

output
```
Integral(x*cos(a + b*log(c*x**n))**p, x)
```

Maxima [F]

$$\int x\cos^p(a+b\log(cx^n))\,dx=\int x\cos(b\log(cx^n)+a)^p\,dx$$

input
```
integrate(x*cos(a+b*log(c*x^n))^p,x, algorithm="maxima")
```

output
```
integrate(x*cos(b*log(c*x^n) + a)^p, x)
```

Giac [F]

$$\int x\cos^p(a+b\log(cx^n))\,dx=\int x\cos(b\log(cx^n)+a)^p\,dx$$

input
```
integrate(x*cos(a+b*log(c*x^n))^p,x, algorithm="giac")
```

output
```
integrate(x*cos(b*log(c*x^n) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cos^p(a+b\log(cx^n))\,dx=\int x\cos\left(a+b\ln\left(c\,x^n\right)\right)^p\,dx$$

input
```
int(x*cos(a + b*log(c*x^n))^p,x)
```

output
```
int(x*cos(a + b*log(c*x^n))^p, x)
```

Reduce [F]

$$\int x\cos^p(a+b\log(cx^n))\,dx=\frac{\cos\left(\log(x^n c)\,b+a\right)^p x^2}{2}+\frac{\left(\int\frac{\cos(\log(x^n c)b+a)^p\sin(\log(x^n c)b+a)x}{\cos(\log(x^n c)b+a)}dx\right)bnp}{2}$$

input
```
int(x*cos(a+b*log(c*x^n))^p,x)
```

output
```
(cos(log(x**n*c)*b + a)**p*x**2 + int((cos(log(x**n*c)*b + a)**p*sin(log(x
**n*c)*b + a)*x)/cos(log(x**n*c)*b + a),x)*b*n*p)/2
```

3.134 $\int \cos^p(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 112

$$\int \cos^p(a+b\log(cx^n))\,dx = \frac{x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{-p}\cos^p(a+b\log(cx^n))\,\text{Hypergeometric2F1}\left(-p,-\frac{i+bnp}{2bn},\frac{1}{2}\left(2-\frac{i}{bn}-p\right),-e^{2ia}(cx^n)\right.}{1-ibnp}$$

output
```
x*cos(a+b*ln(c*x^n))^p*hypergeom([-p, -1/2*(I+b*n*p)/b/n],[1-1/2*I/b/n-1/2
*p],-exp(2*I*a)*(c*x^n)^(2*I*b))/(1-I*b*n*p)/((1+exp(2*I*a)*(c*x^n)^(2*I*b
))^p)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.28

$$\int \cos^p(a+b\log(cx^n))\,dx = \frac{ix\left(e^{-ia}(cx^n)^{-ib}+e^{ia}(cx^n)^{ib}\right)^p\left(2+2e^{2ia}(cx^n)^{2ib}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-\frac{i+bnp}{2bn},1-\frac{i}{2bn}-\frac{p}{2},-e^{2\right.}{i+bnp}$$

input
```
Integrate[Cos[a + b*Log[c*x^n]]^p,x]
```

output
```
(I*x*(1/(E^(I*a)*(c*x^n)^(I*b)) + E^(I*a)*(c*x^n)^(I*b))^p*Hypergeometric2
F1[-p, -1/2*(I + b*n*p)/(b*n), 1 - (I/2)/(b*n) - p/2, -(E^((2*I)*a)*(c*x^n
)^((2*I)*b))])/((I + b*n*p)*(2 + 2*E^((2*I)*a)*(c*x^n)^((2*I)*b))^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4987, 4995, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^p(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{4987}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\cos^p(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{4995}$$

$$\frac{x(cx^n)^{-\frac{1}{n}+ibp}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{-p}\cos^p(a+b\log(cx^n))\int (cx^n)^{-ibp+\frac{1}{n}-1}\left(e^{2ia}(cx^n)^{2ib}+1\right)^p d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-\frac{bnp+i}{2bn},\frac{1}{2}\left(-p-\frac{i}{bn}+2\right),-e^{2ia}(cx^n)^{2ib}\right)\cos^p(a+b\log(cx^n))}{1-ibnp}$$

input
```
Int[Cos[a + b*Log[c*x^n]]^p,x]
```

output
```
(x*Cos[a + b*Log[c*x^n]]^p*Hypergeometric2F1[-p, -1/2*(I + b*n*p)/(b*n), (
2 - I/(b*n) - p)/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((1 - I*b*n*p)*(1 +
 E^((2*I)*a)*(c*x^n)^((2*I)*b))^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 4987
```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cos[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 4995
```
Int[Cos[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol] :
> Simp[Cos[d*(a + b*Log[x])]^p*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p
)   Int[(e*x)^m*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p)), x], x] /; Fr
eeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \cos \left(a+b\ln \left(c\,x^{n}\right)\right)^{p}dx$$

input
```
int(cos(a+b*ln(c*x^n))^p,x)
```

output
```
int(cos(a+b*ln(c*x^n))^p,x)
```

Fricas [F]

$$\int \cos^p\left(a+b\log\left(cx^n\right)\right)\,dx=\int \cos\left(b\log\left(cx^n\right)+a\right)^p\,dx$$

input
```
integrate(cos(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

output
```
integral(cos(b*log(c*x^n) + a)^p, x)
```

Sympy [F]

$$\int \cos^p(a + b\log(cx^n))\,dx = \int \cos^p(a + b\log(cx^n))\,dx$$

input
```
integrate(cos(a+b*ln(c*x**n))**p,x)
```

output
```
Integral(cos(a + b*log(c*x**n))**p, x)
```

Maxima [F]

$$\int \cos^p(a + b\log(cx^n))\,dx = \int \cos(b\log(cx^n) + a)^p\,dx$$

input
```
integrate(cos(a+b*log(c*x^n))^p,x, algorithm="maxima")
```

output
```
integrate(cos(b*log(c*x^n) + a)^p, x)
```

Giac [F]

$$\int \cos^p(a + b\log(cx^n))\,dx = \int \cos(b\log(cx^n) + a)^p\,dx$$

input
```
integrate(cos(a+b*log(c*x^n))^p,x, algorithm="giac")
```

output
```
integrate(cos(b*log(c*x^n) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^p(a+b\log(cx^n))\,dx = \int \cos\left(a+b\,\ln\left(c\,x^n\right)\right)^p\,dx$$

input
```
int(cos(a + b*log(c*x^n))^p,x)
```

output
```
int(cos(a + b*log(c*x^n))^p, x)
```

Reduce [F]

$$\int \cos^p(a+b\log(cx^n))\,dx = \cos(\log(x^n c)\,b+a)^p x + \left(\int \frac{\cos(\log(x^n c)\,b+a)^p \sin(\log(x^n c)\,b+a)}{\cos(\log(x^n c)\,b+a)}dx\right)bnp$$

input
```
int(cos(a+b*log(c*x^n))^p,x)
```

output
```
cos(log(x**n*c)*b + a)**p*x + int((cos(log(x**n*c)*b + a)**p*sin(log(x**n*
c)*b + a))/cos(log(x**n*c)*b + a),x)*b*n*p
```

3.135 $\int x^3 \tan(a + i \log(x)) \, dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int x^3 \tan(a + i \log(x)) \, dx = -ie^{2ia}x^2 + \frac{ix^4}{4} + ie^{4ia} \log\left(e^{2ia} + x^2\right)$$

output
```
-I*exp(2*I*a)*x^2+1/4*I*x^4+I*exp(4*I*a)*ln(exp(2*I*a)+x^2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 132 vs. 2(47) = 94.

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.81

$$\begin{aligned}\int x^3 \tan(a + i \log(x)) \, dx = & \frac{ix^4}{4} - ix^2 \cos(2a) + \arctan\left(\frac{(1 + x^2) \cos(a)}{\sin(a) - x^2 \sin(a)}\right) \cos(4a) \\ & + \frac{1}{2} i \cos(4a) \log\left(1 + x^4 + 2x^2 \cos(2a)\right) \\ & + x^2 \sin(2a) + i \arctan\left(\frac{(1 + x^2) \cos(a)}{\sin(a) - x^2 \sin(a)}\right) \sin(4a) \\ & - \frac{1}{2} \log\left(1 + x^4 + 2x^2 \cos(2a)\right) \sin(4a)\end{aligned}$$

input
```
Integrate[x^3*Tan[a + I*Log[x]],x]
```

output
```
(I/4)*x^4 - I*x^2*Cos[2*a] + ArcTan[((1 + x^2)*Cos[a])/(Sin[a] - x^2*Sin[a
])]*Cos[4*a] + (I/2)*Cos[4*a]*Log[1 + x^4 + 2*x^2*Cos[2*a]] + x^2*Sin[2*a]
 + I*ArcTan[((1 + x^2)*Cos[a])/(Sin[a] - x^2*Sin[a])]*Sin[4*a] - (Log[1 +
x^4 + 2*x^2*Cos[2*a]]*Sin[4*a])/2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5006, 947, 354, 26, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \tan(a+i\log(x))\,dx$$

$$\Big\downarrow \text{ 5006}$$

$$\int \frac{x^3\left(i-\frac{ie^{2ia}}{x^2}\right)}{1+\frac{e^{2ia}}{x^2}}dx$$

$$\Big\downarrow \text{ 947}$$

$$\int \frac{x^3\left(ix^2-ie^{2ia}\right)}{x^2+e^{2ia}}dx$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int -\frac{ix^2\left(e^{2ia}-x^2\right)}{x^2+e^{2ia}}dx^2$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{1}{2}i\int \frac{x^2\left(e^{2ia}-x^2\right)}{x^2+e^{2ia}}dx^2$$

$$\Big\downarrow \text{ 86}$$

$$-\frac{1}{2}i\int \left(-x^2+2e^{2ia}-\frac{2e^{4ia}}{x^2+e^{2ia}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}i\left(2e^{2ia}x^2-2e^{4ia}\log\left(x^2+e^{2ia}\right)-\frac{x^4}{2}\right)$$

input
```
Int[x^3*Tan[a + I*Log[x]],x]
```

output
```
(-1/2*I)*(2*E^((2*I)*a)*x^2 - x^4/2 - 2*E^((4*I)*a)*Log[E^((2*I)*a) + x^2]
)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 947
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

method	result	size
risch	$-i\mathrm{e}^{2ia}x^2+\frac{ix^4}{4}+i\mathrm{e}^{4ia}\ln\left(\mathrm{e}^{2ia}+x^2\right)$	37

input
```
int(x^3*tan(a+I*ln(x)),x,method=_RETURNVERBOSE)
```

output
```
-I*exp(2*I*a)*x^2+1/4*I*x^4+I*exp(4*I*a)*ln(exp(2*I*a)+x^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

$$\int x^3\tan(a+i\log(x))\,dx=\frac{1}{4}i\,x^4-i\,x^2e^{(2i\,a)}+i\,e^{(4i\,a)}\log\left(x^2+e^{(2i\,a)}\right)$$

input
```
integrate(x^3*tan(a+I*log(x)),x, algorithm="fricas")
```

output
```
1/4*I*x^4 - I*x^2*e^(2*I*a) + I*e^(4*I*a)*log(x^2 + e^(2*I*a))
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int x^3 \tan(a + i \log(x)) \, dx = \frac{ix^4}{4} - ix^2e^{2ia} + ie^{4ia} \log \left(x^2 + e^{2ia}\right)$$

input
```
integrate(x**3*tan(a+I*ln(x)),x)
```

output
```
I*x**4/4 - I*x**2*exp(2*I*a) + I*exp(4*I*a)*log(x**2 + exp(2*I*a))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(30) = 60.

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.87

$$\begin{aligned}\int x^3 \tan(a + i \log(x)) \, dx = &\frac{1}{4} i x^4 + x^2(-i \cos(2a) + \sin(2a)) \\ &- (\cos(4a) + i \sin(4a)) \arctan\left(\sin(2a), x^2 + \cos(2a)\right) \\ &+ \frac{1}{2} (i \cos(4a) - \sin(4a)) \log\left(x^4 + 2x^2 \cos(2a) \right. \\ &\left. + \cos(2a)^2 + \sin(2a)^2\right)\end{aligned}$$

input
```
integrate(x^3*tan(a+I*log(x)),x, algorithm="maxima")
```

output
```
1/4*I*x^4 + x^2*(-I*cos(2*a) + sin(2*a)) - (cos(4*a) + I*sin(4*a))*arctan2
(sin(2*a), x^2 + cos(2*a)) + 1/2*(I*cos(4*a) - sin(4*a))*log(x^4 + 2*x^2*c
os(2*a) + cos(2*a)^2 + sin(2*a)^2)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int x^3 \tan(a + i \log(x)) \, dx = \frac{1}{4} i x^4 - i x^2 e^{(2 i a)} - \frac{1}{2} \pi e^{(4 i a)} + i e^{(4 i a)} \log\left(x^2 + e^{(2 i a)}\right)$$

input
```
integrate(x^3*tan(a+I*log(x)),x, algorithm="giac")
```

output
```
1/4*I*x^4 - I*x^2*e^(2*I*a) - 1/2*pi*e^(4*I*a) + I*e^(4*I*a)*log(x^2 + e^(
2*I*a))
```

Mupad [B] (verification not implemented)

Time = 19.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int x^3 \tan(a + i \log(x)) \, dx = e^{a\,4i} \ln\left(x^2 + e^{a\,2i}\right) 1i - x^2 e^{a\,2i} 1i + \frac{x^4 1i}{4}$$

input
```
int(x^3*tan(a + log(x)*1i),x)
```

output
```
exp(a*4i)*log(exp(a*2i) + x^2)*1i - x^2*exp(a*2i)*1i + (x^4*1i)/4
```

Reduce [F]

$$\int x^3 \tan(a + i \log(x)) \, dx = \int \tan(\log(x) i + a) x^3 dx$$

input
```
int(x^3*tan(a+I*log(x)),x)
```

output
```
int(tan(log(x)*i + a)*x**3,x)
```

3.136 $\int x^2 \tan(a + i \log(x)) \, dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int x^2 \tan(a + i \log(x)) \, dx = -2ie^{2ia}x + \frac{ix^3}{3} + 2ie^{3ia} \arctan\left(e^{-ia}x\right)$$

output
```
-2*I*exp(2*I*a)*x+1/3*I*x^3+2*I*exp(3*I*a)*arctan(x/exp(I*a))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int x^2 \tan(a + i \log(x)) \, dx = \frac{ix^3}{3} - 2ix \cos(2a) + 2i \arctan(x \cos(a) - ix \sin(a)) \cos(3a) + 2x \sin(2a) - 2 \arctan(x \cos(a) - ix \sin(a)) \sin(3a)$$

input
```
Integrate[x^2*Tan[a + I*Log[x]],x]
```

output
```
(I/3)*x^3 - (2*I)*x*Cos[2*a] + (2*I)*ArcTan[x*Cos[a] - I*x*Sin[a]]*Cos[3*a
] + 2*x*Sin[2*a] - 2*ArcTan[x*Cos[a] - I*x*Sin[a]]*Sin[3*a]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5006, 947, 363, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \tan(a + i\log(x))\, dx$$

$$\Big\downarrow \text{5006}$$

$$\int \frac{x^2\left(i - \frac{ie^{2ia}}{x^2}\right)}{1 + \frac{e^{2ia}}{x^2}}dx$$

$$\Big\downarrow \text{947}$$

$$\int \frac{x^2(ix^2 - ie^{2ia})}{x^2 + e^{2ia}}dx$$

$$\Big\downarrow \text{363}$$

$$\frac{ix^3}{3} - 2ie^{2ia}\int \frac{x^2}{x^2 + e^{2ia}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{ix^3}{3} - 2ie^{2ia}\left(x - e^{2ia}\int \frac{1}{x^2 + e^{2ia}}dx\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{ix^3}{3} - 2ie^{2ia}\left(x - e^{ia}\arctan\left(e^{-ia}x\right)\right)$$

input
```
Int[x^2*Tan[a + I*Log[x]],x]
```

output
```
(I/3)*x^3 - (2*I)*E^((2*I)*a)*(x - E^(I*a)*ArcTan[x/E^(I*a)])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 947
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{ix^3}{3} - 2i\mathrm{e}^{2ia}x + 2i\arctan\left(x\,\mathrm{e}^{-ia}\right)\mathrm{e}^{3ia}$	33

input
```
int(x^2*tan(a+I*ln(x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*I*x^3-2*I*exp(2*I*a)*x+2*I*arctan(x*exp(-I*a))*exp(3*I*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int x^2 \tan(a+i\log(x))\,dx = \frac{1}{3}i\,x^3 - 2i\,xe^{(2i\,a)} - e^{(3i\,a)}\log\left(x+i\,e^{(i\,a)}\right) + e^{(3i\,a)}\log\left(x-i\,e^{(i\,a)}\right)$$

input
```
integrate(x^2*tan(a+I*log(x)),x, algorithm="fricas")
```

output
```
1/3*I*x^3 - 2*I*x*e^(2*I*a) - e^(3*I*a)*log(x + I*e^(I*a)) + e^(3*I*a)*log
(x - I*e^(I*a))
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int x^2 \tan(a+i\log(x))\,dx = \frac{ix^3}{3} - 2ixe^{2ia} + \left(\log\left(xe^{2ia} - ie^{3ia}\right) - \log\left(xe^{2ia} + ie^{3ia}\right)\right)e^{3ia}$$

input
```
integrate(x**2*tan(a+I*ln(x)),x)
```

output
```
I*x**3/3 - 2*I*x*exp(2*I*a) + (log(x*exp(2*I*a) - I*exp(3*I*a)) - log(x*ex
p(2*I*a) + I*exp(3*I*a)))*exp(3*I*a)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(26) = 52$.

Time = 0.15 (sec) , antiderivative size = 149, normalized size of antiderivative = 3.47

$$\int x^2 \tan(a + i\log(x))\,dx = \frac{1}{3}i\,x^3 - 2\,x(i\,\cos(2\,a) - \sin(2\,a)) - (i\,\cos(3\,a) - \sin(3\,a))\arctan\left(\frac{2\,x\cos(a)}{x^2 + \cos(a)^2 - 2\,x\sin(a) + \sin(a)^2}, \frac{x^2 - \cos(a)^2 - \sin(a)^2}{x^2 + \cos(a)^2 - 2\,x\sin(a) + \sin(a)^2}\right) + \frac{1}{2}(\cos(3\,a) + i\,\sin(3\,a))\log\left(\frac{x^2 + \cos(a)^2 + 2\,x\sin(a) + \sin(a)^2}{x^2 + \cos(a)^2 - 2\,x\sin(a) + \sin(a)^2}\right)$$

input
```
integrate(x^2*tan(a+I*log(x)),x, algorithm="maxima")
```

output
```
1/3*I*x^3 - 2*x*(I*cos(2*a) - sin(2*a)) - (I*cos(3*a) - sin(3*a))*arctan2(
2*x*cos(a)/(x^2 + cos(a)^2 - 2*x*sin(a) + sin(a)^2), (x^2 - cos(a)^2 - sin
(a)^2)/(x^2 + cos(a)^2 - 2*x*sin(a) + sin(a)^2)) + 1/2*(cos(3*a) + I*sin(3
*a))*log((x^2 + cos(a)^2 + 2*x*sin(a) + sin(a)^2)/(x^2 + cos(a)^2 - 2*x*si
n(a) + sin(a)^2))
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.60

$$\int x^2 \tan(a + i\log(x))\,dx = \frac{1}{3}i\,x^3 + 2i\,\arctan\left(xe^{(-i\,a)}\right)e^{(3i\,a)} - 2i\,xe^{(2i\,a)}$$

input
```
integrate(x^2*tan(a+I*log(x)),x, algorithm="giac")
```

output
```
1/3*I*x^3 + 2*I*arctan(x*e^(-I*a))*e^(3*I*a) - 2*I*x*e^(2*I*a)
```

Mupad [B] (verification not implemented)

Time = 19.64 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int x^2 \tan(a + i\log(x))\,dx = \left(e^{a\,2i}\right)^{3/2} \operatorname{atan}\left(\frac{x}{\sqrt{e^{a\,2i}}}\right) 2i + \frac{x^3\,1i}{3} - x\,e^{a\,2i}\,2i$$

input
```
int(x^2*tan(a + log(x)*1i),x)
```

output
```
exp(a*2i)^(3/2)*atan(x/exp(a*2i)^(1/2))*2i + (x^3*1i)/3 - x*exp(a*2i)*2i
```

Reduce [F]

$$\int x^2 \tan(a + i\log(x))\,dx = \int \tan(\log(x)\,i + a)\,x^2 dx$$

input
```
int(x^2*tan(a+I*log(x)),x)
```

output
```
int(tan(log(x)*i + a)*x**2,x)
```

3.137 $\int x\tan(a+i\log(x))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int x\tan(a+i\log(x))\,dx=\frac{ix^2}{2}-ie^{2ia}\log\left(e^{2ia}+x^2\right)$$

output
```
1/2*I*x^2-I*exp(2*I*a)*ln(exp(2*I*a)+x^2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 114 vs. 2(33) = 66.

Time = 0.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.45

$$\begin{aligned}\int x\tan(a+i\log(x))\,dx=&\frac{ix^2}{2}-\arctan\left(\frac{(1+x^2)\cos(a)}{\sin(a)-x^2\sin(a)}\right)\cos(2a)\\&-\frac{1}{2}i\cos(2a)\log\left(1+x^4+2x^2\cos(2a)\right)\\&-i\arctan\left(\frac{(1+x^2)\cos(a)}{\sin(a)-x^2\sin(a)}\right)\sin(2a)\\&+\frac{1}{2}\log\left(1+x^4+2x^2\cos(2a)\right)\sin(2a)\end{aligned}$$

input
```
Integrate[x*Tan[a + I*Log[x]],x]
```

output
```
(I/2)*x^2 - ArcTan[((1 + x^2)*Cos[a])/(Sin[a] - x^2*Sin[a])]*Cos[2*a] - (I
/2)*Cos[2*a]*Log[1 + x^4 + 2*x^2*Cos[2*a]] - I*ArcTan[((1 + x^2)*Cos[a])/(
Sin[a] - x^2*Sin[a])]*Sin[2*a] + (Log[1 + x^4 + 2*x^2*Cos[2*a]]*Sin[2*a])/
2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {5006, 947, 353, 26, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\tan(a+i\log(x))\,dx$$

$$\Big\downarrow \text{5006}$$

$$\int \frac{x\left(i-\frac{ie^{2ia}}{x^2}\right)}{1+\frac{e^{2ia}}{x^2}}dx$$

$$\Big\downarrow \text{947}$$

$$\int \frac{x\left(ix^2-ie^{2ia}\right)}{x^2+e^{2ia}}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int -\frac{i\left(e^{2ia}-x^2\right)}{x^2+e^{2ia}}dx^2$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{2}i\int \frac{e^{2ia}-x^2}{x^2+e^{2ia}}dx^2$$

$$\Big\downarrow \text{49}$$

$$-\frac{1}{2}i\int \left(\frac{2e^{2ia}}{x^2+e^{2ia}}-1\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}i\left(-x^2+2e^{2ia}\log\left(x^2+e^{2ia}\right)\right)$$

input
```
Int[x*Tan[a + I*Log[x]],x]
```

output
```
(-1/2*I)*(-x^2 + 2*E^((2*I)*a)*Log[E^((2*I)*a) + x^2])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 947
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{ix^2}{2} - i\mathrm{e}^{2ia} \ln\left(\mathrm{e}^{2ia} + x^2\right)$	26

input
```
int(x*tan(a+I*ln(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*x^2-I*exp(2*I*a)*ln(exp(2*I*a)+x^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

$$\int x\tan(a + i\log(x))\,dx = \frac{1}{2}i\,x^2 - i\,e^{(2i\,a)}\log\left(x^2 + e^{(2i\,a)}\right)$$

input
```
integrate(x*tan(a+I*log(x)),x, algorithm="fricas")
```

output
```
1/2*I*x^2 - I*e^(2*I*a)*log(x^2 + e^(2*I*a))
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int x\tan(a + i\log(x))\,dx = \frac{ix^2}{2} - ie^{2ia}\log\left(x^2 + e^{2ia}\right)$$

input
```
integrate(x*tan(a+I*ln(x)),x)
```

output
```
I*x**2/2 - I*exp(2*I*a)*log(x**2 + exp(2*I*a))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(21) = 42.

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.12

$$\int x\tan(a+i\log(x))\,dx = \frac{1}{2}i\,x^2+(\cos(2\,a)+i\,\sin(2\,a))\arctan\left(\sin(2\,a)\,,x^2+\cos(2\,a)\right) + \frac{1}{2}\left(-i\,\cos(2\,a)+\sin(2\,a)\right)\log\left(x^4+2\,x^2\cos(2\,a) + \cos(2\,a)^2+\sin(2\,a)^2\right)$$

input
```
integrate(x*tan(a+I*log(x)),x, algorithm="maxima")
```

output
```
1/2*I*x^2 + (cos(2*a) + I*sin(2*a))*arctan2(sin(2*a), x^2 + cos(2*a)) + 1/
2*(-I*cos(2*a) + sin(2*a))*log(x^4 + 2*x^2*cos(2*a) + cos(2*a)^2 + sin(2*a
)^2)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x\tan(a+i\log(x))\,dx = \frac{1}{2}i\,x^2-\frac{1}{2}\,\pi e^{(2i\,a)}-i\,e^{(2i\,a)}\log\left(x^2+e^{(2i\,a)}\right)$$

input
```
integrate(x*tan(a+I*log(x)),x, algorithm="giac")
```

output
```
1/2*I*x^2 - 1/2*pi*e^(2*I*a) - I*e^(2*I*a)*log(x^2 + e^(2*I*a))
```

Mupad [B] (verification not implemented)

Time = 19.61 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int x\tan(a+i\log(x))\,dx = -\mathrm{e}^{a\,2\mathrm{i}}\,\ln\left(x^2+\mathrm{e}^{a\,2\mathrm{i}}\right)\,1\mathrm{i}+\frac{x^2\,1\mathrm{i}}{2}$$

input
```
int(x*tan(a + log(x)*1i),x)
```

output
```
(x^2*1i)/2 - exp(a*2i)*log(exp(a*2i) + x^2)*1i
```

Reduce [F]

$$\int x\tan(a+i\log(x))\,dx = \int \tan\left(\log(x)\,i+a\right)x dx$$

input
```
int(x*tan(a+I*log(x)),x)
```

output
```
int(tan(log(x)*i + a)*x,x)
```

3.138 $\int \tan(a+i\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 27

$$\int \tan(a+i\log(x))\,dx = ix - 2ie^{ia}\arctan\left(e^{-ia}x\right)$$

output
```
I*x-2*I*exp(I*a)*arctan(x/exp(I*a))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \tan(a+i\log(x))\,dx = ix - 2i\arctan(x\cos(a)-ix\sin(a))\cos(a) + 2\arctan(x\cos(a)-ix\sin(a))\sin(a)$$

input
```
Integrate[Tan[a + I*Log[x]],x]
```

output
```
I*x - (2*I)*ArcTan[x*Cos[a] - I*x*Sin[a]]*Cos[a] + 2*ArcTan[x*Cos[a] - I*x
*Sin[a]]*Sin[a]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5002, 898, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(a+i\log(x))\,dx$$

$$\Big\downarrow \text{5002}$$

$$\int \frac{i-\frac{ie^{2ia}}{x^2}}{1+\frac{e^{2ia}}{x^2}}dx$$

$$\Big\downarrow \text{898}$$

$$\int \frac{ix^2-ie^{2ia}}{x^2+e^{2ia}}dx$$

$$\Big\downarrow \text{299}$$

$$ix-2ie^{2ia}\int \frac{1}{x^2+e^{2ia}}dx$$

$$\Big\downarrow \text{216}$$

$$ix-2ie^{ia}\arctan\left(e^{-ia}x\right)$$

input
```
Int[Tan[a + I*Log[x]],x]
```

output
```
I*x - (2*I)*E^(I*a)*ArcTan[x/E^(I*a)]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 898
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

rule 5002
```
Int[Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((I - I*E^(2
*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b, d
, p}, x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
risch	$ix - 2i \arctan\left(x\,\mathrm{e}^{-ia}\right) \mathrm{e}^{ia}$	22

input
```
int(tan(a+I*ln(x)),x,method=_RETURNVERBOSE)
```

output
```
I*x-2*I*arctan(x*exp(-I*a))*exp(I*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \tan(a+i\log(x))\,dx = e^{(i\,a)}\log\left(x+i\,e^{(i\,a)}\right)-e^{(i\,a)}\log\left(x-i\,e^{(i\,a)}\right)+i\,x$$

input
```
integrate(tan(a+I*log(x)),x, algorithm="fricas")
```

output
```
e^(I*a)*log(x + I*e^(I*a)) - e^(I*a)*log(x - I*e^(I*a)) + I*x
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \tan(a+i\log(x))\,dx = ix+\left(-\log\left(x-ie^{ia}\right)+\log\left(x+ie^{ia}\right)\right)e^{ia}$$

input
```
integrate(tan(a+I*ln(x)),x)
```

output
```
I*x + (-log(x - I*exp(I*a)) + log(x + I*exp(I*a)))*exp(I*a)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(17) = 34.

Time = 0.14 (sec) , antiderivative size = 122, normalized size of antiderivative = 4.52

$$\begin{aligned}&\int \tan(a+i\log(x))\,dx\\&=(i\,\cos(a)-\sin(a))\arctan\left(\frac{2\,x\cos(a)}{x^2+\cos(a)^2-2\,x\sin(a)+\sin(a)^2},\frac{x^2-\cos(a)^2-\sin(a)^2}{x^2+\cos(a)^2-2\,x\sin(a)+\sin(a)^2}\right)\\&\quad-\frac{1}{2}\,(\cos(a)+i\,\sin(a))\log\left(\frac{x^2+\cos(a)^2+2\,x\sin(a)+\sin(a)^2}{x^2+\cos(a)^2-2\,x\sin(a)+\sin(a)^2}\right)+i\,x\end{aligned}$$

input
```
integrate(tan(a+I*log(x)),x, algorithm="maxima")
```

output
```
(I*cos(a) - sin(a))*arctan2(2*x*cos(a)/(x^2 + cos(a)^2 - 2*x*sin(a) + sin(
a)^2), (x^2 - cos(a)^2 - sin(a)^2)/(x^2 + cos(a)^2 - 2*x*sin(a) + sin(a)^2
)) - 1/2*(cos(a) + I*sin(a))*log((x^2 + cos(a)^2 + 2*x*sin(a) + sin(a)^2)/
(x^2 + cos(a)^2 - 2*x*sin(a) + sin(a)^2)) + I*x
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \tan(a + i\log(x))\,dx = -2i\arctan\left(xe^{(-ia)}\right)e^{(ia)} + i\,x$$

input
```
integrate(tan(a+I*log(x)),x, algorithm="giac")
```

output
```
-2*I*arctan(x*e^(-I*a))*e^(I*a) + I*x
```

Mupad [B] (verification not implemented)

Time = 19.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \tan(a + i\log(x))\,dx = x\,1\mathrm{i} - \sqrt{\mathrm{e}^{a\,2\mathrm{i}}}\,\mathrm{atan}\left(\frac{x}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}}}\right)\,2\mathrm{i}$$

input
```
int(tan(a + log(x)*1i),x)
```

output
```
x*1i - exp(a*2i)^(1/2)*atan(x/exp(a*2i)^(1/2))*2i
```

Reduce [F]

$$\int \tan(a + i\log(x))\,dx = \int \tan\left(\log(x)\,i + a\right)dx$$

input
```
int(tan(a+I*log(x)),x)
```

output
```
int(tan(log(x)*i + a),x)
```

3.139 $\int \frac{\tan(a+i\log(x))}{x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{\tan(a+i\log(x))}{x}\,dx = i\log(\cos(a+i\log(x)))$$

output
```
I*ln(cos(a+I*ln(x)))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\tan(a+i\log(x))}{x}\,dx = i\log(\cos(a+i\log(x)))$$

input
```
Integrate[Tan[a + I*Log[x]]/x,x]
```

output
```
I*Log[Cos[a + I*Log[x]]]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3039, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(a+i\log(x))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\int \tan(a+i\log(x))d\log(x)$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+i\log(x))d\log(x)$$

$$\Big\downarrow \text{3956}$$

$$i\log(\cos(a+i\log(x)))$$

input
```
Int[Tan[a + I*Log[x]]/x,x]
```

output
```
I*Log[Cos[a + I*Log[x]]]
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

method	result	size
derivativedivides	$-\frac{i \ln\left(1+\tan(a+i \ln(x))^{2}\right)}{2}$	17
default	$-\frac{i \ln\left(1+\tan(a+i \ln(x))^{2}\right)}{2}$	17
norman	$-\frac{i \ln\left(1+\tan(a+i \ln(x))^{2}\right)}{2}$	17
parallelrisch	$-\frac{i \ln\left(1+\tan(a+i \ln(x))^{2}\right)}{2}$	17
risch	$i \ln(x)+2a+i \ln\left(1+\frac{e^{2ia}}{x^{2}}\right)$	25

input
```
int(tan(a+I*ln(x))/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*ln(1+tan(a+I*ln(x))^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tan(a+i\log(x))}{x}\,dx = i\,\log\left(x^{2}+e^{(2i\,a)}\right) - i\,\log\left(x\right)$$

input
```
integrate(tan(a+I*log(x))/x,x, algorithm="fricas")
```

output
```
I*log(x^2 + e^(2*I*a)) - I*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{\tan(a + i\log(x))}{x}\,dx = -i\log(x) + i\log\left(x^2 + e^{2ia}\right)$$

input
```
integrate(tan(a+I*ln(x))/x,x)
```

output
```
-I*log(x) + I*log(x**2 + exp(2*I*a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\tan(a + i\log(x))}{x}\,dx = -i\,\log\left(\sec\left(a + i\,\log(x)\right)\right)$$

input
```
integrate(tan(a+I*log(x))/x,x, algorithm="maxima")
```

output
```
-I*log(sec(a + I*log(x)))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(10) = 20.

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 5.21

$$\int \frac{\tan(a + i\log(x))}{x}\,dx$$
$$= i\,\log\left(\sqrt{-\frac{1}{8}\left(\frac{\left(|x|^2+1\right)^2}{|x|^2} - \frac{\left(|x|^2-1\right)^2}{|x|^2}\right)\cos\left(\pi\mathrm{sgn}(x) + 2\,a\right) + \frac{\left(|x|^2+1\right)^2}{8\,|x|^2} + \frac{\left(|x|^2-1\right)^2}{8\,|x|^2}}\right)$$

input
```
integrate(tan(a+I*log(x))/x,x, algorithm="giac")
```

output
```
I*log(sqrt(-1/8*((abs(x)^2 + 1)^2/abs(x)^2 - (abs(x)^2 - 1)^2/abs(x)^2)*co
s(pi*sgn(x) + 2*a) + 1/8*(abs(x)^2 + 1)^2/abs(x)^2 + 1/8*(abs(x)^2 - 1)^2/
abs(x)^2))
```

Mupad [B] (verification not implemented)

Time = 21.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tan(a+i\log(x))}{x}\,dx = -\frac{\ln\left(\tan(a+\ln(x)\ 1\mathrm{i})^2+1\right)\ 1\mathrm{i}}{2}$$

input
```
int(tan(a + log(x)*1i)/x,x)
```

output
```
-(log(tan(a + log(x)*1i)^2 + 1)*1i)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{\tan(a+i\log(x))}{x}\,dx = -\frac{\log\left(\tan\left(\log(x)\,i+a\right)^2+1\right)i}{2}$$

input
```
int(tan(a+I*log(x))/x,x)
```

output
```
( - log(tan(log(x)*i + a)**2 + 1)*i)/2
```

3.140 $\int \frac{\tan(a+i\log(x))}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{\tan(a+i\log(x))}{x^2}\,dx = \frac{i}{x}+2ie^{-ia}\arctan\left(e^{-ia}x\right)$$

output
```
I/x+2*I*arctan(x/exp(I*a))/exp(I*a)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.52

$$\int \frac{\tan(a+i\log(x))}{x^2}\,dx = \frac{i}{x}+2i\arctan(x\cos(a)-ix\sin(a))\cos(a) + 2\arctan(x\cos(a)-ix\sin(a))\sin(a)$$

input
```
Integrate[Tan[a + I*Log[x]]/x^2,x]
```

output
```
I/x + (2*I)*ArcTan[x*Cos[a] - I*x*Sin[a]]*Cos[a] + 2*ArcTan[x*Cos[a] - I*x
*Sin[a]]*Sin[a]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5006, 947, 359, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(a+i\log(x))}{x^2}\,dx$$

$$\Big\downarrow \text{5006}$$

$$\int \frac{i-\frac{ie^{2ia}}{x^2}}{x^2\left(1+\frac{e^{2ia}}{x^2}\right)}dx$$

$$\Big\downarrow \text{947}$$

$$\int \frac{ix^2-ie^{2ia}}{x^2\left(x^2+e^{2ia}\right)}dx$$

$$\Big\downarrow \text{359}$$

$$2i\int \frac{1}{x^2+e^{2ia}}dx+\frac{i}{x}$$

$$\Big\downarrow \text{216}$$

$$2ie^{-ia}\arctan\left(e^{-ia}x\right)+\frac{i}{x}$$

input
```
Int[Tan[a + I*Log[x]]/x^2,x]
```

output
```
I/x + ((2*I)*ArcTan[x/E^(I*a)])/E^(I*a)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 947
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{i}{x}+2i\arctan\left(x\,{\mathrm e}^{-i a}\right){\mathrm e}^{-i a}$	24

input
```
int(tan(a+I*ln(x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
I/x+2*I*arctan(x*exp(-I*a))*exp(-I*a)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(19) = 38.

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{\tan(a+i\log(x))}{x^2}\,dx = -\frac{\left(x\log\left(x+i\,e^{(i\,a)}\right)-x\log\left(x-i\,e^{(i\,a)}\right)-i\,e^{(i\,a)}\right)e^{(-i\,a)}}{x}$$

input
```
integrate(tan(a+I*log(x))/x^2,x, algorithm="fricas")
```

output
```
-(x*log(x + I*e^(I*a)) - x*log(x - I*e^(I*a)) - I*e^(I*a))*e^(-I*a)/x
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\tan(a+i\log(x))}{x^2}\,dx = \left(\log\left(x-ie^{ia}\right)-\log\left(x+ie^{ia}\right)\right)e^{-ia}+\frac{i}{x}$$

input
```
integrate(tan(a+I*ln(x))/x**2,x)
```

output
```
(log(x - I*exp(I*a)) - log(x + I*exp(I*a)))*exp(-I*a) + I/x
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(19) = 38.

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 4.38

$$\int \frac{\tan(a+i\log(x))}{x^2}\,dx = \frac{2\,x(-i\,\cos(a)-\sin(a))\arctan\left(\frac{2\,x\cos(a)}{x^2+\cos(a)^2-2\,x\sin(a)+\sin(a)^2},\frac{x^2-\cos(a)^2-\sin(a)^2}{x^2+\cos(a)^2-2\,x\sin(a)+\sin(a)^2}\right)+x(\cos(a)-i\,\sin(a}{2\,x}$$

input
```
integrate(tan(a+I*log(x))/x^2,x, algorithm="maxima")
```

output
```
1/2*(2*x*(-I*cos(a) - sin(a))*arctan2(2*x*cos(a)/(x^2 + cos(a)^2 - 2*x*sin
(a) + sin(a)^2), (x^2 - cos(a)^2 - sin(a)^2)/(x^2 + cos(a)^2 - 2*x*sin(a)
+ sin(a)^2)) + x*(cos(a) - I*sin(a))*log((x^2 + cos(a)^2 + 2*x*sin(a) + si
n(a)^2)/(x^2 + cos(a)^2 - 2*x*sin(a) + sin(a)^2)) + 2*I)/x
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{\tan(a + i\log(x))}{x^2} \, dx = 2i \arctan\left(xe^{(-i\,a)}\right)e^{(-i\,a)} + \frac{i}{x}$$

input
```
integrate(tan(a+I*log(x))/x^2,x, algorithm="giac")
```

output
```
2*I*arctan(x*e^(-I*a))*e^(-I*a) + I/x
```

Mupad [B] (verification not implemented)

Time = 19.70 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\tan(a + i\log(x))}{x^2} \, dx = \frac{\mathrm{atan}\left(\frac{x}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}}}\right)\,2\mathrm{i}}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}}} + \frac{1\mathrm{i}}{x}$$

input
```
int(tan(a + log(x)*1i)/x^2,x)
```

output
```
(atan(x/exp(a*2i)^(1/2))*2i)/exp(a*2i)^(1/2) + 1i/x
```

Reduce [F]

$$\int \frac{\tan(a+i\log(x))}{x^2}\,dx = \int \frac{\tan(\log(x)\,i+a)}{x^2}dx$$

input
```
int(tan(a+I*log(x))/x^2,x)
```

output
```
int(tan(log(x)*i + a)/x**2,x)
```

3.141 $\int \frac{\tan(a+i\log(x))}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{\tan(a+i\log(x))}{x^3}\,dx = \frac{i}{2x^2} - ie^{-2ia}\log\left(1+\frac{e^{2ia}}{x^2}\right)$$

output
```
1/2*I/x^2-I*ln(1+exp(2*I*a)/x^2)/exp(2*I*a)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 132 vs. 2(35) = 70.

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 3.77

$$\begin{aligned}\int \frac{\tan(a+i\log(x))}{x^3}\,dx &= \frac{i}{2x^2} - \arctan\left(\frac{(1+x^2)\cos(a)}{\sin(a)-x^2\sin(a)}\right)\cos(2a) \\ &\quad + 2i\cos(2a)\log(x) - \frac{1}{2}i\cos(2a)\log\left(1+x^4+2x^2\cos(2a)\right) \\ &\quad + i\arctan\left(\frac{(1+x^2)\cos(a)}{\sin(a)-x^2\sin(a)}\right)\sin(2a) \\ &\quad + 2\log(x)\sin(2a) - \frac{1}{2}\log\left(1+x^4+2x^2\cos(2a)\right)\sin(2a)\end{aligned}$$

input
```
Integrate[Tan[a + I*Log[x]]/x^3,x]
```

output
```
(I/2)/x^2 - ArcTan[((1 + x^2)*Cos[a])/(Sin[a] - x^2*Sin[a])]*Cos[2*a] + (2
*I)*Cos[2*a]*Log[x] - (I/2)*Cos[2*a]*Log[1 + x^4 + 2*x^2*Cos[2*a]] + I*Arc
Tan[((1 + x^2)*Cos[a])/(Sin[a] - x^2*Sin[a])]*Sin[2*a] + 2*Log[x]*Sin[2*a]
 - (Log[1 + x^4 + 2*x^2*Cos[2*a]]*Sin[2*a])/2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5006, 946, 26, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(a+i\log(x))}{x^3}dx$$

$$\Big\downarrow \text{5006}$$

$$\int \frac{i-\frac{ie^{2ia}}{x^2}}{x^3\left(1+\frac{e^{2ia}}{x^2}\right)}dx$$

$$\Big\downarrow \text{946}$$

$$-\frac{1}{2}\int \frac{i\left(1-\frac{e^{2ia}}{x^2}\right)}{1+\frac{e^{2ia}}{x^2}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{2}i\int \frac{1-\frac{e^{2ia}}{x^2}}{1+\frac{e^{2ia}}{x^2}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{49}$$

$$-\frac{1}{2}i\int \left(\frac{2}{1+\frac{e^{2ia}}{x^2}}-1\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}i\left(-\frac{1}{x^2}+2e^{-2ia}\log\left(1+\frac{e^{2ia}}{x^2}\right)\right)$$

input
```
Int[Tan[a + I*Log[x]]/x^3,x]
```

output
```
(-1/2*I)*(-x^(-2) + (2*Log[1 + E^((2*I)*a)/x^2])/E^((2*I)*a))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

method	result	size
risch	$\frac{i}{2x^2}+2i\mathrm{e}^{-2ia}\ln\left(x\right)-i\mathrm{e}^{-2ia}\ln\left(\mathrm{e}^{2ia}+x^2\right)$	36

input
```
int(tan(a+I*ln(x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*I/x^2+2*I*exp(-2*I*a)*ln(x)-I*exp(-2*I*a)*ln(exp(2*I*a)+x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{\tan(a+i\log(x))}{x^3}\,dx = \frac{\left(-2i\,x^2\log\left(x^2+e^{(2i\,a)}\right)+4i\,x^2\log\left(x\right)+i\,e^{(2i\,a)}\right)e^{(-2i\,a)}}{2\,x^2}$$

input
```
integrate(tan(a+I*log(x))/x^3,x, algorithm="fricas")
```

output
```
1/2*(-2*I*x^2*log(x^2 + e^(2*I*a)) + 4*I*x^2*log(x) + I*e^(2*I*a))*e^(-2*I
*a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{\tan(a+i\log(x))}{x^3}\,dx = 2ie^{-2ia}\log\left(x\right)-ie^{-2ia}\log\left(x^2+e^{2ia}\right)+\frac{i}{2x^2}$$

input
```
integrate(tan(a+I*ln(x))/x**3,x)
```

output
```
2*I*exp(-2*I*a)*log(x) - I*exp(-2*I*a)*log(x**2 + exp(2*I*a)) + I/(2*x**2)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(23) = 46$.

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.69

$$\int \frac{\tan(a+i\log(x))}{x^3}\,dx = -\frac{x^2(i\cos(2a)+\sin(2a))\log\left(x^4+2x^2\cos(2a)+\cos(2a)^2+\sin(2a)^2\right)-2\left((\cos(2a)-i\sin(2a))\cdots\right.}{2x^2}$$

input
```
integrate(tan(a+I*log(x))/x^3,x, algorithm="maxima")
```

output
```
-1/2*(x^2*(I*cos(2*a) + sin(2*a))*log(x^4 + 2*x^2*cos(2*a) + cos(2*a)^2 +
sin(2*a)^2) - 2*((cos(2*a) - I*sin(2*a))*arctan2(sin(2*a), x^2 + cos(2*a))
 + 2*(I*cos(2*a) + sin(2*a))*log(x))*x^2 - I)/x^2
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{\tan(a+i\log(x))}{x^3}\,dx = -\frac{1}{2}\pi e^{(-2ia)} - ie^{(-2ia)}\log\left(x^2+e^{(2ia)}\right) + 2ie^{(-2ia)}\log(x) + \frac{i}{2x^2}$$

input
```
integrate(tan(a+I*log(x))/x^3,x, algorithm="giac")
```

output
```
-1/2*pi*e^(-2*I*a) - I*e^(-2*I*a)*log(x^2 + e^(2*I*a)) + 2*I*e^(-2*I*a)*lo
g(x) + 1/2*I/x^2
```

Mupad [B] (verification not implemented)

Time = 19.53 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{\tan(a+i\log(x))}{x^3}\,dx = -\mathrm{e}^{-a\,2\mathrm{i}}\,\ln\left(x^2+\mathrm{e}^{a\,2\mathrm{i}}\right)\,1\mathrm{i}+\mathrm{e}^{-a\,2\mathrm{i}}\,\ln\left(x\right)\,2\mathrm{i}+\frac{1\mathrm{i}}{2\,x^2}$$

input
```
int(tan(a + log(x)*1i)/x^3,x)
```

output
```
exp(-a*2i)*log(x)*2i - exp(-a*2i)*log(exp(a*2i) + x^2)*1i + 1i/(2*x^2)
```

Reduce [F]

$$\int \frac{\tan(a+i\log(x))}{x^3}\,dx = \int \frac{\tan\left(\log(x)\,i+a\right)}{x^3}dx$$

input
```
int(tan(a+I*log(x))/x^3,x)
```

output
```
int(tan(log(x)*i + a)/x**3,x)
```

3.142 $\int \frac{\tan(a+i\log(x))}{x^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 45

$$\int \frac{\tan(a+i\log(x))}{x^4}\,dx=\frac{i}{3x^3}-\frac{2ie^{-2ia}}{x}-2ie^{-3ia}\arctan\left(e^{-ia}x\right)$$

output
```
1/3*I/x^3-2*I/exp(2*I*a)/x-2*I*arctan(x/exp(I*a))/exp(3*I*a)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.56

$$\int \frac{\tan(a+i\log(x))}{x^4}\,dx=\frac{i}{3x^3}-\frac{2i\cos(2a)}{x}-2i\arctan(x\cos(a)-ix\sin(a))\cos(3a)-\frac{2\sin(2a)}{x}-2\arctan(x\cos(a)-ix\sin(a))\sin(3a)$$

input
```
Integrate[Tan[a + I*Log[x]]/x^4,x]
```

output
```
(I/3)/x^3 - ((2*I)*Cos[2*a])/x - (2*I)*ArcTan[x*Cos[a] - I*x*Sin[a]]*Cos[3
*a] - (2*Sin[2*a])/x - 2*ArcTan[x*Cos[a] - I*x*Sin[a]]*Sin[3*a]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5006, 947, 359, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(a+i\log(x))}{x^4}dx$$

$$\Big\downarrow \text{5006}$$

$$\int \frac{i-\frac{ie^{2ia}}{x^2}}{x^4\left(1+\frac{e^{2ia}}{x^2}\right)}dx$$

$$\Big\downarrow \text{947}$$

$$\int \frac{ix^2-ie^{2ia}}{x^4\left(x^2+e^{2ia}\right)}dx$$

$$\Big\downarrow \text{359}$$

$$2i\int \frac{1}{x^2\left(x^2+e^{2ia}\right)}dx+\frac{i}{3x^3}$$

$$\Big\downarrow \text{264}$$

$$2i\left(-e^{-2ia}\int \frac{1}{x^2+e^{2ia}}dx-\frac{e^{-2ia}}{x}\right)+\frac{i}{3x^3}$$

$$\Big\downarrow \text{216}$$

$$2i\left(-e^{-3ia}\arctan\left(e^{-ia}x\right)-\frac{e^{-2ia}}{x}\right)+\frac{i}{3x^3}$$

input
```
Int[Tan[a + I*Log[x]]/x^4,x]
```

output
```
(I/3)/x^3 + (2*I)*(-(1/(E^((2*I)*a)*x)) - ArcTan[x/E^(I*a)]/E^((3*I)*a))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 947
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{i}{3x^{3}}-2i\arctan\left(x\,{\mathrm e}^{-i a}\right){\mathrm e}^{-3 i a}-\frac{2 i {\mathrm e}^{-2 i a}}{x}$	35

input
```
int(tan(a+I*ln(x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*I/x^3-2*I*arctan(x*exp(-I*a))*exp(-3*I*a)-2*I*exp(-2*I*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \frac{\tan(a+i\log(x))}{x^4}\,dx = \frac{\left(3\,x^3\log\left(x+i\,e^{(i\,a)}\right)-3\,x^3\log\left(x-i\,e^{(i\,a)}\right)-6i\,x^2e^{(i\,a)}+i\,e^{(3i\,a)}\right)e^{(-3i\,a)}}{3\,x^3}$$

input
```
integrate(tan(a+I*log(x))/x^4,x, algorithm="fricas")
```

output
```
1/3*(3*x^3*log(x + I*e^(I*a)) - 3*x^3*log(x - I*e^(I*a)) - 6*I*x^2*e^(I*a)
 + I*e^(3*I*a))*e^(-3*I*a)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \frac{\tan(a+i\log(x))}{x^4}\,dx = \left(-\log\left(x-ie^{ia}\right)+\log\left(x+ie^{ia}\right)\right)e^{-3ia}+\frac{\left(-6ix^2+ie^{2ia}\right)e^{-2ia}}{3x^3}$$

input
```
integrate(tan(a+I*ln(x))/x**4,x)
```

output
```
(-log(x - I*exp(I*a)) + log(x + I*exp(I*a)))*exp(-3*I*a) + (-6*I*x**2 + I*
exp(2*I*a))*exp(-2*I*a)/(3*x**3)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(28) = 56$.

Time = 0.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.47

$$\int \frac{\tan(a+i\log(x))}{x^4}\,dx =$$

$$-\frac{6\,x^3(-i\,\cos(3\,a)-\sin(3\,a))\arctan\left(\frac{2\,x\cos(a)}{x^2+\cos(a)^2-2\,x\sin(a)+\sin(a)^2},\frac{x^2-\cos(a)^2-\sin(a)^2}{x^2+\cos(a)^2-2\,x\sin(a)+\sin(a)^2}\right)+3\,x^3(\cos(3\,a}{6\,x^3}$$

input
```
integrate(tan(a+I*log(x))/x^4,x, algorithm="maxima")
```

output
```
-1/6*(6*x^3*(-I*cos(3*a) - sin(3*a))*arctan2(2*x*cos(a)/(x^2 + cos(a)^2 -
2*x*sin(a) + sin(a)^2), (x^2 - cos(a)^2 - sin(a)^2)/(x^2 + cos(a)^2 - 2*x*
sin(a) + sin(a)^2)) + 3*x^3*(cos(3*a) - I*sin(3*a))*log((x^2 + cos(a)^2 +
2*x*sin(a) + sin(a)^2)/(x^2 + cos(a)^2 - 2*x*sin(a) + sin(a)^2)) + 12*x^2*
(I*cos(2*a) + sin(2*a)) - 2*I)/x^3
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.62

$$\int \frac{\tan(a+i\log(x))}{x^4}\,dx = -2i\arctan\left(xe^{(-i\,a)}\right)e^{(-3i\,a)}-\frac{2i\,e^{(-2i\,a)}}{x}+\frac{i}{3\,x^3}$$

input
```
integrate(tan(a+I*log(x))/x^4,x, algorithm="giac")
```

output
```
-2*I*arctan(x*e^(-I*a))*e^(-3*I*a) - 2*I*e^(-2*I*a)/x + 1/3*I/x^3
```

Mupad [B] (verification not implemented)

Time = 19.44 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int \frac{\tan(a+i\log(x))}{x^4}\,dx = -\frac{\operatorname{atan}\left(\frac{x}{\sqrt{e^{a\,2i}}}\right)\,2i}{\left(e^{a\,2i}\right)^{3/2}} - \frac{x^2\,e^{-a\,2i}\,2i - \frac{1}{3}i}{x^3}$$

input
```
int(tan(a + log(x)*1i)/x^4,x)
```

output
```
- (atan(x/exp(a*2i)^(1/2))*2i)/exp(a*2i)^(3/2) - (x^2*exp(-a*2i)*2i - 1i/3
)/x^3
```

Reduce [F]

$$\int \frac{\tan(a+i\log(x))}{x^4}\,dx = \int \frac{\tan\left(\log(x)\,i+a\right)}{x^4}dx$$

input
```
int(tan(a+I*log(x))/x^4,x)
```

output
```
int(tan(log(x)*i + a)/x**4,x)
```

3.143 $\int x^3 \tan^2(a+i\log(x))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int x^3 \tan^2(a+i\log(x))\,dx = 2e^{2ia}x^2 - \frac{x^4}{4} - \frac{2e^{6ia}}{e^{2ia}+x^2} - 4e^{4ia}\log\left(e^{2ia}+x^2\right)$$

output
```
2*exp(2*I*a)*x^2-1/4*x^4-2*exp(6*I*a)/(exp(2*I*a)+x^2)-4*exp(4*I*a)*ln(exp
(2*I*a)+x^2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 155 vs. 2(63) = 126.

Time = 0.13 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.46

$$\begin{aligned}\int x^3 \tan^2(a+i\log(x))\,dx = & -\frac{x^4}{4} + 2x^2\cos(2a) - 4i\arctan\left(\frac{(1+x^2)\cot(a)}{-1+x^2}\right)\cos(4a) \\ & - 2\cos(4a)\log\left(1+x^4+2x^2\cos(2a)\right) \\ & + 2ix^2\sin(2a) + 4\arctan\left(\frac{(1+x^2)\cot(a)}{-1+x^2}\right)\sin(4a) \\ & - 2i\log\left(1+x^4+2x^2\cos(2a)\right)\sin(4a) \\ & - \frac{2(\cos(5a)+i\sin(5a))}{(1+x^2)\cos(a)-i\left(-1+x^2\right)\sin(a)}\end{aligned}$$

input
```
Integrate[x^3*Tan[a + I*Log[x]]^2,x]
```

output
```
-1/4*x^4 + 2*x^2*Cos[2*a] - (4*I)*ArcTan[((1 + x^2)*Cot[a])/(-1 + x^2)]*Co
s[4*a] - 2*Cos[4*a]*Log[1 + x^4 + 2*x^2*Cos[2*a]] + (2*I)*x^2*Sin[2*a] + 4
*ArcTan[((1 + x^2)*Cot[a])/(-1 + x^2)]*Sin[4*a] - (2*I)*Log[1 + x^4 + 2*x^
2*Cos[2*a]]*Sin[4*a] - (2*(Cos[5*a] + I*Sin[5*a]))/((1 + x^2)*Cos[a] - I*(
-1 + x^2)*Sin[a])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5006, 947, 354, 25, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \tan^2(a+i\log(x))\,dx$$

$$\Big\downarrow \text{5006}$$

$$\int \frac{x^3\left(i-\frac{ie^{2ia}}{x^2}\right)^2}{\left(1+\frac{e^{2ia}}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{947}$$

$$\int \frac{x^3\left(ix^2-ie^{2ia}\right)^2}{\left(x^2+e^{2ia}\right)^2}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int -\frac{x^2\left(e^{2ia}-x^2\right)^2}{\left(x^2+e^{2ia}\right)^2}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{x^2\left(e^{2ia}-x^2\right)^2}{\left(x^2+e^{2ia}\right)^2}dx^2$$

$$\Big\downarrow \text{86}$$

$$-\frac{1}{2}\int\left(x^2-4e^{2ia}+\frac{8e^{4ia}}{x^2+e^{2ia}}-\frac{4e^{6ia}}{(x^2+e^{2ia})^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(4e^{2ia}x^2-\frac{4e^{6ia}}{x^2+e^{2ia}}-8e^{4ia}\log\left(x^2+e^{2ia}\right)-\frac{x^4}{2}\right)$$

input
```
Int[x^3*Tan[a + I*Log[x]]^2,x]
```

output
```
(4*E^((2*I)*a)*x^2 - x^4/2 - (4*E^((6*I)*a))/(E^((2*I)*a) + x^2) - 8*E^((4
*I)*a)*Log[E^((2*I)*a) + x^2])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 947
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{9x^4}{4}+\frac{2x^4}{1+\frac{e^{2ia}}{x^2}}+4\,e^{2ia}x^2-4\,e^{4ia}\ln\left(e^{2ia}+x^2\right)$	52

input
```
int(x^3*tan(a+I*ln(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-9/4*x^4+2*x^4/(1+exp(2*I*a)/x^2)+4*exp(2*I*a)*x^2-4*exp(4*I*a)*ln(exp(2*I
*a)+x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int x^3\tan^2(a+i\log(x))\,dx = -\frac{x^6-7\,x^4e^{(2i\,a)}-8\,x^2e^{(4i\,a)}+16\left(x^2e^{(4i\,a)}+e^{(6i\,a)}\right)\log\left(x^2+e^{(2i\,a)}\right)+8\,e^{(6i\,a)}}{4\left(x^2+e^{(2i\,a)}\right)}$$

input
```
integrate(x^3*tan(a+I*log(x))^2,x, algorithm="fricas")
```

output
```
-1/4*(x^6 - 7*x^4*e^(2*I*a) - 8*x^2*e^(4*I*a) + 16*(x^2*e^(4*I*a) + e^(6*I
*a))*log(x^2 + e^(2*I*a)) + 8*e^(6*I*a))/(x^2 + e^(2*I*a))
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int x^3 \tan^2(a + i\log(x))\,dx = -\frac{x^4}{4} + 2x^2e^{2ia} - 4e^{4ia}\log\left(x^2 + e^{2ia}\right) - \frac{2e^{6ia}}{x^2 + e^{2ia}}$$

input
```
integrate(x**3*tan(a+I*ln(x))**2,x)
```

output
```
-x**4/4 + 2*x**2*exp(2*I*a) - 4*exp(4*I*a)*log(x**2 + exp(2*I*a)) - 2*exp(
6*I*a)/(x**2 + exp(2*I*a))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(46) = 92.

Time = 0.04 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.44

$$\int x^3 \tan^2(a + i\log(x))\,dx =$$
$$-\frac{x^6 - 7\,x^4(\cos(2\,a) + i\,\sin(2\,a)) - 8\,(2\,(-i\,\cos(4\,a) + \sin(4\,a))\arctan(\sin(2\,a)\,, x^2 + \cos(2\,a)) + \cos}{}$$

input
```
integrate(x^3*tan(a+I*log(x))^2,x, algorithm="maxima")
```

output
```
-1/4*(x^6 - 7*x^4*(cos(2*a) + I*sin(2*a)) - 8*(2*(-I*cos(4*a) + sin(4*a))*
arctan2(sin(2*a), x^2 + cos(2*a)) + cos(4*a) + I*sin(4*a))*x^2 - 16*((-I*c
os(2*a) + sin(2*a))*cos(4*a) + (cos(2*a) + I*sin(2*a))*sin(4*a))*arctan2(s
in(2*a), x^2 + cos(2*a)) + 8*(x^2*(cos(4*a) + I*sin(4*a)) + (cos(2*a) + I*
sin(2*a))*cos(4*a) - (-I*cos(2*a) + sin(2*a))*sin(4*a))*log(x^4 + 2*x^2*co
s(2*a) + cos(2*a)^2 + sin(2*a)^2) + 8*cos(6*a) + 8*I*sin(6*a))/(x^2 + cos(
2*a) + I*sin(2*a))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(46) = 92$.

Time = 0.35 (sec) , antiderivative size = 261, normalized size of antiderivative = 4.14

$$\int x^3 \tan^2(a + i\log(x))\,dx = -\frac{x^6}{4\left(x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}\right)} + \frac{3x^4e^{(2ia)}}{2\left(x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}\right)} - \frac{4x^2e^{(4ia)}\log\left(-x^2 - e^{(2ia)}\right)}{x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}} + \frac{17x^2e^{(4ia)}}{4\left(x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}\right)} - \frac{8e^{(6ia)}\log\left(-x^2 - e^{(2ia)}\right)}{x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}} + \frac{e^{(6ia)}}{x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}} - \frac{4e^{(8ia)}\log\left(-x^2 - e^{(2ia)}\right)}{\left(x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}\right)x^2} - \frac{3e^{(8ia)}}{2\left(x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}\right)x^2}$$

input
```
integrate(x^3*tan(a+I*log(x))^2,x, algorithm="giac")
```

output
```
-1/4*x^6/(x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a)) + 3/2*x^4*e^(2*I*a)/(x^2 + e^
(4*I*a)/x^2 + 2*e^(2*I*a)) - 4*x^2*e^(4*I*a)*log(-x^2 - e^(2*I*a))/(x^2 +
e^(4*I*a)/x^2 + 2*e^(2*I*a)) + 17/4*x^2*e^(4*I*a)/(x^2 + e^(4*I*a)/x^2 + 2
*e^(2*I*a)) - 8*e^(6*I*a)*log(-x^2 - e^(2*I*a))/(x^2 + e^(4*I*a)/x^2 + 2*e
^(2*I*a)) + e^(6*I*a)/(x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a)) - 4*e^(8*I*a)*lo
g(-x^2 - e^(2*I*a))/((x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a))*x^2) - 3/2*e^(8*I
*a)/((x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a))*x^2)
```

Mupad [B] (verification not implemented)

Time = 19.76 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^3 \tan^2(a + i\log(x))\,dx = -\frac{2\,e^{a\,6i}}{x^2 + e^{a\,2i}} - 4\,e^{a\,4i}\,\ln\left(x^2 + e^{a\,2i}\right) + 2\,x^2\,e^{a\,2i} - \frac{x^4}{4}$$

input
```
int(x^3*tan(a + log(x)*1i)^2,x)
```

output
```
2*x^2*exp(a*2i) - 4*exp(a*4i)*log(exp(a*2i) + x^2) - (2*exp(a*6i))/(exp(a*
2i) + x^2) - x^4/4
```

Reduce [F]

$$\int x^3 \tan^2(a + i\log(x))\,dx = 4\left(\int \tan(\log(x)\,i + a)\,x^3 dx\right) i - \tan(\log(x)\,i + a)\,i\,x^4 - \frac{x^4}{4}$$

input
```
int(x^3*tan(a+I*log(x))^2,x)
```

output
```
(16*int(tan(log(x)*i + a)*x**3,x)*i - 4*tan(log(x)*i + a)*i*x**4 - x**4)/4
```

3.144 $\int x^2 \tan^2(a + i \log(x)) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 60

$$\int x^2 \tan^2(a + i \log(x)) \, dx = 4e^{2ia}x - \frac{x^3}{3} + \frac{2e^{4ia}x}{e^{2ia} + x^2} - 6e^{3ia} \arctan\left(e^{-ia}x\right)$$

output
```
4*exp(2*I*a)*x-1/3*x^3+2*exp(4*I*a)*x/(exp(2*I*a)+x^2)-6*exp(3*I*a)*arctan
(x/exp(I*a))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.67

$$\begin{aligned}\int x^2 \tan^2(a + i \log(x)) \, dx = & -\frac{x^3}{3} + 4x \cos(2a) - 6 \arctan(x(\cos(a) - i \sin(a))) \cos(3a) \\ & + 4ix \sin(2a) + \frac{2x(\cos(3a) + i \sin(3a))}{(1 + x^2) \cos(a) - i(-1 + x^2) \sin(a)} \\ & - 6i \arctan(x(\cos(a) - i \sin(a))) \sin(3a)\end{aligned}$$

input
```
Integrate[x^2*Tan[a + I*Log[x]]^2,x]
```

output
```
-1/3*x^3 + 4*x*Cos[2*a] - 6*ArcTan[x*(Cos[a] - I*Sin[a])]*Cos[3*a] + (4*I)
*x*Sin[2*a] + (2*x*(Cos[3*a] + I*Sin[3*a]))/((1 + x^2)*Cos[a] - I*(-1 + x^
2)*Sin[a]) - (6*I)*ArcTan[x*(Cos[a] - I*Sin[a])]*Sin[3*a]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5006, 947, 366, 27, 363, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \tan^2(a+i\log(x))\,dx$$

$$\Big\downarrow \text{5006}$$

$$\int \frac{x^2\left(i-\frac{ie^{2ia}}{x^2}\right)^2}{\left(1+\frac{e^{2ia}}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{947}$$

$$\int \frac{x^2\left(ix^2-ie^{2ia}\right)^2}{\left(x^2+e^{2ia}\right)^2}dx$$

$$\Big\downarrow \text{366}$$

$$-\frac{1}{2}e^{-2ia}\int -\frac{2x^2\left(5e^{4ia}-e^{2ia}x^2\right)}{x^2+e^{2ia}}dx-\frac{2e^{2ia}x^3}{x^2+e^{2ia}}$$

$$\Big\downarrow \text{27}$$

$$e^{-2ia}\int \frac{x^2\left(5e^{4ia}-e^{2ia}x^2\right)}{x^2+e^{2ia}}dx-\frac{2e^{2ia}x^3}{x^2+e^{2ia}}$$

$$\Big\downarrow \text{363}$$

$$e^{-2ia}\left(6e^{4ia}\int \frac{x^2}{x^2+e^{2ia}}dx-\frac{1}{3}e^{2ia}x^3\right)-\frac{2e^{2ia}x^3}{x^2+e^{2ia}}$$

$$\Big\downarrow \text{262}$$

$$e^{-2ia}\left(6e^{4ia}\left(x-e^{2ia}\int\frac{1}{x^2+e^{2ia}}dx\right)-\frac{1}{3}e^{2ia}x^3\right)-\frac{2e^{2ia}x^3}{x^2+e^{2ia}}$$

$$\Big\downarrow 216$$

$$e^{-2ia}\left(6e^{4ia}\left(x-e^{ia}\arctan\left(e^{-ia}x\right)\right)-\frac{1}{3}e^{2ia}x^3\right)-\frac{2e^{2ia}x^3}{x^2+e^{2ia}}$$

input
```
Int[x^2*Tan[a + I*Log[x]]^2,x]
```

output
```
(-2*E^((2*I)*a)*x^3)/(E^((2*I)*a) + x^2) + (-1/3*(E^((2*I)*a)*x^3) + 6*E^(
(4*I)*a)*(x - E^(I*a)*ArcTan[x/E^(I*a)]))/E^((2*I)*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 947
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{7x^3}{3}+\frac{2x^3}{1+\frac{e^{2ia}}{x^2}}+6\,e^{2ia}x-6\arctan\left(x\,e^{-ia}\right)e^{3ia}$	48

input
```
int(x^2*tan(a+I*ln(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-7/3*x^3+2*x^3/(1+exp(2*I*a)/x^2)+6*exp(2*I*a)*x-6*arctan(x*exp(-I*a))*exp
(3*I*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

$$\int x^2 \tan^2(a+i\log(x))\,dx = -\frac{x^5 - 11\,x^3e^{(2i\,a)} - 18\,xe^{(4i\,a)} + 9\left(i\,x^2e^{(3i\,a)} + i\,e^{(5i\,a)}\right)\log\left(x + i\,e^{(i\,a)}\right) + 9\left(-i\,x^2e^{(3i\,a)} - i\,e^{(5i\,a)}\right)\log\left(x\right.}{3\left(x^2 + e^{(2i\,a)}\right)}$$

input
```
integrate(x^2*tan(a+I*log(x))^2,x, algorithm="fricas")
```

output
```
-1/3*(x^5 - 11*x^3*e^(2*I*a) - 18*x*e^(4*I*a) + 9*(I*x^2*e^(3*I*a) + I*e^(
5*I*a))*log(x + I*e^(I*a)) + 9*(-I*x^2*e^(3*I*a) - I*e^(5*I*a))*log(x - I*
e^(I*a)))/(x^2 + e^(2*I*a))
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int x^2 \tan^2(a+i\log(x))\,dx = -\frac{x^3}{3} + 4xe^{2ia} + \frac{2xe^{4ia}}{x^2+e^{2ia}} - 3\left(-i\log\left(x - ie^{ia}\right) + i\log\left(x + ie^{ia}\right)\right)e^{3ia}$$

input
```
integrate(x**2*tan(a+I*ln(x))**2,x)
```

output
```
-x**3/3 + 4*x*exp(2*I*a) + 2*x*exp(4*I*a)/(x**2 + exp(2*I*a)) - 3*(-I*log(
x - I*exp(I*a)) + I*log(x + I*exp(I*a)))*exp(3*I*a)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(43) = 86.

Time = 0.14 (sec) , antiderivative size = 254, normalized size of antiderivative = 4.23

$$\int x^2 \tan^2(a + i\log(x))\,dx =$$

$$-\frac{2\,x^5 - 22\,x^3(\cos(2\,a) + i\,\sin(2\,a)) - 36\,x(\cos(4\,a) + i\,\sin(4\,a)) - 18\left(x^2(\cos(3\,a) + i\,\sin(3\,a)) + (\mathrm{co}\right.}{}$$

input
```
integrate(x^2*tan(a+I*log(x))^2,x, algorithm="maxima")
```

output
```
-1/6*(2*x^5 - 22*x^3*(cos(2*a) + I*sin(2*a)) - 36*x*(cos(4*a) + I*sin(4*a)
) - 18*(x^2*(cos(3*a) + I*sin(3*a)) + (cos(2*a) + I*sin(2*a))*cos(3*a) - (
-I*cos(2*a) + sin(2*a))*sin(3*a))*arctan2(2*x*cos(a)/(x^2 + cos(a)^2 - 2*x
*sin(a) + sin(a)^2), (x^2 - cos(a)^2 - sin(a)^2)/(x^2 + cos(a)^2 - 2*x*sin
(a) + sin(a)^2)) + 9*(x^2*(-I*cos(3*a) + sin(3*a)) + (-I*cos(2*a) + sin(2*
a))*cos(3*a) + (cos(2*a) + I*sin(2*a))*sin(3*a))*log((x^2 + cos(a)^2 + 2*x
*sin(a) + sin(a)^2)/(x^2 + cos(a)^2 - 2*x*sin(a) + sin(a)^2)))/(x^2 + cos(
2*a) + I*sin(2*a))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(43) = 86.

Time = 0.39 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.35

$$\begin{aligned}\int x^2 \tan^2(a + i\log(x))\,dx = &-\frac{x^5}{3\left(x^2 + \frac{e^{(4i\,a)}}{x^2} + 2\,e^{(2i\,a)}\right)} + \frac{10\,x^3e^{(2i\,a)}}{3\left(x^2 + \frac{e^{(4i\,a)}}{x^2} + 2\,e^{(2i\,a)}\right)} \\ &- 6\,\arctan\left(xe^{(-i\,a)}\right)e^{(3i\,a)} + \frac{35\,xe^{(4i\,a)}}{3\left(x^2 + \frac{e^{(4i\,a)}}{x^2} + 2\,e^{(2i\,a)}\right)} \\ &+ \frac{2\,xe^{(4i\,a)}}{x^2 + e^{(2i\,a)}} + \frac{8\,e^{(6i\,a)}}{\left(x^2 + \frac{e^{(4i\,a)}}{x^2} + 2\,e^{(2i\,a)}\right)x}\end{aligned}$$

input
```
integrate(x^2*tan(a+I*log(x))^2,x, algorithm="giac")
```

output
```
-1/3*x^5/(x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a)) + 10/3*x^3*e^(2*I*a)/(x^2 + e
^(4*I*a)/x^2 + 2*e^(2*I*a)) - 6*arctan(x*e^(-I*a))*e^(3*I*a) + 35/3*x*e^(4
*I*a)/(x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a)) + 2*x*e^(4*I*a)/(x^2 + e^(2*I*a)
) + 8*e^(6*I*a)/((x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a))*x)
```

Mupad [B] (verification not implemented)

Time = 19.37 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int x^2 \tan^2(a+i\log(x))\,dx = -6\left(\mathrm{e}^{a\,2\mathrm{i}}\right)^{3/2}\mathrm{atan}\left(\frac{x}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}}}\right) - \frac{x^3}{3} + 4\,x\,\mathrm{e}^{a\,2\mathrm{i}} + \frac{2\,x\,\mathrm{e}^{a\,4\mathrm{i}}}{x^2+\mathrm{e}^{a\,2\mathrm{i}}}$$

input
```
int(x^2*tan(a + log(x)*1i)^2,x)
```

output
```
4*x*exp(a*2i) - x^3/3 - 6*exp(a*2i)^(3/2)*atan(x/exp(a*2i)^(1/2)) + (2*x*e
xp(a*4i))/(exp(a*2i) + x^2)
```

Reduce [F]

$$\int x^2 \tan^2(a+i\log(x))\,dx = 3\left(\int \tan\left(\log(x)\,i+a\right)x^2dx\right)i - \tan\left(\log(x)\,i+a\right)i\,x^3 - \frac{x^3}{3}$$

input
```
int(x^2*tan(a+I*log(x))^2,x)
```

output
```
(9*int(tan(log(x)*i + a)*x**2,x)*i - 3*tan(log(x)*i + a)*i*x**3 - x**3)/3
```

3.145 $\int x \tan^2(a+i\log(x))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int x\tan^2(a+i\log(x))\,dx=-\frac{x^2}{2}+\frac{2e^{4ia}}{e^{2ia}+x^2}+2e^{2ia}\log\left(e^{2ia}+x^2\right)$$

output
```
-1/2*x^2+2*exp(4*I*a)/(exp(2*I*a)+x^2)+2*exp(2*I*a)*ln(exp(2*I*a)+x^2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 135 vs. 2(51) = 102.

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.65

$$\begin{aligned}\int x\tan^2(a+i\log(x))\,dx=&-\frac{x^2}{2}+2i\arctan\left(\frac{(1+x^2)\cot(a)}{-1+x^2}\right)\cos(2a)\\&+\cos(2a)\log\left(1+x^4+2x^2\cos(2a)\right)\\&-2\arctan\left(\frac{(1+x^2)\cot(a)}{-1+x^2}\right)\sin(2a)\\&+i\log\left(1+x^4+2x^2\cos(2a)\right)\sin(2a)\\&+\frac{2\cos(3a)+2i\sin(3a)}{(1+x^2)\cos(a)-i(-1+x^2)\sin(a)}\end{aligned}$$

input
```
Integrate[x*Tan[a + I*Log[x]]^2,x]
```

output
```
-1/2*x^2 + (2*I)*ArcTan[((1 + x^2)*Cot[a])/(-1 + x^2)]*Cos[2*a] + Cos[2*a]
*Log[1 + x^4 + 2*x^2*Cos[2*a]] - 2*ArcTan[((1 + x^2)*Cot[a])/(-1 + x^2)]*S
in[2*a] + I*Log[1 + x^4 + 2*x^2*Cos[2*a]]*Sin[2*a] + (2*Cos[3*a] + (2*I)*S
in[3*a])/((1 + x^2)*Cos[a] - I*(-1 + x^2)*Sin[a])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5006, 947, 353, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \tan^2(a + i \log(x))\,dx$$

$$\Big\downarrow \text{5006}$$

$$\int \frac{x\left(i - \frac{ie^{2ia}}{x^2}\right)^2}{\left(1 + \frac{e^{2ia}}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{947}$$

$$\int \frac{x\left(ix^2 - ie^{2ia}\right)^2}{\left(x^2 + e^{2ia}\right)^2}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int -\frac{\left(e^{2ia} - x^2\right)^2}{\left(x^2 + e^{2ia}\right)^2}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{\left(e^{2ia} - x^2\right)^2}{\left(x^2 + e^{2ia}\right)^2}dx^2$$

$$\Big\downarrow \text{49}$$

$$-\frac{1}{2}\int \left(1-\frac{4e^{2ia}}{x^2+e^{2ia}}+\frac{4e^{4ia}}{(x^2+e^{2ia})^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{4e^{4ia}}{x^2+e^{2ia}}+4e^{2ia}\log\left(x^2+e^{2ia}\right)-x^2\right)$$

input
```
Int[x*Tan[a + I*Log[x]]^2,x]
```

output
```
(-x^2 + (4*E^((4*I)*a))/(E^((2*I)*a) + x^2) + 4*E^((2*I)*a)*Log[E^((2*I)*a
) + x^2])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 947
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

method	result	size
risch	$-\frac{5x^2}{2}+\frac{2x^2}{1+\frac{e^{2ia}}{x^2}}+2\,e^{2ia}\ln\left(e^{2ia}+x^2\right)$	42

input
```
int(x*tan(a+I*ln(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-5/2*x^2+2*x^2/(1+exp(2*I*a)/x^2)+2*exp(2*I*a)*ln(exp(2*I*a)+x^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int x\tan^2(a+i\log(x))\,dx = -\frac{x^4+x^2e^{(2i\,a)}-4\left(x^2e^{(2i\,a)}+e^{(4i\,a)}\right)\log\left(x^2+e^{(2i\,a)}\right)-4\,e^{(4i\,a)}}{2\left(x^2+e^{(2i\,a)}\right)}$$

input
```
integrate(x*tan(a+I*log(x))^2,x, algorithm="fricas")
```

output
```
-1/2*(x^4 + x^2*e^(2*I*a) - 4*(x^2*e^(2*I*a) + e^(4*I*a))*log(x^2 + e^(2*I
*a)) - 4*e^(4*I*a))/(x^2 + e^(2*I*a))
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int x\tan^2(a+i\log(x))\,dx=-\frac{x^2}{2}+2e^{2ia}\log\left(x^2+e^{2ia}\right)+\frac{2e^{4ia}}{x^2+e^{2ia}}$$

input
```
integrate(x*tan(a+I*ln(x))**2,x)
```

output
```
-x**2/2 + 2*exp(2*I*a)*log(x**2 + exp(2*I*a)) + 2*exp(4*I*a)/(x**2 + exp(2
*I*a))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(37) = 74.

Time = 0.04 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.63

$$\int x\tan^2(a+i\log(x))\,dx=$$

$$-\frac{x^4+(4\left(-i\,\cos\left(2\,a\right)+\sin\left(2\,a\right)\right)\arctan\left(\sin\left(2\,a\right),x^2+\cos\left(2\,a\right)\right)+\cos\left(2\,a\right)+i\,\sin\left(2\,a\right))x^2+4\left(-i\,\mathrm{c}\right.}{}$$

input
```
integrate(x*tan(a+I*log(x))^2,x, algorithm="maxima")
```

output
```
-1/2*(x^4 + (4*(-I*cos(2*a) + sin(2*a))*arctan2(sin(2*a), x^2 + cos(2*a))
+ cos(2*a) + I*sin(2*a))*x^2 + 4*(-I*cos(2*a)^2 + 2*cos(2*a)*sin(2*a) + I*
sin(2*a)^2)*arctan2(sin(2*a), x^2 + cos(2*a)) - 2*(x^2*(cos(2*a) + I*sin(2
*a)) + cos(2*a)^2 + 2*I*cos(2*a)*sin(2*a) - sin(2*a)^2)*log(x^4 + 2*x^2*co
s(2*a) + cos(2*a)^2 + sin(2*a)^2) - 4*cos(4*a) - 4*I*sin(4*a))/(x^2 + cos(
2*a) + I*sin(2*a))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(37) = 74$.

Time = 0.30 (sec) , antiderivative size = 221, normalized size of antiderivative = 4.33

$$\int x\tan^2(a+i\log(x))\,dx = -\frac{x^4}{2\left(x^2+\frac{e^{(4ia)}}{x^2}+2e^{(2ia)}\right)} + \frac{2x^2e^{(2ia)}\log\left(x^2+e^{(2ia)}\right)}{x^2+\frac{e^{(4ia)}}{x^2}+2e^{(2ia)}} - \frac{5x^2e^{(2ia)}}{2\left(x^2+\frac{e^{(4ia)}}{x^2}+2e^{(2ia)}\right)} + \frac{4e^{(4ia)}\log\left(x^2+e^{(2ia)}\right)}{x^2+\frac{e^{(4ia)}}{x^2}+2e^{(2ia)}} - \frac{3e^{(4ia)}}{2\left(x^2+\frac{e^{(4ia)}}{x^2}+2e^{(2ia)}\right)} + \frac{2e^{(6ia)}\log\left(x^2+e^{(2ia)}\right)}{\left(x^2+\frac{e^{(4ia)}}{x^2}+2e^{(2ia)}\right)x^2} + \frac{e^{(6ia)}}{2\left(x^2+\frac{e^{(4ia)}}{x^2}+2e^{(2ia)}\right)x^2}$$

input
```
integrate(x*tan(a+I*log(x))^2,x, algorithm="giac")
```

output
```
-1/2*x^4/(x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a)) + 2*x^2*e^(2*I*a)*log(x^2 + e
^(2*I*a))/(x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a)) - 5/2*x^2*e^(2*I*a)/(x^2 + e
^(4*I*a)/x^2 + 2*e^(2*I*a)) + 4*e^(4*I*a)*log(x^2 + e^(2*I*a))/(x^2 + e^(4
*I*a)/x^2 + 2*e^(2*I*a)) - 3/2*e^(4*I*a)/(x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a
)) + 2*e^(6*I*a)*log(x^2 + e^(2*I*a))/((x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a))
*x^2) + 1/2*e^(6*I*a)/((x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a))*x^2)
```

Mupad [B] (verification not implemented)

Time = 19.59 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int x\tan^2(a+i\log(x))\,dx = \frac{2\,\mathrm{e}^{a\,4\mathrm{i}}}{x^2+\mathrm{e}^{a\,2\mathrm{i}}} + 2\,\mathrm{e}^{a\,2\mathrm{i}}\,\ln\left(x^2+\mathrm{e}^{a\,2\mathrm{i}}\right) - \frac{x^2}{2}$$

input
```
int(x*tan(a + log(x)*1i)^2,x)
```

output
```
(2*exp(a*4i))/(exp(a*2i) + x^2) + 2*exp(a*2i)*log(exp(a*2i) + x^2) - x^2/2
```

Reduce [F]

$$\int x\tan^2(a+i\log(x))\,dx = 2\left(\int \tan\left(\log(x)\,i+a\right)xdx\right)i - \tan\left(\log(x)\,i+a\right)i\,x^2 - \frac{x^2}{2}$$

input
```
int(x*tan(a+I*log(x))^2,x)
```

output
```
(4*int(tan(log(x)*i + a)*x,x)*i - 2*tan(log(x)*i + a)*i*x**2 - x**2)/2
```

3.146 $\int \tan^2(a+i\log(x))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \tan^2(a+i\log(x))\,dx = -x-\frac{2e^{2ia}x}{e^{2ia}+x^2}+2e^{ia}\arctan\left(e^{-ia}x\right)$$

output
```
-x-2*exp(2*I*a)*x/(exp(2*I*a)+x^2)+2*exp(I*a)*arctan(x/exp(I*a))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.52

$$\int \tan^2(a+i\log(x))\,dx = 2\arctan(x(\cos(a)-i\sin(a)))(\cos(a)+i\sin(a)) + \frac{-x(3+x^2)\cos(a)+ix(-3+x^2)\sin(a)}{(1+x^2)\cos(a)-i(-1+x^2)\sin(a)}$$

input
```
Integrate[Tan[a + I*Log[x]]^2,x]
```

output
```
2*ArcTan[x*(Cos[a] - I*Sin[a])]*(Cos[a] + I*Sin[a]) + (-(x*(3 + x^2)*Cos[a
]) + I*x*(-3 + x^2)*Sin[a])/((1 + x^2)*Cos[a] - I*(-1 + x^2)*Sin[a])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5002, 898, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(a+i\log(x))\,dx$$

$$\Big\downarrow \text{5002}$$

$$\int \frac{\left(i-\frac{ie^{2ia}}{x^2}\right)^2}{\left(1+\frac{e^{2ia}}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{898}$$

$$\int \frac{\left(ix^2-ie^{2ia}\right)^2}{\left(x^2+e^{2ia}\right)^2}dx$$

$$\Big\downarrow \text{300}$$

$$\int \left(-1+\frac{4e^{2ia}x^2}{\left(x^2+e^{2ia}\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2e^{ia}\arctan\left(e^{-ia}x\right)-\frac{2e^{2ia}x}{x^2+e^{2ia}}-x$$

input
```
Int[Tan[a + I*Log[x]]^2,x]
```

output
```
-x - (2*E^((2*I)*a)*x)/(E^((2*I)*a) + x^2) + 2*E^(I*a)*ArcTan[x/E^(I*a)]
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 898
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5002
```
Int[Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((I - I*E^(2
*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b, d
, p}, x]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result	size
risch	$-3x+\frac{2x}{1+\frac{\mathrm{e}^{2ia}}{x^{2}}}+2\arctan\left(x\,\mathrm{e}^{-ia}\right)\mathrm{e}^{ia}$	36

input
```
int(tan(a+I*ln(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-3*x+2*x/(1+exp(2*I*a)/x^2)+2*arctan(x*exp(-I*a))*exp(I*a)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(34) = 68.

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.67

$$\int \tan^2(a+i\log(x))\,dx = -\frac{x^3+3\,xe^{(2i\,a)}-\left(i\,x^2e^{(i\,a)}+i\,e^{(3i\,a)}\right)\log\left(x+i\,e^{(i\,a)}\right)-\left(-i\,x^2e^{(i\,a)}-i\,e^{(3i\,a)}\right)\log\left(x-i\,e^{(i\,a)}\right)}{x^2+e^{(2i\,a)}}$$

input
```
integrate(tan(a+I*log(x))^2,x, algorithm="fricas")
```

output
```
-(x^3 + 3*x*e^(2*I*a) - (I*x^2*e^(I*a) + I*e^(3*I*a))*log(x + I*e^(I*a)) -
 (-I*x^2*e^(I*a) - I*e^(3*I*a))*log(x - I*e^(I*a)))/(x^2 + e^(2*I*a))
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \tan^2(a+i\log(x))\,dx = -x-\frac{2xe^{2ia}}{x^2+e^{2ia}}-\left(i\log\left(x-ie^{ia}\right)-i\log\left(x+ie^{ia}\right)\right)e^{ia}$$

input
```
integrate(tan(a+I*ln(x))**2,x)
```

output
```
-x - 2*x*exp(2*I*a)/(x**2 + exp(2*I*a)) - (I*log(x - I*exp(I*a)) - I*log(x
 + I*exp(I*a)))*exp(I*a)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(34) = 68.

Time = 0.13 (sec) , antiderivative size = 218, normalized size of antiderivative = 4.74

$$\int \tan^2(a + i\log(x))\,dx = -\frac{2x^3 + 6x(\cos(2a) + i\sin(2a)) + 2(x^2(\cos(a) + i\sin(a)) + (\cos(a) + i\sin(a))\cos(2a) - (-i\cos$$

input
```
integrate(tan(a+I*log(x))^2,x, algorithm="maxima")
```

output
```
-1/2*(2*x^3 + 6*x*(cos(2*a) + I*sin(2*a)) + 2*(x^2*(cos(a) + I*sin(a)) + (
cos(a) + I*sin(a))*cos(2*a) - (-I*cos(a) + sin(a))*sin(2*a))*arctan2(2*x*c
os(a)/(x^2 + cos(a)^2 - 2*x*sin(a) + sin(a)^2), (x^2 - cos(a)^2 - sin(a)^2
)/(x^2 + cos(a)^2 - 2*x*sin(a) + sin(a)^2)) + (x^2*(I*cos(a) - sin(a)) + (
I*cos(a) - sin(a))*cos(2*a) - (cos(a) + I*sin(a))*sin(2*a))*log((x^2 + cos
(a)^2 + 2*x*sin(a) + sin(a)^2)/(x^2 + cos(a)^2 - 2*x*sin(a) + sin(a)^2)))/
(x^2 + cos(2*a) + I*sin(2*a))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(34) = 68.

Time = 0.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.48

$$\int \tan^2(a + i\log(x))\,dx = -\frac{x^3}{x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}} + 2\left(\arctan\left(xe^{(-ia)}\right)e^{(-ia)} - \frac{x}{x^2 + e^{(2ia)}}\right)e^{(2ia)} - \frac{6xe^{(2ia)}}{x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}} - \frac{5e^{(4ia)}}{\left(x^2 + \frac{e^{(4ia)}}{x^2} + 2e^{(2ia)}\right)x}$$

input
```
integrate(tan(a+I*log(x))^2,x, algorithm="giac")
```

output
```
-x^3/(x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a)) + 2*(arctan(x*e^(-I*a))*e^(-I*a)
- x/(x^2 + e^(2*I*a)))*e^(2*I*a) - 6*x*e^(2*I*a)/(x^2 + e^(4*I*a)/x^2 + 2*
e^(2*I*a)) - 5*e^(4*I*a)/((x^2 + e^(4*I*a)/x^2 + 2*e^(2*I*a))*x)
```

Mupad [B] (verification not implemented)

Time = 19.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \tan^2(a+i\log(x))\,dx = -x+2\sqrt{e^{a\,2i}}\,\mathrm{atan}\left(\frac{x}{\sqrt{e^{a\,2i}}}\right)-\frac{2\,x\,e^{a\,2i}}{x^2+e^{a\,2i}}$$

input
```
int(tan(a + log(x)*1i)^2,x)
```

output
```
2*exp(a*2i)^(1/2)*atan(x/exp(a*2i)^(1/2)) - x - (2*x*exp(a*2i))/(exp(a*2i)
 + x^2)
```

Reduce [F]

$$\int \tan^2(a+i\log(x))\,dx = \left(\int \tan\left(\log(x)\,i+a\right)dx\right)i-\tan\left(\log(x)\,i+a\right)ix-x$$

input
```
int(tan(a+I*log(x))^2,x)
```

output
```
int(tan(log(x)*i + a),x)*i - tan(log(x)*i + a)*i*x - x
```

3.147 $\int \frac{\tan^2(a+i\log(x))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\tan^2(a+i\log(x))}{x}\,dx = -\log(x) - i\tan(a+i\log(x))$$

output
```
-ln(x)-I*tan(a+I*ln(x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \frac{\tan^2(a+i\log(x))}{x}\,dx = i\arctan(\tan(a+i\log(x))) - i\tan(a+i\log(x))$$

input
```
Integrate[Tan[a + I*Log[x]]^2/x,x]
```

output
```
I*ArcTan[Tan[a + I*Log[x]]] - I*Tan[a + I*Log[x]]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3039, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(a+i\log(x))}{x}\,dx$$

$$\Big\downarrow \text{ 3039}$$

$$\int \tan^2(a+i\log(x))d\log(x)$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(a+i\log(x))^2d\log(x)$$

$$\Big\downarrow \text{ 3954}$$

$$-\int 1d\log(x)-i\tan(a+i\log(x))$$

$$\Big\downarrow \text{ 24}$$

$$-\log(x)-i\tan(a+i\log(x))$$

input
```
Int[Tan[a + I*Log[x]]^2/x,x]
```

output
```
-Log[x] - I*Tan[a + I*Log[x]]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
norman	$-\ln(x) - i\tan(a + i\ln(x))$	17
parallelrisch	$-\ln(x) - i\tan(a + i\ln(x))$	17
risch	$-\ln(x) + \frac{2}{1+\frac{e^{2ia}}{x^2}}$	21
derivativedivides	$-i(\tan(a + i\ln(x)) - \arctan(\tan(a + i\ln(x))))$	24
default	$-i(\tan(a + i\ln(x)) - \arctan(\tan(a + i\ln(x))))$	24

input
```
int(tan(a+I*ln(x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
-ln(x)-I*tan(a+I*ln(x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(14) = 28.

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \frac{\tan^2(a+i\log(x))}{x}\,dx = -\frac{\left(x^2+e^{(2i\,a)}\right)\log\left(x\right)+2\,e^{(2i\,a)}}{x^2+e^{(2i\,a)}}$$

input
```
integrate(tan(a+I*log(x))^2/x,x, algorithm="fricas")
```

output
```
-((x^2 + e^(2*I*a))*log(x) + 2*e^(2*I*a))/(x^2 + e^(2*I*a))
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\tan^2(a+i\log(x))}{x}\,dx = -\log\left(x\right)-\frac{2e^{2ia}}{x^2+e^{2ia}}$$

input
```
integrate(tan(a+I*ln(x))**2/x,x)
```

output
```
-log(x) - 2*exp(2*I*a)/(x**2 + exp(2*I*a))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\tan^2(a+i\log(x))}{x}\,dx = i\,a-\log\left(x\right)-i\,\tan\left(a+i\,\log\left(x\right)\right)$$

input
```
integrate(tan(a+I*log(x))^2/x,x, algorithm="maxima")
```

output
```
I*a - log(x) - I*tan(a + I*log(x))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\tan^2(a+i\log(x))}{x}\,dx = i\,a - \log(x) - i\,\tan(a + i\,\log(x))$$

input
```
integrate(tan(a+I*log(x))^2/x,x, algorithm="giac")
```

output
```
I*a - log(x) - I*tan(a + I*log(x))
```

Mupad [B] (verification not implemented)

Time = 19.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\tan^2(a+i\log(x))}{x}\,dx = -\ln(x) - \tan(a + \ln(x)\ 1\mathrm{i})\ 1\mathrm{i}$$

input
```
int(tan(a + log(x)*1i)^2/x,x)
```

output
```
- tan(a + log(x)*1i)*1i - log(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\tan^2(a+i\log(x))}{x}\,dx = -\log(x) - \tan(\log(x)\,i + a)\,i$$

input
```
int(tan(a+I*log(x))^2/x,x)
```

output
```
- (log(x) + tan(log(x)*i + a)*i)
```

3.148 $\int \frac{\tan^2(a+i\log(x))}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \frac{\tan^2(a+i\log(x))}{x^2}\,dx=\frac{1}{x}+\frac{2x}{e^{2ia}+x^2}+2e^{-ia}\arctan\left(e^{-ia}x\right)$$

output
```
1/x+2*x/(exp(2*I*a)+x^2)+2*arctan(x/exp(I*a))/exp(I*a)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.85

$$\begin{aligned}\int \frac{\tan^2(a+i\log(x))}{x^2}\,dx&=\frac{1}{x}+2\arctan(x(\cos(a)-i\sin(a)))\cos(a)\\&\quad-2i\arctan(x(\cos(a)-i\sin(a)))\sin(a)\\&\quad+\frac{2x(\cos(a)-i\sin(a))}{(1+x^2)\cos(a)-i\left(-1+x^2\right)\sin(a)}\end{aligned}$$

input
```
Integrate[Tan[a + I*Log[x]]^2/x^2,x]
```

output
```
x^(-1) + 2*ArcTan[x*(Cos[a] - I*Sin[a])]*Cos[a] - (2*I)*ArcTan[x*(Cos[a] -
 I*Sin[a])]*Sin[a] + (2*x*(Cos[a] - I*Sin[a]))/((1 + x^2)*Cos[a] - I*(-1 +
 x^2)*Sin[a])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5006, 947, 365, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(a+i\log(x))}{x^2}\,dx$$

$$\Big\downarrow \text{5006}$$

$$\int \frac{\left(i-\frac{ie^{2ia}}{x^2}\right)^2}{x^2\left(1+\frac{e^{2ia}}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{947}$$

$$\int \frac{\left(ix^2-ie^{2ia}\right)^2}{x^2\left(x^2+e^{2ia}\right)^2}dx$$

$$\Big\downarrow \text{365}$$

$$e^{-2ia}\int \frac{5e^{4ia}-e^{2ia}x^2}{\left(x^2+e^{2ia}\right)^2}dx+\frac{e^{2ia}}{x\left(x^2+e^{2ia}\right)}$$

$$\Big\downarrow \text{298}$$

$$e^{-2ia}\left(2e^{2ia}\int \frac{1}{x^2+e^{2ia}}dx+\frac{3e^{2ia}x}{x^2+e^{2ia}}\right)+\frac{e^{2ia}}{x\left(x^2+e^{2ia}\right)}$$

$$\Big\downarrow \text{216}$$

$$e^{-2ia}\left(2e^{ia}\arctan\left(e^{-ia}x\right)+\frac{3e^{2ia}x}{x^2+e^{2ia}}\right)+\frac{e^{2ia}}{x\left(x^2+e^{2ia}\right)}$$

input
```
Int[Tan[a + I*Log[x]]^2/x^2,x]
```

output
```
E^((2*I)*a)/(x*(E^((2*I)*a) + x^2)) + ((3*E^((2*I)*a)*x)/(E^((2*I)*a) + x^
2) + 2*E^(I*a)*ArcTan[x/E^(I*a)])/E^((2*I)*a)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 947
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

method	result	size
risch	$\frac{1}{x}+\frac{2}{x\left(1+\frac{e^{2ia}}{x^2}\right)}+2\arctan\left(x\,e^{-ia}\right)e^{-ia}$	38

input
```
int(tan(a+I*ln(x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x+2/x/(1+exp(2*I*a)/x^2)+2*arctan(x*exp(-I*a))*exp(-I*a)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(30) = 60.

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.00

$$\int \frac{\tan^2(a+i\log(x))}{x^2}\,dx = \frac{3\,x^2e^{(i\,a)}+\left(i\,x^3+i\,xe^{(2i\,a)}\right)\log\left(x+i\,e^{(i\,a)}\right)+\left(-i\,x^3-i\,xe^{(2i\,a)}\right)\log\left(x-i\,e^{(i\,a)}\right)+e^{(3i\,a)}}{x^3e^{(i\,a)}+xe^{(3i\,a)}}$$

input
```
integrate(tan(a+I*log(x))^2/x^2,x, algorithm="fricas")
```

output
```
(3*x^2*e^(I*a) + (I*x^3 + I*x*e^(2*I*a))*log(x + I*e^(I*a)) + (-I*x^3 - I*
x*e^(2*I*a))*log(x - I*e^(I*a)) + e^(3*I*a))/(x^3*e^(I*a) + x*e^(3*I*a))
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

$$\int \frac{\tan^2(a+i\log(x))}{x^2}\,dx = -\frac{-3x^2-e^{2ia}}{x^3+xe^{2ia}} - \left(i\log\left(x-ie^{ia}\right)-i\log\left(x+ie^{ia}\right)\right)e^{-ia}$$

input
```
integrate(tan(a+I*ln(x))**2/x**2,x)
```

output
```
-(-3*x**2 - exp(2*I*a))/(x**3 + x*exp(2*I*a)) - (I*log(x - I*exp(I*a)) - I
*log(x + I*exp(I*a)))*exp(-I*a)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(30) = 60$.

Time = 0.13 (sec) , antiderivative size = 223, normalized size of antiderivative = 5.72

$$\int \frac{\tan^2(a+i\log(x))}{x^2}\,dx$$
$$= \frac{6\,x^2-2\,(x^3(\cos(a)-i\,\sin(a))+((\cos(a)-i\,\sin(a))\cos(2\,a)+(i\,\cos(a)+\sin(a))\sin(2\,a))x)\arctan}{}$$

input
```
integrate(tan(a+I*log(x))^2/x^2,x, algorithm="maxima")
```

output
```
1/2*(6*x^2 - 2*(x^3*(cos(a) - I*sin(a)) + ((cos(a) - I*sin(a))*cos(2*a) +
(I*cos(a) + sin(a))*sin(2*a))*x)*arctan2(2*x*cos(a)/(x^2 + cos(a)^2 - 2*x*
sin(a) + sin(a)^2), (x^2 - cos(a)^2 - sin(a)^2)/(x^2 + cos(a)^2 - 2*x*sin(
a) + sin(a)^2)) + (x^3*(-I*cos(a) - sin(a)) + ((-I*cos(a) - sin(a))*cos(2*
a) + (cos(a) - I*sin(a))*sin(2*a))*x)*log((x^2 + cos(a)^2 + 2*x*sin(a) + s
in(a)^2)/(x^2 + cos(a)^2 - 2*x*sin(a) + sin(a)^2)) + 2*cos(2*a) + 2*I*sin(
2*a))/(x^3 + x*(cos(2*a) + I*sin(2*a)))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(30) = 60.

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.87

$$\int \frac{\tan^2(a + i\log(x))}{x^2} \, dx = 2 \left(\arctan\left(xe^{(-i\,a)}\right) e^{(-3i\,a)} + \frac{xe^{(-2i\,a)}}{x^2 + e^{(2i\,a)}} \right) e^{(2i\,a)} + \frac{5}{x\left(\frac{e^{(2i\,a)}}{x^2} + 1\right)} + \frac{e^{(2i\,a)}}{x^3\left(\frac{e^{(2i\,a)}}{x^2} + 1\right)}$$

input
```
integrate(tan(a+I*log(x))^2/x^2,x, algorithm="giac")
```

output
```
2*(arctan(x*e^(-I*a))*e^(-3*I*a) + x*e^(-2*I*a)/(x^2 + e^(2*I*a)))*e^(2*I*
a) + 5/(x*(e^(2*I*a)/x^2 + 1)) + e^(2*I*a)/(x^3*(e^(2*I*a)/x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 20.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{\tan^2(a + i\log(x))}{x^2} \, dx = \frac{2\,\mathrm{atan}\left(\frac{x}{\sqrt{e^{a\,2i}}}\right)}{\sqrt{e^{a\,2i}}} + \frac{3\,x^2 + e^{a\,2i}}{x^3 + e^{a\,2i}\,x}$$

input
```
int(tan(a + log(x)*1i)^2/x^2,x)
```

output
```
(2*atan(x/exp(a*2i)^(1/2)))/exp(a*2i)^(1/2) + (exp(a*2i) + 3*x^2)/(x^3 + x
*exp(a*2i))
```

Reduce [F]

$$\int \frac{\tan^2(a + i\log(x))}{x^2}\,dx = \frac{-\left(\int \frac{\tan(\log(x)i+a)}{x^2}dx\right)ix - \tan\left(\log(x)\,i + a\right)i + 1}{x}$$

input
```
int(tan(a+I*log(x))^2/x^2,x)
```

output
```
( - int(tan(log(x)*i + a)/x**2,x)*i*x - tan(log(x)*i + a)*i + 1)/x
```

3.149 $\int \frac{\tan^2(a+i\log(x))}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{\tan^2(a+i\log(x))}{x^3}\,dx = -\frac{2e^{-2ia}}{1+\frac{e^{2ia}}{x^2}}+\frac{1}{2x^2}-2e^{-2ia}\log\left(1+\frac{e^{2ia}}{x^2}\right)$$

output
```
-2/exp(2*I*a)/(1+exp(2*I*a)/x^2)+1/2/x^2-2*ln(1+exp(2*I*a)/x^2)/exp(2*I*a)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 150 vs. 2(55) = 110.

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.73

$$\begin{aligned}\int \frac{\tan^2(a+i\log(x))}{x^3}\,dx &= \frac{1}{2x^2}-2i\arctan\left(\frac{(1+x^2)\cot(a)}{-1+x^2}\right)\cos(2a)\\ &+4\cos(2a)\log(x)-\cos(2a)\log\left(1+x^4+2x^2\cos(2a)\right)\\ &+\frac{2\cos(a)-2i\sin(a)}{(1+x^2)\cos(a)-i(-1+x^2)\sin(a)}\\ &-2\arctan\left(\frac{(1+x^2)\cot(a)}{-1+x^2}\right)\sin(2a)-4i\log(x)\sin(2a)\\ &+i\log\left(1+x^4+2x^2\cos(2a)\right)\sin(2a)\end{aligned}$$

input
```
Integrate[Tan[a + I*Log[x]]^2/x^3,x]
```

output
```
1/(2*x^2) - (2*I)*ArcTan[((1 + x^2)*Cot[a])/(-1 + x^2)]*Cos[2*a] + 4*Cos[2
*a]*Log[x] - Cos[2*a]*Log[1 + x^4 + 2*x^2*Cos[2*a]] + (2*Cos[a] - (2*I)*Si
n[a])/((1 + x^2)*Cos[a] - I*(-1 + x^2)*Sin[a]) - 2*ArcTan[((1 + x^2)*Cot[a
])/(-1 + x^2)]*Sin[2*a] - (4*I)*Log[x]*Sin[2*a] + I*Log[1 + x^4 + 2*x^2*Co
s[2*a]]*Sin[2*a]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5006, 946, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(a+i\log(x))}{x^3}\,dx$$

$$\Big\downarrow \text{5006}$$

$$\int \frac{\left(i-\frac{ie^{2ia}}{x^2}\right)^2}{x^3\left(1+\frac{e^{2ia}}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{946}$$

$$-\frac{1}{2}\int -\frac{\left(1-\frac{e^{2ia}}{x^2}\right)^2}{\left(1+\frac{e^{2ia}}{x^2}\right)^2}d\frac{1}{x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{\left(1-\frac{e^{2ia}}{x^2}\right)^2}{\left(1+\frac{e^{2ia}}{x^2}\right)^2}d\frac{1}{x^2}$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(1-\frac{4}{1+\frac{e^{2ia}}{x^2}}+\frac{4}{\left(1+\frac{e^{2ia}}{x^2}\right)^2}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{4e^{-2ia}}{1+\frac{e^{2ia}}{x^2}}-4e^{-2ia}\log\left(1+\frac{e^{2ia}}{x^2}\right)+\frac{1}{x^2}\right)$$

input
```
Int[Tan[a + I*Log[x]]^2/x^3,x]
```

output
```
(-4/(E^((2*I)*a)*(1 + E^((2*I)*a)/x^2)) + x^(-2) - (4*Log[1 + E^((2*I)*a)/
x^2])/E^((2*I)*a))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

method	result	size
risch	$\frac{1}{2x^2}+\frac{2}{x^2\left(1+\frac{e^{2ia}}{x^2}\right)}+4\,e^{-2ia}\ln\left(x\right)-2\,e^{-2ia}\ln\left(e^{2ia}+x^2\right)$	51

input
```
int(tan(a+I*ln(x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/x^2+2/x^2/(1+exp(2*I*a)/x^2)+4*exp(-2*I*a)*ln(x)-2*exp(-2*I*a)*ln(exp(
2*I*a)+x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

$$\int \frac{\tan^2(a+i\log(x))}{x^3}\,dx = \frac{5\,x^2e^{(2i\,a)}-4\left(x^4+x^2e^{(2i\,a)}\right)\log\left(x^2+e^{(2i\,a)}\right)+8\left(x^4+x^2e^{(2i\,a)}\right)\log\left(x\right)+e^{(4i\,a)}}{2\left(x^4e^{(2i\,a)}+x^2e^{(4i\,a)}\right)}$$

input
```
integrate(tan(a+I*log(x))^2/x^3,x, algorithm="fricas")
```

output
```
1/2*(5*x^2*e^(2*I*a) - 4*(x^4 + x^2*e^(2*I*a))*log(x^2 + e^(2*I*a)) + 8*(x
^4 + x^2*e^(2*I*a))*log(x) + e^(4*I*a))/(x^4*e^(2*I*a) + x^2*e^(4*I*a))
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int \frac{\tan^2(a+i\log(x))}{x^3}\,dx = -\frac{-5x^2-e^{2ia}}{2x^4+2x^2e^{2ia}}+4e^{-2ia}\log\left(x\right)-2e^{-2ia}\log\left(x^2+e^{2ia}\right)$$

input
```
integrate(tan(a+I*ln(x))**2/x**3,x)
```

output
```
-(-5*x**2 - exp(2*I*a))/(2*x**4 + 2*x**2*exp(2*I*a)) + 4*exp(-2*I*a)*log(x
) - 2*exp(-2*I*a)*log(x**2 + exp(2*I*a))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^2(a+i\log(x))}{x^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(tan(a+I*log(x))^2/x^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(41) = 82.

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 3.24

$$\int \frac{\tan^2(a+i\log(x))}{x^3}\,dx = -\frac{2\log\left(-x^2-e^{(2ia)}\right)}{\frac{e^{(4ia)}}{x^2}+e^{(2ia)}} + \frac{4\log(x)}{\frac{e^{(4ia)}}{x^2}+e^{(2ia)}} - \frac{2}{\frac{e^{(4ia)}}{x^2}+e^{(2ia)}} - \frac{2e^{(2ia)}\log\left(-x^2-e^{(2ia)}\right)}{x^2\left(\frac{e^{(4ia)}}{x^2}+e^{(2ia)}\right)} + \frac{4e^{(2ia)}\log(x)}{x^2\left(\frac{e^{(4ia)}}{x^2}+e^{(2ia)}\right)} + \frac{e^{(2ia)}}{2x^2\left(\frac{e^{(4ia)}}{x^2}+e^{(2ia)}\right)} + \frac{e^{(4ia)}}{2x^4\left(\frac{e^{(4ia)}}{x^2}+e^{(2ia)}\right)}$$

input
```
integrate(tan(a+I*log(x))^2/x^3,x, algorithm="giac")
```

output
```
-2*log(-x^2 - e^(2*I*a))/(e^(4*I*a)/x^2 + e^(2*I*a)) + 4*log(x)/(e^(4*I*a)
/x^2 + e^(2*I*a)) - 2/(e^(4*I*a)/x^2 + e^(2*I*a)) - 2*e^(2*I*a)*log(-x^2 -
 e^(2*I*a))/(x^2*(e^(4*I*a)/x^2 + e^(2*I*a))) + 4*e^(2*I*a)*log(x)/(x^2*(e
^(4*I*a)/x^2 + e^(2*I*a))) + 1/2*e^(2*I*a)/(x^2*(e^(4*I*a)/x^2 + e^(2*I*a)
)) + 1/2*e^(4*I*a)/(x^4*(e^(4*I*a)/x^2 + e^(2*I*a)))
```

Mupad [B] (verification not implemented)

Time = 20.40 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{\tan^2(a+i\log(x))}{x^3}\,dx = -2\,e^{-a\,2i}\,\ln\left(x^2+e^{a\,2i}\right)+4\,e^{-a\,2i}\,\ln\left(x\right)+\frac{\frac{5\,x^2}{2}+\frac{e^{a\,2i}}{2}}{x^4+e^{a\,2i}\,x^2}$$

input
```
int(tan(a + log(x)*1i)^2/x^3,x)
```

output
```
4*exp(-a*2i)*log(x) - 2*exp(-a*2i)*log(exp(a*2i) + x^2) + (exp(a*2i)/2 + (
5*x^2)/2)/(x^2*exp(a*2i) + x^4)
```

Reduce [F]

$$\int \frac{\tan^2(a+i\log(x))}{x^3}\,dx = \frac{-4\left(\int \frac{\tan(\log(x)i+a)}{x^3}dx\right)i\,x^2-2\tan\left(\log(x)\,i+a\right)i+1}{2x^2}$$

input
```
int(tan(a+I*log(x))^2/x^3,x)
```

output
```
( - 4*int(tan(log(x)*i + a)/x**3,x)*i*x**2 - 2*tan(log(x)*i + a)*i + 1)/(2
*x**2)
```

3.150 $\int (ex)^m \tan(a + i\log(x))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 71

$$\int (ex)^m \tan(a+i\log(x))\,dx = -\frac{i(ex)^{1+m}}{e(1+m)} + \frac{2i(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1-m),\frac{1-m}{2},-\frac{e^{2ia}}{x^2}\right)}{e(1+m)}$$

output
```
-I*(e*x)^(1+m)/e/(1+m)+2*I*(e*x)^(1+m)*hypergeom([1, -1/2-1/2*m],[1/2-1/2*
m],-exp(2*I*a)/x^2)/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.75

$$\int (ex)^m \tan(a+i\log(x))\,dx = \frac{x(ex)^m(\cos(a)-i\sin(a))\left((3+m)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-x^2(\cos(2a)-i\sin(2a))\right)(-i\cos\right.}{(1+m}$$

input
```
Integrate[(e*x)^m*Tan[a + I*Log[x]],x]
```

output
```
(x*(e*x)^m*(Cos[a] - I*Sin[a])*((3 + m)*Hypergeometric2F1[1, (1 + m)/2, (3
 + m)/2, -(x^2*(Cos[2*a] - I*Sin[2*a]))]*((-I)*Cos[a] + Sin[a]) + (1 + m)*
x^2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, -(x^2*(Cos[2*a] - I*Sin[2*a
]))]*(I*Cos[a] + Sin[a])))/((1 + m)*(3 + m))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5006, 959, 862, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \tan(a + i\log(x))\,dx$$

$$\Big\downarrow \text{5006}$$

$$\int \frac{\left(i - \frac{ie^{2ia}}{x^2}\right)(ex)^m}{1 + \frac{e^{2ia}}{x^2}}dx$$

$$\Big\downarrow \text{959}$$

$$2i\int \frac{(ex)^m}{1 + \frac{e^{2ia}}{x^2}}dx - \frac{i(ex)^{m+1}}{e(m+1)}$$

$$\Big\downarrow \text{862}$$

$$-\frac{2i\left(\frac{1}{x}\right)^{m+1}(ex)^{m+1}\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{1+\frac{e^{2ia}}{x^2}}d\frac{1}{x}}{e} - \frac{i(ex)^{m+1}}{e(m+1)}$$

$$\Big\downarrow \text{278}$$

$$\frac{2i(ex)^{m+1}\text{Hypergeometric2F1}\left(1, \frac{1}{2}(-m-1), \frac{1-m}{2}, -\frac{e^{2ia}}{x^2}\right)}{e(m+1)} - \frac{i(ex)^{m+1}}{e(m+1)}$$

input
```
Int[(e*x)^m*Tan[a + I*Log[x]],x]
```

output
```
((-I)*(e*x)^(1 + m))/(e*(1 + m)) + ((2*I)*(e*x)^(1 + m)*Hypergeometric2F1[
1, (-1 - m)/2, (1 - m)/2, -(E^((2*I)*a)/x^2)])/(e*(1 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] && !RationalQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [F]

$$\int (ex)^m \tan(a + i \ln(x)) \, dx$$

input
```
int((e*x)^m*tan(a+I*ln(x)),x)
```

output
```
int((e*x)^m*tan(a+I*ln(x)),x)
```

Fricas [F]

$$\int (ex)^m \tan(a + i\log(x))\, dx = \int (ex)^m \tan(a + i\log(x))\, dx$$

input
```
integrate((e*x)^m*tan(a+I*log(x)),x, algorithm="fricas")
```

output
```
integral((I*x^2 - I*e^(2*I*a))*e^(m*log(e) + m*log(x))/(x^2 + e^(2*I*a)),
x)
```

Sympy [F]

$$\int (ex)^m \tan(a + i\log(x))\, dx = \int (ex)^m \tan(a + i\log(x))\, dx$$

input
```
integrate((e*x)**m*tan(a+I*ln(x)),x)
```

output
```
Integral((e*x)**m*tan(a + I*log(x)), x)
```

Maxima [F]

$$\int (ex)^m \tan(a + i\log(x))\, dx = \int (ex)^m \tan(a + i\log(x))\, dx$$

input
```
integrate((e*x)^m*tan(a+I*log(x)),x, algorithm="maxima")
```

output
```
integrate((e*x)^m*tan(a + I*log(x)), x)
```

Giac [F]

$$\int (ex)^m \tan(a+i\log(x))\,dx = \int (ex)^m \tan\left(a+i\,\log\left(x\right)\right)\,dx$$

input
```
integrate((e*x)^m*tan(a+I*log(x)),x, algorithm="giac")
```

output
```
integrate((e*x)^m*tan(a + I*log(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \tan(a+i\log(x))\,dx = \int \tan(a+\ln\left(x\right)\,1\mathrm{i})\,\left(e\,x\right)^m\,dx$$

input
```
int(tan(a + log(x)*1i)*(e*x)^m,x)
```

output
```
int(tan(a + log(x)*1i)*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \tan(a+i\log(x))\,dx = e^m\left(\int x^m \tan\left(\log(x)\,i+a\right)dx\right)$$

input
```
int((e*x)^m*tan(a+I*log(x)),x)
```

output
```
e**m*int(x**m*tan(log(x)*i + a),x)
```

3.151 $\int (ex)^m \tan^2(a+i\log(x))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 77

$$\int (ex)^m \tan^2(a+i\log(x))\,dx = -\frac{x(ex)^m}{1+m}+\frac{2x(ex)^m}{1+\frac{e^{2ia}}{x^2}} - 2x(ex)^m \text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1-m),\frac{1-m}{2},-\frac{e^{2ia}}{x^2}\right)$$

output
```
-x*(e*x)^m/(1+m)+2*x*(e*x)^m/(1+exp(2*I*a)/x^2)-2*x*(e*x)^m*hypergeom([1,
-1/2-1/2*m],[1/2-1/2*m],-exp(2*I*a)/x^2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.12

$$\int (ex)^m \tan^2(a+i\log(x))\,dx = \frac{x(ex)^m \left(-1+4\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-x^2(\cos(2a)-i\sin(2a))\right)-4\,\text{Hypergeometric2F1}\left(2\right.\right.}{1+m}$$

input
```
Integrate[(e*x)^m*Tan[a + I*Log[x]]^2,x]
```

output
```
(x*(e*x)^m*(-1 + 4*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -(x^2*(Cos[2
*a] - I*Sin[2*a]))] - 4*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -(x^2*(
Cos[2*a] - I*Sin[2*a]))]))/(1 + m)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.58, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5006, 999, 25, 366, 27, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \tan^2(a+i\log(x))\,dx$$

$$\Big\downarrow \text{5006}$$

$$\int \frac{\left(i-\frac{ie^{2ia}}{x^2}\right)^2 (ex)^m}{\left(1+\frac{e^{2ia}}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{999}$$

$$-\left(\frac{1}{x}\right)^m (ex)^m \int -\frac{\left(1-\frac{e^{2ia}}{x^2}\right)^2 \left(\frac{1}{x}\right)^{-m-2}}{\left(1+\frac{e^{2ia}}{x^2}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$\left(\frac{1}{x}\right)^m (ex)^m \int \frac{\left(1-\frac{e^{2ia}}{x^2}\right)^2 \left(\frac{1}{x}\right)^{-m-2}}{\left(1+\frac{e^{2ia}}{x^2}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{366}$$

$$-\left(\frac{1}{x}\right)^m (ex)^m \left(\frac{1}{2}e^{-4ia}\int -\frac{2\left(e^{4ia}(2m+3)+\frac{e^{6ia}}{x^2}\right)\left(\frac{1}{x}\right)^{-m-2}}{1+\frac{e^{2ia}}{x^2}}d\frac{1}{x}-\frac{2\left(\frac{1}{x}\right)^{-m-1}}{1+\frac{e^{2ia}}{x^2}}\right)$$

$$\Big\downarrow \text{27}$$

$$-\left(\frac{1}{x}\right)^m (ex)^m \left(-e^{-4ia}\int \frac{\left(e^{4ia}(2m+3)+\frac{e^{6ia}}{x^2}\right)\left(\frac{1}{x}\right)^{-m-2}}{1+\frac{e^{2ia}}{x^2}}d\frac{1}{x}-\frac{2\left(\frac{1}{x}\right)^{-m-1}}{1+\frac{e^{2ia}}{x^2}}\right)$$

$$\Big\downarrow \text{363}$$

$$-\left(\frac{1}{x}\right)^m (ex)^m \left(-e^{-4ia}\left(2e^{4ia}(m+1)\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{1+\frac{e^{2ia}}{x^2}}d\frac{1}{x}-\frac{e^{4ia}\left(\frac{1}{x}\right)^{-m-1}}{m+1}\right)-\frac{2\left(\frac{1}{x}\right)^{-m-1}}{1+\frac{e^{2ia}}{x^2}}\right)$$

$$\Big\downarrow \text{278}$$

$$-\left(\frac{1}{x}\right)^m (ex)^m \left(-e^{-4ia}\left(-2e^{4ia}\left(\frac{1}{x}\right)^{-m-1}\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-m-1),\frac{1-m}{2},-\frac{e^{2ia}}{x^2}\right)-\frac{e^{4ia}\left(\frac{1}{x}\right)^{-m-1}}{m+1}\right.\right.$$

input
```
Int[(e*x)^m*Tan[a + I*Log[x]]^2,x]
```

output
```
-((x^(-1))^m*(e*x)^m*((-2*(x^(-1))^(-1 - m))/(1 + E^((2*I)*a)/x^2) - (-((E
^((4*I)*a)*(x^(-1))^(-1 - m))/(1 + m)) - 2*E^((4*I)*a)*(x^(-1))^(-1 - m)*H
ypergeometric2F1[1, (-1 - m)/2, (1 - m)/2, -(E^((2*I)*a)/x^2)])/E^((4*I)*a
)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 999
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(-(e*x)^m)*(x^(-1))^m   Subst[Int[(a + b/x^n)^p*(
(c + d/x^n)^q/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q}
, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [F]

$$\int (ex)^m \tan (a + i \ln (x))^2 \, dx$$

input
```
int((e*x)^m*tan(a+I*ln(x))^2,x)
```

output
```
int((e*x)^m*tan(a+I*ln(x))^2,x)
```

Fricas [F]

$$\int (ex)^m \tan^2(a+i\log(x))\,dx = \int (ex)^m \tan\left(a+i\,\log\left(x\right)\right)^2\,dx$$

input
```
integrate((e*x)^m*tan(a+I*log(x))^2,x, algorithm="fricas")
```

output
```
integral(-(x^4 - 2*x^2*e^(2*I*a) + e^(4*I*a))*e^(m*log(e) + m*log(x))/(x^4
 + 2*x^2*e^(2*I*a) + e^(4*I*a)), x)
```

Sympy [F]

$$\int (ex)^m \tan^2(a+i\log(x))\,dx = \int (ex)^m \tan^2\left(a+i\log\left(x\right)\right)dx$$

input
```
integrate((e*x)**m*tan(a+I*ln(x))**2,x)
```

output
```
Integral((e*x)**m*tan(a + I*log(x))**2, x)
```

Maxima [F]

$$\int (ex)^m \tan^2(a+i\log(x))\,dx = \int (ex)^m \tan\left(a+i\,\log\left(x\right)\right)^2\,dx$$

input
```
integrate((e*x)^m*tan(a+I*log(x))^2,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*tan(a + I*log(x))^2, x)
```

Giac [F]

$$\int (ex)^m \tan^2(a+i\log(x))\,dx = \int (ex)^m \tan\left(a+i\,\log\left(x\right)\right)^2\,dx$$

input
```
integrate((e*x)^m*tan(a+I*log(x))^2,x, algorithm="giac")
```

output
```
integrate((e*x)^m*tan(a + I*log(x))^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \tan^2(a+i\log(x))\,dx = \int \tan(a+\ln\left(x\right)\,1\mathrm{i})^2\,\left(e\,x\right)^m\,dx$$

input
```
int(tan(a + log(x)*1i)^2*(e*x)^m,x)
```

output
```
int(tan(a + log(x)*1i)^2*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \tan^2(a+i\log(x))\,dx = \frac{e^m\left(-x^m\tan\left(\log(x)\,i+a\right)imx - x^m\tan\left(\log(x)\,i+a\right)ix - x^m x + \left(\int x^m\tan\left(\log(x)\,i+a\right)dx\right)i\,m^2+2}{m+1}$$

input
```
int((e*x)^m*tan(a+I*log(x))^2,x)
```

output
```
(e**m*( - x**m*tan(log(x)*i + a)*i*m*x - x**m*tan(log(x)*i + a)*i*x - x**m
*x + int(x**m*tan(log(x)*i + a),x)*i*m**2 + 2*int(x**m*tan(log(x)*i + a),x
)*i*m + int(x**m*tan(log(x)*i + a),x)*i))/(m + 1)
```

3.152 $\int (ex)^m \tan^3(a+i\log(x))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 126

$$\int (ex)^m \tan^3(a+i\log(x))\,dx = \frac{ix(ex)^m}{1+m}+\frac{2ix(ex)^m}{\left(1+\frac{e^{2ia}}{x^2}\right)^2}-\frac{i(1-m)x(ex)^m}{1+\frac{e^{2ia}}{x^2}} - \frac{i(3+2m+m^2)\,x(ex)^m \operatorname{Hypergeometric2F1}\left(1,\frac{1}{2}(-1-m),\frac{1-m}{2},-\frac{e^{2ia}}{x^2}\right)}{1+m}$$

output
```
I*x*(e*x)^m/(1+m)+2*I*x*(e*x)^m/(1+exp(2*I*a)/x^2)^2-I*(1-m)*x*(e*x)^m/(1+
exp(2*I*a)/x^2)-I*(m^2+2*m+3)*x*(e*x)^m*hypergeom([1, -1/2-1/2*m],[1/2-1/2
*m],-exp(2*I*a)/x^2)/(1+m)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.99

$$\int (ex)^m \tan^3(a+i\log(x))\,dx = \frac{ix(ex)^m\left(-1+6\operatorname{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-x^2(\cos(2a)-i\sin(2a))\right)-12\operatorname{Hypergeometric2F1}\right.}{1}$$

input
```
Integrate[(e*x)^m*Tan[a + I*Log[x]]^3,x]
```

output
```
(I*x*(e*x)^m*(-1 + 6*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -(x^2*(Cos
[2*a] - I*Sin[2*a]))] - 12*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -(x^
2*(Cos[2*a] - I*Sin[2*a]))] + 8*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2,
 -(x^2*(Cos[2*a] - I*Sin[2*a]))]))/(1 + m)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.79, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {5006, 999, 26, 370, 27, 439, 27, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \tan^3(a+i\log(x))\,dx$$

$$\Big\downarrow \text{5006}$$

$$\int \frac{\left(i-\frac{ie^{2ia}}{x^2}\right)^3 (ex)^m}{\left(1+\frac{e^{2ia}}{x^2}\right)^3}dx$$

$$\Big\downarrow \text{999}$$

$$-\left(\frac{1}{x}\right)^m (ex)^m \int -\frac{i\left(1-\frac{e^{2ia}}{x^2}\right)^3 \left(\frac{1}{x}\right)^{-m-2}}{\left(1+\frac{e^{2ia}}{x^2}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{1}{x}\right)^m (ex)^m \int \frac{\left(1-\frac{e^{2ia}}{x^2}\right)^3 \left(\frac{1}{x}\right)^{-m-2}}{\left(1+\frac{e^{2ia}}{x^2}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{370}$$

$$i\left(\frac{1}{x}\right)^m(ex)^m\left(\frac{\left(1-\frac{e^{2ia}}{x^2}\right)^2\left(\frac{1}{x}\right)^{-m-1}}{2\left(1+\frac{e^{2ia}}{x^2}\right)^2}-\frac{1}{4}e^{-2ia}\int -\frac{2\left(1-\frac{e^{2ia}}{x^2}\right)\left(\frac{e^{4ia}(1-m)}{x^2}+e^{2ia}(m+3)\right)\left(\frac{1}{x}\right)^{-m-2}}{\left(1+\frac{e^{2ia}}{x^2}\right)^2}d\frac{1}{x}\right)$$

$$\Big\downarrow \; 27$$

$$i\left(\frac{1}{x}\right)^m(ex)^m\left(\frac{1}{2}e^{-2ia}\int \frac{\left(1-\frac{e^{2ia}}{x^2}\right)\left(\frac{e^{4ia}(1-m)}{x^2}+e^{2ia}(m+3)\right)\left(\frac{1}{x}\right)^{-m-2}}{\left(1+\frac{e^{2ia}}{x^2}\right)^2}d\frac{1}{x}+\frac{\left(1-\frac{e^{2ia}}{x^2}\right)^2\left(\frac{1}{x}\right)^{-m-1}}{2\left(1+\frac{e^{2ia}}{x^2}\right)^2}\right)$$

$$\Big\downarrow \; 439$$

$$i\left(\frac{1}{x}\right)^m(ex)^m\left(\frac{1}{2}e^{-2ia}\left(\frac{\left(\frac{1}{x}\right)^{-m-1}\left(\frac{e^{4ia}(1-m)}{x^2}+e^{2ia}(m+3)\right)}{1+\frac{e^{2ia}}{x^2}}-\frac{1}{2}e^{-2ia}\int -\frac{2\left(\frac{e^{6ia}(1-m)m}{x^2}+e^{4ia}(m+2)(m+3\right.}{1+\frac{e^{2ia}}{x^2}}\right.\right.$$

$$\Big\downarrow \; 27$$

$$i\left(\frac{1}{x}\right)^m(ex)^m\left(\frac{1}{2}e^{-2ia}\left(e^{-2ia}\int \frac{\left(\frac{e^{6ia}(1-m)m}{x^2}+e^{4ia}(m+2)(m+3)\right)\left(\frac{1}{x}\right)^{-m-2}}{1+\frac{e^{2ia}}{x^2}}d\frac{1}{x}+\frac{\left(\frac{1}{x}\right)^{-m-1}\left(\frac{e^{4ia}(1-m)}{x^2}+e^{2i}\right.}{1+\frac{e^{2ia}}{x^2}}\right.\right.$$

$$\Big\downarrow \; 363$$

$$i\left(\frac{1}{x}\right)^m(ex)^m\left(\frac{1}{2}e^{-2ia}\left(e^{-2ia}\left(2e^{4ia}\left(m^2+2m+3\right)\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{1+\frac{e^{2ia}}{x^2}}d\frac{1}{x}-\frac{e^{4ia}(1-m)m\left(\frac{1}{x}\right)^{-m-1}}{m+1}\right)+\frac{\left(\frac{1}{x}\right)^{-m-1}\left(e\right.}{}\right.\right.$$

$$\Big\downarrow \; 278$$

$$i\left(\frac{1}{x}\right)^m(ex)^m\left(\frac{1}{2}e^{-2ia}\left(e^{-2ia}\left(-\frac{2e^{4ia}\left(m^2+2m+3\right)\left(\frac{1}{x}\right)^{-m-1}\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-m-1),\frac{1-m}{2},-\frac{e^{2ia}}{x^2}\right.}{m+1}\right.\right.\right.$$

input
```
Int[(e*x)^m*Tan[a + I*Log[x]]^3,x]
```

output
```
I*(x^(-1))^m*(e*x)^m*(((1 - E^((2*I)*a)/x^2)^2*(x^(-1))^(-1 - m))/(2*(1 +
E^((2*I)*a)/x^2)^2) + (((E^((2*I)*a)*(3 + m) + (E^((4*I)*a)*(1 - m))/x^2)*
(x^(-1))^(-1 - m))/(1 + E^((2*I)*a)/x^2) + (-((E^((4*I)*a)*(1 - m)*m*(x^(-
1))^(-1 - m))/(1 + m)) - (2*E^((4*I)*a)*(3 + 2*m + m^2)*(x^(-1))^(-1 - m)*
Hypergeometric2F1[1, (-1 - m)/2, (1 - m)/2, -(E^((2*I)*a)/x^2)])/(1 + m))/
E^((2*I)*a))/(2*E^((2*I)*a))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 999
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(-(e*x)^m)*(x^(-1))^m   Subst[Int[(a + b/x^n)^p*(
(c + d/x^n)^q/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q}
, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [F]

$$\int (ex)^m \tan (a+i\ln (x))^3 dx$$

input
```
int((e*x)^m*tan(a+I*ln(x))^3,x)
```

output
```
int((e*x)^m*tan(a+I*ln(x))^3,x)
```

Fricas [F]

$$\int (ex)^m \tan^3(a+i\log(x))\,dx=\int (ex)^m \tan (a+i\ \log (x))^3\ dx$$

input
```
integrate((e*x)^m*tan(a+I*log(x))^3,x, algorithm="fricas")
```

output
```
integral((-I*x^6 + 3*I*x^4*e^(2*I*a) - 3*I*x^2*e^(4*I*a) + I*e^(6*I*a))*e^
(m*log(e) + m*log(x))/(x^6 + 3*x^4*e^(2*I*a) + 3*x^2*e^(4*I*a) + e^(6*I*a)
), x)
```

Sympy [F]

$$\int (ex)^m \tan^3(a + i\log(x))\,dx = \int (ex)^m \tan^3\left(a + i\log\left(x\right)\right)dx$$

input
```
integrate((e*x)**m*tan(a+I*ln(x))**3,x)
```

output
```
Integral((e*x)**m*tan(a + I*log(x))**3, x)
```

Maxima [F]

$$\int (ex)^m \tan^3(a + i\log(x))\,dx = \int (ex)^m \tan\left(a + i\log\left(x\right)\right)^3\,dx$$

input
```
integrate((e*x)^m*tan(a+I*log(x))^3,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*tan(a + I*log(x))^3, x)
```

Giac [F]

$$\int (ex)^m \tan^3(a + i\log(x))\,dx = \int (ex)^m \tan\left(a + i\log\left(x\right)\right)^3\,dx$$

input
```
integrate((e*x)^m*tan(a+I*log(x))^3,x, algorithm="giac")
```

output
```
integrate((e*x)^m*tan(a + I*log(x))^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \tan^3(a+i\log(x))\,dx = \int \tan(a+\ln(x)\ 1\mathrm{i})^3\,(e\,x)^m\,dx$$

input
```
int(tan(a + log(x)*1i)^3*(e*x)^m,x)
```

output
```
int(tan(a + log(x)*1i)^3*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \tan^3(a+i\log(x))\,dx = \frac{e^m\left(-x^m\tan\left(\log(x)\,i+a\right)^2 ix + x^m\tan\left(\log(x)\,i+a\right)mx + x^m\tan\left(\log(x)\,i+a\right)x - x^m ix - \left(\int x^m \tan\right.\right.}{2}$$

input
```
int((e*x)^m*tan(a+I*log(x))^3,x)
```

output
```
(e**m*( - x**m*tan(log(x)*i + a)**2*i*x + x**m*tan(log(x)*i + a)*m*x + x**
m*tan(log(x)*i + a)*x - x**m*i*x - int(x**m*tan(log(x)*i + a),x)*m**2 - 2*
int(x**m*tan(log(x)*i + a),x)*m - 3*int(x**m*tan(log(x)*i + a),x)))/2
```

3.153 $\int \tan^p(a+b\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 142

$$\int \tan^p(a+b\log(x))\,dx = x\left(1-e^{2ia}x^{2ib}\right)^{-p}\left(\frac{i\left(1-e^{2ia}x^{2ib}\right)}{1+e^{2ia}x^{2ib}}\right)^p\left(1+e^{2ia}x^{2ib}\right)^p\operatorname{AppellF1}\left(-\frac{i}{2b},-p,p,1-\frac{i}{2b},e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)$$

output
```
x*(I*(1-exp(2*I*a)*x^(2*I*b))/(1+exp(2*I*a)*x^(2*I*b)))^p*(1+exp(2*I*a)*x^
(2*I*b))^p*AppellF1(-1/2*I/b,-p,p,1-1/2*I/b,exp(2*I*a)*x^(2*I*b),-exp(2*I*
a)*x^(2*I*b))/((1-exp(2*I*a)*x^(2*I*b))^p)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 330 vs. 2(142) = 284.

Time = 0.55 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.32

$$\int \tan^p(a+b\log(x))\,dx$$

$$= \frac{(-i+2b)x\left(-\frac{i(-1+e^{2ia}x^{2ib})}{1+e^{2ia}x^{2ib}}\right)^p \text{AppellF1}\left(-\frac{i}{2b},\right.}{-2be^{2ia}px^{2ib}\,\text{AppellF1}\left(1-\frac{i}{2b},1-p,p,2-\frac{i}{2b},e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)-2be^{2ia}px^{2ib}\,\text{AppellF1}\left(1-\frac{i}{2b},-p,1\right.}$$

input
```
Integrate[Tan[a + b*Log[x]]^p,x]
```

output
```
((-I + 2*b)*x*(((-I)*(-1 + E^((2*I)*a)*x^((2*I)*b)))/(1 + E^((2*I)*a)*x^((
2*I)*b)))^p*AppellF1[(-1/2*I)/b, -p, p, 1 - (I/2)/b, E^((2*I)*a)*x^((2*I)*
b), -(E^((2*I)*a)*x^((2*I)*b))])/(-2*b*E^((2*I)*a)*p*x^((2*I)*b)*AppellF1[
1 - (I/2)/b, 1 - p, p, 2 - (I/2)/b, E^((2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)
*x^((2*I)*b))] - 2*b*E^((2*I)*a)*p*x^((2*I)*b)*AppellF1[1 - (I/2)/b, -p, 1
 + p, 2 - (I/2)/b, E^((2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^((2*I)*b))] +
(-I + 2*b)*AppellF1[(-1/2*I)/b, -p, p, 1 - (I/2)/b, E^((2*I)*a)*x^((2*I)*b
), -(E^((2*I)*a)*x^((2*I)*b))])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5002, 2058, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^p(a+b\log(x))\,dx$$

$$\Big\downarrow \text{ 5002}$$

$$\int \left(\frac{i-ie^{2ia}x^{2ib}}{1+e^{2ia}x^{2ib}}\right)^p dx$$

$$\Big\downarrow \text{ 2058}$$

$$\left(i-ie^{2ia}x^{2ib}\right)^{-p}\left(\frac{i(1-e^{2ia}x^{2ib})}{1+e^{2ia}x^{2ib}}\right)^p\left(1+e^{2ia}x^{2ib}\right)^p\int\left(i-ie^{2ia}x^{2ib}\right)^p\left(e^{2ia}x^{2ib}+1\right)^{-p}dx$$

$$\Big\downarrow 937$$

$$\left(1-e^{2ia}x^{2ib}\right)^{-p}\left(\frac{i(1-e^{2ia}x^{2ib})}{1+e^{2ia}x^{2ib}}\right)^{p}\left(1+e^{2ia}x^{2ib}\right)^{p}\int\left(1-e^{2ia}x^{2ib}\right)^{p}\left(e^{2ia}x^{2ib}+1\right)^{-p}dx$$

$$\Big\downarrow 936$$

$$x\left(1-e^{2ia}x^{2ib}\right)^{-p}\left(\frac{i(1-e^{2ia}x^{2ib})}{1+e^{2ia}x^{2ib}}\right)^{p}\left(1+e^{2ia}x^{2ib}\right)^{p}\text{AppellF1}\left(-\frac{i}{2b},-p,p,1-\frac{i}{2b},e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)$$

input
```
Int[Tan[a + b*Log[x]]^p,x]
```

output
```
(x*((I*(1 - E^((2*I)*a)*x^((2*I)*b)))/(1 + E^((2*I)*a)*x^((2*I)*b)))^p*(1
+ E^((2*I)*a)*x^((2*I)*b))^p*AppellF1[(-1/2*I)/b, -p, p, 1 - (I/2)/b, E^((
2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^((2*I)*b))])/(1 - E^((2*I)*a)*x^((2*I
)*b))^p
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 5002
```
Int[Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((I - I*E^(2
*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b, d
, p}, x]
```

Maple [F]

$$\int \tan(a+b\ln(x))^p\,dx$$

input
```
int(tan(a+b*ln(x))^p,x)
```

output
```
int(tan(a+b*ln(x))^p,x)
```

Fricas [F]

$$\int \tan^p(a+b\log(x))\,dx=\int \tan(b\log(x)+a)^p\,dx$$

input
```
integrate(tan(a+b*log(x))^p,x, algorithm="fricas")
```

output
```
integral(tan(b*log(x) + a)^p, x)
```

Sympy [F]

$$\int \tan^p(a+b\log(x))\,dx=\int \tan^p(a+b\log(x))\,dx$$

input
```
integrate(tan(a+b*ln(x))**p,x)
```

output
```
Integral(tan(a + b*log(x))**p, x)
```

Maxima [F]

$$\int \tan^p(a + b\log(x))\,dx = \int \tan\left(b\log\left(x\right) + a\right)^p\,dx$$

input
```
integrate(tan(a+b*log(x))^p,x, algorithm="maxima")
```

output
```
integrate(tan(b*log(x) + a)^p, x)
```

Giac [F]

$$\int \tan^p(a + b\log(x))\,dx = \int \tan\left(b\log\left(x\right) + a\right)^p\,dx$$

input
```
integrate(tan(a+b*log(x))^p,x, algorithm="giac")
```

output
```
integrate(tan(b*log(x) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^p(a + b\log(x))\,dx = \int \tan(a + b\,\ln\left(x\right))^p\,dx$$

input
```
int(tan(a + b*log(x))^p,x)
```

output
```
int(tan(a + b*log(x))^p, x)
```

Reduce [F]

$$\int \tan^p(a + b\log(x))\,dx = \int \tan\left(\log(x)\,b + a\right)^p\,dx$$

input
```
int(tan(a+b*log(x))^p,x)
```

output
```
int(tan(log(x)*b + a)**p,x)
```

3.154 $\int (ex)^m \tan^p(a+b\log(x))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 162

$$\int (ex)^m \tan^p(a+b\log(x))\,dx$$
$$= \frac{(ex)^{1+m}\left(1-e^{2ia}x^{2ib}\right)^{-p}\left(\frac{i\left(1-e^{2ia}x^{2ib}\right)}{1+e^{2ia}x^{2ib}}\right)^p\left(1+e^{2ia}x^{2ib}\right)^p \text{AppellF1}\left(-\frac{i(1+m)}{2b},-p,p,1-\frac{i(1+m)}{2b},e^{2ia}x^{2ib},-e\right.}{e(1+m)}$$

output
```
(e*x)^(1+m)*(I*(1-exp(2*I*a)*x^(2*I*b))/(1+exp(2*I*a)*x^(2*I*b)))^p*(1+exp
(2*I*a)*x^(2*I*b))^p*AppellF1(-1/2*I*(1+m)/b,-p,p,1-1/2*I*(1+m)/b,exp(2*I*
a)*x^(2*I*b),-exp(2*I*a)*x^(2*I*b))/e/(1+m)/((1-exp(2*I*a)*x^(2*I*b))^p)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.97

$$\int (ex)^m \tan^p(a+b\log(x))\,dx$$
$$= \frac{x(ex)^m\left(1-e^{2ia}x^{2ib}\right)^{-p}\left(-\frac{i\left(-1+e^{2ia}x^{2ib}\right)}{1+e^{2ia}x^{2ib}}\right)^p\left(1+e^{2ia}x^{2ib}\right)^p \text{AppellF1}\left(-\frac{i(1+m)}{2b},-p,p,1-\frac{i(1+m)}{2b},e^{2ia}x^{2ib},-\right.}{1+m}$$

input
```
Integrate[(e*x)^m*Tan[a + b*Log[x]]^p,x]
```

output
```
(x*(e*x)^m*(((-I)*(-1 + E^((2*I)*a)*x^((2*I)*b)))/(1 + E^((2*I)*a)*x^((2*I
)*b)))^p*(1 + E^((2*I)*a)*x^((2*I)*b))^p*AppellF1[((-1/2*I)*(1 + m))/b, -p
, p, 1 - ((I/2)*(1 + m))/b, E^((2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^((2*I
)*b))])/((1 + m)*(1 - E^((2*I)*a)*x^((2*I)*b))^p)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5006, 2058, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \tan^p(a+b\log(x))\,dx$$

$$\Big\downarrow \text{ 5006}$$

$$\int (ex)^m \left(\frac{i-ie^{2ia}x^{2ib}}{1+e^{2ia}x^{2ib}}\right)^p dx$$

$$\Big\downarrow \text{ 2058}$$

$$\left(i-ie^{2ia}x^{2ib}\right)^{-p}\left(\frac{i\left(1-e^{2ia}x^{2ib}\right)}{1+e^{2ia}x^{2ib}}\right)^p\left(1+e^{2ia}x^{2ib}\right)^p\int (ex)^m\left(i-ie^{2ia}x^{2ib}\right)^p\left(e^{2ia}x^{2ib}+1\right)^{-p}dx$$

$$\Big\downarrow \text{ 1013}$$

$$\left(1-e^{2ia}x^{2ib}\right)^{-p}\left(\frac{i\left(1-e^{2ia}x^{2ib}\right)}{1+e^{2ia}x^{2ib}}\right)^p\left(1+e^{2ia}x^{2ib}\right)^p\int (ex)^m\left(1-e^{2ia}x^{2ib}\right)^p\left(e^{2ia}x^{2ib}+1\right)^{-p}dx$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{(ex)^{m+1}\left(1-e^{2ia}x^{2ib}\right)^{-p}\left(\frac{i(1-e^{2ia}x^{2ib})}{1+e^{2ia}x^{2ib}}\right)^p\left(1+e^{2ia}x^{2ib}\right)^p \text{AppellF1}\left(-\frac{i(m+1)}{2b},-p,p,1-\frac{i(m+1)}{2b},e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)}{e(m+1)}$$

input
```
Int[(e*x)^m*Tan[a + b*Log[x]]^p,x]
```

output
```
((e*x)^(1 + m)*((I*(1 - E^((2*I)*a)*x^((2*I)*b)))/(1 + E^((2*I)*a)*x^((2*I
)*b)))^p*(1 + E^((2*I)*a)*x^((2*I)*b))^p*AppellF1[((-1/2*I)*(1 + m))/b, -p
, p, 1 - ((I/2)*(1 + m))/b, E^((2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^((2*I
)*b))])/(e*(1 + m)*(1 - E^((2*I)*a)*x^((2*I)*b))^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [F]

$$\int (ex)^m \tan (a + b\ln (x))^p \, dx$$

input
```
int((e*x)^m*tan(a+b*ln(x))^p,x)
```

output
```
int((e*x)^m*tan(a+b*ln(x))^p,x)
```

Fricas [F]

$$\int (ex)^m \tan^p(a + b\log(x)) \, dx = \int (ex)^m \tan (b\log (x) + a)^p \, dx$$

input
```
integrate((e*x)^m*tan(a+b*log(x))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^m*tan(b*log(x) + a)^p, x)
```

Sympy [F]

$$\int (ex)^m \tan^p(a + b\log(x)) \, dx = \int (ex)^m \tan^p (a + b\log (x)) \, dx$$

input
```
integrate((e*x)**m*tan(a+b*ln(x))**p,x)
```

output
```
Integral((e*x)**m*tan(a + b*log(x))**p, x)
```

Maxima [F]

$$\int (ex)^m \tan^p(a+b\log(x))\,dx = \int (ex)^m \tan\left(b\log\left(x\right)+a\right)^p\,dx$$

input
```
integrate((e*x)^m*tan(a+b*log(x))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*tan(b*log(x) + a)^p, x)
```

Giac [F]

$$\int (ex)^m \tan^p(a+b\log(x))\,dx = \int (ex)^m \tan\left(b\log\left(x\right)+a\right)^p\,dx$$

input
```
integrate((e*x)^m*tan(a+b*log(x))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^m*tan(b*log(x) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \tan^p(a+b\log(x))\,dx = \int \tan(a+b\,\ln\left(x\right))^p\,{\left(e\,x\right)}^m\,dx$$

input
```
int(tan(a + b*log(x))^p*(e*x)^m,x)
```

output
```
int(tan(a + b*log(x))^p*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \tan^p(a + b\log(x))\, dx = e^m \left(\int x^m \tan\left(\log(x)\, b + a\right)^p dx \right)$$

input
```
int((e*x)^m*tan(a+b*log(x))^p,x)
```

output
```
e**m*int(x**m*tan(log(x)*b + a)**p,x)
```

3.155 $\int \tan^p(a+\log(x))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 120

$$\int \tan^p(a+\log(x))\,dx = \left(1-e^{2ia}x^{2i}\right)^{-p}\left(\frac{i(1-e^{2ia}x^{2i})}{1+e^{2ia}x^{2i}}\right)^p\left(1+e^{2ia}x^{2i}\right)^p x\,\text{AppellF1}\left(-\frac{i}{2},-p,p,1-\frac{i}{2},e^{2ia}x^{2i},-e^{2ia}x^{2i}\right)$$

output
```
(I*(1-exp(2*I*a)*x^(2*I))/(1+exp(2*I*a)*x^(2*I)))^p*(1+exp(2*I*a)*x^(2*I))
^p*x*AppellF1(-1/2*I,-p,p,1-1/2*I,exp(2*I*a)*x^(2*I),-exp(2*I*a)*x^(2*I))/
((1-exp(2*I*a)*x^(2*I))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.00

$$\int \tan^p(a+\log(x))\,dx = \frac{(1+2i)\left(-\frac{i(-1+e^{2ia}x^{2i})}{1+e^{2ia}x^{2i}}\right)^p x\,\text{AppellF1}\left(-\frac{i}{2},-p,p,1-\right.}{(1+2i)\,\text{AppellF1}\left(-\frac{i}{2},-p,p,1-\frac{i}{2},e^{2ia}x^{2i},-e^{2ia}x^{2i}\right)-2ie^{2ia}px^{2i}\left(\text{AppellF1}\left(1-\frac{i}{2},1-p,p,2-\frac{i}{2},e^{2ia}\right.\right.}$$

input
```
Integrate[Tan[a + Log[x]]^p,x]
```

output
```
((1 + 2*I)*(((-I)*(-1 + E^((2*I)*a)*x^(2*I)))/(1 + E^((2*I)*a)*x^(2*I)))^p
*x*AppellF1[-1/2*I, -p, p, 1 - I/2, E^((2*I)*a)*x^(2*I), -(E^((2*I)*a)*x^(
2*I))])/((1 + 2*I)*AppellF1[-1/2*I, -p, p, 1 - I/2, E^((2*I)*a)*x^(2*I), -
(E^((2*I)*a)*x^(2*I))] - (2*I)*E^((2*I)*a)*p*x^(2*I)*(AppellF1[1 - I/2, 1
- p, p, 2 - I/2, E^((2*I)*a)*x^(2*I), -(E^((2*I)*a)*x^(2*I))] + AppellF1[1
 - I/2, -p, 1 + p, 2 - I/2, E^((2*I)*a)*x^(2*I), -(E^((2*I)*a)*x^(2*I))]))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5002, 2058, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^p(a+\log(x))\,dx$$

$$\Big\downarrow \text{5002}$$

$$\int \left(\frac{i-ie^{2ia}x^{2i}}{1+e^{2ia}x^{2i}}\right)^p dx$$

$$\Big\downarrow \text{2058}$$

$$\left(i-ie^{2ia}x^{2i}\right)^{-p}\left(\frac{i(1-e^{2ia}x^{2i})}{1+e^{2ia}x^{2i}}\right)^p\left(1+e^{2ia}x^{2i}\right)^p\int \left(i-ie^{2ia}x^{2i}\right)^p\left(e^{2ia}x^{2i}+1\right)^{-p}dx$$

$$\Big\downarrow \text{937}$$

$$\left(1-e^{2ia}x^{2i}\right)^{-p}\left(\frac{i(1-e^{2ia}x^{2i})}{1+e^{2ia}x^{2i}}\right)^p\left(1+e^{2ia}x^{2i}\right)^p\int \left(1-e^{2ia}x^{2i}\right)^p\left(e^{2ia}x^{2i}+1\right)^{-p}dx$$

$$\Big\downarrow \text{936}$$

$$x\left(1-e^{2ia}x^{2i}\right)^{-p}\left(\frac{i(1-e^{2ia}x^{2i})}{1+e^{2ia}x^{2i}}\right)^p\left(1+e^{2ia}x^{2i}\right)^p\text{AppellF1}\left(-\frac{i}{2},-p,p,1-\frac{i}{2},e^{2ia}x^{2i},-e^{2ia}x^{2i}\right)$$

input
```
Int[Tan[a + Log[x]]^p,x]
```

output
```
(((I*(1 - E^((2*I)*a)*x^(2*I)))/(1 + E^((2*I)*a)*x^(2*I)))^p*(1 + E^((2*I)
*a)*x^(2*I))^p*x*AppellF1[-1/2*I, -p, p, 1 - I/2, E^((2*I)*a)*x^(2*I), -(E
^((2*I)*a)*x^(2*I))])/(1 - E^((2*I)*a)*x^(2*I))^p
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 5002
```
Int[Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((I - I*E^(2
*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b, d
, p}, x]
```

Maple [F]

$$\int \tan (a+\ln (x))^p \, dx$$

input
```
int(tan(a+ln(x))^p,x)
```

output
```
int(tan(a+ln(x))^p,x)
```

Fricas [F]

$$\int \tan^p(a+\log(x))\,dx = \int \tan\left(a+\log\left(x\right)\right)^p\,dx$$

input
```
integrate(tan(a+log(x))^p,x, algorithm="fricas")
```

output
```
integral(tan(a + log(x))^p, x)
```

Sympy [F]

$$\int \tan^p(a+\log(x))\,dx = \int \tan^p\left(a+\log\left(x\right)\right)dx$$

input
```
integrate(tan(a+ln(x))**p,x)
```

output
```
Integral(tan(a + log(x))**p, x)
```

Maxima [F]

$$\int \tan^p(a+\log(x))\,dx = \int \tan\left(a+\log\left(x\right)\right)^p\,dx$$

input
```
integrate(tan(a+log(x))^p,x, algorithm="maxima")
```

output
```
integrate(tan(a + log(x))^p, x)
```

Giac [F]

$$\int \tan^p(a+\log(x))\,dx=\int \tan\left(a+\log\left(x\right)\right)^p\,dx$$

input
```
integrate(tan(a+log(x))^p,x, algorithm="giac")
```

output
```
integrate(tan(a + log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^p(a+\log(x))\,dx=\int \tan(a+\ln\left(x\right))^p\,dx$$

input
```
int(tan(a + log(x))^p,x)
```

output
```
int(tan(a + log(x))^p, x)
```

Reduce [F]

$$\int \tan^p(a+\log(x))\,dx=\int \tan\left(\log(x)+a\right)^p\,dx$$

input
```
int(tan(a+log(x))^p,x)
```

output
```
int(tan(log(x) + a)**p,x)
```

3.156 $\int \tan^p(a+2\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 120

$$\int \tan^p(a+2\log(x))\,dx = \left(1-e^{2ia}x^{4i}\right)^{-p}\left(\frac{i(1-e^{2ia}x^{4i})}{1+e^{2ia}x^{4i}}\right)^p\left(1+e^{2ia}x^{4i}\right)^p x\,\text{AppellF1}\left(-\frac{i}{4},-p,p,1-\frac{i}{4},e^{2ia}x^{4i},-e^{2ia}x^{4i}\right)$$

output
```
(I*(1-exp(2*I*a)*x^(4*I))/(1+exp(2*I*a)*x^(4*I)))^p*(1+exp(2*I*a)*x^(4*I))
^p*x*AppellF1(-1/4*I,-p,p,1-1/4*I,exp(2*I*a)*x^(4*I),-exp(2*I*a)*x^(4*I))/
((1-exp(2*I*a)*x^(4*I))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.00

$$\int \tan^p(a+2\log(x))\,dx = \frac{(1+4i)\left(-\frac{i(-1+e^{2ia}x^{4i})}{1+e^{2ia}x^{4i}}\right)^p x\,\text{AppellF1}\left(-\frac{i}{4},-p,p,1-\right.}{(1+4i)\,\text{AppellF1}\left(-\frac{i}{4},-p,p,1-\frac{i}{4},e^{2ia}x^{4i},-e^{2ia}x^{4i}\right)-4ie^{2ia}px^{4i}\left(\text{AppellF1}\left(1-\frac{i}{4},1-p,p,2-\frac{i}{4},e^{2ia}\right.\right.}$$

input
```
Integrate[Tan[a + 2*Log[x]]^p,x]
```

output
```
((1 + 4*I)*(((-I)*(-1 + E^((2*I)*a)*x^(4*I)))/(1 + E^((2*I)*a)*x^(4*I)))^p
*x*AppellF1[-1/4*I, -p, p, 1 - I/4, E^((2*I)*a)*x^(4*I), -(E^((2*I)*a)*x^(
4*I))])/((1 + 4*I)*AppellF1[-1/4*I, -p, p, 1 - I/4, E^((2*I)*a)*x^(4*I), -
(E^((2*I)*a)*x^(4*I))] - (4*I)*E^((2*I)*a)*p*x^(4*I)*(AppellF1[1 - I/4, 1
- p, p, 2 - I/4, E^((2*I)*a)*x^(4*I), -(E^((2*I)*a)*x^(4*I))] + AppellF1[1
 - I/4, -p, 1 + p, 2 - I/4, E^((2*I)*a)*x^(4*I), -(E^((2*I)*a)*x^(4*I))]))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5002, 2058, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^p(a+2\log(x))\,dx$$

$$\Big\downarrow \; 5002$$

$$\int \left(\frac{i-ie^{2ia}x^{4i}}{1+e^{2ia}x^{4i}}\right)^p dx$$

$$\Big\downarrow \; 2058$$

$$\left(i-ie^{2ia}x^{4i}\right)^{-p}\left(\frac{i\left(1-e^{2ia}x^{4i}\right)}{1+e^{2ia}x^{4i}}\right)^p\left(1+e^{2ia}x^{4i}\right)^p\int \left(i-ie^{2ia}x^{4i}\right)^p\left(e^{2ia}x^{4i}+1\right)^{-p}dx$$

$$\Big\downarrow \; 937$$

$$\left(1-e^{2ia}x^{4i}\right)^{-p}\left(\frac{i\left(1-e^{2ia}x^{4i}\right)}{1+e^{2ia}x^{4i}}\right)^p\left(1+e^{2ia}x^{4i}\right)^p\int \left(1-e^{2ia}x^{4i}\right)^p\left(e^{2ia}x^{4i}+1\right)^{-p}dx$$

$$\Big\downarrow \; 936$$

$$x\left(1-e^{2ia}x^{4i}\right)^{-p}\left(\frac{i\left(1-e^{2ia}x^{4i}\right)}{1+e^{2ia}x^{4i}}\right)^p\left(1+e^{2ia}x^{4i}\right)^p \text{AppellF1}\left(-\frac{i}{4},-p,p,1-\frac{i}{4},e^{2ia}x^{4i},-e^{2ia}x^{4i}\right)$$

input
```
Int[Tan[a + 2*Log[x]]^p,x]
```

output
```
(((I*(1 - E^((2*I)*a)*x^(4*I)))/(1 + E^((2*I)*a)*x^(4*I)))^p*(1 + E^((2*I)
*a)*x^(4*I))^p*x*AppellF1[-1/4*I, -p, p, 1 - I/4, E^((2*I)*a)*x^(4*I), -(E
^((2*I)*a)*x^(4*I))])/(1 - E^((2*I)*a)*x^(4*I))^p
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 5002
```
Int[Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((I - I*E^(2
*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b, d
, p}, x]
```

Maple [F]

$$\int \tan \left(a+2\ln \left(x\right)\right)^{p}dx$$

input
```
int(tan(a+2*ln(x))^p,x)
```

output
```
int(tan(a+2*ln(x))^p,x)
```

Fricas [F]

$$\int \tan^p(a + 2\log(x))\, dx = \int \tan\left(a + 2\,\log\left(x\right)\right)^{p}\, dx$$

input
```
integrate(tan(a+2*log(x))^p,x, algorithm="fricas")
```

output
```
integral(tan(a + 2*log(x))^p, x)
```

Sympy [F]

$$\int \tan^p(a + 2\log(x))\, dx = \int \tan^{p}\left(a + 2\log\left(x\right)\right)\, dx$$

input
```
integrate(tan(a+2*ln(x))**p,x)
```

output
```
Integral(tan(a + 2*log(x))**p, x)
```

Maxima [F]

$$\int \tan^p(a + 2\log(x))\, dx = \int \tan\left(a + 2\,\log\left(x\right)\right)^{p}\, dx$$

input
```
integrate(tan(a+2*log(x))^p,x, algorithm="maxima")
```

output
```
integrate(tan(a + 2*log(x))^p, x)
```

Giac [F]

$$\int \tan^p(a+2\log(x))\,dx = \int \tan\left(a+2\,\log\left(x\right)\right)^p\,dx$$

input
```
integrate(tan(a+2*log(x))^p,x, algorithm="giac")
```

output
```
integrate(tan(a + 2*log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^p(a+2\log(x))\,dx = \int \tan(a+2\,\ln\left(x\right))^p\,dx$$

input
```
int(tan(a + 2*log(x))^p,x)
```

output
```
int(tan(a + 2*log(x))^p, x)
```

Reduce [F]

$$\int \tan^p(a+2\log(x))\,dx = \int \tan\left(2\log(x)+a\right)^p\,dx$$

input
```
int(tan(a+2*log(x))^p,x)
```

output
```
int(tan(2*log(x) + a)**p,x)
```

3.157 $\int \tan^p(a+3\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 120

$$\int \tan^p(a+3\log(x))\,dx = \left(1-e^{2ia}x^{6i}\right)^{-p}\left(\frac{i\left(1-e^{2ia}x^{6i}\right)}{1+e^{2ia}x^{6i}}\right)^p\left(1+e^{2ia}x^{6i}\right)^p x\,\text{AppellF1}\left(-\frac{i}{6},-p,p,1-\frac{i}{6},e^{2ia}x^{6i},-e^{2ia}x^{6i}\right)$$

output
```
(I*(1-exp(2*I*a)*x^(6*I))/(1+exp(2*I*a)*x^(6*I)))^p*(1+exp(2*I*a)*x^(6*I))
^p*x*AppellF1(-1/6*I,-p,p,1-1/6*I,exp(2*I*a)*x^(6*I),-exp(2*I*a)*x^(6*I))/
((1-exp(2*I*a)*x^(6*I))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.00

$$\int \tan^p(a+3\log(x))\,dx = \frac{(1+6i)\left(-\frac{i\left(-1+e^{2ia}x^{6i}\right)}{1+e^{2ia}x^{6i}}\right)^p x\,\text{AppellF1}\left(-\frac{i}{6},-p,p,1-\right.}{(1+6i)\,\text{AppellF1}\left(-\frac{i}{6},-p,p,1-\frac{i}{6},e^{2ia}x^{6i},-e^{2ia}x^{6i}\right)-6ie^{2ia}px^{6i}\left(\text{AppellF1}\left(1-\frac{i}{6},1-p,p,2-\frac{i}{6},e^{2ia}\right.\right.}$$

input
```
Integrate[Tan[a + 3*Log[x]]^p,x]
```

output
```
((1 + 6*I)*(((-I)*(-1 + E^((2*I)*a)*x^(6*I)))/(1 + E^((2*I)*a)*x^(6*I)))^p
*x*AppellF1[-1/6*I, -p, p, 1 - I/6, E^((2*I)*a)*x^(6*I), -(E^((2*I)*a)*x^(
6*I))])/((1 + 6*I)*AppellF1[-1/6*I, -p, p, 1 - I/6, E^((2*I)*a)*x^(6*I), -
(E^((2*I)*a)*x^(6*I))] - (6*I)*E^((2*I)*a)*p*x^(6*I)*(AppellF1[1 - I/6, 1
- p, p, 2 - I/6, E^((2*I)*a)*x^(6*I), -(E^((2*I)*a)*x^(6*I))] + AppellF1[1
 - I/6, -p, 1 + p, 2 - I/6, E^((2*I)*a)*x^(6*I), -(E^((2*I)*a)*x^(6*I))]))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5002, 2058, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^p(a+3\log(x))\,dx$$

$$\Big\downarrow \text{5002}$$

$$\int \left(\frac{i-ie^{2ia}x^{6i}}{1+e^{2ia}x^{6i}}\right)^p dx$$

$$\Big\downarrow \text{2058}$$

$$\left(i-ie^{2ia}x^{6i}\right)^{-p}\left(\frac{i\left(1-e^{2ia}x^{6i}\right)}{1+e^{2ia}x^{6i}}\right)^p\left(1+e^{2ia}x^{6i}\right)^p\int \left(i-ie^{2ia}x^{6i}\right)^p\left(e^{2ia}x^{6i}+1\right)^{-p}dx$$

$$\Big\downarrow \text{937}$$

$$\left(1-e^{2ia}x^{6i}\right)^{-p}\left(\frac{i\left(1-e^{2ia}x^{6i}\right)}{1+e^{2ia}x^{6i}}\right)^p\left(1+e^{2ia}x^{6i}\right)^p\int \left(1-e^{2ia}x^{6i}\right)^p\left(e^{2ia}x^{6i}+1\right)^{-p}dx$$

$$\Big\downarrow \text{936}$$

$$x\left(1-e^{2ia}x^{6i}\right)^{-p}\left(\frac{i\left(1-e^{2ia}x^{6i}\right)}{1+e^{2ia}x^{6i}}\right)^p\left(1+e^{2ia}x^{6i}\right)^p\text{AppellF1}\left(-\frac{i}{6},-p,p,1-\frac{i}{6},e^{2ia}x^{6i},-e^{2ia}x^{6i}\right)$$

input
```
Int[Tan[a + 3*Log[x]]^p,x]
```

output
```
(((I*(1 - E^((2*I)*a)*x^(6*I)))/(1 + E^((2*I)*a)*x^(6*I)))^p*(1 + E^((2*I)
*a)*x^(6*I))^p*x*AppellF1[-1/6*I, -p, p, 1 - I/6, E^((2*I)*a)*x^(6*I), -(E
^((2*I)*a)*x^(6*I))])/(1 - E^((2*I)*a)*x^(6*I))^p
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 5002
```
Int[Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((I - I*E^(2
*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b, d
, p}, x]
```

Maple [F]

$$\int \tan \left(a+3 \ln \left(x\right)\right)^{p}d x$$

input
```
int(tan(a+3*ln(x))^p,x)
```

output
```
int(tan(a+3*ln(x))^p,x)
```

Fricas [F]

$$\int \tan^p(a+3\log(x))\,dx = \int \tan\left(a+3\,\log\left(x\right)\right)^{p}\,dx$$

input
```
integrate(tan(a+3*log(x))^p,x, algorithm="fricas")
```

output
```
integral(tan(a + 3*log(x))^p, x)
```

Sympy [F]

$$\int \tan^p(a+3\log(x))\,dx = \int \tan^{p}\left(a+3\log\left(x\right)\right)dx$$

input
```
integrate(tan(a+3*ln(x))**p,x)
```

output
```
Integral(tan(a + 3*log(x))**p, x)
```

Maxima [F]

$$\int \tan^p(a+3\log(x))\,dx = \int \tan\left(a+3\,\log\left(x\right)\right)^{p}\,dx$$

input
```
integrate(tan(a+3*log(x))^p,x, algorithm="maxima")
```

output
```
integrate(tan(a + 3*log(x))^p, x)
```

Giac [F]

$$\int \tan^p(a+3\log(x))\,dx = \int \tan\left(a+3\,\log\left(x\right)\right)^p\,dx$$

input
```
integrate(tan(a+3*log(x))^p,x, algorithm="giac")
```

output
```
integrate(tan(a + 3*log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^p(a+3\log(x))\,dx = \int \tan(a+3\,\ln\left(x\right))^p\,dx$$

input
```
int(tan(a + 3*log(x))^p,x)
```

output
```
int(tan(a + 3*log(x))^p, x)
```

Reduce [F]

$$\int \tan^p(a+3\log(x))\,dx = \int \tan\left(3\log(x)+a\right)^p dx$$

input
```
int(tan(a+3*log(x))^p,x)
```

output
```
int(tan(3*log(x) + a)**p,x)
```

3.158 $\int x^3 \tan(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int x^3 \tan(d(a+b\log(cx^n)))\,dx = -\frac{ix^4}{4} + \frac{1}{2}ix^4 \text{Hypergeometric2F1}\left(1, -\frac{2i}{bdn}, 1-\frac{2i}{bdn}, -e^{2iad}(cx^n)^{2ibd}\right)$$

output
```
-1/4*I*x^4+1/2*I*x^4*hypergeom([1, -2*I/b/d/n],[1-2*I/b/d/n],-exp(2*I*a*d)
*(c*x^n)^(2*I*b*d))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 146 vs. 2(71) = 142.

Time = 5.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.06

$$\int x^3 \tan(d(a+b\log(cx^n)))\,dx = \frac{x^4\left(2ie^{2id(a+b\log(cx^n))}\text{Hypergeometric2F1}\left(1,1-\frac{2i}{bdn},2-\frac{2i}{bdn},-e^{2id(a+b\log(cx^n))}\right)+(-2i+bdn)\,\text{Hypergeo}\right.}{-8-4ibdn}$$

input
```
Integrate[x^3*Tan[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^4*((2*I)*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - (2*I)/
(b*d*n), 2 - (2*I)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] + (-2*I + b*d
*n)*Hypergeometric2F1[1, (-2*I)/(b*d*n), 1 - (2*I)/(b*d*n), -E^((2*I)*d*(a
 + b*Log[c*x^n]))]))/(-8 - (4*I)*b*d*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.51, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5008, 5006, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \tan \left(d\left(a+b \log \left(c x^n\right)\right)\right) \, dx$$

$$\Big\downarrow \text{5008}$$

$$\frac{x^4 (cx^n)^{-4/n} \int (cx^n)^{\frac{4}{n}-1} \tan \left(d\left(a+b \log \left(c x^n\right)\right)\right) d(cx^n)}{n}$$

$$\Big\downarrow \text{5006}$$

$$\frac{x^4 (cx^n)^{-4/n} \int \frac{(cx^n)^{\frac{4}{n}-1} \left(i-ie^{2iad}(cx^n)^{2ibd}\right)}{e^{2iad}(cx^n)^{2ibd}+1} d(cx^n)}{n}$$

$$\Big\downarrow \text{959}$$

$$\frac{x^4 (cx^n)^{-4/n} \left(2i \int \frac{(cx^n)^{\frac{4}{n}-1}}{e^{2iad}(cx^n)^{2ibd}+1} d(cx^n) - \frac{1}{4} i n (cx^n)^{4/n}\right)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^4 (cx^n)^{-4/n} \left(\frac{1}{2} i n (cx^n)^{4/n} \text{Hypergeometric2F1}\left(1, -\frac{2i}{bdn}, 1-\frac{2i}{bdn}, -e^{2iad}(cx^n)^{2ibd}\right) - \frac{1}{4} i n (cx^n)^{4/n}\right)}{n}$$

input
```
Int[x^3*Tan[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^4*((-1/4*I)*n*(c*x^n)^(4/n) + (I/2)*n*(c*x^n)^(4/n)*Hypergeometric2F1[1
, (-2*I)/(b*d*n), 1 - (2*I)/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))]
))/(n*(c*x^n)^(4/n))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5008
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^3 \tan \left(d(a + b \ln \left(c\, x^n\right))\right) dx$$

input
```
int(x^3*tan(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x^3*tan(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int x^3 \tan\left(d(a+b\log\left(cx^n\right))\right) \, dx = \int x^3 \tan\left((b\log\left(cx^n\right)+a)d\right) \, dx$$

input
```
integrate(x^3*tan(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral(x^3*tan(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int x^3 \tan\left(d(a+b\log\left(cx^n\right))\right) \, dx = \int x^3 \tan\left(ad+bd\log\left(cx^n\right)\right) dx$$

input
```
integrate(x**3*tan(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x**3*tan(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x^3 \tan\left(d(a+b\log\left(cx^n\right))\right) \, dx = \int x^3 \tan\left((b\log\left(cx^n\right)+a)d\right) \, dx$$

input
```
integrate(x^3*tan(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(x^3*tan((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int x^3 \tan (d(a+b \log (cx^n))) \, dx = \int x^3 \tan ((b \log (cx^n)+a)d) \, dx$$

input
```
integrate(x^3*tan(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(x^3*tan((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \tan (d(a+b \log (cx^n))) \, dx = \int x^3 \tan(d (a+b \ln (c x^n))) \, dx$$

input
```
int(x^3*tan(d*(a + b*log(c*x^n))),x)
```

output
```
int(x^3*tan(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int x^3 \tan (d(a+b \log (cx^n))) \, dx = \int \tan (\log(x^n c) \, bd+ad) \, x^3 dx$$

input
```
int(x^3*tan(d*(a+b*log(c*x^n))),x)
```

output
```
int(tan(log(x**n*c)*b*d + a*d)*x**3,x)
```

3.159 $\int x^2 \tan(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 75

$$\int x^2 \tan(d(a+b\log(cx^n)))\,dx = -\frac{ix^3}{3} + \frac{2}{3}ix^3 \,\text{Hypergeometric2F1}\left(1, -\frac{3i}{2bdn}, 1 - \frac{3i}{2bdn}, -e^{2iad}(cx^n)^{2ibd}\right)$$

output
```
-1/3*I*x^3+2/3*I*x^3*hypergeom([1, -3/2*I/b/d/n],[1-3/2*I/b/d/n],-exp(2*I*
a*d)*(c*x^n)^(2*I*b*d))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 155 vs. 2(75) = 150.

Time = 4.41 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.07

$$\int x^2 \tan(d(a+b\log(cx^n)))\,dx$$
$$= \frac{x^3\left(3ie^{2id(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1, 1-\frac{3i}{2bdn}, 2-\frac{3i}{2bdn}, -e^{2id(a+b\log(cx^n))}\right) + (-3i+2bdn)\,\text{Hyperg}\right.}{-9-6ibdn}$$

input
```
Integrate[x^2*Tan[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^3*((3*I)*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - ((3*I)
/2)/(b*d*n), 2 - ((3*I)/2)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] + (-3
*I + 2*b*d*n)*Hypergeometric2F1[1, ((-3*I)/2)/(b*d*n), 1 - ((3*I)/2)/(b*d*
n), -E^((2*I)*d*(a + b*Log[c*x^n]))]))/(-9 - (6*I)*b*d*n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.48, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5008, 5006, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \tan (d(a+b\log (cx^n)))\,dx$$

$$\Big\downarrow \text{5008}$$

$$\frac{x^3(cx^n)^{-3/n}\int (cx^n)^{\frac{3}{n}-1}\tan (d(a+b\log (cx^n)))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5006}$$

$$\frac{x^3(cx^n)^{-3/n}\int \frac{(cx^n)^{\frac{3}{n}-1}\left(i-ie^{2iad}(cx^n)^{2ibd}\right)}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{n}$$

$$\Big\downarrow \text{959}$$

$$\frac{x^3(cx^n)^{-3/n}\left(2i\int \frac{(cx^n)^{\frac{3}{n}-1}}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)-\frac{1}{3}in(cx^n)^{3/n}\right)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^3(cx^n)^{-3/n}\left(\frac{2}{3}in(cx^n)^{3/n}\text{Hypergeometric2F1}\left(1,-\frac{3i}{2bdn},1-\frac{3i}{2bdn},-e^{2iad}(cx^n)^{2ibd}\right)-\frac{1}{3}in(cx^n)^{3/n}\right)}{n}$$

input
```
Int[x^2*Tan[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^3*((-1/3*I)*n*(c*x^n)^(3/n) + ((2*I)/3)*n*(c*x^n)^(3/n)*Hypergeometric2
F1[1, ((-3*I)/2)/(b*d*n), 1 - ((3*I)/2)/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^(
(2*I)*b*d))]))/(n*(c*x^n)^(3/n))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5008
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^2 \tan \left(d(a+b \ln \left(c\, x^n\right))\right) dx$$

input
```
int(x^2*tan(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x^2*tan(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int x^2 \tan(d(a + b\log(cx^n)))\,dx = \int x^2 \tan((b\log(cx^n) + a)d)\,dx$$

input
```
integrate(x^2*tan(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral(x^2*tan(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int x^2 \tan(d(a + b\log(cx^n)))\,dx = \int x^2 \tan(ad + bd\log(cx^n))\,dx$$

input
```
integrate(x**2*tan(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x**2*tan(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x^2 \tan(d(a + b\log(cx^n)))\,dx = \int x^2 \tan((b\log(cx^n) + a)d)\,dx$$

input
```
integrate(x^2*tan(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(x^2*tan((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int x^2 \tan \left(d(a+b\log \left(cx^n\right))\right)\,dx=\int x^2 \tan \left((b\log \left(cx^n\right)+a)d\right)\,dx$$

input
```
integrate(x^2*tan(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(x^2*tan((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \tan \left(d(a+b\log \left(cx^n\right))\right)\,dx=\int x^2 \tan\left(d\left(a+b\,\ln\left(c\,x^n\right)\right)\right)\,dx$$

input
```
int(x^2*tan(d*(a + b*log(c*x^n))),x)
```

output
```
int(x^2*tan(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int x^2 \tan \left(d(a+b\log \left(cx^n\right))\right)\,dx=\int \tan \left(\log(x^n c)\,bd+ad\right)x^2 dx$$

input
```
int(x^2*tan(d*(a+b*log(c*x^n))),x)
```

output
```
int(tan(log(x**n*c)*b*d + a*d)*x**2,x)
```

3.160 $\int x \tan(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int x \tan(d(a+b\log(cx^n)))\,dx = -\frac{ix^2}{2} + ix^2 \text{Hypergeometric2F1}\left(1,-\frac{i}{bdn},1-\frac{i}{bdn},-e^{2iad}(cx^n)^{2ibd}\right)$$

output
```
-1/2*I*x^2+I*x^2*hypergeom([1, -I/b/d/n],[1-I/b/d/n],-exp(2*I*a*d)*(c*x^n)
^(2*I*b*d))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 146 vs. 2(69) = 138.

