Computer Algebra
Independent Integration Tests

Summer 2024

4-Trig-functions/4-Miscellaneous/261-4.7

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May 18, 2024 Compiled on May 18, 2024 at 6:10am

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**54**]. This is test number [261].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (54)	0.00 (0)
Maple	100.00 (54)	0.00 (0)
Mupad	100.00 (54)	0.00 (0)
Fricas	98.15 (53)	1.85 (1)
Giac	88.89 (48)	11.11 (6)
Rubi	81.48 (44)	18.52 (10)
Maxima	40.74 (22)	59.26 (32)
Sympy	1.85 (1)	98.15 (53)
Reduce	0.00 (0)	100.00 (54)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

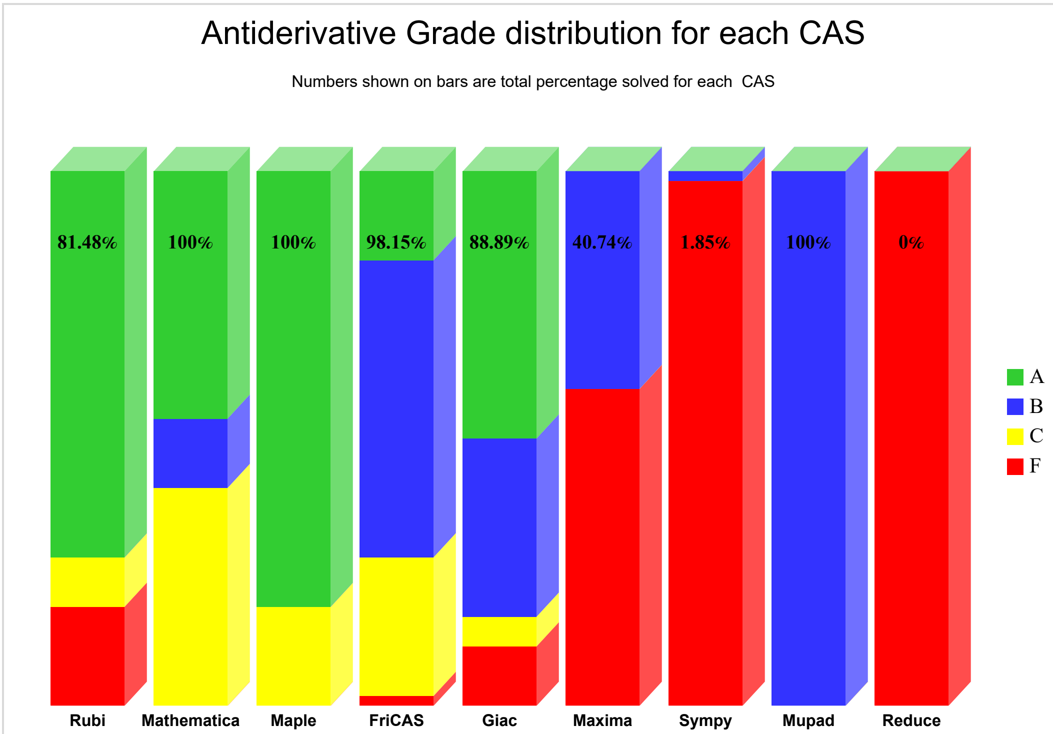
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

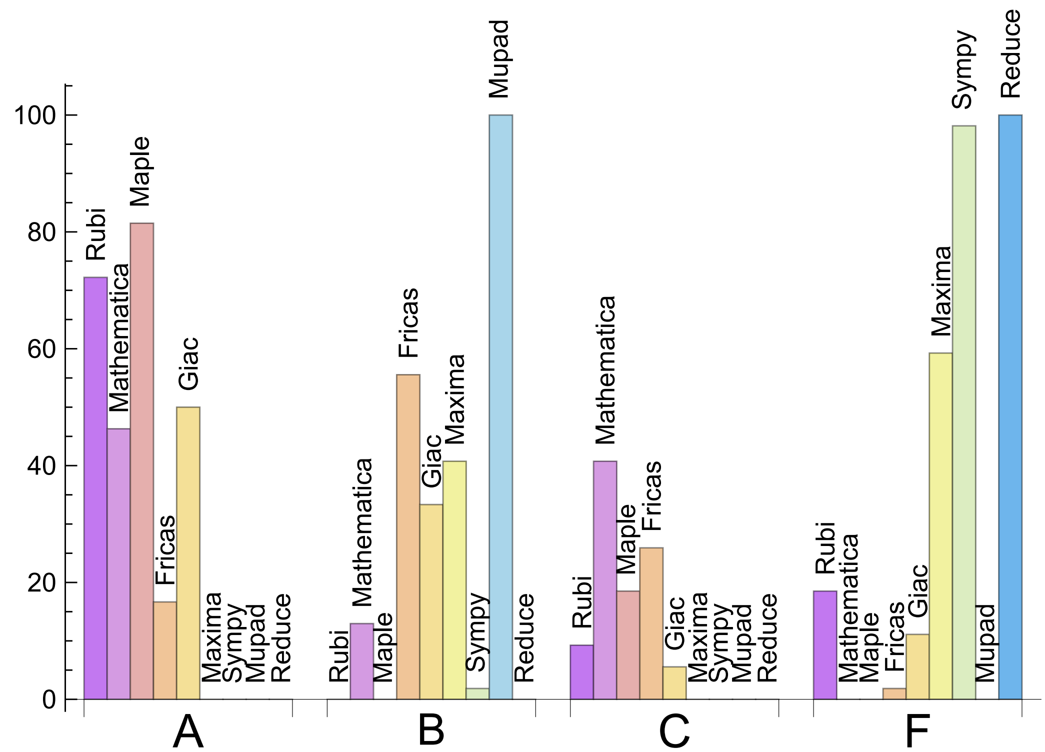
System	% A grade	% B grade	% C grade	% F grade
Maple	81.481	0.000	18.519	0.000
Rubi	72.222	0.000	9.259	18.519
Giac	50.000	33.333	5.556	11.111
Mathematica	46.296	12.963	40.741	0.000
Fricas	16.667	55.556	25.926	1.852
Mupad	0.000	100.000	0.000	0.000
Maxima	0.000	40.741	0.000	59.259
Reduce	0.000	0.000	0.000	100.000
Sympy	0.000	1.852	0.000	98.148

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Mupad	0	0.00	0.00	0.00
Fricas	1	0.00	100.00	0.00
Giac	6	0.00	0.00	100.00
Rubi	10	100.00	0.00	0.00
Maxima	32	21.88	0.00	78.12
Sympy	53	79.25	20.75	0.00
Reduce	54	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Giac	0.14
Fricas	0.39
Sympy	0.95
Maxima	1.22
Rubi	1.45
Mathematica	1.58
Mupad	21.73
Maple	109.77
Reduce	-nan(ind)

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	51.00	3.64	51.00	3.64
Maple	84.59	9.72	56.00	0.85
Giac	167.35	14.86	123.50	0.96
Mathematica	238.07	20.46	193.00	1.60
Rubi	378.23	218.62	132.00	1.10
Mupad	506.78	37.76	308.50	2.07
Fricas	743.38	33.46	300.00	2.07
Maxima	8020.77	57.96	3037.50	52.41
Reduce	-nan(ind)	-nan(ind)	nan	nan

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

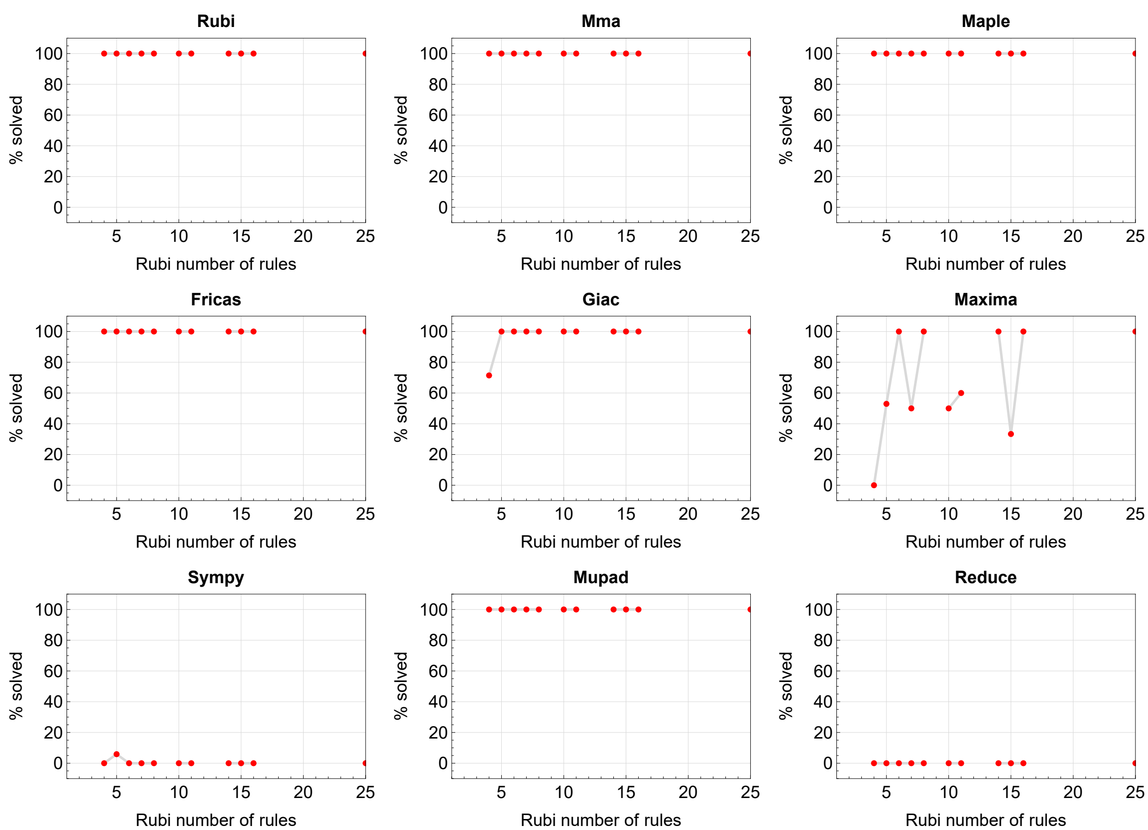


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

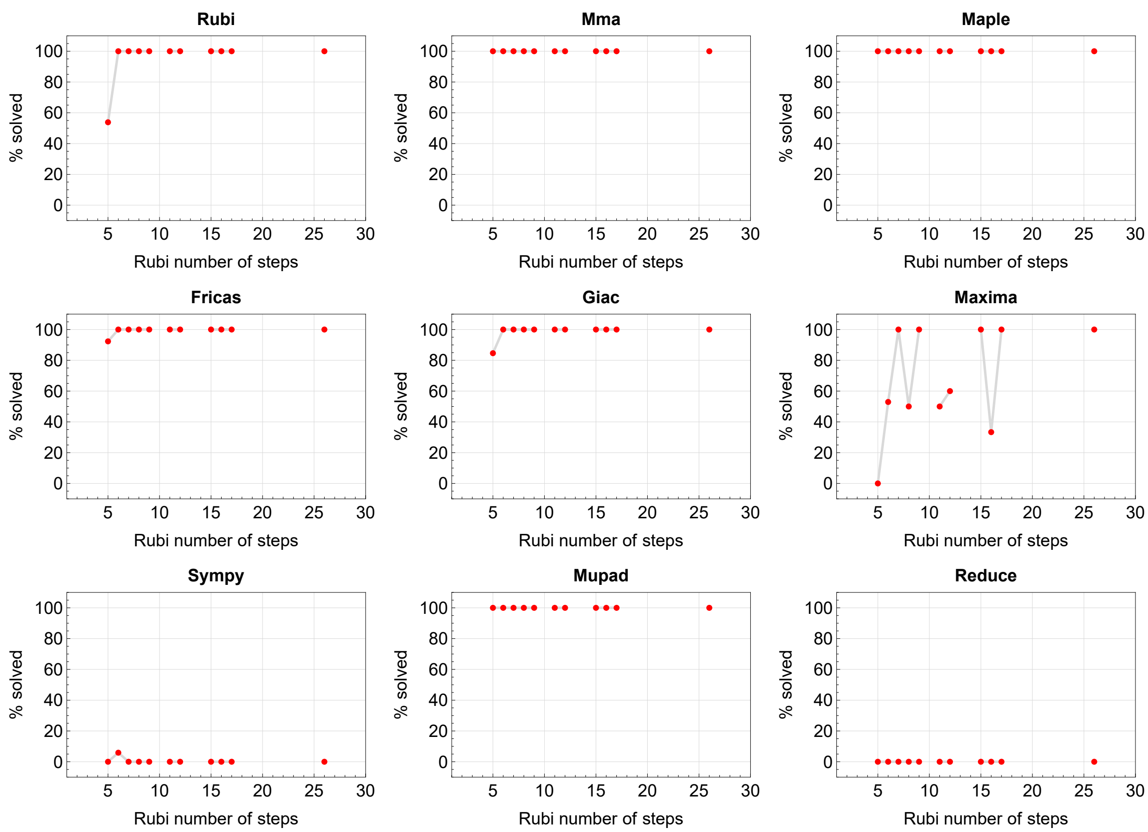


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

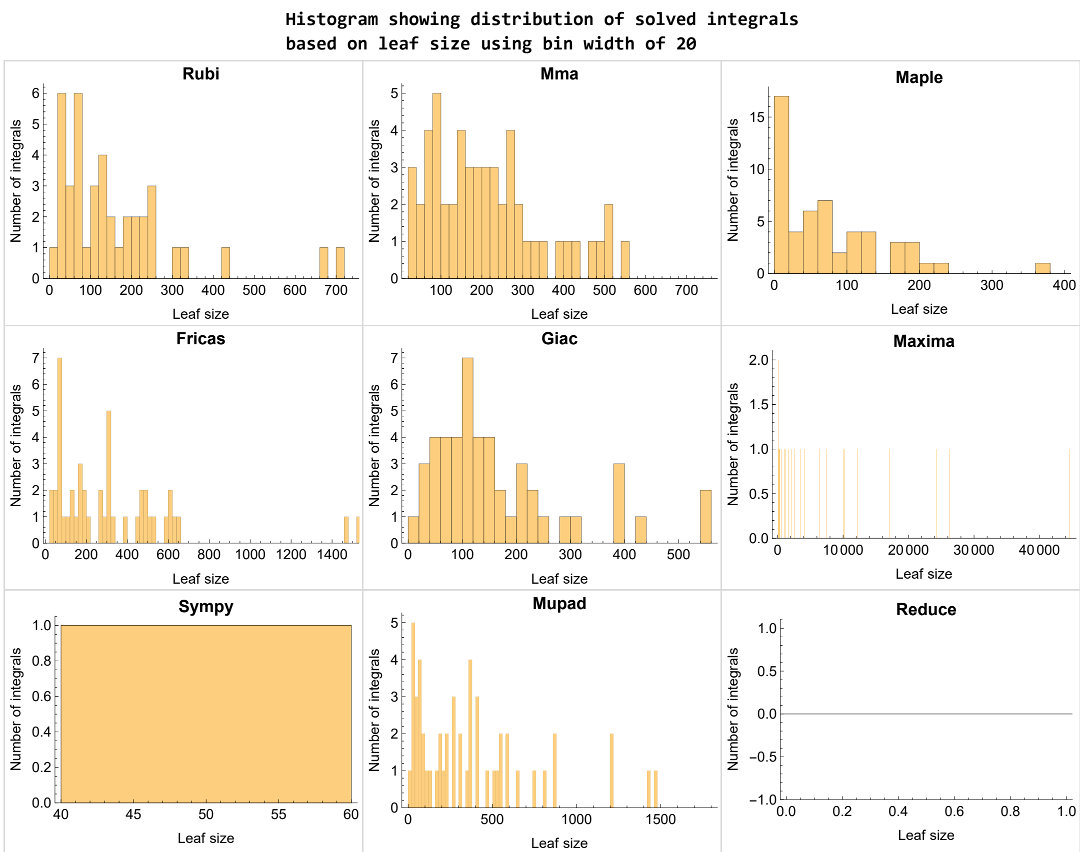


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

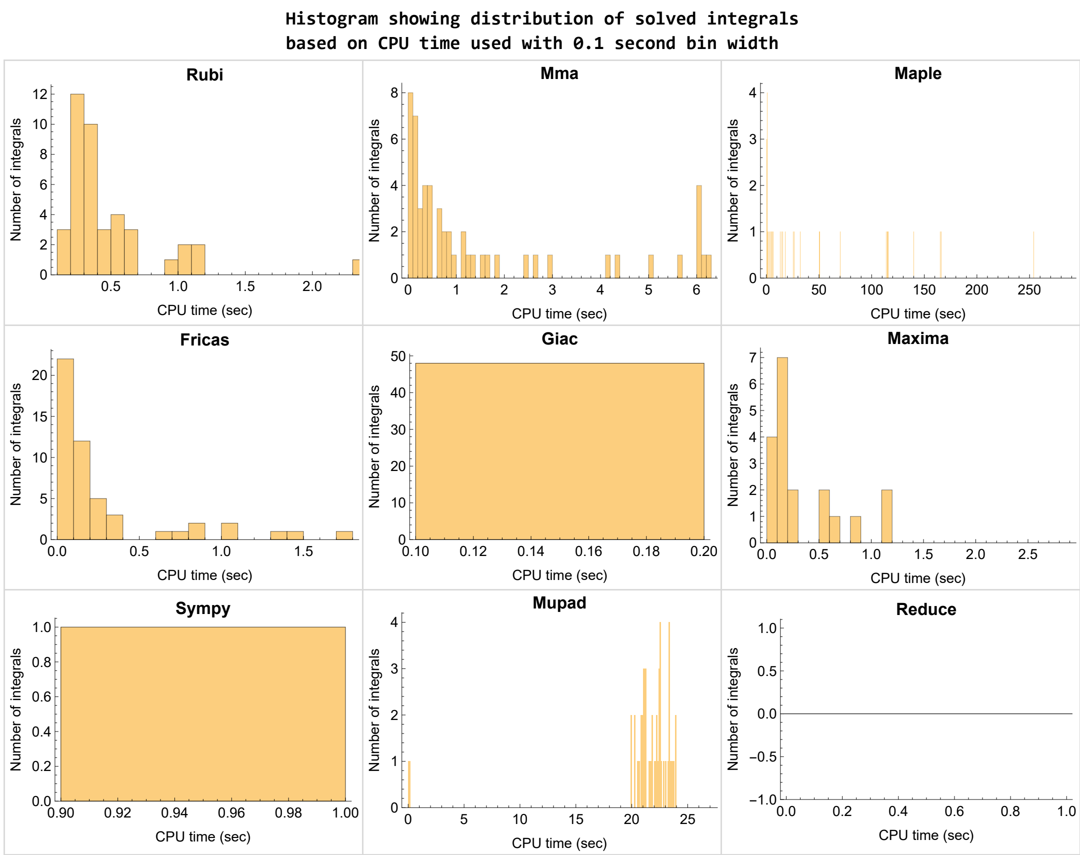


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

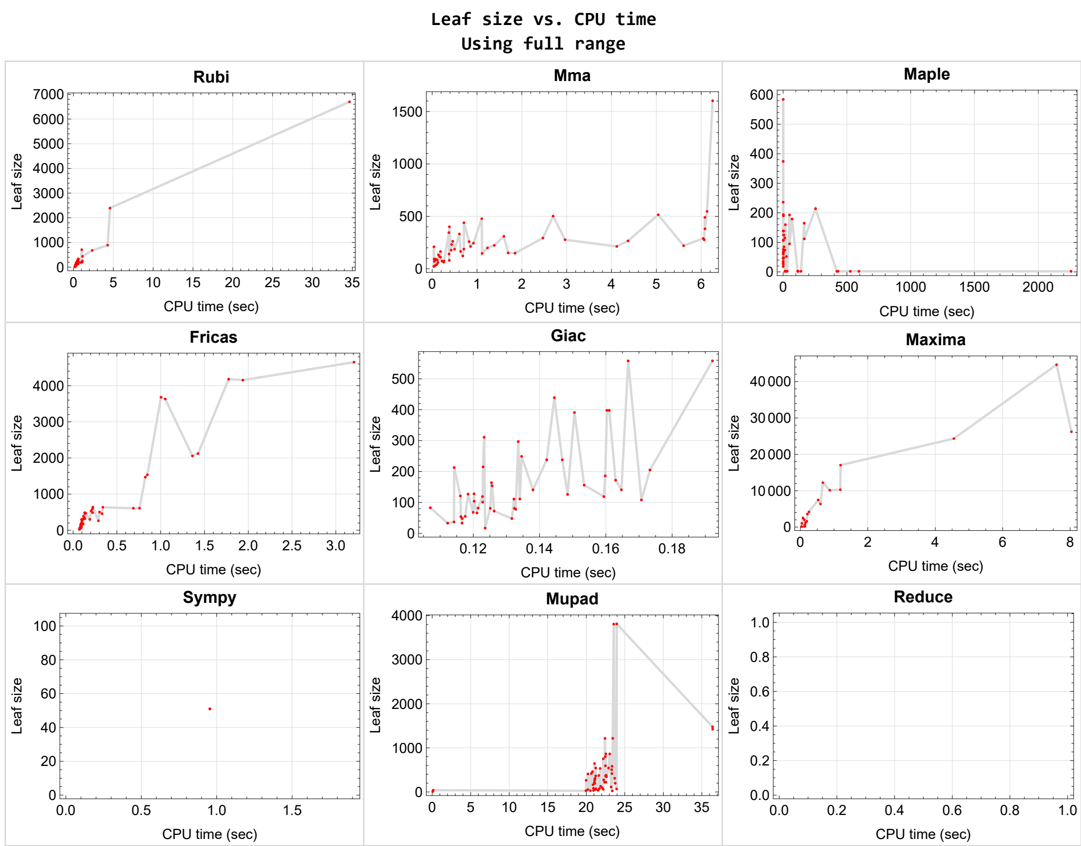


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{6\}$

Mathematica $\{3, 5, 6, 8, 9, 10, 11, 12, 15, 16, 17, 18, 32, 33, 45, 50, 51\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

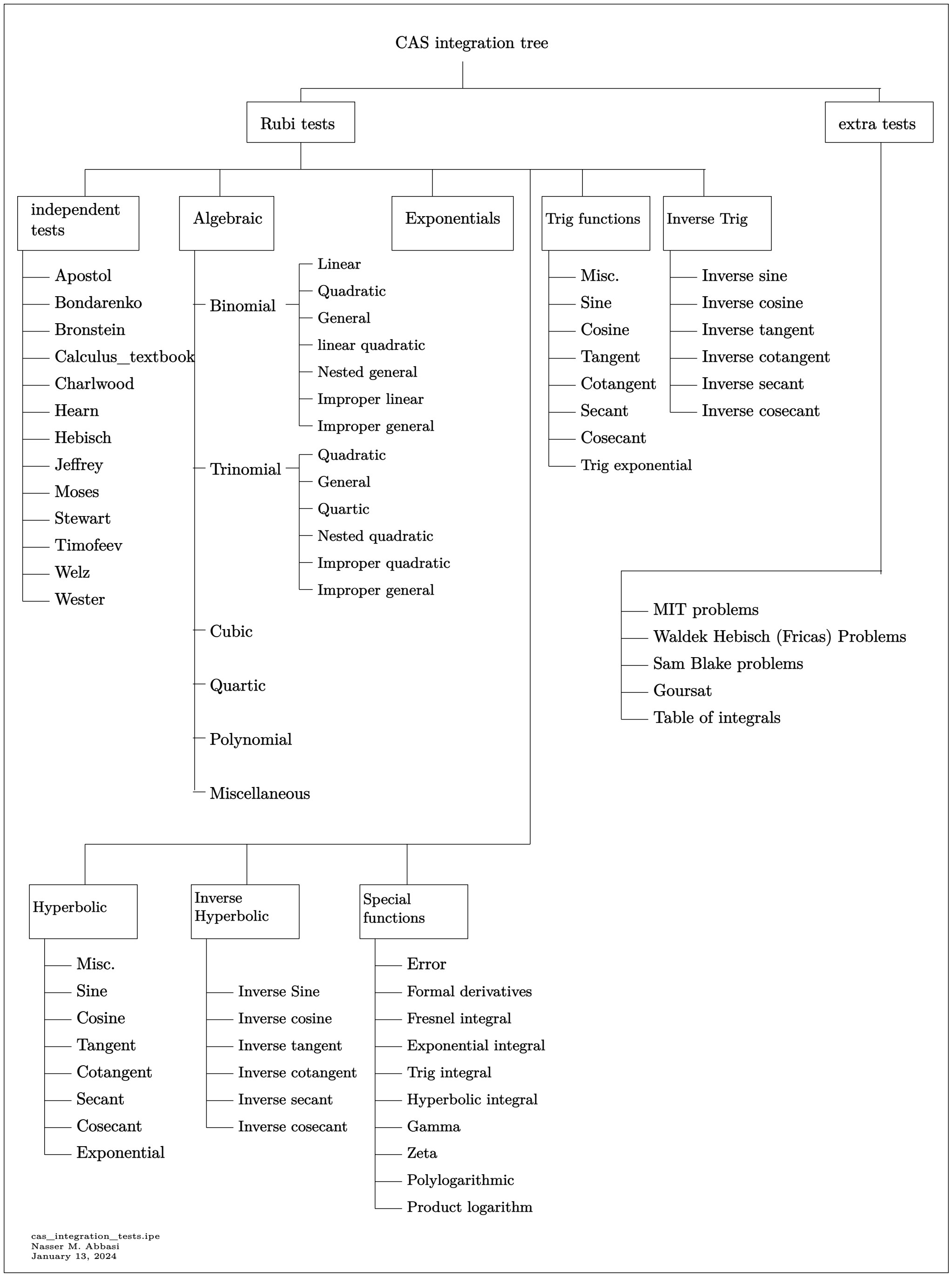
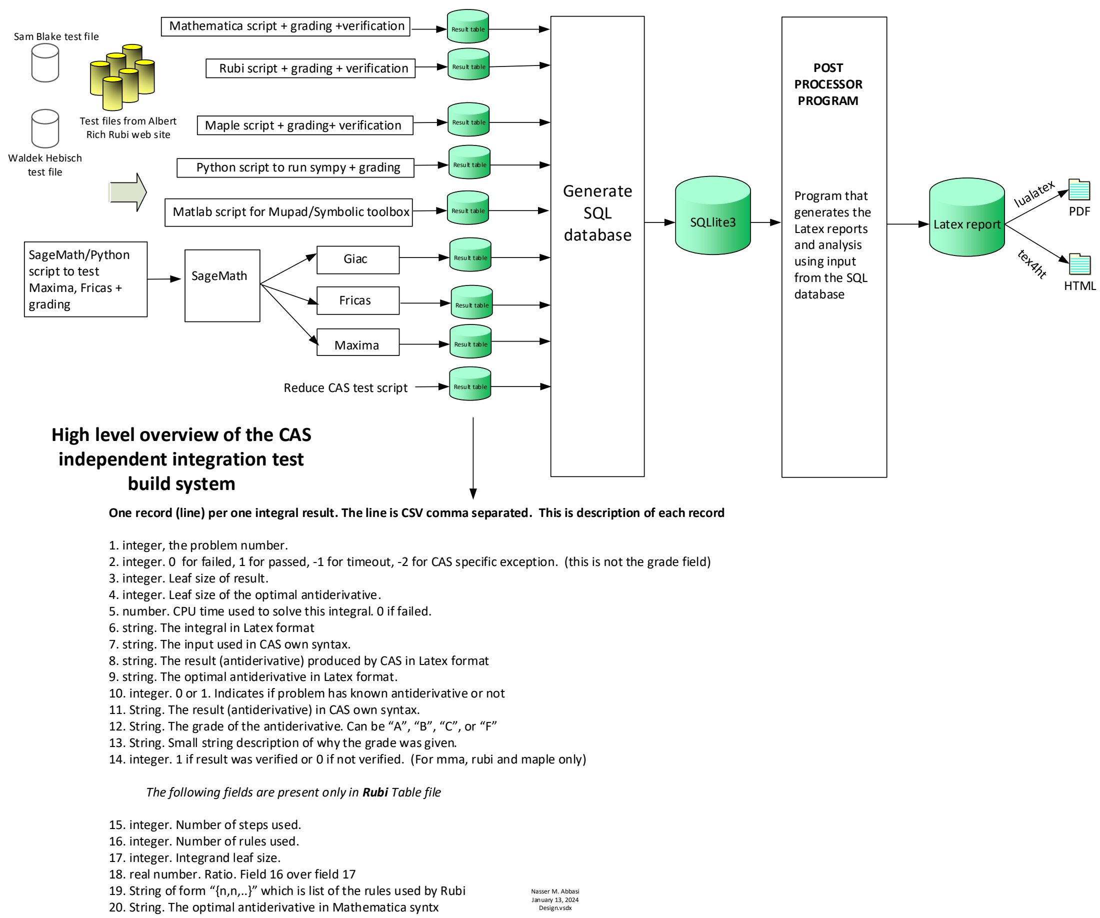


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54 }

B grade { }

C grade { 4, 5, 6, 7, 13 }

F normal fail { 8, 9, 10, 11, 12, 14, 15, 16, 17, 18 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 19, 22, 23, 24, 26, 27, 28, 29, 30, 34, 35, 36, 40, 41, 42, 44, 45, 46, 47, 48, 53, 54 }

B grade { 20, 21, 32, 37, 38, 39, 43 }

C grade { 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 25, 31, 33, 49, 50, 51, 52 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 5, 8, 11, 14, 15, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54 }

B grade { }

C grade { 4, 6, 7, 9, 10, 12, 13, 16, 23, 49 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 21, 22, 23, 24, 35, 36, 50, 51 }

B grade { 2, 3, 19, 20, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54 }

C grade { 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18 }

F normal fail { }

F(-1) timedout fail { 12 }

F(-2) exception fail { }

Maxima

A grade { }

B grade { 19, 20, 21, 22, 23, 24, 25, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 52, 53, 54 }

C grade { }

F normal fail { 1, 7, 13, 43, 49, 50, 51 }

F(-1) timedout fail { }

F(-2) exception fail { 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 26, 27, 28, 29, 30, 44, 45, 46, 47, 48 }

Giac

A grade { 1, 2, 3, 22, 23, 24, 28, 29, 30, 34, 35, 36, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54 }

B grade { 10, 11, 12, 16, 17, 18, 19, 20, 21, 25, 26, 27, 31, 32, 33, 37, 43, 49 }

C grade { 4, 5, 6 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { 7, 8, 9, 13, 14, 15 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Sympy

A grade { }

B grade { 19 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 22, 23, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53 }

F(-1) timedout fail { 6, 12, 18, 20, 21, 24, 30, 36, 42, 48, 54 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	79	93	48	0	73	0	66	13	82
N.S.	1	1.11	1.31	0.68	0.00	1.03	0.00	0.93	0.18	1.15
time (sec)	N/A	0.294	0.075	0.718	0.000	0.081	0.000	0.121	0.159	20.926

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	211	177	2	0	300	0	128	41	269
N.S.	1	1.44	1.20	0.01	0.00	2.04	0.00	0.87	0.28	1.83
time (sec)	N/A	0.457	0.423	14.728	0.000	0.096	0.000	0.120	0.167	21.091

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	329	277	138	0	468	0	164	69	461
N.S.	1	1.46	1.23	0.61	0.00	2.08	0.00	0.73	0.31	2.05
time (sec)	N/A	0.631	6.067	0.601	0.000	0.145	0.000	0.125	0.167	20.806

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F(-2)	C	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	439	186	108	0	292	0	119	27	418
N.S.	1	439.00	186.00	108.00	0.00	292.00	0.00	119.00	27.00	418.00
time (sec)	N/A	1.134	0.500	2.131	0.000	0.098	0.000	0.123	0.155	20.658

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupa
grade	N/A	C	C	A	F(-2)	C	F	C	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	2393	266	2	0	484	0	215	55	594
N.S.	1	2393.00	266.00	2.00	0.00	484.00	0.00	215.00	55.00	594.0
time (sec)	N/A	4.581	4.369	115.719	0.000	0.131	0.000	0.123	0.164	22.48

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F(-2)	C	F(-1)	C	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	6697	547	374	0	640	0	311	83	802
N.S.	1	6697.00	547.00	374.00	0.00	640.00	0.00	311.00	83.00	802.00
time (sec)	N/A	34.644	6.132	0.591	0.000	0.225	0.000	0.123	0.169	22.461

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	C	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	890	116	76	0	607	0	0	13	582
N.S.	1	27.81	3.62	2.38	0.00	18.97	0.00	0.00	0.41	18.19
time (sec)	N/A	4.293	0.166	0.742	0.000	0.688	0.000	0.000	0.152	23.350

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F(-2)	C	F	F(-2)	F	B
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	0	244	2	0	1468	0	0	41	3812
N.S.	1	0.00	1.92	0.02	0.00	11.56	0.00	0.00	0.32	30.02
time (sec)	N/A	0.000	0.921	15.359	0.000	0.824	0.000	0.000	0.158	23.955

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F(-2)	C	F	F(-2)	F	B
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	0	259	236	0	2056	0	0	69	1217
N.S.	1	0.00	1.17	1.07	0.00	9.30	0.00	0.00	0.31	5.51
time (sec)	N/A	0.000	0.824	0.715	0.000	1.362	0.000	0.000	0.164	22.428

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F(-2)	C	F	B	F	B
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	0	401	194	0	3630	0	238	27	544
N.S.	1	0.00	4.46	2.16	0.00	40.33	0.00	2.64	0.30	6.04
time (sec)	N/A	0.000	0.385	2.282	0.000	1.052	0.000	0.142	0.161	21.235

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F(-2)	C	F	B	F	B
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	0	502	2	0	4154	0	398	55	864
N.S.	1	0.00	3.26	0.01	0.00	26.97	0.00	2.58	0.36	5.61
time (sec)	N/A	0.000	2.698	113.989	0.000	1.939	0.000	0.160	0.149	23.048

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F(-2)	F(-1)	F(-1)	B	F	B
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	0	478	584	0	0	0	558	83	1478
N.S.	1	0.00	2.19	2.68	0.00	0.00	0.00	2.56	0.38	6.78
time (sec)	N/A	0.000	1.107	0.864	0.000	0.000	0.000	0.167	0.158	36.395

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	C	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	679	110	62	0	608	0	0	13	507
N.S.	1	21.22	3.44	1.94	0.00	19.00	0.00	0.00	0.41	15.84
time (sec)	N/A	2.351	0.187	0.947	0.000	0.759	0.000	0.000	0.156	23.305

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F(-2)	C	F	F(-2)	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	0	223	2	0	1534	0	0	41	3808
N.S.	1	0.00	2.42	0.02	0.00	16.67	0.00	0.00	0.45	41.39
time (sec)	N/A	0.000	1.385	15.666	0.000	0.848	0.000	0.000	0.144	23.560

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F(-2)	C	F	F(-2)	F	B
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	0	262	2	0	2122	0	0	69	1216
N.S.	1	0.00	1.70	0.01	0.00	13.78	0.00	0.00	0.45	7.90
time (sec)	N/A	0.000	0.460	594.678	0.000	1.427	0.000	0.000	0.157	23.436

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F(-2)	C	F	B	F	B
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	0	345	190	0	3679	0	238	27	544
N.S.	1	0.00	6.63	3.65	0.00	70.75	0.00	4.58	0.52	10.46
time (sec)	N/A	0.000	0.372	2.467	0.000	1.004	0.000	0.147	0.142	22.899

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F(-2)	C	F	B	F	B
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	0	515	2	0	4179	0	398	55	864
N.S.	1	0.00	4.22	0.02	0.00	34.25	0.00	3.26	0.45	7.08
time (sec)	N/A	0.000	5.042	114.889	0.000	1.776	0.000	0.161	0.157	22.546

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F(-2)	C	F(-1)	B	F	B
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	0	439	2	0	4649	0	558	83	1424
N.S.	1	0.00	2.31	0.01	0.00	24.47	0.00	2.94	0.44	7.49
time (sec)	N/A	0.000	0.709	2257.518	0.000	3.208	0.000	0.192	0.162	36.403

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	12	23	18	118	29	51	33	11	12
N.S.	1	0.86	1.64	1.29	8.43	2.07	3.64	2.36	0.79	0.86
time (sec)	N/A	0.192	0.037	0.542	0.042	0.077	0.955	0.112	0.168	0.079

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	51	132	48	2043	75	0	127	35	40
N.S.	1	1.21	3.14	1.14	48.64	1.79	0.00	3.02	0.83	0.95
time (sec)	N/A	0.224	0.141	5.318	0.117	0.090	0.000	0.118	0.151	0.132

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	90	231	2	6347	105	0	213	59	62
N.S.	1	1.29	3.30	0.03	90.67	1.50	0.00	3.04	0.84	0.89
time (sec)	N/A	0.253	0.433	115.482	0.596	0.094	0.000	0.114	0.158	22.200

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	26	165	24	0	17	23	24
N.S.	1	1.00	1.00	1.24	7.86	1.14	0.00	0.81	1.10	1.14
time (sec)	N/A	0.200	0.041	1.303	0.032	0.070	0.000	0.124	0.148	23.370

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	41	41	52	1016	51	0	37	47	37
N.S.	1	0.91	0.91	1.16	22.58	1.13	0.00	0.82	1.04	0.82
time (sec)	N/A	0.210	0.104	26.319	0.044	0.080	0.000	0.114	0.148	21.528

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	67	59	2	2508	73	0	55	71	55
N.S.	1	0.97	0.86	0.03	36.35	1.06	0.00	0.80	1.03	0.80
time (sec)	N/A	0.227	0.129	430.582	0.077	0.095	0.000	0.118	0.159	21.195

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	36	87	46	257	70	0	101	11	74
N.S.	1	1.12	2.72	1.44	8.03	2.19	0.00	3.16	0.34	2.31
time (sec)	N/A	0.243	0.078	0.687	0.136	0.084	0.000	0.123	0.145	21.705

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	154	187	125	0	310	0	297	35	221
N.S.	1	1.21	1.47	0.98	0.00	2.44	0.00	2.34	0.28	1.74
time (sec)	N/A	0.359	0.706	5.685	0.000	0.113	0.000	0.134	0.146	22.383

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	256	293	2	0	533	0	439	59	365
N.S.	1	1.16	1.33	0.01	0.00	2.41	0.00	1.99	0.27	1.65
time (sec)	N/A	0.527	2.470	115.154	0.000	0.206	0.000	0.144	0.151	22.510

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	63	83	63	0	172	0	81	23	128
N.S.	1	0.70	0.92	0.70	0.00	1.91	0.00	0.90	0.26	1.42
time (sec)	N/A	0.336	0.113	1.246	0.000	0.098	0.000	0.132	0.146	21.864

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	128	165	2	0	339	0	111	47	274
N.S.	1	0.83	1.07	0.01	0.00	2.20	0.00	0.72	0.31	1.78
time (sec)	N/A	0.567	0.635	25.681	0.000	0.131	0.000	0.134	0.148	22.263

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	194	289	2	0	502	0	141	71	418
N.S.	1	0.89	1.33	0.01	0.00	2.30	0.00	0.65	0.33	1.92
time (sec)	N/A	0.934	6.046	421.250	0.000	0.302	0.000	0.165	0.152	23.329

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	30	73	34	305	64	0	82	13	67
N.S.	1	0.94	2.28	1.06	9.53	2.00	0.00	2.56	0.41	2.09
time (sec)	N/A	0.214	0.219	0.691	0.128	0.088	0.000	0.121	0.145	23.940

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	B	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	131	277	85	7473	196	0	249	41	220
N.S.	1	1.42	3.01	0.92	81.23	2.13	0.00	2.71	0.45	2.39
time (sec)	N/A	0.398	2.964	6.575	0.529	0.096	0.000	0.135	0.151	22.551

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	B	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	235	1603	2	24329	300	0	391	69	364
N.S.	1	1.53	10.41	0.01	157.98	1.95	0.00	2.54	0.45	2.36
time (sec)	N/A	0.613	6.256	139.911	4.553	0.191	0.000	0.151	0.152	21.223

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	50	73	48	1588	93	0	48	27	53
N.S.	1	0.96	1.40	0.92	30.54	1.79	0.00	0.92	0.52	1.02
time (sec)	N/A	0.252	0.264	1.400	0.191	0.083	0.000	0.132	0.145	20.204

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	139	151	2	10270	171	0	78	55	265
N.S.	1	1.14	1.24	0.02	84.18	1.40	0.00	0.64	0.45	2.17
time (sec)	N/A	0.358	1.696	32.138	1.186	0.115	0.000	0.133	0.150	19.983

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	223	149	2	26212	261	0	108	83	409
N.S.	1	1.17	0.78	0.01	137.96	1.37	0.00	0.57	0.44	2.15
time (sec)	N/A	0.446	1.849	527.004	8.040	0.289	0.000	0.171	0.156	20.220

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	25	70	28	171	50	0	48	11	27
N.S.	1	1.09	3.04	1.22	7.43	2.17	0.00	2.09	0.48	1.17
time (sec)	N/A	0.196	0.047	0.475	0.119	0.078	0.000	0.117	0.145	19.962

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	105	141	67	3567	147	0	83	35	164
N.S.	1	1.75	2.35	1.12	59.45	2.45	0.00	1.38	0.58	2.73
time (sec)	N/A	0.268	0.376	3.229	0.208	0.086	0.000	0.107	0.147	20.870

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	184	213	95	12209	219	0	104	59	307
N.S.	1	1.74	2.01	0.90	115.18	2.07	0.00	0.98	0.56	2.90
time (sec)	N/A	0.343	4.118	50.627	0.669	0.098	0.000	0.120	0.157	21.127

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	26	32	36	655	71	0	33	23	29
N.S.	1	0.76	0.94	1.06	19.26	2.09	0.00	0.97	0.68	0.85
time (sec)	N/A	0.203	0.073	0.843	0.133	0.078	0.000	0.117	0.154	20.534

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	68	80	74	4157	132	0	54	47	74
N.S.	1	0.92	1.08	1.00	56.18	1.78	0.00	0.73	0.64	1.00
time (sec)	N/A	0.263	0.383	13.256	0.254	0.086	0.000	0.116	0.146	21.249

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	108	122	112	10127	186	0	72	71	112
N.S.	1	0.95	1.07	0.98	88.83	1.63	0.00	0.63	0.62	0.98
time (sec)	N/A	0.385	0.684	165.961	0.871	0.106	0.000	0.126	0.153	23.226

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	37	93	46	0	68	0	68	11	34
N.S.	1	1.12	2.82	1.39	0.00	2.06	0.00	2.06	0.33	1.03
time (sec)	N/A	0.235	0.046	0.572	0.000	0.088	0.000	0.120	0.149	20.940

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	155	199	125	0	300	0	121	35	219
N.S.	1	1.22	1.57	0.98	0.00	2.36	0.00	0.95	0.28	1.72
time (sec)	N/A	0.331	1.234	3.306	0.000	0.112	0.000	0.116	0.150	21.052

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	257	381	193	0	486	0	154	59	379
N.S.	1	1.16	1.72	0.87	0.00	2.20	0.00	0.70	0.27	1.71
time (sec)	N/A	0.500	6.085	50.314	0.000	0.222	0.000	0.126	0.144	22.581

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	63	63	60	0	172	0	81	23	95
N.S.	1	0.69	0.69	0.66	0.00	1.89	0.00	0.89	0.25	1.04
time (sec)	N/A	0.309	0.256	0.931	0.000	0.088	0.000	0.125	0.144	22.063

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	133	146	115	0	318	0	111	47	345
N.S.	1	0.84	0.92	0.72	0.00	2.00	0.00	0.70	0.30	2.17
time (sec)	N/A	0.587	1.113	13.339	0.000	0.135	0.000	0.132	0.146	22.624

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	207	220	165	0	456	0	141	71	198
N.S.	1	0.91	0.97	0.73	0.00	2.01	0.00	0.62	0.31	0.87
time (sec)	N/A	1.099	5.608	164.999	0.000	0.331	0.000	0.138	0.148	23.756

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	77	210	67	0	137	0	119	13	193
N.S.	1	0.94	2.56	0.82	0.00	1.67	0.00	1.45	0.16	2.35
time (sec)	N/A	0.326	0.039	0.759	0.000	0.091	0.000	0.159	0.143	21.105

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	A	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	319	332	139	0	388	0	172	41	374
N.S.	1	0.98	1.02	0.43	0.00	1.20	0.00	0.53	0.13	1.15
time (sec)	N/A	0.551	0.609	4.158	0.000	0.119	0.000	0.163	0.146	21.626

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	A	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	729	710	490	179	0	588	0	205	69	534
N.S.	1	0.97	0.67	0.25	0.00	0.81	0.00	0.28	0.09	0.73
time (sec)	N/A	1.039	6.084	70.036	0.000	0.220	0.000	0.173	0.148	21.805

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	105	165	106	1264	276	0	126	27	645
N.S.	1	0.52	0.81	0.52	6.23	1.36	0.00	0.62	0.13	3.18
time (sec)	N/A	0.371	0.185	1.106	0.154	0.103	0.000	0.148	0.153	21.074

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	175	213	160	17029	460	0	156	55	753
N.S.	1	0.66	0.80	0.60	63.78	1.72	0.00	0.58	0.21	2.82
time (sec)	N/A	0.615	0.856	17.886	1.192	0.135	0.000	0.153	0.148	22.216

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	250	309	214	44647	634	0	186	83	310
N.S.	1	0.76	0.93	0.65	134.89	1.92	0.00	0.56	0.25	0.94
time (sec)	N/A	1.121	1.600	254.197	7.599	0.339	0.000	0.160	0.157	23.688

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [33] had the largest ratio of [2.27273000000000014]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.11	11	0.364
2	A	5	4	1.44	11	0.364
3	A	5	4	1.46	11	0.364
4	C	5	4	439.00	11	0.364
5	C	5	4	2393.00	11	0.364
6	C	11	10	6697.00	11	0.909
7	C	5	4	27.81	11	0.364
8	F	0	0	N/A	0.000	N/A
9	F	0	0	N/A	0.000	N/A
10	F	0	0	N/A	0.000	N/A
11	F	0	0	N/A	0.000	N/A
12	F	0	0	N/A	0.000	N/A
13	C	5	4	21.22	11	0.364
14	F	0	0	N/A	0.000	N/A
15	F	0	0	N/A	0.000	N/A
16	F	0	0	N/A	0.000	N/A
17	F	0	0	N/A	0.000	N/A
18	F	0	0	N/A	0.000	N/A
19	A	6	5	0.86	9	0.556
20	A	9	8	1.21	9	0.889

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	12	11	1.29	9	1.222
22	A	6	5	1.00	9	0.556
23	A	6	5	0.91	9	0.556
24	A	6	5	0.97	9	0.556
25	A	6	5	1.12	9	0.556
26	A	6	5	1.21	9	0.556
27	A	6	5	1.16	9	0.556
28	A	8	7	0.70	9	0.778
29	A	12	11	0.83	9	1.222
30	A	16	15	0.89	9	1.667
31	A	7	6	0.94	11	0.545
32	A	17	16	1.42	11	1.455
33	A	26	25	1.53	11	2.273
34	A	8	7	0.96	11	0.636
35	A	12	11	1.14	11	1.000
36	A	15	14	1.17	11	1.273
37	A	6	5	1.09	9	0.556
38	A	11	10	1.75	9	1.111
39	A	15	14	1.74	9	1.556
40	A	6	5	0.76	9	0.556
41	A	6	5	0.92	9	0.556
42	A	6	5	0.95	9	0.556
43	A	6	5	1.12	9	0.556
44	A	6	5	1.22	9	0.556
45	A	6	5	1.16	9	0.556
46	A	8	7	0.69	9	0.778
47	A	12	11	0.84	9	1.222
48	A	16	15	0.91	9	1.667
49	A	6	5	0.94	11	0.455
50	A	6	5	0.98	11	0.455
51	A	6	5	0.97	11	0.455
52	A	8	7	0.52	11	0.636

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
53	A	12	11	0.66	11	1.000
54	A	16	15	0.76	11	1.364

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{1}{\cos(3x)+\sin(2x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 71

$$\begin{aligned}\int \frac{1}{\cos(3x)+\sin(2x)}\,dx = & -\frac{1}{5}\left(1-\sqrt{5}\right)\log\left(1-\sqrt{5}-4\sin(x)\right)\\ & -\frac{1}{5}\left(1+\sqrt{5}\right)\log\left(1+\sqrt{5}-4\sin(x)\right)\\ & +\frac{1}{2}\log(1-\sin(x))-\frac{1}{10}\log(1+\sin(x))\end{aligned}$$

output
```
-1/5*(-5^(1/2)+1)*ln(1-5^(1/2)-4*sin(x))-1/5*(5^(1/2)+1)*ln(1+5^(1/2)-4*si
n(x))+1/2*ln(1-sin(x))-1/10*ln(1+sin(x))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.31

$$\begin{aligned}\int \frac{1}{\cos(3x)+\sin(2x)}\,dx = & \frac{1}{25}\left(25\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-5\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right.\\ & -\sqrt{5}\left(-5+\sqrt{5}\right)\log\left(1-\sqrt{5}-4\sin(x)\right)\\ & \left.-\sqrt{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-4\sin(x)\right)\right)\end{aligned}$$

input
```
Integrate[(Cos[3*x] + Sin[2*x])^(-1),x]
```

output
```
(25*Log[Cos[x/2] - Sin[x/2]] - 5*Log[Cos[x/2] + Sin[x/2]] - Sqrt[5]*(-5 +
Sqrt[5])*Log[1 - Sqrt[5] - 4*Sin[x]] - Sqrt[5]*(5 + Sqrt[5])*Log[1 + Sqrt[
5] - 4*Sin[x]])/25
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4829, 1300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin(2x)+\cos(3x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(2x)+\cos(3x)}dx$$

$$\Big\downarrow \text{4829}$$

$$\int \frac{1}{\left(-4\sin^2(x)+2\sin(x)+1\right)\left(1-\sin^2(x)\right)}d\sin(x)$$

$$\Big\downarrow \text{1300}$$

$$-4\int \left(\frac{1}{40(\sin(x)+1)}-\frac{1}{5\left(\left(1-\sqrt{5}\right)\sin(x)+1\right)}-\frac{1}{5\left(\left(1+\sqrt{5}\right)\sin(x)+1\right)}+\frac{1}{8(1-\sin(x))}\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$-4\left(-\frac{1}{8}\log(1-\sin(x))+\frac{1}{40}\log(\sin(x)+1)-\frac{\log\left(\left(1-\sqrt{5}\right)\sin(x)+1\right)}{5\left(1-\sqrt{5}\right)}-\frac{\log\left(\left(1+\sqrt{5}\right)\sin(x)+1\right)}{5\left(1+\sqrt{5}\right)}\right)$$

input
```
Int[(Cos[3*x] + Sin[2*x])^(-1),x]
```

output
```
-4*(-1/8*Log[1 - Sin[x]] + Log[1 + Sin[x]]/40 - Log[1 + (1 - Sqrt[5])*Sin[
x]]/(5*(1 - Sqrt[5])) - Log[1 + (1 + Sqrt[5])*Sin[x]]/(5*(1 + Sqrt[5])))
```

Defintions of rubi rules used

rule 1300
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand
[(b/2 - r/2 + c*x)^p*(b/2 + r/2 + c*x)^p*(d + f*x^2)^q, x], x], x] /; EqQ[p
, -1] ||  !FractionalPowerFactorQ[r]] /; FreeQ[{a, b, c, d, f}, x] && ILtQ[
p, 0] && IntegerQ[q] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4829
```
Int[(cos[(n_.)*((c_.) + (d_.)*(x_))]*(b_.) + (a_.)*sin[(m_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Sin[
m*ArcSin[x]] + b*Cos[n*ArcSin[x]]]]^p/Sqrt[1 - x^2], x], x, Sin[c + d*x]],
x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[m/2] && Inte
gerQ[(n - 1)/2]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.68

method	result
default	$\frac{\ln(\sin(x)-1)}{2}-\frac{\ln(1+\sin(x))}{10}-\frac{\ln\left(4\sin(x)^2-2\sin(x)-1\right)}{5}+\frac{2\sqrt{5}\,\operatorname{arctanh}\left(\frac{(-2+8\sin(x))\sqrt{5}}{10}\right)}{5}$
risch	$\ln\left(e^{ix}-i\right)-\frac{\ln(e^{ix}+i)}{5}-\frac{\ln\left(e^{2ix}+\frac{i\left(\sqrt{5}-1\right)e^{ix}}{2}-1\right)}{5}+\frac{\ln\left(e^{2ix}+\frac{i\left(\sqrt{5}-1\right)e^{ix}}{2}-1\right)\sqrt{5}}{5}-\frac{\ln\left(e^{2ix}-\frac{i\left(\sqrt{5}+1\right)e^{ix}}{2}-1\right)}{5}-$

input
```
int(1/(cos(3*x)+sin(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(sin(x)-1)-1/10*ln(1+sin(x))-1/5*ln(4*sin(x)^2-2*sin(x)-1)+2/5*5^(1/
2)*arctanh(1/10*(-2+8*sin(x))*5^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int \frac{1}{\cos(3x)+\sin(2x)}\,dx = \frac{1}{5}\sqrt{5}\log\left(\frac{8\cos(x)^2-4(\sqrt{5}-1)\sin(x)+\sqrt{5}-11}{4\cos(x)^2+2\sin(x)-3}\right) - \frac{1}{5}\log\left(4\cos(x)^2+2\sin(x)-3\right) - \frac{1}{10}\log(\sin(x)+1)+\frac{1}{2}\log(-\sin(x)+1)$$

input
```
integrate(1/(cos(3*x)+sin(2*x)),x, algorithm="fricas")
```

output
```
1/5*sqrt(5)*log((8*cos(x)^2 - 4*(sqrt(5) - 1)*sin(x) + sqrt(5) - 11)/(4*co
s(x)^2 + 2*sin(x) - 3)) - 1/5*log(4*cos(x)^2 + 2*sin(x) - 3) - 1/10*log(si
n(x) + 1) + 1/2*log(-sin(x) + 1)
```

Sympy [F]

$$\int \frac{1}{\cos(3x)+\sin(2x)}\,dx = \int \frac{1}{\sin(2x)+\cos(3x)}\,dx$$

input
```
integrate(1/(cos(3*x)+sin(2*x)),x)
```

output
```
Integral(1/(sin(2*x) + cos(3*x)), x)
```

Maxima [F]

$$\int \frac{1}{\cos(3x) + \sin(2x)}\,dx = \int \frac{1}{\cos(3x) + \sin(2x)}\,dx$$

input
```
integrate(1/(cos(3*x)+sin(2*x)),x, algorithm="maxima")
```

output
```
2/5*integrate(-(cos(4*x)*cos(x) - cos(2*x)*cos(x) + cos(x)*sin(3*x) - cos(
3*x)*sin(x) + sin(4*x)*sin(x) - sin(2*x)*sin(x) + cos(x))/(2*(cos(2*x) - s
in(3*x) + sin(x) - 1)*cos(4*x) - cos(4*x)^2 + 2*(cos(x) - sin(2*x))*cos(3*
x) - cos(3*x)^2 - 2*(sin(x) - 1)*cos(2*x) - cos(2*x)^2 - cos(x)^2 + 2*(cos
(3*x) - cos(x) + sin(2*x))*sin(4*x) - sin(4*x)^2 + 2*(cos(2*x) + sin(x) -
1)*sin(3*x) - sin(3*x)^2 + 2*cos(x)*sin(2*x) - sin(2*x)^2 - sin(x)^2 + 2*s
in(x) - 1), x) + 6/5*integrate((cos(3*x)*cos(4/3*arctan2(sin(3*x), cos(3*x
))) - cos(3*x)*cos(2/3*arctan2(sin(3*x), cos(3*x))) + cos(1/3*arctan2(sin(
3*x), cos(3*x)))*sin(3*x) + sin(3*x)*sin(4/3*arctan2(sin(3*x), cos(3*x)))
- sin(3*x)*sin(2/3*arctan2(sin(3*x), cos(3*x))) - cos(3*x)*sin(1/3*arctan2
(sin(3*x), cos(3*x))) + cos(3*x))/(cos(3*x)^2 - 2*(cos(2/3*arctan2(sin(3*x
), cos(3*x))) - sin(3*x) + sin(1/3*arctan2(sin(3*x), cos(3*x))) - 1)*cos(4
/3*arctan2(sin(3*x), cos(3*x))) + cos(4/3*arctan2(sin(3*x), cos(3*x)))^2 -
 2*(sin(3*x) - sin(1/3*arctan2(sin(3*x), cos(3*x))) + 1)*cos(2/3*arctan2(s
in(3*x), cos(3*x))) + cos(2/3*arctan2(sin(3*x), cos(3*x)))^2 - 2*cos(3*x)*
cos(1/3*arctan2(sin(3*x), cos(3*x))) + cos(1/3*arctan2(sin(3*x), cos(3*x))
)^2 + sin(3*x)^2 - 2*(cos(3*x) - cos(1/3*arctan2(sin(3*x), cos(3*x))) + si
n(2/3*arctan2(sin(3*x), cos(3*x))))*sin(4/3*arctan2(sin(3*x), cos(3*x))) +
 sin(4/3*arctan2(sin(3*x), cos(3*x)))^2 + 2*(cos(3*x) - cos(1/3*arctan2(si
n(3*x), cos(3*x))))*sin(2/3*arctan2(sin(3*x), cos(3*x))) + sin(2/3*arct...
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{1}{\cos(3x) + \sin(2x)}\,dx = -\frac{1}{5}\sqrt{5}\log\left(\frac{\left|-2\sqrt{5} + 8\sin(x) - 2\right|}{\left|2\sqrt{5} + 8\sin(x) - 2\right|}\right) - \frac{1}{10}\log(\sin(x) + 1) + \frac{1}{2}\log(-\sin(x) + 1) - \frac{1}{5}\log\left(\left|4\sin(x)^2 - 2\sin(x) - 1\right|\right)$$

input
```
integrate(1/(cos(3*x)+sin(2*x)),x, algorithm="giac")
```

output
```
-1/5*sqrt(5)*log(abs(-2*sqrt(5) + 8*sin(x) - 2)/abs(2*sqrt(5) + 8*sin(x) -
 2)) - 1/10*log(sin(x) + 1) + 1/2*log(-sin(x) + 1) - 1/5*log(abs(4*sin(x)^
2 - 2*sin(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 20.93 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.15

$$\int \frac{1}{\cos(3x)+\sin(2x)}\,dx = \ln\left(\tan\left(\frac{x}{2}\right)-1\right) - \frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{5} - \ln\left(2\tan\left(\frac{x}{2}\right) - 2\sqrt{5}\tan\left(\frac{x}{2}\right) + \tan\left(\frac{x}{2}\right)^2 + 1\right)\left(\frac{\sqrt{5}}{5}+\frac{1}{5}\right) + \ln\left(2\tan\left(\frac{x}{2}\right) + 2\sqrt{5}\tan\left(\frac{x}{2}\right) + \tan\left(\frac{x}{2}\right)^2 + 1\right)\left(\frac{\sqrt{5}}{5}-\frac{1}{5}\right)$$

input
```
int(1/(cos(3*x) + sin(2*x)),x)
```

output
```
log(tan(x/2) - 1) - log(tan(x/2) + 1)/5 - log(2*tan(x/2) - 2*5^(1/2)*tan(x
/2) + tan(x/2)^2 + 1)*(5^(1/2)/5 + 1/5) + log(2*tan(x/2) + 2*5^(1/2)*tan(x
/2) + tan(x/2)^2 + 1)*(5^(1/2)/5 - 1/5)
```

Reduce [F]

$$\int \frac{1}{\cos(3x)+\sin(2x)}\,dx = \int \frac{1}{\cos(3x)+\sin(2x)}\,dx$$

input
```
int(1/(cos(3*x)+sin(2*x)),x)
```

output
```
int(1/(cos(3*x) + sin(2*x)),x)
```

3.2 $\int \frac{1}{(\cos(3x)+\sin(2x))^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 147

$$\begin{aligned}\int \frac{1}{(\cos(3x)+\sin(2x))^3} dx = & -\frac{8}{125}\left(37-17\sqrt{5}\right)\log\left(1-\sqrt{5}-4\sin(x)\right) \\ & -\frac{8}{125}\left(37+17\sqrt{5}\right)\log\left(1+\sqrt{5}-4\sin(x)\right) \\ & +\frac{19}{4}\log(1-\sin(x))-\frac{7}{500}\log(1+\sin(x)) \\ & +\frac{\sec^2(x)(4+5\sin(x))}{10\left(1+2\sin(x)-4\sin^2(x)\right)} \\ & -\frac{37+86\sin(x)}{25\left(1+2\sin(x)-4\sin^2(x)\right)} \\ & +\frac{2\sec(x)\tan(x)}{5\left(1+2\sin(x)-4\sin^2(x)\right)^2}\end{aligned}$$

output
```
-8/125*(37-17*5^(1/2))*ln(1-5^(1/2)-4*sin(x))-8/125*(37+17*5^(1/2))*ln(1+5
^(1/2)-4*sin(x))+19/4*ln(1-sin(x))-7/500*ln(1+sin(x))+sec(x)^2*(4+5*sin(x)
)/(10+20*sin(x)-40*sin(x)^2)-(37+86*sin(x))/(25+50*sin(x)-100*sin(x)^2)+2/
5*sec(x)*tan(x)/(1+2*sin(x)-4*sin(x)^2)^2
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.20

$$\int \frac{1}{(\cos(3x)+\sin(2x))^3}\,dx$$
$$= \frac{23750\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-70\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-32\sqrt{5}(-85+37\sqrt{5})\log\left(1-\sqrt{5}-4\sin(x\right.}{}$$

input
```
Integrate[(Cos[3*x] + Sin[2*x])^(-3),x]
```

output
```
(23750*Log[Cos[x/2] - Sin[x/2]] - 70*Log[Cos[x/2] + Sin[x/2]] - 32*Sqrt[5]
*(-85 + 37*Sqrt[5])*Log[1 - Sqrt[5] - 4*Sin[x]] - 32*Sqrt[5]*(85 + 37*Sqrt
[5])*Log[1 + Sqrt[5] - 4*Sin[x]] - 625/(Cos[x/2] - Sin[x/2])^2 + 5/(Cos[x/
2] + Sin[x/2])^2 + (400*(1 + 6*Sin[x]))/(-1 + 2*Cos[2*x] + 2*Sin[x])^2 - (
80*(31 + 76*Sin[x]))/(-1 + 2*Cos[2*x] + 2*Sin[x]))/2500
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.44, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4829, 1301, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(2x)+\cos(3x))^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(\sin(2x)+\cos(3x))^3}dx$$
$$\Big\downarrow \text{4829}$$
$$\int \frac{1}{\left(-4\sin^2(x)+2\sin(x)+1\right)^3\left(1-\sin^2(x)\right)^2}d\sin(x)$$
$$\Big\downarrow \text{1301}$$

$$\int \left(\frac{16(2\sin(x)+1)}{5\left(-4\sin^2(x)+2\sin(x)+1\right)^3}+\frac{16(148\sin(x)+67)}{125\left(-4\sin^2(x)+2\sin(x)+1\right)}-\frac{16(20\sin(x)+9)}{25\left(-4\sin^2(x)+2\sin(x)+1\right)^2}-\frac{}{4(1-}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{36(1-4\sin(x))}{125\left(-4\sin^2(x)+2\sin(x)+1\right)}-\frac{8(56\sin(x)+11)}{125\left(-4\sin^2(x)+2\sin(x)+1\right)}+\frac{4(6\sin(x)+1)}{25\left(-4\sin^2(x)+2\sin(x)+1\right)^2}-\frac{1}{4(1-\sin(x))}+\frac{1}{500(\sin(x)+1)}-\frac{8}{625}\left(185-104\sqrt{5}\right)\log\left(-4\sin(x)-\sqrt{5}+1\right)-\frac{152\log\left(-4\sin(x)-\sqrt{5}+1\right)}{125\sqrt{5}}-\frac{8}{625}\left(185+104\sqrt{5}\right)\log\left(-4\sin(x)+\sqrt{5}+1\right)+\frac{152\log\left(-4\sin(x)+\sqrt{5}+1\right)}{125\sqrt{5}}+\frac{19}{4}\log(1-\sin(x))-\frac{7}{500}\log(\sin(x)+1)$$

input
```
Int[(Cos[3*x] + Sin[2*x])^(-3),x]
```

output
```
(-152*Log[1 - Sqrt[5] - 4*Sin[x]])/(125*Sqrt[5]) - (8*(185 - 104*Sqrt[5])*
Log[1 - Sqrt[5] - 4*Sin[x]])/625 + (152*Log[1 + Sqrt[5] - 4*Sin[x]])/(125*
Sqrt[5]) - (8*(185 + 104*Sqrt[5])*Log[1 + Sqrt[5] - 4*Sin[x]])/625 + (19*L
og[1 - Sin[x]])/4 - (7*Log[1 + Sin[x]])/500 - 1/(4*(1 - Sin[x])) + 1/(500*
(1 + Sin[x])) + (4*(1 + 6*Sin[x]))/(25*(1 + 2*Sin[x] - 4*Sin[x]^2)^2) - (3
6*(1 - 4*Sin[x]))/(125*(1 + 2*Sin[x] - 4*Sin[x]^2)) - (8*(11 + 56*Sin[x]))
/(125*(1 + 2*Sin[x] - 4*Sin[x]^2))
```

Defintions of rubi rules used

rule 1301
```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> With[{r = Rt[(-a)*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(-r
+ c*x)^p*(r + c*x)^p*(d + e*x + f*x^2)^q, x], x], x] /; EqQ[p, -1] ||  !Fra
ctionalPowerFactorQ[r]] /; FreeQ[{a, c, d, e, f}, x] && ILtQ[p, 0] && Integ
erQ[q] && NiceSqrtQ[(-a)*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4829
```
Int[(cos[(n_.)*((c_.) + (d_.)*(x_))]*(b_.) + (a_.)*sin[(m_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Sin[
m*ArcSin[x]] + b*Cos[n*ArcSin[x]]]]^p/Sqrt[1 - x^2], x], x, Sin[c + d*x]],
x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[m/2] && Inte
gerQ[(n - 1)/2]
```

Maple [A] (verified)

Time = 14.73 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.01

method	result
parallelrisch	0
default	$-\frac{256\left(-\frac{19\sin(x)^3}{4}+\frac{7\sin(x)^2}{16}+\frac{27\sin(x)}{16}+\frac{13}{32}\right)}{125\left(4\sin(x)^2-2\sin(x)-1\right)^2}-\frac{296\ln\left(4\sin(x)^2-2\sin(x)-1\right)}{125}+\frac{272\sqrt{5}\,\operatorname{arctanh}\left(\frac{(-2+8\sin(x))\sqrt{5}}{10}\right)}{125}+$
risch	$\frac{i(-6i e^{10ix}+43\,e^{11ix}+22i e^{8ix}+12\,e^{9ix}+76i e^{6ix}-16\,e^{7ix}+22i e^{4ix}+16\,e^{5ix}-6i e^{2ix}-12\,e^{3ix}-43\,e^{ix})}{25(e^{ix}+i)^2(-2i e^{4ix}+e^{5ix}+2i e^{2ix}-2\,e^{3ix}-i+2\,e^{ix})^2}-\frac{7\ln(e^{ix}+i)}{250}+\frac{19\,\mathrm{l}}{}$

input
```
int(1/(cos(3*x)+sin(2*x))^3,x,method=_RETURNVERBOSE)
```

output
```
0
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(125) = 250.

Time = 0.10 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.04

$$\int \frac{1}{(\cos(3x)+\sin(2x))^3}\,dx$$

$$= \frac{480\cos(x)^4-920\cos(x)^2-1184\left(16\cos(x)^6-28\cos(x)^4+13\cos(x)^2+4\left(4\cos(x)^4-3\cos(x)^2\right)\right.}{}$$

input
```
integrate(1/(cos(3*x)+sin(2*x))^3,x, algorithm="fricas")
```

output
```
1/500*(480*cos(x)^4 - 920*cos(x)^2 - 1184*(16*cos(x)^6 - 28*cos(x)^4 + 13*
cos(x)^2 + 4*(4*cos(x)^4 - 3*cos(x)^2)*sin(x))*log(4*cos(x)^2 + 2*sin(x) -
 3) + 544*(16*sqrt(5)*cos(x)^6 - 28*sqrt(5)*cos(x)^4 + 13*sqrt(5)*cos(x)^2
 + 4*(4*sqrt(5)*cos(x)^4 - 3*sqrt(5)*cos(x)^2)*sin(x))*log((8*cos(x)^2 - 4
*(sqrt(5) - 1)*sin(x) + sqrt(5) - 11)/(4*cos(x)^2 + 2*sin(x) - 3)) - 7*(16
*cos(x)^6 - 28*cos(x)^4 + 13*cos(x)^2 + 4*(4*cos(x)^4 - 3*cos(x)^2)*sin(x)
)*log(sin(x) + 1) + 2375*(16*cos(x)^6 - 28*cos(x)^4 + 13*cos(x)^2 + 4*(4*c
os(x)^4 - 3*cos(x)^2)*sin(x))*log(-sin(x) + 1) - 10*(688*cos(x)^4 - 468*co
s(x)^2 + 15)*sin(x) - 100)/(16*cos(x)^6 - 28*cos(x)^4 + 13*cos(x)^2 + 4*(4
*cos(x)^4 - 3*cos(x)^2)*sin(x))
```

Sympy [F]

$$\int \frac{1}{(\cos(3x)+\sin(2x))^3}\,dx=\int \frac{1}{(\sin{(2x)}+\cos{(3x)})^3}\,dx$$

input
```
integrate(1/(cos(3*x)+sin(2*x))**3,x)
```

output
```
Integral((sin(2*x) + cos(3*x))**(-3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(3x)+\sin(2x))^3}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(1/(cos(3*x)+sin(2*x))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.87

$$\int \frac{1}{(\cos(3x)+\sin(2x))^3}\,dx$$
$$= -\frac{136}{125}\sqrt{5}\log\left(\frac{|-2\sqrt{5}+8\sin(x)-2|}{|2\sqrt{5}+8\sin(x)-2|}\right) - \frac{592\sin(x)^2-63\sin(x)-654}{250\left(\sin(x)^2-1\right)}$$
$$+ \frac{4\left(1776\sin(x)^4-1472\sin(x)^3-472\sin(x)^2+336\sin(x)+85\right)}{125\left(4\sin(x)^2-2\sin(x)-1\right)^2}$$
$$-\frac{7}{500}\log(\sin(x)+1)+\frac{19}{4}\log(-\sin(x)+1)-\frac{296}{125}\log\left(\left|4\sin(x)^2-2\sin(x)-1\right|\right)$$

input
```
integrate(1/(cos(3*x)+sin(2*x))^3,x, algorithm="giac")
```

output
```
-136/125*sqrt(5)*log(abs(-2*sqrt(5) + 8*sin(x) - 2)/abs(2*sqrt(5) + 8*sin(
x) - 2)) - 1/250*(592*sin(x)^2 - 63*sin(x) - 654)/(sin(x)^2 - 1) + 4/125*(
1776*sin(x)^4 - 1472*sin(x)^3 - 472*sin(x)^2 + 336*sin(x) + 85)/(4*sin(x)^
2 - 2*sin(x) - 1)^2 - 7/500*log(sin(x) + 1) + 19/4*log(-sin(x) + 1) - 296/
125*log(abs(4*sin(x)^2 - 2*sin(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 21.09 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.83

$$\int \frac{1}{(\cos(3x)+\sin(2x))^3}\,dx = \frac{19\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{2} - \frac{7\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{250}$$
$$- \frac{\frac{19\tan\left(\frac{x}{2}\right)^{11}}{25}+\frac{548\tan\left(\frac{x}{2}\right)^{10}}{25}+\frac{783\tan\left(\frac{x}{2}\right)^{9}}{25}-\frac{6832\tan\left(\frac{x}{2}\right)^{8}}{25}-\frac{562\tan\left(\frac{x}{2}\right)^{7}}{25}+\frac{12888\tan\left(\frac{x}{2}\right)^{6}}{25}-562\,\mathrm{t}}{\tan\left(\frac{x}{2}\right)^{12}+8\tan\left(\frac{x}{2}\right)^{11}-14\tan\left(\frac{x}{2}\right)^{10}-120\tan\left(\frac{x}{2}\right)^{9}+255\tan\left(\frac{x}{2}\right)^{8}+112\tan\left(\frac{x}{2}\right)^{7}-484\tan\left(\frac{x}{2}\right)^{6}+}$$
$$- \ln\left(2\tan\left(\frac{x}{2}\right)-2\sqrt{5}\tan\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)^2+1\right)\left(\frac{136\sqrt{5}}{125}+\frac{296}{125}\right)$$
$$+ \ln\left(2\tan\left(\frac{x}{2}\right)+2\sqrt{5}\tan\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)^2+1\right)\left(\frac{136\sqrt{5}}{125}-\frac{296}{125}\right)$$

input
```
int(1/(cos(3*x) + sin(2*x))^3,x)
```

output
```
(19*log(tan(x/2) - 1))/2 - (7*log(tan(x/2) + 1))/250 - ((19*tan(x/2))/25 +
 (548*tan(x/2)^2)/25 + (783*tan(x/2)^3)/25 - (6832*tan(x/2)^4)/25 - (562*t
an(x/2)^5)/25 + (12888*tan(x/2)^6)/25 - (562*tan(x/2)^7)/25 - (6832*tan(x/
2)^8)/25 + (783*tan(x/2)^9)/25 + (548*tan(x/2)^10)/25 + (19*tan(x/2)^11)/2
5)/(8*tan(x/2) - 14*tan(x/2)^2 - 120*tan(x/2)^3 + 255*tan(x/2)^4 + 112*tan
(x/2)^5 - 484*tan(x/2)^6 + 112*tan(x/2)^7 + 255*tan(x/2)^8 - 120*tan(x/2)^
9 - 14*tan(x/2)^10 + 8*tan(x/2)^11 + tan(x/2)^12 + 1) - log(2*tan(x/2) - 2
*5^(1/2)*tan(x/2) + tan(x/2)^2 + 1)*((136*5^(1/2))/125 + 296/125) + log(2*
tan(x/2) + 2*5^(1/2)*tan(x/2) + tan(x/2)^2 + 1)*((136*5^(1/2))/125 - 296/1
25)
```

Reduce [F]

$$\int \frac{1}{(\cos(3x)+\sin(2x))^3}\,dx$$
$$= \int \frac{1}{\cos(3x)^3+3\cos(3x)^2\sin(2x)+3\cos(3x)\sin(2x)^2+\sin(2x)^3}dx$$

input
```
int(1/(cos(3*x)+sin(2*x))^3,x)
```

output
```
int(1/(cos(3*x)**3 + 3*cos(3*x)**2*sin(2*x) + 3*cos(3*x)*sin(2*x)**2 + sin
(2*x)**3),x)
```

3.3 $\int \frac{1}{(\cos(3x)+\sin(2x))^5}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 225

$$\begin{aligned}\int \frac{1}{(\cos(3x)+\sin(2x))^5}\,dx =& -\frac{16}{625}\left(1383-619\sqrt{5}\right)\log\left(1-\sqrt{5}-4\sin(x)\right)\\ &-\frac{16}{625}\left(1383+619\sqrt{5}\right)\log\left(1+\sqrt{5}-4\sin(x)\right)\\ &+\frac{1133}{16}\log(1-\sin(x))-\frac{29\log(1+\sin(x))}{10000}\\ &+\frac{\sec^4(x)(4+5\sin(x))}{20\left(1+2\sin(x)-4\sin^2(x)\right)^3}\\ &+\frac{\sec^2(x)(118+127\sin(x))}{40\left(1+2\sin(x)-4\sin^2(x)\right)^3}\\ &-\frac{2269+5014\sin(x)}{300\left(1+2\sin(x)-4\sin^2(x)\right)^3}\\ &+\frac{3367+7754\sin(x)}{300\left(1+2\sin(x)-4\sin^2(x)\right)^2}\\ &-\frac{10951+26994\sin(x)}{500\left(1+2\sin(x)-4\sin^2(x)\right)}\\ &+\frac{\sec^3(x)\tan(x)}{5\left(1+2\sin(x)-4\sin^2(x)\right)^4}\end{aligned}$$

output
```
-16/625*(1383-619*5^(1/2))*ln(1-5^(1/2)-4*sin(x))-16/625*(1383+619*5^(1/2)
)*ln(1+5^(1/2)-4*sin(x))+1133/16*ln(1-sin(x))-29/10000*ln(1+sin(x))+1/20*s
ec(x)^4*(4+5*sin(x))/(1+2*sin(x)-4*sin(x)^2)^3+1/40*sec(x)^2*(118+127*sin(
x))/(1+2*sin(x)-4*sin(x)^2)^3-1/300*(2269+5014*sin(x))/(1+2*sin(x)-4*sin(x
)^2)^3+1/300*(3367+7754*sin(x))/(1+2*sin(x)-4*sin(x)^2)^2-(10951+26994*sin
(x))/(500+1000*sin(x)-2000*sin(x)^2)+1/5*sec(x)^3*tan(x)/(1+2*sin(x)-4*sin
(x)^2)^4
```

Mathematica [A] (warning: unable to verify)

Time = 6.07 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.23

$$\int \frac{1}{(\cos(3x)+\sin(2x))^5}\,dx = \frac{1133}{8}\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\frac{29\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}{5000} - \frac{16\left(-3095+1383\sqrt{5}\right)\log\left(1-\sqrt{5}-4\sin(x)\right)}{625\sqrt{5}} - \frac{16\left(3095+1383\sqrt{5}\right)\log\left(1+\sqrt{5}-4\sin(x)\right)}{625\sqrt{5}} - \frac{1}{16\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)^4} - \frac{63}{16\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)^2} + \frac{1}{50000\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^4} + \frac{23}{50000\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2} + \frac{8(1+4\sin(x))}{25(-1+2\cos(2x)+2\sin(x))^4} - \frac{16(35+92\sin(x))}{375(-1+2\cos(2x)+2\sin(x))^3} + \frac{112(73+194\sin(x))}{1875(-1+2\cos(2x)+2\sin(x))^2} - \frac{16(2789+7468\sin(x))}{3125(-1+2\cos(2x)+2\sin(x))}$$

input
```
Integrate[(Cos[3*x] + Sin[2*x])^(-5),x]
```

output
```
(1133*Log[Cos[x/2] - Sin[x/2]])/8 - (29*Log[Cos[x/2] + Sin[x/2]])/5000 - (
16*(-3095 + 1383*Sqrt[5])*Log[1 - Sqrt[5] - 4*Sin[x]])/(625*Sqrt[5]) - (16
*(3095 + 1383*Sqrt[5])*Log[1 + Sqrt[5] - 4*Sin[x]])/(625*Sqrt[5]) - 1/(16*
(Cos[x/2] - Sin[x/2])^4) - 63/(16*(Cos[x/2] - Sin[x/2])^2) + 1/(50000*(Cos
[x/2] + Sin[x/2])^4) + 23/(50000*(Cos[x/2] + Sin[x/2])^2) + (8*(1 + 4*Sin[
x]))/(25*(-1 + 2*Cos[2*x] + 2*Sin[x])^4) - (16*(35 + 92*Sin[x]))/(375*(-1
+ 2*Cos[2*x] + 2*Sin[x])^3) + (112*(73 + 194*Sin[x]))/(1875*(-1 + 2*Cos[2*
x] + 2*Sin[x])^2) - (16*(2789 + 7468*Sin[x]))/(3125*(-1 + 2*Cos[2*x] + 2*S
in[x]))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.46, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4829, 1301, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(2x)+\cos(3x))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(2x)+\cos(3x))^5}dx$$

$$\Big\downarrow \text{4829}$$

$$\int \frac{1}{\left(-4\sin^2(x)+2\sin(x)+1\right)^5\left(1-\sin^2(x)\right)^3}d\sin(x)$$

$$\Big\downarrow \text{1301}$$

$$-\int \left(-\frac{64(8\sin(x)+3)}{25\left(-4\sin^2(x)+2\sin(x)+1\right)^5}+\frac{1133}{16(1-\sin(x))}+\frac{29}{10000(\sin(x)+1)}-\frac{192(4610\sin(x)+2049)}{3125\left(-4\sin^2(x)+2\sin(x)+\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$
-\frac{10864(1-4\sin(x))}{3125\left(-4\sin^2(x)+2\sin(x)+1\right)}-\frac{32(5092\sin(x)+1055)}{3125\left(-4\sin^2(x)+2\sin(x)+1\right)}+\frac{368(1-4\sin(x))}{375\left(-4\sin^2(x)+2\sin(x)+1\right)^2}+\frac{96(101\sin(x)+22)}{625\left(-4\sin^2(x)+2\sin(x)+1\right)^2}-\frac{112(1-4\sin(x))}{375\left(-4\sin^2(x)+2\sin(x)+1\right)^3}-\frac{64(30\sin(x)+7)}{375\left(-4\sin^2(x)+2\sin(x)+1\right)^3}+\frac{8(4\sin(x)+1)}{25\left(-4\sin^2(x)+2\sin(x)+1\right)^4}-\frac{63}{16(1-\sin(x))}+\frac{23}{50000(\sin(x)+1)}-\frac{1}{16(1-\sin(x))^2}+\frac{1}{50000(\sin(x)+1)^2}-\frac{48\left(11525-6403\sqrt{5}\right)\log\left(-4\sin(x)-\sqrt{5}+1\right)}{15625}-\frac{59744\log\left(-4\sin(x)-\sqrt{5}+1\right)}{3125\sqrt{5}}-\frac{48\left(11525+6403\sqrt{5}\right)\log\left(-4\sin(x)+\sqrt{5}+1\right)}{15625}+\frac{59744\log\left(-4\sin(x)+\sqrt{5}+1\right)}{3125\sqrt{5}}+\frac{1133}{16}\log(1-\sin(x))-\frac{29\log(\sin(x)+1)}{10000}
$$

input
```
Int[(Cos[3*x] + Sin[2*x])^(-5),x]
```

output
```
(-59744*Log[1 - Sqrt[5] - 4*Sin[x]])/(3125*Sqrt[5]) - (48*(11525 - 6403*Sq
rt[5])*Log[1 - Sqrt[5] - 4*Sin[x]])/15625 + (59744*Log[1 + Sqrt[5] - 4*Sin
[x]])/(3125*Sqrt[5]) - (48*(11525 + 6403*Sqrt[5])*Log[1 + Sqrt[5] - 4*Sin[
x]])/15625 + (1133*Log[1 - Sin[x]])/16 - (29*Log[1 + Sin[x]])/10000 - 1/(1
6*(1 - Sin[x])^2) - 63/(16*(1 - Sin[x])) + 1/(50000*(1 + Sin[x])^2) + 23/(
50000*(1 + Sin[x])) + (8*(1 + 4*Sin[x]))/(25*(1 + 2*Sin[x] - 4*Sin[x]^2)^4
) - (112*(1 - 4*Sin[x]))/(375*(1 + 2*Sin[x] - 4*Sin[x]^2)^3) - (64*(7 + 30
*Sin[x]))/(375*(1 + 2*Sin[x] - 4*Sin[x]^2)^3) + (368*(1 - 4*Sin[x]))/(375*
(1 + 2*Sin[x] - 4*Sin[x]^2)^2) + (96*(22 + 101*Sin[x]))/(625*(1 + 2*Sin[x]
 - 4*Sin[x]^2)^2) - (10864*(1 - 4*Sin[x]))/(3125*(1 + 2*Sin[x] - 4*Sin[x]^
2)) - (32*(1055 + 5092*Sin[x]))/(3125*(1 + 2*Sin[x] - 4*Sin[x]^2))
```

Defintions of rubi rules used

rule 1301
```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> With[{r = Rt[(-a)*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[(-r
+ c*x)^p*(r + c*x)^p*(d + e*x + f*x^2)^q, x], x], x] /; EqQ[p, -1] ||  !Fra
ctionalPowerFactorQ[r]] /; FreeQ[{a, c, d, e, f}, x] && ILtQ[p, 0] && Integ
erQ[q] && NiceSqrtQ[(-a)*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4829
```
Int[(cos[(n_.)*((c_.) + (d_.)*(x_))]*(b_.) + (a_.)*sin[(m_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Sin[
m*ArcSin[x]] + b*Cos[n*ArcSin[x]]]]^p/Sqrt[1 - x^2], x], x, Sin[c + d*x]],
x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[m/2] && Inte
gerQ[(n - 1)/2]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.61

$$\frac{1}{50000\left(1+\sin\left(x\right)\right)^{2}}+\frac{23}{50000\left(1+\sin\left(x\right)\right)}-\frac{29\ln\left(1+\sin\left(x\right)\right)}{10000}-\frac{16384\left(-\frac{1867\sin(x)^{7}}{4}+\frac{8413\sin(x)^{6}}{16}+\frac{10853\sin(x)}{48}\right.}{3125\left(4\,s\right.}$$

input
```
int(1/(cos(3*x)+sin(2*x))^5,x)
```

output
```
1/50000/(1+sin(x))^2+23/50000/(1+sin(x))-29/10000*ln(1+sin(x))-16384/3125*
(-1867/4*sin(x)^7+8413/16*sin(x)^6+10853/48*sin(x)^5-51775/192*sin(x)^4-21
55/24*sin(x)^3+4941/128*sin(x)^2+1227/64*sin(x)+4333/2048)/(4*sin(x)^2-2*s
in(x)-1)^4-22128/625*ln(4*sin(x)^2-2*sin(x)-1)+19808/625*5^(1/2)*arctanh(1
/10*(-2+8*sin(x))*5^(1/2))-1/16/(sin(x)-1)^2+63/16/(sin(x)-1)+1133/16*ln(s
in(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(197) = 394$.