Time = 4.45 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.12

$$\int x \tan(d(a+b\log(cx^n)))\,dx = \frac{x^2\left(ie^{2id(a+b\log(cx^n))}\text{Hypergeometric2F1}\left(1,1-\frac{i}{bdn},2-\frac{i}{bdn},-e^{2id(a+b\log(cx^n))}\right)+(-i+bdn)\,\text{Hypergeome}\right.}{-2-2ibdn}$$

input
```
Integrate[x*Tan[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^2*(I*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - I/(b*d*n),
 2 - I/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] + (-I + b*d*n)*Hypergeome
tric2F1[1, (-I)/(b*d*n), 1 - I/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))]))
/(-2 - (2*I)*b*d*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.52, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5008, 5006, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\tan\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

$$\Big\downarrow \text{5008}$$

$$\frac{x^2(cx^n)^{-2/n}\int (cx^n)^{\frac{2}{n}-1}\tan\left(d(a+b\log\left(cx^n\right))\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{5006}$$

$$\frac{x^2(cx^n)^{-2/n}\int \frac{(cx^n)^{\frac{2}{n}-1}\left(i-ie^{2iad}(cx^n)^{2ibd}\right)}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{n}$$

$$\Big\downarrow \text{959}$$

$$\frac{x^2(cx^n)^{-2/n}\left(2i\int \frac{(cx^n)^{\frac{2}{n}-1}}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)-\frac{1}{2}in(cx^n)^{2/n}\right)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2(cx^n)^{-2/n}\left(in(cx^n)^{2/n}\,\text{Hypergeometric2F1}\left(1,-\frac{i}{bdn},1-\frac{i}{bdn},-e^{2iad}(cx^n)^{2ibd}\right)-\frac{1}{2}in(cx^n)^{2/n}\right)}{n}$$

input
```
Int[x*Tan[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^2*((-1/2*I)*n*(c*x^n)^(2/n) + I*n*(c*x^n)^(2/n)*Hypergeometric2F1[1, (-
I)/(b*d*n), 1 - I/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))]))/(n*(c*x
^n)^(2/n))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5008
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x \tan \left(d(a + b \ln \left(c\, x^{n}\right))\right) dx$$

input
```
int(x*tan(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x*tan(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int x\tan\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\tan\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

input
```
integrate(x*tan(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral(x*tan(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int x\tan\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\tan\left(ad+bd\log\left(cx^n\right)\right)dx$$

input
```
integrate(x*tan(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x*tan(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x\tan\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\tan\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

input
```
integrate(x*tan(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(x*tan((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int x\tan\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\tan\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

input
```
integrate(x*tan(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(x*tan((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\tan\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\tan\left(d\left(a+b\,\ln\left(c\,x^n\right)\right)\right)\,dx$$

input
```
int(x*tan(d*(a + b*log(c*x^n))),x)
```

output
```
int(x*tan(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int x\tan\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int\tan\left(\log(x^nc)\,bd+ad\right)xdx$$

input
```
int(x*tan(d*(a+b*log(c*x^n))),x)
```

output
```
int(tan(log(x**n*c)*b*d + a*d)*x,x)
```

3.161 $\int \tan(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \tan(d(a+b\log(cx^n)))\,dx = -ix + 2ix\,\text{Hypergeometric2F1}\left(1,-\frac{i}{2bdn},1-\frac{i}{2bdn},-e^{2iad}(cx^n)^{2ibd}\right)$$

output

```
-I*x+2*I*x*hypergeom([1, -1/2*I/b/d/n],[1-1/2*I/b/d/n],-exp(2*I*a*d)*(c*x^
n)^(2*I*b*d))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 153 vs. 2(67) = 134.

Time = 7.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.28

$$\int \tan(d(a+b\log(cx^n)))\,dx = \frac{x\left(ie^{2id(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1-\frac{i}{2bdn},2-\frac{i}{2bdn},-e^{2id(a+b\log(cx^n))}\right)+(-i+2bdn)\,\text{Hypergeor}\right.}{-1-2ibdn}$$

input

```
Integrate[Tan[d*(a + b*Log[c*x^n])],x]
```

output
```
(x*(I*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - (I/2)/(b*d*n
), 2 - (I/2)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] + (-I + 2*b*d*n)*Hy
pergeometric2F1[1, (-1/2*I)/(b*d*n), 1 - (I/2)/(b*d*n), -E^((2*I)*d*(a + b
*Log[c*x^n]))]))/(-1 - (2*I)*b*d*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.51, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5004, 5006, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan (d(a+b\log (cx^n)))\, dx$$

$$\Big\downarrow \text{5004}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\tan (d(a+b\log (cx^n)))d(cx^n)}{n}$$

$$\Big\downarrow \text{5006}$$

$$\frac{x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}-1}\left(i-ie^{2iad}(cx^n)^{2ibd}\right)}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{n}$$

$$\Big\downarrow \text{959}$$

$$\frac{x(cx^n)^{-1/n}\left(2i\int \frac{(cx^n)^{\frac{1}{n}-1}}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)-in(cx^n)^{\frac{1}{n}}\right)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x(cx^n)^{-1/n}\left(2in(cx^n)^{\frac{1}{n}}\text{Hypergeometric2F1}\left(1,-\frac{i}{2bdn},1-\frac{i}{2bdn},-e^{2iad}(cx^n)^{2ibd}\right)-in(cx^n)^{\frac{1}{n}}\right)}{n}$$

input
```
Int[Tan[d*(a + b*Log[c*x^n])],x]
```

output
```
(x*((-I)*n*(c*x^n)^n^(-1) + (2*I)*n*(c*x^n)^n^(-1)*Hypergeometric2F1[1, (-
1/2*I)/(b*d*n), 1 - (I/2)/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))]))
/(n*(c*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 5004
```
Int[Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Tan[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [F]

$$\int \tan \left(d(a+b\ln \left(c\,x^{n}\right))\right) dx$$

input
```
int(tan(d*(a+b*ln(c*x^n))),x)
```

output
```
int(tan(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int \tan (d(a + b \log (cx^n))) \, dx = \int \tan ((b \log (cx^n) + a)d) \, dx$$

input
```
integrate(tan(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral(tan(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int \tan (d(a + b \log (cx^n))) \, dx = \int \tan (d(a + b \log (cx^n))) \, dx$$

input
```
integrate(tan(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(tan(d*(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \tan (d(a + b \log (cx^n))) \, dx = \int \tan ((b \log (cx^n) + a)d) \, dx$$

input
```
integrate(tan(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(tan((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int \tan \left(d(a+b\log \left(cx^n\right))\right) \, dx = \int \tan \left((b\log \left(cx^n\right)+a)d\right) \, dx$$

input
```
integrate(tan(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(tan((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan \left(d(a+b\log \left(cx^n\right))\right) \, dx = \int \tan(d\left(a+b\,\ln\left(c\,x^n\right)\right)) \, dx$$

input
```
int(tan(d*(a + b*log(c*x^n))),x)
```

output
```
int(tan(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int \tan \left(d(a+b\log \left(cx^n\right))\right) \, dx = \int \tan \left(\log(x^n c)\, bd+ad\right) dx$$

input
```
int(tan(d*(a+b*log(c*x^n))),x)
```

output
```
int(tan(log(x**n*c)*b*d + a*d),x)
```

3.162 $\int \frac{\tan(d(a+b\log(cx^n)))}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 26

$$\int \frac{\tan(d(a+b\log(cx^n)))}{x} dx = -\frac{\log(\cos(ad+bd\log(cx^n)))}{bdn}$$

output
```
-ln(cos(a*d+b*d*ln(c*x^n)))/b/d/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{\tan(d(a+b\log(cx^n)))}{x} dx = -\frac{\log(\cos(d(a+b\log(cx^n))))}{bdn}$$

input
```
Integrate[Tan[d*(a + b*Log[c*x^n])]/x,x]
```

output
```
-(Log[Cos[d*(a + b*Log[c*x^n])]]/(b*d*n))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3039, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan (d(a+b\log (cx^n)))}{x} \, dx$$

$$\Big\downarrow \; 3039$$

$$\frac{\int \tan (d(a+b\log (cx^n)))d\log (cx^n)}{n}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \tan (ad+b\log (cx^n)\,d)d\log (cx^n)}{n}$$

$$\Big\downarrow \; 3956$$

$$-\frac{\log (\cos (ad+bd\log (cx^n)))}{bdn}$$

input
```
Int[Tan[d*(a + b*Log[c*x^n])]/x,x]
```

output
```
-(Log[Cos[a*d + b*d*Log[c*x^n]]]/(b*d*n))
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{\ln\left(1+\tan(d(a+b\ln(c\,x^n)))^2\right)}{2nbd}$
default	$\frac{\ln\left(1+\tan(d(a+b\ln(c\,x^n)))^2\right)}{2nbd}$
parallelrisch	$\frac{\ln\left(1+\tan(d(a+b\ln(c\,x^n)))^2\right)}{2nbd}$
risch	$-i\ln(x)+\frac{2ia}{nb}+\frac{2i\ln(c)}{n}+\frac{2i\ln(x^n)}{n}-\frac{\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}{n}+\frac{\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)}{n}+\frac{\pi}{}$

input
```
int(tan(d*(a+b*ln(c*x^n)))/x,x,method=_RETURNVERBOSE)
```

output
```
1/2/n/b/d*ln(1+tan(d*(a+b*ln(c*x^n)))^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35

$$\int \frac{\tan(d(a+b\log(cx^n)))}{x}\,dx = -\frac{\log\left(\frac{1}{2}\cos\left(2\,bdn\log(x)+2\,bd\log(c)+2\,ad\right)+\frac{1}{2}\right)}{2\,bdn}$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

output
```
-1/2*log(1/2*cos(2*b*d*n*log(x) + 2*b*d*log(c) + 2*a*d) + 1/2)/(b*d*n)
```

Sympy [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int \frac{\tan(d(a+b\log(cx^n)))}{x}\,dx = \begin{cases} \log(x)\tan(ad) & \text{for } b=0 \\ 0 & \text{for } d=0 \\ \log(x)\tan(ad+bd\log(c)) & \text{for } n=0 \\ -\frac{\log(\cos(ad+bd\log(cx^n)))}{bdn} & \text{otherwise} \end{cases}$$

input
```
integrate(tan(d*(a+b*ln(c*x**n)))/x,x)
```

output
```
Piecewise((log(x)*tan(a*d), Eq(b, 0)), (0, Eq(d, 0)), (log(x)*tan(a*d + b*
d*log(c)), Eq(n, 0)), (-log(cos(a*d + b*d*log(c*x**n)))/(b*d*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\tan(d(a+b\log(cx^n)))}{x}\,dx = \frac{\log(\sec((b\log(cx^n)+a)d))}{bdn}$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

output
```
log(sec((b*log(c*x^n) + a)*d))/(b*d*n)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan(d(a+b\log(cx^n)))}{x}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 22.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \frac{\tan\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = \ln\left(x\right)\,1\mathrm{i} - \frac{\ln\left(\mathrm{e}^{a\,d\,2\mathrm{i}}\,\left(c\,x^n\right)^{b\,d\,2\mathrm{i}}+1\right)}{b\,d\,n}$$

input
```
int(tan(d*(a + b*log(c*x^n)))/x,x)
```

output
```
log(x)*1i - log(exp(a*d*2i)*(c*x^n)^(b*d*2i) + 1)/(b*d*n)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\tan\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = \frac{\log\left(\tan\left(\log(x^n c)\,bd+ad\right)^2+1\right)}{2bdn}$$

input
```
int(tan(d*(a+b*log(c*x^n)))/x,x)
```

output
```
log(tan(log(x**n*c)*b*d + a*d)**2 + 1)/(2*b*d*n)
```

3.163 $\int \frac{\tan(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int \frac{\tan(d(a+b\log(cx^n)))}{x^2}\,dx = \frac{i}{x}-\frac{2i\,\text{Hypergeometric2F1}\left(1,\frac{i}{2bdn},1+\frac{i}{2bdn},-e^{2iad}\left(cx^n\right)^{2ibd}\right)}{x}$$

output
```
I/x-2*I*hypergeom([1, 1/2*I/b/d/n],[1+1/2*I/b/d/n],-exp(2*I*a*d)*(c*x^n)^(
2*I*b*d))/x
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 153 vs. 2(71) = 142.

Time = 3.05 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.15

$$\int \frac{\tan(d(a+b\log(cx^n)))}{x^2}\,dx = \frac{-e^{2id(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1+\frac{i}{2bdn},2+\frac{i}{2bdn},-e^{2id(a+b\log(cx^n))}\right)+(1-2ibdn)\,\text{Hypergeome}}{(i+2bdn)x}$$

input
```
Integrate[Tan[d*(a + b*Log[c*x^n])]/x^2,x]
```

output
```
(-(E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + (I/2)/(b*d*n),
2 + (I/2)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))]) + (1 - (2*I)*b*d*n)*H
ypergeometric2F1[1, (I/2)/(b*d*n), 1 + (I/2)/(b*d*n), -E^((2*I)*d*(a + b*L
og[c*x^n]))])/((I + 2*b*d*n)*x)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.48, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5008, 5006, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(d(a+b\log(cx^n)))}{x^2}dx$$

$$\Big\downarrow \text{5008}$$

$$\frac{(cx^n)^{\frac{1}{n}}\int (cx^n)^{-1-\frac{1}{n}}\tan(d(a+b\log(cx^n)))d(cx^n)}{nx}$$

$$\Big\downarrow \text{5006}$$

$$\frac{(cx^n)^{\frac{1}{n}}\int \frac{(cx^n)^{-1-\frac{1}{n}}\left(i-ie^{2iad}(cx^n)^{2ibd}\right)}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{nx}$$

$$\Big\downarrow \text{959}$$

$$\frac{(cx^n)^{\frac{1}{n}}\left(2i\int \frac{(cx^n)^{-1-\frac{1}{n}}}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)+in(cx^n)^{-1/n}\right)}{nx}$$

$$\Big\downarrow \text{888}$$

$$\frac{(cx^n)^{\frac{1}{n}}\left(in(cx^n)^{-1/n}-2in(cx^n)^{-1/n}\text{Hypergeometric2F1}\left(1,\frac{i}{2bdn},1+\frac{i}{2bdn},-e^{2iad}(cx^n)^{2ibd}\right)\right)}{nx}$$

input
```
Int[Tan[d*(a + b*Log[c*x^n])]/x^2,x]
```

output
```
((c*x^n)^n^(-1)*((I*n)/(c*x^n)^n^(-1) - ((2*I)*n*Hypergeometric2F1[1, (I/2
)/(b*d*n), 1 + (I/2)/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/(c*x^
n)^n^(-1)))/(n*x)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5008
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\tan \left(d(a+b\ln \left(c\,x^{n}\right))\right)}{x^{2}}dx$$

input
```
int(tan(d*(a+b*ln(c*x^n)))/x^2,x)
```

output
```
int(tan(d*(a+b*ln(c*x^n)))/x^2,x)
```

Fricas [F]

$$\int \frac{\tan \left(d(a+b\log \left(cx^{n}\right))\right)}{x^{2}}\,dx=\int \frac{\tan \left((b\log \left(cx^{n}\right)+a)d\right)}{x^{2}}\,dx$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

output
```
integral(tan(b*d*log(c*x^n) + a*d)/x^2, x)
```

Sympy [F]

$$\int \frac{\tan \left(d(a+b\log \left(cx^{n}\right))\right)}{x^{2}}\,dx=\int \frac{\tan \left(ad+bd\log \left(cx^{n}\right)\right)}{x^{2}}\,dx$$

input
```
integrate(tan(d*(a+b*ln(c*x**n)))/x**2,x)
```

output
```
Integral(tan(a*d + b*d*log(c*x**n))/x**2, x)
```

Maxima [F]

$$\int \frac{\tan \left(d(a+b\log \left(cx^n\right))\right)}{x^2}\,dx = \int \frac{\tan \left((b\log \left(cx^n\right)+a)d\right)}{x^2}\,dx$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

output
```
integrate(tan((b*log(c*x^n) + a)*d)/x^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan \left(d(a+b\log \left(cx^n\right))\right)}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan \left(d(a+b\log \left(cx^n\right))\right)}{x^2}\,dx = \int \frac{\tan(d\,(a+b\,\ln\left(c\,x^n\right)))}{x^2}\,dx$$

input
```
int(tan(d*(a + b*log(c*x^n)))/x^2,x)
```

output
```
int(tan(d*(a + b*log(c*x^n)))/x^2, x)
```

Reduce [F]

$$\int \frac{\tan(d(a + b\log(cx^n)))}{x^2} dx = \int \frac{\tan(\log(x^n c)bd + ad)}{x^2} dx$$

input
```
int(tan(d*(a+b*log(c*x^n)))/x^2,x)
```

output
```
int(tan(log(x**n*c)*b*d + a*d)/x**2,x)
```

3.164 $\int \frac{\tan(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \frac{\tan(d(a+b\log(cx^n)))}{x^3}\,dx = \frac{i}{2x^2} - \frac{i\,\text{Hypergeometric2F1}\left(1,\frac{i}{bdn},1+\frac{i}{bdn},-e^{2iad}(cx^n)^{2ibd}\right)}{x^2}$$

output
```
1/2*I/x^2-I*hypergeom([1, I/b/d/n],[1+I/b/d/n],-exp(2*I*a*d)*(c*x^n)^(2*I*
b*d))/x^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 147 vs. 2(69) = 138.

Time = 2.68 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.13

$$\int \frac{\tan(d(a+b\log(cx^n)))}{x^3}\,dx = \frac{-e^{2id(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1+\frac{i}{bdn},2+\frac{i}{bdn},-e^{2id(a+b\log(cx^n))}\right)+(1-ibdn)\,\text{Hypergeometri}}{2(i+bdn)x^2}$$

input
```
Integrate[Tan[d*(a + b*Log[c*x^n])]/x^3,x]
```

output
```
(-(E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + I/(b*d*n), 2 +
I/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))]) + (1 - I*b*d*n)*Hypergeometri
c2F1[1, I/(b*d*n), 1 + I/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))])/(2*(I
+ b*d*n)*x^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.52, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5008, 5006, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan (d(a+b\log (cx^n)))}{x^3} dx$$

$$\Big\downarrow \ 5008$$

$$\frac{(cx^n)^{2/n} \int (cx^n)^{-1-\frac{2}{n}} \tan (d(a+b\log (cx^n)))d(cx^n)}{nx^2}$$

$$\Big\downarrow \ 5006$$

$$\frac{(cx^n)^{2/n} \int \frac{(cx^n)^{-1-\frac{2}{n}}\left(i-ie^{2iad}(cx^n)^{2ibd}\right)}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{nx^2}$$

$$\Big\downarrow \ 959$$

$$\frac{(cx^n)^{2/n} \left(2i\int \frac{(cx^n)^{-1-\frac{2}{n}}}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)+\frac{1}{2}in(cx^n)^{-2/n}\right)}{nx^2}$$

$$\Big\downarrow \ 888$$

$$\frac{(cx^n)^{2/n} \left(\frac{1}{2}in(cx^n)^{-2/n}-in(cx^n)^{-2/n}\text{Hypergeometric2F1}\left(1,\frac{i}{bdn},1+\frac{i}{bdn},-e^{2iad}(cx^n)^{2ibd}\right)\right)}{nx^2}$$

input
```
Int[Tan[d*(a + b*Log[c*x^n])]/x^3,x]
```

output
```
((c*x^n)^(2/n)*(((I/2)*n)/(c*x^n)^(2/n) - (I*n*Hypergeometric2F1[1, I/(b*d
*n), 1 + I/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/(c*x^n)^(2/n)))
/(n*x^2)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5008
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\tan(d(a + b\ln(c\,x^n)))}{x^3}dx$$

input
```
int(tan(d*(a+b*ln(c*x^n)))/x^3,x)
```

output
```
int(tan(d*(a+b*ln(c*x^n)))/x^3,x)
```

Fricas [F]

$$\int \frac{\tan(d(a + b\log(cx^n)))}{x^3}\,dx = \int \frac{\tan((b\log(cx^n) + a)d)}{x^3}\,dx$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

output
```
integral(tan(b*d*log(c*x^n) + a*d)/x^3, x)
```

Sympy [F]

$$\int \frac{\tan(d(a + b\log(cx^n)))}{x^3}\,dx = \int \frac{\tan(ad + bd\log(cx^n))}{x^3}\,dx$$

input
```
integrate(tan(d*(a+b*ln(c*x**n)))/x**3,x)
```

output
```
Integral(tan(a*d + b*d*log(c*x**n))/x**3, x)
```

Maxima [F]

$$\int \frac{\tan (d(a+b\log (cx^n)))}{x^3}\,dx=\int \frac{\tan ((b\log (cx^n)+a)d)}{x^3}\,dx$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

output
```
integrate(tan((b*log(c*x^n) + a)*d)/x^3, x)
```

Giac [F]

$$\int \frac{\tan (d(a+b\log (cx^n)))}{x^3}\,dx=\int \frac{\tan ((b\log (cx^n)+a)d)}{x^3}\,dx$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

output
```
integrate(tan((b*log(c*x^n) + a)*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan (d(a+b\log (cx^n)))}{x^3}\,dx=\int \frac{\tan(d\,(a+b\,\ln (c\,x^n)))}{x^3}\,dx$$

input
```
int(tan(d*(a + b*log(c*x^n)))/x^3,x)
```

output
```
int(tan(d*(a + b*log(c*x^n)))/x^3, x)
```

Reduce [F]

$$\int \frac{\tan\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx = \int \frac{\tan\left(\log(x^n c)\,bd+ad\right)}{x^3}dx$$

input
```
int(tan(d*(a+b*log(c*x^n)))/x^3,x)
```

output
```
int(tan(log(x**n*c)*b*d + a*d)/x**3,x)
```

3.165 $\int x^3 \tan^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 159

$$\int x^3 \tan^2(d(a+b\log(cx^n)))\,dx = \frac{(4i-bdn)x^4}{4bdn} + \frac{ix^4\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1+e^{2iad}(cx^n)^{2ibd}\right)} - \frac{2ix^4 \operatorname{Hypergeometric2F1}\left(1,-\frac{2i}{bdn},1-\frac{2i}{bdn},-e^{2iad}(cx^n)^{2ibd}\right)}{bdn}$$

output

```
1/4*(4*I-b*d*n)*x^4/b/d/n+I*x^4*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/(
1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))-2*I*x^4*hypergeom([1, -2*I/b/d/n],[1-2*I
/b/d/n],-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n
```

Mathematica [A] (verified)

Time = 5.24 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.13

$$\int x^3 \tan^2(d(a+b\log(cx^n)))\,dx =$$
$$-\frac{x^4\left(-8e^{2id(a+b\log(cx^n))}\text{Hypergeometric2F1}\left(1,1-\frac{2i}{bdn},2-\frac{2i}{bdn},-e^{2id(a+b\log(cx^n))}\right)+(-2i+bdn)\left(bdn+\right.\right.}{4bdn(-2i+bd}$$

input
```
Integrate[x^3*Tan[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
-1/4*(x^4*(-8*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - (2*I
)/(b*d*n), 2 - (2*I)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] + (-2*I + b
*d*n)*(b*d*n + (4*I)*Hypergeometric2F1[1, (-2*I)/(b*d*n), 1 - (2*I)/(b*d*n
), -E^((2*I)*d*(a + b*Log[c*x^n]))] - 4*Tan[d*(a + b*Log[c*x^n])])))/(b*d*
n*(-2*I + b*d*n))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.34, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5008, 5006, 1004, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \tan^2(d(a+b\log(cx^n)))\,dx$$
$$\Big\downarrow \text{5008}$$
$$\frac{x^4(cx^n)^{-4/n}\int (cx^n)^{\frac{4}{n}-1}\tan^2(d(a+b\log(cx^n)))\,d(cx^n)}{n}$$
$$\Big\downarrow \text{5006}$$
$$\frac{x^4(cx^n)^{-4/n}\int \frac{(cx^n)^{\frac{4}{n}-1}\left(i-ie^{2iad}(cx^n)^{2ibd}\right)^2}{\left(e^{2iad}(cx^n)^{2ibd}+1\right)^2}d(cx^n)}{n}$$

$$\Big\downarrow \text{1004}$$

$$\frac{x^4(cx^n)^{-4/n}\left(\frac{ie^{-2iad}\int -\frac{2(cx^n)^{\frac{4}{n}-1}\left(\frac{e^{2iad}(4-ibdn)}{n}-\frac{e^{4iad}(ibdn+4)(cx^n)^{2ibd}}{n}\right)}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{2bd}+\frac{i(cx^n)^{4/n}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}\right)}{n}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4(cx^n)^{-4/n}\left(\frac{i(cx^n)^{4/n}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\int \frac{(cx^n)^{\frac{4}{n}-1}\left(\frac{e^{2iad}(4-ibdn)}{n}-\frac{e^{4iad}(ibdn+4)(cx^n)^{2ibd}}{n}\right)}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{bd}\right)}{n}$$

$$\Big\downarrow \text{959}$$

$$\frac{x^4(cx^n)^{-4/n}\left(\frac{i(cx^n)^{4/n}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\left(\frac{8e^{2iad}\int \frac{(cx^n)^{\frac{4}{n}-1}}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{n}-\frac{1}{4}e^{2iad}(4+ibdn)(cx^n)^{4/n}\right)}{bd}\right)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^4(cx^n)^{-4/n}\left(\frac{i(cx^n)^{4/n}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\left(2e^{2iad}(cx^n)^{4/n}\text{Hypergeometric2F1}\left(1,-\frac{2i}{bdn},1-\frac{2i}{bdn},-e^{2iad}(cx^n)^{2ibd}\right)-\frac{1}{4}e^{2iad}(4\right.}{bd}}{n}$$

input

```
Int[x^3*Tan[d*(a + b*Log[c*x^n])]^2,x]
```

output

```
(x^4*((I*(c*x^n)^(4/n)*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*(1 +
E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - (I*(-1/4*(E^((2*I)*a*d)*(4 + I*b*d*n
)*(c*x^n)^(4/n)) + 2*E^((2*I)*a*d)*(c*x^n)^(4/n)*Hypergeometric2F1[1, (-2*
I)/(b*d*n), 1 - (2*I)/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))]))/(b*
d*E^((2*I)*a*d))))/(n*(c*x^n)^(4/n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5008
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^3 \tan\left(d(a+b\ln\left(c\,x^n\right))\right)^2 dx$$

input
```
int(x^3*tan(d*(a+b*ln(c*x^n)))^2,x)
```

output
```
int(x^3*tan(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int x^3 \tan^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^3 \tan\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

input
```
integrate(x^3*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

output
```
integral(x^3*tan(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \tan^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \text{Timed out}$$

input
```
integrate(x**3*tan(d*(a+b*ln(c*x**n)))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^3 \tan^2(d(a + b\log(cx^n)))\,dx = \int x^3 \tan((b\log(cx^n) + a)d)^2\,dx$$

input
```
integrate(x^3*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

output
```
-1/4*((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x^4*cos(2*b*d*
log(x^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*
n*x^4*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*x^4 + 2*(b*d*n*cos(2*b*d*log(c
)) - 4*sin(2*b*d*log(c)))*x^4*cos(2*b*d*log(x^n) + 2*a*d) - 2*(b*d*n*sin(2
*b*d*log(c)) + 4*cos(2*b*d*log(c)))*x^4*sin(2*b*d*log(x^n) + 2*a*d) + 32*(
2*b^2*d^2*n^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^
2*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) + b^2*d^2*n^2 + (b^2*d^2*c
os(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) +
 2*a*d)^2 + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^
2*sin(2*b*d*log(x^n) + 2*a*d)^2)*integrate((x^3*cos(2*b*d*log(x^n) + 2*a*d
)*sin(2*b*d*log(c)) + x^3*cos(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d))/(
2*b^2*d^2*n^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^
2*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) + b^2*d^2*n^2 + (b^2*d^2*c
os(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) +
 2*a*d)^2 + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^
2*sin(2*b*d*log(x^n) + 2*a*d)^2), x))/(2*b*d*n*cos(2*b*d*log(c))*cos(2*b*d
*log(x^n) + 2*a*d) - 2*b*d*n*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d)
 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d*log(x^n
) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*sin(2
*b*d*log(x^n) + 2*a*d)^2 + b*d*n)
```

Giac [F]

$$\int x^3 \tan^2(d(a + b\log(cx^n)))\,dx = \int x^3 \tan((b\log(cx^n) + a)d)^2\,dx$$

input
```
integrate(x^3*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

output
```
integrate(x^3*tan((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \tan^2(d(a+b\log(cx^n)))\,dx = \int x^3 \tan\left(d\left(a+b\,\ln\left(c\,x^n\right)\right)\right)^2\,dx$$

input
```
int(x^3*tan(d*(a + b*log(c*x^n)))^2,x)
```

output
```
int(x^3*tan(d*(a + b*log(c*x^n)))^2, x)
```

Reduce [F]

$$\int x^3 \tan^2(d(a+b\log(cx^n)))\,dx = \frac{-16\left(\int \tan(\log(x^n c)\,bd+ad)\,x^3dx\right)+4\tan(\log(x^n c)\,bd+ad)\,x^4-bdn\,x^4}{4bdn}$$

input
```
int(x^3*tan(d*(a+b*log(c*x^n)))^2,x)
```

output
```
( - 16*int(tan(log(x**n*c)*b*d + a*d)*x**3,x) + 4*tan(log(x**n*c)*b*d + a*
d)*x**4 - b*d*n*x**4)/(4*b*d*n)
```

3.166 $\int x^2 \tan^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 163

$$\int x^2 \tan^2(d(a+b\log(cx^n)))\,dx = \frac{(3i-bdn)x^3}{3bdn} + \frac{ix^3\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1+e^{2iad}(cx^n)^{2ibd}\right)} - \frac{2ix^3\,\text{Hypergeometric2F1}\left(1,-\frac{3i}{2bdn},1-\frac{3i}{2bdn},-e^{2iad}(cx^n)^{2ibd}\right)}{bdn}$$

output

```
1/3*(3*I-b*d*n)*x^3/b/d/n+I*x^3*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/(
1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))-2*I*x^3*hypergeom([1, -3/2*I/b/d/n],[1-3
/2*I/b/d/n],-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n
```

Mathematica [A] (verified)

Time = 4.62 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.16

$$\int x^2 \tan^2(d(a+b\log(cx^n)))\,dx =$$
$$-\frac{x^3\left(-9e^{2id(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1-\frac{3i}{2bdn},2-\frac{3i}{2bdn},-e^{2id(a+b\log(cx^n))}\right)+(-3i+2bdn)\left(bd\right.\right.}{3bdn(-3i+2}$$

input
```
Integrate[x^2*Tan[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
-1/3*(x^3*(-9*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - ((3*
I)/2)/(b*d*n), 2 - ((3*I)/2)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] + (
-3*I + 2*b*d*n)*(b*d*n + (3*I)*Hypergeometric2F1[1, ((-3*I)/2)/(b*d*n), 1
- ((3*I)/2)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] - 3*Tan[d*(a + b*Log
[c*x^n])])))/(b*d*n*(-3*I + 2*b*d*n))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.33, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5008, 5006, 1004, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \tan^2(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{5008}$$

$$\frac{x^3(cx^n)^{-3/n}\int (cx^n)^{\frac{3}{n}-1}\tan^2(d(a+b\log(cx^n)))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5006}$$

$$\frac{x^3(cx^n)^{-3/n}\int \frac{(cx^n)^{\frac{3}{n}-1}\left(i-ie^{2iad}(cx^n)^{2ibd}\right)^2}{\left(e^{2iad}(cx^n)^{2ibd}+1\right)^2}d(cx^n)}{n}$$

$$\Big\downarrow \text{1004}$$

$$\frac{x^3(cx^n)^{-3/n}\left(\frac{ie^{-2iad}\int -\frac{2(cx^n)^{\frac{3}{n}-1}\left(\frac{e^{2iad}(3-ibdn)}{n}-\frac{e^{4iad}(ibdn+3)(cx^n)^{2ibd}}{n}\right)}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{2bd}+\frac{i(cx^n)^{3/n}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}\right)}{n}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3(cx^n)^{-3/n}\left(\frac{i(cx^n)^{3/n}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\int \frac{(cx^n)^{\frac{3}{n}-1}\left(\frac{e^{2iad}(3-ibdn)}{n}-\frac{e^{4iad}(ibdn+3)(cx^n)^{2ibd}}{n}\right)}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{bd}\right)}{n}$$

$$\Big\downarrow \text{959}$$

$$\frac{x^3(cx^n)^{-3/n}\left(\frac{i(cx^n)^{3/n}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\left(\frac{6e^{2iad}\int \frac{(cx^n)^{\frac{3}{n}-1}}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{n}-\frac{1}{3}e^{2iad}(3+ibdn)(cx^n)^{3/n}\right)}{bd}\right)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^3(cx^n)^{-3/n}\left(\frac{i(cx^n)^{3/n}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\left(2e^{2iad}(cx^n)^{3/n}\text{ Hypergeometric2F1}\left(1,-\frac{3i}{2bdn},1-\frac{3i}{2bdn},-e^{2iad}(cx^n)^{2ibd}\right)-\frac{1}{3}e^{2iad}\right.}{bd}}{n}$$

input
```
Int[x^2*Tan[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
(x^3*((I*(c*x^n)^(3/n)*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*(1 +
E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - (I*(-1/3*(E^((2*I)*a*d)*(3 + I*b*d*n
)*(c*x^n)^(3/n)) + 2*E^((2*I)*a*d)*(c*x^n)^(3/n)*Hypergeometric2F1[1, ((-3
*I)/2)/(b*d*n), 1 - ((3*I)/2)/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)
)]))/(b*d*E^((2*I)*a*d))))/(n*(c*x^n)^(3/n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5008
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^2 \tan \left(d(a+b\ln \left(c\,x^n\right))\right)^2 dx$$

input
```
int(x^2*tan(d*(a+b*ln(c*x^n)))^2,x)
```

output
```
int(x^2*tan(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int x^2 \tan^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^2 \tan\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

input
```
integrate(x^2*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

output
```
integral(x^2*tan(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int x^2 \tan^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^2 \tan^2\left(ad+bd\log\left(cx^n\right)\right)dx$$

input
```
integrate(x**2*tan(d*(a+b*ln(c*x**n)))**2,x)
```

output
```
Integral(x**2*tan(a*d + b*d*log(c*x**n))**2, x)
```

Maxima [F]

$$\int x^2 \tan^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x^2\tan\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

input
```
integrate(x^2*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

output
```
-1/3*((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x^3*cos(2*b*d*
log(x^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*
n*x^3*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*x^3 + 2*(b*d*n*cos(2*b*d*log(c
)) - 3*sin(2*b*d*log(c)))*x^3*cos(2*b*d*log(x^n) + 2*a*d) - 2*(b*d*n*sin(2
*b*d*log(c)) + 3*cos(2*b*d*log(c)))*x^3*sin(2*b*d*log(x^n) + 2*a*d) + 18*(
2*b^2*d^2*n^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^
2*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) + b^2*d^2*n^2 + (b^2*d^2*c
os(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) +
 2*a*d)^2 + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^
2*sin(2*b*d*log(x^n) + 2*a*d)^2)*integrate((x^2*cos(2*b*d*log(x^n) + 2*a*d
)*sin(2*b*d*log(c)) + x^2*cos(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d))/(
2*b^2*d^2*n^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^
2*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) + b^2*d^2*n^2 + (b^2*d^2*c
os(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) +
 2*a*d)^2 + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^
2*sin(2*b*d*log(x^n) + 2*a*d)^2), x))/(2*b*d*n*cos(2*b*d*log(c))*cos(2*b*d
*log(x^n) + 2*a*d) - 2*b*d*n*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d)
 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d*log(x^n
) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*sin(2
*b*d*log(x^n) + 2*a*d)^2 + b*d*n)
```

Giac [F]

$$\int x^2 \tan^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x^2\tan\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

input
```
integrate(x^2*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

output
```
integrate(x^2*tan((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \tan^2(d(a+b\log(cx^n)))\,dx = \int x^2 \tan(d\,(a+b\,\ln(c\,x^n)))^2\,dx$$

input
```
int(x^2*tan(d*(a + b*log(c*x^n)))^2,x)
```

output
```
int(x^2*tan(d*(a + b*log(c*x^n)))^2, x)
```

Reduce [F]

$$\int x^2 \tan^2(d(a+b\log(cx^n)))\,dx$$
$$= \frac{-9\left(\int \tan(\log(x^n c)\,bd+ad)\,x^2dx\right)+3\tan(\log(x^n c)\,bd+ad)\,x^3-bdn\,x^3}{3bdn}$$

input
```
int(x^2*tan(d*(a+b*log(c*x^n)))^2,x)
```

output
```
( - 9*int(tan(log(x**n*c)*b*d + a*d)*x**2,x) + 3*tan(log(x**n*c)*b*d + a*d
)*x**3 - b*d*n*x**3)/(3*b*d*n)
```

3.167 $\int x \tan^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 159

$$\int x\tan^2(d(a+b\log(cx^n)))\,dx = \frac{(2i-bdn)x^2}{2bdn}+\frac{ix^2\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1+e^{2iad}(cx^n)^{2ibd}\right)} - \frac{2ix^2\,\text{Hypergeometric2F1}\left(1,-\frac{i}{bdn},1-\frac{i}{bdn},-e^{2iad}(cx^n)^{2ibd}\right)}{bdn}$$

output
```
1/2*(2*I-b*d*n)*x^2/b/d/n+I*x^2*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/(
1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))-2*I*x^2*hypergeom([1, -I/b/d/n],[1-I/b/d
/n],-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n
```

Mathematica [A] (verified)

Time = 4.67 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.13

$$\int x\tan^2(d(a+b\log(cx^n)))\,dx = -\frac{x^2\left(-2e^{2id(a+b\log(cx^n))}\text{Hypergeometric2F1}\left(1,1-\frac{i}{bdn},2-\frac{i}{bdn},-e^{2id(a+b\log(cx^n))}\right)+(-i+bdn)\left(bdn+\right.\right.}{2bdn(-i+bdn}$$

input
```
Integrate[x*Tan[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
-1/2*(x^2*(-2*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - I/(b
*d*n), 2 - I/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] + (-I + b*d*n)*(b*d
*n + (2*I)*Hypergeometric2F1[1, (-I)/(b*d*n), 1 - I/(b*d*n), -E^((2*I)*d*(
a + b*Log[c*x^n]))] - 2*Tan[d*(a + b*Log[c*x^n])])))/(b*d*n*(-I + b*d*n))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.34, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {5008, 5006, 1004, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\tan^2(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{5008}$$

$$\frac{x^2(cx^n)^{-2/n}\int (cx^n)^{\frac{2}{n}-1}\tan^2(d(a+b\log(cx^n)))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5006}$$

$$\frac{x^2(cx^n)^{-2/n}\int \frac{(cx^n)^{\frac{2}{n}-1}\left(i-ie^{2iad}(cx^n)^{2ibd}\right)^2}{\left(e^{2iad}(cx^n)^{2ibd}+1\right)^2}d(cx^n)}{n}$$

$$\Big\downarrow \text{1004}$$

$$\frac{x^2 (cx^n)^{-2/n}\left(\frac{ie^{-2iad}\int -\frac{2(cx^n)^{\frac{2}{n}-1}\left(\frac{e^{2iad}(2-ibdn)}{n}-\frac{e^{4iad}(ibdn+2)(cx^n)^{2ibd}}{n}\right)}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{2bd}+\frac{i(cx^n)^{2/n}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}\right)}{n}$$

$$\Big\downarrow \; 27$$

$$\frac{x^2 (cx^n)^{-2/n}\left(\frac{i(cx^n)^{2/n}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\int \frac{(cx^n)^{\frac{2}{n}-1}\left(\frac{e^{2iad}(2-ibdn)}{n}-\frac{e^{4iad}(ibdn+2)(cx^n)^{2ibd}}{n}\right)}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{bd}\right)}{n}$$

$$\Big\downarrow \; 959$$

$$\frac{x^2 (cx^n)^{-2/n}\left(\frac{i(cx^n)^{2/n}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\left(\frac{4e^{2iad}\int \frac{(cx^n)^{\frac{2}{n}-1}}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{n}-\frac{1}{2}e^{2iad}(2+ibdn)(cx^n)^{2/n}\right)}{bd}\right)}{n}$$

$$\Big\downarrow \; 888$$

$$\frac{x^2 (cx^n)^{-2/n}\left(\frac{i(cx^n)^{2/n}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\left(2e^{2iad}(cx^n)^{2/n}\,\text{Hypergeometric2F1}\left(1,-\frac{i}{bdn},1-\frac{i}{bdn},-e^{2iad}(cx^n)^{2ibd}\right)-\frac{1}{2}e^{2iad}(2\right.}{bd}}{n}$$

input
```
Int[x*Tan[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
(x^2*((I*(c*x^n)^(2/n)*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*(1 +
E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - (I*(-1/2*(E^((2*I)*a*d)*(2 + I*b*d*n
)*(c*x^n)^(2/n)) + 2*E^((2*I)*a*d)*(c*x^n)^(2/n)*Hypergeometric2F1[1, (-I)
/(b*d*n), 1 - I/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))]))/(b*d*E^((
2*I)*a*d))))/(n*(c*x^n)^(2/n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5008
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x\tan\left(d(a+b\ln\left(c\,x^{n}\right))\right)^{2}dx$$

input
```
int(x*tan(d*(a+b*ln(c*x^n)))^2,x)
```

output
```
int(x*tan(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int x\tan^{2}\left(d(a+b\log\left(cx^{n}\right))\right)\,dx=\int x\tan\left((b\log\left(cx^{n}\right)+a)d\right)^{2}\,dx$$

input
```
integrate(x*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

output
```
integral(x*tan(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int x\tan^{2}\left(d(a+b\log\left(cx^{n}\right))\right)\,dx=\int x\tan^{2}\left(ad+bd\log\left(cx^{n}\right)\right)dx$$

input
```
integrate(x*tan(d*(a+b*ln(c*x**n)))**2,x)
```

output
```
Integral(x*tan(a*d + b*d*log(c*x**n))**2, x)
```

Maxima [F]

$$\int x\tan^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\tan\left(\left(b\log\left(cx^n\right)+a\right)d\right)^2\,dx$$

input
```
integrate(x*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

output
```
-1/2*((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x^2*cos(2*b*d*
log(x^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*
n*x^2*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*x^2 + 2*(b*d*n*cos(2*b*d*log(c
)) - 2*sin(2*b*d*log(c)))*x^2*cos(2*b*d*log(x^n) + 2*a*d) - 2*(b*d*n*sin(2
*b*d*log(c)) + 2*cos(2*b*d*log(c)))*x^2*sin(2*b*d*log(x^n) + 2*a*d) + 8*(2
*b^2*d^2*n^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2
*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) + b^2*d^2*n^2 + (b^2*d^2*co
s(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) +
2*a*d)^2 + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2
*sin(2*b*d*log(x^n) + 2*a*d)^2)*integrate((x*cos(2*b*d*log(x^n) + 2*a*d)*s
in(2*b*d*log(c)) + x*cos(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d))/(2*b^2
*d^2*n^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*sin
(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) + b^2*d^2*n^2 + (b^2*d^2*cos(2*
b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) + 2*a*
d)^2 + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*sin
(2*b*d*log(x^n) + 2*a*d)^2), x))/(2*b*d*n*cos(2*b*d*log(c))*cos(2*b*d*log(
x^n) + 2*a*d) - 2*b*d*n*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) + (b
*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d*log(x^n) + 2
*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*sin(2*b*d*
log(x^n) + 2*a*d)^2 + b*d*n)
```

Giac [F]

$$\int x\tan^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\tan\left(\left(b\log\left(cx^n\right)+a\right)d\right)^2\,dx$$

input
```
integrate(x*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

output
```
integrate(x*tan((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\tan^2(d(a+b\log(cx^n)))\,dx=\int x\tan(d\,(a+b\,\ln(c\,x^n)))^2\,dx$$

input
```
int(x*tan(d*(a + b*log(c*x^n)))^2,x)
```

output
```
int(x*tan(d*(a + b*log(c*x^n)))^2, x)
```

Reduce [F]

$$\int x\tan^2(d(a+b\log(cx^n)))\,dx$$
$$=\frac{-4\left(\int\tan(\log(x^nc)\,bd+ad)\,xdx\right)+2\tan(\log(x^nc)\,bd+ad)\,x^2-bdn\,x^2}{2bdn}$$

input
```
int(x*tan(d*(a+b*log(c*x^n)))^2,x)
```

output
```
( - 4*int(tan(log(x**n*c)*b*d + a*d)*x,x) + 2*tan(log(x**n*c)*b*d + a*d)*x
**2 - b*d*n*x**2)/(2*b*d*n)
```

3.168 $\int \tan^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 154

$$\int \tan^2(d(a+b\log(cx^n)))\,dx = \frac{(i-bdn)x}{bdn} + \frac{ix\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1+e^{2iad}(cx^n)^{2ibd}\right)} - \frac{2ix\,\text{Hypergeometric2F1}\left(1,-\frac{i}{2bdn},1-\frac{i}{2bdn},-e^{2iad}(cx^n)^{2ibd}\right)}{bdn}$$

output
```
(I-b*d*n)*x/b/d/n+I*x*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/(1+exp(2*I*
a*d)*(c*x^n)^(2*I*b*d))-2*I*x*hypergeom([1, -1/2*I/b/d/n],[1-1/2*I/b/d/n],
-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n
```

Mathematica [A] (verified)

Time = 7.48 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.20

$$\int \tan^2\left(d(a+b\log\left(cx^n\right))\right)\,dx$$
$$= \frac{e^{2id(a+b\log(cx^n))}x\,\text{Hypergeometric2F1}\left(1,1-\frac{i}{2bdn},2-\frac{i}{2bdn},-e^{2id(a+b\log(cx^n))}\right)-(-i+2bdn)x\left(bdn+i\,\text{Hy}\right.}{bdn(-i+2bdn)}$$

input
```
Integrate[Tan[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
(E^((2*I)*d*(a + b*Log[c*x^n]))*x*Hypergeometric2F1[1, 1 - (I/2)/(b*d*n),
2 - (I/2)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] - (-I + 2*b*d*n)*x*(b*
d*n + I*Hypergeometric2F1[1, (-1/2*I)/(b*d*n), 1 - (I/2)/(b*d*n), -E^((2*I
)*d*(a + b*Log[c*x^n]))] - Tan[d*(a + b*Log[c*x^n])]))/(b*d*n*(-I + 2*b*d*
n))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.34, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5004, 5006, 1004, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

$$\Big\downarrow \text{5004}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\tan^2\left(d(a+b\log\left(cx^n\right))\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{5006}$$

$$\frac{x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}-1}\left(i-ie^{2iad}(cx^n)^{2ibd}\right)^2}{\left(e^{2iad}(cx^n)^{2ibd}+1\right)^2}d(cx^n)}{n}$$

$$\Big\downarrow \text{1004}$$

$$\frac{x(cx^n)^{-1/n}\left(\frac{ie^{-2iad}\int -\frac{2(cx^n)^{\frac{1}{n}-1}\left(\frac{e^{2iad}(1-ibdn)}{n}-\frac{e^{4iad}(ibdn+1)(cx^n)^{2ibd}}{n}\right)}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{2bd}+\frac{i(cx^n)^{\frac{1}{n}}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}\right)}{n}$$

$$\Big\downarrow \text{27}$$

$$\frac{x(cx^n)^{-1/n}\left(\frac{i(cx^n)^{\frac{1}{n}}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\int \frac{(cx^n)^{\frac{1}{n}-1}\left(\frac{e^{2iad}(1-ibdn)}{n}-\frac{e^{4iad}(ibdn+1)(cx^n)^{2ibd}}{n}\right)}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{bd}\right)}{n}$$

$$\Big\downarrow \text{959}$$

$$\frac{x(cx^n)^{-1/n}\left(\frac{i(cx^n)^{\frac{1}{n}}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\left(\frac{2e^{2iad}\int \frac{(cx^n)^{\frac{1}{n}-1}}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{n}-e^{2iad}(1+ibdn)(cx^n)^{\frac{1}{n}}\right)}{bd}\right)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x(cx^n)^{-1/n}\left(\frac{i(cx^n)^{\frac{1}{n}}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\left(2e^{2iad}(cx^n)^{\frac{1}{n}}\text{Hypergeometric2F1}\left(1,-\frac{i}{2bdn},1-\frac{i}{2bdn},-e^{2iad}(cx^n)^{2ibd}\right)-e^{2iad}(1+ib\right)}{bd}\right)}{n}$$

input
```
Int[Tan[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
(x*((I*(c*x^n)^n^(-1)*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*(1 + E
^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - (I*(-(E^((2*I)*a*d)*(1 + I*b*d*n)*(c*
x^n)^n^(-1)) + 2*E^((2*I)*a*d)*(c*x^n)^n^(-1)*Hypergeometric2F1[1, (-1/2*I
)/(b*d*n), 1 - (I/2)/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))]))/(b*d
*E^((2*I)*a*d))))/(n*(c*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 5004
```
Int[Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Tan[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [F]

$$\int \tan\left(d(a + b\ln\left(c\,x^{n}\right))\right)^{2}dx$$

input
```
int(tan(d*(a+b*ln(c*x^n)))^2,x)
```

output
```
int(tan(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int \tan^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int \tan\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

output
```
integral(tan(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int \tan^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int \tan^2\left(d(a+b\log\left(cx^n\right))\right)dx$$

input
```
integrate(tan(d*(a+b*ln(c*x**n)))**2,x)
```

output
```
Integral(tan(d*(a + b*log(c*x**n)))**2, x)
```

Maxima [F]

$$\int \tan^2(d(a + b\log(cx^n)))\,dx = \int \tan\left((b\log(cx^n) + a)d\right)^2\,dx$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

output
```
-((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x*cos(2*b*d*log(x^
n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x*si
n(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*x + 2*(b*d*n*cos(2*b*d*log(c)) - sin(2
*b*d*log(c)))*x*cos(2*b*d*log(x^n) + 2*a*d) - 2*(b*d*n*sin(2*b*d*log(c)) +
 cos(2*b*d*log(c)))*x*sin(2*b*d*log(x^n) + 2*a*d) + 2*(2*b^2*d^2*n^2*cos(2
*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*sin(2*b*d*log(c))
*sin(2*b*d*log(x^n) + 2*a*d) + b^2*d^2*n^2 + (b^2*d^2*cos(2*b*d*log(c))^2
+ b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b^2*d^
2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*sin(2*b*d*log(x^n
) + 2*a*d)^2)*integrate((cos(2*b*d*log(x^n) + 2*a*d)*sin(2*b*d*log(c)) + c
os(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d))/(2*b^2*d^2*n^2*cos(2*b*d*log
(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*sin(2*b*d*log(c))*sin(2*b
*d*log(x^n) + 2*a*d) + b^2*d^2*n^2 + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^
2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b^2*d^2*cos(2*
b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*sin(2*b*d*log(x^n) + 2*a*
d)^2), x))/(2*b*d*n*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b*d*
n*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) + (b*d*cos(2*b*d*log(c))^2
 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b*d*cos(2*b
*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*sin(2*b*d*log(x^n) + 2*a*d)^2 +
b*d*n)
```

Giac [F]

$$\int \tan^2(d(a + b\log(cx^n)))\,dx = \int \tan\left((b\log(cx^n) + a)d\right)^2\,dx$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

output
```
integrate(tan((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tan^2(d(a+b\log(cx^n)))\,dx = \int \tan(d\,(a+b\,\ln(c\,x^n)))^2\,dx$$

input
```
int(tan(d*(a + b*log(c*x^n)))^2,x)
```

output
```
int(tan(d*(a + b*log(c*x^n)))^2, x)
```

Reduce [F]

$$\int \tan^2(d(a+b\log(cx^n)))\,dx$$
$$= \frac{-\left(\int \tan(\log(x^n c)\,bd+ad)\,dx\right)+\tan(\log(x^n c)\,bd+ad)\,x-bdnx}{bdn}$$

input
```
int(tan(d*(a+b*log(c*x^n)))^2,x)
```

output
```
( - int(tan(log(x**n*c)*b*d + a*d),x) + tan(log(x**n*c)*b*d + a*d)*x - b*d
*n*x)/(b*d*n)
```

3.169 $\int \frac{\tan^2(d(a+b\log(cx^n)))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 29

$$\int \frac{\tan^2\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx=-\log(x)+\frac{\tan\left(ad+bd\log\left(cx^n\right)\right)}{bdn}$$

output
```
-ln(x)+tan(a*d+b*d*ln(c*x^n))/b/d/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.76

$$\int \frac{\tan^2\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = -\frac{\arctan\left(\tan\left(ad+bd\log\left(cx^n\right)\right)\right)}{bdn}+\frac{\tan\left(ad+bd\log\left(cx^n\right)\right)}{bdn}$$

input
```
Integrate[Tan[d*(a + b*Log[c*x^n])]^2/x,x]
```

output
```
-(ArcTan[Tan[a*d + b*d*Log[c*x^n]]]/(b*d*n)) + Tan[a*d + b*d*Log[c*x^n]]/(
b*d*n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3039, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(d(a+b\log(cx^n)))}{x} \, dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \tan^2(d(a+b\log(cx^n)))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \tan(ad+b\log(cx^n)d)^2d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\frac{\tan(ad+bd\log(cx^n))}{bd} - \int 1d\log(cx^n)}{n}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{\tan(ad+bd\log(cx^n))}{bd} - \log(cx^n)}{n}$$

input
```
Int[Tan[d*(a + b*Log[c*x^n])]^2/x,x]
```

output
```
(-Log[c*x^n] + Tan[a*d + b*d*Log[c*x^n]]/(b*d))/n
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

method	result
parallelrisch	$\frac{-bd\ln(c\,x^n)+\tan(d(a+b\ln(c\,x^n)))}{bdn}$
derivativedivides	$\frac{\tan(d(a+b\ln(c\,x^n)))-\arctan(\tan(d(a+b\ln(c\,x^n))))}{nbd}$
default	$\frac{\tan(d(a+b\ln(c\,x^n)))-\arctan(\tan(d(a+b\ln(c\,x^n))))}{nbd}$
risch	$-\ln(x)+\frac{2i}{dbn\left(c^{2ibd}(x^n)^{2ibd}\mathrm{e}^{d\left(-b\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2+b\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)+b\pi\,\mathrm{csgn}(ic\,x^n)^3-b\pi\,\mathrm{csgn}(\right.}\right.}$

input
```
int(tan(d*(a+b*ln(c*x^n)))^2/x,x,method=_RETURNVERBOSE)
```

output
```
(-b*d*ln(c*x^n)+tan(d*(a+b*ln(c*x^n))))/b/d/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(29) = 58.

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.93

$$\int \frac{\tan^2(d(a+b\log(cx^n)))}{x}\,dx = -\frac{bdn\cos(2\,bdn\log(x)+2\,bd\log(c)+2\,ad)\log(x)+bdn\log(x)-\sin(2\,bdn\log(x)+2\,bd\log(c)+2\,a}{bdn\cos(2\,bdn\log(x)+2\,bd\log(c)+2\,ad)+bdn}$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))^2/x,x, algorithm="fricas")
```

output
```
-(b*d*n*cos(2*b*d*n*log(x) + 2*b*d*log(c) + 2*a*d)*log(x) + b*d*n*log(x) -
 sin(2*b*d*n*log(x) + 2*b*d*log(c) + 2*a*d))/(b*d*n*cos(2*b*d*n*log(x) + 2
*b*d*log(c) + 2*a*d) + b*d*n)
```

Sympy [F]

$$\int \frac{\tan^2(d(a+b\log(cx^n)))}{x}\,dx = \int \frac{\tan^2(ad+bd\log(cx^n))}{x}\,dx$$

input
```
integrate(tan(d*(a+b*ln(c*x**n)))**2/x,x)
```

output
```
Integral(tan(a*d + b*d*log(c*x**n))**2/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(29) = 58.

Time = 0.05 (sec) , antiderivative size = 320, normalized size of antiderivative = 11.03

$$\int \frac{\tan^2(d(a+b\log(cx^n)))}{x}\,dx = -\frac{\left((bd\cos(2\,bd\log(c))^2+bd\sin(2\,bd\log(c))^2\right)n\cos(2\,bd\log(x^n)+2\,ad)^2\log(x)+\left(bd\cos(2\,bd\log(c))^2\right.}{2\,bdn\cos(2\,bd\log(c))\cos(2\,bd\log(x^n)+2\,ad)-2\,bdn\sin(2\,bd}$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))^2/x,x, algorithm="maxima")
```

output
```
-((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d*log(x^n)
 + 2*a*d)^2*log(x) + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n
*log(x)*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*log(x) + 2*(b*d*n*cos(2*b*d*
log(c))*log(x) - sin(2*b*d*log(c)))*cos(2*b*d*log(x^n) + 2*a*d) - 2*(b*d*n
*log(x)*sin(2*b*d*log(c)) + cos(2*b*d*log(c)))*sin(2*b*d*log(x^n) + 2*a*d)
)/(2*b*d*n*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b*d*n*sin(2*b
*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) + (b*d*cos(2*b*d*log(c))^2 + b*d*si
n(2*b*d*log(c))^2)*n*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c)
)^2 + b*d*sin(2*b*d*log(c))^2)*n*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^2(d(a+b\log(cx^n)))}{x}\,dx = \text{Timed out}$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))^2/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 21.73 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{\tan^2(d(a+b\log(cx^n)))}{x}\,dx = -\ln(x) + \frac{2i}{b\,d\,n\,\left(e^{a\,d\,2i}\,(c\,x^n)^{b\,d\,2i}+1\right)}$$

input
```
int(tan(d*(a + b*log(c*x^n)))^2/x,x)
```

output
```
2i/(b*d*n*(exp(a*d*2i)*(c*x^n)^(b*d*2i) + 1)) - log(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{\tan^2(d(a+b\log(cx^n)))}{x}\,dx = \frac{-\log(x^nc)\,bd+\tan(\log(x^nc)\,bd+ad)}{bdn}$$

input
```
int(tan(d*(a+b*log(c*x^n)))^2/x,x)
```

output
```
( - log(x**n*c)*b*d + tan(log(x**n*c)*b*d + a*d))/(b*d*n)
```

3.170 $\int \frac{\tan^2(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 157

$$\int \frac{\tan^2(d(a+b\log(cx^n)))}{x^2}\,dx = \frac{1+\frac{i}{bdn}}{x}+\frac{i\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bdnx\left(1+e^{2iad}(cx^n)^{2ibd}\right)} - \frac{2i\,\text{Hypergeometric2F1}\left(1,\frac{i}{2bdn},1+\frac{i}{2bdn},-e^{2iad}(cx^n)^{2ibd}\right)}{bdnx}$$

output

```
(1+I/b/d/n)/x+I*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/x/(1+exp(2*I*a*d)
*(c*x^n)^(2*I*b*d))-2*I*hypergeom([1, 1/2*I/b/d/n],[1+1/2*I/b/d/n],-exp(2*
I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/x
```

Mathematica [A] (verified)

Time = 3.17 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.17

$$\int \frac{\tan^2(d(a+b\log(cx^n)))}{x^2}\,dx$$
$$= \frac{-e^{2id(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1+\frac{i}{2bdn},2+\frac{i}{2bdn},-e^{2id(a+b\log(cx^n))}\right)+(i+2bdn)\left(bdn-i\,\text{Hype}\right.}{bdn(i+2bdn)x}$$

input
```
Integrate[Tan[d*(a + b*Log[c*x^n])]^2/x^2,x]
```

output
```
(-(E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + (I/2)/(b*d*n),
2 + (I/2)/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))]) + (I + 2*b*d*n)*(b*d*
n - I*Hypergeometric2F1[1, (I/2)/(b*d*n), 1 + (I/2)/(b*d*n), -E^((2*I)*d*(
a + b*Log[c*x^n]))] + Tan[d*(a + b*Log[c*x^n])]))/(b*d*n*(I + 2*b*d*n)*x)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.35, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5008, 5006, 1004, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(d(a+b\log(cx^n)))}{x^2}\,dx$$
$$\Big\downarrow \text{5008}$$
$$\frac{(cx^n)^{\frac{1}{n}}\int (cx^n)^{-1-\frac{1}{n}}\tan^2(d(a+b\log(cx^n)))\,d(cx^n)}{nx}$$
$$\Big\downarrow \text{5006}$$
$$\frac{(cx^n)^{\frac{1}{n}}\int \frac{(cx^n)^{-1-\frac{1}{n}}\left(i-ie^{2iad}(cx^n)^{2ibd}\right)^2}{\left(e^{2iad}(cx^n)^{2ibd}+1\right)^2}d(cx^n)}{nx}$$
$$\Big\downarrow \text{1004}$$

$$\frac{(cx^n)^{\frac{1}{n}}\left(\frac{ie^{-2iad}\int \frac{2(cx^n)^{-1-\frac{1}{n}}\left(\frac{e^{2iad}(ibdn+1)}{n}-\frac{e^{4iad}(1-ibdn)(cx^n)^{2ibd}}{n}\right)}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{2bd}+\frac{i(cx^n)^{-1/n}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}\right)}{nx}$$

$$\Big\downarrow \; 27$$

$$\frac{(cx^n)^{\frac{1}{n}}\left(\frac{ie^{-2iad}\int \frac{(cx^n)^{-1-\frac{1}{n}}\left(\frac{e^{2iad}(ibdn+1)}{n}-\frac{e^{4iad}(1-ibdn)(cx^n)^{2ibd}}{n}\right)}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{bd}+\frac{i(cx^n)^{-1/n}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}\right)}{nx}$$

$$\Big\downarrow \; 959$$

$$\frac{(cx^n)^{\frac{1}{n}}\left(\frac{ie^{-2iad}\left(\frac{2e^{2iad}\int \frac{(cx^n)^{-1-\frac{1}{n}}}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{n}+e^{2iad}(1-ibdn)(cx^n)^{-1/n}\right)}{bd}+\frac{i(cx^n)^{-1/n}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}\right)}{nx}$$

$$\Big\downarrow \; 888$$

$$\frac{(cx^n)^{\frac{1}{n}}\left(\frac{ie^{-2iad}\left(e^{2iad}(1-ibdn)(cx^n)^{-1/n}-2e^{2iad}(cx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(1,\frac{i}{2bdn},1+\frac{i}{2bdn},-e^{2iad}(cx^n)^{2ibd}\right)\right)}{bd}+\frac{i(cx^n)^{-1/n}\left(1-e\ldots\right.}{bd\left(1+e^{2iad}(\ldots\right.}\right.}{nx}$$

input
```
Int[Tan[d*(a + b*Log[c*x^n])]^2/x^2,x]
```

output
```
((c*x^n)^n^(-1)*((I*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*(c*x^n)^
n^(-1)*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) + (I*((E^((2*I)*a*d)*(1 -
I*b*d*n))/(c*x^n)^n^(-1) - (2*E^((2*I)*a*d)*Hypergeometric2F1[1, (I/2)/(b*
d*n), 1 + (I/2)/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/(c*x^n)^n^
(-1)))/(b*d*E^((2*I)*a*d))))/(n*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5008
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\tan\left(d(a+b\ln\left(c\,x^{n}\right))\right)^{2}}{x^{2}}dx$$

input
```
int(tan(d*(a+b*ln(c*x^n)))^2/x^2,x)
```

output
```
int(tan(d*(a+b*ln(c*x^n)))^2/x^2,x)
```

Fricas [F]

$$\int \frac{\tan^{2}\left(d(a+b\log\left(cx^{n}\right))\right)}{x^{2}}\,dx=\int \frac{\tan\left((b\log\left(cx^{n}\right)+a)d\right)^{2}}{x^{2}}\,dx$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="fricas")
```

output
```
integral(tan(b*d*log(c*x^n) + a*d)^2/x^2, x)
```

Sympy [F]

$$\int \frac{\tan^{2}\left(d(a+b\log\left(cx^{n}\right))\right)}{x^{2}}\,dx=\int \frac{\tan^{2}\left(ad+bd\log\left(cx^{n}\right)\right)}{x^{2}}\,dx$$

input
```
integrate(tan(d*(a+b*ln(c*x**n)))**2/x**2,x)
```

output
```
Integral(tan(a*d + b*d*log(c*x**n))**2/x**2, x)
```

Maxima [F]

$$\int \frac{\tan^2(d(a+b\log(cx^n)))}{x^2}\,dx=\int \frac{\tan\left((b\log(cx^n)+a)d\right)^2}{x^2}\,dx$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="maxima")
```

output
```
((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d*log(x^n)
+ 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*sin(2*b
*d*log(x^n) + 2*a*d)^2 + b*d*n + 2*(b*d*n*cos(2*b*d*log(c)) + sin(2*b*d*lo
g(c)))*cos(2*b*d*log(x^n) + 2*a*d) + 2*(2*b^2*d^2*n^2*x*cos(2*b*d*log(c))*
cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*x*sin(2*b*d*log(c))*sin(2*b*d*
log(x^n) + 2*a*d) + b^2*d^2*n^2*x + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2
*sin(2*b*d*log(c))^2)*n^2*x*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b^2*d^2*cos(2
*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*x*sin(2*b*d*log(x^n) + 2
*a*d)^2)*integrate((cos(2*b*d*log(x^n) + 2*a*d)*sin(2*b*d*log(c)) + cos(2*
b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d))/(2*b^2*d^2*n^2*x^2*cos(2*b*d*log(
c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*x^2*sin(2*b*d*log(c))*sin(
2*b*d*log(x^n) + 2*a*d) + b^2*d^2*n^2*x^2 + (b^2*d^2*cos(2*b*d*log(c))^2 +
 b^2*d^2*sin(2*b*d*log(c))^2)*n^2*x^2*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b^2
*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*x^2*sin(2*b*d*
log(x^n) + 2*a*d)^2), x) - 2*(b*d*n*sin(2*b*d*log(c)) - cos(2*b*d*log(c)))
*sin(2*b*d*log(x^n) + 2*a*d))/(2*b*d*n*x*cos(2*b*d*log(c))*cos(2*b*d*log(x
^n) + 2*a*d) - 2*b*d*n*x*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) + (
b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x*cos(2*b*d*log(x^n)
+ 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x*sin(2
*b*d*log(x^n) + 2*a*d)^2 + b*d*n*x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^2(d(a+b\log(cx^n)))}{x^2}\,dx=\text{Timed out}$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2 (d(a + b\log (cx^n)))}{x^2} \, dx = \int \frac{\tan(d\,(a + b\,\ln (c\,x^n)))^2}{x^2} \, dx$$

input
```
int(tan(d*(a + b*log(c*x^n)))^2/x^2,x)
```

output
```
int(tan(d*(a + b*log(c*x^n)))^2/x^2, x)
```

Reduce [F]

$$\int \frac{\tan^2 (d(a + b\log (cx^n)))}{x^2} \, dx = \frac{\left(\int \frac{\tan(\log(x^n c)bd+ad)}{x^2} dx\right) x + \tan (\log(x^n c)\, bd + ad) + bdn}{bdnx}$$

input
```
int(tan(d*(a+b*log(c*x^n)))^2/x^2,x)
```

output
```
(int(tan(log(x**n*c)*b*d + a*d)/x**2,x)*x + tan(log(x**n*c)*b*d + a*d) + b
*d*n)/(b*d*n*x)
```

3.171 $\int \frac{\tan^2(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 156

$$\int \frac{\tan^2(d(a+b\log(cx^n)))}{x^3}\,dx = \frac{1+\frac{2i}{bdn}}{2x^2}+\frac{i\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bdnx^2\left(1+e^{2iad}(cx^n)^{2ibd}\right)} - \frac{2i\,\text{Hypergeometric2F1}\left(1,\frac{i}{bdn},1+\frac{i}{bdn},-e^{2iad}(cx^n)^{2ibd}\right)}{bdnx^2}$$

output
```
1/2*(1+2*I/b/d/n)/x^2+I*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/x^2/(1+ex
p(2*I*a*d)*(c*x^n)^(2*I*b*d))-2*I*hypergeom([1, I/b/d/n],[1+I/b/d/n],-exp(
2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/x^2
```

Mathematica [A] (verified)

Time = 2.89 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.15

$$\int \frac{\tan^2(d(a+b\log(cx^n)))}{x^3}\,dx$$
$$= \frac{-2e^{2id(a+b\log(cx^n))}\text{Hypergeometric2F1}\left(1,1+\frac{i}{bdn},2+\frac{i}{bdn},-e^{2id(a+b\log(cx^n))}\right)+(i+bdn)\left(bdn-2i\,\text{Hype}\right.}{2bdn(i+bdn)x^2}$$

input
```
Integrate[Tan[d*(a + b*Log[c*x^n])]^2/x^3,x]
```

output
```
(-2*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + I/(b*d*n), 2 +
 I/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] + (I + b*d*n)*(b*d*n - (2*I)*
Hypergeometric2F1[1, I/(b*d*n), 1 + I/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^
n]))] + 2*Tan[d*(a + b*Log[c*x^n])]))/(2*b*d*n*(I + b*d*n)*x^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.37, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5008, 5006, 1004, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(d(a+b\log(cx^n)))}{x^3}\,dx$$
$$\Big\downarrow \text{5008}$$
$$\frac{(cx^n)^{2/n}\int (cx^n)^{-1-\frac{2}{n}}\tan^2(d(a+b\log(cx^n)))\,d(cx^n)}{nx^2}$$
$$\Big\downarrow \text{5006}$$
$$\frac{(cx^n)^{2/n}\int \frac{(cx^n)^{-1-\frac{2}{n}}\left(i-ie^{2iad}(cx^n)^{2ibd}\right)^2}{\left(e^{2iad}(cx^n)^{2ibd}+1\right)^2}d(cx^n)}{nx^2}$$
$$\Big\downarrow \text{1004}$$

$$\frac{(cx^n)^{2/n}\left(\frac{ie^{-2iad}\int \frac{2(cx^n)^{-1-\frac{2}{n}}\left(\frac{e^{2iad}(ibdn+2)}{n}-\frac{e^{4iad}(2-ibdn)(cx^n)^{2ibd}}{n}\right)}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{2bd}+\frac{i(cx^n)^{-2/n}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}\right)}{nx^2}$$

$$\Big\downarrow \; 27$$

$$\frac{(cx^n)^{2/n}\left(\frac{ie^{-2iad}\int \frac{(cx^n)^{-1-\frac{2}{n}}\left(\frac{e^{2iad}(ibdn+2)}{n}-\frac{e^{4iad}(2-ibdn)(cx^n)^{2ibd}}{n}\right)}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{bd}+\frac{i(cx^n)^{-2/n}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}\right)}{nx^2}$$

$$\Big\downarrow \; 959$$

$$\frac{(cx^n)^{2/n}\left(\frac{ie^{-2iad}\left(\frac{4e^{2iad}\int \frac{(cx^n)^{-1-\frac{2}{n}}}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{n}+\frac{1}{2}e^{2iad}(2-ibdn)(cx^n)^{-2/n}\right)}{bd}+\frac{i(cx^n)^{-2/n}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}\right)}{nx^2}$$

$$\Big\downarrow \; 888$$

$$\frac{(cx^n)^{2/n}\left(\frac{ie^{-2iad}\left(\frac{1}{2}e^{2iad}(2-ibdn)(cx^n)^{-2/n}-2e^{2iad}(cx^n)^{-2/n}\text{Hypergeometric2F1}\left(1,\frac{i}{bdn},1+\frac{i}{bdn},-e^{2iad}(cx^n)^{2ibd}\right)\right)}{bd}+\frac{i(cx^n)^{-2/n}\left(1-\right.}{bd\left(1+e^{2iac}\right.}\right.}{nx^2}$$

input
```
Int[Tan[d*(a + b*Log[c*x^n])]^2/x^3,x]
```

output
```
((c*x^n)^(2/n)*((I*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*(c*x^n)^(
2/n)*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) + (I*((E^((2*I)*a*d)*(2 - I*
b*d*n))/(2*(c*x^n)^(2/n)) - (2*E^((2*I)*a*d)*Hypergeometric2F1[1, I/(b*d*n
), 1 + I/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/(c*x^n)^(2/n)))/(
b*d*E^((2*I)*a*d))))/(n*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5008
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\tan(d(a+b\ln(c\,x^n)))^2}{x^3}dx$$

input
```
int(tan(d*(a+b*ln(c*x^n)))^2/x^3,x)
```

output
```
int(tan(d*(a+b*ln(c*x^n)))^2/x^3,x)
```

Fricas [F]

$$\int \frac{\tan^2(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\tan((b\log(cx^n)+a)d)^2}{x^3}\,dx$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="fricas")
```

output
```
integral(tan(b*d*log(c*x^n) + a*d)^2/x^3, x)
```

Sympy [F]

$$\int \frac{\tan^2(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\tan^2(ad+bd\log(cx^n))}{x^3}\,dx$$

input
```
integrate(tan(d*(a+b*ln(c*x**n)))**2/x**3,x)
```

output
```
Integral(tan(a*d + b*d*log(c*x**n))**2/x**3, x)
```

Maxima [F]

$$\int \frac{\tan^2(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\tan((b\log(cx^n)+a)d)^2}{x^3}\,dx$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="maxima")
```

output
```
1/2*((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d*log(x
^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*sin
(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n + 2*(b*d*n*cos(2*b*d*log(c)) + 2*sin(2*
b*d*log(c)))*cos(2*b*d*log(x^n) + 2*a*d) + 8*(2*b^2*d^2*n^2*x^2*cos(2*b*d*
log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*x^2*sin(2*b*d*log(c))*
sin(2*b*d*log(x^n) + 2*a*d) + b^2*d^2*n^2*x^2 + (b^2*d^2*cos(2*b*d*log(c))
^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*x^2*cos(2*b*d*log(x^n) + 2*a*d)^2 +
(b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*x^2*sin(2*
b*d*log(x^n) + 2*a*d)^2)*integrate((cos(2*b*d*log(x^n) + 2*a*d)*sin(2*b*d*
log(c)) + cos(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d))/(2*b^2*d^2*n^2*x^
3*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*x^3*sin(2*
b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) + b^2*d^2*n^2*x^3 + (b^2*d^2*cos(2
*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*x^3*cos(2*b*d*log(x^n) +
 2*a*d)^2 + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^
2*x^3*sin(2*b*d*log(x^n) + 2*a*d)^2), x) - 2*(b*d*n*sin(2*b*d*log(c)) - 2*
cos(2*b*d*log(c)))*sin(2*b*d*log(x^n) + 2*a*d))/(2*b*d*n*x^2*cos(2*b*d*log
(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b*d*n*x^2*sin(2*b*d*log(c))*sin(2*b*d
*log(x^n) + 2*a*d) + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n
*x^2*cos(2*b*d*log(x^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*
b*d*log(c))^2)*n*x^2*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*x^2)
```

Giac [F]

$$\int \frac{\tan^2(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\tan((b\log(cx^n)+a)d)^2}{x^3}\,dx$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="giac")
```

output
```
integrate(tan((b*log(c*x^n) + a)*d)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(d(a+b\log(cx^n)))}{x^3}\,dx=\int \frac{\tan(d\,(a+b\,\ln(c\,x^n)))^2}{x^3}\,dx$$

input
```
int(tan(d*(a + b*log(c*x^n)))^2/x^3,x)
```

output
```
int(tan(d*(a + b*log(c*x^n)))^2/x^3, x)
```

Reduce [F]

$$\int \frac{\tan^2(d(a+b\log(cx^n)))}{x^3}\,dx$$
$$=\frac{4\left(\int \frac{\tan(\log(x^n c)bd+ad)}{x^3}dx\right)x^2+2\tan(\log(x^n c)\,bd+ad)+bdn}{2bdn\,x^2}$$

input
```
int(tan(d*(a+b*log(c*x^n)))^2/x^3,x)
```

output
```
(4*int(tan(log(x**n*c)*b*d + a*d)/x**3,x)*x**2 + 2*tan(log(x**n*c)*b*d + a
*d) + b*d*n)/(2*b*d*n*x**2)
```

3.172 $\int \frac{\tan^3(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{\tan^3(a+b\log(cx^n))}{x}\,dx=\frac{\log(\cos(a+b\log(cx^n)))}{bn}+\frac{\tan^2(a+b\log(cx^n))}{2bn}$$

output
```
ln(cos(a+b*ln(c*x^n)))/b/n+1/2*tan(a+b*ln(c*x^n))^2/b/n
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{\tan^3(a+b\log(cx^n))}{x}\,dx=\frac{2\log(\cos(a+b\log(cx^n)))+\sec^2(a+b\log(cx^n))}{2bn}$$

input
```
Integrate[Tan[a + b*Log[c*x^n]]^3/x,x]
```

output
```
(2*Log[Cos[a + b*Log[c*x^n]]] + Sec[a + b*Log[c*x^n]]^2)/(2*b*n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3039, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \tan^3(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \tan(a+b\log(cx^n))^3d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\frac{\tan^2(a+b\log(cx^n))}{2b}-\int \tan(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\tan^2(a+b\log(cx^n))}{2b}-\int \tan(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\frac{\tan^2(a+b\log(cx^n))}{2b}+\frac{\log(\cos(a+b\log(cx^n)))}{b}}{n}$$

input
```
Int[Tan[a + b*Log[c*x^n]]^3/x,x]
```

output
```
(Log[Cos[a + b*Log[c*x^n]]]/b + Tan[a + b*Log[c*x^n]]^2/(2*b))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result
parallelrisch	$\frac{\tan(a+b\ln(c\,x^n))^2-\ln\left(1+\tan(a+b\ln(c\,x^n))^2\right)}{2bn}$
derivativedivides	$\frac{\frac{\tan(a+b\ln(c\,x^n))^2}{2}-\frac{\ln\left(1+\tan(a+b\ln(c\,x^n))^2\right)}{2}}{nb}$
default	$\frac{\frac{\tan(a+b\ln(c\,x^n))^2}{2}-\frac{\ln\left(1+\tan(a+b\ln(c\,x^n))^2\right)}{2}}{nb}$
risch	$i\ln(x)-\frac{2ia}{bn}-\frac{2i\ln(c)}{n}-\frac{2i\ln(x^n)}{n}-\frac{\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)}{n}+\frac{\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}{n}+\frac{\pi\ \mathrm{cs}}{}$

input
```
int(tan(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(tan(a+b*ln(c*x^n))^2-ln(1+tan(a+b*ln(c*x^n))^2))/b/n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

$$\int \frac{\tan^3(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{(\cos(2\,bn\log(x)+2\,b\log(c)+2\,a)+1)\log\left(\frac{1}{2}\,\cos(2\,bn\log(x)+2\,b\log(c)+2\,a)+\frac{1}{2}\right)+2}{2\,(bn\cos(2\,bn\log(x)+2\,b\log(c)+2\,a)+bn)}$$

input
```
integrate(tan(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

output
```
1/2*((cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)*log(1/2*cos(2*b*n*log(x) +
 2*b*log(c) + 2*a) + 1/2) + 2)/(b*n*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) +
 b*n)
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.47

$$\int \frac{\tan^3(a+b\log(cx^n))}{x}\,dx$$
$$= \begin{cases} \log(x)\tan^3(a) & \text{for } b=0 \wedge (b=0 \vee n=0) \\ \log(x)\tan^3(a+b\log(c)) & \text{for } n=0 \\ -\frac{\log(\tan^2(a+b\log(cx^n))+1)}{2bn}+\frac{\tan^2(a+b\log(cx^n))}{2bn} & \text{otherwise} \end{cases}$$

input
```
integrate(tan(a+b*ln(c*x**n))**3/x,x)
```

output
```
Piecewise((log(x)*tan(a)**3, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*ta
n(a + b*log(c))**3, Eq(n, 0)), (-log(tan(a + b*log(c*x**n))**2 + 1)/(2*b*n
) + tan(a + b*log(c*x**n))**2/(2*b*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. $2(41) = 82$.

Time = 0.07 (sec) , antiderivative size = 1242, normalized size of antiderivative = 28.88

$$\int \frac{\tan^3(a + b\log(cx^n))}{x} \, dx = \text{Too large to display}$$

input
```
integrate(tan(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

output
```
1/2*(8*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*cos(2*b*log(x^n) + 2*a)^2 +
 8*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*sin(2*b*log(x^n) + 2*a)^2 + 4*(
(cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*cos(2*
b*log(x^n) + 2*a) + (cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin
(2*b*log(c)))*sin(2*b*log(x^n) + 2*a))*cos(4*b*log(x^n) + 4*a) + 4*cos(2*b
*log(c))*cos(2*b*log(x^n) + 2*a) + ((cos(4*b*log(c))^2 + sin(4*b*log(c))^2
)*cos(4*b*log(x^n) + 4*a)^2 + 4*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*co
s(2*b*log(x^n) + 2*a)^2 + (cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*sin(4*b*
log(x^n) + 4*a)^2 + 4*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*sin(2*b*log(
x^n) + 2*a)^2 + 2*(2*(cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*si
n(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 2*(cos(2*b*log(c))*sin(4*b*log(c)
) - cos(4*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + cos(4*b*log
(c)))*cos(4*b*log(x^n) + 4*a) + 4*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a)
- 2*(2*(cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))
*cos(2*b*log(x^n) + 2*a) - 2*(cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*lo
g(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + sin(4*b*log(c)))*sin(4*b*
log(x^n) + 4*a) - 4*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + 1)*log((cos(
2*a)^2 + sin(2*a)^2)*cos(2*b*log(c))^2 + (cos(2*a)^2 + sin(2*a)^2)*sin(2*b
*log(c))^2 + 2*(cos(2*b*log(c))*cos(2*a) - sin(2*b*log(c))*sin(2*a))*cos(2
*b*log(x^n)) + cos(2*b*log(x^n))^2 - 2*(cos(2*a)*sin(2*b*log(c)) + cos(...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^3(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(tan(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 22.52 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.44

$$\int \frac{\tan^3(a+b\log(cx^n))}{x}\,dx = -\ln(x)\ 1\mathrm{i} - \frac{2}{b\,n\,\left(2\,\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}} + \mathrm{e}^{a\,4\mathrm{i}}\,(c\,x^n)^{b\,4\mathrm{i}} + 1\right)} + \frac{2}{b\,n\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}} + 1\right)} + \frac{\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}} + 1\right)}{b\,n}$$

input
```
int(tan(a + b*log(c*x^n))^3/x,x)
```

output
```
2/(b*n*(exp(a*2i)*(c*x^n)^(b*2i) + 1)) - 2/(b*n*(2*exp(a*2i)*(c*x^n)^(b*2i
) + exp(a*4i)*(c*x^n)^(b*4i) + 1)) - log(x)*1i + log(exp(a*2i)*(c*x^n)^(b*
2i) + 1)/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{\tan^3(a+b\log(cx^n))}{x}\,dx = \frac{-\log\left(\tan(\log(x^nc)\,b+a)^2+1\right)+\tan(\log(x^nc)\,b+a)^2}{2bn}$$

input
```
int(tan(a+b*log(c*x^n))^3/x,x)
```

output

```
( - log(tan(log(x**n*c)*b + a)**2 + 1) + tan(log(x**n*c)*b + a)**2)/(2*b*n
)
```

3.173 $\int \frac{\tan^4(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \frac{\tan^4(a+b\log(cx^n))}{x}\,dx=\log(x)-\frac{\tan(a+b\log(cx^n))}{bn}+\frac{\tan^3(a+b\log(cx^n))}{3bn}$$

output
```
ln(x)-tan(a+b*ln(c*x^n))/b/n+1/3*tan(a+b*ln(c*x^n))^3/b/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.38

$$\int \frac{\tan^4(a+b\log(cx^n))}{x}\,dx=\frac{\arctan(\tan(a+b\log(cx^n)))}{bn}-\frac{\tan(a+b\log(cx^n))}{bn}+\frac{\tan^3(a+b\log(cx^n))}{3bn}$$

input
```
Integrate[Tan[a + b*Log[c*x^n]]^4/x,x]
```

output
```
ArcTan[Tan[a + b*Log[c*x^n]]]/(b*n) - Tan[a + b*Log[c*x^n]]/(b*n) + Tan[a
+ b*Log[c*x^n]]^3/(3*b*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3039, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(a+b\log(cx^n))}{x} \, dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \tan^4(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \tan(a+b\log(cx^n))^4d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\frac{\tan^3(a+b\log(cx^n))}{3b}-\int \tan^2(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\tan^3(a+b\log(cx^n))}{3b}-\int \tan(a+b\log(cx^n))^2d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\int 1d\log(cx^n)+\frac{\tan^3(a+b\log(cx^n))}{3b}-\frac{\tan(a+b\log(cx^n))}{b}}{n}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{\tan^3(a+b\log(cx^n))}{3b}-\frac{\tan(a+b\log(cx^n))}{b}+\log(cx^n)}{n}$$

input
```
Int[Tan[a + b*Log[c*x^n]]^4/x,x]
```

output
```
(Log[c*x^n] - Tan[a + b*Log[c*x^n]]/b + Tan[a + b*Log[c*x^n]]^3/(3*b))/n
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

method	result
parallelrisch	$-\frac{-3\ln(x) b n-\tan(a+b\ln(c\,x^{n}))^{3}+3\tan(a+b\ln(c\,x^{n}))}{3 b n}$
derivativedivides	$\frac{\frac{\tan(a+b\ln(c\,x^{n}))^{3}}{3}-\tan(a+b\ln(c\,x^{n}))+\arctan(\tan(a+b\ln(c\,x^{n})))}{n b}$
default	$\frac{\frac{\tan(a+b\ln(c\,x^{n}))^{3}}{3}-\tan(a+b\ln(c\,x^{n}))+\arctan(\tan(a+b\ln(c\,x^{n})))}{n b}$
risch	$\ln(x)-\frac{4i\left(3(x^{n})^{4ib}c^{4ib}e^{2b\pi\,\operatorname{csgn}(ix^{n})\operatorname{csgn}(ic\,x^{n})\operatorname{csgn}(ic)}e^{-2b\pi\,\operatorname{csgn}(ix^{n})\operatorname{csgn}(ic\,x^{n})^{2}}e^{-2b\pi\,\operatorname{csgn}(ic\,x^{n})^{2}\operatorname{csgn}(ic)}e^{2b\pi}\right.}{3bn\left((x^{n})^{2ib}c^{2ib}e^{-b\pi\,\operatorname{csgn}(ix^{n})\operatorname{csgn}(ic\,x^{n})^{2}}e^{b\pi}\right.}$

input
```
int(tan(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-3*ln(x)*b*n-tan(a+b*ln(c*x^n))^3+3*tan(a+b*ln(c*x^n)))/b/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(43) = 86$.

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.11

$$\int \frac{\tan^4(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{3\,bn\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)^{2}\log\left(x\right)+6\,bn\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)\log\left(x\right)+3\,bn\,\mathrm{l}}{3\left(bn\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)^{2}+2\,bn\cos\left(\right.\right.}$$

input
```
integrate(tan(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

output
```
1/3*(3*b*n*cos(2*b*n*log(x) + 2*b*log(c) + 2*a)^2*log(x) + 6*b*n*cos(2*b*n
*log(x) + 2*b*log(c) + 2*a)*log(x) + 3*b*n*log(x) - 2*(2*cos(2*b*n*log(x)
+ 2*b*log(c) + 2*a) + 1)*sin(2*b*n*log(x) + 2*b*log(c) + 2*a))/(b*n*cos(2*
b*n*log(x) + 2*b*log(c) + 2*a)^2 + 2*b*n*cos(2*b*n*log(x) + 2*b*log(c) + 2
*a) + b*n)
```

Sympy [A] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \frac{\tan^4(a+b\log(cx^n))}{x}\,dx = \begin{cases} \log(x)\tan^4(a) & \text{for } b=0 \wedge (b=0 \vee n=0) \\ \log(x)\tan^4(a+b\log(c)) & \text{for } n=0 \\ \frac{\log(cx^n)}{n} + \frac{\tan^3(a+b\log(cx^n))}{3bn} - \frac{\tan(a+b\log(cx^n))}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(tan(a+b*ln(c*x**n))**4/x,x)
```

output
```
Piecewise((log(x)*tan(a)**4, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*ta
n(a + b*log(c))**4, Eq(n, 0)), (log(c*x**n)/n + tan(a + b*log(c*x**n))**3/
(3*b*n) - tan(a + b*log(c*x**n))/(b*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2171 vs. $2(43) = 86$.

Time = 0.09 (sec) , antiderivative size = 2171, normalized size of antiderivative = 48.24

$$\int \frac{\tan^4(a + b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

input
```
integrate(tan(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

output
```
1/3*(3*(b*cos(6*b*log(c))^2 + b*sin(6*b*log(c))^2)*n*cos(6*b*log(x^n) + 6*
a)^2*log(x) + 27*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*cos(4*b*log
(x^n) + 4*a)^2*log(x) + 27*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*c
os(2*b*log(x^n) + 2*a)^2*log(x) + 3*(b*cos(6*b*log(c))^2 + b*sin(6*b*log(c
))^2)*n*log(x)*sin(6*b*log(x^n) + 6*a)^2 + 27*(b*cos(4*b*log(c))^2 + b*sin
(4*b*log(c))^2)*n*log(x)*sin(4*b*log(x^n) + 4*a)^2 + 27*(b*cos(2*b*log(c))
^2 + b*sin(2*b*log(c))^2)*n*log(x)*sin(2*b*log(x^n) + 2*a)^2 + 3*b*n*log(x
) + 2*(3*b*n*cos(6*b*log(c))*log(x) + 3*(3*(b*cos(6*b*log(c))*cos(4*b*log(
c)) + b*sin(6*b*log(c))*sin(4*b*log(c)))*n*log(x) - 2*cos(4*b*log(c))*sin(
6*b*log(c)) + 2*cos(6*b*log(c))*sin(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) +
 3*(3*(b*cos(6*b*log(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c
)))*n*log(x) - 2*cos(2*b*log(c))*sin(6*b*log(c)) + 2*cos(6*b*log(c))*sin(2
*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 3*(3*(b*cos(4*b*log(c))*sin(6*b*log(
c)) - b*cos(6*b*log(c))*sin(4*b*log(c)))*n*log(x) + 2*cos(6*b*log(c))*cos(
4*b*log(c)) + 2*sin(6*b*log(c))*sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) +
 3*(3*(b*cos(2*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(2*b*log(c
)))*n*log(x) + 2*cos(6*b*log(c))*cos(2*b*log(c)) + 2*sin(6*b*log(c))*sin(2
*b*log(c)))*sin(2*b*log(x^n) + 2*a) - 4*sin(6*b*log(c)))*cos(6*b*log(x^n)
+ 6*a) + 6*(3*b*n*cos(4*b*log(c))*log(x) + 9*(b*cos(4*b*log(c))*cos(2*b*lo
g(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) + 2*a)*lo...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^4(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(tan(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 27.96 (sec) , antiderivative size = 183, normalized size of antiderivative = 4.07

$$\int \frac{\tan^4(a+b\log(cx^n))}{x}\,dx = \ln(x) - \frac{\frac{4i}{3\,b\,n} + \frac{e^{a\,4i}\,(c\,x^n)^{b\,4i}\,4i}{3\,b\,n}}{3\,e^{a\,2i}\,(c\,x^n)^{b\,2i} + 3\,e^{a\,4i}\,(c\,x^n)^{b\,4i} + e^{a\,6i}\,(c\,x^n)^{b\,6i} + 1} - \frac{4i}{3\,b\,n\,\left(e^{a\,2i}\,(c\,x^n)^{b\,2i} + 1\right)} - \frac{e^{a\,2i}\,(c\,x^n)^{b\,2i}\,4i}{3\,b\,n\,\left(2\,e^{a\,2i}\,(c\,x^n)^{b\,2i} + e^{a\,4i}\,(c\,x^n)^{b\,4i} + 1\right)}$$

input
```
int(tan(a + b*log(c*x^n))^4/x,x)
```

output
```
log(x) - (4i/(3*b*n) + (exp(a*4i)*(c*x^n)^(b*4i)*4i)/(3*b*n))/(3*exp(a*2i)
*(c*x^n)^(b*2i) + 3*exp(a*4i)*(c*x^n)^(b*4i) + exp(a*6i)*(c*x^n)^(b*6i) +
1) - 4i/(3*b*n*(exp(a*2i)*(c*x^n)^(b*2i) + 1)) - (exp(a*2i)*(c*x^n)^(b*2i)
*4i)/(3*b*n*(2*exp(a*2i)*(c*x^n)^(b*2i) + exp(a*4i)*(c*x^n)^(b*4i) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{\tan^4(a + b\log(cx^n))}{x}\,dx = \frac{3\log(x^n c)\,b + \tan(\log(x^n c)\,b + a)^3 - 3\tan(\log(x^n c)\,b + a)}{3bn}$$

input
```
int(tan(a+b*log(c*x^n))^4/x,x)
```

output
```
(3*log(x**n*c)*b + tan(log(x**n*c)*b + a)**3 - 3*tan(log(x**n*c)*b + a))/(
3*b*n)
```

3.174 $\int \frac{\tan^5(a+b\log(cx^n))}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int \frac{\tan^5(a+b\log(cx^n))}{x} dx = -\frac{\log(\cos(a+b\log(cx^n)))}{bn} - \frac{\tan^2(a+b\log(cx^n))}{2bn} + \frac{\tan^4(a+b\log(cx^n))}{4bn}$$

output
```
-ln(cos(a+b*ln(c*x^n)))/b/n-1/2*tan(a+b*ln(c*x^n))^2/b/n+1/4*tan(a+b*ln(c*
x^n))^4/b/n
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \frac{\tan^5(a+b\log(cx^n))}{x} dx = -\frac{\log(\cos(a+b\log(cx^n)))}{bn} - \frac{\sec^2(a+b\log(cx^n))}{bn} + \frac{\sec^4(a+b\log(cx^n))}{4bn}$$

input
```
Integrate[Tan[a + b*Log[c*x^n]]^5/x,x]
```

output
```
-(Log[Cos[a + b*Log[c*x^n]]]/(b*n)) - Sec[a + b*Log[c*x^n]]^2/(b*n) + Sec[
a + b*Log[c*x^n]]^4/(4*b*n)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3039, 3042, 3954, 3042, 3954, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(a+b\log(cx^n))}{x}dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \tan^5(a+b\log(cx^n))d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \tan(a+b\log(cx^n))^5d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3954}$$
$$\frac{\frac{\tan^4(a+b\log(cx^n))}{4b}-\int \tan^3(a+b\log(cx^n))d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\frac{\tan^4(a+b\log(cx^n))}{4b}-\int \tan(a+b\log(cx^n))^3d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3954}$$
$$\frac{\int \tan(a+b\log(cx^n))d\log(cx^n)+\frac{\tan^4(a+b\log(cx^n))}{4b}-\frac{\tan^2(a+b\log(cx^n))}{2b}}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \tan(a+b\log(cx^n))d\log(cx^n)+\frac{\tan^4(a+b\log(cx^n))}{4b}-\frac{\tan^2(a+b\log(cx^n))}{2b}}{n}$$
$$\Big\downarrow \text{3956}$$

$$\frac{\frac{\tan^4(a+b\log(cx^n))}{4b} - \frac{\tan^2(a+b\log(cx^n))}{2b} - \frac{\log(\cos(a+b\log(cx^n)))}{b}}{n}$$

input
```
Int[Tan[a + b*Log[c*x^n]]^5/x,x]
```

output
```
(-(Log[Cos[a + b*Log[c*x^n]]]/b) - Tan[a + b*Log[c*x^n]]^2/(2*b) + Tan[a +
 b*Log[c*x^n]]^4/(4*b))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\frac{\tan(a+b\ln(c\,x^n))^4}{4}-\frac{\tan(a+b\ln(c\,x^n))^2}{2}+\frac{\ln\left(1+\tan(a+b\ln(c\,x^n))^2\right)}{2}}{nb}$
default	$\frac{\frac{\tan(a+b\ln(c\,x^n))^4}{4}-\frac{\tan(a+b\ln(c\,x^n))^2}{2}+\frac{\ln\left(1+\tan(a+b\ln(c\,x^n))^2\right)}{2}}{nb}$
parallelrisch	$-\frac{-\tan(a+b\ln(c\,x^n))^4+2\tan(a+b\ln(c\,x^n))^2-2\ln\left(1+\tan(a+b\ln(c\,x^n))^2\right)}{4bn}$
risch	$-i\ln(x)+\frac{2ia}{bn}+\frac{2i\ln(c)}{n}+\frac{2i\ln(x^n)}{n}-\frac{\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}{n}+\frac{\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)}{n}+\frac{\pi}{}$

input
```
int(tan(a+b*ln(c*x^n))^5/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(1/4*tan(a+b*ln(c*x^n))^4-1/2*tan(a+b*ln(c*x^n))^2+1/2*ln(1+tan(a+b*
ln(c*x^n))^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(63) = 126$.

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.93

$$\int \frac{\tan^5(a+b\log(cx^n))}{x}\,dx = -\frac{\left(\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)^2+2\,\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)+1\right)\log\left(\frac{1}{2}\,\cos\left(2\,bn\log\left(\right.\right.\right.}{2\left(bn\cos\left(2\,bn\log\left(x\right)+2\,b\log\left(c\right)+2\,a\right)^2+2\,bn\cos\left(2\,bn\log\right.\right.}$$

input
```
integrate(tan(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

output
```
-1/2*((cos(2*b*n*log(x) + 2*b*log(c) + 2*a)^2 + 2*cos(2*b*n*log(x) + 2*b*l
og(c) + 2*a) + 1)*log(1/2*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1/2) + 4*
cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 2)/(b*n*cos(2*b*n*log(x) + 2*b*log(
c) + 2*a)^2 + 2*b*n*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + b*n)
```

Sympy [A] (verification not implemented)

Time = 4.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.22

$$\int \frac{\tan^5(a+b\log(cx^n))}{x}\,dx = \begin{cases} \log(x)\tan^5(a) & \text{for } b=0 \wedge (b=0 \vee n=0) \\ \log(x)\tan^5(a+b\log(c)) & \text{for } n=0 \\ \frac{\log(\tan^2(a+b\log(cx^n))+1)}{2bn} + \frac{\tan^4(a+b\log(cx^n))}{4bn} - \frac{\tan^2(a+b\log(cx^n))}{2bn} & \text{otherwise} \end{cases}$$

input
```
integrate(tan(a+b*ln(c*x**n))**5/x,x)
```

output
```
Piecewise((log(x)*tan(a)**5, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*ta
n(a + b*log(c))**5, Eq(n, 0)), (log(tan(a + b*log(c*x**n))**2 + 1)/(2*b*n)
 + tan(a + b*log(c*x**n))**4/(4*b*n) - tan(a + b*log(c*x**n))**2/(2*b*n),
True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4466 vs. 2(63) = 126.

Time = 0.13 (sec) , antiderivative size = 4466, normalized size of antiderivative = 66.66

$$\int \frac{\tan^5(a+b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

input
```
integrate(tan(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

output
```
-1/2*(32*(cos(6*b*log(c))^2 + sin(6*b*log(c))^2)*cos(6*b*log(x^n) + 6*a)^2
 + 48*(cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*cos(4*b*log(x^n) + 4*a)^2 +
32*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*cos(2*b*log(x^n) + 2*a)^2 + 32*
(cos(6*b*log(c))^2 + sin(6*b*log(c))^2)*sin(6*b*log(x^n) + 6*a)^2 + 48*(co
s(4*b*log(c))^2 + sin(4*b*log(c))^2)*sin(4*b*log(x^n) + 4*a)^2 + 32*(cos(2
*b*log(c))^2 + sin(2*b*log(c))^2)*sin(2*b*log(x^n) + 2*a)^2 + 8*((cos(8*b*
log(c))*cos(6*b*log(c)) + sin(8*b*log(c))*sin(6*b*log(c)))*cos(6*b*log(x^n
) + 6*a) + (cos(8*b*log(c))*cos(4*b*log(c)) + sin(8*b*log(c))*sin(4*b*log(
c)))*cos(4*b*log(x^n) + 4*a) + (cos(8*b*log(c))*cos(2*b*log(c)) + sin(8*b*
log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + (cos(6*b*log(c))*sin(8*
b*log(c)) - cos(8*b*log(c))*sin(6*b*log(c)))*sin(6*b*log(x^n) + 6*a) + (co
s(4*b*log(c))*sin(8*b*log(c)) - cos(8*b*log(c))*sin(4*b*log(c)))*sin(4*b*l
og(x^n) + 4*a) + (cos(2*b*log(c))*sin(8*b*log(c)) - cos(8*b*log(c))*sin(2*
b*log(c)))*sin(2*b*log(x^n) + 2*a))*cos(8*b*log(x^n) + 8*a) + 8*(10*(cos(6
*b*log(c))*cos(4*b*log(c)) + sin(6*b*log(c))*sin(4*b*log(c)))*cos(4*b*log(
x^n) + 4*a) + 8*(cos(6*b*log(c))*cos(2*b*log(c)) + sin(6*b*log(c))*sin(2*b
*log(c)))*cos(2*b*log(x^n) + 2*a) + 10*(cos(4*b*log(c))*sin(6*b*log(c)) -
cos(6*b*log(c))*sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) + 8*(cos(2*b*log(
c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) +
2*a) + cos(6*b*log(c)))*cos(6*b*log(x^n) + 6*a) + 8*(10*(cos(4*b*log(c)...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^5(a + b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(tan(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 24.25 (sec) , antiderivative size = 247, normalized size of antiderivative = 3.69

$$\int \frac{\tan^5(a+b\log(cx^n))}{x}\,dx$$
$$= \ln(x)\ 1\mathrm{i} + \frac{8}{b\,n\,\left(2\,\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}} + \mathrm{e}^{a\,4\mathrm{i}}\,(c\,x^n)^{b\,4\mathrm{i}} + 1\right)} - \frac{4}{b\,n\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}} + 1\right)}$$
$$+ \frac{4}{b\,n\,\left(4\,\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}} + 6\,\mathrm{e}^{a\,4\mathrm{i}}\,(c\,x^n)^{b\,4\mathrm{i}} + 4\,\mathrm{e}^{a\,6\mathrm{i}}\,(c\,x^n)^{b\,6\mathrm{i}} + \mathrm{e}^{a\,8\mathrm{i}}\,(c\,x^n)^{b\,8\mathrm{i}} + 1\right)}$$
$$- \frac{\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}} + 1\right)}{b\,n} - \frac{8}{b\,n\,\left(3\,\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}} + 3\,\mathrm{e}^{a\,4\mathrm{i}}\,(c\,x^n)^{b\,4\mathrm{i}} + \mathrm{e}^{a\,6\mathrm{i}}\,(c\,x^n)^{b\,6\mathrm{i}} + 1\right)}$$

input
```
int(tan(a + b*log(c*x^n))^5/x,x)
```

output
```
log(x)*1i + 8/(b*n*(2*exp(a*2i)*(c*x^n)^(b*2i) + exp(a*4i)*(c*x^n)^(b*4i)
+ 1)) - 4/(b*n*(exp(a*2i)*(c*x^n)^(b*2i) + 1)) + 4/(b*n*(4*exp(a*2i)*(c*x^
n)^(b*2i) + 6*exp(a*4i)*(c*x^n)^(b*4i) + 4*exp(a*6i)*(c*x^n)^(b*6i) + exp(
a*8i)*(c*x^n)^(b*8i) + 1)) - log(exp(a*2i)*(c*x^n)^(b*2i) + 1)/(b*n) - 8/(
b*n*(3*exp(a*2i)*(c*x^n)^(b*2i) + 3*exp(a*4i)*(c*x^n)^(b*4i) + exp(a*6i)*(
c*x^n)^(b*6i) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int \frac{\tan^5(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{2\log\left(\tan(\log(x^nc)\,b+a)^2+1\right) + \tan(\log(x^nc)\,b+a)^4 - 2\tan(\log(x^nc)\,b+a)^2}{4bn}$$

input
```
int(tan(a+b*log(c*x^n))^5/x,x)
```

output
```
(2*log(tan(log(x**n*c)*b + a)**2 + 1) + tan(log(x**n*c)*b + a)**4 - 2*tan(
log(x**n*c)*b + a)**2)/(4*b*n)
```

3.175 $\int (ex)^m \tan (d(a+b\log (cx^n)))\, dx$

Optimal result

Integrand size = 19, antiderivative size = 101

$$\int (ex)^m \tan (d(a+b\log (cx^n)))\, dx = -\frac{i(ex)^{1+m}}{e(1+m)} + \frac{2i(ex)^{1+m}\, \text{Hypergeometric2F1}\left(1, -\frac{i(1+m)}{2bdn}, 1-\frac{i(1+m)}{2bdn}, -e^{2iad}(cx^n)^{2ibd}\right)}{e(1+m)}$$

output
```
-I*(e*x)^(1+m)/e/(1+m)+2*I*(e*x)^(1+m)*hypergeom([1, -1/2*I*(1+m)/b/d/n],[
1-1/2*I*(1+m)/b/d/n],-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/e/(1+m)
```

Mathematica [A] (verified)

Time = 11.59 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.84

$$\int (ex)^m \tan (d(a+b\log (cx^n)))\, dx = \frac{ix(ex)^m \left(\text{Hypergeometric2F1}\left(1, -\frac{i(1+m)}{2bdn}, 1-\frac{i(1+m)}{2bdn}, -e^{2id(a+b\log(cx^n))}\right) - \frac{e^{2iad}(1+m)(cx^n)^{2ibd}\, \text{Hypergeometric}}{\cdots}\right.}{1+m}$$

input
```
Integrate[(e*x)^m*Tan[d*(a + b*Log[c*x^n])],x]
```

output
```
(I*x*(e*x)^m*(Hypergeometric2F1[1, ((-1/2*I)*(1 + m))/(b*d*n), 1 - ((I/2)*
(1 + m))/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] - (E^((2*I)*a*d)*(1 + m
)*(c*x^n)^((2*I)*b*d)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m + (2*I)*b*d*n)
)/(b*d*n), ((-1/2*I)*(1 + m + (4*I)*b*d*n))/(b*d*n), -(E^((2*I)*a*d)*(c*x^
n)^((2*I)*b*d))])/(1 + m + (2*I)*b*d*n)))/(1 + m)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.36, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {5008, 5006, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \tan (d(a+b\log (cx^n)))\, dx$$

$$\Big\downarrow \text{ 5008}$$

$$\frac{(ex)^{m+1} (cx^n)^{-\frac{m+1}{n}} \int (cx^n)^{\frac{m+1}{n}-1} \tan (d(a+b\log (cx^n)))\,d(cx^n)}{en}$$

$$\Big\downarrow \text{ 5006}$$

$$\frac{(ex)^{m+1} (cx^n)^{-\frac{m+1}{n}} \int \frac{(cx^n)^{\frac{m+1}{n}-1}\left(i-ie^{2iad}(cx^n)^{2ibd}\right)}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{en}$$

$$\Big\downarrow \text{ 959}$$

$$\frac{(ex)^{m+1} (cx^n)^{-\frac{m+1}{n}} \left(2i\int \frac{(cx^n)^{\frac{m+1}{n}-1}}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)-\frac{in(cx^n)^{\frac{m+1}{n}}}{m+1}\right)}{en}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{(ex)^{m+1} (cx^n)^{-\frac{m+1}{n}} \left(\frac{2in(cx^n)^{\frac{m+1}{n}} \text{Hypergeometric2F1}\left(1,-\frac{i(m+1)}{2bdn},1-\frac{i(m+1)}{2bdn},-e^{2iad}(cx^n)^{2ibd}\right)}{m+1}-\frac{in(cx^n)^{\frac{m+1}{n}}}{m+1}\right)}{en}$$

input
```
Int[(e*x)^m*Tan[d*(a + b*Log[c*x^n])],x]
```

output
```
((e*x)^(1 + m)*(((-I)*n*(c*x^n)^((1 + m)/n))/(1 + m) + ((2*I)*n*(c*x^n)^((
1 + m)/n)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m))/(b*d*n), 1 - ((I/2)*(1 +
 m))/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/(1 + m)))/(e*n*(c*x^n
)^((1 + m)/n))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5008
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int (ex)^m \tan(d(a+b\ln(c\,x^n)))\,dx$$

input
```
int((e*x)^m*tan(d*(a+b*ln(c*x^n))),x)
```

output
```
int((e*x)^m*tan(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int (ex)^m \tan(d(a+b\log(cx^n)))\,dx = \int (ex)^m \tan((b\log(cx^n)+a)d)\,dx$$

input
```
integrate((e*x)^m*tan(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral((e*x)^m*tan(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int (ex)^m \tan(d(a+b\log(cx^n)))\,dx = \int (ex)^m \tan(ad+bd\log(cx^n))\,dx$$

input
```
integrate((e*x)**m*tan(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral((e*x)**m*tan(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int (ex)^m \tan (d(a+b\log (cx^n)))\,dx=\int (ex)^m \tan ((b\log (cx^n)+a)d)\,dx$$

input
```
integrate((e*x)^m*tan(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate((e*x)^m*tan((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int (ex)^m \tan (d(a+b\log (cx^n)))\,dx=\int (ex)^m \tan ((b\log (cx^n)+a)d)\,dx$$

input
```
integrate((e*x)^m*tan(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate((e*x)^m*tan((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \tan (d(a+b\log (cx^n)))\,dx=\int \tan(d\,(a+b\,\ln (c\,x^n)))\,(e\,x)^m\,dx$$

input
```
int(tan(d*(a + b*log(c*x^n)))*(e*x)^m,x)
```

output
```
int(tan(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \tan (d(a + b \log (cx^n)))\ dx = e^m \left(\int x^m \tan (\log(x^n c)\, bd + ad)\, dx \right)$$

input
```
int((e*x)^m*tan(d*(a+b*log(c*x^n))),x)
```

output
```
e**m*int(x**m*tan(log(x**n*c)*b*d + a*d),x)
```

3.176 $\int (ex)^m \tan^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 196

$$\int (ex)^m \tan^2(d(a+b\log(cx^n)))\,dx = \frac{(i(1+m)-bdn)(ex)^{1+m}}{bde(1+m)n} + \frac{i(ex)^{1+m}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bden\left(1+e^{2iad}(cx^n)^{2ibd}\right)} - \frac{2i(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,-\frac{i(1+m)}{2bdn},1-\frac{i(1+m)}{2bdn},-e^{2iad}(cx^n)^{2ibd}\right)}{bden}$$

output
```
(I*(1+m)-b*d*n)*(e*x)^(1+m)/b/d/e/(1+m)/n+I*(e*x)^(1+m)*(1-exp(2*I*a*d)*(c
*x^n)^(2*I*b*d))/b/d/e/n/(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))-2*I*(e*x)^(1+m
)*hypergeom([1, -1/2*I*(1+m)/b/d/n],[1-1/2*I*(1+m)/b/d/n],-exp(2*I*a*d)*(c
*x^n)^(2*I*b*d))/b/d/e/n
```

Mathematica [A] (verified)

Time = 15.00 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.78

$$\int (ex)^m \tan^2(d(a+b\log(cx^n)))\,dx = (ex)^m\left(-\frac{x}{1+m} + \frac{ie^{-\frac{(1+2m)(a-bn\log(x)+b\log(cx^n))}{bn}}x^{-2m}\left(-e^{\frac{(1+2m)(a+b\log(cx^n))}{bn}}(1+m+2ibdn)\,\text{Hypergeometric2F1}\left(1,-\frac{i(1+m)}{2bdn},\right.\right.}{}\right.$$

input
```
Integrate[(e*x)^m*Tan[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
(e*x)^m*(-(x/(1 + m)) + (I*(-(E^(((1 + 2*m)*(a + b*Log[c*x^n]))/(b*n))*(1
+ m + (2*I)*b*d*n)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m))/(b*d*n), 1 - ((
I/2)*(1 + m))/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))]) + E^(((1 + 2*m +
(2*I)*b*d*n)*(a - b*n*Log[x] + b*Log[c*x^n]))/(b*n))*(1 + m)*x^(1 + 2*m +
(2*I)*b*d*n)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m + (2*I)*b*d*n))/(b*d*n)
, ((-1/2*I)*(1 + m + (4*I)*b*d*n))/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n])
)] - I*E^(((1 + 2*m)*(a + b*Log[c*x^n]))/(b*n))*(1 + m + (2*I)*b*d*n)*Tan[
d*(a + b*Log[c*x^n])]))/(b*d*E^(((1 + 2*m)*(a - b*n*Log[x] + b*Log[c*x^n])
)/(b*n))*n*(1 + m + (2*I)*b*d*n)*x^(2*m)))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5008, 5006, 1004, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \tan^2(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \; 5008$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\tan^2(d(a+b\log(cx^n)))\,d(cx^n)}{en}$$

$\Big\downarrow$ 5006

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int\frac{(cx^n)^{\frac{m+1}{n}-1}\left(i-ie^{2iad}(cx^n)^{2ibd}\right)^2}{\left(e^{2iad}(cx^n)^{2ibd}+1\right)^2}d(cx^n)}{en}$$

$\Big\downarrow$ 1004

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{ie^{-2iad}\int-\frac{2(cx^n)^{\frac{m+1}{n}-1}\left(\frac{e^{2iad}(m-ibdn+1)}{n}-\frac{e^{4iad}(m+ibdn+1)(cx^n)^{2ibd}}{n}\right)}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{2bd}+\frac{i(cx^n)^{\frac{m+1}{n}}\left(1-e^{2iad}(cx^n)^{2ib}\right.}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}\right.}{en}$$

$\Big\downarrow$ 27

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{i(cx^n)^{\frac{m+1}{n}}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\int\frac{(cx^n)^{\frac{m+1}{n}-1}\left(\frac{e^{2iad}(m-ibdn+1)}{n}-\frac{e^{4iad}(m+ibdn+1)(cx^n)^{2ibd}}{n}\right)}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{bd}\right.}{en}$$

$\Big\downarrow$ 959

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{i(cx^n)^{\frac{m+1}{n}}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\left(\frac{2(m+1)e^{2iad}\int\frac{(cx^n)^{\frac{m+1}{n}-1}}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{n}-\frac{e^{2iad}(ibdn+m+1)(cx^n)^{\frac{m+1}{n}}}{m+1}\right)}{bd}\right.}{en}$$

$\Big\downarrow$ 888

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{i(cx^n)^{\frac{m+1}{n}}\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1+e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\left(2e^{2iad}(cx^n)^{\frac{m+1}{n}}\text{Hypergeometric2F1}\left(1,-\frac{i(m+1)}{2bdn},1-\frac{i(m+1)}{2bdn},-e^{2iad}(\right.\right.}{bd}\right.}{en}$$

input
```
Int[(e*x)^m*Tan[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
((e*x)^(1 + m)*((I*(c*x^n)^((1 + m)/n)*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b
*d)))/(b*d*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - (I*(-((E^((2*I)*a*d)
*(1 + m + I*b*d*n)*(c*x^n)^((1 + m)/n))/(1 + m)) + 2*E^((2*I)*a*d)*(c*x^n)
^((1 + m)/n)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m))/(b*d*n), 1 - ((I/2)*(
1 + m))/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))]))/(b*d*E^((2*I)*a*d
))))/(e*n*(c*x^n)^((1 + m)/n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5008
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int (ex)^m \tan(d(a+b\ln(c\,x^n)))^2 dx$$

input
```
int((e*x)^m*tan(d*(a+b*ln(c*x^n)))^2,x)
```

output
```
int((e*x)^m*tan(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int (ex)^m \tan^2(d(a+b\log(cx^n)))\,dx = \int (ex)^m \tan((b\log(cx^n)+a)d)^2\,dx$$

input
```
integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

output
```
integral((e*x)^m*tan(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int (ex)^m \tan^2(d(a+b\log(cx^n)))\,dx = \int (ex)^m \tan^2(ad+bd\log(cx^n))\,dx$$

input
```
integrate((e*x)**m*tan(d*(a+b*ln(c*x**n)))**2,x)
```

output
```
Integral((e*x)**m*tan(a*d + b*d*log(c*x**n))**2, x)
```

Maxima [F]

$$\int (ex)^m \tan^2 (d(a + b\log (cx^n)))\, dx = \int (ex)^m \tan ((b\log (cx^n) + a)d)^2\, dx$$

input
```
integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

output
```
-((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*e^m*n*x*x^m*cos(2*b*
d*log(x^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2
)*e^m*n*x*x^m*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*e^m*n*x*x^m + 2*(b*d*e^m
*n*cos(2*b*d*log(c)) - e^m*m*sin(2*b*d*log(c)) - e^m*sin(2*b*d*log(c)))*x*
x^m*cos(2*b*d*log(x^n) + 2*a*d) - 2*(b*d*e^m*n*sin(2*b*d*log(c)) + e^m*m*c
os(2*b*d*log(c)) + e^m*cos(2*b*d*log(c)))*x*x^m*sin(2*b*d*log(x^n) + 2*a*d
) + 2*(((b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*e^m*m^
2 + 2*(b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*e^m*m +
(b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*e^m)*n^2*cos(2
*b*d*log(x^n) + 2*a*d)^2 + ((b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b
*d*log(c))^2)*e^m*m^2 + 2*(b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d
*log(c))^2)*e^m*m + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c
))^2)*e^m)*n^2*sin(2*b*d*log(x^n) + 2*a*d)^2 + 2*(b^2*d^2*e^m*m^2*cos(2*b*
d*log(c)) + 2*b^2*d^2*e^m*m*cos(2*b*d*log(c)) + b^2*d^2*e^m*cos(2*b*d*log(
c)))*n^2*cos(2*b*d*log(x^n) + 2*a*d) - 2*(b^2*d^2*e^m*m^2*sin(2*b*d*log(c)
) + 2*b^2*d^2*e^m*m*sin(2*b*d*log(c)) + b^2*d^2*e^m*sin(2*b*d*log(c)))*n^2
*sin(2*b*d*log(x^n) + 2*a*d) + (b^2*d^2*e^m*m^2 + 2*b^2*d^2*e^m*m + b^2*d^
2*e^m)*n^2)*integrate((x^m*cos(2*b*d*log(x^n) + 2*a*d)*sin(2*b*d*log(c)) +
 x^m*cos(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d))/(2*b^2*d^2*n^2*cos(2*b
*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*sin(2*b*d*log(c)...
```

Giac [F]

$$\int (ex)^m \tan^2 (d(a + b\log (cx^n)))\, dx = \int (ex)^m \tan ((b\log (cx^n) + a)d)^2\, dx$$

input
```
integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

output
```
integrate((e*x)^m*tan((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \tan^2 (d(a+b\log (cx^n)))\ dx = \int \tan(d\,(a+b\,\ln (c\,x^n)))^2\,(e\,x)^m\,dx$$

input
```
int(tan(d*(a + b*log(c*x^n)))^2*(e*x)^m,x)
```

output
```
int(tan(d*(a + b*log(c*x^n)))^2*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \tan^2 (d(a+b\log (cx^n)))\ dx = \frac{e^m\,(x^m \tan (\log(x^n c)\,bd+ad)\,mx + x^m \tan (\log(x^n c)\,bd+ad)\,x - x^m bdnx - (\int x^m \tan (\log(x^n c)\,bd+a}{bdn\,(m+1)}$$

input
```
int((e*x)^m*tan(d*(a+b*log(c*x^n)))^2,x)
```

output
```
(e**m*(x**m*tan(log(x**n*c)*b*d + a*d)*m*x + x**m*tan(log(x**n*c)*b*d + a*
d)*x - x**m*b*d*n*x - int(x**m*tan(log(x**n*c)*b*d + a*d),x)*m**2 - 2*int(
x**m*tan(log(x**n*c)*b*d + a*d),x)*m - int(x**m*tan(log(x**n*c)*b*d + a*d)
,x)))/(b*d*n*(m + 1))
```

3.177 $\int (ex)^m \tan^3(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 351

$$\int (ex)^m \tan^3(d(a+b\log(cx^n)))\,dx = -\frac{(i(1+m)-bdn)(1+m+2ibdn)(ex)^{1+m}}{2b^2d^2e(1+m)n^2} - \frac{(ex)^{1+m}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^2}{2bden\left(1+e^{2iad}(cx^n)^{2ibd}\right)^2} - \frac{ie^{-2iad}(ex)^{1+m}\left(\frac{e^{2iad}(1+m-2ibdn)}{n}-\frac{e^{4iad}(1+m+2ibdn)(cx^n)^{2ibd}}{n}\right)}{2b^2d^2en\left(1+e^{2iad}(cx^n)^{2ibd}\right)} + \frac{i(1+2m+m^2-2b^2d^2n^2)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,-\frac{i(1+m)}{2bdn},1-\frac{i(1+m)}{2bdn},-e^{2iad}(cx^n)^{2ibd}\right)}{b^2d^2e(1+m)n^2}$$

output
```
-1/2*(I*(1+m)-b*d*n)*(1+m+2*I*b*d*n)*(e*x)^(1+m)/b^2/d^2/e/(1+m)/n^2-1/2*(
e*x)^(1+m)*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^2/b/d/e/n/(1+exp(2*I*a*d)*(c
*x^n)^(2*I*b*d))^2-1/2*I*(e*x)^(1+m)*(exp(2*I*a*d)*(1+m-2*I*b*d*n)/n-exp(4
*I*a*d)*(1+m+2*I*b*d*n)*(c*x^n)^(2*I*b*d)/n)/b^2/d^2/e/exp(2*I*a*d)/n/(1+e
xp(2*I*a*d)*(c*x^n)^(2*I*b*d))+I*(-2*b^2*d^2*n^2+m^2+2*m+1)*(e*x)^(1+m)*hy
pergeom([1, -1/2*I*(1+m)/b/d/n],[1-1/2*I*(1+m)/b/d/n],-exp(2*I*a*d)*(c*x^n
)^(2*I*b*d))/b^2/d^2/e/(1+m)/n^2
```

Mathematica [A] (verified)

Time = 15.57 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.83

$$\int (ex)^m \tan^3(d(a+b\log(cx^n)))\,dx$$
$$= \frac{x(ex)^m\sec^2(bdn\log(x)+d(a+b(-n\log(x)+\log(cx^n))))}{2bdn}$$
$$-\frac{(1+m)x(ex)^m\sec(d(a+b(-n\log(x)+\log(cx^n))))\sec(bdn\log(x)+d(a+b(-n\log(x)+\log(cx^n))))}{2b^2d^2n^2}$$
$$-\frac{(-1-2m-m^2+2b^2d^2n^2)\,x^{-m}(ex)^m\sec(d(a+b(-n\log(x)+\log(cx^n))))\left(\frac{x^{1+m}\sec(d(a+b\log(cx^n)))\sin(}{1+m}\right.}{}$$
$$-\frac{x(ex)^m\tan(d(a+b(-n\log(x)+\log(cx^n))))}{1+m}$$

input

```
Integrate[(e*x)^m*Tan[d*(a + b*Log[c*x^n])]^3,x]
```

output

```
(x*(e*x)^m*Sec[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]^2)/(2*
b*d*n) - ((1 + m)*x*(e*x)^m*Sec[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sec[
b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sin[b*d*n*Log[x]])/(2
*b^2*d^2*n^2) - ((-1 - 2*m - m^2 + 2*b^2*d^2*n^2)*(e*x)^m*Sec[d*(a + b*(-(
n*Log[x]) + Log[c*x^n]))]*((x^(1 + m)*Sec[d*(a + b*Log[c*x^n])]*Sin[b*d*n*
Log[x]])/(1 + m) - (I*Cos[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*(-(E^((a +
 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n]))/(b*n
))*(1 + m + (2*I)*b*d*n)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m))/(b*d*n),
1 - ((I/2)*(1 + m))/(b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))]) + E^((a*(1
+ 2*m + (2*I)*b*d*n))/(b*n) + (1 + m + (2*I)*b*d*n)*Log[x] + ((1 + 2*m + (
2*I)*b*d*n)*(-(n*Log[x]) + Log[c*x^n]))/n)*(1 + m)*Hypergeometric2F1[1, ((
-1/2*I)*(1 + m + (2*I)*b*d*n))/(b*d*n), ((-1/2*I)*(1 + m + (4*I)*b*d*n))/(
b*d*n), -E^((2*I)*d*(a + b*Log[c*x^n]))] - I*E^((a + 2*a*m + b*(1 + m)*n*L
og[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n]))/(b*n))*(1 + m + (2*I)*b*d*
n)*Tan[d*(a + b*Log[c*x^n])]))/(E^(((1 + 2*m)*(a + b*(-(n*Log[x]) + Log[c*
x^n])))/(b*n))*(1 + m)*(1 + m + (2*I)*b*d*n))))/(2*b^2*d^2*n^2*x^m) - (x*(
e*x)^m*Tan[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))])/(1 + m)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {5008, 5006, 1004, 27, 1064, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \tan^3(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{5008}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\tan^3(d(a+b\log(cx^n)))\,d(cx^n)}{en}$$

$$\Big\downarrow \text{5006}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int \frac{(cx^n)^{\frac{m+1}{n}-1}\left(i-ie^{2iad}(cx^n)^{2ibd}\right)^3}{\left(e^{2iad}(cx^n)^{2ibd}+1\right)^3}d(cx^n)}{en}$$

$$\Big\downarrow \text{1004}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{ie^{-2iad}\int -\frac{2i(cx^n)^{\frac{m+1}{n}-1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)\left(\frac{e^{2iad}(m-2ibdn+1)}{n}-\frac{e^{4iad}(m+2ibdn+1)(cx^n)^{2ibd}}{n}\right)}{\left(e^{2iad}(cx^n)^{2ibd}+1\right)^2}d(cx^n)}{4bd}-\frac{(cx^n)^{m}}{2bd}\right.}{en}$$

$$\Big\downarrow \text{27}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{e^{-2iad}\int \frac{(cx^n)^{\frac{m+1}{n}-1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)\left(\frac{e^{2iad}(m-2ibdn+1)}{n}-\frac{e^{4iad}(m+2ibdn+1)(cx^n)^{2ibd}}{n}\right)}{\left(e^{2iad}(cx^n)^{2ibd}+1\right)^2}d(cx^n)}{2bd}-\frac{(cx^n)^{\frac{m+1}{n}}(}{2bd(1+}\right.}{en}$$

$$\Big\downarrow \text{1064}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{e^{-2iad}\left(\frac{ie^{-2iad}\int \frac{2(cx^n)^{\frac{m+1}{n}-1}\left(\frac{e^{4iad}(m-ibdn+1)(m-2ibdn+1)}{n^2}-\frac{e^{6iad}(m+ibdn+1)(m+2ibdn+1)(cx^n)^{2ibd}}{n^2}\right)}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{2bd}-\cdots\right.}{2bd}\cdots\right.}{en}$$

$\Big\downarrow$ 27

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{e^{-2iad}\left(\frac{ie^{-2iad}\int \frac{(cx^n)^{\frac{m+1}{n}-1}\left(\frac{e^{4iad}(m-ibdn+1)(m-2ibdn+1)}{n^2}-\frac{e^{6iad}(m+ibdn+1)(m+2ibdn+1)(cx^n)^{2ibd}}{n^2}\right)}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{bd}-\cdots\right.}{2bd}\cdots\right.}{en}$$

$\Big\downarrow$ 959

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{e^{-2iad}\left(\frac{ie^{-2iad}\left(\frac{2e^{4iad}\left(-2b^2d^2n^2+m^2+2m+1\right)\int \frac{(cx^n)^{\frac{m+1}{n}-1}}{e^{2iad}(cx^n)^{2ibd}+1}d(cx^n)}{n^2}-\frac{e^{4iad}(ibdn+m+1)(2ibdn+m+1)(cx^n)^{\frac{m+1}{n}}}{(m+1)n}\cdots\right.}{bd}\cdots\right.}{2bd}\cdots\right.}{en}$$

$\Big\downarrow$ 888

$$(ex)^{m+1}\left(cx^n\right)^{-\frac{m+1}{n}} \left(\frac{e^{-2iad}\left(\frac{ie^{-2iad}\left(\frac{2e^{4iad}\left(-2b^2d^2n^2+m^2+2m+1\right)\left(cx^n\right)^{\frac{m+1}{n}} \text{Hypergeometric2F1}\left(1,-\frac{i(m+1)}{2bdn},1-\frac{i(m+1)}{2bdn},-e^{2iad}\left(cx^n\right)^{2ibd}\right.}{(m+1)n} \right.}{bd} \right.}{2bc} \right.$$

input
```
Int[(e*x)^m*Tan[d*(a + b*Log[c*x^n])]^3,x]
```

output
```
((e*x)^(1 + m)*(-1/2*((c*x^n)^((1 + m)/n)*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I
)*b*d))^2)/(b*d*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^2) + (((-I)*(c*x^n
)^((1 + m)/n)*((E^((2*I)*a*d)*(1 + m - (2*I)*b*d*n))/n - (E^((4*I)*a*d)*(1
 + m + (2*I)*b*d*n)*(c*x^n)^((2*I)*b*d))/n))/(b*d*(1 + E^((2*I)*a*d)*(c*x^
n)^((2*I)*b*d))) + (I*(-((E^((4*I)*a*d)*(1 + m + I*b*d*n)*(1 + m + (2*I)*b
*d*n)*(c*x^n)^((1 + m)/n))/((1 + m)*n)) + (2*E^((4*I)*a*d)*(1 + 2*m + m^2
- 2*b^2*d^2*n^2)*(c*x^n)^((1 + m)/n)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m
))/(b*d*n), 1 - ((I/2)*(1 + m))/(b*d*n), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*
d))])/((1 + m)*n)))/(b*d*E^((2*I)*a*d)))/(2*b*d*E^((2*I)*a*d))))/(e*n*(c*x
^n)^((1 + m)/n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1064
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Simp[1/(
a*b*n*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c
*(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m
+ n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && LtQ
[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5008
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int (ex)^m \tan \left(d(a+b\ln \left(c\,x^n\right))\right)^3 dx$$

input
```
int((e*x)^m*tan(d*(a+b*ln(c*x^n)))^3,x)
```

output
```
int((e*x)^m*tan(d*(a+b*ln(c*x^n)))^3,x)
```

Fricas [F]

$$\int (ex)^m \tan^3\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int (ex)^m \tan\left((b\log\left(cx^n\right)+a)d\right)^3\,dx$$

input
```
integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^3,x, algorithm="fricas")
```

output
```
integral((e*x)^m*tan(b*d*log(c*x^n) + a*d)^3, x)
```

Sympy [F]

$$\int (ex)^m \tan^3\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int (ex)^m \tan^3\left(ad+bd\log\left(cx^n\right)\right)dx$$

input
```
integrate((e*x)**m*tan(d*(a+b*ln(c*x**n)))**3,x)
```

output
```
Integral((e*x)**m*tan(a*d + b*d*log(c*x**n))**3, x)
```

Maxima [F]

$$\int (ex)^m \tan^3(d(a+b\log(cx^n)))\,dx = \int (ex)^m \tan((b\log(cx^n)+a)d)^3\,dx$$

input
```
integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^3,x, algorithm="maxima")
```

output
```
(4*(b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*e^m*n*x*x^m*cos(2*b
*d*log(x^n) + 2*a*d)^2 + 4*(b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c)
)^2)*e^m*n*x*x^m*sin(2*b*d*log(x^n) + 2*a*d)^2 + (2*b*d*e^m*n*cos(2*b*d*lo
g(c)) - e^m*m*sin(2*b*d*log(c)) - e^m*sin(2*b*d*log(c)))*x*x^m*cos(2*b*d*l
og(x^n) + 2*a*d) - (2*b*d*e^m*n*sin(2*b*d*log(c)) + e^m*m*cos(2*b*d*log(c)
) + e^m*cos(2*b*d*log(c)))*x*x^m*sin(2*b*d*log(x^n) + 2*a*d) - (((cos(2*b*
d*log(c))*sin(4*b*d*log(c)) - cos(4*b*d*log(c))*sin(2*b*d*log(c)))*e^m*m -
 2*(b*d*cos(4*b*d*log(c))*cos(2*b*d*log(c)) + b*d*sin(4*b*d*log(c))*sin(2*
b*d*log(c)))*e^m*n + (cos(2*b*d*log(c))*sin(4*b*d*log(c)) - cos(4*b*d*log(
c))*sin(2*b*d*log(c)))*e^m)*x*x^m*cos(2*b*d*log(x^n) + 2*a*d) - ((cos(4*b*
d*log(c))*cos(2*b*d*log(c)) + sin(4*b*d*log(c))*sin(2*b*d*log(c)))*e^m*m +
 2*(b*d*cos(2*b*d*log(c))*sin(4*b*d*log(c)) - b*d*cos(4*b*d*log(c))*sin(2*
b*d*log(c)))*e^m*n + (cos(4*b*d*log(c))*cos(2*b*d*log(c)) + sin(4*b*d*log(
c))*sin(2*b*d*log(c)))*e^m)*x*x^m*sin(2*b*d*log(x^n) + 2*a*d) + (e^m*m*sin
(4*b*d*log(c)) + e^m*sin(4*b*d*log(c)))*x*x^m)*cos(4*b*d*log(x^n) + 4*a*d)
 - (2*b^6*d^6*e^m*n^6 - (b^4*d^4*e^m*m^2 + 2*b^4*d^4*e^m*m + b^4*d^4*e^m)*
n^4 + (2*(b^6*d^6*cos(4*b*d*log(c))^2 + b^6*d^6*sin(4*b*d*log(c))^2)*e^m*n
^6 - ((b^4*d^4*cos(4*b*d*log(c))^2 + b^4*d^4*sin(4*b*d*log(c))^2)*e^m*m^2
+ 2*(b^4*d^4*cos(4*b*d*log(c))^2 + b^4*d^4*sin(4*b*d*log(c))^2)*e^m*m + (b
^4*d^4*cos(4*b*d*log(c))^2 + b^4*d^4*sin(4*b*d*log(c))^2)*e^m)*n^4)*cos...
```

Giac [F]

$$\int (ex)^m \tan^3(d(a+b\log(cx^n)))\,dx = \int (ex)^m \tan((b\log(cx^n)+a)d)^3\,dx$$

input
```
integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^3,x, algorithm="giac")
```

output
```
integrate((e*x)^m*tan((b*log(c*x^n) + a)*d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \tan^3(d(a+b\log(cx^n)))\,dx = \int \tan(d\,(a+b\,\ln(c\,x^n)))^3\,(e\,x)^m\,dx$$

input
```
int(tan(d*(a + b*log(c*x^n)))^3*(e*x)^m,x)
```

output
```
int(tan(d*(a + b*log(c*x^n)))^3*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \tan^3(d(a+b\log(cx^n)))\,dx$$
$$= \frac{e^m\left(x^m\tan(\log(x^n c)\,bd+ad)^2 bdnx - x^m\tan(\log(x^n c)\,bd+ad)\,mx - x^m\tan(\log(x^n c)\,bd+ad)\,x + x^m b\right.}{}$$

input
```
int((e*x)^m*tan(d*(a+b*log(c*x^n)))^3,x)
```

output
```
(e**m*(x**m*tan(log(x**n*c)*b*d + a*d)**2*b*d*n*x - x**m*tan(log(x**n*c)*b
*d + a*d)*m*x - x**m*tan(log(x**n*c)*b*d + a*d)*x + x**m*b*d*n*x - 2*int(x
**m*tan(log(x**n*c)*b*d + a*d),x)*b**2*d**2*n**2 + int(x**m*tan(log(x**n*c
)*b*d + a*d),x)*m**2 + 2*int(x**m*tan(log(x**n*c)*b*d + a*d),x)*m + int(x*
*m*tan(log(x**n*c)*b*d + a*d),x)))/(2*b**2*d**2*n**2)
```

3.178 $\int \tan^p(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 190

$$\int \tan^p(d(a+b\log(cx^n)))\,dx = x\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(\frac{i\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{1+e^{2iad}(cx^n)^{2ibd}}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^p \text{AppellF1}\left(-\frac{i}{2bdn},-p,p,1-\frac{i}{2bdn},e^{2iad}(cx^n)^{2ibd},-e^{2iad}(cx^n)^{2ibd}\right)$$

output
```
x*(I*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))
)^p*(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p*AppellF1(-1/2*I/b/d/n,-p,p,1-1/2*
I/b/d/n,exp(2*I*a*d)*(c*x^n)^(2*I*b*d),-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/((
1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 458 vs. $2(190) = 380$.

Time = 1.03 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.41

$$\int \tan^p\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

$$= \frac{(-i+2bdn)x\left(-\frac{i\left(-1\cdots\right.}{1+\cdots}\right.}{-2bde^{2iad}np\left(cx^n\right)^{2ibd}\text{AppellF1}\left(1-\frac{i}{2bdn},1-p,p,2-\frac{i}{2bdn},e^{2iad}\left(cx^n\right)^{2ibd},-e^{2iad}\left(cx^n\right)^{2ibd}\right)-2bde^{2iad}n\cdots}$$

input
```
Integrate[Tan[d*(a + b*Log[c*x^n])]^p,x]
```

output
```
((-I + 2*b*d*n)*x*(((-I)*(-1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(1 + E^
((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))^p*AppellF1[(-1/2*I)/(b*d*n), -p, p, 1 -
(I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^
((2*I)*b*d))])/(-2*b*d*E^((2*I)*a*d)*n*p*(c*x^n)^((2*I)*b*d)*AppellF1[1 -
(I/2)/(b*d*n), 1 - p, p, 2 - (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b
*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))] - 2*b*d*E^((2*I)*a*d)*n*p*(c*x^
n)^((2*I)*b*d)*AppellF1[1 - (I/2)/(b*d*n), -p, 1 + p, 2 - (I/2)/(b*d*n), E
^((2*I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))] +
(-I + 2*b*d*n)*AppellF1[(-1/2*I)/(b*d*n), -p, p, 1 - (I/2)/(b*d*n), E^((2*
I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5004, 5006, 2058, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^p\left(d(a+b\log\left(cx^n\right))\right)\,dx$$

$$\Big\downarrow \ 5004$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\tan^p(d(a+b\log(cx^n)))\,d(cx^n)}{n}$$

$$\Big\downarrow \ 5006$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\left(\frac{i-ie^{2iad}(cx^n)^{2ibd}}{e^{2iad}(cx^n)^{2ibd}+1}\right)^p d(cx^n)}{n}$$

$$\Big\downarrow \ 2058$$

$$\frac{x(cx^n)^{-1/n}\left(i-ie^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(\frac{i\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{1+e^{2iad}(cx^n)^{2ibd}}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^p\int (cx^n)^{\frac{1}{n}-1}\left(i-ie^{2iad}(cx^n)^{2ibd}\right)^{1}}{n}$$

$$\Big\downarrow \ 1013$$

$$\frac{x(cx^n)^{-1/n}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(\frac{i\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{1+e^{2iad}(cx^n)^{2ibd}}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^p\int (cx^n)^{\frac{1}{n}-1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{p}}{n}$$

$$\Big\downarrow \ 1012$$

$$x\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(\frac{i\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{1+e^{2iad}(cx^n)^{2ibd}}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^p \text{AppellF1}\left(-\frac{i}{2bdn},-p,p,1-\frac{i}{2bdn},e^{2}\right.$$

input
```
Int[Tan[d*(a + b*Log[c*x^n])]^p,x]
```

output
```
(x*((I*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(1 + E^((2*I)*a*d)*(c*x^n)
^((2*I)*b*d)))^p*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p*AppellF1[(-1/2*
I)/(b*d*n), -p, p, 1 - (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d), -
(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*
d))^p
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 5004
```
Int[Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Tan[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [F]

$$\int \tan \left(d(a+b\ln \left(c\,x^{n}\right))\right)^{p}dx$$

input
```
int(tan(d*(a+b*ln(c*x^n)))^p,x)
```

output
```
int(tan(d*(a+b*ln(c*x^n)))^p,x)
```

Fricas [F]

$$\int \tan^p(d(a+b\log(cx^n)))\,dx = \int \tan\left((b\log(cx^n)+a)d\right)^p\,dx$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

output
```
integral(tan(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F]

$$\int \tan^p(d(a+b\log(cx^n)))\,dx = \int \tan^p(d(a+b\log(cx^n)))\,dx$$

input
```
integrate(tan(d*(a+b*ln(c*x**n)))**p,x)
```

output
```
Integral(tan(d*(a + b*log(c*x**n)))**p, x)
```

Maxima [F]

$$\int \tan^p(d(a+b\log(cx^n)))\,dx = \int \tan\left((b\log(cx^n)+a)d\right)^p\,dx$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

output
```
integrate(tan((b*log(c*x^n) + a)*d)^p, x)
```

Giac [F(-1)]

Timed out.

$$\int \tan^p\left(d(a+b\log\left(cx^n\right))\right)\,dx = \text{Timed out}$$

input
```
integrate(tan(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \tan^p\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int \tan\left(d\left(a+b\,\ln\left(c\,x^n\right)\right)\right)^p\,dx$$

input
```
int(tan(d*(a + b*log(c*x^n)))^p,x)
```

output
```
int(tan(d*(a + b*log(c*x^n)))^p, x)
```

Reduce [F]

$$\int \tan^p\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int \tan\left(\log(x^n c)\,bd+ad\right)^p dx$$

input
```
int(tan(d*(a+b*log(c*x^n)))^p,x)
```

output
```
int(tan(log(x**n*c)*b*d + a*d)**p,x)
```

3.179 $\int (ex)^m \tan^p(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 210

$$\int (ex)^m \tan^p(d(a+b\log(cx^n)))\,dx = \frac{(ex)^{1+m}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(\frac{i\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{1+e^{2iad}(cx^n)^{2ibd}}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^p \text{AppellF1}\left(-\frac{i(1+m)}{2bdn},-p,p,1-\right.}{e(1+m)}$$

output
```
(e*x)^(1+m)*(I*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/(1+exp(2*I*a*d)*(c*x^n)^
(2*I*b*d)))^p*(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p*AppellF1(-1/2*I*(1+m)/b
/d/n,-p,p,1-1/2*I*(1+m)/b/d/n,exp(2*I*a*d)*(c*x^n)^(2*I*b*d),-exp(2*I*a*d)
*(c*x^n)^(2*I*b*d))/e/(1+m)/((1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p)
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.98

$$\int (ex)^m \tan^p(d(a+b\log(cx^n)))\,dx = \frac{x(ex)^m\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(-\frac{i\left(-1+e^{2iad}(cx^n)^{2ibd}\right)}{1+e^{2iad}(cx^n)^{2ibd}}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^p \text{AppellF1}\left(-\frac{i(1+m)}{2bdn},-p,p,1\right.}{1+m}$$

input
```
Integrate[(e*x)^m*Tan[d*(a + b*Log[c*x^n])]^p,x]
```

output
```
(x*(e*x)^m*(((-I)*(-1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(1 + E^((2*I)*
a*d)*(c*x^n)^((2*I)*b*d)))^p*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p*App
ellF1[((-1/2*I)*(1 + m))/(b*d*n), -p, p, 1 - ((I/2)*(1 + m))/(b*d*n), E^((
2*I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/((1
+ m)*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5008, 5006, 2058, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \tan^p(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{ 5008}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\tan^p(d(a+b\log(cx^n)))\,d(cx^n)}{en}$$

$$\Big\downarrow \text{ 5006}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\left(\frac{i-ie^{2iad}(cx^n)^{2ibd}}{e^{2iad}(cx^n)^{2ibd}+1}\right)^p d(cx^n)}{en}$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(i-ie^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(\frac{i\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{1+e^{2iad}(cx^n)^{2ibd}}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^p\int (cx^n)^{\frac{m+1}{n}-1}\left(i-ie^{2iad}\right.}{en}$$

$$\Big\downarrow \text{ 1013}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(\frac{i\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{1+e^{2iad}(cx^n)^{2ibd}}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^p\int(cx^n)^{\frac{m+1}{n}-1}\left(1-e^{2iad}(\right.}{en}$$

$\big\downarrow$ 1012

$$\frac{(ex)^{m+1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(\frac{i\left(1-e^{2iad}(cx^n)^{2ibd}\right)}{1+e^{2iad}(cx^n)^{2ibd}}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^p\text{AppellF1}\left(-\frac{i(m+1)}{2bdn},-p,p,1-\frac{i(m+1}{2bdn}\right.}{e(m+1)}$$

input
```
Int[(e*x)^m*Tan[d*(a + b*Log[c*x^n])]^p,x]
```

output
```
((e*x)^(1 + m)*((I*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(1 + E^((2*I)*
a*d)*(c*x^n)^((2*I)*b*d)))^p*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p*App
ellF1[((-1/2*I)*(1 + m))/(b*d*n), -p, p, 1 - ((I/2)*(1 + m))/(b*d*n), E^((
2*I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/(e*(
1 + m)*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
 + 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 5006
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Int[(e*x)^m*((I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 + E^(2*I*a*d)*x^(2*I*b*d
)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5008
```
Int[((e_.)*(x_))^(m_.)*Tan[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Tan[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int (ex)^m \tan \left(d(a+b\ln \left(c\,x^n\right))\right)^p dx$$

input
```
int((e*x)^m*tan(d*(a+b*ln(c*x^n)))^p,x)
```

output
```
int((e*x)^m*tan(d*(a+b*ln(c*x^n)))^p,x)
```

Fricas [F]

$$\int (ex)^m \tan^p \left(d(a+b\log \left(cx^n\right))\right)\,dx = \int (ex)^m \tan \left((b\log \left(cx^n\right)+a)d\right)^p\,dx$$

input
```
integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^m*tan(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \tan^p(d(a + b\log(cx^n))) \, dx = \text{Timed out}$$

input
```
integrate((e*x)**m*tan(d*(a+b*ln(c*x**n)))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m \tan^p(d(a + b\log(cx^n))) \, dx = \int (ex)^m \tan((b\log(cx^n) + a)d)^p \, dx$$

input
```
integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*tan((b*log(c*x^n) + a)*d)^p, x)
```

Giac [F(-1)]

Timed out.

$$\int (ex)^m \tan^p(d(a + b\log(cx^n))) \, dx = \text{Timed out}$$

input
```
integrate((e*x)^m*tan(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \tan^p(d(a+b\log(cx^n)))\,dx = \int \tan(d\,(a+b\,\ln(c\,x^n)))^p\,(e\,x)^m\,dx$$

input
```
int(tan(d*(a + b*log(c*x^n)))^p*(e*x)^m,x)
```

output
```
int(tan(d*(a + b*log(c*x^n)))^p*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \tan^p(d(a+b\log(cx^n)))\,dx = e^m\left(\int x^m \tan(\log(x^n c)\,bd+ad)^p dx\right)$$

input
```
int((e*x)^m*tan(d*(a+b*log(c*x^n)))^p,x)
```

output
```
e**m*int(x**m*tan(log(x**n*c)*b*d + a*d)**p,x)
```

3.180 $\int \frac{\tan^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 149

$$\int \frac{\tan^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}}{1+\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn} + \frac{2\tan^{\frac{3}{2}}(a+b\log(cx^n))}{3bn}$$

output
```
-1/2*arctan(-1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n-1/2*arctan(1+
2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n+1/2*arctanh(2^(1/2)*tan(a+b*
ln(c*x^n))^(1/2)/(1+tan(a+b*ln(c*x^n))))*2^(1/2)/b/n+2/3*tan(a+b*ln(c*x^n)
)^(3/2)/b/n
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.81

$$\int \frac{\tan^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{-3\arctan\left(\sqrt[4]{-\tan^2(a+b\log(cx^n))}\right)\sqrt[4]{-\tan(a+b\log(cx^n))}+3\text{arctanh}\left(\sqrt[4]{-\tan^2(a+b\log(cx^n)}\right.}{3bn\sqrt[4]{\tan(a+b\log(cx^n))}}$$

input
```
Integrate[Tan[a + b*Log[c*x^n]]^(5/2)/x,x]
```

output
```
(-3*ArcTan[(-Tan[a + b*Log[c*x^n]]^2)^(1/4)]*(-Tan[a + b*Log[c*x^n]])^(1/4
) + 3*ArcTanh[(-Tan[a + b*Log[c*x^n]]^2)^(1/4)]*(-Tan[a + b*Log[c*x^n]])^(
1/4) + 2*Tan[a + b*Log[c*x^n]]^(7/4))/(3*b*n*Tan[a + b*Log[c*x^n]]^(1/4))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.30, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {3039, 3042, 3954, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$$
$$\Big\downarrow \; 3039$$
$$\frac{\int \tan^{\frac{5}{2}}(a+b\log(cx^n))\,d\log(cx^n)}{n}$$
$$\Big\downarrow \; 3042$$
$$\frac{\int \tan(a+b\log(cx^n))^{5/2}\,d\log(cx^n)}{n}$$
$$\Big\downarrow \; 3954$$

$$\frac{\frac{2\tan^{\frac{3}{2}}(a+b\log(cx^n))}{3b}-\int\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}d\log\left(cx^n\right)}{n}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2\tan^{\frac{3}{2}}(a+b\log(cx^n))}{3b}-\int\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}d\log\left(cx^n\right)}{n}$$

$\Big\downarrow$ 3957

$$\frac{\frac{2\tan^{\frac{3}{2}}(a+b\log(cx^n))}{3b}-\frac{\int\frac{\sqrt{\tan(a+b\log(cx^n))}}{\tan^2(a+b\log(cx^n))+1}d\tan(a+b\log(cx^n))}{b}}{n}$$

$\Big\downarrow$ 266

$$\frac{\frac{2\tan^{\frac{3}{2}}(a+b\log(cx^n))}{3b}-\frac{2\int\frac{\tan(a+b\log(cx^n))}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}}{b}}{n}$$

$\Big\downarrow$ 826

$$\frac{\frac{2\tan^{\frac{3}{2}}(a+b\log(cx^n))}{3b}-\frac{2\left(\frac{1}{2}\int\frac{\tan(a+b\log(cx^n))+1}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}-\frac{1}{2}\int\frac{1-\tan(a+b\log(cx^n))}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}\right)}{b}}{n}$$

$\Big\downarrow$ 1476

$$\frac{\frac{2\tan^{\frac{3}{2}}(a+b\log(cx^n))}{3b}-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}+\frac{1}{2}\int\frac{1}{\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n)}}\right.\right.}{b}}{n}$$

$\Big\downarrow$ 1082

$$\frac{\frac{2\tan^{\frac{3}{2}}(a+b\log(cx^n))}{3b}-\frac{2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-\tan(a+b\log(cx^n))-1}d\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\tan(a+b\log(cx^n))-1}d\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\right.}{b}}{n}$$

$\Big\downarrow$ 217

$$\frac{\frac{2\tan^{\frac{3}{2}}(a+b\log(cx^n))}{3b}-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-\tan(a+b\log(cx^n))}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(c\right.}{b}}{n}$$

$\Big\downarrow$ 1479

$$\frac{\frac{2\tan^{\frac{3}{2}}(a+b\log(cx^n))}{3b}-\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(a+b\log(cx^n))}}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+}}{2\sqrt{2}}\right.\right.}{b}}{n}$$

$\downarrow$ 25

$$\frac{\frac{2\tan^{\frac{3}{2}}(a+b\log(cx^n))}{3b}-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(a+b\log(cx^n))}}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}}{2\sqrt{2}}\right.\right.}{b}}{n}$$

$\downarrow$ 27

$$\frac{\frac{2\tan^{\frac{3}{2}}(a+b\log(cx^n))}{3b}-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(a+b\log(cx^n))}}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}{\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+}\right.\right.}{b}}{n}$$

$\downarrow$ 1103

$$\frac{\frac{2\tan^{\frac{3}{2}}(a+b\log(cx^n))}{3b}-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(\right.}{2\sqrt{2}}\right.\right.}{b}}{n}$$

input
```
Int[Tan[a + b*Log[c*x^n]]^(5/2)/x,x]
```

output
```
((-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/Sqrt[2]) + ArcTan
[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sq
rt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]) - Log[1 + S
qrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]))/2
))/b + (2*Tan[a + b*Log[c*x^n]]^(3/2))/(3*b))/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\frac{2\tan(a+b\ln(c\,x^n))^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(a+b\ln(c\,x^n))-\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}+1}{\tan(a+b\ln(c\,x^n))+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}\right)+2\arctan\left(-\right.\right.}{4}}{nb}$
default	$\frac{\frac{2\tan(a+b\ln(c\,x^n))^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(a+b\ln(c\,x^n))-\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}+1}{\tan(a+b\ln(c\,x^n))+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}\right)+2\arctan\left(-\right.\right.}{4}}{nb}$

input
```
int(tan(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(2/3*tan(a+b*ln(c*x^n))^(3/2)-1/4*2^(1/2)*(ln((tan(a+b*ln(c*x^n))-2^
(1/2)*tan(a+b*ln(c*x^n))^(1/2)+1)/(tan(a+b*ln(c*x^n))+2^(1/2)*tan(a+b*ln(c
*x^n))^(1/2)+1))+2*arctan(1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+
2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. 2(128) = 256.

Time = 0.08 (sec) , antiderivative size = 526, normalized size of antiderivative = 3.53

$$\int \frac{\tan^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

input
```
integrate(tan(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

output
```
-1/12*(6*(sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + sqrt(2))*arctan(s
qrt(2)*sqrt(sin(2*b*n*log(x) + 2*b*log(c) + 2*a)/(cos(2*b*n*log(x) + 2*b*l
og(c) + 2*a) + 1)) + 1) + 6*(sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c) + 2*a)
+ sqrt(2))*arctan(sqrt(2)*sqrt(sin(2*b*n*log(x) + 2*b*log(c) + 2*a)/(cos(2
*b*n*log(x) + 2*b*log(c) + 2*a) + 1)) - 1) - 3*(sqrt(2)*cos(2*b*n*log(x) +
 2*b*log(c) + 2*a) + sqrt(2))*log(((sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c)
+ 2*a) + sqrt(2))*sqrt(sin(2*b*n*log(x) + 2*b*log(c) + 2*a)/(cos(2*b*n*log
(x) + 2*b*log(c) + 2*a) + 1)) + cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + sin
(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*
a) + 1)) + 3*(sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + sqrt(2))*log(
-((sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + sqrt(2))*sqrt(sin(2*b*n*
log(x) + 2*b*log(c) + 2*a)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)) - c
os(2*b*n*log(x) + 2*b*log(c) + 2*a) - sin(2*b*n*log(x) + 2*b*log(c) + 2*a)
 - 1)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)) - 8*sqrt(sin(2*b*n*log(x
) + 2*b*log(c) + 2*a)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1))*sin(2*b*
n*log(x) + 2*b*log(c) + 2*a))/(b*n*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) +
b*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(tan(a+b*ln(c*x**n))**(5/2)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\tan^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\tan(b\log(cx^n)+a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate(tan(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate(tan(b*log(c*x^n) + a)^(5/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(tan(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 21.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.53

$$\int \frac{\tan^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2\tan(a+b\,\ln(c\,x^n))^{3/2}}{3\,b\,n} - \frac{(-1)^{1/4}\,\mathrm{atan}\Big((-1)^{1/4}\,\sqrt{\tan(a+b\,\ln(c\,x^n))}\Big)}{b\,n} + \frac{(-1)^{1/4}\,\mathrm{atanh}\Big((-1)^{1/4}\,\sqrt{\tan(a+b\,\ln(c\,x^n))}\Big)}{b\,n}$$

input
```
int(tan(a + b*log(c*x^n))^(5/2)/x,x)
```

output
```
(2*tan(a + b*log(c*x^n))^(3/2))/(3*b*n) - ((-1)^(1/4)*atan((-1)^(1/4)*tan(
a + b*log(c*x^n))^(1/2)))/(b*n) + ((-1)^(1/4)*atanh((-1)^(1/4)*tan(a + b*l
og(c*x^n))^(1/2)))/(b*n)
```

Reduce [F]

$$\int \frac{\tan^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sqrt{\tan(\log(x^n c)\,b+a)}\,\tan(\log(x^n c)\,b+a)^2}{x}dx$$

input
```
int(tan(a+b*log(c*x^n))^(5/2)/x,x)
```

output
```
int((sqrt(tan(log(x**n*c)*b + a))*tan(log(x**n*c)*b + a)**2)/x,x)
```

3.181 $\int \frac{\tan^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 148

$$\int \frac{\tan^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}}{1+\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn} + \frac{2\sqrt{\tan(a+b\log(cx^n))}}{bn}$$

output
```
-1/2*arctan(-1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n-1/2*arctan(1+
2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n-1/2*arctanh(2^(1/2)*tan(a+b*
ln(c*x^n))^(1/2)/(1+tan(a+b*ln(c*x^n))))*2^(1/2)/b/n+2*tan(a+b*ln(c*x^n))^
(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.19

$$\int \frac{\tan^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$$
$$=\frac{\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+\tan(a+b\log(cx^n))\right)}{2\sqrt{2}}-\frac{\log}{}}{bn}$$

input
```
Integrate[Tan[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
(ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/Sqrt[2] - ArcTan[1 + Sqrt
[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[Tan[a + b*
Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[T
an[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]) + 2*Sqrt[Tan[a
+ b*Log[c*x^n]]])/(b*n)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.30, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {3039, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \tan^{\frac{3}{2}}(a+b\log(cx^n))\,d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \tan(a+b\log(cx^n))^{3/2}\,d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3954}$$

$$\frac{\frac{2\sqrt{\tan(a+b\log(cx^n))}}{b}-\int \frac{1}{\sqrt{\tan(a+b\log(cx^n))}}d\log\left(cx^n\right)}{n}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2\sqrt{\tan(a+b\log(cx^n))}}{b}-\int \frac{1}{\sqrt{\tan(a+b\log(cx^n))}}d\log\left(cx^n\right)}{n}$$

$\Big\downarrow$ 3957

$$\frac{\frac{2\sqrt{\tan(a+b\log(cx^n))}}{b}-\frac{\int \frac{1}{\sqrt{\tan(a+b\log(cx^n))}\left(\tan^2(a+b\log(cx^n))+1\right)}d\tan(a+b\log(cx^n))}{b}}{n}$$

$\Big\downarrow$ 266

$$\frac{\frac{2\sqrt{\tan(a+b\log(cx^n))}}{b}-\frac{2\int \frac{1}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}}{b}}{n}$$

$\Big\downarrow$ 755

$$\frac{\frac{2\sqrt{\tan(a+b\log(cx^n))}}{b}-\frac{2\left(\frac{1}{2}\int \frac{1-\tan(a+b\log(cx^n))}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}+\frac{1}{2}\int \frac{\tan(a+b\log(cx^n))+1}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}\right)}{b}}{n}$$

$\Big\downarrow$ 1476

$$\frac{\frac{2\sqrt{\tan(a+b\log(cx^n))}}{b}-\frac{2\left(\frac{1}{2}\int \frac{1-\tan(a+b\log(cx^n))}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(c\right.}{b}}{n}$$

$\Big\downarrow$ 1082

$$\frac{\frac{2\sqrt{\tan(a+b\log(cx^n))}}{b}-\frac{2\left(\frac{1}{2}\int \frac{1-\tan(a+b\log(cx^n))}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}+\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(a+b\log(cx^n))-1}d\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}}-\frac{\int \frac{}{-\tan(a+}\right.\right.}{b}}{n}$$

$\Big\downarrow$ 217

$$\frac{\frac{2\sqrt{\tan(a+b\log(cx^n))}}{b}-\frac{2\left(\frac{1}{2}\int \frac{1-\tan(a+b\log(cx^n))}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n\right.}{\sqrt{2}}\right.\right.}{b}}{n}$$

$\Big\downarrow$ 1479

$$\frac{2\sqrt{\tan(a+b\log(cx^n))}}{b} - \frac{2\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(a+b\log(cx^n))}}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}}{2\sqrt{2}} - \frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}}}{2\sqrt{2}}\right.\right.}{b}}{n}$$

$\downarrow$ 25

$$\frac{2\sqrt{\tan(a+b\log(cx^n))}}{b} - \frac{2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(a+b\log(cx^n))}}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}}{2\sqrt{2}} + \frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d}{2\sqrt{2}}\right.\right.}{b}}{n}$$

$\downarrow$ 27

$$\frac{2\sqrt{\tan(a+b\log(cx^n))}}{b} - \frac{2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(a+b\log(cx^n))}}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}}{2\sqrt{2}} + \frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}{\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}\right.\right.}{b}}{n}$$

$\downarrow$ 1103

$$\frac{2\sqrt{\tan(a+b\log(cx^n))}}{b} - \frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\sqrt{2}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}}\right) + \frac{1}{2}\left(\frac{\log\left(\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(\right.}{2\sqrt{2}}\right.\right.}{b}}{n}$$

input
```
Int[Tan[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
((-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/Sqrt[2]) + ArcTan
[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[
2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/Sqrt[2] + Log[1 +
Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]))/
2))/b + (2*Sqrt[Tan[a + b*Log[c*x^n]]])/b)/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{2\sqrt{\tan(a+b\ln(c\,x^n))}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(a+b\ln(c\,x^n))+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}+1}{\tan(a+b\ln(c\,x^n))-\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}\right)+2\arctan}{4}}{nb}$
default	$\frac{2\sqrt{\tan(a+b\ln(c\,x^n))}-\frac{\sqrt{2}\left(\ln\left(\frac{\tan(a+b\ln(c\,x^n))+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}+1}{\tan(a+b\ln(c\,x^n))-\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}\right)+2\arctan}{4}}{nb}$

input
```
int(tan(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(2*tan(a+b*ln(c*x^n))^(1/2)-1/4*2^(1/2)*(ln((tan(a+b*ln(c*x^n))+2^(1
/2)*tan(a+b*ln(c*x^n))^(1/2)+1)/(tan(a+b*ln(c*x^n))-2^(1/2)*tan(a+b*ln(c*x
^n))^(1/2)+1))+2*arctan(1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+2^
(1/2)*tan(a+b*ln(c*x^n))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(128) = 256$.

Time = 0.08 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.74

$$\int \frac{\tan^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx =$$

$$-\frac{2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{\sin(2\,bn\log(x)+2\,b\log(c)+2\,a)}{\cos(2\,bn\log(x)+2\,b\log(c)+2\,a)+1}}+1\right)+2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{\sin(2\,bn\log(x)+2\,b\log(c)+2\,a)}{\cos(2\,bn\log(x)+2\,b\log(c)+2\,a)+1}}-1\right.}{}$$

input
```
integrate(tan(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*arctan(sqrt(2)*sqrt(sin(2*b*n*log(x) + 2*b*log(c) + 2*a)/(
cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)) + 1) + 2*sqrt(2)*arctan(sqrt(2)
*sqrt(sin(2*b*n*log(x) + 2*b*log(c) + 2*a)/(cos(2*b*n*log(x) + 2*b*log(c)
+ 2*a) + 1)) - 1) + sqrt(2)*log(((sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c) +
2*a) + sqrt(2))*sqrt(sin(2*b*n*log(x) + 2*b*log(c) + 2*a)/(cos(2*b*n*log(x
) + 2*b*log(c) + 2*a) + 1)) + cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + sin(2
*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a)
 + 1)) - sqrt(2)*log(-((sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + sqr
t(2))*sqrt(sin(2*b*n*log(x) + 2*b*log(c) + 2*a)/(cos(2*b*n*log(x) + 2*b*lo
g(c) + 2*a) + 1)) - cos(2*b*n*log(x) + 2*b*log(c) + 2*a) - sin(2*b*n*log(x
) + 2*b*log(c) + 2*a) - 1)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)) - 8
*sqrt(sin(2*b*n*log(x) + 2*b*log(c) + 2*a)/(cos(2*b*n*log(x) + 2*b*log(c)
+ 2*a) + 1)))/(b*n)
```

Sympy [F]

$$\int \frac{\tan^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx=\int \frac{\tan^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx$$

input
```
integrate(tan(a+b*ln(c*x**n))**(3/2)/x,x)
```

output
```
Integral(tan(a + b*log(c*x**n))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\tan^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx=\int \frac{\tan\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(tan(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(tan(b*log(c*x^n) + a)^(3/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tan^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(tan(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 21.61 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.53

$$\int \frac{\tan^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2\sqrt{\tan(a+b\ln(cx^n))}}{bn} + \frac{(-1)^{1/4}\operatorname{atan}\left((-1)^{1/4}\sqrt{\tan(a+b\ln(cx^n))}\right)1i}{bn} + \frac{(-1)^{1/4}\operatorname{atanh}\left((-1)^{1/4}\sqrt{\tan(a+b\ln(cx^n))}\right)1i}{bn}$$

input
```
int(tan(a + b*log(c*x^n))^(3/2)/x,x)
```

output
```
(2*tan(a + b*log(c*x^n))^(1/2))/(b*n) + ((-1)^(1/4)*atan((-1)^(1/4)*tan(a
+ b*log(c*x^n))^(1/2))*1i)/(b*n) + ((-1)^(1/4)*atanh((-1)^(1/4)*tan(a + b*
log(c*x^n))^(1/2))*1i)/(b*n)
```

Reduce [F]

$$\int \frac{\tan^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2\sqrt{\tan(\log(x^n c)\,b+a)} - \left(\int \frac{\sqrt{\tan(\log(x^n c)b+a)}}{\tan(\log(x^n c)b+a)x}dx\right)bn}{bn}$$

input

```
int(tan(a+b*log(c*x^n))^(3/2)/x,x)
```

output

```
(2*sqrt(tan(log(x**n*c)*b + a)) - int(sqrt(tan(log(x**n*c)*b + a))/(tan(lo
g(x**n*c)*b + a)*x),x)*b*n)/(b*n)
```

3.182 $\int \frac{\sqrt{\tan(a+b\log(cx^n))}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 125

$$\int \frac{\sqrt{\tan(a+b\log(cx^n))}}{x}\,dx = -\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}}{1+\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn}$$

output
```
1/2*arctan(-1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n+1/2*arctan(1+2
^(1/2)*tan(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n-1/2*arctanh(2^(1/2)*tan(a+b*l
n(c*x^n))^(1/2)/(1+tan(a+b*ln(c*x^n))))*2^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{\tan (a+b\log (cx^n))}}{x}\,dx = \frac{\left(\arctan \left(\sqrt[4]{-\tan ^2(a+b\log (cx^n))}\right)-\operatorname{arctanh}\left(\sqrt[4]{-\tan ^2(a+b\log (cx^n))}\right)\right)\sqrt[4]{-\tan (a+b\log (cx^n))}}{bn\sqrt[4]{\tan (a+b\log (cx^n))}}$$

input
```
Integrate[Sqrt[Tan[a + b*Log[c*x^n]]]/x,x]
```

output
```
((ArcTan[(-Tan[a + b*Log[c*x^n]]^2)^(1/4)] - ArcTanh[(-Tan[a + b*Log[c*x^n
]]^2)^(1/4)])*(-Tan[a + b*Log[c*x^n]])^(1/4))/(b*n*Tan[a + b*Log[c*x^n]]^(
1/4))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.36, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.632, Rules used = {3039, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\tan (a+b\log (cx^n))}}{x}dx$$

$$\Big\downarrow \; 3039$$

$$\frac{\int \sqrt{\tan (a+b\log (cx^n))}d\log (cx^n)}{n}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \sqrt{\tan (a+b\log (cx^n))}d\log (cx^n)}{n}$$

$$\Big\downarrow \; 3957$$

$$\frac{\int \frac{\sqrt{\tan(a+b\log(cx^n))}}{\tan^2(a+b\log(cx^n))+1}d\tan\left(a+b\log\left(cx^n\right)\right)}{bn}$$

$$\Big\downarrow \; 266$$

$$\frac{2\int \frac{\tan(a+b\log(cx^n))}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}}{bn}$$

$$\Big\downarrow \; 826$$

$$\frac{2\left(\frac{1}{2}\int \frac{\tan(a+b\log(cx^n))+1}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}-\frac{1}{2}\int \frac{1-\tan(a+b\log(cx^n))}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

$$\Big\downarrow \; 1476$$

$$\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}+\frac{1}{2}\int \frac{1}{\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+\right.\right.}{bn}$$

$$\Big\downarrow \; 1082$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(a+b\log(cx^n))-1}d\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(a+b\log(cx^n))-1}d\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(a+}{\tan^2(a+b\,\mathrm{lo}}\right.}{bn}$$

$$\Big\downarrow \; 217$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(a+b\log(cx^n))}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan\left(a+b\log\left(c\right.\right.}\right.}{bn}$$

$$\Big\downarrow \; 1479$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(a+b\log(cx^n))}}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(c}}{2\sqrt{2}}\right.\right.}{bn}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(a+b\log(cx^n))}}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx}}{2\sqrt{2}}\right.\right.}{bn}$$

$$\Big\downarrow \ 27$$

$$2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(a+b\log(cx^n))}}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}{\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(\right.\right.$$
$$\overline{\qquad bn \qquad}$$

$$\Big\downarrow \ 1103$$

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right.}{2\sqrt{2}}\right.\right.$$
$$\overline{\qquad bn \qquad}$$

input
```
Int[Sqrt[Tan[a + b*Log[c*x^n]]]/x,x]
```

output
```
(2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/Sqrt[2]) + ArcTan[1
 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt
[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]) - Log[1 + Sqr
t[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]))/2))
/(b*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{\sqrt{2}\left(\ln\left(\frac{\tan(a+b\ln(c\,x^n))-\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}+1}{\tan(a+b\ln(c\,x^n))+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan}\right.\right.}{4nb}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{\tan(a+b\ln(c\,x^n))-\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}+1}{\tan(a+b\ln(c\,x^n))+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan}\right.\right.}{4nb}$

input
```
int(tan(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/4/n/b*2^(1/2)*(ln((tan(a+b*ln(c*x^n))-2^(1/2)*tan(a+b*ln(c*x^n))^(1/2)+1
)/(tan(a+b*ln(c*x^n))+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2)+1))+2*arctan(1+2^(1
/2)*tan(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2)
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(107) = 214.

Time = 0.08 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.92

$$\int \frac{\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx$$
$$=\frac{2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{\sin(2\,bn\log(x)+2\,b\log(c)+2\,a)}{\cos(2\,bn\log(x)+2\,b\log(c)+2\,a)+1}}+1\right)+2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{\sin(2\,bn\log(x)+2\,b\log(c)+2\,a)}{\cos(2\,bn\log(x)+2\,b\log(c)+2\,a)+1}}-1\right)}{}$$

input
```
integrate(tan(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(2)*arctan(sqrt(2)*sqrt(sin(2*b*n*log(x) + 2*b*log(c) + 2*a)/(c
os(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)) + 1) + 2*sqrt(2)*arctan(sqrt(2)*
sqrt(sin(2*b*n*log(x) + 2*b*log(c) + 2*a)/(cos(2*b*n*log(x) + 2*b*log(c) +
 2*a) + 1)) - 1) - sqrt(2)*log(((sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c) + 2
*a) + sqrt(2))*sqrt(sin(2*b*n*log(x) + 2*b*log(c) + 2*a)/(cos(2*b*n*log(x)
 + 2*b*log(c) + 2*a) + 1)) + cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + sin(2*
b*n*log(x) + 2*b*log(c) + 2*a) + 1)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a)
+ 1)) + sqrt(2)*log(-((sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + sqrt
(2))*sqrt(sin(2*b*n*log(x) + 2*b*log(c) + 2*a)/(cos(2*b*n*log(x) + 2*b*log
(c) + 2*a) + 1)) - cos(2*b*n*log(x) + 2*b*log(c) + 2*a) - sin(2*b*n*log(x)
 + 2*b*log(c) + 2*a) - 1)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)))/(b*
n)
```

Sympy [F]

$$\int \frac{\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx=\int \frac{\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx$$

input
```
integrate(tan(a+b*ln(c*x**n))**(1/2)/x,x)
```

output
```
Integral(sqrt(tan(a + b*log(c*x**n)))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx=\int \frac{\sqrt{\tan\left(b\log\left(cx^n\right)+a\right)}}{x}\,dx$$

input
```
integrate(tan(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(tan(b*log(c*x^n) + a))/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = \text{Timed out}$$

input
```
integrate(tan(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 20.64 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\sqrt{\tan\left(a+b\,\ln\left(c\,x^n\right)\right)}-1\right)+\operatorname{atan}\left(\sqrt{2}\sqrt{\tan\left(a+b\,\ln\left(c\,x^n\right)\right)}+1\right)\right)}{2\,b\,n} + \frac{\sqrt{2}\left(\ln\left(\sqrt{2}\sqrt{\tan\left(a+b\,\ln\left(c\,x^n\right)\right)}-\tan(a+b\,\ln\left(c\,x^n\right))-1\right)-\ln\left(\tan(a+b\,\ln\left(c\,x^n\right))+\sqrt{2}\sqrt{\tan$$
$$4\,b\,n$$

input
```
int(tan(a + b*log(c*x^n))^(1/2)/x,x)
```

output
```
(2^(1/2)*(atan(2^(1/2)*tan(a + b*log(c*x^n))^(1/2) - 1) + atan(2^(1/2)*tan
(a + b*log(c*x^n))^(1/2) + 1)))/(2*b*n) + (2^(1/2)*(log(2^(1/2)*tan(a + b*
log(c*x^n))^(1/2) - tan(a + b*log(c*x^n)) - 1) - log(tan(a + b*log(c*x^n))
 + 2^(1/2)*tan(a + b*log(c*x^n))^(1/2) + 1)))/(4*b*n)
```

Reduce [F]

$$\int \frac{\sqrt{\tan(a+b\log(cx^n))}}{x}\,dx = \int \frac{\sqrt{\tan(\log(x^n c)\,b+a)}}{x}dx$$

input
```
int(tan(a+b*log(c*x^n))^(1/2)/x,x)
```

output
```
int(sqrt(tan(log(x**n*c)*b + a))/x,x)
```

3.183 $\int \frac{1}{x\sqrt{\tan(a+b\log(cx^n))}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 124

$$\int \frac{1}{x\sqrt{\tan(a+b\log(cx^n))}}\,dx = -\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}}{1+\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn}$$

output
```
1/2*arctan(-1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n+1/2*arctan(1+2
^(1/2)*tan(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n+1/2*arctanh(2^(1/2)*tan(a+b*l
n(c*x^n))^(1/2)/(1+tan(a+b*ln(c*x^n))))*2^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15

$$\int \frac{1}{x\sqrt{\tan(a+b\log(cx^n))}}\,dx$$
$$= \frac{-2\arctan\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)-\log\left(1-\sqrt{2}\sqrt{\ldots}\right.}{2\ldots}$$

input
```
Integrate[1/(x*Sqrt[Tan[a + b*Log[c*x^n]]]),x]
```

output
```
(-2*ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]] + 2*ArcTan[1 + Sqrt[2]
*Sqrt[Tan[a + b*Log[c*x^n]]]] - Log[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]
] + Tan[a + b*Log[c*x^n]]] + Log[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] +
 Tan[a + b*Log[c*x^n]]])/(2*Sqrt[2]*b*n)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.37, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {3039, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\tan(a+b\log(cx^n))}}\,dx$$
$$\Big\downarrow \text{ 3039}$$
$$\frac{\int \frac{1}{\sqrt{\tan(a+b\log(cx^n))}}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int \frac{1}{\sqrt{\tan(a+b\log(cx^n))}}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{ 3957}$$

$$\frac{\int \frac{1}{\sqrt{\tan(a+b\log(cx^n))}(\tan^2(a+b\log(cx^n))+1)}d\tan(a+b\log(cx^n))}{bn}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2\int \frac{1}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}}{bn}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1-\tan(a+b\log(cx^n))}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}+\frac{1}{2}\int \frac{\tan(a+b\log(cx^n))+1}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}\right)}{bn}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1-\tan(a+b\log(cx^n))}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log}\right.\right.}{bn}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1-\tan(a+b\log(cx^n))}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}+\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(a+b\log(cx^n))-1}d\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(a+b\log}}{}\right.\right.}{bn}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{2\left(\frac{1}{2}\int \frac{1-\tan(a+b\log(cx^n))}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n}\right.}{\sqrt{2}}\right.\right.}{bn}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{2\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(a+b\log(cx^n))}}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log}}{2\sqrt{2}}\right.\right.}{bn}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(a+b\log(cx^n))}}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n)}}{2\sqrt{2}}\right.\right.}{bn}$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(a+b\log(cx^n))}}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}{\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a\cdots}\right.\right.$$
$$\overline{\qquad bn \qquad}$$

$$\Big\downarrow \text{1103}$$

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\cdots}{2\sqrt{2}}\right.\right.$$
$$\overline{\qquad bn \qquad}$$

input
```
Int[1/(x*Sqrt[Tan[a + b*Log[c*x^n]]]),x]
```

output
```
(2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/Sqrt[2]) + ArcTan[1
 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]
*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/Sqrt[2] + Log[1 + Sq
rt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]))/2)
)/(b*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{\sqrt{2}\left(\ln\left(\frac{\tan(a+b\ln(c\,x^{n}))+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^{n}))}+1}{\tan(a+b\ln(c\,x^{n}))-\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^{n}))}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^{n}))}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan\right.$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{\tan(a+b\ln(c\,x^{n}))+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^{n}))}+1}{\tan(a+b\ln(c\,x^{n}))-\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^{n}))}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^{n}))}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan\right.$

(denominator of each result: $4nb$)

input
```
int(1/x/tan(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/n/b*2^(1/2)*(ln((tan(a+b*ln(c*x^n))+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2)+1
)/(tan(a+b*ln(c*x^n))-2^(1/2)*tan(a+b*ln(c*x^n))^(1/2)+1))+2*arctan(1+2^(1
/2)*tan(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2)
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(107) = 214.