Time = 0.14 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.08

$$\int \frac{1}{(\cos(3x)+\sin(2x))^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cos(3*x)+sin(2*x))^5,x, algorithm="fricas")
```

output
```
1/30000*(113433600*cos(x)^10 - 282534400*cos(x)^8 + 232826240*cos(x)^6 - 6
4109200*cos(x)^4 - 66000*cos(x)^2 - 1062144*(256*cos(x)^12 - 1152*cos(x)^1
0 + 1840*cos(x)^8 - 1256*cos(x)^6 + 313*cos(x)^4 + 8*(64*cos(x)^10 - 160*c
os(x)^8 + 136*cos(x)^6 - 39*cos(x)^4)*sin(x))*log(4*cos(x)^2 + 2*sin(x) -
3) + 475392*(256*sqrt(5)*cos(x)^12 - 1152*sqrt(5)*cos(x)^10 + 1840*sqrt(5)
*cos(x)^8 - 1256*sqrt(5)*cos(x)^6 + 313*sqrt(5)*cos(x)^4 + 8*(64*sqrt(5)*c
os(x)^10 - 160*sqrt(5)*cos(x)^8 + 136*sqrt(5)*cos(x)^6 - 39*sqrt(5)*cos(x)
^4)*sin(x))*log((8*cos(x)^2 - 4*(sqrt(5) - 1)*sin(x) + sqrt(5) - 11)/(4*co
s(x)^2 + 2*sin(x) - 3)) - 87*(256*cos(x)^12 - 1152*cos(x)^10 + 1840*cos(x)
^8 - 1256*cos(x)^6 + 313*cos(x)^4 + 8*(64*cos(x)^10 - 160*cos(x)^8 + 136*c
os(x)^6 - 39*cos(x)^4)*sin(x))*log(sin(x) + 1) + 2124375*(256*cos(x)^12 -
1152*cos(x)^10 + 1840*cos(x)^8 - 1256*cos(x)^6 + 313*cos(x)^4 + 8*(64*cos(
x)^10 - 160*cos(x)^8 + 136*cos(x)^6 - 39*cos(x)^4)*sin(x))*log(-sin(x) + 1
) - 10*(10365696*cos(x)^10 - 26029952*cos(x)^8 + 22942736*cos(x)^6 - 68770
80*cos(x)^4 + 7875*cos(x)^2 + 450)*sin(x) - 3000)/(256*cos(x)^12 - 1152*co
s(x)^10 + 1840*cos(x)^8 - 1256*cos(x)^6 + 313*cos(x)^4 + 8*(64*cos(x)^10 -
 160*cos(x)^8 + 136*cos(x)^6 - 39*cos(x)^4)*sin(x))
```

Sympy [F]

$$\int \frac{1}{(\cos(3x)+\sin(2x))^5}\,dx = \int \frac{1}{(\sin(2x)+\cos(3x))^5}\,dx$$

input
```
integrate(1/(cos(3*x)+sin(2*x))**5,x)
```

output
```
Integral((sin(2*x) + cos(3*x))**(-5), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(3x) + \sin(2x))^5} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(cos(3*x)+sin(2*x))^5,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.73

$$\begin{aligned}\int \frac{1}{(\cos(3x) + \sin(2x))^5} \, dx = & -\frac{9904}{625}\sqrt{5}\log\left(\frac{\left|-2\sqrt{5} + 8\sin(x) - 2\right|}{\left|2\sqrt{5} + 8\sin(x) - 2\right|}\right) \\ & - \frac{1327680\sin(x)^4 - 98449\sin(x)^3 - 2752224\sin(x)^2 + 101575\sin(x) + 1427668}{25000\left(\sin(x)^2 - 1\right)^2} \\ & + \frac{4\left(44256000\sin(x)^8 - 82776576\sin(x)^7 + 15666816\sin(x)^6 + 41477632\sin(x)^5 - 10516400\sin(x\right.}{9375\left(4\sin(x)^2 - 2\sin(x) - 1\right)^4} \\ & - \frac{29}{10000}\log(\sin(x) + 1) + \frac{1133}{16}\log(-\sin(x) + 1) \\ & - \frac{22128}{625}\log\left(\left|4\sin(x)^2 - 2\sin(x) - 1\right|\right)\end{aligned}$$

input
```
integrate(1/(cos(3*x)+sin(2*x))^5,x, algorithm="giac")
```

output
```
-9904/625*sqrt(5)*log(abs(-2*sqrt(5) + 8*sin(x) - 2)/abs(2*sqrt(5) + 8*sin
(x) - 2)) - 1/25000*(1327680*sin(x)^4 - 98449*sin(x)^3 - 2752224*sin(x)^2
+ 101575*sin(x) + 1427668)/(sin(x)^2 - 1)^2 + 4/9375*(44256000*sin(x)^8 -
82776576*sin(x)^7 + 15666816*sin(x)^6 + 41477632*sin(x)^5 - 10516400*sin(x
)^4 - 9960640*sin(x)^3 + 908664*sin(x)^2 + 1147416*sin(x) + 146877)/(4*sin
(x)^2 - 2*sin(x) - 1)^4 - 29/10000*log(sin(x) + 1) + 1133/16*log(-sin(x) +
 1) - 22128/625*log(abs(4*sin(x)^2 - 2*sin(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 20.81 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.05

$$\int \frac{1}{(\cos(3x)+\sin(2x))^5}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(3*x) + sin(2*x))^5,x)
```

output
```
(1133*log(tan(x/2) - 1))/8 - (29*log(tan(x/2) + 1))/5000 - log(2*tan(x/2)
- 2*5^(1/2)*tan(x/2) + tan(x/2)^2 + 1)*((9904*5^(1/2))/625 + 22128/625) +
log(2*tan(x/2) + 2*5^(1/2)*tan(x/2) + tan(x/2)^2 + 1)*((9904*5^(1/2))/625
- 22128/625) - ((15839*tan(x/2))/500 + (75136*tan(x/2)^2)/125 + (3848627*t
an(x/2)^3)/1500 - (3843304*tan(x/2)^4)/375 - (33523387*tan(x/2)^5)/500 + (
14590144*tan(x/2)^6)/125 + (303233451*tan(x/2)^7)/500 - (444161888*tan(x/2
)^8)/375 - (1081143559*tan(x/2)^9)/750 + (479882176*tan(x/2)^10)/125 + (22
4953597*tan(x/2)^11)/250 - (690373584*tan(x/2)^12)/125 + (224953597*tan(x/
2)^13)/250 + (479882176*tan(x/2)^14)/125 - (1081143559*tan(x/2)^15)/750 -
(444161888*tan(x/2)^16)/375 + (303233451*tan(x/2)^17)/500 + (14590144*tan(
x/2)^18)/125 - (33523387*tan(x/2)^19)/500 - (3843304*tan(x/2)^20)/375 + (3
848627*tan(x/2)^21)/1500 + (75136*tan(x/2)^22)/125 + (15839*tan(x/2)^23)/5
00)/(16*tan(x/2) + 36*tan(x/2)^2 - 464*tan(x/2)^3 - 1214*tan(x/2)^4 + 7664
*tan(x/2)^5 + 8084*tan(x/2)^6 - 71856*tan(x/2)^7 + 53999*tan(x/2)^8 + 1739
84*tan(x/2)^9 - 270264*tan(x/2)^10 - 109344*tan(x/2)^11 + 418716*tan(x/2)^
12 - 109344*tan(x/2)^13 - 270264*tan(x/2)^14 + 173984*tan(x/2)^15 + 53999*
tan(x/2)^16 - 71856*tan(x/2)^17 + 8084*tan(x/2)^18 + 7664*tan(x/2)^19 - 12
14*tan(x/2)^20 - 464*tan(x/2)^21 + 36*tan(x/2)^22 + 16*tan(x/2)^23 + tan(x
/2)^24 + 1)
```

Reduce [F]

$$\int \frac{1}{(\cos(3x)+\sin(2x))^5}\,dx$$
$$= \int \frac{1}{\cos(3x)^5 + 5\cos(3x)^4\sin(2x) + 10\cos(3x)^3\sin(2x)^2 + 10\cos(3x)^2\sin(2x)^3 + 5\cos(3x)\sin(2x)^4 +$$

input
```
int(1/(cos(3*x)+sin(2*x))^5,x)
```

output
```
int(1/(cos(3*x)**5 + 5*cos(3*x)**4*sin(2*x) + 10*cos(3*x)**3*sin(2*x)**2 +
 10*cos(3*x)**2*sin(2*x)**3 + 5*cos(3*x)*sin(2*x)**4 + sin(2*x)**5),x)
```

3.4 $\int \frac{1}{(\cos(3x)+\sin(2x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 1

$$\int \frac{1}{(\cos(3x)+\sin(2x))^2}\,dx = 0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.50 (sec) , antiderivative size = 186, normalized size of antiderivative = 186.00

$$\int \frac{1}{(\cos(3x)+\sin(2x))^2}\,dx = \frac{1}{25}\left(-8\left(-2+\sqrt{5}\right)\sqrt{\frac{10}{5+\sqrt{5}}}\operatorname{arctanh}\left(\frac{4+\left(-1+\sqrt{5}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-4\left(2+\sqrt{5}\right)\sqrt{2\left(5+\sqrt{5}\right)}\operatorname{arctanh}\left(\frac{4-\left(1+\sqrt{5}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{10-2\sqrt{5}}}\right)+\frac{25\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}+\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}+\frac{8(\cos(x)+3\sin(2x))}{-1+2\cos(2x)+2\sin(x)}\right)$$

input
```
Integrate[(Cos[3*x] + Sin[2*x])^(-2),x]
```

output
```
(-8*(-2 + Sqrt[5])*Sqrt[10/(5 + Sqrt[5])]*ArcTanh[(4 + (-1 + Sqrt[5])*Tan[
x/2])/Sqrt[2*(5 + Sqrt[5])]] - 4*(2 + Sqrt[5])*Sqrt[2*(5 + Sqrt[5])]*ArcTa
nh[(4 - (1 + Sqrt[5])*Tan[x/2])/Sqrt[10 - 2*Sqrt[5]]] + (25*Sin[x/2])/(Cos
[x/2] - Sin[x/2]) + Sin[x/2]/(Cos[x/2] + Sin[x/2]) + (8*(Cos[x] + 3*Sin[2*
x]))/(-1 + 2*Cos[2*x] + 2*Sin[x]))/25
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 1.13 (sec) , antiderivative size = 439, normalized size of antiderivative = 439.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4830, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(2x)+\cos(3x))^2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(\sin(2x)+\cos(3x))^2}dx$$

$$\Big\downarrow \text{ 4830}$$

$$2\int \frac{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^5}{\left(-\tan^6\left(\frac{x}{2}\right)-4\tan^5\left(\frac{x}{2}\right)+15\tan^4\left(\frac{x}{2}\right)-15\tan^2\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)+1\right)^2}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{ 2462}$$

$$2\int \left(\frac{4\left(3\tan^2\left(\frac{x}{2}\right)-44\tan\left(\frac{x}{2}\right)+243\right)}{25\left(\tan^4\left(\frac{x}{2}\right)+4\tan^3\left(\frac{x}{2}\right)-14\tan^2\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)+1\right)}+\frac{1}{2\left(\tan\left(\frac{x}{2}\right)-1\right)^2}+\frac{1}{50\left(\tan\left(\frac{x}{2}\right)+1\right)^2}-\frac{6}{5\left(\tan\right.}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(-\frac{2}{25}\sqrt{\frac{2}{5}\left(12665+1499\sqrt{5}\right)}\text{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)-\sqrt{5}+1}{\sqrt{5-2\sqrt{5}}}\right)+\frac{4}{25}\sqrt{1325+218\sqrt{5}}\text{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)-\sqrt{5}+1}{\sqrt{5-2\sqrt{5}}}\right)\right.$$

input
```
Int[(Cos[3*x] + Sin[2*x])^(-2),x]
```

output
```
2*((16*Sqrt[(5 + 2*Sqrt[5])/5]*ArcTanh[(1 - Sqrt[5] + Tan[x/2])/Sqrt[5 - 2
*Sqrt[5]]])/25 + (4*Sqrt[1325 + 218*Sqrt[5]]*ArcTanh[(1 - Sqrt[5] + Tan[x/
2])/Sqrt[5 - 2*Sqrt[5]]])/25 - (2*Sqrt[(2*(12665 + 1499*Sqrt[5]))/5]*ArcTa
nh[(1 - Sqrt[5] + Tan[x/2])/Sqrt[5 - 2*Sqrt[5]]])/25 - (2*Sqrt[(2*(12665 -
 1499*Sqrt[5]))/5]*ArcTanh[(1 + Sqrt[5] + Tan[x/2])/Sqrt[5 + 2*Sqrt[5]]])/
25 + (4*Sqrt[1325 - 218*Sqrt[5]]*ArcTanh[(1 + Sqrt[5] + Tan[x/2])/Sqrt[5 +
 2*Sqrt[5]]])/25 + (16*Sqrt[(5 - 2*Sqrt[5])/5]*ArcTanh[(1 + Sqrt[5] + Tan[
x/2])/Sqrt[5 + 2*Sqrt[5]]])/25 + 1/(2*(1 - Tan[x/2])) - 1/(50*(1 + Tan[x/2
])) + (4*(1 + Sqrt[5])*(1 + (1 - Sqrt[5])*Tan[x/2]))/(25*(1 + 2*(1 - Sqrt[
5])*Tan[x/2] + Tan[x/2]^2)) + (4*(1 - Sqrt[5])*(1 + (1 + Sqrt[5])*Tan[x/2]
))/(25*(1 + 2*(1 + Sqrt[5])*Tan[x/2] + Tan[x/2]^2)))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4830
```
Int[(cos[(n_.)*((c_.) + (d_.)*(x_))]*(b_.) + (a_.)*sin[(m_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[2/d   Subst[Int[Simplify[TrigExpand[a*Sin[
2*m*ArcTan[x]] + b*Cos[2*n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[(1/2)*(c +
d*x)]], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && Intege
rQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 1.

Time = 2.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 108.00

method	result
risch	$\frac{\frac{8\,\mathrm{e}^{5ix}}{5}-\frac{2i\mathrm{e}^{4ix}}{5}+\frac{2\,\mathrm{e}^{3ix}}{5}+\frac{2i\mathrm{e}^{2ix}}{5}-\frac{2\,\mathrm{e}^{ix}}{5}+2i}{-i\mathrm{e}^{5ix}+\mathrm{e}^{6ix}+i\mathrm{e}^{ix}+1}+\left(\sum_{\_R=\mathrm{RootOf}\left(78125\_Z^{4}-130000\_Z^{2}+256\right)}\_R\ln\left(\mathrm{e}^{ix}+\frac{46875\_R^{3}}{3712}-\frac{62}{}\right.\right.$
default	$-\frac{1}{\tan\left(\frac{x}{2}\right)-1}-\frac{1}{25\left(\tan\left(\frac{x}{2}\right)+1\right)}+\frac{-\frac{64\tan\left(\frac{x}{2}\right)^{3}}{25}-\frac{112\tan\left(\frac{x}{2}\right)^{2}}{25}+\frac{128\tan\left(\frac{x}{2}\right)}{25}+\frac{16}{25}}{\tan\left(\frac{x}{2}\right)^{4}+4\tan\left(\frac{x}{2}\right)^{3}-14\tan\left(\frac{x}{2}\right)^{2}+4\tan\left(\frac{x}{2}\right)+1}+\frac{8\left(\frac{1}{2}+\frac{3\sqrt{5}}{10}\right)\operatorname{arctanh}\left(\frac{2\tan\left(\frac{x}{2}\right)-2\sqrt{5}+2}{2\sqrt{5-2\sqrt{5}}}\right)}{5\sqrt{5-2\sqrt{5}}}$

input
```
int(1/(cos(3*x)+sin(2*x))^2,x,method=_RETURNVERBOSE)
```

output
```
2/5*(4*exp(5*I*x)-I*exp(4*I*x)+exp(3*I*x)+I*exp(2*I*x)-exp(I*x)+5*I)/(-I*e
xp(5*I*x)+exp(6*I*x)+I*exp(I*x)+1)+sum(_R*ln(exp(I*x)+46875/3712*_R^3-625/
928*I*_R^2-4975/232*_R+9/29*I),_R=RootOf(78125*_Z^4-130000*_Z^2+256))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.10 (sec) , antiderivative size = 292, normalized size of antiderivative = 292.00

$$\int \frac{1}{(\cos(3x)+\sin(2x))^2}\,dx = \frac{2\sqrt{2}\left(4\cos(x)^3+2\cos(x)\sin(x)-3\cos(x)\right)\sqrt{29\sqrt{5}+65}\log\left(-2\sqrt{2}\sqrt{29\sqrt{5}+65}(3\sqrt{5}-7)\cos(x\right.}{}$$

input
```
integrate(1/(cos(3*x)+sin(2*x))^2,x, algorithm="fricas")
```

output
```
1/50*(2*sqrt(2)*(4*cos(x)^3 + 2*cos(x)*sin(x) - 3*cos(x))*sqrt(29*sqrt(5)
+ 65)*log(-2*sqrt(2)*sqrt(29*sqrt(5) + 65)*(3*sqrt(5) - 7)*cos(x) + 4*(sqr
t(5) + 1)*sin(x) - 16) - 2*sqrt(2)*(4*cos(x)^3 + 2*cos(x)*sin(x) - 3*cos(x
))*sqrt(29*sqrt(5) + 65)*log(-2*sqrt(2)*sqrt(29*sqrt(5) + 65)*(3*sqrt(5) -
 7)*cos(x) - 4*(sqrt(5) + 1)*sin(x) + 16) + (4*cos(x)^3 + 2*cos(x)*sin(x)
- 3*cos(x))*sqrt(-232*sqrt(5) + 520)*log(-(3*sqrt(5) + 7)*sqrt(-232*sqrt(5
) + 520)*cos(x) + 4*(sqrt(5) - 1)*sin(x) + 16) - (4*cos(x)^3 + 2*cos(x)*si
n(x) - 3*cos(x))*sqrt(-232*sqrt(5) + 520)*log(-(3*sqrt(5) + 7)*sqrt(-232*s
qrt(5) + 520)*cos(x) - 4*(sqrt(5) - 1)*sin(x) - 16) + 60*cos(x)^2 + 10*(20
*cos(x)^2 - 3)*sin(x) - 20)/(4*cos(x)^3 + 2*cos(x)*sin(x) - 3*cos(x))
```

Sympy [F]

$$\int \frac{1}{(\cos(3x)+\sin(2x))^2}\,dx = \int \frac{1}{(\sin(2x)+\cos(3x))^2}\,dx$$

input
```
integrate(1/(cos(3*x)+sin(2*x))**2,x)
```

output
```
Integral((sin(2*x) + cos(3*x))**(-2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(3x) + \sin(2x))^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(cos(3*x)+sin(2*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 119.00

$$\int \frac{1}{(\cos(3x) + \sin(2x))^2} \, dx = -\frac{2\left(9\tan\left(\frac{1}{2}x\right)^5 + 24\tan\left(\frac{1}{2}x\right)^4 - 46\tan\left(\frac{1}{2}x\right)^3 - 36\tan\left(\frac{1}{2}x\right)^2 + 25\tan\left(\frac{1}{2}x\right) + 4\right)}{5\left(\tan\left(\frac{1}{2}x\right)^6 + 4\tan\left(\frac{1}{2}x\right)^5 - 15\tan\left(\frac{1}{2}x\right)^4 + 15\tan\left(\frac{1}{2}x\right)^2 - 4\tan\left(\frac{1}{2}x\right) - 1\right)} - 0.0444023282280000\log\left(\tan\left(\frac{1}{2}x\right) + 6.31375151468000\right) + 0.0444023282280000\log\left(\tan\left(\frac{1}{2}x\right) + 0.158384440325000\right) + 1.28919681711200\log\left(\tan\left(\frac{1}{2}x\right) - 0.509525449494000\right) - 1.28919681711200\log\left(\tan\left(\frac{1}{2}x\right) - 1.96261050551000\right)$$

input
```
integrate(1/(cos(3*x)+sin(2*x))^2,x, algorithm="giac")
```

output
```
-2/5*(9*tan(1/2*x)^5 + 24*tan(1/2*x)^4 - 46*tan(1/2*x)^3 - 36*tan(1/2*x)^2
 + 25*tan(1/2*x) + 4)/(tan(1/2*x)^6 + 4*tan(1/2*x)^5 - 15*tan(1/2*x)^4 + 1
5*tan(1/2*x)^2 - 4*tan(1/2*x) - 1) - 0.0444023282280000*log(tan(1/2*x) + 6
.31375151468000) + 0.0444023282280000*log(tan(1/2*x) + 0.158384440325000)
+ 1.28919681711200*log(tan(1/2*x) - 0.509525449494000) - 1.28919681711200*
log(tan(1/2*x) - 1.96261050551000)
```

Mupad [B] (verification not implemented)

Time = 20.66 (sec) , antiderivative size = 418, normalized size of antiderivative = 418.00

$$\int \frac{1}{(\cos(3x) + \sin(2x))^2} \, dx = \text{Too large to display}$$

input
```
int(1/(cos(3*x) + sin(2*x))^2,x)
```

output
```
(10*tan(x/2) - (72*tan(x/2)^2)/5 - (92*tan(x/2)^3)/5 + (48*tan(x/2)^4)/5 +
 (18*tan(x/2)^5)/5 + 8/5)/(4*tan(x/2) - 15*tan(x/2)^2 + 15*tan(x/2)^4 - 4*
tan(x/2)^5 - tan(x/2)^6 + 1) - (4*atanh((712704*(130 - 58*5^(1/2))^(1/2))/
(625*((2850816*tan(x/2))/125 - (6651904*5^(1/2)*tan(x/2))/625 + (2613248*5
^(1/2))/625 - 237568/25)) - (475136*tan(x/2)*(130 - 58*5^(1/2))^(1/2))/(62
5*((2850816*tan(x/2))/125 - (6651904*5^(1/2)*tan(x/2))/625 + (2613248*5^(1
/2))/625 - 237568/25)) - (237568*5^(1/2)*(130 - 58*5^(1/2))^(1/2))/(625*((
2850816*tan(x/2))/125 - (6651904*5^(1/2)*tan(x/2))/625 + (2613248*5^(1/2))
/625 - 237568/25)) + (712704*5^(1/2)*tan(x/2)*(130 - 58*5^(1/2))^(1/2))/(6
25*((2850816*tan(x/2))/125 - (6651904*5^(1/2)*tan(x/2))/625 + (2613248*5^(
1/2))/625 - 237568/25)))*(130 - 58*5^(1/2))^(1/2))/25 - (4*atanh((712704*(
58*5^(1/2) + 130)^(1/2))/(625*((2850816*tan(x/2))/125 + (6651904*5^(1/2)*t
an(x/2))/625 - (2613248*5^(1/2))/625 - 237568/25)) - (475136*tan(x/2)*(58*
5^(1/2) + 130)^(1/2))/(625*((2850816*tan(x/2))/125 + (6651904*5^(1/2)*tan(
x/2))/625 - (2613248*5^(1/2))/625 - 237568/25)) + (237568*5^(1/2)*(58*5^(1
/2) + 130)^(1/2))/(625*((2850816*tan(x/2))/125 + (6651904*5^(1/2)*tan(x/2)
)/625 - (2613248*5^(1/2))/625 - 237568/25)) - (712704*5^(1/2)*tan(x/2)*(58
*5^(1/2) + 130)^(1/2))/(625*((2850816*tan(x/2))/125 + (6651904*5^(1/2)*tan
(x/2))/625 - (2613248*5^(1/2))/625 - 237568/25)))*(58*5^(1/2) + 130)^(1/2)
)/25
```

Reduce [F]

$$\int \frac{1}{(\cos(3x) + \sin(2x))^2} \, dx = \int \frac{1}{\cos(3x)^2 + 2\cos(3x)\sin(2x) + \sin(2x)^2} dx$$

input
```
int(1/(cos(3*x)+sin(2*x))^2,x)
```

output
```
int(1/(cos(3*x)**2 + 2*cos(3*x)*sin(2*x) + sin(2*x)**2),x)
```

3.5 $\int \frac{1}{(\cos(3x)+\sin(2x))^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 1

$$\int \frac{1}{(\cos(3x)+\sin(2x))^4}\,dx=0$$

output

```
0
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 4.37 (sec) , antiderivative size = 266, normalized size of antiderivative = 266.00

$$\int \frac{1}{(\cos(3x)+\sin(2x))^4}\,dx = \frac{192\sqrt{\frac{2}{5+\sqrt{5}}}\left(-825+367\sqrt{5}\right)\operatorname{arctanh}\left(\frac{4+\left(-1+\sqrt{5}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-96\sqrt{2\left(5+\sqrt{5}\right)}\left(367+165\sqrt{5}\right)\operatorname{arctanh}\left(\frac{4-}{}\right.}{}$$

input

```
Integrate[(Cos[3*x] + Sin[2*x])^(-4),x]
```

output
```
(192*Sqrt[2/(5 + Sqrt[5])]*(-825 + 367*Sqrt[5])*ArcTanh[(4 + (-1 + Sqrt[5]
)*Tan[x/2])/Sqrt[2*(5 + Sqrt[5])]] - 96*Sqrt[2*(5 + Sqrt[5])]*(367 + 165*S
qrt[5])*ArcTanh[(4 - (1 + Sqrt[5])*Tan[x/2])/Sqrt[10 - 2*Sqrt[5]]] + (2*(1
360 + 17802*Cos[x] + 34305*Cos[2*x] - 53406*Cos[3*x] + 32985*Cos[4*x] - 11
535*Cos[6*x] + 17802*Cos[7*x] - 12075*Cos[8*x] - 5934*Cos[9*x] + 34395*Sin
[x] - 53406*Sin[2*x] + 35315*Sin[3*x] + 17802*Sin[4*x] + 1110*Sin[5*x] + 5
934*Sin[6*x] + 11565*Sin[7*x] - 17802*Sin[8*x] + 10625*Sin[9*x]))/((Cos[x/
2] - Sin[x/2])^3*(Cos[x/2] + Sin[x/2])^3*(-1 + 2*Cos[2*x] + 2*Sin[x])^3))/
7500
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 4.58 (sec) , antiderivative size = 2393, normalized size of antiderivative = 2393.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4830, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(2x)+\cos(3x))^4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(\sin(2x)+\cos(3x))^4}dx$$

$$\Big\downarrow \text{ 4830}$$

$$2\int \frac{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^{11}}{\left(-\tan^6\left(\frac{x}{2}\right)-4\tan^5\left(\frac{x}{2}\right)+15\tan^4\left(\frac{x}{2}\right)-15\tan^2\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)+1\right)^4}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{ 2462}$$

$$2\int \left(-\frac{16\left(215\tan^2\left(\frac{x}{2}\right)+2012\tan\left(\frac{x}{2}\right)-6585\right)}{625\left(\tan^4\left(\frac{x}{2}\right)+4\tan^3\left(\frac{x}{2}\right)-14\tan^2\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)+1\right)}+\frac{13}{2\left(\tan\left(\frac{x}{2}\right)-1\right)^2}+\frac{1}{250\left(\tan\left(\frac{x}{2}\right)+1\right)^2}-2$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{32}{625}\sqrt{5496395760940885+755342688027722\sqrt{5}}\operatorname{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)-\sqrt{5}+1}{\sqrt{5-2\sqrt{5}}}\right)-\frac{16\sqrt{15903211069805+228}}{}\right.$$

input

```
Int[(Cos[3*x] + Sin[2*x])^(-4),x]
```

output

```
2*((32*(95442211 - 34854353*Sqrt[5])*ArcTanh[(1 - Sqrt[5] + Tan[x/2])/Sqrt
[5 - 2*Sqrt[5]]])/(625*(5 - 2*Sqrt[5])^(3/2)) + (16*(285412903 - 250146680
*Sqrt[5])*ArcTanh[(1 - Sqrt[5] + Tan[x/2])/Sqrt[5 - 2*Sqrt[5]]])/(625*Sqrt
[5*(5 - 2*Sqrt[5])]) + (128*Sqrt[845 + 358*Sqrt[5]]*ArcTanh[(1 - Sqrt[5] +
 Tan[x/2])/Sqrt[5 - 2*Sqrt[5]]])/625 - (128*(6462366 + 833905*Sqrt[5])*Arc
Tanh[(1 - Sqrt[5] + Tan[x/2])/Sqrt[5 - 2*Sqrt[5]]])/(625*Sqrt[5 - 2*Sqrt[5
]]) - (8*Sqrt[2*(37230665 + 1543931*Sqrt[5])]*ArcTanh[(1 - Sqrt[5] + Tan[x
/2])/Sqrt[5 - 2*Sqrt[5]]])/3125 + (96*(4566082 + 2562235*Sqrt[5])*ArcTanh[
(1 - Sqrt[5] + Tan[x/2])/Sqrt[5 - 2*Sqrt[5]]])/(625*Sqrt[5*(5 - 2*Sqrt[5])
]) - (5328*Sqrt[2*(81545744785 + 36416503171*Sqrt[5])]*ArcTanh[(1 - Sqrt[5
] + Tan[x/2])/Sqrt[5 - 2*Sqrt[5]]])/3125 + (64*Sqrt[127439782405 + 5477050
7458*Sqrt[5]]*ArcTanh[(1 - Sqrt[5] + Tan[x/2])/Sqrt[5 - 2*Sqrt[5]]])/3125
- (16*Sqrt[15903211069805 + 2287334301082*Sqrt[5]]*ArcTanh[(1 - Sqrt[5] +
Tan[x/2])/Sqrt[5 - 2*Sqrt[5]]])/3125 + (32*Sqrt[5496395760940885 + 7553426
88027722*Sqrt[5]]*ArcTanh[(1 - Sqrt[5] + Tan[x/2])/Sqrt[5 - 2*Sqrt[5]]])/6
25 - (16*Sqrt[15903211069805 - 2287334301082*Sqrt[5]]*ArcTanh[(1 + Sqrt[5]
 + Tan[x/2])/Sqrt[5 + 2*Sqrt[5]]])/3125 - (8*Sqrt[2*(37230665 - 1543931*Sq
rt[5])]*ArcTanh[(1 + Sqrt[5] + Tan[x/2])/Sqrt[5 + 2*Sqrt[5]]])/3125 + (128
*Sqrt[845 - 358*Sqrt[5]]*ArcTanh[(1 + Sqrt[5] + Tan[x/2])/Sqrt[5 + 2*Sqrt[
5]]])/625 - (128*(6462366 - 833905*Sqrt[5])*ArcTanh[(1 + Sqrt[5] + Tan[...
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4830
```
Int[(cos[(n_.)*((c_.) + (d_.)*(x_))]*(b_.) + (a_.)*sin[(m_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[2/d   Subst[Int[Simplify[TrigExpand[a*Sin[
2*m*ArcTan[x]] + b*Cos[2*n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[(1/2)*(c +
d*x)]], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && Intege
rQ[n]
```

Maple [A] (verified)

Time = 115.72 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result
parallelrisch	0
risch	$\frac{\frac{148\,e^{13ix}}{125}-\frac{8864\,e^{15ix}}{375}-\frac{1172\,e^{ix}}{25}+\frac{68i}{3}+\frac{1088\,e^{9ix}}{375}+\frac{528\,e^{17ix}}{25}-\frac{364\,e^{3ix}}{375}+\frac{17324\,e^{11ix}}{125}-\frac{8176\,e^{7ix}}{125}-\frac{2708ie^{2ix}}{125}-\frac{8836ie^{10ix}}{125}-\frac{148ie^{14}}{125}}{\left(-2ie^{4ix}+e^{5ix}+2ie^{2ix}-2\,e^{3ix}-i+2\,e^{ix}\right)^{3}\left(e^{ix}+i\right)^{3}}$
default	$-\frac{1}{1875\left(\tan\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{1}{1250\left(\tan\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{1}{125\left(\tan\left(\frac{x}{2}\right)+1\right)}-\frac{32\left(290\tan\left(\frac{x}{2}\right)^{11}+3544\tan\left(\frac{x}{2}\right)^{10}+6054\tan\left(\frac{x}{2}\right)^{9}-57\right.}{}$

input
```
int(1/(cos(3*x)+sin(2*x))^4,x,method=_RETURNVERBOSE)
```

output
```
0
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.13 (sec) , antiderivative size = 484, normalized size of antiderivative = 484.00

$$\int \frac{1}{(\cos(3x)+\sin(2x))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cos(3*x)+sin(2*x))^4,x, algorithm="fricas")
```

output
```
-1/3750*(1545600*cos(x)^8 - 2722080*cos(x)^6 + 1114440*cos(x)^4 - 24*sqrt(
2)*(64*cos(x)^9 - 192*cos(x)^7 + 192*cos(x)^5 - 63*cos(x)^3 + 2*(48*cos(x)
^7 - 76*cos(x)^5 + 31*cos(x)^3)*sin(x))*sqrt(219091*sqrt(5) + 489905)*log(
-8*sqrt(2)*sqrt(219091*sqrt(5) + 489905)*(133*sqrt(5) - 298)*cos(x) + 2872
*(sqrt(5) + 1)*sin(x) - 11488) + 24*sqrt(2)*(64*cos(x)^9 - 192*cos(x)^7 +
192*cos(x)^5 - 63*cos(x)^3 + 2*(48*cos(x)^7 - 76*cos(x)^5 + 31*cos(x)^3)*s
in(x))*sqrt(219091*sqrt(5) + 489905)*log(-8*sqrt(2)*sqrt(219091*sqrt(5) +
489905)*(133*sqrt(5) - 298)*cos(x) - 2872*(sqrt(5) + 1)*sin(x) + 11488) -
3*(64*cos(x)^9 - 192*cos(x)^7 + 192*cos(x)^5 - 63*cos(x)^3 + 2*(48*cos(x)^
7 - 76*cos(x)^5 + 31*cos(x)^3)*sin(x))*sqrt(-28043648*sqrt(5) + 62707840)*
log(-(133*sqrt(5) + 298)*sqrt(-28043648*sqrt(5) + 62707840)*cos(x) + 2872*
(sqrt(5) - 1)*sin(x) + 11488) + 3*(64*cos(x)^9 - 192*cos(x)^7 + 192*cos(x)
^5 - 63*cos(x)^3 + 2*(48*cos(x)^7 - 76*cos(x)^5 + 31*cos(x)^3)*sin(x))*sqr
t(-28043648*sqrt(5) + 62707840)*log(-(133*sqrt(5) + 298)*sqrt(-28043648*sq
rt(5) + 62707840)*cos(x) - 2872*(sqrt(5) - 1)*sin(x) - 11488) + 16500*cos(
x)^2 - 10*(272000*cos(x)^8 - 401984*cos(x)^6 + 164256*cos(x)^4 - 1950*cos(
x)^2 - 75)*sin(x) + 500)/(64*cos(x)^9 - 192*cos(x)^7 + 192*cos(x)^5 - 63*c
os(x)^3 + 2*(48*cos(x)^7 - 76*cos(x)^5 + 31*cos(x)^3)*sin(x))
```

Sympy [F]

$$\int \frac{1}{(\cos(3x) + \sin(2x))^4} \, dx = \int \frac{1}{(\sin(2x) + \cos(3x))^4} \, dx$$

input
```
integrate(1/(cos(3*x)+sin(2*x))**4,x)
```

output
```
Integral((sin(2*x) + cos(3*x))**(-4), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(3x)+\sin(2x))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(cos(3*x)+sin(2*x))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.12 (sec) , antiderivative size = 215, normalized size of antiderivative = 215.00

$$\int \frac{1}{(\cos(3x)+\sin(2x))^4}\,dx = -\frac{2\left(5223\tan\left(\frac{1}{2}x\right)^{17}+65820\tan\left(\frac{1}{2}x\right)^{16}+90128\tan\left(\frac{1}{2}x\right)^{15}-1330212\tan\left(\frac{1}{2}x\right)^{14}-1522956\tan\left(\frac{1}{2}\right.\right.}{}$$

$$-0.0293605416398400\log\left(\tan\left(\frac{1}{2}x\right)+6.31375151468000\right)$$
$$+0.0293605416398400\log\left(\tan\left(\frac{1}{2}x\right)+0.158384440325000\right)$$
$$+17.9182387180800\log\left(\tan\left(\frac{1}{2}x\right)-0.509525449494000\right)$$
$$-17.9182387180800\log\left(\tan\left(\frac{1}{2}x\right)-1.96261050551000\right)$$

input
```
integrate(1/(cos(3*x)+sin(2*x))^4,x, algorithm="giac")
```

output
```
-2/375*(5223*tan(1/2*x)^17 + 65820*tan(1/2*x)^16 + 90128*tan(1/2*x)^15 - 1
330212*tan(1/2*x)^14 - 1522956*tan(1/2*x)^13 + 12715668*tan(1/2*x)^12 - 49
99824*tan(1/2*x)^11 - 29883540*tan(1/2*x)^10 + 24059626*tan(1/2*x)^9 + 254
34588*tan(1/2*x)^8 - 28672848*tan(1/2*x)^7 - 5768748*tan(1/2*x)^6 + 122052
36*tan(1/2*x)^5 - 1330212*tan(1/2*x)^4 - 1243056*tan(1/2*x)^3 + 79332*tan(
1/2*x)^2 + 59271*tan(1/2*x) + 4504)/(tan(1/2*x)^6 + 4*tan(1/2*x)^5 - 15*ta
n(1/2*x)^4 + 15*tan(1/2*x)^2 - 4*tan(1/2*x) - 1)^3 - 0.0293605416398400*lo
g(tan(1/2*x) + 6.31375151468000) + 0.0293605416398400*log(tan(1/2*x) + 0.1
58384440325000) + 17.9182387180800*log(tan(1/2*x) - 0.509525449494000) - 1
7.9182387180800*log(tan(1/2*x) - 1.96261050551000)
```

Mupad [B] (verification not implemented)

Time = 22.48 (sec) , antiderivative size = 594, normalized size of antiderivative = 594.00

$$\int \frac{1}{(\cos(3x) + \sin(2x))^4}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(3*x) + sin(2*x))^4,x)
```

output
```
((39514*tan(x/2))/125 + (52888*tan(x/2)^2)/125 - (828704*tan(x/2)^3)/125 -
 (886808*tan(x/2)^4)/125 + (8136824*tan(x/2)^5)/125 - (3845832*tan(x/2)^6)
/125 - (19115232*tan(x/2)^7)/125 + (16956392*tan(x/2)^8)/125 + (48119252*t
an(x/2)^9)/375 - (3984472*tan(x/2)^10)/25 - (3333216*tan(x/2)^11)/125 + (8
477112*tan(x/2)^12)/125 - (1015304*tan(x/2)^13)/125 - (886808*tan(x/2)^14)
/125 + (180256*tan(x/2)^15)/375 + (8776*tan(x/2)^16)/25 + (3482*tan(x/2)^1
7)/125 + 9008/375)/(12*tan(x/2) + 3*tan(x/2)^2 - 296*tan(x/2)^3 + 3048*tan
(x/2)^5 - 4104*tan(x/2)^6 - 5256*tan(x/2)^7 + 12282*tan(x/2)^8 - 12282*tan
(x/2)^10 + 5256*tan(x/2)^11 + 4104*tan(x/2)^12 - 3048*tan(x/2)^13 + 296*ta
n(x/2)^15 - 3*tan(x/2)^16 - 12*tan(x/2)^17 - tan(x/2)^18 + 1) - (16*atanh(
(344600346624*(979810 - 438182*5^(1/2))^(1/2))/(9765625*((122218256269312*
tan(x/2))/1953125 - (273842408783872*5^(1/2)*tan(x/2))/9765625 + (11061671
1266304*5^(1/2))/9765625 - 49507583131648/1953125)) - (229733564416*tan(x/
2)*(979810 - 438182*5^(1/2))^(1/2))/(9765625*((122218256269312*tan(x/2))/1
953125 - (273842408783872*5^(1/2)*tan(x/2))/9765625 + (110616711266304*5^(
1/2))/9765625 - 49507583131648/1953125)) - (114866782208*5^(1/2)*(979810 -
 438182*5^(1/2))^(1/2))/(9765625*((122218256269312*tan(x/2))/1953125 - (27
3842408783872*5^(1/2)*tan(x/2))/9765625 + (110616711266304*5^(1/2))/976562
5 - 49507583131648/1953125)) + (344600346624*5^(1/2)*tan(x/2)*(979810 - 43
8182*5^(1/2))^(1/2))/(9765625*((122218256269312*tan(x/2))/1953125 - (27...
```

Reduce [F]

$$\int \frac{1}{(\cos(3x)+\sin(2x))^4}\,dx$$
$$= \int \frac{1}{\cos(3x)^4+4\cos(3x)^3\sin(2x)+6\cos(3x)^2\sin(2x)^2+4\cos(3x)\sin(2x)^3+\sin(2x)^4}dx$$

input
```
int(1/(cos(3*x)+sin(2*x))^4,x)
```

output
```
int(1/(cos(3*x)**4 + 4*cos(3*x)**3*sin(2*x) + 6*cos(3*x)**2*sin(2*x)**2 +
4*cos(3*x)*sin(2*x)**3 + sin(2*x)**4),x)
```

3.6 $\int \frac{1}{(\cos(3x)+\sin(2x))^6}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 1

$$\int \frac{1}{(\cos(3x)+\sin(2x))^6}\,dx=0$$

output
```
0
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 6.13 (sec) , antiderivative size = 547, normalized size of antiderivative = 547.00

$$\int \frac{1}{(\cos(3x)+\sin(2x))^6}\,dx=\text{Too large to display}$$

input
```
Integrate[(Cos[3*x] + Sin[2*x])^(-6),x]
```

output

```
(-1792*Sqrt[2/(5*(5 - Sqrt[5]))]*(6566 + 2937*Sqrt[5])*ArcTanh[(Sec[x/2]*(
4*Cos[x/2] - Sin[x/2] - Sqrt[5]*Sin[x/2]))/Sqrt[2*(5 - Sqrt[5])]])/15625 -
 (1792*Sqrt[2/(5*(5 + Sqrt[5]))]*(-6566 + 2937*Sqrt[5])*ArcTanh[(Sec[x/2]*
(4*Cos[x/2] - Sin[x/2] + Sqrt[5]*Sin[x/2]))/Sqrt[2*(5 + Sqrt[5])]])/15625
+ 1/(40*(Cos[x/2] - Sin[x/2])^4) + 379/(240*(Cos[x/2] - Sin[x/2])^2) + Sin
[x/2]/(20*(Cos[x/2] - Sin[x/2])^5) + (379*Sin[x/2])/(120*(Cos[x/2] - Sin[x
/2])^3) + (6001*Sin[x/2])/(30*(Cos[x/2] - Sin[x/2])) + Sin[x/2]/(312500*(C
os[x/2] + Sin[x/2])^5) - 1/(625000*(Cos[x/2] + Sin[x/2])^4) + (139*Sin[x/2
])/(1875000*(Cos[x/2] + Sin[x/2])^3) - 139/(3750000*(Cos[x/2] + Sin[x/2])^
2) + (31*Sin[x/2])/(18750*(Cos[x/2] + Sin[x/2])) + (128*(4*Cos[x] + 7*Sin[
2*x]))/(625*(-1 + 2*Cos[2*x] + 2*Sin[x])^5) - (96*(139*Cos[x] + 192*Sin[2*
x]))/(3125*(-1 + 2*Cos[2*x] + 2*Sin[x])^4) - (2624*(3446*Cos[x] + 4473*Sin
[2*x]))/(234375*(-1 + 2*Cos[2*x] + 2*Sin[x])^2) + (512*(18289*Cos[x] + 244
25*Sin[2*x]))/(78125*(-1 + 2*Cos[2*x] + 2*Sin[x])) + (32*(20019*Cos[x] + 2
6527*Sin[2*x]))/(46875*(-1 + 2*Cos[2*x] + 2*Sin[x])^3)
```

Rubi [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 34.64 (sec) , antiderivative size = 6697, normalized size of antiderivative = 6697.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {3042, 4830, 2462, 7239, 2036, 7293, 7239, 2036, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(2x)+\cos(3x))^6} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(2x)+\cos(3x))^6}dx$$

$$\Big\downarrow \text{4830}$$

$$2\int \frac{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^{17}}{\left(-\tan^6\left(\frac{x}{2}\right)-4\tan^5\left(\frac{x}{2}\right)+15\tan^4\left(\frac{x}{2}\right)-15\tan^2\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)+1\right)^6}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{2462}$$

$$2\int \left(-\frac{384\left(20803\tan^2\left(\frac{x}{2}\right)+132116\tan\left(\frac{x}{2}\right)-114597\right)}{78125\left(\tan^4\left(\frac{x}{2}\right)+4\tan^3\left(\frac{x}{2}\right)-14\tan^2\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)+1\right)}+\frac{413}{4\left(\tan\left(\frac{x}{2}\right)-1\right)^2}+\frac{283}{312500\left(\tan\left(\frac{x}{2}\right)+1\right)}\right.$$

$\big\downarrow$ 7239

$$2\int \frac{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^{17}}{\left(1-\tan\left(\frac{x}{2}\right)\right)^6\left(\tan\left(\frac{x}{2}\right)+1\right)^6\left(\tan^4\left(\frac{x}{2}\right)+4\tan^3\left(\frac{x}{2}\right)-14\tan^2\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)+1\right)^6}d\tan\left(\frac{x}{2}\right)$$

$\big\downarrow$ 2036

$$2\int \frac{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^{17}}{\left(1-\tan^2\left(\frac{x}{2}\right)\right)^6\left(\tan^4\left(\frac{x}{2}\right)+4\tan^3\left(\frac{x}{2}\right)-14\tan^2\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)+1\right)^6}d\tan\left(\frac{x}{2}\right)$$

$\big\downarrow$ 7293

$$2\int \left(-\frac{384\left(20803\tan^2\left(\frac{x}{2}\right)+132116\tan\left(\frac{x}{2}\right)-114597\right)}{78125\left(\tan^4\left(\frac{x}{2}\right)+4\tan^3\left(\frac{x}{2}\right)-14\tan^2\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)+1\right)}+\frac{413}{4\left(\tan\left(\frac{x}{2}\right)-1\right)^2}+\frac{283}{312500\left(\tan\left(\frac{x}{2}\right)+1\right)}\right.$$

$\big\downarrow$ 7239

$$2\int \frac{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^{17}}{\left(1-\tan\left(\frac{x}{2}\right)\right)^6\left(\tan\left(\frac{x}{2}\right)+1\right)^6\left(\tan^4\left(\frac{x}{2}\right)+4\tan^3\left(\frac{x}{2}\right)-14\tan^2\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)+1\right)^6}d\tan\left(\frac{x}{2}\right)$$

$\big\downarrow$ 2036

$$2\int \frac{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^{17}}{\left(1-\tan^2\left(\frac{x}{2}\right)\right)^6\left(\tan^4\left(\frac{x}{2}\right)+4\tan^3\left(\frac{x}{2}\right)-14\tan^2\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)+1\right)^6}d\tan\left(\frac{x}{2}\right)$$

$\big\downarrow$ 7293

$$2\int \left(-\frac{384\left(20803\tan^2\left(\frac{x}{2}\right)+132116\tan\left(\frac{x}{2}\right)-114597\right)}{78125\left(\tan^4\left(\frac{x}{2}\right)+4\tan^3\left(\frac{x}{2}\right)-14\tan^2\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)+1\right)}+\frac{413}{4\left(\tan\left(\frac{x}{2}\right)-1\right)^2}+\frac{283}{312500\left(\tan\left(\frac{x}{2}\right)+1\right)}\right.$$

$\big\downarrow$ 2009

input
```
Int[(Cos[3*x] + Sin[2*x])^(-6),x]
```

output
```
2*((192*Sqrt[(2*(1283236265 - 546155621*Sqrt[5]))/5]*ArcTanh[(1 - Sqrt[5]
+ Tan[x/2])/Sqrt[5 - 2*Sqrt[5]]])/78125 + (384*(252458103049835 - 30750708
350822*Sqrt[5])*ArcTanh[(1 - Sqrt[5] + Tan[x/2])/Sqrt[5 - 2*Sqrt[5]]])/(78
125*(5 - 2*Sqrt[5])^(3/2)) - (192*(100122572902055 - 12191554480147*Sqrt[5
])*ArcTanh[(1 - Sqrt[5] + Tan[x/2])/Sqrt[5 - 2*Sqrt[5]]])/(15625*(5 - 2*Sq
rt[5])^(3/2)) - (384*(2146986122415 - 271127200652*Sqrt[5])*ArcTanh[(1 - S
qrt[5] + Tan[x/2])/Sqrt[5 - 2*Sqrt[5]]])/(78125*(5 - 2*Sqrt[5])^(3/2)) - (
384*(4684724605 - 694973687*Sqrt[5])*ArcTanh[(1 - Sqrt[5] + Tan[x/2])/Sqrt
[5 - 2*Sqrt[5]]])/(78125*(5 - 2*Sqrt[5])^(3/2)) - (128*(1207230227055423 -
 1052157602365700*Sqrt[5])*ArcTanh[(1 - Sqrt[5] + Tan[x/2])/Sqrt[5 - 2*Sqr
t[5]]])/(15625*Sqrt[5*(5 - 2*Sqrt[5])]) + (64*(478188055400771 - 416754786
287400*Sqrt[5])*ArcTanh[(1 - Sqrt[5] + Tan[x/2])/Sqrt[5 - 2*Sqrt[5]]])/(31
25*Sqrt[5*(5 - 2*Sqrt[5])]) + (128*(2344698376661 - 2047707259012*Sqrt[5])
*ArcTanh[(1 - Sqrt[5] + Tan[x/2])/Sqrt[5 - 2*Sqrt[5]]])/(3125*Sqrt[5*(5 -
2*Sqrt[5])]) + (128*(36557138261 - 32065231708*Sqrt[5])*ArcTanh[(1 - Sqrt[
5] + Tan[x/2])/Sqrt[5 - 2*Sqrt[5]]])/(15625*Sqrt[5*(5 - 2*Sqrt[5])]) - (25
6*(38783007150929 - 17680045788591*Sqrt[5])*Sqrt[(27365 + 12238*Sqrt[5])/5
]*ArcTanh[(1 - Sqrt[5] + Tan[x/2])/Sqrt[5 - 2*Sqrt[5]]])/15625 + (512*(195
06957513983 - 8892542155366*Sqrt[5])*Sqrt[(27365 + 12238*Sqrt[5])/5]*ArcTa
nh[(1 - Sqrt[5] + Tan[x/2])/Sqrt[5 - 2*Sqrt[5]]])/15625 - (256*(2309081...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2036
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4830
```
Int[(cos[(n_.)*((c_.) + (d_.)*(x_))]*(b_.) + (a_.)*sin[(m_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[2/d   Subst[Int[Simplify[TrigExpand[a*Sin[
2*m*ArcTan[x]] + b*Cos[2*n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[(1/2)*(c +
d*x)]], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && Intege
rQ[n]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.59 (sec) , antiderivative size = 374, normalized size of antiderivative = 374.00

$$-\frac{256\left(-\frac{167530}{3}-\frac{3544418\tan\left(\frac{x}{2}\right)}{3}-6401814\tan\left(\frac{x}{2}\right)^{2}+17962534\tan\left(\frac{x}{2}\right)^{3}+194410616\tan\left(\frac{x}{2}\right)^{4}-\frac{580831840\,\mathrm{ta}}{3}\right.}{}$$

input
```
int(1/(cos(3*x)+sin(2*x))^6,x)
```

output
```
-256/78125*(-167530/3-3544418/3*tan(1/2*x)-6401814*tan(1/2*x)^2+17962534*t
an(1/2*x)^3+194410616*tan(1/2*x)^4-580831840/3*tan(1/2*x)^5-7305842584/3*t
an(1/2*x)^6+11662085264/3*tan(1/2*x)^7+11244480332*tan(1/2*x)^8-1131674524
36/3*tan(1/2*x)^9+39264676220*tan(1/2*x)^10-39613206364/3*tan(1/2*x)^11-11
145371096/3*tan(1/2*x)^12+8320893136/3*tan(1/2*x)^13+477319784/3*tan(1/2*x
)^14-225437760*tan(1/2*x)^15-54674698/3*tan(1/2*x)^16+23176198/3*tan(1/2*x
)^17+1375914*tan(1/2*x)^18+64606*tan(1/2*x)^19)/(tan(1/2*x)^4+4*tan(1/2*x)
^3-14*tan(1/2*x)^2+4*tan(1/2*x)+1)^5-1792/15625*(-3629/2-8119/10*5^(1/2))/
(5-2*5^(1/2))^(1/2)*arctanh(1/2*(2*tan(1/2*x)-2*5^(1/2)+2)/(5-2*5^(1/2))^(
1/2))+1792/15625*(3629/2-8119/10*5^(1/2))/(5+2*5^(1/2))^(1/2)*arctanh(1/2*
(2*tan(1/2*x)+2+2*5^(1/2))/(5+2*5^(1/2))^(1/2))-1/2/(tan(1/2*x)-1)^4-1/5/(
tan(1/2*x)-1)^5-83/12/(tan(1/2*x)-1)^3-79/8/(tan(1/2*x)-1)^2-413/2/(tan(1/
2*x)-1)-1/78125/(tan(1/2*x)+1)^5+1/31250/(tan(1/2*x)+1)^4-7/37500/(tan(1/2
*x)+1)^3+31/125000/(tan(1/2*x)+1)^2-283/156250/(tan(1/2*x)+1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.23 (sec) , antiderivative size = 640, normalized size of antiderivative = 640.00

$$\int \frac{1}{(\cos(3x) + \sin(2x))^6} \, dx = \text{Too large to display}$$

input
```
integrate(1/(cos(3*x)+sin(2*x))^6,x, algorithm="fricas")
```

output
```
-1/468750*(135222722560*cos(x)^14 - 450050539520*cos(x)^12 + 564945510400*
cos(x)^10 - 317707407360*cos(x)^8 + 67470466880*cos(x)^6 + 28000000*cos(x)
^4 - 1344*sqrt(2)*(1024*cos(x)^15 - 6400*cos(x)^13 + 14400*cos(x)^11 - 152
80*cos(x)^9 + 7820*cos(x)^7 - 1563*cos(x)^5 + 2*(1280*cos(x)^13 - 4480*cos
(x)^11 + 5936*cos(x)^9 - 3512*cos(x)^7 + 781*cos(x)^5)*sin(x))*sqrt(279085
621*sqrt(5) + 624054425)*log(-448*sqrt(2)*sqrt(279085621*sqrt(5) + 6240544
25)*(9503*sqrt(5) - 21251)*cos(x) + 15670144*(sqrt(5) + 1)*sin(x) - 626805
76) + 1344*sqrt(2)*(1024*cos(x)^15 - 6400*cos(x)^13 + 14400*cos(x)^11 - 15
280*cos(x)^9 + 7820*cos(x)^7 - 1563*cos(x)^5 + 2*(1280*cos(x)^13 - 4480*co
s(x)^11 + 5936*cos(x)^9 - 3512*cos(x)^7 + 781*cos(x)^5)*sin(x))*sqrt(27908
5621*sqrt(5) + 624054425)*log(-448*sqrt(2)*sqrt(279085621*sqrt(5) + 624054
425)*(9503*sqrt(5) - 21251)*cos(x) - 15670144*(sqrt(5) + 1)*sin(x) + 62680
576) - 3*(1024*cos(x)^15 - 6400*cos(x)^13 + 14400*cos(x)^11 - 15280*cos(x)
^9 + 7820*cos(x)^7 - 1563*cos(x)^5 + 2*(1280*cos(x)^13 - 4480*cos(x)^11 +
5936*cos(x)^9 - 3512*cos(x)^7 + 781*cos(x)^5)*sin(x))*sqrt(-11202720095436
8*sqrt(5) + 250500438630400)*log(-(9503*sqrt(5) + 21251)*sqrt(-11202720095
4368*sqrt(5) + 250500438630400)*cos(x) + 15670144*(sqrt(5) - 1)*sin(x) + 6
2680576) + 3*(1024*cos(x)^15 - 6400*cos(x)^13 + 14400*cos(x)^11 - 15280*co
s(x)^9 + 7820*cos(x)^7 - 1563*cos(x)^5 + 2*(1280*cos(x)^13 - 4480*cos(x)^1
1 + 5936*cos(x)^9 - 3512*cos(x)^7 + 781*cos(x)^5)*sin(x))*sqrt(-1120272...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(\cos(3x) + \sin(2x))^6} \, dx = \text{Timed out}$$

input
```
integrate(1/(cos(3*x)+sin(2*x))**6,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(3x) + \sin(2x))^6} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(cos(3*x)+sin(2*x))^6,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.12 (sec) , antiderivative size = 311, normalized size of antiderivative = 311.00

$$\int \frac{1}{(\cos(3x) + \sin(2x))^6} \, dx = \text{Too large to display}$$

input
```
integrate(1/(cos(3*x)+sin(2*x))^6,x, algorithm="giac")
```

output
```
-2/46875*(9801627*tan(1/2*x)^29 + 207539160*tan(1/2*x)^28 + 1086782010*tan
(1/2*x)^27 - 4091973852*tan(1/2*x)^26 - 39877891959*tan(1/2*x)^25 + 480575
05836*tan(1/2*x)^24 + 611846565540*tan(1/2*x)^23 - 822349312680*tan(1/2*x)
^22 - 4527294198541*tan(1/2*x)^21 + 9882693980368*tan(1/2*x)^20 + 85457576
88614*tan(1/2*x)^19 - 36646833294420*tan(1/2*x)^18 + 7064910503625*tan(1/2
*x)^17 + 60480105024596*tan(1/2*x)^16 - 43130634999688*tan(1/2*x)^15 - 462
65973899504*tan(1/2*x)^14 + 58402675048825*tan(1/2*x)^13 + 9742317629080*t
an(1/2*x)^12 - 35972981583386*tan(1/2*x)^11 + 7234969571388*tan(1/2*x)^10
+ 9848510973859*tan(1/2*x)^9 - 4181395911980*tan(1/2*x)^8 - 850989600860*t
an(1/2*x)^7 + 567880320536*tan(1/2*x)^6 + 50680356857*tan(1/2*x)^5 - 36634
647232*tan(1/2*x)^4 - 3982452230*tan(1/2*x)^3 + 976976500*tan(1/2*x)^2 + 1
90845707*tan(1/2*x) + 9052204)/(tan(1/2*x)^6 + 4*tan(1/2*x)^5 - 15*tan(1/2
*x)^4 + 15*tan(1/2*x)^2 - 4*tan(1/2*x) - 1)^5 - 0.0179538142054400*log(tan
(1/2*x) + 6.31375151468000) + 0.0179538142049280*log(tan(1/2*x) + 0.158384
440325000) + 286.503025128960*log(tan(1/2*x) - 0.509525449494000) - 286.50
3025128960*log(tan(1/2*x) - 1.96261050551000)
```

Mupad [B] (verification not implemented)

Time = 22.46 (sec) , antiderivative size = 802, normalized size of antiderivative = 802.00

$$\int \frac{1}{(\cos(3x)+\sin(2x))^6}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(3*x) + sin(2*x))^6,x)
```

output
```
((381691414*tan(x/2))/46875 + (15631624*tan(x/2)^2)/375 - (1592980892*tan(
x/2)^3)/9375 - (73269294464*tan(x/2)^4)/46875 + (101360713714*tan(x/2)^5)/
46875 + (1135760641072*tan(x/2)^6)/46875 - (340395840344*tan(x/2)^7)/9375
- (1672558364792*tan(x/2)^8)/9375 + (19697021947718*tan(x/2)^9)/46875 + (4
82331304759 2*tan(x/2)^10)/15625 - (71945963166772*tan(x/2)^11)/46875 + (38
96927051632*tan(x/2)^12)/9375 + (4672214003906*tan(x/2)^13)/1875 - (925319
47799008*tan(x/2)^14)/46875 - (86261269999376*tan(x/2)^15)/46875 + (120960
210049192*tan(x/2)^16)/46875 + (37679522686*tan(x/2)^17)/125 - (4886244439
256*tan(x/2)^18)/3125 + (1709151537722 8*tan(x/2)^19)/46875 + (197653879607
36*tan(x/2)^20)/46875 - (9054588397082*tan(x/2)^21)/46875 - (10964657502 4*
tan(x/2)^22)/3125 + (81579542072*tan(x/2)^23)/3125 + (32038337224*tan(x/2)
^24)/15625 - (26585261306*tan(x/2)^25)/15625 - (2727982568*tan(x/2)^26)/15
625 + (144904268*tan(x/2)^27)/3125 + (27671888*tan(x/2)^28)/3125 + (653441
8*tan(x/2)^29)/15625 + 18104408/46875)/(20*tan(x/2) + 85*tan(x/2)^2 - 560*
tan(x/2)^3 - 3595*tan(x/2)^4 + 10004*tan(x/2)^5 + 57425*tan(x/2)^6 - 16160
0*tan(x/2)^7 - 371075*tan(x/2)^8 + 1588100*tan(x/2)^9 - 292495*tan(x/2)^10
 - 4918000*tan(x/2)^11 + 5124625*tan(x/2)^12 + 5671300*tan(x/2)^13 - 11792
275*tan(x/2)^14 + 11792275*tan(x/2)^16 - 5671300*tan(x/2)^17 - 5124625*tan
(x/2)^18 + 4918000*tan(x/2)^19 + 292495*tan(x/2)^20 - 1588100*tan(x/2)^21
+ 371075*tan(x/2)^22 + 161600*tan(x/2)^23 - 57425*tan(x/2)^24 - 10004*t...
```

Reduce [F]

$$\int \frac{1}{(\cos(3x)+\sin(2x))^6}\,dx = \int \frac{1}{\cos(3x)^6+6\cos(3x)^5\sin(2x)+15\cos(3x)^4\sin(2x)^2+20\cos(3x)^3\sin(2x)^3+15\cos(3x)^2\sin(2x)^4$$

input
```
int(1/(cos(3*x)+sin(2*x))^6,x)
```

output
```
int(1/(cos(3*x)**6 + 6*cos(3*x)**5*sin(2*x) + 15*cos(3*x)**4*sin(2*x)**2 +
 20*cos(3*x)**3*sin(2*x)**3 + 15*cos(3*x)**2*sin(2*x)**4 + 6*cos(3*x)*sin(
2*x)**5 + sin(2*x)**6),x)
```

3.7 $\int \frac{1}{\cos(5x)+\sin(2x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \frac{1}{\cos(5x)+\sin(2x)}\,dx = -\frac{1}{6}\text{arctanh}(\cos(x))-\frac{2}{3}\text{arctanh}(2\cos(x))+\frac{\text{arctanh}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

output
```
-1/6*arctanh(cos(x))-2/3*arctanh(2*cos(x))+1/2*arctanh(cos(x)*2^(1/2))*2^(
1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.62

$$\int \frac{1}{\cos(5x)+\sin(2x)}\,dx = \frac{1}{21}\left(-12\log\left(\sec^2\left(\frac{x}{2}\right)\right)-3\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+7\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-14\log(1-2\sin(x))+24\text{RootSum}\left[1-4\#1-32\#1^2+64\#1^3\&,\log\left(-\sec^2\left(\frac{x}{2}\right)\left(-1-4\sin(x)-8\sin(x)\#1+32\sin(x)\#1^2\right)\right)\#1\&\right]\right)$$

input
```
Integrate[(Cos[5*x] + Sin[2*x])^(-1),x]
```

output
```
(-12*Log[Sec[x/2]^2] - 3*Log[Cos[x/2] - Sin[x/2]] + 7*Log[Cos[x/2] + Sin[x
/2]] - 14*Log[1 - 2*Sin[x]] + 24*RootSum[1 - 4*#1 - 32*#1^2 + 64*#1^3 & ,
Log[-(Sec[x/2]^2*(-1 - 4*Sin[x] - 8*Sin[x]*#1 + 32*Sin[x]*#1^2))]*#1 & ])/
21
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 4.29 (sec) , antiderivative size = 890, normalized size of antiderivative = 27.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 4829, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin(2x)+\cos(5x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(2x)+\cos(5x)}dx$$

$$\Big\downarrow \text{4829}$$

$$\int \frac{1}{(1-\sin^2(x))\left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)}d\sin(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{4\left(8\sin^2(x)+12\sin(x)+1\right)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{1}{14(\sin(x)-1)}+\frac{1}{6(\sin(x)+1)}-\frac{4}{3(2\sin(x)-1)}\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{6\ 2^{2/3}\sqrt[3]{7}(1+3i\sqrt{3})\left(2^{2/3}\sqrt[3]{7}-\left(1+3i\sqrt{3}\right)^{2/3}\right)\operatorname{arctanh}\left(\frac{4i\left(1+3i\sqrt{3}\right)^{2/3}(6\sin(x)+1)+\sqrt[3]{14}\left(\sqrt[3]{2}\left(i-3\sqrt{3}\right)+2i\sqrt[3]{7}\right.}{2\sqrt{3\left(-\sqrt[3]{2}7^{2/3}\left(13-3i\sqrt{3}\right)+7\sqrt[3]{7}\left(2+6i\sqrt{3}\right)^{2/3}-\sqrt[3]{1+3i}\right.}}\right.}{\sqrt{-\sqrt[3]{2}7^{2/3}\left(13-3i\sqrt{3}\right)+7\sqrt[3]{7}\left(2+6i\sqrt{3}\right)^{2/3}-\sqrt[3]{1+3i\sqrt{3}}\left(14+42i\sqrt{3}\right)}\left(14\left(1+3i\sqrt{3}\right)^{2/3}+\sqrt[3]{14}\left(14\sqrt[3]{2}+\right.\right.}$$

$$\frac{2}{3}\log(1-2\sin(x))-\frac{1}{14}\log(1-\sin(x))+\frac{1}{6}\log(\sin(x)+1)-$$

$$\frac{2\sqrt[3]{1+3i\sqrt{3}}\left(2\sqrt[3]{2}7^{2/3}+\sqrt[3]{1+3i\sqrt{3}}+\sqrt[3]{7}(2+6i\sqrt{3})^{2/3}\right)\log\left(24(1+3i\sqrt{3})^{2/3}\sin^2(x)+4\ 7^{2/3}\sqrt[3]{2\left(1+3i\right.}\right.}{3\left(14\left(1+3i\right.\right.}$$

$$\frac{4}{21}\log\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)+$$

$$\frac{2(i-3\sqrt{3})\left(2\sqrt[3]{2}7^{2/3}+\sqrt[3]{1+3i\sqrt{3}}+\sqrt[3]{7}(2+6i\sqrt{3})^{2/3}\right)\log\left(\sqrt[3]{14}\left(2\sqrt[3]{7}+\sqrt[3]{2}(1+3i\sqrt{3})^{2/3}\right)-2\sqrt[3]{1+3i\sqrt{3}}\right.}{3\left(7\left(i-3\sqrt{3}\right)\sqrt[3]{1+3i\sqrt{3}}+7i\sqrt[3]{7}\left(2+6i\sqrt{3}\right)^{2/3}-\sqrt[3]{2}7^{2/3}\left(13i+3\sqrt{3}\right)\right)}$$

input
```
Int[(Cos[5*x] + Sin[2*x])^(-1),x]
```

output
```
(6*2^(2/3)*7^(1/3)*(1 + (3*I)*Sqrt[3])*(2^(2/3)*7^(1/3) - (1 + (3*I)*Sqrt[
3])^(2/3))*ArcTanh[(14^(1/3)*(2^(1/3)*(I - 3*Sqrt[3]) + (2*I)*(7 + (21*I)*
Sqrt[3])^(1/3)) + (4*I)*(1 + (3*I)*Sqrt[3])^(2/3)*(1 + 6*Sin[x]))/(2*Sqrt[
3*(-(2^(1/3)*7^(2/3)*(13 - (3*I)*Sqrt[3])) + 7*7^(1/3)*(2 + (6*I)*Sqrt[3])
^(2/3) - (1 + (3*I)*Sqrt[3])^(1/3)*(14 + (42*I)*Sqrt[3]))])])/(Sqrt[-(2^(1
/3)*7^(2/3)*(13 - (3*I)*Sqrt[3])) + 7*7^(1/3)*(2 + (6*I)*Sqrt[3])^(2/3) -
(1 + (3*I)*Sqrt[3])^(1/3)*(14 + (42*I)*Sqrt[3])]*(14*(1 + (3*I)*Sqrt[3])^(
2/3) + 14^(1/3)*(14*2^(1/3) + 7^(1/3)*(1 + (3*I)*Sqrt[3])^(4/3)))) - (2*Lo
g[1 - 2*Sin[x]])/3 - Log[1 - Sin[x]]/14 + Log[1 + Sin[x]]/6 - (2*(1 + (3*I
)*Sqrt[3])^(1/3)*(2*2^(1/3)*7^(2/3) + (1 + (3*I)*Sqrt[3])^(1/3) + 7^(1/3)*
(2 + (6*I)*Sqrt[3])^(2/3))*Log[-4*(1 + (3*I)*Sqrt[3])^(2/3) + 14^(1/3)*(2^
(1/3)*(5 + I*Sqrt[3]) + (1 + I*Sqrt[3])*(7 + (21*I)*Sqrt[3])^(1/3)) + 2*2^
(2/3)*7^(1/3)*Sin[x] + (6*I)*2^(2/3)*Sqrt[3]*7^(1/3)*Sin[x] + 8*(1 + (3*I)
*Sqrt[3])^(2/3)*Sin[x] + 4*7^(2/3)*(2*(1 + (3*I)*Sqrt[3]))^(1/3)*Sin[x] +
24*(1 + (3*I)*Sqrt[3])^(2/3)*Sin[x]^2])/(3*(14*(1 + (3*I)*Sqrt[3])^(2/3) +
 14^(1/3)*(14*2^(1/3) + 7^(1/3)*(1 + (3*I)*Sqrt[3])^(4/3)))) + (4*Log[-1 -
 4*Sin[x] + 4*Sin[x]^2 + 8*Sin[x]^3])/21 + (2*(I - 3*Sqrt[3])*(2*2^(1/3)*7
^(2/3) + (1 + (3*I)*Sqrt[3])^(1/3) + 7^(1/3)*(2 + (6*I)*Sqrt[3])^(2/3))*Lo
g[14^(1/3)*(2*7^(1/3) + 2^(1/3)*(1 + (3*I)*Sqrt[3])^(2/3)) - 2*(1 + (3*I)*
Sqrt[3])^(1/3)*(1 + 6*Sin[x])])/(3*(7*(I - 3*Sqrt[3])*(1 + (3*I)*Sqrt[3...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4829
```
Int[(cos[(n_.)*((c_.) + (d_.)*(x_))]*(b_.) + (a_.)*sin[(m_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Sin[
m*ArcSin[x]] + b*Cos[n*ArcSin[x]]]]^p/Sqrt[1 - x^2], x], x, Sin[c + d*x]],
x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[m/2] && Inte
gerQ[(n - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.74 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.38

method	result
default	$-\frac{\ln(\sin(x)-1)}{14}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(8\_Z^3+4\_Z^2-4\_Z-1\right)}\frac{\left(8\_R^2+12\_R+1\right)\ln\left(\sin(x)-\_R\right)}{6\_R^2+2\_R-1}\right)}{7}-\frac{2\ln(2\sin(x)-1)}{3}+\frac{\ln(1+\mathrm{s}}{6}$
risch	$-\frac{\ln(e^{ix}-i)}{7}+\frac{\ln(e^{ix}+i)}{3}+\left(\sum_{\_R=\mathrm{RootOf}(343\_Z^3-196\_Z^2-28\_Z+8)}\_R\ln\left(e^{2ix}+\left(-\frac{7}{2}i\_R+i\right)e^{ix}-1\right)\right)$

input
```
int(1/(cos(5*x)+sin(2*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/14*ln(sin(x)-1)+1/7*sum((8*_R^2+12*_R+1)/(6*_R^2+2*_R-1)*ln(sin(x)-_R),
_R=RootOf(8*_Z^3+4*_Z^2-4*_Z-1))-2/3*ln(2*sin(x)-1)+1/6*ln(1+sin(x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 607, normalized size of antiderivative = 18.97

$$\int \frac{1}{\cos(5x)+\sin(2x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cos(5*x)+sin(2*x)),x, algorithm="fricas")
```

output
```
1/252*(63*(4/441*I*sqrt(3) + 4/1323)^(1/3)*(-I*sqrt(3) + 1) + 4*(I*sqrt(3)
 + 1)/(4/441*I*sqrt(3) + 4/1323)^(1/3) + 18*sqrt(-1/108*(63*(4/441*I*sqrt(
3) + 4/1323)^(1/3)*(-I*sqrt(3) + 1) + 4*(I*sqrt(3) + 1)/(4/441*I*sqrt(3) +
 4/1323)^(1/3) - 24)^2 - 28*(4/441*I*sqrt(3) + 4/1323)^(1/3)*(-I*sqrt(3) +
 1) - 16/9*(I*sqrt(3) + 1)/(4/441*I*sqrt(3) + 4/1323)^(1/3) + 128/3) + 48)
*log(7/2*(4/441*I*sqrt(3) + 4/1323)^(1/3)*(-I*sqrt(3) + 1) + 2/9*(I*sqrt(3
) + 1)/(4/441*I*sqrt(3) + 4/1323)^(1/3) + sqrt(-1/108*(63*(4/441*I*sqrt(3)
 + 4/1323)^(1/3)*(-I*sqrt(3) + 1) + 4*(I*sqrt(3) + 1)/(4/441*I*sqrt(3) + 4
/1323)^(1/3) - 24)^2 - 28*(4/441*I*sqrt(3) + 4/1323)^(1/3)*(-I*sqrt(3) + 1
) - 16/9*(I*sqrt(3) + 1)/(4/441*I*sqrt(3) + 4/1323)^(1/3) + 128/3) - 8*sin
(x) - 4/3) + 1/252*(63*(4/441*I*sqrt(3) + 4/1323)^(1/3)*(-I*sqrt(3) + 1) +
 4*(I*sqrt(3) + 1)/(4/441*I*sqrt(3) + 4/1323)^(1/3) - 18*sqrt(-1/108*(63*(
4/441*I*sqrt(3) + 4/1323)^(1/3)*(-I*sqrt(3) + 1) + 4*(I*sqrt(3) + 1)/(4/44
1*I*sqrt(3) + 4/1323)^(1/3) - 24)^2 - 28*(4/441*I*sqrt(3) + 4/1323)^(1/3)*
(-I*sqrt(3) + 1) - 16/9*(I*sqrt(3) + 1)/(4/441*I*sqrt(3) + 4/1323)^(1/3) +
 128/3) + 48)*log(-7/2*(4/441*I*sqrt(3) + 4/1323)^(1/3)*(-I*sqrt(3) + 1) -
 2/9*(I*sqrt(3) + 1)/(4/441*I*sqrt(3) + 4/1323)^(1/3) + sqrt(-1/108*(63*(4
/441*I*sqrt(3) + 4/1323)^(1/3)*(-I*sqrt(3) + 1) + 4*(I*sqrt(3) + 1)/(4/441
*I*sqrt(3) + 4/1323)^(1/3) - 24)^2 - 28*(4/441*I*sqrt(3) + 4/1323)^(1/3)*(
-I*sqrt(3) + 1) - 16/9*(I*sqrt(3) + 1)/(4/441*I*sqrt(3) + 4/1323)^(1/3)...
```

Sympy [F]

$$\int \frac{1}{\cos(5x)+\sin(2x)}\,dx=\int \frac{1}{\sin{(2x)}+\cos{(5x)}}\,dx$$

input
```
integrate(1/(cos(5*x)+sin(2*x)),x)
```

output
```
Integral(1/(sin(2*x) + cos(5*x)), x)
```

Maxima [F]

$$\int \frac{1}{\cos(5x) + \sin(2x)}\, dx = \int \frac{1}{\cos(5\,x) + \sin(2\,x)}\, dx$$

input
```
integrate(1/(cos(5*x)+sin(2*x)),x, algorithm="maxima")
```

output
```
-4/21*integrate(((cos(2*x)*sin(8*x) - cos(2*x)*sin(6*x) + cos(2*x)*sin(4*x
) - cos(8*x)*sin(2*x) + cos(6*x)*sin(2*x) - cos(4*x)*sin(2*x) - sin(2*x))*
cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + (cos(2*x)*sin(8*x) - cos(2*x)*sin
(6*x) + cos(2*x)*sin(4*x) - cos(8*x)*sin(2*x) + cos(6*x)*sin(2*x) - cos(4*
x)*sin(2*x) - sin(2*x))*sin(1/2*arctan2(sin(2*x), cos(2*x)))^2 + (cos(6*x)
*cos(2*x) - cos(4*x)*cos(2*x) - (cos(2*x)*sin(8*x) - cos(2*x)*sin(6*x) + c
os(2*x)*sin(4*x) - cos(8*x)*sin(2*x) + cos(6*x)*sin(2*x) - cos(4*x)*sin(2*
x) - sin(2*x))*cos(1/2*arctan2(sin(2*x), cos(2*x))) + sin(6*x)*sin(2*x) -
sin(4*x)*sin(2*x) - (cos(8*x)*cos(2*x) - cos(6*x)*cos(2*x) + cos(4*x)*cos(
2*x) - cos(2*x)^2 + sin(8*x)*sin(2*x) - sin(6*x)*sin(2*x) + sin(4*x)*sin(2
*x) - sin(2*x)^2 + cos(2*x))*sin(1/2*arctan2(sin(2*x), cos(2*x))))*cos(7/2
*arctan2(sin(2*x), cos(2*x))) - (cos(6*x)*cos(2*x) - cos(4*x)*cos(2*x) - (
cos(2*x)*sin(8*x) - cos(2*x)*sin(6*x) + cos(2*x)*sin(4*x) - cos(8*x)*sin(2
*x) + cos(6*x)*sin(2*x) - cos(4*x)*sin(2*x) - sin(2*x))*cos(1/2*arctan2(si
n(2*x), cos(2*x))) + sin(6*x)*sin(2*x) - sin(4*x)*sin(2*x) - (cos(8*x)*cos
(2*x) - cos(6*x)*cos(2*x) + cos(4*x)*cos(2*x) - cos(2*x)^2 + sin(8*x)*sin(
2*x) - sin(6*x)*sin(2*x) + sin(4*x)*sin(2*x) - sin(2*x)^2 + cos(2*x))*sin(
1/2*arctan2(sin(2*x), cos(2*x))))*cos(5/2*arctan2(sin(2*x), cos(2*x))) + (
cos(6*x)*cos(2*x) - cos(4*x)*cos(2*x) - (cos(2*x)*sin(8*x) - cos(2*x)*sin(
6*x) + cos(2*x)*sin(4*x) - cos(8*x)*sin(2*x) + cos(6*x)*sin(2*x) - cos(...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\cos(5x) + \sin(2x)}\, dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(1/(cos(5*x)+sin(2*x)),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: 2*(-
1/28*ln(-sin(sageVARx)+1)+1/12*ln(sin(sageVARx)+1)-1/3*ln(abs(2*sin(sageVA
Rx)-1))+((-7/32768*rootof([[-3,0,35840,0,-66060288],[1,0,-14336,0,51380224
,0,-52613349376]])+
```

Mupad [B] (verification not implemented)

Time = 23.35 (sec) , antiderivative size = 582, normalized size of antiderivative = 18.19

$$\int \frac{1}{\cos(5x) + \sin(2x)} \, dx = \text{Too large to display}$$

input
```
int(1/(cos(5*x) + sin(2*x)),x)
```

output
```
log(tan(x/2) + 1)/3 - log(tan(x/2) - 1)/7 - (2*log(tan(x/2)^2 - 4*tan(x/2)
 + 1))/3 + symsum(log((140737488355328*(11902752*root(z^3 - (4*z^2)/7 - (4
*z)/49 + 8/343, z, k)*cos(x) - 728000*cos(x) - 2080*sin(x) - 14590784*root
(z^3 - (4*z^2)/7 - (4*z)/49 + 8/343, z, k) + 29331680*root(z^3 - (4*z^2)/7
 - (4*z)/49 + 8/343, z, k)*sin(x) - 549239760*root(z^3 - (4*z^2)/7 - (4*z)
/49 + 8/343, z, k)^2 + 108864280*root(z^3 - (4*z^2)/7 - (4*z)/49 + 8/343,
z, k)^3 + 28013641068*root(z^3 - (4*z^2)/7 - (4*z)/49 + 8/343, z, k)^4 + 1
6847690972*root(z^3 - (4*z^2)/7 - (4*z)/49 + 8/343, z, k)^5 - 321476748285
*root(z^3 - (4*z^2)/7 - (4*z)/49 + 8/343, z, k)^6 - 28330551480*root(z^3 -
 (4*z^2)/7 - (4*z)/49 + 8/343, z, k)^7 + 602671372485*root(z^3 - (4*z^2)/7
 - (4*z)/49 + 8/343, z, k)^8 + 592615840*root(z^3 - (4*z^2)/7 - (4*z)/49 +
 8/343, z, k)^2*cos(x) + 152859056*root(z^3 - (4*z^2)/7 - (4*z)/49 + 8/343
, z, k)^3*cos(x) - 32417181972*root(z^3 - (4*z^2)/7 - (4*z)/49 + 8/343, z,
 k)^4*cos(x) - 21798153132*root(z^3 - (4*z^2)/7 - (4*z)/49 + 8/343, z, k)^
5*cos(x) + 390994442041*root(z^3 - (4*z^2)/7 - (4*z)/49 + 8/343, z, k)^6*c
os(x) + 35652852368*root(z^3 - (4*z^2)/7 - (4*z)/49 + 8/343, z, k)^7*cos(x
) - 741082462953*root(z^3 - (4*z^2)/7 - (4*z)/49 + 8/343, z, k)^8*cos(x) +
 54174840*root(z^3 - (4*z^2)/7 - (4*z)/49 + 8/343, z, k)^2*sin(x) - 174956
6588*root(z^3 - (4*z^2)/7 - (4*z)/49 + 8/343, z, k)^3*sin(x) - 1411646320*
root(z^3 - (4*z^2)/7 - (4*z)/49 + 8/343, z, k)^4*sin(x) + 21036147076*r...
```

Reduce [F]

$$\int \frac{1}{\cos(5x) + \sin(2x)}\,dx = \int \frac{1}{\cos(5x) + \sin(2x)}dx$$

input
```
int(1/(cos(5*x)+sin(2*x)),x)
```

output
```
int(1/(cos(5*x) + sin(2*x)),x)
```

3.8 $\int \frac{1}{(\cos(5x)+\sin(2x))^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 127

$$\begin{aligned}\int \frac{1}{(\cos(5x)+\sin(2x))^3} \, dx = & -\frac{7}{144}\text{arctanh}(\cos(x)) - \frac{55}{9}\text{arctanh}(2\cos(x)) \\ & + \frac{139\text{arctanh}\left(\sqrt{2}\cos(x)\right)}{16\sqrt{2}} - \frac{1}{18(1-2\cos(x))^2} \\ & + \frac{35}{54(1-2\cos(x))} - \frac{1}{864(1-\cos(x))} + \frac{1}{864(1+\cos(x))} \\ & + \frac{1}{18(1+2\cos(x))^2} - \frac{35}{54(1+2\cos(x))} \\ & - \frac{19}{16}\cos(x)\sec(2x) + \frac{1}{8}\cos(x)\sec^2(2x)\end{aligned}$$

output
```
-7/144*arctanh(cos(x))-55/9*arctanh(2*cos(x))+139/32*arctanh(cos(x)*2^(1/2
))*2^(1/2)-1/18/(1-2*cos(x))^2+35/(54-108*cos(x))-1/(864-864*cos(x))+1/(86
4+864*cos(x))+1/18/(1+2*cos(x))^2-35/(54+108*cos(x))-19/16*cos(x)*sec(2*x)
+1/8*cos(x)*sec(2*x)^2
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.92 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.92

$$
\begin{aligned}
&\int \frac{1}{(\cos(5x)+\sin(2x))^3}\,dx \\
&= -\frac{19}{686}\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\frac{13}{18}\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-\frac{640}{9}\log(1-2\sin(x)) \\
&\quad+\frac{1}{343}\Bigg(-24272\log\left(\sec^2\left(\frac{x}{2}\right)\right)+256\text{RootSum}\Big[13033-167008\#1-388352\#1^2 \\
&\quad+4096\#1^3\&,\log\left(-\sec^2\left(\frac{x}{2}\right)\left(-2999947-12270846\sin(x)-17960480\sin(x)\#1+190464\sin(x)\#1^2\right)\right) \\
&\quad-\frac{1}{108\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2}+\frac{1}{1372-1372\sin(x)}+\frac{4}{9(1-2\sin(x))^2} \\
&\quad+\frac{248}{-27+54\sin(x)}+\frac{4(-57+40\cos(2x)-92\sin(x))}{49(1-2\cos(2x)+2\sin(x)-2\sin(3x))^2} \\
&\quad+\frac{48(-158+118\cos(2x)-263\sin(x))}{343(-1+2\cos(2x)-2\sin(x)+2\sin(3x))}
\end{aligned}
$$

input
```
Integrate[(Cos[5*x] + Sin[2*x])^(-3),x]
```

output
```
(-19*Log[Cos[x/2] - Sin[x/2]])/686 + (13*Log[Cos[x/2] + Sin[x/2]])/18 - (6
40*Log[1 - 2*Sin[x]])/9 + (-24272*Log[Sec[x/2]^2] + 256*RootSum[13033 - 16
7008*#1 - 388352*#1^2 + 4096*#1^3 & , Log[-(Sec[x/2]^2*(-2999947 - 1227084
6*Sin[x] - 17960480*Sin[x]*#1 + 190464*Sin[x]*#1^2))]*#1 & ])/343 - 1/(108
*(Cos[x/2] + Sin[x/2])^2) + (1372 - 1372*Sin[x])^(-1) + 4/(9*(1 - 2*Sin[x]
)^2) + 248/(-27 + 54*Sin[x]) + (4*(-57 + 40*Cos[2*x] - 92*Sin[x]))/(49*(1
- 2*Cos[2*x] + 2*Sin[x] - 2*Sin[3*x])^2) + (48*(-158 + 118*Cos[2*x] - 263*
Sin[x]))/(343*(-1 + 2*Cos[2*x] - 2*Sin[x] + 2*Sin[3*x]))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(2x)+\cos(5x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(2x)+\cos(5x))^3}dx$$

$$\Big\downarrow \text{4829}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^2\left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{64\left(15\sin^2(x)+17\sin(x)+3\right)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^3}+\frac{128\left(1517\sin^2(x)+1627\sin(x)+153\right)}{343\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{64\left(243\sin^2(x)+\right.}{49\left(8\sin^3(x)+4\sin\right.}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^2\left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{64\left(15\sin^2(x)+17\sin(x)+3\right)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^3}+\frac{128\left(1517\sin^2(x)+1627\sin(x)+153\right)}{343\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{64\left(243\sin^2(x)+\right.}{49\left(8\sin^3(x)+4\sin\right.}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^2\left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{64\left(15\sin^2(x)+17\sin(x)+3\right)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^3}+\frac{128\left(1517\sin^2(x)+1627\sin(x)+153\right)}{343\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{64\left(243\sin^2(x)+\right.}{49\left(8\sin^3(x)+4\sin\right.}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^2 \left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{64(15\sin^2(x)+17\sin(x)+3)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^3}+\frac{128(1517\sin^2(x)+1627\sin(x)+153)}{343\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{64(243\sin^2(x)+}{49\left(8\sin^3(x)+4\sin\right.}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^2 \left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{64(15\sin^2(x)+17\sin(x)+3)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^3}+\frac{128(1517\sin^2(x)+1627\sin(x)+153)}{343\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{64(243\sin^2(x)+}{49\left(8\sin^3(x)+4\sin\right.}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^2 \left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{64(15\sin^2(x)+17\sin(x)+3)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^3}+\frac{128(1517\sin^2(x)+1627\sin(x)+153)}{343\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{64(243\sin^2(x)+}{49\left(8\sin^3(x)+4\sin\right.}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^2 \left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{64(15\sin^2(x)+17\sin(x)+3)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^3}+\frac{128(1517\sin^2(x)+1627\sin(x)+153)}{343\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{64(243\sin^2(x)+}{49\left(8\sin^3(x)+4\sin\right.}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^2 \left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{64(15\sin^2(x)+17\sin(x)+3)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^3}+\frac{128(1517\sin^2(x)+1627\sin(x)+153)}{343\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{64(243\sin^2(x)+}{49\left(8\sin^3(x)+4\sin\right.}$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^2\left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{64(15\sin^2(x)+17\sin(x)+3)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^3}+\frac{128(1517\sin^2(x)+1627\sin(x)+153)}{343\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{64(243\sin^2(x)+}{49\left(8\sin^3(x)+4\sin\right.}$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^2\left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{64(15\sin^2(x)+17\sin(x)+3)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^3}+\frac{128(1517\sin^2(x)+1627\sin(x)+153)}{343\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{64(243\sin^2(x)+}{49\left(8\sin^3(x)+4\sin\right.}$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^2\left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{64(15\sin^2(x)+17\sin(x)+3)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^3}+\frac{128(1517\sin^2(x)+1627\sin(x)+153)}{343\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{64(243\sin^2(x)+}{49\left(8\sin^3(x)+4\sin\right.}$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^2\left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{64(15\sin^2(x)+17\sin(x)+3)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^3}+\frac{128(1517\sin^2(x)+1627\sin(x)+153)}{343\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{64(243\sin^2(x)+}{49\left(8\sin^3(x)+4\sin\right.}$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^2 \left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^3}d\sin(x)$$

$\Big\downarrow$ 2462

$$\int \left(\frac{64\left(15\sin^2(x)+17\sin(x)+3\right)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^3}+\frac{128\left(1517\sin^2(x)+1627\sin(x)+153\right)}{343\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{64\left(243\sin^2(x)+\right.}{49\left(8\sin^3(x)+4\sin\right.}\right.$$

$\Big\downarrow$ 7239

$$\int \frac{1}{\left(1-\sin^2(x)\right)^2 \left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^3}d\sin(x)$$

$\Big\downarrow$ 2462

$$\int \left(\frac{64\left(15\sin^2(x)+17\sin(x)+3\right)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^3}+\frac{128\left(1517\sin^2(x)+1627\sin(x)+153\right)}{343\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{64\left(243\sin^2(x)+\right.}{49\left(8\sin^3(x)+4\sin\right.}\right.$$

$\Big\downarrow$ 7239

$$\int \frac{1}{\left(1-\sin^2(x)\right)^2 \left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^3}d\sin(x)$$

input
```
Int[(Cos[5*x] + Sin[2*x])^(-3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4829
```
Int[(cos[(n_.)*((c_.) + (d_.)*(x_))]*(b_.) + (a_.)*sin[(m_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Sin[
m*ArcSin[x]] + b*Cos[n*ArcSin[x]]]]^p/Sqrt[1 - x^2], x], x, Sin[c + d*x]],
x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[m/2] && Inte
gerQ[(n - 1)/2]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [A] (verified)

Time = 15.36 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.02

method	result
parallelrisch	0
default	$\frac{\frac{90624\sin(x)^5}{343}+\frac{146304\sin(x)^4}{343}+\frac{20544\sin(x)^3}{343}-\frac{56384\sin(x)^2}{343}-\frac{22880\sin(x)}{343}-\frac{2396}{343}}{\left(8\sin(x)^3+4\sin(x)^2-4\sin(x)-1\right)^2}+\frac{16\left(\sum_{\_R=\mathrm{RootOf}\left(8\_Z^3+4\_Z^2-4\_Z-1\right)} \frac{(3 \cdots}{\cdots}\right.}{3 \cdots}$
risch	$\frac{i\left(2912i\mathrm{e}^{18ix}+2561\,\mathrm{e}^{19ix}+2912i\mathrm{e}^{2ix}-667\,\mathrm{e}^{17ix}-1292i\mathrm{e}^{10ix}+2765\,\mathrm{e}^{15ix}+54i\mathrm{e}^{14ix}+96\,\mathrm{e}^{13ix}+5368i\mathrm{e}^{12ix}-5932\,\mathrm{e}^{11ix}+5368i\mathrm{e}\cdots\right.}{147(\mathrm{e}^{ix}-i)^2(i\mathrm{e}^{8ix}+\mathrm{e}^{9ix}-2i\mathrm{e}^{6ix}-\mathrm{e}^{7ix}+2i\mathrm{e}^{4ix}+2\,\mathrm{e}^{5ix}-i\cdots}$

input
```
int(1/(cos(5*x)+sin(2*x))^3,x,method=_RETURNVERBOSE)
```

output
```
0
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 1468, normalized size of antiderivative = 11.56

$$\int \frac{1}{(\cos(5x)+\sin(2x))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cos(5*x)+sin(2*x))^3,x, algorithm="fricas")
```

output
```
-1/605052*(767090688*cos(x)^8 - 1735832448*cos(x)^6 + 1262229024*cos(x)^4
+ 2*(256*cos(x)^10 - 640*cos(x)^8 + 560*cos(x)^6 - 204*cos(x)^4 + 29*cos(x
)^2 + 4*(16*cos(x)^6 - 20*cos(x)^4 + 5*cos(x)^2)*sin(x))*(151263*(61438914
56/51883209*I*sqrt(3) + 2084357543936/155649627)^(1/3)*(-I*sqrt(3) + 1) +
85306624*(I*sqrt(3) + 1)/(6143891456/51883209*I*sqrt(3) + 2084357543936/15
5649627)^(1/3) - 7135968)*log(293/777924*(151263*(6143891456/51883209*I*sq
rt(3) + 2084357543936/155649627)^(1/3)*(-I*sqrt(3) + 1) + 85306624*(I*sqrt
(3) + 1)/(6143891456/51883209*I*sqrt(3) + 2084357543936/155649627)^(1/3) -
 7135968)^2 + 1235173184*(6143891456/51883209*I*sqrt(3) + 2084357543936/15
5649627)^(1/3)*(-I*sqrt(3) + 1) + 307196660006912/441*(I*sqrt(3) + 1)/(614
3891456/51883209*I*sqrt(3) + 2084357543936/155649627)^(1/3) + 1535972864*s
in(x) - 173412965888/3) - 283995768*cos(x)^2 - (5480423424*cos(x)^10 - 137
01058560*cos(x)^8 + 11988426240*cos(x)^6 - 4367212416*cos(x)^4 + (256*cos(
x)^10 - 640*cos(x)^8 + 560*cos(x)^6 - 204*cos(x)^4 + 29*cos(x)^2 + 4*(16*c
os(x)^6 - 20*cos(x)^4 + 5*cos(x)^2)*sin(x))*(151263*(6143891456/51883209*I
*sqrt(3) + 2084357543936/155649627)^(1/3)*(-I*sqrt(3) + 1) + 85306624*(I*s
qrt(3) + 1)/(6143891456/51883209*I*sqrt(3) + 2084357543936/155649627)^(1/3
) - 7135968) + 620829216*cos(x)^2 + 85631616*(16*cos(x)^6 - 20*cos(x)^4 +
5*cos(x)^2)*sin(x) + 882*(256*cos(x)^10 - 640*cos(x)^8 + 560*cos(x)^6 - 20
4*cos(x)^4 + 29*cos(x)^2 + 4*(16*cos(x)^6 - 20*cos(x)^4 + 5*cos(x)^2)*s...
```

Sympy [F]

$$\int \frac{1}{(\cos(5x) + \sin(2x))^3} \, dx = \int \frac{1}{(\sin{(2x)} + \cos{(5x)})^3} \, dx$$

input
```
integrate(1/(cos(5*x)+sin(2*x))**3,x)
```

output
```
Integral((sin(2*x) + cos(5*x))**(-3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(5x)+\sin(2x))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(cos(5*x)+sin(2*x))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(5x)+\sin(2x))^3}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(1/(cos(5*x)+sin(2*x))^3,x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: 2*((
-3216*sin(sageVARx)^2-185*sin(sageVARx)+3374)*1/37044/(sin(sageVARx)^2-1)+
(154368*sin(sageVARx)^8+10337792*sin(sageVARx)^7+5628928*sin(sageVARx)^6-1
2202432*sin(sageVAR
```

Mupad [B] (verification not implemented)

Time = 23.95 (sec) , antiderivative size = 3812, normalized size of antiderivative = 30.02

$$\int \frac{1}{(\cos(5x)+\sin(2x))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(5*x) + sin(2*x))^3,x)
```

output
```
log((tan(x/2) + 1)^(13/18)) - (640*log(tan(x/2)^2 - 4*tan(x/2) + 1))/9 + l
og(1/(tan(x/2) - 1)^(19/686)) - (1151*tan(x/2))/(21*(8*tan(x/2) - 74*tan(x
/2)^2 - 344*tan(x/2)^3 + 2509*tan(x/2)^4 + 960*tan(x/2)^5 - 19256*tan(x/2)
^6 + 1664*tan(x/2)^7 + 63026*tan(x/2)^8 - 2288*tan(x/2)^9 - 92412*tan(x/2)
^10 - 2288*tan(x/2)^11 + 63026*tan(x/2)^12 + 1664*tan(x/2)^13 - 19256*tan(
x/2)^14 + 960*tan(x/2)^15 + 2509*tan(x/2)^16 - 344*tan(x/2)^17 - 74*tan(x/
2)^18 + 8*tan(x/2)^19 + tan(x/2)^20 + 1)) + symsum(log(-(35184372088832*(9
0765549448560551501281361 92*root(z^3 - (24272*z^2)/343 - (2672128*z)/11764
9 + 53383168/40353607, z, k) + 629817864524699162736852992 0*cos(x) + 17602
02297405634484348387328*sin(x) - 20217616625110063036134064128*root(z^3 -
(24272*z^2)/343 - (2672128*z)/117649 + 53383168/40353607, z, k)*cos(x) - 7
8034534172559312435727040512*root(z^3 - (24272*z^2)/343 - (2672128*z)/1176
49 + 53383168/40353607, z, k)*sin(x) + 21218768236281708235274170368 00*roo
t(z^3 - (24272*z^2)/343 - (2672128*z)/117649 + 53383168/40353607, z, k)^2
+ 1290119019714658581791140861952*root(z^3 - (24272*z^2)/343 - (2672128*z)
/117649 + 53383168/40353607, z, k)^3 - 12111407380848387785076052472512*ro
ot(z^3 - (24272*z^2)/343 - (2672128*z)/117649 + 53383168/40353607, z, k)^4
 - 14017247051738107697950366564 32*root(z^3 - (24272*z^2)/343 - (2672128*z
)/117649 + 53383168/40353607, z, k)^5 + 22554291436479188167881996966 3*roo
t(z^3 - (24272*z^2)/343 - (2672128*z)/117649 + 53383168/40353607, z, k)...
```