Time = 0.08 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.94

$$\int \frac{1}{x\sqrt{\tan\left(a+b\log\left(cx^{n}\right)\right)}}\,dx$$
$$= \frac{2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{\sin(2\,bn\log(x)+2\,b\log(c)+2\,a)}{\cos(2\,bn\log(x)+2\,b\log(c)+2\,a)+1}}+1\right)+2\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{\sin(2\,bn\log(x)+2\,b\log(c)+2\,a)}{\cos(2\,bn\log(x)+2\,b\log(c)+2\,a)+1}}-1\right)\cdots}{}$$

input
```
integrate(1/x/tan(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(2)*arctan(sqrt(2)*sqrt(sin(2*b*n*log(x) + 2*b*log(c) + 2*a)/(c
os(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)) + 1) + 2*sqrt(2)*arctan(sqrt(2)*
sqrt(sin(2*b*n*log(x) + 2*b*log(c) + 2*a)/(cos(2*b*n*log(x) + 2*b*log(c) +
 2*a) + 1)) - 1) + sqrt(2)*log(((sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c) + 2
*a) + sqrt(2))*sqrt(sin(2*b*n*log(x) + 2*b*log(c) + 2*a)/(cos(2*b*n*log(x)
 + 2*b*log(c) + 2*a) + 1)) + cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + sin(2*
b*n*log(x) + 2*b*log(c) + 2*a) + 1)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a)
+ 1)) - sqrt(2)*log(-((sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + sqrt
(2))*sqrt(sin(2*b*n*log(x) + 2*b*log(c) + 2*a)/(cos(2*b*n*log(x) + 2*b*log
(c) + 2*a) + 1)) - cos(2*b*n*log(x) + 2*b*log(c) + 2*a) - sin(2*b*n*log(x)
 + 2*b*log(c) + 2*a) - 1)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)))/(b*
n)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{x\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}}\,dx$$

input
```
integrate(1/x/tan(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(tan(a + b*log(c*x**n)))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{x\sqrt{\tan\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

input
```
integrate(1/x/tan(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sqrt(tan(b*log(c*x^n) + a))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\tan(a+b\log(cx^n))}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/tan(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 20.71 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.48

$$\int \frac{1}{x\sqrt{\tan(a+b\log(cx^n))}}\,dx = -\frac{(-1)^{1/4}\,\text{atan}\left((-1)^{1/4}\,\sqrt{\tan(a+b\,\ln(c\,x^n))}\right)\,1\text{i}}{b\,n} - \frac{(-1)^{1/4}\,\text{atanh}\left((-1)^{1/4}\,\sqrt{\tan(a+b\,\ln(c\,x^n))}\right)\,1\text{i}}{b\,n}$$

input
```
int(1/(x*tan(a + b*log(c*x^n))^(1/2)),x)
```

output
```
- ((-1)^(1/4)*atan((-1)^(1/4)*tan(a + b*log(c*x^n))^(1/2))*1i)/(b*n) - ((-
1)^(1/4)*atanh((-1)^(1/4)*tan(a + b*log(c*x^n))^(1/2))*1i)/(b*n)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{\tan(a+b\log(cx^n))}}\,dx = \int \frac{\sqrt{\tan(\log(x^n c)\,b+a)}}{\tan(\log(x^n c)\,b+a)\,x}dx$$

input
```
int(1/x/tan(a+b*log(c*x^n))^(1/2),x)
```

output
```
int(sqrt(tan(log(x**n*c)*b + a))/(tan(log(x**n*c)*b + a)*x),x)
```

3.184 $\int \frac{1}{x\tan^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 147

$$\int \frac{1}{x\tan^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}}{1+\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn} - \frac{2}{bn\sqrt{\tan(a+b\log(cx^n))}}$$

output
```
-1/2*arctan(-1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n-1/2*arctan(1+
2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n+1/2*arctanh(2^(1/2)*tan(a+b*
ln(c*x^n))^(1/2)/(1+tan(a+b*ln(c*x^n))))*2^(1/2)/b/n-2/b/n/tan(a+b*ln(c*x^
n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\tan^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$
$$= \frac{-2-\arctan\left(\sqrt[4]{-\tan^2(a+b\log(cx^n))}\right)\sqrt[4]{-\tan^2(a+b\log(cx^n))}+\operatorname{arctanh}\left(\sqrt[4]{-\tan^2(a+b\log(cx\right.}{bn\sqrt{\tan(a+b\log(cx^n))}}$$

input
```
Integrate[1/(x*Tan[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
(-2 - ArcTan[(-Tan[a + b*Log[c*x^n]]^2)^(1/4)]*(-Tan[a + b*Log[c*x^n]]^2)^
(1/4) + ArcTanh[(-Tan[a + b*Log[c*x^n]]^2)^(1/4)]*(-Tan[a + b*Log[c*x^n]]^
2)^(1/4))/(b*n*Sqrt[Tan[a + b*Log[c*x^n]]])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.31, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {3039, 3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\tan^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$

$$\Big\downarrow \; 3039$$

$$\frac{\int \frac{1}{\tan^{\frac{3}{2}}(a+b\log(cx^n))}\,d\log(cx^n)}{n}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{1}{\tan(a+b\log(cx^n))^{3/2}}\,d\log(cx^n)}{n}$$

$$\Big\downarrow \; 3955$$

$$\frac{-\int \sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}d\log\left(cx^n\right)-\frac{2}{b\sqrt{\tan(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\int \sqrt{\tan\left(a+b\log\left(cx^n\right)\right)}d\log\left(cx^n\right)-\frac{2}{b\sqrt{\tan(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{3957}$$

$$\frac{-\frac{\int \frac{\sqrt{\tan(a+b\log(cx^n))}}{\tan^2(a+b\log(cx^n))+1}d\tan(a+b\log(cx^n))}{b}-\frac{2}{b\sqrt{\tan(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{266}$$

$$\frac{-\frac{2\int \frac{\tan(a+b\log(cx^n))}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}}{b}-\frac{2}{b\sqrt{\tan(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{826}$$

$$\frac{-\frac{2\left(\frac{1}{2}\int \frac{\tan(a+b\log(cx^n))+1}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}-\frac{1}{2}\int \frac{1-\tan(a+b\log(cx^n))}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}\right)}{b}-\frac{2}{b\sqrt{\tan(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{1476}$$

$$\frac{-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}+\frac{1}{2}\int \frac{1}{\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n}\right)\right)}{b}}{n}$$

$$\Big\downarrow \text{1082}$$

$$\frac{-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(a+b\log(cx^n))-1}d\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(a+b\log(cx^n))-1}d\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(a+b\log(cx^n))}{\tan^2(a+b\log(cx^n))+1}d\sqrt{}\right)}{b}}{n}$$

$$\Big\downarrow \text{217}$$

$$\frac{-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\tan(a+b\log(cx^n))}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}\right)}{b}-\frac{2}{b\sqrt{\tan(a+b}}}{n}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(a+b\log(cx^n))}}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}}{2\sqrt{2}}\right)+\ldots}{b}}{n}$$

$\downarrow$ 25

$$-\frac{\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(a+b\log(cx^n))}}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}}{2\sqrt{2}}\right)+\ldots}{b}}{n}$$

$\downarrow$ 27

$$-\frac{\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(a+b\log(cx^n))}}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}{\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}\ldots}{b}}{n}$$

$\downarrow$ 1103

$$-\frac{\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(a+b\log\ldots\right.}{\ldots}\right.\ldots}{b}}{n}$$

input
```
Int[1/(x*Tan[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
((-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/Sqrt[2]) + ArcTan
[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sq
rt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]) - Log[1 + S
qrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]))/2
))/b - 2/(b*Sqrt[Tan[a + b*Log[c*x^n]]]))/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.95

method	result
derivativedivides	$-\frac{\frac{\sqrt{2}\left(\ln\left(\frac{\tan(a+b\ln(c\,x^n))-\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}+1}{\tan(a+b\ln(c\,x^n))+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(a+b\ln(c}{4}}{nb}$
default	$-\frac{\frac{\sqrt{2}\left(\ln\left(\frac{\tan(a+b\ln(c\,x^n))-\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}+1}{\tan(a+b\ln(c\,x^n))+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(a+b\ln(c}{4}}{nb}$

input
```
int(1/x/tan(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(-1/4*2^(1/2)*(ln((tan(a+b*ln(c*x^n))-2^(1/2)*tan(a+b*ln(c*x^n))^(1/
2)+1)/(tan(a+b*ln(c*x^n))+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2)+1))+2*arctan(1+
2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+2^(1/2)*tan(a+b*ln(c*x^n))^(
1/2)))-2/tan(a+b*ln(c*x^n))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. 2(128) = 256.

Time = 0.09 (sec) , antiderivative size = 506, normalized size of antiderivative = 3.44

$$\int \frac{1}{x\tan^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/tan(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*arctan(sqrt(2)*sqrt(sin(2*b*n*log(x) + 2*b*log(c) + 2*a)/(
cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)) + 1)*sin(2*b*n*log(x) + 2*b*log
(c) + 2*a) + 2*sqrt(2)*arctan(sqrt(2)*sqrt(sin(2*b*n*log(x) + 2*b*log(c) +
 2*a)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)) - 1)*sin(2*b*n*log(x) +
2*b*log(c) + 2*a) - sqrt(2)*log(((sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c) +
2*a) + sqrt(2))*sqrt(sin(2*b*n*log(x) + 2*b*log(c) + 2*a)/(cos(2*b*n*log(x
) + 2*b*log(c) + 2*a) + 1)) + cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + sin(2
*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a)
 + 1))*sin(2*b*n*log(x) + 2*b*log(c) + 2*a) + sqrt(2)*log(-((sqrt(2)*cos(2
*b*n*log(x) + 2*b*log(c) + 2*a) + sqrt(2))*sqrt(sin(2*b*n*log(x) + 2*b*log
(c) + 2*a)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)) - cos(2*b*n*log(x)
+ 2*b*log(c) + 2*a) - sin(2*b*n*log(x) + 2*b*log(c) + 2*a) - 1)/(cos(2*b*n
*log(x) + 2*b*log(c) + 2*a) + 1))*sin(2*b*n*log(x) + 2*b*log(c) + 2*a) + 8
*sqrt(sin(2*b*n*log(x) + 2*b*log(c) + 2*a)/(cos(2*b*n*log(x) + 2*b*log(c)
+ 2*a) + 1))*(cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1))/(b*n*sin(2*b*n*lo
g(x) + 2*b*log(c) + 2*a))
```

Sympy [F]

$$\int \frac{1}{x\tan^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{1}{x\tan^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx$$

input
```
integrate(1/x/tan(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Integral(1/(x*tan(a + b*log(c*x**n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\tan^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{1}{x\tan\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/tan(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*tan(b*log(c*x^n) + a)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\tan^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/x/tan(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 20.85 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.54

$$\int \frac{1}{x\tan^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{(-1)^{1/4}\,\text{atanh}\left((-1)^{1/4}\,\sqrt{\tan(a+b\,\ln(c\,x^n))}\right)}{b\,n} - \frac{(-1)^{1/4}\,\text{atan}\left((-1)^{1/4}\,\sqrt{\tan(a+b\,\ln(c\,x^n))}\right)}{b\,n} - \frac{2}{b\,n\,\sqrt{\tan(a+b\,\ln(c\,x^n))}}$$

input
```
int(1/(x*tan(a + b*log(c*x^n))^(3/2)),x)
```

output
```
((-1)^(1/4)*atanh((-1)^(1/4)*tan(a + b*log(c*x^n))^(1/2)))/(b*n) - ((-1)^(
1/4)*atan((-1)^(1/4)*tan(a + b*log(c*x^n))^(1/2)))/(b*n) - 2/(b*n*tan(a +
b*log(c*x^n))^(1/2))
```

Reduce [F]

$$\int \frac{1}{x\tan^{\frac{3}{2}}(a+b\log(cx^n))}\,dx=\int \frac{\sqrt{\tan(\log(x^n c)\,b+a)}}{\tan(\log(x^n c)\,b+a)^2 x}dx$$

input
```
int(1/x/tan(a+b*log(c*x^n))^(3/2),x)
```

output
```
int(sqrt(tan(log(x**n*c)*b + a))/(tan(log(x**n*c)*b + a)**2*x),x)
```

3.185 $\int \frac{1}{x\tan^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 150

$$\int \frac{1}{x\tan^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}}{1+\tan(a+b\log(cx^n))}\right)}{\sqrt{2}bn} - \frac{2}{3bn\tan^{\frac{3}{2}}(a+b\log(cx^n))}$$

output
```
-1/2*arctan(-1+2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n-1/2*arctan(1+
2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n-1/2*arctanh(2^(1/2)*tan(a+b*
ln(c*x^n))^(1/2)/(1+tan(a+b*ln(c*x^n))))*2^(1/2)/b/n-2/3/b/n/tan(a+b*ln(c*
x^n))^(3/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.73

$$\int \frac{1}{x\tan^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$$
$$= \frac{-2+3\arctan\left(\sqrt[4]{-\tan^2(a+b\log(cx^n))}\right)\left(-\tan^2(a+b\log(cx^n))\right)^{3/4}+3\text{arctanh}\left(\sqrt[4]{-\tan^2(a+b\log\right.}{3bn\tan^{\frac{3}{2}}(a+b\log(cx^n))}$$

input
```
Integrate[1/(x*Tan[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
(-2 + 3*ArcTan[(-Tan[a + b*Log[c*x^n]]^2)^(1/4)]*(-Tan[a + b*Log[c*x^n]]^2
)^(3/4) + 3*ArcTanh[(-Tan[a + b*Log[c*x^n]]^2)^(1/4)]*(-Tan[a + b*Log[c*x^
n]]^2)^(3/4))/(3*b*n*Tan[a + b*Log[c*x^n]]^(3/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.29, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {3039, 3042, 3955, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\tan^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$$

$$\Big\downarrow \text{ 3039}$$

$$\frac{\int \frac{1}{\tan^{\frac{5}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{1}{\tan(a+b\log(cx^n))^{5/2}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 3955}$$

$$\frac{-\int \frac{1}{\sqrt{\tan(a+b\log(cx^n))}}d\log(cx^n)-\frac{2}{3b\tan^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$\Big\downarrow$ 3042

$$\frac{-\int \frac{1}{\sqrt{\tan(a+b\log(cx^n))}}d\log(cx^n)-\frac{2}{3b\tan^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$\Big\downarrow$ 3957

$$\frac{-\frac{\int \frac{1}{\sqrt{\tan(a+b\log(cx^n))}\left(\tan^2(a+b\log(cx^n))+1\right)}d\tan(a+b\log(cx^n))}{b}-\frac{2}{3b\tan^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$\Big\downarrow$ 266

$$\frac{-\frac{2\int \frac{1}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}}{b}-\frac{2}{3b\tan^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$\Big\downarrow$ 755

$$\frac{-\frac{2\left(\frac{1}{2}\int \frac{1-\tan(a+b\log(cx^n))}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}+\frac{1}{2}\int \frac{\tan(a+b\log(cx^n))+1}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}\right)}{b}-\frac{2}{3b\tan^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$\Big\downarrow$ 1476

$$\frac{-\frac{2\left(\frac{1}{2}\int \frac{1-\tan(a+b\log(cx^n))}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}+\frac{1}{2}\int \frac{}{\tan(a+b\log(cx}\right.\right.}{b}}{n}$$

$\Big\downarrow$ 1082

$$\frac{-\frac{2\left(\frac{1}{2}\int \frac{1-\tan(a+b\log(cx^n))}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}+\frac{1}{2}\left(\frac{\int \frac{1}{-\tan(a+b\log(cx^n))-1}d\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\tan(a+b\log(cx^n))-1}d\left(\sqrt{2}\sqrt{\tan(\right.}{\sqrt{2}}\right.\right.}{b}}{n}$$

$\Big\downarrow$ 217

$$\frac{-\frac{2\left(\frac{1}{2}\int \frac{1-\tan(a+b\log(cx^n))}{\tan^2(a+b\log(cx^n))+1}d\sqrt{\tan(a+b\log(cx^n))}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}}\right)\right)}{b}-\frac{2}{3b\tan^{\frac{3}{2}}(a+b}}{n}$$

$\Big\downarrow$ 1479

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\tan(a+b\log(cx^n))}}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}}{2\sqrt{2}}\right)}{b}}{n}$$

$\downarrow$ 25

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(a+b\log(cx^n))}}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\right.}{b}}{n}$$

$\downarrow$ 27

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\tan(a+b\log(cx^n))}}{\tan(a+b\log(cx^n))-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}{\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1}d\sqrt{\tan(a+b\log(cx^n))}\right)}{b}}{n}$$

$\downarrow$ 1103

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\tan(a+b\log(cx^n))+\sqrt{2}\sqrt{\tan(a+b\log(cx^n))}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(a+b\log\right.}{}\right.}{b}}{n}$$

input
```
Int[1/(x*Tan[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
((-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/Sqrt[2]) + ArcTan
[1 + Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[
2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/Sqrt[2] + Log[1 +
Sqrt[2]*Sqrt[Tan[a + b*Log[c*x^n]]] + Tan[a + b*Log[c*x^n]]]/(2*Sqrt[2]))/
2))/b - 2/(3*b*Tan[a + b*Log[c*x^n]]^(3/2)))/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

method	result
derivativedivides	$-\frac{\frac{\sqrt{2}\left(\ln\left(\frac{\tan(a+b\ln(c\,x^n))+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}+1}{\tan(a+b\ln(c\,x^n))-\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\right.}{4}}{nb}$
default	$-\frac{\frac{\sqrt{2}\left(\ln\left(\frac{\tan(a+b\ln(c\,x^n))+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}+1}{\tan(a+b\ln(c\,x^n))-\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\,x^n))}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\tan(a+b\ln(c\right.}{4}}{nb}$

input
```
int(1/x/tan(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(-1/4*2^(1/2)*(ln((tan(a+b*ln(c*x^n))+2^(1/2)*tan(a+b*ln(c*x^n))^(1/
2)+1)/(tan(a+b*ln(c*x^n))-2^(1/2)*tan(a+b*ln(c*x^n))^(1/2)+1))+2*arctan(1+
2^(1/2)*tan(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+2^(1/2)*tan(a+b*ln(c*x^n))^(
1/2)))-2/3/tan(a+b*ln(c*x^n))^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 537 vs. 2(128) = 256.

Time = 0.09 (sec) , antiderivative size = 537, normalized size of antiderivative = 3.58

$$\int \frac{1}{x\tan^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/tan(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(6*(sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) - sqrt(2))*arctan(s
qrt(2)*sqrt(sin(2*b*n*log(x) + 2*b*log(c) + 2*a)/(cos(2*b*n*log(x) + 2*b*l
og(c) + 2*a) + 1)) + 1) + 6*(sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c) + 2*a)
- sqrt(2))*arctan(sqrt(2)*sqrt(sin(2*b*n*log(x) + 2*b*log(c) + 2*a)/(cos(2
*b*n*log(x) + 2*b*log(c) + 2*a) + 1)) - 1) + 3*(sqrt(2)*cos(2*b*n*log(x) +
 2*b*log(c) + 2*a) - sqrt(2))*log(((sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c)
+ 2*a) + sqrt(2))*sqrt(sin(2*b*n*log(x) + 2*b*log(c) + 2*a)/(cos(2*b*n*log
(x) + 2*b*log(c) + 2*a) + 1)) + cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + sin
(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*
a) + 1)) - 3*(sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) - sqrt(2))*log(
-((sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + sqrt(2))*sqrt(sin(2*b*n*
log(x) + 2*b*log(c) + 2*a)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)) - c
os(2*b*n*log(x) + 2*b*log(c) + 2*a) - sin(2*b*n*log(x) + 2*b*log(c) + 2*a)
 - 1)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)) - 8*sqrt(sin(2*b*n*log(x
) + 2*b*log(c) + 2*a)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1))*(cos(2*b
*n*log(x) + 2*b*log(c) + 2*a) + 1))/(b*n*cos(2*b*n*log(x) + 2*b*log(c) + 2
*a) - b*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\tan^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/x/tan(a+b*ln(c*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x\tan^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\tan(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/tan(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*tan(b*log(c*x^n) + a)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\tan^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/x/tan(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 22.56 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.52

$$\int \frac{1}{x\tan^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = -\frac{2}{3\,b\,n\,\tan\left(a+b\,\ln\left(c\,x^n\right)\right)^{3/2}} + \frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\tan\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)\,1\mathrm{i}}{b\,n} + \frac{(-1)^{1/4}\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\tan\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)\,1\mathrm{i}}{b\,n}$$

input
```
int(1/(x*tan(a + b*log(c*x^n))^(5/2)),x)
```

output
```
((-1)^(1/4)*atan((-1)^(1/4)*tan(a + b*log(c*x^n))^(1/2))*1i)/(b*n) - 2/(3*
b*n*tan(a + b*log(c*x^n))^(3/2)) + ((-1)^(1/4)*atanh((-1)^(1/4)*tan(a + b*
log(c*x^n))^(1/2))*1i)/(b*n)
```

Reduce [F]

$$\int \frac{1}{x\tan^{\frac{5}{2}}(a+b\log(cx^n))}\,dx=\int \frac{\sqrt{\tan(\log(x^n c)\,b+a)}}{\tan(\log(x^n c)\,b+a)^3 x}dx$$

input
```
int(1/x/tan(a+b*log(c*x^n))^(5/2),x)
```

output
```
int(sqrt(tan(log(x**n*c)*b + a))/(tan(log(x**n*c)*b + a)**3*x),x)
```

3.186 $\int x^3 \cot(a + i\log(x))\, dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int x^3 \cot(a + i\log(x))\, dx = -ie^{2ia}x^2 - \frac{ix^4}{4} - ie^{4ia}\log\left(e^{2ia} - x^2\right)$$

output
```
-I*exp(2*I*a)*x^2-1/4*I*x^4-I*exp(4*I*a)*ln(exp(2*I*a)-x^2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 137 vs. 2(49) = 98.

Time = 0.03 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.80

$$\begin{aligned}\int x^3 \cot(a + i\log(x))\, dx = & -\frac{ix^4}{4} - ix^2\cos(2a) - \arctan\left(\frac{(-1 + x^2)\cos(a)}{-\sin(a) - x^2\sin(a)}\right)\cos(4a) \\ & - \frac{1}{2}i\cos(4a)\log\left(1 + x^4 - 2x^2\cos(2a)\right) + x^2\sin(2a) \\ & - i\arctan\left(\frac{(-1 + x^2)\cos(a)}{-\sin(a) - x^2\sin(a)}\right)\sin(4a) \\ & + \frac{1}{2}\log\left(1 + x^4 - 2x^2\cos(2a)\right)\sin(4a)\end{aligned}$$

input
```
Integrate[x^3*Cot[a + I*Log[x]],x]
```

output
```
(-1/4*I)*x^4 - I*x^2*Cos[2*a] - ArcTan[((-1 + x^2)*Cos[a])/(-Sin[a] - x^2*
Sin[a])]*Cos[4*a] - (I/2)*Cos[4*a]*Log[1 + x^4 - 2*x^2*Cos[2*a]] + x^2*Sin
[2*a] - I*ArcTan[((-1 + x^2)*Cos[a])/(-Sin[a] - x^2*Sin[a])]*Sin[4*a] + (L
og[1 + x^4 - 2*x^2*Cos[2*a]]*Sin[4*a])/2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5007, 947, 354, 26, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cot(a+i\log(x))\,dx$$

$$\Big\downarrow \text{5007}$$

$$\int \frac{x^3\left(-\frac{ie^{2ia}}{x^2}-i\right)}{1-\frac{e^{2ia}}{x^2}}dx$$

$$\Big\downarrow \text{947}$$

$$\int \frac{x^3\left(-ie^{2ia}-ix^2\right)}{x^2-e^{2ia}}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{ix^2\left(x^2+e^{2ia}\right)}{e^{2ia}-x^2}dx^2$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}i\int \frac{x^2\left(x^2+e^{2ia}\right)}{e^{2ia}-x^2}dx^2$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}i\int \left(-x^2-2e^{2ia}+\frac{2e^{4ia}}{e^{2ia}-x^2}\right)dx^2$$

$$\downarrow \text{2009}$$

$$\frac{1}{2}i\left(-2e^{2ia}x^2 - 2e^{4ia}\log\left(-x^2 + e^{2ia}\right) - \frac{x^4}{2}\right)$$

input
```
Int[x^3*Cot[a + I*Log[x]],x]
```

output
```
(I/2)*(-2*E^((2*I)*a)*x^2 - x^4/2 - 2*E^((4*I)*a)*Log[E^((2*I)*a) - x^2])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 947
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

method	result	size
risch	$-i\mathrm{e}^{2ia}x^2-\frac{ix^4}{4}-i\mathrm{e}^{4ia}\ln\left(\mathrm{e}^{2ia}-x^2\right)$	39

input
```
int(x^3*cot(a+I*ln(x)),x,method=_RETURNVERBOSE)
```

output
```
-I*exp(2*I*a)*x^2-1/4*I*x^4-I*exp(4*I*a)*ln(exp(2*I*a)-x^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.65

$$\int x^3\cot(a+i\log(x))\,dx=-\frac{1}{4}i\,x^4-i\,x^2e^{(2i\,a)}-i\,e^{(4i\,a)}\log\left(x^2-e^{(2i\,a)}\right)$$

input
```
integrate(x^3*cot(a+I*log(x)),x, algorithm="fricas")
```

output
```
-1/4*I*x^4 - I*x^2*e^(2*I*a) - I*e^(4*I*a)*log(x^2 - e^(2*I*a))
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int x^3 \cot(a + i\log(x))\, dx = -\frac{ix^4}{4} - ix^2 e^{2ia} - ie^{4ia} \log\left(x^2 - e^{2ia}\right)$$

input
```
integrate(x**3*cot(a+I*ln(x)),x)
```

output
```
-I*x**4/4 - I*x**2*exp(2*I*a) - I*exp(4*I*a)*log(x**2 - exp(2*I*a))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(32) = 64.

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.67

$$\begin{aligned}\int x^3 \cot(a + i\log(x))\, dx = & -\frac{1}{4} i\, x^4 - x^2 (i\, \cos(2\,a) - \sin(2\,a)) \\ & + (\cos(4\,a) + i\, \sin(4\,a)) \arctan(\sin(a), x + \cos(a)) \\ & - (\cos(4\,a) + i\, \sin(4\,a)) \arctan(\sin(a), x - \cos(a)) \\ & - \frac{1}{2} (i\, \cos(4\,a) - \sin(4\,a)) \log\left(x^2 + 2\,x\cos(a) + \cos(a)^2 \right. \\ & \left. + \sin(a)^2\right) - \frac{1}{2} (i\, \cos(4\,a) - \sin(4\,a)) \log\left(x^2 - 2\,x\cos(a) \right. \\ & \left. + \cos(a)^2 + \sin(a)^2\right)\end{aligned}$$

input
```
integrate(x^3*cot(a+I*log(x)),x, algorithm="maxima")
```

output
```
-1/4*I*x^4 - x^2*(I*cos(2*a) - sin(2*a)) + (cos(4*a) + I*sin(4*a))*arctan2
(sin(a), x + cos(a)) - (cos(4*a) + I*sin(4*a))*arctan2(sin(a), x - cos(a))
 - 1/2*(I*cos(4*a) - sin(4*a))*log(x^2 + 2*x*cos(a) + cos(a)^2 + sin(a)^2)
 - 1/2*(I*cos(4*a) - sin(4*a))*log(x^2 - 2*x*cos(a) + cos(a)^2 + sin(a)^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int x^3 \cot(a + i\log(x))\,dx = -\frac{1}{4}i\,x^4 - i\,x^2e^{(2i\,a)} + \frac{1}{2}\,\pi e^{(4i\,a)} - i\,e^{(4i\,a)}\log\left(x + e^{(i\,a)}\right) - i\,e^{(4i\,a)}\log\left(-x + e^{(i\,a)}\right)$$

input
```
integrate(x^3*cot(a+I*log(x)),x, algorithm="giac")
```

output
```
-1/4*I*x^4 - I*x^2*e^(2*I*a) + 1/2*pi*e^(4*I*a) - I*e^(4*I*a)*log(x + e^(I
*a)) - I*e^(4*I*a)*log(-x + e^(I*a))
```

Mupad [B] (verification not implemented)

Time = 19.99 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int x^3 \cot(a + i\log(x))\,dx = -x^2\,\mathrm{e}^{a\,2\mathrm{i}}\,1\mathrm{i} - \ln\left(x^2 - \mathrm{e}^{a\,2\mathrm{i}}\right)\,\mathrm{e}^{a\,4\mathrm{i}}\,1\mathrm{i} - \frac{x^4\,1\mathrm{i}}{4}$$

input
```
int(x^3*cot(a + log(x)*1i),x)
```

output
```
- x^2*exp(a*2i)*1i - log(x^2 - exp(a*2i))*exp(a*4i)*1i - (x^4*1i)/4
```

Reduce [F]

$$\int x^3 \cot(a + i\log(x))\,dx = \int \cot\left(\log(x)\,i + a\right)x^3dx$$

input
```
int(x^3*cot(a+I*log(x)),x)
```

output
```
int(cot(log(x)*i + a)*x**3,x)
```

3.187 $\int x^2 \cot(a + i\log(x))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int x^2 \cot(a + i\log(x))\,dx = -2ie^{2ia}x - \frac{ix^3}{3} + 2ie^{3ia}\text{arctanh}\left(e^{-ia}x\right)$$

output
```
-2*I*exp(2*I*a)*x-1/3*I*x^3+2*I*exp(3*I*a)*arctanh(x/exp(I*a))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int x^2 \cot(a + i\log(x))\,dx = -\frac{ix^3}{3} - 2ix\cos(2a) + 2i\text{arctanh}(x\cos(a) - ix\sin(a))\cos(3a) + 2x\sin(2a) - 2\text{arctanh}(x\cos(a) - ix\sin(a))\sin(3a)$$

input
```
Integrate[x^2*Cot[a + I*Log[x]],x]
```

output
```
(-1/3*I)*x^3 - (2*I)*x*Cos[2*a] + (2*I)*ArcTanh[x*Cos[a] - I*x*Sin[a]]*Cos
[3*a] + 2*x*Sin[2*a] - 2*ArcTanh[x*Cos[a] - I*x*Sin[a]]*Sin[3*a]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {5007, 947, 363, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot(a + i\log(x))\,dx$$

$$\Big\downarrow \text{ 5007}$$

$$\int \frac{x^2\left(-\frac{ie^{2ia}}{x^2}-i\right)}{1-\frac{e^{2ia}}{x^2}}dx$$

$$\Big\downarrow \text{ 947}$$

$$\int \frac{x^2\left(-ie^{2ia}-ix^2\right)}{x^2-e^{2ia}}dx$$

$$\Big\downarrow \text{ 363}$$

$$-2ie^{2ia}\int -\frac{x^2}{e^{2ia}-x^2}dx-\frac{ix^3}{3}$$

$$\Big\downarrow \text{ 25}$$

$$2ie^{2ia}\int \frac{x^2}{e^{2ia}-x^2}dx-\frac{ix^3}{3}$$

$$\Big\downarrow \text{ 262}$$

$$2ie^{2ia}\left(-x+e^{2ia}\int \frac{1}{e^{2ia}-x^2}dx\right)-\frac{ix^3}{3}$$

$$\Big\downarrow \text{ 219}$$

$$2ie^{2ia}\left(-x+e^{ia}\text{arctanh}\left(e^{-ia}x\right)\right)-\frac{ix^3}{3}$$

input
```
Int[x^2*Cot[a + I*Log[x]],x]
```

output
```
(-1/3*I)*x^3 + (2*I)*E^((2*I)*a)*(-x + E^(I*a)*ArcTanh[x/E^(I*a)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 947
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{i x^{3}}{3}-2 i \mathrm{e}^{2 i a} x+2 i \,\mathrm{arctanh}\left(x \,\mathrm{e}^{-i a}\right) \mathrm{e}^{3 i a}$	33

input
```
int(x^2*cot(a+I*ln(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I*x^3-2*I*exp(2*I*a)*x+2*I*arctanh(x*exp(-I*a))*exp(3*I*a)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(26) = 52.

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.81

$$\begin{aligned}\int x^2\cot(a+i\log(x))\,dx =& -\frac{1}{3}i\,x^3-2i\,xe^{(2i\,a)}\\ &-\sqrt{-e^{(6i\,a)}}\log\left(\left(xe^{(2i\,a)}+i\,\sqrt{-e^{(6i\,a)}}\right)e^{(-2i\,a)}\right)\\ &+\sqrt{-e^{(6i\,a)}}\log\left(\left(xe^{(2i\,a)}-i\,\sqrt{-e^{(6i\,a)}}\right)e^{(-2i\,a)}\right)\end{aligned}$$

input
```
integrate(x^2*cot(a+I*log(x)),x, algorithm="fricas")
```

output
```
-1/3*I*x^3 - 2*I*x*e^(2*I*a) - sqrt(-e^(6*I*a))*log((x*e^(2*I*a) + I*sqrt(
-e^(6*I*a)))*e^(-2*I*a)) + sqrt(-e^(6*I*a))*log((x*e^(2*I*a) - I*sqrt(-e^(
6*I*a)))*e^(-2*I*a))
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.47

$$\int x^2\cot(a+i\log(x))\,dx=-\frac{ix^3}{3}-2ixe^{2ia}-\left(i\log\left(xe^{2ia}-e^{3ia}\right)-i\log\left(xe^{2ia}+e^{3ia}\right)\right)e^{3ia}$$

input
```
integrate(x**2*cot(a+I*ln(x)),x)
```

output
```
-I*x**3/3 - 2*I*x*exp(2*I*a) - (I*log(x*exp(2*I*a) - exp(3*I*a)) - I*log(x
*exp(2*I*a) + exp(3*I*a)))*exp(3*I*a)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(26) = 52.

Time = 0.04 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.93

$$\begin{aligned}\int x^2\cot(a+i\log(x))\,dx=&-\frac{1}{3}i\,x^3+2\,x(-i\,\cos(2\,a)+\sin(2\,a))\\&-(\cos(3\,a)+i\,\sin(3\,a))\arctan(\sin(a),x+\cos(a))\\&-(\cos(3\,a)+i\,\sin(3\,a))\arctan(\sin(a),x-\cos(a))\\&+\frac{1}{2}(i\,\cos(3\,a)-\sin(3\,a))\log\left(x^2+2\,x\cos(a)+\cos(a)^2\right.\\&\left.+\sin(a)^2\right)+\frac{1}{2}(-i\,\cos(3\,a)+\sin(3\,a))\log\left(x^2\right.\\&\left.-2\,x\cos(a)+\cos(a)^2+\sin(a)^2\right)\end{aligned}$$

input
```
integrate(x^2*cot(a+I*log(x)),x, algorithm="maxima")
```

output
```
-1/3*I*x^3 + 2*x*(-I*cos(2*a) + sin(2*a)) - (cos(3*a) + I*sin(3*a))*arctan
2(sin(a), x + cos(a)) - (cos(3*a) + I*sin(3*a))*arctan2(sin(a), x - cos(a)
) + 1/2*(I*cos(3*a) - sin(3*a))*log(x^2 + 2*x*cos(a) + cos(a)^2 + sin(a)^2
) + 1/2*(-I*cos(3*a) + sin(3*a))*log(x^2 - 2*x*cos(a) + cos(a)^2 + sin(a)^
2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int x^2 \cot(a + i\log(x))\,dx = -\frac{1}{3}i\,x^3 - 2i\,xe^{(2i\,a)} + i\,e^{(3i\,a)}\log\left(x + e^{(i\,a)}\right) - i\,e^{(3i\,a)}\log\left(-x + e^{(i\,a)}\right)$$

input
```
integrate(x^2*cot(a+I*log(x)),x, algorithm="giac")
```

output
```
-1/3*I*x^3 - 2*I*x*e^(2*I*a) + I*e^(3*I*a)*log(x + e^(I*a)) - I*e^(3*I*a)*
log(-x + e^(I*a))
```

Mupad [B] (verification not implemented)

Time = 19.42 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int x^2 \cot(a + i\log(x))\,dx = -\mathrm{atan}\left(\frac{x}{\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}}}\right)\left(-\mathrm{e}^{a\,2\mathrm{i}}\right)^{3/2}2\mathrm{i} - \frac{x^3\,1\mathrm{i}}{3} - x\,\mathrm{e}^{a\,2\mathrm{i}}\,2\mathrm{i}$$

input
```
int(x^2*cot(a + log(x)*1i),x)
```

output
```
- atan(x/(-exp(a*2i))^(1/2))*(-exp(a*2i))^(3/2)*2i - (x^3*1i)/3 - x*exp(a*
2i)*2i
```

Reduce [F]

$$\int x^2 \cot(a + i\log(x))\,dx = \int \cot(\log(x)\,i + a)\,x^2dx$$

input
```
int(x^2*cot(a+I*log(x)),x)
```

output
```
int(cot(log(x)*i + a)*x**2,x)
```

3.188 $\int x\cot(a+i\log(x))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 35

$$\int x\cot(a+i\log(x))\,dx=-\frac{ix^2}{2}-ie^{2ia}\log\left(e^{2ia}-x^2\right)$$

output
```
-1/2*I*x^2-I*exp(2*I*a)*ln(exp(2*I*a)-x^2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 118 vs. 2(35) = 70.

Time = 0.02 (sec) , antiderivative size = 118, normalized size of antiderivative = 3.37

$$\begin{aligned}\int x\cot(a+i\log(x))\,dx=&-\frac{ix^2}{2}-\arctan\left(\frac{(-1+x^2)\cos(a)}{-\sin(a)-x^2\sin(a)}\right)\cos(2a)\\&-\frac{1}{2}i\cos(2a)\log\left(1+x^4-2x^2\cos(2a)\right)\\&-i\arctan\left(\frac{(-1+x^2)\cos(a)}{-\sin(a)-x^2\sin(a)}\right)\sin(2a)\\&+\frac{1}{2}\log\left(1+x^4-2x^2\cos(2a)\right)\sin(2a)\end{aligned}$$

input
```
Integrate[x*Cot[a + I*Log[x]],x]
```

output
```
(-1/2*I)*x^2 - ArcTan[((-1 + x^2)*Cos[a])/(-Sin[a] - x^2*Sin[a])]*Cos[2*a]
 - (I/2)*Cos[2*a]*Log[1 + x^4 - 2*x^2*Cos[2*a]] - I*ArcTan[((-1 + x^2)*Cos
[a])/(-Sin[a] - x^2*Sin[a])]*Sin[2*a] + (Log[1 + x^4 - 2*x^2*Cos[2*a]]*Sin
[2*a])/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {5007, 947, 353, 26, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cot(a+i\log(x))\,dx$$

$$\Big\downarrow \; 5007$$

$$\int \frac{x\left(-\frac{ie^{2ia}}{x^2}-i\right)}{1-\frac{e^{2ia}}{x^2}}dx$$

$$\Big\downarrow \; 947$$

$$\int \frac{x\left(-ie^{2ia}-ix^2\right)}{x^2-e^{2ia}}dx$$

$$\Big\downarrow \; 353$$

$$\frac{1}{2}\int \frac{i\left(x^2+e^{2ia}\right)}{e^{2ia}-x^2}dx^2$$

$$\Big\downarrow \; 26$$

$$\frac{1}{2}i\int \frac{x^2+e^{2ia}}{e^{2ia}-x^2}dx^2$$

$$\Big\downarrow \; 49$$

$$\frac{1}{2}i\int \left(\frac{2e^{2ia}}{e^{2ia}-x^2}-1\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}i\left(-x^2-2e^{2ia}\log\left(-x^2+e^{2ia}\right)\right)$$

input
```
Int[x*Cot[a + I*Log[x]],x]
```

output
```
(I/2)*(-x^2 - 2*E^((2*I)*a)*Log[E^((2*I)*a) - x^2])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 947
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{i x^2}{2}-i \mathrm{e}^{2 i a} \ln \left(\mathrm{e}^{2 i a}-x^2\right)$	28

input
```
int(x*cot(a+I*ln(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*x^2-I*exp(2*I*a)*ln(exp(2*I*a)-x^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int x \cot (a+i \log (x)) \, dx=-\frac{1}{2} i \, x^2-i \, e^{(2 i \, a)} \log \left(x^2-e^{(2 i \, a)}\right)$$

input
```
integrate(x*cot(a+I*log(x)),x, algorithm="fricas")
```

output
```
-1/2*I*x^2 - I*e^(2*I*a)*log(x^2 - e^(2*I*a))
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x \cot (a+i \log (x)) \, dx=-\frac{i x^2}{2}-i e^{2 i a} \log \left(x^2-e^{2 i a}\right)$$

input
```
integrate(x*cot(a+I*ln(x)),x)
```

output
```
-I*x**2/2 - I*exp(2*I*a)*log(x**2 - exp(2*I*a))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(23) = 46$.

Time = 0.04 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.11

$$\int x\cot(a+i\log(x))\,dx = -\frac{1}{2}i\,x^2 + (\cos(2\,a) + i\,\sin(2\,a))\arctan(\sin(a)\,, x+\cos(a)) - (\cos(2\,a) + i\,\sin(2\,a))\arctan(\sin(a)\,, x-\cos(a)) + \frac{1}{2}(-i\,\cos(2\,a) + \sin(2\,a))\log\left(x^2 + 2\,x\cos(a) + \cos(a)^2 + \sin(a)^2\right) + \frac{1}{2}(-i\,\cos(2\,a) + \sin(2\,a))\log\left(x^2 - 2\,x\cos(a) + \cos(a)^2 + \sin(a)^2\right)$$

input
```
integrate(x*cot(a+I*log(x)),x, algorithm="maxima")
```

output
```
-1/2*I*x^2 + (cos(2*a) + I*sin(2*a))*arctan2(sin(a), x + cos(a)) - (cos(2*
a) + I*sin(2*a))*arctan2(sin(a), x - cos(a)) + 1/2*(-I*cos(2*a) + sin(2*a)
)*log(x^2 + 2*x*cos(a) + cos(a)^2 + sin(a)^2) + 1/2*(-I*cos(2*a) + sin(2*a
))*log(x^2 - 2*x*cos(a) + cos(a)^2 + sin(a)^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int x\cot(a+i\log(x))\,dx = -\frac{1}{2}i\,x^2 + \frac{1}{2}\,\pi e^{(2i\,a)} - i\,e^{(2i\,a)}\log\left(x + e^{(i\,a)}\right) - i\,e^{(2i\,a)}\log\left(-x + e^{(i\,a)}\right)$$

input
```
integrate(x*cot(a+I*log(x)),x, algorithm="giac")
```

output
```
-1/2*I*x^2 + 1/2*pi*e^(2*I*a) - I*e^(2*I*a)*log(x + e^(I*a)) - I*e^(2*I*a)
*log(-x + e^(I*a))
```

Mupad [B] (verification not implemented)

Time = 19.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int x\cot(a+i\log(x))\,dx = -\ln\left(x^2-\mathrm{e}^{a\,2\mathrm{i}}\right)\,\mathrm{e}^{a\,2\mathrm{i}}\,1\mathrm{i}-\frac{x^2\,1\mathrm{i}}{2}$$

input
```
int(x*cot(a + log(x)*1i),x)
```

output
```
- log(x^2 - exp(a*2i))*exp(a*2i)*1i - (x^2*1i)/2
```

Reduce [F]

$$\int x\cot(a+i\log(x))\,dx = \int \cot(\log(x)\,i+a)\,x dx$$

input
```
int(x*cot(a+I*log(x)),x)
```

output
```
int(cot(log(x)*i + a)*x,x)
```

3.189 $\int \cot(a+i\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 27

$$\int \cot(a+i\log(x))\,dx = -ix + 2ie^{ia}\text{arctanh}\left(e^{-ia}x\right)$$

output
```
-I*x+2*I*exp(I*a)*arctanh(x/exp(I*a))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \cot(a+i\log(x))\,dx = -ix + 2i\text{arctanh}(x\cos(a)-ix\sin(a))\cos(a) - 2\text{arctanh}(x\cos(a)-ix\sin(a))\sin(a)$$

input
```
Integrate[Cot[a + I*Log[x]],x]
```

output
```
(-I)*x + (2*I)*ArcTanh[x*Cos[a] - I*x*Sin[a]]*Cos[a] - 2*ArcTanh[x*Cos[a]
- I*x*Sin[a]]*Sin[a]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5003, 898, 299, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(a+i\log(x))\,dx$$

$$\Big\downarrow \; 5003$$

$$\int \frac{-\frac{ie^{2ia}}{x^2}-i}{1-\frac{e^{2ia}}{x^2}}dx$$

$$\Big\downarrow \; 898$$

$$\int \frac{-ie^{2ia}-ix^2}{x^2-e^{2ia}}dx$$

$$\Big\downarrow \; 299$$

$$-2ie^{2ia}\int \frac{1}{x^2-e^{2ia}}dx-ix$$

$$\Big\downarrow \; 220$$

$$2ie^{ia}\text{arctanh}\left(e^{-ia}x\right)-ix$$

input
```
Int[Cot[a + I*Log[x]],x]
```

output
```
(-I)*x + (2*I)*E^(I*a)*ArcTanh[x/E^(I*a)]
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 898
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

rule 5003
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((-I - I*E^(
2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b,
d, p}, x]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
risch	$-ix + 2i \operatorname{arctanh}\left(x\, \mathrm{e}^{-ia}\right) \mathrm{e}^{ia}$	22

input
```
int(cot(a+I*ln(x)),x,method=_RETURNVERBOSE)
```

output
```
-I*x+2*I*arctanh(x*exp(-I*a))*exp(I*a)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(17) = 34.

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \cot(a+i\log(x))\,dx = -\sqrt{-e^{(2i\,a)}}\log\left(x+i\,\sqrt{-e^{(2i\,a)}}\right) + \sqrt{-e^{(2i\,a)}}\log\left(x-i\,\sqrt{-e^{(2i\,a)}}\right) - i\,x$$

input
```
integrate(cot(a+I*log(x)),x, algorithm="fricas")
```

output
```
-sqrt(-e^(2*I*a))*log(x + I*sqrt(-e^(2*I*a))) + sqrt(-e^(2*I*a))*log(x - I
*sqrt(-e^(2*I*a))) - I*x
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \cot(a+i\log(x))\,dx = -ix - \left(i\log\left(x-e^{ia}\right) - i\log\left(x+e^{ia}\right)\right)e^{ia}$$

input
```
integrate(cot(a+I*ln(x)),x)
```

output
```
-I*x - (I*log(x - exp(I*a)) - I*log(x + exp(I*a)))*exp(I*a)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(17) = 34.

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.48

$$\int \cot(a+i\log(x))\,dx = -(\cos(a)+i\sin(a))\arctan(\sin(a),x+\cos(a)) - (\cos(a)+i\sin(a))\arctan(\sin(a),x-\cos(a)) - \frac{1}{2}(-i\cos(a)+\sin(a))\log\left(x^2+2x\cos(a)+\cos(a)^2+\sin(a)^2\right) - \frac{1}{2}(i\cos(a)-\sin(a))\log\left(x^2-2x\cos(a)+\cos(a)^2+\sin(a)^2\right) - ix$$

input
```
integrate(cot(a+I*log(x)),x, algorithm="maxima")
```

output
```
-(cos(a) + I*sin(a))*arctan2(sin(a), x + cos(a)) - (cos(a) + I*sin(a))*arc
tan2(sin(a), x - cos(a)) - 1/2*(-I*cos(a) + sin(a))*log(x^2 + 2*x*cos(a) +
 cos(a)^2 + sin(a)^2) - 1/2*(I*cos(a) - sin(a))*log(x^2 - 2*x*cos(a) + cos
(a)^2 + sin(a)^2) - I*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \cot(a+i\log(x))\,dx = i\,e^{(i\,a)}\log\left(x+e^{(i\,a)}\right) - i\,e^{(i\,a)}\log\left(-x+e^{(i\,a)}\right) - i\,x$$

input
```
integrate(cot(a+I*log(x)),x, algorithm="giac")
```

output
```
I*e^(I*a)*log(x + e^(I*a)) - I*e^(I*a)*log(-x + e^(I*a)) - I*x
```

Mupad [B] (verification not implemented)

Time = 20.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \cot(a+i\log(x))\,dx = -x\,1\mathrm{i} + \operatorname{atan}\left(\frac{x}{\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}}}\right)\,\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}}\,2\mathrm{i}$$

input
```
int(cot(a + log(x)*1i),x)
```

output
```
atan(x/(-exp(a*2i))^(1/2))*(-exp(a*2i))^(1/2)*2i - x*1i
```

Reduce [F]

$$\int \cot(a+i\log(x))\,dx = \int \cot(\log(x)\,i + a)\,dx$$

input
```
int(cot(a+I*log(x)),x)
```

output
```
int(cot(log(x)*i + a),x)
```

3.190 $\int \frac{\cot(a+i\log(x))}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{\cot(a+i\log(x))}{x} \, dx = -i\log(\sin(a+i\log(x)))$$

output
```
-I*ln(sin(a+I*ln(x)))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\cot(a+i\log(x))}{x} \, dx = -i\log(\sin(a+i\log(x)))$$

input
```
Integrate[Cot[a + I*Log[x]]/x,x]
```

output
```
(-I)*Log[Sin[a + I*Log[x]]]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3039, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(a+i\log(x))}{x}\,dx$$

$$\Big\downarrow \text{3039}$$

$$\int \cot(a+i\log(x))d\log(x)$$

$$\Big\downarrow \text{3042}$$

$$\int -\tan\left(a+i\log(x)+\frac{\pi}{2}\right)d\log(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \tan\left(\frac{1}{2}(2a+\pi)+i\log(x)\right)d\log(x)$$

$$\Big\downarrow \text{3956}$$

$$-i\log(-\sin(a+i\log(x)))$$

input
```
Int[Cot[a + I*Log[x]]/x,x]
```

output
```
(-I)*Log[-Sin[a + I*Log[x]]]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

method	result	size
derivativedivides	$\frac{i \ln\left(\cot(a+i\ln(x))^{2}+1\right)}{2}$	17
default	$\frac{i \ln\left(\cot(a+i\ln(x))^{2}+1\right)}{2}$	17
risch	$-i\ln\left(x\right)-2a-i\ln\left(\frac{e^{2ia}}{x^{2}}-1\right)$	25
parallelrisch	$-\frac{i\left(2\ln(\tan(a+i\ln(x)))-\ln\left(\sec(a+i\ln(x))^{2}\right)\right)}{2}$	29
norman	$-i\ln\left(\tan\left(a+i\ln\left(x\right)\right)\right)+\frac{i\ln\left(1+\tan(a+i\ln(x))^{2}\right)}{2}$	30

input
```
int(cot(a+I*ln(x))/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*ln(cot(a+I*ln(x))^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\cot(a+i\log(x))}{x}\,dx = -i\,\log\left(x^2-e^{(2i\,a)}\right)+i\,\log\left(x\right)$$

input
```
integrate(cot(a+I*log(x))/x,x, algorithm="fricas")
```

output
```
-I*log(x^2 - e^(2*I*a)) + I*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{\cot(a+i\log(x))}{x}\,dx = i\log\left(x\right)-i\log\left(x^2-e^{2ia}\right)$$

input
```
integrate(cot(a+I*ln(x))/x,x)
```

output
```
I*log(x) - I*log(x**2 - exp(2*I*a))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\cot(a+i\log(x))}{x}\,dx = -i\,\log\left(\sin\left(a+i\,\log\left(x\right)\right)\right)$$

input
```
integrate(cot(a+I*log(x))/x,x, algorithm="maxima")
```

output
```
-I*log(sin(a + I*log(x)))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(10) = 20$.

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 5.36

$$\int \frac{\cot(a+i\log(x))}{x}\,dx =$$
$$-i\log\left(\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\left(\frac{\left(|x|^2+1\right)^2}{|x|^2}-\frac{\left(|x|^2-1\right)^2}{|x|^2}\right)\cos\left(\pi\mathrm{sgn}\left(x\right)+2\,a\right)+\frac{\left(|x|^2+1\right)^2}{|x|^2}+\frac{\left(|x|^2-1\right)^2}{|x|^2}}\right)$$

input
```
integrate(cot(a+I*log(x))/x,x, algorithm="giac")
```

output
```
-I*log(1/2*sqrt(1/2)*sqrt(((abs(x)^2 + 1)^2/abs(x)^2 - (abs(x)^2 - 1)^2/ab
s(x)^2)*cos(pi*sgn(x) + 2*a) + (abs(x)^2 + 1)^2/abs(x)^2 + (abs(x)^2 - 1)^
2/abs(x)^2))
```

Mupad [B] (verification not implemented)

Time = 19.52 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{\cot(a+i\log(x))}{x}\,dx = -\ln\left(x^2-\mathrm{e}^{a\,2\mathrm{i}}\right)\,1\mathrm{i}+\ln\left(x\right)\,1\mathrm{i}$$

input
```
int(cot(a + log(x)*1i)/x,x)
```

output
```
log(x)*1i - log(x^2 - exp(a*2i))*1i
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \frac{\cot(a + i\log(x))}{x}\,dx = i\left(\log\left(\tan\left(\frac{\log(x)\,i}{2}+\frac{a}{2}\right)^2+1\right)-\log\left(\tan\left(\frac{\log(x)\,i}{2}+\frac{a}{2}\right)\right)\right)$$

input
```
int(cot(a+I*log(x))/x,x)
```

output
```
i*(log(tan((log(x)*i + a)/2)**2 + 1) - log(tan((log(x)*i + a)/2)))
```

3.191 $\int \frac{\cot(a+i\log(x))}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{\cot(a+i\log(x))}{x^2}\,dx=-\frac{i}{x}+2ie^{-ia}\text{arctanh}\left(e^{-ia}x\right)$$

output
```
-I/x+2*I*arctanh(x/exp(I*a))/exp(I*a)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.52

$$\int \frac{\cot(a+i\log(x))}{x^2}\,dx=-\frac{i}{x}+2i\text{arctanh}(x\cos(a)-ix\sin(a))\cos(a)+2\text{arctanh}(x\cos(a)-ix\sin(a))\sin(a)$$

input
```
Integrate[Cot[a + I*Log[x]]/x^2,x]
```

output
```
(-I)/x + (2*I)*ArcTanh[x*Cos[a] - I*x*Sin[a]]*Cos[a] + 2*ArcTanh[x*Cos[a]
- I*x*Sin[a]]*Sin[a]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5007, 947, 359, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(a+i\log(x))}{x^2}\,dx$$

$$\Big\downarrow \text{ 5007}$$

$$\int \frac{-\frac{ie^{2ia}}{x^2}-i}{x^2\left(1-\frac{e^{2ia}}{x^2}\right)}dx$$

$$\Big\downarrow \text{ 947}$$

$$\int \frac{-ie^{2ia}-ix^2}{x^2\left(x^2-e^{2ia}\right)}dx$$

$$\Big\downarrow \text{ 359}$$

$$-2i\int \frac{1}{x^2-e^{2ia}}dx-\frac{i}{x}$$

$$\Big\downarrow \text{ 220}$$

$$2ie^{-ia}\text{arctanh}\left(e^{-ia}x\right)-\frac{i}{x}$$

input
```
Int[Cot[a + I*Log[x]]/x^2,x]
```

output
```
(-I)/x + ((2*I)*ArcTanh[x/E^(I*a)])/E^(I*a)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 947
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{i}{x}+2i \operatorname{arctanh}\left(x \,{\mathrm e}^{-i a}\right) {\mathrm e}^{-i a}$	24

input
```
int(cot(a+I*ln(x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-I/x+2*I*arctanh(x*exp(-I*a))*exp(-I*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \frac{\cot(a+i\log(x))}{x^2}\,dx = \frac{i\,xe^{(-i\,a)}\log\left(x+e^{(i\,a)}\right)-i\,xe^{(-i\,a)}\log\left(x-e^{(i\,a)}\right)-i}{x}$$

input
```
integrate(cot(a+I*log(x))/x^2,x, algorithm="fricas")
```

output
```
(I*x*e^(-I*a)*log(x + e^(I*a)) - I*x*e^(-I*a)*log(x - e^(I*a)) - I)/x
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\cot(a+i\log(x))}{x^2}\,dx = -\left(i\log\left(x-e^{ia}\right)-i\log\left(x+e^{ia}\right)\right)e^{-ia}-\frac{i}{x}$$

input
```
integrate(cot(a+I*ln(x))/x**2,x)
```

output
```
-(I*log(x - exp(I*a)) - I*log(x + exp(I*a)))*exp(-I*a) - I/x
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(19) = 38.

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.41

$$\int \frac{\cot(a+i\log(x))}{x^2}\,dx$$
$$= \frac{x(i\cos(a)+\sin(a))\log\left(x^2+2\,x\cos(a)+\cos(a)^2+\sin(a)^2\right)+x(-i\cos(a)-\sin(a))\log\left(x^2-2\,x\cos\right.}{}$$

input
```
integrate(cot(a+I*log(x))/x^2,x, algorithm="maxima")
```

output
```
1/2*(x*(I*cos(a) + sin(a))*log(x^2 + 2*x*cos(a) + cos(a)^2 + sin(a)^2) + x
*(-I*cos(a) - sin(a))*log(x^2 - 2*x*cos(a) + cos(a)^2 + sin(a)^2) - 2*((co
s(a) - I*sin(a))*arctan2(sin(a), x + cos(a)) + (cos(a) - I*sin(a))*arctan2
(sin(a), x - cos(a)))*x - 2*I)/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{\cot(a+i\log(x))}{x^2}\,dx = i\,e^{(-i\,a)}\log\left(x+e^{(i\,a)}\right) - i\,e^{(-i\,a)}\log\left(-x+e^{(i\,a)}\right) - \frac{i}{x}$$

input
```
integrate(cot(a+I*log(x))/x^2,x, algorithm="giac")
```

output
```
I*e^(-I*a)*log(x + e^(I*a)) - I*e^(-I*a)*log(-x + e^(I*a)) - I/x
```

Mupad [B] (verification not implemented)

Time = 19.93 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\cot(a+i\log(x))}{x^2}\,dx = -\frac{\mathrm{atan}\left(\frac{x}{\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}}}\right)\,2\mathrm{i}}{\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}}} - \frac{1\mathrm{i}}{x}$$

input
```
int(cot(a + log(x)*1i)/x^2,x)
```

output
```
- (atan(x/(-exp(a*2i))^(1/2))*2i)/(-exp(a*2i))^(1/2) - 1i/x
```

Reduce [F]

$$\int \frac{\cot(a + i\log(x))}{x^2} \, dx = \int \frac{\cot(\log(x)\, i + a)}{x^2} dx$$

input
```
int(cot(a+I*log(x))/x^2,x)
```

output
```
int(cot(log(x)*i + a)/x**2,x)
```

3.192 $\int \frac{\cot(a+i\log(x))}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{\cot(a+i\log(x))}{x^3}\,dx = -\frac{i}{2x^2} - ie^{-2ia}\log\left(1-\frac{e^{2ia}}{x^2}\right)$$

output
```
-1/2*I/x^2-I*ln(1-exp(2*I*a)/x^2)/exp(2*I*a)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 136 vs. 2(36) = 72.

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.78

$$\begin{aligned}\int \frac{\cot(a+i\log(x))}{x^3}\,dx = &-\frac{i}{2x^2} - \arctan\left(\frac{(-1+x^2)\cos(a)}{-\sin(a)-x^2\sin(a)}\right)\cos(2a)\\ &+2i\cos(2a)\log(x) - \frac{1}{2}i\cos(2a)\log\left(1+x^4-2x^2\cos(2a)\right)\\ &+i\arctan\left(\frac{(-1+x^2)\cos(a)}{-\sin(a)-x^2\sin(a)}\right)\sin(2a)\\ &+2\log(x)\sin(2a) - \frac{1}{2}\log\left(1+x^4-2x^2\cos(2a)\right)\sin(2a)\end{aligned}$$

input
```
Integrate[Cot[a + I*Log[x]]/x^3,x]
```

output
```
(-1/2*I)/x^2 - ArcTan[((-1 + x^2)*Cos[a])/(-Sin[a] - x^2*Sin[a])]*Cos[2*a]
 + (2*I)*Cos[2*a]*Log[x] - (I/2)*Cos[2*a]*Log[1 + x^4 - 2*x^2*Cos[2*a]] +
I*ArcTan[((-1 + x^2)*Cos[a])/(-Sin[a] - x^2*Sin[a])]*Sin[2*a] + 2*Log[x]*S
in[2*a] - (Log[1 + x^4 - 2*x^2*Cos[2*a]]*Sin[2*a])/2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5007, 946, 26, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(a+i\log(x))}{x^3} \, dx$$

$$\Big\downarrow \text{5007}$$

$$\int \frac{-\frac{ie^{2ia}}{x^2}-i}{x^3\left(1-\frac{e^{2ia}}{x^2}\right)}dx$$

$$\Big\downarrow \text{946}$$

$$-\frac{1}{2}\int -\frac{i\left(1+\frac{e^{2ia}}{x^2}\right)}{1-\frac{e^{2ia}}{x^2}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}i\int \frac{1+\frac{e^{2ia}}{x^2}}{1-\frac{e^{2ia}}{x^2}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}i\int \left(-1-\frac{2}{\frac{e^{2ia}}{x^2}-1}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}i\left(-\frac{1}{x^2}-2e^{-2ia}\log\left(1-\frac{e^{2ia}}{x^2}\right)\right)$$

input
```
Int[Cot[a + I*Log[x]]/x^3,x]
```

output
```
(I/2)*(-x^(-2) - (2*Log[1 - E^((2*I)*a)/x^2])/E^((2*I)*a))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

method	result	size
risch	$-\frac{i}{2x^2}+2i\mathrm{e}^{-2ia}\ln\left(x\right)-i\mathrm{e}^{-2ia}\ln\left(\mathrm{e}^{2ia}-x^2\right)$	38

input
```
int(cot(a+I*ln(x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/x^2+2*I*exp(-2*I*a)*ln(x)-I*exp(-2*I*a)*ln(exp(2*I*a)-x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{\cot(a+i\log(x))}{x^3}\,dx=\frac{\left(-2i\,x^2\log\left(x^2-e^{(2i\,a)}\right)+4i\,x^2\log\left(x\right)-i\,e^{(2i\,a)}\right)e^{(-2i\,a)}}{2\,x^2}$$

input
```
integrate(cot(a+I*log(x))/x^3,x, algorithm="fricas")
```

output
```
1/2*(-2*I*x^2*log(x^2 - e^(2*I*a)) + 4*I*x^2*log(x) - I*e^(2*I*a))*e^(-2*I
*a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{\cot(a+i\log(x))}{x^3}\,dx=2ie^{-2ia}\log\left(x\right)-ie^{-2ia}\log\left(x^2-e^{2ia}\right)-\frac{i}{2x^2}$$

input
```
integrate(cot(a+I*ln(x))/x**3,x)
```

output
```
2*I*exp(-2*I*a)*log(x) - I*exp(-2*I*a)*log(x**2 - exp(2*I*a)) - I/(2*x**2)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(24) = 48.

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.75

$$\int \frac{\cot(a+i\log(x))}{x^3}\,dx = -\frac{x^2(i\cos(2a)+\sin(2a))\log\left(x^2+2x\cos(a)+\cos(a)^2+\sin(a)^2\right)+x^2(i\cos(2a)+\sin(2a))\log\left(x^2\right.}{}$$

input
```
integrate(cot(a+I*log(x))/x^3,x, algorithm="maxima")
```

output
```
-1/2*(x^2*(I*cos(2*a) + sin(2*a))*log(x^2 + 2*x*cos(a) + cos(a)^2 + sin(a)
^2) + x^2*(I*cos(2*a) + sin(2*a))*log(x^2 - 2*x*cos(a) + cos(a)^2 + sin(a)
^2) - 2*((cos(2*a) - I*sin(2*a))*arctan2(sin(a), x + cos(a)) - (cos(2*a) -
 I*sin(2*a))*arctan2(sin(a), x - cos(a)) + 2*(I*cos(2*a) + sin(2*a))*log(x
))*x^2 + I)/x^2
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(24) = 48.

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.36

$$\int \frac{\cot(a+i\log(x))}{x^3}\,dx = \frac{1}{2}\pi e^{(-2ia)} - i\,e^{(-2ia)}\log\left(x+e^{(ia)}\right) + 2i\,e^{(-2ia)}\log(x) - i\,e^{(-2ia)}\log\left(-x+e^{(ia)}\right) - \frac{i}{2x^2}$$

input
```
integrate(cot(a+I*log(x))/x^3,x, algorithm="giac")
```

output
```
1/2*pi*e^(-2*I*a) - I*e^(-2*I*a)*log(x + e^(I*a)) + 2*I*e^(-2*I*a)*log(x)
- I*e^(-2*I*a)*log(-x + e^(I*a)) - 1/2*I/x^2
```

Mupad [B] (verification not implemented)

Time = 19.71 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{\cot(a+i\log(x))}{x^3}\,dx = e^{-a\,2i}\ln(x)\,2i - \ln\left(x^2 - e^{a\,2i}\right)\,e^{-a\,2i}\,1i - \frac{1i}{2\,x^2}$$

input
```
int(cot(a + log(x)*1i)/x^3,x)
```

output
```
exp(-a*2i)*log(x)*2i - log(x^2 - exp(a*2i))*exp(-a*2i)*1i - 1i/(2*x^2)
```

Reduce [F]

$$\int \frac{\cot(a+i\log(x))}{x^3}\,dx = \int \frac{\cot(\log(x)\,i+a)}{x^3}dx$$

input
```
int(cot(a+I*log(x))/x^3,x)
```

output
```
int(cot(log(x)*i + a)/x**3,x)
```

3.193 $\int \frac{\cot(a+i\log(x))}{x^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 45

$$\int \frac{\cot(a+i\log(x))}{x^4}\,dx=-\frac{i}{3x^3}-\frac{2ie^{-2ia}}{x}+2ie^{-3ia}\text{arctanh}\left(e^{-ia}x\right)$$

output
```
-1/3*I/x^3-2*I/exp(2*I*a)/x+2*I*arctanh(x/exp(I*a))/exp(3*I*a)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.56

$$\int \frac{\cot(a+i\log(x))}{x^4}\,dx=-\frac{i}{3x^3}-\frac{2i\cos(2a)}{x}+2i\text{arctanh}(x\cos(a)-ix\sin(a))\cos(3a)-\frac{2\sin(2a)}{x}+2\text{arctanh}(x\cos(a)-ix\sin(a))\sin(3a)$$

input
```
Integrate[Cot[a + I*Log[x]]/x^4,x]
```

output
```
(-1/3*I)/x^3 - ((2*I)*Cos[2*a])/x + (2*I)*ArcTanh[x*Cos[a] - I*x*Sin[a]]*C
os[3*a] - (2*Sin[2*a])/x + 2*ArcTanh[x*Cos[a] - I*x*Sin[a]]*Sin[3*a]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5007, 947, 359, 25, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(a+i\log(x))}{x^4}dx$$
$$\Big\downarrow \text{5007}$$
$$\int \frac{-\frac{ie^{2ia}}{x^2}-i}{x^4\left(1-\frac{e^{2ia}}{x^2}\right)}dx$$
$$\Big\downarrow \text{947}$$
$$\int \frac{-ie^{2ia}-ix^2}{x^4\left(x^2-e^{2ia}\right)}dx$$
$$\Big\downarrow \text{359}$$
$$-2i\int -\frac{1}{x^2\left(e^{2ia}-x^2\right)}dx-\frac{i}{3x^3}$$
$$\Big\downarrow \text{25}$$
$$2i\int \frac{1}{x^2\left(e^{2ia}-x^2\right)}dx-\frac{i}{3x^3}$$
$$\Big\downarrow \text{264}$$
$$2i\left(e^{-2ia}\int \frac{1}{e^{2ia}-x^2}dx-\frac{e^{-2ia}}{x}\right)-\frac{i}{3x^3}$$
$$\Big\downarrow \text{219}$$
$$2i\left(e^{-3ia}\text{arctanh}\left(e^{-ia}x\right)-\frac{e^{-2ia}}{x}\right)-\frac{i}{3x^3}$$

input
```
Int[Cot[a + I*Log[x]]/x^4,x]
```

output
```
(-1/3*I)/x^3 + (2*I)*(-(1/(E^((2*I)*a)*x)) + ArcTanh[x/E^(I*a)]/E^((3*I)*a
))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 947
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
risch	$-\frac{i}{3x^3}+2i\,\mathrm{arctanh}\left(x\,\mathrm{e}^{-ia}\right)\mathrm{e}^{-3ia}-\frac{2i\mathrm{e}^{-2ia}}{x}$	35

input
```
int(cot(a+I*ln(x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/x^3+2*I*arctanh(x*exp(-I*a))*exp(-3*I*a)-2*I*exp(-2*I*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int \frac{\cot(a+i\log(x))}{x^4}\,dx = \frac{\left(3i\,x^3e^{(-i\,a)}\log\left(x+e^{(i\,a)}\right)-3i\,x^3e^{(-i\,a)}\log\left(x-e^{(i\,a)}\right)-6i\,x^2-i\,e^{(2i\,a)}\right)e^{(-2i\,a)}}{3\,x^3}$$

input
```
integrate(cot(a+I*log(x))/x^4,x, algorithm="fricas")
```

output
```
1/3*(3*I*x^3*e^(-I*a)*log(x + e^(I*a)) - 3*I*x^3*e^(-I*a)*log(x - e^(I*a))
 - 6*I*x^2 - I*e^(2*I*a))*e^(-2*I*a)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int \frac{\cot(a+i\log(x))}{x^4}\,dx = -\left(i\log\left(x-e^{ia}\right)-i\log\left(x+e^{ia}\right)\right)e^{-3ia}-\frac{\left(6ix^2+ie^{2ia}\right)e^{-2ia}}{3x^3}$$

input
```
integrate(cot(a+I*ln(x))/x**4,x)
```

output
```
-(I*log(x - exp(I*a)) - I*log(x + exp(I*a)))*exp(-3*I*a) - (6*I*x**2 + I*e
xp(2*I*a))*exp(-2*I*a)/(3*x**3)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(28) = 56.

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.09

$$\int \frac{\cot(a+i\log(x))}{x^4}\,dx = -\frac{3\,x^3(-i\,\cos(3\,a)-\sin(3\,a))\log\left(x^2+2\,x\cos(a)+\cos(a)^2+\sin(a)^2\right)+3\,x^3(i\,\cos(3\,a)+\sin(3\,a))\,\mathrm{lo}}{}$$

input
```
integrate(cot(a+I*log(x))/x^4,x, algorithm="maxima")
```

output
```
-1/6*(3*x^3*(-I*cos(3*a) - sin(3*a))*log(x^2 + 2*x*cos(a) + cos(a)^2 + sin
(a)^2) + 3*x^3*(I*cos(3*a) + sin(3*a))*log(x^2 - 2*x*cos(a) + cos(a)^2 + s
in(a)^2) + 6*((cos(3*a) - I*sin(3*a))*arctan2(sin(a), x + cos(a)) + (cos(3
*a) - I*sin(3*a))*arctan2(sin(a), x - cos(a)))*x^3 + 12*x^2*(I*cos(2*a) +
sin(2*a)) + 2*I)/x^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{\cot(a+i\log(x))}{x^4}\,dx = i\,e^{(-3i\,a)}\log\left(x+e^{(i\,a)}\right) - i\,e^{(-3i\,a)}\log\left(-x+e^{(i\,a)}\right) - \frac{2i\,e^{(-2i\,a)}}{x} - \frac{i}{3\,x^3}$$

input
```
integrate(cot(a+I*log(x))/x^4,x, algorithm="giac")
```

output
```
I*e^(-3*I*a)*log(x + e^(I*a)) - I*e^(-3*I*a)*log(-x + e^(I*a)) - 2*I*e^(-2
*I*a)/x - 1/3*I/x^3
```

Mupad [B] (verification not implemented)

Time = 19.93 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{\cot(a+i\log(x))}{x^4}\,dx=\frac{\operatorname{atan}\left(\frac{x}{\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}}}\right)\,2\mathrm{i}}{\left(-\mathrm{e}^{a\,2\mathrm{i}}\right)^{3/2}}-\frac{2\mathrm{i}\,\mathrm{e}^{-a\,2\mathrm{i}}\,x^2+\frac{1}{3}\mathrm{i}}{x^3}$$

input
```
int(cot(a + log(x)*1i)/x^4,x)
```

output
```
(atan(x/(-exp(a*2i))^(1/2))*2i)/(-exp(a*2i))^(3/2) - (x^2*exp(-a*2i)*2i +
1i/3)/x^3
```

Reduce [F]

$$\int \frac{\cot(a+i\log(x))}{x^4}\,dx=\int \frac{\cot(\log(x)\,i+a)}{x^4}dx$$

input
```
int(cot(a+I*log(x))/x^4,x)
```

output
```
int(cot(log(x)*i + a)/x**4,x)
```

3.194 $\int x^3 \cot^2(a+i\log(x))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int x^3 \cot^2(a+i\log(x))\,dx = -2e^{2ia}x^2 - \frac{x^4}{4} - \frac{2e^{6ia}}{e^{2ia}-x^2} - 4e^{4ia}\log\left(e^{2ia}-x^2\right)$$

output
```
-2*exp(2*I*a)*x^2-1/4*x^4-2*exp(6*I*a)/(exp(2*I*a)-x^2)-4*exp(4*I*a)*ln(ex
p(2*I*a)-x^2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 162 vs. 2(67) = 134.

Time = 0.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.42

$$\begin{aligned}\int x^3 \cot^2(a+i\log(x))\,dx = & -\frac{x^4}{4} - 2x^2\cos(2a) + 4i\arctan\left(\frac{\cot(a)-x^2\cot(a)}{1+x^2}\right)\cos(4a) \\ & - 2\cos(4a)\log\left(1+x^4-2x^2\cos(2a)\right) - 2ix^2\sin(2a) \\ & - 4\arctan\left(\frac{\cot(a)-x^2\cot(a)}{1+x^2}\right)\sin(4a) \\ & - 2i\log\left(1+x^4-2x^2\cos(2a)\right)\sin(4a) \\ & + \frac{2\cos(5a)+2i\sin(5a)}{(-1+x^2)\cos(a)-i(1+x^2)\sin(a)}\end{aligned}$$

input
```
Integrate[x^3*Cot[a + I*Log[x]]^2,x]
```

output
```
-1/4*x^4 - 2*x^2*Cos[2*a] + (4*I)*ArcTan[(Cot[a] - x^2*Cot[a])/(1 + x^2)]*
Cos[4*a] - 2*Cos[4*a]*Log[1 + x^4 - 2*x^2*Cos[2*a]] - (2*I)*x^2*Sin[2*a] -
 4*ArcTan[(Cot[a] - x^2*Cot[a])/(1 + x^2)]*Sin[4*a] - (2*I)*Log[1 + x^4 -
2*x^2*Cos[2*a]]*Sin[4*a] + (2*Cos[5*a] + (2*I)*Sin[5*a])/((-1 + x^2)*Cos[a
] - I*(1 + x^2)*Sin[a])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5007, 947, 354, 25, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cot^2(a+i\log(x))\,dx$$

$$\Big\downarrow \text{5007}$$

$$\int \frac{x^3\left(-\frac{ie^{2ia}}{x^2}-i\right)^2}{\left(1-\frac{e^{2ia}}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{947}$$

$$\int \frac{x^3\left(-ie^{2ia}-ix^2\right)^2}{\left(x^2-e^{2ia}\right)^2}dx$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int -\frac{x^2\left(x^2+e^{2ia}\right)^2}{\left(e^{2ia}-x^2\right)^2}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{x^2\left(x^2+e^{2ia}\right)^2}{\left(e^{2ia}-x^2\right)^2}dx^2$$

$$\Big\downarrow \text{86}$$

$$-\frac{1}{2}\int \left(x^2+4e^{2ia}-\frac{8e^{4ia}}{e^{2ia}-x^2}+\frac{4e^{6ia}}{(e^{2ia}-x^2)^2}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-4e^{2ia}x^2-\frac{4e^{6ia}}{-x^2+e^{2ia}}-8e^{4ia}\log\left(-x^2+e^{2ia}\right)-\frac{x^4}{2}\right)$$

input
```
Int[x^3*Cot[a + I*Log[x]]^2,x]
```

output
```
(-4*E^((2*I)*a)*x^2 - x^4/2 - (4*E^((6*I)*a))/(E^((2*I)*a) - x^2) - 8*E^((
4*I)*a)*Log[E^((2*I)*a) - x^2])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 947
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

method	result	size
risch	$-\frac{9x^4}{4}-\frac{2x^4}{\frac{e^{2ia}}{x^2}-1}-4\,e^{2ia}x^2-4\,e^{4ia}\ln\left(e^{2ia}-x^2\right)$	54

input
```
int(x^3*cot(a+I*ln(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-9/4*x^4-2*x^4/(exp(2*I*a)/x^2-1)-4*exp(2*I*a)*x^2-4*exp(4*I*a)*ln(exp(2*I
*a)-x^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int x^3\cot^2(a+i\log(x))\,dx$$
$$=-\frac{x^6+7\,x^4e^{(2i\,a)}-8\,x^2e^{(4i\,a)}+16\left(x^2e^{(4i\,a)}-e^{(6i\,a)}\right)\log\left(x^2-e^{(2i\,a)}\right)-8\,e^{(6i\,a)}}{4\left(x^2-e^{(2i\,a)}\right)}$$

input
```
integrate(x^3*cot(a+I*log(x))^2,x, algorithm="fricas")
```

output
```
-1/4*(x^6 + 7*x^4*e^(2*I*a) - 8*x^2*e^(4*I*a) + 16*(x^2*e^(4*I*a) - e^(6*I
*a))*log(x^2 - e^(2*I*a)) - 8*e^(6*I*a))/(x^2 - e^(2*I*a))
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int x^3 \cot^2(a+i\log(x))\,dx = -\frac{x^4}{4} - 2x^2e^{2ia} - 4e^{4ia}\log\left(x^2-e^{2ia}\right) + \frac{2e^{6ia}}{x^2-e^{2ia}}$$

input
```
integrate(x**3*cot(a+I*ln(x))**2,x)
```

output
```
-x**4/4 - 2*x**2*exp(2*I*a) - 4*exp(4*I*a)*log(x**2 - exp(2*I*a)) + 2*exp(
6*I*a)/(x**2 - exp(2*I*a))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(50) = 100.

Time = 0.05 (sec) , antiderivative size = 345, normalized size of antiderivative = 5.15

$$\int x^3 \cot^2(a+i\log(x))\,dx = -\frac{x^6 + 7x^4(\cos(2a) + i\sin(2a)) - 8(2(-i\cos(4a) + \sin(4a))\arctan(\sin(a), x+\cos(a)) + 2(i\cos$$

input
```
integrate(x^3*cot(a+I*log(x))^2,x, algorithm="maxima")
```

output
```
-1/4*(x^6 + 7*x^4*(cos(2*a) + I*sin(2*a)) - 8*(2*(-I*cos(4*a) + sin(4*a))*
arctan2(sin(a), x + cos(a)) + 2*(I*cos(4*a) - sin(4*a))*arctan2(sin(a), x
- cos(a)) + cos(4*a) + I*sin(4*a))*x^2 - 16*((I*cos(2*a) - sin(2*a))*cos(4
*a) - (cos(2*a) + I*sin(2*a))*sin(4*a))*arctan2(sin(a), x + cos(a)) - 16*(
(-I*cos(2*a) + sin(2*a))*cos(4*a) + (cos(2*a) + I*sin(2*a))*sin(4*a))*arct
an2(sin(a), x - cos(a)) + 8*(x^2*(cos(4*a) + I*sin(4*a)) - (cos(2*a) + I*s
in(2*a))*cos(4*a) - (I*cos(2*a) - sin(2*a))*sin(4*a))*log(x^2 + 2*x*cos(a)
 + cos(a)^2 + sin(a)^2) + 8*(x^2*(cos(4*a) + I*sin(4*a)) - (cos(2*a) + I*s
in(2*a))*cos(4*a) - (I*cos(2*a) - sin(2*a))*sin(4*a))*log(x^2 - 2*x*cos(a)
 + cos(a)^2 + sin(a)^2) - 8*cos(6*a) - 8*I*sin(6*a))/(x^2 - cos(2*a) - I*s
in(2*a))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(50) = 100$.

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.07

$$\int x^3 \cot^2(a + i\log(x))\,dx = -\frac{x^6}{4\left(x^2 - e^{(2ia)}\right)} - \frac{7\,x^4e^{(2ia)}}{4\left(x^2 - e^{(2ia)}\right)} - \frac{4\,x^2e^{(4ia)}\log\left(-x^2 + e^{(2ia)}\right)}{x^2 - e^{(2ia)}} + \frac{2\,x^2e^{(4ia)}}{x^2 - e^{(2ia)}} + \frac{4\,e^{(6ia)}\log\left(-x^2 + e^{(2ia)}\right)}{x^2 - e^{(2ia)}} + \frac{2\,e^{(6ia)}}{x^2 - e^{(2ia)}}$$

input
```
integrate(x^3*cot(a+I*log(x))^2,x, algorithm="giac")
```

output
```
-1/4*x^6/(x^2 - e^(2*I*a)) - 7/4*x^4*e^(2*I*a)/(x^2 - e^(2*I*a)) - 4*x^2*e
^(4*I*a)*log(-x^2 + e^(2*I*a))/(x^2 - e^(2*I*a)) + 2*x^2*e^(4*I*a)/(x^2 -
e^(2*I*a)) + 4*e^(6*I*a)*log(-x^2 + e^(2*I*a))/(x^2 - e^(2*I*a)) + 2*e^(6*
I*a)/(x^2 - e^(2*I*a))
```

Mupad [B] (verification not implemented)

Time = 19.82 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int x^3 \cot^2(a + i\log(x))\,dx = -2\,x^2\,e^{a\,2i} - \frac{2\,e^{a\,6i}}{e^{a\,2i} - x^2} - 4\,\ln\left(x^2 - e^{a\,2i}\right)\,e^{a\,4i} - \frac{x^4}{4}$$

input
```
int(x^3*cot(a + log(x)*1i)^2,x)
```

output
```
- 2*x^2*exp(a*2i) - (2*exp(a*6i))/(exp(a*2i) - x^2) - 4*log(x^2 - exp(a*2i
))*exp(a*4i) - x^4/4
```

Reduce [F]

$$\int x^3 \cot^2(a + i \log(x)) \, dx = \int \cot \left(\log(x) \, i + a\right)^2 x^3 dx$$

input
```
int(x^3*cot(a+I*log(x))^2,x)
```

output
```
int(cot(log(x)*i + a)**2*x**3,x)
```

3.195 $\int x^2 \cot^2(a+i\log(x))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 62

$$\int x^2 \cot^2(a+i\log(x))\,dx=-4e^{2ia}x-\frac{x^3}{3}-\frac{2e^{4ia}x}{e^{2ia}-x^2}+6e^{3ia}\text{arctanh}\left(e^{-ia}x\right)$$

output
```
-4*exp(2*I*a)*x-1/3*x^3-2*exp(4*I*a)*x/(exp(2*I*a)-x^2)+6*exp(3*I*a)*arcta
nh(x/exp(I*a))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.61

$$\begin{aligned}\int x^2 \cot^2(a+i\log(x))\,dx=&-\frac{x^3}{3}-4x\cos(2a)+6\text{arctanh}(x(\cos(a)-i\sin(a)))\cos(3a)\\&-4ix\sin(2a)+\frac{2x(\cos(3a)+i\sin(3a))}{(-1+x^2)\cos(a)-i(1+x^2)\sin(a)}\\&+6i\text{arctanh}(x(\cos(a)-i\sin(a)))\sin(3a)\end{aligned}$$

input
```
Integrate[x^2*Cot[a + I*Log[x]]^2,x]
```

output
```
-1/3*x^3 - 4*x*Cos[2*a] + 6*ArcTanh[x*(Cos[a] - I*Sin[a])]*Cos[3*a] - (4*I
)*x*Sin[2*a] + (2*x*(Cos[3*a] + I*Sin[3*a]))/((-1 + x^2)*Cos[a] - I*(1 + x
^2)*Sin[a]) + (6*I)*ArcTanh[x*(Cos[a] - I*Sin[a])]*Sin[3*a]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {5007, 947, 366, 27, 363, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^2(a+i\log(x))\,dx$$

$$\Big\downarrow \text{5007}$$

$$\int \frac{x^2\left(-\frac{ie^{2ia}}{x^2}-i\right)^2}{\left(1-\frac{e^{2ia}}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{947}$$

$$\int \frac{x^2\left(-ie^{2ia}-ix^2\right)^2}{\left(x^2-e^{2ia}\right)^2}dx$$

$$\Big\downarrow \text{366}$$

$$\frac{1}{2}e^{-2ia}\int \frac{2x^2\left(e^{2ia}x^2+5e^{4ia}\right)}{e^{2ia}-x^2}dx-\frac{2e^{2ia}x^3}{-x^2+e^{2ia}}$$

$$\Big\downarrow \text{27}$$

$$e^{-2ia}\int \frac{x^2\left(e^{2ia}x^2+5e^{4ia}\right)}{e^{2ia}-x^2}dx-\frac{2e^{2ia}x^3}{-x^2+e^{2ia}}$$

$$\Big\downarrow \text{363}$$

$$e^{-2ia}\left(6e^{4ia}\int \frac{x^2}{e^{2ia}-x^2}dx-\frac{1}{3}e^{2ia}x^3\right)-\frac{2e^{2ia}x^3}{-x^2+e^{2ia}}$$

$$\Big\downarrow \text{262}$$

$$e^{-2ia}\left(6e^{4ia}\left(-x+e^{2ia}\int\frac{1}{e^{2ia}-x^2}dx\right)-\frac{1}{3}e^{2ia}x^3\right)-\frac{2e^{2ia}x^3}{-x^2+e^{2ia}}$$

$$\Big\downarrow \text{219}$$

$$e^{-2ia}\left(6e^{4ia}\left(-x+e^{ia}\text{arctanh}\left(e^{-ia}x\right)\right)-\frac{1}{3}e^{2ia}x^3\right)-\frac{2e^{2ia}x^3}{-x^2+e^{2ia}}$$

input
```
Int[x^2*Cot[a + I*Log[x]]^2,x]
```

output
```
(-2*E^((2*I)*a)*x^3)/(E^((2*I)*a) - x^2) + (-1/3*(E^((2*I)*a)*x^3) + 6*E^(
(4*I)*a)*(-x + E^(I*a)*ArcTanh[x/E^(I*a)]))/E^((2*I)*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 947
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{7x^3}{3}-\frac{2x^3}{\frac{e^{2ia}}{x^2}-1}-6\,e^{2ia}x+6\,\operatorname{arctanh}\left(x\,e^{-ia}\right)e^{3ia}$	48

input
```
int(x^2*cot(a+I*ln(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-7/3*x^3-2*x^3/(exp(2*I*a)/x^2-1)-6*exp(2*I*a)*x+6*arctanh(x*exp(-I*a))*ex
p(3*I*a)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(45) = 90$.

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.65

$$\int x^2 \cot^2(a + i\log(x))\,dx = -\frac{x^5 + 11\,x^3e^{(2i\,a)} - 9\left(x^2 - e^{(2i\,a)}\right)e^{(3i\,a)}\log\left(\left(xe^{(2i\,a)} + e^{(3i\,a)}\right)e^{(-2i\,a)}\right) + 9\left(x^2 - e^{(2i\,a)}\right)e^{(3i\,a)}\log\left(\left(xe^{(2i\,a}\right.\right.}{3\left(x^2 - e^{(2i\,a)}\right)}$$

input
```
integrate(x^2*cot(a+I*log(x))^2,x, algorithm="fricas")
```

output
```
-1/3*(x^5 + 11*x^3*e^(2*I*a) - 9*(x^2 - e^(2*I*a))*e^(3*I*a)*log((x*e^(2*I
*a) + e^(3*I*a))*e^(-2*I*a)) + 9*(x^2 - e^(2*I*a))*e^(3*I*a)*log((x*e^(2*I
*a) - e^(3*I*a))*e^(-2*I*a)) - 18*x*e^(4*I*a))/(x^2 - e^(2*I*a))
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int x^2 \cot^2(a + i\log(x))\,dx = -\frac{x^3}{3} - 4xe^{2ia} + \frac{2xe^{4ia}}{x^2 - e^{2ia}} - 3\left(\log\left(x - e^{ia}\right) - \log\left(x + e^{ia}\right)\right)e^{3ia}$$

input
```
integrate(x**2*cot(a+I*ln(x))**2,x)
```

output
```
-x**3/3 - 4*x*exp(2*I*a) + 2*x*exp(4*I*a)/(x**2 - exp(2*I*a)) - 3*(log(x -
 exp(I*a)) - log(x + exp(I*a)))*exp(3*I*a)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(45) = 90$.

Time = 0.05 (sec) , antiderivative size = 335, normalized size of antiderivative = 5.40

$$\int x^2 \cot^2(a + i\log(x))\,dx = -\frac{2x^5 + 22x^3(\cos(2a) + i\sin(2a)) + 18((-i\cos(3a) + \sin(3a))\arctan(\sin(a), x + \cos(a)) + (-i\cdots}{\cdots}$$

input
```
integrate(x^2*cot(a+I*log(x))^2,x, algorithm="maxima")
```

output
```
-1/6*(2*x^5 + 22*x^3*(cos(2*a) + I*sin(2*a)) + 18*((-I*cos(3*a) + sin(3*a)
)*arctan2(sin(a), x + cos(a)) + (-I*cos(3*a) + sin(3*a))*arctan2(sin(a), x
 - cos(a)))*x^2 - 36*x*(cos(4*a) + I*sin(4*a)) + 18*((I*cos(2*a) - sin(2*a
))*cos(3*a) - (cos(2*a) + I*sin(2*a))*sin(3*a))*arctan2(sin(a), x + cos(a)
) + 18*((I*cos(2*a) - sin(2*a))*cos(3*a) - (cos(2*a) + I*sin(2*a))*sin(3*a
))*arctan2(sin(a), x - cos(a)) - 9*(x^2*(cos(3*a) + I*sin(3*a)) - (cos(2*a
) + I*sin(2*a))*cos(3*a) - (I*cos(2*a) - sin(2*a))*sin(3*a))*log(x^2 + 2*x
*cos(a) + cos(a)^2 + sin(a)^2) + 9*(x^2*(cos(3*a) + I*sin(3*a)) - (cos(2*a
) + I*sin(2*a))*cos(3*a) + (-I*cos(2*a) + sin(2*a))*sin(3*a))*log(x^2 - 2*
x*cos(a) + cos(a)^2 + sin(a)^2))/(x^2 - cos(2*a) - I*sin(2*a))
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.34

$$\int x^2 \cot^2(a + i\log(x))\,dx = -\frac{x^5}{3\,(x^2 - e^{(2ia)})} - \frac{11\,x^3 e^{(2ia)}}{3\,(x^2 - e^{(2ia)})} - \frac{6\arctan\left(\frac{x}{\sqrt{-e^{(2ia)}}}\right)e^{(4ia)}}{\sqrt{-e^{(2ia)}}} + \frac{10\,xe^{(4ia)}}{x^2 - e^{(2ia)}}$$

input
```
integrate(x^2*cot(a+I*log(x))^2,x, algorithm="giac")
```

output
```
-1/3*x^5/(x^2 - e^(2*I*a)) - 11/3*x^3*e^(2*I*a)/(x^2 - e^(2*I*a)) - 6*arct
an(x/sqrt(-e^(2*I*a)))*e^(4*I*a)/sqrt(-e^(2*I*a)) + 10*x*e^(4*I*a)/(x^2 -
e^(2*I*a))
```

Mupad [B] (verification not implemented)

Time = 20.66 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int x^2 \cot^2(a+i\log(x))\,dx = -\left(\mathrm{e}^{a\,2\mathrm{i}}\right)^{3/2}\operatorname{atan}\left(\frac{x\,1\mathrm{i}}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}}}\right)6\mathrm{i} - \frac{x^3}{3} - 4\,x\,\mathrm{e}^{a\,2\mathrm{i}} - \frac{2\,x\,\mathrm{e}^{a\,4\mathrm{i}}}{\mathrm{e}^{a\,2\mathrm{i}} - x^2}$$

input
```
int(x^2*cot(a + log(x)*1i)^2,x)
```

output
```
- exp(a*2i)^(3/2)*atan((x*1i)/exp(a*2i)^(1/2))*6i - x^3/3 - 4*x*exp(a*2i)
- (2*x*exp(a*4i))/(exp(a*2i) - x^2)
```

Reduce [F]

$$\int x^2 \cot^2(a+i\log(x))\,dx = \int \cot\left(\log(x)\,i+a\right)^2 x^2 dx$$

input
```
int(x^2*cot(a+I*log(x))^2,x)
```

output
```
int(cot(log(x)*i + a)**2*x**2,x)
```

3.196 $\int x\cot^2(a+i\log(x))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 55

$$\int x\cot^2(a+i\log(x))\,dx=-\frac{x^2}{2}-\frac{2e^{4ia}}{e^{2ia}-x^2}-2e^{2ia}\log\left(e^{2ia}-x^2\right)$$

output
```
-1/2*x^2-2*exp(4*I*a)/(exp(2*I*a)-x^2)-2*exp(2*I*a)*ln(exp(2*I*a)-x^2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 142 vs. 2(55) = 110.

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.58

$$\begin{aligned}\int x\cot^2(a+i\log(x))\,dx=&-\frac{x^2}{2}+2i\arctan\left(\frac{\cot(a)-x^2\cot(a)}{1+x^2}\right)\cos(2a)\\&-\cos(2a)\log\left(1+x^4-2x^2\cos(2a)\right)\\&-4\arctan\left(\frac{\cot(a)-x^2\cot(a)}{1+x^2}\right)\cos(a)\sin(a)\\&-i\log\left(1+x^4-2x^2\cos(2a)\right)\sin(2a)\\&+\frac{2\cos(3a)+2i\sin(3a)}{(-1+x^2)\cos(a)-i(1+x^2)\sin(a)}\end{aligned}$$

input
```
Integrate[x*Cot[a + I*Log[x]]^2,x]
```

output
```
-1/2*x^2 + (2*I)*ArcTan[(Cot[a] - x^2*Cot[a])/(1 + x^2)]*Cos[2*a] - Cos[2*
a]*Log[1 + x^4 - 2*x^2*Cos[2*a]] - 4*ArcTan[(Cot[a] - x^2*Cot[a])/(1 + x^2
)]*Cos[a]*Sin[a] - I*Log[1 + x^4 - 2*x^2*Cos[2*a]]*Sin[2*a] + (2*Cos[3*a]
+ (2*I)*Sin[3*a])/((-1 + x^2)*Cos[a] - I*(1 + x^2)*Sin[a])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5007, 947, 353, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cot^2(a+i\log(x))\,dx$$

$$\Big\downarrow \text{5007}$$

$$\int \frac{x\left(-\frac{ie^{2ia}}{x^2}-i\right)^2}{\left(1-\frac{e^{2ia}}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{947}$$

$$\int \frac{x\left(-ie^{2ia}-ix^2\right)^2}{\left(x^2-e^{2ia}\right)^2}dx$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int -\frac{\left(x^2+e^{2ia}\right)^2}{\left(e^{2ia}-x^2\right)^2}dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{\left(x^2+e^{2ia}\right)^2}{\left(e^{2ia}-x^2\right)^2}dx^2$$

$$\Big\downarrow \text{49}$$

$$-\frac{1}{2}\int \left(1-\frac{4e^{2ia}}{e^{2ia}-x^2}+\frac{4e^{4ia}}{(e^{2ia}-x^2)^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{4e^{4ia}}{-x^2+e^{2ia}}-4e^{2ia}\log\left(-x^2+e^{2ia}\right)-x^2\right)$$

input
```
Int[x*Cot[a + I*Log[x]]^2,x]
```

output
```
(-x^2 - (4*E^((4*I)*a))/(E^((2*I)*a) - x^2) - 4*E^((2*I)*a)*Log[E^((2*I)*a
) - x^2])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 947
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{5x^2}{2}-\frac{2x^2}{\frac{e^{2ia}}{x^2}-1}-2\,e^{2ia}\ln\left(e^{2ia}-x^2\right)$	44

input
```
int(x*cot(a+I*ln(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-5/2*x^2-2*x^2/(exp(2*I*a)/x^2-1)-2*exp(2*I*a)*ln(exp(2*I*a)-x^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int x\cot^2(a+i\log(x))\,dx$$
$$=-\frac{x^4-x^2e^{(2i\,a)}+4\left(x^2e^{(2i\,a)}-e^{(4i\,a)}\right)\log\left(x^2-e^{(2i\,a)}\right)-4\,e^{(4i\,a)}}{2\left(x^2-e^{(2i\,a)}\right)}$$

input
```
integrate(x*cot(a+I*log(x))^2,x, algorithm="fricas")
```

output
```
-1/2*(x^4 - x^2*e^(2*I*a) + 4*(x^2*e^(2*I*a) - e^(4*I*a))*log(x^2 - e^(2*I
*a)) - 4*e^(4*I*a))/(x^2 - e^(2*I*a))
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int x\cot^2(a+i\log(x))\,dx=-\frac{x^2}{2}-2e^{2ia}\log\left(x^2-e^{2ia}\right)+\frac{2e^{4ia}}{x^2-e^{2ia}}$$

input
```
integrate(x*cot(a+I*ln(x))**2,x)
```

output
```
-x**2/2 - 2*exp(2*I*a)*log(x**2 - exp(2*I*a)) + 2*exp(4*I*a)/(x**2 - exp(2
*I*a))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(41) = 82.

Time = 0.05 (sec) , antiderivative size = 290, normalized size of antiderivative = 5.27

$$\int x\cot^2(a+i\log(x))\,dx=$$

$$-\frac{x^4-(4\,(-i\,\cos{(2\,a)}+\sin{(2\,a)})\arctan{(\sin{(a)}\,,x+\cos{(a)})}+4\,(i\,\cos{(2\,a)}-\sin{(2\,a)})\arctan{(\sin{(a}}}{}$$

input
```
integrate(x*cot(a+I*log(x))^2,x, algorithm="maxima")
```

output
```
-1/2*(x^4 - (4*(-I*cos(2*a) + sin(2*a))*arctan2(sin(a), x + cos(a)) + 4*(I
*cos(2*a) - sin(2*a))*arctan2(sin(a), x - cos(a)) + cos(2*a) + I*sin(2*a))
*x^2 - 4*(I*cos(2*a)^2 - 2*cos(2*a)*sin(2*a) - I*sin(2*a)^2)*arctan2(sin(a
), x + cos(a)) - 4*(-I*cos(2*a)^2 + 2*cos(2*a)*sin(2*a) + I*sin(2*a)^2)*ar
ctan2(sin(a), x - cos(a)) + 2*(x^2*(cos(2*a) + I*sin(2*a)) - cos(2*a)^2 -
2*I*cos(2*a)*sin(2*a) + sin(2*a)^2)*log(x^2 + 2*x*cos(a) + cos(a)^2 + sin(
a)^2) + 2*(x^2*(cos(2*a) + I*sin(2*a)) - cos(2*a)^2 - 2*I*cos(2*a)*sin(2*a
) + sin(2*a)^2)*log(x^2 - 2*x*cos(a) + cos(a)^2 + sin(a)^2) - 4*cos(4*a) -
 4*I*sin(4*a))/(x^2 - cos(2*a) - I*sin(2*a))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(41) = 82$.

Time = 0.18 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.15

$$\int x\cot^2(a+i\log(x))\,dx = -\frac{x^4}{2\left(x^2-e^{(2i\,a)}\right)}-\frac{2\,x^2e^{(2i\,a)}\log\left(-x^2+e^{(2i\,a)}\right)}{x^2-e^{(2i\,a)}}+\frac{x^2e^{(2i\,a)}}{2\left(x^2-e^{(2i\,a)}\right)}+\frac{2\,e^{(4i\,a)}\log\left(-x^2+e^{(2i\,a)}\right)}{x^2-e^{(2i\,a)}}+\frac{2\,e^{(4i\,a)}}{x^2-e^{(2i\,a)}}$$

input
```
integrate(x*cot(a+I*log(x))^2,x, algorithm="giac")
```

output
```
-1/2*x^4/(x^2 - e^(2*I*a)) - 2*x^2*e^(2*I*a)*log(-x^2 + e^(2*I*a))/(x^2 -
e^(2*I*a)) + 1/2*x^2*e^(2*I*a)/(x^2 - e^(2*I*a)) + 2*e^(4*I*a)*log(-x^2 +
e^(2*I*a))/(x^2 - e^(2*I*a)) + 2*e^(4*I*a)/(x^2 - e^(2*I*a))
```

Mupad [B] (verification not implemented)

Time = 20.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int x\cot^2(a+i\log(x))\,dx = -\frac{2\,\mathrm{e}^{a\,4\mathrm{i}}}{\mathrm{e}^{a\,2\mathrm{i}}-x^2}-2\,\ln\left(x^2-\mathrm{e}^{a\,2\mathrm{i}}\right)\,\mathrm{e}^{a\,2\mathrm{i}}-\frac{x^2}{2}$$

input
```
int(x*cot(a + log(x)*1i)^2,x)
```

output
```
- (2*exp(a*4i))/(exp(a*2i) - x^2) - 2*log(x^2 - exp(a*2i))*exp(a*2i) - x^2
/2
```

Reduce [F]

$$\int x\cot^2(a+i\log(x))\,dx=\int \cot\left(\log(x)\,i+a\right)^2 x dx$$

input
```
int(x*cot(a+I*log(x))^2,x)
```

output
```
int(cot(log(x)*i + a)**2*x,x)
```

3.197 $\int \cot^2(a+i\log(x))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 48

$$\int \cot^2(a+i\log(x))\,dx = -x-\frac{2e^{2ia}x}{e^{2ia}-x^2}+2e^{ia}\text{arctanh}\left(e^{-ia}x\right)$$

output
```
-x-2*exp(2*I*a)*x/(exp(2*I*a)-x^2)+2*exp(I*a)*arctanh(x/exp(I*a))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.46

$$\int \cot^2(a+i\log(x))\,dx = 2\text{arctanh}(x(\cos(a)-i\sin(a)))(\cos(a)+i\sin(a)) + \frac{-x(-3+x^2)\cos(a)+ix(3+x^2)\sin(a)}{(-1+x^2)\cos(a)-i(1+x^2)\sin(a)}$$

input
```
Integrate[Cot[a + I*Log[x]]^2,x]
```

output
```
2*ArcTanh[x*(Cos[a] - I*Sin[a])]*(Cos[a] + I*Sin[a]) + (-(x*(-3 + x^2)*Cos
[a]) + I*x*(3 + x^2)*Sin[a])/((-1 + x^2)*Cos[a] - I*(1 + x^2)*Sin[a])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5003, 898, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(a+i\log(x))\,dx$$

$$\Big\downarrow \text{5003}$$

$$\int \frac{\left(-\frac{ie^{2ia}}{x^2}-i\right)^2}{\left(1-\frac{e^{2ia}}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{898}$$

$$\int \frac{\left(-ie^{2ia}-ix^2\right)^2}{\left(x^2-e^{2ia}\right)^2}dx$$

$$\Big\downarrow \text{300}$$

$$\int \left(-1-\frac{4e^{2ia}x^2}{\left(x^2-e^{2ia}\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2e^{ia}\text{arctanh}\left(e^{-ia}x\right)-\frac{2e^{2ia}x}{-x^2+e^{2ia}}-x$$

input
```
Int[Cot[a + I*Log[x]]^2,x]
```

output
```
-x - (2*E^((2*I)*a)*x)/(E^((2*I)*a) - x^2) + 2*E^(I*a)*ArcTanh[x/E^(I*a)]
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 898
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5003
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((-I - I*E^(
2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b,
d, p}, x]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

method	result	size
risch	$-3x-\frac{2x}{\frac{e^{2ia}}{x^{2}}-1}+2\,\operatorname{arctanh}\left(x\,e^{-ia}\right) e^{ia}$	36

input
```
int(cot(a+I*ln(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-3*x-2*x/(exp(2*I*a)/x^2-1)+2*arctanh(x*exp(-I*a))*exp(I*a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.50

$$\int \cot^2(a+i\log(x))\,dx = -\frac{x^3 - \left(x^2 - e^{(2ia)}\right)e^{(ia)}\log\left(x + e^{(ia)}\right) + \left(x^2 - e^{(2ia)}\right)e^{(ia)}\log\left(x - e^{(ia)}\right) - 3\,xe^{(2ia)}}{x^2 - e^{(2ia)}}$$

input
```
integrate(cot(a+I*log(x))^2,x, algorithm="fricas")
```

output
```
-(x^3 - (x^2 - e^(2*I*a))*e^(I*a)*log(x + e^(I*a)) + (x^2 - e^(2*I*a))*e^(
I*a)*log(x - e^(I*a)) - 3*x*e^(2*I*a))/(x^2 - e^(2*I*a))
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \cot^2(a+i\log(x))\,dx = -x + \frac{2xe^{2ia}}{x^2 - e^{2ia}} - \left(\log\left(x - e^{ia}\right) - \log\left(x + e^{ia}\right)\right)e^{ia}$$

input
```
integrate(cot(a+I*ln(x))**2,x)
```

output
```
-x + 2*x*exp(2*I*a)/(x**2 - exp(2*I*a)) - (log(x - exp(I*a)) - log(x + exp
(I*a)))*exp(I*a)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(36) = 72.

Time = 0.04 (sec) , antiderivative size = 270, normalized size of antiderivative = 5.62

$$\int \cot^2(a+i\log(x))\,dx = -\frac{2\left((-i\cos(a) + \sin(a))\arctan(\sin(a), x + \cos(a)) + (-i\cos(a) + \sin(a))\arctan(\sin(a), x - \cos(a\ldots\right.}{}$$

input
```
integrate(cot(a+I*log(x))^2,x, algorithm="maxima")
```

output
```
-1/2*(2*((-I*cos(a) + sin(a))*arctan2(sin(a), x + cos(a)) + (-I*cos(a) + s
in(a))*arctan2(sin(a), x - cos(a)))*x^2 + 2*x^3 - 6*x*(cos(2*a) + I*sin(2*
a)) + 2*((I*cos(a) - sin(a))*cos(2*a) - (cos(a) + I*sin(a))*sin(2*a))*arct
an2(sin(a), x + cos(a)) + 2*((I*cos(a) - sin(a))*cos(2*a) - (cos(a) + I*si
n(a))*sin(2*a))*arctan2(sin(a), x - cos(a)) - (x^2*(cos(a) + I*sin(a)) - (
cos(a) + I*sin(a))*cos(2*a) + (-I*cos(a) + sin(a))*sin(2*a))*log(x^2 + 2*x
*cos(a) + cos(a)^2 + sin(a)^2) + (x^2*(cos(a) + I*sin(a)) - (cos(a) + I*si
n(a))*cos(2*a) - (I*cos(a) - sin(a))*sin(2*a))*log(x^2 - 2*x*cos(a) + cos(
a)^2 + sin(a)^2))/(x^2 - cos(2*a) - I*sin(2*a))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(36) = 72$.

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.65

$$\int \cot^2(a + i\log(x))\,dx = -\frac{x^3}{x^2 - e^{(2i\,a)}} - 2\left(\frac{\arctan\left(\frac{x}{\sqrt{-e^{(2i\,a)}}}\right)}{\sqrt{-e^{(2i\,a)}}} - \frac{x}{x^2 - e^{(2i\,a)}}\right)e^{(2i\,a)} + \frac{5\,xe^{(2i\,a)}}{x^2 - e^{(2i\,a)}}$$

input
```
integrate(cot(a+I*log(x))^2,x, algorithm="giac")
```

output
```
-x^3/(x^2 - e^(2*I*a)) - 2*(arctan(x/sqrt(-e^(2*I*a)))/sqrt(-e^(2*I*a)) -
x/(x^2 - e^(2*I*a)))*e^(2*I*a) + 5*x*e^(2*I*a)/(x^2 - e^(2*I*a))
```

Mupad [B] (verification not implemented)

Time = 19.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \cot^2(a+i\log(x))\,dx = -x+2\,\sqrt{\mathrm{e}^{a\,2\mathrm{i}}}\,\mathrm{atanh}\left(\frac{x}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}}}\right)-\frac{2\,x\,\mathrm{e}^{a\,2\mathrm{i}}}{\mathrm{e}^{a\,2\mathrm{i}}-x^2}$$

input
```
int(cot(a + log(x)*1i)^2,x)
```

output
```
2*exp(a*2i)^(1/2)*atanh(x/exp(a*2i)^(1/2)) - x - (2*x*exp(a*2i))/(exp(a*2i
) - x^2)
```

Reduce [F]

$$\int \cot^2(a+i\log(x))\,dx = \int \cot\left(\log(x)\,i+a\right)^2 dx$$

input
```
int(cot(a+I*log(x))^2,x)
```

output
```
int(cot(log(x)*i + a)**2,x)
```

3.198 $\int \frac{\cot^2(a+i\log(x))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\cot^2(a+i\log(x))}{x}\,dx = i\cot(a+i\log(x)) - \log(x)$$

output
```
I*cot(a+I*ln(x))-ln(x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{\cot^2(a+i\log(x))}{x}\,dx = i\cot(a+i\log(x))\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(a+i\log(x))\right)$$

input
```
Integrate[Cot[a + I*Log[x]]^2/x,x]
```

output
```
I*Cot[a + I*Log[x]]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[a + I*Log[x]]^2]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3039, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(a+i\log(x))}{x}\,dx$$

$$\Big\downarrow \text{3039}$$

$$\int \cot^2(a+i\log(x))d\log(x)$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(a+i\log(x)+\frac{\pi}{2}\right)^2 d\log(x)$$

$$\Big\downarrow \text{3954}$$

$$-\int 1d\log(x)+i\cot(a+i\log(x))$$

$$\Big\downarrow \text{24}$$

$$-\log(x)+i\cot(a+i\log(x))$$

input
```
Int[Cot[a + I*Log[x]]^2/x,x]
```

output
```
I*Cot[a + I*Log[x]] - Log[x]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
parallelrisch	$i\cot\left(a+i\ln\left(x\right)\right)-\ln\left(x\right)$	17
risch	$-\ln\left(x\right)-\frac{2}{\frac{\mathrm{e}^{2ia}}{x^{2}}-1}$	21
derivativedivides	$i\left(\cot\left(a+i\ln\left(x\right)\right)-\frac{\pi}{2}+\operatorname{arccot}\left(\cot\left(a+i\ln\left(x\right)\right)\right)\right)$	25
default	$i\left(\cot\left(a+i\ln\left(x\right)\right)-\frac{\pi}{2}+\operatorname{arccot}\left(\cot\left(a+i\ln\left(x\right)\right)\right)\right)$	25
norman	$\frac{-\ln\left(x\right)\tan\left(a+i\ln\left(x\right)\right)+i}{\tan\left(a+i\ln\left(x\right)\right)}$	27

input
```
int(cot(a+I*ln(x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
I*cot(a+I*ln(x))-ln(x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(14) = 28.

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{\cot^2(a+i\log(x))}{x}\,dx = -\frac{\left(x^2-e^{(2i\,a)}\right)\log\left(x\right)-2\,e^{(2i\,a)}}{x^2-e^{(2i\,a)}}$$

input
```
integrate(cot(a+I*log(x))^2/x,x, algorithm="fricas")
```

output
```
-((x^2 - e^(2*I*a))*log(x) - 2*e^(2*I*a))/(x^2 - e^(2*I*a))
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cot^2(a+i\log(x))}{x}\,dx = -\log\left(x\right)+\frac{2e^{2ia}}{x^2-e^{2ia}}$$

input
```
integrate(cot(a+I*ln(x))**2/x,x)
```

output
```
-log(x) + 2*exp(2*I*a)/(x**2 - exp(2*I*a))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\cot^2(a+i\log(x))}{x}\,dx = i\,a+\frac{i}{\tan\left(a+i\,\log\left(x\right)\right)}-\log\left(x\right)$$

input
```
integrate(cot(a+I*log(x))^2/x,x, algorithm="maxima")
```

output
```
I*a + I/tan(a + I*log(x)) - log(x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(14) = 28.

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \frac{\cot^2(a+i\log(x))}{x}\,dx = i\,a + \frac{i}{2\,\tan\left(\frac{1}{2}\,a + \frac{1}{2}i\,\log\left(x\right)\right)} - \log\left(x\right) - \frac{1}{2}i\,\tan\left(\frac{1}{2}\,a + \frac{1}{2}i\,\log\left(x\right)\right)$$

input
```
integrate(cot(a+I*log(x))^2/x,x, algorithm="giac")
```

output
```
I*a + 1/2*I/tan(1/2*a + 1/2*I*log(x)) - log(x) - 1/2*I*tan(1/2*a + 1/2*I*l
og(x))
```

Mupad [B] (verification not implemented)

Time = 21.45 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\cot^2(a+i\log(x))}{x}\,dx = -\ln\left(x\right) + \cot(a+\ln\left(x\right)\ 1\mathrm{i})\ 1\mathrm{i}$$

input
```
int(cot(a + log(x)*1i)^2/x,x)
```

output
```
cot(a + log(x)*1i)*1i - log(x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\cot^2(a+i\log(x))}{x}\,dx = \cot(\log(x)\,i+a)\,i-\log(x)$$

input
```
int(cot(a+I*log(x))^2/x,x)
```

output
```
cot(log(x)*i + a)*i - log(x)
```

3.199 $\int \frac{\cot^2(a+i\log(x))}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{\cot^2(a+i\log(x))}{x^2}\,dx=\frac{1}{x}-\frac{2x}{e^{2ia}-x^2}-2e^{-ia}\text{arctanh}\left(e^{-ia}x\right)$$

output
```
1/x-2*x/(exp(2*I*a)-x^2)-2*arctanh(x/exp(I*a))/exp(I*a)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.76

$$\begin{aligned}\int \frac{\cot^2(a+i\log(x))}{x^2}\,dx=&\frac{1}{x}-2\text{arctanh}(x(\cos(a)-i\sin(a)))\cos(a)\\&+2i\text{arctanh}(x(\cos(a)-i\sin(a)))\sin(a)\\&+\frac{2x(\cos(a)-i\sin(a))}{(-1+x^2)\cos(a)-i(1+x^2)\sin(a)}\end{aligned}$$

input
```
Integrate[Cot[a + I*Log[x]]^2/x^2,x]
```

output
```
x^(-1) - 2*ArcTanh[x*(Cos[a] - I*Sin[a])]*Cos[a] + (2*I)*ArcTanh[x*(Cos[a]
 - I*Sin[a])]*Sin[a] + (2*x*(Cos[a] - I*Sin[a]))/((-1 + x^2)*Cos[a] - I*(1
 + x^2)*Sin[a])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5007, 947, 365, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(a+i\log(x))}{x^2}dx$$
$$\Big\downarrow \text{5007}$$
$$\int \frac{\left(-\frac{ie^{2ia}}{x^2}-i\right)^2}{x^2\left(1-\frac{e^{2ia}}{x^2}\right)^2}dx$$
$$\Big\downarrow \text{947}$$
$$\int \frac{\left(-ie^{2ia}-ix^2\right)^2}{x^2\left(x^2-e^{2ia}\right)^2}dx$$
$$\Big\downarrow \text{365}$$
$$\frac{e^{2ia}}{x\left(-x^2+e^{2ia}\right)}-e^{-2ia}\int \frac{e^{2ia}x^2+5e^{4ia}}{\left(e^{2ia}-x^2\right)^2}dx$$
$$\Big\downarrow \text{298}$$
$$\frac{e^{2ia}}{x\left(-x^2+e^{2ia}\right)}-e^{-2ia}\left(2e^{2ia}\int \frac{1}{e^{2ia}-x^2}dx+\frac{3e^{2ia}x}{-x^2+e^{2ia}}\right)$$
$$\Big\downarrow \text{219}$$
$$\frac{e^{2ia}}{x\left(-x^2+e^{2ia}\right)}-e^{-2ia}\left(2e^{ia}\text{arctanh}\left(e^{-ia}x\right)+\frac{3e^{2ia}x}{-x^2+e^{2ia}}\right)$$

input
```
Int[Cot[a + I*Log[x]]^2/x^2,x]
```

output
```
E^((2*I)*a)/(x*(E^((2*I)*a) - x^2)) - ((3*E^((2*I)*a)*x)/(E^((2*I)*a) - x^
2) + 2*E^(I*a)*ArcTanh[x/E^(I*a)])/E^((2*I)*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 947
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Int[x^(m + n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[
n]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

method	result	size
risch	$\frac{1}{x}-\frac{2}{x\left(\frac{e^{2ia}}{x^2}-1\right)}-2\,\operatorname{arctanh}\left(x\,e^{-ia}\right)e^{-ia}$	38

input
```
int(cot(a+I*ln(x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/x-2/x/(exp(2*I*a)/x^2-1)-2*arctanh(x*exp(-I*a))*exp(-I*a)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(32) = 64.

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.80

$$\int \frac{\cot^2(a+i\log(x))}{x^2}\,dx = -\frac{\left(x^3-xe^{(2i\,a)}\right)e^{(-i\,a)}\log\left(x+e^{(i\,a)}\right)-\left(x^3-xe^{(2i\,a)}\right)e^{(-i\,a)}\log\left(x-e^{(i\,a)}\right)-3\,x^2+e^{(2i\,a)}}{x^3-xe^{(2i\,a)}}$$

input
```
integrate(cot(a+I*log(x))^2/x^2,x, algorithm="fricas")
```

output
```
-((x^3 - x*e^(2*I*a))*e^(-I*a)*log(x + e^(I*a)) - (x^3 - x*e^(2*I*a))*e^(-
I*a)*log(x - e^(I*a)) - 3*x^2 + e^(2*I*a))/(x^3 - x*e^(2*I*a))
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{\cot^2(a+i\log(x))}{x^2}\,dx = -\frac{-3x^2+e^{2ia}}{x^3-xe^{2ia}} - \left(-\log\left(x-e^{ia}\right)+\log\left(x+e^{ia}\right)\right)e^{-ia}$$

input
```
integrate(cot(a+I*ln(x))**2/x**2,x)
```

output
```
-(-3*x**2 + exp(2*I*a))/(x**3 - x*exp(2*I*a)) - (-log(x - exp(I*a)) + log(
x + exp(I*a)))*exp(-I*a)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(32) = 64.

Time = 0.05 (sec) , antiderivative size = 276, normalized size of antiderivative = 6.73

$$\int \frac{\cot^2(a+i\log(x))}{x^2}\,dx = -\frac{2\left((i\cos(a)+\sin(a))\arctan(\sin(a),x+\cos(a))+(i\cos(a)+\sin(a))\arctan(\sin(a),x-\cos(a))\right)}{}$$

input
```
integrate(cot(a+I*log(x))^2/x^2,x, algorithm="maxima")
```

output
```
-1/2*(2*((I*cos(a) + sin(a))*arctan2(sin(a), x + cos(a)) + (I*cos(a) + sin
(a))*arctan2(sin(a), x - cos(a)))*x^3 + 2*(((-I*cos(a) - sin(a))*cos(2*a)
+ (cos(a) - I*sin(a))*sin(2*a))*arctan2(sin(a), x + cos(a)) + ((-I*cos(a)
- sin(a))*cos(2*a) + (cos(a) - I*sin(a))*sin(2*a))*arctan2(sin(a), x - cos
(a)))*x - 6*x^2 + (x^3*(cos(a) - I*sin(a)) - ((cos(a) - I*sin(a))*cos(2*a)
 + (I*cos(a) + sin(a))*sin(2*a))*x)*log(x^2 + 2*x*cos(a) + cos(a)^2 + sin(
a)^2) - (x^3*(cos(a) - I*sin(a)) - ((cos(a) - I*sin(a))*cos(2*a) - (-I*cos
(a) - sin(a))*sin(2*a))*x)*log(x^2 - 2*x*cos(a) + cos(a)^2 + sin(a)^2) + 2
*cos(2*a) + 2*I*sin(2*a))/(x^3 - x*(cos(2*a) + I*sin(2*a)))
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(32) = 64.

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.12

$$\int \frac{\cot^2(a+i\log(x))}{x^2}\,dx = 2\left(\frac{\arctan\left(\frac{x}{\sqrt{-e^{(2ia)}}}\right)e^{(-2ia)}}{\sqrt{-e^{(2ia)}}} + \frac{xe^{(-2ia)}}{x^2-e^{(2ia)}}\right)e^{(2ia)} + \frac{5x^2}{x^3-xe^{(2ia)}} - \frac{e^{(2ia)}}{x^3-xe^{(2ia)}}$$

input
```
integrate(cot(a+I*log(x))^2/x^2,x, algorithm="giac")
```

output
```
2*(arctan(x/sqrt(-e^(2*I*a)))*e^(-2*I*a)/sqrt(-e^(2*I*a)) + x*e^(-2*I*a)/(
x^2 - e^(2*I*a)))*e^(2*I*a) + 5*x^2/(x^3 - x*e^(2*I*a)) - e^(2*I*a)/(x^3 -
 x*e^(2*I*a))
```

Mupad [B] (verification not implemented)

Time = 21.84 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

$$\int \frac{\cot^2(a+i\log(x))}{x^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{x}{\sqrt{e^{a\,2i}}}\right)}{\sqrt{e^{a\,2i}}} - \frac{e^{a\,2i}-3\,x^2}{x^3-x\,e^{a\,2i}}$$

input
```
int(cot(a + log(x)*1i)^2/x^2,x)
```

output
```
- (2*atanh(x/exp(a*2i)^(1/2)))/exp(a*2i)^(1/2) - (exp(a*2i) - 3*x^2)/(x^3
- x*exp(a*2i))
```

Reduce [F]

$$\int \frac{\cot^2(a + i\log(x))}{x^2}\,dx = \int \frac{\cot\left(\log(x)\,i + a\right)^2}{x^2}dx$$

input
```
int(cot(a+I*log(x))^2/x^2,x)
```

output
```
int(cot(log(x)*i + a)**2/x**2,x)
```

3.200 $\int \frac{\cot^2(a+i\log(x))}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{\cot^2(a+i\log(x))}{x^3} dx = \frac{2e^{-2ia}}{1-\frac{e^{2ia}}{x^2}} + \frac{1}{2x^2} + 2e^{-2ia}\log\left(1-\frac{e^{2ia}}{x^2}\right)$$

output
```
2/exp(2*I*a)/(1-exp(2*I*a)/x^2)+1/2/x^2+2*ln(1-exp(2*I*a)/x^2)/exp(2*I*a)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 153 vs. 2(57) = 114.

Time = 0.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.68

$$\begin{aligned}\int \frac{\cot^2(a+i\log(x))}{x^3} dx = &\frac{1}{2x^2} + \cos(2a)\left(-4\log(x)+\log\left(1+x^4-2x^2\cos(2a)\right)\right) \\ &+ \frac{2\cos(a)}{(-1+x^2)\cos(a)-i(1+x^2)\sin(a)} \\ &+ \frac{2\sin(a)}{i(-1+x^2)\cos(a)+(1+x^2)\sin(a)} \\ &+ \arctan\left(\frac{\cot(a)-x^2\cot(a)}{1+x^2}\right)(-2i\cos(2a)-4\cos(a)\sin(a)) \\ &+ 4i\log(x)\sin(2a)-i\log\left(1+x^4-2x^2\cos(2a)\right)\sin(2a)\end{aligned}$$

input
```
Integrate[Cot[a + I*Log[x]]^2/x^3,x]
```

output
```
1/(2*x^2) + Cos[2*a]*(-4*Log[x] + Log[1 + x^4 - 2*x^2*Cos[2*a]]) + (2*Cos[
a])/((-1 + x^2)*Cos[a] - I*(1 + x^2)*Sin[a]) + (2*Sin[a])/(I*(-1 + x^2)*Co
s[a] + (1 + x^2)*Sin[a]) + ArcTan[(Cot[a] - x^2*Cot[a])/(1 + x^2)]*((-2*I)
*Cos[2*a] - 4*Cos[a]*Sin[a]) + (4*I)*Log[x]*Sin[2*a] - I*Log[1 + x^4 - 2*x
^2*Cos[2*a]]*Sin[2*a]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5007, 946, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(a+i\log(x))}{x^3}dx$$

$$\Big\downarrow \text{5007}$$

$$\int \frac{\left(-\frac{ie^{2ia}}{x^2}-i\right)^2}{x^3\left(1-\frac{e^{2ia}}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{946}$$

$$-\frac{1}{2}\int -\frac{\left(1+\frac{e^{2ia}}{x^2}\right)^2}{\left(1-\frac{e^{2ia}}{x^2}\right)^2}d\frac{1}{x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{\left(1+\frac{e^{2ia}}{x^2}\right)^2}{\left(1-\frac{e^{2ia}}{x^2}\right)^2}d\frac{1}{x^2}$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(1+\frac{4}{\frac{e^{2ia}}{x^2}-1}+\frac{4}{\left(\frac{e^{2ia}}{x^2}-1\right)^2}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{4e^{-2ia}}{1-\frac{e^{2ia}}{x^2}}+4e^{-2ia}\log\left(1-\frac{e^{2ia}}{x^2}\right)+\frac{1}{x^2}\right)$$

input
```
Int[Cot[a + I*Log[x]]^2/x^3,x]
```

output
```
(4/(E^((2*I)*a)*(1 - E^((2*I)*a)/x^2)) + x^(-2) + (4*Log[1 - E^((2*I)*a)/x
^2])/E^((2*I)*a))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

method	result	size
risch	$\frac{1}{2x^{2}}-\frac{2}{x^{2}\left(\frac{\mathrm{e}^{2ia}}{x^{2}}-1\right)}-4\,\mathrm{e}^{-2ia}\ln\left(x\right)+2\,\mathrm{e}^{-2ia}\ln\left(\mathrm{e}^{2ia}-x^{2}\right)$	53

input
```
int(cot(a+I*ln(x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/x^2-2/x^2/(exp(2*I*a)/x^2-1)-4*exp(-2*I*a)*ln(x)+2*exp(-2*I*a)*ln(exp(
2*I*a)-x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.42

$$\int\frac{\cot^{2}(a+i\log(x))}{x^{3}}\,dx =\frac{5\,x^{2}e^{(2i\,a)}+4\left(x^{4}-x^{2}e^{(2i\,a)}\right)\log\left(x^{2}-e^{(2i\,a)}\right)-8\left(x^{4}-x^{2}e^{(2i\,a)}\right)\log\left(x\right)-e^{(4i\,a)}}{2\left(x^{4}e^{(2i\,a)}-x^{2}e^{(4i\,a)}\right)}$$

input
```
integrate(cot(a+I*log(x))^2/x^3,x, algorithm="fricas")
```

output
```
1/2*(5*x^2*e^(2*I*a) + 4*(x^4 - x^2*e^(2*I*a))*log(x^2 - e^(2*I*a)) - 8*(x
^4 - x^2*e^(2*I*a))*log(x) - e^(4*I*a))/(x^4*e^(2*I*a) - x^2*e^(4*I*a))
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int\frac{\cot^{2}(a+i\log(x))}{x^{3}}\,dx=-\frac{-5x^{2}+e^{2ia}}{2x^{4}-2x^{2}e^{2ia}}-4e^{-2ia}\log\left(x\right)+2e^{-2ia}\log\left(x^{2}-e^{2ia}\right)$$

input
```
integrate(cot(a+I*ln(x))**2/x**3,x)
```

output
```
-(-5*x**2 + exp(2*I*a))/(2*x**4 - 2*x**2*exp(2*I*a)) - 4*exp(-2*I*a)*log(x
) + 2*exp(-2*I*a)*log(x**2 - exp(2*I*a))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^2(a+i\log(x))}{x^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cot(a+I*log(x))^2/x^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(42) = 84.

Time = 0.16 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.33

$$\int \frac{\cot^2(a+i\log(x))}{x^3}\,dx = \frac{2\,x^4\log\left(x^2-e^{(2i\,a)}\right)}{x^4e^{(2i\,a)}-x^2e^{(4i\,a)}} - \frac{4\,x^4\log\left(x\right)}{x^4e^{(2i\,a)}-x^2e^{(4i\,a)}} - \frac{2\,x^2e^{(2i\,a)}\log\left(x^2-e^{(2i\,a)}\right)}{x^4e^{(2i\,a)}-x^2e^{(4i\,a)}} + \frac{4\,x^2e^{(2i\,a)}\log\left(x\right)}{x^4e^{(2i\,a)}-x^2e^{(4i\,a)}} + \frac{5\,x^2e^{(2i\,a)}}{2\left(x^4e^{(2i\,a)}-x^2e^{(4i\,a)}\right)} - \frac{e^{(4i\,a)}}{2\left(x^4e^{(2i\,a)}-x^2e^{(4i\,a)}\right)}$$

input
```
integrate(cot(a+I*log(x))^2/x^3,x, algorithm="giac")
```

output
```
2*x^4*log(x^2 - e^(2*I*a))/(x^4*e^(2*I*a) - x^2*e^(4*I*a)) - 4*x^4*log(x)/
(x^4*e^(2*I*a) - x^2*e^(4*I*a)) - 2*x^2*e^(2*I*a)*log(x^2 - e^(2*I*a))/(x^
4*e^(2*I*a) - x^2*e^(4*I*a)) + 4*x^2*e^(2*I*a)*log(x)/(x^4*e^(2*I*a) - x^2
*e^(4*I*a)) + 5/2*x^2*e^(2*I*a)/(x^4*e^(2*I*a) - x^2*e^(4*I*a)) - 1/2*e^(4
*I*a)/(x^4*e^(2*I*a) - x^2*e^(4*I*a))
```

Mupad [B] (verification not implemented)

Time = 22.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{\cot^2(a+i\log(x))}{x^3}\,dx = -4\,\mathrm{e}^{-a\,2\mathrm{i}}\,\ln(x)+2\,\ln\left(x^2-\mathrm{e}^{a\,2\mathrm{i}}\right)\,\mathrm{e}^{-a\,2\mathrm{i}}+\frac{\frac{\mathrm{e}^{a\,2\mathrm{i}}}{2}-\frac{5\,x^2}{2}}{x^2\,\mathrm{e}^{a\,2\mathrm{i}}-x^4}$$

input
```
int(cot(a + log(x)*1i)^2/x^3,x)
```

output
```
2*log(x^2 - exp(a*2i))*exp(-a*2i) - 4*exp(-a*2i)*log(x) + (exp(a*2i)/2 - (
5*x^2)/2)/(x^2*exp(a*2i) - x^4)
```

Reduce [F]

$$\int \frac{\cot^2(a+i\log(x))}{x^3}\,dx = \int \frac{\cot(\log(x)\,i+a)^2}{x^3}dx$$

input
```
int(cot(a+I*log(x))^2/x^3,x)
```

output
```
int(cot(log(x)*i + a)**2/x**3,x)
```

3.201 $\int (ex)^m \cot(a+i\log(x))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int (ex)^m \cot(a+i\log(x))\,dx = \frac{i(ex)^{1+m}}{e(1+m)}-\frac{2i(ex)^{1+m}\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{e^{2ia}}{x^2}\right)}{e(1+m)}$$

output
```
I*(e*x)^(1+m)/e/(1+m)-2*I*(e*x)^(1+m)*hypergeom([1, -1/2-1/2*m],[1/2-1/2*m
],exp(2*I*a)/x^2)/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.47

$$\int (ex)^m \cot(a+i\log(x))\,dx = ix(ex)^m\left(\frac{\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},x^2(\cos(2a)-i\sin(2a))\right)}{1+m}+\frac{x^2\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{2},\frac{5+m}{2},x^2(\cos(2a)-i\sin(2a))\right)(\cos(a)-i\sin(a))^2}{3+m}\right)$$

input
```
Integrate[(e*x)^m*Cot[a + I*Log[x]],x]
```

output
```
I*x*(e*x)^m*(Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, x^2*(Cos[2*a] - I*
Sin[2*a])]/(1 + m) + (x^2*Hypergeometric2F1[1, (3 + m)/2, (5 + m)/2, x^2*(
Cos[2*a] - I*Sin[2*a])]*(Cos[a] - I*Sin[a])^2)/(3 + m))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5007, 959, 862, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \cot(a + i\log(x))\,dx$$

$$\Big\downarrow \text{5007}$$

$$\int \frac{\left(-\frac{ie^{2ia}}{x^2} - i\right)(ex)^m}{1 - \frac{e^{2ia}}{x^2}}dx$$

$$\Big\downarrow \text{959}$$

$$\frac{i(ex)^{m+1}}{e(m+1)} - 2i\int \frac{(ex)^m}{1 - \frac{e^{2ia}}{x^2}}dx$$

$$\Big\downarrow \text{862}$$

$$\frac{2i\left(\frac{1}{x}\right)^{m+1}(ex)^{m+1}\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{1-\frac{e^{2ia}}{x^2}}d\frac{1}{x}}{e} + \frac{i(ex)^{m+1}}{e(m+1)}$$

$$\Big\downarrow \text{278}$$

$$\frac{i(ex)^{m+1}}{e(m+1)} - \frac{2i(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1, \frac{1}{2}(-m-1), \frac{1-m}{2}, \frac{e^{2ia}}{x^2}\right)}{e(m+1)}$$

input
```
Int[(e*x)^m*Cot[a + I*Log[x]],x]
```

output
```
(I*(e*x)^(1 + m))/(e*(1 + m)) - ((2*I)*(e*x)^(1 + m)*Hypergeometric2F1[1,
(-1 - m)/2, (1 - m)/2, E^((2*I)*a)/x^2])/(e*(1 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] && !RationalQ[m]
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [F]

$$\int (ex)^m \cot (a + i \ln (x))\, dx$$

input
```
int((e*x)^m*cot(a+I*ln(x)),x)
```

output
```
int((e*x)^m*cot(a+I*ln(x)),x)
```

Fricas [F]

$$\int (ex)^m \cot(a + i\log(x))\,dx = \int (ex)^m \cot(a + i\log(x))\,dx$$

input
```
integrate((e*x)^m*cot(a+I*log(x)),x, algorithm="fricas")
```

output
```
integral(-(I*x^2 + I*e^(2*I*a))*e^(m*log(e) + m*log(x))/(x^2 - e^(2*I*a)),
 x)
```

Sympy [F]

$$\int (ex)^m \cot(a + i\log(x))\,dx = \int (ex)^m \cot(a + i\log(x))\,dx$$

input
```
integrate((e*x)**m*cot(a+I*ln(x)),x)
```

output
```
Integral((e*x)**m*cot(a + I*log(x)), x)
```

Maxima [F]

$$\int (ex)^m \cot(a + i\log(x))\,dx = \int (ex)^m \cot(a + i\log(x))\,dx$$

input
```
integrate((e*x)^m*cot(a+I*log(x)),x, algorithm="maxima")
```

output
```
integrate((e*x)^m*cot(a + I*log(x)), x)
```

Giac [F]

$$\int (ex)^m \cot(a+i\log(x))\,dx = \int (ex)^m \cot\left(a+i\,\log\left(x\right)\right)\,dx$$

input
```
integrate((e*x)^m*cot(a+I*log(x)),x, algorithm="giac")
```

output
```
integrate((e*x)^m*cot(a + I*log(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \cot(a+i\log(x))\,dx = \int \cot(a+\ln\left(x\right)\,1\mathrm{i})\,\left(e\,x\right)^m\,dx$$

input
```
int(cot(a + log(x)*1i)*(e*x)^m,x)
```

output
```
int(cot(a + log(x)*1i)*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \cot(a+i\log(x))\,dx = e^m \left(\int x^m \cot\left(\log(x)\,i+a\right)dx\right)$$

input
```
int((e*x)^m*cot(a+I*log(x)),x)
```

output
```
e**m*int(x**m*cot(log(x)*i + a),x)
```

3.202 $\int (ex)^m \cot^2(a+i\log(x))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 77

$$\int (ex)^m \cot^2(a+i\log(x))\,dx = -\frac{x(ex)^m}{1+m} + \frac{2x(ex)^m}{1-\frac{e^{2ia}}{x^2}} - 2x(ex)^m \operatorname{Hypergeometric2F1}\left(1, \frac{1}{2}(-1-m), \frac{1-m}{2}, \frac{e^{2ia}}{x^2}\right)$$

output
```
-x*(e*x)^m/(1+m)+2*x*(e*x)^m/(1-exp(2*I*a)/x^2)-2*x*(e*x)^m*hypergeom([1,
-1/2-1/2*m],[1/2-1/2*m],exp(2*I*a)/x^2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

$$\int (ex)^m \cot^2(a+i\log(x))\,dx = \frac{x(ex)^m \left(-1+4\operatorname{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},x^2(\cos(2a)-i\sin(2a))\right)-4\operatorname{Hypergeometric2F1}\left(2,\right.\right.}{1+m}$$

input
```
Integrate[(e*x)^m*Cot[a + I*Log[x]]^2,x]
```

output
```
(x*(e*x)^m*(-1 + 4*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, x^2*(Cos[2*a
] - I*Sin[2*a])] - 4*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, x^2*(Cos[2
*a] - I*Sin[2*a])]))/(1 + m)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.58, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5007, 999, 25, 366, 27, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \cot^2(a+i\log(x))\,dx$$

$$\Big\downarrow \text{5007}$$

$$\int \frac{\left(-\frac{ie^{2ia}}{x^2}-i\right)^2 (ex)^m}{\left(1-\frac{e^{2ia}}{x^2}\right)^2}dx$$

$$\Big\downarrow \text{999}$$

$$-\left(\frac{1}{x}\right)^m (ex)^m \int -\frac{\left(1+\frac{e^{2ia}}{x^2}\right)^2 \left(\frac{1}{x}\right)^{-m-2}}{\left(1-\frac{e^{2ia}}{x^2}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{25}$$

$$\left(\frac{1}{x}\right)^m (ex)^m \int \frac{\left(1+\frac{e^{2ia}}{x^2}\right)^2 \left(\frac{1}{x}\right)^{-m-2}}{\left(1-\frac{e^{2ia}}{x^2}\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{366}$$

$$-\left(\frac{1}{x}\right)^m (ex)^m \left(\frac{1}{2}e^{-4ia}\int -\frac{2\left(e^{4ia}(2m+3)-\frac{e^{6ia}}{x^2}\right)\left(\frac{1}{x}\right)^{-m-2}}{1-\frac{e^{2ia}}{x^2}}d\frac{1}{x}-\frac{2\left(\frac{1}{x}\right)^{-m-1}}{1-\frac{e^{2ia}}{x^2}}\right)$$

$$\Big\downarrow \text{27}$$

$$-\left(\frac{1}{x}\right)^m (ex)^m \left(-e^{-4ia}\int \frac{\left(e^{4ia}(2m+3)-\frac{e^{6ia}}{x^2}\right)\left(\frac{1}{x}\right)^{-m-2}}{1-\frac{e^{2ia}}{x^2}}d\frac{1}{x}-\frac{2\left(\frac{1}{x}\right)^{-m-1}}{1-\frac{e^{2ia}}{x^2}}\right)$$

$$\Big\downarrow \text{363}$$

$$-\left(\frac{1}{x}\right)^m (ex)^m \left(-e^{-4ia}\left(2e^{4ia}(m+1)\int \frac{\left(\frac{1}{x}\right)^{-m-2}}{1-\frac{e^{2ia}}{x^2}}d\frac{1}{x}-\frac{e^{4ia}\left(\frac{1}{x}\right)^{-m-1}}{m+1}\right)-\frac{2\left(\frac{1}{x}\right)^{-m-1}}{1-\frac{e^{2ia}}{x^2}}\right)$$

$$\Big\downarrow \text{278}$$

$$-\left(\frac{1}{x}\right)^m (ex)^m \left(-e^{-4ia}\left(-2e^{4ia}\left(\frac{1}{x}\right)^{-m-1}\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{e^{2ia}}{x^2}\right)-\frac{e^{4ia}\left(\frac{1}{x}\right)^{-m-1}}{m+1}\right.\right.$$

input
```
Int[(e*x)^m*Cot[a + I*Log[x]]^2,x]
```

output
```
-((x^(-1))^m*(e*x)^m*((-2*(x^(-1))^(-1 - m))/(1 - E^((2*I)*a)/x^2) - (-((E
^((4*I)*a)*(x^(-1))^(-1 - m))/(1 + m)) - 2*E^((4*I)*a)*(x^(-1))^(-1 - m)*H
ypergeometric2F1[1, (-1 - m)/2, (1 - m)/2, E^((2*I)*a)/x^2])/E^((4*I)*a))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 999
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(-(e*x)^m)*(x^(-1))^m   Subst[Int[(a + b/x^n)^p*(
(c + d/x^n)^q/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q}
, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [F]

$$\int (ex)^m \cot(a + i \ln(x))^2 \, dx$$

input
```
int((e*x)^m*cot(a+I*ln(x))^2,x)
```

output
```
int((e*x)^m*cot(a+I*ln(x))^2,x)
```

Fricas [F]

$$\int (ex)^m \cot^2(a+i\log(x))\,dx = \int (ex)^m \cot\left(a+i\,\log\left(x\right)\right)^2\,dx$$

input
```
integrate((e*x)^m*cot(a+I*log(x))^2,x, algorithm="fricas")
```

output
```
integral(-(x^4 + 2*x^2*e^(2*I*a) + e^(4*I*a))*e^(m*log(e) + m*log(x))/(x^4
 - 2*x^2*e^(2*I*a) + e^(4*I*a)), x)
```

Sympy [F]

$$\int (ex)^m \cot^2(a+i\log(x))\,dx = \int (ex)^m \cot^2\left(a+i\log\left(x\right)\right)dx$$

input
```
integrate((e*x)**m*cot(a+I*ln(x))**2,x)
```

output
```
Integral((e*x)**m*cot(a + I*log(x))**2, x)
```

Maxima [F]

$$\int (ex)^m \cot^2(a+i\log(x))\,dx = \int (ex)^m \cot\left(a+i\,\log\left(x\right)\right)^2\,dx$$

input
```
integrate((e*x)^m*cot(a+I*log(x))^2,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*cot(a + I*log(x))^2, x)
```

Giac [F]

$$\int (ex)^m \cot^2(a+i\log(x))\,dx = \int (ex)^m \cot\left(a+i\log\left(x\right)\right)^2\,dx$$

input
```
integrate((e*x)^m*cot(a+I*log(x))^2,x, algorithm="giac")
```

output
```
integrate((e*x)^m*cot(a + I*log(x))^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \cot^2(a+i\log(x))\,dx = \int \cot(a+\ln\left(x\right)\,1\mathrm{i})^2\,{(e\,x)}^m\,dx$$

input
```
int(cot(a + log(x)*1i)^2*(e*x)^m,x)
```

output
```
int(cot(a + log(x)*1i)^2*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \cot^2(a+i\log(x))\,dx = e^m \left(\int x^m \cot\left(\log(x)\,i+a\right)^2dx\right)$$

input
```
int((e*x)^m*cot(a+I*log(x))^2,x)
```

output
```
e**m*int(x**m*cot(log(x)*i + a)**2,x)
```

3.203 $\int (ex)^m \cot^3(a+i\log(x))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 127

$$\int (ex)^m \cot^3(a+i\log(x))\,dx = -\frac{ix(ex)^m}{1+m}-\frac{2ix(ex)^m}{\left(1-\frac{e^{2ia}}{x^2}\right)^2}+\frac{i(1-m)x(ex)^m}{1-\frac{e^{2ia}}{x^2}} + \frac{i(3+2m+m^2)\,x(ex)^m \operatorname{Hypergeometric2F1}\left(1,\frac{1}{2}(-1-m),\frac{1-m}{2},\frac{e^{2ia}}{x^2}\right)}{1+m}$$

output
```
-I*x*(e*x)^m/(1+m)-2*I*x*(e*x)^m/(1-exp(2*I*a)/x^2)^2+I*(1-m)*x*(e*x)^m/(1
-exp(2*I*a)/x^2)+I*(m^2+2*m+3)*x*(e*x)^m*hypergeom([1, -1/2-1/2*m],[1/2-1/
2*m],exp(2*I*a)/x^2)/(1+m)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.96

$$\int (ex)^m \cot^3(a+i\log(x))\,dx = -\frac{ix(ex)^m\left(-1+6\operatorname{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},x^2(\cos(2a)-i\sin(2a))\right)-12\operatorname{Hypergeometric2F1}\ldots\right.}{1\ldots}$$

input
```
Integrate[(e*x)^m*Cot[a + I*Log[x]]^3,x]
```

output
```
((-I)*x*(e*x)^m*(-1 + 6*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, x^2*(Co
s[2*a] - I*Sin[2*a])] - 12*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, x^2*
(Cos[2*a] - I*Sin[2*a])] + 8*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, x^
2*(Cos[2*a] - I*Sin[2*a])]))/(1 + m)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.78, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {5007, 999, 26, 370, 27, 439, 27, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \cot^3(a+i\log(x))\,dx$$

$$\Big\downarrow \text{5007}$$

$$\int \frac{\left(-\frac{ie^{2ia}}{x^2}-i\right)^3 (ex)^m}{\left(1-\frac{e^{2ia}}{x^2}\right)^3}dx$$

$$\Big\downarrow \text{999}$$

$$-\left(\frac{1}{x}\right)^m (ex)^m \int \frac{i\left(1+\frac{e^{2ia}}{x^2}\right)^3 \left(\frac{1}{x}\right)^{-m-2}}{\left(1-\frac{e^{2ia}}{x^2}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{1}{x}\right)^m (ex)^m \int \frac{\left(1+\frac{e^{2ia}}{x^2}\right)^3 \left(\frac{1}{x}\right)^{-m-2}}{\left(1-\frac{e^{2ia}}{x^2}\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{370}$$

$$-i\left(\frac{1}{x}\right)^m(ex)^m\left(\frac{1}{4}e^{-2ia}\int\frac{2\left(1+\frac{e^{2ia}}{x^2}\right)\left(e^{2ia}(m+3)-\frac{e^{4ia}(1-m)}{x^2}\right)\left(\frac{1}{x}\right)^{-m-2}}{\left(1-\frac{e^{2ia}}{x^2}\right)^2}d\frac{1}{x}+\frac{\left(1+\frac{e^{2ia}}{x^2}\right)^2\left(\frac{1}{x}\right)^{-m-1}}{2\left(1-\frac{e^{2ia}}{x^2}\right)^2}\right)$$

$\downarrow$ 27

$$-i\left(\frac{1}{x}\right)^m(ex)^m\left(\frac{1}{2}e^{-2ia}\int\frac{\left(1+\frac{e^{2ia}}{x^2}\right)\left(e^{2ia}(m+3)-\frac{e^{4ia}(1-m)}{x^2}\right)\left(\frac{1}{x}\right)^{-m-2}}{\left(1-\frac{e^{2ia}}{x^2}\right)^2}d\frac{1}{x}+\frac{\left(1+\frac{e^{2ia}}{x^2}\right)^2\left(\frac{1}{x}\right)^{-m-1}}{2\left(1-\frac{e^{2ia}}{x^2}\right)^2}\right)$$

$\downarrow$ 439

$$-i\left(\frac{1}{x}\right)^m(ex)^m\left(\frac{1}{2}e^{-2ia}\left(\frac{1}{2}e^{-2ia}\int\frac{2\left(e^{4ia}(m+2)(m+3)-\frac{e^{6ia}(1-m)m}{x^2}\right)\left(\frac{1}{x}\right)^{-m-2}}{1-\frac{e^{2ia}}{x^2}}d\frac{1}{x}+\frac{\left(\frac{1}{x}\right)^{-m-1}\left(e^{2ia}(m+3\right.}{1-\frac{e^{2ia}}{x^2}}\right.\right.$$

$\downarrow$ 27

$$-i\left(\frac{1}{x}\right)^m(ex)^m\left(\frac{1}{2}e^{-2ia}\left(e^{-2ia}\int\frac{\left(e^{4ia}(m+2)(m+3)-\frac{e^{6ia}(1-m)m}{x^2}\right)\left(\frac{1}{x}\right)^{-m-2}}{1-\frac{e^{2ia}}{x^2}}d\frac{1}{x}+\frac{\left(\frac{1}{x}\right)^{-m-1}\left(e^{2ia}(m+3)-\right.}{1-\frac{e^{2ia}}{x^2}}\right.\right.$$

$\downarrow$ 363

$$-i\left(\frac{1}{x}\right)^m(ex)^m\left(\frac{1}{2}e^{-2ia}\left(e^{-2ia}\left(2e^{4ia}\left(m^2+2m+3\right)\int\frac{\left(\frac{1}{x}\right)^{-m-2}}{1-\frac{e^{2ia}}{x^2}}d\frac{1}{x}-\frac{e^{4ia}(1-m)m\left(\frac{1}{x}\right)^{-m-1}}{m+1}\right)+\frac{\left(\frac{1}{x}\right)^{-m-1}}{}\right.\right.$$

$\downarrow$ 278

$$-i\left(\frac{1}{x}\right)^m(ex)^m\left(\frac{1}{2}e^{-2ia}\left(e^{-2ia}\left(-\frac{2e^{4ia}\left(m^2+2m+3\right)\left(\frac{1}{x}\right)^{-m-1}\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-m-1),\frac{1-m}{2},\frac{e^{2ia}}{x^2}\right.}{m+1}\right.\right.\right.$$

input
```
Int[(e*x)^m*Cot[a + I*Log[x]]^3,x]
```

output
```
(-I)*(x^(-1))^m*(e*x)^m*(((1 + E^((2*I)*a)/x^2)^2*(x^(-1))^(-1 - m))/(2*(1
 - E^((2*I)*a)/x^2)^2) + (((E^((2*I)*a)*(3 + m) - (E^((4*I)*a)*(1 - m))/x^
2)*(x^(-1))^(-1 - m))/(1 - E^((2*I)*a)/x^2) + (-((E^((4*I)*a)*(1 - m)*m*(x
^(-1))^(-1 - m))/(1 + m)) - (2*E^((4*I)*a)*(3 + 2*m + m^2)*(x^(-1))^(-1 -
m)*Hypergeometric2F1[1, (-1 - m)/2, (1 - m)/2, E^((2*I)*a)/x^2])/(1 + m))/
E^((2*I)*a))/(2*E^((2*I)*a)))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 999
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(-(e*x)^m)*(x^(-1))^m   Subst[Int[(a + b/x^n)^p*(
(c + d/x^n)^q/x^(m + 2)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q}
, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [F]

$$\int (ex)^m \cot (a + i \ln (x))^3 \, dx$$

input
```
int((e*x)^m*cot(a+I*ln(x))^3,x)
```

output
```
int((e*x)^m*cot(a+I*ln(x))^3,x)
```

Fricas [F]

$$\int (ex)^m \cot^3(a + i \log(x)) \, dx = \int (ex)^m \cot (a + i \log (x))^3 \, dx$$

input
```
integrate((e*x)^m*cot(a+I*log(x))^3,x, algorithm="fricas")
```

output
```
integral(-(-I*x^6 - 3*I*x^4*e^(2*I*a) - 3*I*x^2*e^(4*I*a) - I*e^(6*I*a))*e
^(m*log(e) + m*log(x))/(x^6 - 3*x^4*e^(2*I*a) + 3*x^2*e^(4*I*a) - e^(6*I*a
)), x)
```

Sympy [F]

$$\int (ex)^m \cot^3(a + i\log(x))\, dx = \int (ex)^m \cot^3\left(a + i\log\left(x\right)\right) dx$$

input
```
integrate((e*x)**m*cot(a+I*ln(x))**3,x)
```

output
```
Integral((e*x)**m*cot(a + I*log(x))**3, x)
```

Maxima [F]

$$\int (ex)^m \cot^3(a + i\log(x))\, dx = \int (ex)^m \cot\left(a + i\, \log\left(x\right)\right)^3\, dx$$

input
```
integrate((e*x)^m*cot(a+I*log(x))^3,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*cot(a + I*log(x))^3, x)
```

Giac [F]

$$\int (ex)^m \cot^3(a + i\log(x))\, dx = \int (ex)^m \cot\left(a + i\, \log\left(x\right)\right)^3\, dx$$

input
```
integrate((e*x)^m*cot(a+I*log(x))^3,x, algorithm="giac")
```

output
```
integrate((e*x)^m*cot(a + I*log(x))^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \cot^3(a+i\log(x))\,dx = \int \cot(a+\ln(x)\ 1\mathrm{i})^3\,(e\,x)^m\,dx$$

input
```
int(cot(a + log(x)*1i)^3*(e*x)^m,x)
```

output
```
int(cot(a + log(x)*1i)^3*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \cot^3(a+i\log(x))\,dx = e^m \left(\int x^m \cot(\log(x)\,i+a)^3\,dx\right)$$

input
```
int((e*x)^m*cot(a+I*log(x))^3,x)
```

output
```
e**m*int(x**m*cot(log(x)*i + a)**3,x)
```

3.204 $\int \cot^p(a+b\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 142

$$\int \cot^p(a+b\log(x))\,dx = x\left(1-e^{2ia}x^{2ib}\right)^p\left(1+e^{2ia}x^{2ib}\right)^{-p}\left(-\frac{i\left(1+e^{2ia}x^{2ib}\right)}{1-e^{2ia}x^{2ib}}\right)^p \operatorname{AppellF1}\left(-\frac{i}{2b},p,-p,1-\frac{i}{2b},e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)$$

output
```
x*(1-exp(2*I*a)*x^(2*I*b))^p*(-I*(1+exp(2*I*a)*x^(2*I*b))/(1-exp(2*I*a)*x^
(2*I*b)))^p*AppellF1(-1/2*I/b,p,-p,1-1/2*I/b,exp(2*I*a)*x^(2*I*b),-exp(2*I
*a)*x^(2*I*b))/((1+exp(2*I*a)*x^(2*I*b))^p)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 330 vs. 2(142) = 284.