Reduce [F]

$$\int \frac{1}{(\cos(5x)+\sin(2x))^3}\,dx$$
$$= \int \frac{1}{\cos(5x)^3+3\cos(5x)^2\sin(2x)+3\cos(5x)\sin(2x)^2+\sin(2x)^3}\,dx$$

input
```
int(1/(cos(5*x)+sin(2*x))^3,x)
```

output
```
int(1/(cos(5*x)**3 + 3*cos(5*x)**2*sin(2*x) + 3*cos(5*x)*sin(2*x)**2 + sin
(2*x)**3),x)
```

3.9 $\int \frac{1}{(\cos(5x)+\sin(2x))^5} dx$

Optimal result

Integrand size = 11, antiderivative size = 221

$$\begin{aligned}\int \frac{1}{(\cos(5x)+\sin(2x))^5}\,dx = &-\frac{3889\operatorname{arctanh}(\cos(x))}{186624}-\frac{332929\operatorname{arctanh}(2\cos(x))}{2916}\\ &+\frac{82683\operatorname{arctanh}\left(\sqrt{2}\cos(x)\right)}{512\sqrt{2}}\\ &-\frac{1}{108(1-2\cos(x))^4}+\frac{19}{162(1-2\cos(x))^3}\\ &-\frac{749}{648(1-2\cos(x))^2}+\frac{71551}{5832(1-2\cos(x))}\\ &-\frac{1}{124416(1-\cos(x))^2}-\frac{209}{373248(1-\cos(x))}\\ &+\frac{1}{124416(1+\cos(x))^2}+\frac{209}{373248(1+\cos(x))}\\ &+\frac{1}{108(1+2\cos(x))^4}-\frac{19}{162(1+2\cos(x))^3}\\ &+\frac{749}{648(1+2\cos(x))^2}-\frac{71551}{5832(1+2\cos(x))}\\ &-\frac{11643}{512}\cos(x)\sec(2x)+\frac{681}{256}\cos(x)\sec^2(2x)\\ &-\frac{21}{64}\cos(x)\sec^3(2x)+\frac{1}{32}\cos(x)\sec^4(2x)\end{aligned}$$

output
```
-3889/186624*arctanh(cos(x))-332929/2916*arctanh(2*cos(x))+82683/1024*arct
anh(cos(x)*2^(1/2))*2^(1/2)-1/108/(1-2*cos(x))^4+19/162/(1-2*cos(x))^3-749
/648/(1-2*cos(x))^2+71551/(5832-11664*cos(x))-1/124416/(1-cos(x))^2-209/(3
73248-373248*cos(x))+1/124416/(1+cos(x))^2+209/(373248+373248*cos(x))+1/10
8/(1+2*cos(x))^4-19/162/(1+2*cos(x))^3+749/648/(1+2*cos(x))^2-71551/(5832+
11664*cos(x))-11643/512*cos(x)*sec(2*x)+681/256*cos(x)*sec(2*x)^2-21/64*co
s(x)*sec(2*x)^3+1/32*cos(x)*sec(2*x)^4
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.82 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.17

$$\int \frac{1}{(\cos(5x)+\sin(2x))^5}\,dx$$
$$= \frac{-3070548\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+953225812\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-6069115802624\log(1-2\sin(x}{}$$

input
```
Integrate[(Cos[5*x] + Sin[2*x])^(-5),x]
```

output
```
(-3070548*Log[Cos[x/2] - Sin[x/2]] + 953225812*Log[Cos[x/2] + Sin[x/2]] -
6069115802624*Log[1 - 2*Sin[x]] - 13436928*(451639*Log[Sec[x/2]^2] - 8*Roo
tSum[107476319 - 1002557936*#1 - 231239168*#1^2 + 4096*#1^3 & , Log[-(Sec[
x/2]^2*(-8477353100119 - 37436171566600*Sin[x] - 6312005469856*Sin[x]*#1 +
 111813632*Sin[x]*#1^2))]*#1 & ]) + (21*Sec[x]^4*(14992301988 - 1352488209
40*Cos[2*x] + 50511597564*Cos[4*x] + 118827142092*Cos[6*x] - 118688826492*
Cos[8*x] - 447277004*Cos[10*x] - 7157694992*Cos[12*x] - 5002400916*Cos[14*
x] + 28445689884*Cos[16*x] - 10025130752*Cos[18*x] + 178131036546*Sin[x] +
 25679598098*Sin[3*x] - 60509728422*Sin[5*x] + 19995369696*Sin[7*x] - 1049
47392097*Sin[9*x] + 37686703163*Sin[11*x] + 29725568451*Sin[13*x] - 296580
30003*Sin[15*x] + 2383629525*Sin[17*x] - 8921638271*Sin[19*x]))/(1 - 2*Cos
[2*x] + 2*Cos[4*x] + 2*Sin[x])^4)/392073696
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(2x)+\cos(5x))^5}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(\sin(2x)+\cos(5x))^5}dx$$

$$\Big\downarrow \; 4829$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^3\left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^5}d\sin(x)$$

$$\Big\downarrow \; 2462$$

$$\int \left(\frac{64\left(404\sin^2(x)+454\sin(x)+81\right)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^5}+\frac{64\left(32518008\sin^2(x)+34614094\sin(x)+3256035\right)}{16807\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{256(6444}{2401\,(8\,\mathrm{s}}\right.$$

$$\Big\downarrow \; 7239$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^3\left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^5}d\sin(x)$$

$$\Big\downarrow \; 2462$$

$$\int \left(\frac{64\left(404\sin^2(x)+454\sin(x)+81\right)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^5}+\frac{64\left(32518008\sin^2(x)+34614094\sin(x)+3256035\right)}{16807\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{256(6444}{2401\,(8\,\mathrm{s}}\right.$$

$$\Big\downarrow \; 7239$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^3\left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^5}d\sin(x)$$

$$\Big\downarrow \; 2462$$

$$\int \left(\frac{64\left(404\sin^2(x)+454\sin(x)+81\right)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^5}+\frac{64\left(32518008\sin^2(x)+34614094\sin(x)+3256035\right)}{16807\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{256(6444}{2401\,(8\,\mathrm{s}}\right.$$

$$\Big\downarrow \; 7239$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^3 \left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^5}d\sin(x)$$

$\Big\downarrow$ 2462

$$\int \left(\frac{64\left(404\sin^2(x)+454\sin(x)+81\right)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^5}+\frac{64\left(32518008\sin^2(x)+34614094\sin(x)+3256035\right)}{16807\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{256(6444}{2401\ (8\mathrm{s}}\right.$$

$\Big\downarrow$ 7239

$$\int \frac{1}{\left(1-\sin^2(x)\right)^3 \left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^5}d\sin(x)$$

$\Big\downarrow$ 2462

$$\int \left(\frac{64\left(404\sin^2(x)+454\sin(x)+81\right)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^5}+\frac{64\left(32518008\sin^2(x)+34614094\sin(x)+3256035\right)}{16807\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{256(6444}{2401\ (8\mathrm{s}}\right.$$

$\Big\downarrow$ 7239

$$\int \frac{1}{\left(1-\sin^2(x)\right)^3 \left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^5}d\sin(x)$$

$\Big\downarrow$ 2462

$$\int \left(\frac{64\left(404\sin^2(x)+454\sin(x)+81\right)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^5}+\frac{64\left(32518008\sin^2(x)+34614094\sin(x)+3256035\right)}{16807\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{256(6444}{2401\ (8\mathrm{s}}\right.$$

$\Big\downarrow$ 7239

$$\int \frac{1}{\left(1-\sin^2(x)\right)^3 \left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^5}d\sin(x)$$

$\Big\downarrow$ 2462

$$\int \left(\frac{64\left(404\sin^2(x)+454\sin(x)+81\right)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^5}+\frac{64\left(32518008\sin^2(x)+34614094\sin(x)+3256035\right)}{16807\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{256(6444}{2401\ (8\mathrm{s}}\right.$$

$\Big\downarrow$ 7239

$$\int \frac{1}{\left(1-\sin^2(x)\right)^3 \left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^5}d\sin(x)$$

$\Big\downarrow$ 2462

$$\int \left(\frac{64(404\sin^2(x)+454\sin(x)+81)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^5}+\frac{64(32518008\sin^2(x)+34614094\sin(x)+3256035)}{16807\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{256(6444}{2401\ (8\mathrm{s}}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^3\left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^5}d\sin(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{64(404\sin^2(x)+454\sin(x)+81)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^5}+\frac{64(32518008\sin^2(x)+34614094\sin(x)+3256035)}{16807\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{256(6444}{2401\ (8\mathrm{s}}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^3\left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^5}d\sin(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{64(404\sin^2(x)+454\sin(x)+81)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^5}+\frac{64(32518008\sin^2(x)+34614094\sin(x)+3256035)}{16807\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{256(6444}{2401\ (8\mathrm{s}}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^3\left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^5}d\sin(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{64(404\sin^2(x)+454\sin(x)+81)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^5}+\frac{64(32518008\sin^2(x)+34614094\sin(x)+3256035)}{16807\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{256(6444}{2401\ (8\mathrm{s}}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^3\left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^5}d\sin(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{64(404\sin^2(x)+454\sin(x)+81)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^5}+\frac{64(32518008\sin^2(x)+34614094\sin(x)+3256035)}{16807\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{256(6444}{2401\ (8\mathrm{s}}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^3 \left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^5}d\sin(x)$$

$$\Big\downarrow \text{ 2462}$$

$$\int \left(\frac{64\left(404\sin^2(x)+454\sin(x)+81\right)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^5}+\frac{64\left(32518008\sin^2(x)+34614094\sin(x)+3256035\right)}{16807\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{256(6444\cdots}{2401\left(8\,\mathrm{s}\cdots\right.}\right.$$

$$\Big\downarrow \text{ 7239}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^3 \left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^5}d\sin(x)$$

$$\Big\downarrow \text{ 2462}$$

$$\int \left(\frac{64\left(404\sin^2(x)+454\sin(x)+81\right)}{7\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)^5}+\frac{64\left(32518008\sin^2(x)+34614094\sin(x)+3256035\right)}{16807\left(8\sin^3(x)+4\sin^2(x)-4\sin(x)-1\right)}-\frac{256(6444\cdots}{2401\left(8\,\mathrm{s}\cdots\right.}\right.$$

$$\Big\downarrow \text{ 7239}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^3 \left(16\sin^4(x)-12\sin^2(x)+2\sin(x)+1\right)^5}d\sin(x)$$

input
```
Int[(Cos[5*x] + Sin[2*x])^(-5),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4829
```
Int[(cos[(n_.)*((c_.) + (d_.)*(x_))]*(b_.) + (a_.)*sin[(m_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Sin[
m*ArcSin[x]] + b*Cos[n*ArcSin[x]]]]^p/Sqrt[1 - x^2], x], x, Sin[c + d*x]],
x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[m/2] && Inte
gerQ[(n - 1)/2]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.72 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.07

$$\frac{8}{27\left(2\sin\left(x\right)-1\right)^{4}}+\frac{544}{81\left(2\sin\left(x\right)-1\right)^{3}}+\frac{9056}{81\left(2\sin\left(x\right)-1\right)^{2}}+\frac{1452064}{729\left(2\sin\left(x\right)-1\right)}-\frac{11284576\ln\left(2\sin\left(x\right)-1\right.}{729}$$

input
```
int(1/(cos(5*x)+sin(2*x))^5,x)
```

output
```
8/27/(2*sin(x)-1)^4+544/81/(2*sin(x)-1)^3+9056/81/(2*sin(x)-1)^2+1452064/7
29/(2*sin(x)-1)-11284576/729*ln(2*sin(x)-1)+524288/16807*(480851/4*sin(x)^
11+9996055/32*sin(x)^10+2029375/16*sin(x)^9-72898969/256*sin(x)^8-8098047/
32*sin(x)^7+11246907/256*sin(x)^6+26929133/256*sin(x)^5+88542503/4096*sin(
x)^4-21603091/2048*sin(x)^3-44016103/8192*sin(x)^2-1775379/2048*sin(x)-321
5083/65536)/(8*sin(x)^3+4*sin(x)^2-4*sin(x)-1)^4+288/16807*sum((1806556*_R
^2+2029861*_R+362232)/(6*_R^2+2*_R-1)*ln(sin(x)-_R),_R=RootOf(8*_Z^3+4*_Z^
2-4*_Z-1))-1/3888/(1+sin(x))^2-389/11664/(1+sin(x))+14179/11664*ln(1+sin(x
))+1/268912/(sin(x)-1)^2-9/38416/(sin(x)-1)-1053/268912*ln(sin(x)-1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 2056, normalized size of antiderivative = 9.30

$$\int \frac{1}{(\cos(5x)+\sin(2x))^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cos(5*x)+sin(2*x))^5,x, algorithm="fricas")
```

output
```
-1/470684472048*(3312694838208705331 2*cos(x)^18 - 172570185683432570880*co
s(x)^16 + 374537416757797601280*cos(x)^14 - 438603598203625021440*cos(x)^1
2 + 299653580002679476224*cos(x)^10 - 120005887602794516736*cos(x)^8 + 263
21130059384977152*cos(x)^6 - 2457796064465502768*cos(x)^4 + 5832*(65536*co
s(x)^20 - 327680*cos(x)^18 + 696320*cos(x)^16 - 825344*cos(x)^14 + 603904*
cos(x)^12 - 284800*cos(x)^10 + 86256*cos(x)^8 - 15432*cos(x)^6 + 1241*cos(
x)^4 + 8*(4096*cos(x)^16 - 15360*cos(x)^14 + 23040*cos(x)^12 - 17664*cos(x
)^10 + 7344*cos(x)^8 - 1600*cos(x)^6 + 145*cos(x)^4)*sin(x))*(40353607*(11
25032920858032537 6/678223072849*I*sqrt(3) + 931815687825928996945 92/678223
072849)^(1/3)*(-I*sqrt(3) + 1) + 1074452790721536*(I*sqrt(3) + 1)/(1125032
9208580325376/678223072849*I*sqrt(3) + 931815687825928996945 92/67822307284
9)^(1/3) - 416403931776)*log(58363/23059204*(40353607*(112503292085803253 7
6/678223072849*I*sqrt(3) + 93181568782592899694592/678223072849)^(1/3)*(-I
*sqrt(3) + 1) + 1074452790721536*(I*sqrt(3) + 1)/(11250329208580325376/678
223072849*I*sqrt(3) + 93181568782592899694592/678223072849)^(1/3) - 416403
931776)^2 + 127611403123292256*(11250329208580325376/678223072849*I*sqrt(3
) + 93181568782592899694592/678223072849)^(1/3)*(-I*sqrt(3) + 1) + 1165436
41011578763787675 2384/343*(I*sqrt(3) + 1)/(11250329208580325376/6782230728
49*I*sqrt(3) + 93181568782592899694592/678223072849)^(1/3) + 4687637170241
80224*sin(x) - 1316694816201251589120) + 605165749776*cos(x)^2 - 2916*(...
```

Sympy [F]

$$\int \frac{1}{(\cos(5x)+\sin(2x))^5}\,dx = \int \frac{1}{(\sin{(2x)}+\cos{(5x)})^5}\,dx$$

input
```
integrate(1/(cos(5*x)+sin(2*x))**5,x)
```

output
```
Integral((sin(2*x) + cos(5*x))**(-5), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(5x)+\sin(2x))^5}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(cos(5*x)+sin(2*x))^5,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(5x)+\sin(2x))^5}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(1/(cos(5*x)+sin(2*x))^5,x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: 2*((
-89077056*sin(sageVARx)^4-3291925*sin(sageVARx)^3+181375264*sin(sageVARx)^
2+3343075*sin(sageVARx)-92347900)*1/196036848/(sin(sageVARx)^2-1)^2+(76012
4211200*sin(sageVAR
```

Mupad [B] (verification not implemented)

Time = 22.43 (sec) , antiderivative size = 1217, normalized size of antiderivative = 5.51

$$\int \frac{1}{(\cos(5x)+\sin(2x))^5}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(5*x) + sin(2*x))^5,x)
```

output
```
(14179*log(tan(x/2) + 1))/5832 - (1053*log(tan(x/2) - 1))/134456 - (112845
76*log(tan(x/2)^2 - 4*tan(x/2) + 1))/729 + symsum(log((2199023255552*(3355
8302142494706313979267710332531314776224468449900888064*root(z^3 - (260144
064*z^2)/16807 - (5197260340224*z)/282475249 + 2567380551303168/4747561509
943, z, k)*cos(x) - 1206810284585867320727487656557136524677337776015424880
0640*cos(x) - 890863856345561813414462492104570397111221021823603834880*si
n(x) - 2561108012046848700509957175662882048956774272712939929 6000*root(z^
3 - (260144064*z^2)/16807 - (5197260340224*z)/282475249 + 2567380551303168
/4747561509943, z, k) + 345014685035306038598351697163120842102240499574 97
766674432*root(z^3 - (260144064*z^2)/16807 - (5197260340224*z)/282475249 +
 2567380551303168/4747561509943, z, k)*sin(x) - 304414495847477527982870 27
8435390005304320214291072294846464*root(z^3 - (260144064*z^2)/16807 - (519
7260340224*z)/282475249 + 2567380551303168/4747561509943, z, k)^2 - 391752
9489058679742787198905273888412697431255315380805 6320*root(z^3 - (26014406
4*z^2)/16807 - (5197260340224*z)/282475249 + 2567380551303168/474756150994
3, z, k)^3 + 15891099374947127099944141870820053086008655535491314608 2048*
root(z^3 - (260144064*z^2)/16807 - (5197260340224*z)/282475249 + 256738055
1303168/4747561509943, z, k)^4 + 821882759939232176564869001304867217109 65
619488658442336*root(z^3 - (260144064*z^2)/16807 - (5197260340224*z)/28247
5249 + 2567380551303168/4747561509943, z, k)^5 - 6166384041129098466961...
```

Reduce [F]

$$\int \frac{1}{(\cos(5x)+\sin(2x))^5}\,dx = \int \frac{1}{\cos(5x)^5+5\cos(5x)^4\sin(2x)+10\cos(5x)^3\sin(2x)^2+10\cos(5x)^2\sin(2x)^3+5\cos(5x)\sin(2x)^4+}$$

input
```
int(1/(cos(5*x)+sin(2*x))^5,x)
```

output
```
int(1/(cos(5*x)**5 + 5*cos(5*x)**4*sin(2*x) + 10*cos(5*x)**3*sin(2*x)**2 +
 10*cos(5*x)**2*sin(2*x)**3 + 5*cos(5*x)*sin(2*x)**4 + sin(2*x)**5),x)
```

3.10 $\int \frac{1}{(\cos(5x)+\sin(2x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 90

$$\int \frac{1}{(\cos(5x)+\sin(2x))^2}\,dx = -\frac{3}{4}\operatorname{arctanh}(2\cos(x)\sin(x)) - \frac{\cot(x)}{36} - \frac{4\log\left(\sqrt{3}\cos(x)-\sin(x)\right)}{3\sqrt{3}} + \frac{4\log\left(\sqrt{3}\cos(x)+\sin(x)\right)}{3\sqrt{3}} + \frac{\tan(x)\left(43-25\tan^2(x)\right)}{18\left(3-4\tan^2(x)+\tan^4(x)\right)}$$

output
```
-3/4*arctanh(2*cos(x)*sin(x))-1/36*cot(x)-4/9*ln(3^(1/2)*cos(x)-sin(x))*3^
(1/2)+4/9*ln(3^(1/2)*cos(x)+sin(x))*3^(1/2)+tan(x)*(43-25*tan(x)^2)/(54-72
*tan(x)^2+18*tan(x)^4)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 401, normalized size of antiderivative = 4.46

$$\int \frac{1}{(\cos(5x)+\sin(2x))^2}\,dx = \frac{1}{441}\left(2744\sqrt{3}\text{arctanh}\left(\frac{-2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)\right.$$

$$-378i\text{RootSum}\Bigg[i+\#1-i\#1^2-\#1^3+i\#1^4+\#1^5$$

$$-i\#1^6\&,\frac{10\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)-5i\log\left(1-2\cos(x)\#1+\#1^2\right)-22i\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1-11\log}{}$$

$$+\frac{9\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}+\frac{49\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}+\frac{392\cos(x)}{1-2\sin(x)}$$

$$\left.+\frac{72(9\cos(x)-6\cos(3x)+11\sin(2x))}{-1+2\cos(2x)-2\sin(x)+2\sin(3x)}\right)$$

input
```
Integrate[(Cos[5*x] + Sin[2*x])^(-2),x]
```

output
```
(2744*Sqrt[3]*ArcTanh[(-2 + Tan[x/2])/Sqrt[3]] - (378*I)*RootSum[I + #1 -
I*#1^2 - #1^3 + I*#1^4 + #1^5 - I*#1^6 & , (10*ArcTan[Sin[x]/(Cos[x] - #1)
] - (5*I)*Log[1 - 2*Cos[x]*#1 + #1^2] - (22*I)*ArcTan[Sin[x]/(Cos[x] - #1)
]*#1 - 11*Log[1 - 2*Cos[x]*#1 + #1^2]*#1 - 28*ArcTan[Sin[x]/(Cos[x] - #1)]
*#1^2 + (14*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^2 + (22*I)*ArcTan[Sin[x]/(Co
s[x] - #1)]*#1^3 + 11*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3 + 10*ArcTan[Sin[x]/
(Cos[x] - #1)]*#1^4 - (5*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^4)/(I + 2*#1 -
(3*I)*#1^2 - 4*#1^3 + (5*I)*#1^4 + 6*#1^5) & ] + (9*Sin[x/2])/(Cos[x/2] -
Sin[x/2]) + (49*Sin[x/2])/(Cos[x/2] + Sin[x/2]) + (392*Cos[x])/(1 - 2*Sin[
x]) + (72*(9*Cos[x] - 6*Cos[3*x] + 11*Sin[2*x]))/(-1 + 2*Cos[2*x] - 2*Sin[
x] + 2*Sin[3*x]))/441
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(2x)+\cos(5x))^2} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(\sin(2x)+\cos(5x))^2}dx$$

$$\Big\downarrow \text{ 4830}$$

$$2\int \frac{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^9}{\left(-\tan^{10}\left(\frac{x}{2}\right)-4\tan^9\left(\frac{x}{2}\right)+45\tan^8\left(\frac{x}{2}\right)-8\tan^7\left(\frac{x}{2}\right)-210\tan^6\left(\frac{x}{2}\right)+210\tan^4\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)-45\tan^2\left(\right.\right.}$$

$$\Big\downarrow \text{ 2462}$$

$$2\int \left(\frac{16\tan\left(\frac{x}{2}\right)}{3\left(\tan^2\left(\frac{x}{2}\right)-4\tan\left(\frac{x}{2}\right)+1\right)^2}-\frac{68}{9\left(\tan^2\left(\frac{x}{2}\right)-4\tan\left(\frac{x}{2}\right)+1\right)}+\frac{32\left(13\tan^4\left(\frac{x}{2}\right)+130\tan^3\left(\frac{x}{2}\right)+\right.}{49\left(\tan^6\left(\frac{x}{2}\right)+8\tan^5\left(\frac{x}{2}\right)-13\tan^4\left(\frac{x}{2}\right)-\right.}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{14\log\left(-\tan\left(\frac{x}{2}\right)-\sqrt{3}+2\right)}{3\sqrt{3}}-\frac{14\log\left(-\tan\left(\frac{x}{2}\right)+\sqrt{3}+2\right)}{3\sqrt{3}}+\frac{687104}{21}\int \frac{}{\left(\tan^6\left(\frac{x}{2}\right)+8\tan^5\left(\frac{x}{2}\right)-13\tan^4\left(\frac{x}{2}\right.\right.}\right.$$

input
```
Int[(Cos[5*x] + Sin[2*x])^(-2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4830
```
Int[(cos[(n_.)*((c_.) + (d_.)*(x_))]*(b_.) + (a_.)*sin[(m_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[2/d   Subst[Int[Simplify[TrigExpand[a*Sin[
2*m*ArcTan[x]] + b*Cos[2*n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[(1/2)*(c +
d*x)]], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && Intege
rQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.16

method	result
risch	$\frac{\frac{16\,\mathrm{e}^{9ix}}{21}-\frac{10i\mathrm{e}^{8ix}}{21}+\frac{12\,\mathrm{e}^{7ix}}{7}+\frac{2i\mathrm{e}^{6ix}}{7}-\frac{2\,\mathrm{e}^{5ix}}{7}-\frac{2i\mathrm{e}^{4ix}}{7}-\frac{2\,\mathrm{e}^{3ix}}{7}+\frac{10i\mathrm{e}^{2ix}}{21}+\frac{16\,\mathrm{e}^{ix}}{21}+2i}{\mathrm{e}^{10ix}-i\mathrm{e}^{7ix}+i\mathrm{e}^{3ix}+1}+\frac{28\sqrt{3}\,\ln\left(\mathrm{e}^{ix}-\frac{i}{2}-\frac{\sqrt{3}}{2}\right)}{9}-\frac{28\sqrt{3}\,\ln\left(\mathrm{e}^{ix}-\frac{i}{2}+\frac{\sqrt{3}}{2}\right)}{9}$
default	$-\frac{1}{9\left(\tan\left(\frac{x}{2}\right)+1\right)}-\frac{1}{49\left(\tan\left(\frac{x}{2}\right)-1\right)}+\frac{-\frac{160\tan\left(\frac{x}{2}\right)^{5}}{49}-\frac{1104\tan\left(\frac{x}{2}\right)^{4}}{49}-\frac{1152\tan\left(\frac{x}{2}\right)^{3}}{49}+\frac{480\tan\left(\frac{x}{2}\right)^{2}}{49}+\frac{544\tan\left(\frac{x}{2}\right)}{49}+\frac{48}{49}}{\tan\left(\frac{x}{2}\right)^{6}+8\tan\left(\frac{x}{2}\right)^{5}-13\tan\left(\frac{x}{2}\right)^{4}-48\tan\left(\frac{x}{2}\right)^{3}-13\tan\left(\frac{x}{2}\right)^{2}+8\tan\left(\frac{x}{2}\right)+1}+\frac{48\left(\_R=\right.}{}$

input
```
int(1/(cos(5*x)+sin(2*x))^2,x,method=_RETURNVERBOSE)
```

output
```
2/21*(8*exp(9*I*x)-5*I*exp(8*I*x)+18*exp(7*I*x)+3*I*exp(6*I*x)-3*exp(5*I*x
)-3*I*exp(4*I*x)-3*exp(3*I*x)+5*I*exp(2*I*x)+8*exp(I*x)+21*I)/(exp(10*I*x)
-I*exp(7*I*x)+I*exp(3*I*x)+1)+28/9*3^(1/2)*ln(exp(I*x)-1/2*I-1/2*3^(1/2))-
28/9*3^(1/2)*ln(exp(I*x)-1/2*I+1/2*3^(1/2))+sum(_R*ln(exp(I*x)-93499307419
/3560454144*_R^5-547656095/890113536*I*_R^4+794954293/1030224*_R^3+5572000
7/3090672*I*_R^2-7175315/257556*_R+8531/42926*I),_R=RootOf(1977326743*_Z^6
-58109194080*_Z^4+2339995392*_Z^2-2985984))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 3630, normalized size of antiderivative = 40.33

$$\int \frac{1}{(\cos(5x)+\sin(2x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cos(5*x)+sin(2*x))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(\cos(5x)+\sin(2x))^2}\,dx = \int \frac{1}{(\sin(2x)+\cos(5x))^2}\,dx$$

input
```
integrate(1/(cos(5*x)+sin(2*x))**2,x)
```

output
```
Integral((sin(2*x) + cos(5*x))**(-2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(5x)+\sin(2x))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(cos(5*x)+sin(2*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(72) = 144$.

Time = 0.14 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.64

$$\int \frac{1}{(\cos(5x)+\sin(2x))^2}\,dx = -\frac{28}{9}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\tan\left(\frac{1}{2}x\right)-4\right|}{\left|2\sqrt{3}+2\tan\left(\frac{1}{2}x\right)-4\right|}\right)$$
$$-\frac{2\left(73\tan\left(\frac{1}{2}x\right)^9+384\tan\left(\frac{1}{2}x\right)^8-1436\tan\left(\frac{1}{2}x\right)^7-2724\tan\left(\frac{1}{2}x\right)^6+2470\tan\left(\frac{1}{2}x\right)^5+3156\tan\right.}{21\left(\tan\left(\frac{1}{2}x\right)^{10}+4\tan\left(\frac{1}{2}x\right)^9-45\tan\left(\frac{1}{2}x\right)^8+8\tan\left(\frac{1}{2}x\right)^7+210\tan\left(\frac{1}{2}x\right)^6-210\tan\left(\right.\right.}$$
$$-0.0363219532408571\log\left(\tan\left(\frac{1}{2}x\right)+8.87524547497000\right)$$
$$+0.197492344812286\log\left(\tan\left(\frac{1}{2}x\right)+1.59149088298000\right)$$
$$-0.197492344812286\log\left(\tan\left(\frac{1}{2}x\right)+0.628341645367000\right)$$
$$+0.0363219532408571\log\left(\tan\left(\frac{1}{2}x\right)+0.112672939900000\right)$$
$$-5.41732707074286\log\left(\tan\left(\frac{1}{2}x\right)-0.349915133947000\right)$$
$$+5.41732707074286\log\left(\tan\left(\frac{1}{2}x\right)-2.85783580927000\right)$$

input
```
integrate(1/(cos(5*x)+sin(2*x))^2,x, algorithm="giac")
```

output
```
-28/9*sqrt(3)*log(abs(-2*sqrt(3) + 2*tan(1/2*x) - 4)/abs(2*sqrt(3) + 2*tan
(1/2*x) - 4)) - 2/21*(73*tan(1/2*x)^9 + 384*tan(1/2*x)^8 - 1436*tan(1/2*x)
^7 - 2724*tan(1/2*x)^6 + 2470*tan(1/2*x)^5 + 3156*tan(1/2*x)^4 - 1212*tan(
1/2*x)^3 - 876*tan(1/2*x)^2 + 185*tan(1/2*x) + 28)/(tan(1/2*x)^10 + 4*tan(
1/2*x)^9 - 45*tan(1/2*x)^8 + 8*tan(1/2*x)^7 + 210*tan(1/2*x)^6 - 210*tan(1
/2*x)^4 - 8*tan(1/2*x)^3 + 45*tan(1/2*x)^2 - 4*tan(1/2*x) - 1) - 0.0363219
532408571*log(tan(1/2*x) + 8.87524547497000) + 0.197492344812286*log(tan(1
/2*x) + 1.59149088298000) - 0.197492344812286*log(tan(1/2*x) + 0.628341645
367000) + 0.0363219532408571*log(tan(1/2*x) + 0.112672939900000) - 5.41732
707074286*log(tan(1/2*x) - 0.349915133947000) + 5.41732707074286*log(tan(1
/2*x) - 2.85783580927000)
```

Mupad [B] (verification not implemented)

Time = 21.24 (sec) , antiderivative size = 544, normalized size of antiderivative = 6.04

$$\int \frac{1}{(\cos(5x) + \sin(2x))^2} \, dx = \text{Too large to display}$$

input
```
int(1/(cos(5*x) + sin(2*x))^2,x)
```

output
```
(28*3^(1/2)*log(tan(x/2) + 3^(1/2) - 2))/9 + symsum(log((16267598490940865
595310080*root(z^6 - (1440*z^4)/49 + (974592*z^2)/823543 - 2985984/1977326
743, z, k)^2*tan(x/2))/16807 - (4155082985742892227624 96*tan(x/2))/16807 -
 (895155969527422979250585 6*root(z^6 - (1440*z^4)/49 + (974592*z^2)/823543
 - 2985984/1977326743, z, k))/117649 + (4467521807543688197308416 0*root(z^
6 - (1440*z^4)/49 + (974592*z^2)/823543 - 2985984/1977326743, z, k)^3*tan(
x/2))/2401 - (20039076885035344052879 36*root(z^6 - (1440*z^4)/49 + (974592
*z^2)/823543 - 2985984/1977326743, z, k)^4*tan(x/2))/147 - (93995352480177
163599872*root(z^6 - (1440*z^4)/49 + (974592*z^2)/823543 - 2985984/1977326
743, z, k)^5*tan(x/2))/9 + 456834590888836464640*root(z^6 - (1440*z^4)/49
+ (974592*z^2)/823543 - 2985984/1977326743, z, k)^6*tan(x/2) + 33550309573
5661887488*root(z^6 - (1440*z^4)/49 + (974592*z^2)/823543 - 2985984/197732
6743, z, k)^7*tan(x/2) - (243056904797221523187302 4*root(z^6 - (1440*z^4)/
49 + (974592*z^2)/823543 - 2985984/1977326743, z, k)^2)/16807 + (596916683
2256521743630336*root(z^6 - (1440*z^4)/49 + (974592*z^2)/823543 - 2985984/
1977326743, z, k)^3)/2401 + (319229402799190778576896*root(z^6 - (1440*z^4
)/49 + (974592*z^2)/823543 - 2985984/1977326743, z, k)^4)/147 + (230348635
99711839846400*root(z^6 - (1440*z^4)/49 + (974592*z^2)/823543 - 2985984/19
77326743, z, k)^5)/9 - (21461467738020197171 2*root(z^6 - (1440*z^4)/49 + (
974592*z^2)/823543 - 2985984/1977326743, z, k)^6)/3 - 90517831318333554...
```

Reduce [F]

$$\int \frac{1}{(\cos(5x)+\sin(2x))^2}\,dx = \int \frac{1}{\cos(5x)^2+2\cos(5x)\sin(2x)+\sin(2x)^2}dx$$

input
```
int(1/(cos(5*x)+sin(2*x))^2,x)
```

output
```
int(1/(cos(5*x)**2 + 2*cos(5*x)*sin(2*x) + sin(2*x)**2),x)
```

3.11 $\int \frac{1}{(\cos(5x)+\sin(2x))^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 154

$$\int \frac{1}{(\cos(5x)+\sin(2x))^4}\,dx = -\frac{103}{8}\operatorname{arctanh}(2\cos(x)\sin(x)) - \frac{43\cot(x)}{3888} - \frac{\cot^3(x)}{3888} - \frac{1808\log\left(\sqrt{3}\cos(x)-\sin(x)\right)}{81\sqrt{3}} + \frac{1808\log\left(\sqrt{3}\cos(x)+\sin(x)\right)}{81\sqrt{3}} + \frac{4\tan(x)\left(593-539\tan^2(x)\right)}{81\left(3-4\tan^2(x)+\tan^4(x)\right)^3} - \frac{2\tan(x)\left(3502-1237\tan^2(x)\right)}{243\left(3-4\tan^2(x)+\tan^4(x)\right)^2} + \frac{\tan(x)\left(38097-18413\tan^2(x)\right)}{972\left(3-4\tan^2(x)+\tan^4(x)\right)}$$

output
```
-103/8*arctanh(2*cos(x)*sin(x))-43/3888*cot(x)-1/3888*cot(x)^3-1808/243*ln
(3^(1/2)*cos(x)-sin(x))*3^(1/2)+1808/243*ln(3^(1/2)*cos(x)+sin(x))*3^(1/2)
+4/81*tan(x)*(593-539*tan(x)^2)/(3-4*tan(x)^2+tan(x)^4)^3-2/243*tan(x)*(35
02-1237*tan(x)^2)/(3-4*tan(x)^2+tan(x)^4)^2+tan(x)*(38097-18413*tan(x)^2)/
(2916-3888*tan(x)^2+972*tan(x)^4)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.70 (sec) , antiderivative size = 502, normalized size of antiderivative = 3.26

$$\int \frac{1}{(\cos(5x)+\sin(2x))^4}\,dx$$
$$= \frac{287392\operatorname{arctanh}\left(\frac{-2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{81\sqrt{3}} - \frac{8}{343}i\text{RootSum}\Big[i+\#1-i\#1^2-\#1^3+i\#1^4+\#1^5$$
$$-i\#1^6\&, \frac{68690\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)-34345i\log\left(1-2\cos(x)\#1+\#1^2\right)-154276i\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)}{}$$
$$+\frac{-50806812-46050\cos(x)+47365479\cos(2x)+85384068\cos(4x)+138150\cos(5x)+17105655\cos}{}$$

input
```
Integrate[(Cos[5*x] + Sin[2*x])^(-4),x]
```

output
```
(287392*ArcTanh[(-2 + Tan[x/2])/Sqrt[3]])/(81*Sqrt[3]) - ((8*I)/343)*RootS
um[I + #1 - I*#1^2 - #1^3 + I*#1^4 + #1^5 - I*#1^6 & , (68690*ArcTan[Sin[x
]/(Cos[x] - #1)] - (34345*I)*Log[1 - 2*Cos[x]*#1 + #1^2] - (154276*I)*ArcT
an[Sin[x]/(Cos[x] - #1)]*#1 - 77138*Log[1 - 2*Cos[x]*#1 + #1^2]*#1 - 19244
0*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 + (96220*I)*Log[1 - 2*Cos[x]*#1 + #1^2
]*#1^2 + (154276*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 + 77138*Log[1 - 2*Co
s[x]*#1 + #1^2]*#1^3 + 68690*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^4 - (34345*I)
*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^4)/(I + 2*#1 - (3*I)*#1^2 - 4*#1^3 + (5*I)
*#1^4 + 6*#1^5) & ] + (-50806812 - 46050*Cos[x] + 47365479*Cos[2*x] + 8538
4068*Cos[4*x] + 138150*Cos[5*x] + 17105655*Cos[6*x] + 34460531*Cos[8*x] -
46050*Cos[9*x] - 28082600*Cos[10*x] - 15943473*Cos[12*x] + 16718128*Cos[14
*x] + 15350*Cos[15*x] - 13261101*Sin[x] + 138150*Sin[2*x] + 84763014*Sin[3
*x] + 47550321*Sin[5*x] - 15350*Sin[6*x] - 67506418*Sin[7*x] - 46050*Sin[8
*x] + 15795675*Sin[9*x] + 28666610*Sin[11*x] + 46050*Sin[12*x] + 10801000*
Sin[13*x] + 14931819*Sin[15*x])/(388962*(Cos[x/2] - Sin[x/2])^3*(Cos[x/2]
+ Sin[x/2])^3*(-1 + 2*Sin[x])^3*(1 - 2*Cos[2*x] + 2*Sin[x] - 2*Sin[3*x])^3
)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(2x)+\cos(5x))^4} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(2x)+\cos(5x))^4} dx$$

$$\Big\downarrow \text{4830}$$

$$2\int \frac{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^{19}}{\left(-\tan^{10}\left(\frac{x}{2}\right)-4\tan^9\left(\frac{x}{2}\right)+45\tan^8\left(\frac{x}{2}\right)-8\tan^7\left(\frac{x}{2}\right)-210\tan^6\left(\frac{x}{2}\right)+210\tan^4\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)-45\tan^2\left(\right.\right.}$$

$$\Big\downarrow \text{2462}$$

$$2\int \left(\frac{1024\left(15\tan\left(\frac{x}{2}\right)-4\right)}{9\left(\tan^2\left(\frac{x}{2}\right)-4\tan\left(\frac{x}{2}\right)+1\right)^4}-\frac{362960}{243\left(\tan^2\left(\frac{x}{2}\right)-4\tan\left(\frac{x}{2}\right)+1\right)}+\frac{64\left(56067\tan^4\left(\frac{x}{2}\right)+645324\tan^3\left(\frac{x}{2}\right)+\right.}{2401\left(\tan^6\left(\frac{x}{2}\right)+8\tan^5\left(\frac{x}{2}\right)-13\tan^4\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{71848\log\left(-\tan\left(\frac{x}{2}\right)-\sqrt{3}+2\right)}{81\sqrt{3}}-\frac{71848\log\left(-\tan\left(\frac{x}{2}\right)+\sqrt{3}+2\right)}{81\sqrt{3}}+\frac{316846150348963 84}{7}\int \frac{}{\left(\tan^6\left(\frac{x}{2}\right)+8\mathrm{t}\right.}\right.$$

input
```
Int[(Cos[5*x] + Sin[2*x])^(-4),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4830
```
Int[(cos[(n_.)*((c_.) + (d_.)*(x_))]*(b_.) + (a_.)*sin[(m_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[2/d   Subst[Int[Simplify[TrigExpand[a*Sin[
2*m*ArcTan[x]] + b*Cos[2*n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[(1/2)*(c +
d*x)]], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && Intege
rQ[n]
```

Maple [A] (verified)

Time = 113.99 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.01

method	result
parallelrisch	0
risch	$\frac{\frac{2764i}{9}+\frac{4776608\,e^{ix}}{27783}-\frac{8072668i e^{18ix}}{9261}-\frac{306952\,e^{13ix}}{343}-\frac{9677488\,e^{15ix}}{9261}+\frac{641100\,e^{17ix}}{343}+\frac{8190460i e^{4ix}}{27783}+\frac{3086000i e^{2ix}}{27783}+\frac{641752i e^{10ix}}{343}-\frac{27}{}}{}$
default	$-\frac{1}{7203\left(\tan\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{4802\left(\tan\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{13}{2401\left(\tan\left(\frac{x}{2}\right)-1\right)}-\frac{1}{243\left(\tan\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{1}{162\left(\tan\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{79}{243(\tan(}$

input
```
int(1/(cos(5*x)+sin(2*x))^4,x,method=_RETURNVERBOSE)
```

output
```
0
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.94 (sec) , antiderivative size = 4154, normalized size of antiderivative = 26.97

$$\int \frac{1}{(\cos(5x)+\sin(2x))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cos(5*x)+sin(2*x))^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(\cos(5x)+\sin(2x))^4}\,dx = \int \frac{1}{(\sin(2x)+\cos(5x))^4}\,dx$$

input
```
integrate(1/(cos(5*x)+sin(2*x))**4,x)
```

output
```
Integral((sin(2*x) + cos(5*x))**(-4), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(5x)+\sin(2x))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(cos(5*x)+sin(2*x))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(130) = 260$.