Time = 0.48 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.32

$$\int \cot^p(a+b\log(x))\,dx$$

$$= \frac{(-i+2b)x\left(\frac{i(1+e^{2ia}x^{2ib})}{-1+e^{2ia}x^{2ib}}\right)^p \operatorname{AppellF1}\left(-\frac{i}{2b},p,\right.}{2be^{2ia}px^{2ib}\operatorname{AppellF1}\left(1-\frac{i}{2b},p,1-p,2-\frac{i}{2b},e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)+2be^{2ia}px^{2ib}\operatorname{AppellF1}\left(1-\frac{i}{2b},1+p,-\right.}$$

input
```
Integrate[Cot[a + b*Log[x]]^p,x]
```

output
```
((-I + 2*b)*x*((I*(1 + E^((2*I)*a)*x^((2*I)*b)))/(-1 + E^((2*I)*a)*x^((2*I
)*b)))^p*AppellF1[(-1/2*I)/b, p, -p, 1 - (I/2)/b, E^((2*I)*a)*x^((2*I)*b),
 -(E^((2*I)*a)*x^((2*I)*b))])/(2*b*E^((2*I)*a)*p*x^((2*I)*b)*AppellF1[1 -
(I/2)/b, p, 1 - p, 2 - (I/2)/b, E^((2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^(
(2*I)*b))] + 2*b*E^((2*I)*a)*p*x^((2*I)*b)*AppellF1[1 - (I/2)/b, 1 + p, -p
, 2 - (I/2)/b, E^((2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^((2*I)*b))] + (-I
+ 2*b)*AppellF1[(-1/2*I)/b, p, -p, 1 - (I/2)/b, E^((2*I)*a)*x^((2*I)*b), -
(E^((2*I)*a)*x^((2*I)*b))])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5003, 2058, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^p(a+b\log(x))\,dx$$

$$\Big\downarrow \text{5003}$$

$$\int \left(\frac{-ie^{2ia}x^{2ib}-i}{1-e^{2ia}x^{2ib}}\right)^p dx$$

$$\Big\downarrow \text{2058}$$

$$\left(1-e^{2ia}x^{2ib}\right)^p\left(-ie^{2ia}x^{2ib}-i\right)^{-p}\left(-\frac{i(1+e^{2ia}x^{2ib})}{1-e^{2ia}x^{2ib}}\right)^p\int\left(1-e^{2ia}x^{2ib}\right)^{-p}\left(-ie^{2ia}x^{2ib}-i\right)^p dx$$

$$\Big\downarrow \text{937}$$

$$\left(1-e^{2ia}x^{2ib}\right)^p\left(1+e^{2ia}x^{2ib}\right)^{-p}\left(-\frac{i(1+e^{2ia}x^{2ib})}{1-e^{2ia}x^{2ib}}\right)^p\int\left(1-e^{2ia}x^{2ib}\right)^{-p}\left(e^{2ia}x^{2ib}+1\right)^p dx$$

$$\Big\downarrow \text{936}$$

$$x\left(1-e^{2ia}x^{2ib}\right)^p\left(1+e^{2ia}x^{2ib}\right)^{-p}\left(-\frac{i(1+e^{2ia}x^{2ib})}{1-e^{2ia}x^{2ib}}\right)^p \text{AppellF1}\left(-\frac{i}{2b},p,-p,1-\frac{i}{2b},e^{2ia}x^{2ib},-e^{2ia}x^{2ib}\right)$$

input
```
Int[Cot[a + b*Log[x]]^p,x]
```

output
```
(x*(1 - E^((2*I)*a)*x^((2*I)*b))^p*(((-I)*(1 + E^((2*I)*a)*x^((2*I)*b)))/(
1 - E^((2*I)*a)*x^((2*I)*b)))^p*AppellF1[(-1/2*I)/b, p, -p, 1 - (I/2)/b, E
^((2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^((2*I)*b))])/(1 + E^((2*I)*a)*x^((
2*I)*b))^p
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 5003
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((-I - I*E^(
2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b,
d, p}, x]
```

Maple [F]

$$\int \cot \left(a + b \ln \left(x \right)\right)^{p}dx$$

input
```
int(cot(a+b*ln(x))^p,x)
```

output
```
int(cot(a+b*ln(x))^p,x)
```

Fricas [F]

$$\int \cot^p(a + b\log(x))\, dx = \int \cot \left(b \log \left(x\right) + a\right)^{p}\, dx$$

input
```
integrate(cot(a+b*log(x))^p,x, algorithm="fricas")
```

output
```
integral(cot(b*log(x) + a)^p, x)
```

Sympy [F]

$$\int \cot^p(a+b\log(x))\,dx = \int \cot^p\left(a+b\log\left(x\right)\right)dx$$

input
```
integrate(cot(a+b*ln(x))**p,x)
```

output
```
Integral(cot(a + b*log(x))**p, x)
```

Maxima [F]

$$\int \cot^p(a+b\log(x))\,dx = \int \cot\left(b\log\left(x\right)+a\right)^p\,dx$$

input
```
integrate(cot(a+b*log(x))^p,x, algorithm="maxima")
```

output
```
integrate(cot(b*log(x) + a)^p, x)
```

Giac [F]

$$\int \cot^p(a+b\log(x))\,dx = \int \cot\left(b\log\left(x\right)+a\right)^p\,dx$$

input
```
integrate(cot(a+b*log(x))^p,x, algorithm="giac")
```

output
```
integrate(cot(b*log(x) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^p(a + b\log(x))\,dx = \int \cot(a + b\ \ln(x))^p\,dx$$

input
```
int(cot(a + b*log(x))^p,x)
```

output
```
int(cot(a + b*log(x))^p, x)
```

Reduce [F]

$$\int \cot^p(a + b\log(x))\,dx = \cot(\log(x)\,b + a)^p\,x + \left(\int \frac{\cot(\log(x)\,b + a)^p}{\cot(\log(x)\,b + a)}dx\right)bp + \left(\int \cot(\log(x)\,b + a)^p\cot(\log(x)\,b + a)\,dx\right)bp$$

input
```
int(cot(a+b*log(x))^p,x)
```

output
```
cot(log(x)*b + a)**p*x + int(cot(log(x)*b + a)**p/cot(log(x)*b + a),x)*b*p
 + int(cot(log(x)*b + a)**p*cot(log(x)*b + a),x)*b*p
```

3.205 $\int (ex)^m \cot^p(a+b\log(x))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 162

$$\int (ex)^m \cot^p(a+b\log(x))\,dx = \frac{(ex)^{1+m}\left(1-e^{2ia}x^{2ib}\right)^p\left(1+e^{2ia}x^{2ib}\right)^{-p}\left(-\frac{i(1+e^{2ia}x^{2ib})}{1-e^{2ia}x^{2ib}}\right)^p \operatorname{AppellF1}\left(-\frac{i(1+m)}{2b},p,-p,1-\frac{i(1+m)}{2b},e^{2ia}x^{2ib},-\right.}{e(1+m)}$$

output
```
(e*x)^(1+m)*(1-exp(2*I*a)*x^(2*I*b))^p*(-I*(1+exp(2*I*a)*x^(2*I*b))/(1-exp
(2*I*a)*x^(2*I*b)))^p*AppellF1(-1/2*I*(1+m)/b,p,-p,1-1/2*I*(1+m)/b,exp(2*I
*a)*x^(2*I*b),-exp(2*I*a)*x^(2*I*b))/e/(1+m)/((1+exp(2*I*a)*x^(2*I*b))^p)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.97

$$\int (ex)^m \cot^p(a+b\log(x))\,dx = \frac{x(ex)^m\left(1-e^{2ia}x^{2ib}\right)^p\left(1+e^{2ia}x^{2ib}\right)^{-p}\left(\frac{i(1+e^{2ia}x^{2ib})}{-1+e^{2ia}x^{2ib}}\right)^p \operatorname{AppellF1}\left(-\frac{i(1+m)}{2b},p,-p,1-\frac{i(1+m)}{2b},e^{2ia}x^{2ib},-e^{2}\right.}{1+m}$$

input
```
Integrate[(e*x)^m*Cot[a + b*Log[x]]^p,x]
```

output
```
(x*(e*x)^m*(1 - E^((2*I)*a)*x^((2*I)*b))^p*((I*(1 + E^((2*I)*a)*x^((2*I)*b
)))/(-1 + E^((2*I)*a)*x^((2*I)*b)))^p*AppellF1[((-1/2*I)*(1 + m))/b, p, -p
, 1 - ((I/2)*(1 + m))/b, E^((2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^((2*I)*b
))])/((1 + m)*(1 + E^((2*I)*a)*x^((2*I)*b))^p)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5007, 2058, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \cot^p(a+b\log(x))\,dx$$

$$\Big\downarrow \text{ 5007}$$

$$\int (ex)^m \left(\frac{-ie^{2ia}x^{2ib}-i}{1-e^{2ia}x^{2ib}}\right)^p dx$$

$$\Big\downarrow \text{ 2058}$$

$$\left(1-e^{2ia}x^{2ib}\right)^p \left(-ie^{2ia}x^{2ib}-i\right)^{-p} \left(-\frac{i\left(1+e^{2ia}x^{2ib}\right)}{1-e^{2ia}x^{2ib}}\right)^p \int (ex)^m \left(1-e^{2ia}x^{2ib}\right)^{-p} \left(-ie^{2ia}x^{2ib}-i\right)^p dx$$

$$\Big\downarrow \text{ 1013}$$

$$\left(1-e^{2ia}x^{2ib}\right)^p \left(1+e^{2ia}x^{2ib}\right)^{-p} \left(-\frac{i\left(1+e^{2ia}x^{2ib}\right)}{1-e^{2ia}x^{2ib}}\right)^p \int (ex)^m \left(1-e^{2ia}x^{2ib}\right)^{-p} \left(e^{2ia}x^{2ib}+1\right)^p dx$$

$$\Big\downarrow \text{ 1012}$$

$$\frac{(ex)^{m+1} \left(1-e^{2ia}x^{2ib}\right)^p \left(1+e^{2ia}x^{2ib}\right)^{-p} \left(-\frac{i(1+e^{2ia}x^{2ib})}{1-e^{2ia}x^{2ib}}\right)^p \text{AppellF1}\left(-\frac{i(m+1)}{2b},p,-p,1-\frac{i(m+1)}{2b},e^{2ia}x^{2ib},-e^{2ia}x\right.}{e(m+1)}$$

input
```
Int[(e*x)^m*Cot[a + b*Log[x]]^p,x]
```

output
```
((e*x)^(1 + m)*(1 - E^((2*I)*a)*x^((2*I)*b))^p*(((-I)*(1 + E^((2*I)*a)*x^(
(2*I)*b)))/(1 - E^((2*I)*a)*x^((2*I)*b)))^p*AppellF1[((-1/2*I)*(1 + m))/b,
 p, -p, 1 - ((I/2)*(1 + m))/b, E^((2*I)*a)*x^((2*I)*b), -(E^((2*I)*a)*x^((
2*I)*b))])/(e*(1 + m)*(1 + E^((2*I)*a)*x^((2*I)*b))^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [F]

$$\int (ex)^m \cot (a + b \ln (x))^p \, dx$$

input
```
int((e*x)^m*cot(a+b*ln(x))^p,x)
```

output
```
int((e*x)^m*cot(a+b*ln(x))^p,x)
```

Fricas [F]

$$\int (ex)^m \cot^p(a + b\log(x)) \, dx = \int (ex)^m \cot (b \log (x) + a)^p \, dx$$

input
```
integrate((e*x)^m*cot(a+b*log(x))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^m*cot(b*log(x) + a)^p, x)
```

Sympy [F]

$$\int (ex)^m \cot^p(a + b\log(x)) \, dx = \int (ex)^m \cot^p (a + b \log (x)) \, dx$$

input
```
integrate((e*x)**m*cot(a+b*ln(x))**p,x)
```

output
```
Integral((e*x)**m*cot(a + b*log(x))**p, x)
```

Maxima [F]

$$\int (ex)^m \cot^p(a + b\log(x))\, dx = \int (ex)^m \cot(b\log(x) + a)^p\, dx$$

input
```
integrate((e*x)^m*cot(a+b*log(x))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*cot(b*log(x) + a)^p, x)
```

Giac [F]

$$\int (ex)^m \cot^p(a + b\log(x))\, dx = \int (ex)^m \cot(b\log(x) + a)^p\, dx$$

input
```
integrate((e*x)^m*cot(a+b*log(x))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^m*cot(b*log(x) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \cot^p(a + b\log(x))\, dx = \int \cot(a + b\,\ln(x))^p\,(e\,x)^m\, dx$$

input
```
int(cot(a + b*log(x))^p*(e*x)^m,x)
```

output
```
int(cot(a + b*log(x))^p*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \cot^p(a+b\log(x))\,dx$$
$$= \frac{e^m\left(x^m\cot(\log(x)\,b+a)^p\,x+\left(\int \frac{x^m\cot(\log(x)b+a)^p}{\cot(\log(x)b+a)}dx\right)bp+\left(\int x^m\cot(\log(x)\,b+a)^p\cot(\log(x)\,b+a)\,dx\right.\right.}{m+1}$$

input
```
int((e*x)^m*cot(a+b*log(x))^p,x)
```

output
```
(e**m*(x**m*cot(log(x)*b + a)**p*x + int((x**m*cot(log(x)*b + a)**p)/cot(l
og(x)*b + a),x)*b*p + int(x**m*cot(log(x)*b + a)**p*cot(log(x)*b + a),x)*b
*p))/(m + 1)
```

3.206 $\int \cot^p(a+\log(x))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 120

$$\int \cot^p(a+\log(x))\,dx = \left(1-e^{2ia}x^{2i}\right)^p \left(1+e^{2ia}x^{2i}\right)^{-p} \left(-\frac{i\left(1+e^{2ia}x^{2i}\right)}{1-e^{2ia}x^{2i}}\right)^p x\,\text{AppellF1}\left(-\frac{i}{2},p,-p,1-\frac{i}{2},e^{2ia}x^{2i},-e^{2ia}x^{2i}\right)$$

output
```
(1-exp(2*I*a)*x^(2*I))^p*(-I*(1+exp(2*I*a)*x^(2*I))/(1-exp(2*I*a)*x^(2*I))
)^p*x*AppellF1(-1/2*I,p,-p,1-1/2*I,exp(2*I*a)*x^(2*I),-exp(2*I*a)*x^(2*I))
/((1+exp(2*I*a)*x^(2*I))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.98

$$\int \cot^p(a+\log(x))\,dx = \frac{(2-i)\left(\frac{i\left(1+e^{2ia}x^{2i}\right)}{-1+e^{2ia}x^{2i}}\right)^p x\,\text{AppellF1}\left(-\frac{i}{2},p,-p,1-\frac{i}{2},e\ldots\right.}{(2-i)\,\text{AppellF1}\left(-\frac{i}{2},p,-p,1-\frac{i}{2},e^{2ia}x^{2i},-e^{2ia}x^{2i}\right)+2e^{2ia}px^{2i}\left(\text{AppellF1}\left(1-\frac{i}{2},p,1-p,2-\frac{i}{2},e^{2ia}x^{2}\ldots\right.\right.}$$

input
```
Integrate[Cot[a + Log[x]]^p,x]
```

output
```
((2 - I)*((I*(1 + E^((2*I)*a)*x^(2*I)))/(-1 + E^((2*I)*a)*x^(2*I)))^p*x*Ap
pellF1[-1/2*I, p, -p, 1 - I/2, E^((2*I)*a)*x^(2*I), -(E^((2*I)*a)*x^(2*I))
])/((2 - I)*AppellF1[-1/2*I, p, -p, 1 - I/2, E^((2*I)*a)*x^(2*I), -(E^((2*
I)*a)*x^(2*I))] + 2*E^((2*I)*a)*p*x^(2*I)*(AppellF1[1 - I/2, p, 1 - p, 2 -
 I/2, E^((2*I)*a)*x^(2*I), -(E^((2*I)*a)*x^(2*I))] + AppellF1[1 - I/2, 1 +
 p, -p, 2 - I/2, E^((2*I)*a)*x^(2*I), -(E^((2*I)*a)*x^(2*I))]))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5003, 2058, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^p(a+\log(x))\,dx$$

$$\Big\downarrow \text{5003}$$

$$\int \left(\frac{-ie^{2ia}x^{2i}-i}{1-e^{2ia}x^{2i}}\right)^p dx$$

$$\Big\downarrow \text{2058}$$

$$\left(1-e^{2ia}x^{2i}\right)^p\left(-ie^{2ia}x^{2i}-i\right)^{-p}\left(-\frac{i\left(1+e^{2ia}x^{2i}\right)}{1-e^{2ia}x^{2i}}\right)^p\int \left(1-e^{2ia}x^{2i}\right)^{-p}\left(-ie^{2ia}x^{2i}-i\right)^p dx$$

$$\Big\downarrow \text{937}$$

$$\left(1-e^{2ia}x^{2i}\right)^p\left(1+e^{2ia}x^{2i}\right)^{-p}\left(-\frac{i\left(1+e^{2ia}x^{2i}\right)}{1-e^{2ia}x^{2i}}\right)^p\int \left(1-e^{2ia}x^{2i}\right)^{-p}\left(e^{2ia}x^{2i}+1\right)^p dx$$

$$\Big\downarrow \text{936}$$

$$x\left(1-e^{2ia}x^{2i}\right)^p\left(1+e^{2ia}x^{2i}\right)^{-p}\left(-\frac{i\left(1+e^{2ia}x^{2i}\right)}{1-e^{2ia}x^{2i}}\right)^p \text{AppellF1}\left(-\frac{i}{2},p,-p,1-\frac{i}{2},e^{2ia}x^{2i},-e^{2ia}x^{2i}\right)$$

input
```
Int[Cot[a + Log[x]]^p,x]
```

output
```
((1 - E^((2*I)*a)*x^(2*I))^p*(((-I)*(1 + E^((2*I)*a)*x^(2*I)))/(1 - E^((2*
I)*a)*x^(2*I)))^p*x*AppellF1[-1/2*I, p, -p, 1 - I/2, E^((2*I)*a)*x^(2*I),
-(E^((2*I)*a)*x^(2*I))])/(1 + E^((2*I)*a)*x^(2*I))^p
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 5003
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((-I - I*E^(
2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b,
d, p}, x]
```

Maple [F]

$$\int \cot \left(a+\ln \left(x\right)\right)^{p}dx$$

input
```
int(cot(a+ln(x))^p,x)
```

output
```
int(cot(a+ln(x))^p,x)
```

Fricas [F]

$$\int \cot^p(a+\log(x))\,dx=\int \cot \left(a+\log \left(x\right)\right)^{p}\,dx$$

input
```
integrate(cot(a+log(x))^p,x, algorithm="fricas")
```

output
```
integral(cot(a + log(x))^p, x)
```

Sympy [F]

$$\int \cot^p(a+\log(x))\,dx=\int \cot^{p}\left(a+\log \left(x\right)\right)dx$$

input
```
integrate(cot(a+ln(x))**p,x)
```

output
```
Integral(cot(a + log(x))**p, x)
```

Maxima [F]

$$\int \cot^p(a+\log(x))\,dx = \int \cot\left(a+\log\left(x\right)\right)^p\,dx$$

input
```
integrate(cot(a+log(x))^p,x, algorithm="maxima")
```

output
```
integrate(cot(a + log(x))^p, x)
```

Giac [F]

$$\int \cot^p(a+\log(x))\,dx = \int \cot\left(a+\log\left(x\right)\right)^p\,dx$$

input
```
integrate(cot(a+log(x))^p,x, algorithm="giac")
```

output
```
integrate(cot(a + log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^p(a+\log(x))\,dx = \int \cot(a+\ln\left(x\right))^p\,dx$$

input
```
int(cot(a + log(x))^p,x)
```

output
```
int(cot(a + log(x))^p, x)
```

Reduce [F]

$$\int \cot^p(a+\log(x))\,dx = \cot(\log(x)+a)^p\,x + \left(\int \frac{\cot(\log(x)+a)^p}{\cot(\log(x)+a)}dx\right)p + \left(\int \cot(\log(x)+a)^p \cot(\log(x)+a)\,dx\right)p$$

input
```
int(cot(a+log(x))^p,x)
```

output
```
cot(log(x) + a)**p*x + int(cot(log(x) + a)**p/cot(log(x) + a),x)*p + int(c
ot(log(x) + a)**p*cot(log(x) + a),x)*p
```

3.207 $\int \cot^p(a+2\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 120

$$\int \cot^p(a+2\log(x))\,dx=\left(1-e^{2ia}x^{4i}\right)^p\left(1+e^{2ia}x^{4i}\right)^{-p}\left(-\frac{i\left(1+e^{2ia}x^{4i}\right)}{1-e^{2ia}x^{4i}}\right)^p x\,\text{AppellF1}\left(-\frac{i}{4},p,-p,1-\frac{i}{4},e^{2ia}x^{4i},-e^{2ia}x^{4i}\right)$$

output
```
(1-exp(2*I*a)*x^(4*I))^p*(-I*(1+exp(2*I*a)*x^(4*I))/(1-exp(2*I*a)*x^(4*I))
)^p*x*AppellF1(-1/4*I,p,-p,1-1/4*I,exp(2*I*a)*x^(4*I),-exp(2*I*a)*x^(4*I))
/((1+exp(2*I*a)*x^(4*I))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.98

$$\int \cot^p(a+2\log(x))\,dx$$

$$=\frac{(4-i)\left(\frac{i\left(1+e^{2ia}x^{4i}\right)}{-1+e^{2ia}x^{4i}}\right)^p x\,\text{AppellF1}\left(-\frac{i}{4},p,-p,1-\frac{i}{4},e\right.}{(4-i)\,\text{AppellF1}\left(-\frac{i}{4},p,-p,1-\frac{i}{4},e^{2ia}x^{4i},-e^{2ia}x^{4i}\right)+4e^{2ia}px^{4i}\left(\text{AppellF1}\left(1-\frac{i}{4},p,1-p,2-\frac{i}{4},e^{2ia}x^{4}\right.\right.}$$

input
```
Integrate[Cot[a + 2*Log[x]]^p,x]
```

output
```
((4 - I)*((I*(1 + E^((2*I)*a)*x^(4*I)))/(-1 + E^((2*I)*a)*x^(4*I)))^p*x*Ap
pellF1[-1/4*I, p, -p, 1 - I/4, E^((2*I)*a)*x^(4*I), -(E^((2*I)*a)*x^(4*I))
])/((4 - I)*AppellF1[-1/4*I, p, -p, 1 - I/4, E^((2*I)*a)*x^(4*I), -(E^((2*
I)*a)*x^(4*I))] + 4*E^((2*I)*a)*p*x^(4*I)*(AppellF1[1 - I/4, p, 1 - p, 2 -
 I/4, E^((2*I)*a)*x^(4*I), -(E^((2*I)*a)*x^(4*I))] + AppellF1[1 - I/4, 1 +
 p, -p, 2 - I/4, E^((2*I)*a)*x^(4*I), -(E^((2*I)*a)*x^(4*I))]))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5003, 2058, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^p(a+2\log(x))\,dx$$

$$\Big\downarrow \text{5003}$$

$$\int \left(\frac{-ie^{2ia}x^{4i}-i}{1-e^{2ia}x^{4i}}\right)^p dx$$

$$\Big\downarrow \text{2058}$$

$$\left(1-e^{2ia}x^{4i}\right)^p\left(-ie^{2ia}x^{4i}-i\right)^{-p}\left(-\frac{i\left(1+e^{2ia}x^{4i}\right)}{1-e^{2ia}x^{4i}}\right)^p\int \left(1-e^{2ia}x^{4i}\right)^{-p}\left(-ie^{2ia}x^{4i}-i\right)^p dx$$

$$\Big\downarrow \text{937}$$

$$\left(1-e^{2ia}x^{4i}\right)^p\left(1+e^{2ia}x^{4i}\right)^{-p}\left(-\frac{i\left(1+e^{2ia}x^{4i}\right)}{1-e^{2ia}x^{4i}}\right)^p\int \left(1-e^{2ia}x^{4i}\right)^{-p}\left(e^{2ia}x^{4i}+1\right)^p dx$$

$$\Big\downarrow \text{936}$$

$$x\left(1-e^{2ia}x^{4i}\right)^p\left(1+e^{2ia}x^{4i}\right)^{-p}\left(-\frac{i\left(1+e^{2ia}x^{4i}\right)}{1-e^{2ia}x^{4i}}\right)^p\text{AppellF1}\left(-\frac{i}{4},p,-p,1-\frac{i}{4},e^{2ia}x^{4i},-e^{2ia}x^{4i}\right)$$

input
```
Int[Cot[a + 2*Log[x]]^p,x]
```

output
```
((1 - E^((2*I)*a)*x^(4*I))^p*(((-I)*(1 + E^((2*I)*a)*x^(4*I)))/(1 - E^((2*
I)*a)*x^(4*I)))^p*x*AppellF1[-1/4*I, p, -p, 1 - I/4, E^((2*I)*a)*x^(4*I),
-(E^((2*I)*a)*x^(4*I))])/(1 + E^((2*I)*a)*x^(4*I))^p
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 5003
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((-I - I*E^(
2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b,
d, p}, x]
```

Maple [F]

$$\int \cot\left(a+2\ln\left(x\right)\right)^{p}dx$$

input
```
int(cot(a+2*ln(x))^p,x)
```

output
```
int(cot(a+2*ln(x))^p,x)
```

Fricas [F]

$$\int \cot^p(a+2\log(x))\,dx=\int \cot\left(a+2\,\log\left(x\right)\right)^{p}\,dx$$

input
```
integrate(cot(a+2*log(x))^p,x, algorithm="fricas")
```

output
```
integral(cot(a + 2*log(x))^p, x)
```

Sympy [F]

$$\int \cot^p(a+2\log(x))\,dx=\int \cot^{p}\left(a+2\log\left(x\right)\right)dx$$

input
```
integrate(cot(a+2*ln(x))**p,x)
```

output
```
Integral(cot(a + 2*log(x))**p, x)
```

Maxima [F]

$$\int \cot^p(a+2\log(x))\,dx = \int \cot\left(a+2\,\log\left(x\right)\right)^p\,dx$$

input
```
integrate(cot(a+2*log(x))^p,x, algorithm="maxima")
```

output
```
integrate(cot(a + 2*log(x))^p, x)
```

Giac [F]

$$\int \cot^p(a+2\log(x))\,dx = \int \cot\left(a+2\,\log\left(x\right)\right)^p\,dx$$

input
```
integrate(cot(a+2*log(x))^p,x, algorithm="giac")
```

output
```
integrate(cot(a + 2*log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^p(a+2\log(x))\,dx = \int \cot(a+2\,\ln\left(x\right))^p\,dx$$

input
```
int(cot(a + 2*log(x))^p,x)
```

output
```
int(cot(a + 2*log(x))^p, x)
```

Reduce [F]

$$\int \cot^p(a + 2\log(x))\,dx = \cot(2\log(x) + a)^p\,x + 2\left(\int \frac{\cot(2\log(x) + a)^p}{\cot(2\log(x) + a)}dx\right)p + 2\left(\int \cot(2\log(x) + a)^p \cot(2\log(x) + a)\,dx\right)p$$

input
```
int(cot(a+2*log(x))^p,x)
```

output
```
cot(2*log(x) + a)**p*x + 2*int(cot(2*log(x) + a)**p/cot(2*log(x) + a),x)*p
 + 2*int(cot(2*log(x) + a)**p*cot(2*log(x) + a),x)*p
```

3.208 $\int \cot^p(a+3\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 120

$$\int \cot^p(a+3\log(x))\,dx = \left(1-e^{2ia}x^{6i}\right)^p\left(1+e^{2ia}x^{6i}\right)^{-p}\left(-\frac{i(1+e^{2ia}x^{6i})}{1-e^{2ia}x^{6i}}\right)^p x\,\mathrm{AppellF1}\left(-\frac{i}{6},p,-p,1-\frac{i}{6},e^{2ia}x^{6i},-e^{2ia}x^{6i}\right)$$

output
```
(1-exp(2*I*a)*x^(6*I))^p*(-I*(1+exp(2*I*a)*x^(6*I))/(1-exp(2*I*a)*x^(6*I))
)^p*x*AppellF1(-1/6*I,p,-p,1-1/6*I,exp(2*I*a)*x^(6*I),-exp(2*I*a)*x^(6*I))
/((1+exp(2*I*a)*x^(6*I))^p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.98

$$\int \cot^p(a+3\log(x))\,dx$$

$$= \frac{(6-i)\left(\frac{i(1+e^{2ia}x^{6i})}{-1+e^{2ia}x^{6i}}\right)^p x\,\mathrm{AppellF1}\left(-\frac{i}{6},p,-p,1-\frac{i}{6},e\right.}{(6-i)\,\mathrm{AppellF1}\left(-\frac{i}{6},p,-p,1-\frac{i}{6},e^{2ia}x^{6i},-e^{2ia}x^{6i}\right)+6e^{2ia}px^{6i}\left(\mathrm{AppellF1}\left(1-\frac{i}{6},p,1-p,2-\frac{i}{6},e^{2ia}x^{6}\right.\right.}$$

input
```
Integrate[Cot[a + 3*Log[x]]^p,x]
```

output
```
((6 - I)*((I*(1 + E^((2*I)*a)*x^(6*I)))/(-1 + E^((2*I)*a)*x^(6*I)))^p*x*Ap
pellF1[-1/6*I, p, -p, 1 - I/6, E^((2*I)*a)*x^(6*I), -(E^((2*I)*a)*x^(6*I))
])/((6 - I)*AppellF1[-1/6*I, p, -p, 1 - I/6, E^((2*I)*a)*x^(6*I), -(E^((2*
I)*a)*x^(6*I))] + 6*E^((2*I)*a)*p*x^(6*I)*(AppellF1[1 - I/6, p, 1 - p, 2 -
 I/6, E^((2*I)*a)*x^(6*I), -(E^((2*I)*a)*x^(6*I))] + AppellF1[1 - I/6, 1 +
 p, -p, 2 - I/6, E^((2*I)*a)*x^(6*I), -(E^((2*I)*a)*x^(6*I))]))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5003, 2058, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^p(a+3\log(x))\,dx$$

$$\Big\downarrow \text{5003}$$

$$\int \left(\frac{-ie^{2ia}x^{6i}-i}{1-e^{2ia}x^{6i}}\right)^p dx$$

$$\Big\downarrow \text{2058}$$

$$\left(1-e^{2ia}x^{6i}\right)^p \left(-ie^{2ia}x^{6i}-i\right)^{-p} \left(-\frac{i(1+e^{2ia}x^{6i})}{1-e^{2ia}x^{6i}}\right)^p \int \left(1-e^{2ia}x^{6i}\right)^{-p} \left(-ie^{2ia}x^{6i}-i\right)^p dx$$

$$\Big\downarrow \text{937}$$

$$\left(1-e^{2ia}x^{6i}\right)^p \left(1+e^{2ia}x^{6i}\right)^{-p} \left(-\frac{i(1+e^{2ia}x^{6i})}{1-e^{2ia}x^{6i}}\right)^p \int \left(1-e^{2ia}x^{6i}\right)^{-p} \left(e^{2ia}x^{6i}+1\right)^p dx$$

$$\Big\downarrow \text{936}$$

$$x\left(1-e^{2ia}x^{6i}\right)^p \left(1+e^{2ia}x^{6i}\right)^{-p} \left(-\frac{i(1+e^{2ia}x^{6i})}{1-e^{2ia}x^{6i}}\right)^p \operatorname{AppellF1}\left(-\frac{i}{6},p,-p,1-\frac{i}{6},e^{2ia}x^{6i},-e^{2ia}x^{6i}\right)$$

input
```
Int[Cot[a + 3*Log[x]]^p,x]
```

output
```
((1 - E^((2*I)*a)*x^(6*I))^p*(((-I)*(1 + E^((2*I)*a)*x^(6*I)))/(1 - E^((2*
I)*a)*x^(6*I)))^p*x*AppellF1[-1/6*I, p, -p, 1 - I/6, E^((2*I)*a)*x^(6*I),
-(E^((2*I)*a)*x^(6*I))])/(1 + E^((2*I)*a)*x^(6*I))^p
```

Defintions of rubi rules used

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 5003
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[((-I - I*E^(
2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*d)))^p, x] /; FreeQ[{a, b,
d, p}, x]
```

Maple [F]

$$\int \cot\left(a+3\ln\left(x\right)\right)^{p}dx$$

input
```
int(cot(a+3*ln(x))^p,x)
```

output
```
int(cot(a+3*ln(x))^p,x)
```

Fricas [F]

$$\int \cot^{p}(a+3\log(x))\,dx=\int \cot\left(a+3\,\log\left(x\right)\right)^{p}\,dx$$

input
```
integrate(cot(a+3*log(x))^p,x, algorithm="fricas")
```

output
```
integral(cot(a + 3*log(x))^p, x)
```

Sympy [F]

$$\int \cot^{p}(a+3\log(x))\,dx=\int \cot^{p}\left(a+3\log\left(x\right)\right)dx$$

input
```
integrate(cot(a+3*ln(x))**p,x)
```

output
```
Integral(cot(a + 3*log(x))**p, x)
```

Maxima [F]

$$\int \cot^p(a + 3\log(x))\,dx = \int \cot\left(a + 3\,\log\left(x\right)\right)^p\,dx$$

input
```
integrate(cot(a+3*log(x))^p,x, algorithm="maxima")
```

output
```
integrate(cot(a + 3*log(x))^p, x)
```

Giac [F]

$$\int \cot^p(a + 3\log(x))\,dx = \int \cot\left(a + 3\,\log\left(x\right)\right)^p\,dx$$

input
```
integrate(cot(a+3*log(x))^p,x, algorithm="giac")
```

output
```
integrate(cot(a + 3*log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot^p(a + 3\log(x))\,dx = \int \cot(a + 3\,\ln\left(x\right))^p\,dx$$

input
```
int(cot(a + 3*log(x))^p,x)
```

output
```
int(cot(a + 3*log(x))^p, x)
```

Reduce [F]

$$\int \cot^p(a+3\log(x))\,dx = \cot(3\log(x)+a)^p x + 3\left(\int \frac{\cot(3\log(x)+a)^p}{\cot(3\log(x)+a)}dx\right)p + 3\left(\int \cot(3\log(x)+a)^p \cot(3\log(x)+a)\,dx\right)p$$

input
```
int(cot(a+3*log(x))^p,x)
```

output
```
cot(3*log(x) + a)**p*x + 3*int(cot(3*log(x) + a)**p/cot(3*log(x) + a),x)*p
 + 3*int(cot(3*log(x) + a)**p*cot(3*log(x) + a),x)*p
```

3.209 $\int x^3 \cot(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int x^3 \cot(d(a+b\log(cx^n)))\,dx = \frac{ix^4}{4} - \frac{1}{2}ix^4 \text{Hypergeometric2F1}\left(1, -\frac{2i}{bdn}, 1 - \frac{2i}{bdn}, e^{2iad}(cx^n)^{2ibd}\right)$$

output
```
1/4*I*x^4-1/2*I*x^4*hypergeom([1, -2*I/b/d/n],[1-2*I/b/d/n],exp(2*I*a*d)*(
c*x^n)^(2*I*b*d))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 220 vs. 2(70) = 140.

Time = 3.96 (sec) , antiderivative size = 220, normalized size of antiderivative = 3.14

$$\int x^3 \cot(d(a+b\log(cx^n)))\,dx =$$
$$-\frac{x^4\left(2e^{2id(a+b\log(cx^n))}\text{Hypergeometric2F1}\left(1,1-\frac{2i}{bdn},2-\frac{2i}{bdn},e^{2id(a+b\log(cx^n))}\right)+(-2i+bdn)\left(\cot\left(d(a\right.\right.\right.}{}$$

input
```
Integrate[x^3*Cot[d*(a + b*Log[c*x^n])],x]
```

output
```
-((x^4*(2*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - (2*I)/(b
*d*n), 2 - (2*I)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + (-2*I + b*d*n)
*(Cot[d*(a + b*Log[c*x^n])] - Cot[d*(a - b*n*Log[x] + b*Log[c*x^n])] + I*H
ypergeometric2F1[1, (-2*I)/(b*d*n), 1 - (2*I)/(b*d*n), E^((2*I)*d*(a + b*L
og[c*x^n]))] + Csc[d*(a + b*Log[c*x^n])]*Csc[d*(a - b*n*Log[x] + b*Log[c*x
^n])]*Sin[b*d*n*Log[x]])))/(-8*I + 4*b*d*n))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.51, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5009, 5007, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cot (d(a+b\log (cx^n))) \, dx$$

$$\Big\downarrow \text{ 5009}$$

$$\frac{x^4 (cx^n)^{-4/n} \int (cx^n)^{\frac{4}{n}-1} \cot (d(a+b\log (cx^n))) d(cx^n)}{n}$$

$$\Big\downarrow \text{ 5007}$$

$$\frac{x^4 (cx^n)^{-4/n} \int \frac{(cx^n)^{\frac{4}{n}-1} \left(-ie^{2iad} (cx^n)^{2ibd}-i\right)}{1-e^{2iad} (cx^n)^{2ibd}} d(cx^n)}{n}$$

$$\Big\downarrow \text{ 959}$$

$$\frac{x^4 (cx^n)^{-4/n} \left(\frac{1}{4} in (cx^n)^{4/n}-2i \int \frac{(cx^n)^{\frac{4}{n}-1}}{1-e^{2iad} (cx^n)^{2ibd}} d(cx^n)\right)}{n}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^4 (cx^n)^{-4/n} \left(\frac{1}{4} in (cx^n)^{4/n}-\frac{1}{2} in (cx^n)^{4/n} \text{Hypergeometric2F1}\left(1,-\frac{2i}{bdn},1-\frac{2i}{bdn},e^{2iad} (cx^n)^{2ibd}\right)\right)}{n}$$

input
```
Int[x^3*Cot[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^4*((I/4)*n*(c*x^n)^(4/n) - (I/2)*n*(c*x^n)^(4/n)*Hypergeometric2F1[1, (
-2*I)/(b*d*n), 1 - (2*I)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)]))/(n*
(c*x^n)^(4/n))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5009
```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^3 \cot \left(d(a+b\ln \left(c\,x^n\right))\right) dx$$

input
```
int(x^3*cot(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x^3*cot(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int x^3 \cot (d(a + b \log (cx^n))) \, dx = \int x^3 \cot ((b \log (cx^n) + a)d) \, dx$$

input
```
integrate(x^3*cot(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral(x^3*cot(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int x^3 \cot (d(a + b \log (cx^n))) \, dx = \int x^3 \cot (ad + bd \log (cx^n)) \, dx$$

input
```
integrate(x**3*cot(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x**3*cot(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x^3 \cot (d(a + b \log (cx^n))) \, dx = \int x^3 \cot ((b \log (cx^n) + a)d) \, dx$$

input
```
integrate(x^3*cot(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(x^3*cot((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int x^3 \cot (d(a + b \log (cx^n))) \, dx = \int x^3 \cot ((b \log (cx^n) + a)d) \, dx$$

input
```
integrate(x^3*cot(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(x^3*cot((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \cot (d(a + b \log (cx^n))) \, dx = \int x^3 \cot(d \, (a + b \, \ln (c \, x^n))) \, dx$$

input
```
int(x^3*cot(d*(a + b*log(c*x^n))),x)
```

output
```
int(x^3*cot(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int x^3 \cot (d(a + b \log (cx^n))) \, dx = \int \cot (\log(x^n c) \, bd + ad) \, x^3 dx$$

input
```
int(x^3*cot(d*(a+b*log(c*x^n))),x)
```

output
```
int(cot(log(x**n*c)*b*d + a*d)*x**3,x)
```

3.210 $\int x^2 \cot(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int x^2 \cot(d(a+b\log(cx^n)))\,dx = \frac{ix^3}{3} - \frac{2}{3}ix^3 \text{Hypergeometric2F1}\left(1,-\frac{3i}{2bdn},1-\frac{3i}{2bdn},e^{2iad}(cx^n)^{2ibd}\right)$$

output
```
1/3*I*x^3-2/3*I*x^3*hypergeom([1, -3/2*I/b/d/n],[1-3/2*I/b/d/n],exp(2*I*a*
d)*(c*x^n)^(2*I*b*d))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 229 vs. 2(74) = 148.

Time = 4.13 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.09

$$\int x^2 \cot(d(a+b\log(cx^n)))\,dx = -\frac{x^3\left(3e^{2id(a+b\log(cx^n))}\text{Hypergeometric2F1}\left(1,1-\frac{3i}{2bdn},2-\frac{3i}{2bdn},e^{2id(a+b\log(cx^n))}\right)+(-3i+2bdn)\left(\cot(d\right.\right.}{}$$

input
```
Integrate[x^2*Cot[d*(a + b*Log[c*x^n])],x]
```

output
```
-((x^3*(3*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - ((3*I)/2
)/(b*d*n), 2 - ((3*I)/2)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + (-3*I
+ 2*b*d*n)*(Cot[d*(a + b*Log[c*x^n])] - Cot[d*(a - b*n*Log[x] + b*Log[c*x^
n])] + I*Hypergeometric2F1[1, ((-3*I)/2)/(b*d*n), 1 - ((3*I)/2)/(b*d*n), E
^((2*I)*d*(a + b*Log[c*x^n]))] + Csc[d*(a + b*Log[c*x^n])]*Csc[d*(a - b*n*
Log[x] + b*Log[c*x^n])]*Sin[b*d*n*Log[x]])))/(-9*I + 6*b*d*n))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.49, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5009, 5007, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot (d(a+b\log (cx^n)))\,dx$$

$$\Big\downarrow \text{5009}$$

$$\frac{x^3 (cx^n)^{-3/n} \int (cx^n)^{\frac{3}{n}-1} \cot (d(a+b\log (cx^n)))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5007}$$

$$\frac{x^3 (cx^n)^{-3/n} \int \frac{(cx^n)^{\frac{3}{n}-1}\left(-ie^{2iad}(cx^n)^{2ibd}-i\right)}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{n}$$

$$\Big\downarrow \text{959}$$

$$\frac{x^3 (cx^n)^{-3/n} \left(\frac{1}{3}in(cx^n)^{3/n}-2i\int \frac{(cx^n)^{\frac{3}{n}-1}}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)\right)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^3 (cx^n)^{-3/n} \left(\frac{1}{3}in(cx^n)^{3/n}-\frac{2}{3}in(cx^n)^{3/n}\text{Hypergeometric2F1}\left(1,-\frac{3i}{2bdn},1-\frac{3i}{2bdn},e^{2iad}(cx^n)^{2ibd}\right)\right)}{n}$$

input
```
Int[x^2*Cot[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^3*((I/3)*n*(c*x^n)^(3/n) - ((2*I)/3)*n*(c*x^n)^(3/n)*Hypergeometric2F1[
1, ((-3*I)/2)/(b*d*n), 1 - ((3*I)/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)
*b*d)]))/(n*(c*x^n)^(3/n))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5009
```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^2 \cot \left(d(a + b\ln \left(c\, x^n\right))\right) dx$$

input
```
int(x^2*cot(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x^2*cot(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int x^2 \cot (d(a+b\log (cx^n)))\ dx = \int x^2 \cot ((b\log (cx^n)+a)d)\ dx$$

input
```
integrate(x^2*cot(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral(x^2*cot(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int x^2 \cot (d(a+b\log (cx^n)))\ dx = \int x^2 \cot (ad+bd\log (cx^n))\,dx$$

input
```
integrate(x**2*cot(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x**2*cot(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x^2 \cot (d(a+b\log (cx^n)))\ dx = \int x^2 \cot ((b\log (cx^n)+a)d)\ dx$$

input
```
integrate(x^2*cot(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(x^2*cot((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int x^2 \cot (d(a+b\log (cx^n))) \, dx = \int x^2 \cot ((b\log (cx^n) + a)d) \, dx$$

input
```
integrate(x^2*cot(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(x^2*cot((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot (d(a+b\log (cx^n))) \, dx = \int x^2 \cot(d\,(a+b\,\ln (c\,x^n))) \, dx$$

input
```
int(x^2*cot(d*(a + b*log(c*x^n))),x)
```

output
```
int(x^2*cot(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int x^2 \cot (d(a+b\log (cx^n))) \, dx = \int \cot (\log(x^n c)\, bd + ad)\, x^2 dx$$

input
```
int(x^2*cot(d*(a+b*log(c*x^n))),x)
```

output
```
int(cot(log(x**n*c)*b*d + a*d)*x**2,x)
```

3.211 $\int x \cot(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int x \cot(d(a+b\log(cx^n)))\,dx = \frac{ix^2}{2} - ix^2\,\text{Hypergeometric2F1}\left(1,-\frac{i}{bdn},1-\frac{i}{bdn},e^{2iad}(cx^n)^{2ibd}\right)$$

output
```
1/2*I*x^2-I*x^2*hypergeom([1, -I/b/d/n],[1-I/b/d/n],exp(2*I*a*d)*(c*x^n)^(
2*I*b*d))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 219 vs. 2(68) = 136.

Time = 4.11 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.22

$$\int x \cot(d(a+b\log(cx^n)))\,dx = -\frac{x^2\left(e^{2id(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1-\frac{i}{bdn},2-\frac{i}{bdn},e^{2id(a+b\log(cx^n))}\right)+(-i+bdn)\left(\cot(d(a+b\cdots\right.\right.}{\cdots}$$

input
```
Integrate[x*Cot[d*(a + b*Log[c*x^n])],x]
```

output
```
-((x^2*(E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - I/(b*d*n),
 2 - I/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + (-I + b*d*n)*(Cot[d*(a +
 b*Log[c*x^n])] - Cot[d*(a - b*n*Log[x] + b*Log[c*x^n])] + I*Hypergeometri
c2F1[1, (-I)/(b*d*n), 1 - I/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + Csc
[d*(a + b*Log[c*x^n])]*Csc[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sin[b*d*n*Lo
g[x]])))/(-2*I + 2*b*d*n))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.53, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5009, 5007, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cot (d(a+b\log (cx^n)))\, dx$$

$$\Big\downarrow \text{5009}$$

$$\frac{x^2 (cx^n)^{-2/n} \int (cx^n)^{\frac{2}{n}-1} \cot (d(a+b\log (cx^n)))d(cx^n)}{n}$$

$$\Big\downarrow \text{5007}$$

$$\frac{x^2 (cx^n)^{-2/n} \int \frac{(cx^n)^{\frac{2}{n}-1}\left(-ie^{2iad}(cx^n)^{2ibd}-i\right)}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{n}$$

$$\Big\downarrow \text{959}$$

$$\frac{x^2 (cx^n)^{-2/n} \left(\frac{1}{2}in(cx^n)^{2/n}-2i\int \frac{(cx^n)^{\frac{2}{n}-1}}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)\right)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2 (cx^n)^{-2/n} \left(\frac{1}{2}in(cx^n)^{2/n}-in(cx^n)^{2/n}\text{Hypergeometric2F1}\left(1,-\frac{i}{bdn},1-\frac{i}{bdn},e^{2iad}(cx^n)^{2ibd}\right)\right)}{n}$$

input
```
Int[x*Cot[d*(a + b*Log[c*x^n])],x]
```

output
```
(x^2*((I/2)*n*(c*x^n)^(2/n) - I*n*(c*x^n)^(2/n)*Hypergeometric2F1[1, (-I)/
(b*d*n), 1 - I/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)]))/(n*(c*x^n)^(2
/n))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5009
```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x \cot \left(d(a + b \ln \left(c\, x^{n}\right))\right) dx$$

input
```
int(x*cot(d*(a+b*ln(c*x^n))),x)
```

output
```
int(x*cot(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int x\cot\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\cot\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

input
```
integrate(x*cot(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral(x*cot(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int x\cot\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\cot\left(ad+bd\log\left(cx^n\right)\right)dx$$

input
```
integrate(x*cot(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(x*cot(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x\cot\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\cot\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

input
```
integrate(x*cot(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(x*cot((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int x\cot\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\cot\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

input
```
integrate(x*cot(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(x*cot((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\cot\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\cot\left(d\left(a+b\,\ln\left(c\,x^n\right)\right)\right)\,dx$$

input
```
int(x*cot(d*(a + b*log(c*x^n))),x)
```

output
```
int(x*cot(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int x\cot\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int\cot\left(\log(x^n c)\,bd+ad\right)x\,dx$$

input
```
int(x*cot(d*(a+b*log(c*x^n))),x)
```

output
```
int(cot(log(x**n*c)*b*d + a*d)*x,x)
```

3.212 $\int \cot (d(a+b\log (cx^n)))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 66

$$\int \cot (d(a+b\log (cx^n)))\,dx=ix-2ix\,\text{Hypergeometric2F1}\left(1,-\frac{i}{2bdn},1-\frac{i}{2bdn},e^{2iad}(cx^n)^{2ibd}\right)$$

output
```
I*x-2*I*x*hypergeom([1, -1/2*I/b/d/n],[1-1/2*I/b/d/n],exp(2*I*a*d)*(c*x^n)
^(2*I*b*d))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 141 vs. 2(66) = 132.

Time = 7.42 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.14

$$\int \cot (d(a+b\log (cx^n)))\,dx = x\left(-\frac{e^{2id(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1-\frac{i}{2bdn},2-\frac{i}{2bdn},e^{2id(a+b\log(cx^n))}\right)}{-i+2bdn}-i\,\text{Hypergeometric2F1}\left(1,-\frac{i}{2bdn},1-\frac{i}{2bdn},e^{2id(a+b\log(cx^n))}\right)\right)$$

input
```
Integrate[Cot[d*(a + b*Log[c*x^n])],x]
```

output
```
x*(-((E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - (I/2)/(b*d*n
), 2 - (I/2)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))])/(-I + 2*b*d*n)) - I
*Hypergeometric2F1[1, (-1/2*I)/(b*d*n), 1 - (I/2)/(b*d*n), E^((2*I)*d*(a +
 b*Log[c*x^n]))])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.52, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5005, 5007, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot \left(d(a+b\log \left(cx^n\right))\right) \, dx$$

$$\Big\downarrow \text{5005}$$

$$\frac{x(cx^n)^{-1/n} \int (cx^n)^{\frac{1}{n}-1} \cot \left(d(a+b\log \left(cx^n\right))\right) d(cx^n)}{n}$$

$$\Big\downarrow \text{5007}$$

$$\frac{x(cx^n)^{-1/n} \int \frac{(cx^n)^{\frac{1}{n}-1}\left(-ie^{2iad}(cx^n)^{2ibd}-i\right)}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{n}$$

$$\Big\downarrow \text{959}$$

$$\frac{x(cx^n)^{-1/n}\left(in(cx^n)^{\frac{1}{n}}-2i\int \frac{(cx^n)^{\frac{1}{n}-1}}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)\right)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x(cx^n)^{-1/n}\left(in(cx^n)^{\frac{1}{n}}-2in(cx^n)^{\frac{1}{n}}\text{Hypergeometric2F1}\left(1,-\frac{i}{2bdn},1-\frac{i}{2bdn},e^{2iad}(cx^n)^{2ibd}\right)\right)}{n}$$

input
```
Int[Cot[d*(a + b*Log[c*x^n])],x]
```

output
```
(x*(I*n*(c*x^n)^n^(-1) - (2*I)*n*(c*x^n)^n^(-1)*Hypergeometric2F1[1, (-1/2
*I)/(b*d*n), 1 - (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)]))/(n*(c
*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 5005
```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cot[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [F]

$$\int \cot\left(d(a+b\ln\left(c\,x^{n}\right))\right)dx$$

input
```
int(cot(d*(a+b*ln(c*x^n))),x)
```

output
```
int(cot(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int \cot (d(a+b\log (cx^n)))\,dx=\int \cot ((b\log (cx^n)+a)d)\,dx$$

input
```
integrate(cot(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral(cot(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int \cot (d(a+b\log (cx^n)))\,dx=\int \cot (d(a+b\log (cx^n)))\,dx$$

input
```
integrate(cot(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral(cot(d*(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \cot (d(a+b\log (cx^n)))\,dx=\int \cot ((b\log (cx^n)+a)d)\,dx$$

input
```
integrate(cot(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate(cot((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int \cot (d(a+b\log (cx^n)))\ dx=\int \cot ((b\log (cx^n)+a)d)\ dx$$

input
```
integrate(cot(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate(cot((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cot (d(a+b\log (cx^n)))\ dx=\int \cot(d\,(a+b\,\ln (c\,x^n)))\ dx$$

input
```
int(cot(d*(a + b*log(c*x^n))),x)
```

output
```
int(cot(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int \cot (d(a+b\log (cx^n)))\ dx=\int \cot (\log(x^n c)\,bd+ad)\,dx$$

input
```
int(cot(d*(a+b*log(c*x^n))),x)
```

output
```
int(cot(log(x**n*c)*b*d + a*d),x)
```

3.213 $\int \frac{\cot(d(a+b\log(cx^n)))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 25

$$\int \frac{\cot(d(a+b\log(cx^n)))}{x}\,dx=\frac{\log(\sin(ad+bd\log(cx^n)))}{bdn}$$

output
```
ln(sin(a*d+b*d*ln(c*x^n)))/b/d/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\cot(d(a+b\log(cx^n)))}{x}\,dx=\frac{\log(\sin(ad+bd\log(cx^n)))}{bdn}$$

input
```
Integrate[Cot[d*(a + b*Log[c*x^n])]/x,x]
```

output
```
Log[Sin[a*d + b*d*Log[c*x^n]]]/(b*d*n)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3039, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot (d(a+b\log (cx^n)))}{x} \, dx$$

$$\Big\downarrow \; 3039$$

$$\frac{\int \cot (d(a+b\log (cx^n)))d\log (cx^n)}{n}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int -\tan \left(ad+b\log (cx^n)d+\frac{\pi}{2}\right)d\log (cx^n)}{n}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \tan \left(\frac{1}{2}(2ad+\pi)+bd\log (cx^n)\right)d\log (cx^n)}{n}$$

$$\Big\downarrow \; 3956$$

$$\frac{\log (-\sin (ad+bd\log (cx^n)))}{bdn}$$

input
```
Int[Cot[d*(a + b*Log[c*x^n])]/x,x]
```

output
```
Log[-Sin[a*d + b*d*Log[c*x^n]]]/(b*d*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

method	result
derivativedivides	$-\frac{\ln\left(\cot(d(a+b\ln(c\,x^{n})))^{2}+1\right)}{2nbd}$
default	$-\frac{\ln\left(\cot(d(a+b\ln(c\,x^{n})))^{2}+1\right)}{2nbd}$
parallelrisch	$\frac{\ln(\tan(d(a+b\ln(c\,x^{n}))))+\ln\left(\frac{1}{\sqrt{\sec(d(a+b\ln(c\,x^{n})))^{2}}}\right)}{bdn}$
risch	$i\ln\left(x\right)-\frac{2ia}{bn}-\frac{2i\ln(c)}{n}-\frac{2i\ln(x^{n})}{n}+\frac{\pi\ \mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})^{2}}{n}-\frac{\pi\ \mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})\,\mathrm{csgn}(ic)}{n}-\frac{\pi\ \mathrm{cs}}{}$

input
```
int(cot(d*(a+b*ln(c*x^n)))/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2/n/b/d*ln(cot(d*(a+b*ln(c*x^n)))^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \frac{\cot(d(a+b\log(cx^n)))}{x}\,dx = \frac{\log\left(-\frac{1}{2}\cos(2\,bdn\log(x)+2\,bd\log(c)+2\,ad)+\frac{1}{2}\right)}{2\,bdn}$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
```

output
```
1/2*log(-1/2*cos(2*b*d*n*log(x) + 2*b*d*log(c) + 2*a*d) + 1/2)/(b*d*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(20) = 40.

Time = 1.43 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{\cot(d(a+b\log(cx^n)))}{x}\,dx = \begin{cases} \log(x)\cot(ad) & \text{for } b=0 \\ \tilde{\infty}\log(x) & \text{for } d=0 \\ \log(x)\cot(ad+bd\log(c)) & \text{for } n=0 \\ \frac{\log(\sin(ad+bd\log(cx^n)))}{bdn} & \text{otherwise} \end{cases}$$

input
```
integrate(cot(d*(a+b*ln(c*x**n)))/x,x)
```

output
```
Piecewise((log(x)*cot(a*d), Eq(b, 0)), (zoo*log(x), Eq(d, 0)), (log(x)*cot
(a*d + b*d*log(c)), Eq(n, 0)), (log(sin(a*d + b*d*log(c*x**n)))/(b*d*n), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\cot (d(a+b\log (cx^n)))}{x}\, dx = \frac{\log (\sin ((b\log (cx^n)+a)d))}{bdn}$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
```

output
```
log(sin((b*log(c*x^n) + a)*d))/(b*d*n)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot (d(a+b\log (cx^n)))}{x}\, dx = \text{Timed out}$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 23.55 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{\cot (d(a+b\log (cx^n)))}{x}\, dx = -\ln (x)\ 1\mathrm{i} + \frac{\ln \left(\mathrm{e}^{a\,d\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,d\,2\mathrm{i}}-1\right)}{b\,d\,n}$$

input
```
int(cot(d*(a + b*log(c*x^n)))/x,x)
```

output
```
log(exp(a*d*2i)*(c*x^n)^(b*d*2i) - 1)/(b*d*n) - log(x)*1i
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{\cot(d(a+b\log(cx^n)))}{x}\,dx$$
$$= \frac{-\log\left(\tan\left(\frac{\log(x^n c)bd}{2}+\frac{ad}{2}\right)^2+1\right)+\log\left(\tan\left(\frac{\log(x^n c)bd}{2}+\frac{ad}{2}\right)\right)}{bdn}$$

input
```
int(cot(d*(a+b*log(c*x^n)))/x,x)
```

output
```
( - log(tan((log(x**n*c)*b*d + a*d)/2)**2 + 1) + log(tan((log(x**n*c)*b*d
+ a*d)/2)))/(b*d*n)
```

3.214 $\int \frac{\cot(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{\cot(d(a+b\log(cx^n)))}{x^2}\,dx = -\frac{i}{x}+\frac{2i\,\text{Hypergeometric2F1}\left(1,\frac{i}{2bdn},1+\frac{i}{2bdn},e^{2iad}(cx^n)^{2ibd}\right)}{x}$$

output
```
-I/x+2*I*hypergeom([1, 1/2*I/b/d/n],[1+1/2*I/b/d/n],exp(2*I*a*d)*(c*x^n)^(
2*I*b*d))/x
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 217 vs. 2(70) = 140.

Time = 3.47 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.10

$$\int \frac{\cot(d(a+b\log(cx^n)))}{x^2}\,dx = \frac{\cot(d(a+b\log(cx^n)))-\cot(d(a-bn\log(x)+b\log(cx^n)))-\frac{e^{2id(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1+\frac{i}{2bdn},2+\frac{i}{2b\cdots}\right.}{i+2bdn}}{\cdots}$$

input
```
Integrate[Cot[d*(a + b*Log[c*x^n])]/x^2,x]
```

output
```
(Cot[d*(a + b*Log[c*x^n])] - Cot[d*(a - b*n*Log[x] + b*Log[c*x^n])] - (E^(
(2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + (I/2)/(b*d*n), 2 + (I
/2)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))])/(I + 2*b*d*n) + I*Hypergeome
tric2F1[1, (I/2)/(b*d*n), 1 + (I/2)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n])
)] + Csc[d*(a + b*Log[c*x^n])]*Csc[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sin[
b*d*n*Log[x]])/x
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.49, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5009, 5007, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot (d(a+b\log (cx^n)))}{x^2} dx$$

$$\Big\downarrow \text{ 5009}$$

$$\frac{(cx^n)^{\frac{1}{n}} \int (cx^n)^{-1-\frac{1}{n}} \cot (d(a+b\log (cx^n)))d(cx^n)}{nx}$$

$$\Big\downarrow \text{ 5007}$$

$$\frac{(cx^n)^{\frac{1}{n}} \int \frac{(cx^n)^{-1-\frac{1}{n}}\left(-ie^{2iad}(cx^n)^{2ibd}-i\right)}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{nx}$$

$$\Big\downarrow \text{ 959}$$

$$\frac{(cx^n)^{\frac{1}{n}} \left(-2i\int \frac{(cx^n)^{-1-\frac{1}{n}}}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)-in(cx^n)^{-1/n}\right)}{nx}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{(cx^n)^{\frac{1}{n}} \left(2in(cx^n)^{-1/n}\text{Hypergeometric2F1}\left(1,\frac{i}{2bdn},1+\frac{i}{2bdn},e^{2iad}(cx^n)^{2ibd}\right)-in(cx^n)^{-1/n}\right)}{nx}$$

input
```
Int[Cot[d*(a + b*Log[c*x^n])]/x^2,x]
```

output
```
((c*x^n)^n^(-1)*(((-I)*n)/(c*x^n)^n^(-1) + ((2*I)*n*Hypergeometric2F1[1, (
I/2)/(b*d*n), 1 + (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(c*x^
n)^n^(-1)))/(n*x)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5009
```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\cot(d(a+b\ln(c\,x^n)))}{x^2}dx$$

input
```
int(cot(d*(a+b*ln(c*x^n)))/x^2,x)
```

output
```
int(cot(d*(a+b*ln(c*x^n)))/x^2,x)
```

Fricas [F]

$$\int \frac{\cot(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\cot((b\log(cx^n)+a)d)}{x^2}\,dx$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

output
```
integral(cot(b*d*log(c*x^n) + a*d)/x^2, x)
```

Sympy [F]

$$\int \frac{\cot(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\cot(ad+bd\log(cx^n))}{x^2}\,dx$$

input
```
integrate(cot(d*(a+b*ln(c*x**n)))/x**2,x)
```

output
```
Integral(cot(a*d + b*d*log(c*x**n))/x**2, x)
```

Maxima [F]

$$\int \frac{\cot(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\cot((b\log(cx^n)+a)d)}{x^2}\,dx$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

output
```
integrate(cot((b*log(c*x^n) + a)*d)/x^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot(d(a+b\log(cx^n)))}{x^2}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\cot(d\,(a+b\,\ln(c\,x^n)))}{x^2}\,dx$$

input
```
int(cot(d*(a + b*log(c*x^n)))/x^2,x)
```

output
```
int(cot(d*(a + b*log(c*x^n)))/x^2, x)
```

Reduce [F]

$$\int \frac{\cot (d(a + b\log (cx^n)))}{x^2} dx = \int \frac{\cot (\log(x^n c) bd + ad)}{x^2} dx$$

input
```
int(cot(d*(a+b*log(c*x^n)))/x^2,x)
```

output
```
int(cot(log(x**n*c)*b*d + a*d)/x**2,x)
```

3.215 $\int \frac{\cot(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 68

$$\int \frac{\cot(d(a+b\log(cx^n)))}{x^3}\,dx = -\frac{i}{2x^2} + \frac{i\,\text{Hypergeometric2F1}\left(1,\frac{i}{bdn},1+\frac{i}{bdn},e^{2iad}(cx^n)^{2ibd}\right)}{x^2}$$

output
```
-1/2*I/x^2+I*hypergeom([1, I/b/d/n],[1+I/b/d/n],exp(2*I*a*d)*(c*x^n)^(2*I*
b*d))/x^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 211 vs. 2(68) = 136.

Time = 2.97 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.10

$$\int \frac{\cot(d(a+b\log(cx^n)))}{x^3}\,dx = \frac{\cot(d(a+b\log(cx^n))) - \cot(d(a-bn\log(x)+b\log(cx^n))) - \frac{e^{2id(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1+\frac{i}{bdn},2+\frac{i}{bdn}\right.}{i+bdn}}{}$$

input
```
Integrate[Cot[d*(a + b*Log[c*x^n])]/x^3,x]
```

output
```
(Cot[d*(a + b*Log[c*x^n])] - Cot[d*(a - b*n*Log[x] + b*Log[c*x^n])] - (E^(
(2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + I/(b*d*n), 2 + I/(b*d
*n), E^((2*I)*d*(a + b*Log[c*x^n]))])/(I + b*d*n) + I*Hypergeometric2F1[1,
 I/(b*d*n), 1 + I/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + Csc[d*(a + b*
Log[c*x^n])]*Csc[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sin[b*d*n*Log[x]])/(2*
x^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.53, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5009, 5007, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot (d(a+b\log (cx^n)))}{x^3}dx$$

$$\Big\downarrow \ 5009$$

$$\frac{(cx^n)^{2/n}\int (cx^n)^{-1-\frac{2}{n}}\cot (d(a+b\log (cx^n)))d(cx^n)}{nx^2}$$

$$\Big\downarrow \ 5007$$

$$\frac{(cx^n)^{2/n}\int \frac{(cx^n)^{-1-\frac{2}{n}}\left(-ie^{2iad}(cx^n)^{2ibd}-i\right)}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{nx^2}$$

$$\Big\downarrow \ 959$$

$$\frac{(cx^n)^{2/n}\left(-2i\int \frac{(cx^n)^{-1-\frac{2}{n}}}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)-\frac{1}{2}in(cx^n)^{-2/n}\right)}{nx^2}$$

$$\Big\downarrow \ 888$$

$$\frac{(cx^n)^{2/n}\left(in(cx^n)^{-2/n}\text{Hypergeometric2F1}\left(1,\frac{i}{bdn},1+\frac{i}{bdn},e^{2iad}(cx^n)^{2ibd}\right)-\frac{1}{2}in(cx^n)^{-2/n}\right)}{nx^2}$$

input
```
Int[Cot[d*(a + b*Log[c*x^n])]/x^3,x]
```

output
```
((c*x^n)^(2/n)*(((-1/2*I)*n)/(c*x^n)^(2/n) + (I*n*Hypergeometric2F1[1, I/(
b*d*n), 1 + I/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(c*x^n)^(2/n)))
/(n*x^2)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5009
```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\cot(d(a + b\ln(c\,x^n)))}{x^3}dx$$

input
```
int(cot(d*(a+b*ln(c*x^n)))/x^3,x)
```

output
```
int(cot(d*(a+b*ln(c*x^n)))/x^3,x)
```

Fricas [F]

$$\int \frac{\cot(d(a + b\log(cx^n)))}{x^3}\,dx = \int \frac{\cot((b\log(cx^n) + a)d)}{x^3}\,dx$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

output
```
integral(cot(b*d*log(c*x^n) + a*d)/x^3, x)
```

Sympy [F]

$$\int \frac{\cot(d(a + b\log(cx^n)))}{x^3}\,dx = \int \frac{\cot(ad + bd\log(cx^n))}{x^3}\,dx$$

input
```
integrate(cot(d*(a+b*ln(c*x**n)))/x**3,x)
```

output
```
Integral(cot(a*d + b*d*log(c*x**n))/x**3, x)
```

Maxima [F]

$$\int \frac{\cot\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx=\int \frac{\cot\left((b\log\left(cx^n\right)+a)d\right)}{x^3}\,dx$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

output
```
integrate(cot((b*log(c*x^n) + a)*d)/x^3, x)
```

Giac [F]

$$\int \frac{\cot\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx=\int \frac{\cot\left((b\log\left(cx^n\right)+a)d\right)}{x^3}\,dx$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

output
```
integrate(cot((b*log(c*x^n) + a)*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx=\int \frac{\cot(d\left(a+b\,\ln\left(c\,x^n\right)\right))}{x^3}\,dx$$

input
```
int(cot(d*(a + b*log(c*x^n)))/x^3,x)
```

output
```
int(cot(d*(a + b*log(c*x^n)))/x^3, x)
```

Reduce [F]

$$\int \frac{\cot(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\cot(\log(x^n c)\,bd+ad)}{x^3}dx$$

input
```
int(cot(d*(a+b*log(c*x^n)))/x^3,x)
```

output
```
int(cot(log(x**n*c)*b*d + a*d)/x**3,x)
```

3.216 $\int x^3 \cot^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 158

$$\int x^3 \cot^2(d(a+b\log(cx^n)))\,dx = \frac{(4i-bdn)x^4}{4bdn} + \frac{ix^4\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1-e^{2iad}(cx^n)^{2ibd}\right)} - \frac{2ix^4\,\text{Hypergeometric2F1}\left(1,-\frac{2i}{bdn},1-\frac{2i}{bdn},e^{2iad}(cx^n)^{2ibd}\right)}{bdn}$$

output

```
1/4*(4*I-b*d*n)*x^4/b/d/n+I*x^4*(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/(
1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))-2*I*x^4*hypergeom([1, -2*I/b/d/n],[1-2*I
/b/d/n],exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n
```

Mathematica [A] (verified)

Time = 3.57 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.11

$$\int x^3 \cot^2 (d(a + b \log (cx^n))) \, dx =$$
$$-\frac{x^4 \left(8e^{2id(a+b\log(cx^n))} \text{Hypergeometric2F1}\left(1, 1 - \frac{2i}{bdn}, 2 - \frac{2i}{bdn}, e^{2id(a+b\log(cx^n))}\right) + (-2i + bdn)\left(bdn + 4\,\text{c}\right.}{4bdn(-2i + bdn)}$$

input
```
Integrate[x^3*Cot[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
-1/4*(x^4*(8*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - (2*I)
/(b*d*n), 2 - (2*I)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + (-2*I + b*d
*n)*(b*d*n + 4*Cot[d*(a + b*Log[c*x^n])] + (4*I)*Hypergeometric2F1[1, (-2*
I)/(b*d*n), 1 - (2*I)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))])))/(b*d*n*(
-2*I + b*d*n))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.34, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5009, 5007, 1004, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cot^2 (d(a + b \log (cx^n))) \, dx$$
$$\Big\downarrow \; 5009$$
$$\frac{x^4 (cx^n)^{-4/n} \int (cx^n)^{\frac{4}{n}-1} \cot^2 (d(a + b \log (cx^n))) \, d(cx^n)}{n}$$
$$\Big\downarrow \; 5007$$
$$\frac{x^4 (cx^n)^{-4/n} \int \frac{(cx^n)^{\frac{4}{n}-1} \left(-ie^{2iad} (cx^n)^{2ibd} - i\right)^2}{\left(1 - e^{2iad} (cx^n)^{2ibd}\right)^2} d(cx^n)}{n}$$

$$\Big\downarrow \ 1004$$

$$\frac{x^4(cx^n)^{-4/n}\left(\frac{i(cx^n)^{4/n}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\int\frac{2(cx^n)^{\frac{4}{n}-1}\left(\frac{e^{4iad}(ibdn+4)(cx^n)^{2ibd}}{n}+\frac{e^{2iad}(4-ibdn)}{n}\right)}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{2bd}\right)}{n}$$

$$\Big\downarrow \ 27$$

$$\frac{x^4(cx^n)^{-4/n}\left(\frac{i(cx^n)^{4/n}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\int\frac{(cx^n)^{\frac{4}{n}-1}\left(\frac{e^{4iad}(ibdn+4)(cx^n)^{2ibd}}{n}+\frac{e^{2iad}(4-ibdn)}{n}\right)}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{bd}\right)}{n}$$

$$\Big\downarrow \ 959$$

$$\frac{x^4(cx^n)^{-4/n}\left(\frac{i(cx^n)^{4/n}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\left(\frac{8e^{2iad}\int\frac{(cx^n)^{\frac{4}{n}-1}}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{n}-\frac{1}{4}e^{2iad}(4+ibdn)(cx^n)^{4/n}\right)}{bd}\right)}{n}$$

$$\Big\downarrow \ 888$$

$$\frac{x^4(cx^n)^{-4/n}\left(\frac{i(cx^n)^{4/n}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\left(2e^{2iad}(cx^n)^{4/n}\text{ Hypergeometric2F1}\left(1,-\frac{2i}{bdn},1-\frac{2i}{bdn},e^{2iad}(cx^n)^{2ibd}\right)-\frac{1}{4}e^{2iad}(4+\right.}{bd}}{n}$$

input
```
Int[x^3*Cot[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
(x^4*((I*(c*x^n)^(4/n)*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*(1 -
E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - (I*(-1/4*(E^((2*I)*a*d)*(4 + I*b*d*n
)*(c*x^n)^(4/n)) + 2*E^((2*I)*a*d)*(c*x^n)^(4/n)*Hypergeometric2F1[1, (-2*
I)/(b*d*n), 1 - (2*I)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)]))/(b*d*E
^((2*I)*a*d))))/(n*(c*x^n)^(4/n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5009
```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^3 \cot\left(d(a+b\ln\left(c\,x^n\right))\right)^2 dx$$

input
```
int(x^3*cot(d*(a+b*ln(c*x^n)))^2,x)
```

output
```
int(x^3*cot(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int x^3 \cot^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^3 \cot\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

input
```
integrate(x^3*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

output
```
integral(x^3*cot(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int x^3 \cot^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^3 \cot^2\left(ad+bd\log\left(cx^n\right)\right)dx$$

input
```
integrate(x**3*cot(d*(a+b*ln(c*x**n)))**2,x)
```

output
```
Integral(x**3*cot(a*d + b*d*log(c*x**n))**2, x)
```

Maxima [F]

$$\int x^3 \cot^2(d(a+b\log(cx^n)))\,dx = \int x^3 \cot((b\log(cx^n)+a)d)^2\,dx$$

input
```
integrate(x^3*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

output
```
1/4*((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x^4*cos(2*b*d*l
og(x^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n
*x^4*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*x^4 - 2*(b*d*n*cos(2*b*d*log(c)
) - 4*sin(2*b*d*log(c)))*x^4*cos(2*b*d*log(x^n) + 2*a*d) + 2*(b*d*n*sin(2*
b*d*log(c)) + 4*cos(2*b*d*log(c)))*x^4*sin(2*b*d*log(x^n) + 2*a*d) - 16*(2
*b^2*d^2*n^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2
*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) - b^2*d^2*n^2 - (b^2*d^2*co
s(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) +
2*a*d)^2 - (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2
*sin(2*b*d*log(x^n) + 2*a*d)^2)*integrate((x^3*cos(b*d*log(x^n) + a*d)*sin
(b*d*log(c)) + x^3*cos(b*d*log(c))*sin(b*d*log(x^n) + a*d))/(2*b^2*d^2*n^2
*cos(b*d*log(c))*cos(b*d*log(x^n) + a*d) - 2*b^2*d^2*n^2*sin(b*d*log(c))*s
in(b*d*log(x^n) + a*d) + b^2*d^2*n^2 + (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^
2*sin(b*d*log(c))^2)*n^2*cos(b*d*log(x^n) + a*d)^2 + (b^2*d^2*cos(b*d*log(
c))^2 + b^2*d^2*sin(b*d*log(c))^2)*n^2*sin(b*d*log(x^n) + a*d)^2), x) + 16
*(2*b^2*d^2*n^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*
n^2*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) - b^2*d^2*n^2 - (b^2*d^2
*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n)
 + 2*a*d)^2 - (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*
n^2*sin(2*b*d*log(x^n) + 2*a*d)^2)*integrate(-(x^3*cos(b*d*log(x^n) + a...
```

Giac [F(-1)]

Timed out.

$$\int x^3 \cot^2(d(a+b\log(cx^n)))\,dx = \text{Timed out}$$

input
```
integrate(x^3*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \cot^2(d(a+b\log(cx^n)))\,dx = \int x^3 \cot(d\,(a+b\,\ln(c\,x^n)))^2\,dx$$

input
```
int(x^3*cot(d*(a + b*log(c*x^n)))^2,x)
```

output
```
int(x^3*cot(d*(a + b*log(c*x^n)))^2, x)
```

Reduce [F]

$$\int x^3 \cot^2(d(a+b\log(cx^n)))\,dx = \int \cot(\log(x^n c)\,bd+ad)^2 x^3 dx$$

input
```
int(x^3*cot(d*(a+b*log(c*x^n)))^2,x)
```

output
```
int(cot(log(x**n*c)*b*d + a*d)**2*x**3,x)
```

3.217 $\int x^2 \cot^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 162

$$\begin{aligned}\int x^2\cot^2(d(a+b\log(cx^n)))\,dx &= \frac{(3i-bdn)x^3}{3bdn}+\frac{ix^3\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1-e^{2iad}(cx^n)^{2ibd}\right)}\\&\quad-\frac{2ix^3\,\text{Hypergeometric2F1}\left(1,-\frac{3i}{2bdn},1-\frac{3i}{2bdn},e^{2iad}(cx^n)^{2ibd}\right)}{bdn}\end{aligned}$$

output
```
1/3*(3*I-b*d*n)*x^3/b/d/n+I*x^3*(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/(
1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))-2*I*x^3*hypergeom([1, -3/2*I/b/d/n],[1-3
/2*I/b/d/n],exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n
```

Mathematica [A] (verified)

Time = 3.84 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.14

$$\int x^2 \cot^2(d(a+b\log(cx^n)))\,dx = -\frac{x^3\left(9e^{2id(a+b\log(cx^n))}\text{Hypergeometric2F1}\left(1,1-\frac{3i}{2bdn},2-\frac{3i}{2bdn},e^{2id(a+b\log(cx^n))}\right)+(-3i+2bdn)\left(bdn+\right.\right.}{3bdn(-3i+2bd}$$

input
```
Integrate[x^2*Cot[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
-1/3*(x^3*(9*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - ((3*I
)/2)/(b*d*n), 2 - ((3*I)/2)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + (-3
*I + 2*b*d*n)*(b*d*n + 3*Cot[d*(a + b*Log[c*x^n])] + (3*I)*Hypergeometric2
F1[1, ((-3*I)/2)/(b*d*n), 1 - ((3*I)/2)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x
^n]))])))/(b*d*n*(-3*I + 2*b*d*n))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.33, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5009, 5007, 1004, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cot^2(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \; 5009$$

$$\frac{x^3(cx^n)^{-3/n}\int (cx^n)^{\frac{3}{n}-1}\cot^2(d(a+b\log(cx^n)))\,d(cx^n)}{n}$$

$$\Big\downarrow \; 5007$$

$$\frac{x^3(cx^n)^{-3/n}\int \frac{(cx^n)^{\frac{3}{n}-1}\left(-ie^{2iad}(cx^n)^{2ibd}-i\right)^2}{\left(1-e^{2iad}(cx^n)^{2ibd}\right)^2}d(cx^n)}{n}$$

$$\Big\downarrow \text{1004}$$

$$\frac{x^3(cx^n)^{-3/n}\left(\frac{i(cx^n)^{3/n}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\int\frac{2(cx^n)^{\frac{3}{n}-1}\left(\frac{e^{4iad}(ibdn+3)(cx^n)^{2ibd}}{n}+\frac{e^{2iad}(3-ibdn)}{n}\right)}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{2bd}\right)}{n}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3(cx^n)^{-3/n}\left(\frac{i(cx^n)^{3/n}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\int\frac{(cx^n)^{\frac{3}{n}-1}\left(\frac{e^{4iad}(ibdn+3)(cx^n)^{2ibd}}{n}+\frac{e^{2iad}(3-ibdn)}{n}\right)}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{bd}\right)}{n}$$

$$\Big\downarrow \text{959}$$

$$\frac{x^3(cx^n)^{-3/n}\left(\frac{i(cx^n)^{3/n}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\left(\frac{6e^{2iad}\int\frac{(cx^n)^{\frac{3}{n}-1}}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{n}-\frac{1}{3}e^{2iad}(3+ibdn)(cx^n)^{3/n}\right)}{bd}\right)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^3(cx^n)^{-3/n}\left(\frac{i(cx^n)^{3/n}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\left(2e^{2iad}(cx^n)^{3/n}\text{Hypergeometric2F1}\left(1,-\frac{3i}{2bdn},1-\frac{3i}{2bdn},e^{2iad}(cx^n)^{2ibd}\right)-\frac{1}{3}e^{2iad}(3\right.}{bd}}{n}$$

input
```
Int[x^2*Cot[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
(x^3*((I*(c*x^n)^(3/n)*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*(1 -
E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - (I*(-1/3*(E^((2*I)*a*d)*(3 + I*b*d*n
)*(c*x^n)^(3/n)) + 2*E^((2*I)*a*d)*(c*x^n)^(3/n)*Hypergeometric2F1[1, ((-3
*I)/2)/(b*d*n), 1 - ((3*I)/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])
)/(b*d*E^((2*I)*a*d))))/(n*(c*x^n)^(3/n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5009
```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^2\cot\left(d(a+b\ln\left(c\,x^n\right))\right)^2dx$$

input
```
int(x^2*cot(d*(a+b*ln(c*x^n)))^2,x)
```

output
```
int(x^2*cot(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int x^2\cot^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x^2\cot\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

input
```
integrate(x^2*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

output
```
integral(x^2*cot(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int x^2\cot^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x^2\cot^2\left(ad+bd\log\left(cx^n\right)\right)dx$$

input
```
integrate(x**2*cot(d*(a+b*ln(c*x**n)))**2,x)
```

output
```
Integral(x**2*cot(a*d + b*d*log(c*x**n))**2, x)
```

Maxima [F]

$$\int x^2 \cot^2(d(a+b\log(cx^n)))\,dx = \int x^2 \cot((b\log(cx^n)+a)d)^2\,dx$$

input
```
integrate(x^2*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

output
```
1/3*((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x^3*cos(2*b*d*l
og(x^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n
*x^3*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*x^3 - 2*(b*d*n*cos(2*b*d*log(c)
) - 3*sin(2*b*d*log(c)))*x^3*cos(2*b*d*log(x^n) + 2*a*d) + 2*(b*d*n*sin(2*
b*d*log(c)) + 3*cos(2*b*d*log(c)))*x^3*sin(2*b*d*log(x^n) + 2*a*d) - 9*(2*
b^2*d^2*n^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*
sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) - b^2*d^2*n^2 - (b^2*d^2*cos
(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) + 2
*a*d)^2 - (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*
sin(2*b*d*log(x^n) + 2*a*d)^2)*integrate((x^2*cos(b*d*log(x^n) + a*d)*sin(
b*d*log(c)) + x^2*cos(b*d*log(c))*sin(b*d*log(x^n) + a*d))/(2*b^2*d^2*n^2*
cos(b*d*log(c))*cos(b*d*log(x^n) + a*d) - 2*b^2*d^2*n^2*sin(b*d*log(c))*si
n(b*d*log(x^n) + a*d) + b^2*d^2*n^2 + (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2
*sin(b*d*log(c))^2)*n^2*cos(b*d*log(x^n) + a*d)^2 + (b^2*d^2*cos(b*d*log(c
))^2 + b^2*d^2*sin(b*d*log(c))^2)*n^2*sin(b*d*log(x^n) + a*d)^2), x) + 9*(
2*b^2*d^2*n^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^
2*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) - b^2*d^2*n^2 - (b^2*d^2*c
os(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) +
 2*a*d)^2 - (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^
2*sin(2*b*d*log(x^n) + 2*a*d)^2)*integrate(-(x^2*cos(b*d*log(x^n) + a*d...
```

Giac [F(-1)]

Timed out.

$$\int x^2 \cot^2(d(a+b\log(cx^n)))\,dx = \text{Timed out}$$

input
```
integrate(x^2*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cot^2(d(a+b\log(cx^n)))\,dx = \int x^2 \cot(d\,(a+b\,\ln(c\,x^n)))^2\,dx$$

input
```
int(x^2*cot(d*(a + b*log(c*x^n)))^2,x)
```

output
```
int(x^2*cot(d*(a + b*log(c*x^n)))^2, x)
```

Reduce [F]

$$\int x^2 \cot^2(d(a+b\log(cx^n)))\,dx = \int \cot(\log(x^n c)\,bd+ad)^2 x^2 dx$$

input
```
int(x^2*cot(d*(a+b*log(c*x^n)))^2,x)
```

output
```
int(cot(log(x**n*c)*b*d + a*d)**2*x**2,x)
```

3.218 $\int x \cot^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 158

$$\int x\cot^2(d(a+b\log(cx^n)))\,dx = \frac{(2i-bdn)x^2}{2bdn}+\frac{ix^2\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1-e^{2iad}(cx^n)^{2ibd}\right)} - \frac{2ix^2\,\text{Hypergeometric2F1}\left(1,-\frac{i}{bdn},1-\frac{i}{bdn},e^{2iad}(cx^n)^{2ibd}\right)}{bdn}$$

output
```
1/2*(2*I-b*d*n)*x^2/b/d/n+I*x^2*(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/(
1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))-2*I*x^2*hypergeom([1, -I/b/d/n],[1-I/b/d
/n],exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n
```

Mathematica [A] (verified)

Time = 3.87 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.11

$$\int x\cot^2(d(a+b\log(cx^n)))\,dx = -\frac{x^2\left(2e^{2id(a+b\log(cx^n))}\text{Hypergeometric2F1}\left(1,1-\frac{i}{bdn},2-\frac{i}{bdn},e^{2id(a+b\log(cx^n))}\right)+(-i+bdn)\left(bdn+2\cot\ldots\right.\right.}{2bdn(-i+bdn)}$$

input
```
Integrate[x*Cot[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
-1/2*(x^2*(2*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - I/(b*
d*n), 2 - I/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + (-I + b*d*n)*(b*d*n
 + 2*Cot[d*(a + b*Log[c*x^n])] + (2*I)*Hypergeometric2F1[1, (-I)/(b*d*n),
1 - I/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))])))/(b*d*n*(-I + b*d*n))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.34, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {5009, 5007, 1004, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cot^2(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{5009}$$

$$\frac{x^2(cx^n)^{-2/n}\int (cx^n)^{\frac{2}{n}-1}\cot^2(d(a+b\log(cx^n)))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5007}$$

$$\frac{x^2(cx^n)^{-2/n}\int \frac{(cx^n)^{\frac{2}{n}-1}\left(-ie^{2iad}(cx^n)^{2ibd}-i\right)^2}{\left(1-e^{2iad}(cx^n)^{2ibd}\right)^2}d(cx^n)}{n}$$

$$\Big\downarrow \text{1004}$$

$$\frac{x^2 (cx^n)^{-2/n} \left(\frac{i(cx^n)^{2/n}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)} - \frac{ie^{-2iad}\int \frac{2(cx^n)^{\frac{2}{n}-1}\left(\frac{e^{4iad}(ibdn+2)(cx^n)^{2ibd}}{n}+\frac{e^{2iad}(2-ibdn)}{n}\right)}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{2bd}\right)}{n}$$

$$\Big\downarrow \; 27$$

$$\frac{x^2 (cx^n)^{-2/n} \left(\frac{i(cx^n)^{2/n}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)} - \frac{ie^{-2iad}\int \frac{(cx^n)^{\frac{2}{n}-1}\left(\frac{e^{4iad}(ibdn+2)(cx^n)^{2ibd}}{n}+\frac{e^{2iad}(2-ibdn)}{n}\right)}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{bd}\right)}{n}$$

$$\Big\downarrow \; 959$$

$$\frac{x^2 (cx^n)^{-2/n} \left(\frac{i(cx^n)^{2/n}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)} - \frac{ie^{-2iad}\left(\frac{4e^{2iad}\int \frac{(cx^n)^{\frac{2}{n}-1}}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{n}-\frac{1}{2}e^{2iad}(2+ibdn)(cx^n)^{2/n}\right)}{bd}\right)}{n}$$

$$\Big\downarrow \; 888$$

$$\frac{x^2 (cx^n)^{-2/n} \left(\frac{i(cx^n)^{2/n}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)} - \frac{ie^{-2iad}\left(2e^{2iad}(cx^n)^{2/n}\,\text{Hypergeometric2F1}\left(1,-\frac{i}{bdn},1-\frac{i}{bdn},e^{2iad}(cx^n)^{2ibd}\right)-\frac{1}{2}e^{2iad}(2+\right.}{bd}\right.}{n}$$

input
```
Int[x*Cot[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
(x^2*((I*(c*x^n)^(2/n)*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*(1 -
E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - (I*(-1/2*(E^((2*I)*a*d)*(2 + I*b*d*n
)*(c*x^n)^(2/n)) + 2*E^((2*I)*a*d)*(c*x^n)^(2/n)*Hypergeometric2F1[1, (-I)
/(b*d*n), 1 - I/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)]))/(b*d*E^((2*I
)*a*d))))/(n*(c*x^n)^(2/n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5009
```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x\cot\left(d(a+b\ln\left(c\,x^{n}\right))\right)^{2}dx$$

input
```
int(x*cot(d*(a+b*ln(c*x^n)))^2,x)
```

output
```
int(x*cot(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int x\cot^{2}\left(d(a+b\log\left(cx^{n}\right))\right)\,dx=\int x\cot\left((b\log\left(cx^{n}\right)+a)d\right)^{2}\,dx$$

input
```
integrate(x*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

output
```
integral(x*cot(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int x\cot^{2}\left(d(a+b\log\left(cx^{n}\right))\right)\,dx=\int x\cot^{2}\left(ad+bd\log\left(cx^{n}\right)\right)dx$$

input
```
integrate(x*cot(d*(a+b*ln(c*x**n)))**2,x)
```

output
```
Integral(x*cot(a*d + b*d*log(c*x**n))**2, x)
```

Maxima [F]

$$\int x\cot^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\cot\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

input
```
integrate(x*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

output
```
1/2*((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x^2*cos(2*b*d*l
og(x^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n
*x^2*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*x^2 - 2*(b*d*n*cos(2*b*d*log(c)
) - 2*sin(2*b*d*log(c)))*x^2*cos(2*b*d*log(x^n) + 2*a*d) + 2*(b*d*n*sin(2*
b*d*log(c)) + 2*cos(2*b*d*log(c)))*x^2*sin(2*b*d*log(x^n) + 2*a*d) - 4*(2*
b^2*d^2*n^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*
sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) - b^2*d^2*n^2 - (b^2*d^2*cos
(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) + 2
*a*d)^2 - (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*
sin(2*b*d*log(x^n) + 2*a*d)^2)*integrate((x*cos(b*d*log(x^n) + a*d)*sin(b*
d*log(c)) + x*cos(b*d*log(c))*sin(b*d*log(x^n) + a*d))/(2*b^2*d^2*n^2*cos(
b*d*log(c))*cos(b*d*log(x^n) + a*d) - 2*b^2*d^2*n^2*sin(b*d*log(c))*sin(b*
d*log(x^n) + a*d) + b^2*d^2*n^2 + (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin
(b*d*log(c))^2)*n^2*cos(b*d*log(x^n) + a*d)^2 + (b^2*d^2*cos(b*d*log(c))^2
 + b^2*d^2*sin(b*d*log(c))^2)*n^2*sin(b*d*log(x^n) + a*d)^2), x) + 4*(2*b^
2*d^2*n^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*si
n(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) - b^2*d^2*n^2 - (b^2*d^2*cos(2
*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) + 2*a
*d)^2 - (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*si
n(2*b*d*log(x^n) + 2*a*d)^2)*integrate(-(x*cos(b*d*log(x^n) + a*d)*sin(...
```

Giac [F(-1)]

Timed out.

$$\int x\cot^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\text{Timed out}$$

input
```
integrate(x*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x \cot^2\left(d(a+b\log\left(cx^n\right))\right) \, dx = \int x \cot(d\left(a+b\,\ln\left(c\,x^n\right)\right))^2 \,dx$$

input
```
int(x*cot(d*(a + b*log(c*x^n)))^2,x)
```

output
```
int(x*cot(d*(a + b*log(c*x^n)))^2, x)
```

Reduce [F]

$$\int x \cot^2\left(d(a+b\log\left(cx^n\right))\right) \, dx = \int \cot\left(\log(x^n c)\,bd+ad\right)^2 x dx$$

input
```
int(x*cot(d*(a+b*log(c*x^n)))^2,x)
```

output
```
int(cot(log(x**n*c)*b*d + a*d)**2*x,x)
```

3.219 $\int \cot^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 153

$$\int \cot^2(d(a+b\log(cx^n)))\,dx = \frac{(i-bdn)x}{bdn}+\frac{ix\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bdn\left(1-e^{2iad}(cx^n)^{2ibd}\right)} - \frac{2ix\,\text{Hypergeometric2F1}\left(1,-\frac{i}{2bdn},1-\frac{i}{2bdn},e^{2iad}(cx^n)^{2ibd}\right)}{bdn}$$

output
```
(I-b*d*n)*x/b/d/n+I*x*(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/(1-exp(2*I*
a*d)*(c*x^n)^(2*I*b*d))-2*I*x*hypergeom([1, -1/2*I/b/d/n],[1-1/2*I/b/d/n],
exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n
```

Mathematica [A] (verified)

Time = 7.37 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.16

$$\int \cot^2(d(a+b\log(cx^n)))\,dx = -\frac{x\left(e^{2id(a+b\log(cx^n))}\text{Hypergeometric2F1}\left(1,1-\frac{i}{2bdn},2-\frac{i}{2bdn},e^{2id(a+b\log(cx^n))}\right)+(-i+2bdn)\left(bdn+\cot\right.\right.}{bdn(-i+2bdn)}$$

input
```
Integrate[Cot[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
-((x*(E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - (I/2)/(b*d*n
), 2 - (I/2)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + (-I + 2*b*d*n)*(b*
d*n + Cot[d*(a + b*Log[c*x^n])] + I*Hypergeometric2F1[1, (-1/2*I)/(b*d*n),
 1 - (I/2)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))])))/(b*d*n*(-I + 2*b*d*
n)))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.35, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5005, 5007, 1004, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{5005}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\cot^2(d(a+b\log(cx^n)))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5007}$$

$$\frac{x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}-1}\left(-ie^{2iad}(cx^n)^{2ibd}-i\right)^2}{\left(1-e^{2iad}(cx^n)^{2ibd}\right)^2}d(cx^n)}{n}$$

$$\Big\downarrow \; 1004$$

$$\frac{x(cx^n)^{-1/n}\left(\frac{i(cx^n)^{\frac{1}{n}}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\int \frac{2(cx^n)^{\frac{1}{n}-1}\left(\frac{e^{4iad}(ibdn+1)(cx^n)^{2ibd}}{n}+\frac{e^{2iad}(1-ibdn)}{n}\right)}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{2bd}\right)}{n}$$

$$\Big\downarrow \; 27$$

$$\frac{x(cx^n)^{-1/n}\left(\frac{i(cx^n)^{\frac{1}{n}}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\int \frac{(cx^n)^{\frac{1}{n}-1}\left(\frac{e^{4iad}(ibdn+1)(cx^n)^{2ibd}}{n}+\frac{e^{2iad}(1-ibdn)}{n}\right)}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{bd}\right)}{n}$$

$$\Big\downarrow \; 959$$

$$\frac{x(cx^n)^{-1/n}\left(\frac{i(cx^n)^{\frac{1}{n}}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\left(\frac{2e^{2iad}\int \frac{(cx^n)^{\frac{1}{n}-1}}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{n}-e^{2iad}(1+ibdn)(cx^n)^{\frac{1}{n}}\right)}{bd}\right)}{n}$$

$$\Big\downarrow \; 888$$

$$\frac{x(cx^n)^{-1/n}\left(\frac{i(cx^n)^{\frac{1}{n}}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\left(2e^{2iad}(cx^n)^{\frac{1}{n}}\text{Hypergeometric2F1}\left(1,-\frac{i}{2bdn},1-\frac{i}{2bdn},e^{2iad}(cx^n)^{2ibd}\right)-e^{2iad}(1+ibd\right.}{bd}}{n}$$

input
```
Int[Cot[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
(x*((I*(c*x^n)^n^(-1)*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*(1 - E
^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - (I*(-(E^((2*I)*a*d)*(1 + I*b*d*n)*(c*
x^n)^n^(-1)) + 2*E^((2*I)*a*d)*(c*x^n)^n^(-1)*Hypergeometric2F1[1, (-1/2*I
)/(b*d*n), 1 - (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)]))/(b*d*E^
((2*I)*a*d))))/(n*(c*x^n)^n^(-1))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 5005
```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cot[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [F]

$$\int \cot\left(d(a+b\ln\left(c\,x^{n}\right))\right)^{2}dx$$

input
```
int(cot(d*(a+b*ln(c*x^n)))^2,x)
```

output
```
int(cot(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int \cot^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int \cot\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

output
```
integral(cot(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int \cot^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int \cot^2\left(d(a+b\log\left(cx^n\right))\right)dx$$

input
```
integrate(cot(d*(a+b*ln(c*x**n)))**2,x)
```

output
```
Integral(cot(d*(a + b*log(c*x**n)))**2, x)
```

Maxima [F]

$$\int \cot^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int \cot\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

output
```
((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x*cos(2*b*d*log(x^n
) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*x*sin
(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*x - 2*(b*d*n*cos(2*b*d*log(c)) - sin(2*
b*d*log(c)))*x*cos(2*b*d*log(x^n) + 2*a*d) + 2*(b*d*n*sin(2*b*d*log(c)) +
cos(2*b*d*log(c)))*x*sin(2*b*d*log(x^n) + 2*a*d) - (2*b^2*d^2*n^2*cos(2*b*
d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*sin(2*b*d*log(c))*si
n(2*b*d*log(x^n) + 2*a*d) - b^2*d^2*n^2 - (b^2*d^2*cos(2*b*d*log(c))^2 + b
^2*d^2*sin(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^2 - (b^2*d^2*c
os(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*sin(2*b*d*log(x^n) +
 2*a*d)^2)*integrate((cos(b*d*log(x^n) + a*d)*sin(b*d*log(c)) + cos(b*d*lo
g(c))*sin(b*d*log(x^n) + a*d))/(2*b^2*d^2*n^2*cos(b*d*log(c))*cos(b*d*log(
x^n) + a*d) - 2*b^2*d^2*n^2*sin(b*d*log(c))*sin(b*d*log(x^n) + a*d) + b^2*
d^2*n^2 + (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin(b*d*log(c))^2)*n^2*cos(
b*d*log(x^n) + a*d)^2 + (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin(b*d*log(c
))^2)*n^2*sin(b*d*log(x^n) + a*d)^2), x) + (2*b^2*d^2*n^2*cos(2*b*d*log(c)
)*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*sin(2*b*d*log(c))*sin(2*b*d*
log(x^n) + 2*a*d) - b^2*d^2*n^2 - (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*s
in(2*b*d*log(c))^2)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^2 - (b^2*d^2*cos(2*b*d
*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*sin(2*b*d*log(x^n) + 2*a*d)^
2)*integrate(-(cos(b*d*log(x^n) + a*d)*sin(b*d*log(c)) + cos(b*d*log(c)...
```

Giac [F(-1)]

Timed out.

$$\int \cot^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \text{Timed out}$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \cot^2(d(a+b\log(cx^n)))\,dx = \int \cot(d\,(a+b\,\ln(c\,x^n)))^2\,dx$$

input
```
int(cot(d*(a + b*log(c*x^n)))^2,x)
```

output
```
int(cot(d*(a + b*log(c*x^n)))^2, x)
```

Reduce [F]

$$\int \cot^2(d(a+b\log(cx^n)))\,dx = \int \cot(\log(x^n c)\,bd+ad)^2dx$$

input
```
int(cot(d*(a+b*log(c*x^n)))^2,x)
```

output
```
int(cot(log(x**n*c)*b*d + a*d)**2,x)
```

3.220 $\int \frac{\cot^2(d(a+b\log(cx^n)))}{x} dx$

Optimal result

Integrand size = 19, antiderivative size = 30

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x} dx = -\frac{\cot(ad+bd\log(cx^n))}{bdn} - \log(x)$$

output
```
-cot(a*d+b*d*ln(c*x^n))/b/d/n-ln(x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.70

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x} dx = -\frac{\cot(ad+bd\log(cx^n))\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(ad+bd\log(cx^n))\right)}{bdn}$$

input
```
Integrate[Cot[d*(a + b*Log[c*x^n])]^2/x,x]
```

output
```
-((Cot[a*d + b*d*Log[c*x^n]]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[a*d + b*
d*Log[c*x^n]]^2])/(b*d*n))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3039, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x}dx$$

$$\Big\downarrow \text{ 3039}$$

$$\frac{\int \cot^2(d(a+b\log(cx^n)))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \tan\left(ad+b\log(cx^n)d+\frac{\pi}{2}\right)^2d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{-\int 1d\log(cx^n)-\frac{\cot(ad+bd\log(cx^n))}{bd}}{n}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{-\frac{\cot(ad+bd\log(cx^n))}{bd}-\log(cx^n)}{n}$$

input
```
Int[Cot[d*(a + b*Log[c*x^n])]^2/x,x]
```

output
```
(-(Cot[a*d + b*d*Log[c*x^n]]/(b*d)) - Log[c*x^n])/n
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

method	result
parallelrisch	$\frac{-\ln(x) d b n-\cot(d(a+b\ln(c\,x^{n})))}{b d n}$
derivativedivides	$\frac{-\cot(d(a+b\ln(c\,x^{n})))+\frac{\pi}{2}-\operatorname{arccot}(\cot(d(a+b\ln(c\,x^{n}))))}{n b d}$
default	$\frac{-\cot(d(a+b\ln(c\,x^{n})))+\frac{\pi}{2}-\operatorname{arccot}(\cot(d(a+b\ln(c\,x^{n}))))}{n b d}$
risch	$-\ln(x)-\frac{2i}{d b n\left(c^{2ibd}(x^{n})^{2ibd}\mathrm{e}^{d\left(-b\pi\ \operatorname{csgn}(ix^{n})\operatorname{csgn}(ic\,x^{n})^{2}+b\pi\ \operatorname{csgn}(ix^{n})\operatorname{csgn}(ic\,x^{n})\operatorname{csgn}(ic)+b\pi\operatorname{csgn}(ic\,x^{n})^{3}-b\pi\operatorname{csgn}(\right.}\right.}$

input
```
int(cot(d*(a+b*ln(c*x^n)))^2/x,x,method=_RETURNVERBOSE)
```

output
```
(-ln(x)*d*b*n-cot(d*(a+b*ln(c*x^n))))/b/d/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(30) = 60$.

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.60

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x}\,dx = -\frac{bdn\log(x)\sin(2\,bdn\log(x)+2\,bd\log(c)+2\,ad)+\cos(2\,bdn\log(x)+2\,bd\log(c)+2\,ad)+1}{bdn\sin(2\,bdn\log(x)+2\,bd\log(c)+2\,ad)}$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))^2/x,x, algorithm="fricas")
```

output
```
-(b*d*n*log(x)*sin(2*b*d*n*log(x) + 2*b*d*log(c) + 2*a*d) + cos(2*b*d*n*lo
g(x) + 2*b*d*log(c) + 2*a*d) + 1)/(b*d*n*sin(2*b*d*n*log(x) + 2*b*d*log(c)
 + 2*a*d))
```

Sympy [F]

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x}\,dx = \int \frac{\cot^2(ad+bd\log(cx^n))}{x}\,dx$$

input
```
integrate(cot(d*(a+b*ln(c*x**n)))**2/x,x)
```

output
```
Integral(cot(a*d + b*d*log(c*x**n))**2/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(30) = 60$.

Time = 0.10 (sec) , antiderivative size = 322, normalized size of antiderivative = 10.73

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x}\,dx = \frac{\left(bd\cos(2\,bd\log(c))^2+bd\sin(2\,bd\log(c))^2\right)n\cos(2\,bd\log(x^n)+2\,ad)^2\log(x)+\left(bd\cos(2\,bd\log(c))^2+\right.}{2\,bdn\cos(2\,bd\log(c))\cos(2\,bd\log(x^n)+2\,ad)-2\,bdn\sin(2\,bd\log}$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))^2/x,x, algorithm="maxima")
```

output
```
((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d*log(x^n)
+ 2*a*d)^2*log(x) + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*
log(x)*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n*log(x) - 2*(b*d*n*cos(2*b*d*l
og(c))*log(x) - sin(2*b*d*log(c)))*cos(2*b*d*log(x^n) + 2*a*d) + 2*(b*d*n*
log(x)*sin(2*b*d*log(c)) + cos(2*b*d*log(c)))*sin(2*b*d*log(x^n) + 2*a*d))
/(2*b*d*n*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b*d*n*sin(2*b*
d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) - (b*d*cos(2*b*d*log(c))^2 + b*d*sin
(2*b*d*log(c))^2)*n*cos(2*b*d*log(x^n) + 2*a*d)^2 - (b*d*cos(2*b*d*log(c))
^2 + b*d*sin(2*b*d*log(c))^2)*n*sin(2*b*d*log(x^n) + 2*a*d)^2 - b*d*n)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))^2/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 24.92 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x}\,dx = -\ln(x) - \frac{2i}{b\,d\,n\,\left(e^{a\,d\,2i}\,(c\,x^n)^{b\,d\,2i}-1\right)}$$

input
```
int(cot(d*(a + b*log(c*x^n)))^2/x,x)
```

output
```
- log(x) - 2i/(b*d*n*(exp(a*d*2i)*(c*x^n)^(b*d*2i) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x}\,dx = \frac{-\cot(\log(x^nc)\,bd+ad)-\log(x^nc)\,bd}{bdn}$$

input
```
int(cot(d*(a+b*log(c*x^n)))^2/x,x)
```

output
```
( - (cot(log(x**n*c)*b*d + a*d) + log(x**n*c)*b*d))/(b*d*n)
```

3.221 $\int \frac{\cot^2(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 156

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x^2}\,dx = \frac{1+\frac{i}{bdn}}{x}+\frac{i\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bdnx\left(1-e^{2iad}(cx^n)^{2ibd}\right)} - \frac{2i\,\text{Hypergeometric2F1}\left(1,\frac{i}{2bdn},1+\frac{i}{2bdn},e^{2iad}(cx^n)^{2ibd}\right)}{bdnx}$$

output
```
(1+I/b/d/n)/x+I*(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/x/(1-exp(2*I*a*d)
*(c*x^n)^(2*I*b*d))-2*I*hypergeom([1, 1/2*I/b/d/n],[1+1/2*I/b/d/n],exp(2*I
*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/x
```

Mathematica [A] (verified)

Time = 3.26 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.16

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x^2}\,dx = \frac{e^{2id(a+b\log(cx^n))}\text{Hypergeometric2F1}\left(1,1+\frac{i}{2bdn},2+\frac{i}{2bdn},e^{2id(a+b\log(cx^n))}\right)+(i+2bdn)\left(bdn-\cot\left(d(a+\right.\right.}{bdn(i+2bdn)x}$$

input
```
Integrate[Cot[d*(a + b*Log[c*x^n])]^2/x^2,x]
```

output
```
(E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + (I/2)/(b*d*n), 2
+ (I/2)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + (I + 2*b*d*n)*(b*d*n -
Cot[d*(a + b*Log[c*x^n])] - I*Hypergeometric2F1[1, (I/2)/(b*d*n), 1 + (I/2
)/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))]))/(b*d*n*(I + 2*b*d*n)*x)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.35, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5009, 5007, 1004, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x^2}dx$$

$$\Big\downarrow \text{5009}$$

$$\frac{(cx^n)^{\frac{1}{n}}\int (cx^n)^{-1-\frac{1}{n}}\cot^2(d(a+b\log(cx^n)))d(cx^n)}{nx}$$

$$\Big\downarrow \text{5007}$$

$$\frac{(cx^n)^{\frac{1}{n}}\int \frac{(cx^n)^{-1-\frac{1}{n}}\left(-ie^{2iad}(cx^n)^{2ibd}-i\right)^2}{\left(1-e^{2iad}(cx^n)^{2ibd}\right)^2}d(cx^n)}{nx}$$

$$\Big\downarrow \text{1004}$$

$$\frac{(cx^n)^{\frac{1}{n}}\left(\frac{i(cx^n)^{-1/n}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\int -\frac{2(cx^n)^{-1-\frac{1}{n}}\left(\frac{e^{4iad}(1-ibdn)(cx^n)^{2ibd}}{n}+\frac{e^{2iad}(ibdn+1)}{n}\right)}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{2bd}\right)}{nx}$$

$$\Big\downarrow \; 27$$

$$\frac{(cx^n)^{\frac{1}{n}}\left(\frac{ie^{-2iad}\int \frac{(cx^n)^{-1-\frac{1}{n}}\left(\frac{e^{4iad}(1-ibdn)(cx^n)^{2ibd}}{n}+\frac{e^{2iad}(ibdn+1)}{n}\right)}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{bd}+\frac{i(cx^n)^{-1/n}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}\right)}{nx}$$

$$\Big\downarrow \; 959$$

$$\frac{(cx^n)^{\frac{1}{n}}\left(\frac{ie^{-2iad}\left(\frac{2e^{2iad}\int \frac{(cx^n)^{-1-\frac{1}{n}}}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{n}+e^{2iad}(1-ibdn)(cx^n)^{-1/n}\right)}{bd}+\frac{i(cx^n)^{-1/n}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}\right)}{nx}$$

$$\Big\downarrow \; 888$$

$$\frac{(cx^n)^{\frac{1}{n}}\left(\frac{ie^{-2iad}\left(e^{2iad}(1-ibdn)(cx^n)^{-1/n}-2e^{2iad}(cx^n)^{-1/n}\,\text{Hypergeometric2F1}\left(1,\frac{i}{2bdn},1+\frac{i}{2bdn},e^{2iad}(cx^n)^{2ibd}\right)\right)}{bd}+\frac{i(cx^n)^{-1/n}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}\right)}{nx}$$

input
```
Int[Cot[d*(a + b*Log[c*x^n])]^2/x^2,x]
```

output
```
((c*x^n)^n^(-1)*((I*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*(c*x^n)^
n^(-1)*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) + (I*((E^((2*I)*a*d)*(1 -
I*b*d*n))/(c*x^n)^n^(-1) - (2*E^((2*I)*a*d)*Hypergeometric2F1[1, (I/2)/(b*
d*n), 1 + (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(c*x^n)^n^(-1
)))/(b*d*E^((2*I)*a*d))))/(n*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5009
```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\cot \left(d(a+b\ln \left(c\,x^{n}\right))\right)^{2}}{x^{2}}dx$$

input
```
int(cot(d*(a+b*ln(c*x^n)))^2/x^2,x)
```

output
```
int(cot(d*(a+b*ln(c*x^n)))^2/x^2,x)
```

Fricas [F]

$$\int \frac{\cot^2 \left(d(a+b\log \left(cx^n\right))\right)}{x^2}\,dx=\int \frac{\cot \left((b\log \left(cx^n\right)+a)d\right)^2}{x^2}\,dx$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="fricas")
```

output
```
integral(cot(b*d*log(c*x^n) + a*d)^2/x^2, x)
```

Sympy [F]

$$\int \frac{\cot^2 \left(d(a+b\log \left(cx^n\right))\right)}{x^2}\,dx=\int \frac{\cot^2 \left(ad+bd\log \left(cx^n\right)\right)}{x^2}\,dx$$

input
```
integrate(cot(d*(a+b*ln(c*x**n)))**2/x**2,x)
```

output
```
Integral(cot(a*d + b*d*log(c*x**n))**2/x**2, x)
```

Maxima [F]

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x^2}\,dx=\int \frac{\cot((b\log(cx^n)+a)d)^2}{x^2}\,dx$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="maxima")
```

output
```
-((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d*log(x^n)
 + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*sin(2*
b*d*log(x^n) + 2*a*d)^2 + b*d*n - 2*(b*d*n*cos(2*b*d*log(c)) + sin(2*b*d*l
og(c)))*cos(2*b*d*log(x^n) + 2*a*d) - (2*b^2*d^2*n^2*x*cos(2*b*d*log(c))*c
os(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*x*sin(2*b*d*log(c))*sin(2*b*d*l
og(x^n) + 2*a*d) - b^2*d^2*n^2*x - (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*
sin(2*b*d*log(c))^2)*n^2*x*cos(2*b*d*log(x^n) + 2*a*d)^2 - (b^2*d^2*cos(2*
b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*x*sin(2*b*d*log(x^n) + 2*
a*d)^2)*integrate((cos(b*d*log(x^n) + a*d)*sin(b*d*log(c)) + cos(b*d*log(c
))*sin(b*d*log(x^n) + a*d))/(2*b^2*d^2*n^2*x^2*cos(b*d*log(c))*cos(b*d*log
(x^n) + a*d) - 2*b^2*d^2*n^2*x^2*sin(b*d*log(c))*sin(b*d*log(x^n) + a*d) +
 b^2*d^2*n^2*x^2 + (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*sin(b*d*log(c))^2)
*n^2*x^2*cos(b*d*log(x^n) + a*d)^2 + (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*
sin(b*d*log(c))^2)*n^2*x^2*sin(b*d*log(x^n) + a*d)^2), x) + (2*b^2*d^2*n^2
*x*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*x*sin(2*b
*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) - b^2*d^2*n^2*x - (b^2*d^2*cos(2*b*
d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*x*cos(2*b*d*log(x^n) + 2*a*
d)^2 - (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*x*s
in(2*b*d*log(x^n) + 2*a*d)^2)*integrate(-(cos(b*d*log(x^n) + a*d)*sin(b*d*
log(c)) + cos(b*d*log(c))*sin(b*d*log(x^n) + a*d))/(2*b^2*d^2*n^2*x^2*c...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x^2}\,dx=\text{Timed out}$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\cot(d\,(a+b\,\ln(c\,x^n)))^2}{x^2}\,dx$$

input
```
int(cot(d*(a + b*log(c*x^n)))^2/x^2,x)
```

output
```
int(cot(d*(a + b*log(c*x^n)))^2/x^2, x)
```

Reduce [F]

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\cot(\log(x^nc)\,bd+ad)^2}{x^2}dx$$

input
```
int(cot(d*(a+b*log(c*x^n)))^2/x^2,x)
```

output
```
int(cot(log(x**n*c)*b*d + a*d)**2/x**2,x)
```

3.222 $\int \frac{\cot^2(d(a+b\log(cx^n)))}{x^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 155

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x^3} dx = \frac{1+\frac{2i}{bdn}}{2x^2} + \frac{i\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bdnx^2\left(1-e^{2iad}(cx^n)^{2ibd}\right)} - \frac{2i\,\text{Hypergeometric2F1}\left(1,\frac{i}{bdn},1+\frac{i}{bdn},e^{2iad}(cx^n)^{2ibd}\right)}{bdnx^2}$$

output

```
1/2*(1+2*I/b/d/n)/x^2+I*(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/x^2/(1-ex
p(2*I*a*d)*(c*x^n)^(2*I*b*d))-2*I*hypergeom([1, I/b/d/n],[1+I/b/d/n],exp(2
*I*a*d)*(c*x^n)^(2*I*b*d))/b/d/n/x^2
```

Mathematica [A] (verified)

Time = 2.88 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.13

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x^3} dx = \frac{2e^{2id(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1+\frac{i}{bdn},2+\frac{i}{bdn},e^{2id(a+b\log(cx^n))}\right)+(i+bdn)\left(bdn-2\cot\left(d(a+\right.\right.}{2bdn(i+bdn)x^2}$$

input
```
Integrate[Cot[d*(a + b*Log[c*x^n])]^2/x^3,x]
```

output
```
(2*E^((2*I)*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + I/(b*d*n), 2 +
I/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + (I + b*d*n)*(b*d*n - 2*Cot[d*
(a + b*Log[c*x^n])] - (2*I)*Hypergeometric2F1[1, I/(b*d*n), 1 + I/(b*d*n),
 E^((2*I)*d*(a + b*Log[c*x^n]))]))/(2*b*d*n*(I + b*d*n)*x^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.37, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5009, 5007, 1004, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x^3}dx$$

$$\Big\downarrow \text{5009}$$

$$\frac{(cx^n)^{2/n}\int (cx^n)^{-1-\frac{2}{n}}\cot^2(d(a+b\log(cx^n)))d(cx^n)}{nx^2}$$

$$\Big\downarrow \text{5007}$$

$$\frac{(cx^n)^{2/n}\int \frac{(cx^n)^{-1-\frac{2}{n}}\left(-ie^{2iad}(cx^n)^{2ibd}-i\right)^2}{\left(1-e^{2iad}(cx^n)^{2ibd}\right)^2}d(cx^n)}{nx^2}$$

$$\Big\downarrow \text{1004}$$

$$\frac{(cx^n)^{2/n}\left(\frac{i(cx^n)^{-2/n}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\int -\frac{2(cx^n)^{-1-\frac{2}{n}}\left(\frac{e^{4iad}(2-ibdn)(cx^n)^{2ibd}}{n}+\frac{e^{2iad}(ibdn+2)}{n}\right)}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{2bd}\right)}{nx^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(cx^n)^{2/n}\left(\frac{ie^{-2iad}\int \frac{(cx^n)^{-1-\frac{2}{n}}\left(\frac{e^{4iad}(2-ibdn)(cx^n)^{2ibd}}{n}+\frac{e^{2iad}(ibdn+2)}{n}\right)}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{bd}+\frac{i(cx^n)^{-2/n}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}\right)}{nx^2}$$

$$\Big\downarrow \; 959$$

$$\frac{(cx^n)^{2/n}\left(\frac{ie^{-2iad}\left(\frac{4e^{2iad}\int \frac{(cx^n)^{-1-\frac{2}{n}}}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{n}+\frac{1}{2}e^{2iad}(2-ibdn)(cx^n)^{-2/n}\right)}{bd}+\frac{i(cx^n)^{-2/n}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}\right)}{nx^2}$$

$$\Big\downarrow \; 888$$

$$\frac{(cx^n)^{2/n}\left(\frac{ie^{-2iad}\left(\frac{1}{2}e^{2iad}(2-ibdn)(cx^n)^{-2/n}-2e^{2iad}(cx^n)^{-2/n}\,\text{Hypergeometric2F1}\left(1,\frac{i}{bdn},1+\frac{i}{bdn},e^{2iad}(cx^n)^{2ibd}\right)\right)}{bd}+\frac{i(cx^n)^{-2/n}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}\right)}{nx^2}$$

input
```
Int[Cot[d*(a + b*Log[c*x^n])]^2/x^3,x]
```

output
```
((c*x^n)^(2/n)*((I*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(b*d*(c*x^n)^(
2/n)*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) + (I*((E^((2*I)*a*d)*(2 - I*
b*d*n))/(2*(c*x^n)^(2/n)) - (2*E^((2*I)*a*d)*Hypergeometric2F1[1, I/(b*d*n
), 1 + I/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(c*x^n)^(2/n)))/(b*d
*E^((2*I)*a*d))))/(n*x^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5009
```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\cot \left(d(a+b\ln \left(c\,x^{n}\right))\right)^{2}}{x^{3}}dx$$

input
```
int(cot(d*(a+b*ln(c*x^n)))^2/x^3,x)
```

output
```
int(cot(d*(a+b*ln(c*x^n)))^2/x^3,x)
```

Fricas [F]

$$\int \frac{\cot^2\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx=\int \frac{\cot\left((b\log\left(cx^n\right)+a)d\right)^2}{x^3}\,dx$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="fricas")
```

output
```
integral(cot(b*d*log(c*x^n) + a*d)^2/x^3, x)
```

Sympy [F]

$$\int \frac{\cot^2\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx=\int \frac{\cot^2\left(ad+bd\log\left(cx^n\right)\right)}{x^3}\,dx$$

input
```
integrate(cot(d*(a+b*ln(c*x**n)))**2/x**3,x)
```

output
```
Integral(cot(a*d + b*d*log(c*x**n))**2/x**3, x)
```

Maxima [F]

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\cot((b\log(cx^n)+a)d)^2}{x^3}\,dx$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="maxima")
```

output
```
-1/2*((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*cos(2*b*d*log(
x^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*n*si
n(2*b*d*log(x^n) + 2*a*d)^2 + b*d*n - 2*(b*d*n*cos(2*b*d*log(c)) + 2*sin(2
*b*d*log(c)))*cos(2*b*d*log(x^n) + 2*a*d) - 4*(2*b^2*d^2*n^2*x^2*cos(2*b*d
*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2*b^2*d^2*n^2*x^2*sin(2*b*d*log(c))
*sin(2*b*d*log(x^n) + 2*a*d) - b^2*d^2*n^2*x^2 - (b^2*d^2*cos(2*b*d*log(c)
)^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*x^2*cos(2*b*d*log(x^n) + 2*a*d)^2 -
 (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*x^2*sin(2
*b*d*log(x^n) + 2*a*d)^2)*integrate((cos(b*d*log(x^n) + a*d)*sin(b*d*log(c
)) + cos(b*d*log(c))*sin(b*d*log(x^n) + a*d))/(2*b^2*d^2*n^2*x^3*cos(b*d*l
og(c))*cos(b*d*log(x^n) + a*d) - 2*b^2*d^2*n^2*x^3*sin(b*d*log(c))*sin(b*d
*log(x^n) + a*d) + b^2*d^2*n^2*x^3 + (b^2*d^2*cos(b*d*log(c))^2 + b^2*d^2*
sin(b*d*log(c))^2)*n^2*x^3*cos(b*d*log(x^n) + a*d)^2 + (b^2*d^2*cos(b*d*lo
g(c))^2 + b^2*d^2*sin(b*d*log(c))^2)*n^2*x^3*sin(b*d*log(x^n) + a*d)^2), x
) + 4*(2*b^2*d^2*n^2*x^2*cos(2*b*d*log(c))*cos(2*b*d*log(x^n) + 2*a*d) - 2
*b^2*d^2*n^2*x^2*sin(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) - b^2*d^2*n
^2*x^2 - (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*n^2*x
^2*cos(2*b*d*log(x^n) + 2*a*d)^2 - (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*
sin(2*b*d*log(c))^2)*n^2*x^2*sin(2*b*d*log(x^n) + 2*a*d)^2)*integrate(-(co
s(b*d*log(x^n) + a*d)*sin(b*d*log(c)) + cos(b*d*log(c))*sin(b*d*log(x^n...
```

Giac [F]

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\cot((b\log(cx^n)+a)d)^2}{x^3}\,dx$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="giac")
```

output
```
integrate(cot((b*log(c*x^n) + a)*d)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x^3}\,dx=\int \frac{\cot(d\,(a+b\,\ln(c\,x^n)))^2}{x^3}\,dx$$

input
```
int(cot(d*(a + b*log(c*x^n)))^2/x^3,x)
```

output
```
int(cot(d*(a + b*log(c*x^n)))^2/x^3, x)
```

Reduce [F]

$$\int \frac{\cot^2(d(a+b\log(cx^n)))}{x^3}\,dx=\int \frac{\cot(\log(x^n c)\,bd+ad)^2}{x^3}dx$$

input
```
int(cot(d*(a+b*log(c*x^n)))^2/x^3,x)
```

output
```
int(cot(log(x**n*c)*b*d + a*d)**2/x**3,x)
```

3.223 $\int \frac{\cot^3(a+b\log(cx^n))}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \frac{\cot^3(a+b\log(cx^n))}{x} dx = -\frac{\cot^2(a+b\log(cx^n))}{2bn} - \frac{\log(\sin(a+b\log(cx^n)))}{bn}$$

output
```
-1/2*cot(a+b*ln(c*x^n))^2/b/n-ln(sin(a+b*ln(c*x^n)))/b/n
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{\cot^3(a+b\log(cx^n))}{x} dx = -\frac{\csc^2(a+b\log(cx^n))}{2bn} - \frac{\log(\sin(a+b\log(cx^n)))}{bn}$$

input
```
Integrate[Cot[a + b*Log[c*x^n]]^3/x,x]
```

output
```
-1/2*Csc[a + b*Log[c*x^n]]^2/(b*n) - Log[Sin[a + b*Log[c*x^n]]]/(b*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {3039, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \cot^3(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\tan\left(a+b\log(cx^n)+\frac{\pi}{2}\right)^3d\log(cx^n)}{n}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \tan\left(\frac{1}{2}(2a+\pi)+b\log(cx^n)\right)^3d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\int -\cot(a+b\log(cx^n))d\log(cx^n)-\frac{\cot^2(a+b\log(cx^n))}{2b}}{n}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\int \cot(a+b\log(cx^n))d\log(cx^n)-\frac{\cot^2(a+b\log(cx^n))}{2b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\int -\tan\left(a+b\log(cx^n)+\frac{\pi}{2}\right)d\log(cx^n)-\frac{\cot^2(a+b\log(cx^n))}{2b}}{n}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \tan\left(\frac{1}{2}(2a+\pi)+b\log(cx^n)\right)d\log(cx^n)-\frac{\cot^2(a+b\log(cx^n))}{2b}}{n}$$

$$\Big\downarrow \text{3956}$$

$$\frac{-\frac{\log(-\sin(a+b\log(cx^n)))}{b}-\frac{\cot^2(a+b\log(cx^n))}{2b}}{n}$$

input
```
Int[Cot[a + b*Log[c*x^n]]^3/x,x]
```

output
```
(-1/2*Cot[a + b*Log[c*x^n]]^2/b - Log[-Sin[a + b*Log[c*x^n]]]/b)/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-\frac{\cot(a+b\ln(c\,x^{n}))^{2}}{2}+\frac{\ln\left(\cot(a+b\ln(c\,x^{n}))^{2}+1\right)}{2}}{nb}$
default	$\frac{-\frac{\cot(a+b\ln(c\,x^{n}))^{2}}{2}+\frac{\ln\left(\cot(a+b\ln(c\,x^{n}))^{2}+1\right)}{2}}{nb}$
parallelrisch	$\frac{-2\ln(\tan(a+b\ln(c\,x^{n})))+\ln\left(\sec(a+b\ln(c\,x^{n}))^{2}\right)-\cot(a+b\ln(c\,x^{n}))^{2}}{2bn}$
risch	$-i\ln(x)+\frac{2ia}{bn}+\frac{2i\ln(c)}{n}+\frac{2i\ln(x^{n})}{n}-\frac{\pi\,\mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})^{2}}{n}+\frac{\pi\,\mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})\,\mathrm{csgn}(ic)}{n}+\frac{\pi}{}$

input
```
int(cot(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(-1/2*cot(a+b*ln(c*x^n))^2+1/2*ln(cot(a+b*ln(c*x^n))^2+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.59

$$\int \frac{\cot^3(a+b\log(cx^n))}{x}\,dx = -\frac{(\cos(2\,bn\log(x)+2\,b\log(c)+2\,a)-1)\log\left(-\frac{1}{2}\cos(2\,bn\log(x)+2\,b\log(c)+2\,a)+\frac{1}{2}\right)-2}{2\,(bn\cos(2\,bn\log(x)+2\,b\log(c)+2\,a)-bn)}$$

input
```
integrate(cot(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

output
```
-1/2*((cos(2*b*n*log(x) + 2*b*log(c) + 2*a) - 1)*log(-1/2*cos(2*b*n*log(x)
 + 2*b*log(c) + 2*a) + 1/2) - 2)/(b*n*cos(2*b*n*log(x) + 2*b*log(c) + 2*a)
 - b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(36) = 72$.

Time = 4.53 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.20

$$\int \frac{\cot^3(a+b\log(cx^n))}{x}\,dx = \begin{cases} \tilde{\infty}\log(x) & \text{for } a=0 \wedge b=0 \wedge n=0 \\ \log(x)\cot^3(a) & \text{for } b=0 \\ \log(x)\cot^3(a+b\log(c)) & \text{for } n=0 \\ \tilde{\infty}\log(x) & \text{for } a=-b\log(cx^n) \\ \frac{\log(\tan^2(a+b\log(cx^n))+1)}{2bn} - \frac{\log(\tan(a+b\log(cx^n)))}{bn} - \frac{1}{2bn\tan^2(a+b\log(cx^n))} & \text{otherwise} \end{cases}$$

input
```
integrate(cot(a+b*ln(c*x**n))**3/x,x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)*cot(a)**3,
 Eq(b, 0)), (log(x)*cot(a + b*log(c))**3, Eq(n, 0)), (zoo*log(x), Eq(a, -b
*log(c*x**n))), (log(tan(a + b*log(c*x**n))**2 + 1)/(2*b*n) - log(tan(a +
b*log(c*x**n)))/(b*n) - 1/(2*b*n*tan(a + b*log(c*x**n))**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1713 vs. $2(42) = 84$.

Time = 0.28 (sec) , antiderivative size = 1713, normalized size of antiderivative = 38.93

$$\int \frac{\cot^3(a+b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

input
```
integrate(cot(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

output
```
-1/2*(8*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*cos(2*b*log(x^n) + 2*a)^2
+ 8*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*sin(2*b*log(x^n) + 2*a)^2 - 4*
((cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*cos(2
*b*log(x^n) + 2*a) + (cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*si
n(2*b*log(c)))*sin(2*b*log(x^n) + 2*a))*cos(4*b*log(x^n) + 4*a) - 4*cos(2*
b*log(c))*cos(2*b*log(x^n) + 2*a) + ((cos(4*b*log(c))^2 + sin(4*b*log(c))^
2)*cos(4*b*log(x^n) + 4*a)^2 + 4*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*c
os(2*b*log(x^n) + 2*a)^2 + (cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*sin(4*b
*log(x^n) + 4*a)^2 + 4*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*sin(2*b*log
(x^n) + 2*a)^2 - 2*(2*(cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*log(c))*s
in(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 2*(cos(2*b*log(c))*sin(4*b*log(c
)) - cos(4*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) - cos(4*b*lo
g(c)))*cos(4*b*log(x^n) + 4*a) - 4*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a)
 + 2*(2*(cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c))
)*cos(2*b*log(x^n) + 2*a) - 2*(cos(4*b*log(c))*cos(2*b*log(c)) + sin(4*b*l
og(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) - sin(4*b*log(c)))*sin(4*b
*log(x^n) + 4*a) + 4*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + 1)*log((cos
(a)^2 + sin(a)^2)*cos(b*log(c))^2 + (cos(a)^2 + sin(a)^2)*sin(b*log(c))^2
+ 2*(cos(b*log(c))*cos(a) - sin(b*log(c))*sin(a))*cos(b*log(x^n)) + cos(b*
log(x^n))^2 - 2*(cos(a)*sin(b*log(c)) + cos(b*log(c))*sin(a))*sin(b*log...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^3(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(cot(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 22.48 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.41

$$\int \frac{\cot^3(a+b\log(cx^n))}{x}\,dx = \ln(x)\ 1\mathrm{i} + \frac{2}{b\,n\,\left(1+\mathrm{e}^{a\,4\mathrm{i}}\,(c\,x^n)^{b\,4\mathrm{i}}-2\,\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}}\right)} + \frac{2}{b\,n\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}}-1\right)} - \frac{\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}}-1\right)}{b\,n}$$

input
```
int(cot(a + b*log(c*x^n))^3/x,x)
```

output
```
log(x)*1i + 2/(b*n*(exp(a*4i)*(c*x^n)^(b*4i) - 2*exp(a*2i)*(c*x^n)^(b*2i)
+ 1)) + 2/(b*n*(exp(a*2i)*(c*x^n)^(b*2i) - 1)) - log(exp(a*2i)*(c*x^n)^(b*
2i) - 1)/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.27

$$\int \frac{\cot^3(a+b\log(cx^n))}{x}\,dx = \frac{4\log\left(\tan\left(\frac{\log(x^nc)b}{2}+\frac{a}{2}\right)^2+1\right)\sin(\log(x^nc)\,b+a)^2 - 4\log\left(\tan\left(\frac{\log(x^nc)b}{2}+\frac{a}{2}\right)\right)\sin(\log(x^nc)\,b+a)^2 +}{4\sin(\log(x^nc)\,b+a)^2bn}$$

input
```
int(cot(a+b*log(c*x^n))^3/x,x)
```

output
```
(4*log(tan((log(x**n*c)*b + a)/2)**2 + 1)*sin(log(x**n*c)*b + a)**2 - 4*lo
g(tan((log(x**n*c)*b + a)/2))*sin(log(x**n*c)*b + a)**2 + sin(log(x**n*c)*
b + a)**2 - 2)/(4*sin(log(x**n*c)*b + a)**2*b*n)
```

3.224 $\int \frac{\cot^4(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \frac{\cot^4(a+b\log(cx^n))}{x}\,dx=\frac{\cot(a+b\log(cx^n))}{bn}-\frac{\cot^3(a+b\log(cx^n))}{3bn}+\log(x)$$

output
```
cot(a+b*ln(c*x^n))/b/n-1/3*cot(a+b*ln(c*x^n))^3/b/n+ln(x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{\cot^4(a+b\log(cx^n))}{x}\,dx$$
$$=-\frac{\cot^3(a+b\log(cx^n))\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(a+b\log(cx^n))\right)}{3bn}$$

input
```
Integrate[Cot[a + b*Log[c*x^n]]^4/x,x]
```

output
```
-1/3*(Cot[a + b*Log[c*x^n]]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[a + b*
Log[c*x^n]]^2])/(b*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3039, 3042, 3954, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \cot^4(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \tan\left(a+b\log(cx^n)+\frac{\pi}{2}\right)^4d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3954}$$

$$\frac{-\int \cot^2(a+b\log(cx^n))d\log(cx^n)-\frac{\cot^3(a+b\log(cx^n))}{3b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\int \tan\left(a+b\log(cx^n)+\frac{\pi}{2}\right)^2d\log(cx^n)-\frac{\cot^3(a+b\log(cx^n))}{3b}}{n}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\int 1d\log(cx^n)-\frac{\cot^3(a+b\log(cx^n))}{3b}+\frac{\cot(a+b\log(cx^n))}{b}}{n}$$

$$\Big\downarrow \text{24}$$

$$\frac{-\frac{\cot^3(a+b\log(cx^n))}{3b}+\frac{\cot(a+b\log(cx^n))}{b}+\log(cx^n)}{n}$$

input
```
Int[Cot[a + b*Log[c*x^n]]^4/x,x]
```

output
```
(Cot[a + b*Log[c*x^n]]/b - Cot[a + b*Log[c*x^n]]^3/(3*b) + Log[c*x^n])/n
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{-\cot\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}+3\ln\left(x\right) b n+3\cot\left(a+b\ln\left(c\,x^{n}\right)\right)}{3 b n}$
derivativedivides	$\frac{-\frac{\cot\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}}{3}+\cot\left(a+b\ln\left(c\,x^{n}\right)\right)-\frac{\pi}{2}+\operatorname{arccot}\left(\cot\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{n b}$
default	$\frac{-\frac{\cot\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}}{3}+\cot\left(a+b\ln\left(c\,x^{n}\right)\right)-\frac{\pi}{2}+\operatorname{arccot}\left(\cot\left(a+b\ln\left(c\,x^{n}\right)\right)\right)}{n b}$
risch	$\ln\left(x\right)+\frac{4 i \left(3 \left(x^{n}\right)^{4 i b} c^{4 i b} {\mathrm e}^{2 b \pi \,\operatorname{csgn}\left(i x^{n}\right) \operatorname{csgn}\left(i c\,x^{n}\right) \operatorname{csgn}\left(i c\right)} {\mathrm e}^{-2 b \pi \,\operatorname{csgn}\left(i x^{n}\right) \operatorname{csgn}\left(i c\,x^{n}\right)^{2}} {\mathrm e}^{-2 b \pi \,\operatorname{csgn}\left(i c\,x^{n}\right)^{2} \operatorname{csgn}\left(i c\right)} {\mathrm e}^{2 b \pi}\cdots}{3 b n \left(\left(x^{n}\right)^{2 i b} c^{2 i b} {\mathrm e}^{-b \pi \,\operatorname{csgn}\left(i x^{n}\right) \operatorname{csgn}\left(i c\,x^{n}\right)^{2}} {\mathrm e}^{b \pi}\cdots}$

input
```
int(cot(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-cot(a+b*ln(c*x^n))^3+3*ln(x)*b*n+3*cot(a+b*ln(c*x^n)))/b/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(42) = 84$.

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 3.00

$$\int \frac{\cot^4(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{4\cos(2bn\log(x)+2b\log(c)+2a)^2+3(bn\cos(2bn\log(x)+2b\log(c)+2a)\log(x)-bn\log(x))\sin}{3(bn\cos(2bn\log(x)+2b\log(c)+2a)-bn)\sin(2bn}$$

input
```
integrate(cot(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

output
```
1/3*(4*cos(2*b*n*log(x) + 2*b*log(c) + 2*a)^2 + 3*(b*n*cos(2*b*n*log(x) +
2*b*log(c) + 2*a)*log(x) - b*n*log(x))*sin(2*b*n*log(x) + 2*b*log(c) + 2*a
) + 2*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) - 2)/((b*n*cos(2*b*n*log(x) + 2
*b*log(c) + 2*a) - b*n)*sin(2*b*n*log(x) + 2*b*log(c) + 2*a))
```

Sympy [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.48

$$\int \frac{\cot^4(a+b\log(cx^n))}{x}\,dx$$
$$= \begin{cases} \log(x)\cot^4(a) & \text{for } b=0 \wedge (b=0 \vee n=0) \\ \log(x)\cot^4(a+b\log(c)) & \text{for } n=0 \\ \frac{\log(cx^n)}{n} - \frac{\cot^3(a+b\log(cx^n))}{3bn} + \frac{\cot(a+b\log(cx^n))}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(cot(a+b*ln(c*x**n))**4/x,x)
```

output
```
Piecewise((log(x)*cot(a)**4, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*co
t(a + b*log(c))**4, Eq(n, 0)), (log(c*x**n)/n - cot(a + b*log(c*x**n))**3/
(3*b*n) + cot(a + b*log(c*x**n))/(b*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2172 vs. $2(42) = 84$.

Time = 0.19 (sec) , antiderivative size = 2172, normalized size of antiderivative = 49.36

$$\int \frac{\cot^4(a + b\log(cx^n))}{x} \, dx = \text{Too large to display}$$

input
```
integrate(cot(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

output
```
1/3*(3*(b*cos(6*b*log(c))^2 + b*sin(6*b*log(c))^2)*n*cos(6*b*log(x^n) + 6*
a)^2*log(x) + 27*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*cos(4*b*log
(x^n) + 4*a)^2*log(x) + 27*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*c
os(2*b*log(x^n) + 2*a)^2*log(x) + 3*(b*cos(6*b*log(c))^2 + b*sin(6*b*log(c
))^2)*n*log(x)*sin(6*b*log(x^n) + 6*a)^2 + 27*(b*cos(4*b*log(c))^2 + b*sin
(4*b*log(c))^2)*n*log(x)*sin(4*b*log(x^n) + 4*a)^2 + 27*(b*cos(2*b*log(c))
^2 + b*sin(2*b*log(c))^2)*n*log(x)*sin(2*b*log(x^n) + 2*a)^2 + 3*b*n*log(x
) - 2*(3*b*n*cos(6*b*log(c))*log(x) + 3*(3*(b*cos(6*b*log(c))*cos(4*b*log(
c)) + b*sin(6*b*log(c))*sin(4*b*log(c)))*n*log(x) - 2*cos(4*b*log(c))*sin(
6*b*log(c)) + 2*cos(6*b*log(c))*sin(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) -
 3*(3*(b*cos(6*b*log(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c
)))*n*log(x) - 2*cos(2*b*log(c))*sin(6*b*log(c)) + 2*cos(6*b*log(c))*sin(2
*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 3*(3*(b*cos(4*b*log(c))*sin(6*b*log(
c)) - b*cos(6*b*log(c))*sin(4*b*log(c)))*n*log(x) + 2*cos(6*b*log(c))*cos(
4*b*log(c)) + 2*sin(6*b*log(c))*sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) -
 3*(3*(b*cos(2*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(2*b*log(c
)))*n*log(x) + 2*cos(6*b*log(c))*cos(2*b*log(c)) + 2*sin(6*b*log(c))*sin(2
*b*log(c)))*sin(2*b*log(x^n) + 2*a) - 4*sin(6*b*log(c)))*cos(6*b*log(x^n)
+ 6*a) + 6*(3*b*n*cos(4*b*log(c))*log(x) - 9*(b*cos(4*b*log(c))*cos(2*b*lo
g(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)))*n*cos(2*b*log(x^n) + 2*a)*lo...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^4(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(cot(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 28.50 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.14

$$\int \frac{\cot^4(a+b\log(cx^n))}{x}\,dx = \ln(x) + \frac{\frac{4i}{3\,b\,n} + \frac{e^{a\,4i}\,(c\,x^n)^{b\,4i}\,4i}{3\,b\,n}}{3\,e^{a\,2i}\,(c\,x^n)^{b\,2i} - 3\,e^{a\,4i}\,(c\,x^n)^{b\,4i} + e^{a\,6i}\,(c\,x^n)^{b\,6i} - 1} + \frac{4i}{3\,b\,n\,\left(e^{a\,2i}\,(c\,x^n)^{b\,2i} - 1\right)} + \frac{e^{a\,2i}\,(c\,x^n)^{b\,2i}\,4i}{3\,b\,n\,\left(1 + e^{a\,4i}\,(c\,x^n)^{b\,4i} - 2\,e^{a\,2i}\,(c\,x^n)^{b\,2i}\right)}$$

input
```
int(cot(a + b*log(c*x^n))^4/x,x)
```

output
```
log(x) + (4i/(3*b*n) + (exp(a*4i)*(c*x^n)^(b*4i)*4i)/(3*b*n))/(3*exp(a*2i)
*(c*x^n)^(b*2i) - 3*exp(a*4i)*(c*x^n)^(b*4i) + exp(a*6i)*(c*x^n)^(b*6i) -
1) + 4i/(3*b*n*(exp(a*2i)*(c*x^n)^(b*2i) - 1)) + (exp(a*2i)*(c*x^n)^(b*2i)
*4i)/(3*b*n*(exp(a*4i)*(c*x^n)^(b*4i) - 2*exp(a*2i)*(c*x^n)^(b*2i) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{\cot^4(a+b\log(cx^n))}{x}\,dx = \frac{-\cot(\log(x^n c)\,b+a)^3+3\cot(\log(x^n c)\,b+a)+3\log(x^n c)\,b}{3bn}$$

input
```
int(cot(a+b*log(c*x^n))^4/x,x)
```

output
```
( - cot(log(x**n*c)*b + a)**3 + 3*cot(log(x**n*c)*b + a) + 3*log(x**n*c)*b
)/(3*b*n)
```

3.225 $\int \frac{\cot^5(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \frac{\cot^5(a+b\log(cx^n))}{x}\,dx = \frac{\cot^2(a+b\log(cx^n))}{2bn} - \frac{\cot^4(a+b\log(cx^n))}{4bn} + \frac{\log(\sin(a+b\log(cx^n)))}{bn}$$

output
```
1/2*cot(a+b*ln(c*x^n))^2/b/n-1/4*cot(a+b*ln(c*x^n))^4/b/n+ln(sin(a+b*ln(c*
x^n)))/b/n
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int \frac{\cot^5(a+b\log(cx^n))}{x}\,dx = \frac{\csc^2(a+b\log(cx^n))}{bn} - \frac{\csc^4(a+b\log(cx^n))}{4bn} + \frac{\log(\sin(a+b\log(cx^n)))}{bn}$$

input
```
Integrate[Cot[a + b*Log[c*x^n]]^5/x,x]
```

output
```
Csc[a + b*Log[c*x^n]]^2/(b*n) - Csc[a + b*Log[c*x^n]]^4/(4*b*n) + Log[Sin[
a + b*Log[c*x^n]]]/(b*n)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {3039, 3042, 25, 3954, 25, 3042, 25, 3954, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \cot^5(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\tan\left(a+b\log(cx^n)+\frac{\pi}{2}\right)^5d\log(cx^n)}{n}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \tan\left(\frac{1}{2}(2a+\pi)+b\log(cx^n)\right)^5d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3954}$$

$$\frac{\int -\cot^3(a+b\log(cx^n))d\log(cx^n)-\frac{\cot^4(a+b\log(cx^n))}{4b}}{n}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\int \cot^3(a+b\log(cx^n))d\log(cx^n)-\frac{\cot^4(a+b\log(cx^n))}{4b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\int -\tan\left(a+b\log(cx^n)+\frac{\pi}{2}\right)^3d\log(cx^n)-\frac{\cot^4(a+b\log(cx^n))}{4b}}{n}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \tan\left(\frac{1}{2}(2a+\pi)+b\log\left(cx^n\right)\right)^3 d\log\left(cx^n\right)-\frac{\cot^4(a+b\log(cx^n))}{4b}}{n}$$
$$\Big\downarrow \text{3954}$$
$$\frac{-\int -\cot\left(a+b\log\left(cx^n\right)\right)d\log\left(cx^n\right)-\frac{\cot^4(a+b\log(cx^n))}{4b}+\frac{\cot^2(a+b\log(cx^n))}{2b}}{n}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \cot\left(a+b\log\left(cx^n\right)\right)d\log\left(cx^n\right)-\frac{\cot^4(a+b\log(cx^n))}{4b}+\frac{\cot^2(a+b\log(cx^n))}{2b}}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int -\tan\left(a+b\log\left(cx^n\right)+\frac{\pi}{2}\right)d\log\left(cx^n\right)-\frac{\cot^4(a+b\log(cx^n))}{4b}+\frac{\cot^2(a+b\log(cx^n))}{2b}}{n}$$
$$\Big\downarrow \text{25}$$
$$\frac{-\int \tan\left(\frac{1}{2}(2a+\pi)+b\log\left(cx^n\right)\right)d\log\left(cx^n\right)-\frac{\cot^4(a+b\log(cx^n))}{4b}+\frac{\cot^2(a+b\log(cx^n))}{2b}}{n}$$
$$\Big\downarrow \text{3956}$$
$$\frac{\frac{\log(-\sin(a+b\log(cx^n)))}{b}-\frac{\cot^4(a+b\log(cx^n))}{4b}+\frac{\cot^2(a+b\log(cx^n))}{2b}}{n}$$

input
```
Int[Cot[a + b*Log[c*x^n]]^5/x,x]
```

output
```
(Cot[a + b*Log[c*x^n]]^2/(2*b) - Cot[a + b*Log[c*x^n]]^4/(4*b) + Log[-Sin[
a + b*Log[c*x^n]]]/b)/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{-\frac{\cot(a+b\ln(c\,x^n))^4}{4}+\frac{\cot(a+b\ln(c\,x^n))^2}{2}-\frac{\ln\left(\cot(a+b\ln(c\,x^n))^2+1\right)}{2}}{nb}$
default	$\frac{-\frac{\cot(a+b\ln(c\,x^n))^4}{4}+\frac{\cot(a+b\ln(c\,x^n))^2}{2}-\frac{\ln\left(\cot(a+b\ln(c\,x^n))^2+1\right)}{2}}{nb}$
parallelrisch	$\frac{-\cot(a+b\ln(c\,x^n))^4+4\ln(\tan(a+b\ln(c\,x^n)))-2\ln\left(\sec(a+b\ln(c\,x^n))^2\right)+2\cot(a+b\ln(c\,x^n))^2}{4bn}$
risch	$i\ln\left(x\right)-\frac{2ia}{bn}-\frac{2i\ln(c)}{n}-\frac{2i\ln(x^n)}{n}+\frac{\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}{n}-\frac{\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)}{n}-\frac{\pi\ \mathrm{cs}}{}$

input
```
int(cot(a+b*ln(c*x^n))^5/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(-1/4*cot(a+b*ln(c*x^n))^4+1/2*cot(a+b*ln(c*x^n))^2-1/2*ln(cot(a+b*l
n(c*x^n))^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(62) = 124$.

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.95

$$\int \frac{\cot^5(a+b\log(cx^n))}{x}\,dx = \frac{\left(\cos(2\,bn\log(x)+2\,b\log(c)+2\,a)^2-2\,\cos(2\,bn\log(x)+2\,b\log(c)+2\,a)+1\right)\log\left(-\frac{1}{2}\,\cos(2\,bn\log(\right.}{2\left(bn\cos(2\,bn\log(x)+2\,b\log(c)+2\,a)^2-2\,bn\cos(2\,bn\log(\right.}$$

input
```
integrate(cot(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

output
```
1/2*((cos(2*b*n*log(x) + 2*b*log(c) + 2*a)^2 - 2*cos(2*b*n*log(x) + 2*b*lo
g(c) + 2*a) + 1)*log(-1/2*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1/2) - 4*
cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 2)/(b*n*cos(2*b*n*log(x) + 2*b*log(
c) + 2*a)^2 - 2*b*n*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(53) = 106$.

Time = 20.70 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.77

$$\int \frac{\cot^5(a+b\log(cx^n))}{x}\,dx = \begin{cases} \tilde{\infty}\log(x) & \text{for } a=0\wedge b \\ \log(x)\cot^5(a) & \text{for } b=0 \\ \log(x)\cot^5(a+b\log(c)) & \text{for } n=0 \\ \tilde{\infty}\log(x) & \text{for } a=-b\log \\ -\frac{\log(\tan^2(a+b\log(cx^n))+1)}{2bn}+\frac{\log(\tan(a+b\log(cx^n)))}{bn}+\frac{1}{2bn\tan^2(a+b\log(cx^n))}-\frac{1}{4bn\tan^4(a+b\log(cx^n))} & \text{otherwise} \end{cases}$$

input
```
integrate(cot(a+b*ln(c*x**n))**5/x,x)
```

output
```
Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)*cot(a)**5,
 Eq(b, 0)), (log(x)*cot(a + b*log(c))**5, Eq(n, 0)), (zoo*log(x), Eq(a, -b
*log(c*x**n))), (-log(tan(a + b*log(c*x**n))**2 + 1)/(2*b*n) + log(tan(a +
 b*log(c*x**n)))/(b*n) + 1/(2*b*n*tan(a + b*log(c*x**n))**2) - 1/(4*b*n*ta
n(a + b*log(c*x**n))**4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5998 vs. $2(62) = 124$.

Time = 0.25 (sec) , antiderivative size = 5998, normalized size of antiderivative = 90.88

$$\int \frac{\cot^5(a + b\log(cx^n))}{x} \, dx = \text{Too large to display}$$

input
```
integrate(cot(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

output
```
1/2*(32*(cos(6*b*log(c))^2 + sin(6*b*log(c))^2)*cos(6*b*log(x^n) + 6*a)^2
+ 48*(cos(4*b*log(c))^2 + sin(4*b*log(c))^2)*cos(4*b*log(x^n) + 4*a)^2 + 3
2*(cos(2*b*log(c))^2 + sin(2*b*log(c))^2)*cos(2*b*log(x^n) + 2*a)^2 + 32*(
cos(6*b*log(c))^2 + sin(6*b*log(c))^2)*sin(6*b*log(x^n) + 6*a)^2 + 48*(cos
(4*b*log(c))^2 + sin(4*b*log(c))^2)*sin(4*b*log(x^n) + 4*a)^2 + 32*(cos(2*
b*log(c))^2 + sin(2*b*log(c))^2)*sin(2*b*log(x^n) + 2*a)^2 - 8*((cos(8*b*l
og(c))*cos(6*b*log(c)) + sin(8*b*log(c))*sin(6*b*log(c)))*cos(6*b*log(x^n)
 + 6*a) - (cos(8*b*log(c))*cos(4*b*log(c)) + sin(8*b*log(c))*sin(4*b*log(c
)))*cos(4*b*log(x^n) + 4*a) + (cos(8*b*log(c))*cos(2*b*log(c)) + sin(8*b*l
og(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + (cos(6*b*log(c))*sin(8*b
*log(c)) - cos(8*b*log(c))*sin(6*b*log(c)))*sin(6*b*log(x^n) + 6*a) - (cos
(4*b*log(c))*sin(8*b*log(c)) - cos(8*b*log(c))*sin(4*b*log(c)))*sin(4*b*lo
g(x^n) + 4*a) + (cos(2*b*log(c))*sin(8*b*log(c)) - cos(8*b*log(c))*sin(2*b
*log(c)))*sin(2*b*log(x^n) + 2*a))*cos(8*b*log(x^n) + 8*a) - 8*(10*(cos(6*
b*log(c))*cos(4*b*log(c)) + sin(6*b*log(c))*sin(4*b*log(c)))*cos(4*b*log(x
^n) + 4*a) - 8*(cos(6*b*log(c))*cos(2*b*log(c)) + sin(6*b*log(c))*sin(2*b*
log(c)))*cos(2*b*log(x^n) + 2*a) + 10*(cos(4*b*log(c))*sin(6*b*log(c)) - c
os(6*b*log(c))*sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) - 8*(cos(2*b*log(c
))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2
*a) + cos(6*b*log(c)))*cos(6*b*log(x^n) + 6*a) - 8*(10*(cos(4*b*log(c))...
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^5(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(cot(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 24.46 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.73

$$\begin{aligned}
\int \frac{\cot^5(a+b\log(cx^n))}{x}\,dx &= -\ln(x)\,1\mathrm{i} - \frac{8}{b\,n\,\left(1+\mathrm{e}^{a\,4\mathrm{i}}\,(c\,x^n)^{b\,4\mathrm{i}} - 2\,\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}}\right)} - \frac{4}{b\,n\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}} - 1\right)} \\
&\quad - \frac{4}{b\,n\,\left(1+6\,\mathrm{e}^{a\,4\mathrm{i}}\,(c\,x^n)^{b\,4\mathrm{i}} - 4\,\mathrm{e}^{a\,6\mathrm{i}}\,(c\,x^n)^{b\,6\mathrm{i}} + \mathrm{e}^{a\,8\mathrm{i}}\,(c\,x^n)^{b\,8\mathrm{i}} - 4\,\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}}\right)} \\
&\quad + \frac{\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}} - 1\right)}{b\,n} - \frac{8}{b\,n\,\left(3\,\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}} - 3\,\mathrm{e}^{a\,4\mathrm{i}}\,(c\,x^n)^{b\,4\mathrm{i}} + \mathrm{e}^{a\,6\mathrm{i}}\,(c\,x^n)^{b\,6\mathrm{i}} - 1\right)}
\end{aligned}$$

input
```
int(cot(a + b*log(c*x^n))^5/x,x)
```

output
```
log(exp(a*2i)*(c*x^n)^(b*2i) - 1)/(b*n) - 8/(b*n*(exp(a*4i)*(c*x^n)^(b*4i)
 - 2*exp(a*2i)*(c*x^n)^(b*2i) + 1)) - 4/(b*n*(exp(a*2i)*(c*x^n)^(b*2i) - 1
)) - 4/(b*n*(6*exp(a*4i)*(c*x^n)^(b*4i) - 4*exp(a*2i)*(c*x^n)^(b*2i) - 4*e
xp(a*6i)*(c*x^n)^(b*6i) + exp(a*8i)*(c*x^n)^(b*8i) + 1)) - log(x)*1i - 8/(
b*n*(3*exp(a*2i)*(c*x^n)^(b*2i) - 3*exp(a*4i)*(c*x^n)^(b*4i) + exp(a*6i)*(
c*x^n)^(b*6i) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.77

$$\int \frac{\cot^5(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{-32\log\left(\tan\left(\frac{\log(x^nc)b}{2}+\frac{a}{2}\right)^2+1\right)\sin(\log(x^nc)\,b+a)^4+32\log\left(\tan\left(\frac{\log(x^nc)b}{2}+\frac{a}{2}\right)\right)\sin(\log(x^nc)\,b+a}{32\sin(\log(x^nc)\,b+a)^4bn}$$

input
```
int(cot(a+b*log(c*x^n))^5/x,x)
```

output
```
( - 32*log(tan((log(x**n*c)*b + a)/2)**2 + 1)*sin(log(x**n*c)*b + a)**4 +
32*log(tan((log(x**n*c)*b + a)/2))*sin(log(x**n*c)*b + a)**4 - 13*sin(log(
x**n*c)*b + a)**4 + 32*sin(log(x**n*c)*b + a)**2 - 8)/(32*sin(log(x**n*c)*
b + a)**4*b*n)
```

3.226 $\int (ex)^m \cot (d(a+b\log (cx^n)))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 100

$$\int (ex)^m \cot (d(a+b\log (cx^n)))\,dx = \frac{i(ex)^{1+m}}{e(1+m)} - \frac{2i(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,-\frac{i(1+m)}{2bdn},1-\frac{i(1+m)}{2bdn},e^{2iad}(cx^n)^{2ibd}\right)}{e(1+m)}$$

output
```
I*(e*x)^(1+m)/e/(1+m)-2*I*(e*x)^(1+m)*hypergeom([1, -1/2*I*(1+m)/b/d/n],[1
-1/2*I*(1+m)/b/d/n],exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/e/(1+m)
```

Mathematica [A] (verified)

Time = 9.98 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.82

$$\int (ex)^m \cot (d(a+b\log (cx^n)))\,dx = -\frac{ix(ex)^m\left(\text{Hypergeometric2F1}\left(1,-\frac{i(1+m)}{2bdn},1-\frac{i(1+m)}{2bdn},e^{2id(a+b\log(cx^n))}\right)+\frac{e^{2iad}(1+m)(cx^n)^{2ibd}\,\text{Hypergeometric}}{}\right.}{1+m}$$

input
```
Integrate[(e*x)^m*Cot[d*(a + b*Log[c*x^n])],x]
```

output
```
((-I)*x*(e*x)^m*(Hypergeometric2F1[1, ((-1/2*I)*(1 + m))/(b*d*n), 1 - ((I/
2)*(1 + m))/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + (E^((2*I)*a*d)*(1 +
 m)*(c*x^n)^((2*I)*b*d)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m + (2*I)*b*d*
n))/(b*d*n), ((-1/2*I)*(1 + m + (4*I)*b*d*n))/(b*d*n), E^((2*I)*a*d)*(c*x^
n)^((2*I)*b*d)])/(1 + m + (2*I)*b*d*n)))/(1 + m)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.36, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {5009, 5007, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \cot (d(a+b\log (cx^n)))\, dx$$

$$\Big\downarrow \text{5009}$$

$$\frac{(ex)^{m+1} (cx^n)^{-\frac{m+1}{n}} \int (cx^n)^{\frac{m+1}{n}-1} \cot (d(a+b\log (cx^n)))d(cx^n)}{en}$$

$$\Big\downarrow \text{5007}$$

$$\frac{(ex)^{m+1} (cx^n)^{-\frac{m+1}{n}} \int \frac{(cx^n)^{\frac{m+1}{n}-1}\left(-ie^{2iad}(cx^n)^{2ibd}-i\right)}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{en}$$

$$\Big\downarrow \text{959}$$

$$\frac{(ex)^{m+1} (cx^n)^{-\frac{m+1}{n}} \left(\frac{in(cx^n)^{\frac{m+1}{n}}}{m+1}-2i\int \frac{(cx^n)^{\frac{m+1}{n}-1}}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)\right)}{en}$$

$$\Big\downarrow \text{888}$$

$$\frac{(ex)^{m+1} (cx^n)^{-\frac{m+1}{n}} \left(\frac{in(cx^n)^{\frac{m+1}{n}}}{m+1}-\frac{2in(cx^n)^{\frac{m+1}{n}}\text{Hypergeometric2F1}\left(1,-\frac{i(m+1)}{2bdn},1-\frac{i(m+1)}{2bdn},e^{2iad}(cx^n)^{2ibd}\right)}{m+1}\right)}{en}$$

input
```
Int[(e*x)^m*Cot[d*(a + b*Log[c*x^n])],x]
```

output
```
((e*x)^(1 + m)*((I*n*(c*x^n)^((1 + m)/n))/(1 + m) - ((2*I)*n*(c*x^n)^((1 +
 m)/n)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m))/(b*d*n), 1 - ((I/2)*(1 + m)
)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(1 + m)))/(e*n*(c*x^n)^((1
+ m)/n))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5009
```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int (ex)^m \cot (d(a + b\ln (c\,x^n)))\,dx$$

input
```
int((e*x)^m*cot(d*(a+b*ln(c*x^n))),x)
```

output
```
int((e*x)^m*cot(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int (ex)^m \cot (d(a + b\log (cx^n)))\ dx = \int (ex)^m \cot ((b\log (cx^n) + a)d)\ dx$$

input
```
integrate((e*x)^m*cot(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral((e*x)^m*cot(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int (ex)^m \cot (d(a + b\log (cx^n)))\ dx = \int (ex)^m \cot (ad + bd\log (cx^n))\,dx$$

input
```
integrate((e*x)**m*cot(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral((e*x)**m*cot(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int (ex)^m \cot (d(a+b\log (cx^n)))\,dx = \int (ex)^m \cot ((b\log (cx^n)+a)d)\,dx$$

input
```
integrate((e*x)^m*cot(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate((e*x)^m*cot((b*log(c*x^n) + a)*d), x)
```

Giac [F(-1)]

Timed out.

$$\int (ex)^m \cot (d(a+b\log (cx^n)))\,dx = \text{Timed out}$$

input
```
integrate((e*x)^m*cot(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \cot (d(a+b\log (cx^n)))\,dx = \int \cot(d\,(a+b\,\ln (c\,x^n)))\,(e\,x)^m\,dx$$

input
```
int(cot(d*(a + b*log(c*x^n)))*(e*x)^m,x)
```

output
```
int(cot(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \cot \left(d(a+b\log \left(cx^n\right))\right) \, dx = e^m \left(\int x^m \cot \left(\log(x^n c)\, bd + ad\right) dx\right)$$

input
```
int((e*x)^m*cot(d*(a+b*log(c*x^n))),x)
```

output
```
e**m*int(x**m*cot(log(x**n*c)*b*d + a*d),x)
```

3.227 $\int (ex)^m \cot^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 195

$$\int (ex)^m \cot^2(d(a+b\log(cx^n)))\,dx = \frac{(i(1+m)-bdn)(ex)^{1+m}}{bde(1+m)n} + \frac{i(ex)^{1+m}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bden\left(1-e^{2iad}(cx^n)^{2ibd}\right)} - \frac{2i(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,-\frac{i(1+m)}{2bdn},1-\frac{i(1+m)}{2bdn},e^{2iad}(cx^n)^{2ibd}\right)}{bden}$$

output

```
(I*(1+m)-b*d*n)*(e*x)^(1+m)/b/d/e/(1+m)/n+I*(e*x)^(1+m)*(1+exp(2*I*a*d)*(c
*x^n)^(2*I*b*d))/b/d/e/n/(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))-2*I*(e*x)^(1+m
)*hypergeom([1, -1/2*I*(1+m)/b/d/n],[1-1/2*I*(1+m)/b/d/n],exp(2*I*a*d)*(c*
x^n)^(2*I*b*d))/b/d/e/n
```

Mathematica [A] (verified)

Time = 13.40 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.77

$$\int (ex)^m \cot^2\left(d(a+b\log(cx^n))\right)\,dx = (ex)^m\left(-\frac{x}{1+m} + \frac{ie^{-\frac{(1+2m)(a-bn\log(x)+b\log(cx^n))}{bn}}x^{-2m}\left(ie^{\frac{(1+2m)(a+b\log(cx^n))}{bn}}(1+m+2ibdn)\cot\left(d(a+b\log(cx^n))\right)-e^{\frac{(1+2m)(a+}{b}}\right.}{}\right.$$

input
```
Integrate[(e*x)^m*Cot[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
(e*x)^m*(-(x/(1 + m)) + (I*(I*E^(((1 + 2*m)*(a + b*Log[c*x^n]))/(b*n))*(1
+ m + (2*I)*b*d*n)*Cot[d*(a + b*Log[c*x^n])] - E^(((1 + 2*m)*(a + b*Log[c*
x^n]))/(b*n))*(1 + m + (2*I)*b*d*n)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m)
)/(b*d*n), 1 - ((I/2)*(1 + m))/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] -
E^(((1 + 2*m + (2*I)*b*d*n)*(a - b*n*Log[x] + b*Log[c*x^n]))/(b*n))*(1 + m
)*x^(1 + 2*m + (2*I)*b*d*n)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m + (2*I)*
b*d*n))/(b*d*n), ((-1/2*I)*(1 + m + (4*I)*b*d*n))/(b*d*n), E^((2*I)*d*(a +
 b*Log[c*x^n]))]))/(b*d*E^(((1 + 2*m)*(a - b*n*Log[x] + b*Log[c*x^n]))/(b*
n))*n*(1 + m + (2*I)*b*d*n)*x^(2*m)))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5009, 5007, 1004, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \cot^2\left(d(a+b\log(cx^n))\right)\,dx$$

$$\Big\downarrow \text{ 5009}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\cot^2\left(d(a+b\log(cx^n))\right)d(cx^n)}{en}$$

$$\Big\downarrow \; 5007$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int \frac{(cx^n)^{\frac{m+1}{n}-1}\left(-ie^{2iad}(cx^n)^{2ibd}-i\right)^2}{\left(1-e^{2iad}(cx^n)^{2ibd}\right)^2}d(cx^n)}{en}$$

$$\Big\downarrow \; 1004$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{i(cx^n)^{\frac{m+1}{n}}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\int \frac{2(cx^n)^{\frac{m+1}{n}-1}\left(\frac{e^{4iad}(m+ibdn+1)(cx^n)^{2ibd}}{n}+\frac{e^{2iad}(m-ibdn+1)}{n}\right)}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n}{2bd}\right.}{en}$$

$$\Big\downarrow \; 27$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{i(cx^n)^{\frac{m+1}{n}}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\int \frac{(cx^n)^{\frac{m+1}{n}-1}\left(\frac{e^{4iad}(m+ibdn+1)(cx^n)^{2ibd}}{n}+\frac{e^{2iad}(m-ibdn+1)}{n}\right)}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{bd}\right.}{en}$$

$$\Big\downarrow \; 959$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{i(cx^n)^{\frac{m+1}{n}}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\left(\frac{2(m+1)e^{2iad}\int \frac{(cx^n)^{\frac{m+1}{n}-1}}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{n}-\frac{e^{2iad}(ibdn+m+1)(cx^n)^{\frac{m+1}{n}}}{m+1}\right)}{bd}\right.}{en}$$

$$\Big\downarrow \; 888$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{i(cx^n)^{\frac{m+1}{n}}\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)}-\frac{ie^{-2iad}\left(2e^{2iad}(cx^n)^{\frac{m+1}{n}}\text{Hypergeometric2F1}\left(1,-\frac{i(m+1)}{2bdn},1-\frac{i(m+1)}{2bdn},e^{2iad}(c\right.\right.}{bd}\right.}{en}$$

input
```
Int[(e*x)^m*Cot[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
((e*x)^(1 + m)*((I*(c*x^n)^((1 + m)/n)*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b
*d)))/(b*d*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))) - (I*(-((E^((2*I)*a*d)
*(1 + m + I*b*d*n)*(c*x^n)^((1 + m)/n))/(1 + m)) + 2*E^((2*I)*a*d)*(c*x^n)
^((1 + m)/n)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m))/(b*d*n), 1 - ((I/2)*(
1 + m))/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)]))/(b*d*E^((2*I)*a*d)))
)/(e*n*(c*x^n)^((1 + m)/n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5009
```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int (ex)^m \cot \left(d(a + b\ln \left(c\,x^n\right))\right)^2 dx$$

input
```
int((e*x)^m*cot(d*(a+b*ln(c*x^n)))^2,x)
```

output
```
int((e*x)^m*cot(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int (ex)^m \cot^2\left(d(a + b\log\left(cx^n\right))\right)\,dx = \int (ex)^m \cot\left((b\log\left(cx^n\right) + a)d\right)^2\,dx$$

input
```
integrate((e*x)^m*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

output
```
integral((e*x)^m*cot(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int (ex)^m \cot^2\left(d(a + b\log\left(cx^n\right))\right)\,dx = \int (ex)^m \cot^2\left(ad + bd\log\left(cx^n\right)\right)dx$$

input
```
integrate((e*x)**m*cot(d*(a+b*ln(c*x**n)))**2,x)
```

output
```
Integral((e*x)**m*cot(a*d + b*d*log(c*x**n))**2, x)
```

Maxima [F]

$$\int (ex)^m \cot^2(d(a+b\log(cx^n)))\,dx = \int (ex)^m \cot((b\log(cx^n)+a)d)^2\,dx$$

input
```
integrate((e*x)^m*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

output
```
-((b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*e^m*n*x*x^m*cos(2*b*
d*log(x^n) + 2*a*d)^2 + (b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2
)*e^m*n*x*x^m*sin(2*b*d*log(x^n) + 2*a*d)^2 + b*d*e^m*n*x*x^m - 2*(b*d*e^m
*n*cos(2*b*d*log(c)) - e^m*m*sin(2*b*d*log(c)) - e^m*sin(2*b*d*log(c)))*x*
x^m*cos(2*b*d*log(x^n) + 2*a*d) + 2*(b*d*e^m*n*sin(2*b*d*log(c)) + e^m*m*c
os(2*b*d*log(c)) + e^m*cos(2*b*d*log(c)))*x*x^m*sin(2*b*d*log(x^n) + 2*a*d
) + (((b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*e^m*m^2
+ 2*(b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*e^m*m + (b
^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*e^m)*n^2*cos(2*b
*d*log(x^n) + 2*a*d)^2 + ((b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d
*log(c))^2)*e^m*m^2 + 2*(b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*l
og(c))^2)*e^m*m + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))
^2)*e^m)*n^2*sin(2*b*d*log(x^n) + 2*a*d)^2 - 2*(b^2*d^2*e^m*m^2*cos(2*b*d*
log(c)) + 2*b^2*d^2*e^m*m*cos(2*b*d*log(c)) + b^2*d^2*e^m*cos(2*b*d*log(c)
))*n^2*cos(2*b*d*log(x^n) + 2*a*d) + 2*(b^2*d^2*e^m*m^2*sin(2*b*d*log(c))
+ 2*b^2*d^2*e^m*m*sin(2*b*d*log(c)) + b^2*d^2*e^m*sin(2*b*d*log(c)))*n^2*s
in(2*b*d*log(x^n) + 2*a*d) + (b^2*d^2*e^m*m^2 + 2*b^2*d^2*e^m*m + b^2*d^2*
e^m)*n^2)*integrate((x^m*cos(b*d*log(x^n) + a*d)*sin(b*d*log(c)) + x^m*cos
(b*d*log(c))*sin(b*d*log(x^n) + a*d))/(2*b^2*d^2*n^2*cos(b*d*log(c))*cos(b
*d*log(x^n) + a*d) - 2*b^2*d^2*n^2*sin(b*d*log(c))*sin(b*d*log(x^n) + a...
```

Giac [F(-1)]

Timed out.

$$\int (ex)^m \cot^2(d(a+b\log(cx^n)))\,dx = \text{Timed out}$$

input
```
integrate((e*x)^m*cot(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \cot^2(d(a+b\log(cx^n)))\,dx = \int \cot(d\,(a+b\,\ln(c\,x^n)))^2\,(e\,x)^m\,dx$$

input
```
int(cot(d*(a + b*log(c*x^n)))^2*(e*x)^m,x)
```

output
```
int(cot(d*(a + b*log(c*x^n)))^2*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \cot^2(d(a+b\log(cx^n)))\,dx = e^m\left(\int x^m\cot(\log(x^n c)\,bd+ad)^2dx\right)$$

input
```
int((e*x)^m*cot(d*(a+b*log(c*x^n)))^2,x)
```

output
```
e**m*int(x**m*cot(log(x**n*c)*b*d + a*d)**2,x)
```

3.228 $\int (ex)^m \cot^3(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 350

$$\int (ex)^m \cot^3(d(a+b\log(cx^n)))\,dx = \frac{(i(1+m)-bdn)(1+m+2ibdn)(ex)^{1+m}}{2b^2d^2e(1+m)n^2} + \frac{(ex)^{1+m}\left(1+e^{2iad}(cx^n)^{2ibd}\right)^2}{2bden\left(1-e^{2iad}(cx^n)^{2ibd}\right)^2} + \frac{ie^{-2iad}(ex)^{1+m}\left(\frac{e^{2iad}(1+m-2ibdn)}{n}+\frac{e^{4iad}(1+m+2ibdn)(cx^n)^{2ibd}}{n}\right)}{2b^2d^2en\left(1-e^{2iad}(cx^n)^{2ibd}\right)} - \frac{i(1+2m+m^2-2b^2d^2n^2)(ex)^{1+m}\operatorname{Hypergeometric2F1}\left(1,-\frac{i(1+m)}{2bdn},1-\frac{i(1+m)}{2bdn},e^{2iad}(cx^n)^{2ibd}\right)}{b^2d^2e(1+m)n^2}$$

output

```
1/2*(I*(1+m)-b*d*n)*(1+m+2*I*b*d*n)*(e*x)^(1+m)/b^2/d^2/e/(1+m)/n^2+1/2*(e
*x)^(1+m)*(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^2/b/d/e/n/(1-exp(2*I*a*d)*(c*
x^n)^(2*I*b*d))^2+1/2*I*(e*x)^(1+m)*(exp(2*I*a*d)*(1+m-2*I*b*d*n)/n+exp(4*
I*a*d)*(1+m+2*I*b*d*n)*(c*x^n)^(2*I*b*d)/n)/b^2/d^2/e/exp(2*I*a*d)/n/(1-ex
p(2*I*a*d)*(c*x^n)^(2*I*b*d))-I*(-2*b^2*d^2*n^2+m^2+2*m+1)*(e*x)^(1+m)*hyp
ergeom([1, -1/2*I*(1+m)/b/d/n],[1-1/2*I*(1+m)/b/d/n],exp(2*I*a*d)*(c*x^n)^
(2*I*b*d))/b^2/d^2/e/(1+m)/n^2
```

Mathematica [A] (verified)

Time = 13.84 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.83

$$\int (ex)^m \cot^3(d(a+b\log(cx^n)))\,dx = -\frac{x(ex)^m\cot(d(a+b(-n\log(x)+\log(cx^n))))}{1+m} - \frac{x(ex)^m\csc^2(bdn\log(x)+d(a+b(-n\log(x)+\log(cx^n))))}{2bdn} + \frac{(1+m)x(ex)^m\csc(d(a+b(-n\log(x)+\log(cx^n))))\csc(bdn\log(x)+d(a+b(-n\log(x)+\log(cx^n)))}{2b^2d^2n^2} + \frac{(-1-2m-m^2+2b^2d^2n^2)\,x^{-m}(ex)^m\csc(d(a+b(-n\log(x)+\log(cx^n))))\left(\frac{x^{1+m}\csc(d(a+b\log(cx^n)))\sin(}{1+m}\right.}{}$$

input
```
Integrate[(e*x)^m*Cot[d*(a + b*Log[c*x^n])]^3,x]
```

output
```
-((x*(e*x)^m*Cot[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))])/(1 + m)) - (x*(e*x
)^m*Csc[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]^2)/(2*b*d*n)
+ ((1 + m)*x*(e*x)^m*Csc[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Csc[b*d*n*L
og[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sin[b*d*n*Log[x]])/(2*b^2*d^
2*n^2) + ((-1 - 2*m - m^2 + 2*b^2*d^2*n^2)*(e*x)^m*Csc[d*(a + b*(-(n*Log[x
]) + Log[c*x^n]))]*((x^(1 + m)*Csc[d*(a + b*Log[c*x^n])]*Sin[b*d*n*Log[x]]
)/(1 + m) - (I*(I*E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Lo
g[x]) + Log[c*x^n]))/(b*n))*(1 + m + (2*I)*b*d*n)*Cot[d*(a + b*Log[c*x^n])
] - E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*
x^n]))/(b*n))*(1 + m + (2*I)*b*d*n)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m)
)/(b*d*n), 1 - ((I/2)*(1 + m))/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] -
E^((a*(1 + 2*m + (2*I)*b*d*n))/(b*n) + (1 + m + (2*I)*b*d*n)*Log[x] + ((1
+ 2*m + (2*I)*b*d*n)*(-(n*Log[x]) + Log[c*x^n]))/n)*(1 + m)*Hypergeometric
2F1[1, ((-1/2*I)*(1 + m + (2*I)*b*d*n))/(b*d*n), ((-1/2*I)*(1 + m + (4*I)*
b*d*n))/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))])*Sin[d*(a + b*(-(n*Log[x]
) + Log[c*x^n]))])/(E^(((1 + 2*m)*(a + b*(-(n*Log[x]) + Log[c*x^n])))/(b*n
))*(1 + m)*(1 + m + (2*I)*b*d*n))))/(2*b^2*d^2*n^2*x^m)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5009, 5007, 1004, 27, 1064, 27, 959, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \cot^3(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{ 5009}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\cot^3(d(a+b\log(cx^n)))\,d(cx^n)}{en}$$

$$\Big\downarrow \text{ 5007}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int \frac{(cx^n)^{\frac{m+1}{n}-1}\left(-ie^{2iad}(cx^n)^{2ibd}-i\right)^3}{\left(1-e^{2iad}(cx^n)^{2ibd}\right)^3}d(cx^n)}{en}$$

$$\Big\downarrow \text{ 1004}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{(cx^n)^{\frac{m+1}{n}}\left(1+e^{2iad}(cx^n)^{2ibd}\right)^2}{2bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)^2}-\frac{ie^{-2iad}\int -\frac{2i(cx^n)^{\frac{m+1}{n}-1}\left(e^{2iad}(cx^n)^{2ibd}+1\right)\left(\frac{e^{4iad}(m+2ibdn+1)(cx^n)^{2ibd}}{n}+e^{2}\right.}{\left(1-e^{2iad}(cx^n)^{2ibd}\right)^2}}{4bd}\right.}{en}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{(cx^n)^{\frac{m+1}{n}}\left(1+e^{2iad}(cx^n)^{2ibd}\right)^2}{2bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)^2}-\frac{e^{-2iad}\int \frac{(cx^n)^{\frac{m+1}{n}-1}\left(e^{2iad}(cx^n)^{2ibd}+1\right)\left(\frac{e^{4iad}(m+2ibdn+1)(cx^n)^{2ibd}}{n}+e^{2iad}(\right.}{\left(1-e^{2iad}(cx^n)^{2ibd}\right)^2}}{2bd}\right.}{en}$$

$$\Big\downarrow \text{ 1064}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{(cx^n)^{\frac{m+1}{n}}\left(1+e^{2iad}(cx^n)^{2ibd}\right)^2}{2bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)^2}-\frac{e^{-2iad}\left(-\frac{ie^{-2iad}\int -\frac{2(cx^n)^{\frac{m+1}{n}-1}\left(\frac{e^{6iad}(m+ibdn+1)(m+2ibdn+1)(cx^n)^{2ibd}}{n^2}+\cdots\right.}{1-e^{2iad}(cx^n)^{2ibd}}}{2bd}\cdots\right.}{\cdots}\cdots\right)}{en\cdots}$$

$$\downarrow 27$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{(cx^n)^{\frac{m+1}{n}}\left(1+e^{2iad}(cx^n)^{2ibd}\right)^2}{2bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)^2}-\frac{e^{-2iad}\left(\frac{ie^{-2iad}\int \frac{(cx^n)^{\frac{m+1}{n}-1}\left(\frac{e^{6iad}(m+ibdn+1)(m+2ibdn+1)(cx^n)^{2ibd}}{n^2}+e^{4iad}\cdots\right.}{1-e^{2iad}(cx^n)^{2ibd}}}{bd}\cdots\right.}{\cdots}\cdots\right)}{en\cdots}$$

$$\downarrow 959$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{(cx^n)^{\frac{m+1}{n}}\left(1+e^{2iad}(cx^n)^{2ibd}\right)^2}{2bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)^2}-\frac{e^{-2iad}\left(\frac{ie^{-2iad}\left(\frac{2e^{4iad}\left(-2b^2d^2n^2+m^2+2m+1\right)\int \frac{(cx^n)^{\frac{m+1}{n}-1}}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{n^2}-\frac{e^{4i\cdots}}{\cdots}\cdots\right.}{bd}\cdots\right.}{\cdots}\cdots\right)}{en\cdots}$$

$$\downarrow 888$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(\frac{(cx^n)^{\frac{m+1}{n}}\left(1+e^{2iad}(cx^n)^{2ibd}\right)^2}{2bd\left(1-e^{2iad}(cx^n)^{2ibd}\right)^2}-\frac{e^{-2iad}\left(\frac{ie^{-2iad}\left(\frac{2e^{4iad}\left(-2b^2d^2n^2+m^2+2m+1\right)(cx^n)^{\frac{m+1}{n}}\text{Hypergeometric2F1}\left(1\right.}{(m+1)n}\right.}{}\right.}{}\right.}{}$$

input
```
Int[(e*x)^m*Cot[d*(a + b*Log[c*x^n])]^3,x]
```

output
```
((e*x)^(1 + m)*(((c*x^n)^((1 + m)/n)*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d
))^2)/(2*b*d*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^2) - (((-I)*(c*x^n)^(
(1 + m)/n)*((E^((2*I)*a*d)*(1 + m - (2*I)*b*d*n))/n + (E^((4*I)*a*d)*(1 +
m + (2*I)*b*d*n)*(c*x^n)^((2*I)*b*d))/n))/(b*d*(1 - E^((2*I)*a*d)*(c*x^n)^
((2*I)*b*d))) + (I*(-((E^((4*I)*a*d)*(1 + m + I*b*d*n)*(1 + m + (2*I)*b*d*
n)*(c*x^n)^((1 + m)/n))/((1 + m)*n)) + (2*E^((4*I)*a*d)*(1 + 2*m + m^2 - 2
*b^2*d^2*n^2)*(c*x^n)^((1 + m)/n)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m))/
(b*d*n), 1 - ((I/2)*(1 + m))/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/
((1 + m)*n)))/(b*d*E^((2*I)*a*d)))/(2*b*d*E^((2*I)*a*d))))/(e*n*(c*x^n)^((
1 + m)/n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 959
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1))   Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
 n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

rule 1004
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Simp[1/(a*b*n*(p + 1))   Int
[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c
*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && Lt
Q[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

rule 1064
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Simp[1/(
a*b*n*(p + 1))   Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c
*(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m
+ n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && LtQ
[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5009
```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int (ex)^m \cot \left(d(a+b\ln \left(c\,x^n\right))\right)^3 dx$$

input
```
int((e*x)^m*cot(d*(a+b*ln(c*x^n)))^3,x)
```

output
```
int((e*x)^m*cot(d*(a+b*ln(c*x^n)))^3,x)
```

Fricas [F]

$$\int (ex)^m \cot^3\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int (ex)^m \cot\left((b\log\left(cx^n\right)+a)d\right)^3\,dx$$

input
```
integrate((e*x)^m*cot(d*(a+b*log(c*x^n)))^3,x, algorithm="fricas")
```

output
```
integral((e*x)^m*cot(b*d*log(c*x^n) + a*d)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \cot^3\left(d(a+b\log\left(cx^n\right))\right)\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m*cot(d*(a+b*ln(c*x**n)))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m \cot^3(d(a+b\log(cx^n)))\,dx = \int (ex)^m \cot\left((b\log(cx^n)+a)d\right)^3\,dx$$

input
```
integrate((e*x)^m*cot(d*(a+b*log(c*x^n)))^3,x, algorithm="maxima")
```

output
```
(4*(b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c))^2)*e^m*n*x*x^m*cos(2*b
*d*log(x^n) + 2*a*d)^2 + 4*(b*d*cos(2*b*d*log(c))^2 + b*d*sin(2*b*d*log(c)
)^2)*e^m*n*x*x^m*sin(2*b*d*log(x^n) + 2*a*d)^2 - (2*b*d*e^m*n*cos(2*b*d*lo
g(c)) - e^m*m*sin(2*b*d*log(c)) - e^m*sin(2*b*d*log(c)))*x*x^m*cos(2*b*d*l
og(x^n) + 2*a*d) + (2*b*d*e^m*n*sin(2*b*d*log(c)) + e^m*m*cos(2*b*d*log(c)
) + e^m*cos(2*b*d*log(c)))*x*x^m*sin(2*b*d*log(x^n) + 2*a*d) + (((cos(2*b*
d*log(c))*sin(4*b*d*log(c)) - cos(4*b*d*log(c))*sin(2*b*d*log(c)))*e^m*m -
 2*(b*d*cos(4*b*d*log(c))*cos(2*b*d*log(c)) + b*d*sin(4*b*d*log(c))*sin(2*
b*d*log(c)))*e^m*n + (cos(2*b*d*log(c))*sin(4*b*d*log(c)) - cos(4*b*d*log(
c))*sin(2*b*d*log(c)))*e^m)*x*x^m*cos(2*b*d*log(x^n) + 2*a*d) - ((cos(4*b*
d*log(c))*cos(2*b*d*log(c)) + sin(4*b*d*log(c))*sin(2*b*d*log(c)))*e^m*m +
 2*(b*d*cos(2*b*d*log(c))*sin(4*b*d*log(c)) - b*d*cos(4*b*d*log(c))*sin(2*
b*d*log(c)))*e^m*n + (cos(4*b*d*log(c))*cos(2*b*d*log(c)) + sin(4*b*d*log(
c))*sin(2*b*d*log(c)))*e^m)*x*x^m*sin(2*b*d*log(x^n) + 2*a*d) - (e^m*m*sin
(4*b*d*log(c)) + e^m*sin(4*b*d*log(c)))*x*x^m)*cos(4*b*d*log(x^n) + 4*a*d)
 - 2*(2*b^6*d^6*e^m*n^6 - (b^4*d^4*e^m*m^2 + 2*b^4*d^4*e^m*m + b^4*d^4*e^m
)*n^4 + (2*(b^6*d^6*cos(4*b*d*log(c))^2 + b^6*d^6*sin(4*b*d*log(c))^2)*e^m
*n^6 - ((b^4*d^4*cos(4*b*d*log(c))^2 + b^4*d^4*sin(4*b*d*log(c))^2)*e^m*m^
2 + 2*(b^4*d^4*cos(4*b*d*log(c))^2 + b^4*d^4*sin(4*b*d*log(c))^2)*e^m*m +
(b^4*d^4*cos(4*b*d*log(c))^2 + b^4*d^4*sin(4*b*d*log(c))^2)*e^m)*n^4)*c...
```

Giac [F(-1)]

Timed out.

$$\int (ex)^m \cot^3(d(a+b\log(cx^n)))\,dx = \text{Timed out}$$

input
```
integrate((e*x)^m*cot(d*(a+b*log(c*x^n)))^3,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \cot^3(d(a+b\log(cx^n)))\,dx = \int \cot(d\,(a+b\,\ln(c\,x^n)))^3\,(e\,x)^m\,dx$$

input
```
int(cot(d*(a + b*log(c*x^n)))^3*(e*x)^m,x)
```

output
```
int(cot(d*(a + b*log(c*x^n)))^3*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \cot^3(d(a+b\log(cx^n)))\,dx = e^m\left(\int x^m \cot(\log(x^n c)\,bd+ad)^3 dx\right)$$

input
```
int((e*x)^m*cot(d*(a+b*log(c*x^n)))^3,x)
```

output
```
e**m*int(x**m*cot(log(x**n*c)*b*d + a*d)**3,x)
```

3.229 $\int \cot^p(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 190

$$\int \cot^p(d(a+b\log(cx^n)))\,dx = x\left(1-e^{2iad}(cx^n)^{2ibd}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(-\frac{i\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{1-e^{2iad}(cx^n)^{2ibd}}\right)^p \operatorname{AppellF1}\left(-\frac{i}{2bdn},p,-p,1-\frac{i}{2bdn},e^{2iad}(cx^n)^{2ibd},-e^{2iad}(cx^n)^{2ibd}\right)$$

output
```
x*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p*(-I*(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*
d))/(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d)))^p*AppellF1(-1/2*I/b/d/n,p,-p,1-1/2
*I/b/d/n,exp(2*I*a*d)*(c*x^n)^(2*I*b*d),-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/(
(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 458 vs. 2(190) = 380.

Time = 0.92 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.41

$$\int \cot^p(d(a + b\log(cx^n)))\,dx$$

$$= \frac{(-i + 2bdn)x\left(\frac{i(1+e^{2}\cdots}{-1+e^{\cdots}}\right.}{2bde^{2iad}np\,(cx^n)^{2ibd}\,\text{AppellF1}\left(1 - \frac{i}{2bdn}, p, 1 - p, 2 - \frac{i}{2bdn}, e^{2iad}(cx^n)^{2ibd}, -e^{2iad}(cx^n)^{2ibd}\right) + 2bde^{2iad}np\,(\cdots}$$

input
```
Integrate[Cot[d*(a + b*Log[c*x^n])]^p,x]
```

output
```
((-I + 2*b*d*n)*x*((I*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(-1 + E^((2
*I)*a*d)*(c*x^n)^((2*I)*b*d)))^p*AppellF1[(-1/2*I)/(b*d*n), p, -p, 1 - (I/
2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2
*I)*b*d))])/(2*b*d*E^((2*I)*a*d)*n*p*(c*x^n)^((2*I)*b*d)*AppellF1[1 - (I/2
)/(b*d*n), p, 1 - p, 2 - (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d),
 -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))] + 2*b*d*E^((2*I)*a*d)*n*p*(c*x^n)^(
(2*I)*b*d)*AppellF1[1 - (I/2)/(b*d*n), 1 + p, -p, 2 - (I/2)/(b*d*n), E^((2
*I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))] + (-I
+ 2*b*d*n)*AppellF1[(-1/2*I)/(b*d*n), p, -p, 1 - (I/2)/(b*d*n), E^((2*I)*a
*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5005, 5007, 2058, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^p(d(a + b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{5005}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\cot^p\left(d(a+b\log\left(cx^n\right))\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{5007}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\left(\frac{-ie^{2iad}(cx^n)^{2ibd}-i}{1-e^{2iad}(cx^n)^{2ibd}}\right)^p d(cx^n)}{n}$$

$$\Big\downarrow \text{2058}$$

$$\frac{x(cx^n)^{-1/n}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^p\left(-ie^{2iad}(cx^n)^{2ibd}-i\right)^{-p}\left(-\frac{i\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{1-e^{2iad}(cx^n)^{2ibd}}\right)^p\int (cx^n)^{\frac{1}{n}-1}\left(1-e^{2iad}(cx^n)^{2ib}\right.}{n}$$

$$\Big\downarrow \text{1013}$$

$$\frac{x(cx^n)^{-1/n}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(-\frac{i\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{1-e^{2iad}(cx^n)^{2ibd}}\right)^p\int (cx^n)^{\frac{1}{n}-1}\left(1-e^{2iad}(cx^n)^{2ibd}\right.}{n}$$

$$\Big\downarrow \text{1012}$$

$$x\left(1-e^{2iad}(cx^n)^{2ibd}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(-\frac{i\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{1-e^{2iad}\left(cx^n\right)^{2ibd}}\right)^p \text{AppellF1}\left(-\frac{i}{2bdn},p,-p,1-\frac{i}{2bdn},\right.$$

input
```
Int[Cot[d*(a + b*Log[c*x^n])]^p,x]
```

output
```
(x*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p*(((-I)*(1 + E^((2*I)*a*d)*(c*
x^n)^((2*I)*b*d)))/(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))^p*AppellF1[(-1
/2*I)/(b*d*n), p, -p, 1 - (I/2)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)
, -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)
*b*d))^p
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 5005
```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cot[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Maple [F]

$$\int \cot \left(d(a+b\ln \left(c\,x^{n}\right))\right)^{p}dx$$

input
```
int(cot(d*(a+b*ln(c*x^n)))^p,x)
```

output
```
int(cot(d*(a+b*ln(c*x^n)))^p,x)
```

Fricas [F]

$$\int \cot^p(d(a+b\log(cx^n)))\,dx=\int \cot((b\log(cx^n)+a)d)^p\,dx$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

output
```
integral(cot(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F]

$$\int \cot^p(d(a+b\log(cx^n)))\,dx=\int \cot^p(d(a+b\log(cx^n)))\,dx$$

input
```
integrate(cot(d*(a+b*ln(c*x**n)))**p,x)
```

output
```
Integral(cot(d*(a + b*log(c*x**n)))**p, x)
```

Maxima [F]

$$\int \cot^p(d(a+b\log(cx^n)))\,dx=\int \cot((b\log(cx^n)+a)d)^p\,dx$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

output
```
integrate(cot((b*log(c*x^n) + a)*d)^p, x)
```

Giac [F(-1)]

Timed out.

$$\int \cot^p(d(a+b\log(cx^n)))\,dx = \text{Timed out}$$

input
```
integrate(cot(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \cot^p(d(a+b\log(cx^n)))\,dx = \int \cot(d\,(a+b\,\ln(c\,x^n)))^p\,dx$$

input
```
int(cot(d*(a + b*log(c*x^n)))^p,x)
```

output
```
int(cot(d*(a + b*log(c*x^n)))^p, x)
```

Reduce [F]

$$\int \cot^p(d(a+b\log(cx^n)))\,dx = \cot(\log(x^n c)\,bd+ad)^p x + \left(\int \frac{\cot(\log(x^n c)\,bd+ad)^p}{\cot(\log(x^n c)\,bd+ad)}dx\right) bdnp + \left(\int \cot(\log(x^n c)\,bd+ad)^p \cot(\log(x^n c)\,bd + ad)\,dx\right) bdnp$$

input
```
int(cot(d*(a+b*log(c*x^n)))^p,x)
```

output
```
cot(log(x**n*c)*b*d + a*d)**p*x + int(cot(log(x**n*c)*b*d + a*d)**p/cot(lo
g(x**n*c)*b*d + a*d),x)*b*d*n*p + int(cot(log(x**n*c)*b*d + a*d)**p*cot(lo
g(x**n*c)*b*d + a*d),x)*b*d*n*p
```

3.230 $\int (ex)^m \cot^p(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 210

$$\int (ex)^m \cot^p(d(a+b\log(cx^n)))\,dx = \frac{(ex)^{1+m}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(-\frac{i\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{1-e^{2iad}(cx^n)^{2ibd}}\right)^p \operatorname{AppellF1}\left(-\frac{i(1+m)}{2bdn},p,-p,1\right.}{e(1+m)}$$

output
```
(e*x)^(1+m)*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p*(-I*(1+exp(2*I*a*d)*(c*x^
n)^(2*I*b*d))/(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d)))^p*AppellF1(-1/2*I*(1+m)/
b/d/n,p,-p,1-1/2*I*(1+m)/b/d/n,exp(2*I*a*d)*(c*x^n)^(2*I*b*d),-exp(2*I*a*d
)*(c*x^n)^(2*I*b*d))/e/(1+m)/((1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.98

$$\int (ex)^m \cot^p(d(a+b\log(cx^n)))\,dx = \frac{x(ex)^m\left(1-e^{2iad}(cx^n)^{2ibd}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(\frac{i\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{-1+e^{2iad}(cx^n)^{2ibd}}\right)^p \operatorname{AppellF1}\left(-\frac{i(1+m)}{2bdn},p,-p,1-\right.}{1+m}$$

input
```
Integrate[(e*x)^m*Cot[d*(a + b*Log[c*x^n])]^p,x]
```

output
```
(x*(e*x)^m*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p*((I*(1 + E^((2*I)*a*d
)*(c*x^n)^((2*I)*b*d)))/(-1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))^p*Appell
F1[((-1/2*I)*(1 + m))/(b*d*n), p, -p, 1 - ((I/2)*(1 + m))/(b*d*n), E^((2*I
)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/((1 + m
)*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5009, 5007, 2058, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \cot^p(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \ 5009$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\cot^p(d(a+b\log(cx^n)))\,d(cx^n)}{en}$$

$$\Big\downarrow \ 5007$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\left(\frac{-ie^{2iad}(cx^n)^{2ibd}-i}{1-e^{2iad}(cx^n)^{2ibd}}\right)^p d(cx^n)}{en}$$

$$\Big\downarrow \ 2058$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^p\left(-ie^{2iad}(cx^n)^{2ibd}-i\right)^{-p}\left(-\frac{i\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{1-e^{2iad}(cx^n)^{2ibd}}\right)^p\int (cx^n)^{\frac{m+1}{n}-1}\left(1-e\right.}{en}$$

$$\Big\downarrow \ 1013$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(-\frac{i\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{1-e^{2iad}(cx^n)^{2ibd}}\right)^p\int(cx^n)^{\frac{m+1}{n}-1}\left(1-e^{2ia}\right.}{en}$$

$\downarrow$ 1012

$$\frac{(ex)^{m+1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^{-p}\left(-\frac{i\left(1+e^{2iad}(cx^n)^{2ibd}\right)}{1-e^{2iad}(cx^n)^{2ibd}}\right)^p\text{AppellF1}\left(-\frac{i(m+1)}{2bdn},p,-p,1-\frac{i(m}{2b}\right.}{e(m+1)}$$

input
```
Int[(e*x)^m*Cot[d*(a + b*Log[c*x^n])]^p,x]
```

output
```
((e*x)^(1 + m)*(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p*(((-I)*(1 + E^((2
*I)*a*d)*(c*x^n)^((2*I)*b*d)))/(1 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))^p*
AppellF1[((-1/2*I)*(1 + m))/(b*d*n), p, -p, 1 - ((I/2)*(1 + m))/(b*d*n), E
^((2*I)*a*d)*(c*x^n)^((2*I)*b*d), -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/(
e*(1 + m)*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p)
```

Defintions of rubi rules used

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
 + 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 5007
```
Int[Cot[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Int[(e*x)^m*((-I - I*E^(2*I*a*d)*x^(2*I*b*d))/(1 - E^(2*I*a*d)*x^(2*I*b*
d)))^p, x] /; FreeQ[{a, b, d, e, m, p}, x]
```

rule 5009
```
Int[Cot[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Cot[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int (ex)^m \cot \left(d(a+b\ln \left(c\,x^n\right))\right)^p dx$$

input
```
int((e*x)^m*cot(d*(a+b*ln(c*x^n)))^p,x)
```

output
```
int((e*x)^m*cot(d*(a+b*ln(c*x^n)))^p,x)
```

Fricas [F]

$$\int (ex)^m \cot^p \left(d(a+b\log \left(cx^n\right))\right) \, dx = \int (ex)^m \cot \left((b\log \left(cx^n\right)+a)d\right)^p \, dx$$

input
```
integrate((e*x)^m*cot(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^m*cot(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F]

$$\int (ex)^m \cot^p(d(a+b\log(cx^n)))\,dx = \int (ex)^m \cot^p(ad+bd\log(cx^n))\,dx$$

input
```
integrate((e*x)**m*cot(d*(a+b*ln(c*x**n)))**p,x)
```

output
```
Integral((e*x)**m*cot(a*d + b*d*log(c*x**n))**p, x)
```

Maxima [F]

$$\int (ex)^m \cot^p(d(a+b\log(cx^n)))\,dx = \int (ex)^m \cot((b\log(cx^n)+a)d)^p\,dx$$

input
```
integrate((e*x)^m*cot(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*cot((b*log(c*x^n) + a)*d)^p, x)
```

Giac [F(-1)]

Timed out.

$$\int (ex)^m \cot^p(d(a+b\log(cx^n)))\,dx = \text{Timed out}$$

input
```
integrate((e*x)^m*cot(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \cot^p(d(a+b\log(cx^n)))\,dx = \int \cot(d\,(a+b\,\ln(c\,x^n)))^p\,(e\,x)^m\,dx$$

input
```
int(cot(d*(a + b*log(c*x^n)))^p*(e*x)^m,x)
```

output
```
int(cot(d*(a + b*log(c*x^n)))^p*(e*x)^m, x)
```

Reduce [F]

$$\int (ex)^m \cot^p(d(a+b\log(cx^n)))\,dx$$
$$= \frac{e^m\left(x^m\cot(\log(x^nc)\,bd+ad)^p x + \left(\int \frac{x^m\cot(\log(x^nc)bd+ad)^p}{\cot(\log(x^nc)bd+ad)}dx\right)bdnp + \left(\int x^m\cot(\log(x^nc)\,bd+ad)^p\cot(\mathrm{lo}\right.\right.}{m+1}$$

input
```
int((e*x)^m*cot(d*(a+b*log(c*x^n)))^p,x)
```

output
```
(e**m*(x**m*cot(log(x**n*c)*b*d + a*d)**p*x + int((x**m*cot(log(x**n*c)*b*
d + a*d)**p)/cot(log(x**n*c)*b*d + a*d),x)*b*d*n*p + int(x**m*cot(log(x**n
*c)*b*d + a*d)**p*cot(log(x**n*c)*b*d + a*d),x)*b*d*n*p))/(m + 1)
```

3.231 $\int \frac{\cot^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 150

$$\int \frac{\cot^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}}{1+\cot(a+b\log(cx^n))}\right)}{\sqrt{2}bn} - \frac{2\cot^{\frac{3}{2}}(a+b\log(cx^n))}{3bn}$$

output

```
1/2*arctan(-1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n+1/2*arctan(1+2
^(1/2)*cot(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n-1/2*arctanh(2^(1/2)*cot(a+b*l
n(c*x^n))^(1/2)/(1+cot(a+b*ln(c*x^n))))*2^(1/2)/b/n-2/3*cot(a+b*ln(c*x^n))
^(3/2)/b/n
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.81

$$\int \frac{\cot^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{-3\arctan\left(\sqrt[4]{-\cot^2(a+b\log(cx^n))}\right)\sqrt[4]{-\cot(a+b\log(cx^n))}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(a+b\log(cx^n}\right.}{3bn\sqrt[4]{\cot(a+b\log(cx^n))}}$$

input
```
Integrate[Cot[a + b*Log[c*x^n]]^(5/2)/x,x]
```

output
```
-1/3*(-3*ArcTan[(-Cot[a + b*Log[c*x^n]]^2)^(1/4)]*(-Cot[a + b*Log[c*x^n]])
^(1/4) + 3*ArcTanh[(-Cot[a + b*Log[c*x^n]]^2)^(1/4)]*(-Cot[a + b*Log[c*x^n
]])^(1/4) + 2*Cot[a + b*Log[c*x^n]]^(7/4))/(b*n*Cot[a + b*Log[c*x^n]]^(1/4
))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.29, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {3039, 3042, 3954, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$$

$$\Big\downarrow \text{ 3039}$$

$$\frac{\int \cot^{\frac{5}{2}}(a+b\log(cx^n))\,d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(-\tan\left(a+b\log(cx^n)+\frac{\pi}{2}\right)\right)^{5/2}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{ 3954}$$

$$\frac{-\int \sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}d\log\left(cx^n\right)-\frac{2\cot^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

$\Big\downarrow$ 3042

$$\frac{-\int \sqrt{-\tan\left(a+b\log\left(cx^n\right)+\frac{\pi}{2}\right)}d\log\left(cx^n\right)-\frac{2\cot^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

$\Big\downarrow$ 3957

$$\frac{\frac{\int \frac{\sqrt{\cot(a+b\log(cx^n))}}{\cot^2(a+b\log(cx^n))+1}d\cot(a+b\log(cx^n))}{b}-\frac{2\cot^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

$\Big\downarrow$ 266

$$\frac{\frac{2\int \frac{\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}}{b}-\frac{2\cot^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

$\Big\downarrow$ 826

$$\frac{\frac{2\left(\frac{1}{2}\int \frac{\cot(a+b\log(cx^n))+1}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}-\frac{1}{2}\int \frac{1-\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}\right)}{b}-\frac{2\cot^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

$\Big\downarrow$ 1476

$$\frac{\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}+\frac{1}{2}\int \frac{1}{\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}\right)}{b}}{n}$$

$\Big\downarrow$ 1082

$$\frac{\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\cot(a+b\log(cx^n))-1}d\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\cot(a+b\log(cx^n))-1}d\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(}}{b}}{n}$$

$\Big\downarrow$ 217

$$\frac{\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}\right)}{b}-\frac{2\cot^{\frac{3}{2}}(a+b\log(}{3b}}{n}$$

$\Big\downarrow$ 1479

$$\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(a+b\log(cx^n))}}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\right.}{b}}{n}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(a+b\log(cx^n))}}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\right.}{b}}{n}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(a+b\log(cx^n))}}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}{\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}\right)+}{b}}{n}$$

$$\Big\downarrow \; 1103$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(a+b\log(c\right.}{}\right.}{b}}{n}$$

input
```
Int[Cot[a + b*Log[c*x^n]]^(5/2)/x,x]
```

output
```
((-2*Cot[a + b*Log[c*x^n]]^(3/2))/(3*b) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[C
ot[a + b*Log[c*x^n]]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*
x^n]]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] + Cot[a
+ b*Log[c*x^n]]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]
 + Cot[a + b*Log[c*x^n]]]/(2*Sqrt[2]))/2))/b)/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{-\frac{2\cot\left(a+b\ln\left(c\,x^{n}\right)\right)^{\frac{3}{2}}}{3}+\frac{\sqrt{2}\,\left(\ln\left(\frac{\cot\left(a+b\ln\left(c\,x^{n}\right)\right)-\sqrt{2}\,\sqrt{\cot\left(a+b\ln\left(c\,x^{n}\right)\right)}+1}{\cot\left(a+b\ln\left(c\,x^{n}\right)\right)+\sqrt{2}\,\sqrt{\cot\left(a+b\ln\left(c\,x^{n}\right)\right)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cot\left(a+b\ln\left(c\,x^{n}\right)\right)}\right)+2\arctan\left(\right.}{4}}{nb}$
default	$\frac{-\frac{2\cot\left(a+b\ln\left(c\,x^{n}\right)\right)^{\frac{3}{2}}}{3}+\frac{\sqrt{2}\,\left(\ln\left(\frac{\cot\left(a+b\ln\left(c\,x^{n}\right)\right)-\sqrt{2}\,\sqrt{\cot\left(a+b\ln\left(c\,x^{n}\right)\right)}+1}{\cot\left(a+b\ln\left(c\,x^{n}\right)\right)+\sqrt{2}\,\sqrt{\cot\left(a+b\ln\left(c\,x^{n}\right)\right)}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cot\left(a+b\ln\left(c\,x^{n}\right)\right)}\right)+2\arctan\left(\right.}{4}}{nb}$

input
```
int(cot(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(-2/3*cot(a+b*ln(c*x^n))^(3/2)+1/4*2^(1/2)*(ln((cot(a+b*ln(c*x^n))-2
^(1/2)*cot(a+b*ln(c*x^n))^(1/2)+1)/(cot(a+b*ln(c*x^n))+2^(1/2)*cot(a+b*ln(
c*x^n))^(1/2)+1))+2*arctan(1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))+2*arctan(-1
+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. 2(128) = 256.

Time = 0.08 (sec) , antiderivative size = 605, normalized size of antiderivative = 4.03

$$\int \frac{\cot^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

input
```
integrate(cot(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

output
```
-1/12*(6*sqrt(2)*arctan((sqrt(2)*sqrt((cos(2*b*n*log(x) + 2*b*log(c) + 2*a
) + 1)/sin(2*b*n*log(x) + 2*b*log(c) + 2*a))*sin(2*b*n*log(x) + 2*b*log(c)
 + 2*a) + cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/(cos(2*b*n*log(x) + 2*
b*log(c) + 2*a) + 1))*sin(2*b*n*log(x) + 2*b*log(c) + 2*a) + 6*sqrt(2)*arc
tan((sqrt(2)*sqrt((cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/sin(2*b*n*log
(x) + 2*b*log(c) + 2*a))*sin(2*b*n*log(x) + 2*b*log(c) + 2*a) - cos(2*b*n*
log(x) + 2*b*log(c) + 2*a) - 1)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)
)*sin(2*b*n*log(x) + 2*b*log(c) + 2*a) + 3*sqrt(2)*log((sqrt(2)*sqrt((cos(
2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/sin(2*b*n*log(x) + 2*b*log(c) + 2*a)
)*sin(2*b*n*log(x) + 2*b*log(c) + 2*a) + cos(2*b*n*log(x) + 2*b*log(c) + 2
*a) + sin(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/(cos(2*b*n*log(x) + 2*b*lo
g(c) + 2*a) + 1))*sin(2*b*n*log(x) + 2*b*log(c) + 2*a) - 3*sqrt(2)*log(-(s
qrt(2)*sqrt((cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/sin(2*b*n*log(x) +
2*b*log(c) + 2*a))*sin(2*b*n*log(x) + 2*b*log(c) + 2*a) - cos(2*b*n*log(x)
 + 2*b*log(c) + 2*a) - sin(2*b*n*log(x) + 2*b*log(c) + 2*a) - 1)/(cos(2*b*
n*log(x) + 2*b*log(c) + 2*a) + 1))*sin(2*b*n*log(x) + 2*b*log(c) + 2*a) +
8*sqrt((cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/sin(2*b*n*log(x) + 2*b*l
og(c) + 2*a))*(cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1))/(b*n*sin(2*b*n*l
og(x) + 2*b*log(c) + 2*a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{5}{2}}\left(a + b\log\left(cx^{n}\right)\right)}{x}\,dx = \text{Timed out}$$

input
```
integrate(cot(a+b*ln(c*x**n))**(5/2)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cot^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\cot(b\log(cx^n)+a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate(cot(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate(cot(b*log(c*x^n) + a)^(5/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(cot(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 20.84 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.53

$$\int \frac{\cot^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\cot(a+b\,\ln(c\,x^n))}\right)}{b\,n} - \frac{2\cot(a+b\,\ln(c\,x^n))^{3/2}}{3\,b\,n} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\cot(a+b\,\ln(c\,x^n))}\right)}{b\,n}$$

input
```
int(cot(a + b*log(c*x^n))^(5/2)/x,x)
```

output
```
((-1)^(1/4)*atan((-1)^(1/4)*cot(a + b*log(c*x^n))^(1/2)))/(b*n) - (2*cot(a
 + b*log(c*x^n))^(3/2))/(3*b*n) - ((-1)^(1/4)*atanh((-1)^(1/4)*cot(a + b*l
og(c*x^n))^(1/2)))/(b*n)
```

Reduce [F]

$$\int \frac{\cot^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx=\int \frac{\sqrt{\cot\left(\log\left(x^nc\right)b+a\right)}\,\cot\left(\log(x^nc)\,b+a\right)^2}{x}dx$$

input
```
int(cot(a+b*log(c*x^n))^(5/2)/x,x)
```

output
```
int((sqrt(cot(log(x**n*c)*b + a))*cot(log(x**n*c)*b + a)**2)/x,x)
```

3.232 $\int \frac{\cot^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 147

$$\int \frac{\cot^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}}{1+\cot(a+b\log(cx^n))}\right)}{\sqrt{2}bn} - \frac{2\sqrt{\cot(a+b\log(cx^n))}}{bn}$$

output
```
1/2*arctan(-1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n+1/2*arctan(1+2
^(1/2)*cot(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n+1/2*arctanh(2^(1/2)*cot(a+b*l
n(c*x^n))^(1/2)/(1+cot(a+b*ln(c*x^n))))*2^(1/2)/b/n-2*cot(a+b*ln(c*x^n))^(
1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.20

$$\int \frac{\cot^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx =$$
$$-\frac{\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}}-\frac{\arctan\left(1+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}}+2\sqrt{\cot(a+b\log(cx^n))}+\frac{\log\left(1-\sqrt{2}\sqrt{\cot(a+b\right.}{}}{bn}$$

input
```
Integrate[Cot[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
-((ArcTan[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]]/Sqrt[2] - ArcTan[1 + Sq
rt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]]/Sqrt[2] + 2*Sqrt[Cot[a + b*Log[c*x^n]]]
 + Log[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] + Cot[a + b*Log[c*x^n]]]/(2
*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] + Cot[a + b*Log[c*
x^n]]]/(2*Sqrt[2]))/(b*n))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.31, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {3039, 3042, 3954, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \cot^{\frac{3}{2}}(a+b\log(cx^n))\,d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \left(-\tan\left(a+b\log(cx^n)+\frac{\pi}{2}\right)\right)^{3/2}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3954}$$

$$\frac{-\int \frac{1}{\sqrt{\cot(a+b\log(cx^n))}}d\log\left(cx^n\right)-\frac{2\sqrt{\cot(a+b\log(cx^n))}}{b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\int \frac{1}{\sqrt{-\tan\left(a+b\log(cx^n)+\frac{\pi}{2}\right)}}d\log\left(cx^n\right)-\frac{2\sqrt{\cot(a+b\log(cx^n))}}{b}}{n}$$

$$\Big\downarrow \text{3957}$$

$$\frac{\frac{\int \frac{1}{\sqrt{\cot(a+b\log(cx^n))}\left(\cot^2(a+b\log(cx^n))+1\right)}d\cot(a+b\log(cx^n))}{b}-\frac{2\sqrt{\cot(a+b\log(cx^n))}}{b}}{n}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{2\int \frac{1}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}}{b}-\frac{2\sqrt{\cot(a+b\log(cx^n))}}{b}}{n}$$

$$\Big\downarrow \text{755}$$

$$\frac{\frac{2\left(\frac{1}{2}\int \frac{1-\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}+\frac{1}{2}\int \frac{\cot(a+b\log(cx^n))+1}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}\right)}{b}-\frac{2\sqrt{\cot(a+b\log(cx^n))}}{b}}{n}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{2\left(\frac{1}{2}\int \frac{1-\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}+\frac{1}{2}\int \frac{}{\cot(a+b\log(cx^n))\cdot}\right.\right.}{b}}{n}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{2\left(\frac{1}{2}\int \frac{1-\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}+\frac{1}{2}\left(\frac{\int \frac{1}{-\cot(a+b\log(cx^n))-1}d\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\cot(a+b\log(cx^n))-1}d\left(\sqrt{2}\sqrt{\cot(a+b\right.}{\sqrt{2}}\right.\right.}{b}}{n}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{2\left(\frac{1}{2}\int \frac{1-\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}}\right)\right)}{b}-\frac{2\sqrt{\cot(a+b\log(}{b}}{n}$$

$$\Big\downarrow \text{1479}$$

$$\frac{\frac{2\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(a+b\log(cx^n))}}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}\right)+\frac{1}{2}\cdots}{b}}{n}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(a+b\log(cx^n))}}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\text{arc}\cdots}{}\right.\cdots}{b}}{n}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(a+b\log(cx^n))}}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}{\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}\right)+\frac{1}{2}\cdots}{b}}{n}$$

$$\Big\downarrow \ 1103$$

$$\frac{\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(a+b\log(c\cdots\right.}{}\right.\cdots}{b}}{n}$$

input
```
Int[Cot[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
((-2*Sqrt[Cot[a + b*Log[c*x^n]]])/b + (2*(((-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[a
 + b*Log[c*x^n]]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]
]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] + Cot[a
 + b*Log[c*x^n]]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] +
Cot[a + b*Log[c*x^n]]]/(2*Sqrt[2]))/2))/b)/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{-2\sqrt{\cot(a+b\ln(c\,x^{n}))}+\frac{\sqrt{2}\left(\ln\left(\frac{\cot(a+b\ln(c\,x^{n}))+\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^{n}))}+1}{\cot(a+b\ln(c\,x^{n}))-\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^{n}))}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^{n}))}\right)+2\arctan}{4}}{nb}$
default	$\frac{-2\sqrt{\cot(a+b\ln(c\,x^{n}))}+\frac{\sqrt{2}\left(\ln\left(\frac{\cot(a+b\ln(c\,x^{n}))+\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^{n}))}+1}{\cot(a+b\ln(c\,x^{n}))-\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^{n}))}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^{n}))}\right)+2\arctan}{4}}{nb}$

input
```
int(cot(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(-2*cot(a+b*ln(c*x^n))^(1/2)+1/4*2^(1/2)*(ln((cot(a+b*ln(c*x^n))+2^(
1/2)*cot(a+b*ln(c*x^n))^(1/2)+1)/(cot(a+b*ln(c*x^n))-2^(1/2)*cot(a+b*ln(c*
x^n))^(1/2)+1))+2*arctan(1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+2
^(1/2)*cot(a+b*ln(c*x^n))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. 2(128) = 256.

Time = 0.08 (sec) , antiderivative size = 504, normalized size of antiderivative = 3.43

$$\int \frac{\cot^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

input
```
integrate(cot(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*arctan((sqrt(2)*sqrt((cos(2*b*n*log(x) + 2*b*log(c) + 2*a)
 + 1)/sin(2*b*n*log(x) + 2*b*log(c) + 2*a))*sin(2*b*n*log(x) + 2*b*log(c)
+ 2*a) + cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/(cos(2*b*n*log(x) + 2*b
*log(c) + 2*a) + 1)) + 2*sqrt(2)*arctan((sqrt(2)*sqrt((cos(2*b*n*log(x) +
2*b*log(c) + 2*a) + 1)/sin(2*b*n*log(x) + 2*b*log(c) + 2*a))*sin(2*b*n*log
(x) + 2*b*log(c) + 2*a) - cos(2*b*n*log(x) + 2*b*log(c) + 2*a) - 1)/(cos(2
*b*n*log(x) + 2*b*log(c) + 2*a) + 1)) - sqrt(2)*log((sqrt(2)*sqrt((cos(2*b
*n*log(x) + 2*b*log(c) + 2*a) + 1)/sin(2*b*n*log(x) + 2*b*log(c) + 2*a))*s
in(2*b*n*log(x) + 2*b*log(c) + 2*a) + cos(2*b*n*log(x) + 2*b*log(c) + 2*a)
 + sin(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/(cos(2*b*n*log(x) + 2*b*log(c
) + 2*a) + 1)) + sqrt(2)*log(-(sqrt(2)*sqrt((cos(2*b*n*log(x) + 2*b*log(c)
 + 2*a) + 1)/sin(2*b*n*log(x) + 2*b*log(c) + 2*a))*sin(2*b*n*log(x) + 2*b*
log(c) + 2*a) - cos(2*b*n*log(x) + 2*b*log(c) + 2*a) - sin(2*b*n*log(x) +
2*b*log(c) + 2*a) - 1)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)) + 8*sqr
t((cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/sin(2*b*n*log(x) + 2*b*log(c)
 + 2*a)))/(b*n)
```

Sympy [F]

$$\int \frac{\cot^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\cot^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$$

input
```
integrate(cot(a+b*ln(c*x**n))**(3/2)/x,x)
```

output
```
Integral(cot(a + b*log(c*x**n))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\cot^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\cot(b\log(cx^n)+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(cot(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(cot(b*log(c*x^n) + a)^(3/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\cot^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(cot(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 20.72 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.54

$$\int \frac{\cot^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = -\frac{2\sqrt{\cot(a + b\ln(c\,x^n))}}{b\,n} - \frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{1/4}\sqrt{\cot(a + b\ln(c\,x^n))}\right)\,1\mathrm{i}}{b\,n} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left((-1)^{1/4}\sqrt{\cot(a + b\ln(c\,x^n))}\right)\,1\mathrm{i}}{b\,n}$$

input
```
int(cot(a + b*log(c*x^n))^(3/2)/x,x)
```

output
```
- (2*cot(a + b*log(c*x^n))^(1/2))/(b*n) - ((-1)^(1/4)*atan((-1)^(1/4)*cot(
a + b*log(c*x^n))^(1/2))*1i)/(b*n) - ((-1)^(1/4)*atanh((-1)^(1/4)*cot(a +
b*log(c*x^n))^(1/2))*1i)/(b*n)
```

Reduce [F]

$$\int \frac{\cot^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{-2\sqrt{\cot(\log(x^n c)\,b+a)} - \left(\int \frac{\sqrt{\cot(\log(x^n c)b+a)}}{\cot(\log(x^n c)b+a)x}dx\right) bn}{bn}$$

input
```
int(cot(a+b*log(c*x^n))^(3/2)/x,x)
```

output
```
( - 2*sqrt(cot(log(x**n*c)*b + a)) - int(sqrt(cot(log(x**n*c)*b + a))/(cot
(log(x**n*c)*b + a)*x),x)*b*n)/(b*n)
```

3.233 $\int \frac{\sqrt{\cot(a+b\log(cx^n))}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 124

$$\int \frac{\sqrt{\cot(a+b\log(cx^n))}}{x}\,dx = \frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}}{1+\cot(a+b\log(cx^n))}\right)}{\sqrt{2}bn}$$

output
```
-1/2*arctan(-1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n-1/2*arctan(1+
2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n+1/2*arctanh(2^(1/2)*cot(a+b*
ln(c*x^n))^(1/2)/(1+cot(a+b*ln(c*x^n))))*2^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{\cot(a+b\log(cx^n))}}{x}\,dx$$
$$=\frac{\left(-\arctan\left(\sqrt[4]{-\cot^2(a+b\log(cx^n))}\right)+\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(a+b\log(cx^n))}\right)\right)\sqrt[4]{-\cot(a+b\log(cx^n))}}{bn\sqrt[4]{\cot(a+b\log(cx^n))}}$$

input
```
Integrate[Sqrt[Cot[a + b*Log[c*x^n]]]/x,x]
```

output
```
((-ArcTan[(-Cot[a + b*Log[c*x^n]]^2)^(1/4)] + ArcTanh[(-Cot[a + b*Log[c*x^
n]]^2)^(1/4)])*(-Cot[a + b*Log[c*x^n]])^(1/4))/(b*n*Cot[a + b*Log[c*x^n]]^
(1/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.37, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {3039, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cot(a+b\log(cx^n))}}{x}\,dx$$
$$\Big\downarrow \text{ 3039}$$
$$\frac{\int \sqrt{\cot(a+b\log(cx^n))}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\int \sqrt{-\tan\left(a+b\log(cx^n)+\frac{\pi}{2}\right)}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{ 3957}$$

$$-\frac{\int \frac{\sqrt{\cot(a+b\log(cx^n))}}{\cot^2(a+b\log(cx^n))+1}d\cot(a+b\log(cx^n))}{bn}$$

$\big\downarrow$ 266

$$-\frac{2\int \frac{\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}}{bn}$$

$\big\downarrow$ 826

$$-\frac{2\left(\frac{1}{2}\int \frac{\cot(a+b\log(cx^n))+1}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}-\frac{1}{2}\int \frac{1-\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}\right)}{bn}$$

$\big\downarrow$ 1476

$$-\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}+\frac{1}{2}\int \frac{1}{\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\cdots}\right.\right.}{bn}$$

$\big\downarrow$ 1082

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\cot(a+b\log(cx^n))-1}d\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\cot(a+b\log(cx^n))-1}d\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\cot(a\cdots}{\cot^2(a+b\cdots}\right.}{bn}$$

$\big\downarrow$ 217

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(\cdots}\right.}{bn}$$

$\big\downarrow$ 1479

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(a+b\log(cx^n))}}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(\cdots}}{2\sqrt{2}}\right.\right.}{bn}$$

$\big\downarrow$ 25

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(a+b\log(cx^n))}}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(c\cdots}}{2\sqrt{2}}\right.\right.}{bn}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(a+b\log(cx^n))}}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}{\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(\right.\right.}{bn}$$

$$\Big\downarrow \; 1103$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n\right.}{2\sqrt{2}}\right.}{bn}$$

input
```
Int[Sqrt[Cot[a + b*Log[c*x^n]]]/x,x]
```

output
```
(-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]]/Sqrt[2]) + ArcTan[
1 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqr
t[Cot[a + b*Log[c*x^n]]] + Cot[a + b*Log[c*x^n]]]/(2*Sqrt[2]) - Log[1 + Sq
rt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] + Cot[a + b*Log[c*x^n]]]/(2*Sqrt[2]))/2)
)/(b*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

method	result
derivativedivides	$-\frac{\sqrt{2}\left(\ln\left(\frac{\cot(a+b\ln(c\,x^{n}))-\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^{n}))}+1}{\cot(a+b\ln(c\,x^{n}))+\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^{n}))}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^{n}))}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\mathrm{co}\right.\right.}{4nb}$
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{\cot(a+b\ln(c\,x^{n}))-\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^{n}))}+1}{\cot(a+b\ln(c\,x^{n}))+\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^{n}))}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^{n}))}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\mathrm{co}\right.\right.}{4nb}$

input
```
int(cot(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/4/n/b*2^(1/2)*(ln((cot(a+b*ln(c*x^n))-2^(1/2)*cot(a+b*ln(c*x^n))^(1/2)+
1)/(cot(a+b*ln(c*x^n))+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2)+1))+2*arctan(1+2^(
1/2)*cot(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. 2(107) = 214.

Time = 0.08 (sec) , antiderivative size = 463, normalized size of antiderivative = 3.73

$$\int \frac{\sqrt{\cot(a+b\log(cx^n))}}{x}\,dx = \text{Too large to display}$$

input
```
integrate(cot(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(2)*arctan((sqrt(2)*sqrt((cos(2*b*n*log(x) + 2*b*log(c) + 2*a)
+ 1)/sin(2*b*n*log(x) + 2*b*log(c) + 2*a))*sin(2*b*n*log(x) + 2*b*log(c) +
 2*a) + cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/(cos(2*b*n*log(x) + 2*b*
log(c) + 2*a) + 1)) + 2*sqrt(2)*arctan((sqrt(2)*sqrt((cos(2*b*n*log(x) + 2
*b*log(c) + 2*a) + 1)/sin(2*b*n*log(x) + 2*b*log(c) + 2*a))*sin(2*b*n*log(
x) + 2*b*log(c) + 2*a) - cos(2*b*n*log(x) + 2*b*log(c) + 2*a) - 1)/(cos(2*
b*n*log(x) + 2*b*log(c) + 2*a) + 1)) + sqrt(2)*log((sqrt(2)*sqrt((cos(2*b*
n*log(x) + 2*b*log(c) + 2*a) + 1)/sin(2*b*n*log(x) + 2*b*log(c) + 2*a))*si
n(2*b*n*log(x) + 2*b*log(c) + 2*a) + cos(2*b*n*log(x) + 2*b*log(c) + 2*a)
+ sin(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/(cos(2*b*n*log(x) + 2*b*log(c)
 + 2*a) + 1)) - sqrt(2)*log(-(sqrt(2)*sqrt((cos(2*b*n*log(x) + 2*b*log(c)
+ 2*a) + 1)/sin(2*b*n*log(x) + 2*b*log(c) + 2*a))*sin(2*b*n*log(x) + 2*b*l
og(c) + 2*a) - cos(2*b*n*log(x) + 2*b*log(c) + 2*a) - sin(2*b*n*log(x) + 2
*b*log(c) + 2*a) - 1)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)))/(b*n)
```

Sympy [F]

$$\int \frac{\sqrt{\cot \left(a + b \log \left(c x^{n}\right)\right)}}{x}\, dx = \int \frac{\sqrt{\cot \left(a + b \log \left(c x^{n}\right)\right)}}{x}\, dx$$

input
```
integrate(cot(a+b*ln(c*x**n))**(1/2)/x,x)
```

output
```
Integral(sqrt(cot(a + b*log(c*x**n)))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{\cot \left(a + b \log \left(c x^{n}\right)\right)}}{x}\, dx = \int \frac{\sqrt{\cot \left(b \log \left(c x^{n}\right) + a\right)}}{x}\, dx$$

input
```
integrate(cot(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(cot(b*log(c*x^n) + a))/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cot(a+b\log(cx^n))}}{x}\,dx = \text{Timed out}$$

input
```
integrate(cot(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 20.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{\cot(a+b\log(cx^n))}}{x}\,dx = \frac{(-1)^{1/4}\operatorname{atanh}\left((-1)^{1/4}\sqrt{\cot(a+b\ln(cx^n))}\right)}{bn} - \frac{(-1)^{1/4}\operatorname{atan}\left((-1)^{1/4}\sqrt{\cot(a+b\ln(cx^n))}\right)}{bn}$$

input
```
int(cot(a + b*log(c*x^n))^(1/2)/x,x)
```

output
```
((-1)^(1/4)*atanh((-1)^(1/4)*cot(a + b*log(c*x^n))^(1/2)))/(b*n) - ((-1)^(
1/4)*atan((-1)^(1/4)*cot(a + b*log(c*x^n))^(1/2)))/(b*n)
```

Reduce [F]

$$\int \frac{\sqrt{\cot(a+b\log(cx^n))}}{x}\,dx = \int \frac{\sqrt{\cot(\log(x^nc)\,b+a)}}{x}\,dx$$

input
```
int(cot(a+b*log(c*x^n))^(1/2)/x,x)
```

output
```
int(sqrt(cot(log(x**n*c)*b + a))/x,x)
```

3.234 $\int \frac{1}{x\sqrt{\cot(a+b\log(cx^n))}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 125

$$\int \frac{1}{x\sqrt{\cot(a+b\log(cx^n))}}\,dx = \frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\arctan\left(1+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}}{1+\cot(a+b\log(cx^n))}\right)}{\sqrt{2}bn}$$

output
```
-1/2*arctan(-1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n-1/2*arctan(1+
2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n-1/2*arctanh(2^(1/2)*cot(a+b*
ln(c*x^n))^(1/2)/(1+cot(a+b*ln(c*x^n))))*2^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}}\,dx$$
$$= \frac{2\arctan\left(1-\sqrt{2}\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}\right)-2\arctan\left(1+\sqrt{2}\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}\right)+\log\left(1-\sqrt{2}\sqrt{\mathrm{c}\cdots\right.}{2\sqrt{\cdots}}$$

input
```
Integrate[1/(x*Sqrt[Cot[a + b*Log[c*x^n]]]),x]
```

output
```
(2*ArcTan[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]] - 2*ArcTan[1 + Sqrt[2]*
Sqrt[Cot[a + b*Log[c*x^n]]]] + Log[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]
 + Cot[a + b*Log[c*x^n]]] - Log[1 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] +
Cot[a + b*Log[c*x^n]]])/(2*Sqrt[2]*b*n)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.36, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {3039, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}}\,dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\sqrt{\cot(a+b\log(cx^n))}}d\log\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{-\tan\left(a+b\log(cx^n)+\frac{\pi}{2}\right)}}d\log\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{3957}$$

$$-\frac{\int \frac{1}{\sqrt{\cot(a+b\log(cx^n))}(\cot^2(a+b\log(cx^n))+1)}d\cot(a+b\log(cx^n))}{bn}$$

$$\Big\downarrow \text{ 266}$$

$$-\frac{2\int \frac{1}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}}{bn}$$

$$\Big\downarrow \text{ 755}$$

$$-\frac{2\left(\frac{1}{2}\int \frac{1-\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}+\frac{1}{2}\int \frac{\cot(a+b\log(cx^n))+1}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}\right)}{bn}$$

$$\Big\downarrow \text{ 1476}$$

$$-\frac{2\left(\frac{1}{2}\int \frac{1-\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log\right.}{bn}$$

$$\Big\downarrow \text{ 1082}$$

$$-\frac{2\left(\frac{1}{2}\int \frac{1-\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}+\frac{1}{2}\left(\frac{\int \frac{1}{-\cot(a+b\log(cx^n))-1}d\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\cot(a+b\log\right.\right.}{bn}$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{2\left(\frac{1}{2}\int \frac{1-\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx\right.}{\sqrt{2}}\right.\right.}{bn}$$

$$\Big\downarrow \text{ 1479}$$

$$-\frac{2\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(a+b\log(cx^n))}}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log}{2\sqrt{2}}\right.\right.}{bn}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(a+b\log(cx^n))}}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n}{2\sqrt{2}}\right.\right.}{bn}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(a+b\log(cx^n))}}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}{\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a}{bn}$$

$$\Big\downarrow \; 1103$$

$$-\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n}{2\sqrt{2}}\right.}{bn}$$

input
```
Int[1/(x*Sqrt[Cot[a + b*Log[c*x^n]]]),x]
```

output
```
(-2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]]/Sqrt[2]) + ArcTan[
1 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2
]*Sqrt[Cot[a + b*Log[c*x^n]]] + Cot[a + b*Log[c*x^n]]]/Sqrt[2] + Log[1 + S
qrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] + Cot[a + b*Log[c*x^n]]]/(2*Sqrt[2]))/2
))/(b*n)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

method	result
derivativedivides	$-\frac{\sqrt{2}\left(\ln\left(\frac{\cot(a+b\ln(cx^n))+\sqrt{2}\sqrt{\cot(a+b\ln(cx^n))}+1}{\cot(a+b\ln(cx^n))-\sqrt{2}\sqrt{\cot(a+b\ln(cx^n))}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(a+b\ln(cx^n))}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\mathrm{co}\right.\right.}{4nb}$
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{\cot(a+b\ln(cx^n))+\sqrt{2}\sqrt{\cot(a+b\ln(cx^n))}+1}{\cot(a+b\ln(cx^n))-\sqrt{2}\sqrt{\cot(a+b\ln(cx^n))}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{\cot(a+b\ln(cx^n))}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{\mathrm{co}\right.\right.}{4nb}$

input
```
int(1/x/cot(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/n/b*2^(1/2)*(ln((cot(a+b*ln(c*x^n))+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2)+
1)/(cot(a+b*ln(c*x^n))-2^(1/2)*cot(a+b*ln(c*x^n))^(1/2)+1))+2*arctan(1+2^(
1/2)*cot(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. 2(107) = 214.

Time = 0.08 (sec) , antiderivative size = 463, normalized size of antiderivative = 3.70

$$\int \frac{1}{x\sqrt{\cot(a+b\log(cx^n))}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/cot(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(2)*arctan((sqrt(2)*sqrt((cos(2*b*n*log(x) + 2*b*log(c) + 2*a)
+ 1)/sin(2*b*n*log(x) + 2*b*log(c) + 2*a))*sin(2*b*n*log(x) + 2*b*log(c) +
 2*a) + cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/(cos(2*b*n*log(x) + 2*b*
log(c) + 2*a) + 1)) + 2*sqrt(2)*arctan((sqrt(2)*sqrt((cos(2*b*n*log(x) + 2
*b*log(c) + 2*a) + 1)/sin(2*b*n*log(x) + 2*b*log(c) + 2*a))*sin(2*b*n*log(
x) + 2*b*log(c) + 2*a) - cos(2*b*n*log(x) + 2*b*log(c) + 2*a) - 1)/(cos(2*
b*n*log(x) + 2*b*log(c) + 2*a) + 1)) - sqrt(2)*log((sqrt(2)*sqrt((cos(2*b*
n*log(x) + 2*b*log(c) + 2*a) + 1)/sin(2*b*n*log(x) + 2*b*log(c) + 2*a))*si
n(2*b*n*log(x) + 2*b*log(c) + 2*a) + cos(2*b*n*log(x) + 2*b*log(c) + 2*a)
+ sin(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/(cos(2*b*n*log(x) + 2*b*log(c)
 + 2*a) + 1)) + sqrt(2)*log(-(sqrt(2)*sqrt((cos(2*b*n*log(x) + 2*b*log(c)
+ 2*a) + 1)/sin(2*b*n*log(x) + 2*b*log(c) + 2*a))*sin(2*b*n*log(x) + 2*b*l
og(c) + 2*a) - cos(2*b*n*log(x) + 2*b*log(c) + 2*a) - sin(2*b*n*log(x) + 2
*b*log(c) + 2*a) - 1)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)))/(b*n)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}}\,dx=\int \frac{1}{x\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}}\,dx$$

input
```
integrate(1/x/cot(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(cot(a + b*log(c*x**n)))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}}\,dx=\int \frac{1}{x\sqrt{\cot\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

input
```
integrate(1/x/cot(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sqrt(cot(b*log(c*x^n) + a))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\cot(a+b\log(cx^n))}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/cot(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 20.63 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.46

$$\int \frac{1}{x\sqrt{\cot(a+b\log(cx^n))}}\,dx = \frac{(-1)^{1/4}\operatorname{atan}\left((-1)^{1/4}\sqrt{\cot(a+b\ln(c\,x^n))}\right)\,1\mathrm{i}}{b\,n} + \frac{(-1)^{1/4}\operatorname{atanh}\left((-1)^{1/4}\sqrt{\cot(a+b\ln(c\,x^n))}\right)\,1\mathrm{i}}{b\,n}$$

input
```
int(1/(x*cot(a + b*log(c*x^n))^(1/2)),x)
```

output
```
((-1)^(1/4)*atan((-1)^(1/4)*cot(a + b*log(c*x^n))^(1/2))*1i)/(b*n) + ((-1)
^(1/4)*atanh((-1)^(1/4)*cot(a + b*log(c*x^n))^(1/2))*1i)/(b*n)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{\cot(a+b\log(cx^n))}}\,dx = \int \frac{\sqrt{\cot(\log(x^n c)\,b+a)}}{\cot(\log(x^n c)\,b+a)\,x}\,dx$$

input
```
int(1/x/cot(a+b*log(c*x^n))^(1/2),x)
```

output
```
int(sqrt(cot(log(x**n*c)*b + a))/(cot(log(x**n*c)*b + a)*x),x)
```

3.235 $\int \frac{1}{x\cot^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 148

$$\int \frac{1}{x\cot^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = -\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}bn} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}}{1+\cot(a+b\log(cx^n))}\right)}{\sqrt{2}bn} + \frac{2}{bn\sqrt{\cot(a+b\log(cx^n))}}$$

output
```
1/2*arctan(-1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n+1/2*arctan(1+2
^(1/2)*cot(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n-1/2*arctanh(2^(1/2)*cot(a+b*l
n(c*x^n))^(1/2)/(1+cot(a+b*ln(c*x^n))))*2^(1/2)/b/n+2/b/n/cot(a+b*ln(c*x^n
))^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\cot^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{2+\arctan\left(\sqrt[4]{-\cot^2(a+b\log(cx^n))}\right)\sqrt[4]{-\cot^2(a+b\log(cx^n))}-\operatorname{arctanh}\left(\sqrt[4]{-\cot^2(a+b\log(cx^n))}\right.}{bn\sqrt{\cot(a+b\log(cx^n))}}$$

input
```
Integrate[1/(x*Cot[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
(2 + ArcTan[(-Cot[a + b*Log[c*x^n]]^2)^(1/4)]*(-Cot[a + b*Log[c*x^n]]^2)^(
1/4) - ArcTanh[(-Cot[a + b*Log[c*x^n]]^2)^(1/4)]*(-Cot[a + b*Log[c*x^n]]^2
)^(1/4))/(b*n*Sqrt[Cot[a + b*Log[c*x^n]]])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.30, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {3039, 3042, 3955, 3042, 3957, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\cot^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\cot^{\frac{3}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\left(-\tan\left(a+b\log(cx^n)+\frac{\pi}{2}\right)\right)^{3/2}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\frac{2}{b\sqrt{\cot(a+b\log(cx^n))}}-\int \sqrt{\cot\left(a+b\log\left(cx^n\right)\right)}d\log\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{2}{b\sqrt{\cot(a+b\log(cx^n))}}-\int \sqrt{-\tan\left(a+b\log\left(cx^n\right)+\frac{\pi}{2}\right)}d\log\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{ 3957}$$

$$\frac{\frac{\int \frac{\sqrt{\cot(a+b\log(cx^n))}}{\cot^2(a+b\log(cx^n))+1}d\cot(a+b\log(cx^n))}{b}+\frac{2}{b\sqrt{\cot(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{\frac{2\int \frac{\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}}{b}+\frac{2}{b\sqrt{\cot(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{ 826}$$

$$\frac{\frac{2\left(\frac{1}{2}\int \frac{\cot(a+b\log(cx^n))+1}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}-\frac{1}{2}\int \frac{1-\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}\right)}{b}+\frac{2}{b\sqrt{\cot(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}+\frac{1}{2}\int \frac{1}{\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}\right)\cdots\right.}{b}}{n}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\cot(a+b\log(cx^n))-1}d\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-\cot(a+b\log(cx^n))-1}d\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(\cdots}\right.}{b}}{n}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}\right)}{b}+\frac{2}{b\sqrt{\cot(a+b\log(\cdots}}}{n}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(a+b\log(cx^n))}}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\cdots\right.}{b}}{n}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(a+b\log(cx^n))}}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\cdots\right.}{b}}{n}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(a+b\log(cx^n))}}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}{\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}\right)+\cdots}{b}}{n}$$

$$\Big\downarrow \; 1103$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(a+b\log(c\cdots\right.}{\cdots}\right.}{b}}{n}$$

input
```
Int[1/(x*Cot[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
(2/(b*Sqrt[Cot[a + b*Log[c*x^n]]]) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[a
+ b*Log[c*x^n]]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]
]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] + Cot[a + b*L
og[c*x^n]]]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] + Co
t[a + b*Log[c*x^n]]]/(2*Sqrt[2]))/2))/b)/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{\sqrt{2}\left(\ln\left(\frac{\cot(a+b\ln(c\,x^n))-\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^n))}+1}{\cot(a+b\ln(c\,x^n))+\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^n))}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^n))}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^n}$
default	$\frac{\frac{\sqrt{2}\left(\ln\left(\frac{\cot(a+b\ln(c\,x^n))-\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^n))}+1}{\cot(a+b\ln(c\,x^n))+\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^n))}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^n))}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^n}$

input
```
int(1/x/cot(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(1/4*2^(1/2)*(ln((cot(a+b*ln(c*x^n))-2^(1/2)*cot(a+b*ln(c*x^n))^(1/2
)+1)/(cot(a+b*ln(c*x^n))+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2)+1))+2*arctan(1+2
^(1/2)*cot(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+2^(1/2)*cot(a+b*ln(c*x^n))^(1
/2)))+2/cot(a+b*ln(c*x^n))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. 2(128) = 256.

Time = 0.09 (sec) , antiderivative size = 623, normalized size of antiderivative = 4.21

$$\int \frac{1}{x\cot^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/cot(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(2*(sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + sqrt(2))*arctan((s
qrt(2)*sqrt((cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/sin(2*b*n*log(x) +
2*b*log(c) + 2*a))*sin(2*b*n*log(x) + 2*b*log(c) + 2*a) + cos(2*b*n*log(x)
 + 2*b*log(c) + 2*a) + 1)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)) + 2*
(sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + sqrt(2))*arctan((sqrt(2)*s
qrt((cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/sin(2*b*n*log(x) + 2*b*log(
c) + 2*a))*sin(2*b*n*log(x) + 2*b*log(c) + 2*a) - cos(2*b*n*log(x) + 2*b*l
og(c) + 2*a) - 1)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)) + (sqrt(2)*c
os(2*b*n*log(x) + 2*b*log(c) + 2*a) + sqrt(2))*log((sqrt(2)*sqrt((cos(2*b*
n*log(x) + 2*b*log(c) + 2*a) + 1)/sin(2*b*n*log(x) + 2*b*log(c) + 2*a))*si
n(2*b*n*log(x) + 2*b*log(c) + 2*a) + cos(2*b*n*log(x) + 2*b*log(c) + 2*a)
+ sin(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/(cos(2*b*n*log(x) + 2*b*log(c)
 + 2*a) + 1)) - (sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + sqrt(2))*l
og(-(sqrt(2)*sqrt((cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/sin(2*b*n*log
(x) + 2*b*log(c) + 2*a))*sin(2*b*n*log(x) + 2*b*log(c) + 2*a) - cos(2*b*n*
log(x) + 2*b*log(c) + 2*a) - sin(2*b*n*log(x) + 2*b*log(c) + 2*a) - 1)/(co
s(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)) - 8*sqrt((cos(2*b*n*log(x) + 2*b*
log(c) + 2*a) + 1)/sin(2*b*n*log(x) + 2*b*log(c) + 2*a))*sin(2*b*n*log(x)
+ 2*b*log(c) + 2*a))/(b*n*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + b*n)
```

Sympy [F]

$$\int \frac{1}{x\cot^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\cot^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$

input
```
integrate(1/x/cot(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Integral(1/(x*cot(a + b*log(c*x**n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\cot^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\cot(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/cot(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*cot(b*log(c*x^n) + a)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\cot^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/x/cot(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 20.82 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.53

$$\int \frac{1}{x\cot^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{2}{b\,n\,\sqrt{\cot(a+b\,\ln(c\,x^n))}} + \frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\cot(a+b\,\ln(c\,x^n))}\right)}{b\,n} - \frac{(-1)^{1/4}\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\cot(a+b\,\ln(c\,x^n))}\right)}{b\,n}$$

input
```
int(1/(x*cot(a + b*log(c*x^n))^(3/2)),x)
```

output
```
2/(b*n*cot(a + b*log(c*x^n))^(1/2)) + ((-1)^(1/4)*atan((-1)^(1/4)*cot(a +
b*log(c*x^n))^(1/2)))/(b*n) - ((-1)^(1/4)*atanh((-1)^(1/4)*cot(a + b*log(c
*x^n))^(1/2)))/(b*n)
```

Reduce [F]

$$\int \frac{1}{x\cot^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{\sqrt{\cot(\log(x^n c)\,b+a)}}{\cot(\log(x^n c)\,b+a)^2 x}dx$$

input
```
int(1/x/cot(a+b*log(c*x^n))^(3/2),x)
```

output
```
int(sqrt(cot(log(x**n*c)*b + a))/(cot(log(x**n*c)*b + a)**2*x),x)
```

3.236 $\int \frac{1}{x\cot^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 149

$$\int \frac{1}{x\cot^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = -\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\arctan\left(1+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}bn} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}}{1+\cot(a+b\log(cx^n))}\right)}{\sqrt{2}bn} + \frac{2}{3bn\cot^{\frac{3}{2}}(a+b\log(cx^n))}$$

output
```
1/2*arctan(-1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n+1/2*arctan(1+2
^(1/2)*cot(a+b*ln(c*x^n))^(1/2))*2^(1/2)/b/n+1/2*arctanh(2^(1/2)*cot(a+b*l
n(c*x^n))^(1/2)/(1+cot(a+b*ln(c*x^n))))*2^(1/2)/b/n+2/3/b/n/cot(a+b*ln(c*x
^n))^(3/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.73

$$\int \frac{1}{x \cot^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = -\frac{-2+3\arctan\left(\sqrt[4]{-\cot^2(a+b\log(cx^n))}\right)(-\cot^2(a+b\log(cx^n)))^{3/4}+3\text{arctanh}\left(\sqrt[4]{-\cot^2(a+b\log(cx^n))}\right)(-\cot^2(a+b\log(cx^n)))^{3/4}}{3bn\cot^{\frac{3}{2}}(a+b\log(cx^n))}$$

input
```
Integrate[1/(x*Cot[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
-1/3*(-2 + 3*ArcTan[(-Cot[a + b*Log[c*x^n]]^2)^(1/4)]*(-Cot[a + b*Log[c*x^
n]]^2)^(3/4) + 3*ArcTanh[(-Cot[a + b*Log[c*x^n]]^2)^(1/4)]*(-Cot[a + b*Log
[c*x^n]]^2)^(3/4))/(b*n*Cot[a + b*Log[c*x^n]]^(3/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.30, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {3039, 3042, 3955, 3042, 3957, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\cot^{\frac{5}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\cot^{\frac{5}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\left(-\tan\left(a+b\log(cx^n)+\frac{\pi}{2}\right)\right)^{5/2}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3955}$$

$$\frac{\frac{2}{3b\cot^{\frac{3}{2}}(a+b\log(cx^n))}-\int\frac{1}{\sqrt{\cot(a+b\log(cx^n))}}d\log(cx^n)}{n}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2}{3b\cot^{\frac{3}{2}}(a+b\log(cx^n))}-\int\frac{1}{\sqrt{-\tan\left(a+b\log(cx^n)+\frac{\pi}{2}\right)}}d\log(cx^n)}{n}$$

$\Big\downarrow$ 3957

$$\frac{\frac{\int\frac{1}{\sqrt{\cot(a+b\log(cx^n))}\left(\cot^2(a+b\log(cx^n))+1\right)}d\cot(a+b\log(cx^n))}{b}+\frac{2}{3b\cot^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$\Big\downarrow$ 266

$$\frac{\frac{2\int\frac{1}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}}{b}+\frac{2}{3b\cot^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$\Big\downarrow$ 755

$$\frac{\frac{2\left(\frac{1}{2}\int\frac{1-\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}+\frac{1}{2}\int\frac{\cot(a+b\log(cx^n))+1}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}\right)}{b}+\frac{2}{3b\cot^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$\Big\downarrow$ 1476

$$\frac{\frac{2\left(\frac{1}{2}\int\frac{1-\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}+\frac{1}{2}\int\frac{}{\cot(a+b\log(cx^n))\cdot}\right.}{b}}{n}$$

$\Big\downarrow$ 1082

$$\frac{\frac{2\left(\frac{1}{2}\int\frac{1-\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}+\frac{1}{2}\left(\frac{\int\frac{1}{-\cot(a+b\log(cx^n))-1}d\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-\cot(a+b\log(cx^n))-1}d\left(\sqrt{2}\sqrt{\cot(a+b\right.}}{\sqrt{2}}\right.\right.}{b}}{n}$$

$\Big\downarrow$ 217

$$\frac{\frac{2\left(\frac{1}{2}\int\frac{1-\cot(a+b\log(cx^n))}{\cot^2(a+b\log(cx^n))+1}d\sqrt{\cot(a+b\log(cx^n))}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}}\right)\right)}{b}+\frac{2}{3b\cot^{\frac{3}{2}}(a+b\log}}{n}$$

$\Big\downarrow$ 1479

$$2\left(\frac{\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{\cot(a+b\log(cx^n))}}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}\right)+\frac{1}{2}}{b}\right. \over n$$

$$\Big\downarrow \; 25$$

$$2\left(\frac{\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(a+b\log(cx^n))}}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\text{arc}}{}\right.}{b}\right. \over n$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{\cot(a+b\log(cx^n))}}{\cot(a+b\log(cx^n))-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}{\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1}d\sqrt{\cot(a+b\log(cx^n))}\right)+\frac{1}{2}}{b}\right. \over n$$

$$\Big\downarrow \; 1103$$

$$2\left(\frac{\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\cot(a+b\log(cx^n))+\sqrt{2}\sqrt{\cot(a+b\log(cx^n))}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(a+b\log(c\right.}{}\right.}{b}\right. \over n$$

input
```
Int[1/(x*Cot[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
(2/(3*b*Cot[a + b*Log[c*x^n]]^(3/2)) + (2*((-(ArcTan[1 - Sqrt[2]*Sqrt[Cot[
a + b*Log[c*x^n]]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n
]]]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] + Cot[
a + b*Log[c*x^n]]]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[Cot[a + b*Log[c*x^n]]] +
 Cot[a + b*Log[c*x^n]]]/(2*Sqrt[2]))/2))/b)/n
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3955
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Simp[1/b^2   Int[(b*Tan[c + d*x])^(n + 2), x]
, x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

rule 3957
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\frac{2}{3\cot(a+b\ln(c\,x^n))^{\frac{3}{2}}}+\frac{\sqrt{2}\left(\ln\left(\frac{\cot(a+b\ln(c\,x^n))+\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^n))}+1}{\cot(a+b\ln(c\,x^n))-\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^n))}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^n))}\right)+2\arctan\left(-1\cdots\right)\right)}{4}}{nb}$
default	$\frac{\frac{2}{3\cot(a+b\ln(c\,x^n))^{\frac{3}{2}}}+\frac{\sqrt{2}\left(\ln\left(\frac{\cot(a+b\ln(c\,x^n))+\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^n))}+1}{\cot(a+b\ln(c\,x^n))-\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^n))}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{\cot(a+b\ln(c\,x^n))}\right)+2\arctan\left(-1\cdots\right)\right)}{4}}{nb}$

input
```
int(1/x/cot(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(2/3/cot(a+b*ln(c*x^n))^(3/2)+1/4*2^(1/2)*(ln((cot(a+b*ln(c*x^n))+2^
(1/2)*cot(a+b*ln(c*x^n))^(1/2)+1)/(cot(a+b*ln(c*x^n))-2^(1/2)*cot(a+b*ln(c
*x^n))^(1/2)+1))+2*arctan(1+2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))+2*arctan(-1+
2^(1/2)*cot(a+b*ln(c*x^n))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 626 vs. 2(128) = 256.

Time = 0.09 (sec) , antiderivative size = 626, normalized size of antiderivative = 4.20

$$\int \frac{1}{x\cot^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/cot(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

output
```
-1/12*(6*(sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + sqrt(2))*arctan((
sqrt(2)*sqrt((cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/sin(2*b*n*log(x) +
 2*b*log(c) + 2*a))*sin(2*b*n*log(x) + 2*b*log(c) + 2*a) + cos(2*b*n*log(x
) + 2*b*log(c) + 2*a) + 1)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)) + 6
*(sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + sqrt(2))*arctan((sqrt(2)*
sqrt((cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/sin(2*b*n*log(x) + 2*b*log
(c) + 2*a))*sin(2*b*n*log(x) + 2*b*log(c) + 2*a) - cos(2*b*n*log(x) + 2*b*
log(c) + 2*a) - 1)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)) - 3*(sqrt(2
)*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + sqrt(2))*log((sqrt(2)*sqrt((cos(2
*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/sin(2*b*n*log(x) + 2*b*log(c) + 2*a))
*sin(2*b*n*log(x) + 2*b*log(c) + 2*a) + cos(2*b*n*log(x) + 2*b*log(c) + 2*
a) + sin(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/(cos(2*b*n*log(x) + 2*b*log
(c) + 2*a) + 1)) + 3*(sqrt(2)*cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + sqrt(
2))*log(-(sqrt(2)*sqrt((cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)/sin(2*b*
n*log(x) + 2*b*log(c) + 2*a))*sin(2*b*n*log(x) + 2*b*log(c) + 2*a) - cos(2
*b*n*log(x) + 2*b*log(c) + 2*a) - sin(2*b*n*log(x) + 2*b*log(c) + 2*a) - 1
)/(cos(2*b*n*log(x) + 2*b*log(c) + 2*a) + 1)) + 8*sqrt((cos(2*b*n*log(x) +
 2*b*log(c) + 2*a) + 1)/sin(2*b*n*log(x) + 2*b*log(c) + 2*a))*(cos(2*b*n*l
og(x) + 2*b*log(c) + 2*a) - 1))/(b*n*cos(2*b*n*log(x) + 2*b*log(c) + 2*a)
+ b*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\cot^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/x/cot(a+b*ln(c*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x\cot^{\frac{5}{2}}(a+b\log(cx^n))}\,dx=\int \frac{1}{x\cot(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/cot(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*cot(b*log(c*x^n) + a)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\cot^{\frac{5}{2}}(a+b\log(cx^n))}\,dx=\text{Timed out}$$

input
```
integrate(1/x/cot(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 21.87 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.54

$$\int \frac{1}{x\cot^{\frac{5}{2}}(a+b\log(cx^n))}\,dx=\frac{2}{3\,b\,n\,\cot\left(a+b\,\ln\left(c\,x^n\right)\right)^{3/2}}-\frac{(-1)^{1/4}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{\cot\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)\,1\mathrm{i}}{b\,n}-\frac{(-1)^{1/4}\,\mathrm{atanh}\left((-1)^{1/4}\,\sqrt{\cot\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)\,1\mathrm{i}}{b\,n}$$

input
```
int(1/(x*cot(a + b*log(c*x^n))^(5/2)),x)
```

output
```
2/(3*b*n*cot(a + b*log(c*x^n))^(3/2)) - ((-1)^(1/4)*atan((-1)^(1/4)*cot(a
+ b*log(c*x^n))^(1/2))*1i)/(b*n) - ((-1)^(1/4)*atanh((-1)^(1/4)*cot(a + b*
log(c*x^n))^(1/2))*1i)/(b*n)
```

Reduce [F]

$$\int \frac{1}{x\cot^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{\sqrt{\cot(\log(x^n c)\,b+a)}}{\cot(\log(x^n c)\,b+a)^3 x}dx$$

input
```
int(1/x/cot(a+b*log(c*x^n))^(5/2),x)
```

output
```
int(sqrt(cot(log(x**n*c)*b + a))/(cot(log(x**n*c)*b + a)**3*x),x)
```

3.237 $\int x^2 \sec(a + b\log(cx^n)) \, dx$

Optimal result

Integrand size = 15, antiderivative size = 87

$$\int x^2 \sec(a + b\log(cx^n)) \, dx = \frac{2e^{ia}x^3(cx^n)^{ib} \text{Hypergeometric2F1}\left(1, \frac{1}{2}\left(1 - \frac{3i}{bn}\right), \frac{3}{2}\left(1 - \frac{i}{bn}\right), -e^{2ia}(cx^n)^{2ib}\right)}{3 + ibn}$$

output
```
2*exp(I*a)*x^3*(c*x^n)^(I*b)*hypergeom([1, 1/2-3/2*I/b/n],[3/2-3/2*I/b/n],
-exp(2*I*a)*(c*x^n)^(2*I*b))/(3+I*b*n)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int x^2 \sec(a + b\log(cx^n)) \, dx = -\frac{2ie^{ia}x^3(cx^n)^{ib} \text{Hypergeometric2F1}\left(1, \frac{1}{2} - \frac{3i}{2bn}, \frac{3}{2} - \frac{3i}{2bn}, -e^{2i(a+b\log(cx^n))}\right)}{-3i + bn}$$

input
```
Integrate[x^2*Sec[a + b*Log[c*x^n]],x]
```

output
```
((-2*I)*E^(I*a)*x^3*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 - ((3*I)/2)/(b*
n), 3/2 - ((3*I)/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))])/(-3*I + b*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5020, 5016, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sec(a + b\log(cx^n))\,dx$$

$$\Big\downarrow \text{5020}$$

$$\frac{x^3 (cx^n)^{-3/n} \int (cx^n)^{\frac{3}{n}-1} \sec(a + b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5016}$$

$$\frac{2e^{ia} x^3 (cx^n)^{-3/n} \int \frac{(cx^n)^{ib+\frac{3}{n}-1}}{e^{2ia}(cx^n)^{2ib}+1}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{2e^{ia} x^3 (cx^n)^{ib} \text{Hypergeometric2F1}\left(1, \frac{1}{2}\left(1-\frac{3i}{bn}\right), \frac{3}{2}\left(1-\frac{i}{bn}\right), -e^{2ia}(cx^n)^{2ib}\right)}{3+ibn}$$

input
```
Int[x^2*Sec[a + b*Log[c*x^n]],x]
```

output
```
(2*E^(I*a)*x^3*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 - (3*I)/(b*n))/2, (3*
(1 - I/(b*n)))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(3 + I*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^2 \sec\left(a + b\ln\left(c\,x^n\right)\right) dx$$

input
```
int(x^2*sec(a+b*ln(c*x^n)),x)
```

output
```
int(x^2*sec(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int x^2 \sec\left(a + b\log\left(cx^n\right)\right)\, dx = \int x^2 \sec\left(b\log\left(cx^n\right) + a\right)\, dx$$

input
```
integrate(x^2*sec(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
integral(x^2*sec(b*log(c*x^n) + a), x)
```

Sympy [F]

$$\int x^2 \sec(a + b\log(cx^n)) \, dx = \int x^2 \sec(a + b\log(cx^n)) \, dx$$

input
```
integrate(x**2*sec(a+b*ln(c*x**n)),x)
```

output
```
Integral(x**2*sec(a + b*log(c*x**n)), x)
```

Maxima [F]

$$\int x^2 \sec(a + b\log(cx^n)) \, dx = \int x^2 \sec(b\log(cx^n) + a) \, dx$$

input
```
integrate(x^2*sec(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
integrate(x^2*sec(b*log(c*x^n) + a), x)
```

Giac [F]

$$\int x^2 \sec(a + b\log(cx^n)) \, dx = \int x^2 \sec(b\log(cx^n) + a) \, dx$$

input
```
integrate(x^2*sec(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
integrate(x^2*sec(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sec(a + b\log(cx^n))\,dx = \int \frac{x^2}{\cos(a + b\ln(c\,x^n))}\,dx$$

input
```
int(x^2/cos(a + b*log(c*x^n)),x)
```

output
```
int(x^2/cos(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x^2 \sec(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
int(x^2*sec(a+b*log(c*x^n)),x)
```

output
```
(24*cos(log(x**n*c)*b + a)*int(x**2/(2*tan((log(x**n*c)*b + a)/2)**4*b**2*
n**2 - 9*tan((log(x**n*c)*b + a)/2)**4 - 4*tan((log(x**n*c)*b + a)/2)**2*b
**2*n**2 + 18*tan((log(x**n*c)*b + a)/2)**2 + 2*b**2*n**2 - 9),x)*b**4*n**
4 - 108*cos(log(x**n*c)*b + a)*int(x**2/(2*tan((log(x**n*c)*b + a)/2)**4*b
**2*n**2 - 9*tan((log(x**n*c)*b + a)/2)**4 - 4*tan((log(x**n*c)*b + a)/2)*
*2*b**2*n**2 + 18*tan((log(x**n*c)*b + a)/2)**2 + 2*b**2*n**2 - 9),x)*b**2
*n**2 + 72*cos(log(x**n*c)*b + a)*int((tan((log(x**n*c)*b + a)/2)*x**2)/(2
*tan((log(x**n*c)*b + a)/2)**4*b**2*n**2 - 9*tan((log(x**n*c)*b + a)/2)**4
 - 4*tan((log(x**n*c)*b + a)/2)**2*b**2*n**2 + 18*tan((log(x**n*c)*b + a)/
2)**2 + 2*b**2*n**2 - 9),x)*b**3*n**3 - 324*cos(log(x**n*c)*b + a)*int((ta
n((log(x**n*c)*b + a)/2)*x**2)/(2*tan((log(x**n*c)*b + a)/2)**4*b**2*n**2
- 9*tan((log(x**n*c)*b + a)/2)**4 - 4*tan((log(x**n*c)*b + a)/2)**2*b**2*n
**2 + 18*tan((log(x**n*c)*b + a)/2)**2 + 2*b**2*n**2 - 9),x)*b*n + 2*cos(l
og(x**n*c)*b + a)*sec(log(x**n*c)*b + a)*b**2*n**2*x**3 - 9*cos(log(x**n*c
)*b + a)*sec(log(x**n*c)*b + a)*x**3 - cos(log(x**n*c)*b + a)*b**2*n**2*x*
*3 - 3*sin(log(x**n*c)*b + a)*b*n*x**3 - 2*b**2*n**2*x**3)/(3*cos(log(x**n
*c)*b + a)*(2*b**2*n**2 - 9))
```

3.238 $\int x \sec(a + b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 87

$$\int x\sec(a+b\log(cx^n))\,dx = \frac{2e^{ia}x^2(cx^n)^{ib}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1-\frac{2i}{bn}\right),\frac{1}{2}\left(3-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{2+ibn}$$

output
```
2*exp(I*a)*x^2*(c*x^n)^(I*b)*hypergeom([1, 1/2-I/b/n],[3/2-I/b/n],-exp(2*I
*a)*(c*x^n)^(2*I*b))/(2+I*b*n)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int x\sec(a+b\log(cx^n))\,dx = -\frac{2ie^{ia}x^2(cx^n)^{ib}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{i}{bn},\frac{3}{2}-\frac{i}{bn},-e^{2i(a+b\log(cx^n))}\right)}{-2i+bn}$$

input
```
Integrate[x*Sec[a + b*Log[c*x^n]],x]
```

output
```
((-2*I)*E^(I*a)*x^2*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 - I/(b*n), 3/2
- I/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))])/(-2*I + b*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5020, 5016, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sec (a+b \log (cx^n)) \, dx$$

$$\Big\downarrow \text{ 5020}$$

$$\frac{x^2 (cx^n)^{-2/n} \int (cx^n)^{\frac{2}{n}-1} \sec (a+b \log (cx^n)) d(cx^n)}{n}$$

$$\Big\downarrow \text{ 5016}$$

$$\frac{2e^{ia} x^2 (cx^n)^{-2/n} \int \frac{(cx^n)^{ib+\frac{2}{n}-1}}{e^{2ia} (cx^n)^{2ib}+1} d(cx^n)}{n}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{2e^{ia} x^2 (cx^n)^{ib} \text{Hypergeometric2F1}\left(1, \frac{1}{2}\left(1-\frac{2i}{bn}\right), \frac{1}{2}\left(3-\frac{2i}{bn}\right), -e^{2ia} (cx^n)^{2ib}\right)}{2+ibn}$$

input
```
Int[x*Sec[a + b*Log[c*x^n]],x]
```

output
```
(2*E^(I*a)*x^2*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 - (2*I)/(b*n))/2, (3
- (2*I)/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(2 + I*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x \sec \left(a + b \ln \left(c\, x^{n}\right)\right) dx$$

input
```
int(x*sec(a+b*ln(c*x^n)),x)
```

output
```
int(x*sec(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int x \sec \left(a + b \log \left(cx^{n}\right)\right)\, dx = \int x \sec \left(b \log \left(cx^{n}\right) + a\right)\, dx$$

input
```
integrate(x*sec(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
integral(x*sec(b*log(c*x^n) + a), x)
```

Sympy [F]

$$\int x\sec(a+b\log(cx^n))\,dx=\int x\sec(a+b\log(cx^n))\,dx$$

input
```
integrate(x*sec(a+b*ln(c*x**n)),x)
```

output
```
Integral(x*sec(a + b*log(c*x**n)), x)
```

Maxima [F]

$$\int x\sec(a+b\log(cx^n))\,dx=\int x\sec(b\log(cx^n)+a)\,dx$$

input
```
integrate(x*sec(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
integrate(x*sec(b*log(c*x^n) + a), x)
```

Giac [F]

$$\int x\sec(a+b\log(cx^n))\,dx=\int x\sec(b\log(cx^n)+a)\,dx$$

input
```
integrate(x*sec(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
integrate(x*sec(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \sec (a + b \log (cx^n)) \, dx = \int \frac{x}{\cos (a + b \ln (c x^n))} \, dx$$

input
```
int(x/cos(a + b*log(c*x^n)),x)
```

output
```
int(x/cos(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x \sec (a + b \log (cx^n)) \, dx = \text{Too large to display}$$

input
```
int(x*sec(a+b*log(c*x^n)),x)
```

output
```
(16*cos(log(x**n*c)*b + a)*int((tan((log(x**n*c)*b + a)/2)*x)/(tan((log(x*
*n*c)*b + a)/2)**4*b**2*n**2 - 2*tan((log(x**n*c)*b + a)/2)**4 - 2*tan((lo
g(x**n*c)*b + a)/2)**2*b**2*n**2 + 4*tan((log(x**n*c)*b + a)/2)**2 + b**2*
n**2 - 2),x)*b**3*n**3 - 32*cos(log(x**n*c)*b + a)*int((tan((log(x**n*c)*b
 + a)/2)*x)/(tan((log(x**n*c)*b + a)/2)**4*b**2*n**2 - 2*tan((log(x**n*c)*
b + a)/2)**4 - 2*tan((log(x**n*c)*b + a)/2)**2*b**2*n**2 + 4*tan((log(x**n
*c)*b + a)/2)**2 + b**2*n**2 - 2),x)*b*n + 8*cos(log(x**n*c)*b + a)*int(x/
(tan((log(x**n*c)*b + a)/2)**4*b**2*n**2 - 2*tan((log(x**n*c)*b + a)/2)**4
 - 2*tan((log(x**n*c)*b + a)/2)**2*b**2*n**2 + 4*tan((log(x**n*c)*b + a)/2
)**2 + b**2*n**2 - 2),x)*b**4*n**4 - 16*cos(log(x**n*c)*b + a)*int(x/(tan(
(log(x**n*c)*b + a)/2)**4*b**2*n**2 - 2*tan((log(x**n*c)*b + a)/2)**4 - 2*
tan((log(x**n*c)*b + a)/2)**2*b**2*n**2 + 4*tan((log(x**n*c)*b + a)/2)**2
+ b**2*n**2 - 2),x)*b**2*n**2 + 2*cos(log(x**n*c)*b + a)*sec(log(x**n*c)*b
 + a)*b**2*n**2*x**2 - 4*cos(log(x**n*c)*b + a)*sec(log(x**n*c)*b + a)*x**
2 - cos(log(x**n*c)*b + a)*b**2*n**2*x**2 - 2*sin(log(x**n*c)*b + a)*b*n*x
**2 - 2*b**2*n**2*x**2)/(4*cos(log(x**n*c)*b + a)*(b**2*n**2 - 2))
```

3.239 $\int \sec(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 85

$$\int \sec(a+b\log(cx^n))\,dx = \frac{2e^{ia}x(cx^n)^{ib}\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1-\frac{i}{bn}\right),\frac{1}{2}\left(3-\frac{i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{1+ibn}$$

output
```
2*exp(I*a)*x*(c*x^n)^(I*b)*hypergeom([1, 1/2-1/2*I/b/n],[3/2-1/2*I/b/n],-e
xp(2*I*a)*(c*x^n)^(2*I*b))/(1+I*b*n)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

$$\int \sec(a+b\log(cx^n))\,dx = -\frac{2ie^{ia}x(cx^n)^{ib}\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{i}{2bn},\frac{3}{2}-\frac{i}{2bn},-e^{2i(a+b\log(cx^n))}\right)}{-i+bn}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]],x]
```

output
```
((-2*I)*E^(I*a)*x*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 - (I/2)/(b*n), 3/
2 - (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))])/(-I + b*n)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5014, 5016, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec (a+b\log (cx^n))\,dx$$

$$\Big\downarrow \text{ 5014}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\sec (a+b\log (cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{ 5016}$$

$$\frac{2e^{ia}x(cx^n)^{-1/n}\int \frac{(cx^n)^{ib+\frac{1}{n}-1}}{e^{2ia}(cx^n)^{2ib}+1}d(cx^n)}{n}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{2e^{ia}x(cx^n)^{ib}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1-\frac{i}{bn}\right),\frac{1}{2}\left(3-\frac{i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{n\left(\frac{1}{n}+ib\right)}$$

input
```
Int[Sec[a + b*Log[c*x^n]],x]
```

output
```
(2*E^(I*a)*x*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 - I/(b*n))/2, (3 - I/(b
*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((I*b + n^(-1))*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5014
```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [F]

$$\int \sec \left(a+b\ln \left(c\,x^{n}\right)\right)dx$$

input
```
int(sec(a+b*ln(c*x^n)),x)
```

output
```
int(sec(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int \sec \left(a+b\log \left(cx^{n}\right)\right)\,dx=\int \sec \left(b\log \left(cx^{n}\right)+a\right)\,dx$$

input
```
integrate(sec(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
integral(sec(b*log(c*x^n) + a), x)
```

Sympy [F]

$$\int \sec\left(a + b\log\left(cx^n\right)\right)\,dx = \int \sec\left(a + b\log\left(cx^n\right)\right)dx$$

input
```
integrate(sec(a+b*ln(c*x**n)),x)
```

output
```
Integral(sec(a + b*log(c*x**n)), x)
```

Maxima [F]

$$\int \sec\left(a + b\log\left(cx^n\right)\right)\,dx = \int \sec\left(b\log\left(cx^n\right) + a\right)\,dx$$

input
```
integrate(sec(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
integrate(sec(b*log(c*x^n) + a), x)
```

Giac [F]

$$\int \sec\left(a + b\log\left(cx^n\right)\right)\,dx = \int \sec\left(b\log\left(cx^n\right) + a\right)\,dx$$

input
```
integrate(sec(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
integrate(sec(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec(a+b\log(cx^n))\,dx=\int \frac{1}{\cos(a+b\,\ln(c\,x^n))}\,dx$$

input
```
int(1/cos(a + b*log(c*x^n)),x)
```

output
```
int(1/cos(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int \sec(a+b\log(cx^n))\,dx=\text{Too large to display}$$

input
```
int(sec(a+b*log(c*x^n)),x)
```

output
```
(8*cos(log(x**n*c)*b + a)*int(tan((log(x**n*c)*b + a)/2)/(2*tan((log(x**n*
c)*b + a)/2)**4*b**2*n**2 - tan((log(x**n*c)*b + a)/2)**4 - 4*tan((log(x**
n*c)*b + a)/2)**2*b**2*n**2 + 2*tan((log(x**n*c)*b + a)/2)**2 + 2*b**2*n**
2 - 1),x)*b**3*n**3 - 4*cos(log(x**n*c)*b + a)*int(tan((log(x**n*c)*b + a)
/2)/(2*tan((log(x**n*c)*b + a)/2)**4*b**2*n**2 - tan((log(x**n*c)*b + a)/2
)**4 - 4*tan((log(x**n*c)*b + a)/2)**2*b**2*n**2 + 2*tan((log(x**n*c)*b +
a)/2)**2 + 2*b**2*n**2 - 1),x)*b*n + 8*cos(log(x**n*c)*b + a)*int(1/(2*tan
((log(x**n*c)*b + a)/2)**4*b**2*n**2 - tan((log(x**n*c)*b + a)/2)**4 - 4*t
an((log(x**n*c)*b + a)/2)**2*b**2*n**2 + 2*tan((log(x**n*c)*b + a)/2)**2 +
 2*b**2*n**2 - 1),x)*b**4*n**4 - 4*cos(log(x**n*c)*b + a)*int(1/(2*tan((lo
g(x**n*c)*b + a)/2)**4*b**2*n**2 - tan((log(x**n*c)*b + a)/2)**4 - 4*tan((
log(x**n*c)*b + a)/2)**2*b**2*n**2 + 2*tan((log(x**n*c)*b + a)/2)**2 + 2*b
**2*n**2 - 1),x)*b**2*n**2 + 2*cos(log(x**n*c)*b + a)*sec(log(x**n*c)*b +
a)*b**2*n**2*x - cos(log(x**n*c)*b + a)*sec(log(x**n*c)*b + a)*x - cos(log
(x**n*c)*b + a)*b**2*n**2*x - sin(log(x**n*c)*b + a)*b*n*x - 2*b**2*n**2*x
)/(cos(log(x**n*c)*b + a)*(2*b**2*n**2 - 1))
```

3.240 $\int \frac{\sec(a+b\log(cx^n))}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{\sec(a+b\log(cx^n))}{x} dx = \frac{\operatorname{arctanh}(\sin(a+b\log(cx^n)))}{bn}$$

output
```
arctanh(sin(a+b*ln(c*x^n)))/b/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\sec(a+b\log(cx^n))}{x} dx = \frac{\coth^{-1}(\sin(a+b\log(cx^n)))}{bn}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]]/x,x]
```

output
```
ArcCoth[Sin[a + b*Log[c*x^n]]]/(b*n)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3039, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec (a+b\log (cx^n))}{x}dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \sec (a+b\log (cx^n))d\log (cx^n)}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \csc \left(a+b\log (cx^n)+\frac{\pi}{2}\right)d\log (cx^n)}{n}$$
$$\Big\downarrow \text{4257}$$
$$\frac{\text{arctanh}(\sin (a+b\log (cx^n)))}{bn}$$

input
```
Int[Sec[a + b*Log[c*x^n]]/x,x]
```

output
```
ArcTanh[Sin[a + b*Log[c*x^n]]]/(b*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

method	result
derivativedivides	$\frac{\ln(\sec(a+b\ln(c\,x^n))+\tan(a+b\ln(c\,x^n)))}{nb}$
default	$\frac{\ln(\sec(a+b\ln(c\,x^n))+\tan(a+b\ln(c\,x^n)))}{nb}$
parallelrisch	$\frac{-\ln\left(\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)-1\right)+\ln\left(\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)+1\right)}{nb}$
risch	$\frac{\ln\left(c^{ib}(x^n)^{ib}\mathrm{e}^{-\frac{b\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}{2}}\mathrm{e}^{\frac{b\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)}{2}}\mathrm{e}^{\frac{b\pi\,\mathrm{csgn}(ic\,x^n)^3}{2}}\mathrm{e}^{-\frac{b\pi\,\mathrm{csgn}(ic\,x^n)^2\,\mathrm{csgn}(ic)}{2}}\mathrm{e}^{ia}+\right.}{bn}$

input
```
int(sec(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*ln(sec(a+b*ln(c*x^n))+tan(a+b*ln(c*x^n)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(19) = 38$.

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.26

$$\int \frac{\sec(a+b\log(cx^n))}{x}\,dx = \frac{\log(\sin(bn\log(x)+b\log(c)+a)+1)-\log(-\sin(bn\log(x)+b\log(c)+a)+1)}{2\,bn}$$

input
```
integrate(sec(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

output
```
1/2*(log(sin(b*n*log(x) + b*log(c) + a) + 1) - log(-sin(b*n*log(x) + b*log
(c) + a) + 1))/(b*n)
```

Sympy [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.68

$$\int \frac{\sec (a+b\log (cx^n))}{x}\,dx=-\begin{cases} -\log (x)\sec (a) & \text{for } b=0\\ -\log (x)\sec (a+b\log (c)) & \text{for } n=0\\ -\frac{\log (\tan (a+b\log (cx^n))+\sec (a+b\log (cx^n)))}{bn} & \text{otherwise}\end{cases}$$

input
```
integrate(sec(a+b*ln(c*x**n))/x,x)
```

output
```
-Piecewise((-log(x)*sec(a), Eq(b, 0)), (-log(x)*sec(a + b*log(c)), Eq(n, 0
)), (-log(tan(a + b*log(c*x**n)) + sec(a + b*log(c*x**n)))/(b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \frac{\sec (a+b\log (cx^n))}{x}\,dx=\frac{\log (\sec (b\log (cx^n)+a)+\tan (b\log (cx^n)+a))}{bn}$$

input
```
integrate(sec(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

output
```
log(sec(b*log(c*x^n) + a) + tan(b*log(c*x^n) + a))/(b*n)
```

Giac [F]

$$\int \frac{\sec (a+b\log (cx^n))}{x}\,dx=\int \frac{\sec (b\log (cx^n)+a)}{x}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))/x,x, algorithm="giac")
```

output
```
integrate(sec(b*log(c*x^n) + a)/x, x)
```

Mupad [B] (verification not implemented)

Time = 21.47 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.47

$$\int \frac{\sec (a+b\log (cx^n))}{x}\,dx=-\frac{\ln \left(\frac{2\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}-2\mathrm{i}}{x}\right)}{b\,n}+\frac{\ln \left(\frac{2\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}+2\mathrm{i}}{x}\right)}{b\,n}$$

input
```
int(1/(x*cos(a + b*log(c*x^n))),x)
```

output
```
log((2*exp(a*1i)*(c*x^n)^(b*1i) + 2i)/x)/(b*n) - log((2*exp(a*1i)*(c*x^n)^
(b*1i) - 2i)/x)/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.32

$$\int \frac{\sec (a+b\log (cx^n))}{x}\,dx = \frac{-\log\left(\tan\left(\frac{\log(x^n c)b}{2}+\frac{a}{2}\right)-1\right)+\log\left(\tan\left(\frac{\log(x^n c)b}{2}+\frac{a}{2}\right)+1\right)}{bn}$$

input
```
int(sec(a+b*log(c*x^n))/x,x)
```

output
```
( - log(tan((log(x**n*c)*b + a)/2) - 1) + log(tan((log(x**n*c)*b + a)/2) +
 1))/(b*n)
```

3.241 $\int \frac{\sec(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 87

$$\int \frac{\sec(a+b\log(cx^n))}{x^2}\,dx = -\frac{2e^{ia}(cx^n)^{ib}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{i}{bn}\right),\frac{1}{2}\left(3+\frac{i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{(1-ibn)x}$$

output
```
-2*exp(I*a)*(c*x^n)^(I*b)*hypergeom([1, 1/2+1/2*I/b/n],[3/2+1/2*I/b/n],-ex
p(2*I*a)*(c*x^n)^(2*I*b))/(1-I*b*n)/x
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int \frac{\sec(a+b\log(cx^n))}{x^2}\,dx = \frac{2e^{ia}(cx^n)^{ib}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{i}{2bn},\frac{3}{2}+\frac{i}{2bn},-e^{2i(a+b\log(cx^n))}\right)}{(-1+ibn)x}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]]/x^2,x]
```

output
```
(2*E^(I*a)*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 + (I/2)/(b*n), 3/2 + (I/
2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))])/((-1 + I*b*n)*x)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5020, 5016, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(a+b\log(cx^n))}{x^2}dx$$

$$\Big\downarrow \text{5020}$$

$$\frac{(cx^n)^{\frac{1}{n}}\int (cx^n)^{-1-\frac{1}{n}}\sec(a+b\log(cx^n))d(cx^n)}{nx}$$

$$\Big\downarrow \text{5016}$$

$$\frac{2e^{ia}(cx^n)^{\frac{1}{n}}\int \frac{(cx^n)^{ib-\frac{1}{n}-1}}{e^{2ia}(cx^n)^{2ib}+1}d(cx^n)}{nx}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2e^{ia}(cx^n)^{ib}\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{i}{bn}\right),\frac{1}{2}\left(3+\frac{i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{x(1-ibn)}$$

input
```
Int[Sec[a + b*Log[c*x^n]]/x^2,x]
```

output
```
(-2*E^(I*a)*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 + I/(b*n))/2, (3 + I/(b*
n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((1 - I*b*n)*x)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\sec(a+b\ln(c\,x^n))}{x^2}dx$$

input
```
int(sec(a+b*ln(c*x^n))/x^2,x)
```

output
```
int(sec(a+b*ln(c*x^n))/x^2,x)
```

Fricas [F]

$$\int \frac{\sec(a+b\log(cx^n))}{x^2}\,dx=\int \frac{\sec(b\log(cx^n)+a)}{x^2}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))/x^2,x, algorithm="fricas")
```

output
```
integral(sec(b*log(c*x^n) + a)/x^2, x)
```

Sympy [F]

$$\int \frac{\sec(a+b\log(cx^n))}{x^2}\,dx=\int \frac{\sec(a+b\log(cx^n))}{x^2}\,dx$$

input
```
integrate(sec(a+b*ln(c*x**n))/x**2,x)
```

output
```
Integral(sec(a + b*log(c*x**n))/x**2, x)
```

Maxima [F]

$$\int \frac{\sec(a+b\log(cx^n))}{x^2}\,dx=\int \frac{\sec(b\log(cx^n)+a)}{x^2}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))/x^2,x, algorithm="maxima")
```

output
```
integrate(sec(b*log(c*x^n) + a)/x^2, x)
```

Giac [F]

$$\int \frac{\sec(a+b\log(cx^n))}{x^2}\,dx=\int \frac{\sec(b\log(cx^n)+a)}{x^2}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))/x^2,x, algorithm="giac")
```

output
```
integrate(sec(b*log(c*x^n) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(a+b\log(cx^n))}{x^2}\,dx = \int \frac{1}{x^2\,\cos(a+b\,\ln(c\,x^n))}\,dx$$

input
```
int(1/(x^2*cos(a + b*log(c*x^n))),x)
```

output
```
int(1/(x^2*cos(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int \frac{\sec(a+b\log(cx^n))}{x^2}\,dx = \text{Too large to display}$$

input
```
int(sec(a+b*log(c*x^n))/x^2,x)
```

output
```
( - 8*cos(log(x**n*c)*b + a)*int(tan((log(x**n*c)*b + a)/2)/(2*tan((log(x*
*n*c)*b + a)/2)**4*b**2*n**2*x**2 - tan((log(x**n*c)*b + a)/2)**4*x**2 - 4
*tan((log(x**n*c)*b + a)/2)**2*b**2*n**2*x**2 + 2*tan((log(x**n*c)*b + a)/
2)**2*x**2 + 2*b**2*n**2*x**2 - x**2),x)*b**3*n**3*x + 4*cos(log(x**n*c)*b
 + a)*int(tan((log(x**n*c)*b + a)/2)/(2*tan((log(x**n*c)*b + a)/2)**4*b**2
*n**2*x**2 - tan((log(x**n*c)*b + a)/2)**4*x**2 - 4*tan((log(x**n*c)*b + a
)/2)**2*b**2*n**2*x**2 + 2*tan((log(x**n*c)*b + a)/2)**2*x**2 + 2*b**2*n**
2*x**2 - x**2),x)*b*n*x + 8*cos(log(x**n*c)*b + a)*int(1/(2*tan((log(x**n*
c)*b + a)/2)**4*b**2*n**2*x**2 - tan((log(x**n*c)*b + a)/2)**4*x**2 - 4*ta
n((log(x**n*c)*b + a)/2)**2*b**2*n**2*x**2 + 2*tan((log(x**n*c)*b + a)/2)*
*2*x**2 + 2*b**2*n**2*x**2 - x**2),x)*b**4*n**4*x - 4*cos(log(x**n*c)*b +
a)*int(1/(2*tan((log(x**n*c)*b + a)/2)**4*b**2*n**2*x**2 - tan((log(x**n*c
)*b + a)/2)**4*x**2 - 4*tan((log(x**n*c)*b + a)/2)**2*b**2*n**2*x**2 + 2*t
an((log(x**n*c)*b + a)/2)**2*x**2 + 2*b**2*n**2*x**2 - x**2),x)*b**2*n**2*
x - 2*cos(log(x**n*c)*b + a)*sec(log(x**n*c)*b + a)*b**2*n**2 + cos(log(x*
*n*c)*b + a)*sec(log(x**n*c)*b + a) + cos(log(x**n*c)*b + a)*b**2*n**2 - s
in(log(x**n*c)*b + a)*b*n + 2*b**2*n**2)/(cos(log(x**n*c)*b + a)*x*(2*b**2
*n**2 - 1))
```

3.242 $\int \frac{\sec(a+b\log(cx^n))}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 87

$$\int \frac{\sec(a+b\log(cx^n))}{x^3}\,dx = -\frac{2e^{ia}(cx^n)^{ib}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{2i}{bn}\right),\frac{1}{2}\left(3+\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{(2-ibn)x^2}$$

output
```
-2*exp(I*a)*(c*x^n)^(I*b)*hypergeom([1, 1/2+I/b/n],[3/2+I/b/n],-exp(2*I*a)
*(c*x^n)^(2*I*b))/(2-I*b*n)/x^2
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \frac{\sec(a+b\log(cx^n))}{x^3}\,dx = \frac{2e^{ia}(cx^n)^{ib}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{i}{bn},\frac{3}{2}+\frac{i}{bn},-e^{2i(a+b\log(cx^n))}\right)}{(-2+ibn)x^2}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]]/x^3,x]
```

output
```
(2*E^(I*a)*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 + I/(b*n), 3/2 + I/(b*n)
, -E^((2*I)*(a + b*Log[c*x^n]))])/((-2 + I*b*n)*x^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5020, 5016, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec (a+b\log (cx^n))}{x^3}dx$$

$$\Big\downarrow \text{ 5020}$$

$$\frac{(cx^n)^{2/n}\int (cx^n)^{-1-\frac{2}{n}}\sec (a+b\log (cx^n))d(cx^n)}{nx^2}$$

$$\Big\downarrow \text{ 5016}$$

$$\frac{2e^{ia}(cx^n)^{2/n}\int \frac{(cx^n)^{ib-\frac{2}{n}-1}}{e^{2ia}(cx^n)^{2ib}+1}d(cx^n)}{nx^2}$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{2e^{ia}(cx^n)^{ib}\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{2i}{bn}\right),\frac{1}{2}\left(3+\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{x^2(2-ibn)}$$

input
```
Int[Sec[a + b*Log[c*x^n]]/x^3,x]
```

output
```
(-2*E^(I*a)*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 + (2*I)/(b*n))/2, (3 + (
2*I)/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 - I*b*n)*x^2)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\sec \left(a +b \ln \left(c \,x^{n}\right)\right)}{x^{3}}d x$$

input
```
int(sec(a+b*ln(c*x^n))/x^3,x)
```

output
```
int(sec(a+b*ln(c*x^n))/x^3,x)
```

Fricas [F]

$$\int \frac{\sec (a+b\log (cx^n))}{x^3} \, dx=\int \frac{\sec \left(b \log \left(c x^{n}\right) + a\right)}{x^{3}} \,d x$$

input
```
integrate(sec(a+b*log(c*x^n))/x^3,x, algorithm="fricas")
```

output
```
integral(sec(b*log(c*x^n) + a)/x^3, x)
```

Sympy [F]

$$\int \frac{\sec(a+b\log(cx^n))}{x^3}\,dx=\int \frac{\sec(a+b\log(cx^n))}{x^3}\,dx$$

input
```
integrate(sec(a+b*ln(c*x**n))/x**3,x)
```

output
```
Integral(sec(a + b*log(c*x**n))/x**3, x)
```

Maxima [F]

$$\int \frac{\sec(a+b\log(cx^n))}{x^3}\,dx=\int \frac{\sec(b\log(cx^n)+a)}{x^3}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))/x^3,x, algorithm="maxima")
```

output
```
integrate(sec(b*log(c*x^n) + a)/x^3, x)
```

Giac [F]

$$\int \frac{\sec(a+b\log(cx^n))}{x^3}\,dx=\int \frac{\sec(b\log(cx^n)+a)}{x^3}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))/x^3,x, algorithm="giac")
```

output
```
integrate(sec(b*log(c*x^n) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec (a+b\log (cx^n))}{x^3}\,dx=\int \frac{1}{x^3\,\cos \left (a+b\,\ln \left (c\,x^n\right)\right)} \,dx$$

input
```
int(1/(x^3*cos(a + b*log(c*x^n))),x)
```

output
```
int(1/(x^3*cos(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int \frac{\sec (a+b\log (cx^n))}{x^3}\,dx=\text{Too large to display}$$

input
```
int(sec(a+b*log(c*x^n))/x^3,x)
```

output
```
( - 16*cos(log(x**n*c)*b + a)*int(tan((log(x**n*c)*b + a)/2)/(tan((log(x**
n*c)*b + a)/2)**4*b**2*n**2*x**3 - 2*tan((log(x**n*c)*b + a)/2)**4*x**3 -
2*tan((log(x**n*c)*b + a)/2)**2*b**2*n**2*x**3 + 4*tan((log(x**n*c)*b + a)
/2)**2*x**3 + b**2*n**2*x**3 - 2*x**3),x)*b**3*n**3*x**2 + 32*cos(log(x**n
*c)*b + a)*int(tan((log(x**n*c)*b + a)/2)/(tan((log(x**n*c)*b + a)/2)**4*b
**2*n**2*x**3 - 2*tan((log(x**n*c)*b + a)/2)**4*x**3 - 2*tan((log(x**n*c)*
b + a)/2)**2*b**2*n**2*x**3 + 4*tan((log(x**n*c)*b + a)/2)**2*x**3 + b**2*
n**2*x**3 - 2*x**3),x)*b*n*x**2 + 8*cos(log(x**n*c)*b + a)*int(1/(tan((log
(x**n*c)*b + a)/2)**4*b**2*n**2*x**3 - 2*tan((log(x**n*c)*b + a)/2)**4*x**
3 - 2*tan((log(x**n*c)*b + a)/2)**2*b**2*n**2*x**3 + 4*tan((log(x**n*c)*b
+ a)/2)**2*x**3 + b**2*n**2*x**3 - 2*x**3),x)*b**4*n**4*x**2 - 16*cos(log(
x**n*c)*b + a)*int(1/(tan((log(x**n*c)*b + a)/2)**4*b**2*n**2*x**3 - 2*tan
((log(x**n*c)*b + a)/2)**4*x**3 - 2*tan((log(x**n*c)*b + a)/2)**2*b**2*n**
2*x**3 + 4*tan((log(x**n*c)*b + a)/2)**2*x**3 + b**2*n**2*x**3 - 2*x**3),x
)*b**2*n**2*x**2 - 2*cos(log(x**n*c)*b + a)*sec(log(x**n*c)*b + a)*b**2*n*
*2 + 4*cos(log(x**n*c)*b + a)*sec(log(x**n*c)*b + a) + cos(log(x**n*c)*b +
 a)*b**2*n**2 - 2*sin(log(x**n*c)*b + a)*b*n + 2*b**2*n**2)/(4*cos(log(x**
n*c)*b + a)*x**2*(b**2*n**2 - 2))
```

3.243 $\int x^2 \sec^2(a + b\log(cx^n))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 87

$$\int x^2 \sec^2(a + b\log(cx^n))\,dx = \frac{4e^{2ia}x^3(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2-\frac{3i}{bn}\right),\frac{1}{2}\left(4-\frac{3i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{3+2ibn}$$

output
```
4*exp(2*I*a)*x^3*(c*x^n)^(2*I*b)*hypergeom([2, 1-3/2*I/b/n],[2-3/2*I/b/n],
-exp(2*I*a)*(c*x^n)^(2*I*b))/(3+2*I*b*n)
```

Mathematica [A] (verified)

Time = 3.91 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.84

$$\int x^2 \sec^2(a + b\log(cx^n))\,dx = \frac{x^3\left(3e^{2ia}(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(1,1-\frac{3i}{2bn},2-\frac{3i}{2bn},-e^{2i(a+b\log(cx^n))}\right)+(-3i+2bn)\left(-i\,\text{Hypergeor}\right.\right.}{bn(-3i+2bn)}$$

input
```
Integrate[x^2*Sec[a + b*Log[c*x^n]]^2,x]
```

output
```
(x^3*(3*E^((2*I)*a)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[1, 1 - ((3*I)/2)/(
b*n), 2 - ((3*I)/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] + (-3*I + 2*b*n)
*((-I)*Hypergeometric2F1[1, ((-3*I)/2)/(b*n), 1 - ((3*I)/2)/(b*n), -E^((2*
I)*(a + b*Log[c*x^n]))] + Tan[a + b*Log[c*x^n]])))/(b*n*(-3*I + 2*b*n))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5020, 5016, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sec^2(a + b\log(cx^n))\,dx$$

$$\Big\downarrow \text{5020}$$

$$\frac{x^3(cx^n)^{-3/n}\int (cx^n)^{\frac{3}{n}-1}\sec^2(a + b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5016}$$

$$\frac{4e^{2ia}x^3(cx^n)^{-3/n}\int \frac{(cx^n)^{2ib+\frac{3}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^2}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{4e^{2ia}x^3(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(2, \frac{1}{2}\left(2 - \frac{3i}{bn}\right), \frac{1}{2}\left(4 - \frac{3i}{bn}\right), -e^{2ia}(cx^n)^{2ib}\right)}{3 + 2ibn}$$

input
```
Int[x^2*Sec[a + b*Log[c*x^n]]^2,x]
```

output
```
(4*E^((2*I)*a)*x^3*(c*x^n)^((2*I)*b)*Hypergeometric2F1[2, (2 - (3*I)/(b*n)
)/2, (4 - (3*I)/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(3 + (2*I)*b*
n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^2 \sec\left(a+b\ln\left(c\,x^n\right)\right)^2 dx$$

input
```
int(x^2*sec(a+b*ln(c*x^n))^2,x)
```

output
```
int(x^2*sec(a+b*ln(c*x^n))^2,x)
```

Fricas [F]

$$\int x^2 \sec^2\left(a+b\log\left(cx^n\right)\right)\,dx=\int x^2 \sec\left(b\log\left(cx^n\right)+a\right)^2\,dx$$

input
```
integrate(x^2*sec(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
integral(x^2*sec(b*log(c*x^n) + a)^2, x)
```

Sympy [F]

$$\int x^2 \sec^2(a + b \log(cx^n)) \, dx = \int x^2 \sec^2(a + b \log(cx^n)) \, dx$$

input
```
integrate(x**2*sec(a+b*ln(c*x**n))**2,x)
```

output
```
Integral(x**2*sec(a + b*log(c*x**n))**2, x)
```

Maxima [F]

$$\int x^2 \sec^2(a + b \log(cx^n)) \, dx = \int x^2 \sec(b \log(cx^n) + a)^2 \, dx$$

input
```
integrate(x^2*sec(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
2*(x^3*cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + x^3*cos(2*b*log(c))*sin(2
*b*log(x^n) + 2*a) - 3*(2*b^2*n^2*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a)
- 2*b^2*n^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2*cos(2*b*log(c))
^2 + b^2*sin(2*b*log(c))^2)*n^2*cos(2*b*log(x^n) + 2*a)^2 + (b^2*cos(2*b*l
og(c))^2 + b^2*sin(2*b*log(c))^2)*n^2*sin(2*b*log(x^n) + 2*a)^2 + b^2*n^2)
*integrate((x^2*cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + x^2*cos(2*b*log(
c))*sin(2*b*log(x^n) + 2*a))/(2*b^2*n^2*cos(2*b*log(c))*cos(2*b*log(x^n) +
 2*a) - 2*b^2*n^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2*cos(2*b*l
og(c))^2 + b^2*sin(2*b*log(c))^2)*n^2*cos(2*b*log(x^n) + 2*a)^2 + (b^2*cos
(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2*sin(2*b*log(x^n) + 2*a)^2 + b^
2*n^2), x))/(2*b*n*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) + (b*cos(2*b*lo
g(c))^2 + b*sin(2*b*log(c))^2)*n*cos(2*b*log(x^n) + 2*a)^2 - 2*b*n*sin(2*b
*log(c))*sin(2*b*log(x^n) + 2*a) + (b*cos(2*b*log(c))^2 + b*sin(2*b*log(c)
)^2)*n*sin(2*b*log(x^n) + 2*a)^2 + b*n)
```

Giac [F]

$$\int x^2 \sec^2(a+b\log(cx^n))\,dx = \int x^2 \sec(b\log(cx^n)+a)^2\,dx$$

input
```
integrate(x^2*sec(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
integrate(x^2*sec(b*log(c*x^n) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sec^2(a+b\log(cx^n))\,dx = \int \frac{x^2}{\cos(a+b\,\ln(c\,x^n))^2}\,dx$$

input
```
int(x^2/cos(a + b*log(c*x^n))^2,x)
```

output
```
int(x^2/cos(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int x^2 \sec^2(a+b\log(cx^n))\,dx = \text{too large to display}$$

input
```
int(x^2*sec(a+b*log(c*x^n))^2,x)
```

output
```
( - 42*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*tan((log(x**n*c)*b +
a)/2)**4*b**3*n**3*x**3 - 54*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)
*tan((log(x**n*c)*b + a)/2)**4*b*n*x**3 + 84*cos(log(x**n*c)*b + a)*sin(lo
g(x**n*c)*b + a)*tan((log(x**n*c)*b + a)/2)**2*b**3*n**3*x**3 + 108*cos(lo
g(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*tan((log(x**n*c)*b + a)/2)**2*b*n*
x**3 - 42*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*b**3*n**3*x**3 - 5
4*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*b*n*x**3 - 252*cos(log(x**
n*c)*b + a)*tan((log(x**n*c)*b + a)/2)**4*b**2*n**2*x**3 + 504*cos(log(x**
n*c)*b + a)*tan((log(x**n*c)*b + a)/2)**2*b**2*n**2*x**3 - 252*cos(log(x**
n*c)*b + a)*b**2*n**2*x**3 - 16128*int((tan((log(x**n*c)*b + a)/2)**3*x**2
)/(14*tan((log(x**n*c)*b + a)/2)**6*b**4*n**4 - 135*tan((log(x**n*c)*b + a
)/2)**6*b**2*n**2 + 81*tan((log(x**n*c)*b + a)/2)**6 - 42*tan((log(x**n*c)
*b + a)/2)**4*b**4*n**4 + 405*tan((log(x**n*c)*b + a)/2)**4*b**2*n**2 - 24
3*tan((log(x**n*c)*b + a)/2)**4 + 42*tan((log(x**n*c)*b + a)/2)**2*b**4*n*
*4 - 405*tan((log(x**n*c)*b + a)/2)**2*b**2*n**2 + 243*tan((log(x**n*c)*b
+ a)/2)**2 - 14*b**4*n**4 + 135*b**2*n**2 - 81),x)*sin(log(x**n*c)*b + a)*
*2*tan((log(x**n*c)*b + a)/2)**4*b**7*n**7 + 155520*int((tan((log(x**n*c)*
b + a)/2)**3*x**2)/(14*tan((log(x**n*c)*b + a)/2)**6*b**4*n**4 - 135*tan((
log(x**n*c)*b + a)/2)**6*b**2*n**2 + 81*tan((log(x**n*c)*b + a)/2)**6 - 42
*tan((log(x**n*c)*b + a)/2)**4*b**4*n**4 + 405*tan((log(x**n*c)*b + a)/...
```

3.244 $\int x \sec^2(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int x\sec^2(a+b\log(cx^n))\,dx = \frac{2e^{2ia}x^2(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(2,1-\frac{i}{bn},2-\frac{i}{bn},-e^{2ia}(cx^n)^{2ib}\right)}{1+ibn}$$

output
```
2*exp(2*I*a)*x^2*(c*x^n)^(2*I*b)*hypergeom([2, 1-I/b/n],[2-I/b/n],-exp(2*I
*a)*(c*x^n)^(2*I*b))/(1+I*b*n)
```

Mathematica [A] (verified)

Time = 3.72 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.89

$$\int x\sec^2(a+b\log(cx^n))\,dx = \frac{x^2\left(e^{2ia}(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(1,1-\frac{i}{bn},2-\frac{i}{bn},-e^{2i(a+b\log(cx^n))}\right)+(-i+bn)\left(-i\,\text{Hypergeometric}\right.\right.}{bn(-i+bn)}$$

input
```
Integrate[x*Sec[a + b*Log[c*x^n]]^2,x]
```

output
```
(x^2*(E^((2*I)*a)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[1, 1 - I/(b*n), 2 -
I/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] + (-I + b*n)*((-I)*Hypergeometric2
F1[1, (-I)/(b*n), 1 - I/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] + Tan[a + b*
Log[c*x^n]])))/(b*n*(-I + b*n))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5020, 5016, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sec^2(a + b\log(cx^n))\,dx$$

$$\Big\downarrow \text{5020}$$

$$\frac{x^2(cx^n)^{-2/n}\int (cx^n)^{\frac{2}{n}-1}\sec^2(a + b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5016}$$

$$\frac{4e^{2ia}x^2(cx^n)^{-2/n}\int \frac{(cx^n)^{2ib+\frac{2}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^2}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{2e^{2ia}x^2(cx^n)^{-\frac{2}{n}+2\left(\frac{1}{n}+ib\right)}\text{Hypergeometric2F1}\left(2,1-\frac{i}{bn},2-\frac{i}{bn},-e^{2ia}(cx^n)^{2ib}\right)}{1+ibn}$$

input
```
Int[x*Sec[a + b*Log[c*x^n]]^2,x]
```

output
```
(2*E^((2*I)*a)*x^2*(c*x^n)^(2*(I*b + n^(-1)) - 2/n)*Hypergeometric2F1[2, 1
 - I/(b*n), 2 - I/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(1 + I*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x\sec\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}dx$$

input
```
int(x*sec(a+b*ln(c*x^n))^2,x)
```

output
```
int(x*sec(a+b*ln(c*x^n))^2,x)
```

Fricas [F]

$$\int x\sec^{2}\left(a+b\log\left(cx^{n}\right)\right)\,dx=\int x\sec\left(b\log\left(cx^{n}\right)+a\right)^{2}\,dx$$

input
```
integrate(x*sec(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
integral(x*sec(b*log(c*x^n) + a)^2, x)
```

Sympy [F]

$$\int x\sec^2(a+b\log(cx^n))\,dx = \int x\sec^2(a+b\log(cx^n))\,dx$$

input
```
integrate(x*sec(a+b*ln(c*x**n))**2,x)
```

output
```
Integral(x*sec(a + b*log(c*x**n))**2, x)
```

Maxima [F]

$$\int x\sec^2(a+b\log(cx^n))\,dx = \int x\sec(b\log(cx^n)+a)^2\,dx$$

input
```
integrate(x*sec(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
2*(x^2*cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + x^2*cos(2*b*log(c))*sin(2
*b*log(x^n) + 2*a) - 2*(2*b^2*n^2*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a)
- 2*b^2*n^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2*cos(2*b*log(c))
^2 + b^2*sin(2*b*log(c))^2)*n^2*cos(2*b*log(x^n) + 2*a)^2 + (b^2*cos(2*b*l
og(c))^2 + b^2*sin(2*b*log(c))^2)*n^2*sin(2*b*log(x^n) + 2*a)^2 + b^2*n^2)
*integrate((x*cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + x*cos(2*b*log(c))*
sin(2*b*log(x^n) + 2*a))/(2*b^2*n^2*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a
) - 2*b^2*n^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2*cos(2*b*log(c
))^2 + b^2*sin(2*b*log(c))^2)*n^2*cos(2*b*log(x^n) + 2*a)^2 + (b^2*cos(2*b
*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2*sin(2*b*log(x^n) + 2*a)^2 + b^2*n^
2), x))/(2*b*n*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) + (b*cos(2*b*log(c)
)^2 + b*sin(2*b*log(c))^2)*n*cos(2*b*log(x^n) + 2*a)^2 - 2*b*n*sin(2*b*log
(c))*sin(2*b*log(x^n) + 2*a) + (b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)
*n*sin(2*b*log(x^n) + 2*a)^2 + b*n)
```

Giac [F]

$$\int x\sec^2\left(a+b\log\left(cx^n\right)\right)\,dx=\int x\sec\left(b\log\left(cx^n\right)+a\right)^2\,dx$$

input
```
integrate(x*sec(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
integrate(x*sec(b*log(c*x^n) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sec^2\left(a+b\log\left(cx^n\right)\right)\,dx=\int\frac{x}{\cos\left(a+b\ln\left(c\,x^n\right)\right)^2}\,dx$$

input
```
int(x/cos(a + b*log(c*x^n))^2,x)
```

output
```
int(x/cos(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int x\sec^2\left(a+b\log\left(cx^n\right)\right)\,dx=\text{too large to display}$$

input
```
int(x*sec(a+b*log(c*x^n))^2,x)
```

output

```
( - 14*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*tan((log(x**n*c)*b +
a)/2)**4*b**3*n**3*x**2 - 8*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*
tan((log(x**n*c)*b + a)/2)**4*b*n*x**2 + 28*cos(log(x**n*c)*b + a)*sin(log
(x**n*c)*b + a)*tan((log(x**n*c)*b + a)/2)**2*b**3*n**3*x**2 + 16*cos(log(
x**n*c)*b + a)*sin(log(x**n*c)*b + a)*tan((log(x**n*c)*b + a)/2)**2*b*n*x*
*2 - 14*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*b**3*n**3*x**2 - 8*c
os(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*b*n*x**2 - 56*cos(log(x**n*c)
*b + a)*tan((log(x**n*c)*b + a)/2)**4*b**2*n**2*x**2 + 112*cos(log(x**n*c)
*b + a)*tan((log(x**n*c)*b + a)/2)**2*b**2*n**2*x**2 - 56*cos(log(x**n*c)*
b + a)*b**2*n**2*x**2 - 1792*int((tan((log(x**n*c)*b + a)/2)**3*x)/(7*tan(
(log(x**n*c)*b + a)/2)**6*b**4*n**4 - 30*tan((log(x**n*c)*b + a)/2)**6*b**
2*n**2 + 8*tan((log(x**n*c)*b + a)/2)**6 - 21*tan((log(x**n*c)*b + a)/2)**
4*b**4*n**4 + 90*tan((log(x**n*c)*b + a)/2)**4*b**2*n**2 - 24*tan((log(x**
n*c)*b + a)/2)**4 + 21*tan((log(x**n*c)*b + a)/2)**2*b**4*n**4 - 90*tan((l
og(x**n*c)*b + a)/2)**2*b**2*n**2 + 24*tan((log(x**n*c)*b + a)/2)**2 - 7*b
**4*n**4 + 30*b**2*n**2 - 8),x)*sin(log(x**n*c)*b + a)**2*tan((log(x**n*c)
*b + a)/2)**4*b**7*n**7 + 7680*int((tan((log(x**n*c)*b + a)/2)**3*x)/(7*ta
n((log(x**n*c)*b + a)/2)**6*b**4*n**4 - 30*tan((log(x**n*c)*b + a)/2)**6*b
**2*n**2 + 8*tan((log(x**n*c)*b + a)/2)**6 - 21*tan((log(x**n*c)*b + a)/2)
**4*b**4*n**4 + 90*tan((log(x**n*c)*b + a)/2)**4*b**2*n**2 - 24*tan((lo...
```

3.245 $\int \sec^2(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 85

$$\int \sec^2(a+b\log(cx^n))\,dx = \frac{4e^{2ia}x(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2-\frac{i}{bn}\right),\frac{1}{2}\left(4-\frac{i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{1+2ibn}$$

output
```
4*exp(2*I*a)*x*(c*x^n)^(2*I*b)*hypergeom([2, 1-1/2*I/b/n],[2-1/2*I/b/n],-e
xp(2*I*a)*(c*x^n)^(2*I*b))/(1+2*I*b*n)
```

Mathematica [A] (verified)

Time = 4.50 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.73

$$\int \sec^2(a+b\log(cx^n))\,dx = \frac{x\left(\frac{e^{2ia}(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(1,1-\frac{i}{2bn},2-\frac{i}{2bn},-e^{2i(a+b\log(cx^n))}\right)}{-i+2bn}-i\,\text{Hypergeometric2F1}\left(1,-\frac{i}{2bn},1-\frac{i}{2bn},-e^{2i(a}\right.\right.}{bn}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]]^2,x]
```

output
```
(x*((E^((2*I)*a)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[1, 1 - (I/2)/(b*n), 2
 - (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))])/(-I + 2*b*n) - I*Hypergeom
etric2F1[1, (-1/2*I)/(b*n), 1 - (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))
] + Tan[a + b*Log[c*x^n]]))/(b*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5014, 5016, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{5014}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\sec^2(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5016}$$

$$\frac{4e^{2ia}x(cx^n)^{-1/n}\int \frac{(cx^n)^{2ib+\frac{1}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^2}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{4e^{2ia}x(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2-\frac{i}{bn}\right),\frac{1}{2}\left(4-\frac{i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{n\left(\frac{1}{n}+2ib\right)}$$

input
```
Int[Sec[a + b*Log[c*x^n]]^2,x]
```

output
```
(4*E^((2*I)*a)*x*(c*x^n)^((2*I)*b)*Hypergeometric2F1[2, (2 - I/(b*n))/2, (
4 - I/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(((2*I)*b + n^(-1))*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5014
```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [F]

$$\int \sec \left(a +b \ln \left(c \,x^{n}\right)\right)^{2}d x$$

input
```
int(sec(a+b*ln(c*x^n))^2,x)
```

output
```
int(sec(a+b*ln(c*x^n))^2,x)
```

Fricas [F]

$$\int \sec^2\left(a+b\log\left(cx^n\right)\right)\,dx=\int \sec\left(b\log\left(cx^n\right)+a\right)^2\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
integral(sec(b*log(c*x^n) + a)^2, x)
```

Sympy [F]

$$\int \sec^2 (a + b\log (cx^n))\, dx = \int \sec^2 (a + b\log (cx^n))\, dx$$

input
```
integrate(sec(a+b*ln(c*x**n))**2,x)
```

output
```
Integral(sec(a + b*log(c*x**n))**2, x)
```

Maxima [F]

$$\int \sec^2 (a + b\log (cx^n))\, dx = \int \sec (b\log (cx^n) + a)^2\, dx$$

input
```
integrate(sec(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
2*(x*cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + x*cos(2*b*log(c))*sin(2*b*l
og(x^n) + 2*a) - (2*b^2*n^2*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) - 2*b^
2*n^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2*cos(2*b*log(c))^2 + b
^2*sin(2*b*log(c))^2)*n^2*cos(2*b*log(x^n) + 2*a)^2 + (b^2*cos(2*b*log(c))
^2 + b^2*sin(2*b*log(c))^2)*n^2*sin(2*b*log(x^n) + 2*a)^2 + b^2*n^2)*integ
rate((cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + cos(2*b*log(c))*sin(2*b*lo
g(x^n) + 2*a))/(2*b^2*n^2*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) - 2*b^2*
n^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2*cos(2*b*log(c))^2 + b^2
*sin(2*b*log(c))^2)*n^2*cos(2*b*log(x^n) + 2*a)^2 + (b^2*cos(2*b*log(c))^2
 + b^2*sin(2*b*log(c))^2)*n^2*sin(2*b*log(x^n) + 2*a)^2 + b^2*n^2), x))/(2
*b*n*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) + (b*cos(2*b*log(c))^2 + b*si
n(2*b*log(c))^2)*n*cos(2*b*log(x^n) + 2*a)^2 - 2*b*n*sin(2*b*log(c))*sin(2
*b*log(x^n) + 2*a) + (b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*sin(2*b
*log(x^n) + 2*a)^2 + b*n)
```

Giac [F]

$$\int \sec^2(a+b\log(cx^n))\,dx = \int \sec(b\log(cx^n)+a)^2\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
integrate(sec(b*log(c*x^n) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^2(a+b\log(cx^n))\,dx = \int \frac{1}{\cos(a+b\,\ln(c\,x^n))^2}\,dx$$

input
```
int(1/cos(a + b*log(c*x^n))^2,x)
```

output
```
int(1/cos(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int \sec^2(a+b\log(cx^n))\,dx = \text{too large to display}$$

input
```
int(sec(a+b*log(c*x^n))^2,x)
```

output
```
( - 14*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*tan((log(x**n*c)*b +
a)/2)**4*b**3*n**3*x - 2*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*tan
((log(x**n*c)*b + a)/2)**4*b*n*x + 28*cos(log(x**n*c)*b + a)*sin(log(x**n*
c)*b + a)*tan((log(x**n*c)*b + a)/2)**2*b**3*n**3*x + 4*cos(log(x**n*c)*b
+ a)*sin(log(x**n*c)*b + a)*tan((log(x**n*c)*b + a)/2)**2*b*n*x - 14*cos(l
og(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*b**3*n**3*x - 2*cos(log(x**n*c)*b
 + a)*sin(log(x**n*c)*b + a)*b*n*x - 28*cos(log(x**n*c)*b + a)*tan((log(x*
*n*c)*b + a)/2)**4*b**2*n**2*x + 56*cos(log(x**n*c)*b + a)*tan((log(x**n*c
)*b + a)/2)**2*b**2*n**2*x - 28*cos(log(x**n*c)*b + a)*b**2*n**2*x - 1792*
int(tan((log(x**n*c)*b + a)/2)**3/(14*tan((log(x**n*c)*b + a)/2)**6*b**4*n
**4 - 15*tan((log(x**n*c)*b + a)/2)**6*b**2*n**2 + tan((log(x**n*c)*b + a)
/2)**6 - 42*tan((log(x**n*c)*b + a)/2)**4*b**4*n**4 + 45*tan((log(x**n*c)*
b + a)/2)**4*b**2*n**2 - 3*tan((log(x**n*c)*b + a)/2)**4 + 42*tan((log(x**
n*c)*b + a)/2)**2*b**4*n**4 - 45*tan((log(x**n*c)*b + a)/2)**2*b**2*n**2 +
 3*tan((log(x**n*c)*b + a)/2)**2 - 14*b**4*n**4 + 15*b**2*n**2 - 1),x)*sin
(log(x**n*c)*b + a)**2*tan((log(x**n*c)*b + a)/2)**4*b**7*n**7 + 1920*int(
tan((log(x**n*c)*b + a)/2)**3/(14*tan((log(x**n*c)*b + a)/2)**6*b**4*n**4
- 15*tan((log(x**n*c)*b + a)/2)**6*b**2*n**2 + tan((log(x**n*c)*b + a)/2)*
*6 - 42*tan((log(x**n*c)*b + a)/2)**4*b**4*n**4 + 45*tan((log(x**n*c)*b +
a)/2)**4*b**2*n**2 - 3*tan((log(x**n*c)*b + a)/2)**4 + 42*tan((log(x**n...
```

3.246 $\int \frac{\sec^2(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 18

$$\int \frac{\sec^2(a+b\log(cx^n))}{x}\,dx = \frac{\tan(a+b\log(cx^n))}{bn}$$

output
```
tan(a+b*ln(c*x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(a+b\log(cx^n))}{x}\,dx = \frac{\tan(a+b\log(cx^n))}{bn}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]]^2/x,x]
```

output
```
Tan[a + b*Log[c*x^n]]/(b*n)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3039, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \sec^2(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(a+b\log(cx^n)+\frac{\pi}{2}\right)^2d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int 1d(-\tan(a+b\log(cx^n)))}{bn}$$

$$\Big\downarrow \text{24}$$

$$\frac{\tan(a+b\log(cx^n))}{bn}$$

input
```
Int[Sec[a + b*Log[c*x^n]]^2/x,x]
```

output
```
Tan[a + b*Log[c*x^n]]/(b*n)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\tan(a+b\ln(c\,x^{n}))}{bn}$
default	$\frac{\tan(a+b\ln(c\,x^{n}))}{bn}$
parallelrisch	$\frac{\sin(a+b\ln(c\,x^{n}))}{\cos(a+b\ln(c\,x^{n}))bn}$
risch	$\frac{2i}{bn\left(\left(x^{n}\right)^{2ib}c^{2ib}\mathrm{e}^{-b\pi\ \operatorname{csgn}(ix^{n})\operatorname{csgn}(ic\,x^{n})^{2}}\mathrm{e}^{b\pi\ \operatorname{csgn}(ix^{n})\operatorname{csgn}(ic\,x^{n})\operatorname{csgn}(ic)}\mathrm{e}^{b\pi\operatorname{csgn}(ic\,x^{n})^{3}}\mathrm{e}^{-b\pi\operatorname{csgn}(ic\,x^{n})^{2}\operatorname{csgn}(ic)}\mathrm{e}^{2ia}+\right.}$

input
```
int(sec(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

output
```
tan(a+b*ln(c*x^n))/b/n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.83

$$\int \frac{\sec^2(a+b\log(cx^n))}{x}\,dx=\frac{\sin(bn\log(x)+b\log(c)+a)}{bn\cos(bn\log(x)+b\log(c)+a)}$$

input
```
integrate(sec(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

output
```
sin(b*n*log(x) + b*log(c) + a)/(b*n*cos(b*n*log(x) + b*log(c) + a))
```

Sympy [F]

$$\int \frac{\sec^2(a+b\log(cx^n))}{x}\,dx=\int \frac{\sec^2(a+b\log(cx^n))}{x}\,dx$$

input
```
integrate(sec(a+b*ln(c*x**n))**2/x,x)
```

output
```
Integral(sec(a + b*log(c*x**n))**2/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(18) = 36.

Time = 0.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 9.17

$$\int \frac{\sec^2(a+b\log(cx^n))}{x}\,dx$$
$$=\frac{2\left(\cos(2b\log(x^n)+2a)\sin(2b\right.}{2bn\cos(2b\log(c))\cos(2b\log(x^n)+2a)+\left(b\cos(2b\log(c))^2+b\sin(2b\log(c))^2\right)n\cos(2b\log(x^n)+2}$$

input
```
integrate(sec(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

output
```
2*(cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + cos(2*b*log(c))*sin(2*b*log(x
^n) + 2*a))/(2*b*n*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) + (b*cos(2*b*lo
g(c))^2 + b*sin(2*b*log(c))^2)*n*cos(2*b*log(x^n) + 2*a)^2 - 2*b*n*sin(2*b
*log(c))*sin(2*b*log(x^n) + 2*a) + (b*cos(2*b*log(c))^2 + b*sin(2*b*log(c)
)^2)*n*sin(2*b*log(x^n) + 2*a)^2 + b*n)
```

Giac [F]

$$\int \frac{\sec^2(a+b\log(cx^n))}{x}\,dx=\int \frac{\sec(b\log(cx^n)+a)^2}{x}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

output
```
integrate(sec(b*log(c*x^n) + a)^2/x, x)
```

Mupad [B] (verification not implemented)

Time = 21.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.61

$$\int \frac{\sec^2(a+b\log(cx^n))}{x}\,dx=\frac{2\mathrm{i}}{b\,n\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}+1\right)}$$

input
```
int(1/(x*cos(a + b*log(c*x^n))^2),x)
```

output
```
2i/(b*n*(exp(a*2i)*(c*x^n)^(b*2i) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{\sec^2(a + b\log(cx^n))}{x}\,dx = \frac{\sin(\log(x^n c)\,b + a)}{\cos(\log(x^n c)\,b + a)\,bn}$$

input
```
int(sec(a+b*log(c*x^n))^2/x,x)
```

output
```
sin(log(x**n*c)*b + a)/(cos(log(x**n*c)*b + a)*b*n)
```

3.247 $\int \frac{\sec^2(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 87

$$\int \frac{\sec^2(a+b\log(cx^n))}{x^2}\,dx = -\frac{4e^{2ia}(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2+\frac{i}{bn}\right),\frac{1}{2}\left(4+\frac{i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{(1-2ibn)x}$$

output
```
-4*exp(2*I*a)*(c*x^n)^(2*I*b)*hypergeom([2, 1+1/2*I/b/n],[2+1/2*I/b/n],-ex
p(2*I*a)*(c*x^n)^(2*I*b))/(1-2*I*b*n)/x
```

Mathematica [A] (verified)

Time = 2.77 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.84

$$\int \frac{\sec^2(a+b\log(cx^n))}{x^2}\,dx = \frac{-e^{2ia}(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(1,1+\frac{i}{2bn},2+\frac{i}{2bn},-e^{2i(a+b\log(cx^n))}\right)+(1-2ibn)\left(\text{Hypergeometric2F}\right.}{bn(i+2bn)x}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]]^2/x^2,x]
```

output
```
(-(E^((2*I)*a)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[1, 1 + (I/2)/(b*n), 2 +
 (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))]) + (1 - (2*I)*b*n)*(Hypergeom
etric2F1[1, (I/2)/(b*n), 1 + (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] +
 I*Tan[a + b*Log[c*x^n]]))/(b*n*(I + 2*b*n)*x)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5020, 5016, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(a+b\log(cx^n))}{x^2}\,dx$$
$$\Big\downarrow \; 5020$$
$$\frac{(cx^n)^{\frac{1}{n}}\int (cx^n)^{-1-\frac{1}{n}}\sec^2(a+b\log(cx^n))\,d(cx^n)}{nx}$$
$$\Big\downarrow \; 5016$$
$$\frac{4e^{2ia}(cx^n)^{\frac{1}{n}}\int \frac{(cx^n)^{2ib-\frac{1}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^2}d(cx^n)}{nx}$$
$$\Big\downarrow \; 888$$
$$-\frac{4e^{2ia}(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2+\frac{i}{bn}\right),\frac{1}{2}\left(4+\frac{i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{x(1-2ibn)}$$

input
```
Int[Sec[a + b*Log[c*x^n]]^2/x^2,x]
```

output
```
(-4*E^((2*I)*a)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[2, (2 + I/(b*n))/2, (4
 + I/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((1 - (2*I)*b*n)*x)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\sec \left(a+b\ln \left(c\,x^{n}\right)\right)^{2}}{x^{2}}dx$$

input
```
int(sec(a+b*ln(c*x^n))^2/x^2,x)
```

output
```
int(sec(a+b*ln(c*x^n))^2/x^2,x)
```

Fricas [F]

$$\int \frac{\sec^2\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx=\int \frac{\sec\left(b\log\left(cx^n\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^2/x^2,x, algorithm="fricas")
```

output
```
integral(sec(b*log(c*x^n) + a)^2/x^2, x)
```

Sympy [F]

$$\int \frac{\sec^2\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx=\int \frac{\sec^2\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx$$

input
```
integrate(sec(a+b*ln(c*x**n))**2/x**2,x)
```

output
```
Integral(sec(a + b*log(c*x**n))**2/x**2, x)
```

Maxima [F]

$$\int \frac{\sec^2\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx=\int \frac{\sec\left(b\log\left(cx^n\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^2/x^2,x, algorithm="maxima")
```

output
```
2*((2*b^2*n^2*x*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) - 2*b^2*n^2*x*sin(
2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2*cos(2*b*log(c))^2 + b^2*sin(2*b
*log(c))^2)*n^2*x*cos(2*b*log(x^n) + 2*a)^2 + (b^2*cos(2*b*log(c))^2 + b^2
*sin(2*b*log(c))^2)*n^2*x*sin(2*b*log(x^n) + 2*a)^2 + b^2*n^2*x)*integrate
((cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + cos(2*b*log(c))*sin(2*b*log(x^
n) + 2*a))/(2*b^2*n^2*x^2*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) - 2*b^2*
n^2*x^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2*cos(2*b*log(c))^2 +
 b^2*sin(2*b*log(c))^2)*n^2*x^2*cos(2*b*log(x^n) + 2*a)^2 + (b^2*cos(2*b*l
og(c))^2 + b^2*sin(2*b*log(c))^2)*n^2*x^2*sin(2*b*log(x^n) + 2*a)^2 + b^2*
n^2*x^2), x) + cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + cos(2*b*log(c))*s
in(2*b*log(x^n) + 2*a))/(2*b*n*x*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) +
 (b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*x*cos(2*b*log(x^n) + 2*a)^2
 - 2*b*n*x*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b*cos(2*b*log(c))^2
+ b*sin(2*b*log(c))^2)*n*x*sin(2*b*log(x^n) + 2*a)^2 + b*n*x)
```

Giac [F]

$$\int \frac{\sec^2(a+b\log(cx^n))}{x^2}\,dx = \int \frac{\sec(b\log(cx^n)+a)^2}{x^2}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^2/x^2,x, algorithm="giac")
```

output
```
integrate(sec(b*log(c*x^n) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(a+b\log(cx^n))}{x^2}\,dx = \int \frac{1}{x^2\cos(a+b\ln(c\,x^n))^2}\,dx$$

input
```
int(1/(x^2*cos(a + b*log(c*x^n))^2),x)
```

output
```
int(1/(x^2*cos(a + b*log(c*x^n))^2), x)
```

Reduce [F]

$$\int \frac{\sec^2(a+b\log(cx^n))}{x^2}\,dx = \text{too large to display}$$

input
```
int(sec(a+b*log(c*x^n))^2/x^2,x)
```

output

```
( - 14*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*tan((log(x**n*c)*b +
a)/2)**4*b**3*n**3 - 2*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*tan((
log(x**n*c)*b + a)/2)**4*b*n + 28*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b
 + a)*tan((log(x**n*c)*b + a)/2)**2*b**3*n**3 + 4*cos(log(x**n*c)*b + a)*s
in(log(x**n*c)*b + a)*tan((log(x**n*c)*b + a)/2)**2*b*n - 14*cos(log(x**n*
c)*b + a)*sin(log(x**n*c)*b + a)*b**3*n**3 - 2*cos(log(x**n*c)*b + a)*sin(
log(x**n*c)*b + a)*b*n + 28*cos(log(x**n*c)*b + a)*tan((log(x**n*c)*b + a)
/2)**4*b**2*n**2 - 56*cos(log(x**n*c)*b + a)*tan((log(x**n*c)*b + a)/2)**2
*b**2*n**2 + 28*cos(log(x**n*c)*b + a)*b**2*n**2 + 1792*int(tan((log(x**n*
c)*b + a)/2)**3/(14*tan((log(x**n*c)*b + a)/2)**6*b**4*n**4*x**2 - 15*tan(
(log(x**n*c)*b + a)/2)**6*b**2*n**2*x**2 + tan((log(x**n*c)*b + a)/2)**6*x
**2 - 42*tan((log(x**n*c)*b + a)/2)**4*b**4*n**4*x**2 + 45*tan((log(x**n*c
)*b + a)/2)**4*b**2*n**2*x**2 - 3*tan((log(x**n*c)*b + a)/2)**4*x**2 + 42*
tan((log(x**n*c)*b + a)/2)**2*b**4*n**4*x**2 - 45*tan((log(x**n*c)*b + a)/
2)**2*b**2*n**2*x**2 + 3*tan((log(x**n*c)*b + a)/2)**2*x**2 - 14*b**4*n**4
*x**2 + 15*b**2*n**2*x**2 - x**2),x)*sin(log(x**n*c)*b + a)**2*tan((log(x*
*n*c)*b + a)/2)**4*b**7*n**7*x - 1920*int(tan((log(x**n*c)*b + a)/2)**3/(1
4*tan((log(x**n*c)*b + a)/2)**6*b**4*n**4*x**2 - 15*tan((log(x**n*c)*b + a
)/2)**6*b**2*n**2*x**2 + tan((log(x**n*c)*b + a)/2)**6*x**2 - 42*tan((log(
x**n*c)*b + a)/2)**4*b**4*n**4*x**2 + 45*tan((log(x**n*c)*b + a)/2)**4*...
```

3.248 $\int \frac{\sec^2(a+b\log(cx^n))}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 79

$$\int \frac{\sec^2(a+b\log(cx^n))}{x^3}\,dx = -\frac{2e^{2ia}(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(2,1+\frac{i}{bn},2+\frac{i}{bn},-e^{2ia}(cx^n)^{2ib}\right)}{(1-ibn)x^2}$$

output
```
-2*exp(2*I*a)*(c*x^n)^(2*I*b)*hypergeom([2, 1+I/b/n],[2+I/b/n],-exp(2*I*a)
*(c*x^n)^(2*I*b))/(1-I*b*n)/x^2
```

Mathematica [A] (verified)

Time = 2.68 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.90

$$\int \frac{\sec^2(a+b\log(cx^n))}{x^3}\,dx = \frac{-e^{2ia}(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(1,1+\frac{i}{bn},2+\frac{i}{bn},-e^{2i(a+b\log(cx^n))}\right)+(i+bn)\left(-i\,\text{Hypergeometric2F1}\right.}{bn(i+bn)x^2}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]]^2/x^3,x]
```

output
```
(-(E^((2*I)*a)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[1, 1 + I/(b*n), 2 + I/(
b*n), -E^((2*I)*(a + b*Log[c*x^n]))]) + (I + b*n)*((-I)*Hypergeometric2F1[
1, I/(b*n), 1 + I/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] + Tan[a + b*Log[c*
x^n]]))/(b*n*(I + b*n)*x^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5020, 5016, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(a+b\log(cx^n))}{x^3}\,dx$$

$$\Big\downarrow \text{5020}$$

$$\frac{(cx^n)^{2/n}\int (cx^n)^{-1-\frac{2}{n}}\sec^2(a+b\log(cx^n))\,d(cx^n)}{nx^2}$$

$$\Big\downarrow \text{5016}$$

$$\frac{4e^{2ia}(cx^n)^{2/n}\int \frac{(cx^n)^{2ib-\frac{2}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^2}d(cx^n)}{nx^2}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2e^{2ia}(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(2,1+\frac{i}{bn},2+\frac{i}{bn},-e^{2ia}(cx^n)^{2ib}\right)}{x^2(1-ibn)}$$

input
```
Int[Sec[a + b*Log[c*x^n]]^2/x^3,x]
```

output
```
(-2*E^((2*I)*a)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[2, 1 + I/(b*n), 2 + I/
(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((1 - I*b*n)*x^2)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\sec \left(a+b\ln \left(c\,x^{n}\right)\right)^{2}}{x^{3}}dx$$

input
```
int(sec(a+b*ln(c*x^n))^2/x^3,x)
```

output
```
int(sec(a+b*ln(c*x^n))^2/x^3,x)
```

Fricas [F]

$$\int \frac{\sec^2\left(a+b\log \left(cx^n\right)\right)}{x^3}\, dx=\int \frac{\sec \left(b \log \left(cx^{n}\right)+a\right)^{2}}{x^{3}}\, dx$$

input
```
integrate(sec(a+b*log(c*x^n))^2/x^3,x, algorithm="fricas")
```

output
```
integral(sec(b*log(c*x^n) + a)^2/x^3, x)
```

Sympy [F]

$$\int \frac{\sec^2(a+b\log(cx^n))}{x^3}\,dx = \int \frac{\sec^2(a+b\log(cx^n))}{x^3}\,dx$$

input
```
integrate(sec(a+b*ln(c*x**n))**2/x**3,x)
```

output
```
Integral(sec(a + b*log(c*x**n))**2/x**3, x)
```

Maxima [F]

$$\int \frac{\sec^2(a+b\log(cx^n))}{x^3}\,dx = \int \frac{\sec(b\log(cx^n)+a)^2}{x^3}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^2/x^3,x, algorithm="maxima")
```

output
```
2*(2*(2*b^2*n^2*x^2*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) - 2*b^2*n^2*x^
2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2*cos(2*b*log(c))^2 + b^2*s
in(2*b*log(c))^2)*n^2*x^2*cos(2*b*log(x^n) + 2*a)^2 + (b^2*cos(2*b*log(c))
^2 + b^2*sin(2*b*log(c))^2)*n^2*x^2*sin(2*b*log(x^n) + 2*a)^2 + b^2*n^2*x^
2)*integrate((cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + cos(2*b*log(c))*si
n(2*b*log(x^n) + 2*a))/(2*b^2*n^2*x^3*cos(2*b*log(c))*cos(2*b*log(x^n) + 2
*a) - 2*b^2*n^2*x^3*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b^2*cos(2*b
*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2*x^3*cos(2*b*log(x^n) + 2*a)^2 + (b
^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2*x^3*sin(2*b*log(x^n) + 2
*a)^2 + b^2*n^2*x^3), x) + cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + cos(2
*b*log(c))*sin(2*b*log(x^n) + 2*a))/(2*b*n*x^2*cos(2*b*log(c))*cos(2*b*log
(x^n) + 2*a) + (b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*x^2*cos(2*b*l
og(x^n) + 2*a)^2 - 2*b*n*x^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b*
cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*x^2*sin(2*b*log(x^n) + 2*a)^2 +
 b*n*x^2)
```

Giac [F]

$$\int \frac{\sec^2(a+b\log(cx^n))}{x^3}\,dx = \int \frac{\sec(b\log(cx^n)+a)^2}{x^3}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^2/x^3,x, algorithm="giac")
```

output
```
integrate(sec(b*log(c*x^n) + a)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(a+b\log(cx^n))}{x^3}\,dx = \int \frac{1}{x^3\cos(a+b\ln(c\,x^n))^2}\,dx$$

input
```
int(1/(x^3*cos(a + b*log(c*x^n))^2),x)
```

output
```
int(1/(x^3*cos(a + b*log(c*x^n))^2), x)
```

Reduce [F]

$$\int \frac{\sec^2(a+b\log(cx^n))}{x^3}\,dx = \text{too large to display}$$

input
```
int(sec(a+b*log(c*x^n))^2/x^3,x)
```

output
```
( - 14*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*tan((log(x**n*c)*b +
a)/2)**4*b**3*n**3 - 8*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)*tan((
log(x**n*c)*b + a)/2)**4*b*n + 28*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b
 + a)*tan((log(x**n*c)*b + a)/2)**2*b**3*n**3 + 16*cos(log(x**n*c)*b + a)*
sin(log(x**n*c)*b + a)*tan((log(x**n*c)*b + a)/2)**2*b*n - 14*cos(log(x**n
*c)*b + a)*sin(log(x**n*c)*b + a)*b**3*n**3 - 8*cos(log(x**n*c)*b + a)*sin
(log(x**n*c)*b + a)*b*n + 56*cos(log(x**n*c)*b + a)*tan((log(x**n*c)*b + a
)/2)**4*b**2*n**2 - 112*cos(log(x**n*c)*b + a)*tan((log(x**n*c)*b + a)/2)*
*2*b**2*n**2 + 56*cos(log(x**n*c)*b + a)*b**2*n**2 + 1792*int(tan((log(x**
n*c)*b + a)/2)**3/(7*tan((log(x**n*c)*b + a)/2)**6*b**4*n**4*x**3 - 30*tan
((log(x**n*c)*b + a)/2)**6*b**2*n**2*x**3 + 8*tan((log(x**n*c)*b + a)/2)**
6*x**3 - 21*tan((log(x**n*c)*b + a)/2)**4*b**4*n**4*x**3 + 90*tan((log(x**
n*c)*b + a)/2)**4*b**2*n**2*x**3 - 24*tan((log(x**n*c)*b + a)/2)**4*x**3 +
 21*tan((log(x**n*c)*b + a)/2)**2*b**4*n**4*x**3 - 90*tan((log(x**n*c)*b +
 a)/2)**2*b**2*n**2*x**3 + 24*tan((log(x**n*c)*b + a)/2)**2*x**3 - 7*b**4*
n**4*x**3 + 30*b**2*n**2*x**3 - 8*x**3),x)*sin(log(x**n*c)*b + a)**2*tan((
log(x**n*c)*b + a)/2)**4*b**7*n**7*x**2 - 7680*int(tan((log(x**n*c)*b + a)
/2)**3/(7*tan((log(x**n*c)*b + a)/2)**6*b**4*n**4*x**3 - 30*tan((log(x**n*
c)*b + a)/2)**6*b**2*n**2*x**3 + 8*tan((log(x**n*c)*b + a)/2)**6*x**3 - 21
*tan((log(x**n*c)*b + a)/2)**4*b**4*n**4*x**3 + 90*tan((log(x**n*c)*b +...
```

3.249 $\int x \sec^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 87

$$\int x\sec^3(a+b\log(cx^n))\,dx = \frac{8e^{3ia}x^2(cx^n)^{3ib}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3-\frac{2i}{bn}\right),\frac{1}{2}\left(5-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{2+3ibn}$$

output
```
8*exp(3*I*a)*x^2*(c*x^n)^(3*I*b)*hypergeom([3, 3/2-I/b/n],[5/2-I/b/n],-exp
(2*I*a)*(c*x^n)^(2*I*b))/(2+3*I*b*n)
```

Mathematica [A] (verified)

Time = 4.67 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.36

$$\int x\sec^3(a+b\log(cx^n))\,dx = \frac{x^2\left(2e^{ia}(2-ibn)(cx^n)^{ib}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{i}{bn},\frac{3}{2}-\frac{i}{bn},-e^{2i(a+b\log(cx^n))}\right)+\sec(a+b\log(cx^n))\left(\right.\right.}{2b^2n^2}$$

input
```
Integrate[x*Sec[a + b*Log[c*x^n]]^3,x]
```

output
```
(x^2*(2*E^(I*a)*(2 - I*b*n)*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 - I/(b*
n), 3/2 - I/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] + Sec[a + b*Log[c*x^n]]*
(-2 + b*n*Tan[a + b*Log[c*x^n]])))/(2*b^2*n^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5020, 5016, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sec^3(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \; 5020$$

$$\frac{x^2(cx^n)^{-2/n}\int (cx^n)^{\frac{2}{n}-1}\sec^3(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \; 5016$$

$$\frac{8e^{3ia}x^2(cx^n)^{-2/n}\int \frac{(cx^n)^{3ib+\frac{2}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^3}d(cx^n)}{n}$$

$$\Big\downarrow \; 888$$

$$\frac{8e^{3ia}x^2(cx^n)^{3ib}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3-\frac{2i}{bn}\right),\frac{1}{2}\left(5-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{2+3ibn}$$

input
```
Int[x*Sec[a + b*Log[c*x^n]]^3,x]
```

output
```
(8*E^((3*I)*a)*x^2*(c*x^n)^((3*I)*b)*Hypergeometric2F1[3, (3 - (2*I)/(b*n)
)/2, (5 - (2*I)/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(2 + (3*I)*b*
n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x\sec\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}dx$$

input
```
int(x*sec(a+b*ln(c*x^n))^3,x)
```

output
```
int(x*sec(a+b*ln(c*x^n))^3,x)
```

Fricas [F]

$$\int x\sec^3\left(a+b\log\left(cx^n\right)\right)\,dx=\int x\sec\left(b\log\left(cx^n\right)+a\right)^3\,dx$$

input
```
integrate(x*sec(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
integral(x*sec(b*log(c*x^n) + a)^3, x)
```

Sympy [F]

$$\int x\sec^3\left(a+b\log\left(cx^n\right)\right)\,dx=\int x\sec^3\left(a+b\log\left(cx^n\right)\right)dx$$

input
```
integrate(x*sec(a+b*ln(c*x**n))**3,x)
```

output
```
Integral(x*sec(a + b*log(c*x**n))**3, x)
```

Maxima [F]

$$\int x\sec^3\left(a+b\log\left(cx^n\right)\right)\,dx=\int x\sec\left(b\log\left(cx^n\right)+a\right)^3\,dx$$

input
```
integrate(x*sec(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
-((b*n*sin(b*log(c)) + 2*cos(b*log(c)))*x^2*cos(b*log(x^n) + a) + (b*n*cos
(b*log(c)) - 2*sin(b*log(c)))*x^2*sin(b*log(x^n) + a) + (((b*cos(3*b*log(c
))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(3*b*log(c)))*n + 2*cos(4*b*log(
c))*cos(3*b*log(c)) + 2*sin(4*b*log(c))*sin(3*b*log(c)))*x^2*cos(3*b*log(x
^n) + 3*a) - ((b*cos(b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(b*l
og(c)))*n - 2*cos(4*b*log(c))*cos(b*log(c)) - 2*sin(4*b*log(c))*sin(b*log(
c)))*x^2*cos(b*log(x^n) + a) - ((b*cos(4*b*log(c))*cos(3*b*log(c)) + b*sin
(4*b*log(c))*sin(3*b*log(c)))*n - 2*cos(3*b*log(c))*sin(4*b*log(c)) + 2*co
s(4*b*log(c))*sin(3*b*log(c)))*x^2*sin(3*b*log(x^n) + 3*a) + ((b*cos(4*b*l
og(c))*cos(b*log(c)) + b*sin(4*b*log(c))*sin(b*log(c)))*n + 2*cos(b*log(c)
)*sin(4*b*log(c)) - 2*cos(4*b*log(c))*sin(b*log(c)))*x^2*sin(b*log(x^n) +
a))*cos(4*b*log(x^n) + 4*a) - (2*((b*cos(2*b*log(c))*sin(3*b*log(c)) - b*c
os(3*b*log(c))*sin(2*b*log(c)))*n - 2*cos(3*b*log(c))*cos(2*b*log(c)) - 2*
sin(3*b*log(c))*sin(2*b*log(c)))*x^2*cos(2*b*log(x^n) + 2*a) - 2*((b*cos(3
*b*log(c))*cos(2*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c)))*n + 2*cos(
2*b*log(c))*sin(3*b*log(c)) - 2*cos(3*b*log(c))*sin(2*b*log(c)))*x^2*sin(2
*b*log(x^n) + 2*a) + (b*n*sin(3*b*log(c)) - 2*cos(3*b*log(c)))*x^2)*cos(3*
b*log(x^n) + 3*a) - 2*(((b*cos(b*log(c))*sin(2*b*log(c)) - b*cos(2*b*log(c
))*sin(b*log(c)))*n - 2*cos(2*b*log(c))*cos(b*log(c)) - 2*sin(2*b*log(c))*
sin(b*log(c)))*x^2*cos(b*log(x^n) + a) - ((b*cos(2*b*log(c))*cos(b*log(...
```

Giac [F]

$$\int x\sec^3(a+b\log(cx^n))\,dx = \int x\sec(b\log(cx^n)+a)^3\,dx$$

input
```
integrate(x*sec(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
integrate(x*sec(b*log(c*x^n) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sec^3(a+b\log(cx^n))\,dx = \int \frac{x}{\cos(a+b\ln(c\,x^n))^3}\,dx$$

input
```
int(x/cos(a + b*log(c*x^n))^3,x)
```

output
```
int(x/cos(a + b*log(c*x^n))^3, x)
```

Reduce [F]

$$\int x\sec^3(a+b\log(cx^n))\,dx = \text{too large to display}$$

input
```
int(x*sec(a+b*log(c*x^n))^3,x)
```

output
```
( - 11664*cos(log(x**n*c)*b + a)*int((tan((log(x**n*c)*b + a)/2)**3*x)/(27
*tan((log(x**n*c)*b + a)/2)**8*b**6*n**6 - 182*tan((log(x**n*c)*b + a)/2)*
*8*b**4*n**4 + 98*tan((log(x**n*c)*b + a)/2)**8*b**2*n**2 - 8*tan((log(x**
n*c)*b + a)/2)**8 - 108*tan((log(x**n*c)*b + a)/2)**6*b**6*n**6 + 728*tan(
(log(x**n*c)*b + a)/2)**6*b**4*n**4 - 392*tan((log(x**n*c)*b + a)/2)**6*b*
*2*n**2 + 32*tan((log(x**n*c)*b + a)/2)**6 + 162*tan((log(x**n*c)*b + a)/2
)**4*b**6*n**6 - 1092*tan((log(x**n*c)*b + a)/2)**4*b**4*n**4 + 588*tan((l
og(x**n*c)*b + a)/2)**4*b**2*n**2 - 48*tan((log(x**n*c)*b + a)/2)**4 - 108
*tan((log(x**n*c)*b + a)/2)**2*b**6*n**6 + 728*tan((log(x**n*c)*b + a)/2)*
*2*b**4*n**4 - 392*tan((log(x**n*c)*b + a)/2)**2*b**2*n**2 + 32*tan((log(x
**n*c)*b + a)/2)**2 + 27*b**6*n**6 - 182*b**4*n**4 + 98*b**2*n**2 - 8),x)*
sin(log(x**n*c)*b + a)**2*tan((log(x**n*c)*b + a)/2)**6*b**13*n**13 - 3801
6*cos(log(x**n*c)*b + a)*int((tan((log(x**n*c)*b + a)/2)**3*x)/(27*tan((lo
g(x**n*c)*b + a)/2)**8*b**6*n**6 - 182*tan((log(x**n*c)*b + a)/2)**8*b**4*
n**4 + 98*tan((log(x**n*c)*b + a)/2)**8*b**2*n**2 - 8*tan((log(x**n*c)*b +
 a)/2)**8 - 108*tan((log(x**n*c)*b + a)/2)**6*b**6*n**6 + 728*tan((log(x**
n*c)*b + a)/2)**6*b**4*n**4 - 392*tan((log(x**n*c)*b + a)/2)**6*b**2*n**2
+ 32*tan((log(x**n*c)*b + a)/2)**6 + 162*tan((log(x**n*c)*b + a)/2)**4*b**
6*n**6 - 1092*tan((log(x**n*c)*b + a)/2)**4*b**4*n**4 + 588*tan((log(x**n*
c)*b + a)/2)**4*b**2*n**2 - 48*tan((log(x**n*c)*b + a)/2)**4 - 108*tan(...
```

3.250 $\int \sec^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 85

$$\int \sec^3(a+b\log(cx^n))\,dx = \frac{8e^{3ia}x(cx^n)^{3ib}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3-\frac{i}{bn}\right),\frac{1}{2}\left(5-\frac{i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{1+3ibn}$$

output
```
8*exp(3*I*a)*x*(c*x^n)^(3*I*b)*hypergeom([3, 3/2-1/2*I/b/n],[5/2-1/2*I/b/n
],-exp(2*I*a)*(c*x^n)^(2*I*b))/(1+3*I*b*n)
```

Mathematica [A] (verified)

Time = 4.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.41

$$\int \sec^3(a+b\log(cx^n))\,dx = \frac{x\left(2e^{ia}(1-ibn)(cx^n)^{ib}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{i}{2bn},\frac{3}{2}-\frac{i}{2bn},-e^{2i(a+b\log(cx^n))}\right)+\sec(a+b\log(cx^n))\right.}{2b^2n^2}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]]^3,x]
```

output
```
(x*(2*E^(I*a)*(1 - I*b*n)*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 - (I/2)/(
b*n), 3/2 - (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] + Sec[a + b*Log[c*
x^n]]*(-1 + b*n*Tan[a + b*Log[c*x^n]])))/(2*b^2*n^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5014, 5016, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(a+b\log(cx^n))\,dx$$
$$\Big\downarrow \text{5014}$$
$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\sec^3(a+b\log(cx^n))\,d(cx^n)}{n}$$
$$\Big\downarrow \text{5016}$$
$$\frac{8e^{3ia}x(cx^n)^{-1/n}\int \frac{(cx^n)^{3ib+\frac{1}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^3}d(cx^n)}{n}$$
$$\Big\downarrow \text{888}$$
$$\frac{8e^{3ia}x(cx^n)^{3ib}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3-\frac{i}{bn}\right),\frac{1}{2}\left(5-\frac{i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{n\left(\frac{1}{n}+3ib\right)}$$

input
```
Int[Sec[a + b*Log[c*x^n]]^3,x]
```

output
```
(8*E^((3*I)*a)*x*(c*x^n)^((3*I)*b)*Hypergeometric2F1[3, (3 - I/(b*n))/2, (
5 - I/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(((3*I)*b + n^(-1))*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5014
```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [F]

$$\int \sec \left(a+b\ln \left(c\,x^{n}\right)\right)^{3}dx$$

input
```
int(sec(a+b*ln(c*x^n))^3,x)
```

output
```
int(sec(a+b*ln(c*x^n))^3,x)
```

Fricas [F]

$$\int \sec^3\left(a+b\log\left(cx^n\right)\right)\,dx=\int \sec\left(b\log\left(cx^n\right)+a\right)^3\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
integral(sec(b*log(c*x^n) + a)^3, x)
```

Sympy [F]

$$\int \sec^3(a + b\log(cx^n))\,dx = \int \sec^3(a + b\log(cx^n))\,dx$$

input
```
integrate(sec(a+b*ln(c*x**n))**3,x)
```

output
```
Integral(sec(a + b*log(c*x**n))**3, x)
```

Maxima [F]

$$\int \sec^3(a + b\log(cx^n))\,dx = \int \sec(b\log(cx^n) + a)^3\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
-((b*n*sin(b*log(c)) + cos(b*log(c)))*x*cos(b*log(x^n) + a) + (b*n*cos(b*l
og(c)) - sin(b*log(c)))*x*sin(b*log(x^n) + a) + (((b*cos(3*b*log(c))*sin(4
*b*log(c)) - b*cos(4*b*log(c))*sin(3*b*log(c)))*n + cos(4*b*log(c))*cos(3*
b*log(c)) + sin(4*b*log(c))*sin(3*b*log(c)))*x*cos(3*b*log(x^n) + 3*a) - (
(b*cos(b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(b*log(c)))*n - co
s(4*b*log(c))*cos(b*log(c)) - sin(4*b*log(c))*sin(b*log(c)))*x*cos(b*log(x
^n) + a) - ((b*cos(4*b*log(c))*cos(3*b*log(c)) + b*sin(4*b*log(c))*sin(3*b
*log(c)))*n - cos(3*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin(3*b*lo
g(c)))*x*sin(3*b*log(x^n) + 3*a) + ((b*cos(4*b*log(c))*cos(b*log(c)) + b*s
in(4*b*log(c))*sin(b*log(c)))*n + cos(b*log(c))*sin(4*b*log(c)) - cos(4*b*
log(c))*sin(b*log(c)))*x*sin(b*log(x^n) + a))*cos(4*b*log(x^n) + 4*a) - (2
*((b*cos(2*b*log(c))*sin(3*b*log(c)) - b*cos(3*b*log(c))*sin(2*b*log(c)))*
n - cos(3*b*log(c))*cos(2*b*log(c)) - sin(3*b*log(c))*sin(2*b*log(c)))*x*c
os(2*b*log(x^n) + 2*a) - 2*((b*cos(3*b*log(c))*cos(2*b*log(c)) + b*sin(3*b
*log(c))*sin(2*b*log(c)))*n + cos(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*lo
g(c))*sin(2*b*log(c)))*x*sin(2*b*log(x^n) + 2*a) + (b*n*sin(3*b*log(c)) -
cos(3*b*log(c)))*x)*cos(3*b*log(x^n) + 3*a) - 2*(((b*cos(b*log(c))*sin(2*b
*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)))*n - cos(2*b*log(c))*cos(b*log(
c)) - sin(2*b*log(c))*sin(b*log(c)))*x*cos(b*log(x^n) + a) - ((b*cos(2*b*l
og(c))*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c)))*n + cos(b*log(c...
```

Giac [F]

$$\int \sec^3(a+b\log(cx^n))\,dx = \int \sec(b\log(cx^n)+a)^3\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
integrate(sec(b*log(c*x^n) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^3(a+b\log(cx^n))\,dx = \int \frac{1}{\cos(a+b\,\ln(c\,x^n))^3}\,dx$$

input
```
int(1/cos(a + b*log(c*x^n))^3,x)
```

output
```
int(1/cos(a + b*log(c*x^n))^3, x)
```

Reduce [F]

$$\int \sec^3(a+b\log(cx^n))\,dx = \text{too large to display}$$

input
```
int(sec(a+b*log(c*x^n))^3,x)
```

output
```
( - 186624*cos(log(x**n*c)*b + a)*int(tan((log(x**n*c)*b + a)/2)**3/(216*t
an((log(x**n*c)*b + a)/2)**8*b**6*n**6 - 364*tan((log(x**n*c)*b + a)/2)**8
*b**4*n**4 + 49*tan((log(x**n*c)*b + a)/2)**8*b**2*n**2 - tan((log(x**n*c)
*b + a)/2)**8 - 864*tan((log(x**n*c)*b + a)/2)**6*b**6*n**6 + 1456*tan((lo
g(x**n*c)*b + a)/2)**6*b**4*n**4 - 196*tan((log(x**n*c)*b + a)/2)**6*b**2*
n**2 + 4*tan((log(x**n*c)*b + a)/2)**6 + 1296*tan((log(x**n*c)*b + a)/2)**
4*b**6*n**6 - 2184*tan((log(x**n*c)*b + a)/2)**4*b**4*n**4 + 294*tan((log(
x**n*c)*b + a)/2)**4*b**2*n**2 - 6*tan((log(x**n*c)*b + a)/2)**4 - 864*tan
((log(x**n*c)*b + a)/2)**2*b**6*n**6 + 1456*tan((log(x**n*c)*b + a)/2)**2*
b**4*n**4 - 196*tan((log(x**n*c)*b + a)/2)**2*b**2*n**2 + 4*tan((log(x**n*
c)*b + a)/2)**2 + 216*b**6*n**6 - 364*b**4*n**4 + 49*b**2*n**2 - 1),x)*sin
(log(x**n*c)*b + a)**2*tan((log(x**n*c)*b + a)/2)**6*b**13*n**13 - 152064*
cos(log(x**n*c)*b + a)*int(tan((log(x**n*c)*b + a)/2)**3/(216*tan((log(x**
n*c)*b + a)/2)**8*b**6*n**6 - 364*tan((log(x**n*c)*b + a)/2)**8*b**4*n**4
+ 49*tan((log(x**n*c)*b + a)/2)**8*b**2*n**2 - tan((log(x**n*c)*b + a)/2)*
*8 - 864*tan((log(x**n*c)*b + a)/2)**6*b**6*n**6 + 1456*tan((log(x**n*c)*b
 + a)/2)**6*b**4*n**4 - 196*tan((log(x**n*c)*b + a)/2)**6*b**2*n**2 + 4*ta
n((log(x**n*c)*b + a)/2)**6 + 1296*tan((log(x**n*c)*b + a)/2)**4*b**6*n**6
 - 2184*tan((log(x**n*c)*b + a)/2)**4*b**4*n**4 + 294*tan((log(x**n*c)*b +
 a)/2)**4*b**2*n**2 - 6*tan((log(x**n*c)*b + a)/2)**4 - 864*tan((log(x*...
```

3.251 $\int \frac{\sec^3(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \frac{\sec^3(a+b\log(cx^n))}{x}\,dx = \frac{\operatorname{arctanh}(\sin(a+b\log(cx^n)))}{2bn} + \frac{\sec(a+b\log(cx^n))\tan(a+b\log(cx^n))}{2bn}$$

output
```
1/2*arctanh(sin(a+b*ln(c*x^n)))/b/n+1/2*sec(a+b*ln(c*x^n))*tan(a+b*ln(c*x^
n))/b/n
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{\sec^3(a+b\log(cx^n))}{x}\,dx = \frac{\operatorname{arctanh}(\sin(a+b\log(cx^n)))}{2bn} + \frac{\sec(a+b\log(cx^n))\tan(a+b\log(cx^n))}{2bn}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]]^3/x,x]
```

output
```
ArcTanh[Sin[a + b*Log[c*x^n]]]/(2*b*n) + (Sec[a + b*Log[c*x^n]]*Tan[a + b*
Log[c*x^n]])/(2*b*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3039, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \sec^3(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(a+b\log(cx^n)+\frac{\pi}{2}\right)^3d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{1}{2}\int \sec(a+b\log(cx^n))d\log(cx^n)+\frac{\tan(a+b\log(cx^n))\sec(a+b\log(cx^n))}{2b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}\int \csc\left(a+b\log(cx^n)+\frac{\pi}{2}\right)d\log(cx^n)+\frac{\tan(a+b\log(cx^n))\sec(a+b\log(cx^n))}{2b}}{n}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{\text{arctanh}(\sin(a+b\log(cx^n)))}{2b}+\frac{\tan(a+b\log(cx^n))\sec(a+b\log(cx^n))}{2b}}{n}$$

input
```
Int[Sec[a + b*Log[c*x^n]]^3/x,x]
```

output
```
(ArcTanh[Sin[a + b*Log[c*x^n]]]/(2*b) + (Sec[a + b*Log[c*x^n]]*Tan[a + b*L
og[c*x^n]])/(2*b))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 5.54 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{\frac{\sec(a+b\ln(c\,x^n))\tan(a+b\ln(c\,x^n))}{2}+\frac{\ln(\sec(a+b\ln(c\,x^n))+\tan(a+b\ln(c\,x^n)))}{2}}{nb}$
default	$\frac{\frac{\sec(a+b\ln(c\,x^n))\tan(a+b\ln(c\,x^n))}{2}+\frac{\ln(\sec(a+b\ln(c\,x^n))+\tan(a+b\ln(c\,x^n)))}{2}}{nb}$
parallelrisch	$\frac{(-\cos(2b\ln(c\,x^n)+2a)-1)\ln\left(\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)-1\right)+(\cos(2b\ln(c\,x^n)+2a)+1)\ln\left(\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)+1\right)+2\sin(}{2bn(\cos(2b\ln(c\,x^n)+2a)+1)}$
risch	$-\frac{ic^{ib}(x^n)^{ib}\left(c^{2ib}(x^n)^{2ib}\mathrm{e}^{-\frac{3b\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}{2}}\mathrm{e}^{\frac{3b\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)}{2}}\mathrm{e}^{\frac{3b\pi\,\mathrm{csgn}(ic\,x^n)^3}{2}}\mathrm{e}^{-\frac{3b\pi\,\mathrm{csgn}(ic}{}}\right.}{bn\left((x^n)^{2ib}c^{2ib}\mathrm{e}^{-b\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}\mathrm{e}^{b\pi\ \mathrm{csgn}(ix^n)\,\mathrm{cs}}\right.}$

input
```
int(sec(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(1/2*sec(a+b*ln(c*x^n))*tan(a+b*ln(c*x^n))+1/2*ln(sec(a+b*ln(c*x^n))
+tan(a+b*ln(c*x^n))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.82

$$\int \frac{\sec^3(a+b\log(cx^n))}{x}\,dx = \frac{\cos(bn\log(x)+b\log(c)+a)^2\log(\sin(bn\log(x)+b\log(c)+a)+1) - \cos(bn\log(x)+b\log(c)+a)^2}{4\,bn\cos(bn\log(x)+b\log(c)+}$$

input
```
integrate(sec(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

output
```
1/4*(cos(b*n*log(x) + b*log(c) + a)^2*log(sin(b*n*log(x) + b*log(c) + a) +
 1) - cos(b*n*log(x) + b*log(c) + a)^2*log(-sin(b*n*log(x) + b*log(c) + a)
 + 1) + 2*sin(b*n*log(x) + b*log(c) + a))/(b*n*cos(b*n*log(x) + b*log(c) +
 a)^2)
```

Sympy [F]

$$\int \frac{\sec^3(a+b\log(cx^n))}{x}\,dx = \int \frac{\sec^3(a+b\log(cx^n))}{x}\,dx$$

input
```
integrate(sec(a+b*ln(c*x**n))**3/x,x)
```

output
```
Integral(sec(a + b*log(c*x**n))**3/x, x)
```

Maxima [F]

$$\int \frac{\sec^3(a+b\log(cx^n))}{x}\,dx = \int \frac{\sec(b\log(cx^n)+a)^3}{x}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

output
```
-(((cos(3*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(3*b*log(c)))*cos
(3*b*log(x^n) + 3*a) - (cos(b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*si
n(b*log(c)))*cos(b*log(x^n) + a) - (cos(4*b*log(c))*cos(3*b*log(c)) + sin(
4*b*log(c))*sin(3*b*log(c)))*sin(3*b*log(x^n) + 3*a) + (cos(4*b*log(c))*co
s(b*log(c)) + sin(4*b*log(c))*sin(b*log(c)))*sin(b*log(x^n) + a))*cos(4*b*
log(x^n) + 4*a) - (2*(cos(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*log(c))*si
n(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) - 2*(cos(3*b*log(c))*cos(2*b*log(c)
) + sin(3*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + sin(3*b*log
(c)))*cos(3*b*log(x^n) + 3*a) - 2*((cos(b*log(c))*sin(2*b*log(c)) - cos(2*
b*log(c))*sin(b*log(c)))*cos(b*log(x^n) + a) - (cos(2*b*log(c))*cos(b*log(
c)) + sin(2*b*log(c))*sin(b*log(c)))*sin(b*log(x^n) + a))*cos(2*b*log(x^n)
 + 2*a) - (4*b*n*cos(2*b*log(c))*cos(b*log(c))*cos(2*b*log(x^n) + 2*a) - 4
*b*n*cos(b*log(c))*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b*cos(4*b*lo
g(c))^2*cos(b*log(c)) + b*cos(b*log(c))*sin(4*b*log(c))^2)*n*cos(4*b*log(x
^n) + 4*a)^2 + 4*(b*cos(2*b*log(c))^2*cos(b*log(c)) + b*cos(b*log(c))*sin(
2*b*log(c))^2)*n*cos(2*b*log(x^n) + 2*a)^2 + (b*cos(4*b*log(c))^2*cos(b*lo
g(c)) + b*cos(b*log(c))*sin(4*b*log(c))^2)*n*sin(4*b*log(x^n) + 4*a)^2 + 4
*(b*cos(2*b*log(c))^2*cos(b*log(c)) + b*cos(b*log(c))*sin(2*b*log(c))^2)*n
*sin(2*b*log(x^n) + 2*a)^2 + b*n*cos(b*log(c)) + 2*(b*n*cos(4*b*log(c))*co
s(b*log(c)) + 2*(b*cos(4*b*log(c))*cos(2*b*log(c))*cos(b*log(c)) + b*co...
```

Giac [F]

$$\int \frac{\sec^3(a+b\log(cx^n))}{x}\,dx = \int \frac{\sec(b\log(cx^n)+a)^3}{x}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

output
```
integrate(sec(b*log(c*x^n) + a)^3/x, x)
```

Mupad [B] (verification not implemented)

Time = 23.17 (sec) , antiderivative size = 178, normalized size of antiderivative = 3.24

$$\int \frac{\sec^3(a+b\log(cx^n))}{x}\,dx = \frac{\ln\left(-\frac{1\mathrm{i}}{x}-\frac{\mathrm{e}^{a\,1\mathrm{i}}\,(c\,x^n)^{b\,1\mathrm{i}}}{x}\right)}{2\,b\,n}-\frac{\ln\left(\frac{1\mathrm{i}}{x}-\frac{\mathrm{e}^{a\,1\mathrm{i}}\,(c\,x^n)^{b\,1\mathrm{i}}}{x}\right)}{2\,b\,n}+\frac{\mathrm{e}^{a\,1\mathrm{i}}\,(c\,x^n)^{b\,1\mathrm{i}}\,2\mathrm{i}}{b\,n\,\left(2\,\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}}+\mathrm{e}^{a\,4\mathrm{i}}\,(c\,x^n)^{b\,4\mathrm{i}}+1\right)}-\frac{\mathrm{e}^{a\,1\mathrm{i}}\,(c\,x^n)^{b\,1\mathrm{i}}\,1\mathrm{i}}{b\,n\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}}+1\right)}$$

input
```
int(1/(x*cos(a + b*log(c*x^n))^3),x)
```

output
```
log(- 1i/x - (exp(a*1i)*(c*x^n)^(b*1i))/x)/(2*b*n) - log(1i/x - (exp(a*1i)
*(c*x^n)^(b*1i))/x)/(2*b*n) + (exp(a*1i)*(c*x^n)^(b*1i)*2i)/(b*n*(2*exp(a*
2i)*(c*x^n)^(b*2i) + exp(a*4i)*(c*x^n)^(b*4i) + 1)) - (exp(a*1i)*(c*x^n)^(
b*1i)*1i)/(b*n*(exp(a*2i)*(c*x^n)^(b*2i) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.51

$$\int \frac{\sec^3(a+b\log(cx^n))}{x}\,dx = \frac{-\log\left(\tan\left(\frac{\log(x^nc)b}{2}+\frac{a}{2}\right)-1\right)\sin\left(\log(x^nc)\,b+a\right)^2+\log\left(\tan\left(\frac{\log(x^nc)b}{2}+\frac{a}{2}\right)-1\right)+\log\left(\tan\left(\frac{\log(x^nc)b}{2}\right.\right.}{2bn\left(\sin\left(\log\left(x^nc\right)b+a\right)^2\right.}$$

input
```
int(sec(a+b*log(c*x^n))^3/x,x)
```

output
```
( - log(tan((log(x**n*c)*b + a)/2) - 1)*sin(log(x**n*c)*b + a)**2 + log(ta
n((log(x**n*c)*b + a)/2) - 1) + log(tan((log(x**n*c)*b + a)/2) + 1)*sin(lo
g(x**n*c)*b + a)**2 - log(tan((log(x**n*c)*b + a)/2) + 1) - sin(log(x**n*c
)*b + a))/(2*b*n*(sin(log(x**n*c)*b + a)**2 - 1))
```

3.252 $\int \frac{\sec^3(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 87

$$\int \frac{\sec^3(a+b\log(cx^n))}{x^2}\,dx = -\frac{8e^{3ia}(cx^n)^{3ib}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3+\frac{i}{bn}\right),\frac{1}{2}\left(5+\frac{i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{(1-3ibn)x}$$

output
```
-8*exp(3*I*a)*(c*x^n)^(3*I*b)*hypergeom([3, 3/2+1/2*I/b/n],[5/2+1/2*I/b/n]
,-exp(2*I*a)*(c*x^n)^(2*I*b))/(1-3*I*b*n)/x
```

Mathematica [A] (verified)

Time = 4.40 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.41

$$\int \frac{\sec^3(a+b\log(cx^n))}{x^2}\,dx = \frac{-2ie^{ia}(-i+bn)(cx^n)^{ib}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{i}{2bn},\frac{3}{2}+\frac{i}{2bn},-e^{2i(a+b\log(cx^n))}\right)+\sec(a+b\log(cx^n))}{2b^2n^2x}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]]^3/x^2,x]
```

output
```
((-2*I)*E^(I*a)*(-I + b*n)*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 + (I/2)/
(b*n), 3/2 + (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] + Sec[a + b*Log[c
*x^n]]*(1 + b*n*Tan[a + b*Log[c*x^n]]))/(2*b^2*n^2*x)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5020, 5016, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(a+b\log(cx^n))}{x^2}dx$$

$$\Big\downarrow \text{ 5020}$$

$$\frac{(cx^n)^{\frac{1}{n}}\int (cx^n)^{-1-\frac{1}{n}}\sec^3(a+b\log(cx^n))d(cx^n)}{nx}$$

$$\Big\downarrow \text{ 5016}$$

$$\frac{8e^{3ia}(cx^n)^{\frac{1}{n}}\int \frac{(cx^n)^{3ib-\frac{1}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^3}d(cx^n)}{nx}$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{8e^{3ia}(cx^n)^{3ib}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3+\frac{i}{bn}\right),\frac{1}{2}\left(5+\frac{i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{x(1-3ibn)}$$

input
```
Int[Sec[a + b*Log[c*x^n]]^3/x^2,x]
```

output
```
(-8*E^((3*I)*a)*(c*x^n)^((3*I)*b)*Hypergeometric2F1[3, (3 + I/(b*n))/2, (5
 + I/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((1 - (3*I)*b*n)*x)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\sec \left(a+b\ln \left(c\,x^{n}\right)\right)^{3}}{x^{2}}dx$$

input
```
int(sec(a+b*ln(c*x^n))^3/x^2,x)
```

output
```
int(sec(a+b*ln(c*x^n))^3/x^2,x)
```

Fricas [F]

$$\int \frac{\sec^3\left(a+b\log\left(cx^n\right)\right)}{x^2}\,dx=\int \frac{\sec\left(b\log\left(cx^n\right)+a\right)^3}{x^2}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^3/x^2,x, algorithm="fricas")
```

output
```
integral(sec(b*log(c*x^n) + a)^3/x^2, x)
```

Sympy [F]

$$\int \frac{\sec^3(a+b\log(cx^n))}{x^2}\,dx=\int \frac{\sec^3(a+b\log(cx^n))}{x^2}\,dx$$

input
```
integrate(sec(a+b*ln(c*x**n))**3/x**2,x)
```

output
```
Integral(sec(a + b*log(c*x**n))**3/x**2, x)
```

Maxima [F]

$$\int \frac{\sec^3(a+b\log(cx^n))}{x^2}\,dx=\int \frac{\sec(b\log(cx^n)+a)^3}{x^2}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^3/x^2,x, algorithm="maxima")
```

output
```
-((((b*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(3*b*log(c))
)*n - cos(4*b*log(c))*cos(3*b*log(c)) - sin(4*b*log(c))*sin(3*b*log(c)))*c
os(3*b*log(x^n) + 3*a) - ((b*cos(b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log
(c))*sin(b*log(c)))*n + cos(4*b*log(c))*cos(b*log(c)) + sin(4*b*log(c))*si
n(b*log(c)))*cos(b*log(x^n) + a) - ((b*cos(4*b*log(c))*cos(3*b*log(c)) + b
*sin(4*b*log(c))*sin(3*b*log(c)))*n + cos(3*b*log(c))*sin(4*b*log(c)) - co
s(4*b*log(c))*sin(3*b*log(c)))*sin(3*b*log(x^n) + 3*a) + ((b*cos(4*b*log(c
))*cos(b*log(c)) + b*sin(4*b*log(c))*sin(b*log(c)))*n - cos(b*log(c))*sin(
4*b*log(c)) + cos(4*b*log(c))*sin(b*log(c)))*sin(b*log(x^n) + a))*cos(4*b*
log(x^n) + 4*a) - (b*n*sin(3*b*log(c)) + 2*((b*cos(2*b*log(c))*sin(3*b*log
(c)) - b*cos(3*b*log(c))*sin(2*b*log(c)))*n + cos(3*b*log(c))*cos(2*b*log(
c)) + sin(3*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) - 2*((b*cos
(3*b*log(c))*cos(2*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c)))*n - cos(
2*b*log(c))*sin(3*b*log(c)) + cos(3*b*log(c))*sin(2*b*log(c)))*sin(2*b*log
(x^n) + 2*a) + cos(3*b*log(c)))*cos(3*b*log(x^n) + 3*a) - 2*(((b*cos(b*log
(c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)))*n + cos(2*b*log(c)
)*cos(b*log(c)) + sin(2*b*log(c))*sin(b*log(c)))*cos(b*log(x^n) + a) - ((b
*cos(2*b*log(c))*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c)))*n - cos(
b*log(c))*sin(2*b*log(c)) + cos(2*b*log(c))*sin(b*log(c)))*sin(b*log(x^n)
+ a))*cos(2*b*log(x^n) + 2*a) + (b*n*sin(b*log(c)) - cos(b*log(c)))*cos...
```