Time = 0.16 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.58

$$\int \frac{1}{(\cos(5x) + \sin(2x))^4} \, dx = \text{Too large to display}$$

input
```
integrate(1/(cos(5*x)+sin(2*x))^4,x, algorithm="giac")
```

output
```
-143696/243*sqrt(3)*log(abs(-2*sqrt(3) + 2*tan(1/2*x) - 4)/abs(2*sqrt(3) +
 2*tan(1/2*x) - 4)) - 2/27783*(13636631*tan(1/2*x)^29 + 204766588*tan(1/2*
x)^28 - 553680134*tan(1/2*x)^27 - 15018201276*tan(1/2*x)^26 + 18647592429*
tan(1/2*x)^25 + 420080085120*tan(1/2*x)^24 - 718319708124*tan(1/2*x)^23 -
4206791944584*tan(1/2*x)^22 + 7146174304719*tan(1/2*x)^21 + 22393376841796
*tan(1/2*x)^20 - 30960170007130*tan(1/2*x)^19 - 70996981497940*tan(1/2*x)^
18 + 67072099281005*tan(1/2*x)^17 + 131929900367416*tan(1/2*x)^16 - 811159
14524552*tan(1/2*x)^15 - 144156490971824*tan(1/2*x)^14 + 59297186514317*ta
n(1/2*x)^13 + 92704275150980*tan(1/2*x)^12 - 28321773645658*tan(1/2*x)^11
- 34467255328676*tan(1/2*x)^10 + 9389108574703*tan(1/2*x)^9 + 706790571412
8*tan(1/2*x)^8 - 2026230242268*tan(1/2*x)^7 - 693687479112*tan(1/2*x)^6 +
227742142605*tan(1/2*x)^5 + 20915657532*tan(1/2*x)^4 - 8078777862*tan(1/2*
x)^3 - 455196620*tan(1/2*x)^2 + 104666039*tan(1/2*x) + 7585784)/(tan(1/2*x
)^10 + 4*tan(1/2*x)^9 - 45*tan(1/2*x)^8 + 8*tan(1/2*x)^7 + 210*tan(1/2*x)^
6 - 210*tan(1/2*x)^4 - 8*tan(1/2*x)^3 + 45*tan(1/2*x)^2 - 4*tan(1/2*x) - 1
)^3 - 0.0292685064921866*log(tan(1/2*x) + 8.87524547497000) + 0.6450220540
46647*log(tan(1/2*x) + 1.59149088298000) - 0.645022054029154*log(tan(1/2*x
) + 0.628341645367000) + 0.0292685064952187*log(tan(1/2*x) + 0.11267293990
0000) - 1024.25938420408*log(tan(1/2*x) - 0.349915133947000) + 1024.259384
20408*log(tan(1/2*x) - 2.85783580927000)
```

Mupad [B] (verification not implemented)

Time = 23.05 (sec) , antiderivative size = 864, normalized size of antiderivative = 5.61

$$\int \frac{1}{(\cos(5x) + \sin(2x))^4} dx = \text{Too large to display}$$

input
```
int(1/(cos(5*x) + sin(2*x))^4,x)
```

output
```
((209332078*tan(x/2))/27783 - (910393240*tan(x/2)^2)/27783 - (5385851908*t
an(x/2)^3)/9261 + (663989128*tan(x/2)^4)/441 + (151828095070*tan(x/2)^5)/9
261 - (17128085904*tan(x/2)^6)/343 - (1350820161512*tan(x/2)^7)/9261 + (67
3133877536*tan(x/2)^8)/1323 + (18778217149406*tan(x/2)^9)/27783 - (6893451
0657352*tan(x/2)^10)/27783 - (56643547291316*tan(x/2)^11)/27783 + (1854085
50301960*tan(x/2)^12)/27783 + (118594373028634*tan(x/2)^13)/27783 - (28831
2981943648*tan(x/2)^14)/27783 - (162231829049104*tan(x/2)^15)/27783 + (263
859800734832*tan(x/2)^16)/27783 + (19163456937430*tan(x/2)^17)/3969 - (141
993962995880*tan(x/2)^18)/27783 - (8845762859180*tan(x/2)^19)/3969 + (4478
6753683592*tan(x/2)^20)/27783 + (529346244794*tan(x/2)^21)/1029 - (9348426
54352*tan(x/2)^22)/3087 - (478879805416*tan(x/2)^23)/9261 + (280053390080*
tan(x/2)^24)/9261 + (12431728286*tan(x/2)^25)/9261 - (10012134184*tan(x/2)
^26)/9261 - (158194324*tan(x/2)^27)/3969 + (409533176*tan(x/2)^28)/27783 +
 (27273262*tan(x/2)^29)/27783 + 15171568/27783)/(12*tan(x/2) - 87*tan(x/2)
^2 - 992*tan(x/2)^3 + 4737*tan(x/2)^4 + 27564*tan(x/2)^5 - 146823*tan(x/2)
^6 - 172416*tan(x/2)^7 + 1486293*tan(x/2)^8 + 295676*tan(x/2)^9 - 7495683*
tan(x/2)^10 + 347808*tan(x/2)^11 + 21580005*tan(x/2)^12 - 1024932*tan(x/2)
^13 - 36395235*tan(x/2)^14 + 36395235*tan(x/2)^16 + 1024932*tan(x/2)^17 -
21580005*tan(x/2)^18 - 347808*tan(x/2)^19 + 7495683*tan(x/2)^20 - 295676*t
an(x/2)^21 - 1486293*tan(x/2)^22 + 172416*tan(x/2)^23 + 146823*tan(x/2)...
```

Reduce [F]

$$\int \frac{1}{(\cos(5x) + \sin(2x))^4} dx$$
$$= \int \frac{1}{\cos(5x)^4 + 4\cos(5x)^3 \sin(2x) + 6\cos(5x)^2 \sin(2x)^2 + 4\cos(5x)\sin(2x)^3 + \sin(2x)^4} dx$$

input
```
int(1/(cos(5*x)+sin(2*x))^4,x)
```

output
```
int(1/(cos(5*x)**4 + 4*cos(5*x)**3*sin(2*x) + 6*cos(5*x)**2*sin(2*x)**2 +
4*cos(5*x)*sin(2*x)**3 + sin(2*x)**4),x)
```

3.12 $\int \frac{1}{(\cos(5x)+\sin(2x))^6}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 218

$$\begin{aligned}\int \frac{1}{(\cos(5x)+\sin(2x))^6}\,dx &= -\frac{33243}{128}\text{arctanh}(2\cos(x)\sin(x)) - \frac{715\cot(x)}{139968} \\ &\quad - \frac{11\cot^3(x)}{69984} - \frac{\cot^5(x)}{233280} - \frac{109312\log\left(\sqrt{3}\cos(x)-\sin(x)\right)}{243\sqrt{3}} \\ &\quad + \frac{109312\log\left(\sqrt{3}\cos(x)+\sin(x)\right)}{243\sqrt{3}} \\ &\quad + \frac{32\tan(x)\left(16627-16465\tan^2(x)\right)}{405\left(3-4\tan^2(x)+\tan^4(x)\right)^5} \\ &\quad - \frac{2\tan(x)\left(76795+61567\tan^2(x)\right)}{405\left(3-4\tan^2(x)+\tan^4(x)\right)^4} \\ &\quad + \frac{\tan(x)\left(778363-606643\tan^2(x)\right)}{2430\left(3-4\tan^2(x)+\tan^4(x)\right)^3} \\ &\quad - \frac{\tan(x)\left(91032631-43437157\tan^2(x)\right)}{174960\left(3-4\tan^2(x)+\tan^4(x)\right)^2} \\ &\quad + \frac{\tan(x)\left(111184863-53226455\tan^2(x)\right)}{139968\left(3-4\tan^2(x)+\tan^4(x)\right)}\end{aligned}$$

output
```
-33243/128*arctanh(2*cos(x)*sin(x))-715/139968*cot(x)-11/69984*cot(x)^3-1/
233280*cot(x)^5-109312/729*ln(3^(1/2)*cos(x)-sin(x))*3^(1/2)+109312/729*ln
(3^(1/2)*cos(x)+sin(x))*3^(1/2)+32/405*tan(x)*(16627-16465*tan(x)^2)/(3-4*
tan(x)^2+tan(x)^4)^5-2/405*tan(x)*(76795+61567*tan(x)^2)/(3-4*tan(x)^2+tan
(x)^4)^4+1/2430*tan(x)*(778363-606643*tan(x)^2)/(3-4*tan(x)^2+tan(x)^4)^3-
1/174960*tan(x)*(91032631-43437157*tan(x)^2)/(3-4*tan(x)^2+tan(x)^4)^2+tan
(x)*(111184863-53226455*tan(x)^2)/(419904-559872*tan(x)^2+139968*tan(x)^4)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.11 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.19

$$\int \frac{1}{(\cos(5x)+\sin(2x))^6}\,dx$$

$$= \frac{17013137182720\sqrt{3}\text{arctanh}\left(\frac{-2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right) - 699840i\text{RootSum}\left[i + \#1 - i\#1^2 - \#1^3 + i\#1^4 + \#1^5 - i\#\right.}{}$$

input
```
Integrate[(Cos[5*x] + Sin[2*x])^(-6),x]
```

output
```
(17013137182720*Sqrt[3]*ArcTanh[(-2 + Tan[x/2])/Sqrt[3]] - (699840*I)*Root
Sum[I + #1 - I*#1^2 - #1^3 + I*#1^4 + #1^5 - I*#1^6 & , (32920146*ArcTan[S
in[x]/(Cos[x] - #1)] - (16460073*I)*Log[1 - 2*Cos[x]*#1 + #1^2] - (7397036
2*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 - 36985181*Log[1 - 2*Cos[x]*#1 + #1^2
]*#1 - 92239970*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 + (46119985*I)*Log[1 - 2
*Cos[x]*#1 + #1^2]*#1^2 + (73970362*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 +
 36985181*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3 + 32920146*ArcTan[Sin[x]/(Cos[x
] - #1)]*#1^4 - (16460073*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^4)/(I + 2*#1 -
 (3*I)*#1^2 - 4*#1^3 + (5*I)*#1^4 + 6*#1^5) & ] + (3*Sec[x]^5*(-1921829222
123 + 617908487785*Cos[2*x] + 2410067577070*Cos[4*x] + 832867940630*Cos[6*
x] + 640082095910*Cos[8*x] - 1775875210157*Cos[10*x] - 442786557220*Cos[12
*x] + 1152877214505*Cos[14*x] + 71920822420*Cos[16*x] - 31579364460*Cos[18
*x] - 153758708336*Cos[20*x] - 140817863340*Cos[22*x] + 51684808280*Cos[24
*x] + 128408941550*Sin[x] + 2897605989350*Sin[3*x] + 785145965921*Sin[5*x]
 - 2882604224865*Sin[7*x] + 141794432890*Sin[9*x] + 898151545730*Sin[11*x]
 + 627827561740*Sin[13*x] + 550510980460*Sin[15*x] - 519369889645*Sin[17*x
] - 95260796820*Sin[19*x] + 192101628320*Sin[21*x] + 33425806160*Sin[23*x]
 + 45921631644*Sin[25*x]))/(1 - 2*Cos[2*x] + 2*Cos[4*x] + 2*Sin[x])^5)/612
61515
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(2x)+\cos(5x))^6} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(\sin(2x)+\cos(5x))^6} dx$$

$$\Big\downarrow \; 4830$$

$$2\int \frac{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^{29}}{\left(-\tan^{10}\left(\frac{x}{2}\right)-4\tan^9\left(\frac{x}{2}\right)+45\tan^8\left(\frac{x}{2}\right)-8\tan^7\left(\frac{x}{2}\right)-210\tan^6\left(\frac{x}{2}\right)+210\tan^4\left(\frac{x}{2}\right)+8\tan^3\left(\frac{x}{2}\right)-45\tan^2\left(\right.\right.}$$

$$\Big\downarrow \; 2462$$

$$2\int\left(\frac{65536\left(209\tan\left(\frac{x}{2}\right)-56\right)}{27\left(\tan^2\left(\frac{x}{2}\right)-4\tan\left(\frac{x}{2}\right)+1\right)^6}-\frac{768522368}{2187\left(\tan^2\left(\frac{x}{2}\right)-4\tan\left(\frac{x}{2}\right)+1\right)}+\frac{128\left(322988095\tan^4\left(\frac{x}{2}\right)+37251206\right.}{117649\left(\tan^6\left(\frac{x}{2}\right)+8\tan^5\right.}\right.$$

$\downarrow$ 2009

$$2\left(\frac{50613248\log\left(-\tan\left(\frac{x}{2}\right)-\sqrt{3}+2\right)}{243\sqrt{3}}-\frac{50613248\log\left(-\tan\left(\frac{x}{2}\right)+\sqrt{3}+2\right)}{243\sqrt{3}}+\frac{123396853066885034874712883}{21}\right.$$

input
```
Int[(Cos[5*x] + Sin[2*x])^(-6),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4830
```
Int[(cos[(n_.)*((c_.) + (d_.)*(x_))]*(b_.) + (a_.)*sin[(m_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[2/d   Subst[Int[Simplify[TrigExpand[a*Sin[
2*m*ArcTan[x]] + b*Cos[2*n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[(1/2)*(c +
d*x)]], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && Intege
rQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.86 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.68

Expression too large to display

input
```
int(1/(cos(5*x)+sin(2*x))^6,x)
```

output
```
-1/3645/(tan(1/2*x)+1)^5+1/1458/(tan(1/2*x)+1)^4-163/8748/(tan(1/2*x)+1)^3
+53/1944/(tan(1/2*x)+1)^2-5093/4374/(tan(1/2*x)+1)+256/117649*(1139608324*
tan(1/2*x)+181317662625834*tan(1/2*x)^8+111701990444896/3*tan(1/2*x)^7-213
633013222/3*tan(1/2*x)^4-56009237441236/15*tan(1/2*x)^5-30410542112408/3*t
an(1/2*x)^6+324943703648/3*tan(1/2*x)^3+17681293036*tan(1/2*x)^2+930083521
2690970/3*tan(1/2*x)^12-3568418391473792/3*tan(1/2*x)^11-6092129246546524/
5*tan(1/2*x)^10-53732817434796*tan(1/2*x)^9+19856315688316396/3*tan(1/2*x)
^13+1262608900769456/3*tan(1/2*x)^14-48148710706309824/5*tan(1/2*x)^15-278
56977579524294/3*tan(1/2*x)^16-2342101723253980/3*tan(1/2*x)^17+1003157840
5774460/3*tan(1/2*x)^18+4606807164518240/3*tan(1/2*x)^19+136727106/5-32260
0338203844*tan(1/2*x)^21-1344782502216014/5*tan(1/2*x)^20-45791476*tan(1/2
*x)^29-1957675846*tan(1/2*x)^28-95242165568/3*tan(1/2*x)^27-650619023804/3
*tan(1/2*x)^26-407268246316/3*tan(1/2*x)^25+17860224325562/3*tan(1/2*x)^24
+69630966616544/3*tan(1/2*x)^23-33601935843720*tan(1/2*x)^22)/(tan(1/2*x)^
6+8*tan(1/2*x)^5-13*tan(1/2*x)^4-48*tan(1/2*x)^3-13*tan(1/2*x)^2+8*tan(1/2
*x)+1)^5+384/16807*sum((13199839*_R^4+147940724*_R^3+289760846*_R^2+147940
724*_R+13199839)/(3*_R^5+20*_R^4-26*_R^3-72*_R^2-13*_R+4)*ln(tan(1/2*x)-_R
),_R=RootOf(_Z^6+8*_Z^5-13*_Z^4-48*_Z^3-13*_Z^2+8*_Z+1))-1/588245/(tan(1/2
*x)-1)^5-1/235298/(tan(1/2*x)-1)^4-83/1411788/(tan(1/2*x)-1)^3-79/941192/(
tan(1/2*x)-1)^2-389/235298/(tan(1/2*x)-1)-256/2187*(1113407*tan(1/2*x)^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(\cos(5x)+\sin(2x))^6}\,dx = \text{Timed out}$$

input
```
integrate(1/(cos(5*x)+sin(2*x))^6,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(\cos(5x) + \sin(2x))^6} \, dx = \text{Timed out}$$

input
```
integrate(1/(cos(5*x)+sin(2*x))**6,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(5x) + \sin(2x))^6} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(cos(5*x)+sin(2*x))^6,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(188) = 376$.

Time = 0.17 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.56

$$\int \frac{1}{(\cos(5x) + \sin(2x))^6} \, dx = \text{Too large to display}$$

input
```
integrate(1/(cos(5*x)+sin(2*x))^6,x, algorithm="giac")
```

output
```
-101226496/729*sqrt(3)*log(abs(-2*sqrt(3) + 2*tan(1/2*x) - 4)/abs(2*sqrt(3
) + 2*tan(1/2*x) - 4)) - 2/20420505*(2348072128025*tan(1/2*x)^49 + 5409679
0137800*tan(1/2*x)^48 + 13549152904760*tan(1/2*x)^47 - 6772739536801060*ta
n(1/2*x)^46 - 16732042947484412*tan(1/2*x)^45 + 413382950827430660*tan(1/2
*x)^44 + 860079438673998120*tan(1/2*x)^43 - 15446974901266156660*tan(1/2*x
)^42 - 12771099815592707870*tan(1/2*x)^41 + 346843324808711498884*tan(1/2*
x)^40 - 128616301300448545880*tan(1/2*x)^39 - 4261744436703269881340*tan(1
/2*x)^38 + 4087003099790949762260*tan(1/2*x)^37 + 32321841562360517280860*
tan(1/2*x)^36 - 39300124477699353280392*tan(1/2*x)^35 - 166464525360402634
977420*tan(1/2*x)^34 + 210043995965200725907815*tan(1/2*x)^33 + 6152633010
71302396476740*tan(1/2*x)^32 - 712900843785776889116240*tan(1/2*x)^31 - 16
68740428753947443177832*tan(1/2*x)^30 + 1622442220962425618684040*tan(1/2*
x)^29 + 3324257502931363090223400*tan(1/2*x)^28 - 255948656991511101952408
0*tan(1/2*x)^27 - 4839760037792021284833800*tan(1/2*x)^26 + 28751551212638
24099638940*tan(1/2*x)^25 + 5128039883029583065246440*tan(1/2*x)^24 - 2362
769526621783819276080*tan(1/2*x)^23 - 3940075773402854604723320*tan(1/2*x)
^22 + 1464953473634168374848840*tan(1/2*x)^21 + 2183930134431309922808888*
tan(1/2*x)^20 - 706814016978668423208080*tan(1/2*x)^19 - 86501756106011011
0030360*tan(1/2*x)^18 + 269100700664240373408375*tan(1/2*x)^17 + 240215839
105178851609040*tan(1/2*x)^16 - 790609482275616099921640*tan(1/2*x)^15 -...
```

Mupad [B] (verification not implemented)

Time = 36.40 (sec) , antiderivative size = 1478, normalized size of antiderivative = 6.78

$$\int \frac{1}{(\cos(5x)+\sin(2x))^6}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(5*x) + sin(2*x))^6,x)
```

output
```
symsum(log(1247391969916038497823910618605161911090410346908745728 - root(
z^6 - (114375530107022229504*z^4)/1977326743 + (86370274106142777748767280
98816*z^2)/27368747340080916343 - 57301151339511992506470305855843598336/3
78818692265664781682717625943, z, k)*(585418123911245063659126011685586187
73539640919797530624*exp(x*1i) - root(z^6 - (114375530107022229504*z^4)/19
77326743 + (8637027410614277774876728098816*z^2)/27368747340080916343 - 57
301151339511992506470305855843598336/378818692265664781682717625943, z, k)
*(exp(x*1i)*2940635903824389143597992720127636661043972641128448i + root(z
^6 - (114375530107022229504*z^4)/1977326743 + (863702741061427777487672809
8816*z^2)/27368747340080916343 - 57301151339511992506470305855843598336/37
8818692265664781682717625943, z, k)*(1516893653059746760696739190383151125
8935529652210368512*exp(x*1i) - root(z^6 - (114375530107022229504*z^4)/197
7326743 + (8637027410614277774876728098816*z^2)/27368747340080916343 - 573
01151339511992506470305855843598336/378818692265664781682717625943, z, k)*
(exp(x*1i)*538997260307260682351482578655848285668872040742912i - root(z^6
 - (114375530107022229504*z^4)/1977326743 + (86370274106142777748767280988
16*z^2)/27368747340080916343 - 57301151339511992506470305855843598336/3788
18692265664781682717625943, z, k)*(root(z^6 - (114375530107022229504*z^4)/
1977326743 + (8637027410614277774876728098816*z^2)/27368747340080916343 -
57301151339511992506470305855843598336/378818692265664781682717625943, ...
```

Reduce [F]

$$\int \frac{1}{(\cos(5x)+\sin(2x))^6}\,dx$$
$$= \int \frac{1}{\cos(5x)^6 + 6\cos(5x)^5\sin(2x) + 15\cos(5x)^4\sin(2x)^2 + 20\cos(5x)^3\sin(2x)^3 + 15\cos(5x)^2\sin(2x)^4}$$

input
```
int(1/(cos(5*x)+sin(2*x))^6,x)
```

output
```
int(1/(cos(5*x)**6 + 6*cos(5*x)**5*sin(2*x) + 15*cos(5*x)**4*sin(2*x)**2 +
 20*cos(5*x)**3*sin(2*x)**3 + 15*cos(5*x)**2*sin(2*x)**4 + 6*cos(5*x)*sin(
2*x)**5 + sin(2*x)**6),x)
```

3.13 $\int \frac{1}{\cos(5x)+\sin(4x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \frac{1}{\cos(5x)+\sin(4x)}\,dx = -\frac{1}{8}\text{arctanh}(\cos(x)) + \frac{\text{arctanh}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}} - \frac{\sec(x)}{8}$$

output
```
-1/8*arctanh(cos(x))+1/4*arctanh(cos(x)*2^(1/2))*2^(1/2)-1/8*sec(x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 3.44

$$\int \frac{1}{\cos(5x)+\sin(4x)}\,dx = \frac{1}{9}\left(-6\log\left(\sec^2\left(\frac{x}{2}\right)\right) - 9\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right) + \log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right) - 2\log(1-2\sin(x)) + 192\text{RootSum}\left[1-27648\#1^2 + 884736\#1^3\&, \log\left(-\sec^2\left(\frac{x}{2}\right)\left(-1-4\sin(x)-384\sin(x)\#1+18432\sin(x)\#1^2\right)\right)\#1\&\right]\right)$$

input
```
Integrate[(Cos[5*x] + Sin[4*x])^(-1),x]
```

output
```
(-6*Log[Sec[x/2]^2] - 9*Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]
] - 2*Log[1 - 2*Sin[x]] + 192*RootSum[1 - 27648*#1^2 + 884736*#1^3 & , Log
[-(Sec[x/2]^2*(-1 - 4*Sin[x] - 384*Sin[x]*#1 + 18432*Sin[x]*#1^2))]*#1 & ]
)/9
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.35 (sec) , antiderivative size = 679, normalized size of antiderivative = 21.22, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4829, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin(4x)+\cos(5x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(4x)+\cos(5x)}dx$$

$$\Big\downarrow \text{4829}$$

$$\int \frac{1}{(1-\sin^2(x))\left(16\sin^4(x)-8\sin^3(x)-12\sin^2(x)+4\sin(x)+1\right)}d\sin(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{16\sin(x)(\sin(x)+1)}{3\left(8\sin^3(x)-6\sin(x)-1\right)}-\frac{1}{2(\sin(x)-1)}-\frac{4}{9(2\sin(x)-1)}+\frac{1}{18(\sin(x)+1)}\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\,2^{2/3}(-\sqrt{3}+i)\left(2^{2/3}-(1+i\sqrt{3})^{2/3}\right)\operatorname{arctanh}\left(\frac{8\left(1+i\sqrt{3}\right)^{2/3}\sin(x)+2\sqrt[3]{2+2i\sqrt{3}}+2^{2/3}\left(1+i\sqrt{3}\right)}{2\sqrt{3\left(\sqrt[3]{2}-i\sqrt[3]{2}\sqrt{3}+2\left(1+i\sqrt{3}\right)^{4/3}-\left(2+2i\sqrt{3}\right)^{2/3}\right)}}\right)}{3\left(2\,2^{2/3}+2\,(1+i\sqrt{3})^{2/3}+\sqrt[3]{2}\,(1+i\sqrt{3})^{4/3}\right)\sqrt{\sqrt[3]{2}-i\sqrt[3]{2}\sqrt{3}+2\,(1+i\sqrt{3})^{4/3}-(2+2i\sqrt{3})^{2/3}}}+$$

$$\frac{2}{9}\log\left(8\sin^3(x)-6\sin(x)-1\right)-$$

$$\frac{2\sqrt[3]{1+i\sqrt{3}}\left(2\sqrt[3]{2}+\sqrt[3]{1+i\sqrt{3}}+(2+2i\sqrt{3})^{2/3}\right)\log\left(8(1+i\sqrt{3})^{2/3}\sin^2(x)+2\,2^{2/3}\left(1+i\sqrt{3}+2^{2/3}\sqrt[3]{1+i\sqrt{}}\right.\right.}{9\left(2\,2^{2/3}+2\,(1+i\sqrt{3})^{2/3}+\sqrt[3]{2}\,(1+i\sqrt{3})^{4/3}\right)}$$

$$\frac{2}{9}\log(1-2\sin(x))-\frac{1}{2}\log(1-\sin(x))+\frac{1}{18}\log(\sin(x)+1)+$$

$$\frac{4\sqrt[3]{1+i\sqrt{3}}\left(2\sqrt[3]{2}+\sqrt[3]{1+i\sqrt{3}}+(2+2i\sqrt{3})^{2/3}\right)\log\left(-4\sqrt[3]{1+i\sqrt{3}}\sin(x)+(2+2i\sqrt{3})^{2/3}+2\sqrt[3]{2}\right)}{9\left(2\,2^{2/3}+2\,(1+i\sqrt{3})^{2/3}+\sqrt[3]{2}\,(1+i\sqrt{3})^{4/3}\right)}$$

input
```
Int[(Cos[5*x] + Sin[4*x])^(-1),x]
```

output
```
(-2*2^(2/3)*(I - Sqrt[3])*(2^(2/3) - (1 + I*Sqrt[3])^(2/3))*ArcTanh[(2^(2/
3)*(1 + I*Sqrt[3]) + 2*(2 + (2*I)*Sqrt[3])^(1/3) + 8*(1 + I*Sqrt[3])^(2/3)
*Sin[x])/(2*Sqrt[3*(2^(1/3) - I*2^(1/3)*Sqrt[3] + 2*(1 + I*Sqrt[3])^(4/3)
- (2 + (2*I)*Sqrt[3])^(2/3))])])/(3*(2*2^(2/3) + 2*(1 + I*Sqrt[3])^(2/3) +
 2^(1/3)*(1 + I*Sqrt[3])^(4/3))*Sqrt[2^(1/3) - I*2^(1/3)*Sqrt[3] + 2*(1 +
I*Sqrt[3])^(4/3) - (2 + (2*I)*Sqrt[3])^(2/3)]) - (2*Log[1 - 2*Sin[x]])/9 -
 Log[1 - Sin[x]]/2 + Log[1 + Sin[x]]/18 + (4*(1 + I*Sqrt[3])^(1/3)*(2*2^(1
/3) + (1 + I*Sqrt[3])^(1/3) + (2 + (2*I)*Sqrt[3])^(2/3))*Log[2*2^(1/3) + (
2 + (2*I)*Sqrt[3])^(2/3) - 4*(1 + I*Sqrt[3])^(1/3)*Sin[x]])/(9*(2*2^(2/3)
+ 2*(1 + I*Sqrt[3])^(2/3) + 2^(1/3)*(1 + I*Sqrt[3])^(4/3))) - (2*(1 + I*Sq
rt[3])^(1/3)*(2*2^(1/3) + (1 + I*Sqrt[3])^(1/3) + (2 + (2*I)*Sqrt[3])^(2/3
))*Log[2*2^(2/3) - 2*(1 + I*Sqrt[3])^(2/3) + 2^(1/3)*(1 + I*Sqrt[3])^(4/3)
 + 2*2^(2/3)*(1 + I*Sqrt[3] + 2^(2/3)*(1 + I*Sqrt[3])^(1/3))*Sin[x] + 8*(1
 + I*Sqrt[3])^(2/3)*Sin[x]^2])/(9*(2*2^(2/3) + 2*(1 + I*Sqrt[3])^(2/3) + 2
^(1/3)*(1 + I*Sqrt[3])^(4/3))) + (2*Log[-1 - 6*Sin[x] + 8*Sin[x]^3])/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4829
```
Int[(cos[(n_.)*((c_.) + (d_.)*(x_))]*(b_.) + (a_.)*sin[(m_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Sin[
m*ArcSin[x]] + b*Cos[n*ArcSin[x]]]]^p/Sqrt[1 - x^2], x], x, Sin[c + d*x]],
x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[m/2] && Inte
gerQ[(n - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.95 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.94

method	result
default	$-\frac{\ln(\sin(x)-1)}{2}-\frac{2\ln(2\sin(x)-1)}{9}+\frac{\ln(1+\sin(x))}{18}+\frac{8\left(\sum_{\_R=\mathrm{RootOf}\left(8\_Z^3-6\_Z-1\right)}\frac{\left(\_R+1\right)\_R\ln\left(\sin(x)-\_R\right)}{4\_R^2-1}\right)}{9}$
risch	$-\ln\left(\mathrm{e}^{ix}-i\right)+\frac{\ln(\mathrm{e}^{ix}+i)}{9}-\frac{2\ln\left(-i\mathrm{e}^{ix}+\mathrm{e}^{2ix}-1\right)}{9}+\left(\sum_{\_R=\mathrm{RootOf}\left(729\_Z^3-486\_Z^2+8\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(-\frac{9}{2}i\_$

input
```
int(1/(cos(5*x)+sin(4*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(sin(x)-1)-2/9*ln(2*sin(x)-1)+1/18*ln(1+sin(x))+8/9*sum((_R+1)*_R/(
4*_R^2-1)*ln(sin(x)-_R),_R=RootOf(8*_Z^3-6*_Z-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 608, normalized size of antiderivative = 19.00

$$\int \frac{1}{\cos(5x) + \sin(4x)} \, dx = \text{Too large to display}$$

input
```
integrate(1/(cos(5*x)+sin(4*x)),x, algorithm="fricas")
```

output
```
1/324*(81*(4/729*I*sqrt(3) + 4/729)^(1/3)*(-I*sqrt(3) + 1) + 4*(I*sqrt(3)
+ 1)/(4/729*I*sqrt(3) + 4/729)^(1/3) + 54*sqrt(-1/972*(81*(4/729*I*sqrt(3)
 + 4/729)^(1/3)*(-I*sqrt(3) + 1) + 4*(I*sqrt(3) + 1)/(4/729*I*sqrt(3) + 4/
729)^(1/3) - 36)^2 - 6*(4/729*I*sqrt(3) + 4/729)^(1/3)*(-I*sqrt(3) + 1) -
8/27*(I*sqrt(3) + 1)/(4/729*I*sqrt(3) + 4/729)^(1/3) + 20/3) + 72)*log(9/2
*(4/729*I*sqrt(3) + 4/729)^(1/3)*(-I*sqrt(3) + 1) + 2/9*(I*sqrt(3) + 1)/(4
/729*I*sqrt(3) + 4/729)^(1/3) + 3*sqrt(-1/972*(81*(4/729*I*sqrt(3) + 4/729
)^(1/3)*(-I*sqrt(3) + 1) + 4*(I*sqrt(3) + 1)/(4/729*I*sqrt(3) + 4/729)^(1/
3) - 36)^2 - 6*(4/729*I*sqrt(3) + 4/729)^(1/3)*(-I*sqrt(3) + 1) - 8/27*(I*
sqrt(3) + 1)/(4/729*I*sqrt(3) + 4/729)^(1/3) + 20/3) - 8*sin(x)) + 1/324*(
81*(4/729*I*sqrt(3) + 4/729)^(1/3)*(-I*sqrt(3) + 1) + 4*(I*sqrt(3) + 1)/(4
/729*I*sqrt(3) + 4/729)^(1/3) - 54*sqrt(-1/972*(81*(4/729*I*sqrt(3) + 4/72
9)^(1/3)*(-I*sqrt(3) + 1) + 4*(I*sqrt(3) + 1)/(4/729*I*sqrt(3) + 4/729)^(1
/3) - 36)^2 - 6*(4/729*I*sqrt(3) + 4/729)^(1/3)*(-I*sqrt(3) + 1) - 8/27*(I
*sqrt(3) + 1)/(4/729*I*sqrt(3) + 4/729)^(1/3) + 20/3) + 72)*log(-9/2*(4/72
9*I*sqrt(3) + 4/729)^(1/3)*(-I*sqrt(3) + 1) - 2/9*(I*sqrt(3) + 1)/(4/729*I
*sqrt(3) + 4/729)^(1/3) + 3*sqrt(-1/972*(81*(4/729*I*sqrt(3) + 4/729)^(1/3
)*(-I*sqrt(3) + 1) + 4*(I*sqrt(3) + 1)/(4/729*I*sqrt(3) + 4/729)^(1/3) - 3
6)^2 - 6*(4/729*I*sqrt(3) + 4/729)^(1/3)*(-I*sqrt(3) + 1) - 8/27*(I*sqrt(3
) + 1)/(4/729*I*sqrt(3) + 4/729)^(1/3) + 20/3) + 8*sin(x)) - 1/162*(81*...
```

Sympy [F]

$$\int \frac{1}{\cos(5x)+\sin(4x)}\,dx=\int \frac{1}{\sin{(4x)}+\cos{(5x)}}\,dx$$

input
```
integrate(1/(cos(5*x)+sin(4*x)),x)
```

output
```
Integral(1/(sin(4*x) + cos(5*x)), x)
```

Maxima [F]

$$\int \frac{1}{\cos(5x)+\sin(4x)}\,dx=\int \frac{1}{\cos{(5\,x)}+\sin{(4\,x)}}\,dx$$

input
```
integrate(1/(cos(5*x)+sin(4*x)),x, algorithm="maxima")
```

output
```
4/9*integrate((cos(6*x)*cos(3*x) - cos(3*x)*cos(8/3*arctan2(sin(3*x), cos(
3*x))) - cos(3*x)*cos(4/3*arctan2(sin(3*x), cos(3*x))) + cos(3*x)*cos(2/3*
arctan2(sin(3*x), cos(3*x))) + cos(7/3*arctan2(sin(3*x), cos(3*x)))*sin(3*
x) - cos(5/3*arctan2(sin(3*x), cos(3*x)))*sin(3*x) - cos(1/3*arctan2(sin(3
*x), cos(3*x)))*sin(3*x) + sin(6*x)*sin(3*x) - sin(3*x)*sin(8/3*arctan2(si
n(3*x), cos(3*x))) - cos(3*x)*sin(7/3*arctan2(sin(3*x), cos(3*x))) + cos(3
*x)*sin(5/3*arctan2(sin(3*x), cos(3*x))) - sin(3*x)*sin(4/3*arctan2(sin(3*
x), cos(3*x))) + sin(3*x)*sin(2/3*arctan2(sin(3*x), cos(3*x))) + cos(3*x)*
sin(1/3*arctan2(sin(3*x), cos(3*x))) - cos(3*x))/(2*(sin(3*x) + 1)*cos(6*x
) - cos(6*x)^2 - cos(3*x)^2 + 2*(cos(6*x) - cos(4/3*arctan2(sin(3*x), cos(
3*x))) + cos(2/3*arctan2(sin(3*x), cos(3*x))) - sin(3*x) - sin(7/3*arctan2
(sin(3*x), cos(3*x))) + sin(5/3*arctan2(sin(3*x), cos(3*x))) + sin(1/3*arc
tan2(sin(3*x), cos(3*x))) - 1)*cos(8/3*arctan2(sin(3*x), cos(3*x))) - cos(
8/3*arctan2(sin(3*x), cos(3*x)))^2 - 2*(cos(3*x) - cos(5/3*arctan2(sin(3*x
), cos(3*x))) - cos(1/3*arctan2(sin(3*x), cos(3*x))) + sin(6*x) - sin(4/3*
arctan2(sin(3*x), cos(3*x))) + sin(2/3*arctan2(sin(3*x), cos(3*x))))*cos(7
/3*arctan2(sin(3*x), cos(3*x))) - cos(7/3*arctan2(sin(3*x), cos(3*x)))^2 +
 2*(cos(3*x) - cos(1/3*arctan2(sin(3*x), cos(3*x))) + sin(6*x) - sin(4/3*a
rctan2(sin(3*x), cos(3*x))) + sin(2/3*arctan2(sin(3*x), cos(3*x))))*cos(5/
3*arctan2(sin(3*x), cos(3*x))) - cos(5/3*arctan2(sin(3*x), cos(3*x)))^2...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\cos(5x) + \sin(4x)}\, dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(1/(cos(5*x)+sin(4*x)),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: 2*(-
1/4*ln(-sin(sageVARx)+1)+1/36*ln(sin(sageVARx)+1)-1/9*ln(abs(2*sin(sageVAR
x)-1))+((-1/98304*rootof([[-3,0,46080,0,-113246208],[1,0,-18432,0,84934656
,0,-86973087744]])+
```

Mupad [B] (verification not implemented)

Time = 23.30 (sec) , antiderivative size = 507, normalized size of antiderivative = 15.84

$$\int \frac{1}{\cos(5x)+\sin(4x)}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(5*x) + sin(4*x)),x)
```

output
```
log(tan(x/2) + 1)/9 - log(tan(x/2) - 1) - (2*log(tan(x/2)^2 - 4*tan(x/2) +
 1))/9 + symsum(log((562949953421312*(2928640*root(z^3 - (2*z^2)/3 + 8/729
, z, k) - 179088*cos(x) - 23296*sin(x) - 3744936*root(z^3 - (2*z^2)/3 + 8/
729, z, k)*cos(x) + 5983744*root(z^3 - (2*z^2)/3 + 8/729, z, k)*sin(x) - 1
40095872*root(z^3 - (2*z^2)/3 + 8/729, z, k)^2 - 109095660*root(z^3 - (2*z
^2)/3 + 8/729, z, k)^3 + 11742411969*root(z^3 - (2*z^2)/3 + 8/729, z, k)^4
 - 7810568694*root(z^3 - (2*z^2)/3 + 8/729, z, k)^5 - 244216507572*root(z^
3 - (2*z^2)/3 + 8/729, z, k)^6 + 20246839218*root(z^3 - (2*z^2)/3 + 8/729,
 z, k)^7 + 530216028495*root(z^3 - (2*z^2)/3 + 8/729, z, k)^8 + 140962140*
root(z^3 - (2*z^2)/3 + 8/729, z, k)^2*cos(x) + 132253884*root(z^3 - (2*z^2
)/3 + 8/729, z, k)^3*cos(x) - 12897031329*root(z^3 - (2*z^2)/3 + 8/729, z,
 k)^4*cos(x) + 10366524342*root(z^3 - (2*z^2)/3 + 8/729, z, k)^5*cos(x) +
275492873304*root(z^3 - (2*z^2)/3 + 8/729, z, k)^6*cos(x) - 42376042458*ro
ot(z^3 - (2*z^2)/3 + 8/729, z, k)^7*cos(x) - 572478342579*root(z^3 - (2*z^
2)/3 + 8/729, z, k)^8*cos(x) + 14119866*root(z^3 - (2*z^2)/3 + 8/729, z, k
)^2*sin(x) - 732819960*root(z^3 - (2*z^2)/3 + 8/729, z, k)^3*sin(x) - 2398
555071*root(z^3 - (2*z^2)/3 + 8/729, z, k)^4*sin(x) + 19796177250*root(z^3
 - (2*z^2)/3 + 8/729, z, k)^5*sin(x) + 98538963534*root(z^3 - (2*z^2)/3 +
8/729, z, k)^6*sin(x) + 25019179398*root(z^3 - (2*z^2)/3 + 8/729, z, k)^7*
sin(x) - 337338020601*root(z^3 - (2*z^2)/3 + 8/729, z, k)^8*sin(x) + 30...
```