Giac [F]

$$\int \frac{\sec^3(a+b\log(cx^n))}{x^2}\,dx=\int \frac{\sec(b\log(cx^n)+a)^3}{x^2}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^3/x^2,x, algorithm="giac")
```

output
```
integrate(sec(b*log(c*x^n) + a)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(a+b\log(cx^n))}{x^2}\,dx=\int \frac{1}{x^2\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^3}\,dx$$

input
```
int(1/(x^2*cos(a + b*log(c*x^n))^3),x)
```

output
```
int(1/(x^2*cos(a + b*log(c*x^n))^3), x)
```

Reduce [F]

$$\int \frac{\sec^3(a+b\log(cx^n))}{x^2}\,dx=\text{too large to display}$$

input
```
int(sec(a+b*log(c*x^n))^3/x^2,x)
```

output
```
(186624*cos(log(x**n*c)*b + a)*int(tan((log(x**n*c)*b + a)/2)**3/(216*tan(
(log(x**n*c)*b + a)/2)**8*b**6*n**6*x**2 - 364*tan((log(x**n*c)*b + a)/2)*
*8*b**4*n**4*x**2 + 49*tan((log(x**n*c)*b + a)/2)**8*b**2*n**2*x**2 - tan(
(log(x**n*c)*b + a)/2)**8*x**2 - 864*tan((log(x**n*c)*b + a)/2)**6*b**6*n*
*6*x**2 + 1456*tan((log(x**n*c)*b + a)/2)**6*b**4*n**4*x**2 - 196*tan((log
(x**n*c)*b + a)/2)**6*b**2*n**2*x**2 + 4*tan((log(x**n*c)*b + a)/2)**6*x**
2 + 1296*tan((log(x**n*c)*b + a)/2)**4*b**6*n**6*x**2 - 2184*tan((log(x**n
*c)*b + a)/2)**4*b**4*n**4*x**2 + 294*tan((log(x**n*c)*b + a)/2)**4*b**2*n
**2*x**2 - 6*tan((log(x**n*c)*b + a)/2)**4*x**2 - 864*tan((log(x**n*c)*b +
 a)/2)**2*b**6*n**6*x**2 + 1456*tan((log(x**n*c)*b + a)/2)**2*b**4*n**4*x*
*2 - 196*tan((log(x**n*c)*b + a)/2)**2*b**2*n**2*x**2 + 4*tan((log(x**n*c)
*b + a)/2)**2*x**2 + 216*b**6*n**6*x**2 - 364*b**4*n**4*x**2 + 49*b**2*n**
2*x**2 - x**2),x)*sin(log(x**n*c)*b + a)**2*tan((log(x**n*c)*b + a)/2)**6*
b**13*n**13*x + 152064*cos(log(x**n*c)*b + a)*int(tan((log(x**n*c)*b + a)/
2)**3/(216*tan((log(x**n*c)*b + a)/2)**8*b**6*n**6*x**2 - 364*tan((log(x**
n*c)*b + a)/2)**8*b**4*n**4*x**2 + 49*tan((log(x**n*c)*b + a)/2)**8*b**2*n
**2*x**2 - tan((log(x**n*c)*b + a)/2)**8*x**2 - 864*tan((log(x**n*c)*b + a
)/2)**6*b**6*n**6*x**2 + 1456*tan((log(x**n*c)*b + a)/2)**6*b**4*n**4*x**2
 - 196*tan((log(x**n*c)*b + a)/2)**6*b**2*n**2*x**2 + 4*tan((log(x**n*c)*b
 + a)/2)**6*x**2 + 1296*tan((log(x**n*c)*b + a)/2)**4*b**6*n**6*x**2 - ...
```

3.253 $\int \frac{\sec^3(a+b\log(cx^n))}{x^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 87

$$\int \frac{\sec^3(a+b\log(cx^n))}{x^3} dx = -\frac{8e^{3ia}(cx^n)^{3ib} \text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3+\frac{2i}{bn}\right),\frac{1}{2}\left(5+\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{(2-3ibn)x^2}$$

output
```
-8*exp(3*I*a)*(c*x^n)^(3*I*b)*hypergeom([3, 3/2+I/b/n],[5/2+I/b/n],-exp(2*
I*a)*(c*x^n)^(2*I*b))/(2-3*I*b*n)/x^2
```

Mathematica [A] (verified)

Time = 4.55 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.37

$$\int \frac{\sec^3(a+b\log(cx^n))}{x^3} dx = \frac{-2ie^{ia}(-2i+bn)(cx^n)^{ib} \text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{i}{bn},\frac{3}{2}+\frac{i}{bn},-e^{2i(a+b\log(cx^n))}\right)+\sec(a+b\log(cx^n))}{2b^2n^2x^2}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]]^3/x^3,x]
```

output
```
((-2*I)*E^(I*a)*(-2*I + b*n)*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 + I/(b
*n), 3/2 + I/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] + Sec[a + b*Log[c*x^n]]
*(2 + b*n*Tan[a + b*Log[c*x^n]]))/(2*b^2*n^2*x^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5020, 5016, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(a+b\log(cx^n))}{x^3}\,dx$$

$$\Big\downarrow \text{5020}$$

$$\frac{(cx^n)^{2/n}\int (cx^n)^{-1-\frac{2}{n}}\sec^3(a+b\log(cx^n))\,d(cx^n)}{nx^2}$$

$$\Big\downarrow \text{5016}$$

$$\frac{8e^{3ia}(cx^n)^{2/n}\int \frac{(cx^n)^{3ib-\frac{2}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^3}d(cx^n)}{nx^2}$$

$$\Big\downarrow \text{888}$$

$$-\frac{8e^{3ia}(cx^n)^{3ib}\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3+\frac{2i}{bn}\right),\frac{1}{2}\left(5+\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{x^2(2-3ibn)}$$

input
```
Int[Sec[a + b*Log[c*x^n]]^3/x^3,x]
```

output
```
(-8*E^((3*I)*a)*(c*x^n)^((3*I)*b)*Hypergeometric2F1[3, (3 + (2*I)/(b*n))/2
, (5 + (2*I)/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 - (3*I)*b*n)
*x^2)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\sec\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}}{x^{3}}dx$$

input
```
int(sec(a+b*ln(c*x^n))^3/x^3,x)
```

output
```
int(sec(a+b*ln(c*x^n))^3/x^3,x)
```

Fricas [F]

$$\int \frac{\sec^3\left(a+b\log\left(cx^n\right)\right)}{x^3}\,dx=\int \frac{\sec\left(b\log\left(cx^n\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^3/x^3,x, algorithm="fricas")
```

output
```
integral(sec(b*log(c*x^n) + a)^3/x^3, x)
```

Sympy [F]

$$\int \frac{\sec^3(a + b\log(cx^n))}{x^3} dx = \int \frac{\sec^3(a + b\log(cx^n))}{x^3} dx$$

input
```
integrate(sec(a+b*ln(c*x**n))**3/x**3,x)
```

output
```
Integral(sec(a + b*log(c*x**n))**3/x**3, x)
```

Maxima [F]

$$\int \frac{\sec^3(a + b\log(cx^n))}{x^3} dx = \int \frac{\sec(b\log(cx^n) + a)^3}{x^3} dx$$

input
```
integrate(sec(a+b*log(c*x^n))^3/x^3,x, algorithm="maxima")
```

output
```
-((((b*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(3*b*log(c))
)*n - 2*cos(4*b*log(c))*cos(3*b*log(c)) - 2*sin(4*b*log(c))*sin(3*b*log(c)
))*cos(3*b*log(x^n) + 3*a) - ((b*cos(b*log(c))*sin(4*b*log(c)) - b*cos(4*b
*log(c))*sin(b*log(c)))*n + 2*cos(4*b*log(c))*cos(b*log(c)) + 2*sin(4*b*lo
g(c))*sin(b*log(c)))*cos(b*log(x^n) + a) - ((b*cos(4*b*log(c))*cos(3*b*log
(c)) + b*sin(4*b*log(c))*sin(3*b*log(c)))*n + 2*cos(3*b*log(c))*sin(4*b*lo
g(c)) - 2*cos(4*b*log(c))*sin(3*b*log(c)))*sin(3*b*log(x^n) + 3*a) + ((b*c
os(4*b*log(c))*cos(b*log(c)) + b*sin(4*b*log(c))*sin(b*log(c)))*n - 2*cos(
b*log(c))*sin(4*b*log(c)) + 2*cos(4*b*log(c))*sin(b*log(c)))*sin(b*log(x^n
) + a))*cos(4*b*log(x^n) + 4*a) - (b*n*sin(3*b*log(c)) + 2*((b*cos(2*b*log
(c))*sin(3*b*log(c)) - b*cos(3*b*log(c))*sin(2*b*log(c)))*n + 2*cos(3*b*lo
g(c))*cos(2*b*log(c)) + 2*sin(3*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n
) + 2*a) - 2*((b*cos(3*b*log(c))*cos(2*b*log(c)) + b*sin(3*b*log(c))*sin(2
*b*log(c)))*n - 2*cos(2*b*log(c))*sin(3*b*log(c)) + 2*cos(3*b*log(c))*sin(
2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + 2*cos(3*b*log(c)))*cos(3*b*log(x^n)
 + 3*a) - 2*(((b*cos(b*log(c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*l
og(c)))*n + 2*cos(2*b*log(c))*cos(b*log(c)) + 2*sin(2*b*log(c))*sin(b*log(
c)))*cos(b*log(x^n) + a) - ((b*cos(2*b*log(c))*cos(b*log(c)) + b*sin(2*b*l
og(c))*sin(b*log(c)))*n - 2*cos(b*log(c))*sin(2*b*log(c)) + 2*cos(2*b*log(
c))*sin(b*log(c)))*sin(b*log(x^n) + a))*cos(2*b*log(x^n) + 2*a) + (b*n*...
```

Giac [F]

$$\int \frac{\sec^3(a+b\log(cx^n))}{x^3}\,dx=\int \frac{\sec(b\log(cx^n)+a)^3}{x^3}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^3/x^3,x, algorithm="giac")
```

output
```
integrate(sec(b*log(c*x^n) + a)^3/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(a+b\log(cx^n))}{x^3}\,dx = \int \frac{1}{x^3\cos(a+b\ln(c\,x^n))^3}\,dx$$

input
```
int(1/(x^3*cos(a + b*log(c*x^n))^3),x)
```

output
```
int(1/(x^3*cos(a + b*log(c*x^n))^3), x)
```

Reduce [F]

$$\int \frac{\sec^3(a+b\log(cx^n))}{x^3}\,dx = \text{too large to display}$$

input
```
int(sec(a+b*log(c*x^n))^3/x^3,x)
```

output
```
(11664*cos(log(x**n*c)*b + a)*int(tan((log(x**n*c)*b + a)/2)**3/(27*tan((l
og(x**n*c)*b + a)/2)**8*b**6*n**6*x**3 - 182*tan((log(x**n*c)*b + a)/2)**8
*b**4*n**4*x**3 + 98*tan((log(x**n*c)*b + a)/2)**8*b**2*n**2*x**3 - 8*tan(
(log(x**n*c)*b + a)/2)**8*x**3 - 108*tan((log(x**n*c)*b + a)/2)**6*b**6*n*
*6*x**3 + 728*tan((log(x**n*c)*b + a)/2)**6*b**4*n**4*x**3 - 392*tan((log(
x**n*c)*b + a)/2)**6*b**2*n**2*x**3 + 32*tan((log(x**n*c)*b + a)/2)**6*x**
3 + 162*tan((log(x**n*c)*b + a)/2)**4*b**6*n**6*x**3 - 1092*tan((log(x**n*
c)*b + a)/2)**4*b**4*n**4*x**3 + 588*tan((log(x**n*c)*b + a)/2)**4*b**2*n*
*2*x**3 - 48*tan((log(x**n*c)*b + a)/2)**4*x**3 - 108*tan((log(x**n*c)*b +
 a)/2)**2*b**6*n**6*x**3 + 728*tan((log(x**n*c)*b + a)/2)**2*b**4*n**4*x**
3 - 392*tan((log(x**n*c)*b + a)/2)**2*b**2*n**2*x**3 + 32*tan((log(x**n*c)
*b + a)/2)**2*x**3 + 27*b**6*n**6*x**3 - 182*b**4*n**4*x**3 + 98*b**2*n**2
*x**3 - 8*x**3),x)*sin(log(x**n*c)*b + a)**2*tan((log(x**n*c)*b + a)/2)**6
*b**13*n**13*x**2 + 38016*cos(log(x**n*c)*b + a)*int(tan((log(x**n*c)*b +
a)/2)**3/(27*tan((log(x**n*c)*b + a)/2)**8*b**6*n**6*x**3 - 182*tan((log(x
**n*c)*b + a)/2)**8*b**4*n**4*x**3 + 98*tan((log(x**n*c)*b + a)/2)**8*b**2
*n**2*x**3 - 8*tan((log(x**n*c)*b + a)/2)**8*x**3 - 108*tan((log(x**n*c)*b
 + a)/2)**6*b**6*n**6*x**3 + 728*tan((log(x**n*c)*b + a)/2)**6*b**4*n**4*x
**3 - 392*tan((log(x**n*c)*b + a)/2)**6*b**2*n**2*x**3 + 32*tan((log(x**n*
c)*b + a)/2)**6*x**3 + 162*tan((log(x**n*c)*b + a)/2)**4*b**6*n**6*x**3...
```

3.254 $\int x \sec^4(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int x \sec^4(a+b\log(cx^n))\,dx = \frac{8e^{4ia}x^2(cx^n)^{4ib}\,\text{Hypergeometric2F1}\left(4,2-\frac{i}{bn},3-\frac{i}{bn},-e^{2ia}(cx^n)^{2ib}\right)}{1+2ibn}$$

output
```
8*exp(4*I*a)*x^2*(c*x^n)^(4*I*b)*hypergeom([4, 2-I/b/n],[3-I/b/n],-exp(2*I
*a)*(c*x^n)^(2*I*b))/(1+2*I*b*n)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 204 vs. 2(79) = 158.

Time = 9.04 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.58

$$\int x \sec^4(a+b\log(cx^n))\,dx = \frac{x^2\left(2e^{2ia}(i+bn)\,(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(1,1-\frac{i}{bn},2-\frac{i}{bn},-e^{2i(a+b\log(cx^n))}\right)-2i(1+b^2n^2)\,\text{Hyperge}\right.}{}$$

input
```
Integrate[x*Sec[a + b*Log[c*x^n]]^4,x]
```

output
```
(x^2*(2*E^((2*I)*a)*(I + b*n)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[1, 1 - I
/(b*n), 2 - I/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] - (2*I)*(1 + b^2*n^2)*
Hypergeometric2F1[1, (-I)/(b*n), 1 - I/(b*n), -E^((2*I)*(a + b*Log[c*x^n])
)] + Sec[a + b*Log[c*x^n]]^2*(-(b*n) + (1 + 2*b^2*n^2 + (1 + b^2*n^2)*Cos[
2*(a + b*Log[c*x^n])])*Tan[a + b*Log[c*x^n]])))/(3*b^3*n^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5020, 5016, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sec^4(a+b\log(cx^n))\,dx$$
$$\Big\downarrow \text{5020}$$
$$\frac{x^2(cx^n)^{-2/n}\int (cx^n)^{\frac{2}{n}-1}\sec^4(a+b\log(cx^n))\,d(cx^n)}{n}$$
$$\Big\downarrow \text{5016}$$
$$\frac{16e^{4ia}x^2(cx^n)^{-2/n}\int \frac{(cx^n)^{4ib+\frac{2}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^4}d(cx^n)}{n}$$
$$\Big\downarrow \text{888}$$
$$\frac{8e^{4ia}x^2(cx^n)^{-\frac{2}{n}+2\left(\frac{1}{n}+2ib\right)}\text{Hypergeometric2F1}\left(4,2-\frac{i}{bn},3-\frac{i}{bn},-e^{2ia}(cx^n)^{2ib}\right)}{1+2ibn}$$

input
```
Int[x*Sec[a + b*Log[c*x^n]]^4,x]
```

output
```
(8*E^((4*I)*a)*x^2*(c*x^n)^(2*((2*I)*b + n^(-1)) - 2/n)*Hypergeometric2F1[
4, 2 - I/(b*n), 3 - I/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(1 + (2*I)
*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x\sec \left(a+b\ln \left(c\,x^{n}\right)\right)^{4}dx$$

input
```
int(x*sec(a+b*ln(c*x^n))^4,x)
```

output
```
int(x*sec(a+b*ln(c*x^n))^4,x)
```

Fricas [F]

$$\int x\sec^4(a+b\log(cx^n))\,dx=\int x\sec(b\log(cx^n)+a)^4\,dx$$

input
```
integrate(x*sec(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

output
```
integral(x*sec(b*log(c*x^n) + a)^4, x)
```

Sympy [F]

$$\int x\sec^4(a+b\log(cx^n))\,dx=\int x\sec^4(a+b\log(cx^n))\,dx$$

input
```
integrate(x*sec(a+b*ln(c*x**n))**4,x)
```

output
```
Integral(x*sec(a + b*log(c*x**n))**4, x)
```

Maxima [F]

$$\int x\sec^4(a+b\log(cx^n))\,dx=\int x\sec(b\log(cx^n)+a)^4\,dx$$

input
```
integrate(x*sec(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

output
```
-4/3*(3*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*x^2*cos(4*b*log(x^n)
 + 4*a)^2 + 3*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*x^2*cos(2*b*lo
g(x^n) + 2*a)^2 + 3*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*x^2*sin(
4*b*log(x^n) + 4*a)^2 + 3*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*x^
2*sin(2*b*log(x^n) + 2*a)^2 + (b*n*cos(2*b*log(c)) - sin(2*b*log(c)))*x^2*
cos(2*b*log(x^n) + 2*a) - (b*n*sin(2*b*log(c)) + cos(2*b*log(c)))*x^2*sin(
2*b*log(x^n) + 2*a) + (((b*cos(6*b*log(c))*cos(4*b*log(c)) + b*sin(6*b*log
(c))*sin(4*b*log(c)))*n - cos(4*b*log(c))*sin(6*b*log(c)) + cos(6*b*log(c)
)*sin(4*b*log(c)))*x^2*cos(4*b*log(x^n) + 4*a) - (3*(b^2*cos(2*b*log(c))*s
in(6*b*log(c)) - b^2*cos(6*b*log(c))*sin(2*b*log(c)))*n^2 - (b*cos(6*b*log
(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n + 2*cos(2*b*lo
g(c))*sin(6*b*log(c)) - 2*cos(6*b*log(c))*sin(2*b*log(c)))*x^2*cos(2*b*log
(x^n) + 2*a) + ((b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin
(4*b*log(c)))*n + cos(6*b*log(c))*cos(4*b*log(c)) + sin(6*b*log(c))*sin(4*
b*log(c)))*x^2*sin(4*b*log(x^n) + 4*a) + (3*(b^2*cos(6*b*log(c))*cos(2*b*l
og(c)) + b^2*sin(6*b*log(c))*sin(2*b*log(c)))*n^2 + (b*cos(2*b*log(c))*sin
(6*b*log(c)) - b*cos(6*b*log(c))*sin(2*b*log(c)))*n + 2*cos(6*b*log(c))*co
s(2*b*log(c)) + 2*sin(6*b*log(c))*sin(2*b*log(c)))*x^2*sin(2*b*log(x^n) +
2*a) - (b^2*n^2*sin(6*b*log(c)) + sin(6*b*log(c)))*x^2)*cos(6*b*log(x^n) +
 6*a) - (3*(3*(b^2*cos(2*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c)...
```

Giac [F]

$$\int x\sec^4\left(a+b\log\left(cx^n\right)\right)\,dx=\int x\sec\left(b\log\left(cx^n\right)+a\right)^4\,dx$$

input
```
integrate(x*sec(a+b*log(c*x^n))^4,x, algorithm="giac")
```

output
```
integrate(x*sec(b*log(c*x^n) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sec^4(a+b\log(cx^n))\,dx=\int \frac{x}{\cos(a+b\ln(c\,x^n))^4}\,dx$$

input
```
int(x/cos(a + b*log(c*x^n))^4,x)
```

output
```
int(x/cos(a + b*log(c*x^n))^4, x)
```

Reduce [F]

$$\int x\sec^4(a+b\log(cx^n))\,dx=\text{too large to display}$$

input
```
int(x*sec(a+b*log(c*x^n))^4,x)
```

output
```
( - 996*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*
b + a)/2)**8*b**7*n**7*x**2 - 3076*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*
b + a)**3*tan((log(x**n*c)*b + a)/2)**8*b**5*n**5*x**2 - 2144*cos(log(x**n
*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**8*b**3*n*
*3*x**2 - 64*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**
n*c)*b + a)/2)**8*b*n*x**2 + 3984*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b
 + a)**3*tan((log(x**n*c)*b + a)/2)**6*b**7*n**7*x**2 + 12304*cos(log(x**n
*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**6*b**5*n*
*5*x**2 + 8576*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x
**n*c)*b + a)/2)**6*b**3*n**3*x**2 + 256*cos(log(x**n*c)*b + a)*sin(log(x*
*n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**6*b*n*x**2 - 5976*cos(log(x**n
*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**4*b**7*n*
*7*x**2 - 18456*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(
x**n*c)*b + a)/2)**4*b**5*n**5*x**2 - 12864*cos(log(x**n*c)*b + a)*sin(log
(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**4*b**3*n**3*x**2 - 384*cos(
log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**4
*b*n*x**2 + 3984*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log
(x**n*c)*b + a)/2)**2*b**7*n**7*x**2 + 12304*cos(log(x**n*c)*b + a)*sin(lo
g(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**2*b**5*n**5*x**2 + 8576*co
s(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/...
```

3.255 $\int \sec^4(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 85

$$\int \sec^4(a+b\log(cx^n))\,dx = \frac{16e^{4ia}x(cx^n)^{4ib}\,\text{Hypergeometric2F1}\left(4,\frac{1}{2}\left(4-\frac{i}{bn}\right),\frac{1}{2}\left(6-\frac{i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{1+4ibn}$$

output
```
16*exp(4*I*a)*x*(c*x^n)^(4*I*b)*hypergeom([4, 2-1/2*I/b/n],[3-1/2*I/b/n],-
exp(2*I*a)*(c*x^n)^(2*I*b))/(1+4*I*b*n)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 213 vs. 2(85) = 170.

Time = 7.50 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.51

$$\int \sec^4(a+b\log(cx^n))\,dx = \frac{x\Big(2e^{2ia}(i+2bn)(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(1,1-\frac{i}{2bn},2-\frac{i}{2bn},-e^{2i(a+b\log(cx^n))}\right)-2i(1+4b^2n^2)\,\text{Hype}}{}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]]^4,x]
```

output
```
(x*(2*E^((2*I)*a)*(I + 2*b*n)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[1, 1 - (
I/2)/(b*n), 2 - (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] - (2*I)*(1 + 4
*b^2*n^2)*Hypergeometric2F1[1, (-1/2*I)/(b*n), 1 - (I/2)/(b*n), -E^((2*I)*
(a + b*Log[c*x^n]))] + Sec[a + b*Log[c*x^n]]^2*(-2*b*n + (1 + 8*b^2*n^2 +
(1 + 4*b^2*n^2)*Cos[2*(a + b*Log[c*x^n])])*Tan[a + b*Log[c*x^n]])))/(12*b^
3*n^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5014, 5016, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{5014}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\sec^4(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5016}$$

$$\frac{16e^{4ia}x(cx^n)^{-1/n}\int \frac{(cx^n)^{4ib+\frac{1}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^4}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{16e^{4ia}x(cx^n)^{4ib}\,\text{Hypergeometric2F1}\left(4,\frac{1}{2}\left(4-\frac{i}{bn}\right),\frac{1}{2}\left(6-\frac{i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{n\left(\frac{1}{n}+4ib\right)}$$

input
```
Int[Sec[a + b*Log[c*x^n]]^4,x]
```

output
```
(16*E^((4*I)*a)*x*(c*x^n)^((4*I)*b)*Hypergeometric2F1[4, (4 - I/(b*n))/2,
(6 - I/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(((4*I)*b + n^(-1))*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5014
```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [F]

$$\int \sec \left(a+b\ln \left(c\,x^{n}\right)\right)^{4}dx$$

input
```
int(sec(a+b*ln(c*x^n))^4,x)
```

output
```
int(sec(a+b*ln(c*x^n))^4,x)
```

Fricas [F]

$$\int \sec^4(a + b\log(cx^n))\,dx = \int \sec(b\log(cx^n) + a)^4\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

output
```
integral(sec(b*log(c*x^n) + a)^4, x)
```

Sympy [F]

$$\int \sec^4(a + b\log(cx^n))\,dx = \int \sec^4(a + b\log(cx^n))\,dx$$

input
```
integrate(sec(a+b*ln(c*x**n))**4,x)
```

output
```
Integral(sec(a + b*log(c*x**n))**4, x)
```

Maxima [F]

$$\int \sec^4(a + b\log(cx^n))\,dx = \int \sec(b\log(cx^n) + a)^4\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

output
```
-1/3*(6*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*x*cos(4*b*log(x^n) +
 4*a)^2 + 6*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*x*cos(2*b*log(x^
n) + 2*a)^2 + 6*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*x*sin(4*b*lo
g(x^n) + 4*a)^2 + 6*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*x*sin(2*
b*log(x^n) + 2*a)^2 + (2*b*n*cos(2*b*log(c)) - sin(2*b*log(c)))*x*cos(2*b*
log(x^n) + 2*a) - (2*b*n*sin(2*b*log(c)) + cos(2*b*log(c)))*x*sin(2*b*log(
x^n) + 2*a) + ((2*(b*cos(6*b*log(c))*cos(4*b*log(c)) + b*sin(6*b*log(c))*s
in(4*b*log(c)))*n - cos(4*b*log(c))*sin(6*b*log(c)) + cos(6*b*log(c))*sin(
4*b*log(c)))*x*cos(4*b*log(x^n) + 4*a) - 2*(6*(b^2*cos(2*b*log(c))*sin(6*b
*log(c)) - b^2*cos(6*b*log(c))*sin(2*b*log(c)))*n^2 - (b*cos(6*b*log(c))*c
os(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n + cos(2*b*log(c))*si
n(6*b*log(c)) - cos(6*b*log(c))*sin(2*b*log(c)))*x*cos(2*b*log(x^n) + 2*a)
 + (2*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*log(c
)))*n + cos(6*b*log(c))*cos(4*b*log(c)) + sin(6*b*log(c))*sin(4*b*log(c)))
*x*sin(4*b*log(x^n) + 4*a) + 2*(6*(b^2*cos(6*b*log(c))*cos(2*b*log(c)) + b
^2*sin(6*b*log(c))*sin(2*b*log(c)))*n^2 + (b*cos(2*b*log(c))*sin(6*b*log(c
)) - b*cos(6*b*log(c))*sin(2*b*log(c)))*n + cos(6*b*log(c))*cos(2*b*log(c)
) + sin(6*b*log(c))*sin(2*b*log(c)))*x*sin(2*b*log(x^n) + 2*a) - (4*b^2*n^
2*sin(6*b*log(c)) + sin(6*b*log(c)))*x)*cos(6*b*log(x^n) + 6*a) - (3*(12*(
b^2*cos(2*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(2*b*log(c...
```

Giac [F]

$$\int \sec^4(a+b\log(cx^n))\,dx=\int \sec(b\log(cx^n)+a)^4\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^4,x, algorithm="giac")
```

output
```
integrate(sec(b*log(c*x^n) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^4(a+b\log(cx^n))\,dx = \int \frac{1}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^4}\,dx$$

input
```
int(1/cos(a + b*log(c*x^n))^4,x)
```

output
```
int(1/cos(a + b*log(c*x^n))^4, x)
```

Reduce [F]

$$\int \sec^4(a+b\log(cx^n))\,dx = \text{too large to display}$$

input
```
int(sec(a+b*log(c*x^n))^4,x)
```

output
```
( - 3984*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)
*b + a)/2)**8*b**7*n**7*x - 3076*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b
+ a)**3*tan((log(x**n*c)*b + a)/2)**8*b**5*n**5*x - 536*cos(log(x**n*c)*b
+ a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**8*b**3*n**3*x -
 4*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a
)/2)**8*b*n*x + 15936*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan
((log(x**n*c)*b + a)/2)**6*b**7*n**7*x + 12304*cos(log(x**n*c)*b + a)*sin(
log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**6*b**5*n**5*x + 2144*cos
(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**
6*b**3*n**3*x + 16*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((l
og(x**n*c)*b + a)/2)**6*b*n*x - 23904*cos(log(x**n*c)*b + a)*sin(log(x**n*
c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**4*b**7*n**7*x - 18456*cos(log(x**
n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**4*b**5*n
**5*x - 3216*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**
n*c)*b + a)/2)**4*b**3*n**3*x - 24*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*
b + a)**3*tan((log(x**n*c)*b + a)/2)**4*b*n*x + 15936*cos(log(x**n*c)*b +
a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**2*b**7*n**7*x + 1
2304*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b +
 a)/2)**2*b**5*n**5*x + 2144*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)
**3*tan((log(x**n*c)*b + a)/2)**2*b**3*n**3*x + 16*cos(log(x**n*c)*b + ...
```

3.256 $\int \frac{\sec^4(a+b\log(cx^n))}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{\sec^4(a+b\log(cx^n))}{x} dx = \frac{\tan(a+b\log(cx^n))}{bn} + \frac{\tan^3(a+b\log(cx^n))}{3bn}$$

output
```
tan(a+b*ln(c*x^n))/b/n+1/3*tan(a+b*ln(c*x^n))^3/b/n
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{\sec^4(a+b\log(cx^n))}{x} dx = \frac{\tan(a+b\log(cx^n)) + \frac{1}{3}\tan^3(a+b\log(cx^n))}{bn}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]]^4/x,x]
```

output
```
(Tan[a + b*Log[c*x^n]] + Tan[a + b*Log[c*x^n]]^3/3)/(b*n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3039, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(a+b\log(cx^n))}{x}dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \sec^4(a+b\log(cx^n))d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \csc\left(a+b\log(cx^n)+\frac{\pi}{2}\right)^4d\log(cx^n)}{n}$$
$$\Big\downarrow \text{4254}$$
$$-\frac{\int \left(\tan^2(a+b\log(cx^n))+1\right)d(-\tan(a+b\log(cx^n)))}{bn}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{-\frac{1}{3}\tan^3(a+b\log(cx^n))-\tan(a+b\log(cx^n))}{bn}$$

input
```
Int[Sec[a + b*Log[c*x^n]]^4/x,x]
```

output
```
-((-Tan[a + b*Log[c*x^n]] - Tan[a + b*Log[c*x^n]]^3/3)/(b*n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 17.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result
derivativedivides	$-\frac{\left(-\frac{2}{3}-\frac{\sec(a+b\ln(c\,x^n))^2}{3}\right)\tan(a+b\ln(c\,x^n))}{nb}$
default	$-\frac{\left(-\frac{2}{3}-\frac{\sec(a+b\ln(c\,x^n))^2}{3}\right)\tan(a+b\ln(c\,x^n))}{nb}$
parallelrisch	$\frac{-6\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^5+4\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^3-6\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)}{3bn\left(\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)-1\right)^3\left(\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)+1\right)^3}$
risch	$\frac{4i\left(3(x^n)^{2ib}c^{2ib}e^{-b\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}e^{b\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)}e^{b\pi\,\mathrm{csgn}(ic\,x^n)^3}e^{-b\pi\,\mathrm{csgn}(ic\,x^n)^2\,\mathrm{csgn}(ic)}e^{2ia}\right.}{3bn\left((x^n)^{2ib}c^{2ib}e^{-b\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}e^{b\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)}e^{b\pi\,\mathrm{csgn}(ic\,x^n)^3}e^{-b\pi\,\mathrm{csgn}(ic\,x^n)^2\,\mathrm{csgn}(ic)}e^{2ia}+\right.}$

input
```
int(sec(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
```

output
```
-1/n/b*(-2/3-1/3*sec(a+b*ln(c*x^n))^2)*tan(a+b*ln(c*x^n))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

$$\int \frac{\sec^4(a+b\log(cx^n))}{x}\,dx = \frac{\left(2\cos(bn\log(x)+b\log(c)+a)^2+1\right)\sin(bn\log(x)+b\log(c)+a)}{3\,bn\cos(bn\log(x)+b\log(c)+a)^3}$$

input
```
integrate(sec(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

output
```
1/3*(2*cos(b*n*log(x) + b*log(c) + a)^2 + 1)*sin(b*n*log(x) + b*log(c) + a
)/(b*n*cos(b*n*log(x) + b*log(c) + a)^3)
```

Sympy [F]

$$\int \frac{\sec^4(a+b\log(cx^n))}{x}\,dx = \int \frac{\sec^4(a+b\log(cx^n))}{x}\,dx$$

input
```
integrate(sec(a+b*ln(c*x**n))**4/x,x)
```

output
```
Integral(sec(a + b*log(c*x**n))**4/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1323 vs. 2(40) = 80.

Time = 0.14 (sec) , antiderivative size = 1323, normalized size of antiderivative = 31.50

$$\int \frac{\sec^4(a+b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

input
```
integrate(sec(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

output
```
4/3*((3*(cos(2*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(2*b*log(c))
)*cos(2*b*log(x^n) + 2*a) - 3*(cos(6*b*log(c))*cos(2*b*log(c)) + sin(6*b*l
og(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + sin(6*b*log(c)))*cos(6*b
*log(x^n) + 6*a) + 3*(3*(cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))
*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) - 3*(cos(4*b*log(c))*cos(2*b*log
(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) + sin(4*b*
log(c)))*cos(4*b*log(x^n) + 4*a) + (3*(cos(6*b*log(c))*cos(2*b*log(c)) + s
in(6*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 3*(cos(2*b*log(c
))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2
*a) + cos(6*b*log(c)))*sin(6*b*log(x^n) + 6*a) + 3*(3*(cos(4*b*log(c))*cos
(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) +
3*(cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))*sin(
2*b*log(x^n) + 2*a) + cos(4*b*log(c)))*sin(4*b*log(x^n) + 4*a))/((b*cos(6*
b*log(c))^2 + b*sin(6*b*log(c))^2)*n*cos(6*b*log(x^n) + 6*a)^2 + 9*(b*cos(
4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*cos(4*b*log(x^n) + 4*a)^2 + 6*b*n*c
os(2*b*log(c))*cos(2*b*log(x^n) + 2*a) + 9*(b*cos(2*b*log(c))^2 + b*sin(2*
b*log(c))^2)*n*cos(2*b*log(x^n) + 2*a)^2 + (b*cos(6*b*log(c))^2 + b*sin(6*
b*log(c))^2)*n*sin(6*b*log(x^n) + 6*a)^2 + 9*(b*cos(4*b*log(c))^2 + b*sin(
4*b*log(c))^2)*n*sin(4*b*log(x^n) + 4*a)^2 - 6*b*n*sin(2*b*log(c))*sin(2*b
*log(x^n) + 2*a) + 9*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*sin(...
```

Giac [F]

$$\int \frac{\sec^4(a+b\log(cx^n))}{x}\,dx = \int \frac{\sec(b\log(cx^n)+a)^4}{x}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

output
```
integrate(sec(b*log(c*x^n) + a)^4/x, x)
```

Mupad [B] (verification not implemented)

Time = 30.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \frac{\sec^4(a+b\log(cx^n))}{x}\,dx = \frac{4\left(e^{a\,2i}\,(c\,x^n)^{b\,2i}\,3i+1i\right)}{3\,b\,n\left(e^{a\,2i}\,(c\,x^n)^{b\,2i}+1\right)^3}$$

input
```
int(1/(x*cos(a + b*log(c*x^n))^4),x)
```

output
```
(4*(exp(a*2i)*(c*x^n)^(b*2i)*3i + 1i))/(3*b*n*(exp(a*2i)*(c*x^n)^(b*2i) +
1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.57

$$\int \frac{\sec^4(a+b\log(cx^n))}{x}\,dx = \frac{\sin(\log(x^n c)\,b+a)\left(2\sin(\log(x^n c)\,b+a)^2-3\right)}{3\cos(\log(x^n c)\,b+a)\,bn\left(\sin(\log(x^n c)\,b+a)^2-1\right)}$$

input
```
int(sec(a+b*log(c*x^n))^4/x,x)
```

output
```
(sin(log(x**n*c)*b + a)*(2*sin(log(x**n*c)*b + a)**2 - 3))/(3*cos(log(x**n
*c)*b + a)*b*n*(sin(log(x**n*c)*b + a)**2 - 1))
```

3.257 $\int \frac{\sec^4(a+b\log(cx^n))}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 87

$$\int \frac{\sec^4(a+b\log(cx^n))}{x^2}\,dx = -\frac{16e^{4ia}(cx^n)^{4ib}\,\text{Hypergeometric2F1}\left(4,\frac{1}{2}\left(4+\frac{i}{bn}\right),\frac{1}{2}\left(6+\frac{i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{(1-4ibn)x}$$

output
```
-16*exp(4*I*a)*(c*x^n)^(4*I*b)*hypergeom([4, 2+1/2*I/b/n],[3+1/2*I/b/n],-e
xp(2*I*a)*(c*x^n)^(2*I*b))/(1-4*I*b*n)/x
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 215 vs. 2(87) = 174.

Time = 6.75 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.47

$$\int \frac{\sec^4(a+b\log(cx^n))}{x^2}\,dx = \frac{-2e^{2ia}(-i+2bn)(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(1,1+\frac{i}{2bn},2+\frac{i}{2bn},-e^{2i(a+b\log(cx^n))}\right)-2i(1+4b^2n^2)\,\text{Hyp}}{}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]]^4/x^2,x]
```

output
```
(-2*E^((2*I)*a)*(-I + 2*b*n)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[1, 1 + (I
/2)/(b*n), 2 + (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] - (2*I)*(1 + 4*
b^2*n^2)*Hypergeometric2F1[1, (I/2)/(b*n), 1 + (I/2)/(b*n), -E^((2*I)*(a +
 b*Log[c*x^n]))] + Sec[a + b*Log[c*x^n]]^2*(2*b*n + (1 + 8*b^2*n^2 + (1 +
4*b^2*n^2)*Cos[2*(a + b*Log[c*x^n])])*Tan[a + b*Log[c*x^n]]))/(12*b^3*n^3*
x)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5020, 5016, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(a+b\log(cx^n))}{x^2}dx$$

$$\Big\downarrow \text{ 5020}$$

$$\frac{(cx^n)^{\frac{1}{n}}\int (cx^n)^{-1-\frac{1}{n}}\sec^4(a+b\log(cx^n))d(cx^n)}{nx}$$

$$\Big\downarrow \text{ 5016}$$

$$\frac{16e^{4ia}(cx^n)^{\frac{1}{n}}\int \frac{(cx^n)^{4ib-\frac{1}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^4}d(cx^n)}{nx}$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{16e^{4ia}(cx^n)^{4ib}\text{Hypergeometric2F1}\left(4,\frac{1}{2}\left(4+\frac{i}{bn}\right),\frac{1}{2}\left(6+\frac{i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{x(1-4ibn)}$$

input
```
Int[Sec[a + b*Log[c*x^n]]^4/x^2,x]
```

output
```
(-16*E^((4*I)*a)*(c*x^n)^((4*I)*b)*Hypergeometric2F1[4, (4 + I/(b*n))/2, (
6 + I/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((1 - (4*I)*b*n)*x)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\sec \left(a+b\ln \left(c\,x^{n}\right)\right)^{4}}{x^{2}}dx$$

input
```
int(sec(a+b*ln(c*x^n))^4/x^2,x)
```

output
```
int(sec(a+b*ln(c*x^n))^4/x^2,x)
```

Fricas [F]

$$\int \frac{\sec^4(a+b\log(cx^n))}{x^2} dx = \int \frac{\sec(b\log(cx^n)+a)^4}{x^2} dx$$

input
```
integrate(sec(a+b*log(c*x^n))^4/x^2,x, algorithm="fricas")
```

output
```
integral(sec(b*log(c*x^n) + a)^4/x^2, x)
```

Sympy [F]

$$\int \frac{\sec^4(a+b\log(cx^n))}{x^2} dx = \int \frac{\sec^4(a+b\log(cx^n))}{x^2} dx$$

input
```
integrate(sec(a+b*ln(c*x**n))**4/x**2,x)
```

output
```
Integral(sec(a + b*log(c*x**n))**4/x**2, x)
```

Maxima [F]

$$\int \frac{\sec^4(a+b\log(cx^n))}{x^2} dx = \int \frac{\sec(b\log(cx^n)+a)^4}{x^2} dx$$

input
```
integrate(sec(a+b*log(c*x^n))^4/x^2,x, algorithm="maxima")
```

output
```
1/3*(6*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*cos(4*b*log(x^n) + 4*
a)^2 + 6*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*cos(2*b*log(x^n) +
2*a)^2 + 6*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*sin(4*b*log(x^n)
+ 4*a)^2 + 6*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*sin(2*b*log(x^n
) + 2*a)^2 + (4*b^2*n^2*sin(6*b*log(c)) + (2*(b*cos(6*b*log(c))*cos(4*b*lo
g(c)) + b*sin(6*b*log(c))*sin(4*b*log(c)))*n + cos(4*b*log(c))*sin(6*b*log
(c)) - cos(6*b*log(c))*sin(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) + 2*(6*(b^
2*cos(2*b*log(c))*sin(6*b*log(c)) - b^2*cos(6*b*log(c))*sin(2*b*log(c)))*n
^2 + (b*cos(6*b*log(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)
))*n + cos(2*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(2*b*log(c)))*
cos(2*b*log(x^n) + 2*a) + (2*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*
b*log(c))*sin(4*b*log(c)))*n - cos(6*b*log(c))*cos(4*b*log(c)) - sin(6*b*l
og(c))*sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) - 2*(6*(b^2*cos(6*b*log(c)
)*cos(2*b*log(c)) + b^2*sin(6*b*log(c))*sin(2*b*log(c)))*n^2 - (b*cos(2*b*
log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(2*b*log(c)))*n + cos(6*b*l
og(c))*cos(2*b*log(c)) + sin(6*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n)
 + 2*a) + sin(6*b*log(c)))*cos(6*b*log(x^n) + 6*a) + (12*b^2*n^2*sin(4*b*l
og(c)) + 2*b*n*cos(4*b*log(c)) + 3*(12*(b^2*cos(2*b*log(c))*sin(4*b*log(c)
) - b^2*cos(4*b*log(c))*sin(2*b*log(c)))*n^2 + 4*(b*cos(4*b*log(c))*cos(2*
b*log(c)) + b*sin(4*b*log(c))*sin(2*b*log(c)))*n + cos(2*b*log(c))*sin(...
```

Giac [F]

$$\int \frac{\sec^4(a+b\log(cx^n))}{x^2}\,dx=\int \frac{\sec(b\log(cx^n)+a)^4}{x^2}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^4/x^2,x, algorithm="giac")
```

output
```
integrate(sec(b*log(c*x^n) + a)^4/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(a+b\log(cx^n))}{x^2}\,dx = \int \frac{1}{x^2\cos(a+b\ln(c\,x^n))^4}\,dx$$

input
```
int(1/(x^2*cos(a + b*log(c*x^n))^4),x)
```

output
```
int(1/(x^2*cos(a + b*log(c*x^n))^4), x)
```

Reduce [F]

$$\int \frac{\sec^4(a+b\log(cx^n))}{x^2}\,dx = \text{too large to display}$$

input
```
int(sec(a+b*log(c*x^n))^4/x^2,x)
```

output
```
( - 3984*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)
*b + a)/2)**8*b**7*n**7 - 3076*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b +
a)**3*tan((log(x**n*c)*b + a)/2)**8*b**5*n**5 - 536*cos(log(x**n*c)*b + a)
*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**8*b**3*n**3 - 4*cos
(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**
8*b*n + 15936*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x*
*n*c)*b + a)/2)**6*b**7*n**7 + 12304*cos(log(x**n*c)*b + a)*sin(log(x**n*c
)*b + a)**3*tan((log(x**n*c)*b + a)/2)**6*b**5*n**5 + 2144*cos(log(x**n*c)
*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**6*b**3*n**3
+ 16*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b +
 a)/2)**6*b*n - 23904*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan
((log(x**n*c)*b + a)/2)**4*b**7*n**7 - 18456*cos(log(x**n*c)*b + a)*sin(lo
g(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**4*b**5*n**5 - 3216*cos(log
(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**4*b*
*3*n**3 - 24*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**
n*c)*b + a)/2)**4*b*n + 15936*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a
)**3*tan((log(x**n*c)*b + a)/2)**2*b**7*n**7 + 12304*cos(log(x**n*c)*b + a
)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**2*b**5*n**5 + 2144
*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/
2)**2*b**3*n**3 + 16*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*t...
```

3.258 $\int \frac{\sec^4(a+b\log(cx^n))}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 79

$$\int \frac{\sec^4(a+b\log(cx^n))}{x^3}\,dx = -\frac{8e^{4ia}(cx^n)^{4ib}\,\text{Hypergeometric2F1}\left(4,2+\frac{i}{bn},3+\frac{i}{bn},-e^{2ia}(cx^n)^{2ib}\right)}{(1-2ibn)x^2}$$

output

```
-8*exp(4*I*a)*(c*x^n)^(4*I*b)*hypergeom([4, 2+I/b/n],[3+I/b/n],-exp(2*I*a)
*(c*x^n)^(2*I*b))/(1-2*I*b*n)/x^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 203 vs. 2(79) = 158.

Time = 6.60 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.57

$$\int \frac{\sec^4(a+b\log(cx^n))}{x^3}\,dx = \frac{-2e^{2ia}(-i+bn)(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(1,1+\frac{i}{bn},2+\frac{i}{bn},-e^{2i(a+b\log(cx^n))}\right)-2i(1+b^2n^2)\,\text{Hyperge}}{}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]]^4/x^3,x]
```

output
```
(-2*E^((2*I)*a)*(-I + b*n)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[1, 1 + I/(b
*n), 2 + I/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] - (2*I)*(1 + b^2*n^2)*Hyp
ergeometric2F1[1, I/(b*n), 1 + I/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] + S
ec[a + b*Log[c*x^n]]^2*(b*n + (1 + 2*b^2*n^2 + (1 + b^2*n^2)*Cos[2*(a + b*
Log[c*x^n])])*Tan[a + b*Log[c*x^n]]))/(3*b^3*n^3*x^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5020, 5016, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(a+b\log(cx^n))}{x^3}dx$$
$$\Big\downarrow \text{5020}$$
$$\frac{(cx^n)^{2/n}\int (cx^n)^{-1-\frac{2}{n}}\sec^4(a+b\log(cx^n))d(cx^n)}{nx^2}$$
$$\Big\downarrow \text{5016}$$
$$\frac{16e^{4ia}(cx^n)^{2/n}\int \frac{(cx^n)^{4ib-\frac{2}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^4}d(cx^n)}{nx^2}$$
$$\Big\downarrow \text{888}$$
$$-\frac{8e^{4ia}(cx^n)^{4ib}\text{Hypergeometric2F1}\left(4,2+\frac{i}{bn},3+\frac{i}{bn},-e^{2ia}(cx^n)^{2ib}\right)}{x^2(1-2ibn)}$$

input
```
Int[Sec[a + b*Log[c*x^n]]^4/x^3,x]
```

output
```
(-8*E^((4*I)*a)*(c*x^n)^((4*I)*b)*Hypergeometric2F1[4, 2 + I/(b*n), 3 + I/
(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((1 - (2*I)*b*n)*x^2)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\sec \left(a + b \ln \left(c\, x^{n}\right)\right)^{4}}{x^{3}}dx$$

input
```
int(sec(a+b*ln(c*x^n))^4/x^3,x)
```

output
```
int(sec(a+b*ln(c*x^n))^4/x^3,x)
```

Fricas [F]

$$\int \frac{\sec^4(a + b\log(cx^n))}{x^3} dx = \int \frac{\sec(b\log(cx^n) + a)^4}{x^3} dx$$

input
```
integrate(sec(a+b*log(c*x^n))^4/x^3,x, algorithm="fricas")
```

output
```
integral(sec(b*log(c*x^n) + a)^4/x^3, x)
```

Sympy [F]

$$\int \frac{\sec^4(a + b\log(cx^n))}{x^3} dx = \int \frac{\sec^4(a + b\log(cx^n))}{x^3} dx$$

input
```
integrate(sec(a+b*ln(c*x**n))**4/x**3,x)
```

output
```
Integral(sec(a + b*log(c*x**n))**4/x**3, x)
```

Maxima [F]

$$\int \frac{\sec^4(a + b\log(cx^n))}{x^3} dx = \int \frac{\sec(b\log(cx^n) + a)^4}{x^3} dx$$

input
```
integrate(sec(a+b*log(c*x^n))^4/x^3,x, algorithm="maxima")
```

output
```
4/3*(3*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*cos(4*b*log(x^n) + 4*
a)^2 + 3*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*cos(2*b*log(x^n) +
2*a)^2 + 3*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*sin(4*b*log(x^n)
+ 4*a)^2 + 3*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*sin(2*b*log(x^n
) + 2*a)^2 + (b^2*n^2*sin(6*b*log(c)) + ((b*cos(6*b*log(c))*cos(4*b*log(c)
) + b*sin(6*b*log(c))*sin(4*b*log(c)))*n + cos(4*b*log(c))*sin(6*b*log(c))
 - cos(6*b*log(c))*sin(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) + (3*(b^2*cos(
2*b*log(c))*sin(6*b*log(c)) - b^2*cos(6*b*log(c))*sin(2*b*log(c)))*n^2 + (
b*cos(6*b*log(c))*cos(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n +
 2*cos(2*b*log(c))*sin(6*b*log(c)) - 2*cos(6*b*log(c))*sin(2*b*log(c)))*co
s(2*b*log(x^n) + 2*a) + ((b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*lo
g(c))*sin(4*b*log(c)))*n - cos(6*b*log(c))*cos(4*b*log(c)) - sin(6*b*log(c
))*sin(4*b*log(c)))*sin(4*b*log(x^n) + 4*a) - (3*(b^2*cos(6*b*log(c))*cos(
2*b*log(c)) + b^2*sin(6*b*log(c))*sin(2*b*log(c)))*n^2 - (b*cos(2*b*log(c)
)*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(2*b*log(c)))*n + 2*cos(6*b*log(c
))*cos(2*b*log(c)) + 2*sin(6*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) +
 2*a) + sin(6*b*log(c)))*cos(6*b*log(x^n) + 6*a) + (3*b^2*n^2*sin(4*b*log(
c)) + b*n*cos(4*b*log(c)) + 3*(3*(b^2*cos(2*b*log(c))*sin(4*b*log(c)) - b^
2*cos(4*b*log(c))*sin(2*b*log(c)))*n^2 + 2*(b*cos(4*b*log(c))*cos(2*b*log(
c)) + b*sin(4*b*log(c))*sin(2*b*log(c)))*n + cos(2*b*log(c))*sin(4*b*lo...
```

Giac [F]

$$\int \frac{\sec^4(a+b\log(cx^n))}{x^3}\,dx=\int \frac{\sec(b\log(cx^n)+a)^4}{x^3}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^4/x^3,x, algorithm="giac")
```

output
```
integrate(sec(b*log(c*x^n) + a)^4/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(a + b\log(cx^n))}{x^3}\,dx = \int \frac{1}{x^3 \cos\left(a + b\,\ln\left(c\,x^n\right)\right)^4}\,dx$$

input
```
int(1/(x^3*cos(a + b*log(c*x^n))^4),x)
```

output
```
int(1/(x^3*cos(a + b*log(c*x^n))^4), x)
```

Reduce [F]

$$\int \frac{\sec^4(a + b\log(cx^n))}{x^3}\,dx = \text{too large to display}$$

input
```
int(sec(a+b*log(c*x^n))^4/x^3,x)
```

output
```
( - 996*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*
b + a)/2)**8*b**7*n**7 - 3076*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a
)**3*tan((log(x**n*c)*b + a)/2)**8*b**5*n**5 - 2144*cos(log(x**n*c)*b + a)
*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**8*b**3*n**3 - 64*co
s(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)*
*8*b*n + 3984*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x*
*n*c)*b + a)/2)**6*b**7*n**7 + 12304*cos(log(x**n*c)*b + a)*sin(log(x**n*c
)*b + a)**3*tan((log(x**n*c)*b + a)/2)**6*b**5*n**5 + 8576*cos(log(x**n*c)
*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**6*b**3*n**3
+ 256*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b
+ a)/2)**6*b*n - 5976*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan
((log(x**n*c)*b + a)/2)**4*b**7*n**7 - 18456*cos(log(x**n*c)*b + a)*sin(lo
g(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**4*b**5*n**5 - 12864*cos(lo
g(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**4*b
**3*n**3 - 384*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x
**n*c)*b + a)/2)**4*b*n + 3984*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b +
a)**3*tan((log(x**n*c)*b + a)/2)**2*b**7*n**7 + 12304*cos(log(x**n*c)*b +
a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)/2)**2*b**5*n**5 + 857
6*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3*tan((log(x**n*c)*b + a)
/2)**2*b**3*n**3 + 256*cos(log(x**n*c)*b + a)*sin(log(x**n*c)*b + a)**3...
```

3.259 $\int \left(-\left(\left(1+b^2n^2\right)\sec\left(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\sec^3\left(a\right.\right.$

Optimal result

Integrand size = 44, antiderivative size = 41

$$\int \left(-\left(\left(1+b^2n^2\right)\sec\left(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\sec^3\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$
$$=-x\sec\left(a+b\log\left(cx^n\right)\right)+bnx\sec\left(a+b\log\left(cx^n\right)\right)\tan\left(a+b\log\left(cx^n\right)\right)$$

output
```
-x*sec(a+b*ln(c*x^n))+b*n*x*sec(a+b*ln(c*x^n))*tan(a+b*ln(c*x^n))
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int \left(-\left(\left(1+b^2n^2\right)\sec\left(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\sec^3\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$
$$=x\sec\left(a+b\log\left(cx^n\right)\right)\left(-1+bn\tan\left(a+b\log\left(cx^n\right)\right)\right)$$

input
```
Integrate[-((1 + b^2*n^2)*Sec[a + b*Log[c*x^n]]) + 2*b^2*n^2*Sec[a + b*Log
[c*x^n]]^3,x]
```

output
```
x*Sec[a + b*Log[c*x^n]]*(-1 + b*n*Tan[a + b*Log[c*x^n]])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 175, normalized size of antiderivative = 4.27, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2b^2n^2\sec^3(a+b\log(cx^n))-\left(b^2n^2+1\right)\sec(a+b\log(cx^n))\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{16e^{3ia}b^2n^2x(cx^n)^{3ib}\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3-\frac{i}{bn}\right),\frac{1}{2}\left(5-\frac{i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{1+3ibn}-2e^{ia}x(1-ibn)\left(cx^n\right)^{ib}\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1-\frac{i}{bn}\right),\frac{1}{2}\left(3-\frac{i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)$$

input
```
Int[-((1 + b^2*n^2)*Sec[a + b*Log[c*x^n]]) + 2*b^2*n^2*Sec[a + b*Log[c*x^n
]]^3,x]
```

output
```
-2*E^(I*a)*(1 - I*b*n)*x*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 - I/(b*n))/
2, (3 - I/(b*n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))] + (16*b^2*E^((3*I)*a
)*n^2*x*(c*x^n)^((3*I)*b)*Hypergeometric2F1[3, (3 - I/(b*n))/2, (5 - I/(b*
n))/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(1 + (3*I)*b*n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 30.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

method	result
parallelrisch	$\sec\left(a+b\ln\left(c\,x^{n}\right)\right)\left(\tan\left(a+b\ln\left(c\,x^{n}\right)\right)bn-1\right)x$
risch	$-\frac{2i\left(x^{n}\right)^{ib}c^{ib}x\left(nb\,c^{2ib}\left(x^{n}\right)^{2ib}\mathrm{e}^{\frac{3b\pi\,\mathrm{csgn}\left(ic\,x^{n}\right)^{3}}{2}}\mathrm{e}^{-\frac{3b\pi\,\mathrm{csgn}\left(ic\,x^{n}\right)^{2}\mathrm{csgn}\left(ic\right)}{2}}\mathrm{e}^{-\frac{3b\pi\,\mathrm{csgn}\left(ix^{n}\right)\mathrm{csgn}\left(ic\,x^{n}\right)^{2}}{2}}\mathrm{e}^{\frac{3b\pi\,\mathrm{csgn}\left(ix^{n}\right)\mathrm{csgn}\left(ic\right.}{2}}\right.}{}$

input
```
int(-(b^2*n^2+1)*sec(a+b*ln(c*x^n))+2*b^2*n^2*sec(a+b*ln(c*x^n))^3,x,metho
d=_RETURNVERBOSE)
```

output
```
sec(a+b*ln(c*x^n))*(tan(a+b*ln(c*x^n))*b*n-1)*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

$$\int\left(-\left(\left(1+b^2n^2\right)\sec\left(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\sec^3\left(a+b\log\left(cx^n\right)\right)\right)\,dx = \frac{bnx\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-x\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2}$$

input
```
integrate(-(b^2*n^2+1)*sec(a+b*log(c*x^n))+2*b^2*n^2*sec(a+b*log(c*x^n))^3
,x, algorithm="fricas")
```

output
```
(b*n*x*sin(b*n*log(x) + b*log(c) + a) - x*cos(b*n*log(x) + b*log(c) + a))/
cos(b*n*log(x) + b*log(c) + a)^2
```

Sympy [F]

$$\int \left(-\left(\left(1+b^2n^2\right)\sec\left(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\sec^3\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$
$$=\int \left(2b^2n^2\sec^2\left(a+b\log\left(cx^n\right)\right)-b^2n^2-1\right)\sec\left(a+b\log\left(cx^n\right)\right)dx$$

input
```
integrate(-(b**2*n**2+1)*sec(a+b*ln(c*x**n))+2*b**2*n**2*sec(a+b*ln(c*x**n
))**3,x)
```

output
```
Integral((2*b**2*n**2*sec(a + b*log(c*x**n))**2 - b**2*n**2 - 1)*sec(a + b
*log(c*x**n)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1696 vs. $2(41) = 82$.

Time = 0.77 (sec) , antiderivative size = 1696, normalized size of antiderivative = 41.37

$$\int \left(-\left(\left(1+b^2n^2\right)\sec\left(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\sec^3\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$
$$=\text{Too large to display}$$

input
```
integrate(-(b^2*n^2+1)*sec(a+b*log(c*x^n))+2*b^2*n^2*sec(a+b*log(c*x^n))^3
,x, algorithm="maxima")
```

output
```
-2*((b*n*sin(b*log(c)) + cos(b*log(c)))*x*cos(b*log(x^n) + a) + (b*n*cos(b
*log(c)) - sin(b*log(c)))*x*sin(b*log(x^n) + a) + (((b*cos(3*b*log(c))*sin
(4*b*log(c)) - b*cos(4*b*log(c))*sin(3*b*log(c)))*n + cos(4*b*log(c))*cos(
3*b*log(c)) + sin(4*b*log(c))*sin(3*b*log(c)))*x*cos(3*b*log(x^n) + 3*a) -
 ((b*cos(b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(b*log(c)))*n -
cos(4*b*log(c))*cos(b*log(c)) - sin(4*b*log(c))*sin(b*log(c)))*x*cos(b*log
(x^n) + a) - ((b*cos(4*b*log(c))*cos(3*b*log(c)) + b*sin(4*b*log(c))*sin(3
*b*log(c)))*n - cos(3*b*log(c))*sin(4*b*log(c)) + cos(4*b*log(c))*sin(3*b*
log(c)))*x*sin(3*b*log(x^n) + 3*a) + ((b*cos(4*b*log(c))*cos(b*log(c)) + b
*sin(4*b*log(c))*sin(b*log(c)))*n + cos(b*log(c))*sin(4*b*log(c)) - cos(4*
b*log(c))*sin(b*log(c)))*x*sin(b*log(x^n) + a))*cos(4*b*log(x^n) + 4*a) -
(2*((b*cos(2*b*log(c))*sin(3*b*log(c)) - b*cos(3*b*log(c))*sin(2*b*log(c))
)*n - cos(3*b*log(c))*cos(2*b*log(c)) - sin(3*b*log(c))*sin(2*b*log(c)))*x
*cos(2*b*log(x^n) + 2*a) - 2*((b*cos(3*b*log(c))*cos(2*b*log(c)) + b*sin(3
*b*log(c))*sin(2*b*log(c)))*n + cos(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*
log(c))*sin(2*b*log(c)))*x*sin(2*b*log(x^n) + 2*a) + (b*n*sin(3*b*log(c))
- cos(3*b*log(c)))*x)*cos(3*b*log(x^n) + 3*a) - 2*(((b*cos(b*log(c))*sin(2
*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)))*n - cos(2*b*log(c))*cos(b*lo
g(c)) - sin(2*b*log(c))*sin(b*log(c)))*x*cos(b*log(x^n) + a) - ((b*cos(2*b
*log(c))*cos(b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c)))*n + cos(b*log...
```

Giac [F]

$$\int \left(-\left(\left(1+b^2n^2\right)\sec\left(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\sec^3\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$
$$=\int 2\,b^2n^2\sec\left(b\log\left(cx^n\right)+a\right)^3-\left(b^2n^2+1\right)\sec\left(b\log\left(cx^n\right)+a\right)\,dx$$

input
```
integrate(-(b^2*n^2+1)*sec(a+b*log(c*x^n))+2*b^2*n^2*sec(a+b*log(c*x^n))^3
,x, algorithm="giac")
```

output
```
integrate(2*b^2*n^2*sec(b*log(c*x^n) + a)^3 - (b^2*n^2 + 1)*sec(b*log(c*x^
n) + a), x)
```

Mupad [B] (verification not implemented)

Time = 21.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.12

$$\int \left(-\left(\left(1+b^2n^2\right)\sec\left(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\sec^3\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$
$$=\frac{2\,x\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}\,\left(-1+b\,n\,1\mathrm{i}\right)-2\,x\,\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}\,\left(1+b\,n\,1\mathrm{i}\right)}{{\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}+1\right)}^2}$$

input
```
int((2*b^2*n^2)/cos(a + b*log(c*x^n))^3 - (b^2*n^2 + 1)/cos(a + b*log(c*x^
n)),x)
```

output
```
(2*x*exp(a*1i)*(c*x^n)^(b*1i)*(b*n*1i - 1) - 2*x*exp(a*1i)*exp(a*2i)*(c*x^
n)^(b*1i)*(c*x^n)^(b*2i)*(b*n*1i + 1))/(exp(a*2i)*(c*x^n)^(b*2i) + 1)^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 295, normalized size of antiderivative = 7.20

$$\int \left(-\left(\left(1+b^2n^2\right)\sec\left(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\sec^3\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$
$$=\frac{x\left(2\cos\left(\log(x^nc)\,b+a\right)\sec\left(\log(x^nc)\,b+a\right)^3\sin\left(\log(x^nc)\,b+a\right)^2b^2n^2-2\cos\left(\log(x^nc)\,b+a\right)\sec\left(\log(x^n\right.\right.}{}$$

input
```
int(-(b^2*n^2+1)*sec(a+b*log(c*x^n))+2*b^2*n^2*sec(a+b*log(c*x^n))^3,x)
```

output
```
(x*(2*cos(log(x**n*c)*b + a)*sec(log(x**n*c)*b + a)**3*sin(log(x**n*c)*b +
 a)**2*b**2*n**2 - 2*cos(log(x**n*c)*b + a)*sec(log(x**n*c)*b + a)**3*b**2
*n**2 - cos(log(x**n*c)*b + a)*sec(log(x**n*c)*b + a)*sin(log(x**n*c)*b +
a)**2*b**2*n**2 - cos(log(x**n*c)*b + a)*sec(log(x**n*c)*b + a)*sin(log(x*
*n*c)*b + a)**2 + cos(log(x**n*c)*b + a)*sec(log(x**n*c)*b + a)*b**2*n**2
+ cos(log(x**n*c)*b + a)*sec(log(x**n*c)*b + a) - cos(log(x**n*c)*b + a)*s
in(log(x**n*c)*b + a)*b*n + sin(log(x**n*c)*b + a)**2*b**2*n**2 + b**2*n**
2))/(cos(log(x**n*c)*b + a)*(sin(log(x**n*c)*b + a)**2 - 1))
```

3.260 $\int x^m \sec^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 110

$$\int x^m \sec^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\,dx = \frac{x^{1+m}\sec\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)}{2(1+m)} + \frac{x^{1+m}\sec\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\tan\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)}{2\sqrt{-(1+m)^2}}$$

output

```
x^(1+m)*sec(a+2*ln(c*x^(1/2*(-(1+m)^2)^(1/2))))/(2+2*m)+1/2*x^(1+m)*sec(a+
2*ln(c*x^(1/2*(-(1+m)^2)^(1/2))))*tan(a+2*ln(c*x^(1/2*(-(1+m)^2)^(1/2))))/
(-(1+m)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.80

$$\int x^m \sec^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\,dx$$
$$=\frac{x^{1+m}\left((1+m)\cos\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)-\sqrt{-(1+m)^2}\sin\left(a+2\log\left(cx\right.\right.\right.}{2(1+m)^2\left(\cos\left(\frac{a}{2}+\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)-\sin\left(\frac{a}{2}+\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\right)^2\left(\cos\left(\frac{a}{2}+\log\left(cx^{\frac{1}{2}\sqrt{-(1+m}}\right.\right.\right.}$$

input
```
Integrate[x^m*Sec[a + 2*Log[c*x^(Sqrt[-(1 + m)^2]/2)]]^3,x]
```

output
```
(x^(1 + m)*((1 + m)*Cos[a + 2*Log[c*x^(Sqrt[-(1 + m)^2]/2)]] - Sqrt[-(1 +
m)^2]*Sin[a + 2*Log[c*x^(Sqrt[-(1 + m)^2]/2)]]))/(2*(1 + m)^2*(Cos[a/2 + L
og[c*x^(Sqrt[-(1 + m)^2]/2)]] - Sin[a/2 + Log[c*x^(Sqrt[-(1 + m)^2]/2)]])^
2*(Cos[a/2 + Log[c*x^(Sqrt[-(1 + m)^2]/2)]] + Sin[a/2 + Log[c*x^(Sqrt[-(1
+ m)^2]/2)]])^2)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.46 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.37, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {5020, 5016, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sec^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)\right)\,dx$$

$\big\downarrow$ 5020

$$\frac{2x^{m+1}\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)^{-\frac{2(m+1)}{\sqrt{-(m+1)^2}}}\int\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)^{\frac{2(m+1)}{\sqrt{-(m+1)^2}}-1}\sec^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)\right)d\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right.}{\sqrt{-(m+1)^2}}$$

$\big\downarrow$ 5016

$$\frac{16e^{3ia}x^{m+1}\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)^{-\frac{2(m+1)}{\sqrt{-(m+1)^2}}}\int \frac{\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)^{\frac{2(m+1)}{\sqrt{-(m+1)^2}}-(1-6i)}}{\left(e^{2ia}\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)^{4i}+1\right)^3}d\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)}{\sqrt{-(m+1)^2}}$$

$\Big\downarrow$ 888

$$\frac{8e^{3ia}x^{m+1}\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)^{6i}\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3-\frac{i(m+1)}{\sqrt{-(m+1)^2}}\right),\frac{1}{2}\left(5-\frac{i(m+1)}{\sqrt{-(m+1)^2}}\right),-e^{2ia}\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right.\right.}{\sqrt{-(m+1)^2}\left(\frac{m+1}{\sqrt{-(m+1)^2}}+3i\right)}$$

input
```
Int[x^m*Sec[a + 2*Log[c*x^(Sqrt[-(1 + m)^2]/2)]]^3,x]
```

output
```
(8*E^((3*I)*a)*x^(1 + m)*(c*x^(Sqrt[-(1 + m)^2]/2))^(6*I)*Hypergeometric2F
1[3, (3 - (I*(1 + m))/Sqrt[-(1 + m)^2])/2, (5 - (I*(1 + m))/Sqrt[-(1 + m)^
2])/2, -(E^((2*I)*a)*(c*x^(Sqrt[-(1 + m)^2]/2))^(4*I))])/(Sqrt[-(1 + m)^2]
*(3*I + (1 + m)/Sqrt[-(1 + m)^2]))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 302.69 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.88

method	result	size
parallelrisch	$\frac{x^{1+m}\left(-\sqrt{-(1+m)^{2}}\,\sin\left(a+2\ln\left(c\,x^{\frac{\sqrt{-(1+m)^{2}}}{2}}\right)\right)+\cos\left(a+2\ln\left(c\,x^{\frac{\sqrt{-(1+m)^{2}}}{2}}\right)\right)(1+m)\right)}{(1+m)^{2}\left(\cos\left(2a+4\ln\left(c\,x^{\frac{\sqrt{-(1+m)^{2}}}{2}}\right)\right)+1\right)}$	97

input
```
int(x^m*sec(a+2*ln(c*x^(1/2*(-(1+m)^2)^(1/2))))^3,x,method=_RETURNVERBOSE)
```

output
```
x^(1+m)*(-(-(1+m)^2)^(1/2)*sin(a+2*ln(c*x^(1/2*(-(1+m)^2)^(1/2))))+cos(a+2
*ln(c*x^(1/2*(-(1+m)^2)^(1/2))))*(1+m))/(1+m)^2/(cos(2*a+4*ln(c*x^(1/2*(-(
1+m)^2)^(1/2))))+1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.74

$$\int x^m \sec^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\,dx$$
$$= -\frac{2\left(2\,x^2x^{2\,m}e^{(3i\,a+6i\,\log(c))}+e^{(5i\,a+10i\,\log(c))}\right)}{(m+1)x^4x^{4\,m}+2\,(m+1)x^2x^{2\,m}e^{(2i\,a+4i\,\log(c))}+(m+1)e^{(4i\,a+8i\,\log(c))}}$$

input
```
integrate(x^m*sec(a+2*log(c*x^(1/2*(-(1+m)^2)^(1/2))))^3,x, algorithm="fri
cas")
```

output
```
-2*(2*x^2*x^(2*m)*e^(3*I*a + 6*I*log(c)) + e^(5*I*a + 10*I*log(c)))/((m +
1)*x^4*x^(4*m) + 2*(m + 1)*x^2*x^(2*m)*e^(2*I*a + 4*I*log(c)) + (m + 1)*e^
(4*I*a + 8*I*log(c)))
```

Sympy [F(-1)]

Timed out.

$$\int x^m \sec^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x**m*sec(a+2*ln(c*x**(1/2*(-(1+m)**2)**(1/2))))**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 976 vs. $2(92) = 184$.

Time = 0.25 (sec) , antiderivative size = 976, normalized size of antiderivative = 8.87

$$\int x^m \sec^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x^m*sec(a+2*log(c*x^(1/2*(-(1+m)^2)^(1/2))))^3,x, algorithm="max
ima")
```

output
```
2*((cos(a)*cos(2*log(c)) - sin(a)*sin(2*log(c)))*x*e^(m*log(x) + 14*arctan
2(sin(1/2*m*log(x)), cos(1/2*m*log(x))) + 14*arctan2(sin(1/2*log(x)), cos(
1/2*log(x)))) + 2*(((cos(2*a)*cos(a) + sin(2*a)*sin(a))*cos(2*log(c)) + (c
os(a)*sin(2*a) - cos(2*a)*sin(a))*sin(2*log(c)))*cos(4*log(c)) - ((cos(a)*
sin(2*a) - cos(2*a)*sin(a))*cos(2*log(c)) - (cos(2*a)*cos(a) + sin(2*a)*si
n(a))*sin(2*log(c)))*sin(4*log(c)))*x*e^(m*log(x) + 10*arctan2(sin(1/2*m*l
og(x)), cos(1/2*m*log(x))) + 10*arctan2(sin(1/2*log(x)), cos(1/2*log(x))))
 + (((cos(4*a)*cos(a) + sin(4*a)*sin(a))*cos(2*log(c)) + (cos(a)*sin(4*a)
- cos(4*a)*sin(a))*sin(2*log(c)))*cos(8*log(c)) - ((cos(a)*sin(4*a) - cos(
4*a)*sin(a))*cos(2*log(c)) - (cos(4*a)*cos(a) + sin(4*a)*sin(a))*sin(2*log
(c)))*sin(8*log(c)))*x*e^(m*log(x) + 6*arctan2(sin(1/2*m*log(x)), cos(1/2*
m*log(x))) + 6*arctan2(sin(1/2*log(x)), cos(1/2*log(x)))))/((cos(4*a)^2 +
sin(4*a)^2)*cos(8*log(c))^2 + (cos(4*a)^2 + sin(4*a)^2)*sin(8*log(c))^2 +
((cos(4*a)^2 + sin(4*a)^2)*cos(8*log(c))^2 + (cos(4*a)^2 + sin(4*a)^2)*sin
(8*log(c))^2)*m + (m + 1)*e^(16*arctan2(sin(1/2*m*log(x)), cos(1/2*m*log(x
))) + 16*arctan2(sin(1/2*log(x)), cos(1/2*log(x)))) + 4*((cos(2*a)*cos(4*l
og(c)) - sin(2*a)*sin(4*log(c)))*m + cos(2*a)*cos(4*log(c)) - sin(2*a)*sin
(4*log(c)))*e^(12*arctan2(sin(1/2*m*log(x)), cos(1/2*m*log(x))) + 12*arcta
n2(sin(1/2*log(x)), cos(1/2*log(x)))) + 2*(2*(cos(2*a)^2 + sin(2*a)^2)*cos
(4*log(c))^2 + 2*(cos(2*a)^2 + sin(2*a)^2)*sin(4*log(c))^2 + (2*(cos(2*...
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.28 (sec) , antiderivative size = 834, normalized size of antiderivative = 7.58

$$\int x^m \sec^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^m*sec(a+2*log(c*x^(1/2*(-(1+m)^2)^(1/2))))^3,x, algorithm="gia
c")
```

output
```
c^(6*I)*m*x*x^m*x^abs(m + 1)*e^(3*I*a)/(c^(8*I)*m^2*e^(4*I*a) + 2*c^(8*I)*
m*e^(4*I*a) + c^(8*I)*e^(4*I*a) + 2*c^(4*I)*m^2*x^(2*abs(m + 1))*e^(2*I*a)
 + 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) + 2*c^(4*I)*x^(2*abs(m + 1))*e^(
2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(m + 1))) -
 c^(6*I)*x*x^m*x^abs(m + 1)*abs(m + 1)*e^(3*I*a)/(c^(8*I)*m^2*e^(4*I*a) +
2*c^(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) + 2*c^(4*I)*m^2*x^(2*abs(m + 1))
*e^(2*I*a) + 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) + 2*c^(4*I)*x^(2*abs(m
 + 1))*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(
m + 1))) + c^(6*I)*x*x^m*x^abs(m + 1)*e^(3*I*a)/(c^(8*I)*m^2*e^(4*I*a) + 2
*c^(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) + 2*c^(4*I)*m^2*x^(2*abs(m + 1))*
e^(2*I*a) + 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) + 2*c^(4*I)*x^(2*abs(m
+ 1))*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(m
 + 1))) + c^(2*I)*m*x*x^m*x^(3*abs(m + 1))*e^(I*a)/(c^(8*I)*m^2*e^(4*I*a)
+ 2*c^(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) + 2*c^(4*I)*m^2*x^(2*abs(m + 1
))*e^(2*I*a) + 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) + 2*c^(4*I)*x^(2*abs
(m + 1))*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*ab
s(m + 1))) + c^(2*I)*x*x^m*x^(3*abs(m + 1))*abs(m + 1)*e^(I*a)/(c^(8*I)*m^
2*e^(4*I*a) + 2*c^(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) + 2*c^(4*I)*m^2*x^
(2*abs(m + 1))*e^(2*I*a) + 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) + 2*c^(4
*I)*x^(2*abs(m + 1))*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m ...
```

Mupad [B] (verification not implemented)

Time = 25.44 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.60

$$\int x^m \sec^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\,dx = \frac{\frac{x^{m+1}\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^{\frac{\sqrt{-m^2-2\,m-1}}{2}}\right)}^{2\mathrm{i}}\,\left(m\,1\mathrm{i}+\sqrt{-{(m+1)}^2}+1\mathrm{i}\right)}{\sqrt{-{(m+1)}^2}}-\frac{x^{m+1}\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^{\frac{\sqrt{-m^2-2\,m-1}}{2}}\right)}^{6\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{e}^{a\,2\mathrm{i}}\,\sqrt{-{(m+1)}^2}+m\,\mathrm{e}^{a\,2\mathrm{i}}\,1\mathrm{i}\right)}{\sqrt{-{(m+1)}^2}}}{(m+1)\,{\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^{\frac{\sqrt{-m^2-2\,m-1}}{2}}\right)}^{4\mathrm{i}}+1\right)}^2}$$

input
```
int(x^m/cos(a + 2*log(c*x^((-(m + 1)^2)^(1/2)/2)))^3,x)
```

output
```
((x^(m + 1)*exp(a*1i)*(c*x^((- 2*m - m^2 - 1)^(1/2)/2))^2i*(m*1i + (-(m +
1)^2)^(1/2) + 1i))/(-(m + 1)^2)^(1/2) - (x^(m + 1)*exp(a*1i)*(c*x^((- 2*m
- m^2 - 1)^(1/2)/2))^6i*(exp(a*2i)*1i - exp(a*2i)*(-(m + 1)^2)^(1/2) + m*e
xp(a*2i)*1i))/(-(m + 1)^2)^(1/2))/((m + 1)*(exp(a*2i)*(c*x^((- 2*m - m^2 -
 1)^(1/2)/2))^4i + 1)^2)
```

Reduce [F]

$$\int x^m \sec^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\,dx=\int x^m \sec\left(2\log\left(x^{\frac{m}{2}+\frac{1}{2}}c\right)+a\right)^3dx$$

input
```
int(x^m*sec(a+2*log(c*x^(1/2*(-(1+m)^2)^(1/2))))^3,x)
```

output
```
int(x**m*sec(2*log(x**((m + 1)/2)*c) + a)**3,x)
```

3.261 $\int x \sec^3\left(a+2\log\left(cx^i\right)\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int x \sec^3\left(a+2\log\left(cx^i\right)\right)\,dx=\frac{e^{ia}\left(cx^i\right)^{2i}x^2}{\left(1+e^{2ia}\left(cx^i\right)^{4i}\right)^2}$$

output
```
exp(I*a)*(c*x^I)^(2*I)*x^2/(1+exp(2*I*a)*(c*x^I)^(4*I))^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 127 vs. 2(45) = 90.

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.82

$$\int x \sec^3\left(a+2\log\left(cx^i\right)\right)\,dx =$$
$$-\frac{\sec^2\left(a+2\log\left(cx^i\right)\right)\left((1+2x^4)\cos\left(a+2\log\left(cx^i\right)-2i\log(x)\right)+i(1-2x^4)\sin\left(a+2\log\left(cx^i\right)-2i\,\mathrm{lo}\right.}{4x^4}$$

input
```
Integrate[x*Sec[a + 2*Log[c*x^I]]^3,x]
```

output
```
-1/4*(Sec[a + 2*Log[c*x^I]]^2*((1 + 2*x^4)*Cos[a + 2*Log[c*x^I] - (2*I)*Lo
g[x]] + I*(1 - 2*x^4)*Sin[a + 2*Log[c*x^I] - (2*I)*Log[x]])*(Cos[2*(a + 2*
Log[c*x^I] - (2*I)*Log[x])] + I*Sin[2*(a + 2*Log[c*x^I] - (2*I)*Log[x])]))
/x^4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5020, 5016, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sec^3\left(a+2\log\left(cx^i\right)\right)\,dx$$

$$\Big\downarrow \text{ 5020}$$

$$-ix^2\left(cx^i\right)^{2i}\int \left(cx^i\right)^{-1-2i}\sec^3\left(a+2\log\left(cx^i\right)\right)d\left(cx^i\right)$$

$$\Big\downarrow \text{ 5016}$$

$$-8ie^{3ia}x^2\left(cx^i\right)^{2i}\int \frac{\left(cx^i\right)^{-1+4i}}{\left(e^{2ia}\left(cx^i\right)^{4i}+1\right)^3}d\left(cx^i\right)$$

$$\Big\downarrow \text{ 793}$$

$$\frac{e^{ia}x^2\left(cx^i\right)^{2i}}{\left(1+e^{2ia}\left(cx^i\right)^{4i}\right)^2}$$

input
```
Int[x*Sec[a + 2*Log[c*x^I]]^3,x]
```

output
```
(E^(I*a)*(c*x^I)^(2*I)*x^2)/(1 + E^((2*I)*a)*(c*x^I)^(4*I))^2
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 209, normalized size of antiderivative = 4.64

$$\frac{x^{2} c^{2i} \left(x^{i}\right)^{2i} \mathrm{e}^{-\operatorname{csgn}\left(i x^{i}\right) \pi \operatorname{csgn}\left(i c\,x^{i}\right)^{2}+\operatorname{csgn}\left(i x^{i}\right) \pi \operatorname{csgn}\left(i c\,x^{i}\right) \operatorname{csgn}\left(i c\right)+\pi \operatorname{csgn}\left(i c\,x^{i}\right)^{3}-\pi \operatorname{csgn}\left(i c\,x^{i}\right)^{2} \operatorname{csgn}\left(i c\right)+i a}}{\left(\left(x^{i}\right)^{4i} c^{4i} \mathrm{e}^{-2 \operatorname{csgn}\left(i x^{i}\right) \pi \operatorname{csgn}\left(i c\,x^{i}\right)^{2}} \mathrm{e}^{2 \operatorname{csgn}\left(i x^{i}\right) \pi \operatorname{csgn}\left(i c\,x^{i}\right) \operatorname{csgn}\left(i c\right)} \mathrm{e}^{2 \pi \operatorname{csgn}\left(i c\,x^{i}\right)^{3}} \mathrm{e}^{-2 \pi \operatorname{csgn}\left(i c\,x^{i}\right)^{2} \operatorname{csgn}\left(i c\right)} \mathrm{e}^{2 i a}+1\right)^{2}}$$

input
```
int(x*sec(a+2*ln(c*x^I))^3,x)
```

output
```
x^2*c^(2*I)*(x^I)^(2*I)*exp(-csgn(I*x^I)*Pi*csgn(I*c*x^I)^2+csgn(I*x^I)*Pi
*csgn(I*c*x^I)*csgn(I*c)+Pi*csgn(I*c*x^I)^3-Pi*csgn(I*c*x^I)^2*csgn(I*c)+I
*a)/(((x^I)^(2*I))^2*(c^(2*I))^2*exp(-2*csgn(I*x^I)*Pi*csgn(I*c*x^I)^2)*ex
p(2*csgn(I*x^I)*Pi*csgn(I*c*x^I)*csgn(I*c))*exp(2*Pi*csgn(I*c*x^I)^3)*exp(
-2*Pi*csgn(I*c*x^I)^2*csgn(I*c))*exp(2*I*a)+1)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int x\sec^3\left(a+2\log\left(cx^i\right)\right)\,dx = -\frac{2\,x^4e^{(3i\,a+6i\,\log(c))}+e^{(5i\,a+10i\,\log(c))}}{x^8+2\,x^4e^{(2i\,a+4i\,\log(c))}+e^{(4i\,a+8i\,\log(c))}}$$

input
```
integrate(x*sec(a+2*log(c*x^I))^3,x, algorithm="fricas")
```

output
```
-(2*x^4*e^(3*I*a + 6*I*log(c)) + e^(5*I*a + 10*I*log(c)))/(x^8 + 2*x^4*e^(
2*I*a + 4*I*log(c)) + e^(4*I*a + 8*I*log(c)))
```

Sympy [F]

$$\int x\sec^3\left(a+2\log\left(cx^i\right)\right)\,dx = \int x\sec^3\left(a+2\log\left(cx^i\right)\right)dx$$

input
```
integrate(x*sec(a+2*ln(c*x**I))**3,x)
```

output
```
Integral(x*sec(a + 2*log(c*x**I))**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(31) = 62.

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.09

$$\int x\sec^3\left(a+2\log\left(cx^i\right)\right)\,dx$$

$$= \frac{((\cos(a)+i\,\sin(a))\cos(2\,\log(c))-(-i\,\cos(}{(\cos(4\,a)+i\,\sin(4\,a))\cos(8\,\log(c))+2\,((\cos(2\,a)+i\,\sin(2\,a))\cos(4\,\log(c))-(-i\,\cos(2\,a)+\sin(2}$$

input
```
integrate(x*sec(a+2*log(c*x^I))^3,x, algorithm="maxima")
```

output
```
((cos(a) + I*sin(a))*cos(2*log(c)) - (-I*cos(a) + sin(a))*sin(2*log(c)))*x
^2*e^(6*arctan2(sin(log(x)), cos(log(x))))/((cos(4*a) + I*sin(4*a))*cos(8*
log(c)) + 2*((cos(2*a) + I*sin(2*a))*cos(4*log(c)) - (-I*cos(2*a) + sin(2*
a))*sin(4*log(c)))*e^(4*arctan2(sin(log(x)), cos(log(x)))) + (I*cos(4*a) -
 sin(4*a))*sin(8*log(c)) + e^(8*arctan2(sin(log(x)), cos(log(x)))))
```

Giac [F]

$$\int x\sec^3\left(a+2\log\left(cx^i\right)\right)\,dx=\int x\sec\left(a+2\,\log\left(cx^i\right)\right)^3\,dx$$

input
```
integrate(x*sec(a+2*log(c*x^I))^3,x, algorithm="giac")
```

output
```
integrate(x*sec(a + 2*log(c*x^I))^3, x)
```

Mupad [B] (verification not implemented)

Time = 22.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int x\sec^3\left(a+2\log\left(cx^i\right)\right)\,dx=\frac{x^2\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^{1\mathrm{i}}\right)}^{2\mathrm{i}}}{2\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^{1\mathrm{i}}\right)}^{4\mathrm{i}}+\mathrm{e}^{a\,4\mathrm{i}}\,{\left(c\,x^{1\mathrm{i}}\right)}^{8\mathrm{i}}+1}$$

input
```
int(x/cos(a + 2*log(c*x^1i))^3,x)
```

output
```
(x^2*exp(a*1i)*(c*x^1i)^2i)/(2*exp(a*2i)*(c*x^1i)^4i + exp(a*4i)*(c*x^1i)^
8i + 1)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.09

$$\int x \sec^3\left(a+2\log\left(cx^i\right)\right)\,dx$$
$$= \frac{x^2\Big(2\cos\left(2\log(x^ic)+a\right)\sec\left(2\log(x^ic)+a\right)^3\sin\left(2\log(x^ic)+a\right)^2-2\cos\left(2\log(x^ic)+a\right)\sec\left(2\log(x^ic)\ \cdots}{4\cos\left(2\log\left(x^ic\right)+a\right)\Big(\sin\left(2\log\left(x^ic\right)\ \cdots}$$

input
```
int(x*sec(a+2*log(c*x^I))^3,x)
```

output
```
(x**2*(2*cos(2*log(x**i*c) + a)*sec(2*log(x**i*c) + a)**3*sin(2*log(x**i*c
) + a)**2 - 2*cos(2*log(x**i*c) + a)*sec(2*log(x**i*c) + a)**3 + cos(2*log
(x**i*c) + a)*sin(2*log(x**i*c) + a)*i + sin(2*log(x**i*c) + a)**2 + 1))/(
4*cos(2*log(x**i*c) + a)*(sin(2*log(x**i*c) + a)**2 - 1))
```

3.262 $\int \sec^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \sec^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx=\frac{1}{2}x\sec\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)-\frac{1}{2}ix\sec\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\tan\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)$$

output
```
1/2*x*sec(a+2*ln(c*x^(1/2*I)))-1/2*I*x*sec(a+2*ln(c*x^(1/2*I)))*tan(a+2*ln
(c*x^(1/2*I)))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 137 vs. 2(58) = 116.

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.36

$$\int \sec^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx=-\frac{\sec^2\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\left((1+2x^2)\cos\left(a+2\log\left(cx^{\frac{i}{2}}\right)-i\log(x)\right)+i(1-2x^2)\sin\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right.\right.}{2x^2}$$

input
```
Integrate[Sec[a + 2*Log[c*x^(I/2)]]^3,x]
```

output
```
-1/2*(Sec[a + 2*Log[c*x^(I/2)]]^2*((1 + 2*x^2)*Cos[a + 2*Log[c*x^(I/2)] -
I*Log[x]] + I*(1 - 2*x^2)*Sin[a + 2*Log[c*x^(I/2)] - I*Log[x]])*(Cos[2*(a
+ 2*Log[c*x^(I/2)] - I*Log[x])] + I*Sin[2*(a + 2*Log[c*x^(I/2)] - I*Log[x]
)]))/x^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5014, 5016, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)dx$$

$$\Big\downarrow \text{5014}$$

$$-2ix\left(cx^{\frac{i}{2}}\right)^{2i}\int \left(cx^{\frac{i}{2}}\right)^{-1-2i}\sec^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)d\left(cx^{\frac{i}{2}}\right)$$

$$\Big\downarrow \text{5016}$$

$$-16ie^{3ia}x\left(cx^{\frac{i}{2}}\right)^{2i}\int \frac{\left(cx^{\frac{i}{2}}\right)^{-1+4i}}{\left(e^{2ia}\left(cx^{\frac{i}{2}}\right)^{4i}+1\right)^3}d\left(cx^{\frac{i}{2}}\right)$$

$$\Big\downarrow \text{793}$$

$$\frac{2e^{ia}x\left(cx^{\frac{i}{2}}\right)^{2i}}{\left(1+e^{2ia}\left(cx^{\frac{i}{2}}\right)^{4i}\right)^2}$$

input
```
Int[Sec[a + 2*Log[c*x^(I/2)]]^3,x]
```

output
```
(2*E^(I*a)*(c*x^(I/2))^(2*I)*x)/(1 + E^((2*I)*a)*(c*x^(I/2))^(4*I))^2
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 5014
```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 119.79 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.57

method	result
parallelrisch	$-\frac{\sec\left(a+2\ln\left(c\,x^{\frac{i}{2}}\right)\right)\left(i\tan\left(a+2\ln\left(c\,x^{\frac{i}{2}}\right)\right)-1\right)x}{2}$
risch	$\frac{2x\left(x^{\frac{i}{2}}\right)^{2i}c^{2i}\mathrm{e}^{-\operatorname{csgn}\left(ix^{\frac{i}{2}}\right)\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)^{2}\pi+\operatorname{csgn}\left(ix^{\frac{i}{2}}\right)\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)\operatorname{csgn}(ic)\pi+\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)^{3}\pi-\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)^{2}\operatorname{csgn}(ic)\pi+ia\cdots}}{\left(c^{4i}\left(x^{\frac{i}{2}}\right)^{4i}\mathrm{e}^{-2\operatorname{csgn}\left(ix^{\frac{i}{2}}\right)\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)^{2}\pi}\mathrm{e}^{2\operatorname{csgn}\left(ix^{\frac{i}{2}}\right)\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)\operatorname{csgn}(ic)\pi}\mathrm{e}^{2\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)^{3}\pi}\mathrm{e}^{-2\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)^{2}\operatorname{csgn}(ic)\pi}\mathrm{e}^{2ia\cdots}\cdots\right.}$

input
```
int(sec(a+2*ln(c*x^(1/2*I)))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*sec(a+2*ln(c*x^(1/2*I)))*(I*tan(a+2*ln(c*x^(1/2*I)))-1)*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \sec^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx = -\frac{2\left(2\,x^2e^{(3i\,a+6i\,\log(c))}+e^{(5i\,a+10i\,\log(c))}\right)}{x^4+2\,x^2e^{(2i\,a+4i\,\log(c))}+e^{(4i\,a+8i\,\log(c))}}$$

input
```
integrate(sec(a+2*log(c*x^(1/2*I)))^3,x, algorithm="fricas")
```

output
```
-2*(2*x^2*e^(3*I*a + 6*I*log(c)) + e^(5*I*a + 10*I*log(c)))/(x^4 + 2*x^2*e
^(2*I*a + 4*I*log(c)) + e^(4*I*a + 8*I*log(c)))
```

Sympy [F]

$$\int \sec^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx = \int \sec^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx$$

input
```
integrate(sec(a+2*ln(c*x**(1/2*I)))**3,x)
```

output
```
Integral(sec(a + 2*log(c*x**(I/2)))**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(40) = 80.

Time = 0.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.60

$$\int \sec^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx$$

$$= \frac{2\left(\left(\cos\left(a\right)+i\,\sin\left(a\right)\right)\cos\left(2\,\log\left(c\right)\right)+\left(i\,\mathrm{co}\right.\right.}{\left(\cos\left(4\,a\right)+i\,\sin\left(4\,a\right)\right)\cos\left(8\,\log\left(c\right)\right)+2\left(\left(\cos\left(2\,a\right)+i\,\sin\left(2\,a\right)\right)\cos\left(4\,\log\left(c\right)\right)-\left(-i\,\cos\left(2\,a\right)+\sin\left(2\,a\right.\right.\right.}$$

input
```
integrate(sec(a+2*log(c*x^(1/2*I)))^3,x, algorithm="maxima")
```

output
```
2*((cos(a) + I*sin(a))*cos(2*log(c)) + (I*cos(a) - sin(a))*sin(2*log(c)))*
x*e^(6*arctan2(sin(1/2*log(x)), cos(1/2*log(x))))/((cos(4*a) + I*sin(4*a))
*cos(8*log(c)) + 2*((cos(2*a) + I*sin(2*a))*cos(4*log(c)) - (-I*cos(2*a) +
 sin(2*a))*sin(4*log(c)))*e^(4*arctan2(sin(1/2*log(x)), cos(1/2*log(x))))
+ (I*cos(4*a) - sin(4*a))*sin(8*log(c)) + e^(8*arctan2(sin(1/2*log(x)), co
s(1/2*log(x)))))
```

Giac [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.28

$$\int \sec^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx = -\frac{2\,c^{10i}e^{(5i\,a)}}{c^{8i}e^{(4i\,a)}+2\,c^{4i}x^2e^{(2i\,a)}+x^4}-\frac{4\,c^{6i}x^2e^{(3i\,a)}}{c^{8i}e^{(4i\,a)}+2\,c^{4i}x^2e^{(2i\,a)}+x^4}$$

input
```
integrate(sec(a+2*log(c*x^(1/2*I)))^3,x, algorithm="giac")
```

output
```
-2*c^(10*I)*e^(5*I*a)/(c^(8*I)*e^(4*I*a) + 2*c^(4*I)*x^2*e^(2*I*a) + x^4)
- 4*c^(6*I)*x^2*e^(3*I*a)/(c^(8*I)*e^(4*I*a) + 2*c^(4*I)*x^2*e^(2*I*a) + x
^4)
```

Mupad [B] (verification not implemented)

Time = 22.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \sec^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx = \frac{2\,x\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^{\frac{1}{2}\mathrm{i}}\right)}^{2\mathrm{i}}}{2\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^{\frac{1}{2}\mathrm{i}}\right)}^{4\mathrm{i}}+\mathrm{e}^{a\,4\mathrm{i}}\,{\left(c\,x^{\frac{1}{2}\mathrm{i}}\right)}^{8\mathrm{i}}+1}$$

input
```
int(1/cos(a + 2*log(c*x^(1i/2)))^3,x)
```

output
```
(2*x*exp(a*1i)*(c*x^(1i/2))^2i)/(2*exp(a*2i)*(c*x^(1i/2))^4i + exp(a*4i)*(
c*x^(1i/2))^8i + 1)
```

Reduce [F]

$$\int \sec^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx=-3\left(\int \sec\left(2\log\left(x^{\frac{i}{2}}c\right)+a\right)^3\tan\left(2\log\left(x^{\frac{i}{2}}c\right)+a\right)dx\right)i+\sec\left(2\log\left(x^{\frac{i}{2}}c\right)+a\right)^3x$$

input

```
int(sec(a+2*log(c*x^(1/2*I)))^3,x)
```

output

```
 - 3*int(sec(2*log(x**(i/2)*c) + a)**3*tan(2*log(x**(i/2)*c) + a),x)*i + s
ec(2*log(x**(i/2)*c) + a)**3*x
```

3.263 $\int \sec^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \sec^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx=\frac{2e^{3ia}\left(cx^{-\frac{i}{2}}\right)^{6i}x}{\left(1+e^{2ia}\left(cx^{-\frac{i}{2}}\right)^{4i}\right)^2}$$

output
```
2*exp(3*I*a)*(c/(x^(1/2*I)))^(6*I)*x/(1+exp(2*I*a)*(c/(x^(1/2*I)))^(4*I))^
2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 139 vs. 2(48) = 96.

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.90

$$\int \sec^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx$$
$$=\frac{\sec^2\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\left((1+2x^2)\cos\left(a+2\log\left(cx^{-\frac{i}{2}}\right)+i\log(x)\right)+i(-1+2x^2)\sin\left(a+2\log\left(cx^{-\right.\right.\right.}\right.}{4}$$

input
```
Integrate[Sec[a + 2*Log[c/x^(I/2)]]^3,x]
```

output
```
(Sec[a + 2*Log[c/x^(I/2)]]^2*((1 + 2*x^2)*Cos[a + 2*Log[c/x^(I/2)] + I*Log
[x]] + I*(-1 + 2*x^2)*Sin[a + 2*Log[c/x^(I/2)] + I*Log[x]])*(-2*Cos[2*(a +
 2*Log[c/x^(I/2)] + I*Log[x])] + (2*I)*Sin[2*(a + 2*Log[c/x^(I/2)] + I*Log
[x])]))/(4*x^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5014, 5016, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)dx$$

$$\Big\downarrow \text{5014}$$

$$2ix\left(cx^{-\frac{i}{2}}\right)^{-2i}\int \left(cx^{-\frac{i}{2}}\right)^{-1+2i}\sec^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)d\left(cx^{-\frac{i}{2}}\right)$$

$$\Big\downarrow \text{5016}$$

$$16ie^{3ia}x\left(cx^{-\frac{i}{2}}\right)^{-2i}\int \frac{\left(cx^{-\frac{i}{2}}\right)^{-1+8i}}{\left(e^{2ia}\left(cx^{-\frac{i}{2}}\right)^{4i}+1\right)^3}d\left(cx^{-\frac{i}{2}}\right)$$

$$\Big\downarrow \text{796}$$

$$\frac{2e^{3ia}x\left(cx^{-\frac{i}{2}}\right)^{6i}}{\left(1+e^{2ia}\left(cx^{-\frac{i}{2}}\right)^{4i}\right)^2}$$

input
```
Int[Sec[a + 2*Log[c/x^(I/2)]]^3,x]
```

output
```
(2*E^((3*I)*a)*(c/x^(I/2))^(6*I)*x)/(1 + E^((2*I)*a)*(c/x^(I/2))^(4*I))^2
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 5014
```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 96.67 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

method	result
parallelrisch	$\frac{\left(i \sin\left(a+2\ln\left(c\,x^{-\frac{i}{2}}\right)\right)+\cos\left(a+2\ln\left(c\,x^{-\frac{i}{2}}\right)\right)\right)x}{\cos\left(2a+4\ln\left(c\,x^{-\frac{i}{2}}\right)\right)+1}$
risch	$\frac{2x\,c^{6i}\left(x^{\frac{i}{2}}\right)^{-6i} \mathrm{e}^{3\,\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^{3}\pi-3\,\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^{2}\mathrm{csgn}(ic)\pi-3\,\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^{2}\mathrm{csgn}\left(ix^{-\frac{i}{2}}\right)\pi+3\,\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)\mathrm{csgn}(ic)\,\mathrm{csgn}}}{\left(c^{4i}\left(x^{\frac{i}{2}}\right)^{-4i}\mathrm{e}^{2\,\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^{3}\pi}\mathrm{e}^{-2\,\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^{2}\mathrm{csgn}(ic)\pi}\mathrm{e}^{-2\,\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^{2}\mathrm{csgn}\left(ix^{-\frac{i}{2}}\right)\pi}\mathrm{e}^{2\,\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)\mathrm{csgn}(ic)\,\mathrm{csgn}\left(i\right.}\right.}$

input
```
int(sec(a+2*ln(c/(x^(1/2*I))))^3,x,method=_RETURNVERBOSE)
```

output
```
(I*sin(a+2*ln(c*x^(-1/2*I)))+cos(a+2*ln(c*x^(-1/2*I))))/(cos(2*a+4*ln(c*x^
(-1/2*I)))+1)*x
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(27) = 54$.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \sec^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx = -\frac{2\left(2\,x^2e^{(2i\,a+4i\,\log(c))}+1\right)}{x^4e^{(5i\,a+10i\,\log(c))}+2\,x^2e^{(3i\,a+6i\,\log(c))}+e^{(i\,a+2i\,\log(c))}}$$

input
```
integrate(sec(a+2*log(c/(x^(1/2*I))))^3,x, algorithm="fricas")
```

output
```
-2*(2*x^2*e^(2*I*a + 4*I*log(c)) + 1)/(x^4*e^(5*I*a + 10*I*log(c)) + 2*x^2
*e^(3*I*a + 6*I*log(c)) + e^(I*a + 2*I*log(c)))
```

Sympy [F]

$$\int \sec^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx = \int \sec^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx$$

input
```
integrate(sec(a+2*ln(c/(x**(1/2*I))))**3,x)
```

output
```
Integral(sec(a + 2*log(c/x**(I/2)))**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(27) = 54$.

Time = 0.12 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.38

$$\int \sec^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx$$

$$= \frac{2\left(\left(\cos\left(3\,a\right)+i\,\sin\left(3\,a\right)\right)\cos\left(6\,\log\left(c\right)\right)+\left(i\,c\right.\right.}{\left(\left(\cos\left(4\,a\right)+i\,\sin\left(4\,a\right)\right)\cos\left(8\,\log\left(c\right)\right)-\left(-i\,\cos\left(4\,a\right)+\sin\left(4\,a\right)\right)\sin\left(8\,\log\left(c\right)\right)\right)e^{\left(8\,\arctan\left(\sin\left(\frac{1}{2}\,\log\left(x\right)\right),\cos\left(\frac{1}{2}\right.\right.\right.}}$$

input
```
integrate(sec(a+2*log(c/(x^(1/2*I))))^3,x, algorithm="maxima")
```

output
```
2*((cos(3*a) + I*sin(3*a))*cos(6*log(c)) + (I*cos(3*a) - sin(3*a))*sin(6*l
og(c)))*x*e^(6*arctan2(sin(1/2*log(x)), cos(1/2*log(x))))/(((cos(4*a) + I*
sin(4*a))*cos(8*log(c)) - (-I*cos(4*a) + sin(4*a))*sin(8*log(c)))*e^(8*arc
tan2(sin(1/2*log(x)), cos(1/2*log(x)))) + 2*((cos(2*a) + I*sin(2*a))*cos(4
*log(c)) + (I*cos(2*a) - sin(2*a))*sin(4*log(c)))*e^(4*arctan2(sin(1/2*log
(x)), cos(1/2*log(x)))) + 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(27) = 54.

Time = 0.98 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.73

$$\int \sec^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx = -\frac{4\,c^{4i}x^2e^{(2i\,a)}}{c^{10i}x^4e^{(5i\,a)}+2\,c^{6i}x^2e^{(3i\,a)}+c^{2i}e^{(i\,a)}} - \frac{2}{c^{10i}x^4e^{(5i\,a)}+2\,c^{6i}x^2e^{(3i\,a)}+c^{2i}e^{(i\,a)}}$$

input
```
integrate(sec(a+2*log(c/(x^(1/2*I))))^3,x, algorithm="giac")
```

output
```
-4*c^(4*I)*x^2*e^(2*I*a)/(c^(10*I)*x^4*e^(5*I*a) + 2*c^(6*I)*x^2*e^(3*I*a)
 + c^(2*I)*e^(I*a)) - 2/(c^(10*I)*x^4*e^(5*I*a) + 2*c^(6*I)*x^2*e^(3*I*a)
+ c^(2*I)*e^(I*a))
```

Mupad [B] (verification not implemented)

Time = 24.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \sec^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx = \frac{2\,x\,\mathrm{e}^{a\,3\mathrm{i}}\,{\left(\frac{c}{x^{\frac{1}{2}\mathrm{i}}}\right)}^{6\mathrm{i}}}{{\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(\frac{c}{x^{\frac{1}{2}\mathrm{i}}}\right)}^{4\mathrm{i}}+1\right)}^2}$$

input
```
int(1/cos(a + 2*log(c/x^(1i/2)))^3,x)
```

output
```
(2*x*exp(a*3i)*(c/x^(1i/2))^6i)/(exp(a*2i)*(c/x^(1i/2))^4i + 1)^2
```

Reduce [F]

$$\int \sec^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx = 3\left(\int \sec\left(2\log\left(\frac{c}{x^{\frac{i}{2}}}\right)+a\right)^3\tan\left(2\log\left(\frac{c}{x^{\frac{i}{2}}}\right)+a\right)dx\right)i+\sec\left(2\log\left(\frac{c}{x^{\frac{i}{2}}}\right)+a\right)^3 x$$

input
```
int(sec(a+2*log(c/(x^(1/2*I))))^3,x)
```

output
```
3*int(sec(2*log(c/x**(i/2)) + a)**3*tan(2*log(c/x**(i/2)) + a),x)*i + sec(
2*log(c/x**(i/2)) + a)**3*x
```

3.264 $\int \sec^p\left(a+\frac{i\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \sec^p\left(a+\frac{i\log\left(cx^n\right)}{n(-2+p)}\right)\,dx = \frac{e^{-2ia}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}}\left(1+e^{2ia}(cx^n)^{\frac{2}{n(2-p)}}\right)\sec^p\left(a-\frac{i\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

output
```
1/2*(2-p)*x*(1+exp(2*I*a)*(c*x^n)^(2/n/(2-p)))*sec(a-I*ln(c*x^n)/n/(2-p))^
p/exp(2*I*a)/(1-p)/((c*x^n)^(2/n/(2-p)))
```

Mathematica [A] (warning: unable to verify)

Time = 1.18 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.23

$$\int \sec^p\left(a+\frac{i\log\left(cx^n\right)}{n(-2+p)}\right)\,dx = \frac{2^{-1+p}e^{-ia}(-2+p)x(cx^n)^{\frac{1}{n(-2+p)}}\left(\frac{e^{\frac{ia(2+p)}{-2+p}}(cx^n)^{\frac{1}{n(-2+p)}}}{e^{\frac{2iap}{-2+p}}+e^{\frac{4ia}{-2+p}}(cx^n)^{\frac{2}{n(-2+p)}}}\right)^{-1+p}}{-1+p}$$

input
```
Integrate[Sec[a + (I*Log[c*x^n])/(n*(-2 + p))]^p,x]
```

output
```
(2^(-1 + p)*(-2 + p)*x*(c*x^n)^(1/(n*(-2 + p)))*((E^((I*a*(2 + p))/(-2 + p
))*(c*x^n)^(1/(n*(-2 + p))))/(E^(((2*I)*a*p)/(-2 + p)) + E^(((4*I)*a)/(-2
+ p))*(c*x^n)^(2/(n*(-2 + p)))))^(-1 + p))/(E^(I*a)*(-1 + p))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5014, 5018, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^p\left(a+\frac{i\log\left(cx^n\right)}{n(p-2)}\right)dx$$

$$\Big\downarrow \text{ 5014}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\sec^p\left(a-\frac{i\log(cx^n)}{n(2-p)}\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{ 5018}$$

$$\frac{x(cx^n)^{-\frac{p}{n(2-p)}-\frac{1}{n}}\left(1+e^{2ia}(cx^n)^{\frac{2}{n(2-p)}}\right)^p\sec^p\left(a-\frac{i\log(cx^n)}{n(2-p)}\right)\int (cx^n)^{\frac{p}{2n-np}+\frac{1}{n}-1}\left(e^{2ia}(cx^n)^{\frac{2}{n(2-p)}}+1\right)^{-p}d(cx^n)}{n}$$

$$\Big\downarrow \text{ 793}$$

$$\frac{e^{-2ia}(2-p)x(cx^n)^{-\frac{p}{n(2-p)}-\frac{1}{n}}\left(1+e^{2ia}(cx^n)^{\frac{2}{n(2-p)}}\right)\sec^p\left(a-\frac{i\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

input
```
Int[Sec[a + (I*Log[c*x^n])/(n*(-2 + p))]^p,x]
```

output
```
((2 - p)*x*(c*x^n)^(-n^(-1) - p/(n*(2 - p)))*(1 + E^((2*I)*a)*(c*x^n)^(2/(
n*(2 - p))))*Sec[a - (I*Log[c*x^n])/(n*(2 - p))]^p)/(2*E^((2*I)*a)*(1 - p)
)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 5014
```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5018
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \sec\left(a+\frac{i\ln\left(c\,x^{n}\right)}{n\left(-2+p\right)}\right)^{p}dx$$

input
```
int(sec(a+I*ln(c*x^n)/n/(-2+p))^p,x)
```

output
```
int(sec(a+I*ln(c*x^n)/n/(-2+p))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.57

$$\int \sec^p\left(a+\frac{i\log\left(cx^n\right)}{n(-2+p)}\right)\,dx = \frac{\left((p-2)xe^{\left(\frac{2\,(i\,anp-2i\,an-n\,\log(x)-\log(c))}{np-2\,n}\right)}+(p-2)x\right)\left(\frac{2\,e^{\left(\frac{i\,anp-2i\,an-n\,\log(x)-\log(c)}{np-2\,n}\right)}}{e^{\left(\frac{2\,(i\,anp-2i\,an-n\,\log(x)-\log(c))}{np-2\,n}\right)}+1}\right)^{p}e^{\left(-\frac{2\,(i\,anp-2i\,an-n\,\log(x)-\log(c))}{np-2\,n}\right)}}{2\,(p-1)}$$

input
```
integrate(sec(a+I*log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

output
```
1/2*((p - 2)*x*e^(2*(I*a*n*p - 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n)) +
 (p - 2)*x)*(2*e^((I*a*n*p - 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n))/(e^
(2*(I*a*n*p - 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n)) + 1))^p*e^(-2*(I*a
*n*p - 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n))/(p - 1)
```

Sympy [F]

$$\int \sec^p\left(a+\frac{i\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\int \sec^p\left(a+\frac{i\log\left(cx^n\right)}{n\left(p-2\right)}\right)\,dx$$

input
```
integrate(sec(a+I*ln(c*x**n)/n/(-2+p))**p,x)
```

output
```
Integral(sec(a + I*log(c*x**n)/(n*(p - 2)))**p, x)
```

Maxima [F]

$$\int \sec^p\left(a+\frac{i\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\int \sec\left(a+\frac{i\log\left(cx^n\right)}{n(p-2)}\right)^p\,dx$$

input
```
integrate(sec(a+I*log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
```

output
```
integrate(sec(a + I*log(c*x^n)/(n*(p - 2)))^p, x)
```

Giac [F]

$$\int \sec^p\left(a+\frac{i\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\int \sec\left(a+\frac{i\log\left(cx^n\right)}{n(p-2)}\right)^p\,dx$$

input
```
integrate(sec(a+I*log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
```

output
```
integrate(sec(a + I*log(c*x^n)/(n*(p - 2)))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^p\left(a+\frac{i\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\int \left(\frac{1}{\cos\left(a+\frac{\ln(c\,x^n)\,1\mathrm{i}}{n\,(p-2)}\right)}\right)^p\,dx$$

input
```
int((1/cos(a + (log(c*x^n)*1i)/(n*(p - 2))))^p,x)
```

output
```
int((1/cos(a + (log(c*x^n)*1i)/(n*(p - 2))))^p, x)
```

Reduce [F]

$$\int \sec^p\left(a+\frac{i\log\left(cx^n\right)}{n(-2+p)}\right)\,dx$$
$$=\frac{\sec\left(\frac{\log(x^nc)i+anp-2an}{np-2n}\right)^p px-2\sec\left(\frac{\log(x^nc)i+anp-2an}{np-2n}\right)^p x-\left(\int \sec\left(\frac{\log(x^nc)i+anp-2an}{np-2n}\right)^p\tan\left(\frac{\log(x^nc)i+anp-2an}{np-2n}\right.\right.}{p-2}$$

input
```
int(sec(a+I*log(c*x^n)/n/(-2+p))^p,x)
```

output
```
(sec((log(x**n*c)*i + a*n*p - 2*a*n)/(n*p - 2*n))**p*p*x - 2*sec((log(x**n
*c)*i + a*n*p - 2*a*n)/(n*p - 2*n))**p*x - int(sec((log(x**n*c)*i + a*n*p
- 2*a*n)/(n*p - 2*n))**p*tan((log(x**n*c)*i + a*n*p - 2*a*n)/(n*p - 2*n)),
x)*i*p)/(p - 2)
```

3.265 $\int \sec^p\left(a-\frac{i\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \sec^p\left(a-\frac{i\log(cx^n)}{n(-2+p)}\right)\,dx=\frac{(2-p)x\left(1+e^{2ia}(cx^n)^{-\frac{2}{n(2-p)}}\right)\sec^p\left(a+\frac{i\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

output
```
(2-p)*x*(1+exp(2*I*a)/((c*x^n)^(2/n/(2-p))))*sec(a+I*ln(c*x^n)/n/(2-p))^p/
(2-2*p)
```

Mathematica [A] (warning: unable to verify)

Time = 1.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.67

$$\int \sec^p\left(a-\frac{i\log(cx^n)}{n(-2+p)}\right)\,dx=\frac{2^{-1+p}e^{ia}(-2+p)x(cx^n)^{\frac{1}{n(-2+p)}}\left(\frac{e^{\frac{ia(2+p)}{-2+p}}(cx^n)^{\frac{1}{n(-2+p)}}}{e^{\frac{4ia}{-2+p}}+e^{\frac{2iap}{-2+p}}(cx^n)^{\frac{2}{n(-2+p)}}}\right)^{-1+p}}{-1+p}$$

input
```
Integrate[Sec[a - (I*Log[c*x^n])/(n*(-2 + p))]^p,x]
```

output
```
(2^(-1 + p)*E^(I*a)*(-2 + p)*x*(c*x^n)^(1/(n*(-2 + p)))*((E^((I*a*(2 + p))
/(-2 + p))*(c*x^n)^(1/(n*(-2 + p))))/(E^(((4*I)*a)/(-2 + p)) + E^(((2*I)*a
*p)/(-2 + p))*(c*x^n)^(2/(n*(-2 + p)))))^(-1 + p))/(-1 + p)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.59, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5014, 5018, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^p\left(a-\frac{i\log\left(cx^n\right)}{n(p-2)}\right)dx$$

$$\Big\downarrow \text{5014}$$

$$\frac{x\left(cx^n\right)^{-1/n}\int \left(cx^n\right)^{\frac{1}{n}-1}\sec^p\left(a+\frac{i\log(cx^n)}{n(2-p)}\right)d\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{5018}$$

$$\frac{x\left(cx^n\right)^{\frac{p}{n(2-p)}-\frac{1}{n}}\left(1+e^{2ia}\left(cx^n\right)^{-\frac{2}{n(2-p)}}\right)^p\sec^p\left(a+\frac{i\log(cx^n)}{n(2-p)}\right)\int \left(cx^n\right)^{\frac{1-\frac{p}{2-p}}{n}-1}\left(e^{2ia}\left(cx^n\right)^{-\frac{2}{n(2-p)}}+1\right)^{-p}d\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{796}$$

$$\frac{(2-p)x\left(cx^n\right)^{\frac{2(1-p)}{n(2-p)}+\frac{p}{n(2-p)}-\frac{1}{n}}\left(1+e^{2ia}\left(cx^n\right)^{-\frac{2}{n(2-p)}}\right)\sec^p\left(a+\frac{i\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

input
```
Int[Sec[a - (I*Log[c*x^n])/(n*(-2 + p))]^p,x]
```

output
```
((2 - p)*x*(c*x^n)^(-n^(-1) + (2*(1 - p))/(n*(2 - p)) + p/(n*(2 - p)))*(1
+ E^((2*I)*a)/(c*x^n)^(2/(n*(2 - p))))*Sec[a + (I*Log[c*x^n])/(n*(2 - p))]
^p)/(2*(1 - p))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 5014
```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5018
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \sec\left(a-\frac{i\ln\left(c\,x^{n}\right)}{n\left(-2+p\right)}\right)^{p}dx$$

input
```
int(sec(a-I*ln(c*x^n)/n/(-2+p))^p,x)
```

output
```
int(sec(a-I*ln(c*x^n)/n/(-2+p))^p,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(55) = 110.

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.13

$$\int \sec^{p}\left(a-\frac{i\log\left(cx^{n}\right)}{n(-2+p)}\right)\,dx = \frac{\left((p-2)xe^{\left(\frac{2\left(-i\,anp+2i\,an-n\log(x)-\log(c)\right)}{np-2\,n}\right)}+(p-2)x\right)\left(\frac{2\,e^{\left(\frac{-i\,anp+2i\,an-n\log(x)-\log(c)}{np-2\,n}\right)}}{e^{\left(\frac{2\left(-i\,anp+2i\,an-n\log(x)-\log(c)\right)}{np-2\,n}\right)}+1}\right)^{p}e^{\left(-\frac{2\left(-i\,anp+2i\,an-n\log(x)-\right.}{np-2\,n}\right.}}{2\left(p-1\right)}$$

input
```
integrate(sec(a-I*log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

output
```
1/2*((p - 2)*x*e^(2*(-I*a*n*p + 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n))
+ (p - 2)*x)*(2*e^((-I*a*n*p + 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n))/(
e^(2*(-I*a*n*p + 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n)) + 1))^p*e^(-2*(
-I*a*n*p + 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n))/(p - 1)
```

Sympy [F]

$$\int \sec^p\left(a - \frac{i\log\left(cx^n\right)}{n(-2+p)}\right)\,dx = \int \sec^p\left(a - \frac{i\log\left(cx^n\right)}{n\left(p-2\right)}\right)dx$$

input
```
integrate(sec(a-I*ln(c*x**n)/n/(-2+p))**p,x)
```

output
```
Integral(sec(a - I*log(c*x**n)/(n*(p - 2)))**p, x)
```

Maxima [F]

$$\int \sec^p\left(a - \frac{i\log\left(cx^n\right)}{n(-2+p)}\right)\,dx = \int \sec\left(a - \frac{i\log\left(cx^n\right)}{n(p-2)}\right)^p\,dx$$

input
```
integrate(sec(a-I*log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
```

output
```
integrate(sec(-a + I*log(c*x^n)/(n*(p - 2)))^p, x)
```

Giac [F]

$$\int \sec^p\left(a-\frac{i\log(cx^n)}{n(-2+p)}\right)\,dx=\int \sec\left(a-\frac{i\log(cx^n)}{n(p-2)}\right)^p\,dx$$

input
```
integrate(sec(a-I*log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
```

output
```
integrate(sec(a - I*log(c*x^n)/(n*(p - 2)))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^p\left(a-\frac{i\log(cx^n)}{n(-2+p)}\right)\,dx=\int \left(\frac{1}{\cos\left(a-\frac{\ln(c\,x^n)\,1\mathrm{i}}{n\,(p-2)}\right)}\right)^p\,dx$$

input
```
int((1/cos(a - (log(c*x^n)*1i)/(n*(p - 2))))^p,x)
```

output
```
int((1/cos(a - (log(c*x^n)*1i)/(n*(p - 2))))^p, x)
```

Reduce [F]

$$\int \sec^p\left(a-\frac{i\log(cx^n)}{n(-2+p)}\right)\,dx$$
$$=\frac{\sec\left(\frac{\log(x^nc)i-anp+2an}{np-2n}\right)^p px-2\sec\left(\frac{\log(x^nc)i-anp+2an}{np-2n}\right)^p x-\left(\int \sec\left(\frac{\log(x^nc)i-anp+2an}{np-2n}\right)^p\tan\left(\frac{\log(x^nc)i-anp+2an}{np-2n}\right.\right.}{p-2}$$

input
```
int(sec(a-I*log(c*x^n)/n/(-2+p))^p,x)
```

output
```
(sec((log(x**n*c)*i - a*n*p + 2*a*n)/(n*p - 2*n))**p*p*x - 2*sec((log(x**n
*c)*i - a*n*p + 2*a*n)/(n*p - 2*n))**p*x - int(sec((log(x**n*c)*i - a*n*p
+ 2*a*n)/(n*p - 2*n))**p*tan((log(x**n*c)*i - a*n*p + 2*a*n)/(n*p - 2*n)),
x)*i*p)/(p - 2)
```

3.266 $\int \sqrt{\sec(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \sqrt{\sec(a+b\log(cx^n))}\,dx = \frac{2x\sqrt{1+e^{2ia}(cx^n)^{2ib}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}\left(1-\frac{2i}{bn}\right),\frac{1}{4}\left(5-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sec(a+b\log(cx^n))}}{2+ibn}$$

output
```
2*x*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)*hypergeom([1/2, 1/4-1/2*I/b/n],[5
/4-1/2*I/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))*sec(a+b*ln(c*x^n))^(1/2)/(2+I*b
*n)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91

$$\int \sqrt{\sec(a+b\log(cx^n))}\,dx = -\frac{2i\left(1+e^{2i(a+b\log(cx^n))}\right)x\,\text{Hypergeometric2F1}\left(1,\frac{3}{4}-\frac{i}{2bn},\frac{5}{4}-\frac{i}{2bn},-e^{2i(a+b\log(cx^n))}\right)\sqrt{\sec(a+b\log(cx^n))}}{-2i+bn}$$

input
```
Integrate[Sqrt[Sec[a + b*Log[c*x^n]]],x]
```

output
```
((-2*I)*(1 + E^((2*I)*(a + b*Log[c*x^n])))*x*Hypergeometric2F1[1, 3/4 - (I
/2)/(b*n), 5/4 - (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))]*Sqrt[Sec[a +
b*Log[c*x^n]]])/(-2*I + b*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5014, 5018, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(a+b\log(cx^n))}\,dx$$

$$\Big\downarrow \text{5014}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\sqrt{\sec(a+b\log(cx^n))}d(cx^n)}{n}$$

$$\Big\downarrow \text{5018}$$

$$\frac{x(cx^n)^{-\frac{1}{n}-\frac{ib}{2}}\sqrt{1+e^{2ia}(cx^n)^{2ib}}\sqrt{\sec(a+b\log(cx^n))}\int \frac{(cx^n)^{\frac{ib}{2}+\frac{1}{n}-1}}{\sqrt{e^{2ia}(cx^n)^{2ib}+1}}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x\sqrt{1+e^{2ia}(cx^n)^{2ib}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}\left(1-\frac{2i}{bn}\right),\frac{1}{4}\left(5-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sec(a+b\log(cx^n))}}{2+ibn}$$

input
```
Int[Sqrt[Sec[a + b*Log[c*x^n]]],x]
```

output
```
(2*x*Sqrt[1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Hypergeometric2F1[1/2, (1 - (
2*I)/(b*n))/4, (5 - (2*I)/(b*n))/4, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))]*Sqrt
[Sec[a + b*Log[c*x^n]]])/(2 + I*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5014
```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5018
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \sqrt{\sec \left(a+b \ln \left(c \,x^{n}\right)\right)}dx$$

input
```
int(sec(a+b*ln(c*x^n))^(1/2),x)
```

output
```
int(sec(a+b*ln(c*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\sec \left(a+b \log \left(cx^n\right)\right)}\, dx=\text{Exception raised: TypeError}$$

input
```
integrate(sec(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{\sec\left(a+b\log\left(cx^{n}\right)\right)}\,dx=\int \sqrt{\sec\left(a+b\log\left(cx^{n}\right)\right)}\,dx$$

input
```
integrate(sec(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(sqrt(sec(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \sqrt{\sec\left(a+b\log\left(cx^{n}\right)\right)}\,dx=\int \sqrt{\sec\left(b\log\left(cx^{n}\right)+a\right)}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(b*log(c*x^n) + a)), x)
```

Giac [F]

$$\int \sqrt{\sec\left(a+b\log\left(cx^{n}\right)\right)}\,dx=\int \sqrt{\sec\left(b\log\left(cx^{n}\right)+a\right)}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sec(b*log(c*x^n) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sec (a+b\log (cx^n))}\,dx=\int \sqrt{\frac{1}{\cos \left(a+b\,\ln \left(c\,x^n\right)\right)}}\,dx$$

input
```
int((1/cos(a + b*log(c*x^n)))^(1/2),x)
```

output
```
int((1/cos(a + b*log(c*x^n)))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\sec (a+b\log (cx^n))}\,dx=\sqrt{\sec (\log (x^n c)\,b+a)}\,x-\frac{\left(\int \sqrt{\sec (\log (x^n c)\,b+a)}\,\tan (\log (x^n c)\,b+a)\,dx\right) bn}{2}$$

input
```
int(sec(a+b*log(c*x^n))^(1/2),x)
```

output
```
(2*sqrt(sec(log(x**n*c)*b + a))*x - int(sqrt(sec(log(x**n*c)*b + a))*tan(l
og(x**n*c)*b + a),x)*b*n)/2
```

3.267 $\int \frac{\sqrt{\sec(a+b\log(cx^n))}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{\sqrt{\sec (a+b\log (cx^n))}}{x}\,dx = \frac{2\sqrt{\cos (a+b\log (cx^n))}\,\text{EllipticF}\left(\frac{1}{2}(a+b\log (cx^n)),2\right)\sqrt{\sec (a+b\log (cx^n))}}{bn}$$

output
```
2*cos(a+b*ln(c*x^n))^(1/2)*InverseJacobiAM(1/2*a+1/2*b*ln(c*x^n),2^(1/2))*
sec(a+b*ln(c*x^n))^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\sec (a+b\log (cx^n))}}{x}\,dx = \frac{2\sqrt{\cos (a+b\log (cx^n))}\,\text{EllipticF}\left(\frac{1}{2}(a+b\log (cx^n)),2\right)\sqrt{\sec (a+b\log (cx^n))}}{bn}$$

input
```
Integrate[Sqrt[Sec[a + b*Log[c*x^n]]]/x,x]
```

output
```
(2*Sqrt[Cos[a + b*Log[c*x^n]]]*EllipticF[(a + b*Log[c*x^n])/2, 2]*Sqrt[Sec
[a + b*Log[c*x^n]]])/(b*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3039, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(a+b\log(cx^n))}}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \sqrt{\sec(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\csc\left(a+b\log(cx^n)+\frac{\pi}{2}\right)}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\sqrt{\sec(a+b\log(cx^n))}\sqrt{\cos(a+b\log(cx^n))}\int \frac{1}{\sqrt{\cos(a+b\log(cx^n))}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(a+b\log(cx^n))}\sqrt{\cos(a+b\log(cx^n))}\int \frac{1}{\sqrt{\sin\left(a+b\log(cx^n)+\frac{\pi}{2}\right)}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\sec(a+b\log(cx^n))}\sqrt{\cos(a+b\log(cx^n))}\operatorname{EllipticF}\left(\frac{1}{2}(a+b\log(cx^n)),2\right)}{bn}$$

input
```
Int[Sqrt[Sec[a + b*Log[c*x^n]]]/x,x]
```

output
```
(2*Sqrt[Cos[a + b*Log[c*x^n]]]*EllipticF[(a + b*Log[c*x^n])/2, 2]*Sqrt[Sec
[a + b*Log[c*x^n]]])/(b*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(51) = 102$.

Time = 0.80 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.35

method	result
derivativedivides	$-\frac{2\sqrt{\left(2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{\frac{1}{2}-\frac{\cos\left(a+2b\ln\left(\sqrt{c\,x^n}\right)\right)}{2}}\,\sqrt{-2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+1}\ \mathrm{Elliptic}}{n\sqrt{-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\ \sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-}}$
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{\frac{1}{2}-\frac{\cos\left(a+2b\ln\left(\sqrt{c\,x^n}\right)\right)}{2}}\,\sqrt{-2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+1}\ \mathrm{Elliptic}}{n\sqrt{-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\ \sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-}}$

input
```
int(sec(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2/n*((2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/
2)*(sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cos(1/2*a+1/2*b*ln(c*x^n))^2+1
)^(1/2)/(-2*sin(1/2*a+1/2*b*ln(c*x^n))^4+sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/
2)*EllipticF(cos(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/sin(1/2*a+1/2*b*ln(c*x^n)
)/(2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{\sec (a+b\log (cx^n))}}{x}\,dx = \frac{-i\sqrt{2}\text{weierstrassPInverse}(-4,0,\cos (bn\log (x)+b\log (c)+a)+i\,\sin (bn\log (x)+b\log (c)+a))+i\sqrt{}}{bn}$$

input
```
integrate(sec(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b*n*log(x) + b*log(c) + a) + I*
sin(b*n*log(x) + b*log(c) + a)) + I*sqrt(2)*weierstrassPInverse(-4, 0, cos
(b*n*log(x) + b*log(c) + a) - I*sin(b*n*log(x) + b*log(c) + a)))/(b*n)
```

Sympy [F]

$$\int \frac{\sqrt{\sec (a+b\log (cx^n))}}{x}\,dx = \int \frac{\sqrt{\sec (a+b\log (cx^n))}}{x}\,dx$$

input
```
integrate(sec(a+b*ln(c*x**n))**(1/2)/x,x)
```

output
```
Integral(sqrt(sec(a + b*log(c*x**n)))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = \int \frac{\sqrt{\sec\left(b\log\left(cx^n\right)+a\right)}}{x}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(b*log(c*x^n) + a))/x, x)
```

Giac [F]

$$\int \frac{\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = \int \frac{\sqrt{\sec\left(b\log\left(cx^n\right)+a\right)}}{x}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(sec(b*log(c*x^n) + a))/x, x)
```

Mupad [B] (verification not implemented)

Time = 19.81 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = \frac{2\sqrt{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}\,\sqrt{\frac{1}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}}\;\mathrm{F}\left(\frac{a}{2}+\frac{b\,\ln\left(c\,x^n\right)}{2}\middle|2\right)}{b\,n}$$

input
```
int((1/cos(a + b*log(c*x^n)))^(1/2)/x,x)
```

output
```
(2*cos(a + b*log(c*x^n))^(1/2)*(1/cos(a + b*log(c*x^n)))^(1/2)*ellipticF(a
/2 + (b*log(c*x^n))/2, 2))/(b*n)
```

Reduce [F]

$$\int \frac{\sqrt{\sec(a+b\log(cx^n))}}{x}\,dx = \int \frac{\sqrt{\sec(\log(x^n c)\,b+a)}}{x}dx$$

input
```
int(sec(a+b*log(c*x^n))^(1/2)/x,x)
```

output
```
int(sqrt(sec(log(x**n*c)*b + a))/x,x)
```

3.268 $\int \sec^{\frac{3}{2}}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \sec^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \frac{2x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i}{bn}\right),\frac{1}{4}\left(7-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)\sec^{\frac{3}{2}}(a+b\log(cx$$
$$\overline{2+3ibn}$$

output
```
2*x*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)*hypergeom([3/2, 3/4-1/2*I/b/n],[7
/4-1/2*I/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))*sec(a+b*ln(c*x^n))^(3/2)/(2+3*I
*b*n)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 415 vs. 2(109) = 218.

Time = 4.34 (sec) , antiderivative size = 415, normalized size of antiderivative = 3.81

$$\int \sec^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \frac{\sqrt{2}x^{1-ibn}\left(-\left(\left(4+b^2n^2\right)x^{2ibn}\sqrt{\frac{e^{ia}(cx^n)^{ib}}{1+e^{2ia}(cx^n)^{2ib}}}\sqrt{1+e^{2ia}\left(cx^n\right)^{2ib}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4}-\frac{i}{2bn},\frac{7}{4}-\frac{i}{2bn},\right.\right.\right.}{}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]]^(3/2),x]
```

output
```
(Sqrt[2]*x^(1 - I*b*n)*(-((4 + b^2*n^2)*x^((2*I)*b*n)*Sqrt[(E^(I*a)*(c*x^n
)^(I*b))/(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))]*Sqrt[1 + E^((2*I)*a)*(c*x^n)
^((2*I)*b)]*Hypergeometric2F1[1/2, 3/4 - (I/2)/(b*n), 7/4 - (I/2)/(b*n), -
(E^((2*I)*a)*(c*x^n)^((2*I)*b))]) + (-2*I + 3*b*n)*((2*I - b*n)*Sqrt[(E^(I
*a)*(c*x^n)^(I*b))/(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))]*Sqrt[1 + E^((2*I)*
a)*(c*x^n)^((2*I)*b)]*Hypergeometric2F1[1/2, -1/4*(2*I + b*n)/(b*n), 3/4 -
 (I/2)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))] + Sqrt[2]*x^(I*b*n)*Sqrt[S
ec[a + b*Log[c*x^n]]]*(b*n*Cos[b*n*Log[x]] - 2*Sin[b*n*Log[x]]))))/(b*n*(-
2*I + 3*b*n)*(-2*Cos[a - b*n*Log[x] + b*Log[c*x^n]] + b*n*Sin[a - b*n*Log[
x] + b*Log[c*x^n]]))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5014, 5018, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{3}{2}}(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{ 5014}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\sec^{\frac{3}{2}}(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{ 5018}$$

$$\frac{x(cx^n)^{-\frac{1}{n}-\frac{3ib}{2}}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\sec^{\frac{3}{2}}(a+b\log(cx^n))\int \frac{(cx^n)^{\frac{3ib}{2}+\frac{1}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^{3/2}}d(cx^n)}{n}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{2x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i}{bn}\right),\frac{1}{4}\left(7-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)\sec^{\frac{3}{2}}(a+b\log(cx^n))}{2+3ibn}$$

input
```
Int[Sec[a + b*Log[c*x^n]]^(3/2),x]
```

output
```
(2*x*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2)*Hypergeometric2F1[3/2, (3 -
 (2*I)/(b*n))/4, (7 - (2*I)/(b*n))/4, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))]*Se
c[a + b*Log[c*x^n]]^(3/2))/(2 + (3*I)*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5014
```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5018
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \sec \left(a+b \ln \left(c \,x^{n}\right)\right)^{\frac{3}{2}}d x$$

input
```
int(sec(a+b*ln(c*x^n))^(3/2),x)
```

output
```
int(sec(a+b*ln(c*x^n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sec^{\frac{3}{2}}(a + b\log(cx^n)) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sec^{\frac{3}{2}}(a + b\log(cx^n)) \, dx = \int \sec^{\frac{3}{2}}(a + b\log(cx^n)) \, dx$$

input
```
integrate(sec(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Integral(sec(a + b*log(c*x**n))**(3/2), x)
```

Maxima [F]

$$\int \sec^{\frac{3}{2}}(a + b\log(cx^n)) \, dx = \int \sec(b\log(cx^n) + a)^{\frac{3}{2}} \, dx$$

input
```
integrate(sec(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(b*log(c*x^n) + a)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \text{Timed out}$$

input
```
integrate(sec(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \int \left(\frac{1}{\cos(a+b\,\ln(c\,x^n))}\right)^{3/2}\,dx$$

input
```
int((1/cos(a + b*log(c*x^n)))^(3/2),x)
```

output
```
int((1/cos(a + b*log(c*x^n)))^(3/2), x)
```

Reduce [F]

$$\begin{aligned}&\int \sec^{\frac{3}{2}}(a+b\log(cx^n))\,dx\\&=\sqrt{\sec(\log(x^nc)\,b+a)}\,\sec(\log(x^nc)\,b+a)\,x\\&\quad-\frac{3\left(\int\sqrt{\sec(\log(x^nc)\,b+a)}\,\sec(\log(x^nc)\,b+a)\tan(\log(x^nc)\,b+a)\,dx\right)bn}{2}\end{aligned}$$

input
```
int(sec(a+b*log(c*x^n))^(3/2),x)
```

output
```
(2*sqrt(sec(log(x**n*c)*b + a))*sec(log(x**n*c)*b + a)*x - 3*int(sqrt(sec(
log(x**n*c)*b + a))*sec(log(x**n*c)*b + a)*tan(log(x**n*c)*b + a),x)*b*n)/
2
```

3.269 $\int \frac{\sec^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{\sec^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2\sqrt{\cos(a+b\log(cx^n))}E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)\sqrt{\sec(a+b\log(cx^n))}}{bn} + \frac{2\sqrt{\sec(a+b\log(cx^n))}\sin(a+b\log(cx^n))}{bn}$$

output
```
-2*cos(a+b*ln(c*x^n))^(1/2)*EllipticE(sin(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*
sec(a+b*ln(c*x^n))^(1/2)/b/n+2*sec(a+b*ln(c*x^n))^(1/2)*sin(a+b*ln(c*x^n))
/b/n
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \frac{\sec^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2\sqrt{\sec(a+b\log(cx^n))}\left(-\sqrt{\cos(a+b\log(cx^n))}E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)+\sin(a+b\log(cx^n))\right)}{bn}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
(2*Sqrt[Sec[a + b*Log[c*x^n]]]*(-(Sqrt[Cos[a + b*Log[c*x^n]]]*EllipticE[(a
 + b*Log[c*x^n])/2, 2]) + Sin[a + b*Log[c*x^n]]))/(b*n)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3039, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{3}{2}}(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \sec^{\frac{3}{2}}(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(a+b\log(cx^n)+\frac{\pi}{2}\right)^{3/2}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{2\sin(a+b\log(cx^n))\sqrt{\sec(a+b\log(cx^n))}}{b}-\int \frac{1}{\sqrt{\sec(a+b\log(cx^n))}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sin(a+b\log(cx^n))\sqrt{\sec(a+b\log(cx^n))}}{b}-\int \frac{1}{\sqrt{\csc\left(a+b\log(cx^n)+\frac{\pi}{2}\right)}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{2\sin(a+b\log(cx^n))\sqrt{\sec(a+b\log(cx^n))}}{b}-\sqrt{\sec(a+b\log(cx^n))}\sqrt{\cos(a+b\log(cx^n))}\int \sqrt{\cos(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2\sin(a+b\log(cx^n))\sqrt{\sec(a+b\log(cx^n))}}{b}-\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\int\sqrt{\sin\left(a+b\log\left(cx^n\right)+\frac{\pi}{2}\right)}d\log}{n}$$

$$\Big\downarrow \; 3119$$

$$\frac{\frac{2\sin(a+b\log(cx^n))\sqrt{\sec(a+b\log(cx^n))}}{b}-\frac{2\sqrt{\sec(a+b\log(cx^n))}\sqrt{\cos(a+b\log(cx^n))}E\left(\frac{1}{2}(a+b\log(cx^n))|2\right)}{b}}{n}$$

input
```
Int[Sec[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
((-2*Sqrt[Cos[a + b*Log[c*x^n]]]*EllipticE[(a + b*Log[c*x^n])/2, 2]*Sqrt[S
ec[a + b*Log[c*x^n]]])/b + (2*Sqrt[Sec[a + b*Log[c*x^n]]]*Sin[a + b*Log[c*
x^n]])/b)/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(85) = 170$.

Time = 0.98 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.81

method	result
derivativedivides	$-\frac{2\left(-2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos\left(a+2b\ln\left(\sqrt{c}\right.\right.}{2}}\right.}{n\sqrt{-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right.}}$
default	$-\frac{2\left(-2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+\sqrt{\frac{1}{2}-\frac{\cos\left(a+2b\ln\left(\sqrt{c}\right.\right.}{2}}\right.}{n\sqrt{-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right.}}$

input
```
int(sec(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2/n*(-2*cos(1/2*a+1/2*b*ln(c*x^n))*(-2*sin(1/2*a+1/2*b*ln(c*x^n))^4+sin(1
/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*sin(1/2*a+1/2*b*ln(c*x^n))^2+(sin(1/2*a+1/2
*b*ln(c*x^n))^2)^(1/2)*(-1+2*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*sin(1
/2*a+1/2*b*ln(c*x^n))^4+sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*EllipticE(cos(
1/2*a+1/2*b*ln(c*x^n)),2^(1/2)))/(-2*sin(1/2*a+1/2*b*ln(c*x^n))^4+sin(1/2*
a+1/2*b*ln(c*x^n))^2)^(1/2)/sin(1/2*a+1/2*b*ln(c*x^n))/(2*cos(1/2*a+1/2*b*
ln(c*x^n))^2-1)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

$$\int \frac{\sec^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{-i\sqrt{2}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bn\log(x)+b\log(c)+a)+i\,\sin(bn\log(x)$$

input
```
integrate(sec(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*n*log(
x) + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a))) + I*sqrt(2)*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*n*log(x) + b*log(c) + a)
 - I*sin(b*n*log(x) + b*log(c) + a))) + 2*sin(b*n*log(x) + b*log(c) + a)/s
qrt(cos(b*n*log(x) + b*log(c) + a)))/(b*n)
```

Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sec^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$$

input
```
integrate(sec(a+b*ln(c*x**n))**(3/2)/x,x)
```

output
```
Integral(sec(a + b*log(c*x**n))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sec(b\log(cx^n)+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(sec(b*log(c*x^n) + a)^(3/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(sec(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\left(\frac{1}{\cos(a+b\,\ln(c\,x^n))}\right)^{3/2}}{x}\,dx$$

input
```
int((1/cos(a + b*log(c*x^n)))^(3/2)/x,x)
```

output
```
int((1/cos(a + b*log(c*x^n)))^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sqrt{\sec(\log(x^n c)\,b+a)}\,\sec(\log(x^n c)\,b+a)}{x}dx$$

input
```
int(sec(a+b*log(c*x^n))^(3/2)/x,x)
```

output
```
int((sqrt(sec(log(x**n*c)*b + a))*sec(log(x**n*c)*b + a))/x,x)
```

3.270 $\int \sec^{\frac{5}{2}}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \sec^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \frac{2x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i}{bn}\right),\frac{1}{4}\left(9-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)\sec^{\frac{5}{2}}(a+b\log(cx$$
$$2+5ibn$$

output
```
2*x*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(5/2)*hypergeom([5/2, 5/4-1/2*I/b/n],[9
/4-1/2*I/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))*sec(a+b*ln(c*x^n))^(5/2)/(2+5*I
*b*n)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$\int \sec^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \frac{2x\sqrt{\sec(a+b\log(cx^n))}\left(-2+(2-ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right)\text{Hypergeometric2F1}\left(1,\frac{3}{4}-\frac{i}{2bn},\frac{5}{4}-\frac{i}{2bn},-e\right.\right.}{3b^2n^2}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]]^(5/2),x]
```

output
```
(2*x*Sqrt[Sec[a + b*Log[c*x^n]]]*(-2 + (2 - I*b*n)*(1 + E^((2*I)*a)*(c*x^n
)^((2*I)*b))*Hypergeometric2F1[1, 3/4 - (I/2)/(b*n), 5/4 - (I/2)/(b*n), -E
^((2*I)*(a + b*Log[c*x^n]))] + b*n*Tan[a + b*Log[c*x^n]]))/(3*b^2*n^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5014, 5018, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{\frac{5}{2}}(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{5014}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\sec^{\frac{5}{2}}(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5018}$$

$$\frac{x(cx^n)^{-\frac{1}{n}-\frac{5ib}{2}}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\sec^{\frac{5}{2}}(a+b\log(cx^n))\int \frac{(cx^n)^{\frac{5ib}{2}+\frac{1}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^{5/2}}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i}{bn}\right),\frac{1}{4}\left(9-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)\sec^{\frac{5}{2}}(a+b\log(cx^n))}{2+5ibn}$$

input
```
Int[Sec[a + b*Log[c*x^n]]^(5/2),x]
```

output
```
(2*x*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^(5/2)*Hypergeometric2F1[5/2, (5 -
 (2*I)/(b*n))/4, (9 - (2*I)/(b*n))/4, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))]*Se
c[a + b*Log[c*x^n]]^(5/2))/(2 + (5*I)*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5014
```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5018
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \sec \left(a+b\ln \left(c\,x^{n}\right)\right)^{\frac{5}{2}}dx$$

input
```
int(sec(a+b*ln(c*x^n))^(5/2),x)
```

output
```
int(sec(a+b*ln(c*x^n))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sec^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(sec(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \text{Timed out}$$

input
```
integrate(sec(a+b*ln(c*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sec^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \int \sec(b\log(cx^n)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(b*log(c*x^n) + a)^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \text{Timed out}$$

input
```
integrate(sec(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \sec^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \int \left(\frac{1}{\cos(a+b\,\ln(c\,x^n))}\right)^{5/2}dx$$

input
```
int((1/cos(a + b*log(c*x^n)))^(5/2),x)
```

output
```
int((1/cos(a + b*log(c*x^n)))^(5/2), x)
```

Reduce [F]

$$\int \sec^{\frac{5}{2}}(a+b\log(cx^n))\,dx$$
$$= \sqrt{\sec(\log(x^nc)\,b+a)}\sec(\log(x^nc)\,b+a)^2x - \frac{5\left(\int \sqrt{\sec(\log(x^nc)\,b+a)}\sec(\log(x^nc)\,b+a)^2\tan(\log(x^nc)\,b+a)\,dx\right)bn}{2}$$

input
```
int(sec(a+b*log(c*x^n))^(5/2),x)
```

output
```
(2*sqrt(sec(log(x**n*c)*b + a))*sec(log(x**n*c)*b + a)**2*x - 5*int(sqrt(s
ec(log(x**n*c)*b + a))*sec(log(x**n*c)*b + a)**2*tan(log(x**n*c)*b + a),x)
*b*n)/2
```

3.271 $\int \frac{\sec^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 93

$$\int \frac{\sec^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2\sqrt{\cos(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}(a+b\log(cx^n)),2\right)\sqrt{\sec(a+b\log(cx^n))}}{3bn} + \frac{2\sec^{\frac{3}{2}}(a+b\log(cx^n))\sin(a+b\log(cx^n))}{3bn}$$

output
```
2/3*cos(a+b*ln(c*x^n))^(1/2)*InverseJacobiAM(1/2*a+1/2*b*ln(c*x^n),2^(1/2)
)*sec(a+b*ln(c*x^n))^(1/2)/b/n+2/3*sec(a+b*ln(c*x^n))^(3/2)*sin(a+b*ln(c*x
^n))/b/n
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.74

$$\int \frac{\sec^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2\sec^{\frac{3}{2}}(a+b\log(cx^n))\left(\cos^{\frac{3}{2}}(a+b\log(cx^n))\,\text{EllipticF}\left(\frac{1}{2}(a+b\log(cx^n)),2\right)+\sin(a+b\log(cx^n))\right)}{3bn}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]]^(5/2)/x,x]
```

output
```
(2*Sec[a + b*Log[c*x^n]]^(3/2)*(Cos[a + b*Log[c*x^n]]^(3/2)*EllipticF[(a +
 b*Log[c*x^n])/2, 2] + Sin[a + b*Log[c*x^n]]))/(3*b*n)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3039, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \sec^{\frac{5}{2}}(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(a+b\log(cx^n)+\frac{\pi}{2}\right)^{5/2}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{1}{3}\int \sqrt{\sec(a+b\log(cx^n))}d\log(cx^n)+\frac{2\sin(a+b\log(cx^n))\sec^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}\int \sqrt{\csc\left(a+b\log(cx^n)+\frac{\pi}{2}\right)}d\log(cx^n)+\frac{2\sin(a+b\log(cx^n))\sec^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{1}{3}\sqrt{\sec(a+b\log(cx^n))}\sqrt{\cos(a+b\log(cx^n))}\int \frac{1}{\sqrt{\cos(a+b\log(cx^n))}}d\log(cx^n)+\frac{2\sin(a+b\log(cx^n))\sec^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\int\frac{1}{\sqrt{\sin\left(a+b\log(cx^n)+\frac{\pi}{2}\right)}}d\log\left(cx^n\right)+\frac{2\sin(a+b\log(cx^n))\sec^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2\sin(a+b\log(cx^n))\sec^{\frac{3}{2}}(a+b\log(cx^n))}{3b}+\frac{2\sqrt{\sec(a+b\log(cx^n))}\sqrt{\cos(a+b\log(cx^n))}\operatorname{EllipticF}\left(\frac{1}{2}(a+b\log(cx^n)),2\right)}{3b}}{n}$$

input
```
Int[Sec[a + b*Log[c*x^n]]^(5/2)/x,x]
```

output
```
((2*Sqrt[Cos[a + b*Log[c*x^n]]]*EllipticF[(a + b*Log[c*x^n])/2, 2]*Sqrt[Se
c[a + b*Log[c*x^n]]])/(3*b) + (2*Sec[a + b*Log[c*x^n]]^(3/2)*Sin[a + b*Log
[c*x^n]])/(3*b))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(84) = 168$.

Time = 21.80 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.13

method	result
derivativedivides	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos\left(a+2b\ln\left(\sqrt{c\,x^{n}}\right)\right)}{2}}\,\sqrt{-1+2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\text{EllipticF}\left(\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right),\sqrt{2}\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\right.}{3n\sqrt{-2\sin\left(\right.}}$
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos\left(a+2b\ln\left(\sqrt{c\,x^{n}}\right)\right)}{2}}\,\sqrt{-1+2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\text{EllipticF}\left(\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right),\sqrt{2}\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\right.}{3n\sqrt{-2\sin\left(\right.}}$

input
```
int(sec(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2/3/n*(-2*(sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-1+2*sin(1/2*a+1/2*b*ln(c
*x^n))^2)^(1/2)*EllipticF(cos(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*sin(1/2*a+1/
2*b*ln(c*x^n))^2-2*sin(1/2*a+1/2*b*ln(c*x^n))^2*cos(1/2*a+1/2*b*ln(c*x^n))
+(sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-1+2*sin(1/2*a+1/2*b*ln(c*x^n))^2)^
(1/2)*EllipticF(cos(1/2*a+1/2*b*ln(c*x^n)),2^(1/2)))*((2*cos(1/2*a+1/2*b*l
n(c*x^n))^2-1)*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(-2*sin(1/2*a+1/2*b*ln(
c*x^n))^4+sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(2*cos(1/2*a+1/2*b*ln(c*x^n)
)^2-1)^(3/2)/sin(1/2*a+1/2*b*ln(c*x^n))/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.56

$$\int \frac{\sec^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{-i\sqrt{2}\cos(bn\log(x)+b\log(c)+a)\,\text{weierstrassPInverse}(-4,0,\cos(bn\log(x)+b\log(c)+a)+i\sin(b$$

input
```
integrate(sec(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2)*cos(b*n*log(x) + b*log(c) + a)*weierstrassPInverse(-4, 0,
cos(b*n*log(x) + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a)) + I*sqr
t(2)*cos(b*n*log(x) + b*log(c) + a)*weierstrassPInverse(-4, 0, cos(b*n*log
(x) + b*log(c) + a) - I*sin(b*n*log(x) + b*log(c) + a)) + 2*sin(b*n*log(x)
 + b*log(c) + a)/sqrt(cos(b*n*log(x) + b*log(c) + a)))/(b*n*cos(b*n*log(x)
 + b*log(c) + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(sec(a+b*ln(c*x**n))**(5/2)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sec(b\log(cx^n)+a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate(sec(b*log(c*x^n) + a)^(5/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(sec(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\left(\frac{1}{\cos(a+b\,\ln(c\,x^n))}\right)^{5/2}}{x}\,dx$$

input
```
int((1/cos(a + b*log(c*x^n)))^(5/2)/x,x)
```

output
```
int((1/cos(a + b*log(c*x^n)))^(5/2)/x, x)
```

Reduce [F]

$$\int \frac{\sec^{\frac{5}{2}}(a + b\log(cx^n))}{x}\,dx = \int \frac{\sqrt{\sec(\log(x^n c)\,b + a)}\,\sec(\log(x^n c)\,b + a)^2}{x}dx$$

input
```
int(sec(a+b*log(c*x^n))^(5/2)/x,x)
```

output
```
int((sqrt(sec(log(x**n*c)*b + a))*sec(log(x**n*c)*b + a)**2)/x,x)
```

3.272 $\int \frac{1}{\sqrt{\sec(a+b\log(cx^n))}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \frac{1}{\sqrt{\sec(a+b\log(cx^n))}}\,dx = \frac{2x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2i+bn}{4bn},\frac{1}{4}\left(3-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{(2-ibn)\sqrt{1+e^{2ia}(cx^n)^{2ib}}\sqrt{\sec(a+b\log(cx^n))}}$$

output
```
2*x*hypergeom([-1/2, -1/4*(2*I+b*n)/b/n],[3/4-1/2*I/b/n],-exp(2*I*a)*(c*x^
n)^(2*I*b))/(2-I*b*n)/(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)/sec(a+b*ln(c*x^
n))^(1/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 380 vs. 2(110) = 220.

Time = 3.23 (sec) , antiderivative size = 380, normalized size of antiderivative = 3.45

$$\int \frac{1}{\sqrt{\sec (a+b\log (cx^n))}}\,dx$$
$$= \frac{2be^{2ia}nx(cx^n)^{2ib}\left((2i+bn)x^{2ibn}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4}-\frac{i}{2bn},\frac{7}{4}-\frac{i}{2bn},-e^{2ia}(cx^n)^{2ib}\right)+(-2i+3bn)\right.}{(2i+bn)(-2i+3bn)\sqrt{1+e^{2ia}\left(cx^n\right)^{2ib}}\sqrt{\frac{e^{ia}(cx^n)^{ib}}{2+2e^{2ia}(cx^n)^{2ib}}}\left((-2+ibn)x^{2ibn}-\right.}$$
$$-\frac{2x\cos (a-bn\log(x)+b\log (cx^n))}{\sqrt{\sec (a+b\log (cx^n))}\,(-2\cos (a-bn\log(x)+b\log (cx^n))+bn\sin (a-bn\log(x)+b\log (cx^n)))}$$

input
```
Integrate[1/Sqrt[Sec[a + b*Log[c*x^n]]],x]
```

output
```
(2*b*E^((2*I)*a)*n*x*(c*x^n)^((2*I)*b)*((2*I + b*n)*x^((2*I)*b*n)*Hypergeo
metric2F1[1/2, 3/4 - (I/2)/(b*n), 7/4 - (I/2)/(b*n), -(E^((2*I)*a)*(c*x^n)
^((2*I)*b))] + (-2*I + 3*b*n)*Hypergeometric2F1[1/2, -1/4*(2*I + b*n)/(b*n
), 3/4 - (I/2)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))]))/((2*I + b*n)*(-2
*I + 3*b*n)*Sqrt[1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[(E^(I*a)*(c*x^n)^
(I*b))/(2 + 2*E^((2*I)*a)*(c*x^n)^((2*I)*b))]*((-2 + I*b*n)*x^((2*I)*b*n)
- I*E^((2*I)*a)*(-2*I + b*n)*(c*x^n)^((2*I)*b))) - (2*x*Cos[a - b*n*Log[x]
 + b*Log[c*x^n]])/(Sqrt[Sec[a + b*Log[c*x^n]]]*(-2*Cos[a - b*n*Log[x] + b*
Log[c*x^n]] + b*n*Sin[a - b*n*Log[x] + b*Log[c*x^n]]))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5014, 5018, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec (a+b\log (cx^n))}}\,dx$$
$$\Big\downarrow \; 5014$$

$$\frac{x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}-1}}{\sqrt{\sec(a+b\log(cx^n))}}d(cx^n)}{n}$$

$$\Big\downarrow \text{ 5018}$$

$$\frac{x(cx^n)^{-\frac{1}{n}+\frac{ib}{2}}\int (cx^n)^{-\frac{ib}{2}+\frac{1}{n}-1}\sqrt{e^{2ia}(cx^n)^{2ib}+1}d(cx^n)}{n\sqrt{1+e^{2ia}(cx^n)^{2ib}}\sqrt{\sec(a+b\log(cx^n))}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{2x \text{ Hypergeometric2F1}\left(-\frac{1}{2},-\frac{bn+2i}{4bn},\frac{1}{4}\left(3-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{(2-ibn)\sqrt{1+e^{2ia}(cx^n)^{2ib}}\sqrt{\sec(a+b\log(cx^n))}}$$

input
```
Int[1/Sqrt[Sec[a + b*Log[c*x^n]]],x]
```

output
```
(2*x*Hypergeometric2F1[-1/2, -1/4*(2*I + b*n)/(b*n), (3 - (2*I)/(b*n))/4,
-(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 - I*b*n)*Sqrt[1 + E^((2*I)*a)*(c*x^
n)^((2*I)*b)]*Sqrt[Sec[a + b*Log[c*x^n]]])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5014
```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5018
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\sqrt{\sec \left(a+b\ln \left(c\,x^{n}\right)\right)}}dx$$

input
```
int(1/sec(a+b*ln(c*x^n))^(1/2),x)
```

output
```
int(1/sec(a+b*ln(c*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\sec \left(a+b\log \left(cx^{n}\right)\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/sec(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sec \left(a+b\log \left(cx^{n}\right)\right)}}\,dx = \int \frac{1}{\sqrt{\sec \left(a+b\log \left(cx^{n}\right)\right)}}\,dx$$

input
```
integrate(1/sec(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(1/sqrt(sec(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{\sqrt{\sec\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

input
```
integrate(1/sec(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(sec(b*log(c*x^n) + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{\sqrt{\sec\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

input
```
integrate(1/sec(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(sec(b*log(c*x^n) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)}}}\,dx$$

input
```
int(1/(1/cos(a + b*log(c*x^n)))^(1/2),x)
```

output
```
int(1/(1/cos(a + b*log(c*x^n)))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\sec (a+b\log (cx^n))}}\,dx=\int \frac{\sqrt{\sec (\log (x^n c)\,b+a)}}{\sec (\log (x^n c)\,b+a)}dx$$

input
```
int(1/sec(a+b*log(c*x^n))^(1/2),x)
```

output
```
int(sqrt(sec(log(x**n*c)*b + a))/sec(log(x**n*c)*b + a),x)
```

3.273 $\int \frac{1}{x\sqrt{\sec(a+b\log(cx^n))}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{1}{x\sqrt{\sec(a+b\log(cx^n))}}\,dx = \frac{2\sqrt{\cos(a+b\log(cx^n))}E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)\sqrt{\sec(a+b\log(cx^n))}}{bn}$$

output
```
2*cos(a+b*ln(c*x^n))^(1/2)*EllipticE(sin(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*s
ec(a+b*ln(c*x^n))^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\sec(a+b\log(cx^n))}}\,dx = \frac{2E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{bn\sqrt{\cos(a+b\log(cx^n))}\sqrt{\sec(a+b\log(cx^n))}}$$

input
```
Integrate[1/(x*Sqrt[Sec[a + b*Log[c*x^n]]]),x]
```

output
```
(2*EllipticE[(a + b*Log[c*x^n])/2, 2])/(b*n*Sqrt[Cos[a + b*Log[c*x^n]]]*Sq
rt[Sec[a + b*Log[c*x^n]]])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3039, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\sec(a+b\log(cx^n))}}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\sqrt{\sec(a+b\log(cx^n))}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{\csc\left(a+b\log(cx^n)+\frac{\pi}{2}\right)}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\sqrt{\sec(a+b\log(cx^n))}\sqrt{\cos(a+b\log(cx^n))}\int \sqrt{\cos(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(a+b\log(cx^n))}\sqrt{\cos(a+b\log(cx^n))}\int \sqrt{\sin\left(a+b\log(cx^n)+\frac{\pi}{2}\right)}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sqrt{\sec(a+b\log(cx^n))}\sqrt{\cos(a+b\log(cx^n))}E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)}{bn}$$

input
```
Int[1/(x*Sqrt[Sec[a + b*Log[c*x^n]]]),x]
```

output
```
(2*Sqrt[Cos[a + b*Log[c*x^n]]]*EllipticE[(a + b*Log[c*x^n])/2, 2]*Sqrt[Sec
[a + b*Log[c*x^n]]])/(b*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(52) = 104$.

Time = 1.37 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.35

method	result
derivativedivides	$\frac{2\sqrt{\left(2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{\frac{1}{2}-\frac{\cos\left(a+2b\ln\left(\sqrt{c\,x^n}\right)\right)}{2}}\,\sqrt{-2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+1}\ \mathrm{EllipticE}\Big(}{n\sqrt{-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\ \sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1}}$
default	$\frac{2\sqrt{\left(2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{\frac{1}{2}-\frac{\cos\left(a+2b\ln\left(\sqrt{c\,x^n}\right)\right)}{2}}\,\sqrt{-2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+1}\ \mathrm{EllipticE}\Big(}{n\sqrt{-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\ \sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1}}$

input
```
int(1/x/sec(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/n*((2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2
)*(sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cos(1/2*a+1/2*b*ln(c*x^n))^2+1)
^(1/2)*EllipticE(cos(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/(-2*sin(1/2*a+1/2*b*l
n(c*x^n))^4+sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sin(1/2*a+1/2*b*ln(c*x^n))
/(2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.56

$$\int \frac{1}{x\sqrt{\sec(a+b\log(cx^n))}}\,dx$$
$$= \frac{i\sqrt{2}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bn\log(x)+b\log(c)+a)+i\sin(bn\log(x)-$$

input
```
integrate(1/x/sec(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
(I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*n*log(x
) + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a))) - I*sqrt(2)*weierst
rassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*n*log(x) + b*log(c) + a)
- I*sin(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{\sec(a+b\log(cx^n))}}\,dx = \int \frac{1}{x\sqrt{\sec(a+b\log(cx^n))}}\,dx$$

input
```
integrate(1/x/sec(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(sec(a + b*log(c*x**n)))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{\sec(a+b\log(cx^n))}}\,dx = \int \frac{1}{x\sqrt{\sec(b\log(cx^n)+a)}}\,dx$$

input
```
integrate(1/x/sec(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sqrt(sec(b*log(c*x^n) + a))), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{\sec(a+b\log(cx^n))}}\,dx = \int \frac{1}{x\sqrt{\sec(b\log(cx^n)+a)}}\,dx$$

input
```
integrate(1/x/sec(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(x*sqrt(sec(b*log(c*x^n) + a))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\sec(a+b\log(cx^n))}}\,dx = \int \frac{1}{x\sqrt{\frac{1}{\cos(a+b\,\ln(c\,x^n))}}}\,dx$$

input
```
int(1/(x*(1/cos(a + b*log(c*x^n)))^(1/2)),x)
```

output
```
int(1/(x*(1/cos(a + b*log(c*x^n)))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{\sec\left(a+b\log\left(cx^{n}\right)\right)}}\,dx=\int \frac{\sqrt{\sec\left(\log\left(x^{n}c\right)b+a\right)}}{\sec\left(\log\left(x^{n}c\right)b+a\right)x}dx$$

input
```
int(1/x/sec(a+b*log(c*x^n))^(1/2),x)
```

output
```
int(sqrt(sec(log(x**n*c)*b + a))/(sec(log(x**n*c)*b + a)*x),x)
```

3.274 $\int \frac{1}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \frac{1}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{2x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i}{bn}\right),\frac{1}{4}\left(1-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{(2-3ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\sec^{\frac{3}{2}}(a+b\log(cx^n))}$$

output
```
2*x*hypergeom([-3/2, -3/4-1/2*I/b/n],[1/4-1/2*I/b/n],-exp(2*I*a)*(c*x^n)^(
2*I*b))/(2-3*I*b*n)/(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)/sec(a+b*ln(c*x^n)
)^(3/2)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.54

$$\int \frac{1}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{2x\left(3b^2n^2\left(1+e^{2ia}(cx^n)^{2ib}\right)\text{Hypergeometric2F1}\left(1,\frac{3}{4}-\frac{i}{2bn},\frac{5}{4}-\frac{i}{2bn},-e^{2i(a+b\log(cx^n))}\right)\sec^2(a+b\log(cx\right.}{(2+3ibn)(-2i+bn)(2i+3bn)\sec^{\frac{3}{2}}(a+b\log(cx^n))}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]]^(-3/2),x]
```

output
```
(2*x*(3*b^2*n^2*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))*Hypergeometric2F1[1, 3
/4 - (I/2)/(b*n), 5/4 - (I/2)/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))]*Sec[a
+ b*Log[c*x^n]]^2 + (2 + I*b*n)*(2 + 3*b*n*Tan[a + b*Log[c*x^n]])))/((2 +
(3*I)*b*n)*(-2*I + b*n)*(2*I + 3*b*n)*Sec[a + b*Log[c*x^n]]^(3/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5014, 5018, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}dx$$
$$\Big\downarrow \text{ 5014}$$
$$\frac{x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}-1}}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}d(cx^n)}{n}$$
$$\Big\downarrow \text{ 5018}$$
$$\frac{x(cx^n)^{-\frac{1}{n}+\frac{3ib}{2}}\int (cx^n)^{-\frac{3ib}{2}+\frac{1}{n}-1}\left(e^{2ia}(cx^n)^{2ib}+1\right)^{3/2}d(cx^n)}{n\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\sec^{\frac{3}{2}}(a+b\log(cx^n))}$$
$$\Big\downarrow \text{ 888}$$
$$\frac{2x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i}{bn}\right),\frac{1}{4}\left(1-\frac{2i}{bn}\right),-e^{2ia}(cx^n)^{2ib}\right)}{(2-3ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\sec^{\frac{3}{2}}(a+b\log(cx^n))}$$

input
```
Int[Sec[a + b*Log[c*x^n]]^(-3/2),x]
```

output
```
(2*x*Hypergeometric2F1[-3/2, (-3 - (2*I)/(b*n))/4, (1 - (2*I)/(b*n))/4, -(
E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 - (3*I)*b*n)*(1 + E^((2*I)*a)*(c*x^n)
^((2*I)*b))^(3/2)*Sec[a + b*Log[c*x^n]]^(3/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5014
```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5018
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\sec \left(a+b\ln \left(c\,x^{n}\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/sec(a+b*ln(c*x^n))^(3/2),x)
```

output
```
int(1/sec(a+b*ln(c*x^n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/sec(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$

input
```
integrate(1/sec(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Integral(sec(a + b*log(c*x**n))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{\sec(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(sec(b*log(c*x^n) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{\sec(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/sec(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate(sec(b*log(c*x^n) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{\left(\frac{1}{\cos(a+b\ln(c\,x^n))}\right)^{3/2}}\,dx$$

input
```
int(1/(1/cos(a + b*log(c*x^n)))^(3/2),x)
```

output
```
int(1/(1/cos(a + b*log(c*x^n)))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{\sqrt{\sec(\log(x^n c)\,b+a)}}{\sec(\log(x^n c)\,b+a)^2}\,dx$$

input
```
int(1/sec(a+b*log(c*x^n))^(3/2),x)
```

output
```
int(sqrt(sec(log(x**n*c)*b + a))/sec(log(x**n*c)*b + a)**2,x)
```

3.275 $\int \frac{1}{x \sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 93

$$\int \frac{1}{x \sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{2\sqrt{\cos(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}(a+b\log(cx^n)),2\right)\sqrt{\sec(a+b\log(cx^n))}}{3bn} + \frac{2\sin(a+b\log(cx^n))}{3bn\sqrt{\sec(a+b\log(cx^n))}}$$

output
```
2/3*cos(a+b*ln(c*x^n))^(1/2)*InverseJacobiAM(1/2*a+1/2*b*ln(c*x^n),2^(1/2)
)*sec(a+b*ln(c*x^n))^(1/2)/b/n+2/3*sin(a+b*ln(c*x^n))/b/n/sec(a+b*ln(c*x^n
))^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.77

$$\int \frac{1}{x \sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\sqrt{\sec(a+b\log(cx^n))}\left(2\sqrt{\cos(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}(a+b\log(cx^n)),2\right)+\sin(2(a+b\log(cx^n))\right.}{3bn}$$

input
```
Integrate[1/(x*Sec[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
(Sqrt[Sec[a + b*Log[c*x^n]]]*(2*Sqrt[Cos[a + b*Log[c*x^n]]]*EllipticF[(a +
 b*Log[c*x^n])/2, 2] + Sin[2*(a + b*Log[c*x^n])]))/(3*b*n)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3039, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sec^{\frac{3}{2}}(a+b\log(cx^n))}dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \frac{1}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \frac{1}{\csc\left(a+b\log(cx^n)+\frac{\pi}{2}\right)^{3/2}}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{4256}$$
$$\frac{\frac{1}{3}\int \sqrt{\sec(a+b\log(cx^n))}d\log(cx^n)+\frac{2\sin(a+b\log(cx^n))}{3b\sqrt{\sec(a+b\log(cx^n))}}}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\frac{1}{3}\int \sqrt{\csc\left(a+b\log(cx^n)+\frac{\pi}{2}\right)}d\log(cx^n)+\frac{2\sin(a+b\log(cx^n))}{3b\sqrt{\sec(a+b\log(cx^n))}}}{n}$$
$$\Big\downarrow \text{4258}$$
$$\frac{\frac{1}{3}\sqrt{\sec(a+b\log(cx^n))}\sqrt{\cos(a+b\log(cx^n))}\int \frac{1}{\sqrt{\cos(a+b\log(cx^n))}}d\log(cx^n)+\frac{2\sin(a+b\log(cx^n))}{3b\sqrt{\sec(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\int\frac{1}{\sqrt{\sin\left(a+b\log\left(cx^n\right)+\frac{\pi}{2}\right)}}d\log\left(cx^n\right)+\frac{2\sin(a+b\log(cx^n))}{3b\sqrt{\sec(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2\sin(a+b\log(cx^n))}{3b\sqrt{\sec(a+b\log(cx^n))}}+\frac{2\sqrt{\sec(a+b\log(cx^n))}\sqrt{\cos(a+b\log(cx^n))}\operatorname{EllipticF}\left(\frac{1}{2}(a+b\log(cx^n)),2\right)}{3b}}{n}$$

input
```
Int[1/(x*Sec[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
((2*Sqrt[Cos[a + b*Log[c*x^n]]]*EllipticF[(a + b*Log[c*x^n])/2, 2]*Sqrt[Se
c[a + b*Log[c*x^n]]])/(3*b) + (2*Sin[a + b*Log[c*x^n]])/(3*b*Sqrt[Sec[a +
b*Log[c*x^n]]]))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(84) = 168$.

Time = 2.01 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.66

method	result
derivativedivides	$-\frac{2\sqrt{\left(2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\left(4\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right.}{3n\sqrt{-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)}}$
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\left(4\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\right.}{3n\sqrt{-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)}}$

input
```
int(1/x/sec(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/n*((2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*(4*cos(1/2*a+1/2*b*ln(c*x^n))*sin(1/2*a+1/2*b*ln(c*x^n))^4-2*sin(1/2*
a+1/2*b*ln(c*x^n))^2*cos(1/2*a+1/2*b*ln(c*x^n))+(sin(1/2*a+1/2*b*ln(c*x^n)
)^2)^(1/2)*(-1+2*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*EllipticF(cos(1/2*a+1
/2*b*ln(c*x^n)),2^(1/2)))/(-2*sin(1/2*a+1/2*b*ln(c*x^n))^4+sin(1/2*a+1/2*b
*ln(c*x^n))^2)^(1/2)/sin(1/2*a+1/2*b*ln(c*x^n))/(2*cos(1/2*a+1/2*b*ln(c*x^
n))^2-1)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.15

$$\int \frac{1}{x\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{2\sqrt{\cos(bn\log(x)+b\log(c)+a)}\sin(bn\log(x)+b\log(c)+a) - i\sqrt{2}\text{weierstrassPInverse}(-4,0,\cos(b$$

input
```
integrate(1/x/sec(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(cos(b*n*log(x) + b*log(c) + a))*sin(b*n*log(x) + b*log(c) + a)
 - I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b*n*log(x) + b*log(c) + a) + I
*sin(b*n*log(x) + b*log(c) + a)) + I*sqrt(2)*weierstrassPInverse(-4, 0, co
s(b*n*log(x) + b*log(c) + a) - I*sin(b*n*log(x) + b*log(c) + a)))/(b*n)
```

Sympy [F]

$$\int \frac{1}{x\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$

input
```
integrate(1/x/sec(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Integral(1/(x*sec(a + b*log(c*x**n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\sec(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/sec(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sec(b*log(c*x^n) + a)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{x\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\sec(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/sec(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(x*sec(b*log(c*x^n) + a)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\left(\frac{1}{\cos(a+b\ln(cx^n))}\right)^{3/2}}\,dx$$

input
```
int(1/(x*(1/cos(a + b*log(c*x^n)))^(3/2)),x)
```

output
```
int(1/(x*(1/cos(a + b*log(c*x^n)))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x \sec^{\frac{3}{2}}(a + b\log(cx^n))}\,dx = \int \frac{\sqrt{\sec(\log(x^n c)\,b + a)}}{\sec(\log(x^n c)\,b + a)^2 x}dx$$

input
```
int(1/x/sec(a+b*log(c*x^n))^(3/2),x)
```

output
```
int(sqrt(sec(log(x**n*c)*b + a))/(sec(log(x**n*c)*b + a)**2*x),x)
```

3.276 $\int \frac{1}{\sec^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \frac{1}{\sec^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{2x\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4}\left(-5-\frac{2i}{bn}\right),-\frac{2i+bn}{4bn},-e^{2ia}(cx^n)^{2ib}\right)}{(2-5ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\sec^{\frac{5}{2}}(a+b\log(cx^n))}$$

output
```
2*x*hypergeom([-5/2, -5/4-1/2*I/b/n],[-1/4*(2*I+b*n)/b/n],-exp(2*I*a)*(c*x
^n)^(2*I*b))/(2-5*I*b*n)/(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(5/2)/sec(a+b*ln(c
*x^n))^(5/2)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 867 vs. 2(110) = 220.

Time = 7.94 (sec) , antiderivative size = 867, normalized size of antiderivative = 7.88

$$\int \frac{1}{\sec^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]]^(-5/2),x]
```

output
```
(30*b^3*E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*n^3*x*((2*I + b*n)*x^
((2*I)*b*n)*Hypergeometric2F1[1/2, 3/4 - (I/2)/(b*n), 7/4 - (I/2)/(b*n), -
(E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*x^((2*I)*b*n))] + (-2*I + 3*
b*n)*Hypergeometric2F1[1/2, -1/4*(2*I + b*n)/(b*n), 3/4 - (I/2)/(b*n), -(E
^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])))*x^((2*I)*b*n))]))/((2 - (5*I)*
b*n)*(2*I + b*n)*(-2*I + 3*b*n)*(-2*I + 5*b*n)*(-2*I - b*n + E^((2*I)*(a +
 b*(-(n*Log[x]) + Log[c*x^n])))*(-2*I + b*n))*Sqrt[1 + E^((2*I)*(a + b*(-(
n*Log[x]) + Log[c*x^n])))*x^((2*I)*b*n)]*Sqrt[(E^(I*(a + b*(-(n*Log[x]) +
Log[c*x^n])))*x^(I*b*n))/(2 + 2*E^((2*I)*(a + b*(-(n*Log[x]) + Log[c*x^n])
))*x^((2*I)*b*n))]) + Sqrt[Sec[a + b*n*Log[x] + b*(-(n*Log[x]) + Log[c*x^n
])]]*(-1/4*(x*Cos[b*n*Log[x]]*(12 + 55*b^2*n^2 + 12*Cos[2*(a + b*(-(n*Log[
x]) + Log[c*x^n]))] + 65*b^2*n^2*Cos[2*(a + b*(-(n*Log[x]) + Log[c*x^n]))]
 + 4*b*n*Sin[2*(a + b*(-(n*Log[x]) + Log[c*x^n]))]))/((-2*I + 5*b*n)*(2*I
+ 5*b*n)*(-2*Cos[a + b*(-(n*Log[x]) + Log[c*x^n])] + b*n*Sin[a + b*(-(n*Lo
g[x]) + Log[c*x^n])])) + (x*Sin[b*n*Log[x]]*(-16*b*n - 4*b*n*Cos[2*(a + b*
(-(n*Log[x]) + Log[c*x^n]))] + 12*Sin[2*(a + b*(-(n*Log[x]) + Log[c*x^n]))
] + 65*b^2*n^2*Sin[2*(a + b*(-(n*Log[x]) + Log[c*x^n]))]))/(4*(-2*I + 5*b*
n)*(2*I + 5*b*n)*(-2*Cos[a + b*(-(n*Log[x]) + Log[c*x^n])] + b*n*Sin[a + b
*(-(n*Log[x]) + Log[c*x^n])])) + (x*Sin[3*b*n*Log[x]]*(5*b*n*Cos[3*(a + b*
(-(n*Log[x]) + Log[c*x^n]))] - 2*Sin[3*(a + b*(-(n*Log[x]) + Log[c*x^n]...
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5014, 5018, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^{\frac{5}{2}}(a + b\log(cx^n))} dx$$

$$\Big\downarrow \text{5014}$$

$$\frac{x(cx^n)^{-1/n}\int\frac{(cx^n)^{\frac{1}{n}-1}}{\sec^{\frac{5}{2}}(a+b\log(cx^n))}d(cx^n)}{n}$$

$$\Big\downarrow \text{5018}$$

$$\frac{x(cx^n)^{-\frac{1}{n}+\frac{5ib}{2}}\int(cx^n)^{-\frac{5ib}{2}+\frac{1}{n}-1}\left(e^{2ia}(cx^n)^{2ib}+1\right)^{5/2}d(cx^n)}{n\left(1+e^{2ia}\left(cx^n\right)^{2ib}\right)^{5/2}\sec^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4}\left(-5-\frac{2i}{bn}\right),-\frac{bn+2i}{4bn},-e^{2ia}(cx^n)^{2ib}\right)}{(2-5ibn)\left(1+e^{2ia}\left(cx^n\right)^{2ib}\right)^{5/2}\sec^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

input
```
Int[Sec[a + b*Log[c*x^n]]^(-5/2),x]
```

output
```
(2*x*Hypergeometric2F1[-5/2, (-5 - (2*I)/(b*n))/4, -1/4*(2*I + b*n)/(b*n),
 -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 - (5*I)*b*n)*(1 + E^((2*I)*a)*(c*x
^n)^((2*I)*b))^(5/2)*Sec[a + b*Log[c*x^n]]^(5/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5014
```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5018
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\sec\left(a+b\ln\left(c\,x^{n}\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/sec(a+b*ln(c*x^n))^(5/2),x)
```

output
```
int(1/sec(a+b*ln(c*x^n))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sec^{\frac{5}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/sec(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/sec(a+b*ln(c*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{\sec(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(sec(b*log(c*x^n) + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{\sec(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/sec(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

output
```
integrate(sec(b*log(c*x^n) + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sec^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{\left(\frac{1}{\cos(a+b\,\ln(c\,x^n))}\right)^{5/2}}\,dx$$

input
```
int(1/(1/cos(a + b*log(c*x^n)))^(5/2),x)
```

output
```
int(1/(1/cos(a + b*log(c*x^n)))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\sec^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{\sqrt{\sec(\log(x^n c)\,b+a)}}{\sec(\log(x^n c)\,b+a)^3}dx$$

input

```
int(1/sec(a+b*log(c*x^n))^(5/2),x)
```

output

```
int(sqrt(sec(log(x**n*c)*b + a))/sec(log(x**n*c)*b + a)**3,x)
```

3.277 $\int \frac{1}{x\sec^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 93

$$\int \frac{1}{x\sec^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{6\sqrt{\cos(a+b\log(cx^n))}E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)\sqrt{\sec(a+b\log(cx^n))}}{5bn} + \frac{2\sin(a+b\log(cx^n))}{5bn\sec^{\frac{3}{2}}(a+b\log(cx^n))}$$

output
```
6/5*cos(a+b*ln(c*x^n))^(1/2)*EllipticE(sin(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))
*sec(a+b*ln(c*x^n))^(1/2)/b/n+2/5*sin(a+b*ln(c*x^n))/b/n/sec(a+b*ln(c*x^n)
)^(3/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sec^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\sqrt{\sec(a+b\log(cx^n))}\left(12\sqrt{\cos(a+b\log(cx^n))}E\left(\frac{1}{2}(a+b\log(cx^n))\middle|2\right)+\sin(a+b\log(cx^n))+\sin(3(\cdots\right.}{10bn}$$

input
```
Integrate[1/(x*Sec[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
(Sqrt[Sec[a + b*Log[c*x^n]]]*(12*Sqrt[Cos[a + b*Log[c*x^n]]]*EllipticE[(a
+ b*Log[c*x^n])/2, 2] + Sin[a + b*Log[c*x^n]] + Sin[3*(a + b*Log[c*x^n])])
)/(10*b*n)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3039, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sec^{\frac{5}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\sec^{\frac{5}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\csc\left(a+b\log(cx^n)+\frac{\pi}{2}\right)^{5/2}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4256}$$

$$\frac{\frac{3}{5}\int \frac{1}{\sqrt{\sec(a+b\log(cx^n))}}d\log(cx^n)+\frac{2\sin(a+b\log(cx^n))}{5b\sec^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(a+b\log(cx^n)+\frac{\pi}{2}\right)}}d\log(cx^n)+\frac{2\sin(a+b\log(cx^n))}{5b\sec^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{3}{5}\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\int\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}d\log\left(cx^n\right)+\frac{2\sin(a+b\log(cx^n))}{5b\sec^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cos\left(a+b\log\left(cx^n\right)\right)}\int\sqrt{\sin\left(a+b\log\left(cx^n\right)+\frac{\pi}{2}\right)}d\log\left(cx^n\right)+\frac{2\sin(a+b\log(cx^n))}{5b\sec^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{2\sin(a+b\log(cx^n))}{5b\sec^{\frac{3}{2}}(a+b\log(cx^n))}+\frac{6\sqrt{\sec(a+b\log(cx^n))}\sqrt{\cos(a+b\log(cx^n))}E\left(\frac{1}{2}(a+b\log(cx^n))|2\right)}{5b}}{n}$$

input
```
Int[1/(x*Sec[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
((6*Sqrt[Cos[a + b*Log[c*x^n]]]*EllipticE[(a + b*Log[c*x^n])/2, 2]*Sqrt[Se
c[a + b*Log[c*x^n]]])/(5*b) + (2*Sin[a + b*Log[c*x^n]])/(5*b*Sec[a + b*Log
[c*x^n]]^(3/2)))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(85) = 170$.

Time = 3.37 (sec) , antiderivative size = 280, normalized size of antiderivative = 3.01

method	result
derivativedivides	$-\frac{2\sqrt{\left(2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}-1\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\left(-8\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{6}+8\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^{n}}{2}\right.\right.}{5n\sqrt{-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right.}}$
default	$-\frac{2\sqrt{\left(2\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}-1\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\left(-8\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{6}+8\cos\left(\frac{a}{2}+\frac{b\ln(c\,x^{n}}{2}\right.\right.}{5n\sqrt{-2\sin\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right.}}$

input
```
int(1/x/sec(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/n*((2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*(-8*cos(1/2*a+1/2*b*ln(c*x^n))*sin(1/2*a+1/2*b*ln(c*x^n))^6+8*cos(1/2
*a+1/2*b*ln(c*x^n))*sin(1/2*a+1/2*b*ln(c*x^n))^4-2*sin(1/2*a+1/2*b*ln(c*x^
n))^2*cos(1/2*a+1/2*b*ln(c*x^n))-3*(sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-
1+2*sin(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*EllipticE(cos(1/2*a+1/2*b*ln(c*x^n
)),2^(1/2)))/(-2*sin(1/2*a+1/2*b*ln(c*x^n))^4+sin(1/2*a+1/2*b*ln(c*x^n))^2
)^(1/2)/sin(1/2*a+1/2*b*ln(c*x^n))/(2*cos(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2
)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\sec^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$$
$$= \frac{2\cos(bn\log(x)+b\log(c)+a)^{\frac{3}{2}}\sin(bn\log(x)+b\log(c)+a)+3i\sqrt{2}\text{weierstrassZeta}(-4,0,\text{weierstra}$$

input
```
integrate(1/x/sec(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

output
```
1/5*(2*cos(b*n*log(x) + b*log(c) + a)^(3/2)*sin(b*n*log(x) + b*log(c) + a)
 + 3*I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*n*l
og(x) + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a))) - 3*I*sqrt(2)*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*n*log(x) + b*log(c)
 + a) - I*sin(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\sec^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/x/sec(a+b*ln(c*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x\sec^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\sec(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/sec(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sec(b*log(c*x^n) + a)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{x\sec^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\sec(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/sec(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/(x*sec(b*log(c*x^n) + a)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sec^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\left(\frac{1}{\cos(a+b\,\ln(c\,x^n))}\right)^{5/2}}\,dx$$

input
```
int(1/(x*(1/cos(a + b*log(c*x^n)))^(5/2)),x)
```

output
```
int(1/(x*(1/cos(a + b*log(c*x^n)))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{x \sec^{\frac{5}{2}}(a + b \log(cx^n))} dx = \int \frac{\sqrt{\sec(\log(x^n c) b + a)}}{\sec(\log(x^n c) b + a)^3 x} dx$$

input
```
int(1/x/sec(a+b*log(c*x^n))^(5/2),x)
```

output
```
int(sqrt(sec(log(x**n*c)*b + a))/(sec(log(x**n*c)*b + a)**3*x),x)
```

3.278 $\int x^m \sec^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 102

$$\int x^m \sec^3(a+b\log(cx^n))\,dx = \frac{8e^{3ia}x^{1+m}(cx^n)^{3ib}\,\text{Hypergeometric2F1}\left(3,-\frac{i(1+m)-3bn}{2bn},-\frac{i(1+m)-5bn}{2bn},-e^{2ia}(cx^n)^{2ib}\right)}{1+m+3ibn}$$

output
```
8*exp(3*I*a)*x^(1+m)*(c*x^n)^(3*I*b)*hypergeom([3, -1/2*(I*(1+m)-3*b*n)/b/
n],[-1/2*(I*(1+m)-5*b*n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(1+m+3*I*b*n)
```

Mathematica [A] (verified)

Time = 5.63 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.36

$$\int x^m \sec^3(a+b\log(cx^n))\,dx = \frac{x^{1+m}\left(4e^{ia}(1+m-ibn)(cx^n)^{ib}\,\text{Hypergeometric2F1}\left(1,\frac{-i-im+bn}{2bn},-\frac{i(1+m+3ibn)}{2bn},-e^{2i(a+b\log(cx^n))}\right)-2\sec\right.}{4b^2n^2}$$

input
```
Integrate[x^m*Sec[a + b*Log[c*x^n]]^3,x]
```

output
```
(x^(1 + m)*(4*E^(I*a)*(1 + m - I*b*n)*(c*x^n)^(I*b)*Hypergeometric2F1[1, (
-I - I*m + b*n)/(2*b*n), ((-1/2*I)*(1 + m + (3*I)*b*n))/(b*n), -E^((2*I)*(
a + b*Log[c*x^n]))] - 2*Sec[a + b*Log[c*x^n]]*(1 + m - b*n*Tan[a + b*Log[c
*x^n]])))/(4*b^2*n^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5020, 5016, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sec^3(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{5020}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\sec^3(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5016}$$

$$\frac{8e^{3ia}x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int \frac{(cx^n)^{3ib+\frac{m+1}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^3}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{8e^{3ia}x^{m+1}(cx^n)^{-\frac{m+1}{n}+\frac{3ibn+m+1}{n}}\text{Hypergeometric2F1}\left(3,-\frac{i(m+1)-3bn}{2bn},-\frac{i(m+1)-5bn}{2bn},-e^{2ia}(cx^n)^{2ib}\right)}{3ibn+m+1}$$

input
```
Int[x^m*Sec[a + b*Log[c*x^n]]^3,x]
```

output
```
(8*E^((3*I)*a)*x^(1 + m)*(c*x^n)^(-((1 + m)/n) + (1 + m + (3*I)*b*n)/n)*Hy
pergeometric2F1[3, -1/2*(I*(1 + m) - 3*b*n)/(b*n), -1/2*(I*(1 + m) - 5*b*n
)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(1 + m + (3*I)*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^m \sec \left(a+b\ln \left(c\,x^n\right)\right)^3 dx$$

input
```
int(x^m*sec(a+b*ln(c*x^n))^3,x)
```

output
```
int(x^m*sec(a+b*ln(c*x^n))^3,x)
```

Fricas [F]

$$\int x^m \sec^3\left(a+b\log\left(cx^n\right)\right)\,dx=\int x^m \sec\left(b\log\left(cx^n\right)+a\right)^3\,dx$$

input
```
integrate(x^m*sec(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
integral(x^m*sec(b*log(c*x^n) + a)^3, x)
```

Sympy [F]

$$\int x^m \sec^3(a + b\log(cx^n))\,dx = \int x^m \sec^3(a + b\log(cx^n))\,dx$$

input
```
integrate(x**m*sec(a+b*ln(c*x**n))**3,x)
```

output
```
Integral(x**m*sec(a + b*log(c*x**n))**3, x)
```

Maxima [F]

$$\int x^m \sec^3(a + b\log(cx^n))\,dx = \int x^m \sec(b\log(cx^n) + a)^3\,dx$$

input
```
integrate(x^m*sec(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
-((b*n*sin(b*log(c)) + m*cos(b*log(c)) + cos(b*log(c)))*x*x^m*cos(b*log(x^
n) + a) + (b*n*cos(b*log(c)) - m*sin(b*log(c)) - sin(b*log(c)))*x*x^m*sin(
b*log(x^n) + a) + (((cos(4*b*log(c))*cos(3*b*log(c)) + sin(4*b*log(c))*sin
(3*b*log(c)))*m + (b*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*s
in(3*b*log(c)))*n + cos(4*b*log(c))*cos(3*b*log(c)) + sin(4*b*log(c))*sin(
3*b*log(c)))*x*x^m*cos(3*b*log(x^n) + 3*a) + ((cos(4*b*log(c))*cos(b*log(c
)) + sin(4*b*log(c))*sin(b*log(c)))*m - (b*cos(b*log(c))*sin(4*b*log(c)) -
 b*cos(4*b*log(c))*sin(b*log(c)))*n + cos(4*b*log(c))*cos(b*log(c)) + sin(
4*b*log(c))*sin(b*log(c)))*x*x^m*cos(b*log(x^n) + a) + ((cos(3*b*log(c))*s
in(4*b*log(c)) - cos(4*b*log(c))*sin(3*b*log(c)))*m - (b*cos(4*b*log(c))*c
os(3*b*log(c)) + b*sin(4*b*log(c))*sin(3*b*log(c)))*n + cos(3*b*log(c))*si
n(4*b*log(c)) - cos(4*b*log(c))*sin(3*b*log(c)))*x*x^m*sin(3*b*log(x^n) +
3*a) + ((cos(b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(b*log(c)))*m
+ (b*cos(4*b*log(c))*cos(b*log(c)) + b*sin(4*b*log(c))*sin(b*log(c)))*n +
cos(b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(b*log(c)))*x*x^m*sin(b
*log(x^n) + a))*cos(4*b*log(x^n) + 4*a) + (2*((cos(3*b*log(c))*cos(2*b*log
(c)) + sin(3*b*log(c))*sin(2*b*log(c)))*m - (b*cos(2*b*log(c))*sin(3*b*log
(c)) - b*cos(3*b*log(c))*sin(2*b*log(c)))*n + cos(3*b*log(c))*cos(2*b*log(
c)) + sin(3*b*log(c))*sin(2*b*log(c)))*x*x^m*cos(2*b*log(x^n) + 2*a) + 2*(
(cos(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*log(c)))*m +...
```

Giac [F]

$$\int x^m \sec^3(a+b\log(cx^n))\,dx=\int x^m \sec(b\log(cx^n)+a)^3\,dx$$

input
```
integrate(x^m*sec(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
integrate(x^m*sec(b*log(c*x^n) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sec^3(a+b\log(cx^n))\,dx=\int \frac{x^m}{\cos(a+b\ln(c\,x^n))^3}\,dx$$

input
```
int(x^m/cos(a + b*log(c*x^n))^3,x)
```

output
```
int(x^m/cos(a + b*log(c*x^n))^3, x)
```

Reduce [F]

$$\int x^m \sec^3(a+b\log(cx^n))\,dx=\text{too large to display}$$

input
```
int(x^m*sec(a+b*log(c*x^n))^3,x)
```

output

```
(216*x**m*cos(log(x**n*c)*b + a)*sec(log(x**n*c)*b + a)**3*sin(log(x**n*c)
*b + a)**2*b**6*n**6*x - 364*x**m*cos(log(x**n*c)*b + a)*sec(log(x**n*c)*b
 + a)**3*sin(log(x**n*c)*b + a)**2*b**4*m**2*n**4*x - 728*x**m*cos(log(x**
n*c)*b + a)*sec(log(x**n*c)*b + a)**3*sin(log(x**n*c)*b + a)**2*b**4*m*n**
4*x - 364*x**m*cos(log(x**n*c)*b + a)*sec(log(x**n*c)*b + a)**3*sin(log(x*
*n*c)*b + a)**2*b**4*n**4*x + 49*x**m*cos(log(x**n*c)*b + a)*sec(log(x**n*
c)*b + a)**3*sin(log(x**n*c)*b + a)**2*b**2*m**4*n**2*x + 196*x**m*cos(log
(x**n*c)*b + a)*sec(log(x**n*c)*b + a)**3*sin(log(x**n*c)*b + a)**2*b**2*m
**3*n**2*x + 294*x**m*cos(log(x**n*c)*b + a)*sec(log(x**n*c)*b + a)**3*sin
(log(x**n*c)*b + a)**2*b**2*m**2*n**2*x + 196*x**m*cos(log(x**n*c)*b + a)*
sec(log(x**n*c)*b + a)**3*sin(log(x**n*c)*b + a)**2*b**2*m*n**2*x + 49*x**
m*cos(log(x**n*c)*b + a)*sec(log(x**n*c)*b + a)**3*sin(log(x**n*c)*b + a)*
*2*b**2*n**2*x - x**m*cos(log(x**n*c)*b + a)*sec(log(x**n*c)*b + a)**3*sin
(log(x**n*c)*b + a)**2*m**6*x - 6*x**m*cos(log(x**n*c)*b + a)*sec(log(x**n
*c)*b + a)**3*sin(log(x**n*c)*b + a)**2*m**5*x - 15*x**m*cos(log(x**n*c)*b
 + a)*sec(log(x**n*c)*b + a)**3*sin(log(x**n*c)*b + a)**2*m**4*x - 20*x**m
*cos(log(x**n*c)*b + a)*sec(log(x**n*c)*b + a)**3*sin(log(x**n*c)*b + a)**
2*m**3*x - 15*x**m*cos(log(x**n*c)*b + a)*sec(log(x**n*c)*b + a)**3*sin(lo
g(x**n*c)*b + a)**2*m**2*x - 6*x**m*cos(log(x**n*c)*b + a)*sec(log(x**n*c)
*b + a)**3*sin(log(x**n*c)*b + a)**2*m*x - x**m*cos(log(x**n*c)*b + a)*...
```

3.279 $\int x^m \sec^2(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 102

$$\int x^m \sec^2(a+b\log(cx^n))\,dx = \frac{4e^{2ia}x^{1+m}(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(2,-\frac{i(1+m)-2bn}{2bn},-\frac{i(1+m)-4bn}{2bn},-e^{2ia}(cx^n)^{2ib}\right)}{1+m+2ibn}$$

output
```
4*exp(2*I*a)*x^(1+m)*(c*x^n)^(2*I*b)*hypergeom([2, -1/2*(I*(1+m)-2*b*n)/b/
n],[-1/2*(I*(1+m)-4*b*n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(1+m+2*I*b*n)
```

Mathematica [A] (verified)

Time = 14.64 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.94

$$\int x^m \sec^2(a+b\log(cx^n))\,dx = -\frac{ix^{1+m}\left((1+m+2ibn)\,\text{Hypergeometric2F1}\left(1,-\frac{i(1+m)}{2bn},1-\frac{i(1+m)}{2bn},-e^{2i(a+b\log(cx^n))}\right)-e^{2ia}(1+m)\,(c\ldots\right.}{bn\ldots}$$

input
```
Integrate[x^m*Sec[a + b*Log[c*x^n]]^2,x]
```

output
```
((-I)*x^(1 + m)*((1 + m + (2*I)*b*n)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m
))/(b*n), 1 - ((I/2)*(1 + m))/(b*n), -E^((2*I)*(a + b*Log[c*x^n]))] - E^((
2*I)*a)*(1 + m)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m +
(2*I)*b*n))/(b*n), ((-1/2*I)*(1 + m + (4*I)*b*n))/(b*n), -E^((2*I)*(a + b*
Log[c*x^n]))] + I*(1 + m + (2*I)*b*n)*Tan[a + b*Log[c*x^n]]))/(b*n*(1 + m
+ (2*I)*b*n))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5020, 5016, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sec^2(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{5020}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\sec^2(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5016}$$

$$\frac{4e^{2ia}x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int \frac{(cx^n)^{2ib+\frac{m+1}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^2}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{4e^{2ia}x^{m+1}(cx^n)^{-\frac{m+1}{n}+\frac{2ibn+m+1}{n}}\text{Hypergeometric2F1}\left(2,-\frac{i(m+1)-2bn}{2bn},-\frac{i(m+1)-4bn}{2bn},-e^{2ia}(cx^n)^{2ib}\right)}{2ibn+m+1}$$

input
```
Int[x^m*Sec[a + b*Log[c*x^n]]^2,x]
```

output
```
(4*E^((2*I)*a)*x^(1 + m)*(c*x^n)^(-((1 + m)/n) + (1 + m + (2*I)*b*n)/n)*Hy
pergeometric2F1[2, -1/2*(I*(1 + m) - 2*b*n)/(b*n), -1/2*(I*(1 + m) - 4*b*n
)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(1 + m + (2*I)*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^m \sec(a + b\ln(c\,x^n))^2 dx$$

input
```
int(x^m*sec(a+b*ln(c*x^n))^2,x)
```

output
```
int(x^m*sec(a+b*ln(c*x^n))^2,x)
```

Fricas [F]

$$\int x^m \sec^2(a + b\log(cx^n)) \, dx = \int x^m \sec(b\log(cx^n) + a)^2 \, dx$$

input
```
integrate(x^m*sec(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
integral(x^m*sec(b*log(c*x^n) + a)^2, x)
```

Sympy [F]

$$\int x^m \sec^2(a + b\log(cx^n)) \, dx = \int x^m \sec^2(a + b\log(cx^n)) \, dx$$

input
```
integrate(x**m*sec(a+b*ln(c*x**n))**2,x)
```

output
```
Integral(x**m*sec(a + b*log(c*x**n))**2, x)
```

Maxima [F]

$$\int x^m \sec^2(a + b\log(cx^n)) \, dx = \int x^m \sec(b\log(cx^n) + a)^2 \, dx$$

input
```
integrate(x^m*sec(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
2*(x*x^m*cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + x*x^m*cos(2*b*log(c))*s
in(2*b*log(x^n) + 2*a) - ((b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2 +
 (b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*m)*n^2*cos(2*b*log(x^n) +
 2*a)^2 + (b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2 + (b^2*cos(2*b*lo
g(c))^2 + b^2*sin(2*b*log(c))^2)*m)*n^2*sin(2*b*log(x^n) + 2*a)^2 + 2*(b^2
*m*cos(2*b*log(c)) + b^2*cos(2*b*log(c)))*n^2*cos(2*b*log(x^n) + 2*a) - 2*
(b^2*m*sin(2*b*log(c)) + b^2*sin(2*b*log(c)))*n^2*sin(2*b*log(x^n) + 2*a)
+ (b^2*m + b^2)*n^2)*integrate((x^m*cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)
) + x^m*cos(2*b*log(c))*sin(2*b*log(x^n) + 2*a))/(2*b^2*n^2*cos(2*b*log(c)
)*cos(2*b*log(x^n) + 2*a) - 2*b^2*n^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2
*a) + (b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2*cos(2*b*log(x^n)
 + 2*a)^2 + (b^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2*sin(2*b*lo
g(x^n) + 2*a)^2 + b^2*n^2), x))/(2*b*n*cos(2*b*log(c))*cos(2*b*log(x^n) +
2*a) + (b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*cos(2*b*log(x^n) + 2*
a)^2 - 2*b*n*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) + (b*cos(2*b*log(c))^
2 + b*sin(2*b*log(c))^2)*n*sin(2*b*log(x^n) + 2*a)^2 + b*n)
```

Giac [F]

$$\int x^m \sec^2(a+b\log(cx^n))\,dx=\int x^m \sec(b\log(cx^n)+a)^2\,dx$$

input
```
integrate(x^m*sec(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
integrate(x^m*sec(b*log(c*x^n) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sec^2(a+b\log(cx^n))\,dx=\int \frac{x^m}{\cos\left(a+b\,\ln\left(c\,x^n\right)\right)^2}\,dx$$

input
```
int(x^m/cos(a + b*log(c*x^n))^2,x)
```

output
```
int(x^m/cos(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int x^m \sec^2\left(a + b \log\left(cx^n\right)\right) \, dx = \text{too large to display}$$

input
```
int(x^m*sec(a+b*log(c*x^n))^2,x)
```

output
```
too large to display
```

3.280 $\int x^m \sec(a + b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 103

$$\int x^m \sec(a + b\log(cx^n))\,dx = \frac{2e^{ia}x^{1+m}(cx^n)^{ib}\,\text{Hypergeometric2F1}\left(1, -\frac{i+im-bn}{2bn}, -\frac{i(1+m)-3bn}{2bn}, -e^{2ia}(cx^n)^{2ib}\right)}{1+m+ibn}$$

output
```
2*exp(I*a)*x^(1+m)*(c*x^n)^(I*b)*hypergeom([1, -1/2*(I+I*m-b*n)/b/n],[-1/2
*(I*(1+m)-3*b*n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(1+m+I*b*n)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.96

$$\int x^m \sec(a + b\log(cx^n))\,dx = \frac{2e^{ia}x^{1+m}(cx^n)^{ib}\,\text{Hypergeometric2F1}\left(1, \frac{-i-im+bn}{2bn}, -\frac{i(1+m+3ibn)}{2bn}, -e^{2i(a+b\log(cx^n))}\right)}{1+m+ibn}$$

input
```
Integrate[x^m*Sec[a + b*Log[c*x^n]],x]
```

output
```
(2*E^(I*a)*x^(1 + m)*(c*x^n)^(I*b)*Hypergeometric2F1[1, (-I - I*m + b*n)/(
2*b*n), ((-1/2*I)*(1 + m + (3*I)*b*n))/(b*n), -E^((2*I)*(a + b*Log[c*x^n])
)])/(1 + m + I*b*n)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5020, 5016, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sec(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{5020}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\sec(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5016}$$

$$\frac{2e^{ia}x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int \frac{(cx^n)^{ib+\frac{m+1}{n}-1}}{e^{2ia}(cx^n)^{2ib}+1}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{2e^{ia}x^{m+1}(cx^n)^{-\frac{m+1}{n}+\frac{ibn+m+1}{n}}\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1-\frac{i(m+1)}{bn}\right),-\frac{i(m+1)-3bn}{2bn},-e^{2ia}(cx^n)^{2ib}\right)}{ibn+m+1}$$

input
```
Int[x^m*Sec[a + b*Log[c*x^n]],x]
```

output
```
(2*E^(I*a)*x^(1 + m)*(c*x^n)^(-((1 + m)/n) + (1 + m + I*b*n)/n)*Hypergeome
tric2F1[1, (1 - (I*(1 + m))/(b*n))/2, -1/2*(I*(1 + m) - 3*b*n)/(b*n), -(E^
((2*I)*a)*(c*x^n)^((2*I)*b))])/(1 + m + I*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5016
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[2^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*
b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^m \sec \left(a+b\ln \left(c\,x^n\right)\right) dx$$

input
```
int(x^m*sec(a+b*ln(c*x^n)),x)
```

output
```
int(x^m*sec(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int x^m \sec \left(a+b\log \left(cx^n\right)\right)\,dx=\int x^m \sec \left(b\log \left(cx^n\right)+a\right)\,dx$$

input
```
integrate(x^m*sec(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
integral(x^m*sec(b*log(c*x^n) + a), x)
```

Sympy [F]

$$\int x^m \sec(a + b\log(cx^n))\, dx = \int x^m \sec(a + b\log(cx^n))\, dx$$

input
```
integrate(x**m*sec(a+b*ln(c*x**n)),x)
```

output
```
Integral(x**m*sec(a + b*log(c*x**n)), x)
```

Maxima [F]

$$\int x^m \sec(a + b\log(cx^n))\, dx = \int x^m \sec(b\log(cx^n) + a)\, dx$$

input
```
integrate(x^m*sec(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
integrate(x^m*sec(b*log(c*x^n) + a), x)
```

Giac [F]

$$\int x^m \sec(a + b\log(cx^n))\, dx = \int x^m \sec(b\log(cx^n) + a)\, dx$$

input
```
integrate(x^m*sec(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
integrate(x^m*sec(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sec(a + b\log(cx^n)) \, dx = \int \frac{x^m}{\cos(a + b\ln(c\,x^n))} \, dx$$

input
```
int(x^m/cos(a + b*log(c*x^n)),x)
```

output
```
int(x^m/cos(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x^m \sec(a + b\log(cx^n)) \, dx = \text{too large to display}$$

input
```
int(x^m*sec(a+b*log(c*x^n)),x)
```

output
```
(2*x**m*cos(log(x**n*c)*b + a)*sec(log(x**n*c)*b + a)*b**2*n**2*x - x**m*c
os(log(x**n*c)*b + a)*sec(log(x**n*c)*b + a)*m**2*x - 2*x**m*cos(log(x**n*
c)*b + a)*sec(log(x**n*c)*b + a)*m*x - x**m*cos(log(x**n*c)*b + a)*sec(log
(x**n*c)*b + a)*x - x**m*cos(log(x**n*c)*b + a)*b**2*n**2*x + 8*cos(log(x*
*n*c)*b + a)*int(x**m/(2*tan((log(x**n*c)*b + a)/2)**4*b**2*n**2 - tan((lo
g(x**n*c)*b + a)/2)**4*m**2 - 2*tan((log(x**n*c)*b + a)/2)**4*m - tan((log
(x**n*c)*b + a)/2)**4 - 4*tan((log(x**n*c)*b + a)/2)**2*b**2*n**2 + 2*tan(
(log(x**n*c)*b + a)/2)**2*m**2 + 4*tan((log(x**n*c)*b + a)/2)**2*m + 2*tan
((log(x**n*c)*b + a)/2)**2 + 2*b**2*n**2 - m**2 - 2*m - 1),x)*b**4*m*n**4
+ 8*cos(log(x**n*c)*b + a)*int(x**m/(2*tan((log(x**n*c)*b + a)/2)**4*b**2*
n**2 - tan((log(x**n*c)*b + a)/2)**4*m**2 - 2*tan((log(x**n*c)*b + a)/2)**
4*m - tan((log(x**n*c)*b + a)/2)**4 - 4*tan((log(x**n*c)*b + a)/2)**2*b**2
*n**2 + 2*tan((log(x**n*c)*b + a)/2)**2*m**2 + 4*tan((log(x**n*c)*b + a)/2
)**2*m + 2*tan((log(x**n*c)*b + a)/2)**2 + 2*b**2*n**2 - m**2 - 2*m - 1),x
)*b**4*n**4 - 4*cos(log(x**n*c)*b + a)*int(x**m/(2*tan((log(x**n*c)*b + a)
/2)**4*b**2*n**2 - tan((log(x**n*c)*b + a)/2)**4*m**2 - 2*tan((log(x**n*c)
*b + a)/2)**4*m - tan((log(x**n*c)*b + a)/2)**4 - 4*tan((log(x**n*c)*b + a
)/2)**2*b**2*n**2 + 2*tan((log(x**n*c)*b + a)/2)**2*m**2 + 4*tan((log(x**n
*c)*b + a)/2)**2*m + 2*tan((log(x**n*c)*b + a)/2)**2 + 2*b**2*n**2 - m**2
- 2*m - 1),x)*b**2*m**3*n**2 - 12*cos(log(x**n*c)*b + a)*int(x**m/(2*ta...
```

3.281 $\int x^m \sec^{\frac{5}{2}}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int x^m \sec^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \frac{2x^{1+m}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{2i+2im-5bn}{4bn},-\frac{2i+2im-9bn}{4bn},-e^{2ia}(cx^n)^{2ib}\right)\sec^{\frac{5}{2}}(a+b\ldots}{2+2m+5ibn}$$

output
```
2*x^(1+m)*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(5/2)*hypergeom([5/2, -1/4*(2*I+2
*I*m-5*b*n)/b/n],[-1/4*(2*I+2*I*m-9*b*n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))
*sec(a+b*ln(c*x^n))^(5/2)/(2+2*m+5*I*b*n)
```

Mathematica [A] (warning: unable to verify)

Time = 1.60 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.40

$$\int x^m \sec^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \frac{2x^{1+m}\sqrt{\sec(a+b\log(cx^n))}\left((4+8m+4m^2+b^2n^2)\left(1+e^{2ia}(cx^n)^{2ib}\right)\text{Hypergeometric2F1}\left(1,-\frac{2i+2im\ldots}{4b\ldots}\right.\right.}{3b^2n^2(2+2m+\ldots}$$

input
```
Integrate[x^m*Sec[a + b*Log[c*x^n]]^(5/2),x]
```

output
```
(2*x^(1 + m)*Sqrt[Sec[a + b*Log[c*x^n]]]*((4 + 8*m + 4*m^2 + b^2*n^2)*(1 +
 E^((2*I)*a)*(c*x^n)^((2*I)*b))*Hypergeometric2F1[1, -1/4*(2*I + (2*I)*m -
 3*b*n)/(b*n), -1/4*(2*I + (2*I)*m - 5*b*n)/(b*n), -E^((2*I)*(a + b*Log[c*
x^n]))] - (2 + 2*m + I*b*n)*(2 + 2*m - b*n*Tan[a + b*Log[c*x^n]])))/(3*b^2
*n^2*(2 + 2*m + I*b*n))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5020, 5018, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sec^{\frac{5}{2}}(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{5020}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\sec^{\frac{5}{2}}(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5018}$$

$$\frac{x^{m+1}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}(cx^n)^{-\frac{m+1}{n}-\frac{5ib}{2}}\sec^{\frac{5}{2}}(a+b\log(cx^n))\int \frac{(cx^n)^{\frac{5ib}{2}+\frac{m+1}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^{5/2}}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^{m+1}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i(m+1)}{bn}\right),-\frac{2im-9bn+2i}{4bn},-e^{2ia}(cx^n)^{2ib}\right)\sec^{\frac{5}{2}}(a+b\log$$
$$5ibn+2m+2$$

input
```
Int[x^m*Sec[a + b*Log[c*x^n]]^(5/2),x]
```

output
```
(2*x^(1 + m)*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^(5/2)*Hypergeometric2F1[5
/2, (5 - ((2*I)*(1 + m))/(b*n))/4, -1/4*(2*I + (2*I)*m - 9*b*n)/(b*n), -(E
^((2*I)*a)*(c*x^n)^((2*I)*b))]*Sec[a + b*Log[c*x^n]]^(5/2))/(2 + 2*m + (5*
I)*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5018
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^m \sec(a + b\ln(c\,x^n))^{\frac{5}{2}}dx$$

input
```
int(x^m*sec(a+b*ln(c*x^n))^(5/2),x)
```

output
```
int(x^m*sec(a+b*ln(c*x^n))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^m \sec^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*sec(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \sec^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \text{Timed out}$$

input
```
integrate(x**m*sec(a+b*ln(c*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^m \sec^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \int x^m \sec(b\log(cx^n)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate(x^m*sec(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^m*sec(b*log(c*x^n) + a)^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int x^m \sec^{\frac{5}{2}}(a + b\log(cx^n)) \, dx = \text{Timed out}$$

input
```
integrate(x^m*sec(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sec^{\frac{5}{2}}(a + b\log(cx^n)) \, dx = \int x^m \left(\frac{1}{\cos(a + b\ln(c\,x^n))}\right)^{5/2} dx$$

input
```
int(x^m*(1/cos(a + b*log(c*x^n)))^(5/2),x)
```

output
```
int(x^m*(1/cos(a + b*log(c*x^n)))^(5/2), x)
```

Reduce [F]

$$\int x^m \sec^{\frac{5}{2}}(a + b\log(cx^n)) \, dx$$
$$= \frac{2x^m\sqrt{\sec(\log(x^nc)\,b + a)}\sec(\log(x^nc)\,b + a)^2 x - 5\left(\int x^m\sqrt{\sec(\log(x^nc)\,b + a)}\sec(\log(x^nc)\,b + a)^2 \mathrm{ta}\right.}{2m + 2}$$

input
```
int(x^m*sec(a+b*log(c*x^n))^(5/2),x)
```

output
```
(2*x**m*sqrt(sec(log(x**n*c)*b + a))*sec(log(x**n*c)*b + a)**2*x - 5*int(x
**m*sqrt(sec(log(x**n*c)*b + a))*sec(log(x**n*c)*b + a)**2*tan(log(x**n*c)
*b + a),x)*b*n)/(2*(m + 1))
```

3.282 $\int x^m \sec^{\frac{3}{2}}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int x^m \sec^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \frac{2x^{1+m}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{2i+2im-3bn}{4bn},-\frac{2i+2im-7bn}{4bn},-e^{2ia}(cx^n)^{2ib}\right)\sec^{\frac{3}{2}}(a+b\ldots}{2+2m+3ibn}$$

output
```
2*x^(1+m)*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)*hypergeom([3/2, -1/4*(2*I+2
*I*m-3*b*n)/b/n],[-1/4*(2*I+2*I*m-7*b*n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))
*sec(a+b*ln(c*x^n))^(3/2)/(2+2*m+3*I*b*n)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 470 vs. 2(130) = 260.

Time = 7.53 (sec) , antiderivative size = 470, normalized size of antiderivative = 3.62

$$\int x^m \sec^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \frac{\sqrt{2}x^{1+m-ibn}\left(-\left((4+8m+4m^2+b^2n^2)\,x^{2ibn}\sqrt{\frac{e^{ia}(cx^n)^{ib}}{1+e^{2ia}(cx^n)^{2ib}}}\sqrt{1+e^{2ia}(cx^n)^{2ib}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\ldots\right.\right.\right.}{\ldots}$$

input
```
Integrate[x^m*Sec[a + b*Log[c*x^n]]^(3/2),x]
```

output
```
(Sqrt[2]*x^(1 + m - I*b*n)*(-((4 + 8*m + 4*m^2 + b^2*n^2)*x^((2*I)*b*n)*Sq
rt[(E^(I*a)*(c*x^n)^(I*b))/(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))]*Sqrt[1 + E
^((2*I)*a)*(c*x^n)^((2*I)*b)]*Hypergeometric2F1[1/2, ((-1/2*I)*(1 + m + ((
3*I)/2)*b*n))/(b*n), -1/4*(2*I + (2*I)*m - 7*b*n)/(b*n), -(E^((2*I)*a)*(c*
x^n)^((2*I)*b))]) + (2 + 2*m + (3*I)*b*n)*((2 + 2*m + I*b*n)*Sqrt[(E^(I*a)
*(c*x^n)^(I*b))/(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))]*Sqrt[1 + E^((2*I)*a)*
(c*x^n)^((2*I)*b)]*Hypergeometric2F1[1/2, -1/4*(2*I + (2*I)*m + b*n)/(b*n)
, -1/4*(2*I + (2*I)*m - 3*b*n)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))] -
I*Sqrt[2]*x^(I*b*n)*Sqrt[Sec[a + b*Log[c*x^n]]]*(b*n*Cos[b*n*Log[x]] - 2*(
1 + m)*Sin[b*n*Log[x]]))))/(b*n*(-2*I - (2*I)*m + 3*b*n)*(-2*(1 + m)*Cos[a
 - b*n*Log[x] + b*Log[c*x^n]] + b*n*Sin[a - b*n*Log[x] + b*Log[c*x^n]]))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5020, 5018, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sec^{\frac{3}{2}}(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{5020}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\sec^{\frac{3}{2}}(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5018}$$

$$\frac{x^{m+1}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}(cx^n)^{-\frac{m+1}{n}-\frac{3ib}{2}}\sec^{\frac{3}{2}}(a+b\log(cx^n))\int \frac{(cx^n)^{\frac{3ib}{2}+\frac{m+1}{n}-1}}{\left(e^{2ia}(cx^n)^{2ib}+1\right)^{3/2}}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^{m+1}\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i(m+1)}{bn}\right),-\frac{2im-7bn+2i}{4bn},-e^{2ia}(cx^n)^{2ib}\right)\sec^{\frac{3}{2}}(a+b\log}{3ibn+2m+2}$$

input
```
Int[x^m*Sec[a + b*Log[c*x^n]]^(3/2),x]
```

output
```
(2*x^(1 + m)*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2)*Hypergeometric2F1[3
/2, (3 - ((2*I)*(1 + m))/(b*n))/4, -1/4*(2*I + (2*I)*m - 7*b*n)/(b*n), -(E
^((2*I)*a)*(c*x^n)^((2*I)*b))]*Sec[a + b*Log[c*x^n]]^(3/2))/(2 + 2*m + (3*
I)*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5018
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^m \sec\left(a+b\ln\left(c\,x^n\right)\right)^{\frac{3}{2}}dx$$

input
```
int(x^m*sec(a+b*ln(c*x^n))^(3/2),x)
```

output
```
int(x^m*sec(a+b*ln(c*x^n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^m \sec^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*sec(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \sec^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)\,dx = \text{Timed out}$$

input
```
integrate(x**m*sec(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^m \sec^{\frac{3}{2}}(a + b\log(cx^n))\,dx = \int x^m \sec(b\log(cx^n) + a)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^m*sec(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^m*sec(b*log(c*x^n) + a)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int x^m \sec^{\frac{3}{2}}(a + b\log(cx^n))\,dx = \text{Timed out}$$

input
```
integrate(x^m*sec(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sec^{\frac{3}{2}}(a + b\log(cx^n))\,dx = \int x^m \left(\frac{1}{\cos(a + b\ln(c\,x^n))}\right)^{3/2} dx$$

input
```
int(x^m*(1/cos(a + b*log(c*x^n)))^(3/2),x)
```

output
```
int(x^m*(1/cos(a + b*log(c*x^n)))^(3/2), x)
```

Reduce [F]

$$\int x^m \sec^{\frac{3}{2}}(a+b\log(cx^n))\,dx$$
$$= \frac{2x^m\sqrt{\sec(\log(x^n c)\,b+a)}\sec(\log(x^n c)\,b+a)\,x - 3\Big(\int x^m\sqrt{\sec(\log(x^n c)\,b+a)}\sec(\log(x^n c)\,b+a)\tan}{2m+2}$$

input
```
int(x^m*sec(a+b*log(c*x^n))^(3/2),x)
```

output
```
(2*x**m*sqrt(sec(log(x**n*c)*b + a))*sec(log(x**n*c)*b + a)*x - 3*int(x**m
*sqrt(sec(log(x**n*c)*b + a))*sec(log(x**n*c)*b + a)*tan(log(x**n*c)*b + a
),x)*b*n)/(2*(m + 1))
```

3.283 $\int x^m \sqrt{\sec (a+b \log (c x^n))} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int x^m \sqrt{\sec (a+b \log (c x^n))} \, dx = \frac{2x^{1+m}\sqrt{1+e^{2ia}(cx^n)^{2ib}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{2i+2im-bn}{4bn},-\frac{2i+2im-5bn}{4bn},-e^{2ia}(cx^n)^{2ib}\right)\sqrt{\sec (a+b\log}}{2+2m+ibn}$$

output
```
2*x^(1+m)*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)*hypergeom([1/2, -1/4*(2*I+2
*I*m-b*n)/b/n],[-1/4*(2*I+2*I*m-5*b*n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))*s
ec(a+b*ln(c*x^n))^(1/2)/(2+2*m+I*b*n)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

$$\int x^m \sqrt{\sec (a+b \log (c x^n))} \, dx = \frac{2\left(1+e^{2i(a+b\log(cx^n))}\right)x^{1+m}\,\text{Hypergeometric2F1}\left(1,-\frac{2i+2im-3bn}{4bn},-\frac{2i+2im-5bn}{4bn},-e^{2i(a+b\log(cx^n))}\right)\sqrt{\sec (a+}}{2+2m+ibn}$$

input
```
Integrate[x^m*Sqrt[Sec[a + b*Log[c*x^n]]],x]
```

output
```
(2*(1 + E^((2*I)*(a + b*Log[c*x^n])))*x^(1 + m)*Hypergeometric2F1[1, -1/4*
(2*I + (2*I)*m - 3*b*n)/(b*n), -1/4*(2*I + (2*I)*m - 5*b*n)/(b*n), -E^((2*
I)*(a + b*Log[c*x^n]))]*Sqrt[Sec[a + b*Log[c*x^n]]])/(2 + 2*m + I*b*n)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5020, 5018, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{\sec (a+b \log (cx^n))} \, dx$$

$$\Big\downarrow \text{ 5020}$$

$$\frac{x^{m+1} (cx^n)^{-\frac{m+1}{n}} \int (cx^n)^{\frac{m+1}{n}-1} \sqrt{\sec (a+b \log (cx^n))} d(cx^n)}{n}$$

$$\Big\downarrow \text{ 5018}$$

$$\frac{x^{m+1} \sqrt{1+e^{2ia} (cx^n)^{2ib}} (cx^n)^{-\frac{m+1}{n}-\frac{ib}{2}} \sqrt{\sec (a+b \log (cx^n))} \int \frac{(cx^n)^{\frac{ib}{2}+\frac{m+1}{n}-1}}{\sqrt{e^{2ia} (cx^n)^{2ib}+1}} d(cx^n)}{n}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{2x^{m+1} \sqrt{1+e^{2ia} (cx^n)^{2ib}} \text{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{2im-bn+2i}{4bn}, -\frac{2im-5bn+2i}{4bn}, -e^{2ia} (cx^n)^{2ib}\right) \sqrt{\sec (a+b \log (cx^n}}{ibn+2m+2}$$

input
```
Int[x^m*Sqrt[Sec[a + b*Log[c*x^n]]],x]
```

output
```
(2*x^(1 + m)*Sqrt[1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Hypergeometric2F1[1/2
, -1/4*(2*I + (2*I)*m - b*n)/(b*n), -1/4*(2*I + (2*I)*m - 5*b*n)/(b*n), -(
E^((2*I)*a)*(c*x^n)^((2*I)*b))]*Sqrt[Sec[a + b*Log[c*x^n]]])/(2 + 2*m + I*
b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5018
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^m \sqrt{\sec (a+b\ln (c\,x^n))}dx$$

input
```
int(x^m*sec(a+b*ln(c*x^n))^(1/2),x)
```

output
```
int(x^m*sec(a+b*ln(c*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^m \sqrt{\sec (a+b\log (cx^n))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*sec(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^m \sqrt{\sec (a+b \log (c x^n))} \, dx = \int x^m \sqrt{\sec (a+b \log (c x^n))} \, dx$$

input
```
integrate(x**m*sec(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(x**m*sqrt(sec(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int x^m \sqrt{\sec (a+b \log (c x^n))} \, dx = \int x^m \sqrt{\sec (b \log (c x^n) + a)} \, dx$$

input
```
integrate(x^m*sec(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m*sqrt(sec(b*log(c*x^n) + a)), x)
```

Giac [F]

$$\int x^m \sqrt{\sec (a+b \log (c x^n))} \, dx = \int x^m \sqrt{\sec (b \log (c x^n) + a)} \, dx$$

input
```
integrate(x^m*sec(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m*sqrt(sec(b*log(c*x^n) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sqrt{\sec(a + b\log(cx^n))}\,dx = \int x^m \sqrt{\frac{1}{\cos(a + b\ln(c\,x^n))}}\,dx$$

input
```
int(x^m*(1/cos(a + b*log(c*x^n)))^(1/2),x)
```

output
```
int(x^m*(1/cos(a + b*log(c*x^n)))^(1/2), x)
```

Reduce [F]

$$\int x^m \sqrt{\sec(a + b\log(cx^n))}\,dx = \frac{2x^m \sqrt{\sec(\log(x^n c)\,b + a)}\,x - \left(\int x^m \sqrt{\sec(\log(x^n c)\,b + a)}\,\tan(\log(x^n c)\,b + a)\,dx\right) bn}{2m + 2}$$

input
```
int(x^m*sec(a+b*log(c*x^n))^(1/2),x)
```

output
```
(2*x**m*sqrt(sec(log(x**n*c)*b + a))*x - int(x**m*sqrt(sec(log(x**n*c)*b +
 a))*tan(log(x**n*c)*b + a),x)*b*n)/(2*(m + 1))
```

3.284 $\int \frac{x^m}{\sqrt{\sec(a+b\log(cx^n))}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 129

$$\int \frac{x^m}{\sqrt{\sec(a+b\log(cx^n))}}\,dx = \frac{2x^{1+m}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2i+2im+bn}{4bn},-\frac{2i+2im-3bn}{4bn},-e^{2ia}(cx^n)^{2ib}\right)}{(2+2m-ibn)\sqrt{1+e^{2ia}(cx^n)^{2ib}}\sqrt{\sec(a+b\log(cx^n))}}$$

output
```
2*x^(1+m)*hypergeom([-1/2, -1/4*(2*I+2*I*m+b*n)/b/n],[-1/4*(2*I+2*I*m-3*b*
n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(2+2*m-I*b*n)/(1+exp(2*I*a)*(c*x^n)^(
2*I*b))^(1/2)/sec(a+b*ln(c*x^n))^(1/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 437 vs. 2(129) = 258.

Time = 5.11 (sec) , antiderivative size = 437, normalized size of antiderivative = 3.39

$$\int \frac{x^m}{\sqrt{\sec(a+b\log(cx^n))}}\,dx =$$
$$-\frac{2be^{2i(a-bn\log(x)+b\log(cx^n))}nx^{1+m}\left((2i+2im+bn)x^{2ibn}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{i\left(1+m+\frac{3ibn}{2}\right)}{2bn},-\frac{2i+2im-}{4bn}\right.\right.}{(2+2m-ibn)(2+2m+3ibn)\left(2+2m-ibn+e^{2i(a-bn}\right.}$$
$$+\frac{2x^{1+m}\cos(a-bn\log(x)+b\log(cx^n))}{\sqrt{\sec(a+b\log(cx^n))}\left(2(1+m)\cos(a-bn\log(x)+b\log(cx^n))-bn\sin(a-bn\log(x)+b\log(cx^n)\right)}$$

input
```
Integrate[x^m/Sqrt[Sec[a + b*Log[c*x^n]]],x]
```

output
```
(-2*b*E^((2*I)*(a - b*n*Log[x] + b*Log[c*x^n]))*n*x^(1 + m)*((2*I + (2*I)*
m + b*n)*x^((2*I)*b*n)*Hypergeometric2F1[1/2, ((-1/2*I)*(1 + m + ((3*I)/2)
*b*n))/(b*n), -1/4*(2*I + (2*I)*m - 7*b*n)/(b*n), -(E^((2*I)*a)*(c*x^n)^((
2*I)*b))] + (-2*I - (2*I)*m + 3*b*n)*Hypergeometric2F1[1/2, -1/4*(2*I + (2
*I)*m + b*n)/(b*n), -1/4*(2*I + (2*I)*m - 3*b*n)/(b*n), -(E^((2*I)*a)*(c*x
^n)^((2*I)*b))]))/((2 + 2*m - I*b*n)*(2 + 2*m + (3*I)*b*n)*(2 + 2*m - I*b*
n + E^((2*I)*(a - b*n*Log[x] + b*Log[c*x^n]))*(2 + 2*m + I*b*n))*Sqrt[1 +
E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[(E^(I*a)*(c*x^n)^(I*b))/(2 + 2*E^((2*I
)*a)*(c*x^n)^((2*I)*b))]) + (2*x^(1 + m)*Cos[a - b*n*Log[x] + b*Log[c*x^n]
])/(Sqrt[Sec[a + b*Log[c*x^n]]]*(2*(1 + m)*Cos[a - b*n*Log[x] + b*Log[c*x^
n]] - b*n*Sin[a - b*n*Log[x] + b*Log[c*x^n]]))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5020, 5018, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{\sec(a+b\log(cx^n))}}\,dx$$

$$\Big\downarrow \text{ 5020}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int\frac{(cx^n)^{\frac{m+1}{n}-1}}{\sqrt{\sec(a+b\log(cx^n))}}d(cx^n)}{n}$$

$$\Big\downarrow \text{ 5018}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}+\frac{ib}{2}}\int(cx^n)^{-\frac{ib}{2}+\frac{m+1}{n}-1}\sqrt{e^{2ia}\left(cx^n\right)^{2ib}+1}d(cx^n)}{n\sqrt{1+e^{2ia}\left(cx^n\right)^{2ib}}\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{2x^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-\frac{2i(m+1)}{bn}-1\right),-\frac{2im-3bn+2i}{4bn},-e^{2ia}(cx^n)^{2ib}\right)}{(-ibn+2m+2)\sqrt{1+e^{2ia}\left(cx^n\right)^{2ib}}\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}$$

input
```
Int[x^m/Sqrt[Sec[a + b*Log[c*x^n]]],x]
```

output
```
(2*x^(1 + m)*Hypergeometric2F1[-1/2, (-1 - ((2*I)*(1 + m))/(b*n))/4, -1/4*
(2*I + (2*I)*m - 3*b*n)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 + 2*
m - I*b*n)*Sqrt[1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[Sec[a + b*Log[c*x^
n]]])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5018
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{x^m}{\sqrt{\sec\left(a+b\ln\left(c\,x^n\right)\right)}}dx$$

input
```
int(x^m/sec(a+b*ln(c*x^n))^(1/2),x)
```

output
```
int(x^m/sec(a+b*ln(c*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^m}{\sqrt{\sec\left(a+b\log\left(cx^n\right)\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/sec(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{x^m}{\sqrt{\sec(a+b\log(cx^n))}}\,dx = \int \frac{x^m}{\sqrt{\sec(a+b\log(cx^n))}}\,dx$$

input
```
integrate(x**m/sec(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(x**m/sqrt(sec(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \frac{x^m}{\sqrt{\sec(a+b\log(cx^n))}}\,dx = \int \frac{x^m}{\sqrt{\sec(b\log(cx^n)+a)}}\,dx$$

input
```
integrate(x^m/sec(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/sqrt(sec(b*log(c*x^n) + a)), x)
```

Giac [F]

$$\int \frac{x^m}{\sqrt{\sec(a+b\log(cx^n))}}\,dx = \int \frac{x^m}{\sqrt{\sec(b\log(cx^n)+a)}}\,dx$$

input
```
integrate(x^m/sec(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/sqrt(sec(b*log(c*x^n) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{\sec(a+b\log(cx^n))}}\,dx = \int \frac{x^m}{\sqrt{\frac{1}{\cos(a+b\ln(c\,x^n))}}}\,dx$$

input
```
int(x^m/(1/cos(a + b*log(c*x^n)))^(1/2),x)
```

output
```
int(x^m/(1/cos(a + b*log(c*x^n)))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^m}{\sqrt{\sec(a+b\log(cx^n))}}\,dx = \int \frac{x^m\sqrt{\sec(\log(x^nc)\,b+a)}}{\sec(\log(x^nc)\,b+a)}\,dx$$

input
```
int(x^m/sec(a+b*log(c*x^n))^(1/2),x)
```

output
```
int((x**m*sqrt(sec(log(x**n*c)*b + a)))/sec(log(x**n*c)*b + a),x)
```

3.285 $\int \frac{x^m}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{x^m}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{2x^{1+m}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{2i+2im+3bn}{4bn},-\frac{2i+2im-bn}{4bn},-e^{2ia}(cx^n)^{2ib}\right)}{(2+2m-3ibn)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\sec^{\frac{3}{2}}(a+b\log(cx^n))}$$

output
```
2*x^(1+m)*hypergeom([-3/2, -1/4*(2*I+2*I*m+3*b*n)/b/n],[-1/4*(2*I+2*I*m-b*
n)/b/n],-exp(2*I*a)*(c*x^n)^(2*I*b))/(2+2*m-3*I*b*n)/(1+exp(2*I*a)*(c*x^n)
^(2*I*b))^(3/2)/sec(a+b*ln(c*x^n))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.77 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.55

$$\int \frac{x^m}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{2x^{1+m}\left(3b^2n^2\left(1+e^{2ia}(cx^n)^{2ib}\right)\text{Hypergeometric2F1}\left(1,-\frac{2i+2im-3bn}{4bn},-\frac{2i+2im-5bn}{4bn},-e^{2i(a+b\log(cx^n))}\right)\sec^2(\cdots}{(2+2m+ibn)(2+2m-3ibn)(2+2m+3ibn)\,\text{se}\cdots}$$

input
```
Integrate[x^m/Sec[a + b*Log[c*x^n]]^(3/2),x]
```

output
```
(2*x^(1 + m)*(3*b^2*n^2*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))*Hypergeometric
2F1[1, -1/4*(2*I + (2*I)*m - 3*b*n)/(b*n), -1/4*(2*I + (2*I)*m - 5*b*n)/(b
*n), -E^((2*I)*(a + b*Log[c*x^n]))]*Sec[a + b*Log[c*x^n]]^2 + (2 + 2*m + I
*b*n)*(2 + 2*m + 3*b*n*Tan[a + b*Log[c*x^n]])))/((2 + 2*m + I*b*n)*(2 + 2*
m - (3*I)*b*n)*(2 + 2*m + (3*I)*b*n)*Sec[a + b*Log[c*x^n]]^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5020, 5018, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{5020}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int \frac{(cx^n)^{\frac{m+1}{n}-1}}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}d(cx^n)}{n}$$

$$\Big\downarrow \text{5018}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}+\frac{3ib}{2}}\int (cx^n)^{-\frac{3ib}{2}+\frac{m+1}{n}-1}\left(e^{2ia}(cx^n)^{2ib}+1\right)^{3/2}d(cx^n)}{n\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\sec^{\frac{3}{2}}(a+b\log(cx^n))}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^{m+1}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4}\left(-\frac{2i(m+1)}{bn}-3\right),-\frac{2im-bn+2i}{4bn},-e^{2ia}(cx^n)^{2ib}\right)}{(-3ibn+2m+2)\left(1+e^{2ia}(cx^n)^{2ib}\right)^{3/2}\sec^{\frac{3}{2}}(a+b\log(cx^n))}$$

input
```
Int[x^m/Sec[a + b*Log[c*x^n]]^(3/2),x]
```

output
```
(2*x^(1 + m)*Hypergeometric2F1[-3/2, (-3 - ((2*I)*(1 + m))/(b*n))/4, -1/4*
(2*I + (2*I)*m - b*n)/(b*n), -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/((2 + 2*m
- (3*I)*b*n)*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2)*Sec[a + b*Log[c*x^n
]]^(3/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5018
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{x^m}{\sec\left(a+b\ln\left(c\,x^n\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m/sec(a+b*ln(c*x^n))^(3/2),x)
```

output
```
int(x^m/sec(a+b*ln(c*x^n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^m}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/sec(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{x^m}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{x^m}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$

input
```
integrate(x**m/sec(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Integral(x**m/sec(a + b*log(c*x**n))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^m}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{x^m}{\sec(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/sec(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^m/sec(b*log(c*x^n) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^m}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{x^m}{\sec(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/sec(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m/sec(b*log(c*x^n) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{x^m}{\left(\frac{1}{\cos(a+b\,\ln(c\,x^n))}\right)^{3/2}}\,dx$$

input
```
int(x^m/(1/cos(a + b*log(c*x^n)))^(3/2),x)
```

output
```
int(x^m/(1/cos(a + b*log(c*x^n)))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^m}{\sec^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{x^m\sqrt{\sec(\log(x^nc)\,b+a)}}{\sec(\log(x^nc)\,b+a)^2}dx$$

input
```
int(x^m/sec(a+b*log(c*x^n))^(3/2),x)
```

output
```
int((x**m*sqrt(sec(log(x**n*c)*b + a)))/sec(log(x**n*c)*b + a)**2,x)
```

3.286 $\int (ex)^m \sec^p(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 139

$$\int (ex)^m \sec^p(d(a+b\log(cx^n)))\,dx = \frac{(ex)^{1+m}\left(1+e^{2iad}(cx^n)^{2ibd}\right)^p \text{Hypergeometric2F1}\left(p,-\frac{i+im-bdnp}{2bdn},\frac{1}{2}\left(2-\frac{i(1+m)}{bdn}+p\right),-e^{2iad}(cx^n)^{2ibd}\right) \mathrm{s}}{e(1+m+ibdnp)}$$

output
```
(e*x)^(1+m)*(1+exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p*hypergeom([p, -1/2*(I+I*m
-b*d*n*p)/b/d/n],[1-1/2*I*(1+m)/b/d/n+1/2*p],-exp(2*I*a*d)*(c*x^n)^(2*I*b*
d))*sec(d*(a+b*ln(c*x^n)))^p/e/(1+m+I*b*d*n*p)
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.22

$$\int (ex)^m \sec^p(d(a+b\log(cx^n)))\,dx = \frac{2^p x(ex)^m\left(\frac{e^{iad}(cx^n)^{ibd}}{1+e^{2iad}(cx^n)^{2ibd}}\right)^p\left(1+e^{2iad}(cx^n)^{2ibd}\right)^p \text{Hypergeometric2F1}\left(p,-\frac{i(1+m+ibdnp)}{2bdn},\frac{1}{2}\left(2-\frac{i(1+m)}{bdn}+\right.\right.}{1+m+ibdnp}$$

input
```
Integrate[(e*x)^m*Sec[d*(a + b*Log[c*x^n])]^p,x]
```

output
```
(2^p*x*(e*x)^m*((E^(I*a*d)*(c*x^n)^(I*b*d))/(1 + E^((2*I)*a*d)*(c*x^n)^((2
*I)*b*d)))^p*(1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p*Hypergeometric2F1[p
, ((-1/2*I)*(1 + m + I*b*d*n*p))/(b*d*n), (2 - (I*(1 + m))/(b*d*n) + p)/2,
 -(E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))])/(1 + m + I*b*d*n*p)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5020, 5018, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \sec^p(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{5020}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\sec^p(d(a+b\log(cx^n)))\,d(cx^n)}{en}$$

$$\Big\downarrow \text{5018}$$

$$\frac{(ex)^{m+1}\left(1+e^{2iad}(cx^n)^{2ibd}\right)^p(cx^n)^{-\frac{m+1}{n}-ibdp}\sec^p(d(a+b\log(cx^n)))\int (cx^n)^{\frac{m+1}{n}+ibdp-1}\left(e^{2iad}(cx^n)^{2ibd}+1\right)^{-p}}{en}$$

$$\Big\downarrow \text{888}$$

$$\frac{(ex)^{m+1}\left(1+e^{2iad}(cx^n)^{2ibd}\right)^p(cx^n)^{\frac{ibdnp+m+1}{n}-ibdp-\frac{m+1}{n}}\text{Hypergeometric2F1}\left(p,\frac{1}{2}\left(p-\frac{i(m+1)}{bdn}\right),\frac{1}{2}\left(-\frac{i(m+1)}{bdn}+p\right.\right.}{e(ibdnp+m+1)}$$

input
```
Int[(e*x)^m*Sec[d*(a + b*Log[c*x^n])]^p,x]
```

output
```
((e*x)^(1 + m)*(c*x^n)^(-((1 + m)/n) - I*b*d*p + (1 + m + I*b*d*n*p)/n)*(1
 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p*Hypergeometric2F1[p, (((-I)*(1 + m
))/(b*d*n) + p)/2, (2 - (I*(1 + m))/(b*d*n) + p)/2, -(E^((2*I)*a*d)*(c*x^n
)^((2*I)*b*d))]*Sec[d*(a + b*Log[c*x^n])]^p)/(e*(1 + m + I*b*d*n*p))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5018
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int (ex)^m \sec\left(d(a + b\ln\left(c\,x^n\right))\right)^p dx$$

input
```
int((e*x)^m*sec(d*(a+b*ln(c*x^n)))^p,x)
```

output
```
int((e*x)^m*sec(d*(a+b*ln(c*x^n)))^p,x)
```

Fricas [F]

$$\int (ex)^m \sec^p\left(d(a + b\log\left(cx^n\right))\right)\,dx = \int (ex)^m \sec\left((b\log\left(cx^n\right) + a)d\right)^p\,dx$$

input
```
integrate((e*x)^m*sec(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^m*sec(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F]

$$\int (ex)^m \sec^p(d(a+b\log(cx^n)))\,dx = \int (ex)^m \sec^p(ad+bd\log(cx^n))\,dx$$

input
```
integrate((e*x)**m*sec(d*(a+b*ln(c*x**n)))**p,x)
```

output
```
Integral((e*x)**m*sec(a*d + b*d*log(c*x**n))**p, x)
```

Maxima [F]

$$\int (ex)^m \sec^p(d(a+b\log(cx^n)))\,dx = \int (ex)^m \sec((b\log(cx^n)+a)d)^p\,dx$$

input
```
integrate((e*x)^m*sec(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*sec((b*log(c*x^n) + a)*d)^p, x)
```

Giac [F]

$$\int (ex)^m \sec^p(d(a+b\log(cx^n)))\,dx = \int (ex)^m \sec((b\log(cx^n)+a)d)^p\,dx$$

input
```
integrate((e*x)^m*sec(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^m*sec((b*log(c*x^n) + a)*d)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \sec^p(d(a+b\log(cx^n)))\,dx = \int (e\,x)^m \left(\frac{1}{\cos(d\,(a+b\,\ln(c\,x^n)))}\right)^p dx$$

input
```
int((e*x)^m*(1/cos(d*(a + b*log(c*x^n))))^p,x)
```

output
```
int((e*x)^m*(1/cos(d*(a + b*log(c*x^n))))^p, x)
```

Reduce [F]

$$\int (ex)^m \sec^p(d(a+b\log(cx^n)))\,dx$$
$$= \frac{e^m\left(x^m\sec(\log(x^n c)\,bd+ad)^p x-\left(\int x^m\sec(\log(x^n c)\,bd+ad)^p\tan(\log(x^n c)\,bd+ad)\,dx\right)bdnp\right)}{m+1}$$

input
```
int((e*x)^m*sec(d*(a+b*log(c*x^n)))^p,x)
```

output
```
(e**m*(x**m*sec(log(x**n*c)*b*d + a*d)**p*x - int(x**m*sec(log(x**n*c)*b*d
 + a*d)**p*tan(log(x**n*c)*b*d + a*d),x)*b*d*n*p))/(m + 1)
```

3.287 $\int x \sec^p(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 106

$$\int x\sec^p(a+b\log(cx^n))\,dx = \frac{x^2\left(1+e^{2ia}(cx^n)^{2ib}\right)^p \text{Hypergeometric2F1}\left(p,\frac{1}{2}\left(-\frac{2i}{bn}+p\right),\frac{1}{2}\left(2-\frac{2i}{bn}+p\right),-e^{2ia}(cx^n)^{2ib}\right)\sec^p(a+b\log(cx^n))}{2+ibnp}$$

output
```
x^2*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^p*hypergeom([p, -I/b/n+1/2*p],[1-I/b/n+
1/2*p],-exp(2*I*a)*(c*x^n)^(2*I*b))*sec(a+b*ln(c*x^n))^p/(2+I*b*n*p)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.34

$$\int x\sec^p(a+b\log(cx^n))\,dx = -\frac{i2^p x^2\left(\frac{e^{ia}(cx^n)^{ib}}{1+e^{2ia}(cx^n)^{2ib}}\right)^p\left(1+e^{2ia}(cx^n)^{2ib}\right)^p \text{Hypergeometric2F1}\left(-\frac{i}{bn}+\frac{p}{2},p,1-\frac{i}{bn}+\frac{p}{2},-e^{2ia}(cx^n)^{2ib}\right)}{-2i+bnp}$$

input
```
Integrate[x*Sec[a + b*Log[c*x^n]]^p,x]
```

output
```
((-I)*2^p*x^2*((E^(I*a)*(c*x^n)^(I*b))/(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))
)^p*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^p*Hypergeometric2F1[(-I)/(b*n) + p
/2, p, 1 - I/(b*n) + p/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(-2*I + b*n*p
)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5020, 5018, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sec^p(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{5020}$$

$$\frac{x^2(cx^n)^{-2/n}\int (cx^n)^{\frac{2}{n}-1}\sec^p(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5018}$$

$$\frac{x^2(cx^n)^{-\frac{2}{n}-ibp}\left(1+e^{2ia}(cx^n)^{2ib}\right)^p\sec^p(a+b\log(cx^n))\int (cx^n)^{ibp+\frac{2}{n}-1}\left(e^{2ia}(cx^n)^{2ib}+1\right)^{-p}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2\left(1+e^{2ia}(cx^n)^{2ib}\right)^p\text{Hypergeometric2F1}\left(p,\frac{1}{2}\left(p-\frac{2i}{bn}\right),\frac{1}{2}\left(p-\frac{2i}{bn}+2\right),-e^{2ia}(cx^n)^{2ib}\right)\sec^p(a+b\log(cx^n))}{2+ibnp}$$

input
```
Int[x*Sec[a + b*Log[c*x^n]]^p,x]
```

output
```
(x^2*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^p*Hypergeometric2F1[p, ((-2*I)/(b
*n) + p)/2, (2 - (2*I)/(b*n) + p)/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))]*Sec
[a + b*Log[c*x^n]]^p)/(2 + I*b*n*p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5018
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 5020
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Sec[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x\sec\left(a+b\ln\left(c\,x^{n}\right)\right)^{p}dx$$

input
```
int(x*sec(a+b*ln(c*x^n))^p,x)
```

output
```
int(x*sec(a+b*ln(c*x^n))^p,x)
```

Fricas [F]

$$\int x\sec^{p}\left(a+b\log\left(cx^{n}\right)\right)\,dx=\int x\sec\left(b\log\left(cx^{n}\right)+a\right)^{p}\,dx$$

input
```
integrate(x*sec(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

output
```
integral(x*sec(b*log(c*x^n) + a)^p, x)
```

Sympy [F]

$$\int x\sec^p\left(a+b\log\left(cx^n\right)\right)\,dx=\int x\sec^p\left(a+b\log\left(cx^n\right)\right)dx$$

input
```
integrate(x*sec(a+b*ln(c*x**n))**p,x)
```

output
```
Integral(x*sec(a + b*log(c*x**n))**p, x)
```

Maxima [F]

$$\int x\sec^p\left(a+b\log\left(cx^n\right)\right)\,dx=\int x\sec\left(b\log\left(cx^n\right)+a\right)^p\,dx$$

input
```
integrate(x*sec(a+b*log(c*x^n))^p,x, algorithm="maxima")
```

output
```
integrate(x*sec(b*log(c*x^n) + a)^p, x)
```

Giac [F]

$$\int x\sec^p\left(a+b\log\left(cx^n\right)\right)\,dx=\int x\sec\left(b\log\left(cx^n\right)+a\right)^p\,dx$$

input
```
integrate(x*sec(a+b*log(c*x^n))^p,x, algorithm="giac")
```

output
```
integrate(x*sec(b*log(c*x^n) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int x \sec^p(a + b\log(cx^n))\, dx = \int x \left(\frac{1}{\cos(a + b\ln(c x^n))}\right)^p dx$$

input
```
int(x*(1/cos(a + b*log(c*x^n)))^p,x)
```

output
```
int(x*(1/cos(a + b*log(c*x^n)))^p, x)
```

Reduce [F]

$$\int x \sec^p(a + b\log(cx^n))\, dx = \frac{\sec(\log(x^n c)\, b + a)^p x^2}{2} - \frac{\left(\int \sec(\log(x^n c)\, b + a)^p \tan(\log(x^n c)\, b + a)\, x dx\right) bnp}{2}$$

input
```
int(x*sec(a+b*log(c*x^n))^p,x)
```

output
```
(sec(log(x**n*c)*b + a)**p*x**2 - int(sec(log(x**n*c)*b + a)**p*tan(log(x*
*n*c)*b + a)*x,x)*b*n*p)/2
```

3.288 $\int \sec^p(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 107

$$\int \sec^p(a+b\log(cx^n))\,dx = \frac{x\left(1+e^{2ia}(cx^n)^{2ib}\right)^p \text{Hypergeometric2F1}\left(p,-\frac{i-bnp}{2bn},\frac{1}{2}\left(2-\frac{i}{bn}+p\right),-e^{2ia}(cx^n)^{2ib}\right)\sec^p(a+b\log(cx^n))}{1+ibnp}$$

output
```
x*(1+exp(2*I*a)*(c*x^n)^(2*I*b))^p*hypergeom([p, -1/2*(I-b*n*p)/b/n],[1-1/
2*I/b/n+1/2*p],-exp(2*I*a)*(c*x^n)^(2*I*b))*sec(a+b*ln(c*x^n))^p/(1+I*b*n*
p)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.33

$$\int \sec^p(a+b\log(cx^n))\,dx = -\frac{i2^p x\left(\frac{e^{ia}(cx^n)^{ib}}{1+e^{2ia}(cx^n)^{2ib}}\right)^p\left(1+e^{2ia}(cx^n)^{2ib}\right)^p \text{Hypergeometric2F1}\left(p,\frac{-i+bnp}{2bn},\frac{1}{2}\left(2-\frac{i}{bn}+p\right),-e^{2ia}(cx^n)^{2ib}\right)}{-i+bnp}$$

input
```
Integrate[Sec[a + b*Log[c*x^n]]^p,x]
```

output
```
((-I)*2^p*x*((E^(I*a)*(c*x^n)^(I*b))/(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))^
p*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^p*Hypergeometric2F1[p, (-I + b*n*p)/
(2*b*n), (2 - I/(b*n) + p)/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))])/(-I + b*n
*p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5014, 5018, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^p(a + b\log(cx^n)) \, dx$$

$$\Big\downarrow \text{5014}$$

$$\frac{x(cx^n)^{-1/n} \int (cx^n)^{\frac{1}{n}-1} \sec^p(a + b\log(cx^n)) \, d(cx^n)}{n}$$

$$\Big\downarrow \text{5018}$$

$$\frac{x(cx^n)^{-\frac{1}{n}-ibp}\left(1 + e^{2ia}(cx^n)^{2ib}\right)^p \sec^p(a + b\log(cx^n)) \int (cx^n)^{ibp+\frac{1}{n}-1}\left(e^{2ia}(cx^n)^{2ib} + 1\right)^{-p} d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x\left(1 + e^{2ia}(cx^n)^{2ib}\right)^p \text{Hypergeometric2F1}\left(p, -\frac{i-bnp}{2bn}, \frac{1}{2}\left(p - \frac{i}{bn} + 2\right), -e^{2ia}(cx^n)^{2ib}\right) \sec^p(a + b\log(cx^n))}{n\left(\frac{1}{n} + ibp\right)}$$

input
```
Int[Sec[a + b*Log[c*x^n]]^p,x]
```

output
```
(x*(1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))^p*Hypergeometric2F1[p, -1/2*(I - b*
n*p)/(b*n), (2 - I/(b*n) + p)/2, -(E^((2*I)*a)*(c*x^n)^((2*I)*b))]*Sec[a +
 b*Log[c*x^n]]^p)/(n*(n^(-1) + I*b*p))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5014
```
Int[Sec[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Sec[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5018
```
Int[((e_.)*(x_))^(m_.)*Sec[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
:> Simp[Sec[d*(a + b*Log[x])]^p*((1 + E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 + E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \sec \left(a +b \ln \left(c \,x^{n}\right)\right)^{p}d x$$

input
```
int(sec(a+b*ln(c*x^n))^p,x)
```

output
```
int(sec(a+b*ln(c*x^n))^p,x)
```

Fricas [F]

$$\int \sec^{p}\left(a+b\log\left(cx^{n}\right)\right)\,dx=\int \sec\left(b\log\left(cx^{n}\right)+a\right)^{p}\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

output
```
integral(sec(b*log(c*x^n) + a)^p, x)
```

Sympy [F]

$$\int \sec^p\left(a+b\log\left(cx^n\right)\right)\,dx=\int \sec^p\left(a+b\log\left(cx^n\right)\right)dx$$

input
```
integrate(sec(a+b*ln(c*x**n))**p,x)
```

output
```
Integral(sec(a + b*log(c*x**n))**p, x)
```

Maxima [F]

$$\int \sec^p\left(a+b\log\left(cx^n\right)\right)\,dx=\int \sec\left(b\log\left(cx^n\right)+a\right)^p\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^p,x, algorithm="maxima")
```

output
```
integrate(sec(b*log(c*x^n) + a)^p, x)
```

Giac [F]

$$\int \sec^p\left(a+b\log\left(cx^n\right)\right)\,dx=\int \sec\left(b\log\left(cx^n\right)+a\right)^p\,dx$$

input
```
integrate(sec(a+b*log(c*x^n))^p,x, algorithm="giac")
```

output
```
integrate(sec(b*log(c*x^n) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sec^p(a+b\log(cx^n))\,dx=\int\left(\frac{1}{\cos(a+b\,\ln(c\,x^n))}\right)^p\,dx$$

input
```
int((1/cos(a + b*log(c*x^n)))^p,x)
```

output
```
int((1/cos(a + b*log(c*x^n)))^p, x)
```

Reduce [F]

$$\int \sec^p(a+b\log(cx^n))\,dx=\sec(\log(x^n c)\,b+a)^p x-\left(\int \sec(\log(x^n c)\,b+a)^p\tan(\log(x^n c)\,b+a)\,dx\right)bnp$$

input
```
int(sec(a+b*log(c*x^n))^p,x)
```

output
```
sec(log(x**n*c)*b + a)**p*x - int(sec(log(x**n*c)*b + a)**p*tan(log(x**n*c
)*b + a),x)*b*n*p
```

3.289 $\int x^2 \csc(a + b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 86

$$\int x^2 \csc(a + b\log(cx^n))\,dx = \frac{2e^{ia}x^3(cx^n)^{ib}\,\text{Hypergeometric2F1}\left(1, \frac{1}{2}\left(1 - \frac{3i}{bn}\right), \frac{3}{2}\left(1 - \frac{i}{bn}\right), e^{2ia}(cx^n)^{2ib}\right)}{3i - bn}$$

output
```
2*exp(I*a)*x^3*(c*x^n)^(I*b)*hypergeom([1, 1/2-3/2*I/b/n],[3/2-3/2*I/b/n],
exp(2*I*a)*(c*x^n)^(2*I*b))/(3*I-b*n)
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int x^2 \csc(a + b\log(cx^n))\,dx = -\frac{2e^{ia}x^3(cx^n)^{ib}\,\text{Hypergeometric2F1}\left(1, \frac{1}{2} - \frac{3i}{2bn}, \frac{3}{2} - \frac{3i}{2bn}, e^{2i(a+b\log(cx^n))}\right)}{-3i + bn}$$

input
```
Integrate[x^2*Csc[a + b*Log[c*x^n]],x]
```

output
```
(-2*E^(I*a)*x^3*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 - ((3*I)/2)/(b*n),
3/2 - ((3*I)/2)/(b*n), E^((2*I)*(a + b*Log[c*x^n]))])/(-3*I + b*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5021, 5017, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \csc(a + b\log(cx^n))\, dx$$

$$\Big\downarrow \text{5021}$$

$$\frac{x^3(cx^n)^{-3/n}\int (cx^n)^{\frac{3}{n}-1}\csc(a + b\log(cx^n))\, d(cx^n)}{n}$$

$$\Big\downarrow \text{5017}$$

$$-\frac{2ie^{ia}x^3(cx^n)^{-3/n}\int \frac{(cx^n)^{ib+\frac{3}{n}-1}}{1-e^{2ia}(cx^n)^{2ib}}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2ie^{ia}x^3(cx^n)^{ib}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1-\frac{3i}{bn}\right),\frac{3}{2}\left(1-\frac{i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{3+ibn}$$

input
```
Int[x^2*Csc[a + b*Log[c*x^n]],x]
```

output
```
((-2*I)*E^(I*a)*x^3*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 - (3*I)/(b*n))/2
, (3*(1 - I/(b*n)))/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/(3 + I*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5017
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[(-2*I)^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^
(2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5021
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^2 \csc\left(a + b\ln\left(c\,x^n\right)\right) dx$$

input
```
int(x^2*csc(a+b*ln(c*x^n)),x)
```

output
```
int(x^2*csc(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int x^2 \csc\left(a + b\log\left(cx^n\right)\right)\, dx = \int x^2 \csc\left(b\log\left(cx^n\right) + a\right)\, dx$$

input
```
integrate(x^2*csc(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
integral(x^2*csc(b*log(c*x^n) + a), x)
```

Sympy [F]

$$\int x^2 \csc(a + b \log(cx^n)) \, dx = \int x^2 \csc(a + b \log(cx^n)) \, dx$$

input
```
integrate(x**2*csc(a+b*ln(c*x**n)),x)
```

output
```
Integral(x**2*csc(a + b*log(c*x**n)), x)
```

Maxima [F]

$$\int x^2 \csc(a + b \log(cx^n)) \, dx = \int x^2 \csc(b \log(cx^n) + a) \, dx$$

input
```
integrate(x^2*csc(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
integrate(x^2*csc(b*log(c*x^n) + a), x)
```

Giac [F]

$$\int x^2 \csc(a + b \log(cx^n)) \, dx = \int x^2 \csc(b \log(cx^n) + a) \, dx$$

input
```
integrate(x^2*csc(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
integrate(x^2*csc(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \csc (a + b \log (cx^n)) \, dx = \int \frac{x^2}{\sin (a + b \ln (c x^n))} \, dx$$

input
```
int(x^2/sin(a + b*log(c*x^n)),x)
```

output
```
int(x^2/sin(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x^2 \csc (a + b \log (cx^n)) \, dx = \int \csc (\log(x^n c) \, b + a) \, x^2 dx$$

input
```
int(x^2*csc(a+b*log(c*x^n)),x)
```

output
```
int(csc(log(x**n*c)*b + a)*x**2,x)
```

3.290 $\int x \csc (a+b \log (c x^n)) \, dx$

Optimal result

Integrand size = 13, antiderivative size = 86

$$\int x \csc (a+b \log (c x^n)) \, dx = \frac{2 e^{i a} x^2 (c x^n)^{i b} \text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1-\frac{2 i}{b n}\right),\frac{1}{2}\left(3-\frac{2 i}{b n}\right),e^{2 i a} (c x^n)^{2 i b}\right)}{2 i-b n}$$

output
```
2*exp(I*a)*x^2*(c*x^n)^(I*b)*hypergeom([1, 1/2-I/b/n],[3/2-I/b/n],exp(2*I*
a)*(c*x^n)^(2*I*b))/(2*I-b*n)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91

$$\int x \csc (a+b \log (c x^n)) \, dx = -\frac{2 e^{i a} x^2 (c x^n)^{i b} \text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{i}{b n},\frac{3}{2}-\frac{i}{b n},e^{2 i (a+b \log (c x^n))}\right)}{-2 i+b n}$$

input
```
Integrate[x*Csc[a + b*Log[c*x^n]],x]
```

output
```
(-2*E^(I*a)*x^2*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 - I/(b*n), 3/2 - I/
(b*n), E^((2*I)*(a + b*Log[c*x^n]))])/(-2*I + b*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5021, 5017, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\csc\left(a+b\log\left(cx^n\right)\right)dx$$

$$\Big\downarrow \text{5021}$$

$$\frac{x^2\left(cx^n\right)^{-2/n}\int \left(cx^n\right)^{\frac{2}{n}-1}\csc\left(a+b\log\left(cx^n\right)\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{5017}$$

$$-\frac{2ie^{ia}x^2\left(cx^n\right)^{-2/n}\int \frac{\left(cx^n\right)^{ib+\frac{2}{n}-1}}{1-e^{2ia}\left(cx^n\right)^{2ib}}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2ie^{ia}x^2\left(cx^n\right)^{ib}\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1-\frac{2i}{bn}\right),\frac{1}{2}\left(3-\frac{2i}{bn}\right),e^{2ia}\left(cx^n\right)^{2ib}\right)}{2+ibn}$$

input
```
Int[x*Csc[a + b*Log[c*x^n]],x]
```

output
```
((-2*I)*E^(I*a)*x^2*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 - (2*I)/(b*n))/2
, (3 - (2*I)/(b*n))/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/(2 + I*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5017
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[(-2*I)^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^
(2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5021
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x \csc \left(a + b \ln \left(c\, x^{n}\right)\right) dx$$

input
```
int(x*csc(a+b*ln(c*x^n)),x)
```

output
```
int(x*csc(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int x \csc \left(a + b \log \left(cx^{n}\right)\right)\, dx = \int x \csc \left(b \log \left(cx^{n}\right) + a\right)\, dx$$

input
```
integrate(x*csc(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
integral(x*csc(b*log(c*x^n) + a), x)
```

Sympy [F]

$$\int x\csc\left(a+b\log\left(cx^n\right)\right)\,dx=\int x\csc\left(a+b\log\left(cx^n\right)\right)dx$$

input
```
integrate(x*csc(a+b*ln(c*x**n)),x)
```

output
```
Integral(x*csc(a + b*log(c*x**n)), x)
```

Maxima [F]

$$\int x\csc\left(a+b\log\left(cx^n\right)\right)\,dx=\int x\csc\left(b\log\left(cx^n\right)+a\right)\,dx$$

input
```
integrate(x*csc(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
integrate(x*csc(b*log(c*x^n) + a), x)
```

Giac [F]

$$\int x\csc\left(a+b\log\left(cx^n\right)\right)\,dx=\int x\csc\left(b\log\left(cx^n\right)+a\right)\,dx$$

input
```
integrate(x*csc(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
integrate(x*csc(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\csc\left(a+b\log\left(cx^n\right)\right)\,dx=\int\frac{x}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}\,dx$$

input
```
int(x/sin(a + b*log(c*x^n)),x)
```

output
```
int(x/sin(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int x\csc\left(a+b\log\left(cx^n\right)\right)\,dx=\int\csc\left(\log(x^nc)\,b+a\right)x\,dx$$

input
```
int(x*csc(a+b*log(c*x^n)),x)
```

output
```
int(csc(log(x**n*c)*b + a)*x,x)
```

3.291 $\int \csc (a+b \log (c x^n)) \, dx$

Optimal result

Integrand size = 11, antiderivative size = 84

$$\int \csc (a+b \log (c x^n)) \, dx = \frac{2 e^{i a} x (c x^n)^{i b} \text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1-\frac{i}{b n}\right),\frac{1}{2}\left(3-\frac{i}{b n}\right),e^{2 i a} (c x^n)^{2 i b}\right)}{i-b n}$$

output
```
2*exp(I*a)*x*(c*x^n)^(I*b)*hypergeom([1, 1/2-1/2*I/b/n],[3/2-1/2*I/b/n],ex
p(2*I*a)*(c*x^n)^(2*I*b))/(I-b*n)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \csc (a+b \log (c x^n)) \, dx = -\frac{2 e^{i a} x (c x^n)^{i b} \text{Hypergeometric2F1}\left(1,\frac{1}{2}-\frac{i}{2 b n},\frac{3}{2}-\frac{i}{2 b n},e^{2 i (a+b \log (c x^n))}\right)}{-i+b n}$$

input
```
Integrate[Csc[a + b*Log[c*x^n]],x]
```

output
```
(-2*E^(I*a)*x*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 - (I/2)/(b*n), 3/2 -
(I/2)/(b*n), E^((2*I)*(a + b*Log[c*x^n]))])/(-I + b*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5015, 5017, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a + b\log(cx^n))\,dx$$

$$\Big\downarrow \text{ 5015}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\csc(a + b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{ 5017}$$

$$-\frac{2ie^{ia}x(cx^n)^{-1/n}\int \frac{(cx^n)^{ib+\frac{1}{n}-1}}{1-e^{2ia}(cx^n)^{2ib}}d(cx^n)}{n}$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{2ie^{ia}x(cx^n)^{ib}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1-\frac{i}{bn}\right),\frac{1}{2}\left(3-\frac{i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{n\left(\frac{1}{n}+ib\right)}$$

input
```
Int[Csc[a + b*Log[c*x^n]],x]
```

output
```
((-2*I)*E^(I*a)*x*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 - I/(b*n))/2, (3 -
I/(b*n))/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((I*b + n^(-1))*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5015
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5017
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[(-2*I)^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^
(2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [F]

$$\int \csc \left(a+b \ln \left(c \,x^{n}\right)\right)d x$$

input
```
int(csc(a+b*ln(c*x^n)),x)
```

output
```
int(csc(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int \csc \left(a+b \log \left(cx^{n}\right)\right)\, dx=\int \csc \left(b \log \left(cx^{n}\right)+a\right)\, dx$$

input
```
integrate(csc(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
integral(csc(b*log(c*x^n) + a), x)
```

Sympy [F]

$$\int \csc\left(a + b\log\left(cx^n\right)\right)\,dx = \int \csc\left(a + b\log\left(cx^n\right)\right)dx$$

input
```
integrate(csc(a+b*ln(c*x**n)),x)
```

output
```
Integral(csc(a + b*log(c*x**n)), x)
```

Maxima [F]

$$\int \csc\left(a + b\log\left(cx^n\right)\right)\,dx = \int \csc\left(b\log\left(cx^n\right) + a\right)\,dx$$

input
```
integrate(csc(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
integrate(csc(b*log(c*x^n) + a), x)
```

Giac [F]

$$\int \csc\left(a + b\log\left(cx^n\right)\right)\,dx = \int \csc\left(b\log\left(cx^n\right) + a\right)\,dx$$

input
```
integrate(csc(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
integrate(csc(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc \left(a+b\log \left(cx^{n}\right)\right)\,dx=\int \frac{1}{\sin \left(a+b\,\ln \left(c\,x^{n}\right)\right)}\,dx$$

input
```
int(1/sin(a + b*log(c*x^n)),x)
```

output
```
int(1/sin(a + b*log(c*x^n)), x)
```

Reduce [F]

$$\int \csc \left(a+b\log \left(cx^{n}\right)\right)\,dx=\int \csc \left(\log(x^{n}c)\,b+a\right)dx$$

input
```
int(csc(a+b*log(c*x^n)),x)
```

output
```
int(csc(log(x**n*c)*b + a),x)
```

3.292 $\int \frac{\csc(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{\csc(a+b\log(cx^n))}{x}\,dx=-\frac{\operatorname{arctanh}(\cos(a+b\log(cx^n)))}{bn}$$

output
```
-arctanh(cos(a+b*ln(c*x^n)))/b/n
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\csc(a+b\log(cx^n))}{x}\,dx=-\frac{\operatorname{arctanh}(\cos(a+b\log(cx^n)))}{bn}$$

input
```
Integrate[Csc[a + b*Log[c*x^n]]/x,x]
```

output
```
-(ArcTanh[Cos[a + b*Log[c*x^n]]]/(b*n))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3039, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc (a+b \log (c x^n))}{x} \, dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \csc (a+b \log (c x^n))d\log (c x^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc (a+b \log (c x^n))d\log (c x^n)}{n}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\operatorname{arctanh}(\cos (a+b \log (c x^n)))}{b n}$$

input
```
Int[Csc[a + b*Log[c*x^n]]/x,x]
```

output
```
-(ArcTanh[Cos[a + b*Log[c*x^n]]]/(b*n))
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

method	result
parallelrisch	$\frac{\ln\left(\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)\right)}{bn}$
derivativedivides	$-\frac{\ln(\csc(a+b\ln(c\,x^{n}))+\cot(a+b\ln(c\,x^{n})))}{nb}$
default	$-\frac{\ln(\csc(a+b\ln(c\,x^{n}))+\cot(a+b\ln(c\,x^{n})))}{nb}$
risch	$-\frac{\ln\left(c^{ib}(x^{n})^{ib}\mathrm{e}^{-\frac{b\pi\ \mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})^{2}}{2}}\mathrm{e}^{\frac{b\pi\ \mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})\,\mathrm{csgn}(ic)}{2}}\mathrm{e}^{\frac{b\pi\,\mathrm{csgn}(ic\,x^{n})^{3}}{2}}\mathrm{e}^{-\frac{b\pi\,\mathrm{csgn}(ic\,x^{n})^{2}\,\mathrm{csgn}(ic)}{2}}\mathrm{e}^{i}\right.}{bn}$

input
```
int(csc(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
```

output
```
ln(tan(1/2*a+b*ln((c*x^n)^(1/2))))/b/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(20) = 40.

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.25

$$\int \frac{\csc\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx = -\frac{\log\left(\frac{1}{2}\,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\,\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\frac{1}{2}\right)}{2\,bn}$$

input
```
integrate(csc(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

output
```
-1/2*(log(1/2*cos(b*n*log(x) + b*log(c) + a) + 1/2) - log(-1/2*cos(b*n*log
(x) + b*log(c) + a) + 1/2))/(b*n)
```

Sympy [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.45

$$\int \frac{\csc (a+b\log (cx^n))}{x}\,dx = -\begin{cases} -\log (x)\csc (a) & \text{for } b=0 \\ -\log (x)\csc (a+b\log (c)) & \text{for } n=0 \\ \frac{\log (\cot (a+b\log (cx^n))+\csc (a+b\log (cx^n)))}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(csc(a+b*ln(c*x**n))/x,x)
```

output
```
-Piecewise((-log(x)*csc(a), Eq(b, 0)), (-log(x)*csc(a + b*log(c)), Eq(n, 0
)), (log(cot(a + b*log(c*x**n)) + csc(a + b*log(c*x**n)))/(b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{\csc (a+b\log (cx^n))}{x}\,dx = -\frac{\log (\cot (b\log (cx^n)+a)+\csc (b\log (cx^n)+a))}{bn}$$

input
```
integrate(csc(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

output
```
-log(cot(b*log(c*x^n) + a) + csc(b*log(c*x^n) + a))/(b*n)
```

Giac [F]

$$\int \frac{\csc (a+b\log (cx^n))}{x}\,dx = \int \frac{\csc (b\log (cx^n)+a)}{x}\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))/x,x, algorithm="giac")
```

output
```
integrate(csc(b*log(c*x^n) + a)/x, x)
```

Mupad [B] (verification not implemented)

Time = 24.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.40

$$\int \frac{\csc(a+b\log(cx^n))}{x}\,dx = \frac{\ln\left(\frac{e^{a\,1i}\,(c\,x^n)^{b\,1i}\,2i-2i}{x}\right)}{b\,n} - \frac{\ln\left(\frac{e^{a\,1i}\,(c\,x^n)^{b\,1i}\,2i+2i}{x}\right)}{b\,n}$$

input
```
int(1/(x*sin(a + b*log(c*x^n))),x)
```

output
```
log((exp(a*1i)*(c*x^n)^(b*1i)*2i - 2i)/x)/(b*n) - log((exp(a*1i)*(c*x^n)^(
b*1i)*2i + 2i)/x)/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc(a+b\log(cx^n))}{x}\,dx = \frac{\log\left(\tan\left(\frac{\log(x^n c)b}{2}+\frac{a}{2}\right)\right)}{bn}$$

input
```
int(csc(a+b*log(c*x^n))/x,x)
```

output
```
log(tan((log(x**n*c)*b + a)/2))/(b*n)
```

3.293 $\int \frac{\csc(a+b\log(cx^n))}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 85

$$\int \frac{\csc(a+b\log(cx^n))}{x^2} dx = -\frac{2e^{ia}(cx^n)^{ib}\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{i}{bn}\right),\frac{1}{2}\left(3+\frac{i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{(i+bn)x}$$

output
```
-2*exp(I*a)*(c*x^n)^(I*b)*hypergeom([1, 1/2+1/2*I/b/n],[3/2+1/2*I/b/n],exp
(2*I*a)*(c*x^n)^(2*I*b))/(I+b*n)/x
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int \frac{\csc(a+b\log(cx^n))}{x^2} dx = -\frac{2e^{ia}(cx^n)^{ib}\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{i}{2bn},\frac{3}{2}+\frac{i}{2bn},e^{2i(a+b\log(cx^n))}\right)}{(i+bn)x}$$

input
```
Integrate[Csc[a + b*Log[c*x^n]]/x^2,x]
```

output
```
(-2*E^(I*a)*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 + (I/2)/(b*n), 3/2 + (I
/2)/(b*n), E^((2*I)*(a + b*Log[c*x^n]))])/((I + b*n)*x)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5021, 5017, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+b\log(cx^n))}{x^2}dx$$

$$\Big\downarrow \text{5021}$$

$$\frac{(cx^n)^{\frac{1}{n}}\int (cx^n)^{-1-\frac{1}{n}}\csc(a+b\log(cx^n))d(cx^n)}{nx}$$

$$\Big\downarrow \text{5017}$$

$$-\frac{2ie^{ia}(cx^n)^{\frac{1}{n}}\int \frac{(cx^n)^{ib-\frac{1}{n}-1}}{1-e^{2ia}(cx^n)^{2ib}}d(cx^n)}{nx}$$

$$\Big\downarrow \text{888}$$

$$\frac{2ie^{ia}(cx^n)^{ib}\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{i}{bn}\right),\frac{1}{2}\left(3+\frac{i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{x(1-ibn)}$$

input
```
Int[Csc[a + b*Log[c*x^n]]/x^2,x]
```

output
```
((2*I)*E^(I*a)*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 + I/(b*n))/2, (3 + I/
(b*n))/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((1 - I*b*n)*x)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5017
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[(-2*I)^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^
(2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5021
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\csc(a+b\ln(c\,x^n))}{x^2}dx$$

input
```
int(csc(a+b*ln(c*x^n))/x^2,x)
```

output
```
int(csc(a+b*ln(c*x^n))/x^2,x)
```

Fricas [F]

$$\int \frac{\csc(a+b\log(cx^n))}{x^2}\,dx = \int \frac{\csc(b\log(cx^n)+a)}{x^2}\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))/x^2,x, algorithm="fricas")
```

output
```
integral(csc(b*log(c*x^n) + a)/x^2, x)
```

Sympy [F]

$$\int \frac{\csc(a+b\log(cx^n))}{x^2}\,dx = \int \frac{\csc(a+b\log(cx^n))}{x^2}\,dx$$

input
```
integrate(csc(a+b*ln(c*x**n))/x**2,x)
```

output
```
Integral(csc(a + b*log(c*x**n))/x**2, x)
```

Maxima [F]

$$\int \frac{\csc(a+b\log(cx^n))}{x^2}\,dx = \int \frac{\csc(b\log(cx^n)+a)}{x^2}\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))/x^2,x, algorithm="maxima")
```

output
```
integrate(csc(b*log(c*x^n) + a)/x^2, x)
```

Giac [F]

$$\int \frac{\csc(a+b\log(cx^n))}{x^2}\,dx = \int \frac{\csc(b\log(cx^n)+a)}{x^2}\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))/x^2,x, algorithm="giac")
```

output
```
integrate(csc(b*log(c*x^n) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a + b\log(cx^n))}{x^2} \, dx = \int \frac{1}{x^2 \sin(a + b \ln(c x^n))} \, dx$$

input
```
int(1/(x^2*sin(a + b*log(c*x^n))),x)
```

output
```
int(1/(x^2*sin(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int \frac{\csc(a + b\log(cx^n))}{x^2} \, dx = \int \frac{\csc(\log(x^n c) b + a)}{x^2} dx$$

input
```
int(csc(a+b*log(c*x^n))/x^2,x)
```

output
```
int(csc(log(x**n*c)*b + a)/x**2,x)
```

3.294 $\int \frac{\csc(a+b\log(cx^n))}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 85

$$\int \frac{\csc(a+b\log(cx^n))}{x^3}\,dx = -\frac{2e^{ia}(cx^n)^{ib}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{2i}{bn}\right),\frac{1}{2}\left(3+\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{(2i+bn)x^2}$$

output
```
-2*exp(I*a)*(c*x^n)^(I*b)*hypergeom([1, 1/2+I/b/n],[3/2+I/b/n],exp(2*I*a)*
(c*x^n)^(2*I*b))/(2*I+b*n)/x^2
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{\csc(a+b\log(cx^n))}{x^3}\,dx = -\frac{2e^{ia}(cx^n)^{ib}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{i}{bn},\frac{3}{2}+\frac{i}{bn},e^{2i(a+b\log(cx^n))}\right)}{(2i+bn)x^2}$$

input
```
Integrate[Csc[a + b*Log[c*x^n]]/x^3,x]
```

output
```
(-2*E^(I*a)*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 + I/(b*n), 3/2 + I/(b*n
), E^((2*I)*(a + b*Log[c*x^n]))])/((2*I + b*n)*x^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5021, 5017, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+b\log(cx^n))}{x^3}dx$$

$$\Big\downarrow \text{5021}$$

$$\frac{(cx^n)^{2/n}\int (cx^n)^{-1-\frac{2}{n}}\csc(a+b\log(cx^n))d(cx^n)}{nx^2}$$

$$\Big\downarrow \text{5017}$$

$$-\frac{2ie^{ia}(cx^n)^{2/n}\int \frac{(cx^n)^{ib-\frac{2}{n}-1}}{1-e^{2ia}(cx^n)^{2ib}}d(cx^n)}{nx^2}$$

$$\Big\downarrow \text{888}$$

$$\frac{2ie^{ia}(cx^n)^{ib}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{2i}{bn}\right),\frac{1}{2}\left(3+\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{x^2(2-ibn)}$$

input
```
Int[Csc[a + b*Log[c*x^n]]/x^3,x]
```

output
```
((2*I)*E^(I*a)*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 + (2*I)/(b*n))/2, (3
+ (2*I)/(b*n))/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((2 - I*b*n)*x^2)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5017
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[(-2*I)^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^
(2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5021
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{\csc\left(a+b\ln\left(c\,x^{n}\right)\right)}{x^{3}}dx$$

input
```
int(csc(a+b*ln(c*x^n))/x^3,x)
```

output
```
int(csc(a+b*ln(c*x^n))/x^3,x)
```

Fricas [F]

$$\int \frac{\csc\left(a+b\log\left(cx^{n}\right)\right)}{x^{3}}\,dx=\int \frac{\csc\left(b\log\left(cx^{n}\right)+a\right)}{x^{3}}\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))/x^3,x, algorithm="fricas")
```

output
```
integral(csc(b*log(c*x^n) + a)/x^3, x)
```

Sympy [F]

$$\int \frac{\csc(a+b\log(cx^n))}{x^3}\,dx = \int \frac{\csc(a+b\log(cx^n))}{x^3}\,dx$$

input
```
integrate(csc(a+b*ln(c*x**n))/x**3,x)
```

output
```
Integral(csc(a + b*log(c*x**n))/x**3, x)
```

Maxima [F]

$$\int \frac{\csc(a+b\log(cx^n))}{x^3}\,dx = \int \frac{\csc(b\log(cx^n)+a)}{x^3}\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))/x^3,x, algorithm="maxima")
```

output
```
integrate(csc(b*log(c*x^n) + a)/x^3, x)
```

Giac [F]

$$\int \frac{\csc(a+b\log(cx^n))}{x^3}\,dx = \int \frac{\csc(b\log(cx^n)+a)}{x^3}\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))/x^3,x, algorithm="giac")
```

output
```
integrate(csc(b*log(c*x^n) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a + b\log(cx^n))}{x^3} dx = \int \frac{1}{x^3 \sin(a + b\ln(c x^n))} dx$$

input
```
int(1/(x^3*sin(a + b*log(c*x^n))),x)
```

output
```
int(1/(x^3*sin(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int \frac{\csc(a + b\log(cx^n))}{x^3} dx = \int \frac{\csc(\log(x^n c) b + a)}{x^3} dx$$

input
```
int(csc(a+b*log(c*x^n))/x^3,x)
```

output
```
int(csc(log(x**n*c)*b + a)/x**3,x)
```

3.295 $\int \csc^2(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 84

$$\int \csc^2(a+b\log(cx^n))\,dx = -\frac{4e^{2ia}x(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2-\frac{i}{bn}\right),\frac{1}{2}\left(4-\frac{i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{1+2ibn}$$

output
```
-4*exp(2*I*a)*x*(c*x^n)^(2*I*b)*hypergeom([2, 1-1/2*I/b/n],[2-1/2*I/b/n],e
xp(2*I*a)*(c*x^n)^(2*I*b))/(1+2*I*b*n)
```

Mathematica [A] (verified)

Time = 3.86 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.74

$$\int \csc^2(a+b\log(cx^n))\,dx = \frac{x\left(-\cot(a+b\log(cx^n))-\frac{e^{2ia}(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(1,1-\frac{i}{2bn},2-\frac{i}{2bn},e^{2i(a+b\log(cx^n))}\right)}{-i+2bn}-i\,\text{Hypergeometric2F1}\right.}{bn}$$

input
```
Integrate[Csc[a + b*Log[c*x^n]]^2,x]
```

output
```
(x*(-Cot[a + b*Log[c*x^n]] - (E^((2*I)*a)*(c*x^n)^((2*I)*b)*Hypergeometric
2F1[1, 1 - (I/2)/(b*n), 2 - (I/2)/(b*n), E^((2*I)*(a + b*Log[c*x^n]))])/(-
I + 2*b*n) - I*Hypergeometric2F1[1, (-1/2*I)/(b*n), 1 - (I/2)/(b*n), E^((2
*I)*(a + b*Log[c*x^n]))]))/(b*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5015, 5017, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{5015}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\csc^2(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5017}$$

$$-\frac{4e^{2ia}x(cx^n)^{-1/n}\int \frac{(cx^n)^{2ib+\frac{1}{n}-1}}{\left(1-e^{2ia}(cx^n)^{2ib}\right)^2}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$-\frac{4e^{2ia}x(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2-\frac{i}{bn}\right),\frac{1}{2}\left(4-\frac{i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{n\left(\frac{1}{n}+2ib\right)}$$

input
```
Int[Csc[a + b*Log[c*x^n]]^2,x]
```

output
```
(-4*E^((2*I)*a)*x*(c*x^n)^((2*I)*b)*Hypergeometric2F1[2, (2 - I/(b*n))/2,
(4 - I/(b*n))/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/(((2*I)*b + n^(-1))*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5015
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5017
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[(-2*I)^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^
(2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [F]

$$\int \csc\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}dx$$

input
```
int(csc(a+b*ln(c*x^n))^2,x)
```

output
```
int(csc(a+b*ln(c*x^n))^2,x)
```

Fricas [F]

$$\int \csc^2\left(a+b\log\left(cx^n\right)\right)\,dx=\int \csc\left(b\log\left(cx^n\right)+a\right)^2\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
integral(csc(b*log(c*x^n) + a)^2, x)
```

Sympy [F]

$$\int \csc^2(a+b\log(cx^n))\,dx = \int \csc^2(a+b\log(cx^n))\,dx$$

input
```
integrate(csc(a+b*ln(c*x**n))**2,x)
```

output
```
Integral(csc(a + b*log(c*x**n))**2, x)
```

Maxima [F]

$$\int \csc^2(a+b\log(cx^n))\,dx = \int \csc(b\log(cx^n)+a)^2\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
(2*x*cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + 2*x*cos(2*b*log(c))*sin(2*b
*log(x^n) + 2*a) - (2*b^2*n^2*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) - 2*
b^2*n^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) - (b^2*cos(2*b*log(c))^2 +
 b^2*sin(2*b*log(c))^2)*n^2*cos(2*b*log(x^n) + 2*a)^2 - (b^2*cos(2*b*log(c
))^2 + b^2*sin(2*b*log(c))^2)*n^2*sin(2*b*log(x^n) + 2*a)^2 - b^2*n^2)*int
egrate((cos(b*log(x^n) + a)*sin(b*log(c)) + cos(b*log(c))*sin(b*log(x^n) +
 a))/(2*b^2*n^2*cos(b*log(c))*cos(b*log(x^n) + a) - 2*b^2*n^2*sin(b*log(c)
)*sin(b*log(x^n) + a) + (b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))^2)*n^2*co
s(b*log(x^n) + a)^2 + (b^2*cos(b*log(c))^2 + b^2*sin(b*log(c))^2)*n^2*sin(
b*log(x^n) + a)^2 + b^2*n^2), x) + (2*b^2*n^2*cos(2*b*log(c))*cos(2*b*log(
x^n) + 2*a) - 2*b^2*n^2*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a) - (b^2*cos
(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2*cos(2*b*log(x^n) + 2*a)^2 - (b
^2*cos(2*b*log(c))^2 + b^2*sin(2*b*log(c))^2)*n^2*sin(2*b*log(x^n) + 2*a)^
2 - b^2*n^2)*integrate(-(cos(b*log(x^n) + a)*sin(b*log(c)) + cos(b*log(c))
*sin(b*log(x^n) + a))/(2*b^2*n^2*cos(b*log(c))*cos(b*log(x^n) + a) - 2*b^2
*n^2*sin(b*log(c))*sin(b*log(x^n) + a) - (b^2*cos(b*log(c))^2 + b^2*sin(b*
log(c))^2)*n^2*cos(b*log(x^n) + a)^2 - (b^2*cos(b*log(c))^2 + b^2*sin(b*lo
g(c))^2)*n^2*sin(b*log(x^n) + a)^2 - b^2*n^2), x))/(2*b*n*cos(2*b*log(c))*
cos(2*b*log(x^n) + 2*a) - (b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*co
s(2*b*log(x^n) + 2*a)^2 - 2*b*n*sin(2*b*log(c))*sin(2*b*log(x^n) + 2*a)...
```

Giac [F]

$$\int \csc^2(a+b\log(cx^n))\,dx = \int \csc(b\log(cx^n)+a)^2\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
integrate(csc(b*log(c*x^n) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^2(a+b\log(cx^n))\,dx = \int \frac{1}{\sin(a+b\,\ln(c\,x^n))^2}\,dx$$

input
```
int(1/sin(a + b*log(c*x^n))^2,x)
```

output
```
int(1/sin(a + b*log(c*x^n))^2, x)
```

Reduce [F]

$$\int \csc^2(a+b\log(cx^n))\,dx = \int \csc(\log(x^n c)\,b+a)^2dx$$

input
```
int(csc(a+b*log(c*x^n))^2,x)
```

output
```
int(csc(log(x**n*c)*b + a)**2,x)
```

3.296 $\int \frac{\csc^2(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \frac{\csc^2(a+b\log(cx^n))}{x}\,dx = -\frac{\cot(a+b\log(cx^n))}{bn}$$

output
```
-cot(a+b*ln(c*x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(a+b\log(cx^n))}{x}\,dx = -\frac{\cot(a+b\log(cx^n))}{bn}$$

input
```
Integrate[Csc[a + b*Log[c*x^n]]^2/x,x]
```

output
```
-(Cot[a + b*Log[c*x^n]]/(b*n))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3039, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+b\log(cx^n))}{x} \, dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \csc^2(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc(a+b\log(cx^n))^2d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int 1d\cot(a+b\log(cx^n))}{bn}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\cot(a+b\log(cx^n))}{bn}$$

input
```
Int[Csc[a + b*Log[c*x^n]]^2/x,x]
```

output
```
-(Cot[a + b*Log[c*x^n]]/(b*n))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result
derivativedivides	$-\frac{\cot(a+b\ln(c\,x^{n}))}{bn}$
default	$-\frac{\cot(a+b\ln(c\,x^{n}))}{bn}$
parallelrisch	$\frac{\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)-\cot\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)}{2bn}$
risch	$-\frac{2i}{bn\left(\left(x^{n}\right)^{2ib}c^{2ib}\mathrm{e}^{-b\pi\ \mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})^{2}}\mathrm{e}^{b\pi\ \mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})\,\mathrm{csgn}(ic)}\mathrm{e}^{b\pi\,\mathrm{csgn}(ic\,x^{n})^{3}}\mathrm{e}^{-b\pi\,\mathrm{csgn}(ic\,x^{n})^{2}\,\mathrm{csgn}(ic)}\mathrm{e}^{2ia}\right.}$

input
```
int(csc(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

output
```
-cot(a+b*ln(c*x^n))/b/n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{\csc^2(a+b\log(cx^n))}{x}\,dx = -\frac{\cos(bn\log(x)+b\log(c)+a)}{bn\sin(bn\log(x)+b\log(c)+a)}$$

input
```
integrate(csc(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

output
```
-cos(b*n*log(x) + b*log(c) + a)/(b*n*sin(b*n*log(x) + b*log(c) + a))
```

Sympy [F]

$$\int \frac{\csc^2(a+b\log(cx^n))}{x}\,dx = \int \frac{\csc^2(a+b\log(cx^n))}{x}\,dx$$

input
```
integrate(csc(a+b*ln(c*x**n))**2/x,x)
```

output
```
Integral(csc(a + b*log(c*x**n))**2/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(19) = 38.

Time = 0.05 (sec) , antiderivative size = 168, normalized size of antiderivative = 8.84

$$\int \frac{\csc^2(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{2(\cos(2b\log(x^n)+2a)\sin(2b}{2bn\cos(2b\log(c))\cos(2b\log(x^n)+2a) - \left(b\cos(2b\log(c))^2 + b\sin(2b\log(c))^2\right)n\cos(2b\log(x^n)+2}$$

input
```
integrate(csc(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

output
```
2*(cos(2*b*log(x^n) + 2*a)*sin(2*b*log(c)) + cos(2*b*log(c))*sin(2*b*log(x
^n) + 2*a))/(2*b*n*cos(2*b*log(c))*cos(2*b*log(x^n) + 2*a) - (b*cos(2*b*lo
g(c))^2 + b*sin(2*b*log(c))^2)*n*cos(2*b*log(x^n) + 2*a)^2 - 2*b*n*sin(2*b
*log(c))*sin(2*b*log(x^n) + 2*a) - (b*cos(2*b*log(c))^2 + b*sin(2*b*log(c)
)^2)*n*sin(2*b*log(x^n) + 2*a)^2 - b*n)
```

Giac [F]

$$\int \frac{\csc^2(a+b\log(cx^n))}{x}\,dx = \int \frac{\csc(b\log(cx^n)+a)^2}{x}\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

output
```
integrate(csc(b*log(c*x^n) + a)^2/x, x)
```

Mupad [B] (verification not implemented)

Time = 23.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{\csc^2(a+b\log(cx^n))}{x}\,dx = -\frac{2\mathrm{i}}{b\,n\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}-1\right)}$$

input
```
int(1/(x*sin(a + b*log(c*x^n))^2),x)
```

output
```
-2i/(b*n*(exp(a*2i)*(c*x^n)^(b*2i) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int \frac{\csc^2(a+b\log(cx^n))}{x}\,dx = -\frac{\cos(\log(x^nc)\,b+a)}{\sin(\log(x^nc)\,b+a)\,bn}$$

input
```
int(csc(a+b*log(c*x^n))^2/x,x)
```

output
```
( - cos(log(x**n*c)*b + a))/(sin(log(x**n*c)*b + a)*b*n)
```

3.297 $\int \csc^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 84

$$\int \csc^3(a+b\log(cx^n))\,dx = -\frac{8e^{3ia}x(cx^n)^{3ib}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3-\frac{i}{bn}\right),\frac{1}{2}\left(5-\frac{i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{i-3bn}$$

output
```
-8*exp(3*I*a)*x*(c*x^n)^(3*I*b)*hypergeom([3, 3/2-1/2*I/b/n],[5/2-1/2*I/b/
n],exp(2*I*a)*(c*x^n)^(2*I*b))/(I-3*b*n)
```

Mathematica [A] (verified)

Time = 4.62 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.39

$$\int \csc^3(a+b\log(cx^n))\,dx = -\frac{x\Big((1+bn\cot(a+b\log(cx^n)))\csc(a+b\log(cx^n))+2e^{ia}(i+bn)(cx^n)^{ib}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}-\right.}{2b^2n^2}$$

input
```
Integrate[Csc[a + b*Log[c*x^n]]^3,x]
```

output
```
-1/2*(x*((1 + b*n*Cot[a + b*Log[c*x^n]])*Csc[a + b*Log[c*x^n]] + 2*E^(I*a)
*(I + b*n)*(c*x^n)^(I*b)*Hypergeometric2F1[1, 1/2 - (I/2)/(b*n), 3/2 - (I/
2)/(b*n), E^((2*I)*(a + b*Log[c*x^n]))]))/(b^2*n^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5015, 5017, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{5015}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\csc^3(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5017}$$

$$\frac{8ie^{3ia}x(cx^n)^{-1/n}\int \frac{(cx^n)^{3ib+\frac{1}{n}-1}}{\left(1-e^{2ia}(cx^n)^{2ib}\right)^3}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{8ie^{3ia}x(cx^n)^{3ib}\,\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3-\frac{i}{bn}\right),\frac{1}{2}\left(5-\frac{i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{n\left(\frac{1}{n}+3ib\right)}$$

input
```
Int[Csc[a + b*Log[c*x^n]]^3,x]
```

output
```
((8*I)*E^((3*I)*a)*x*(c*x^n)^((3*I)*b)*Hypergeometric2F1[3, (3 - I/(b*n))/
2, (5 - I/(b*n))/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/(((3*I)*b + n^(-1))*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5015
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5017
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[(-2*I)^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^
(2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [F]

$$\int \csc \left(a+b \ln \left(c \,x^{n}\right)\right)^{3}d x$$

input
```
int(csc(a+b*ln(c*x^n))^3,x)
```

output
```
int(csc(a+b*ln(c*x^n))^3,x)
```

Fricas [F]

$$\int \csc^3\left(a+b\log\left(cx^n\right)\right)\,dx=\int \csc\left(b\log\left(cx^n\right)+a\right)^3\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
integral(csc(b*log(c*x^n) + a)^3, x)
```

Sympy [F]

$$\int \csc^3(a + b\log(cx^n))\, dx = \int \csc^3(a + b\log(cx^n))\, dx$$

input
```
integrate(csc(a+b*ln(c*x**n))**3,x)
```

output
```
Integral(csc(a + b*log(c*x**n))**3, x)
```

Maxima [F]

$$\int \csc^3(a + b\log(cx^n))\, dx = \int \csc(b\log(cx^n) + a)^3\, dx$$

input
```
integrate(csc(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
-((b*n*cos(b*log(c)) - sin(b*log(c)))*x*cos(b*log(x^n) + a) - (b*n*sin(b*l
og(c)) + cos(b*log(c)))*x*sin(b*log(x^n) + a) + (((b*cos(4*b*log(c))*cos(3
*b*log(c)) + b*sin(4*b*log(c))*sin(3*b*log(c)))*n - cos(3*b*log(c))*sin(4*
b*log(c)) + cos(4*b*log(c))*sin(3*b*log(c)))*x*cos(3*b*log(x^n) + 3*a) + (
(b*cos(4*b*log(c))*cos(b*log(c)) + b*sin(4*b*log(c))*sin(b*log(c)))*n + co
s(b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(b*log(c)))*x*cos(b*log(x
^n) + a) + ((b*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(3*b
*log(c)))*n + cos(4*b*log(c))*cos(3*b*log(c)) + sin(4*b*log(c))*sin(3*b*lo
g(c)))*x*sin(3*b*log(x^n) + 3*a) + ((b*cos(b*log(c))*sin(4*b*log(c)) - b*c
os(4*b*log(c))*sin(b*log(c)))*n - cos(4*b*log(c))*cos(b*log(c)) - sin(4*b*
log(c))*sin(b*log(c)))*x*sin(b*log(x^n) + a))*cos(4*b*log(x^n) + 4*a) - (2
*((b*cos(3*b*log(c))*cos(2*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c)))*
n + cos(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*log(c)))*x*c
os(2*b*log(x^n) + 2*a) + 2*((b*cos(2*b*log(c))*sin(3*b*log(c)) - b*cos(3*b
*log(c))*sin(2*b*log(c)))*n - cos(3*b*log(c))*cos(2*b*log(c)) - sin(3*b*lo
g(c))*sin(2*b*log(c)))*x*sin(2*b*log(x^n) + 2*a) - (b*n*cos(3*b*log(c)) +
sin(3*b*log(c)))*x)*cos(3*b*log(x^n) + 3*a) - 2*(((b*cos(2*b*log(c))*cos(b
*log(c)) + b*sin(2*b*log(c))*sin(b*log(c)))*n + cos(b*log(c))*sin(2*b*log(
c)) - cos(2*b*log(c))*sin(b*log(c)))*x*cos(b*log(x^n) + a) + ((b*cos(b*log
(c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)))*n - cos(2*b*log...
```

Giac [F]

$$\int \csc^3(a + b\log(cx^n)) \, dx = \int \csc(b\log(cx^n) + a)^3 \, dx$$

input
```
integrate(csc(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
integrate(csc(b*log(c*x^n) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^3(a + b\log(cx^n)) \, dx = \int \frac{1}{\sin(a + b\ln(c x^n))^3} \, dx$$

input
```
int(1/sin(a + b*log(c*x^n))^3,x)
```

output
```
int(1/sin(a + b*log(c*x^n))^3, x)
```

Reduce [F]

$$\int \csc^3(a + b\log(cx^n)) \, dx = \int \csc(\log(x^n c) b + a)^3 dx$$

input
```
int(csc(a+b*log(c*x^n))^3,x)
```

output
```
int(csc(log(x**n*c)*b + a)**3,x)
```

3.298 $\int \frac{\csc^3(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \frac{\csc^3(a+b\log(cx^n))}{x}\,dx = -\frac{\operatorname{arctanh}(\cos(a+b\log(cx^n)))}{2bn} - \frac{\cot(a+b\log(cx^n))\csc(a+b\log(cx^n))}{2bn}$$

output
```
-1/2*arctanh(cos(a+b*ln(c*x^n)))/b/n-1/2*cot(a+b*ln(c*x^n))*csc(a+b*ln(c*x
^n))/b/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.95

$$\int \frac{\csc^3(a+b\log(cx^n))}{x}\,dx = -\frac{\csc^2\left(\frac{1}{2}(a+b\log(cx^n))\right)}{8bn} - \frac{\log\left(\cos\left(\frac{1}{2}(a+b\log(cx^n))\right)\right)}{2bn} + \frac{\log\left(\sin\left(\frac{1}{2}(a+b\log(cx^n))\right)\right)}{2bn} + \frac{\sec^2\left(\frac{1}{2}(a+b\log(cx^n))\right)}{8bn}$$

input
```
Integrate[Csc[a + b*Log[c*x^n]]^3/x,x]
```

output
```
-1/8*Csc[(a + b*Log[c*x^n])/2]^2/(b*n) - Log[Cos[(a + b*Log[c*x^n])/2]]/(2
*b*n) + Log[Sin[(a + b*Log[c*x^n])/2]]/(2*b*n) + Sec[(a + b*Log[c*x^n])/2]
^2/(8*b*n)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3039, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+b\log(cx^n))}{x}\,dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \csc^3(a+b\log(cx^n))\,d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc(a+b\log(cx^n))^3\,d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{1}{2}\int \csc(a+b\log(cx^n))\,d\log(cx^n)-\frac{\cot(a+b\log(cx^n))\csc(a+b\log(cx^n))}{2b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}\int \csc(a+b\log(cx^n))\,d\log(cx^n)-\frac{\cot(a+b\log(cx^n))\csc(a+b\log(cx^n))}{2b}}{n}$$

$$\Big\downarrow \text{4257}$$

$$\frac{-\frac{\text{arctanh}(\cos(a+b\log(cx^n)))}{2b}-\frac{\cot(a+b\log(cx^n))\csc(a+b\log(cx^n))}{2b}}{n}$$

input
```
Int[Csc[a + b*Log[c*x^n]]^3/x,x]
```

output
```
(-1/2*ArcTanh[Cos[a + b*Log[c*x^n]]]/b - (Cot[a + b*Log[c*x^n]]*Csc[a + b*
Log[c*x^n]])/(2*b))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{-\frac{\csc(a+b\ln(c\,x^n))\cot(a+b\ln(c\,x^n))}{2}+\frac{\ln(\csc(a+b\ln(c\,x^n))-\cot(a+b\ln(c\,x^n)))}{2}}{nb}$
default	$\frac{-\frac{\csc(a+b\ln(c\,x^n))\cot(a+b\ln(c\,x^n))}{2}+\frac{\ln(\csc(a+b\ln(c\,x^n))-\cot(a+b\ln(c\,x^n)))}{2}}{nb}$
parallelrisch	$\frac{-\cot\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^2+\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^2+4\ln\left(\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)\right)}{8bn}$
risch	$\frac{(x^n)^{ib}c^{ib}\left(c^{2ib}(x^n)^{2ib}\mathrm{e}^{-\frac{3b\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}{2}}\mathrm{e}^{\frac{3b\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)}{2}}\mathrm{e}^{\frac{3b\pi\,\mathrm{csgn}(ic\,x^n)^3}{2}}\mathrm{e}^{-\frac{3b\pi\,\mathrm{csgn}(ic\,x^n)}{2}}\right.}{bn\left((x^n)^{2ib}c^{2ib}\mathrm{e}^{-b\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}\mathrm{e}^{b\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(\right.}$

input
```
int(csc(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(-1/2*csc(a+b*ln(c*x^n))*cot(a+b*ln(c*x^n))+1/2*ln(csc(a+b*ln(c*x^n)
)-cot(a+b*ln(c*x^n))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(51) = 102.

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.00

$$\int \frac{\csc^3(a+b\log(cx^n))}{x}\,dx = -\frac{(\cos(bn\log(x)+b\log(c)+a)^2-1)\log\left(\frac{1}{2}\cos(bn\log(x)+b\log(c)+a)+\frac{1}{2}\right)-(\cos(bn\log(x)+b}{4\,(bn\cos(bn\log(x)+b}$$

input
```
integrate(csc(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

output
```
-1/4*((cos(b*n*log(x) + b*log(c) + a)^2 - 1)*log(1/2*cos(b*n*log(x) + b*lo
g(c) + a) + 1/2) - (cos(b*n*log(x) + b*log(c) + a)^2 - 1)*log(-1/2*cos(b*n
*log(x) + b*log(c) + a) + 1/2) - 2*cos(b*n*log(x) + b*log(c) + a))/(b*n*co
s(b*n*log(x) + b*log(c) + a)^2 - b*n)
```

Sympy [F]

$$\int \frac{\csc^3(a+b\log(cx^n))}{x}\,dx = \int \frac{\csc^3(a+b\log(cx^n))}{x}\,dx$$

input
```
integrate(csc(a+b*ln(c*x**n))**3/x,x)
```

output
```
Integral(csc(a + b*log(c*x**n))**3/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2168 vs. $2(51) = 102$.

Time = 0.13 (sec) , antiderivative size = 2168, normalized size of antiderivative = 39.42

$$\int \frac{\csc^3(a + b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

input
```
integrate(csc(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

output
```
1/4*(4*((cos(4*b*log(c))*cos(3*b*log(c)) + sin(4*b*log(c))*sin(3*b*log(c))
)*cos(3*b*log(x^n) + 3*a) + (cos(4*b*log(c))*cos(b*log(c)) + sin(4*b*log(c
))*sin(b*log(c)))*cos(b*log(x^n) + a) + (cos(3*b*log(c))*sin(4*b*log(c)) -
 cos(4*b*log(c))*sin(3*b*log(c)))*sin(3*b*log(x^n) + 3*a) + (cos(b*log(c))
*sin(4*b*log(c)) - cos(4*b*log(c))*sin(b*log(c)))*sin(b*log(x^n) + a))*cos
(4*b*log(x^n) + 4*a) - 4*(2*(cos(3*b*log(c))*cos(2*b*log(c)) + sin(3*b*log
(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 2*(cos(2*b*log(c))*sin(3*b
*log(c)) - cos(3*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) - cos(
3*b*log(c)))*cos(3*b*log(x^n) + 3*a) - 8*((cos(2*b*log(c))*cos(b*log(c)) +
 sin(2*b*log(c))*sin(b*log(c)))*cos(b*log(x^n) + a) + (cos(b*log(c))*sin(2
*b*log(c)) - cos(2*b*log(c))*sin(b*log(c)))*sin(b*log(x^n) + a))*cos(2*b*l
og(x^n) + 2*a) + 4*cos(b*log(c))*cos(b*log(x^n) + a) - ((cos(4*b*log(c))^2
 + sin(4*b*log(c))^2)*cos(4*b*log(x^n) + 4*a)^2 + 4*(cos(2*b*log(c))^2 + s
in(2*b*log(c))^2)*cos(2*b*log(x^n) + 2*a)^2 + (cos(4*b*log(c))^2 + sin(4*b
*log(c))^2)*sin(4*b*log(x^n) + 4*a)^2 + 4*(cos(2*b*log(c))^2 + sin(2*b*log
(c))^2)*sin(2*b*log(x^n) + 2*a)^2 - 2*(2*(cos(4*b*log(c))*cos(2*b*log(c))
+ sin(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 2*(cos(2*b*lo
g(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n)
+ 2*a) - cos(4*b*log(c)))*cos(4*b*log(x^n) + 4*a) - 4*cos(2*b*log(c))*cos(
2*b*log(x^n) + 2*a) + 2*(2*(cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*l...
```

Giac [F]

$$\int \frac{\csc^3(a+b\log(cx^n))}{x}\,dx = \int \frac{\csc(b\log(cx^n)+a)^3}{x}\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

output
```
integrate(csc(b*log(c*x^n) + a)^3/x, x)
```

Mupad [B] (verification not implemented)

Time = 27.70 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.22

$$\int \frac{\csc^3(a+b\log(cx^n))}{x}\,dx = -\frac{\ln\left(-\frac{1\mathrm{i}}{x}-\frac{\mathrm{e}^{a\,1\mathrm{i}}\,(c\,x^n)^{b\,1\mathrm{i}}\,1\mathrm{i}}{x}\right)}{2\,b\,n}+\frac{\ln\left(\frac{1\mathrm{i}}{x}-\frac{\mathrm{e}^{a\,1\mathrm{i}}\,(c\,x^n)^{b\,1\mathrm{i}}\,1\mathrm{i}}{x}\right)}{2\,b\,n}+\frac{2\,\mathrm{e}^{a\,1\mathrm{i}}\,(c\,x^n)^{b\,1\mathrm{i}}}{b\,n\,\left(1+\mathrm{e}^{a\,4\mathrm{i}}\,(c\,x^n)^{b\,4\mathrm{i}}-2\,\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}}\right)}+\frac{\mathrm{e}^{a\,1\mathrm{i}}\,(c\,x^n)^{b\,1\mathrm{i}}}{b\,n\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,(c\,x^n)^{b\,2\mathrm{i}}-1\right)}$$

input
```
int(1/(x*sin(a + b*log(c*x^n))^3),x)
```

output
```
log(1i/x - (exp(a*1i)*(c*x^n)^(b*1i)*1i)/x)/(2*b*n) - log(- 1i/x - (exp(a*
1i)*(c*x^n)^(b*1i)*1i)/x)/(2*b*n) + (2*exp(a*1i)*(c*x^n)^(b*1i))/(b*n*(exp
(a*4i)*(c*x^n)^(b*4i) - 2*exp(a*2i)*(c*x^n)^(b*2i) + 1)) + (exp(a*1i)*(c*x
^n)^(b*1i))/(b*n*(exp(a*2i)*(c*x^n)^(b*2i) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

$$\int \frac{\csc^3(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{-\cos(\log(x^nc)\,b+a)+\log\left(\tan\left(\frac{\log(x^nc)b}{2}+\frac{a}{2}\right)\right)\sin(\log(x^nc)\,b+a)^2}{2\sin(\log(x^nc)\,b+a)^2bn}$$

input
```
int(csc(a+b*log(c*x^n))^3/x,x)
```

output
```
( - cos(log(x**n*c)*b + a) + log(tan((log(x**n*c)*b + a)/2))*sin(log(x**n*
c)*b + a)**2)/(2*sin(log(x**n*c)*b + a)**2*b*n)
```

3.299 $\int \csc^4(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 84

$$\int \csc^4(a+b\log(cx^n))\,dx = \frac{16e^{4ia}x(cx^n)^{4ib}\,\text{Hypergeometric2F1}\left(4,\frac{1}{2}\left(4-\frac{i}{bn}\right),\frac{1}{2}\left(6-\frac{i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{1+4ibn}$$

output
```
16*exp(4*I*a)*x*(c*x^n)^(4*I*b)*hypergeom([4, 2-1/2*I/b/n],[3-1/2*I/b/n],e
xp(2*I*a)*(c*x^n)^(2*I*b))/(1+4*I*b*n)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 221 vs. 2(84) = 168.

Time = 9.40 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.63

$$\int \csc^4(a+b\log(cx^n))\,dx = \frac{x\left(-4e^{2ia}(i+2bn)(cx^n)^{2ib}\,\text{Hypergeometric2F1}\left(1,1-\frac{i}{2bn},2-\frac{i}{2bn},e^{2i(a+b\log(cx^n))}\right)-4i(1+4b^2n^2)\,\text{Hype}\right.}{}$$

input
```
Integrate[Csc[a + b*Log[c*x^n]]^4,x]
```

output
```
(x*(-4*E^((2*I)*a)*(I + 2*b*n)*(c*x^n)^((2*I)*b)*Hypergeometric2F1[1, 1 -
(I/2)/(b*n), 2 - (I/2)/(b*n), E^((2*I)*(a + b*Log[c*x^n]))] - (4*I)*(1 + 4
*b^2*n^2)*Hypergeometric2F1[1, (-1/2*I)/(b*n), 1 - (I/2)/(b*n), E^((2*I)*(
a + b*Log[c*x^n]))] + Csc[a + b*Log[c*x^n]]^3*(-((1 + 12*b^2*n^2)*Cos[a +
b*Log[c*x^n]]) + (1 + 4*b^2*n^2)*Cos[3*(a + b*Log[c*x^n])] - 4*b*n*Sin[a +
 b*Log[c*x^n]])))/(24*b^3*n^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5015, 5017, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^4\left(a+b\log\left(cx^n\right)\right)dx$$

$$\Big\downarrow \text{ 5015}$$

$$\frac{x\left(cx^n\right)^{-1/n}\int \left(cx^n\right)^{\frac{1}{n}-1}\csc^4\left(a+b\log\left(cx^n\right)\right)d\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{ 5017}$$

$$\frac{16e^{4ia}x\left(cx^n\right)^{-1/n}\int \frac{\left(cx^n\right)^{4ib+\frac{1}{n}-1}}{\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^4}d\left(cx^n\right)}{n}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{16e^{4ia}x\left(cx^n\right)^{4ib}\text{Hypergeometric2F1}\left(4,\frac{1}{2}\left(4-\frac{i}{bn}\right),\frac{1}{2}\left(6-\frac{i}{bn}\right),e^{2ia}\left(cx^n\right)^{2ib}\right)}{n\left(\frac{1}{n}+4ib\right)}$$

input
```
Int[Csc[a + b*Log[c*x^n]]^4,x]
```

output
```
(16*E^((4*I)*a)*x*(c*x^n)^((4*I)*b)*Hypergeometric2F1[4, (4 - I/(b*n))/2,
(6 - I/(b*n))/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/(((4*I)*b + n^(-1))*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5015
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5017
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[(-2*I)^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^
(2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [F]

$$\int \csc \left(a + b \ln \left(c\, x^{n}\right)\right)^{4} dx$$

input
```
int(csc(a+b*ln(c*x^n))^4,x)
```

output
```
int(csc(a+b*ln(c*x^n))^4,x)
```

Fricas [F]

$$\int \csc^4(a + b\log(cx^n))\, dx = \int \csc(b\log(cx^n) + a)^4\, dx$$

input
```
integrate(csc(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

output
```
integral(csc(b*log(c*x^n) + a)^4, x)
```

Sympy [F]

$$\int \csc^4(a + b\log(cx^n))\, dx = \int \csc^4(a + b\log(cx^n))\, dx$$

input
```
integrate(csc(a+b*ln(c*x**n))**4,x)
```

output
```
Integral(csc(a + b*log(c*x**n))**4, x)
```

Maxima [F]

$$\int \csc^4(a + b\log(cx^n))\, dx = \int \csc(b\log(cx^n) + a)^4\, dx$$

input
```
integrate(csc(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

output
```
1/3*(6*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*x*cos(4*b*log(x^n) +
4*a)^2 + 6*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*x*cos(2*b*log(x^n
) + 2*a)^2 + 6*(b*cos(4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*x*sin(4*b*log
(x^n) + 4*a)^2 + 6*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*x*sin(2*b
*log(x^n) + 2*a)^2 - (2*b*n*cos(2*b*log(c)) - sin(2*b*log(c)))*x*cos(2*b*l
og(x^n) + 2*a) + (2*b*n*sin(2*b*log(c)) + cos(2*b*log(c)))*x*sin(2*b*log(x
^n) + 2*a) - ((2*(b*cos(6*b*log(c))*cos(4*b*log(c)) + b*sin(6*b*log(c))*si
n(4*b*log(c)))*n - cos(4*b*log(c))*sin(6*b*log(c)) + cos(6*b*log(c))*sin(4
*b*log(c)))*x*cos(4*b*log(x^n) + 4*a) + 2*(6*(b^2*cos(2*b*log(c))*sin(6*b*
log(c)) - b^2*cos(6*b*log(c))*sin(2*b*log(c)))*n^2 - (b*cos(6*b*log(c))*co
s(2*b*log(c)) + b*sin(6*b*log(c))*sin(2*b*log(c)))*n + cos(2*b*log(c))*sin
(6*b*log(c)) - cos(6*b*log(c))*sin(2*b*log(c)))*x*cos(2*b*log(x^n) + 2*a)
+ (2*(b*cos(4*b*log(c))*sin(6*b*log(c)) - b*cos(6*b*log(c))*sin(4*b*log(c)
))*n + cos(6*b*log(c))*cos(4*b*log(c)) + sin(6*b*log(c))*sin(4*b*log(c)))*
x*sin(4*b*log(x^n) + 4*a) - 2*(6*(b^2*cos(6*b*log(c))*cos(2*b*log(c)) + b^
2*sin(6*b*log(c))*sin(2*b*log(c)))*n^2 + (b*cos(2*b*log(c))*sin(6*b*log(c)
) - b*cos(6*b*log(c))*sin(2*b*log(c)))*n + cos(6*b*log(c))*cos(2*b*log(c))
 + sin(6*b*log(c))*sin(2*b*log(c)))*x*sin(2*b*log(x^n) + 2*a) - (4*b^2*n^2
*sin(6*b*log(c)) + sin(6*b*log(c)))*x)*cos(6*b*log(x^n) + 6*a) + (3*(12*(b
^2*cos(2*b*log(c))*sin(4*b*log(c)) - b^2*cos(4*b*log(c))*sin(2*b*log(c)...
```

Giac [F]

$$\int \csc^4(a+b\log(cx^n))\,dx = \int \csc(b\log(cx^n)+a)^4\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))^4,x, algorithm="giac")
```

output
```
integrate(csc(b*log(c*x^n) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^4\left(a+b\log\left(cx^n\right)\right)\,dx=\int \frac{1}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)^4}\,dx$$

input
```
int(1/sin(a + b*log(c*x^n))^4,x)
```

output
```
int(1/sin(a + b*log(c*x^n))^4, x)
```

Reduce [F]

$$\int \csc^4\left(a+b\log\left(cx^n\right)\right)\,dx=\int \csc\left(\log(x^n c)\,b+a\right)^4dx$$

input
```
int(csc(a+b*log(c*x^n))^4,x)
```

output
```
int(csc(log(x**n*c)*b + a)**4,x)
```

3.300 $\int \frac{\csc^4(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{\csc^4(a+b\log(cx^n))}{x}\,dx = -\frac{\cot(a+b\log(cx^n))}{bn} - \frac{\cot^3(a+b\log(cx^n))}{3bn}$$

output
```
-cot(a+b*ln(c*x^n))/b/n-1/3*cot(a+b*ln(c*x^n))^3/b/n
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

$$\int \frac{\csc^4(a+b\log(cx^n))}{x}\,dx = -\frac{2\cot(a+b\log(cx^n))}{3bn} - \frac{\cot(a+b\log(cx^n))\csc^2(a+b\log(cx^n))}{3bn}$$

input
```
Integrate[Csc[a + b*Log[c*x^n]]^4/x,x]
```

output
```
(-2*Cot[a + b*Log[c*x^n]])/(3*b*n) - (Cot[a + b*Log[c*x^n]]*Csc[a + b*Log[
c*x^n]]^2)/(3*b*n)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3039, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^4(a+b\log(cx^n))}{x}dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \csc^4(a+b\log(cx^n))d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \csc(a+b\log(cx^n))^4d\log(cx^n)}{n}$$
$$\Big\downarrow \text{4254}$$
$$-\frac{\int \left(\cot^2(a+b\log(cx^n))+1\right)d\cot(a+b\log(cx^n))}{bn}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\frac{1}{3}\cot^3(a+b\log(cx^n))+\cot(a+b\log(cx^n))}{bn}$$

input
```
Int[Csc[a + b*Log[c*x^n]]^4/x,x]
```

output
```
-((Cot[a + b*Log[c*x^n]] + Cot[a + b*Log[c*x^n]]^3/3)/(b*n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 4.66 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{\left(-\frac{2}{3}-\frac{\csc(a+b\ln(c\,x^{n}))^{2}}{3}\right)\cot(a+b\ln(c\,x^{n}))}{nb}$
default	$\frac{\left(-\frac{2}{3}-\frac{\csc(a+b\ln(c\,x^{n}))^{2}}{3}\right)\cot(a+b\ln(c\,x^{n}))}{nb}$
parallelrisch	$\frac{-\cot\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)^{3}+\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)^{3}+9\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)-9\cot\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)}{24bn}$
risch	$\frac{4i\left(3(x^{n})^{2ib}c^{2ib}\mathrm{e}^{-b\pi\,\mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})^{2}}\mathrm{e}^{b\pi\,\mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})\,\mathrm{csgn}(ic)}\mathrm{e}^{b\pi\,\mathrm{csgn}(ic\,x^{n})^{3}}\mathrm{e}^{-b\pi\,\mathrm{csgn}(ic\,x^{n})^{2}\,\mathrm{csgn}(ic)}\mathrm{e}^{2ia}\cdots\right.}{3bn\left((x^{n})^{2ib}c^{2ib}\mathrm{e}^{-b\pi\,\mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})^{2}}\mathrm{e}^{b\pi\,\mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})\,\mathrm{csgn}(ic)}\mathrm{e}^{b\pi\,\mathrm{csgn}(ic\,x^{n})^{3}}\mathrm{e}^{-b\pi\,\mathrm{csgn}(ic\,x^{n})^{2}\,\mathrm{csgn}(ic)}\mathrm{e}^{2ia}-\cdots\right.}$

input
```
int(csc(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(-2/3-1/3*csc(a+b*ln(c*x^n))^2)*cot(a+b*ln(c*x^n))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{\csc^4(a+b\log(cx^n))}{x}\,dx = -\frac{2\cos(bn\log(x)+b\log(c)+a)^3-3\cos(bn\log(x)+b\log(c)+a)}{3\left(bn\cos(bn\log(x)+b\log(c)+a)^2-bn\right)\sin(bn\log(x)+b\log(c)+a)}$$

input
```
integrate(csc(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

output
```
-1/3*(2*cos(b*n*log(x) + b*log(c) + a)^3 - 3*cos(b*n*log(x) + b*log(c) + a
))/((b*n*cos(b*n*log(x) + b*log(c) + a)^2 - b*n)*sin(b*n*log(x) + b*log(c)
 + a))
```

Sympy [F]

$$\int \frac{\csc^4(a+b\log(cx^n))}{x}\,dx = \int \frac{\csc^4(a+b\log(cx^n))}{x}\,dx$$

input
```
integrate(csc(a+b*ln(c*x**n))**4/x,x)
```

output
```
Integral(csc(a + b*log(c*x**n))**4/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1332 vs. 2(41) = 82.

Time = 0.07 (sec) , antiderivative size = 1332, normalized size of antiderivative = 30.98

$$\int \frac{\csc^4(a+b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

input
```
integrate(csc(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

output
```
4/3*((3*(cos(2*b*log(c))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(2*b*log(c))
)*cos(2*b*log(x^n) + 2*a) - 3*(cos(6*b*log(c))*cos(2*b*log(c)) + sin(6*b*l
og(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) - sin(6*b*log(c)))*cos(6*b
*log(x^n) + 6*a) - 3*(3*(cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))
*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) - 3*(cos(4*b*log(c))*cos(2*b*log
(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2*a) - sin(4*b*
log(c)))*cos(4*b*log(x^n) + 4*a) + (3*(cos(6*b*log(c))*cos(2*b*log(c)) + s
in(6*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) + 3*(cos(2*b*log(c
))*sin(6*b*log(c)) - cos(6*b*log(c))*sin(2*b*log(c)))*sin(2*b*log(x^n) + 2
*a) - cos(6*b*log(c)))*sin(6*b*log(x^n) + 6*a) - 3*(3*(cos(4*b*log(c))*cos
(2*b*log(c)) + sin(4*b*log(c))*sin(2*b*log(c)))*cos(2*b*log(x^n) + 2*a) +
3*(cos(2*b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(2*b*log(c)))*sin(
2*b*log(x^n) + 2*a) - cos(4*b*log(c)))*sin(4*b*log(x^n) + 4*a))/((b*cos(6*
b*log(c))^2 + b*sin(6*b*log(c))^2)*n*cos(6*b*log(x^n) + 6*a)^2 + 9*(b*cos(
4*b*log(c))^2 + b*sin(4*b*log(c))^2)*n*cos(4*b*log(x^n) + 4*a)^2 - 6*b*n*c
os(2*b*log(c))*cos(2*b*log(x^n) + 2*a) + 9*(b*cos(2*b*log(c))^2 + b*sin(2*
b*log(c))^2)*n*cos(2*b*log(x^n) + 2*a)^2 + (b*cos(6*b*log(c))^2 + b*sin(6*
b*log(c))^2)*n*sin(6*b*log(x^n) + 6*a)^2 + 9*(b*cos(4*b*log(c))^2 + b*sin(
4*b*log(c))^2)*n*sin(4*b*log(x^n) + 4*a)^2 + 6*b*n*sin(2*b*log(c))*sin(2*b
*log(x^n) + 2*a) + 9*(b*cos(2*b*log(c))^2 + b*sin(2*b*log(c))^2)*n*sin(...
```

Giac [F]

$$\int \frac{\csc^4(a+b\log(cx^n))}{x}\,dx=\int \frac{\csc(b\log(cx^n)+a)^4}{x}\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

output
```
integrate(csc(b*log(c*x^n) + a)^4/x, x)
```

Mupad [B] (verification not implemented)

Time = 34.46 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{\csc^4(a+b\log(cx^n))}{x}\,dx = \frac{4\left(e^{a\,2i}\left(c\,x^n\right)^{b\,2i}\,3i-i\right)}{3\,b\,n\left(e^{a\,2i}\left(c\,x^n\right)^{b\,2i}-1\right)^3}$$

input
```
int(1/(x*sin(a + b*log(c*x^n))^4),x)
```

output
```
(4*(exp(a*2i)*(c*x^n)^(b*2i)*3i - 1i))/(3*b*n*(exp(a*2i)*(c*x^n)^(b*2i) -
1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{\csc^4(a+b\log(cx^n))}{x}\,dx = \frac{\cos(\log(x^n c)\,b+a)\left(-2\sin(\log(x^n c)\,b+a)^2-1\right)}{3\sin(\log(x^n c)\,b+a)^3 bn}$$

input
```
int(csc(a+b*log(c*x^n))^4/x,x)
```

output
```
(cos(log(x**n*c)*b + a)*( - 2*sin(log(x**n*c)*b + a)**2 - 1))/(3*sin(log(x
**n*c)*b + a)**3*b*n)
```

3.301 $\int (-((1+b^2n^2)\csc(a+b\log(cx^n)))+2b^2n^2\csc^3(a$

Optimal result

Integrand size = 44, antiderivative size = 42

$$\int \left(-\left(\left(1+b^2n^2\right)\csc\left(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\csc^3\left(a+b\log\left(cx^n\right)\right)\right)\,dx = -x\csc\left(a+b\log\left(cx^n\right)\right)-bnx\cot\left(a+b\log\left(cx^n\right)\right)\csc\left(a+b\log\left(cx^n\right)\right)$$

output
```
-x*csc(a+b*ln(c*x^n))-b*n*x*cot(a+b*ln(c*x^n))*csc(a+b*ln(c*x^n))
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \left(-\left(\left(1+b^2n^2\right)\csc\left(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\csc^3\left(a+b\log\left(cx^n\right)\right)\right)\,dx = -x(1+bn\cot\left(a+b\log\left(cx^n\right)\right))\csc\left(a+b\log\left(cx^n\right)\right)$$

input
```
Integrate[-((1 + b^2*n^2)*Csc[a + b*Log[c*x^n]]) + 2*b^2*n^2*Csc[a + b*Log
[c*x^n]]^3,x]
```

output
```
-(x*(1 + b*n*Cot[a + b*Log[c*x^n]])*Csc[a + b*Log[c*x^n]])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 172, normalized size of antiderivative = 4.10, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2b^2n^2\csc^3(a+b\log(cx^n))-\left(b^2n^2+1\right)\csc(a+b\log(cx^n))\right)dx$$

$$\big\downarrow \text{2009}$$

$$2e^{ia}x(bn+i)(cx^n)^{ib}\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1-\frac{i}{bn}\right),\frac{1}{2}\left(3-\frac{i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)-\frac{16e^{3ia}b^2n^2x(cx^n)^{3ib}\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3-\frac{i}{bn}\right),\frac{1}{2}\left(5-\frac{i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{-3bn+i}$$

input
```
Int[-((1 + b^2*n^2)*Csc[a + b*Log[c*x^n]]) + 2*b^2*n^2*Csc[a + b*Log[c*x^n
]]^3,x]
```

output
```
2*E^(I*a)*(I + b*n)*x*(c*x^n)^(I*b)*Hypergeometric2F1[1, (1 - I/(b*n))/2,
(3 - I/(b*n))/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)] - (16*b^2*E^((3*I)*a)*n^2*
x*(c*x^n)^((3*I)*b)*Hypergeometric2F1[3, (3 - I/(b*n))/2, (5 - I/(b*n))/2,
 E^((2*I)*a)*(c*x^n)^((2*I)*b)])/(I - 3*b*n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 14.75 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.90

method	result
parallelrisch	$-\frac{x\left(-bn\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)^{2}+2\tan\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)+\cot\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)^{2}bn+2\cot\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)\right)}{4}$
risch	$\frac{2(x^{n})^{ib}c^{ib}x\left(nb\,c^{2ib}(x^{n})^{2ib}\mathrm{e}^{\frac{3b\pi\,\mathrm{csgn}(ic\,x^{n})^{3}}{2}}\mathrm{e}^{-\frac{3b\pi\,\mathrm{csgn}(ic\,x^{n})^{2}\mathrm{csgn}(ic)}{2}}\mathrm{e}^{-\frac{3b\pi\,\mathrm{csgn}(ix^{n})\mathrm{csgn}(ic\,x^{n})^{2}}{2}}\mathrm{e}^{\frac{3b\pi\,\mathrm{csgn}(ix^{n})\mathrm{csgn}(ic\,x^{n})}{2}}\right.}{}$

input
```
int(-(b^2*n^2+1)*csc(a+b*ln(c*x^n))+2*b^2*n^2*csc(a+b*ln(c*x^n))^3,x,metho
d=_RETURNVERBOSE)
```

output
```
-1/4*x*(-b*n*tan(1/2*a+b*ln((c*x^n)^(1/2)))^2+2*tan(1/2*a+b*ln((c*x^n)^(1/
2)))+cot(1/2*a+b*ln((c*x^n)^(1/2)))^2*b*n+2*cot(1/2*a+b*ln((c*x^n)^(1/2)))
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int \left(-\left(\left(1+b^2n^2\right)\csc\left(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\csc^3\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$
$$=\frac{bnx\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+x\sin\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{\cos\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2-1}$$

input
```
integrate(-(b^2*n^2+1)*csc(a+b*log(c*x^n))+2*b^2*n^2*csc(a+b*log(c*x^n))^3
,x, algorithm="fricas")
```

output
```
(b*n*x*cos(b*n*log(x) + b*log(c) + a) + x*sin(b*n*log(x) + b*log(c) + a))/
(cos(b*n*log(x) + b*log(c) + a)^2 - 1)
```

Sympy [F]

$$\int \left(-\left(\left(1+b^2n^2\right)\csc\left(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\csc^3\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$
$$=\int \left(2b^2n^2\csc^2\left(a+b\log\left(cx^n\right)\right)-b^2n^2-1\right)\csc\left(a+b\log\left(cx^n\right)\right)dx$$

input
```
integrate(-(b**2*n**2+1)*csc(a+b*ln(c*x**n))+2*b**2*n**2*csc(a+b*ln(c*x**n
))**3,x)
```

output
```
Integral((2*b**2*n**2*csc(a + b*log(c*x**n))**2 - b**2*n**2 - 1)*csc(a + b
*log(c*x**n)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1701 vs. $2(42) = 84$.

Time = 0.27 (sec) , antiderivative size = 1701, normalized size of antiderivative = 40.50

$$\int \left(-\left(\left(1+b^2n^2\right)\csc\left(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\csc^3\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$
$$=\text{Too large to display}$$

input
```
integrate(-(b^2*n^2+1)*csc(a+b*log(c*x^n))+2*b^2*n^2*csc(a+b*log(c*x^n))^3
,x, algorithm="maxima")
```

output
```
2*((b*n*cos(b*log(c)) - sin(b*log(c)))*x*cos(b*log(x^n) + a) - (b*n*sin(b*
log(c)) + cos(b*log(c)))*x*sin(b*log(x^n) + a) + (((b*cos(4*b*log(c))*cos(
3*b*log(c)) + b*sin(4*b*log(c))*sin(3*b*log(c)))*n - cos(3*b*log(c))*sin(4
*b*log(c)) + cos(4*b*log(c))*sin(3*b*log(c)))*x*cos(3*b*log(x^n) + 3*a) +
((b*cos(4*b*log(c))*cos(b*log(c)) + b*sin(4*b*log(c))*sin(b*log(c)))*n + c
os(b*log(c))*sin(4*b*log(c)) - cos(4*b*log(c))*sin(b*log(c)))*x*cos(b*log(
x^n) + a) + ((b*cos(3*b*log(c))*sin(4*b*log(c)) - b*cos(4*b*log(c))*sin(3*
b*log(c)))*n + cos(4*b*log(c))*cos(3*b*log(c)) + sin(4*b*log(c))*sin(3*b*l
og(c)))*x*sin(3*b*log(x^n) + 3*a) + ((b*cos(b*log(c))*sin(4*b*log(c)) - b*
cos(4*b*log(c))*sin(b*log(c)))*n - cos(4*b*log(c))*cos(b*log(c)) - sin(4*b
*log(c))*sin(b*log(c)))*x*sin(b*log(x^n) + a))*cos(4*b*log(x^n) + 4*a) - (
2*((b*cos(3*b*log(c))*cos(2*b*log(c)) + b*sin(3*b*log(c))*sin(2*b*log(c)))
*n + cos(2*b*log(c))*sin(3*b*log(c)) - cos(3*b*log(c))*sin(2*b*log(c)))*x*
cos(2*b*log(x^n) + 2*a) + 2*((b*cos(2*b*log(c))*sin(3*b*log(c)) - b*cos(3*
b*log(c))*sin(2*b*log(c)))*n - cos(3*b*log(c))*cos(2*b*log(c)) - sin(3*b*l
og(c))*sin(2*b*log(c)))*x*sin(2*b*log(x^n) + 2*a) - (b*n*cos(3*b*log(c)) +
 sin(3*b*log(c)))*x)*cos(3*b*log(x^n) + 3*a) - 2*(((b*cos(2*b*log(c))*cos(
b*log(c)) + b*sin(2*b*log(c))*sin(b*log(c)))*n + cos(b*log(c))*sin(2*b*log
(c)) - cos(2*b*log(c))*sin(b*log(c)))*x*cos(b*log(x^n) + a) + ((b*cos(b*lo
g(c))*sin(2*b*log(c)) - b*cos(2*b*log(c))*sin(b*log(c)))*n - cos(2*b*lo...
```

Giac [F]

$$\int \left(-\left(\left(1+b^2n^2\right)\csc\left(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\csc^3\left(a+b\log\left(cx^n\right)\right)\right)\,dx$$
$$=\int 2\,b^2n^2\csc\left(b\log\left(cx^n\right)+a\right)^3-\left(b^2n^2+1\right)\csc\left(b\log\left(cx^n\right)+a\right)\,dx$$

input
```
integrate(-(b^2*n^2+1)*csc(a+b*log(c*x^n))+2*b^2*n^2*csc(a+b*log(c*x^n))^3
,x, algorithm="giac")
```

output
```
integrate(2*b^2*n^2*csc(b*log(c*x^n) + a)^3 - (b^2*n^2 + 1)*csc(b*log(c*x^
n) + a), x)
```

Mupad [B] (verification not implemented)

Time = 23.73 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.02

$$\int \left(-\left(\left(1+b^2n^2\right)\csc\left(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\csc^3\left(a+b\log\left(cx^n\right)\right)\right)\,dx = \frac{2\,x\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}\,\left(b\,n+1\mathrm{i}\right)+2\,x\,\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,1\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}\,\left(b\,n-\mathrm{i}\right)}{{\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^n\right)}^{b\,2\mathrm{i}}-1\right)}^2}$$

input
```
int((2*b^2*n^2)/sin(a + b*log(c*x^n))^3 - (b^2*n^2 + 1)/sin(a + b*log(c*x^
n)),x)
```

output
```
(2*x*exp(a*1i)*(c*x^n)^(b*1i)*(b*n + 1i) + 2*x*exp(a*1i)*exp(a*2i)*(c*x^n)
^(b*1i)*(c*x^n)^(b*2i)*(b*n - 1i))/(exp(a*2i)*(c*x^n)^(b*2i) - 1)^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \left(-\left(\left(1+b^2n^2\right)\csc\left(a+b\log\left(cx^n\right)\right)\right)+2b^2n^2\csc^3\left(a+b\log\left(cx^n\right)\right)\right)\,dx = -\frac{x(\cos\left(\log(x^n c)\,b+a\right)bn+\sin\left(\log(x^n c)\,b+a\right))}{\sin\left(\log\left(x^n c\right)b+a\right)^2}$$

input
```
int(-(b^2*n^2+1)*csc(a+b*log(c*x^n))+2*b^2*n^2*csc(a+b*log(c*x^n))^3,x)
```

output
```
( - x*(cos(log(x**n*c)*b + a)*b*n + sin(log(x**n*c)*b + a)))/sin(log(x**n*
c)*b + a)**2
```

3.302 $\int x^m \csc^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\,dx$

Optimal result

Integrand size = 31, antiderivative size = 110

$$\int x^m \csc^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\,dx = \frac{x^{1+m}\csc\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)}{2(1+m)} - \frac{x^{1+m}\cot\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\csc\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)}{2\sqrt{-(1+m)^2}}$$

output
```
x^(1+m)*csc(a+2*ln(c*x^(1/2*(-(1+m)^2)^(1/2))))/(2+2*m)-1/2*x^(1+m)*cot(a+
2*ln(c*x^(1/2*(-(1+m)^2)^(1/2))))*csc(a+2*ln(c*x^(1/2*(-(1+m)^2)^(1/2))))/
(-(1+m)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int x^m \csc^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\,dx$$
$$=\frac{x^{1+m}\left(1+m+\sqrt{-(1+m)^2}\cot\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\right)\csc\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)}{2(1+m)^2}$$

input
```
Integrate[x^m*Csc[a + 2*Log[c*x^(Sqrt[-(1 + m)^2]/2)]]^3,x]
```

output
```
(x^(1 + m)*(1 + m + Sqrt[-(1 + m)^2]*Cot[a + 2*Log[c*x^(Sqrt[-(1 + m)^2]/2
)]])*Csc[a + 2*Log[c*x^(Sqrt[-(1 + m)^2]/2)]])/(2*(1 + m)^2)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.43 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.38, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {5021, 5017, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \csc^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)\right)\,dx$$

$$\Big\downarrow \ 5021$$

$$\frac{2x^{m+1}\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)^{-\frac{2(m+1)}{\sqrt{-(m+1)^2}}}\int\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)^{\frac{2(m+1)}{\sqrt{-(m+1)^2}}-1}\csc^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)\right)d\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right.}{\sqrt{-(m+1)^2}}$$

$$\Big\downarrow \ 5017$$

$$\frac{16ie^{3ia}x^{m+1}\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)^{-\frac{2(m+1)}{\sqrt{-(m+1)^2}}}\int\frac{\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)^{\frac{2(m+1)}{\sqrt{-(m+1)^2}}-(1-6i)}}{\left(1-e^{2ia}\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)^{4i}\right)^3}d\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)}{\sqrt{-(m+1)^2}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{8ie^{3ia}x^{m+1}\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right)^{6i}\text{Hypergeometric2F1}\left(3,\frac{1}{2}\left(3-\frac{i(m+1)}{\sqrt{-(m+1)^2}}\right),\frac{1}{2}\left(5-\frac{i(m+1)}{\sqrt{-(m+1)^2}}\right),e^{2ia}\left(cx^{\frac{1}{2}\sqrt{-(m+1)^2}}\right.\right.}{\sqrt{-(m+1)^2}\left(\frac{m+1}{\sqrt{-(m+1)^2}}+3i\right)}$$

input
```
Int[x^m*Csc[a + 2*Log[c*x^(Sqrt[-(1 + m)^2]/2)]]^3,x]
```

output
```
((8*I)*E^((3*I)*a)*x^(1 + m)*(c*x^(Sqrt[-(1 + m)^2]/2))^(6*I)*Hypergeometr
ic2F1[3, (3 - (I*(1 + m))/Sqrt[-(1 + m)^2])/2, (5 - (I*(1 + m))/Sqrt[-(1 +
 m)^2])/2, E^((2*I)*a)*(c*x^(Sqrt[-(1 + m)^2]/2))^(4*I)])/(Sqrt[-(1 + m)^2
]*(3*I + (1 + m)/Sqrt[-(1 + m)^2]))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5017
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[(-2*I)^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^
(2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5021
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [A] (verified)

Time = 170.40 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

method	result
parallelrisch	$-\frac{x^{1+m}\left(\left(\tan\left(\frac{a}{2}+\ln\left(c\,x^{\frac{\sqrt{-(1+m)^{2}}}{2}}\right)\right)-\cot\left(\frac{a}{2}+\ln\left(c\,x^{\frac{\sqrt{-(1+m)^{2}}}{2}}\right)\right)\right)\sqrt{-(1+m)^{2}}-2m-2\right)\left(\tan\left(\frac{a}{2}+\ln\left(c\,x^{\sqrt{-(}}\right.\right.\right.}{8(1+m)^{2}}$

input
```
int(x^m*csc(a+2*ln(c*x^(1/2*(-(1+m)^2)^(1/2))))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*x^(1+m)*((tan(1/2*a+ln(c*x^(1/2*(-(1+m)^2)^(1/2))))-cot(1/2*a+ln(c*x^
(1/2*(-(1+m)^2)^(1/2)))))*(-(1+m)^2)^(1/2)-2*m-2)*(tan(1/2*a+ln(c*x^(1/2*(
-(1+m)^2)^(1/2))))+cot(1/2*a+ln(c*x^(1/2*(-(1+m)^2)^(1/2)))))/(1+m)^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75

$$\int x^m \csc^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\,dx$$
$$= -\frac{2\left(2i\,x^2x^{2\,m}e^{(3i\,a+6i\,\log(c))}-i\,e^{(5i\,a+10i\,\log(c))}\right)}{(m+1)x^4x^{4\,m}-2\,(m+1)x^2x^{2\,m}e^{(2i\,a+4i\,\log(c))}+(m+1)e^{(4i\,a+8i\,\log(c))}}$$

input
```
integrate(x^m*csc(a+2*log(c*x^(1/2*(-(1+m)^2)^(1/2))))^3,x, algorithm="fri
cas")
```

output
```
-2*(2*I*x^2*x^(2*m)*e^(3*I*a + 6*I*log(c)) - I*e^(5*I*a + 10*I*log(c)))/((
m + 1)*x^4*x^(4*m) - 2*(m + 1)*x^2*x^(2*m)*e^(2*I*a + 4*I*log(c)) + (m + 1
)*e^(4*I*a + 8*I*log(c)))
```

Sympy [F(-1)]

Timed out.

$$\int x^m \csc^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x**m*csc(a+2*ln(c*x**(1/2*(-(1+m)**2)**(1/2))))**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(92) = 184$.

Time = 0.13 (sec) , antiderivative size = 974, normalized size of antiderivative = 8.85

$$\int x^m \csc^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x^m*csc(a+2*log(c*x^(1/2*(-(1+m)^2)^(1/2))))^3,x, algorithm="max
ima")
```

output
```
2*((cos(2*log(c))*sin(a) + cos(a)*sin(2*log(c)))*x*e^(m*log(x) + 14*arctan
2(sin(1/2*m*log(x)), cos(1/2*m*log(x))) + 14*arctan2(sin(1/2*log(x)), cos(
1/2*log(x)))) + 2*(((cos(a)*sin(2*a) - cos(2*a)*sin(a))*cos(2*log(c)) - (c
os(2*a)*cos(a) + sin(2*a)*sin(a))*sin(2*log(c)))*cos(4*log(c)) + ((cos(2*a
)*cos(a) + sin(2*a)*sin(a))*cos(2*log(c)) + (cos(a)*sin(2*a) - cos(2*a)*si
n(a))*sin(2*log(c)))*sin(4*log(c)))*x*e^(m*log(x) + 10*arctan2(sin(1/2*m*l
og(x)), cos(1/2*m*log(x))) + 10*arctan2(sin(1/2*log(x)), cos(1/2*log(x))))
 - (((cos(a)*sin(4*a) - cos(4*a)*sin(a))*cos(2*log(c)) - (cos(4*a)*cos(a)
+ sin(4*a)*sin(a))*sin(2*log(c)))*cos(8*log(c)) + ((cos(4*a)*cos(a) + sin(
4*a)*sin(a))*cos(2*log(c)) + (cos(a)*sin(4*a) - cos(4*a)*sin(a))*sin(2*log
(c)))*sin(8*log(c)))*x*e^(m*log(x) + 6*arctan2(sin(1/2*m*log(x)), cos(1/2*
m*log(x))) + 6*arctan2(sin(1/2*log(x)), cos(1/2*log(x)))))/((cos(4*a)^2 +
sin(4*a)^2)*cos(8*log(c))^2 + (cos(4*a)^2 + sin(4*a)^2)*sin(8*log(c))^2 +
((cos(4*a)^2 + sin(4*a)^2)*cos(8*log(c))^2 + (cos(4*a)^2 + sin(4*a)^2)*sin
(8*log(c))^2)*m + (m + 1)*e^(16*arctan2(sin(1/2*m*log(x)), cos(1/2*m*log(x
))) + 16*arctan2(sin(1/2*log(x)), cos(1/2*log(x)))) - 4*((cos(2*a)*cos(4*l
og(c)) - sin(2*a)*sin(4*log(c)))*m + cos(2*a)*cos(4*log(c)) - sin(2*a)*sin
(4*log(c)))*e^(12*arctan2(sin(1/2*m*log(x)), cos(1/2*m*log(x))) + 12*arcta
n2(sin(1/2*log(x)), cos(1/2*log(x)))) + 2*(2*(cos(2*a)^2 + sin(2*a)^2)*cos
(4*log(c))^2 + 2*(cos(2*a)^2 + sin(2*a)^2)*sin(4*log(c))^2 + (2*(cos(2*...
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.19 (sec) , antiderivative size = 839, normalized size of antiderivative = 7.63

$$\int x^m \csc^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^m*csc(a+2*log(c*x^(1/2*(-(1+m)^2)^(1/2))))^3,x, algorithm="gia
c")
```

output
```
I*c^(6*I)*m*x*x^m*x^abs(m + 1)*e^(3*I*a)/(c^(8*I)*m^2*e^(4*I*a) + 2*c^(8*I
)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) - 2*c^(4*I)*m^2*x^(2*abs(m + 1))*e^(2*I*
a) - 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) - 2*c^(4*I)*x^(2*abs(m + 1))*e
^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*abs(m + 1)))
 - I*c^(6*I)*x*x^m*x^abs(m + 1)*abs(m + 1)*e^(3*I*a)/(c^(8*I)*m^2*e^(4*I*a
) + 2*c^(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) - 2*c^(4*I)*m^2*x^(2*abs(m +
 1))*e^(2*I*a) - 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) - 2*c^(4*I)*x^(2*a
bs(m + 1))*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4*
abs(m + 1))) + I*c^(6*I)*x*x^m*x^abs(m + 1)*e^(3*I*a)/(c^(8*I)*m^2*e^(4*I*
a) + 2*c^(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) - 2*c^(4*I)*m^2*x^(2*abs(m
+ 1))*e^(2*I*a) - 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) - 2*c^(4*I)*x^(2*
abs(m + 1))*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) + x^(4
*abs(m + 1))) - I*c^(2*I)*m*x*x^m*x^(3*abs(m + 1))*e^(I*a)/(c^(8*I)*m^2*e^
(4*I*a) + 2*c^(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) - 2*c^(4*I)*m^2*x^(2*a
bs(m + 1))*e^(2*I*a) - 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a) - 2*c^(4*I)*
x^(2*abs(m + 1))*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x^(4*abs(m + 1)) +
 x^(4*abs(m + 1))) - I*c^(2*I)*x*x^m*x^(3*abs(m + 1))*abs(m + 1)*e^(I*a)/(
c^(8*I)*m^2*e^(4*I*a) + 2*c^(8*I)*m*e^(4*I*a) + c^(8*I)*e^(4*I*a) - 2*c^(4
*I)*m^2*x^(2*abs(m + 1))*e^(2*I*a) - 4*c^(4*I)*m*x^(2*abs(m + 1))*e^(2*I*a
) - 2*c^(4*I)*x^(2*abs(m + 1))*e^(2*I*a) + m^2*x^(4*abs(m + 1)) + 2*m*x...
```

Mupad [B] (verification not implemented)

Time = 26.31 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.55

$$\int x^m \csc^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\,dx = \frac{\frac{x^{m+1}\,\mathrm{e}^{a\,1\mathrm{i}}\,\left(c\,x^{\frac{\sqrt{-m^2-2\,m-1}}{2}}\right)^{6\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}}+\mathrm{e}^{a\,2\mathrm{i}}\,\sqrt{-{(m+1)}^2}\,1\mathrm{i}+m\,\mathrm{e}^{a\,2\mathrm{i}}\right)}{\sqrt{-{(m+1)}^2}}+\frac{x^{m+1}\,\mathrm{e}^{a\,1\mathrm{i}}\,\left(c\,x^{\frac{\sqrt{-m^2-2\,m-1}}{2}}\right)^{2\mathrm{i}}\,\left(m+1-\sqrt{-{(m+1)}^2}\,1\mathrm{i}\right)}{\sqrt{-{(m+1)}^2}}}{(m+1)\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\left(c\,x^{\frac{\sqrt{-m^2-2\,m-1}}{2}}\right)^{4\mathrm{i}}-1\right)^2}$$

input
```
int(x^m/sin(a + 2*log(c*x^((-(m + 1)^2)^(1/2)/2)))^3,x)
```

output
```
((x^(m + 1)*exp(a*1i)*(c*x^((- 2*m - m^2 - 1)^(1/2)/2))^6i*(exp(a*2i) + ex
p(a*2i)*(-(m + 1)^2)^(1/2)*1i + m*exp(a*2i)))/(-(m + 1)^2)^(1/2) + (x^(m +
 1)*exp(a*1i)*(c*x^((- 2*m - m^2 - 1)^(1/2)/2))^2i*(m - (-(m + 1)^2)^(1/2)
*1i + 1))/(-(m + 1)^2)^(1/2))/((m + 1)*(exp(a*2i)*(c*x^((- 2*m - m^2 - 1)^
(1/2)/2))^4i - 1)^2)
```

Reduce [F]

$$\int x^m \csc^3\left(a+2\log\left(cx^{\frac{1}{2}\sqrt{-(1+m)^2}}\right)\right)\,dx=\int x^m\csc\left(2\log\left(x^{\frac{m}{2}+\frac{1}{2}}c\right)+a\right)^3dx$$

input
```
int(x^m*csc(a+2*log(c*x^(1/2*(-(1+m)^2)^(1/2))))^3,x)
```

output
```
int(x**m*csc(2*log(x**((m + 1)/2)*c) + a)**3,x)
```

3.303 $\int x \csc^3\left(a+2\log\left(cx^i\right)\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int x\csc^3\left(a+2\log\left(cx^i\right)\right)\,dx=-\frac{ie^{ia}\left(cx^i\right)^{2i}x^2}{\left(1-e^{2ia}\left(cx^i\right)^{4i}\right)^2}$$

output
```
-I*exp(I*a)*(c*x^I)^(2*I)*x^2/(1-exp(2*I*a)*(c*x^I)^(4*I))^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 127 vs. 2(49) = 98.