Reduce [F]

$$\int \frac{1}{\cos(5x)+\sin(4x)}\,dx = \int \frac{1}{\cos(5x)+\sin(4x)}dx$$

input
```
int(1/(cos(5*x)+sin(4*x)),x)
```

output
```
int(1/(cos(5*x) + sin(4*x)),x)
```

3.14 $\int \frac{1}{(\cos(5x)+\sin(4x))^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 92

$$\int \frac{1}{(\cos(5x)+\sin(4x))^3}\,dx = -\frac{19\text{arctanh}(\cos(x))}{1024}+\frac{31\text{arctanh}\left(\sqrt{2}\cos(x)\right)}{128\sqrt{2}} - \frac{105\sec(x)}{1024}-\frac{43\sec^3(x)}{3072}-\frac{3\sec^5(x)}{1280}+\frac{\sec^5(x)\sec(2x)}{1024} + \frac{1}{256}\sec^5(x)\sec^2(2x)-\frac{\csc^2(x)\sec^5(x)\sec^2(2x)}{1024}$$

output

```
-19/1024*arctanh(cos(x))+31/256*arctanh(cos(x)*2^(1/2))*2^(1/2)-105/1024*s
ec(x)-43/3072*sec(x)^3-3/1280*sec(x)^5+1/1024*sec(x)^5*sec(2*x)+1/256*sec(
x)^5*sec(2*x)^2-1/1024*csc(x)^2*sec(x)^5*sec(2*x)^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.39 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.42

$$\int \frac{1}{(\cos(5x)+\sin(4x))^3}\,dx = \frac{-88938\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+42\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-480\log(1-2\sin(x))-1152(39\log\left(\sec^2\right.}{}$$

input
```
Integrate[(Cos[5*x] + Sin[4*x])^(-3),x]
```

output
```
(-88938*Log[Cos[x/2] - Sin[x/2]] + 42*Log[Cos[x/2] + Sin[x/2]] - 480*Log[1
 - 2*Sin[x]] - 1152*(39*Log[Sec[x/2]^2] - 2*RootSum[269 + 5976*#1 - 269568
*#1^2 + 13824*#1^3 & , Log[-(Sec[x/2]^2*(-39007 - 109078*Sin[x] - 2595120*
Sin[x]*#1 + 133632*Sin[x]*#1^2))]*#1 & ]) + 729/(Cos[x/2] - Sin[x/2])^2 -
(Cos[x/2] + Sin[x/2])^(-2) + 48/(1 - 2*Sin[x])^2 + 128/(-1 + 2*Sin[x]) + (
864*(-4 + 3*Cos[2*x] - 6*Sin[x]))/(1 + 2*Sin[3*x])^2 - (288*(-35 + 26*Cos[
2*x] - 50*Sin[x]))/(1 + 2*Sin[3*x]))/2916
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(4x)+\cos(5x))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(4x)+\cos(5x))^3}dx$$

$$\Big\downarrow \text{4829}$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^2\left(16\sin^4(x)-8\sin^3(x)-12\sin^2(x)+4\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{32\left(16\sin^2(x)+15\sin(x)+2\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^3}+\frac{64\left(52\sin^2(x)+51\sin(x)+11\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{16\left(140\sin^2(x)+138\sin(x)+31\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)^2}-\frac{}{4(}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(1-2\sin(x))^3\left(1-\sin^2(x)\right)^2\left(-8\sin^3(x)+6\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{32\left(16\sin^2(x)+15\sin(x)+2\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^3}+\frac{64\left(52\sin^2(x)+51\sin(x)+11\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{16\left(140\sin^2(x)+138\sin(x)+31\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)^2}-\frac{}{4(}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(1-2\sin(x))^3\left(1-\sin^2(x)\right)^2\left(-8\sin^3(x)+6\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{32\left(16\sin^2(x)+15\sin(x)+2\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^3}+\frac{64\left(52\sin^2(x)+51\sin(x)+11\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{16\left(140\sin^2(x)+138\sin(x)+31\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)^2}-\frac{}{4(}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(1-2\sin(x))^3\left(1-\sin^2(x)\right)^2\left(-8\sin^3(x)+6\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{32\left(16\sin^2(x)+15\sin(x)+2\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^3}+\frac{64\left(52\sin^2(x)+51\sin(x)+11\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{16\left(140\sin^2(x)+138\sin(x)+31\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)^2}-\frac{}{4(}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(1-2\sin(x))^3\left(1-\sin^2(x)\right)^2\left(-8\sin^3(x)+6\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{32\left(16\sin^2(x)+15\sin(x)+2\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^3}+\frac{64\left(52\sin^2(x)+51\sin(x)+11\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{16\left(140\sin^2(x)+138\sin(x)+31\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)^2}-\frac{}{4(}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(1-2\sin(x))^3\left(1-\sin^2(x)\right)^2\left(-8\sin^3(x)+6\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{32\left(16\sin^2(x)+15\sin(x)+2\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^3}+\frac{64\left(52\sin^2(x)+51\sin(x)+11\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{16\left(140\sin^2(x)+138\sin(x)+31\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)^2}-\frac{}{4(}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(1-2\sin(x))^3\left(1-\sin^2(x)\right)^2\left(-8\sin^3(x)+6\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{32\left(16\sin^2(x)+15\sin(x)+2\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^3}+\frac{64\left(52\sin^2(x)+51\sin(x)+11\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{16\left(140\sin^2(x)+138\sin(x)+31\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)^2}-\frac{\ }{4(\ldots}\right.$$

$$\Big\downarrow \text{ 7239}$$

$$\int \frac{1}{(1-2\sin(x))^3\left(1-\sin^2(x)\right)^2\left(-8\sin^3(x)+6\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{32\left(16\sin^2(x)+15\sin(x)+2\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^3}+\frac{64\left(52\sin^2(x)+51\sin(x)+11\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{16\left(140\sin^2(x)+138\sin(x)+31\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)^2}-\frac{\ }{4(\ldots}\right.$$

$$\Big\downarrow \text{ 7239}$$

$$\int \frac{1}{(1-2\sin(x))^3\left(1-\sin^2(x)\right)^2\left(-8\sin^3(x)+6\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{32\left(16\sin^2(x)+15\sin(x)+2\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^3}+\frac{64\left(52\sin^2(x)+51\sin(x)+11\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{16\left(140\sin^2(x)+138\sin(x)+31\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)^2}-\frac{\ }{4(\ldots}\right.$$

$$\Big\downarrow \text{ 7239}$$

$$\int \frac{1}{(1-2\sin(x))^3\left(1-\sin^2(x)\right)^2\left(-8\sin^3(x)+6\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{32\left(16\sin^2(x)+15\sin(x)+2\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^3}+\frac{64\left(52\sin^2(x)+51\sin(x)+11\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{16\left(140\sin^2(x)+138\sin(x)+31\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)^2}-\frac{\ }{4(\ldots}\right.$$

$$\Big\downarrow \text{ 7239}$$

$$\int \frac{1}{(1-2\sin(x))^3\left(1-\sin^2(x)\right)^2\left(-8\sin^3(x)+6\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{32\left(16\sin^2(x)+15\sin(x)+2\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^3}+\frac{64\left(52\sin^2(x)+51\sin(x)+11\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{16\left(140\sin^2(x)+138\sin(x)+31\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)^2}-\frac{}{4(}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(1-2\sin(x))^3\left(1-\sin^2(x)\right)^2\left(-8\sin^3(x)+6\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{32\left(16\sin^2(x)+15\sin(x)+2\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^3}+\frac{64\left(52\sin^2(x)+51\sin(x)+11\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{16\left(140\sin^2(x)+138\sin(x)+31\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)^2}-\frac{}{4(}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(1-2\sin(x))^3\left(1-\sin^2(x)\right)^2\left(-8\sin^3(x)+6\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{32\left(16\sin^2(x)+15\sin(x)+2\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^3}+\frac{64\left(52\sin^2(x)+51\sin(x)+11\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{16\left(140\sin^2(x)+138\sin(x)+31\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)^2}-\frac{}{4(}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(1-2\sin(x))^3\left(1-\sin^2(x)\right)^2\left(-8\sin^3(x)+6\sin(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{32\left(16\sin^2(x)+15\sin(x)+2\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^3}+\frac{64\left(52\sin^2(x)+51\sin(x)+11\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{16\left(140\sin^2(x)+138\sin(x)+31\right)}{27\left(8\sin^3(x)-6\sin(x)-1\right)^2}-\frac{}{4(}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(1-2\sin(x))^3\left(1-\sin^2(x)\right)^2\left(-8\sin^3(x)+6\sin(x)+1\right)^3}d\sin(x)$$

input
```
Int[(Cos[5*x] + Sin[4*x])^(-3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4829
```
Int[(cos[(n_.)*((c_.) + (d_.)*(x_))]*(b_.) + (a_.)*sin[(m_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Sin[
m*ArcSin[x]] + b*Cos[n*ArcSin[x]]]]^p/Sqrt[1 - x^2], x], x, Sin[c + d*x]],
x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[m/2] && Inte
gerQ[(n - 1)/2]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 15.67 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.02

method	result
parallelrisch	0
default	$-\frac{1}{2916(1+\sin(x))}+\frac{7\ln(1+\sin(x))}{972}+\frac{-\frac{3328\sin(x)^5}{81}-\frac{3200\sin(x)^4}{81}+\frac{640\sin(x)^3}{27}+\frac{2672\sin(x)^2}{81}+\frac{688\sin(x)}{81}+\frac{16}{27}}{\left(8\sin(x)^3-6\sin(x)-1\right)^2}+\frac{16\left(\_R=\right.}{}$
risch	$-\frac{i\left(-2i e^{18ix}+47\,e^{19ix}-2i e^{2ix}+4\,e^{17ix}+88i e^{10ix}-8\,e^{15ix}-14i e^{14ix}+12\,e^{13ix}+10i e^{12ix}-8\,e^{11ix}+10i e^{8ix}+8\,e^{9ix}+6i e^{4ix}-1\right.}{27(e^{3ix}-i)^2(-i e^{6ix}+e^{7ix}+i e^{4ix}+e^{3ix}+i-e^{ix})^2}$

input
```
int(1/(cos(5*x)+sin(4*x))^3,x,method=_RETURNVERBOSE)
```

output
```
0
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 1534, normalized size of antiderivative = 16.67

$$\int \frac{1}{(\cos(5x) + \sin(4x))^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/(cos(5*x)+sin(4*x))^3,x, algorithm="fricas")
```

output
```
-1/236196*(1119744*cos(x)^8 - 3079296*cos(x)^6 + 3149280*cos(x)^4 + 2*(256
*cos(x)^10 - 704*cos(x)^8 + 688*cos(x)^6 - 280*cos(x)^4 + 41*cos(x)^2 + 8*
(32*cos(x)^8 - 56*cos(x)^6 + 30*cos(x)^4 - 5*cos(x)^2)*sin(x))*(59049*(998
5792/14348907*I*sqrt(3) + 1933752064/14348907)^(1/3)*(-I*sqrt(3) + 1) + 15
52192*(I*sqrt(3) + 1)/(9985792/14348907*I*sqrt(3) + 1933752064/14348907)^(
1/3) - 606528)*log(61/236196*(59049*(9985792/14348907*I*sqrt(3) + 19337520
64/14348907)^(1/3)*(-I*sqrt(3) + 1) + 1552192*(I*sqrt(3) + 1)/(9985792/143
48907*I*sqrt(3) + 1933752064/14348907)^(1/3) - 606528)^2 + 27961524*(99857
92/14348907*I*sqrt(3) + 1933752064/14348907)^(1/3)*(-I*sqrt(3) + 1) + 5953
5876352/81*(I*sqrt(3) + 1)/(9985792/14348907*I*sqrt(3) + 1933752064/143489
07)^(1/3) + 4992896*sin(x) - 284375168) - 1329696*cos(x)^2 - (465813504*co
s(x)^10 - 1280987136*cos(x)^8 + 1251873792*cos(x)^6 - 509483520*cos(x)^4 +
 (256*cos(x)^10 - 704*cos(x)^8 + 688*cos(x)^6 - 280*cos(x)^4 + 41*cos(x)^2
 + 8*(32*cos(x)^8 - 56*cos(x)^6 + 30*cos(x)^4 - 5*cos(x)^2)*sin(x))*(59049
*(9985792/14348907*I*sqrt(3) + 1933752064/14348907)^(1/3)*(-I*sqrt(3) + 1)
 + 1552192*(I*sqrt(3) + 1)/(9985792/14348907*I*sqrt(3) + 1933752064/143489
07)^(1/3) - 606528) + 74602944*cos(x)^2 + 14556672*(32*cos(x)^8 - 56*cos(x
)^6 + 30*cos(x)^4 - 5*cos(x)^2)*sin(x) + 1458*(256*cos(x)^10 - 704*cos(x)^
8 + 688*cos(x)^6 - 280*cos(x)^4 + 41*cos(x)^2 + 8*(32*cos(x)^8 - 56*cos(x)
^6 + 30*cos(x)^4 - 5*cos(x)^2)*sin(x))*sqrt(-1/708588*(59049*(9985792/1...
```

Sympy [F]

$$\int \frac{1}{(\cos(5x)+\sin(4x))^3}\,dx = \int \frac{1}{(\sin(4x)+\cos(5x))^3}\,dx$$

input
```
integrate(1/(cos(5*x)+sin(4*x))**3,x)
```

output
```
Integral((sin(4*x) + cos(5*x))**(-3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(5x)+\sin(4x))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(cos(5*x)+sin(4*x))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(5x)+\sin(4x))^3}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(1/(cos(5*x)+sin(4*x))^3,x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: 2*((
11112*sin(sageVARx)^2-365*sin(sageVARx)-11476)*1/2916/(sin(sageVARx)^2-1)+
(-1066752*sin(sageVARx)^8+951040*sin(sageVARx)^7+1336768*sin(sageVARx)^6-1
185216*sin(sageVARx
```

Mupad [B] (verification not implemented)

Time = 23.56 (sec) , antiderivative size = 3808, normalized size of antiderivative = 41.39

$$\int \frac{1}{(\cos(5x)+\sin(4x))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(5*x) + sin(4*x))^3,x)
```

output
```
log((tan(x/2) + 1)^(7/486)) - (61*log(tan(x/2) - 1))/2 + log(1/(tan(x/2)^2
 - 4*tan(x/2) + 1)^(40/243)) + symsum(log((140737488355328*(56773075803816
0310272*root(z^3 - (416*z^2)/27 + (5312*z)/19683 + 137728/14348907, z, k)
- 7876471068796846080*cos(x) + 14048411516620636 16*sin(x) - 90263756714586
2332416*root(z^3 - (416*z^2)/27 + (5312*z)/19683 + 137728/14348907, z, k)*
cos(x) - 182569497549826129920*root(z^3 - (416*z^2)/27 + (5312*z)/19683 +
137728/14348907, z, k)*sin(x) - 9041331297779142005 1456*root(z^3 - (416*z^
2)/27 + (5312*z)/19683 + 137728/14348907, z, k)^2 - 307615448746217568516 2
496*root(z^3 - (416*z^2)/27 + (5312*z)/19683 + 137728/14348907, z, k)^3 +
113345302030483110355175652*root(z^3 - (416*z^2)/27 + (5312*z)/19683 + 137
728/14348907, z, k)^4 + 749811600252164300513936184*root(z^3 - (416*z^2)/2
7 + (5312*z)/19683 + 137728/14348907, z, k)^5 - 1904056561245119976909905 6
7*root(z^3 - (416*z^2)/27 + (5312*z)/19683 + 137728/14348907, z, k)^6 - 12
754708563904991971 2*root(z^3 - (416*z^2)/27 + (5312*z)/19683 + 137728/1434
8907, z, k)^7 + 598994668650558152104380*root(z^3 - (416*z^2)/27 + (5312*z
)/19683 + 137728/14348907, z, k)^8 + 791830396110287841085 44*root(z^3 - (4
16*z^2)/27 + (5312*z)/19683 + 137728/14348907, z, k)^2*cos(x) + 4029718528
254535928065152*root(z^3 - (416*z^2)/27 + (5312*z)/19683 + 137728/14348907
, z, k)^3*cos(x) - 12288435439469186755087963 2*root(z^3 - (416*z^2)/27 + (
5312*z)/19683 + 137728/14348907, z, k)^4*cos(x) - 841003008403170140972...
```

Reduce [F]

$$\int \frac{1}{(\cos(5x)+\sin(4x))^3}\,dx = \int \frac{1}{\cos(5x)^3+3\cos(5x)^2\sin(4x)+3\cos(5x)\sin(4x)^2+\sin(4x)^3}\,dx$$

input
```
int(1/(cos(5*x)+sin(4*x))^3,x)
```

output
```
int(1/(cos(5*x)**3 + 3*cos(5*x)**2*sin(4*x) + 3*cos(5*x)*sin(4*x)**2 + sin
(4*x)**3),x)
```

3.15 $\int \frac{1}{(\cos(5x)+\sin(4x))^5} dx$

Optimal result

Integrand size = 11, antiderivative size = 154

$$\int \frac{1}{(\cos(5x)+\sin(4x))^5} dx = -\frac{1063\text{arctanh}(\cos(x))}{262144} + \frac{3603\text{arctanh}\left(\sqrt{2}\cos(x)\right)}{16384\sqrt{2}} - \frac{27761\sec(x)}{262144} - \frac{13349\sec^3(x)}{786432} - \frac{6143\sec^5(x)}{1310720} - \frac{635\sec^7(x)}{458752} - \frac{1477\sec^9(x)}{4718592} + \frac{59\sec^9(x)\sec(2x)}{524288} + \frac{159\sec^9(x)\sec^2(2x)}{262144} + \frac{149\sec^9(x)\sec^3(2x)}{393216} + \frac{45\sec^9(x)\sec^4(2x)}{131072} - \frac{37\csc^2(x)\sec^9(x)\sec^4(2x)}{262144} - \frac{\csc^4(x)\sec^9(x)\sec^4(2x)}{131072}$$

output
```
-1063/262144*arctanh(cos(x))+3603/32768*arctanh(cos(x)*2^(1/2))*2^(1/2)-27
761/262144*sec(x)-13349/786432*sec(x)^3-6143/1310720*sec(x)^5-635/458752*s
ec(x)^7-1477/4718592*sec(x)^9+59/524288*sec(x)^9*sec(2*x)+159/262144*sec(x
)^9*sec(2*x)^2+149/393216*sec(x)^9*sec(2*x)^3+45/131072*sec(x)^9*sec(2*x)^
4-37/262144*csc(x)^2*sec(x)^9*sec(2*x)^4-1/131072*csc(x)^4*sec(x)^9*sec(2*
x)^4
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.46 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.70

$$\int \frac{1}{(\cos(5x)+\sin(4x))^5}\,dx = \frac{-4147531776\log\left(\sec^2\left(\frac{x}{2}\right)\right)-8292605364\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+15796\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-123}{}$$

input
```
Integrate[(Cos[5*x] + Sin[4*x])^(-5),x]
```

output
```
(-4147531776*Log[Sec[x/2]^2] - 8292605364*Log[Cos[x/2] - Sin[x/2]] + 15796
*Log[Cos[x/2] + Sin[x/2]] - 1236992*Log[1 - 2*Sin[x]] + 49152*RootSum[8516
86039 + 1303482600*#1 - 1166493312*#1^2 + 13824*#1^3 & , Log[-(Sec[x/2]^2*
(-22330596672539 - 51662025250178*Sin[x] - 48704028454896*Sin[x]*#1 + 5772
04992*Sin[x]*#1^2))]*#1 & ] + (9*Sec[x]^4*(42765300 + 302016*Cos[2*x] - 41
2656*Cos[4*x] + 302112*Cos[6*x] - 27872708*Cos[8*x] + 27805588*Cos[10*x] -
 114192*Cos[12*x] + 104480*Cos[14*x] - 58960*Cos[16*x] - 14277396*Cos[18*x
] - 41817870*Sin[x] + 367056*Sin[3*x] - 358512*Sin[5*x] + 228432*Sin[7*x]
+ 57060192*Sin[9*x] + 97552*Sin[11*x] - 123632*Sin[13*x] + 84240*Sin[15*x]
 - 6962869*Sin[17*x] + 6932549*Sin[19*x]))/(1 - 2*Cos[2*x] + 2*Cos[4*x] -
2*Sin[x] + 2*Sin[3*x])^4)/5668704
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(4x)+\cos(5x))^5}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(\sin(4x)+\cos(5x))^5}dx$$

$$\Big\downarrow \; 4829$$

$$\int \frac{1}{\left(1-\sin^2(x)\right)^3 \left(16\sin^4(x)-8\sin^3(x)-12\sin^2(x)+4\sin(x)+1\right)^5}d\sin(x)$$

$\downarrow$ 2462

$$\int \left(\frac{64\left(196\sin^2(x)+190\sin(x)+37\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^4}+\frac{64\left(66672\sin^2(x)+65510\sin(x)+14357\right)}{729\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{64\left(46068\sin^2(x)+4513\right.}{729\left(8\sin^3(x)-6\sin\right.}\right.$$

$\downarrow$ 7239

$$\int \frac{1}{(1-2\sin(x))^5 \left(1-\sin^2(x)\right)^3 \left(-8\sin^3(x)+6\sin(x)+1\right)^5}d\sin(x)$$

$\downarrow$ 7293

$$\int \left(\frac{64\left(196\sin^2(x)+190\sin(x)+37\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^4}+\frac{64\left(66672\sin^2(x)+65510\sin(x)+14357\right)}{729\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{64\left(46068\sin^2(x)+4513\right.}{729\left(8\sin^3(x)-6\sin\right.}\right.$$

$\downarrow$ 7239

$$\int \frac{1}{(1-2\sin(x))^5 \left(1-\sin^2(x)\right)^3 \left(-8\sin^3(x)+6\sin(x)+1\right)^5}d\sin(x)$$

$\downarrow$ 7293

$$\int \left(\frac{64\left(196\sin^2(x)+190\sin(x)+37\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^4}+\frac{64\left(66672\sin^2(x)+65510\sin(x)+14357\right)}{729\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{64\left(46068\sin^2(x)+4513\right.}{729\left(8\sin^3(x)-6\sin\right.}\right.$$

$\downarrow$ 7239

$$\int \frac{1}{(1-2\sin(x))^5 \left(1-\sin^2(x)\right)^3 \left(-8\sin^3(x)+6\sin(x)+1\right)^5}d\sin(x)$$

$\downarrow$ 7293

$$\int \left(\frac{64\left(196\sin^2(x)+190\sin(x)+37\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^4}+\frac{64\left(66672\sin^2(x)+65510\sin(x)+14357\right)}{729\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{64\left(46068\sin^2(x)+4513\right.}{729\left(8\sin^3(x)-6\sin\right.}\right.$$

$\downarrow$ 7239

$$\int \frac{1}{(1-2\sin(x))^5 \left(1-\sin^2(x)\right)^3 \left(-8\sin^3(x)+6\sin(x)+1\right)^5}d\sin(x)$$

$\downarrow$ 7293

$$\int \left(\frac{64\left(196\sin^2(x)+190\sin(x)+37\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^4}+\frac{64\left(66672\sin^2(x)+65510\sin(x)+14357\right)}{729\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{64\left(46068\sin^2(x)+4513\right.}{729\left(8\sin^3(x)-6\sin\right.}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(1-2\sin(x))^5\left(1-\sin^2(x)\right)^3\left(-8\sin^3(x)+6\sin(x)+1\right)^5}d\sin(x)$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{64\left(196\sin^2(x)+190\sin(x)+37\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^4}+\frac{64\left(66672\sin^2(x)+65510\sin(x)+14357\right)}{729\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{64\left(46068\sin^2(x)+4513\right.}{729\left(8\sin^3(x)-6\sin\right.}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(1-2\sin(x))^5\left(1-\sin^2(x)\right)^3\left(-8\sin^3(x)+6\sin(x)+1\right)^5}d\sin(x)$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{64\left(196\sin^2(x)+190\sin(x)+37\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^4}+\frac{64\left(66672\sin^2(x)+65510\sin(x)+14357\right)}{729\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{64\left(46068\sin^2(x)+4513\right.}{729\left(8\sin^3(x)-6\sin\right.}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(1-2\sin(x))^5\left(1-\sin^2(x)\right)^3\left(-8\sin^3(x)+6\sin(x)+1\right)^5}d\sin(x)$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{64\left(196\sin^2(x)+190\sin(x)+37\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^4}+\frac{64\left(66672\sin^2(x)+65510\sin(x)+14357\right)}{729\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{64\left(46068\sin^2(x)+4513\right.}{729\left(8\sin^3(x)-6\sin\right.}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(1-2\sin(x))^5\left(1-\sin^2(x)\right)^3\left(-8\sin^3(x)+6\sin(x)+1\right)^5}d\sin(x)$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{64\left(196\sin^2(x)+190\sin(x)+37\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^4}+\frac{64\left(66672\sin^2(x)+65510\sin(x)+14357\right)}{729\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{64\left(46068\sin^2(x)+4513\right.}{729\left(8\sin^3(x)-6\sin\right.}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(1-2\sin(x))^5 \left(1-\sin^2(x)\right)^3 \left(-8\sin^3(x)+6\sin(x)+1\right)^5}d\sin(x)$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{64\left(196\sin^2(x)+190\sin(x)+37\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^4}+\frac{64\left(66672\sin^2(x)+65510\sin(x)+14357\right)}{729\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{64\left(46068\sin^2(x)+4513\right.}{729\left(8\sin^3(x)-6\sin\right.}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(1-2\sin(x))^5 \left(1-\sin^2(x)\right)^3 \left(-8\sin^3(x)+6\sin(x)+1\right)^5}d\sin(x)$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{64\left(196\sin^2(x)+190\sin(x)+37\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^4}+\frac{64\left(66672\sin^2(x)+65510\sin(x)+14357\right)}{729\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{64\left(46068\sin^2(x)+4513\right.}{729\left(8\sin^3(x)-6\sin\right.}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(1-2\sin(x))^5 \left(1-\sin^2(x)\right)^3 \left(-8\sin^3(x)+6\sin(x)+1\right)^5}d\sin(x)$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{64\left(196\sin^2(x)+190\sin(x)+37\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^4}+\frac{64\left(66672\sin^2(x)+65510\sin(x)+14357\right)}{729\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{64\left(46068\sin^2(x)+4513\right.}{729\left(8\sin^3(x)-6\sin\right.}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(1-2\sin(x))^5 \left(1-\sin^2(x)\right)^3 \left(-8\sin^3(x)+6\sin(x)+1\right)^5}d\sin(x)$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{64\left(196\sin^2(x)+190\sin(x)+37\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^4}+\frac{64\left(66672\sin^2(x)+65510\sin(x)+14357\right)}{729\left(8\sin^3(x)-6\sin(x)-1\right)}+\frac{64\left(46068\sin^2(x)+4513\right.}{729\left(8\sin^3(x)-6\sin\right.}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(1-2\sin(x))^5 \left(1-\sin^2(x)\right)^3 \left(-8\sin^3(x)+6\sin(x)+1\right)^5}d\sin(x)$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{64\left(196\sin^2(x)+190\sin(x)+37\right)}{9\left(8\sin^3(x)-6\sin(x)-1\right)^4} + \frac{64\left(66672\sin^2(x)+65510\sin(x)+14357\right)}{729\left(8\sin^3(x)-6\sin(x)-1\right)} + \frac{64\left(46068\sin^2(x)+45130\right.}{729\left(8\sin^3(x)-6\sin\right.}\right.$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{(1-2\sin(x))^5\left(1-\sin^2(x)\right)^3\left(-8\sin^3(x)+6\sin(x)+1\right)^5}d\sin(x)$$

input
```
Int[(Cos[5*x] + Sin[4*x])^(-5),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4829
```
Int[(cos[(n_.)*((c_.) + (d_.)*(x_))]*(b_.) + (a_.)*sin[(m_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Sin[
m*ArcSin[x]] + b*Cos[n*ArcSin[x]]]]^p/Sqrt[1 - x^2], x], x, Sin[c + d*x]],
x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[m/2] && Inte
gerQ[(n - 1)/2]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 594.68 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.01

method	result
parallelrisch	0
default	$-\frac{1}{944784(1+\sin(x))^2}-\frac{209}{2834352(1+\sin(x))}+\frac{3949\ln(1+\sin(x))}{2834352}+\frac{-\frac{7589920768\sin(x)^{11}}{59049}-\frac{7284604928\sin(x)^{10}}{59049}+\frac{15777529}{5}}{}$
risch	$-\frac{i(-6932549\,\mathrm{e}^{ix}+302016i\mathrm{e}^{18ix}-228432\,\mathrm{e}^{13ix}+358512\,\mathrm{e}^{15ix}-367056\,\mathrm{e}^{17ix}-58960i\mathrm{e}^{4ix}-14277396i\mathrm{e}^{2ix}+27805588i\mathrm{e}^{10ix}-}{}$

input
```
int(1/(cos(5*x)+sin(4*x))^5,x,method=_RETURNVERBOSE)
```

output
```
0
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.43 (sec) , antiderivative size = 2122, normalized size of antiderivative = 13.78

$$\int \frac{1}{(\cos(5x)+\sin(4x))^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cos(5*x)+sin(4*x))^5,x, algorithm="fricas")
```

output
```
-1/502096953744*(1491781604010098688*cos(x)^18 - 6711477101007077376*cos(x
)^16 + 12580064525026344960*cos(x)^14 - 12714176602821500928*cos(x)^12 + 7
476036712105789440*cos(x)^10 - 2557995116180228352*cos(x)^8 + 472145715040
915008*cos(x)^6 - 36398752187700384*cos(x)^4 + 8*(65536*cos(x)^20 - 425984
*cos(x)^18 + 1142784*cos(x)^16 - 1665024*cos(x)^14 + 1447680*cos(x)^12 - 7
71968*cos(x)^10 + 247456*cos(x)^8 - 43760*cos(x)^6 + 3281*cos(x)^4 + 16*(8
192*cos(x)^18 - 36864*cos(x)^16 + 69120*cos(x)^14 - 69888*cos(x)^12 + 4115
2*cos(x)^10 - 14136*cos(x)^8 + 2630*cos(x)^6 - 205*cos(x)^4)*sin(x))*(3138
1059609*(2926915967063031808/5559060566555523*I*sqrt(3) + 806361130699389
0807808/5559060566555523)^(1/3)*(-I*sqrt(3) + 1) + 1866464456667136*(I*sqr
t(3) + 1)/(2926915967063031808/5559060566555523*I*sqrt(3) + 80636113069993
890807808/5559060566555523)^(1/3) - 15306725240064)*log(266579/12552423843
6*(31381059609*(2926915967063031808/5559060566555523*I*sqrt(3) + 806361130
69993890807808/5559060566555523)^(1/3)*(-I*sqrt(3) + 1) + 1866464456667136
*(I*sqrt(3) + 1)/(2926915967063031808/5559060566555523*I*sqrt(3) + 8063611
3069993890807808/5559060566555523)^(1/3) - 15306725240064)^2 + 30604945803
27546528*(2926915967063031808/5559060566555523*I*sqrt(3) + 806361130699938
90807808/5559060566555523)^(1/3)*(-I*sqrt(3) + 1) + 107487084248369405908 4
5657088/59049*(I*sqrt(3) + 1)/(2926915967063031808/5559060566555523*I*sqrt
(3) + 80636113069993890807808/5559060566555523)^(1/3) + 182932247941439...
```

Sympy [F]

$$\int \frac{1}{(\cos(5x)+\sin(4x))^5}\,dx = \int \frac{1}{(\sin(4x)+\cos(5x))^5}\,dx$$

input
```
integrate(1/(cos(5*x)+sin(4*x))**5,x)
```

output
```
Integral((sin(4*x) + cos(5*x))**(-5), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(5x)+\sin(4x))^5}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(cos(5*x)+sin(4*x))^5,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(5x)+\sin(4x))^5}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(1/(cos(5*x)+sin(4*x))^5,x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: 2*((
777430272*sin(sageVARx)^4-17980525*sin(sageVARx)^3-1572752288*sin(sageVARx
)^2+18157675*sin(sageVARx)+795499160)*1/2834352/(sin(sageVARx)^2-1)^2+(-66
34071654400*sin(sag
```

Mupad [B] (verification not implemented)

Time = 23.44 (sec) , antiderivative size = 1216, normalized size of antiderivative = 7.90

$$\int \frac{1}{(\cos(5x)+\sin(4x))^5}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(5*x) + sin(4*x))^5,x)
```

output
```
(3949*log(tan(x/2) + 1))/1417176 - (11703*log(tan(x/2) - 1))/8 - (38656*lo
g(tan(x/2)^2 - 4*tan(x/2) + 1))/177147 + symsum(log((8796093022208*(197852
47954114645806985184845140560244969046016*sin(x) - 36163269112055873708639
92759016553285993627648 0*cos(x) - 827702993595112616136722276938960035637 6
239931392*root(z^3 - (59264*z^2)/81 + (74153676800*z)/10460353203 + 223264
385007616/5559060566555523, z, k) + 561131139172005331899684260930828004 96
68041211904*root(z^3 - (59264*z^2)/81 + (74153676800*z)/10460353203 + 2232
64385007616/5559060566555523, z, k)*cos(x) - 751245097217175446805316580 23
0649288083911789772 8*root(z^3 - (59264*z^2)/81 + (74153676800*z)/104603532
03 + 223264385007616/5559060566555523, z, k)*sin(x) - 501959090309062741 34
83968508646945045343513278939136*root(z^3 - (59264*z^2)/81 + (74153676800*
z)/10460353203 + 223264385007616/5559060566555523, z, k)^2 + 4246724871726
16956800597420559285409243812491561533 44*root(z^3 - (59264*z^2)/81 + (7415
3676800*z)/10460353203 + 223264385007616/5559060566555523, z, k)^3 + 24130
94880440511693423489953439598930184795160129998745 6*root(z^3 - (59264*z^2)
/81 + (74153676800*z)/10460353203 + 223264385007616/5559060566555523, z, k
)^4 + 1090912575805093587459279624914524853375674306975231390 08*root(z^3 -
 (59264*z^2)/81 + (74153676800*z)/10460353203 + 223264385007616/5559060566
555523, z, k)^5 - 555954874424632137152452856127196972464769411669929903*r
oot(z^3 - (59264*z^2)/81 + (74153676800*z)/10460353203 + 2232643850076 1...
```

Reduce [F]

$$\int \frac{1}{(\cos(5x)+\sin(4x))^5}\,dx$$
$$= \int \frac{1}{\cos(5x)^5+5\cos(5x)^4\sin(4x)+10\cos(5x)^3\sin(4x)^2+10\cos(5x)^2\sin(4x)^3+5\cos(5x)\sin(4x)^4+}$$

input
```
int(1/(cos(5*x)+sin(4*x))^5,x)
```

output
```
int(1/(cos(5*x)**5 + 5*cos(5*x)**4*sin(4*x) + 10*cos(5*x)**3*sin(4*x)**2 +
 10*cos(5*x)**2*sin(4*x)**3 + 5*cos(5*x)*sin(4*x)**4 + sin(4*x)**5),x)
```

3.16 $\int \frac{1}{(\cos(5x)+\sin(4x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 52

$$\int \frac{1}{(\cos(5x)+\sin(4x))^2}\,dx = -\frac{1}{16}\operatorname{arctanh}(2\cos(x)\sin(x)) - \frac{\cot(x)}{32} + \frac{5\tan(x)}{32} + \frac{\tan^3(x)}{48} + \frac{\csc(x)\sec^5(x)}{64\,(1-\tan^2(x))}$$

output
```
-1/16*arctanh(2*cos(x)*sin(x))-1/32*cot(x)+5/32*tan(x)+1/48*tan(x)^3+csc(x
)*sec(x)^5/(64-64*tan(x)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.37 (sec) , antiderivative size = 345, normalized size of antiderivative = 6.63

$$\int \frac{1}{(\cos(5x)+\sin(4x))^2}\,dx = \frac{1}{81}\left(8\sqrt{3}\text{arctanh}\left(\frac{-2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)+2i\text{RootSum}\left[i+\#1^3 -i\#1^6\&, \frac{22\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)-11i\log\left(1-2\cos(x)\#1+\#1^2\right)-40i\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1-20\text{lo}\right.\right.$$
$$\left.+\frac{81\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}+\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}+\frac{8\cos(x)}{1-2\sin(x)}+\frac{24(4\cos(x)-3\cos(3x)+5\sin(2x))}{1+2\sin(3x)}\right)$$

input
```
Integrate[(Cos[5*x] + Sin[4*x])^(-2),x]
```

output
```
(8*Sqrt[3]*ArcTanh[(-2 + Tan[x/2])/Sqrt[3]] + (2*I)*RootSum[I + #1^3 - I*#
1^6 & , (22*ArcTan[Sin[x]/(Cos[x] - #1)] - (11*I)*Log[1 - 2*Cos[x]*#1 + #1
^2] - (40*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 - 20*Log[1 - 2*Cos[x]*#1 + #1
^2]*#1 - 54*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 + (27*I)*Log[1 - 2*Cos[x]*#1
 + #1^2]*#1^2 + (40*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 + 20*Log[1 - 2*Co
s[x]*#1 + #1^2]*#1^3 + 22*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^4 - (11*I)*Log[1
 - 2*Cos[x]*#1 + #1^2]*#1^4)/(I*#1^2 + 2*#1^5) & ] + (81*Sin[x/2])/(Cos[x/
2] - Sin[x/2]) + Sin[x/2]/(Cos[x/2] + Sin[x/2]) + (8*Cos[x])/(1 - 2*Sin[x]
) + (24*(4*Cos[x] - 3*Cos[3*x] + 5*Sin[2*x]))/(1 + 2*Sin[3*x]))/81
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(4x)+\cos(5x))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(4x)+\cos(5x))^2}dx$$

$$\Big\downarrow \text{4830}$$

$$2\int \frac{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^9}{\left(-\tan^{10}\left(\frac{x}{2}\right)-8\tan^9\left(\frac{x}{2}\right)+45\tan^8\left(\frac{x}{2}\right)+48\tan^7\left(\frac{x}{2}\right)-210\tan^6\left(\frac{x}{2}\right)+210\tan^4\left(\frac{x}{2}\right)-48\tan^3\left(\frac{x}{2}\right)-45\tan$$

$$\Big\downarrow \text{2462}$$

$$2\int \left(\frac{16\tan\left(\frac{x}{2}\right)}{27\left(\tan^2\left(\frac{x}{2}\right)-4\tan\left(\frac{x}{2}\right)+1\right)^2}+\frac{4}{81\left(\tan^2\left(\frac{x}{2}\right)-4\tan\left(\frac{x}{2}\right)+1\right)}+\frac{4\left(\tan^4\left(\frac{x}{2}\right)-28\tan^3\left(\frac{x}{2}\right)+16\right.}{9\left(\tan^6\left(\frac{x}{2}\right)+12\tan^5\left(\frac{x}{2}\right)+3\tan^4\left(\frac{x}{2}\right)\right.}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{2\log\left(-\tan\left(\frac{x}{2}\right)-\sqrt{3}+2\right)}{27\sqrt{3}}-\frac{2\log\left(-\tan\left(\frac{x}{2}\right)+\sqrt{3}+2\right)}{27\sqrt{3}}+\frac{715264}{3}\int \frac{}{\left(\tan^6\left(\frac{x}{2}\right)+12\tan^5\left(\frac{x}{2}\right)+3\tan^4\left(\frac{x}{2}\right)\right.}\right.$$

input
```
Int[(Cos[5*x] + Sin[4*x])^(-2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4830
```
Int[(cos[(n_.)*((c_.) + (d_.)*(x_))]*(b_.) + (a_.)*sin[(m_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[2/d   Subst[Int[Simplify[TrigExpand[a*Sin[
2*m*ArcTan[x]] + b*Cos[2*n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[(1/2)*(c +
d*x)]], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && Intege
rQ[n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.47 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.65

method	result
risch	$\frac{\frac{16\,e^{9ix}}{9}-\frac{2i e^{8ix}}{9}+\frac{2\,e^{7ix}}{9}+\frac{2i e^{6ix}}{9}-\frac{2\,e^{5ix}}{9}-\frac{2i e^{4ix}}{9}+\frac{2\,e^{3ix}}{9}+\frac{2i e^{2ix}}{9}-\frac{2\,e^{ix}}{9}+2i}{-i e^{9ix}+e^{10ix}+i e^{ix}+1}+\left(\sum_{\_R=\mathrm{RootOf}(94143178827\_Z^6-507606934032\_\right.$
default	$\frac{-\frac{64\tan\left(\frac{x}{2}\right)^5}{27}-\frac{368\tan\left(\frac{x}{2}\right)^4}{27}-\frac{320\tan\left(\frac{x}{2}\right)^3}{27}+\frac{416\tan\left(\frac{x}{2}\right)^2}{27}+\frac{256\tan\left(\frac{x}{2}\right)}{27}+\frac{16}{27}}{\tan\left(\frac{x}{2}\right)^6+12\tan\left(\frac{x}{2}\right)^5+3\tan\left(\frac{x}{2}\right)^4-40\tan\left(\frac{x}{2}\right)^3+3\tan\left(\frac{x}{2}\right)^2+12\tan\left(\frac{x}{2}\right)+1}+\frac{4\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6+12\_Z^5+3\_Z^4-40\_Z^3+3\_Z^2+\right.}\right.}{}$

input
```
int(1/(cos(5*x)+sin(4*x))^2,x,method=_RETURNVERBOSE)
```

output
```
2/9*(8*exp(9*I*x)-I*exp(8*I*x)+exp(7*I*x)+I*exp(6*I*x)-exp(5*I*x)-I*exp(4*
I*x)+exp(3*I*x)+I*exp(2*I*x)-exp(I*x)+9*I)/(-I*exp(9*I*x)+exp(10*I*x)+I*ex
p(I*x)+1)+sum(_R*ln(exp(I*x)-5965888110111/130316288*_R^5+12526595811/3257
9072*I*_R^4+2010526865001/8144768*_R^3-2110877091/1018096*I*_R^2-33777405/
254524*_R+104321/127262*I),_R=RootOf(94143178827*_Z^6-507606934032*_Z^4+17
1320832*_Z^2-4096))+4/81*3^(1/2)*ln(exp(I*x)-1/2*I-1/2*3^(1/2))-4/81*3^(1/
2)*ln(exp(I*x)-1/2*I+1/2*3^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 3679, normalized size of antiderivative = 70.75

$$\int \frac{1}{(\cos(5x) + \sin(4x))^2} \, dx = \text{Too large to display}$$

input
```
integrate(1/(cos(5*x)+sin(4*x))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(\cos(5x) + \sin(4x))^2} \, dx = \int \frac{1}{(\sin(4x) + \cos(5x))^2} \, dx$$

input
```
integrate(1/(cos(5*x)+sin(4*x))**2,x)
```

output
```
Integral((sin(4*x) + cos(5*x))**(-2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(5x) + \sin(4x))^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(cos(5*x)+sin(4*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(40) = 80$.

Time = 0.15 (sec) , antiderivative size = 238, normalized size of antiderivative = 4.58

$$
\begin{aligned}
\int \frac{1}{(\cos(5x)+\sin(4x))^2}\,dx &= -\frac{4}{81}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\tan\left(\frac{1}{2}x\right)-4\right|}{\left|2\sqrt{3}+2\tan\left(\frac{1}{2}x\right)-4\right|}\right) \\
&- \frac{2\left(17\tan\left(\frac{1}{2}x\right)^9+80\tan\left(\frac{1}{2}x\right)^8-364\tan\left(\frac{1}{2}x\right)^7-712\tan\left(\frac{1}{2}x\right)^6+1094\tan\left(\frac{1}{2}x\right)^5+968\tan\left(\frac{1}{2}x\right.\right.}{9\left(\tan\left(\frac{1}{2}x\right)^{10}+8\tan\left(\frac{1}{2}x\right)^9-45\tan\left(\frac{1}{2}x\right)^8-48\tan\left(\frac{1}{2}x\right)^7+210\tan\left(\frac{1}{2}x\right)^6-210\tan\left(\frac{1}{2}x\right)^4\right.} \\
&- 0.0176530836122963\log\left(\tan\left(\frac{1}{2}x\right)+11.4300523028000\right) \\
&- 0.00508872774952593\log\left(\tan\left(\frac{1}{2}x\right)+2.14450692051000\right) \\
&+ 0.00508872774952593\log\left(\tan\left(\frac{1}{2}x\right)+0.466307658155000\right) \\
&+ 0.0176530836122963\log\left(\tan\left(\frac{1}{2}x\right)+0.0874886635259000\right) \\
&+ 2.32196543262222\log\left(\tan\left(\frac{1}{2}x\right)-0.700207538210000\right) \\
&- 2.32196543262222\log\left(\tan\left(\frac{1}{2}x\right)-1.42814800674000\right)
\end{aligned}
$$

input

```
integrate(1/(cos(5*x)+sin(4*x))^2,x, algorithm="giac")
```

output

```
-4/81*sqrt(3)*log(abs(-2*sqrt(3) + 2*tan(1/2*x) - 4)/abs(2*sqrt(3) + 2*tan
(1/2*x) - 4)) - 2/9*(17*tan(1/2*x)^9 + 80*tan(1/2*x)^8 - 364*tan(1/2*x)^7
- 712*tan(1/2*x)^6 + 1094*tan(1/2*x)^5 + 968*tan(1/2*x)^4 - 748*tan(1/2*x)
^3 - 280*tan(1/2*x)^2 + 81*tan(1/2*x) + 8)/(tan(1/2*x)^10 + 8*tan(1/2*x)^9
 - 45*tan(1/2*x)^8 - 48*tan(1/2*x)^7 + 210*tan(1/2*x)^6 - 210*tan(1/2*x)^4
 + 48*tan(1/2*x)^3 + 45*tan(1/2*x)^2 - 8*tan(1/2*x) - 1) - 0.0176530836122
963*log(tan(1/2*x) + 11.4300523028000) - 0.00508872774952593*log(tan(1/2*x
) + 2.14450692051000) + 0.00508872774952593*log(tan(1/2*x) + 0.46630765815
5000) + 0.0176530836122963*log(tan(1/2*x) + 0.0874886635259000) + 2.321965
43262222*log(tan(1/2*x) - 0.700207538210000) - 2.32196543262222*log(tan(1/
2*x) - 1.42814800674000)
```

Mupad [B] (verification not implemented)

Time = 22.90 (sec) , antiderivative size = 544, normalized size of antiderivative = 10.46

$$\int \frac{1}{(\cos(5x)+\sin(4x))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(5*x) + sin(4*x))^2,x)
```

output
```
(4*3^(1/2)*log(tan(x/2) + 3^(1/2) - 2))/81 + symsum(log((33577757957763849
25696*root(z^6 - (11792*z^4)/2187 + (8704*z^2)/4782969 - 4096/94143178827,
 z, k)^2*tan(x/2))/2187 - (2536427310135063347 2*tan(x/2))/531441 - (532577
677534325374976*root(z^6 - (11792*z^4)/2187 + (8704*z^2)/4782969 - 4096/94
143178827, z, k))/531441 + (620437051584808170 2912*root(z^6 - (11792*z^4)/
2187 + (8704*z^2)/4782969 - 4096/94143178827, z, k)^3*tan(x/2))/81 - 11417
12045134321811456*root(z^6 - (11792*z^4)/2187 + (8704*z^2)/4782969 - 4096/
94143178827, z, k)^4*tan(x/2) - 2124584861718560322 3552*root(z^6 - (11792*
z^4)/2187 + (8704*z^2)/4782969 - 4096/94143178827, z, k)^5*tan(x/2) + 1072
966004197394743296*root(z^6 - (11792*z^4)/2187 + (8704*z^2)/4782969 - 4096
/94143178827, z, k)^6*tan(x/2) + 4319334531957283356672*root(z^6 - (11792*
z^4)/2187 + (8704*z^2)/4782969 - 4096/94143178827, z, k)^7*tan(x/2) + (185
73745583201399603 2*root(z^6 - (11792*z^4)/2187 + (8704*z^2)/4782969 - 4096
/94143178827, z, k)^2)/6561 + (6254734270480965959 68*root(z^6 - (11792*z^4
)/2187 + (8704*z^2)/4782969 - 4096/94143178827, z, k)^3)/81 + 248732118569
812230144*root(z^6 - (11792*z^4)/2187 + (8704*z^2)/4782969 - 4096/94143178
827, z, k)^4 + 1865156497001259466752*root(z^6 - (11792*z^4)/2187 + (8704*
z^2)/4782969 - 4096/94143178827, z, k)^5 - 6403631014723694100 48*root(z^6
- (11792*z^4)/2187 + (8704*z^2)/4782969 - 4096/94143178827, z, k)^6 - 6170
47790279611908096*root(z^6 - (11792*z^4)/2187 + (8704*z^2)/4782969 - 40...
```