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.59

$$\int x\csc^3\left(a+2\log\left(cx^i\right)\right)\,dx$$
$$=\frac{\csc^2\left(a+2\log\left(cx^i\right)\right)\left(i(-1+2x^4)\cos\left(a+2\log\left(cx^i\right)-2i\log(x)\right)+(1+2x^4)\sin\left(a+2\log\left(cx^i\right)-2i\log\right.\right.}{4x^4}$$

input
```
Integrate[x*Csc[a + 2*Log[c*x^I]]^3,x]
```

output
```
(Csc[a + 2*Log[c*x^I]]^2*(I*(-1 + 2*x^4)*Cos[a + 2*Log[c*x^I] - (2*I)*Log[
x]] + (1 + 2*x^4)*Sin[a + 2*Log[c*x^I] - (2*I)*Log[x]])*(Cos[2*(a + 2*Log[
c*x^I] - (2*I)*Log[x])] + I*Sin[2*(a + 2*Log[c*x^I] - (2*I)*Log[x])]))/(4*
x^4)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5021, 5017, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \csc^3\left(a+2\log\left(cx^i\right)\right)\,dx$$

$$\Big\downarrow \; 5021$$

$$-ix^2\left(cx^i\right)^{2i}\int \left(cx^i\right)^{-1-2i}\csc^3\left(a+2\log\left(cx^i\right)\right)d\left(cx^i\right)$$

$$\Big\downarrow \; 5017$$

$$8e^{3ia}x^2\left(cx^i\right)^{2i}\int \frac{\left(cx^i\right)^{-1+4i}}{\left(1-e^{2ia}\left(cx^i\right)^{4i}\right)^3}d\left(cx^i\right)$$

$$\Big\downarrow \; 793$$

$$-\frac{ie^{ia}x^2\left(cx^i\right)^{2i}}{\left(1-e^{2ia}\left(cx^i\right)^{4i}\right)^2}$$

input
```
Int[x*Csc[a + 2*Log[c*x^I]]^3,x]
```

output
```
((-I)*E^(I*a)*(c*x^I)^(2*I)*x^2)/(1 - E^((2*I)*a)*(c*x^I)^(4*I))^2
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 5017
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[(-2*I)^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^
(2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5021
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 211, normalized size of antiderivative = 4.31

$$-\frac{i x^{2} c^{2i} \left(x^{i}\right)^{2i} \mathrm{e}^{-\operatorname{csgn}\left(i x^{i}\right) \pi \operatorname{csgn}\left(i c\, x^{i}\right)^{2}+\operatorname{csgn}\left(i x^{i}\right) \pi \operatorname{csgn}\left(i c\, x^{i}\right) \operatorname{csgn}\left(i c\right)+\pi \operatorname{csgn}\left(i c\, x^{i}\right)^{3}-\pi \operatorname{csgn}\left(i c\, x^{i}\right)^{2} \operatorname{csgn}\left(i c\right)+i a}}{\left(\left(x^{i}\right)^{4i} c^{4i} \mathrm{e}^{-2 \operatorname{csgn}\left(i x^{i}\right) \pi \operatorname{csgn}\left(i c\, x^{i}\right)^{2}} \mathrm{e}^{2 \operatorname{csgn}\left(i x^{i}\right) \pi \operatorname{csgn}\left(i c\, x^{i}\right) \operatorname{csgn}\left(i c\right)} \mathrm{e}^{2 \pi \operatorname{csgn}\left(i c\, x^{i}\right)^{3}} \mathrm{e}^{-2 \pi \operatorname{csgn}\left(i c\, x^{i}\right)^{2} \operatorname{csgn}\left(i c\right)} \mathrm{e}^{2 i a}-1\right)^{2}}$$

input
```
int(x*csc(a+2*ln(c*x^I))^3,x)
```

output
```
-I*x^2*c^(2*I)*(x^I)^(2*I)*exp(-csgn(I*x^I)*Pi*csgn(I*c*x^I)^2+csgn(I*x^I)
*Pi*csgn(I*c*x^I)*csgn(I*c)+Pi*csgn(I*c*x^I)^3-Pi*csgn(I*c*x^I)^2*csgn(I*c
)+I*a)/(((x^I)^(2*I))^2*(c^(2*I))^2*exp(-2*csgn(I*x^I)*Pi*csgn(I*c*x^I)^2)
*exp(2*csgn(I*x^I)*Pi*csgn(I*c*x^I)*csgn(I*c))*exp(2*Pi*csgn(I*c*x^I)^3)*e
xp(-2*Pi*csgn(I*c*x^I)^2*csgn(I*c))*exp(2*I*a)-1)^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int x\csc^3\left(a+2\log\left(cx^i\right)\right)\,dx=\frac{-2i\,x^4e^{(3i\,a+6i\,\log(c))}+i\,e^{(5i\,a+10i\,\log(c))}}{x^8-2\,x^4e^{(2i\,a+4i\,\log(c))}+e^{(4i\,a+8i\,\log(c))}}$$

input
```
integrate(x*csc(a+2*log(c*x^I))^3,x, algorithm="fricas")
```

output
```
(-2*I*x^4*e^(3*I*a + 6*I*log(c)) + I*e^(5*I*a + 10*I*log(c)))/(x^8 - 2*x^4
*e^(2*I*a + 4*I*log(c)) + e^(4*I*a + 8*I*log(c)))
```

Sympy [F]

$$\int x\csc^3\left(a+2\log\left(cx^i\right)\right)\,dx=\int x\csc^3\left(a+2\log\left(cx^i\right)\right)dx$$

input
```
integrate(x*csc(a+2*ln(c*x**I))**3,x)
```

output
```
Integral(x*csc(a + 2*log(c*x**I))**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(32) = 64.

Time = 0.05 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.84

$$\int x\csc^3\left(a+2\log\left(cx^i\right)\right)\,dx$$
$$=\frac{((-i\,\cos(a)+\sin(a))\cos(2\,\log(c))+(\cos(a)}{(\cos(4\,a)+i\,\sin(4\,a))\cos(8\,\log(c))-2\,((\cos(2\,a)+i\,\sin(2\,a))\cos(4\,\log(c))+(i\,\cos(2\,a)-\sin(2\,a)}$$

input
```
integrate(x*csc(a+2*log(c*x^I))^3,x, algorithm="maxima")
```

output
```
((-I*cos(a) + sin(a))*cos(2*log(c)) + (cos(a) + I*sin(a))*sin(2*log(c)))*x
^2*e^(6*arctan2(sin(log(x)), cos(log(x))))/((cos(4*a) + I*sin(4*a))*cos(8*
log(c)) - 2*((cos(2*a) + I*sin(2*a))*cos(4*log(c)) + (I*cos(2*a) - sin(2*a
))*sin(4*log(c)))*e^(4*arctan2(sin(log(x)), cos(log(x)))) + (I*cos(4*a) -
sin(4*a))*sin(8*log(c)) + e^(8*arctan2(sin(log(x)), cos(log(x)))))
```

Giac [F]

$$\int x \csc^3 \left(a + 2 \log \left(cx^i\right)\right) \, dx = \int x \csc \left(a + 2 \, \log \left(cx^i\right)\right)^3 \, dx$$

input
```
integrate(x*csc(a+2*log(c*x^I))^3,x, algorithm="giac")
```

output
```
integrate(x*csc(a + 2*log(c*x^I))^3, x)
```

Mupad [B] (verification not implemented)

Time = 22.74 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int x \csc^3 \left(a + 2 \log \left(cx^i\right)\right) \, dx = -\frac{x^2 \, e^{a \, 1i} \left(c \, x^{1i}\right)^{2i} \, 1i}{1 + e^{a \, 4i} \left(c \, x^{1i}\right)^{8i} - 2 \, e^{a \, 2i} \left(c \, x^{1i}\right)^{4i}}$$

input
```
int(x/sin(a + 2*log(c*x^1i))^3,x)
```

output
```
-(x^2*exp(a*1i)*(c*x^1i)^2i*1i)/(exp(a*4i)*(c*x^1i)^8i - 2*exp(a*2i)*(c*x^
1i)^4i + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int x \csc^3\left(a+2\log\left(cx^i\right)\right)\,dx = \frac{x^2(\cos\left(2\log(x^ic)+a\right)i+\sin\left(2\log(x^ic)+a\right))}{4\sin\left(2\log\left(x^ic\right)+a\right)^2}$$

input
```
int(x*csc(a+2*log(c*x^I))^3,x)
```

output
```
(x**2*(cos(2*log(x**i*c) + a)*i + sin(2*log(x**i*c) + a)))/(4*sin(2*log(x*
*i*c) + a)**2)
```

3.304 $\int \csc^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \csc^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx=\frac{1}{2}x\csc\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)+\frac{1}{2}ix\cot\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\csc\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)$$

output
```
1/2*x*csc(a+2*ln(c*x^(1/2*I)))+1/2*I*x*cot(a+2*ln(c*x^(1/2*I)))*csc(a+2*ln
(c*x^(1/2*I)))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 137 vs. 2(58) = 116.

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.36

$$\int \csc^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx=\frac{\csc^2\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\left(i(-1+2x^2)\cos\left(a+2\log\left(cx^{\frac{i}{2}}\right)-i\log(x)\right)+(1+2x^2)\sin\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right.\right.}{2x^2}$$

input
```
Integrate[Csc[a + 2*Log[c*x^(I/2)]]^3,x]
```

output
```
(Csc[a + 2*Log[c*x^(I/2)]]^2*(I*(-1 + 2*x^2)*Cos[a + 2*Log[c*x^(I/2)] - I*
Log[x]] + (1 + 2*x^2)*Sin[a + 2*Log[c*x^(I/2)] - I*Log[x]])*(Cos[2*(a + 2*
Log[c*x^(I/2)] - I*Log[x])] + I*Sin[2*(a + 2*Log[c*x^(I/2)] - I*Log[x])]))
/(2*x^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5015, 5017, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)dx$$

$$\Big\downarrow \text{5015}$$

$$-2ix\left(cx^{\frac{i}{2}}\right)^{2i}\int \left(cx^{\frac{i}{2}}\right)^{-1-2i}\csc^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)d\left(cx^{\frac{i}{2}}\right)$$

$$\Big\downarrow \text{5017}$$

$$16e^{3ia}x\left(cx^{\frac{i}{2}}\right)^{2i}\int \frac{\left(cx^{\frac{i}{2}}\right)^{-1+4i}}{\left(1-e^{2ia}\left(cx^{\frac{i}{2}}\right)^{4i}\right)^3}d\left(cx^{\frac{i}{2}}\right)$$

$$\Big\downarrow \text{793}$$

$$-\frac{2ie^{ia}x\left(cx^{\frac{i}{2}}\right)^{2i}}{\left(1-e^{2ia}\left(cx^{\frac{i}{2}}\right)^{4i}\right)^2}$$

input
```
Int[Csc[a + 2*Log[c*x^(I/2)]]^3,x]
```

output
```
((-2*I)*E^(I*a)*(c*x^(I/2))^(2*I)*x)/(1 - E^((2*I)*a)*(c*x^(I/2))^(4*I))^2
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 5015
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5017
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[(-2*I)^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^
(2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 92.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

method	result
parallelrisch	$\frac{x\left(i\cot\left(\frac{a}{2}+\ln\left(c\,x^{\frac{i}{2}}\right)\right)^{2}-i\tan\left(\frac{a}{2}+\ln\left(c\,x^{\frac{i}{2}}\right)\right)^{2}+2\cot\left(\frac{a}{2}+\ln\left(c\,x^{\frac{i}{2}}\right)\right)+2\tan\left(\frac{a}{2}+\ln\left(c\,x^{\frac{i}{2}}\right)\right)\right)}{8}$
risch	$-\frac{2ix\left(x^{\frac{i}{2}}\right)^{2i}c^{2i}\mathrm{e}^{-\operatorname{csgn}\left(ix^{\frac{i}{2}}\right)\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)^{2}\pi+\operatorname{csgn}\left(ix^{\frac{i}{2}}\right)\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)\operatorname{csgn}(ic)\pi+\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)^{3}\pi-\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)^{2}\operatorname{csgn}(ic)\pi}}{\left(c^{4i}\left(x^{\frac{i}{2}}\right)^{4i}\mathrm{e}^{-2\operatorname{csgn}\left(ix^{\frac{i}{2}}\right)\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)^{2}\pi}\mathrm{e}^{2\operatorname{csgn}\left(ix^{\frac{i}{2}}\right)\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)\operatorname{csgn}(ic)\pi}\mathrm{e}^{2\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)^{3}\pi}\mathrm{e}^{-2\operatorname{csgn}\left(ic\,x^{\frac{i}{2}}\right)^{2}\operatorname{csgn}(ic)\pi}\mathrm{e}}$

input
```
int(csc(a+2*ln(c*x^(1/2*I)))^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(I*cot(1/2*a+ln(c*x^(1/2*I)))^2-I*tan(1/2*a+ln(c*x^(1/2*I)))^2+2*cot
(1/2*a+ln(c*x^(1/2*I)))+2*tan(1/2*a+ln(c*x^(1/2*I))))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \csc^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx = -\frac{2\left(2i\,x^2e^{(3i\,a+6i\,\log(c))}-i\,e^{(5i\,a+10i\,\log(c))}\right)}{x^4-2\,x^2e^{(2i\,a+4i\,\log(c))}+e^{(4i\,a+8i\,\log(c))}}$$

input
```
integrate(csc(a+2*log(c*x^(1/2*I)))^3,x, algorithm="fricas")
```

output
```
-2*(2*I*x^2*e^(3*I*a + 6*I*log(c)) - I*e^(5*I*a + 10*I*log(c)))/(x^4 - 2*x
^2*e^(2*I*a + 4*I*log(c)) + e^(4*I*a + 8*I*log(c)))
```

Sympy [F]

$$\int \csc^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx = \int \csc^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx$$

input
```
integrate(csc(a+2*ln(c*x**(1/2*I)))**3,x)
```

output
```
Integral(csc(a + 2*log(c*x**(I/2)))**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(40) = 80.

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.64

$$\int \csc^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx =$$

$$-\frac{2\left(\left(i\,\cos\left(a\right)-\sin\left(a\right)\right)\cos\left(2\,\log\left(c\right)\right)-(\mathrm{co}}{\left(\cos\left(4\,a\right)+i\,\sin\left(4\,a\right)\right)\cos\left(8\,\log\left(c\right)\right)-2\left(\left(\cos\left(2\,a\right)+i\,\sin\left(2\,a\right)\right)\cos\left(4\,\log\left(c\right)\right)+\left(i\,\cos\left(2\,a\right)-\sin\left(2\right.\right.\right.}$$

input
```
integrate(csc(a+2*log(c*x^(1/2*I)))^3,x, algorithm="maxima")
```

output
```
-2*((I*cos(a) - sin(a))*cos(2*log(c)) - (cos(a) + I*sin(a))*sin(2*log(c)))
*x*e^(6*arctan2(sin(1/2*log(x)), cos(1/2*log(x))))/((cos(4*a) + I*sin(4*a)
)*cos(8*log(c)) - 2*((cos(2*a) + I*sin(2*a))*cos(4*log(c)) + (I*cos(2*a) -
 sin(2*a))*sin(4*log(c)))*e^(4*arctan2(sin(1/2*log(x)), cos(1/2*log(x)))
+ (I*cos(4*a) - sin(4*a))*sin(8*log(c)) + e^(8*arctan2(sin(1/2*log(x)), co
s(1/2*log(x)))))
```

Giac [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.28

$$\int \csc^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx=\frac{2i\,c^{10i}e^{(5i\,a)}}{c^{8i}e^{(4i\,a)}-2\,c^{4i}x^2e^{(2i\,a)}+x^4}-\frac{4i\,c^{6i}x^2e^{(3i\,a)}}{c^{8i}e^{(4i\,a)}-2\,c^{4i}x^2e^{(2i\,a)}+x^4}$$

input
```
integrate(csc(a+2*log(c*x^(1/2*I)))^3,x, algorithm="giac")
```

output
```
2*I*c^(10*I)*e^(5*I*a)/(c^(8*I)*e^(4*I*a) - 2*c^(4*I)*x^2*e^(2*I*a) + x^4)
 - 4*I*c^(6*I)*x^2*e^(3*I*a)/(c^(8*I)*e^(4*I*a) - 2*c^(4*I)*x^2*e^(2*I*a)
+ x^4)
```

Mupad [B] (verification not implemented)

Time = 22.72 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \csc^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx=-\frac{x\,\mathrm{e}^{a\,1\mathrm{i}}\,{\left(c\,x^{\frac{1}{2}\mathrm{i}}\right)}^{2\mathrm{i}}\,2\mathrm{i}}{1+\mathrm{e}^{a\,4\mathrm{i}}\,{\left(c\,x^{\frac{1}{2}\mathrm{i}}\right)}^{8\mathrm{i}}-2\,\mathrm{e}^{a\,2\mathrm{i}}\,{\left(c\,x^{\frac{1}{2}\mathrm{i}}\right)}^{4\mathrm{i}}}$$

input
```
int(1/sin(a + 2*log(c*x^(1i/2)))^3,x)
```

output
```
-(x*exp(a*1i)*(c*x^(1i/2))^2i*2i)/(exp(a*4i)*(c*x^(1i/2))^8i - 2*exp(a*2i)
*(c*x^(1i/2))^4i + 1)
```

Reduce [F]

$$\int \csc^3\left(a+2\log\left(cx^{\frac{i}{2}}\right)\right)\,dx=\int \csc\left(2\log\left(x^{\frac{i}{2}}c\right)+a\right)^3dx$$

input
```
int(csc(a+2*log(c*x^(1/2*I)))^3,x)
```

output
```
int(csc(2*log(x**(i/2)*c) + a)**3,x)
```

3.305 $\int \csc^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 51

$$\int \csc^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx=\frac{2ie^{3ia}\left(cx^{-\frac{i}{2}}\right)^{6i}x}{\left(1-e^{2ia}\left(cx^{-\frac{i}{2}}\right)^{4i}\right)^2}$$

output
```
2*I*exp(3*I*a)*(c/(x^(1/2*I)))^(6*I)*x/(1-exp(2*I*a)*(c/(x^(1/2*I)))^(4*I)
)^2
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 137 vs. 2(51) = 102.

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.69

$$\int \csc^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx=$$

$$-\frac{\csc^2\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\left(\left(-1+2x^2\right)\cos\left(a+2\log\left(cx^{-\frac{i}{2}}\right)+i\log(x)\right)+i(1+2x^2)\sin\left(a+2\log\left(c\right.\right.\right.}{2}$$

input
```
Integrate[Csc[a + 2*Log[c/x^(I/2)]]^3,x]
```

output
```
-1/2*(Csc[a + 2*Log[c/x^(I/2)]]^2*((-1 + 2*x^2)*Cos[a + 2*Log[c/x^(I/2)] +
 I*Log[x]] + I*(1 + 2*x^2)*Sin[a + 2*Log[c/x^(I/2)] + I*Log[x]])*(I*Cos[2*
(a + 2*Log[c/x^(I/2)] + I*Log[x])] + Sin[2*(a + 2*Log[c/x^(I/2)] + I*Log[x
])]))/x^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5015, 5017, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)dx$$

$$\Big\downarrow \text{5015}$$

$$2ix\left(cx^{-\frac{i}{2}}\right)^{-2i}\int \left(cx^{-\frac{i}{2}}\right)^{-1+2i}\csc^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)d\left(cx^{-\frac{i}{2}}\right)$$

$$\Big\downarrow \text{5017}$$

$$-16e^{3ia}x\left(cx^{-\frac{i}{2}}\right)^{-2i}\int \frac{\left(cx^{-\frac{i}{2}}\right)^{-1+8i}}{\left(1-e^{2ia}\left(cx^{-\frac{i}{2}}\right)^{4i}\right)^3}d\left(cx^{-\frac{i}{2}}\right)$$

$$\Big\downarrow \text{796}$$

$$\frac{2ie^{3ia}x\left(cx^{-\frac{i}{2}}\right)^{6i}}{\left(1-e^{2ia}\left(cx^{-\frac{i}{2}}\right)^{4i}\right)^2}$$

input
```
Int[Csc[a + 2*Log[c/x^(I/2)]]^3,x]
```

output
```
((2*I)*E^((3*I)*a)*(c/x^(I/2))^(6*I)*x)/(1 - E^((2*I)*a)*(c/x^(I/2))^(4*I)
)^2
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 5015
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5017
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[(-2*I)^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^
(2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Maple [A] (verified)

Time = 109.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.31

method	result
parallelrisch	$\frac{x\left(i\tan\left(\frac{a}{2}+\ln\left(c\,x^{-\frac{i}{2}}\right)\right)^{2}-i\cot\left(\frac{a}{2}+\ln\left(c\,x^{-\frac{i}{2}}\right)\right)^{2}+2\tan\left(\frac{a}{2}+\ln\left(c\,x^{-\frac{i}{2}}\right)\right)+2\cot\left(\frac{a}{2}+\ln\left(c\,x^{-\frac{i}{2}}\right)\right)\right)}{8}$
risch	$\frac{2ix\,c^{6i}\left(x^{\frac{i}{2}}\right)^{-6i}\mathrm{e}^{3\,\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^{3}\pi-3\,\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^{2}\mathrm{csgn}(ic)\pi-3\,\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^{2}\mathrm{csgn}\left(ix^{-\frac{i}{2}}\right)\pi+3\,\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)\mathrm{csgn}(ic)\,\mathrm{csgn}}}{\left(c^{4i}\left(x^{\frac{i}{2}}\right)^{-4i}\mathrm{e}^{2\,\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^{3}\pi}\mathrm{e}^{-2\,\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^{2}\mathrm{csgn}(ic)\pi}\mathrm{e}^{-2\,\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)^{2}\mathrm{csgn}\left(ix^{-\frac{i}{2}}\right)\pi}\mathrm{e}^{2\,\mathrm{csgn}\left(ic\,x^{-\frac{i}{2}}\right)\mathrm{csgn}(ic)\,\mathrm{csgn}\left(i\right.}\right.}$

input
```
int(csc(a+2*ln(c/(x^(1/2*I))))^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(I*tan(1/2*a+ln(c*x^(-1/2*I)))^2-I*cot(1/2*a+ln(c*x^(-1/2*I)))^2+2*t
an(1/2*a+ln(c*x^(-1/2*I)))+2*cot(1/2*a+ln(c*x^(-1/2*I))))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(27) = 54.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12

$$\int \csc^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx = -\frac{2\left(-2i\,x^2e^{(2i\,a+4i\,\log(c))}+i\right)}{x^4e^{(5i\,a+10i\,\log(c))}-2\,x^2e^{(3i\,a+6i\,\log(c))}+e^{(i\,a+2i\,\log(c))}}$$

input
```
integrate(csc(a+2*log(c/(x^(1/2*I))))^3,x, algorithm="fricas")
```

output
```
-2*(-2*I*x^2*e^(2*I*a + 4*I*log(c)) + I)/(x^4*e^(5*I*a + 10*I*log(c)) - 2*
x^2*e^(3*I*a + 6*I*log(c)) + e^(I*a + 2*I*log(c)))
```

Sympy [F]

$$\int \csc^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx = \int \csc^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx$$

input
```
integrate(csc(a+2*ln(c/(x**(1/2*I))))**3,x)
```

output
```
Integral(csc(a + 2*log(c/x**(I/2)))**3, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(27) = 54.

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.18

$$\int \csc^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx$$

$$= \frac{2\left(\left(i\,\cos\left(3\,a\right)-\sin\left(3\,a\right)\right)\cos\left(6\,\log\left(c\right)\right)-(\mathrm{c}\right.}{\left(\left(\cos\left(4\,a\right)+i\,\sin\left(4\,a\right)\right)\cos\left(8\,\log\left(c\right)\right)-\left(-i\,\cos\left(4\,a\right)+\sin\left(4\,a\right)\right)\sin\left(8\,\log\left(c\right)\right)\right)e^{\left(8\,\arctan\left(\sin\left(\frac{1}{2}\,\log(x)\right),\cos\left(\frac{1}{2}\right.\right.\right.}}$$

input
```
integrate(csc(a+2*log(c/(x^(1/2*I))))^3,x, algorithm="maxima")
```

output
```
2*((I*cos(3*a) - sin(3*a))*cos(6*log(c)) - (cos(3*a) + I*sin(3*a))*sin(6*l
og(c)))*x*e^(6*arctan2(sin(1/2*log(x)), cos(1/2*log(x))))/(((cos(4*a) + I*
sin(4*a))*cos(8*log(c)) - (-I*cos(4*a) + sin(4*a))*sin(8*log(c)))*e^(8*arc
tan2(sin(1/2*log(x)), cos(1/2*log(x)))) - 2*((cos(2*a) + I*sin(2*a))*cos(4
*log(c)) - (-I*cos(2*a) + sin(2*a))*sin(4*log(c)))*e^(4*arctan2(sin(1/2*lo
g(x)), cos(1/2*log(x)))) + 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(27) = 54$.

Time = 1.00 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \csc^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx = \frac{4i\,c^{4i}x^2e^{(2i\,a)}}{c^{10i}x^4e^{(5i\,a)}-2\,c^{6i}x^2e^{(3i\,a)}+c^{2i}e^{(i\,a)}} - \frac{2i}{c^{10i}x^4e^{(5i\,a)}-2\,c^{6i}x^2e^{(3i\,a)}+c^{2i}e^{(i\,a)}}$$

input
```
integrate(csc(a+2*log(c/(x^(1/2*I))))^3,x, algorithm="giac")
```

output
```
4*I*c^(4*I)*x^2*e^(2*I*a)/(c^(10*I)*x^4*e^(5*I*a) - 2*c^(6*I)*x^2*e^(3*I*a
) + c^(2*I)*e^(I*a)) - 2*I/(c^(10*I)*x^4*e^(5*I*a) - 2*c^(6*I)*x^2*e^(3*I*
a) + c^(2*I)*e^(I*a))
```

Mupad [B] (verification not implemented)

Time = 24.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75

$$\int \csc^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx = \frac{x\,\mathrm{e}^{a\,3\mathrm{i}}\,{\left(\frac{c}{x^{\frac{1}{2}\mathrm{i}}}\right)}^{6\mathrm{i}}\,2\mathrm{i}}{{\left(\mathrm{e}^{a\,2\mathrm{i}}\,{\left(\frac{c}{x^{\frac{1}{2}\mathrm{i}}}\right)}^{4\mathrm{i}}-1\right)}^2}$$

input
```
int(1/sin(a + 2*log(c/x^(1i/2)))^3,x)
```

output
```
(x*exp(a*3i)*(c/x^(1i/2))^6i*2i)/(exp(a*2i)*(c/x^(1i/2))^4i - 1)^2
```

Reduce [F]

$$\int \csc^3\left(a+2\log\left(cx^{-\frac{i}{2}}\right)\right)\,dx=\int \csc\left(2\log\left(\frac{c}{x^{\frac{i}{2}}}\right)+a\right)^3dx$$

input
```
int(csc(a+2*log(c/(x^(1/2*I))))^3,x)
```

output
```
int(csc(2*log(c/x**(i/2)) + a)**3,x)
```

3.306 $\int \csc^p\left(a+\frac{i\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \csc^p\left(a+\frac{i\log(cx^n)}{n(-2+p)}\right)\,dx = -\frac{e^{-2ia}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}}\left(1-e^{2ia}(cx^n)^{\frac{2}{n(2-p)}}\right)\csc^p\left(a-\frac{i\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

output
```
-1/2*(2-p)*x*(1-exp(2*I*a)*(c*x^n)^(2/n/(2-p)))*csc(a-I*ln(c*x^n)/n/(2-p))
^p/exp(2*I*a)/(1-p)/((c*x^n)^(2/n/(2-p)))
```

Mathematica [A] (warning: unable to verify)

Time = 1.23 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.61

$$\int \csc^p\left(a+\frac{i\log(cx^n)}{n(-2+p)}\right)\,dx = \frac{2^{-1+p}e^{-\frac{2iap}{-2+p}}(-2+p)x\left(e^{\frac{2iap}{-2+p}}-e^{\frac{4ia}{-2+p}}(cx^n)^{\frac{2}{n(-2+p)}}\right)\left(-\frac{ie^{\frac{ia(2+p)}{-2+p}}(cx^n)^{\frac{1}{n(-2+p)}}}{-e^{\frac{2iap}{-2+p}}+e^{\frac{4ia}{-2+p}}(cx^n)^{\frac{2}{n(-2+p)}}}\right)^p}{-1+p}$$

input
```
Integrate[Csc[a + (I*Log[c*x^n])/(n*(-2 + p))]^p,x]
```

output
```
(2^(-1 + p)*(-2 + p)*x*(E^(((2*I)*a*p)/(-2 + p)) - E^(((4*I)*a)/(-2 + p))*
(c*x^n)^(2/(n*(-2 + p))))*(((-I)*E^((I*a*(2 + p))/(-2 + p))*(c*x^n)^(1/(n*
(-2 + p))))/(-E^(((2*I)*a*p)/(-2 + p)) + E^(((4*I)*a)/(-2 + p))*(c*x^n)^(2
/(n*(-2 + p)))))^p)/(E^(((2*I)*a*p)/(-2 + p))*(-1 + p))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5015, 5019, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^p\left(a+\frac{i\log(cx^n)}{n(p-2)}\right)dx$$

$$\Big\downarrow \text{5015}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\csc^p\left(a-\frac{i\log(cx^n)}{n(2-p)}\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{5019}$$

$$\frac{x(cx^n)^{-\frac{p}{n(2-p)}-\frac{1}{n}}\left(1-e^{2ia}(cx^n)^{\frac{2}{n(2-p)}}\right)^p\csc^p\left(a-\frac{i\log(cx^n)}{n(2-p)}\right)\int (cx^n)^{\frac{p}{2n-np}+\frac{1}{n}-1}\left(1-e^{2ia}(cx^n)^{\frac{2}{n(2-p)}}\right)^{-p}d(cx^n)}{n}$$

$$\Big\downarrow \text{793}$$

$$-\frac{e^{-2ia}(2-p)x(cx^n)^{-\frac{p}{n(2-p)}-\frac{1}{n}}\left(1-e^{2ia}(cx^n)^{\frac{2}{n(2-p)}}\right)\csc^p\left(a-\frac{i\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

input
```
Int[Csc[a + (I*Log[c*x^n])/(n*(-2 + p))]^p,x]
```

output
```
-1/2*((2 - p)*x*(c*x^n)^(-n^(-1) - p/(n*(2 - p)))*(1 - E^((2*I)*a)*(c*x^n)
^(2/(n*(2 - p))))*Csc[a - (I*Log[c*x^n])/(n*(2 - p))]^p)/(E^((2*I)*a)*(1 -
 p))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 5015
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5019
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \csc\left(a+\frac{i\ln\left(c\,x^{n}\right)}{n\left(-2+p\right)}\right)^{p}dx$$

input
```
int(csc(a+I*ln(c*x^n)/n/(-2+p))^p,x)
```

output
```
int(csc(a+I*ln(c*x^n)/n/(-2+p))^p,x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.56

$$\int \csc^{p}\left(a+\frac{i\log\left(cx^{n}\right)}{n(-2+p)}\right)\,dx \\ = \frac{\left((p-2)xe^{\left(\frac{2\,(i\,anp-2i\,an-n\,\log(x)-\log(c))}{np-2\,n}\right)}-(p-2)x\right)\left(\frac{2i\,e^{\left(\frac{i\,anp-2i\,an-n\,\log(x)-\log(c)}{np-2\,n}\right)}}{e^{\left(\frac{2\,(i\,anp-2i\,an-n\,\log(x)-\log(c))}{np-2\,n}\right)}-1}\right)^{p}e^{\left(-\frac{2\,(i\,anp-2i\,an-n\,\log(x)-\log(c}{np-2\,n}}}{2\,(p-1)}$$

input
```
integrate(csc(a+I*log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

output
```
1/2*((p - 2)*x*e^(2*(I*a*n*p - 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n)) -
 (p - 2)*x)*(2*I*e^((I*a*n*p - 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n))/(
e^(2*(I*a*n*p - 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n)) - 1))^p*e^(-2*(I
*a*n*p - 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n))/(p - 1)
```

Sympy [F]

$$\int \csc^p\left(a+\frac{i\log(cx^n)}{n(-2+p)}\right)\,dx=\int \csc^p\left(a+\frac{i\log(cx^n)}{n\,(p-2)}\right)\,dx$$

input
```
integrate(csc(a+I*ln(c*x**n)/n/(-2+p))**p,x)
```

output
```
Integral(csc(a + I*log(c*x**n)/(n*(p - 2)))**p, x)
```

Maxima [F]

$$\int \csc^p\left(a+\frac{i\log(cx^n)}{n(-2+p)}\right)\,dx=\int \csc\left(a+\frac{i\,\log(cx^n)}{n(p-2)}\right)^p\,dx$$

input
```
integrate(csc(a+I*log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
```

output
```
integrate(csc(a + I*log(c*x^n)/(n*(p - 2)))^p, x)
```

Giac [F]

$$\int \csc^p\left(a+\frac{i\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\int \csc\left(a+\frac{i\log\left(cx^n\right)}{n(p-2)}\right)^p\,dx$$

input
```
integrate(csc(a+I*log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
```

output
```
integrate(csc(a + I*log(c*x^n)/(n*(p - 2)))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^p\left(a+\frac{i\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\int \left(\frac{1}{\sin\left(a+\frac{\ln(c\,x^n)\,1\mathrm{i}}{n\,(p-2)}\right)}\right)^p\,dx$$

input
```
int((1/sin(a + (log(c*x^n)*1i)/(n*(p - 2))))^p,x)
```

output
```
int((1/sin(a + (log(c*x^n)*1i)/(n*(p - 2))))^p, x)
```

Reduce [F]

$$\int \csc^p\left(a+\frac{i\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\int \csc\left(\frac{\log(x^n c)\,i+anp-2an}{np-2n}\right)^p dx$$

input
```
int(csc(a+I*log(c*x^n)/n/(-2+p))^p,x)
```

output
```
int(csc((log(x**n*c)*i + a*n*p - 2*a*n)/(n*p - 2*n))**p,x)
```

3.307 $\int \csc^p\left(a-\frac{i\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 71

$$\int \csc^p\left(a-\frac{i\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\frac{(2-p)x\left(1-e^{2ia}\left(cx^n\right)^{-\frac{2}{n(2-p)}}\right)\csc^p\left(a+\frac{i\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

output
```
(2-p)*x*(1-exp(2*I*a)/((c*x^n)^(2/n/(2-p))))*csc(a+I*ln(c*x^n)/n/(2-p))^p/
(2-2*p)
```

Mathematica [A] (verified)

Time = 1.96 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.80

$$\int \csc^p\left(a-\frac{i\log\left(cx^n\right)}{n(-2+p)}\right)\,dx =\frac{2^{-1+p}(-2+p)x\left(\frac{ie^{ia}(cx^n)^{\frac{1}{n(-2+p)}}}{-1+e^{2ia}(cx^n)^{\frac{2}{n(-2+p)}}}\right)^p\left(1+e^{2ia}(cx^n)^{\frac{2}{n(-2+p)}}\left(-1+\left(1-e^{-2ia}(cx^n)^{-\frac{2}{n(-2+p)}}\right)^p\right)\right)}{-1+p}$$

input
```
Integrate[Csc[a - (I*Log[c*x^n])/(n*(-2 + p))]^p,x]
```

output
```
(2^(-1 + p)*(-2 + p)*x*((I*E^(I*a)*(c*x^n)^(1/(n*(-2 + p))))/(-1 + E^((2*I
)*a)*(c*x^n)^(2/(n*(-2 + p)))))^p*(1 + E^((2*I)*a)*(c*x^n)^(2/(n*(-2 + p))
)*(-1 + (1 - 1/(E^((2*I)*a)*(c*x^n)^(2/(n*(-2 + p)))))^p)))/(-1 + p)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.58, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {5015, 5019, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^p\left(a-\frac{i \log \left(c x^n\right)}{n (p-2)}\right) \, dx$$

$$\Big\downarrow \text{5015}$$

$$\frac{x \left(c x^n\right)^{-1/n} \int \left(c x^n\right)^{\frac{1}{n}-1} \csc^p\left(a+\frac{i \log \left(c x^n\right)}{n (2-p)}\right)d\left(c x^n\right)}{n}$$

$$\Big\downarrow \text{5019}$$

$$\frac{x \left(c x^n\right)^{\frac{p}{n (2-p)}-\frac{1}{n}} \left(1-e^{2 i a} \left(c x^n\right)^{-\frac{2}{n (2-p)}}\right)^p \csc^p\left(a+\frac{i \log \left(c x^n\right)}{n (2-p)}\right) \int \left(c x^n\right)^{\frac{1-\frac{p}{2-p}}{n}-1} \left(1-e^{2 i a} \left(c x^n\right)^{-\frac{2}{n (2-p)}}\right)^{-p}d\left(c x^n\right)}{n}$$

$$\Big\downarrow \text{796}$$

$$\frac{(2-p) x \left(c x^n\right)^{\frac{2 (1-p)}{n (2-p)}+\frac{p}{n (2-p)}-\frac{1}{n}} \left(1-e^{2 i a} \left(c x^n\right)^{-\frac{2}{n (2-p)}}\right) \csc^p\left(a+\frac{i \log \left(c x^n\right)}{n (2-p)}\right)}{2 (1-p)}$$

input
```
Int[Csc[a - (I*Log[c*x^n])/(n*(-2 + p))]^p,x]
```

output
```
((2 - p)*x*(c*x^n)^(-n^(-1) + (2*(1 - p))/(n*(2 - p)) + p/(n*(2 - p)))*(1
- E^((2*I)*a)/(c*x^n)^(2/(n*(2 - p))))*Csc[a + (I*Log[c*x^n])/(n*(2 - p))]
^p)/(2*(1 - p))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 5015
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5019
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \csc\left(a - \frac{i \ln\left(c\, x^{n}\right)}{n\left(-2+p\right)}\right)^{p} dx$$

input
```
int(csc(a-I*ln(c*x^n)/n/(-2+p))^p,x)
```

output
```
int(csc(a-I*ln(c*x^n)/n/(-2+p))^p,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(55) = 110$.

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.11

$$\int \csc^{p}\left(a - \frac{i \log\left(cx^{n}\right)}{n(-2+p)}\right)\, dx = \frac{\left((p-2)xe^{\left(\frac{2\left(-i\,anp+2i\,an-n\log(x)-\log(c)\right)}{np-2\,n}\right)} - (p-2)x\right)\left(-\frac{2i\,e^{\left(\frac{-i\,anp+2i\,an-n\log(x)-\log(c)}{np-2\,n}\right)}}{e^{\left(\frac{2\left(-i\,anp+2i\,an-n\log(x)-\log(c)\right)}{np-2\,n}\right)}-1}\right)^{p} e^{\left(-\frac{2\left(-i\,anp+2i\,an-n\log(\right.}{np-2\,n}\right.}}{2\left(p-1\right)}$$

input
```
integrate(csc(a-I*log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

output
```
1/2*((p - 2)*x*e^(2*(-I*a*n*p + 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n))
- (p - 2)*x)*(-2*I*e^((-I*a*n*p + 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n)
)/(e^(2*(-I*a*n*p + 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n)) - 1))^p*e^(-
2*(-I*a*n*p + 2*I*a*n - n*log(x) - log(c))/(n*p - 2*n))/(p - 1)
```

Sympy [F]

$$\int \csc^p\left(a-\frac{i\log(cx^n)}{n(-2+p)}\right)\,dx=\int \csc^p\left(a-\frac{i\log(cx^n)}{n\,(p-2)}\right)\,dx$$

input
```
integrate(csc(a-I*ln(c*x**n)/n/(-2+p))**p,x)
```

output
```
Integral(csc(a - I*log(c*x**n)/(n*(p - 2)))**p, x)
```

Maxima [F]

$$\int \csc^p\left(a-\frac{i\log(cx^n)}{n(-2+p)}\right)\,dx=\int \csc\left(a-\frac{i\log(cx^n)}{n(p-2)}\right)^p\,dx$$

input
```
integrate(csc(a-I*log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
```

output
```
integrate((-csc(-a + I*log(c*x^n)/(n*(p - 2))))^p, x)
```

Giac [F]

$$\int \csc^p\left(a-\frac{i\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\int \csc\left(a-\frac{i\log\left(cx^n\right)}{n(p-2)}\right)^p\,dx$$

input
```
integrate(csc(a-I*log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
```

output
```
integrate(csc(a - I*log(c*x^n)/(n*(p - 2)))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^p\left(a-\frac{i\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=\int \left(\frac{1}{\sin\left(a-\frac{\ln(c\,x^n)\,1\mathrm{i}}{n\,(p-2)}\right)}\right)^p\,dx$$

input
```
int((1/sin(a - (log(c*x^n)*1i)/(n*(p - 2))))^p,x)
```

output
```
int((1/sin(a - (log(c*x^n)*1i)/(n*(p - 2))))^p, x)
```

Reduce [F]

$$\int \csc^p\left(a-\frac{i\log\left(cx^n\right)}{n(-2+p)}\right)\,dx=(-1)^p\left(\int \csc\left(\frac{\log(x^nc)\,i-anp+2an}{np-2n}\right)^pdx\right)$$

input
```
int(csc(a-I*log(c*x^n)/n/(-2+p))^p,x)
```

output
```
( - 1)**p*int(csc((log(x**n*c)*i - a*n*p + 2*a*n)/(n*p - 2*n))**p,x)
```

3.308 $\int \sqrt{\csc (a+b\log (cx^n))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \sqrt{\csc (a+b\log (cx^n))}\,dx = \frac{2x\sqrt{1-e^{2ia}\left(cx^n\right)^{2ib}}\sqrt{\csc (a+b\log (cx^n))}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}\left(1-\frac{2i}{bn}\right),\frac{1}{4}\left(5-\frac{2i}{bn}\right),e^{2ia}\left(cx^n\right)^{2ib}\right)}{2+ibn}$$

output
```
2*x*(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)*csc(a+b*ln(c*x^n))^(1/2)*hypergeo
m([1/2, 1/4-1/2*I/b/n],[5/4-1/2*I/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))/(2+I*b*
n)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06

$$\int \sqrt{\csc (a+b\log (cx^n))}\,dx = \frac{2ie^{-2ia}\left(-1+e^{2i(a+b\log (cx^n))}\right)x\left(cx^n\right)^{-2ib}\sqrt{\csc (a+b\log (cx^n))}\,\text{Hypergeometric2F1}\left(1,\frac{3}{4}+\frac{i}{2bn},\frac{5}{4}+\frac{i}{2bn},e\right.}{2i+bn}$$

input
```
Integrate[Sqrt[Csc[a + b*Log[c*x^n]]],x]
```

output
```
((2*I)*(-1 + E^((2*I)*(a + b*Log[c*x^n])))*x*Sqrt[Csc[a + b*Log[c*x^n]]]*H
ypergeometric2F1[1, 3/4 + (I/2)/(b*n), 5/4 + (I/2)/(b*n), E^((-2*I)*(a + b
*Log[c*x^n]))])/(E^((2*I)*a)*(2*I + b*n)*(c*x^n)^((2*I)*b))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5015, 5019, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\csc(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{5015}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\sqrt{\csc(a+b\log(cx^n))}d(cx^n)}{n}$$

$$\Big\downarrow \text{5019}$$

$$\frac{x(cx^n)^{-\frac{1}{n}-\frac{ib}{2}}\sqrt{1-e^{2ia}(cx^n)^{2ib}}\sqrt{\csc(a+b\log(cx^n))}\int \frac{(cx^n)^{\frac{ib}{2}+\frac{1}{n}-1}}{\sqrt{1-e^{2ia}(cx^n)^{2ib}}}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x\sqrt{1-e^{2ia}(cx^n)^{2ib}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}\left(1-\frac{2i}{bn}\right),\frac{1}{4}\left(5-\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)\sqrt{\csc(a+b\log(cx^n))}}{2+ibn}$$

input
```
Int[Sqrt[Csc[a + b*Log[c*x^n]]],x]
```

output
```
(2*x*Sqrt[1 - E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[Csc[a + b*Log[c*x^n]]]*H
ypergeometric2F1[1/2, (1 - (2*I)/(b*n))/4, (5 - (2*I)/(b*n))/4, E^((2*I)*a
)*(c*x^n)^((2*I)*b)])/(2 + I*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5015
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5019
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \sqrt{\csc \left(a+b\ln \left(c\,x^{n}\right)\right)}dx$$

input
```
int(csc(a+b*ln(c*x^n))^(1/2),x)
```

output
```
int(csc(a+b*ln(c*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\csc \left(a+b\log \left(cx^{n}\right)\right)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(csc(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{\csc\left(a+b\log\left(cx^{n}\right)\right)}\,dx=\int \sqrt{\csc\left(a+b\log\left(cx^{n}\right)\right)}\,dx$$

input
```
integrate(csc(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(sqrt(csc(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \sqrt{\csc\left(a+b\log\left(cx^{n}\right)\right)}\,dx=\int \sqrt{\csc\left(b\log\left(cx^{n}\right)+a\right)}\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(csc(b*log(c*x^n) + a)), x)
```

Giac [F]

$$\int \sqrt{\csc\left(a+b\log\left(cx^{n}\right)\right)}\,dx=\int \sqrt{\csc\left(b\log\left(cx^{n}\right)+a\right)}\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(csc(b*log(c*x^n) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\csc(a+b\log(cx^n))}\,dx = \int \sqrt{\frac{1}{\sin(a+b\ln(c\,x^n))}}\,dx$$

input
```
int((1/sin(a + b*log(c*x^n)))^(1/2),x)
```

output
```
int((1/sin(a + b*log(c*x^n)))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\csc(a+b\log(cx^n))}\,dx = \int \sqrt{\csc(\log(x^n c)\,b+a)}dx$$

input
```
int(csc(a+b*log(c*x^n))^(1/2),x)
```

output
```
int(sqrt(csc(log(x**n*c)*b + a)),x)
```

3.309 $\int \frac{\sqrt{\csc(a+b\log(cx^n))}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 60

$$\int \frac{\sqrt{\csc(a+b\log(cx^n))}}{x}\,dx = \frac{2\sqrt{\csc(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{4}(2a-\pi+2b\log(cx^n)),2\right)\sqrt{\sin(a+b\log(cx^n))}}{bn}$$

output
```
2*csc(a+b*ln(c*x^n))^(1/2)*InverseJacobiAM(1/2*a-1/4*Pi+1/2*b*ln(c*x^n),2^
(1/2))*sin(a+b*ln(c*x^n))^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{\csc(a+b\log(cx^n))}}{x}\,dx = -\frac{2\sqrt{\csc(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2b\log(cx^n)),2\right)\sqrt{\sin(a+b\log(cx^n))}}{bn}$$

input
```
Integrate[Sqrt[Csc[a + b*Log[c*x^n]]]/x,x]
```

output
```
(-2*Sqrt[Csc[a + b*Log[c*x^n]]]*EllipticF[(-2*a + Pi - 2*b*Log[c*x^n])/4,
2]*Sqrt[Sin[a + b*Log[c*x^n]]])/(b*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3039, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\csc(a+b\log(cx^n))}}{x} \, dx$$

$$\Big\downarrow \; 3039$$

$$\frac{\int \sqrt{\csc(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \sqrt{\csc(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \; 4258$$

$$\frac{\sqrt{\sin(a+b\log(cx^n))}\sqrt{\csc(a+b\log(cx^n))}\int \frac{1}{\sqrt{\sin(a+b\log(cx^n))}}d\log(cx^n)}{n}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{\sin(a+b\log(cx^n))}\sqrt{\csc(a+b\log(cx^n))}\int \frac{1}{\sqrt{\sin(a+b\log(cx^n))}}d\log(cx^n)}{n}$$

$$\Big\downarrow \; 3120$$

$$\frac{2\sqrt{\sin(a+b\log(cx^n))}\sqrt{\csc(a+b\log(cx^n))}\text{EllipticF}\left(\frac{1}{2}\left(a+b\log(cx^n)-\frac{\pi}{2}\right),2\right)}{bn}$$

input
```
Int[Sqrt[Csc[a + b*Log[c*x^n]]]/x,x]
```

output
```
(2*Sqrt[Csc[a + b*Log[c*x^n]]]*EllipticF[(a - Pi/2 + b*Log[c*x^n])/2, 2]*S
qrt[Sin[a + b*Log[c*x^n]]])/(b*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.70

method	result	siz
derivativedivides	$\frac{\sqrt{\sin(a+b\ln(c\,x^{n}))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^{n}))+2}\,\sqrt{-\sin(a+b\ln(c\,x^{n}))}\ \text{EllipticF}\left(\sqrt{\sin(a+b\ln(c\,x^{n}))+1},\frac{\sqrt{2}}{2}\right)}{n\cos(a+b\ln(c\,x^{n}))\sqrt{\sin(a+b\ln(c\,x^{n}))}\,b}$	10
default	$\frac{\sqrt{\sin(a+b\ln(c\,x^{n}))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^{n}))+2}\,\sqrt{-\sin(a+b\ln(c\,x^{n}))}\ \text{EllipticF}\left(\sqrt{\sin(a+b\ln(c\,x^{n}))+1},\frac{\sqrt{2}}{2}\right)}{n\cos(a+b\ln(c\,x^{n}))\sqrt{\sin(a+b\ln(c\,x^{n}))}\,b}$	10

input
```
int(csc(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin(a+b
*ln(c*x^n)))^(1/2)*EllipticF((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2))/cos
(a+b*ln(c*x^n))/sin(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{\csc(a+b\log(cx^n))}}{x}\,dx = \frac{-i\sqrt{2i}\text{weierstrassPInverse}(4,0,\cos(bn\log(x)+b\log(c)+a)+i\sin(bn\log(x)+b\log(c)+a))+i\sqrt{-}}{bn}$$

input
```
integrate(csc(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

output
```
(-I*sqrt(2*I)*weierstrassPInverse(4, 0, cos(b*n*log(x) + b*log(c) + a) + I
*sin(b*n*log(x) + b*log(c) + a)) + I*sqrt(-2*I)*weierstrassPInverse(4, 0,
cos(b*n*log(x) + b*log(c) + a) - I*sin(b*n*log(x) + b*log(c) + a)))/(b*n)
```

Sympy [F]

$$\int \frac{\sqrt{\csc(a+b\log(cx^n))}}{x}\,dx = \int \frac{\sqrt{\csc(a+b\log(cx^n))}}{x}\,dx$$

input
```
integrate(csc(a+b*ln(c*x**n))**(1/2)/x,x)
```

output
```
Integral(sqrt(csc(a + b*log(c*x**n)))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{\csc(a+b\log(cx^n))}}{x}\,dx = \int \frac{\sqrt{\csc(b\log(cx^n)+a)}}{x}\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(csc(b*log(c*x^n) + a))/x, x)
```

Giac [F]

$$\int \frac{\sqrt{\csc(a+b\log(cx^n))}}{x}\,dx = \int \frac{\sqrt{\csc(b\log(cx^n)+a)}}{x}\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(csc(b*log(c*x^n) + a))/x, x)
```

Mupad [B] (verification not implemented)

Time = 22.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{\csc(a+b\log(cx^n))}}{x}\,dx = -\frac{2\sqrt{\sin(a+b\ln(c\,x^n))}\,\mathrm{F}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin(a+b\ln(c\,x^n))}}{2}\right)\middle|\,2\right)\,\sqrt{\cos(a+b\ln(c\,x^n))^2}\,\sqrt{\frac{1}{\sin(a+b\ln(c\,x^n))}}}{b\,n\,\cos(a+b\ln(c\,x^n))}$$

input
```
int((1/sin(a + b*log(c*x^n)))^(1/2)/x,x)
```

output
```
-(2*sin(a + b*log(c*x^n))^(1/2)*ellipticF(asin((2^(1/2)*(1 - sin(a + b*log
(c*x^n)))^(1/2))/2), 2)*(cos(a + b*log(c*x^n))^2)^(1/2)*(1/sin(a + b*log(c
*x^n)))^(1/2))/(b*n*cos(a + b*log(c*x^n)))
```

Reduce [F]

$$\int \frac{\sqrt{\csc(a+b\log(cx^n))}}{x}\,dx = \int \frac{\sqrt{\csc(\log(x^n c)\,b+a)}}{x}dx$$

input
```
int(csc(a+b*log(c*x^n))^(1/2)/x,x)
```

output
```
int(sqrt(csc(log(x**n*c)*b + a))/x,x)
```

3.310 $\int \csc^{\frac{3}{2}}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \csc^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \frac{2x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}(a+b\log(cx^n))\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i}{bn}\right),\frac{1}{4}\left(7-\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{2+3ibn}$$

output
```
2*x*(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)*csc(a+b*ln(c*x^n))^(3/2)*hypergeo
m([3/2, 3/4-1/2*I/b/n],[7/4-1/2*I/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))/(2+3*I*
b*n)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 411 vs. 2(109) = 218.

Time = 4.54 (sec) , antiderivative size = 411, normalized size of antiderivative = 3.77

$$\int \csc^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \frac{x\left((4+b^2n^2)\,x^{ibn}\sqrt{2-2e^{2ia}(cx^n)^{2ib}}\sqrt{\frac{ie^{ia}(cx^n)^{ib}}{-1+e^{2ia}(cx^n)^{2ib}}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4}-\frac{i}{2bn},\frac{7}{4}-\frac{i}{2bn},e^{2ia}(cx^n)^{2}\right.\right.}{}$$

input
```
Integrate[Csc[a + b*Log[c*x^n]]^(3/2),x]
```

output
```
(x*((4 + b^2*n^2)*x^(I*b*n)*Sqrt[2 - 2*E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt
[(I*E^(I*a)*(c*x^n)^(I*b))/(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))]*Hypergeom
etric2F1[1/2, 3/4 - (I/2)/(b*n), 7/4 - (I/2)/(b*n), E^((2*I)*a)*(c*x^n)^((
2*I)*b)] - ((-2*I + 3*b*n)*((2*I - b*n)*Sqrt[2 - 2*E^((2*I)*a)*(c*x^n)^((2
*I)*b)]*Sqrt[(I*E^(I*a)*(c*x^n)^(I*b))/(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)
)]*Hypergeometric2F1[1/2, -1/4*(2*I + b*n)/(b*n), 3/4 - (I/2)/(b*n), E^((2
*I)*a)*(c*x^n)^((2*I)*b)] + 2*x^(I*b*n)*Sqrt[Csc[a + b*Log[c*x^n]]]*(b*n*C
os[b*n*Log[x]] - 2*Sin[b*n*Log[x]])))/x^(I*b*n)))/(b*n*(-2*I + 3*b*n)*(b*n
*Cos[a - b*n*Log[x] + b*Log[c*x^n]] + 2*Sin[a - b*n*Log[x] + b*Log[c*x^n]]
))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5015, 5019, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^{\frac{3}{2}}(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{5015}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\csc^{\frac{3}{2}}(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5019}$$

$$\frac{x(cx^n)^{-\frac{1}{n}-\frac{3ib}{2}}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}(a+b\log(cx^n))\int \frac{(cx^n)^{\frac{3ib}{2}+\frac{1}{n}-1}}{\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i}{bn}\right),\frac{1}{4}\left(7-\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)\csc^{\frac{3}{2}}(a+b\log(cx^n))}{2+3ibn}$$

input
```
Int[Csc[a + b*Log[c*x^n]]^(3/2),x]
```

output
```
(2*x*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2)*Csc[a + b*Log[c*x^n]]^(3/2)
*Hypergeometric2F1[3/2, (3 - (2*I)/(b*n))/4, (7 - (2*I)/(b*n))/4, E^((2*I)
*a)*(c*x^n)^((2*I)*b)])/(2 + (3*I)*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5015
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5019
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \csc\left(a+b\ln\left(c\,x^{n}\right)\right)^{\frac{3}{2}}dx$$

input
```
int(csc(a+b*ln(c*x^n))^(3/2),x)
```

output
```
int(csc(a+b*ln(c*x^n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \csc^{\frac{3}{2}}(a + b\log(cx^n)) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(csc(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \csc^{\frac{3}{2}}(a + b\log(cx^n)) \, dx = \int \csc^{\frac{3}{2}}(a + b\log(cx^n)) \, dx$$

input
```
integrate(csc(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Integral(csc(a + b*log(c*x**n))**(3/2), x)
```

Maxima [F]

$$\int \csc^{\frac{3}{2}}(a + b\log(cx^n)) \, dx = \int \csc(b\log(cx^n) + a)^{\frac{3}{2}} \, dx$$

input
```
integrate(csc(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*log(c*x^n) + a)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \csc^{\frac{3}{2}}(a + b\log(cx^n)) \, dx = \text{Timed out}$$

input
```
integrate(csc(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \csc^{\frac{3}{2}}(a + b\log(cx^n)) \, dx = \int \left(\frac{1}{\sin(a + b\ln(c\,x^n))}\right)^{3/2} dx$$

input
```
int((1/sin(a + b*log(c*x^n)))^(3/2),x)
```

output
```
int((1/sin(a + b*log(c*x^n)))^(3/2), x)
```

Reduce [F]

$$\int \csc^{\frac{3}{2}}(a + b\log(cx^n)) \, dx = \int \sqrt{\csc(\log(x^n c)\,b + a)}\,\csc(\log(x^n c)\,b + a)\,dx$$

input
```
int(csc(a+b*log(c*x^n))^(3/2),x)
```

output
```
int(sqrt(csc(log(x**n*c)*b + a))*csc(log(x**n*c)*b + a),x)
```

3.311 $\int \frac{\csc^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 95

$$\int \frac{\csc^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2\cos(a+b\log(cx^n))\sqrt{\csc(a+b\log(cx^n))}}{bn} - \frac{2\sqrt{\csc(a+b\log(cx^n))}E\left(\frac{1}{4}(2a-\pi+2b\log(cx^n))\middle|2\right)\sqrt{\sin(a+b\log(cx^n))}}{bn}$$

output
```
-2*cos(a+b*ln(c*x^n))*csc(a+b*ln(c*x^n))^(1/2)/b/n+2*csc(a+b*ln(c*x^n))^(1
/2)*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*ln(c*x^n)),2^(1/2))*sin(a+b*ln(c*x^n)
)^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.76

$$\int \frac{\csc^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2\sqrt{\csc(a+b\log(cx^n))}\left(\cos(a+b\log(cx^n)) - E\left(\frac{1}{4}(-2a+\pi-2b\log(cx^n))\middle|2\right)\sqrt{\sin(a+b\log(cx^n)}\right.}{bn}$$

input
```
Integrate[Csc[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
(-2*Sqrt[Csc[a + b*Log[c*x^n]]]*(Cos[a + b*Log[c*x^n]] - EllipticE[(-2*a +
 Pi - 2*b*Log[c*x^n])/4, 2]*Sqrt[Sin[a + b*Log[c*x^n]]]))/(b*n)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3039, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^{\frac{3}{2}}(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \csc^{\frac{3}{2}}(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc(a+b\log(cx^n))^{3/2}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4255}$$

$$\frac{-\int \frac{1}{\sqrt{\csc(a+b\log(cx^n))}}d\log(cx^n)-\frac{2\cos(a+b\log(cx^n))\sqrt{\csc(a+b\log(cx^n))}}{b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\int \frac{1}{\sqrt{\csc(a+b\log(cx^n))}}d\log(cx^n)-\frac{2\cos(a+b\log(cx^n))\sqrt{\csc(a+b\log(cx^n))}}{b}}{n}$$

$$\Big\downarrow \text{4258}$$

$$\frac{-\sqrt{\sin(a+b\log(cx^n))}\sqrt{\csc(a+b\log(cx^n))}\int \sqrt{\sin(a+b\log(cx^n))}d\log(cx^n)-\frac{2\cos(a+b\log(cx^n))\sqrt{\csc(a+b\log(cx}{b}}{n}$$

$$\downarrow \text{3042}$$

$$\frac{-\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}\int\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}d\log\left(cx^n\right)-\frac{2\cos(a+b\log(cx^n))\sqrt{\csc(a+b\log(cx^n))}}{b}}{n}$$

$$\downarrow \text{3119}$$

$$\frac{-\frac{2\cos(a+b\log(cx^n))\sqrt{\csc(a+b\log(cx^n))}}{b}-\frac{2\sqrt{\sin(a+b\log(cx^n))}\sqrt{\csc(a+b\log(cx^n))}E\left(\frac{1}{2}\left(a+b\log(cx^n)-\frac{\pi}{2}\right)|2\right)}{b}}{n}$$

input
```
Int[Csc[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
((-2*Cos[a + b*Log[c*x^n]]*Sqrt[Csc[a + b*Log[c*x^n]]])/b - (2*Sqrt[Csc[a
+ b*Log[c*x^n]]]*EllipticE[(a - Pi/2 + b*Log[c*x^n])/2, 2]*Sqrt[Sin[a + b*
Log[c*x^n]]])/b)/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(88) = 176.

Time = 0.68 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.00

method	result
derivativedivides	$\frac{2\sqrt{\sin(a+b\ln(cx^n))+1}\,\sqrt{-2\sin(a+b\ln(cx^n))+2}\,\sqrt{-\sin(a+b\ln(cx^n))}\,\operatorname{EllipticE}\left(\sqrt{\sin(a+b\ln(cx^n))+1},\frac{\sqrt{2}}{2}\right)-\sqrt{\sin}}{n\cos(a+b\ln(c}$
default	$\frac{2\sqrt{\sin(a+b\ln(cx^n))+1}\,\sqrt{-2\sin(a+b\ln(cx^n))+2}\,\sqrt{-\sin(a+b\ln(cx^n))}\,\operatorname{EllipticE}\left(\sqrt{\sin(a+b\ln(cx^n))+1},\frac{\sqrt{2}}{2}\right)-\sqrt{\sin}}{n\cos(a+b\ln(c}$

input
```
int(csc(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(2*(sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin(
a+b*ln(c*x^n)))^(1/2)*EllipticE((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2))-
(sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin(a+b*ln(
c*x^n)))^(1/2)*EllipticF((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2))-2*cos(a
+b*ln(c*x^n))^2)/cos(a+b*ln(c*x^n))/sin(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.17

$$\int \frac{\csc^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{\sqrt{2i}\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(bn\log(x)+b\log(c)+a)+i\,\sin(bn\log(x)+}{}$$

input
```
integrate(csc(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

output
```
-(sqrt(2*I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*n*log(x)
 + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a))) + sqrt(-2*I)*weierst
rassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*n*log(x) + b*log(c) + a) -
I*sin(b*n*log(x) + b*log(c) + a))) + 2*cos(b*n*log(x) + b*log(c) + a)/sqrt
(sin(b*n*log(x) + b*log(c) + a)))/(b*n)
```

Sympy [F]

$$\int \frac{\csc^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx=\int \frac{\csc^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx$$

input
```
integrate(csc(a+b*ln(c*x**n))**(3/2)/x,x)
```

output
```
Integral(csc(a + b*log(c*x**n))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\csc^{\frac{3}{2}}\left(a+b\log\left(cx^{n}\right)\right)}{x}\,dx=\int \frac{\csc\left(b\log\left(cx^{n}\right)+a\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(csc(b*log(c*x^n) + a)^(3/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\csc^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(csc(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\left(\frac{1}{\sin(a+b\,\ln(c\,x^n))}\right)^{3/2}}{x}\,dx$$

input
```
int((1/sin(a + b*log(c*x^n)))^(3/2)/x,x)
```

output
```
int((1/sin(a + b*log(c*x^n)))^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{\csc^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sqrt{\csc(\log(x^nc)\,b+a)}\,\csc(\log(x^nc)\,b+a)}{x}\,dx$$

input
```
int(csc(a+b*log(c*x^n))^(3/2)/x,x)
```

output
```
int((sqrt(csc(log(x**n*c)*b + a))*csc(log(x**n*c)*b + a))/x,x)
```

3.312 $\int \csc^{\frac{5}{2}}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \csc^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \frac{2x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}\csc^{\frac{5}{2}}(a+b\log(cx^n))\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i}{bn}\right),\frac{1}{4}\left(9-\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{2+5ibn}$$

output
```
2*x*(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(5/2)*csc(a+b*ln(c*x^n))^(5/2)*hypergeo
m([5/2, 5/4-1/2*I/b/n],[9/4-1/2*I/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))/(2+5*I*
b*n)
```

Mathematica [A] (warning: unable to verify)

Time = 1.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.60

$$\int \csc^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \frac{2e^{-2i(a-bn\log(x)+b\log(cx^n))}x^{1-2ibn}\sqrt{\csc(a+b\log(cx^n))}\left(-e^{2ia}(cx^n)^{2ib}(2+bn\cot(a+b\log(cx^n)))+(2+\cdots\right.}{3b^2n^2}$$

input
```
Integrate[Csc[a + b*Log[c*x^n]]^(5/2),x]
```

output
```
(2*x^(1 - (2*I)*b*n)*Sqrt[Csc[a + b*Log[c*x^n]]]*(-(E^((2*I)*a)*(c*x^n)^((
2*I)*b)*(2 + b*n*Cot[a + b*Log[c*x^n]])) + (2 + I*b*n)*(-1 + E^((2*I)*a)*(
c*x^n)^((2*I)*b))*Hypergeometric2F1[1, 3/4 + (I/2)/(b*n), 5/4 + (I/2)/(b*n
), E^((-2*I)*(a + b*Log[c*x^n]))]))/(3*b^2*E^((2*I)*(a - b*n*Log[x] + b*Lo
g[c*x^n]))*n^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5015, 5019, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^{\frac{5}{2}}(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{5015}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\csc^{\frac{5}{2}}(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5019}$$

$$\frac{x(cx^n)^{-\frac{1}{n}-\frac{5ib}{2}}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}\csc^{\frac{5}{2}}(a+b\log(cx^n))\int \frac{(cx^n)^{\frac{5ib}{2}+\frac{1}{n}-1}}{\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i}{bn}\right),\frac{1}{4}\left(9-\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)\csc^{\frac{5}{2}}(a+b\log(cx^n))}{2+5ibn}$$

input
```
Int[Csc[a + b*Log[c*x^n]]^(5/2),x]
```

output
```
(2*x*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^(5/2)*Csc[a + b*Log[c*x^n]]^(5/2)
*Hypergeometric2F1[5/2, (5 - (2*I)/(b*n))/4, (9 - (2*I)/(b*n))/4, E^((2*I)
*a)*(c*x^n)^((2*I)*b)])/(2 + (5*I)*b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5015
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5019
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \csc \left(a+b \ln \left(c \,x^{n}\right)\right)^{\frac{5}{2}}d x$$

input
```
int(csc(a+b*ln(c*x^n))^(5/2),x)
```

output
```
int(csc(a+b*ln(c*x^n))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \csc^{\frac{5}{2}}(a+b\log(cx^n))\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(csc(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \csc^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \text{Timed out}$$

input
```
integrate(csc(a+b*ln(c*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \csc^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \int \csc(b\log(cx^n)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*log(c*x^n) + a)^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \csc^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \text{Timed out}$$

input
```
integrate(csc(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \csc^{\frac{5}{2}}(a+b\log(cx^n))\,dx=\int\left(\frac{1}{\sin(a+b\ln(c\,x^n))}\right)^{5/2}dx$$

input
```
int((1/sin(a + b*log(c*x^n)))^(5/2),x)
```

output
```
int((1/sin(a + b*log(c*x^n)))^(5/2), x)
```

Reduce [F]

$$\int \csc^{\frac{5}{2}}(a+b\log(cx^n))\,dx=\int\sqrt{\csc(\log(x^nc)\,b+a)}\,\csc(\log(x^nc)\,b+a)^2dx$$

input
```
int(csc(a+b*log(c*x^n))^(5/2),x)
```

output
```
int(sqrt(csc(log(x**n*c)*b + a))*csc(log(x**n*c)*b + a)**2,x)
```

3.313 $\int \frac{\csc^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 99

$$\int \frac{\csc^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2\cos(a+b\log(cx^n))\csc^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{2\sqrt{\csc(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{4}(2a-\pi+2b\log(cx^n)),2\right)\sqrt{\sin(a+b\log(cx^n))}}{3bn}$$

output
```
-2/3*cos(a+b*ln(c*x^n))*csc(a+b*ln(c*x^n))^(3/2)/b/n+2/3*csc(a+b*ln(c*x^n)
)^(1/2)*InverseJacobiAM(1/2*a-1/4*Pi+1/2*b*ln(c*x^n),2^(1/2))*sin(a+b*ln(c
*x^n))^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74

$$\int \frac{\csc^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2\csc^{\frac{3}{2}}(a+b\log(cx^n))\left(\cos(a+b\log(cx^n))+\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2b\log(cx^n)),2\right)\sin^{\frac{3}{2}}(a+b\log}{3bn}$$

input
```
Integrate[Csc[a + b*Log[c*x^n]]^(5/2)/x,x]
```

output
```
(-2*Csc[a + b*Log[c*x^n]]^(3/2)*(Cos[a + b*Log[c*x^n]] + EllipticF[(-2*a +
 Pi - 2*b*Log[c*x^n])/4, 2]*Sin[a + b*Log[c*x^n]]^(3/2)))/(3*b*n)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3039, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^{\frac{5}{2}}(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \csc^{\frac{5}{2}}(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc(a+b\log(cx^n))^{5/2}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4255}$$

$$\frac{\frac{1}{3}\int \sqrt{\csc(a+b\log(cx^n))}d\log(cx^n)-\frac{2\cos(a+b\log(cx^n))\csc^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}\int \sqrt{\csc(a+b\log(cx^n))}d\log(cx^n)-\frac{2\cos(a+b\log(cx^n))\csc^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{1}{3}\sqrt{\sin(a+b\log(cx^n))}\sqrt{\csc(a+b\log(cx^n))}\int \frac{1}{\sqrt{\sin(a+b\log(cx^n))}}d\log(cx^n)-\frac{2\cos(a+b\log(cx^n))\csc^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}\int\frac{1}{\sqrt{\sin(a+b\log(cx^n))}}d\log\left(cx^n\right)-\frac{2\cos(a+b\log(cx^n))\csc^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2\sqrt{\sin(a+b\log(cx^n))}\sqrt{\csc(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}\left(a+b\log(cx^n)-\frac{\pi}{2}\right),2\right)}{3b}-\frac{2\cos(a+b\log(cx^n))\csc^{\frac{3}{2}}(a+b\log(cx^n))}{3b}}{n}$$

input
```
Int[Csc[a + b*Log[c*x^n]]^(5/2)/x,x]
```

output
```
((-2*Cos[a + b*Log[c*x^n]]*Csc[a + b*Log[c*x^n]]^(3/2))/(3*b) + (2*Sqrt[Cs
c[a + b*Log[c*x^n]]]*EllipticF[(a - Pi/2 + b*Log[c*x^n])/2, 2]*Sqrt[Sin[a
+ b*Log[c*x^n]]])/(3*b))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\,\operatorname{EllipticF}\left(\sqrt{\sin(a+b\ln(c\,x^n))+1},\frac{\sqrt{2}}{2}\right)\sin(a+}{3n\sin(a+b\ln(c\,x^n))^{\frac{3}{2}}\cos(a+b\ln(c\,x^n))b}$
default	$\frac{\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\,\operatorname{EllipticF}\left(\sqrt{\sin(a+b\ln(c\,x^n))+1},\frac{\sqrt{2}}{2}\right)\sin(a+}{3n\sin(a+b\ln(c\,x^n))^{\frac{3}{2}}\cos(a+b\ln(c\,x^n))b}$

input
```
int(csc(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/3/n/sin(a+b*ln(c*x^n))^(3/2)*((sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*l
n(c*x^n))+2)^(1/2)*(-sin(a+b*ln(c*x^n)))^(1/2)*EllipticF((sin(a+b*ln(c*x^n
))+1)^(1/2),1/2*2^(1/2))*sin(a+b*ln(c*x^n))-2*cos(a+b*ln(c*x^n))^2)/cos(a+
b*ln(c*x^n))/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.46

$$\int \frac{\csc^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{-i\sqrt{2i}\sin(bn\log(x)+b\log(c)+a)\,\text{weierstrassPInverse}(4,0,\cos(bn\log(x)+b\log(c)+a)+i\,\sin(bn}{}$$

input
```
integrate(csc(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

output
```
1/3*(-I*sqrt(2*I)*sin(b*n*log(x) + b*log(c) + a)*weierstrassPInverse(4, 0,
 cos(b*n*log(x) + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a)) + I*sq
rt(-2*I)*sin(b*n*log(x) + b*log(c) + a)*weierstrassPInverse(4, 0, cos(b*n*
log(x) + b*log(c) + a) - I*sin(b*n*log(x) + b*log(c) + a)) - 2*cos(b*n*log
(x) + b*log(c) + a)/sqrt(sin(b*n*log(x) + b*log(c) + a)))/(b*n*sin(b*n*log
(x) + b*log(c) + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^{\frac{5}{2}}(a + b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(csc(a+b*ln(c*x**n))**(5/2)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\csc^{\frac{5}{2}}(a + b\log(cx^n))}{x}\,dx = \int \frac{\csc(b\log(cx^n) + a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate(csc(b*log(c*x^n) + a)^(5/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\csc^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(csc(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\left(\frac{1}{\sin(a+b\,\ln(c\,x^n))}\right)^{5/2}}{x}\,dx$$

input
```
int((1/sin(a + b*log(c*x^n)))^(5/2)/x,x)
```

output
```
int((1/sin(a + b*log(c*x^n)))^(5/2)/x, x)
```

Reduce [F]

$$\int \frac{\csc^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sqrt{\csc(\log(x^n c)\,b+a)}\,\csc(\log(x^n c)\,b+a)^2}{x}dx$$

input
```
int(csc(a+b*log(c*x^n))^(5/2)/x,x)
```

output
```
int((sqrt(csc(log(x**n*c)*b + a))*csc(log(x**n*c)*b + a)**2)/x,x)
```

3.314 $\int \frac{1}{\sqrt{\csc(a+b\log(cx^n))}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \frac{1}{\sqrt{\csc(a+b\log(cx^n))}}\,dx = \frac{2x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2i+bn}{4bn},\frac{1}{4}\left(3-\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{(2-ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}\sqrt{\csc(a+b\log(cx^n))}}$$

output
```
2*x*hypergeom([-1/2, -1/4*(2*I+b*n)/b/n],[3/4-1/2*I/b/n],exp(2*I*a)*(c*x^n
)^(2*I*b))/(2-I*b*n)/(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)/csc(a+b*ln(c*x^n
))^(1/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 377 vs. 2(110) = 220.

Time = 3.07 (sec) , antiderivative size = 377, normalized size of antiderivative = 3.43

$$\int \frac{1}{\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}}\,dx =$$

$$-\frac{2be^{ia}nx(cx^n)^{ib}\sqrt{2-2e^{2ia}\left(cx^n\right)^{2ib}}\sqrt{\frac{ie^{ia}(cx^n)^{ib}}{-1+e^{2ia}(cx^n)^{2ib}}}\left((2i+bn)x^{2ibn}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4}-\frac{i}{2bn},\frac{7}{4}-\right.\right.}{(2i+bn)(-2i+3bn)\left((2i+bn)x^{2ibn}-\right.}$$

$$+\frac{2x\sin\left(a-bn\log(x)+b\log\left(cx^n\right)\right)}{\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}\left(bn\cos\left(a-bn\log(x)+b\log\left(cx^n\right)\right)+2\sin\left(a-bn\log(x)+b\log\left(cx^n\right)\right)\right)}$$

input
```
Integrate[1/Sqrt[Csc[a + b*Log[c*x^n]]],x]
```

output
```
(-2*b*E^(I*a)*n*x*(c*x^n)^(I*b)*Sqrt[2 - 2*E^((2*I)*a)*(c*x^n)^((2*I)*b)]*
Sqrt[(I*E^(I*a)*(c*x^n)^(I*b))/(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))]*((2*I
 + b*n)*x^((2*I)*b*n)*Hypergeometric2F1[1/2, 3/4 - (I/2)/(b*n), 7/4 - (I/2
)/(b*n), E^((2*I)*a)*(c*x^n)^((2*I)*b)] + (-2*I + 3*b*n)*Hypergeometric2F1
[1/2, -1/4*(2*I + b*n)/(b*n), 3/4 - (I/2)/(b*n), E^((2*I)*a)*(c*x^n)^((2*I
)*b)]))/((2*I + b*n)*(-2*I + 3*b*n)*((2*I + b*n)*x^((2*I)*b*n) + E^((2*I)*
a)*(-2*I + b*n)*(c*x^n)^((2*I)*b))) + (2*x*Sin[a - b*n*Log[x] + b*Log[c*x^
n]])/(Sqrt[Csc[a + b*Log[c*x^n]]]*(b*n*Cos[a - b*n*Log[x] + b*Log[c*x^n]]
+ 2*Sin[a - b*n*Log[x] + b*Log[c*x^n]]))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5015, 5019, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}}dx$$

$$\Big\downarrow \text{5015}$$

$$\frac{x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}-1}}{\sqrt{\csc(a+b\log(cx^n))}}d(cx^n)}{n}$$

$$\Big\downarrow \text{ 5019}$$

$$\frac{x(cx^n)^{-\frac{1}{n}+\frac{ib}{2}}\int (cx^n)^{-\frac{ib}{2}+\frac{1}{n}-1}\sqrt{1-e^{2ia}(cx^n)^{2ib}}d(cx^n)}{n\sqrt{1-e^{2ia}(cx^n)^{2ib}}\sqrt{\csc(a+b\log(cx^n))}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{2x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{bn+2i}{4bn},\frac{1}{4}\left(3-\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{(2-ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}\sqrt{\csc(a+b\log(cx^n))}}$$

input
```
Int[1/Sqrt[Csc[a + b*Log[c*x^n]]],x]
```

output
```
(2*x*Hypergeometric2F1[-1/2, -1/4*(2*I + b*n)/(b*n), (3 - (2*I)/(b*n))/4,
E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((2 - I*b*n)*Sqrt[1 - E^((2*I)*a)*(c*x^n)^
((2*I)*b)]*Sqrt[Csc[a + b*Log[c*x^n]]])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5015
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5019
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\sqrt{\csc\left(a+b\ln\left(c\,x^{n}\right)\right)}}dx$$

input
```
int(1/csc(a+b*ln(c*x^n))^(1/2),x)
```

output
```
int(1/csc(a+b*ln(c*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\csc\left(a+b\log\left(cx^{n}\right)\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/csc(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\csc\left(a+b\log\left(cx^{n}\right)\right)}}\,dx = \int \frac{1}{\sqrt{\csc\left(a+b\log\left(cx^{n}\right)\right)}}\,dx$$

input
```
integrate(1/csc(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(1/sqrt(csc(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}}\,dx=\int \frac{1}{\sqrt{\csc\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

input
```
integrate(1/csc(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(csc(b*log(c*x^n) + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}}\,dx=\int \frac{1}{\sqrt{\csc\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

input
```
integrate(1/csc(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(csc(b*log(c*x^n) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}}\,dx=\int \frac{1}{\sqrt{\frac{1}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}}}\,dx$$

input
```
int(1/(1/sin(a + b*log(c*x^n)))^(1/2),x)
```

output
```
int(1/(1/sin(a + b*log(c*x^n)))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{\sqrt{\csc\left(\log\left(x^n c\right)b+a\right)}}{\csc\left(\log\left(x^n c\right)b+a\right)}dx$$

input
```
int(1/csc(a+b*log(c*x^n))^(1/2),x)
```

output
```
int(sqrt(csc(log(x**n*c)*b + a))/csc(log(x**n*c)*b + a),x)
```

3.315 $\int \frac{1}{x\sqrt{\csc(a+b\log(cx^n))}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 60

$$\int \frac{1}{x\sqrt{\csc(a+b\log(cx^n))}}\,dx = \frac{2\sqrt{\csc(a+b\log(cx^n))}E\left(\frac{1}{4}(2a-\pi+2b\log(cx^n))\middle|2\right)\sqrt{\sin(a+b\log(cx^n))}}{bn}$$

output
```
-2*csc(a+b*ln(c*x^n))^(1/2)*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*ln(c*x^n)),2^
(1/2))*sin(a+b*ln(c*x^n))^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\sqrt{\csc(a+b\log(cx^n))}}\,dx = -\frac{2\sqrt{\csc(a+b\log(cx^n))}E\left(\frac{1}{4}(-2a+\pi-2b\log(cx^n))\middle|2\right)\sqrt{\sin(a+b\log(cx^n))}}{bn}$$

input
```
Integrate[1/(x*Sqrt[Csc[a + b*Log[c*x^n]]]),x]
```

output
```
(-2*Sqrt[Csc[a + b*Log[c*x^n]]]*EllipticE[(-2*a + Pi - 2*b*Log[c*x^n])/4,
2]*Sqrt[Sin[a + b*Log[c*x^n]]])/(b*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3039, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\csc(a+b\log(cx^n))}}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\sqrt{\csc(a+b\log(cx^n))}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{\csc(a+b\log(cx^n))}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\sqrt{\sin(a+b\log(cx^n))}\sqrt{\csc(a+b\log(cx^n))}\int \sqrt{\sin(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(a+b\log(cx^n))}\sqrt{\csc(a+b\log(cx^n))}\int \sqrt{\sin(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sqrt{\sin(a+b\log(cx^n))}\sqrt{\csc(a+b\log(cx^n))}E\left(\frac{1}{2}\left(a+b\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{bn}$$

input
```
Int[1/(x*Sqrt[Csc[a + b*Log[c*x^n]]]),x]
```

output
```
(2*Sqrt[Csc[a + b*Log[c*x^n]]]*EllipticE[(a - Pi/2 + b*Log[c*x^n])/2, 2]*S
qrt[Sin[a + b*Log[c*x^n]]])/(b*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(55) = 110$.

Time = 0.69 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.15

method	result
derivativedivides	$-\frac{\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{\sin(a+b\ln(c\,x^n))+1},\frac{\sqrt{2}}{2}\right)-\right.}{n\cos(a+b\ln(c\,x^n))\sqrt{\sin(a+b\ln(c\,x^n))}\,b}$
default	$-\frac{\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{\sin(a+b\ln(c\,x^n))+1},\frac{\sqrt{2}}{2}\right)-\right.}{n\cos(a+b\ln(c\,x^n))\sqrt{\sin(a+b\ln(c\,x^n))}\,b}$

input
```
int(1/x/csc(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/n*(sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin(a+
b*ln(c*x^n)))^(1/2)*(2*EllipticE((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2))
-EllipticF((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2)))/cos(a+b*ln(c*x^n))/s
in(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.37

$$\int \frac{1}{x\sqrt{\csc(a+b\log(cx^n))}}\,dx = \frac{\sqrt{2i}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(bn\log(x)+b\log(c)+a)+i\sin(bn\log(x)+b\log(c)+a)))}{}$$

input
```
integrate(1/x/csc(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(2*I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*n*log(x)
+ b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a))) + sqrt(-2*I)*weierstr
assZeta(4, 0, weierstrassPInverse(4, 0, cos(b*n*log(x) + b*log(c) + a) - I
*sin(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{\csc(a+b\log(cx^n))}}\,dx = \int \frac{1}{x\sqrt{\csc(a+b\log(cx^n))}}\,dx$$

input
```
integrate(1/x/csc(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(csc(a + b*log(c*x**n)))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}}\,dx=\int \frac{1}{x\sqrt{\csc\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

input
```
integrate(1/x/csc(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sqrt(csc(b*log(c*x^n) + a))), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}}\,dx=\int \frac{1}{x\sqrt{\csc\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

input
```
integrate(1/x/csc(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(x*sqrt(csc(b*log(c*x^n) + a))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}}\,dx=\int \frac{1}{x\,\sqrt{\frac{1}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}}}\,dx$$

input
```
int(1/(x*(1/sin(a + b*log(c*x^n)))^(1/2)),x)
```

output
```
int(1/(x*(1/sin(a + b*log(c*x^n)))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{\csc\left(a+b\log\left(cx^{n}\right)\right)}}\,dx=\int \frac{\sqrt{\csc\left(\log\left(x^{n}c\right)b+a\right)}}{\csc\left(\log\left(x^{n}c\right)b+a\right)x}dx$$

input
```
int(1/x/csc(a+b*log(c*x^n))^(1/2),x)
```

output
```
int(sqrt(csc(log(x**n*c)*b + a))/(csc(log(x**n*c)*b + a)*x),x)
```

3.316 $\int \frac{1}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 109

$$\int \frac{1}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{2x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i}{bn}\right),\frac{1}{4}\left(1-\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{(2-3ibn)\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}(a+b\log(cx^n))}$$

output
```
2*x*hypergeom([-3/2, -3/4-1/2*I/b/n],[1/4-1/2*I/b/n],exp(2*I*a)*(c*x^n)^(2
*I*b))/(2-3*I*b*n)/(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)/csc(a+b*ln(c*x^n))
^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.49 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.71

$$\int \frac{1}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{2ix\left((2-ibn)(-2+3bn\cot(a+b\log(cx^n)))-3b^2e^{-2ia}n^2(cx^n)^{-2ib}\left(-1+e^{2ia}(cx^n)^{2ib}\right)\csc^2(a+b\log(\right.}{(2i-3bn)(2i+bn)(2i+3bn)\csc^{\frac{3}{2}}(a+b\log(}$$

input
```
Integrate[Csc[a + b*Log[c*x^n]]^(-3/2),x]
```

output
```
((2*I)*x*((2 - I*b*n)*(-2 + 3*b*n*Cot[a + b*Log[c*x^n]]) - (3*b^2*n^2*(-1
+ E^((2*I)*a)*(c*x^n)^((2*I)*b))*Csc[a + b*Log[c*x^n]]^2*Hypergeometric2F1
[1, 3/4 + (I/2)/(b*n), 5/4 + (I/2)/(b*n), E^((-2*I)*(a + b*Log[c*x^n]))])/
(E^((2*I)*a)*(c*x^n)^((2*I)*b))))/((2*I - 3*b*n)*(2*I + b*n)*(2*I + 3*b*n)
*Csc[a + b*Log[c*x^n]]^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5015, 5019, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{5015}$$

$$\frac{x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}-1}}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}d(cx^n)}{n}$$

$$\Big\downarrow \text{5019}$$

$$\frac{x(cx^n)^{-\frac{1}{n}+\frac{3ib}{2}}\int (cx^n)^{-\frac{3ib}{2}+\frac{1}{n}-1}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}d(cx^n)}{n\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}(a+b\log(cx^n))}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4}\left(-3-\frac{2i}{bn}\right),\frac{1}{4}\left(1-\frac{2i}{bn}\right),e^{2ia}(cx^n)^{2ib}\right)}{(2-3ibn)\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}(a+b\log(cx^n))}$$

input
```
Int[Csc[a + b*Log[c*x^n]]^(-3/2),x]
```

output
```
(2*x*Hypergeometric2F1[-3/2, (-3 - (2*I)/(b*n))/4, (1 - (2*I)/(b*n))/4, E^
((2*I)*a)*(c*x^n)^((2*I)*b)])/((2 - (3*I)*b*n)*(1 - E^((2*I)*a)*(c*x^n)^((
2*I)*b))^(3/2)*Csc[a + b*Log[c*x^n]]^(3/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5015
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5019
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\csc \left(a+b \ln \left(c\,x^{n}\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/csc(a+b*ln(c*x^n))^(3/2),x)
```

output
```
int(1/csc(a+b*ln(c*x^n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/csc(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$

input
```
integrate(1/csc(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Integral(csc(a + b*log(c*x**n))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{\csc(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/csc(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*log(c*x^n) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{\csc(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/csc(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate(csc(b*log(c*x^n) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{\left(\frac{1}{\sin(a+b\ln(cx^n))}\right)^{3/2}}\,dx$$

input
```
int(1/(1/sin(a + b*log(c*x^n)))^(3/2),x)
```

output
```
int(1/(1/sin(a + b*log(c*x^n)))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{\sqrt{\csc(\log(x^n c)\,b+a)}}{\csc(\log(x^n c)\,b+a)^2}\,dx$$

input
```
int(1/csc(a+b*log(c*x^n))^(3/2),x)
```

output
```
int(sqrt(csc(log(x**n*c)*b + a))/csc(log(x**n*c)*b + a)**2,x)
```

3.317 $\int \frac{1}{x \csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 99

$$\int \frac{1}{x \csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = -\frac{2\cos(a+b\log(cx^n))}{3bn\sqrt{\csc(a+b\log(cx^n))}} + \frac{2\sqrt{\csc(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{4}(2a-\pi+2b\log(cx^n)),2\right)\sqrt{\sin(a+b\log(cx^n))}}{3bn}$$

output
```
-2/3*cos(a+b*ln(c*x^n))/b/n/csc(a+b*ln(c*x^n))^(1/2)+2/3*csc(a+b*ln(c*x^n)
)^(1/2)*InverseJacobiAM(1/2*a-1/4*Pi+1/2*b*ln(c*x^n),2^(1/2))*sin(a+b*ln(c
*x^n))^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.77

$$\int \frac{1}{x \csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = -\frac{\sqrt{\csc(a+b\log(cx^n))}\Big(2\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2b\log(cx^n)),2\right)\sqrt{\sin(a+b\log(cx^n))}+\sin(2(a+}{3bn}$$

input
```
Integrate[1/(x*Csc[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
-1/3*(Sqrt[Csc[a + b*Log[c*x^n]]]*(2*EllipticF[(-2*a + Pi - 2*b*Log[c*x^n]
)/4, 2]*Sqrt[Sin[a + b*Log[c*x^n]]] + Sin[2*(a + b*Log[c*x^n])]))/(b*n)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3039, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\csc^{\frac{3}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\csc(a+b\log(cx^n))^{3/2}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4256}$$

$$\frac{\frac{1}{3}\int \sqrt{\csc(a+b\log(cx^n))}d\log(cx^n)-\frac{2\cos(a+b\log(cx^n))}{3b\sqrt{\csc(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}\int \sqrt{\csc(a+b\log(cx^n))}d\log(cx^n)-\frac{2\cos(a+b\log(cx^n))}{3b\sqrt{\csc(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{1}{3}\sqrt{\sin(a+b\log(cx^n))}\sqrt{\csc(a+b\log(cx^n))}\int \frac{1}{\sqrt{\sin(a+b\log(cx^n))}}d\log(cx^n)-\frac{2\cos(a+b\log(cx^n))}{3b\sqrt{\csc(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}\int\frac{1}{\sqrt{\sin(a+b\log(cx^n))}}d\log\left(cx^n\right)-\frac{2\cos(a+b\log(cx^n))}{3b\sqrt{\csc(a+b\log(cx^n))}}}{n}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2\sqrt{\sin(a+b\log(cx^n))}\sqrt{\csc(a+b\log(cx^n))}\operatorname{EllipticF}\left(\frac{1}{2}\left(a+b\log(cx^n)-\frac{\pi}{2}\right),2\right)}{3b}-\frac{2\cos(a+b\log(cx^n))}{3b\sqrt{\csc(a+b\log(cx^n))}}}{n}$$

input
```
Int[1/(x*Csc[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
((-2*Cos[a + b*Log[c*x^n]])/(3*b*Sqrt[Csc[a + b*Log[c*x^n]]]) + (2*Sqrt[Cs
c[a + b*Log[c*x^n]]]*EllipticF[(a - Pi/2 + b*Log[c*x^n])/2, 2]*Sqrt[Sin[a
+ b*Log[c*x^n]]])/(3*b))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
 + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{\frac{\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\ \mathrm{EllipticF}\left(\sqrt{\sin(a+b\ln(c\,x^n))+1},\frac{\sqrt{2}}{2}\right)}{3}-\frac{2\cos(a+b\ln(c\,x^n))}{3}}{n\cos(a+b\ln(c\,x^n))\sqrt{\sin(a+b\ln(c\,x^n))}\,b}$
default	$\frac{\frac{\sqrt{\sin(a+b\ln(c\,x^n))+1}\,\sqrt{-2\sin(a+b\ln(c\,x^n))+2}\,\sqrt{-\sin(a+b\ln(c\,x^n))}\ \mathrm{EllipticF}\left(\sqrt{\sin(a+b\ln(c\,x^n))+1},\frac{\sqrt{2}}{2}\right)}{3}-\frac{2\cos(a+b\ln(c\,x^n))}{3}}{n\cos(a+b\ln(c\,x^n))\sqrt{\sin(a+b\ln(c\,x^n))}\,b}$

input
```
int(1/x/csc(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/n*(1/3*(sin(a+b*ln(c*x^n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-si
n(a+b*ln(c*x^n)))^(1/2)*EllipticF((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2)
)-2/3*cos(a+b*ln(c*x^n))^2*sin(a+b*ln(c*x^n)))/cos(a+b*ln(c*x^n))/sin(a+b*
ln(c*x^n))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08

$$\int \frac{1}{x \csc^{\frac{3}{2}}(a + b\log(cx^n))}\,dx = -\frac{2\cos(bn\log(x) + b\log(c) + a)\sqrt{\sin(bn\log(x) + b\log(c) + a)} + i\sqrt{2i}\text{weierstrassPInverse}(4, 0, \cos(}{}$$

input
```
integrate(1/x/csc(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(2*cos(b*n*log(x) + b*log(c) + a)*sqrt(sin(b*n*log(x) + b*log(c) + a)
) + I*sqrt(2*I)*weierstrassPInverse(4, 0, cos(b*n*log(x) + b*log(c) + a) +
 I*sin(b*n*log(x) + b*log(c) + a)) - I*sqrt(-2*I)*weierstrassPInverse(4, 0
, cos(b*n*log(x) + b*log(c) + a) - I*sin(b*n*log(x) + b*log(c) + a)))/(b*n
)
```

Sympy [F]

$$\int \frac{1}{x\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$

input
```
integrate(1/x/csc(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Integral(1/(x*csc(a + b*log(c*x**n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\csc(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/csc(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*csc(b*log(c*x^n) + a)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{x\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\csc(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/csc(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(x*csc(b*log(c*x^n) + a)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\left(\frac{1}{\sin(a+b\ln(c\,x^n))}\right)^{3/2}}\,dx$$

input
```
int(1/(x*(1/sin(a + b*log(c*x^n)))^(3/2)),x)
```

output
```
int(1/(x*(1/sin(a + b*log(c*x^n)))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{\sqrt{\csc(\log(x^nc)\,b+a)}}{\csc(\log(x^nc)\,b+a)^2\,x}\,dx$$

input
```
int(1/x/csc(a+b*log(c*x^n))^(3/2),x)
```

output
```
int(sqrt(csc(log(x**n*c)*b + a))/(csc(log(x**n*c)*b + a)**2*x),x)
```

3.318 $\int \frac{1}{\csc^{\frac{5}{2}}(a+b\log(cx^n))} dx$

Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \frac{1}{\csc^{\frac{5}{2}}(a+b\log(cx^n))} dx = \frac{2x \,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4}\left(-5-\frac{2i}{bn}\right),-\frac{2i+bn}{4bn},e^{2ia}(cx^n)^{2ib}\right)}{(2-5ibn)\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}\csc^{\frac{5}{2}}(a+b\log(cx^n))}$$

output
```
2*x*hypergeom([-5/2, -5/4-1/2*I/b/n],[-1/4*(2*I+b*n)/b/n],exp(2*I*a)*(c*x^
n)^(2*I*b))/(2-5*I*b*n)/(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(5/2)/csc(a+b*ln(c*
x^n))^(5/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 579 vs. 2(110) = 220.

Time = 6.91 (sec) , antiderivative size = 579, normalized size of antiderivative = 5.26

$$\int \frac{1}{\csc^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$$
$$= \frac{x\left(-\frac{60b^3e^{ia}n^3(cx^n)^{ib}\sqrt{2-2e^{2ia}(cx^n)^{2ib}}\sqrt{\frac{ie^{ia}(cx^n)^{ib}}{-1+e^{2ia}(cx^n)^{2ib}}}\left((2i+bn)x^{2ibn}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4}-\frac{i}{2bn},\frac{7}{4}-\frac{i}{2bn},e^{2ia}(cx^n)^{2ib}\right)+(-2i+3b\right.}{(2i+bn)(-2i+3bn)\left((2i+bn)x^{2ibn}+e^{2ia}(-2i+bn)(cx^n)^{2ib}\right)}\right.}{}$$

input
```
Integrate[Csc[a + b*Log[c*x^n]]^(-5/2),x]
```

output
```
(x*((-60*b^3*E^(I*a)*n^3*(c*x^n)^(I*b)*Sqrt[2 - 2*E^((2*I)*a)*(c*x^n)^((2*
I)*b)]*Sqrt[(I*E^(I*a)*(c*x^n)^(I*b))/(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))
]*((2*I + b*n)*x^((2*I)*b*n)*Hypergeometric2F1[1/2, 3/4 - (I/2)/(b*n), 7/4
 - (I/2)/(b*n), E^((2*I)*a)*(c*x^n)^((2*I)*b)] + (-2*I + 3*b*n)*Hypergeome
tric2F1[1/2, -1/4*(2*I + b*n)/(b*n), 3/4 - (I/2)/(b*n), E^((2*I)*a)*(c*x^n
)^((2*I)*b)]))/((2*I + b*n)*(-2*I + 3*b*n)*((2*I + b*n)*x^((2*I)*b*n) + E^
((2*I)*a)*(-2*I + b*n)*(c*x^n)^((2*I)*b))) + (4*b*n*Cos[a - b*n*Log[x] + b
*Log[c*x^n]] - 12*b*n*Cos[a + b*n*Log[x] + b*Log[c*x^n]] + 8*b*n*Cos[b*n*L
og[x] - 3*(a + b*Log[c*x^n])] + 8*Sin[a - b*n*Log[x] + b*Log[c*x^n]] + 60*
b^2*n^2*Sin[a - b*n*Log[x] + b*Log[c*x^n]] + 4*Sin[a + b*n*Log[x] + b*Log[
c*x^n]] - 5*b^2*n^2*Sin[a + b*n*Log[x] + b*Log[c*x^n]] - 4*Sin[3*a - b*n*L
og[x] + 3*b*Log[c*x^n]] - 5*b^2*n^2*Sin[3*a - b*n*Log[x] + 3*b*Log[c*x^n]]
)/(Sqrt[Csc[a + b*Log[c*x^n]]]*(b*n*Cos[a - b*n*Log[x] + b*Log[c*x^n]] + 2
*Sin[a - b*n*Log[x] + b*Log[c*x^n]]))))/(2*(4 + 25*b^2*n^2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5015, 5019, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\csc^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$$

$$\Big\downarrow \text{5015}$$

$$\frac{x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}-1}}{\csc^{\frac{5}{2}}(a+b\log(cx^n))}d(cx^n)}{n}$$

$$\Big\downarrow \text{5019}$$

$$\frac{x(cx^n)^{-\frac{1}{n}+\frac{5ib}{2}}\int (cx^n)^{-\frac{5ib}{2}+\frac{1}{n}-1}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}d(cx^n)}{n\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{5/2}\csc^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4}\left(-5-\frac{2i}{bn}\right),-\frac{bn+2i}{4bn},e^{2ia}(cx^n)^{2ib}\right)}{(2-5ibn)\left(1-e^{2ia}\left(cx^n\right)^{2ib}\right)^{5/2}\csc^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}$$

input

```
Int[Csc[a + b*Log[c*x^n]]^(-5/2),x]
```

output

```
(2*x*Hypergeometric2F1[-5/2, (-5 - (2*I)/(b*n))/4, -1/4*(2*I + b*n)/(b*n),
 E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((2 - (5*I)*b*n)*(1 - E^((2*I)*a)*(c*x^n)
^((2*I)*b))^(5/2)*Csc[a + b*Log[c*x^n]]^(5/2))
```

Defintions of rubi rules used

rule 888

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5015

```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5019
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\csc\left(a+b\ln\left(c\,x^{n}\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/csc(a+b*ln(c*x^n))^(5/2),x)
```

output
```
int(1/csc(a+b*ln(c*x^n))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\csc^{\frac{5}{2}}\left(a+b\log\left(cx^{n}\right)\right)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/csc(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\csc^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/csc(a+b*ln(c*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\csc^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{\csc(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/csc(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(csc(b*log(c*x^n) + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\csc^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{\csc(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/csc(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

output
```
integrate(csc(b*log(c*x^n) + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\csc^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{\left(\frac{1}{\sin(a+b\,\ln(c\,x^n))}\right)^{5/2}}\,dx$$

input
```
int(1/(1/sin(a + b*log(c*x^n)))^(5/2),x)
```

output
```
int(1/(1/sin(a + b*log(c*x^n)))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\csc^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{\sqrt{\csc(\log(x^n c)\,b+a)}}{\csc(\log(x^n c)\,b+a)^3}dx$$

input
```
int(1/csc(a+b*log(c*x^n))^(5/2),x)
```

output
```
int(sqrt(csc(log(x**n*c)*b + a))/csc(log(x**n*c)*b + a)**3,x)
```

3.319 $\int \frac{1}{x \csc^{\frac{5}{2}}(a+b\log(cx^n))} dx$

Optimal result

Integrand size = 19, antiderivative size = 99

$$\int \frac{1}{x \csc^{\frac{5}{2}}(a+b\log(cx^n))} dx = -\frac{2\cos(a+b\log(cx^n))}{5bn\csc^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{6\sqrt{\csc(a+b\log(cx^n))}E\left(\frac{1}{4}(2a-\pi+2b\log(cx^n))\middle|2\right)\sqrt{\sin(a+b\log(cx^n))}}{5bn}$$

output
```
-2/5*cos(a+b*ln(c*x^n))/b/n/csc(a+b*ln(c*x^n))^(3/2)-6/5*csc(a+b*ln(c*x^n)
)^(1/2)*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*ln(c*x^n)),2^(1/2))*sin(a+b*ln(c*
x^n))^(1/2)/b/n
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

$$\int \frac{1}{x \csc^{\frac{5}{2}}(a+b\log(cx^n))} dx = -\frac{2\sqrt{\csc(a+b\log(cx^n))}\left(3E\left(\frac{1}{4}(-2a+\pi-2b\log(cx^n))\middle|2\right)\sqrt{\sin(a+b\log(cx^n))}+\cos(a+b\log(cx^n

input
```
Integrate[1/(x*Csc[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
(-2*Sqrt[Csc[a + b*Log[c*x^n]]]*(3*EllipticE[(-2*a + Pi - 2*b*Log[c*x^n])/
4, 2]*Sqrt[Sin[a + b*Log[c*x^n]]] + Cos[a + b*Log[c*x^n]]*Sin[a + b*Log[c*
x^n]]^2))/(5*b*n)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3039, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\csc^{\frac{5}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\csc^{\frac{5}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\csc(a+b\log(cx^n))^{5/2}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{4256}$$

$$\frac{\frac{3}{5}\int \frac{1}{\sqrt{\csc(a+b\log(cx^n))}}d\log(cx^n)-\frac{2\cos(a+b\log(cx^n))}{5b\csc^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{5}\int \frac{1}{\sqrt{\csc(a+b\log(cx^n))}}d\log(cx^n)-\frac{2\cos(a+b\log(cx^n))}{5b\csc^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$$\Big\downarrow \text{4258}$$

$$\frac{\frac{3}{5}\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}\int\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}d\log\left(cx^n\right)-\frac{2\cos(a+b\log(cx^n))}{5b\csc^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$\Big\downarrow$ 3042

$$\frac{\frac{3}{5}\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}\int\sqrt{\sin\left(a+b\log\left(cx^n\right)\right)}d\log\left(cx^n\right)-\frac{2\cos(a+b\log(cx^n))}{5b\csc^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$\Big\downarrow$ 3119

$$\frac{\frac{6\sqrt{\sin(a+b\log(cx^n))}\sqrt{\csc(a+b\log(cx^n))}E\left(\frac{1}{2}\left(a+b\log(cx^n)-\frac{\pi}{2}\right)|2\right)}{5b}-\frac{2\cos(a+b\log(cx^n))}{5b\csc^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

input
```
Int[1/(x*Csc[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
((-2*Cos[a + b*Log[c*x^n]])/(5*b*Csc[a + b*Log[c*x^n]]^(3/2)) + (6*Sqrt[Cs
c[a + b*Log[c*x^n]]]*EllipticE[(a - Pi/2 + b*Log[c*x^n])/2, 2]*Sqrt[Sin[a
+ b*Log[c*x^n]]])/(5*b))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(88) = 176$.

Time = 0.68 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.07

method	result
derivativedivides	$\frac{\frac{2\sin(a+b\ln(cx^n))^4}{5}-\frac{2\sin(a+b\ln(cx^n))^2}{5}-\frac{6\sqrt{\sin(a+b\ln(cx^n))+1}\,\sqrt{-2\sin(a+b\ln(cx^n))+2}\,\sqrt{-\sin(a+b\ln(cx^n))}\ \text{EllipticE}\left(\sqrt{\text{si}\right.}{5}}{n\cos(a+b\ln(cx^n}$
default	$\frac{\frac{2\sin(a+b\ln(cx^n))^4}{5}-\frac{2\sin(a+b\ln(cx^n))^2}{5}-\frac{6\sqrt{\sin(a+b\ln(cx^n))+1}\,\sqrt{-2\sin(a+b\ln(cx^n))+2}\,\sqrt{-\sin(a+b\ln(cx^n))}\ \text{EllipticE}\left(\sqrt{\text{si}\right.}{5}}{n\cos(a+b\ln(cx^n}$

input
```
int(1/x/csc(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/n*(2/5*sin(a+b*ln(c*x^n))^4-2/5*sin(a+b*ln(c*x^n))^2-6/5*(sin(a+b*ln(c*x
^n))+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin(a+b*ln(c*x^n)))^(1/2)*
EllipticE((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2))+3/5*(sin(a+b*ln(c*x^n)
)+1)^(1/2)*(-2*sin(a+b*ln(c*x^n))+2)^(1/2)*(-sin(a+b*ln(c*x^n)))^(1/2)*Ell
ipticF((sin(a+b*ln(c*x^n))+1)^(1/2),1/2*2^(1/2)))/cos(a+b*ln(c*x^n))/sin(a
+b*ln(c*x^n))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.31

$$\int \frac{1}{x\csc^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$$
$$= \frac{3\sqrt{2i}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos(bn\log(x)+b\log(c)+a)+i\sin(bn\log(x)+b$$

input
```
integrate(1/x/csc(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

output
```
1/5*(3*sqrt(2*I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*n*l
og(x) + b*log(c) + a) + I*sin(b*n*log(x) + b*log(c) + a))) + 3*sqrt(-2*I)*
weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*n*log(x) + b*log(c)
+ a) - I*sin(b*n*log(x) + b*log(c) + a))) + 2*(cos(b*n*log(x) + b*log(c) +
 a)^3 - cos(b*n*log(x) + b*log(c) + a))/sqrt(sin(b*n*log(x) + b*log(c) + a
)))/(b*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\csc^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

input
```
integrate(1/x/csc(a+b*ln(c*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x\csc^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{1}{x\csc\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/csc(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*csc(b*log(c*x^n) + a)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{x\csc^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{1}{x\csc\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/csc(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/(x*csc(b*log(c*x^n) + a)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\csc^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{1}{x\,{\left(\frac{1}{\sin\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}^{5/2}}\,dx$$

input
```
int(1/(x*(1/sin(a + b*log(c*x^n)))^(5/2)),x)
```

output
```
int(1/(x*(1/sin(a + b*log(c*x^n)))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{x \csc^{\frac{5}{2}}(a + b \log(cx^n))} \, dx = \int \frac{\sqrt{\csc(\log(x^n c) b + a)}}{\csc(\log(x^n c) b + a)^3 x} dx$$

input
```
int(1/x/csc(a+b*log(c*x^n))^(5/2),x)
```

output
```
int(sqrt(csc(log(x**n*c)*b + a))/(csc(log(x**n*c)*b + a)**3*x),x)
```

3.320 $\int (ex)^m \csc^3(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 122

$$\int (ex)^m \csc^3(d(a+b\log(cx^n)))\,dx = -\frac{8e^{3iad}(ex)^{1+m}(cx^n)^{3ibd}\,\text{Hypergeometric2F1}\left(3,-\frac{i(1+m)-3bdn}{2bdn},-\frac{i(1+m)-5bdn}{2bdn},e^{2iad}(cx^n)^{2ibd}\right)}{e(i(1+m)-3bdn)}$$

output
```
-8*exp(3*I*a*d)*(e*x)^(1+m)*(c*x^n)^(3*I*b*d)*hypergeom([3, -1/2*(I*(1+m)-
3*b*d*n)/b/d/n],[-1/2*(I*(1+m)-5*b*d*n)/b/d/n],exp(2*I*a*d)*(c*x^n)^(2*I*b
*d))/e/(I*(1+m)-3*b*d*n)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 367 vs. 2(122) = 244.

Time = 1.69 (sec) , antiderivative size = 367, normalized size of antiderivative = 3.01

$$\int (ex)^m \csc^3(d(a+b\log(cx^n)))\,dx$$

$$= \frac{x(ex)^m\left(-bdn\csc^2\left(\frac{1}{2}d(a+b\log(cx^n))\right)-4(1+m)\csc(d(a-bn\log(x)+b\log(cx^n)))+bdn\sec^2\left(\frac{1}{2}d(\right.\right.}{}$$

input
```
Integrate[(e*x)^m*Csc[d*(a + b*Log[c*x^n])]^3,x]
```

output
```
(x*(e*x)^m*(-(b*d*n*Csc[(d*(a + b*Log[c*x^n]))/2]^2) - 4*(1 + m)*Csc[d*(a
- b*n*Log[x] + b*Log[c*x^n])] + b*d*n*Sec[(d*(a + b*Log[c*x^n]))/2]^2 + 2*
(1 + m)*Csc[(d*(a + b*Log[c*x^n]))/2]*Csc[(d*(a - b*n*Log[x] + b*Log[c*x^n
]))/2]*Sin[(b*d*n*Log[x])/2] - 2*(1 + m)*Sec[(d*(a + b*Log[c*x^n]))/2]*Sec
[(d*(a - b*n*Log[x] + b*Log[c*x^n]))/2]*Sin[(b*d*n*Log[x])/2] + 8*(1 + m -
 I*b*d*n)*x^(I*b*d*n)*Hypergeometric2F1[1, (-I - I*m + b*d*n)/(2*b*d*n), (
(-1/2*I)*(1 + m + (3*I)*b*d*n))/(b*d*n), x^((2*I)*b*d*n)*(Cos[2*d*(a - b*n
*Log[x] + b*Log[c*x^n])] + I*Sin[2*d*(a - b*n*Log[x] + b*Log[c*x^n])])]*((
-I)*Cos[d*(a - b*n*Log[x] + b*Log[c*x^n])] + Sin[d*(a - b*n*Log[x] + b*Log
[c*x^n])])))/(8*b^2*d^2*n^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5021, 5017, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \csc^3(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{5021}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\csc^3(d(a+b\log(cx^n)))\,d(cx^n)}{en}$$

$$\Big\downarrow \text{5017}$$

$$\frac{8ie^{3iad}(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int \frac{(cx^n)^{3ibd+\frac{m+1}{n}-1}}{\left(1-e^{2iad}(cx^n)^{2ibd}\right)^3}d(cx^n)}{en}$$

$$\Big\downarrow \text{888}$$

$$\frac{8ie^{3iad}(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}+\frac{3ibdn+m+1}{n}}\text{Hypergeometric2F1}\left(3,-\frac{i(m+1)-3bdn}{2bdn},-\frac{i(m+1)-5bdn}{2bdn},e^{2iad}(cx^n)^{2ibd}\right)}{e(3ibdn+m+1)}$$

input
```
Int[(e*x)^m*Csc[d*(a + b*Log[c*x^n])]^3,x]
```

output
```
((8*I)*E^((3*I)*a*d)*(e*x)^(1 + m)*(c*x^n)^(-((1 + m)/n) + (1 + m + (3*I)*
b*d*n)/n)*Hypergeometric2F1[3, -1/2*(I*(1 + m) - 3*b*d*n)/(b*d*n), -1/2*(I
*(1 + m) - 5*b*d*n)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(e*(1 + m
 + (3*I)*b*d*n))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5017
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[(-2*I)^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^
(2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5021
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int (ex)^m \csc (d(a+b\ln (c\,x^n)))^3 dx$$

input
```
int((e*x)^m*csc(d*(a+b*ln(c*x^n)))^3,x)
```

output
```
int((e*x)^m*csc(d*(a+b*ln(c*x^n)))^3,x)
```

Fricas [F]

$$\int (ex)^m \csc^3(d(a+b\log(cx^n)))\,dx = \int (ex)^m \csc\left((b\log(cx^n)+a)d\right)^3\,dx$$

input
```
integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^3,x, algorithm="fricas")
```

output
```
integral((e*x)^m*csc(b*d*log(c*x^n) + a*d)^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \csc^3(d(a+b\log(cx^n)))\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m*csc(d*(a+b*ln(c*x**n)))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m \csc^3(d(a+b\log(cx^n)))\,dx = \int (ex)^m \csc\left((b\log(cx^n)+a)d\right)^3\,dx$$

input
```
integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^3,x, algorithm="maxima")
```

output
```
-((b*d*e^m*n*cos(b*d*log(c)) - e^m*m*sin(b*d*log(c)) - e^m*sin(b*d*log(c))
)*x*x^m*cos(b*d*log(x^n) + a*d) - (b*d*e^m*n*sin(b*d*log(c)) + e^m*m*cos(b
*d*log(c)) + e^m*cos(b*d*log(c)))*x*x^m*sin(b*d*log(x^n) + a*d) - (((cos(3
*b*d*log(c))*sin(4*b*d*log(c)) - cos(4*b*d*log(c))*sin(3*b*d*log(c)))*e^m*
m - (b*d*cos(4*b*d*log(c))*cos(3*b*d*log(c)) + b*d*sin(4*b*d*log(c))*sin(3
*b*d*log(c)))*e^m*n + (cos(3*b*d*log(c))*sin(4*b*d*log(c)) - cos(4*b*d*log
(c))*sin(3*b*d*log(c)))*e^m)*x*x^m*cos(3*b*d*log(x^n) + 3*a*d) - ((cos(b*d
*log(c))*sin(4*b*d*log(c)) - cos(4*b*d*log(c))*sin(b*d*log(c)))*e^m*m + (b
*d*cos(4*b*d*log(c))*cos(b*d*log(c)) + b*d*sin(4*b*d*log(c))*sin(b*d*log(c
)))*e^m*n + (cos(b*d*log(c))*sin(4*b*d*log(c)) - cos(4*b*d*log(c))*sin(b*d
*log(c)))*e^m)*x*x^m*cos(b*d*log(x^n) + a*d) - ((cos(4*b*d*log(c))*cos(3*b
*d*log(c)) + sin(4*b*d*log(c))*sin(3*b*d*log(c)))*e^m*m + (b*d*cos(3*b*d*l
og(c))*sin(4*b*d*log(c)) - b*d*cos(4*b*d*log(c))*sin(3*b*d*log(c)))*e^m*n
+ (cos(4*b*d*log(c))*cos(3*b*d*log(c)) + sin(4*b*d*log(c))*sin(3*b*d*log(c
)))*e^m)*x*x^m*sin(3*b*d*log(x^n) + 3*a*d) + ((cos(4*b*d*log(c))*cos(b*d*l
og(c)) + sin(4*b*d*log(c))*sin(b*d*log(c)))*e^m*m - (b*d*cos(b*d*log(c))*s
in(4*b*d*log(c)) - b*d*cos(4*b*d*log(c))*sin(b*d*log(c)))*e^m*n + (cos(4*b
*d*log(c))*cos(b*d*log(c)) + sin(4*b*d*log(c))*sin(b*d*log(c)))*e^m)*x*x^m
*sin(b*d*log(x^n) + a*d))*cos(4*b*d*log(x^n) + 4*a*d) - (2*((cos(2*b*d*log
(c))*sin(3*b*d*log(c)) - cos(3*b*d*log(c))*sin(2*b*d*log(c)))*e^m*m + (...
```

Giac [F]

$$\int (ex)^m \csc^3 (d(a + b\log(cx^n)))\ dx = \int (ex)^m \csc((b\log(cx^n) + a)d)^3\ dx$$

input
```
integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^3,x, algorithm="giac")
```

output
```
integrate((e*x)^m*csc((b*log(c*x^n) + a)*d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \csc^3(d(a+b\log(cx^n)))\,dx = \int \frac{(e\,x)^m}{\sin\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)^3}\,dx$$

input
```
int((e*x)^m/sin(d*(a + b*log(c*x^n)))^3,x)
```

output
```
int((e*x)^m/sin(d*(a + b*log(c*x^n)))^3, x)
```

Reduce [F]

$$\int (ex)^m \csc^3(d(a+b\log(cx^n)))\,dx = e^m\left(\int x^m \csc\left(\log(x^n c)\,bd+ad\right)^3 dx\right)$$

input
```
int((e*x)^m*csc(d*(a+b*log(c*x^n)))^3,x)
```

output
```
e**m*int(x**m*csc(log(x**n*c)*b*d + a*d)**3,x)
```

3.321 $\int (ex)^m \csc^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 119

$$\int (ex)^m \csc^2(d(a+b\log(cx^n)))\,dx = -\frac{4e^{2iad}(ex)^{1+m}(cx^n)^{2ibd}\,\text{Hypergeometric2F1}\left(2,-\frac{i(1+m)-2bdn}{2bdn},-\frac{i(1+m)-4bdn}{2bdn},e^{2iad}(cx^n)^{2ibd}\right)}{e(1+m+2ibdn)}$$

output
```
-4*exp(2*I*a*d)*(e*x)^(1+m)*(c*x^n)^(2*I*b*d)*hypergeom([2, -1/2*(I*(1+m)-
2*b*d*n)/b/d/n],[-1/2*(I*(1+m)-4*b*d*n)/b/d/n],exp(2*I*a*d)*(c*x^n)^(2*I*b
*d))/e/(1+m+2*I*b*d*n)
```

Mathematica [A] (verified)

Time = 13.02 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.89

$$\int (ex)^m \csc^2(d(a+b\log(cx^n)))\,dx = -\frac{x(ex)^m\left((1+m+2ibdn)\cot(d(a+b\log(cx^n)))+i(1+m+2ibdn)\,\text{Hypergeometric2F1}\left(1,-\frac{i(1+m)}{2bdn}\right.\right.}{}$$

input
```
Integrate[(e*x)^m*Csc[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
-((x*(e*x)^m*((1 + m + (2*I)*b*d*n)*Cot[d*(a + b*Log[c*x^n])] + I*(1 + m +
 (2*I)*b*d*n)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m))/(b*d*n), 1 - ((I/2)*
(1 + m))/(b*d*n), E^((2*I)*d*(a + b*Log[c*x^n]))] + I*E^((2*I)*a*d)*(1 + m
)*(c*x^n)^((2*I)*b*d)*Hypergeometric2F1[1, ((-1/2*I)*(1 + m + (2*I)*b*d*n)
)/(b*d*n), ((-1/2*I)*(1 + m + (4*I)*b*d*n))/(b*d*n), E^((2*I)*a*d)*(c*x^n)
^((2*I)*b*d)]))/(b*d*n*(1 + m + (2*I)*b*d*n)))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5021, 5017, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \csc^2(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{5021}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\csc^2(d(a+b\log(cx^n)))d(cx^n)}{en}$$

$$\Big\downarrow \text{5017}$$

$$-\frac{4e^{2iad}(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int \frac{(cx^n)^{2ibd+\frac{m+1}{n}-1}}{\left(1-e^{2iad}(cx^n)^{2ibd}\right)^2}d(cx^n)}{en}$$

$$\Big\downarrow \text{888}$$

$$-\frac{4e^{2iad}(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}+\frac{2ibdn+m+1}{n}}\text{Hypergeometric2F1}\left(2,-\frac{i(m+1)-2bdn}{2bdn},-\frac{i(m+1)-4bdn}{2bdn},e^{2iad}(cx^n)^{2ibd}\right)}{e(2ibdn+m+1)}$$

input
```
Int[(e*x)^m*Csc[d*(a + b*Log[c*x^n])]^2,x]
```

output
```
(-4*E^((2*I)*a*d)*(e*x)^(1 + m)*(c*x^n)^(-((1 + m)/n) + (1 + m + (2*I)*b*d
*n)/n)*Hypergeometric2F1[2, -1/2*(I*(1 + m) - 2*b*d*n)/(b*d*n), -1/2*(I*(1
 + m) - 4*b*d*n)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(e*(1 + m +
(2*I)*b*d*n))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5017
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[(-2*I)^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^
(2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5021
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int (ex)^m \csc \left(d(a+b\ln \left(c\,x^n\right))\right)^2dx$$

input
```
int((e*x)^m*csc(d*(a+b*ln(c*x^n)))^2,x)
```

output
```
int((e*x)^m*csc(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int (ex)^m \csc^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int (ex)^m \csc\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

input
```
integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

output
```
integral((e*x)^m*csc(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int (ex)^m \csc^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int (ex)^m \csc^2\left(ad+bd\log\left(cx^n\right)\right)dx$$

input
```
integrate((e*x)**m*csc(d*(a+b*ln(c*x**n)))**2,x)
```

output
```
Integral((e*x)**m*csc(a*d + b*d*log(c*x**n))**2, x)
```

Maxima [F]

$$\int (ex)^m \csc^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int (ex)^m \csc\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

input
```
integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

output
```
(2*e^m*x*x^m*cos(2*b*d*log(x^n) + 2*a*d)*sin(2*b*d*log(c)) + 2*e^m*x*x^m*c
os(2*b*d*log(c))*sin(2*b*d*log(x^n) + 2*a*d) + (((b^2*d^2*cos(2*b*d*log(c)
)^2 + b^2*d^2*sin(2*b*d*log(c))^2)*e^m*m + (b^2*d^2*cos(2*b*d*log(c))^2 +
b^2*d^2*sin(2*b*d*log(c))^2)*e^m)*n^2*cos(2*b*d*log(x^n) + 2*a*d)^2 + ((b^
2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*e^m*m + (b^2*d^2*
cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*e^m)*n^2*sin(2*b*d*log(
x^n) + 2*a*d)^2 - 2*(b^2*d^2*e^m*m*cos(2*b*d*log(c)) + b^2*d^2*e^m*cos(2*b
*d*log(c)))*n^2*cos(2*b*d*log(x^n) + 2*a*d) + 2*(b^2*d^2*e^m*m*sin(2*b*d*l
og(c)) + b^2*d^2*e^m*sin(2*b*d*log(c)))*n^2*sin(2*b*d*log(x^n) + 2*a*d) +
(b^2*d^2*e^m*m + b^2*d^2*e^m)*n^2)*integrate((x^m*cos(b*d*log(x^n) + a*d)*
sin(b*d*log(c)) + x^m*cos(b*d*log(c))*sin(b*d*log(x^n) + a*d))/(2*b^2*d^2*
n^2*cos(b*d*log(c))*cos(b*d*log(x^n) + a*d) - 2*b^2*d^2*n^2*sin(b*d*log(c)
)*sin(b*d*log(x^n) + a*d) + b^2*d^2*n^2 + (b^2*d^2*cos(b*d*log(c))^2 + b^2
*d^2*sin(b*d*log(c))^2)*n^2*cos(b*d*log(x^n) + a*d)^2 + (b^2*d^2*cos(b*d*l
og(c))^2 + b^2*d^2*sin(b*d*log(c))^2)*n^2*sin(b*d*log(x^n) + a*d)^2), x) -
 (((b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*e^m*m + (b^
2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c))^2)*e^m)*n^2*cos(2*b*
d*log(x^n) + 2*a*d)^2 + ((b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*
log(c))^2)*e^m*m + (b^2*d^2*cos(2*b*d*log(c))^2 + b^2*d^2*sin(2*b*d*log(c)
)^2)*e^m)*n^2*sin(2*b*d*log(x^n) + 2*a*d)^2 - 2*(b^2*d^2*e^m*m*cos(2*b*...
```

Giac [F]

$$\int (ex)^m \csc^2(d(a+b\log(cx^n)))\,dx = \int (ex)^m \csc((b\log(cx^n)+a)d)^2\,dx$$

input
```
integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

output
```
integrate((e*x)^m*csc((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \csc^2(d(a+b\log(cx^n)))\,dx = \int \frac{(e\,x)^m}{\sin\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)^2}\,dx$$

input
```
int((e*x)^m/sin(d*(a + b*log(c*x^n)))^2,x)
```

output
```
int((e*x)^m/sin(d*(a + b*log(c*x^n)))^2, x)
```

Reduce [F]

$$\int (ex)^m \csc^2(d(a+b\log(cx^n)))\,dx = e^m\left(\int x^m \csc\left(\log(x^n c)\,bd+ad\right)^2 dx\right)$$

input
```
int((e*x)^m*csc(d*(a+b*log(c*x^n)))^2,x)
```

output
```
e**m*int(x**m*csc(log(x**n*c)*b*d + a*d)**2,x)
```

3.322 $\int (ex)^m \csc (d(a+b\log (cx^n)))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 123

$$\int (ex)^m \csc (d(a+b\log (cx^n)))\,dx = \frac{2e^{iad}(ex)^{1+m}\left(cx^n\right)^{ibd}\,\text{Hypergeometric2F1}\left(1,-\frac{i+im-bdn}{2bdn},-\frac{i(1+m)-3bdn}{2bdn},e^{2iad}\left(cx^n\right)^{2ibd}\right)}{e(i(1+m)-bdn)}$$

output
```
2*exp(I*a*d)*(e*x)^(1+m)*(c*x^n)^(I*b*d)*hypergeom([1, -1/2*(I+I*m-b*d*n)/
b/d/n],[-1/2*(I*(1+m)-3*b*d*n)/b/d/n],exp(2*I*a*d)*(c*x^n)^(2*I*b*d))/e/(I
*(1+m)-b*d*n)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.47

$$\int (ex)^m \csc (d(a+b\log (cx^n)))\,dx = \frac{2x^{1+ibdn}(ex)^m\,\text{Hypergeometric2F1}\left(1,\frac{-i-im+bdn}{2bdn},-\frac{i(1+m+3ibdn)}{2bdn},x^{2ibdn}(\cos (2d(a+b(-n\log(x)+\log (cx\ldots\right.}{}$$

input
```
Integrate[(e*x)^m*Csc[d*(a + b*Log[c*x^n])],x]
```

output
```
(2*x^(1 + I*b*d*n)*(e*x)^m*Hypergeometric2F1[1, (-I - I*m + b*d*n)/(2*b*d*
n), ((-1/2*I)*(1 + m + (3*I)*b*d*n))/(b*d*n), x^((2*I)*b*d*n)*(Cos[2*d*(a
+ b*(-(n*Log[x]) + Log[c*x^n]))] + I*Sin[2*d*(a + b*(-(n*Log[x]) + Log[c*x
^n]))])]*((-I)*Cos[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))] + Sin[d*(a + b*(-
(n*Log[x]) + Log[c*x^n]))]))/(1 + m + I*b*d*n)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5021, 5017, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \csc (d(a+b\log (cx^n)))\, dx$$

$$\Big\downarrow \text{5021}$$

$$\frac{(ex)^{m+1} (cx^n)^{-\frac{m+1}{n}} \int (cx^n)^{\frac{m+1}{n}-1} \csc (d(a+b\log (cx^n)))\, d(cx^n)}{en}$$

$$\Big\downarrow \text{5017}$$

$$-\frac{2ie^{iad}(ex)^{m+1} (cx^n)^{-\frac{m+1}{n}} \int \frac{(cx^n)^{ibd+\frac{m+1}{n}-1}}{1-e^{2iad}(cx^n)^{2ibd}}d(cx^n)}{en}$$

$$\Big\downarrow \text{888}$$

$$-\frac{2ie^{iad}(ex)^{m+1} (cx^n)^{-\frac{m+1}{n}+\frac{ibdn+m+1}{n}} \text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1-\frac{i(m+1)}{bdn}\right),-\frac{i(m+1)-3bdn}{2bdn},e^{2iad}(cx^n)^{2ibd}\right)}{e(ibdn+m+1)}$$

input
```
Int[(e*x)^m*Csc[d*(a + b*Log[c*x^n])],x]
```

output
```
((-2*I)*E^(I*a*d)*(e*x)^(1 + m)*(c*x^n)^(-((1 + m)/n) + (1 + m + I*b*d*n)/
n)*Hypergeometric2F1[1, (1 - (I*(1 + m))/(b*d*n))/2, -1/2*(I*(1 + m) - 3*b
*d*n)/(b*d*n), E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(e*(1 + m + I*b*d*n))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5017
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[(-2*I)^p*E^(I*a*d*p)   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^
(2*I*b*d))^p), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

rule 5021
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int (ex)^m \csc \left(d(a + b\ln \left(c\,x^n\right))\right) dx$$

input
```
int((e*x)^m*csc(d*(a+b*ln(c*x^n))),x)
```

output
```
int((e*x)^m*csc(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int (ex)^m \csc \left(d(a + b\log \left(cx^n\right))\right)\,dx = \int (ex)^m \csc \left((b\log \left(cx^n\right) + a)d\right)\,dx$$

input
```
integrate((e*x)^m*csc(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

output
```
integral((e*x)^m*csc(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int (ex)^m \csc(d(a+b\log(cx^n)))\,dx = \int (ex)^m \csc(ad+bd\log(cx^n))\,dx$$

input
```
integrate((e*x)**m*csc(d*(a+b*ln(c*x**n))),x)
```

output
```
Integral((e*x)**m*csc(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int (ex)^m \csc(d(a+b\log(cx^n)))\,dx = \int (ex)^m \csc((b\log(cx^n)+a)d)\,dx$$

input
```
integrate((e*x)^m*csc(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

output
```
integrate((e*x)^m*csc((b*log(c*x^n) + a)*d), x)
```

Giac [F]

$$\int (ex)^m \csc(d(a+b\log(cx^n)))\,dx = \int (ex)^m \csc((b\log(cx^n)+a)d)\,dx$$

input
```
integrate((e*x)^m*csc(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

output
```
integrate((e*x)^m*csc((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \csc(d(a+b\log(cx^n)))\,dx = \int \frac{(e\,x)^m}{\sin\left(d\,\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}\,dx$$

input
```
int((e*x)^m/sin(d*(a + b*log(c*x^n))),x)
```

output
```
int((e*x)^m/sin(d*(a + b*log(c*x^n))), x)
```

Reduce [F]

$$\int (ex)^m \csc(d(a+b\log(cx^n)))\,dx = e^m\left(\int x^m \csc(\log(x^n c)\,bd+ad)\,dx\right)$$

input
```
int((e*x)^m*csc(d*(a+b*log(c*x^n))),x)
```

output
```
e**m*int(x**m*csc(log(x**n*c)*b*d + a*d),x)
```

3.323 $\int x^m \csc^{\frac{5}{2}}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int x^m \csc^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \frac{2x^{1+m}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}\csc^{\frac{5}{2}}(a+b\log(cx^n))\,\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{2i+2im-5bn}{4bn},-\frac{2i+2im-9bn}{4bn},e^{2ia}(\ldots\right.}{2+2m+5ibn}$$

output
```
2*x^(1+m)*(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(5/2)*csc(a+b*ln(c*x^n))^(5/2)*hy
pergeom([5/2, -1/4*(2*I+2*I*m-5*b*n)/b/n],[-1/4*(2*I+2*I*m-9*b*n)/b/n],exp
(2*I*a)*(c*x^n)^(2*I*b))/(2+2*m+5*I*b*n)
```

Mathematica [A] (warning: unable to verify)

Time = 2.18 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.27

$$\int x^m \csc^{\frac{5}{2}}(a+b\log(cx^n))\,dx = \frac{2x^{1+m}\sqrt{\csc(a+b\log(cx^n))}\left(-2-2m-bn\cot(a+b\log(cx^n))+e^{-2ia}(2+2m+ibn)(cx^n)^{-2ib}\left(-1+\ldots\right.\right.}{3b^2n^2}$$

input
```
Integrate[x^m*Csc[a + b*Log[c*x^n]]^(5/2),x]
```

output
```
(2*x^(1 + m)*Sqrt[Csc[a + b*Log[c*x^n]]]*(-2 - 2*m - b*n*Cot[a + b*Log[c*x
^n]] + ((2 + 2*m + I*b*n)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))*Hypergeomet
ric2F1[1, (2*I + (2*I)*m + 3*b*n)/(4*b*n), (2*I + (2*I)*m + 5*b*n)/(4*b*n)
, E^((-2*I)*(a + b*Log[c*x^n]))])/(E^((2*I)*a)*(c*x^n)^((2*I)*b))))/(3*b^2
*n^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5021, 5019, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \csc^{\frac{5}{2}}(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{5021}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int(cx^n)^{\frac{m+1}{n}-1}\csc^{\frac{5}{2}}(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5019}$$

$$\frac{x^{m+1}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}(cx^n)^{-\frac{m+1}{n}-\frac{5ib}{2}}\csc^{\frac{5}{2}}(a+b\log(cx^n))\int\frac{(cx^n)^{\frac{5ib}{2}+\frac{m+1}{n}-1}}{\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^{m+1}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1}{4}\left(5-\frac{2i(m+1)}{bn}\right),-\frac{2im-9bn+2i}{4bn},e^{2ia}(cx^n)^{2ib}\right)\csc^{\frac{5}{2}}(a+b\log$$
$$5ibn+2m+2$$

input
```
Int[x^m*Csc[a + b*Log[c*x^n]]^(5/2),x]
```

output
```
(2*x^(1 + m)*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^(5/2)*Csc[a + b*Log[c*x^n
]]^(5/2)*Hypergeometric2F1[5/2, (5 - ((2*I)*(1 + m))/(b*n))/4, -1/4*(2*I +
 (2*I)*m - 9*b*n)/(b*n), E^((2*I)*a)*(c*x^n)^((2*I)*b)])/(2 + 2*m + (5*I)*
b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5019
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 5021
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^m \csc\left(a+b\ln\left(c\,x^n\right)\right)^{\frac{5}{2}}dx$$

input
```
int(x^m*csc(a+b*ln(c*x^n))^(5/2),x)
```

output
```
int(x^m*csc(a+b*ln(c*x^n))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^m \csc^{\frac{5}{2}}(a + b \log(cx^n)) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*csc(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \csc^{\frac{5}{2}}(a + b \log(cx^n)) \, dx = \text{Timed out}$$

input
```
integrate(x**m*csc(a+b*ln(c*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^m \csc^{\frac{5}{2}}(a + b \log(cx^n)) \, dx = \int x^m \csc(b \log(cx^n) + a)^{\frac{5}{2}} \, dx$$

input
```
integrate(x^m*csc(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^m*csc(b*log(c*x^n) + a)^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int x^m \csc^{\frac{5}{2}}(a + b\log(cx^n))\,dx = \text{Timed out}$$

input
```
integrate(x^m*csc(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^m \csc^{\frac{5}{2}}(a + b\log(cx^n))\,dx = \int x^m \left(\frac{1}{\sin(a + b\ln(c\,x^n))}\right)^{5/2} dx$$

input
```
int(x^m*(1/sin(a + b*log(c*x^n)))^(5/2),x)
```

output
```
int(x^m*(1/sin(a + b*log(c*x^n)))^(5/2), x)
```

Reduce [F]

$$\int x^m \csc^{\frac{5}{2}}(a + b\log(cx^n))\,dx = \int x^m \sqrt{\csc(\log(x^n c)\,b + a)}\,\csc(\log(x^n c)\,b + a)^2 dx$$

input
```
int(x^m*csc(a+b*log(c*x^n))^(5/2),x)
```

output
```
int(x**m*sqrt(csc(log(x**n*c)*b + a))*csc(log(x**n*c)*b + a)**2,x)
```

3.324 $\int x^m \csc^{\frac{3}{2}}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int x^m \csc^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \frac{2x^{1+m}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}(a+b\log(cx^n))\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{2i+2im-3bn}{4bn},-\frac{2i+2im-7bn}{4bn},e^{2ia}(\ldots\right.}{2+2m+3ibn}$$

output
```
2*x^(1+m)*(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(3/2)*csc(a+b*ln(c*x^n))^(3/2)*hy
pergeom([3/2, -1/4*(2*I+2*I*m-3*b*n)/b/n],[-1/4*(2*I+2*I*m-7*b*n)/b/n],exp
(2*I*a)*(c*x^n)^(2*I*b))/(2+2*m+3*I*b*n)
```

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 466 vs. 2(130) = 260.

Time = 7.18 (sec) , antiderivative size = 466, normalized size of antiderivative = 3.58

$$\int x^m \csc^{\frac{3}{2}}(a+b\log(cx^n))\,dx = \frac{x^{1+m-ibn}\left(\left(4+8m+4m^2+b^2n^2\right)x^{2ibn}\sqrt{2-2e^{2ia}(cx^n)^{2ib}}\sqrt{\frac{ie^{ia}(cx^n)^{ib}}{-1+e^{2ia}(cx^n)^{2ib}}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{i(1\ldots}{\ldots}\right.\right.}{\ldots}$$

input
```
Integrate[x^m*Csc[a + b*Log[c*x^n]]^(3/2),x]
```

output
```
(x^(1 + m - I*b*n)*((4 + 8*m + 4*m^2 + b^2*n^2)*x^((2*I)*b*n)*Sqrt[2 - 2*E
^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[(I*E^(I*a)*(c*x^n)^(I*b))/(-1 + E^((2*I
)*a)*(c*x^n)^((2*I)*b))]*Hypergeometric2F1[1/2, ((-1/2*I)*(1 + m + ((3*I)/
2)*b*n))/(b*n), -1/4*(2*I + (2*I)*m - 7*b*n)/(b*n), E^((2*I)*a)*(c*x^n)^((
2*I)*b)] + (-2*I - (2*I)*m + 3*b*n)*((-2*I - (2*I)*m + b*n)*Sqrt[2 - 2*E^(
(2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[(I*E^(I*a)*(c*x^n)^(I*b))/(-1 + E^((2*I)*
a)*(c*x^n)^((2*I)*b))]*Hypergeometric2F1[1/2, -1/4*(2*I + (2*I)*m + b*n)/(
b*n), -1/4*(2*I + (2*I)*m - 3*b*n)/(b*n), E^((2*I)*a)*(c*x^n)^((2*I)*b)] -
 2*x^(I*b*n)*Sqrt[Csc[a + b*Log[c*x^n]]]*(b*n*Cos[b*n*Log[x]] - 2*(1 + m)*
Sin[b*n*Log[x]]))))/(b*n*(-2*I - (2*I)*m + 3*b*n)*(b*n*Cos[a - b*n*Log[x]
+ b*Log[c*x^n]] + 2*(1 + m)*Sin[a - b*n*Log[x] + b*Log[c*x^n]]))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5021, 5019, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \csc^{\frac{3}{2}}(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{5021}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\csc^{\frac{3}{2}}(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5019}$$

$$\frac{x^{m+1}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}(cx^n)^{-\frac{m+1}{n}-\frac{3ib}{2}}\csc^{\frac{3}{2}}(a+b\log(cx^n))\int \frac{(cx^n)^{\frac{3ib}{2}+\frac{m+1}{n}-1}}{\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^{m+1}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1}{4}\left(3-\frac{2i(m+1)}{bn}\right),-\frac{2im-7bn+2i}{4bn},e^{2ia}(cx^n)^{2ib}\right)\csc^{\frac{3}{2}}\left(a+b\log\left(\right.\right.}{3ibn+2m+2}$$

input
```
Int[x^m*Csc[a + b*Log[c*x^n]]^(3/2),x]
```

output
```
(2*x^(1 + m)*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2)*Csc[a + b*Log[c*x^n
]]^(3/2)*Hypergeometric2F1[3/2, (3 - ((2*I)*(1 + m))/(b*n))/4, -1/4*(2*I +
 (2*I)*m - 7*b*n)/(b*n), E^((2*I)*a)*(c*x^n)^((2*I)*b)])/(2 + 2*m + (3*I)*
b*n)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5019
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 5021
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^m \csc\left(a + b\ln\left(c\,x^n\right)\right)^{\frac{3}{2}}dx$$

input
```
int(x^m*csc(a+b*ln(c*x^n))^(3/2),x)
```

output
```
int(x^m*csc(a+b*ln(c*x^n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^m \csc^{\frac{3}{2}}\left(a + b\log\left(cx^n\right)\right)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*csc(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \csc^{\frac{3}{2}}\left(a + b\log\left(cx^n\right)\right)\,dx = \text{Timed out}$$

input
```
integrate(x**m*csc(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^m \csc^{\frac{3}{2}}(a+b\log(cx^n))\,dx=\int x^m \csc(b\log(cx^n)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^m*csc(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^m*csc(b*log(c*x^n) + a)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int x^m \csc^{\frac{3}{2}}(a+b\log(cx^n))\,dx=\text{Timed out}$$

input
```
integrate(x^m*csc(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int x^m \csc^{\frac{3}{2}}(a+b\log(cx^n))\,dx=\int x^m\left(\frac{1}{\sin(a+b\,\ln(c\,x^n))}\right)^{3/2}dx$$

input
```
int(x^m*(1/sin(a + b*log(c*x^n)))^(3/2),x)
```

output
```
int(x^m*(1/sin(a + b*log(c*x^n)))^(3/2), x)
```

Reduce [F]

$$\int x^m \csc^{\frac{3}{2}}(a + b\log(cx^n))\,dx = \int x^m \sqrt{\csc(\log(x^n c)\,b + a)}\,\csc(\log(x^n c)\,b + a)\,dx$$

input
```
int(x^m*csc(a+b*log(c*x^n))^(3/2),x)
```

output
```
int(x**m*sqrt(csc(log(x**n*c)*b + a))*csc(log(x**n*c)*b + a),x)
```

3.325 $\int x^m \sqrt{\csc(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int x^m \sqrt{\csc(a+b\log(cx^n))}\,dx = \frac{2x^{1+m}\sqrt{1-e^{2ia}(cx^n)^{2ib}}\sqrt{\csc(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{2i+2im-bn}{4bn},-\frac{2i+2im-5bn}{4bn},e^{2ia}(cx\ldots\right.}{2+2m+ibn}$$

output
```
2*x^(1+m)*(1-exp(2*I*a)*(c*x^n)^(2*I*b))^(1/2)*csc(a+b*ln(c*x^n))^(1/2)*hy
pergeom([1/2, -1/4*(2*I+2*I*m-b*n)/b/n],[-1/4*(2*I+2*I*m-5*b*n)/b/n],exp(2
*I*a)*(c*x^n)^(2*I*b))/(2+2*m+I*b*n)
```

Mathematica [A] (warning: unable to verify)

Time = 0.67 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.06

$$\int x^m \sqrt{\csc(a+b\log(cx^n))}\,dx = \frac{2e^{-2ia}x^{1+m}(cx^n)^{-2ib}\left(-1+e^{2ia}(cx^n)^{2ib}\right)\sqrt{\csc(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,\frac{2i+2im+3bn}{4bn},\frac{2i+2i\ldots}{4b\ldots}\right.}{2+2m-ibn}$$

input
```
Integrate[x^m*Sqrt[Csc[a + b*Log[c*x^n]]],x]
```

output
```
(2*x^(1 + m)*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))*Sqrt[Csc[a + b*Log[c*x^n
]]]*Hypergeometric2F1[1, (2*I + (2*I)*m + 3*b*n)/(4*b*n), (2*I + (2*I)*m +
 5*b*n)/(4*b*n), E^((-2*I)*(a + b*Log[c*x^n]))])/(E^((2*I)*a)*(2 + 2*m - I
*b*n)*(c*x^n)^((2*I)*b))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5021, 5019, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{\csc(a + b\log(cx^n))}\,dx$$

$$\Big\downarrow \text{5021}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}} \int (cx^n)^{\frac{m+1}{n}-1}\sqrt{\csc(a + b\log(cx^n))}d(cx^n)}{n}$$

$$\Big\downarrow \text{5019}$$

$$\frac{x^{m+1}\sqrt{1-e^{2ia}(cx^n)^{2ib}}(cx^n)^{-\frac{m+1}{n}-\frac{ib}{2}}\sqrt{\csc(a + b\log(cx^n))}\int \frac{(cx^n)^{\frac{ib}{2}+\frac{m+1}{n}-1}}{\sqrt{1-e^{2ia}(cx^n)^{2ib}}}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^{m+1}\sqrt{1-e^{2ia}(cx^n)^{2ib}}\,\text{Hypergeometric2F1}\left(\frac{1}{2}, -\frac{2im-bn+2i}{4bn}, -\frac{2im-5bn+2i}{4bn}, e^{2ia}(cx^n)^{2ib}\right)\sqrt{\csc(a + b\log(cx^n))}}{ibn+2m+2}$$

input
```
Int[x^m*Sqrt[Csc[a + b*Log[c*x^n]]],x]
```

output
```
(2*x^(1 + m)*Sqrt[1 - E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[Csc[a + b*Log[c*
x^n]]]*Hypergeometric2F1[1/2, -1/4*(2*I + (2*I)*m - b*n)/(b*n), -1/4*(2*I
+ (2*I)*m - 5*b*n)/(b*n), E^((2*I)*a)*(c*x^n)^((2*I)*b)])/(2 + 2*m + I*b*n
)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5019
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 5021
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x^m \sqrt{\csc\left(a+b\ln\left(c\,x^n\right)\right)}dx$$

input
```
int(x^m*csc(a+b*ln(c*x^n))^(1/2),x)
```

output
```
int(x^m*csc(a+b*ln(c*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^m \sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*csc(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^m \sqrt{\csc(a+b\log(cx^n))}\,dx = \int x^m \sqrt{\csc(a+b\log(cx^n))}\,dx$$

input
```
integrate(x**m*csc(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(x**m*sqrt(csc(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int x^m \sqrt{\csc(a+b\log(cx^n))}\,dx = \int x^m \sqrt{\csc(b\log(cx^n)+a)}\,dx$$

input
```
integrate(x^m*csc(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m*sqrt(csc(b*log(c*x^n) + a)), x)
```

Giac [F]

$$\int x^m \sqrt{\csc(a+b\log(cx^n))}\,dx = \int x^m \sqrt{\csc(b\log(cx^n)+a)}\,dx$$

input
```
integrate(x^m*csc(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m*sqrt(csc(b*log(c*x^n) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sqrt{\csc(a+b\log(cx^n))}\,dx = \int x^m \sqrt{\frac{1}{\sin(a+b\ln(c\,x^n))}}\,dx$$

input
```
int(x^m*(1/sin(a + b*log(c*x^n)))^(1/2),x)
```

output
```
int(x^m*(1/sin(a + b*log(c*x^n)))^(1/2), x)
```

Reduce [F]

$$\int x^m \sqrt{\csc(a+b\log(cx^n))}\,dx = \int x^m \sqrt{\csc(\log(x^n c)\,b+a)}dx$$

input
```
int(x^m*csc(a+b*log(c*x^n))^(1/2),x)
```

output
```
int(x**m*sqrt(csc(log(x**n*c)*b + a)),x)
```

3.326 $\int \frac{x^m}{\sqrt{\csc(a+b\log(cx^n))}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 129

$$\int \frac{x^m}{\sqrt{\csc(a+b\log(cx^n))}}\,dx = \frac{2x^{1+m}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{2i+2im+bn}{4bn},-\frac{2i+2im-3bn}{4bn},e^{2ia}(cx^n)^{2ib}\right)}{(2+2m-ibn)\sqrt{1-e^{2ia}(cx^n)^{2ib}}\sqrt{\csc(a+b\log(cx^n))}}$$

output
```
2*x^(1+m)*hypergeom([-1/2, -1/4*(2*I+2*I*m+b*n)/b/n],[-1/4*(2*I+2*I*m-3*b*
n)/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))/(2+2*m-I*b*n)/(1-exp(2*I*a)*(c*x^n)^(2
*I*b))^(1/2)/csc(a+b*ln(c*x^n))^(1/2)
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 441 vs. 2(129) = 258.

Time = 5.47 (sec) , antiderivative size = 441, normalized size of antiderivative = 3.42

$$\int \frac{x^m}{\sqrt{\csc(a+b\log(cx^n))}}\,dx =$$

$$-\frac{2be^{ia}nx^{1+m}(cx^n)^{ib}\sqrt{2-2e^{2ia}(cx^n)^{2ib}}\sqrt{\frac{ie^{ia}(cx^n)^{ib}}{-1+e^{2ia}(cx^n)^{2ib}}}\Big((2i+2im+bn)x^{2ibn}\,\text{Hypergeometric2F1}\Big(\frac{1}{2},}{(2+2m-ibn)(2+2m+3ibn}$$

$$+\frac{2x^{1+m}\sin(a-bn\log(x)+b\log(cx^n))}{\sqrt{\csc(a+b\log(cx^n))}\,(bn\cos(a-bn\log(x)+b\log(cx^n))+2(1+m)\sin(a-bn\log(x)+b\log(cx^n)}$$

input
```
Integrate[x^m/Sqrt[Csc[a + b*Log[c*x^n]]],x]
```

output
```
(-2*b*E^(I*a)*n*x^(1 + m)*(c*x^n)^(I*b)*Sqrt[2 - 2*E^((2*I)*a)*(c*x^n)^((2
*I)*b)]*Sqrt[(I*E^(I*a)*(c*x^n)^(I*b))/(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)
)]*((2*I + (2*I)*m + b*n)*x^((2*I)*b*n)*Hypergeometric2F1[1/2, ((-1/2*I)*(
1 + m + ((3*I)/2)*b*n))/(b*n), -1/4*(2*I + (2*I)*m - 7*b*n)/(b*n), E^((2*I
)*a)*(c*x^n)^((2*I)*b)] + (-2*I - (2*I)*m + 3*b*n)*Hypergeometric2F1[1/2,
-1/4*(2*I + (2*I)*m + b*n)/(b*n), -1/4*(2*I + (2*I)*m - 3*b*n)/(b*n), E^((
2*I)*a)*(c*x^n)^((2*I)*b)]))/((2 + 2*m - I*b*n)*(2 + 2*m + (3*I)*b*n)*((2*
I + (2*I)*m + b*n)*x^((2*I)*b*n) + E^((2*I)*a)*(-2*I - (2*I)*m + b*n)*(c*x
^n)^((2*I)*b))) + (2*x^(1 + m)*Sin[a - b*n*Log[x] + b*Log[c*x^n]])/(Sqrt[C
sc[a + b*Log[c*x^n]]]*(b*n*Cos[a - b*n*Log[x] + b*Log[c*x^n]] + 2*(1 + m)*
Sin[a - b*n*Log[x] + b*Log[c*x^n]]))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5021, 5019, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{\csc(a+b\log(cx^n))}}\,dx$$

$$\Big\downarrow \text{5021}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int\frac{(cx^n)^{\frac{m+1}{n}-1}}{\sqrt{\csc(a+b\log(cx^n))}}d(cx^n)}{n}$$

$\Big\downarrow$ 5019

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}+\frac{ib}{2}}\int(cx^n)^{-\frac{ib}{2}+\frac{m+1}{n}-1}\sqrt{1-e^{2ia}(cx^n)^{2ib}}d(cx^n)}{n\sqrt{1-e^{2ia}(cx^n)^{2ib}}\sqrt{\csc(a+b\log(cx^n))}}$$

$\Big\downarrow$ 888

$$\frac{2x^{m+1}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4}\left(-\frac{2i(m+1)}{bn}-1\right),-\frac{2im-3bn+2i}{4bn},e^{2ia}(cx^n)^{2ib}\right)}{(-ibn+2m+2)\sqrt{1-e^{2ia}(cx^n)^{2ib}}\sqrt{\csc(a+b\log(cx^n))}}$$

input
```
Int[x^m/Sqrt[Csc[a + b*Log[c*x^n]]],x]
```

output
```
(2*x^(1 + m)*Hypergeometric2F1[-1/2, (-1 - ((2*I)*(1 + m))/(b*n))/4, -1/4*
(2*I + (2*I)*m - 3*b*n)/(b*n), E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((2 + 2*m -
 I*b*n)*Sqrt[1 - E^((2*I)*a)*(c*x^n)^((2*I)*b)]*Sqrt[Csc[a + b*Log[c*x^n]]
])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5019
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 5021
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{x^m}{\sqrt{\csc\left(a+b\ln\left(c\,x^n\right)\right)}}dx$$

input
```
int(x^m/csc(a+b*ln(c*x^n))^(1/2),x)
```

output
```
int(x^m/csc(a+b*ln(c*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^m}{\sqrt{\csc\left(a+b\log\left(cx^n\right)\right)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/csc(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{x^m}{\sqrt{\csc(a+b\log(cx^n))}}\,dx = \int \frac{x^m}{\sqrt{\csc(a+b\log(cx^n))}}\,dx$$

input
```
integrate(x**m/csc(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(x**m/sqrt(csc(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \frac{x^m}{\sqrt{\csc(a+b\log(cx^n))}}\,dx = \int \frac{x^m}{\sqrt{\csc(b\log(cx^n)+a)}}\,dx$$

input
```
integrate(x^m/csc(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/sqrt(csc(b*log(c*x^n) + a)), x)
```

Giac [F]

$$\int \frac{x^m}{\sqrt{\csc(a+b\log(cx^n))}}\,dx = \int \frac{x^m}{\sqrt{\csc(b\log(cx^n)+a)}}\,dx$$

input
```
integrate(x^m/csc(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/sqrt(csc(b*log(c*x^n) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{\csc(a+b\log(cx^n))}}\,dx = \int \frac{x^m}{\sqrt{\frac{1}{\sin(a+b\ln(c\,x^n))}}}\,dx$$

input
```
int(x^m/(1/sin(a + b*log(c*x^n)))^(1/2),x)
```

output
```
int(x^m/(1/sin(a + b*log(c*x^n)))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^m}{\sqrt{\csc(a+b\log(cx^n))}}\,dx = \int \frac{x^m\sqrt{\csc(\log(x^n c)\,b+a)}}{\csc(\log(x^n c)\,b+a)}\,dx$$

input
```
int(x^m/csc(a+b*log(c*x^n))^(1/2),x)
```

output
```
int((x**m*sqrt(csc(log(x**n*c)*b + a)))/csc(log(x**n*c)*b + a),x)
```

3.327 $\int \frac{x^m}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{x^m}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{2x^{1+m}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{2i+2im+3bn}{4bn},-\frac{2i+2im-bn}{4bn},e^{2ia}(cx^n)^{2ib}\right)}{(2+2m-3ibn)\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}(a+b\log(cx^n))}$$

output
```
2*x^(1+m)*hypergeom([-3/2, -1/4*(2*I+2*I*m+3*b*n)/b/n],[-1/4*(2*I+2*I*m-b*
n)/b/n],exp(2*I*a)*(c*x^n)^(2*I*b))/(2+2*m-3*I*b*n)/(1-exp(2*I*a)*(c*x^n)^
(2*I*b))^(3/2)/csc(a+b*ln(c*x^n))^(3/2)
```

Mathematica [A] (warning: unable to verify)

Time = 1.60 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.68

$$\int \frac{x^m}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{2x^{1+m}\left((2+2m-ibn)\,(2+2m-3bn\cot(a+b\log(cx^n)))+3b^2e^{-2ia}n^2(cx^n)^{-2ib}\left(-1+e^{2ia}(cx^n)^{2ib}\right)\text{c}\ldots}{(2+2m-ibn)(2+2m-3ibn)(2+2m+3i\ldots}$$

input
```
Integrate[x^m/Csc[a + b*Log[c*x^n]]^(3/2),x]
```

output
```
(2*x^(1 + m)*((2 + 2*m - I*b*n)*(2 + 2*m - 3*b*n*Cot[a + b*Log[c*x^n]]) +
(3*b^2*n^2*(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b))*Csc[a + b*Log[c*x^n]]^2*Hy
pergeometric2F1[1, (2*I + (2*I)*m + 3*b*n)/(4*b*n), (2*I + (2*I)*m + 5*b*n
)/(4*b*n), E^((-2*I)*(a + b*Log[c*x^n]))])/(E^((2*I)*a)*(c*x^n)^((2*I)*b))
))/((2 + 2*m - I*b*n)*(2 + 2*m - (3*I)*b*n)*(2 + 2*m + (3*I)*b*n)*Csc[a +
b*Log[c*x^n]]^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5021, 5019, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{5021}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}}\int \frac{(cx^n)^{\frac{m+1}{n}-1}}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}d(cx^n)}{n}$$

$$\Big\downarrow \text{5019}$$

$$\frac{x^{m+1}(cx^n)^{-\frac{m+1}{n}+\frac{3ib}{2}}\int (cx^n)^{-\frac{3ib}{2}+\frac{m+1}{n}-1}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}d(cx^n)}{n\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}(a+b\log(cx^n))}$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^{m+1}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4}\left(-\frac{2i(m+1)}{bn}-3\right),-\frac{2im-bn+2i}{4bn},e^{2ia}(cx^n)^{2ib}\right)}{(-3ibn+2m+2)\left(1-e^{2ia}(cx^n)^{2ib}\right)^{3/2}\csc^{\frac{3}{2}}(a+b\log(cx^n))}$$

input
```
Int[x^m/Csc[a + b*Log[c*x^n]]^(3/2),x]
```

output
```
(2*x^(1 + m)*Hypergeometric2F1[-3/2, (-3 - ((2*I)*(1 + m))/(b*n))/4, -1/4*
(2*I + (2*I)*m - b*n)/(b*n), E^((2*I)*a)*(c*x^n)^((2*I)*b)])/((2 + 2*m - (
3*I)*b*n)*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^(3/2)*Csc[a + b*Log[c*x^n]]^
(3/2))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5019
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 5021
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int \frac{x^m}{\csc\left(a+b\ln\left(c\,x^n\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m/csc(a+b*ln(c*x^n))^(3/2),x)
```

output
```
int(x^m/csc(a+b*ln(c*x^n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^m}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/csc(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{x^m}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{x^m}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$

input
```
integrate(x**m/csc(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Integral(x**m/csc(a + b*log(c*x**n))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^m}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{x^m}{\csc(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/csc(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^m/csc(b*log(c*x^n) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^m}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{x^m}{\csc\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/csc(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m/csc(b*log(c*x^n) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{x^m}{\left(\frac{1}{\sin(a+b\,\ln(c\,x^n))}\right)^{3/2}}\,dx$$

input
```
int(x^m/(1/sin(a + b*log(c*x^n)))^(3/2),x)
```

output
```
int(x^m/(1/sin(a + b*log(c*x^n)))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^m}{\csc^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{x^m\sqrt{\csc\left(\log\left(x^nc\right)b+a\right)}}{\csc\left(\log\left(x^nc\right)b+a\right)^2}dx$$

input
```
int(x^m/csc(a+b*log(c*x^n))^(3/2),x)
```

output
```
int((x**m*sqrt(csc(log(x**n*c)*b + a)))/csc(log(x**n*c)*b + a)**2,x)
```

3.328 $\int (ex)^m \csc^p(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 139

$$\int (ex)^m \csc^p(d(a+b\log(cx^n)))\,dx = \frac{(ex)^{1+m}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^p \csc^p(d(a+b\log(cx^n)))\,\text{Hypergeometric2F1}\left(p,-\frac{i+im-bdnp}{2bdn},\frac{1}{2}\left(2-\frac{i(1+m)}{bdn}\right.\right.}{e(1+m+ibdnp)}$$

output
```
(e*x)^(1+m)*(1-exp(2*I*a*d)*(c*x^n)^(2*I*b*d))^p*csc(d*(a+b*ln(c*x^n)))^p*
hypergeom([p, -1/2*(I+I*m-b*d*n*p)/b/d/n],[1-1/2*I*(1+m)/b/d/n+1/2*p],exp(
2*I*a*d)*(c*x^n)^(2*I*b*d))/e/(1+m+I*b*d*n*p)
```

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.22

$$\int (ex)^m \csc^p(d(a+b\log(cx^n)))\,dx = \frac{x(ex)^m\left(2-2e^{2iad}(cx^n)^{2ibd}\right)^p\left(\frac{ie^{iad}(cx^n)^{ibd}}{-1+e^{2iad}(cx^n)^{2ibd}}\right)^p \text{Hypergeometric2F1}\left(p,-\frac{i(1+m+ibdnp)}{2bdn},\frac{1}{2}\left(2-\frac{i(1+m)}{bdn}+\right.\right.}{1+m+ibdnp}$$

input
```
Integrate[(e*x)^m*Csc[d*(a + b*Log[c*x^n])]^p,x]
```

output
```
(x*(e*x)^m*(2 - 2*E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p*((I*E^(I*a*d)*(c*x^
n)^(I*b*d))/(-1 + E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)))^p*Hypergeometric2F1[
p, ((-1/2*I)*(1 + m + I*b*d*n*p))/(b*d*n), (2 - (I*(1 + m))/(b*d*n) + p)/2
, E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(1 + m + I*b*d*n*p)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5021, 5019, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^m \csc^p(d(a+b\log(cx^n)))\,dx$$

$$\Big\downarrow \text{5021}$$

$$\frac{(ex)^{m+1}(cx^n)^{-\frac{m+1}{n}}\int (cx^n)^{\frac{m+1}{n}-1}\csc^p(d(a+b\log(cx^n)))\,d(cx^n)}{en}$$

$$\Big\downarrow \text{5019}$$

$$\frac{(ex)^{m+1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^p(cx^n)^{-\frac{m+1}{n}-ibdp}\csc^p(d(a+b\log(cx^n)))\int (cx^n)^{\frac{m+1}{n}+ibdp-1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^{-p}$$
$$en$$

$$\Big\downarrow \text{888}$$

$$\frac{(ex)^{m+1}\left(1-e^{2iad}(cx^n)^{2ibd}\right)^p(cx^n)^{\frac{ibdnp+m+1}{n}-ibdp-\frac{m+1}{n}}\text{Hypergeometric2F1}\left(p,\frac{1}{2}\left(p-\frac{i(m+1)}{bdn}\right),\frac{1}{2}\left(-\frac{i(m+1)}{bdn}+p\right.\right.$$
$$e(ibdnp+m+1)$$

input
```
Int[(e*x)^m*Csc[d*(a + b*Log[c*x^n])]^p,x]
```

output
```
((e*x)^(1 + m)*(c*x^n)^(-((1 + m)/n) - I*b*d*p + (1 + m + I*b*d*n*p)/n)*(1
 - E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d))^p*Csc[d*(a + b*Log[c*x^n])]^p*Hyperg
eometric2F1[p, (((-I)*(1 + m))/(b*d*n) + p)/2, (2 - (I*(1 + m))/(b*d*n) +
p)/2, E^((2*I)*a*d)*(c*x^n)^((2*I)*b*d)])/(e*(1 + m + I*b*d*n*p))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5019
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 5021
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int (ex)^m \csc (d(a+b\ln (c\,x^n)))^p dx$$

input
```
int((e*x)^m*csc(d*(a+b*ln(c*x^n)))^p,x)
```

output
```
int((e*x)^m*csc(d*(a+b*ln(c*x^n)))^p,x)
```

Fricas [F]

$$\int (ex)^m \csc^p(d(a+b\log(cx^n)))\ dx = \int (ex)^m \csc((b\log(cx^n)+a)d)^p\ dx$$

input
```
integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

output
```
integral((e*x)^m*csc(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F]

$$\int (ex)^m \csc^p (d(a+b\log(cx^n)))\ dx = \int (ex)^m \csc^p (ad+bd\log(cx^n))\ dx$$

input
```
integrate((e*x)**m*csc(d*(a+b*ln(c*x**n)))**p,x)
```

output
```
Integral((e*x)**m*csc(a*d + b*d*log(c*x**n))**p, x)
```

Maxima [F]

$$\int (ex)^m \csc^p (d(a+b\log(cx^n)))\ dx = \int (ex)^m \csc\left((b\log(cx^n)+a)d\right)^p\ dx$$

input
```
integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
```

output
```
integrate((e*x)^m*csc((b*log(c*x^n) + a)*d)^p, x)
```

Giac [F]

$$\int (ex)^m \csc^p (d(a+b\log(cx^n)))\ dx = \int (ex)^m \csc\left((b\log(cx^n)+a)d\right)^p\ dx$$

input
```
integrate((e*x)^m*csc(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
```

output
```
integrate((e*x)^m*csc((b*log(c*x^n) + a)*d)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \csc^p(d(a+b\log(cx^n)))\,dx = \int (e\,x)^m \left(\frac{1}{\sin(d\,(a+b\,\ln(c\,x^n)))}\right)^p dx$$

input
```
int((e*x)^m*(1/sin(d*(a + b*log(c*x^n))))^p,x)
```

output
```
int((e*x)^m*(1/sin(d*(a + b*log(c*x^n))))^p, x)
```

Reduce [F]

$$\int (ex)^m \csc^p(d(a+b\log(cx^n)))\,dx = e^m \left(\int x^m \csc(\log(x^n c)\,bd+ad)^p dx\right)$$

input
```
int((e*x)^m*csc(d*(a+b*log(c*x^n)))^p,x)
```

output
```
e**m*int(x**m*csc(log(x**n*c)*b*d + a*d)**p,x)
```

3.329 $\int x \csc^p(a + b\log(cx^n))\, dx$

Optimal result

Integrand size = 15, antiderivative size = 106

$$\int x \csc^p(a + b\log(cx^n))\, dx = \frac{x^2\left(1 - e^{2ia}(cx^n)^{2ib}\right)^p \csc^p(a + b\log(cx^n))\,\text{Hypergeometric2F1}\left(p, \frac{1}{2}\left(-\frac{2i}{bn} + p\right), \frac{1}{2}\left(2 - \frac{2i}{bn} + p\right), e^{2ia}(cx^n\right.}{2 + ibnp}$$

output
```
x^2*(1-exp(2*I*a)*(c*x^n)^(2*I*b))^p*csc(a+b*ln(c*x^n))^p*hypergeom([p, -I
/b/n+1/2*p],[1-I/b/n+1/2*p],exp(2*I*a)*(c*x^n)^(2*I*b))/(2+I*b*n*p)
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.34

$$\int x \csc^p(a + b\log(cx^n))\, dx = -\frac{ix^2\left(2 - 2e^{2ia}(cx^n)^{2ib}\right)^p \left(\frac{ie^{ia}(cx^n)^{ib}}{-1+e^{2ia}(cx^n)^{2ib}}\right)^p \text{Hypergeometric2F1}\left(-\frac{i}{bn} + \frac{p}{2}, p, 1 - \frac{i}{bn} + \frac{p}{2}, e^{2ia}(cx^n)^{2ib}\right)}{-2i + bnp}$$

input
```
Integrate[x*Csc[a + b*Log[c*x^n]]^p,x]
```

output
```
((-I)*x^2*(2 - 2*E^((2*I)*a)*(c*x^n)^((2*I)*b))^p*((I*E^(I*a)*(c*x^n)^(I*b
))/(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))^p*Hypergeometric2F1[(-I)/(b*n) +
p/2, p, 1 - I/(b*n) + p/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/(-2*I + b*n*p)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5021, 5019, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \csc^p(a + b\log(cx^n)) \, dx$$

$$\Big\downarrow \text{5021}$$

$$\frac{x^2(cx^n)^{-2/n} \int (cx^n)^{\frac{2}{n}-1} \csc^p(a + b\log(cx^n)) d(cx^n)}{n}$$

$$\Big\downarrow \text{5019}$$

$$\frac{x^2(cx^n)^{-\frac{2}{n}-ibp}\left(1 - e^{2ia}(cx^n)^{2ib}\right)^p \csc^p(a + b\log(cx^n)) \int (cx^n)^{ibp+\frac{2}{n}-1}\left(1 - e^{2ia}(cx^n)^{2ib}\right)^{-p} d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2\left(1 - e^{2ia}(cx^n)^{2ib}\right)^p \text{Hypergeometric2F1}\left(p, \frac{1}{2}\left(p - \frac{2i}{bn}\right), \frac{1}{2}\left(p - \frac{2i}{bn} + 2\right), e^{2ia}(cx^n)^{2ib}\right) \csc^p(a + b\log(cx^n))}{2 + ibnp}$$

input
```
Int[x*Csc[a + b*Log[c*x^n]]^p,x]
```

output
```
(x^2*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^p*Csc[a + b*Log[c*x^n]]^p*Hyperge
ometric2F1[p, ((-2*I)/(b*n) + p)/2, (2 - (2*I)/(b*n) + p)/2, E^((2*I)*a)*(
c*x^n)^((2*I)*b)])/(2 + I*b*n*p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5019
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

rule 5021
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n))   Subst[Int[x
^((m + 1)/n - 1)*Csc[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b,
 c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Maple [F]

$$\int x\csc \left(a+b\ln \left(c\,x^{n}\right)\right)^{p}dx$$

input
```
int(x*csc(a+b*ln(c*x^n))^p,x)
```

output
```
int(x*csc(a+b*ln(c*x^n))^p,x)
```

Fricas [F]

$$\int x\csc^{p}\left(a+b\log\left(cx^{n}\right)\right)\,dx=\int x\csc\left(b\log\left(cx^{n}\right)+a\right)^{p}\,dx$$

input
```
integrate(x*csc(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

output
```
integral(x*csc(b*log(c*x^n) + a)^p, x)
```

Sympy [F]

$$\int x\csc^p(a+b\log(cx^n))\,dx=\int x\csc^p(a+b\log(cx^n))\,dx$$

input
```
integrate(x*csc(a+b*ln(c*x**n))**p,x)
```

output
```
Integral(x*csc(a + b*log(c*x**n))**p, x)
```

Maxima [F]

$$\int x\csc^p(a+b\log(cx^n))\,dx=\int x\csc(b\log(cx^n)+a)^p\,dx$$

input
```
integrate(x*csc(a+b*log(c*x^n))^p,x, algorithm="maxima")
```

output
```
integrate(x*csc(b*log(c*x^n) + a)^p, x)
```

Giac [F]

$$\int x\csc^p(a+b\log(cx^n))\,dx=\int x\csc(b\log(cx^n)+a)^p\,dx$$

input
```
integrate(x*csc(a+b*log(c*x^n))^p,x, algorithm="giac")
```

output
```
integrate(x*csc(b*log(c*x^n) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int x \csc^p(a+b\log(cx^n))\,dx = \int x\left(\frac{1}{\sin(a+b\ln(c\,x^n))}\right)^p dx$$

input
```
int(x*(1/sin(a + b*log(c*x^n)))^p,x)
```

output
```
int(x*(1/sin(a + b*log(c*x^n)))^p, x)
```

Reduce [F]

$$\int x \csc^p(a+b\log(cx^n))\,dx = \int \csc(\log(x^n c)\,b+a)^p x dx$$

input
```
int(x*csc(a+b*log(c*x^n))^p,x)
```

output
```
int(csc(log(x**n*c)*b + a)**p*x,x)
```

3.330 $\int \csc^p(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 107

$$\int \csc^p(a+b\log(cx^n))\,dx = \frac{x\left(1-e^{2ia}(cx^n)^{2ib}\right)^p \csc^p(a+b\log(cx^n))\,\text{Hypergeometric2F1}\left(p,-\frac{i-bnp}{2bn},\frac{1}{2}\left(2-\frac{i}{bn}+p\right),e^{2ia}(cx^n)^{2ib}\right)}{1+ibnp}$$

output
```
x*(1-exp(2*I*a)*(c*x^n)^(2*I*b))^p*csc(a+b*ln(c*x^n))^p*hypergeom([p, -1/2
*(I-b*n*p)/b/n],[1-1/2*I/b/n+1/2*p],exp(2*I*a)*(c*x^n)^(2*I*b))/(1+I*b*n*p
)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.33

$$\int \csc^p(a+b\log(cx^n))\,dx = -\frac{ix\left(2-2e^{2ia}(cx^n)^{2ib}\right)^p\left(\frac{ie^{ia}(cx^n)^{ib}}{-1+e^{2ia}(cx^n)^{2ib}}\right)^p \text{Hypergeometric2F1}\left(p,\frac{-i+bnp}{2bn},\frac{1}{2}\left(2-\frac{i}{bn}+p\right),e^{2ia}(cx^n)^{2ib}\right)}{-i+bnp}$$

input
```
Integrate[Csc[a + b*Log[c*x^n]]^p,x]
```

output
```
((-I)*x*(2 - 2*E^((2*I)*a)*(c*x^n)^((2*I)*b))^p*((I*E^(I*a)*(c*x^n)^(I*b))
/(-1 + E^((2*I)*a)*(c*x^n)^((2*I)*b)))^p*Hypergeometric2F1[p, (-I + b*n*p)
/(2*b*n), (2 - I/(b*n) + p)/2, E^((2*I)*a)*(c*x^n)^((2*I)*b)])/(-I + b*n*p
)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5015, 5019, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^p(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{5015}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\csc^p(a+b\log(cx^n))\,d(cx^n)}{n}$$

$$\Big\downarrow \text{5019}$$

$$\frac{x(cx^n)^{-\frac{1}{n}-ibp}\left(1-e^{2ia}(cx^n)^{2ib}\right)^p\csc^p(a+b\log(cx^n))\int (cx^n)^{ibp+\frac{1}{n}-1}\left(1-e^{2ia}(cx^n)^{2ib}\right)^{-p}d(cx^n)}{n}$$

$$\Big\downarrow \text{888}$$

$$\frac{x\left(1-e^{2ia}(cx^n)^{2ib}\right)^p\text{Hypergeometric2F1}\left(p,-\frac{i-bnp}{2bn},\frac{1}{2}\left(p-\frac{i}{bn}+2\right),e^{2ia}(cx^n)^{2ib}\right)\csc^p(a+b\log(cx^n))}{n\left(\frac{1}{n}+ibp\right)}$$

input
```
Int[Csc[a + b*Log[c*x^n]]^p,x]
```

output
```
(x*(1 - E^((2*I)*a)*(c*x^n)^((2*I)*b))^p*Csc[a + b*Log[c*x^n]]^p*Hypergeom
etric2F1[p, -1/2*(I - b*n*p)/(b*n), (2 - I/(b*n) + p)/2, E^((2*I)*a)*(c*x^
n)^((2*I)*b)])/(n*(n^(-1) + I*b*p))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 5015
```
Int[Csc[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Si
mp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Csc[d*(a + b*Log[x])]^p, x],
 x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

rule 5019
```
Int[Csc[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Csc[d*(a + b*Log[x])]^p*((1 - E^(2*I*a*d)*x^(2*I*b*d))^p/x^(I*b*d*p
))   Int[(e*x)^m*(x^(I*b*d*p)/(1 - E^(2*I*a*d)*x^(2*I*b*d))^p), x], x] /; F
reeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \csc \left(a+b\ln \left(c\,x^{n}\right)\right)^{p}dx$$

input
```
int(csc(a+b*ln(c*x^n))^p,x)
```

output
```
int(csc(a+b*ln(c*x^n))^p,x)
```

Fricas [F]

$$\int \csc^{p}\left(a+b\log \left(cx^{n}\right)\right)\,dx=\int \csc \left(b\log \left(cx^{n}\right)+a\right)^{p}\,dx$$

input
```
integrate(csc(a+b*log(c*x^n))^p,x, algorithm="fricas")
```

output
```
integral(csc(b*log(c*x^n) + a)^p, x)
```

Sympy [F]

$$\int \csc^p(a + b\log(cx^n))\, dx = \int \csc^p(a + b\log(cx^n))\, dx$$

input
```
integrate(csc(a+b*ln(c*x**n))**p,x)
```

output
```
Integral(csc(a + b*log(c*x**n))**p, x)
```

Maxima [F]

$$\int \csc^p(a + b\log(cx^n))\, dx = \int \csc(b\log(cx^n) + a)^p\, dx$$

input
```
integrate(csc(a+b*log(c*x^n))^p,x, algorithm="maxima")
```

output
```
integrate(csc(b*log(c*x^n) + a)^p, x)
```

Giac [F]

$$\int \csc^p(a + b\log(cx^n))\, dx = \int \csc(b\log(cx^n) + a)^p\, dx$$

input
```
integrate(csc(a+b*log(c*x^n))^p,x, algorithm="giac")
```

output
```
integrate(csc(b*log(c*x^n) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \csc^p(a + b\log(cx^n))\ dx = \int \left(\frac{1}{\sin(a + b\ \ln(c\,x^n))}\right)^p dx$$

input
```
int((1/sin(a + b*log(c*x^n)))^p,x)
```

output
```
int((1/sin(a + b*log(c*x^n)))^p, x)
```

Reduce [F]

$$\int \csc^p(a + b\log(cx^n))\ dx = \int \csc(\log(x^n c)\,b + a)^p dx$$

input
```
int(csc(a+b*log(c*x^n))^p,x)
```

output
```
int(csc(log(x**n*c)*b + a)**p,x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                    convert(leaf_count_result,string)," vs. $2 (",
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

def is_atom(expn):
  try:
    if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
      return True
    else:
      return False

  except AttributeError as error:
    return False

def expnType(expn):
  debug=False
  if debug:
    print("expn=",expn,"type(expn)=",type(expn))

  if is_atom(expn):
    return 1
  elif isinstance(expn,list):
    return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
  elif is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
      return 1
    else:
      return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):  #type(expn,'`^`')
    if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
      return expnType(expn.args[0])  #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
      else:
        return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    else:
      return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file