Reduce [F]

$$\int \frac{1}{(\cos(5x)+\sin(4x))^2}\,dx = \int \frac{1}{\cos(5x)^2+2\cos(5x)\sin(4x)+\sin(4x)^2}dx$$

input
```
int(1/(cos(5*x)+sin(4*x))^2,x)
```

output
```
int(1/(cos(5*x)**2 + 2*cos(5*x)*sin(4*x) + sin(4*x)**2),x)
```

3.17 $\int \frac{1}{(\cos(5x)+\sin(4x))^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 122

$$\begin{aligned}\int \frac{1}{(\cos(5x)+\sin(4x))^4}\,dx = &-\frac{9}{128}\operatorname{arctanh}(2\cos(x)\sin(x)) - \frac{37\cot(x)}{4096} \\ &- \frac{11\cot^3(x)}{12288} + \frac{289\tan(x)}{2048} + \frac{199\tan^3(x)}{6144} \\ &+ \frac{163\tan^5(x)}{20480} + \frac{29\tan^7(x)}{28672} + \frac{\csc^3(x)\sec^{13}(x)}{12288\left(1-\tan^2(x)\right)^3} \\ &- \frac{\csc^3(x)\sec^{11}(x)}{12288\left(1-\tan^2(x)\right)^2} + \frac{5\csc^3(x)\sec^9(x)}{6144\left(1-\tan^2(x)\right)}\end{aligned}$$

output

```
-9/128*arctanh(2*cos(x)*sin(x))-37/4096*cot(x)-11/12288*cot(x)^3+289/2048*
tan(x)+199/6144*tan(x)^3+163/20480*tan(x)^5+29/28672*tan(x)^7+1/12288*csc(
x)^3*sec(x)^13/(1-tan(x)^2)^3-1/12288*csc(x)^3*sec(x)^11/(1-tan(x)^2)^2+5*
csc(x)^3*sec(x)^9/(6144-6144*tan(x)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 5.04 (sec) , antiderivative size = 515, normalized size of antiderivative = 4.22

$$\int \frac{1}{(\cos(5x)+\sin(4x))^4}\,dx$$

$$= \frac{316872i + 13696\sqrt{3}\operatorname{arctanh}\left(\frac{-2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right) + 32i\operatorname{RootSum}\left[i + \#1^3 - i\#1^6\&, \frac{57926\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right) - 28963i\ldots}{\ldots}\right.}{\ldots}$$

input
```
Integrate[(Cos[5*x] + Sin[4*x])^(-4),x]
```

output
```
(316872*I + 13696*Sqrt[3]*ArcTanh[(-2 + Tan[x/2])/Sqrt[3]] + (32*I)*RootSu
m[I + #1^3 - I*#1^6 & , (57926*ArcTan[Sin[x]/(Cos[x] - #1)] - (28963*I)*Lo
g[1 - 2*Cos[x]*#1 + #1^2] - (108818*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 - 5
4409*Log[1 - 2*Cos[x]*#1 + #1^2]*#1 - 146610*ArcTan[Sin[x]/(Cos[x] - #1)]*
#1^2 + (73305*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^2 + (108818*I)*ArcTan[Sin[
x]/(Cos[x] - #1)]*#1^3 + 54409*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3 + 57926*Ar
cTan[Sin[x]/(Cos[x] - #1)]*#1^4 - (28963*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1
^4)/(I*#1^2 + 2*#1^5) & ] + 6561/(Cos[x/2] - Sin[x/2])^2 + (13122*Sin[x/2]
)/(Cos[x/2] - Sin[x/2])^3 + (3201768*Sin[x/2])/(Cos[x/2] - Sin[x/2]) + (2*
Sin[x/2])/(Cos[x/2] + Sin[x/2])^3 - (Cos[x/2] + Sin[x/2])^(-2) + (168*Sin[
x/2])/(Cos[x/2] + Sin[x/2]) - (1088*Cos[x])/(1 - 2*Sin[x])^2 + (8256*Cos[x
])/(1 - 2*Sin[x]) - (384*Cos[x])/(-1 + 2*Sin[x])^3 + (10368*(35*Cos[x] - 2
3*Cos[3*x] + 43*Sin[2*x]))/(1 + 2*Sin[3*x])^3 + (1728*(-723*Cos[x] + 427*C
os[3*x] - 821*Sin[2*x]))/(1 + 2*Sin[3*x])^2 + (192*(21739*Cos[x] - 12987*C
os[3*x] + 24878*Sin[2*x]))/(1 + 2*Sin[3*x]))/78732
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(4x)+\cos(5x))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(4x)+\cos(5x))^4}dx$$

$$\Big\downarrow \text{4830}$$

$$2\int \frac{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^{19}}{\left(-\tan^{10}\left(\frac{x}{2}\right)-8\tan^9\left(\frac{x}{2}\right)+45\tan^8\left(\frac{x}{2}\right)+48\tan^7\left(\frac{x}{2}\right)-210\tan^6\left(\frac{x}{2}\right)+210\tan^4\left(\frac{x}{2}\right)-48\tan^3\left(\frac{x}{2}\right)-45\tan\right.}$$

$$\Big\downarrow \text{2462}$$

$$2\int \left(\frac{1024\left(15\tan\left(\frac{x}{2}\right)-4\right)}{729\left(\tan^2\left(\frac{x}{2}\right)-4\tan\left(\frac{x}{2}\right)+1\right)^4}-\frac{1520}{19683\left(\tan^2\left(\frac{x}{2}\right)-4\tan\left(\frac{x}{2}\right)+1\right)}-\frac{16\left(295\tan^4\left(\frac{x}{2}\right)+4660\tan^3\left(\frac{x}{2}\right.\right.}{243\left(\tan^6\left(\frac{x}{2}\right)+12\tan^5\left(\frac{x}{2}\right)+3\tan\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{856\log\left(-\tan\left(\frac{x}{2}\right)-\sqrt{3}+2\right)}{6561\sqrt{3}}-\frac{856\log\left(-\tan\left(\frac{x}{2}\right)+\sqrt{3}+2\right)}{6561\sqrt{3}}+\frac{634670224974964326 4}{9}\int\frac{}{\left(\tan^6\left(\frac{x}{2}\right)+12\tan\right.}\right.$$

input
```
Int[(Cos[5*x] + Sin[4*x])^(-4),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4830
```
Int[(cos[(n_.)*((c_.) + (d_.)*(x_))]*(b_.) + (a_.)*sin[(m_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[2/d   Subst[Int[Simplify[TrigExpand[a*Sin[
2*m*ArcTan[x]] + b*Cos[2*n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[(1/2)*(c +
d*x)]], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && Intege
rQ[n]
```

Maple [A] (verified)

Time = 114.89 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.02

method	result
parallelrisch	0
risch	$\frac{\frac{652i}{9}-\frac{106756\,e^{ix}}{729}-\frac{159380ie^{18ix}}{729}+\frac{3232\,e^{13ix}}{2187}-\frac{1184\,e^{15ix}}{729}+\frac{3232\,e^{17ix}}{2187}+\frac{1148ie^{4ix}}{2187}-\frac{155564ie^{2ix}}{2187}+\frac{959704ie^{10ix}}{2187}+\frac{1436ie^{8ix}}{2187}-\frac{484ie^{\cdots}}{729}\cdots}{\cdots}$
default	$-\frac{32\left(-\frac{6736}{27}-\frac{249910\tan\left(\frac{x}{2}\right)}{27}-\frac{3238904\tan\left(\frac{x}{2}\right)^{2}}{27}-\frac{15729764\tan\left(\frac{x}{2}\right)^{3}}{27}-\frac{6320408\tan\left(\frac{x}{2}\right)^{4}}{27}+\frac{109380812\tan\left(\frac{x}{2}\right)^{5}}{27}+\frac{122921288\tan\left(\frac{x}{2}\right)\cdots}{27}\cdots\right.}{\cdots}$

input
```
int(1/(cos(5*x)+sin(4*x))^4,x,method=_RETURNVERBOSE)
```

output
```
0
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.78 (sec) , antiderivative size = 4179, normalized size of antiderivative = 34.25

$$\int \frac{1}{(\cos(5x)+\sin(4x))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cos(5*x)+sin(4*x))^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(\cos(5x)+\sin(4x))^4}\,dx = \int \frac{1}{(\sin{(4x)}+\cos{(5x)})^4}\,dx$$

input
```
integrate(1/(cos(5*x)+sin(4*x))**4,x)
```

output
```
Integral((sin(4*x) + cos(5*x))**(-4), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(5x)+\sin(4x))^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(cos(5*x)+sin(4*x))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(96) = 192$.

Time = 0.16 (sec) , antiderivative size = 398, normalized size of antiderivative = 3.26

$$\int \frac{1}{(\cos(5x)+\sin(4x))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cos(5*x)+sin(4*x))^4,x, algorithm="giac")
```

output
```
-1712/19683*sqrt(3)*log(abs(-2*sqrt(3) + 2*tan(1/2*x) - 4)/abs(2*sqrt(3) +
 2*tan(1/2*x) - 4)) - 2/2187*(84779*tan(1/2*x)^29 + 2089080*tan(1/2*x)^28
+ 6842722*tan(1/2*x)^27 - 141493912*tan(1/2*x)^26 - 496870231*tan(1/2*x)^2
5 + 4630724320*tan(1/2*x)^24 + 5060118068*tan(1/2*x)^23 - 57259181968*tan(
1/2*x)^22 - 8100002429*tan(1/2*x)^21 + 334520168968*tan(1/2*x)^20 - 116212
308802*tan(1/2*x)^19 - 1023875673160*tan(1/2*x)^18 + 669078512041*tan(1/2*
x)^17 + 1689393065296*tan(1/2*x)^16 - 1481034238120*tan(1/2*x)^15 - 150989
3990240*tan(1/2*x)^14 + 1663359582505*tan(1/2*x)^13 + 688806280328*tan(1/2
*x)^12 - 1009432595266*tan(1/2*x)^11 - 124324145576*tan(1/2*x)^10 + 329728
991491*tan(1/2*x)^9 - 6204582592*tan(1/2*x)^8 - 56361572812*tan(1/2*x)^7 +
 4829210608*tan(1/2*x)^6 + 4545990185*tan(1/2*x)^5 - 483937288*tan(1/2*x)^
4 - 137922206*tan(1/2*x)^3 + 6693768*tan(1/2*x)^2 + 2023595*tan(1/2*x) + 8
0784)/(tan(1/2*x)^10 + 8*tan(1/2*x)^9 - 45*tan(1/2*x)^8 - 48*tan(1/2*x)^7
+ 210*tan(1/2*x)^6 - 210*tan(1/2*x)^4 + 48*tan(1/2*x)^3 + 45*tan(1/2*x)^2
- 8*tan(1/2*x) - 1)^3 - 0.0111144805997257*log(tan(1/2*x) + 11.43005230280
00) - 0.00202751790850175*log(tan(1/2*x) + 2.14450692051000) + 0.002027517
90841030*log(tan(1/2*x) + 0.466307658155000) + 0.0111144805996952*log(tan(
1/2*x) + 0.0874886635259000) + 103.219357306813*log(tan(1/2*x) - 0.7002075
38210000) - 103.219357306813*log(tan(1/2*x) - 1.42814800674000)
```

Mupad [B] (verification not implemented)

Time = 22.55 (sec) , antiderivative size = 864, normalized size of antiderivative = 7.08

$$\int \frac{1}{(\cos(5x)+\sin(4x))^4}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(5*x) + sin(4*x))^4,x)
```

output
```
(1712*3^(1/2)*log(tan(x/2) + 3^(1/2) - 2))/19683 + symsum(log((93041736942
579793841828316456241O7008*root(z^6 - (1375889754368*z^4)/129140163 + (226
79855871164416*z^2)/16677181699666569 - 104871273876793851904/193832456676
80019896796723, z, k)^2*tan(x/2))/1853020188851841 - (7065938372581153630l
6303278174075O929920*tan(x/2))/23929932923061752959O083 - (248738930312971
44062793996703369684582 4*root(z^6 - (1375889754368*z^4)/129140163 + (22679
855871164416*z^2)/16677181699666569 - 104871273876793851904/19383245667680
019896796723, z, k))/109418989131512359209 + (140021946390804972182725283 7
2334592*root(z^6 - (1375889754368*z^4)/129140163 + (22679855871164416*z^2)
/16677181699666569 - 104871273876793851904/19383245667680019896796723, z,
k)^3*tan(x/2))/31381059609 - (51396907458973907160183630462976*root(z^6 -
(1375889754368*z^4)/129140163 + (22679855871164416*z^2)/16677181699666569
- 104871273876793851904/19383245667680019896796723, z, k)^4*tan(x/2))/1434
8907 - (30581771712029100737225031680*root(z^6 - (1375889754368*z^4)/12914
0163 + (22679855871164416*z^2)/16677181699666569 - 104871273876793851904/1
9383245667680019896796723, z, k)^5*tan(x/2))/729 + 386764334035475512688 64
*root(z^6 - (1375889754368*z^4)/129140163 + (22679855871164416*z^2)/166771
81699666569 - 104871273876793851904/19383245667680019896796723, z, k)^6*ta
n(x/2) + 431933453195728335667 2*root(z^6 - (1375889754368*z^4)/129140163 +
 (22679855871164416*z^2)/16677181699666569 - 104871273876793851904/1938...
```

Reduce [F]

$$\int \frac{1}{(\cos(5x)+\sin(4x))^4}\,dx = \int \frac{1}{\cos(5x)^4+4\cos(5x)^3\sin(4x)+6\cos(5x)^2\sin(4x)^2+4\cos(5x)\sin(4x)^3+\sin(4x)^4}dx$$

input
```
int(1/(cos(5*x)+sin(4*x))^4,x)
```

output
```
int(1/(cos(5*x)**4 + 4*cos(5*x)**3*sin(4*x) + 6*cos(5*x)**2*sin(4*x)**2 +
4*cos(5*x)*sin(4*x)**3 + sin(4*x)**4),x)
```

3.18 $\int \frac{1}{(\cos(5x)+\sin(4x))^6}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 190

$$\int \frac{1}{(\cos(5x)+\sin(4x))^6}\,dx = -\frac{637\text{arctanh}(2\cos(x)\sin(x))}{8192} - \frac{3955\cot(x)}{1048576} - \frac{2131\cot^3(x)}{3145728} - \frac{173\cot^5(x)}{2621440} + \frac{158879\tan(x)}{1048576} + \frac{128051\tan^3(x)}{3145728} + \frac{78617\tan^5(x)}{5242880} + \frac{4751\tan^7(x)}{1048576} + \frac{8501\tan^9(x)}{9437184} + \frac{983\tan^{11}(x)}{11534336} + \frac{\csc^5(x)\sec^{21}(x)}{1310720\left(1-\tan^2(x)\right)^5} - \frac{3\csc^5(x)\sec^{19}(x)}{5242880\left(1-\tan^2(x)\right)^4} + \frac{61\csc^5(x)\sec^{17}(x)}{15728640\left(1-\tan^2(x)\right)^3} - \frac{97\csc^5(x)\sec^{15}(x)}{10485760\left(1-\tan^2(x)\right)^2} + \frac{443\csc^5(x)\sec^{13}(x)}{6291456\left(1-\tan^2(x)\right)}$$

output
```
-637/8192*arctanh(2*cos(x)*sin(x))-3955/1048576*cot(x)-2131/3145728*cot(x)
^3-173/2621440*cot(x)^5+158879/1048576*tan(x)+128051/3145728*tan(x)^3+7861
7/5242880*tan(x)^5+4751/1048576*tan(x)^7+8501/9437184*tan(x)^9+983/1153433
6*tan(x)^11+1/1310720*csc(x)^5*sec(x)^21/(1-tan(x)^2)^5-3/5242880*csc(x)^5
*sec(x)^19/(1-tan(x)^2)^4+61/15728640*csc(x)^5*sec(x)^17/(1-tan(x)^2)^3-97
/10485760*csc(x)^5*sec(x)^15/(1-tan(x)^2)^2+443*csc(x)^5*sec(x)^13/(629145
6-6291456*tan(x)^2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.71 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.31

$$\int \frac{1}{(\cos(5x)+\sin(4x))^6}\,dx$$

$$= \frac{1568000\sqrt{3}\operatorname{arctanh}\left(\frac{-2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right) + 2240i\operatorname{RootSum}\left[i + \#1^3 - i\#1^6\&, \frac{2871766\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right) - 1435883i\log(\right.}{}\right.}{}$$

input
```
Integrate[(Cos[5*x] + Sin[4*x])^(-6),x]
```

output
```
(1568000*Sqrt[3]*ArcTanh[(-2 + Tan[x/2])/Sqrt[3]] + (2240*I)*RootSum[I + #
1^3 - I*#1^6 & , (2871766*ArcTan[Sin[x]/(Cos[x] - #1)] - (1435883*I)*Log[1
 - 2*Cos[x]*#1 + #1^2] - (5397130*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 - 269
8565*Log[1 - 2*Cos[x]*#1 + #1^2]*#1 - 7271532*ArcTan[Sin[x]/(Cos[x] - #1)]
*#1^2 + (3635766*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^2 + (5397130*I)*ArcTan[
Sin[x]/(Cos[x] - #1)]*#1^3 + 2698565*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3 + 28
71766*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^4 - (1435883*I)*Log[1 - 2*Cos[x]*#1
+ #1^2]*#1^4)/(I*#1^2 + 2*#1^5) & ] + (9*Sec[x]^5*(369912 + 683354970*Cos[
2*x] - 1397328660*Cos[4*x] - 2079944950*Cos[6*x] + 460950*Cos[8*x] - 43353
6*Cos[10*x] + 1040066515*Cos[12*x] + 698762430*Cos[14*x] - 341889585*Cos[1
6*x] - 108940*Cos[18*x] - 68318421*Cos[20*x] + 139743270*Cos[22*x] + 20796
0045*Cos[24*x] + 711810*Sin[x] - 2080097220*Sin[3*x] - 1397410614*Sin[5*x]
 + 683564550*Sin[7*x] + 465730*Sin[9*x] + 341626245*Sin[11*x] - 698684940*
Sin[13*x] - 1039886629*Sin[15*x] + 110220*Sin[17*x] - 98700*Sin[19*x] + 20
8013905*Sin[21*x] + 139765050*Sin[23*x] - 68398425*Sin[25*x]))/(-1 + 2*Cos
[2*x] - 2*Cos[4*x] + 2*Sin[x] - 2*Sin[3*x])^5)/5314410
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(4x)+\cos(5x))^6}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(\sin(4x)+\cos(5x))^6}dx$$

$$\Big\downarrow \text{ 4830}$$

$$2\int \frac{\left(\tan^2\left(\frac{x}{2}\right)+1\right)^{29}}{\left(-\tan^{10}\left(\frac{x}{2}\right)-8\tan^9\left(\frac{x}{2}\right)+45\tan^8\left(\frac{x}{2}\right)+48\tan^7\left(\frac{x}{2}\right)-210\tan^6\left(\frac{x}{2}\right)+210\tan^4\left(\frac{x}{2}\right)-48\tan^3\left(\frac{x}{2}\right)-45\tan\right.}$$

$$\Big\downarrow \text{ 2462}$$

$$2\int \left(\frac{65536\left(209\tan\left(\frac{x}{2}\right)-56\right)}{19683\left(\tan^2\left(\frac{x}{2}\right)-4\tan\left(\frac{x}{2}\right)+1\right)^6}-\frac{331328}{1594323\left(\tan^2\left(\frac{x}{2}\right)-4\tan\left(\frac{x}{2}\right)+1\right)}-\frac{1024\left(2225\tan^4\left(\frac{x}{2}\right)+31323\right.}{2187\left(\tan^6\left(\frac{x}{2}\right)+12\tan^5\left(\frac{x}{2}\right)\right.}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{39200\log\left(-\tan\left(\frac{x}{2}\right)-\sqrt{3}+2\right)}{177147\sqrt{3}}-\frac{39200\log\left(-\tan\left(\frac{x}{2}\right)+\sqrt{3}+2\right)}{177147\sqrt{3}}+\frac{56297367981513695706973755932672}{27}\right.$$

input
```
Int[(Cos[5*x] + Sin[4*x])^(-6),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4830
```
Int[(cos[(n_.)*((c_.) + (d_.)*(x_))]*(b_.) + (a_.)*sin[(m_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[2/d   Subst[Int[Simplify[TrigExpand[a*Sin[
2*m*ArcTan[x]] + b*Cos[2*n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[(1/2)*(c +
d*x)]], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && Intege
rQ[n]
```

Maple [A] (verified)

Time = 2257.52 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.01

method	result	size
parallelrisch	0	2
risch	Expression too large to display	494
default	Expression too large to display	580

input
```
int(1/(cos(5*x)+sin(4*x))^6,x,method=_RETURNVERBOSE)
```

output
```
0
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.21 (sec) , antiderivative size = 4649, normalized size of antiderivative = 24.47

$$\int \frac{1}{(\cos(5x) + \sin(4x))^6} \, dx = \text{Too large to display}$$

input
```
integrate(1/(cos(5*x)+sin(4*x))^6,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(\cos(5x) + \sin(4x))^6} \, dx = \text{Timed out}$$

input
```
integrate(1/(cos(5*x)+sin(4*x))**6,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(5x) + \sin(4x))^6} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(cos(5*x)+sin(4*x))^6,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(150) = 300$.

Time = 0.19 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.94

$$\int \frac{1}{(\cos(5x) + \sin(4x))^6} \, dx = \text{Too large to display}$$

input
```
integrate(1/(cos(5*x)+sin(4*x))^6,x, algorithm="giac")
```

output
```
-78400/531441*sqrt(3)*log(abs(-2*sqrt(3) + 2*tan(1/2*x) - 4)/abs(2*sqrt(3)
 + 2*tan(1/2*x) - 4)) - 2/295245*(570879245*tan(1/2*x)^49 + 23435482320*ta
n(1/2*x)^48 + 261760494200*tan(1/2*x)^47 - 1044525945800*tan(1/2*x)^46 - 3
0941381898172*tan(1/2*x)^45 - 8631530312440*tan(1/2*x)^44 + 16079637123166
80*tan(1/2*x)^43 + 633749329566680*tan(1/2*x)^42 - 47684055834309030*tan(1
/2*x)^41 + 26328493931909320*tan(1/2*x)^40 + 748820798895545640*tan(1/2*x)
^39 - 950770122126857720*tan(1/2*x)^38 - 6690753840451590220*tan(1/2*x)^37
 + 12335535845921782840*tan(1/2*x)^36 + 35561269336653928280*tan(1/2*x)^35
 - 88103415001950478680*tan(1/2*x)^34 - 109819358579340807325*tan(1/2*x)^3
3 + 390657633726427449480*tan(1/2*x)^32 + 155773364304800690480*tan(1/2*x)
^31 - 1129415837133505053648*tan(1/2*x)^30 + 134596391753975528840*tan(1/2
*x)^29 + 2170692932524957437520*tan(1/2*x)^28 - 1057481831338684890160*tan
(1/2*x)^27 - 2773487144406606245520*tan(1/2*x)^26 + 2243019453400555433964
*tan(1/2*x)^25 + 2290042927496543814480*tan(1/2*x)^24 - 273669099592372697
0160*tan(1/2*x)^23 - 1100216199124955722480*tan(1/2*x)^22 + 21506065009765
24488840*tan(1/2*x)^21 + 162387425897044906352*tan(1/2*x)^20 - 11220875887
21029229520*tan(1/2*x)^19 + 142875405732537734480*tan(1/2*x)^18 + 38905254
9763681952675*tan(1/2*x)^17 - 105589715296256443680*tan(1/2*x)^16 - 879751
05882599815720*tan(1/2*x)^15 + 34597964386356642840*tan(1/2*x)^14 + 123665
35412686729780*tan(1/2*x)^13 - 6542492768242677720*tan(1/2*x)^12 - 9613...
```

Mupad [B] (verification not implemented)

Time = 36.40 (sec) , antiderivative size = 1424, normalized size of antiderivative = 7.49

$$\int \frac{1}{(\cos(5x)+\sin(4x))^6}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(5*x) + sin(4*x))^6,x)
```

output
```
(78400*3^(1/2)*log(exp(x*1i)*27646681785151430519823444522041344 00i - (784
00*3^(1/2)*((78400*3^(1/2)*(exp(x*1i)*638336698300929854077909893928429748
748288i - (78400*3^(1/2)*(267007703491546346947879946801148921045319680*ex
p(x*1i) + (78400*3^(1/2)*((78400*3^(1/2)*((78400*3^(1/2)*(exp(x*1i)*130449
1175732155376976616258798333603226112i + 175744772433937284288990442630691
9947277760))/531441 - 825250767075807219521923887737795486133098496 0*exp(x
*1i) + 538644182134455576713606782902090866508777062 4i))/531441 - exp(x*1i
)*183264243132658712063172825700342639482450739 2i + 244272807471831843209 5
3992675402059931451392))/531441 + 191924369645357376435893207099242602792 2
22720i))/531441 + 160231630443098134997872331998304139214848 0))/531441 - 4
519411994710268730896802454229766635 52*exp(x*1i) + 180472251543563377241 93
79328408935727104i))/531441 - 129704685189129702224555472322560000 0))/5314
41 - (78400*3^(1/2)*log(exp(x*1i)*27646681785151430519823444522041344 00i -
 (78400*3^(1/2)*(451941199471026873089680245422976663552*exp(x*1i) + (7840
0*3^(1/2)*(exp(x*1i)*638336698300929854077909893928429748748288i + (78400*
3^(1/2)*(267007703491546346947879946801148921045319680*exp(x*1i) - (78400*
3^(1/2)*((78400*3^(1/2)*(825250767075807219521923887737795486133098496 0*ex
p(x*1i) + (78400*3^(1/2)*(exp(x*1i)*1304491175732155376976616258798333 6032
26112i + 1757447724339372842889904426306919947277760))/531441 - 5386441821
34455576713606782902090866508777062 4i))/531441 - exp(x*1i)*183264243132...
```

Reduce [F]

$$\int \frac{1}{(\cos(5x)+\sin(4x))^6}\,dx$$
$$= \int \frac{1}{\cos(5x)^6 + 6\cos(5x)^5\sin(4x) + 15\cos(5x)^4\sin(4x)^2 + 20\cos(5x)^3\sin(4x)^3 + 15\cos(5x)^2\sin(4x)^4}$$

input
```
int(1/(cos(5*x)+sin(4*x))^6,x)
```

output
```
int(1/(cos(5*x)**6 + 6*cos(5*x)**5*sin(4*x) + 15*cos(5*x)**4*sin(4*x)**2 +
 20*cos(5*x)**3*sin(4*x)**3 + 15*cos(5*x)**2*sin(4*x)**4 + 6*cos(5*x)*sin(
4*x)**5 + sin(4*x)**6),x)
```

3.19 $\int \frac{1}{\sin(x)+\sin(3x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \frac{1}{\sin(x)+\sin(3x)}\,dx=-\frac{1}{4}\text{arctanh}(\cos(x))+\frac{\sec(x)}{4}$$

output
```
-1/4*arctanh(cos(x))+1/4*sec(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \frac{1}{\sin(x)+\sin(3x)}\,dx=\frac{1}{4}\left(-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)+\sec(x)\right)$$

input
```
Integrate[(Sin[x] + Sin[3*x])^(-1),x]
```

output
```
(-Log[Cos[x/2]] + Log[Sin[x/2]] + Sec[x])/4
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 4824, 27, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin(x)+\sin(3x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(x)+\sin(3x)}dx$$
$$\Big\downarrow \text{4824}$$
$$-\int \frac{\sec^2(x)}{4\left(1-\cos^2(x)\right)}d\cos(x)$$
$$\Big\downarrow \text{27}$$
$$-\frac{1}{4}\int \frac{\sec^2(x)}{1-\cos^2(x)}d\cos(x)$$
$$\Big\downarrow \text{264}$$
$$\frac{1}{4}\left(\sec(x)-\int \frac{1}{1-\cos^2(x)}d\cos(x)\right)$$
$$\Big\downarrow \text{219}$$
$$\frac{1}{4}(\sec(x)-\text{arctanh}(\cos(x)))$$

input
```
Int[(Sin[x] + Sin[3*x])^(-1),x]
```

output
```
(-ArcTanh[Cos[x]] + Sec[x])/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4824
```
Int[((a_.)*sin[(m_.)*((c_.) + (d_.)*(x_))] + (b_.)*sin[(n_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Simplify[TrigExpand[a*
Sin[m*ArcCos[x]] + b*Sin[n*ArcCos[x]]]]^p/Sqrt[1 - x^2], x], x, Cos[c + d*x
]], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[(m - 1)/
2] && IntegerQ[(n - 1)/2]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

method	result	size
default	$\frac{1}{4\cos(x)}+\frac{\ln(-\cot(x)+\csc(x))}{4}$	18
risch	$\frac{e^{ix}}{2\,e^{2ix}+2}-\frac{\ln(e^{ix}+1)}{4}+\frac{\ln(e^{ix}-1)}{4}$	38

input
```
int(1/(sin(x)+sin(3*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4/cos(x)+1/4*ln(-cot(x)+csc(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(10) = 20$.

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.07

$$\int \frac{1}{\sin(x) + \sin(3x)}\,dx = -\frac{\cos{(x)}\log{\left(\frac{1}{2}\cos{(x)} + \frac{1}{2}\right)} - \cos{(x)}\log{\left(-\frac{1}{2}\cos{(x)} + \frac{1}{2}\right)} - 2}{8\cos{(x)}}$$

input
```
integrate(1/(sin(x)+sin(3*x)),x, algorithm="fricas")
```

output
```
-1/8*(cos(x)*log(1/2*cos(x) + 1/2) - cos(x)*log(-1/2*cos(x) + 1/2) - 2)/co
s(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(10) = 20$.

Time = 0.95 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.64

$$\int \frac{1}{\sin(x) + \sin(3x)}\,dx = \frac{\log{\left(\tan{\left(\frac{x}{2}\right)}\right)}\tan^{2}{\left(\frac{x}{2}\right)}}{4\tan^{2}{\left(\frac{x}{2}\right)} - 4} - \frac{\log{\left(\tan{\left(\frac{x}{2}\right)}\right)}}{4\tan^{2}{\left(\frac{x}{2}\right)} - 4} - \frac{2}{4\tan^{2}{\left(\frac{x}{2}\right)} - 4}$$

input
```
integrate(1/(sin(x)+sin(3*x)),x)
```

output
```
log(tan(x/2))*tan(x/2)**2/(4*tan(x/2)**2 - 4) - log(tan(x/2))/(4*tan(x/2)*
*2 - 4) - 2/(4*tan(x/2)**2 - 4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 8.43

$$\int \frac{1}{\sin(x)+\sin(3x)}\,dx = \frac{4\cos(2x)\cos(x) - \left(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1\right)\log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right) + \cdots}{8\left(\cos(2x)^2 + \cdots\right.}$$

input
```
integrate(1/(sin(x)+sin(3*x)),x, algorithm="maxima")
```

output
```
1/8*(4*cos(2*x)*cos(x) - (cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(co
s(x)^2 + sin(x)^2 + 2*cos(x) + 1) + (cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x)
+ 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 4*sin(2*x)*sin(x) + 4*cos(x
))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(10) = 20$.

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.36

$$\int \frac{1}{\sin(x)+\sin(3x)}\,dx = \frac{1}{2\left(\frac{\cos(x)-1}{\cos(x)+1}+1\right)} + \frac{1}{8}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

input
```
integrate(1/(sin(x)+sin(3*x)),x, algorithm="giac")
```

output
```
1/2/((cos(x) - 1)/(cos(x) + 1) + 1) + 1/8*log(-(cos(x) - 1)/(cos(x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sin(x)+\sin(3x)}\,dx = \frac{1}{4\cos(x)} - \frac{\operatorname{atanh}(\cos(x))}{4}$$

input
```
int(1/(sin(3*x) + sin(x)),x)
```

output
```
1/(4*cos(x)) - atanh(cos(x))/4
```

Reduce [F]

$$\int \frac{1}{\sin(x)+\sin(3x)}\,dx = \int \frac{1}{\sin(3x)+\sin(x)}dx$$

input
```
int(1/(sin(x)+sin(3*x)),x)
```

output
```
int(1/(sin(3*x) + sin(x)),x)
```

3.20 $\int \frac{1}{(\sin(x)+\sin(3x))^3} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 42

$$\int \frac{1}{(\sin(x)+\sin(3x))^3} \, dx = -\frac{7}{128}\operatorname{arctanh}(\cos(x)) + \frac{7\sec(x)}{128} + \frac{7\sec^3(x)}{384} + \frac{7\sec^5(x)}{640} - \frac{1}{128}\csc^2(x)\sec^5(x)$$

output
```
-7/128*arctanh(cos(x))+7/128*sec(x)+7/384*sec(x)^3+7/640*sec(x)^5-1/128*cs
c(x)^2*sec(x)^5
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 132 vs. 2(42) = 84.

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 3.14

$$\int \frac{1}{(\sin(x)+\sin(3x))^3} \, dx = -\frac{\csc^2(x)\left(-412+462\cos(2x)+700\cos(4x)+210\cos(6x)+525\cos(x)\log\left(\cos\left(\frac{x}{2}\right)\right)-105\cos(3x)\log\right.}{}$$

input
```
Integrate[(Sin[x] + Sin[3*x])^(-3),x]
```

output
```
-1/122880*(Csc[x]^2*(-412 + 462*Cos[2*x] + 700*Cos[4*x] + 210*Cos[6*x] + 5
25*Cos[x]*Log[Cos[x/2]] - 105*Cos[3*x]*Log[Cos[x/2]] - 315*Cos[5*x]*Log[Co
s[x/2]] - 105*Cos[7*x]*Log[Cos[x/2]] - 525*Cos[x]*Log[Sin[x/2]] + 105*Cos[
3*x]*Log[Sin[x/2]] + 315*Cos[5*x]*Log[Sin[x/2]] + 105*Cos[7*x]*Log[Sin[x/2
]])*Sec[x]^5)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {3042, 4824, 27, 253, 264, 264, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(x)+\sin(3x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(x)+\sin(3x))^3}dx$$

$$\Big\downarrow \text{4824}$$

$$-\int \frac{\sec^6(x)}{64\left(1-\cos^2(x)\right)^2}d\cos(x)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{64}\int \frac{\sec^6(x)}{\left(1-\cos^2(x)\right)^2}d\cos(x)$$

$$\Big\downarrow \text{253}$$

$$\frac{1}{64}\left(-\frac{7}{2}\int \frac{\sec^6(x)}{1-\cos^2(x)}d\cos(x)-\frac{\sec^5(x)}{2\left(1-\cos^2(x)\right)}\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{64}\left(-\frac{7}{2}\left(\int \frac{\sec^4(x)}{1-\cos^2(x)}d\cos(x)-\frac{\sec^5(x)}{5}\right)-\frac{\sec^5(x)}{2\left(1-\cos^2(x)\right)}\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{64}\left(-\frac{7}{2}\left(\int \frac{\sec^2(x)}{1-\cos^2(x)}d\cos(x)-\frac{1}{5}\sec^5(x)-\frac{\sec^3(x)}{3}\right)-\frac{\sec^5(x)}{2\left(1-\cos^2(x)\right)}\right)$$

$\Big\downarrow$ 264

$$\frac{1}{64}\left(-\frac{7}{2}\left(\int \frac{1}{1-\cos^2(x)}d\cos(x)-\frac{1}{5}\sec^5(x)-\frac{\sec^3(x)}{3}-\sec(x)\right)-\frac{\sec^5(x)}{2\left(1-\cos^2(x)\right)}\right)$$

$\Big\downarrow$ 219

$$\frac{1}{64}\left(-\frac{7}{2}\left(\text{arctanh}(\cos(x))-\frac{1}{5}\sec^5(x)-\frac{\sec^3(x)}{3}-\sec(x)\right)-\frac{\sec^5(x)}{2\left(1-\cos^2(x)\right)}\right)$$

input
```
Int[(Sin[x] + Sin[3*x])^(-3),x]
```

output
```
(-1/2*Sec[x]^5/(1 - Cos[x]^2) - (7*(ArcTanh[Cos[x]] - Sec[x] - Sec[x]^3/3
- Sec[x]^5/5))/2)/64
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4824
```
Int[((a_.)*sin[(m_.)*((c_.) + (d_.)*(x_))] + (b_.)*sin[(n_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Simplify[TrigExpand[a*
Sin[m*ArcCos[x]] + b*Sin[n*ArcCos[x]]]]^p/Sqrt[1 - x^2], x], x, Cos[c + d*x
]], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[(m - 1)/
2] && IntegerQ[(n - 1)/2]
```

Maple [A] (verified)

Time = 5.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

method	result	size
default	$\frac{1}{320 \sin(x)^{2} \cos(x)^{5}}+\frac{7}{960 \sin(x)^{2} \cos(x)^{3}}-\frac{7}{384 \sin(x)^{2} \cos(x)}+\frac{7}{128 \cos(x)}+\frac{7 \ln(-\cot(x)+\csc(x))}{128}$	48
risch	$\frac{105 \,\mathrm{e}^{13 i x}+350 \,\mathrm{e}^{11 i x}+231 \,\mathrm{e}^{9 i x}-412 \,\mathrm{e}^{7 i x}+231 \,\mathrm{e}^{5 i x}+350 \,\mathrm{e}^{3 i x}+105 \,\mathrm{e}^{i x}}{960 (\mathrm{e}^{2 i x}+1)^{5} (\mathrm{e}^{2 i x}-1)^{2}}-\frac{7 \ln(\mathrm{e}^{i x}+1)}{128}+\frac{7 \ln(\mathrm{e}^{i x}-1)}{128}$	92

input
```
int(1/(sin(x)+sin(3*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/320/sin(x)^2/cos(x)^5+7/960/sin(x)^2/cos(x)^3-7/384/sin(x)^2/cos(x)+7/12
8/cos(x)+7/128*ln(-cot(x)+csc(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(32) = 64.

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.79

$$\int \frac{1}{(\sin(x)+\sin(3x))^3}\,dx = \frac{210 \cos(x)^{6} - 140 \cos(x)^{4} - 28 \cos(x)^{2} - 105 \left(\cos(x)^{7} - \cos(x)^{5}\right) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) + 105 \left(\cos(x)^{7}\right.}{3840 \left(\cos(x)^{7} - \cos(x)^{5}\right)}$$

input
```
integrate(1/(sin(x)+sin(3*x))^3,x, algorithm="fricas")
```

output
```
1/3840*(210*cos(x)^6 - 140*cos(x)^4 - 28*cos(x)^2 - 105*(cos(x)^7 - cos(x)
^5)*log(1/2*cos(x) + 1/2) + 105*(cos(x)^7 - cos(x)^5)*log(-1/2*cos(x) + 1/
2) - 12)/(cos(x)^7 - cos(x)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(\sin(x)+\sin(3x))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(sin(x)+sin(3*x))**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2043 vs. $2(32) = 64$.

Time = 0.12 (sec) , antiderivative size = 2043, normalized size of antiderivative = 48.64

$$\int \frac{1}{(\sin(x)+\sin(3x))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(sin(x)+sin(3*x))^3,x, algorithm="maxima")
```

output
```
1/3840*(4*(105*cos(13*x) + 350*cos(11*x) + 231*cos(9*x) - 412*cos(7*x) + 2
31*cos(5*x) + 350*cos(3*x) + 105*cos(x))*cos(14*x) + 420*(3*cos(12*x) + co
s(10*x) - 5*cos(8*x) - 5*cos(6*x) + cos(4*x) + 3*cos(2*x) + 1)*cos(13*x) +
 12*(350*cos(11*x) + 231*cos(9*x) - 412*cos(7*x) + 231*cos(5*x) + 350*cos(
3*x) + 105*cos(x))*cos(12*x) + 1400*(cos(10*x) - 5*cos(8*x) - 5*cos(6*x) +
 cos(4*x) + 3*cos(2*x) + 1)*cos(11*x) + 4*(231*cos(9*x) - 412*cos(7*x) + 2
31*cos(5*x) + 350*cos(3*x) + 105*cos(x))*cos(10*x) - 924*(5*cos(8*x) + 5*c
os(6*x) - cos(4*x) - 3*cos(2*x) - 1)*cos(9*x) + 20*(412*cos(7*x) - 231*cos
(5*x) - 350*cos(3*x) - 105*cos(x))*cos(8*x) + 1648*(5*cos(6*x) - cos(4*x)
- 3*cos(2*x) - 1)*cos(7*x) - 140*(33*cos(5*x) + 50*cos(3*x) + 15*cos(x))*c
os(6*x) + 924*(cos(4*x) + 3*cos(2*x) + 1)*cos(5*x) + 140*(10*cos(3*x) + 3*
cos(x))*cos(4*x) + 1400*(3*cos(2*x) + 1)*cos(3*x) + 1260*cos(2*x)*cos(x) -
 105*(2*(3*cos(12*x) + cos(10*x) - 5*cos(8*x) - 5*cos(6*x) + cos(4*x) + 3*
cos(2*x) + 1)*cos(14*x) + cos(14*x)^2 + 6*(cos(10*x) - 5*cos(8*x) - 5*cos(
6*x) + cos(4*x) + 3*cos(2*x) + 1)*cos(12*x) + 9*cos(12*x)^2 - 2*(5*cos(8*x
) + 5*cos(6*x) - cos(4*x) - 3*cos(2*x) - 1)*cos(10*x) + cos(10*x)^2 + 10*(
5*cos(6*x) - cos(4*x) - 3*cos(2*x) - 1)*cos(8*x) + 25*cos(8*x)^2 - 10*(cos
(4*x) + 3*cos(2*x) + 1)*cos(6*x) + 25*cos(6*x)^2 + 2*(3*cos(2*x) + 1)*cos(
4*x) + cos(4*x)^2 + 9*cos(2*x)^2 + 2*(3*sin(12*x) + sin(10*x) - 5*sin(8*x)
 - 5*sin(6*x) + sin(4*x) + 3*sin(2*x))*sin(14*x) + sin(14*x)^2 + 6*(sin...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(32) = 64$.

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 3.02

$$\int \frac{1}{(\sin(x)+\sin(3x))^3}\,dx = -\frac{\left(\frac{14(\cos(x)-1)}{\cos(x)+1}-1\right)(\cos(x)+1)}{512(\cos(x)-1)} - \frac{\cos(x)-1}{512(\cos(x)+1)} + \frac{\frac{100(\cos(x)-1)}{\cos(x)+1}+\frac{170(\cos(x)-1)^2}{(\cos(x)+1)^2}+\frac{120(\cos(x)-1)^3}{(\cos(x)+1)^3}+\frac{45(\cos(x)-1)^4}{(\cos(x)+1)^4}+29}{240\left(\frac{\cos(x)-1}{\cos(x)+1}+1\right)^5} + \frac{7}{256}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

input
```
integrate(1/(sin(x)+sin(3*x))^3,x, algorithm="giac")
```

output
```
-1/512*(14*(cos(x) - 1)/(cos(x) + 1) - 1)*(cos(x) + 1)/(cos(x) - 1) - 1/51
2*(cos(x) - 1)/(cos(x) + 1) + 1/240*(100*(cos(x) - 1)/(cos(x) + 1) + 170*(
cos(x) - 1)^2/(cos(x) + 1)^2 + 120*(cos(x) - 1)^3/(cos(x) + 1)^3 + 45*(cos
(x) - 1)^4/(cos(x) + 1)^4 + 29)/((cos(x) - 1)/(cos(x) + 1) + 1)^5 + 7/256*
log(-(cos(x) - 1)/(cos(x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{1}{(\sin(x)+\sin(3x))^3}\,dx = \frac{-\frac{7\cos(x)^6}{128}+\frac{7\cos(x)^4}{192}+\frac{7\cos(x)^2}{960}+\frac{1}{320}}{\cos(x)^5-\cos(x)^7}-\frac{7\,\mathrm{atanh}(\cos(x))}{128}$$

input
```
int(1/(sin(3*x) + sin(x))^3,x)
```

output
```
((7*cos(x)^2)/960 + (7*cos(x)^4)/192 - (7*cos(x)^6)/128 + 1/320)/(cos(x)^5
 - cos(x)^7) - (7*atanh(cos(x)))/128
```

Reduce [F]

$$\int \frac{1}{(\sin(x)+\sin(3x))^3}\,dx = \int \frac{1}{\sin(3x)^3+3\sin(3x)^2\sin(x)+3\sin(3x)\sin(x)^2+\sin(x)^3}dx$$

input
```
int(1/(sin(x)+sin(3*x))^3,x)
```

output
```
int(1/(sin(3*x)**3 + 3*sin(3*x)**2*sin(x) + 3*sin(3*x)*sin(x)**2 + sin(x)*
*3),x)
```

3.21 $\int \frac{1}{(\sin(x)+\sin(3x))^5}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 70

$$\int \frac{1}{(\sin(x)+\sin(3x))^5}\,dx = -\frac{143\operatorname{arctanh}(\cos(x))}{8192}+\frac{143\sec(x)}{8192}+\frac{143\sec^3(x)}{24576}+\frac{143\sec^5(x)}{40960}+\frac{143\sec^7(x)}{57344}+\frac{143\sec^9(x)}{73728}-\frac{13\csc^2(x)\sec^9(x)}{8192}-\frac{\csc^4(x)\sec^9(x)}{4096}$$

output
```
-143/8192*arctanh(cos(x))+143/8192*sec(x)+143/24576*sec(x)^3+143/40960*sec
(x)^5+143/57344*sec(x)^7+143/73728*sec(x)^9-13/8192*csc(x)^2*sec(x)^9-1/40
96*csc(x)^4*sec(x)^9
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 231 vs. 2(70) = 140.

Time = 0.43 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.30

$$\int \frac{1}{(\sin(x)+\sin(3x))^5}\,dx = -\frac{\csc^4(x)\left(1726400\cos(2x)+2730442\cos(4x)+3\left(-404948+205920\cos(6x)-236236\cos(8x)-1601\right.\right.}{}$$

input
```
Integrate[(Sin[x] + Sin[3*x])^(-5),x]
```

output
```
-1/10569646080*(Csc[x]^4*(1726400*Cos[2*x] + 2730442*Cos[4*x] + 3*(-404948
 + 205920*Cos[6*x] - 236236*Cos[8*x] - 160160*Cos[10*x] - 30030*Cos[12*x]
+ 540540*Cos[x]*Log[Cos[x/2]] - 135135*Cos[3*x]*Log[Cos[x/2]] - 435435*Cos
[5*x]*Log[Cos[x/2]] - 150150*Cos[7*x]*Log[Cos[x/2]] + 90090*Cos[9*x]*Log[C
os[x/2]] + 75075*Cos[11*x]*Log[Cos[x/2]] + 15015*Cos[13*x]*Log[Cos[x/2]] -
 540540*Cos[x]*Log[Sin[x/2]] + 135135*Cos[3*x]*Log[Sin[x/2]] + 435435*Cos[
5*x]*Log[Sin[x/2]] + 150150*Cos[7*x]*Log[Sin[x/2]] - 90090*Cos[9*x]*Log[Si
n[x/2]] - 75075*Cos[11*x]*Log[Sin[x/2]] - 15015*Cos[13*x]*Log[Sin[x/2]]))*
Sec[x]^9)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.222$, Rules used = {3042, 4824, 27, 253, 253, 264, 264, 264, 264, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(x)+\sin(3x))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(x)+\sin(3x))^5}dx$$

$$\Big\downarrow \text{4824}$$

$$-\int \frac{\sec^{10}(x)}{1024\left(1-\cos^2(x)\right)^3}d\cos(x)$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sec^{10}(x)}{\left(1-\cos^2(x)\right)^3}d\cos(x)}{1024}$$

$$\Big\downarrow \text{253}$$

$$\frac{-\frac{13}{4}\int \frac{\sec^{10}(x)}{(1-\cos^2(x))^2}d\cos(x)-\frac{\sec^9(x)}{4(1-\cos^2(x))^2}}{1024}$$

$$\Big\downarrow \ 253$$

$$\frac{-\frac{13}{4}\left(\frac{11}{2}\int \frac{\sec^{10}(x)}{1-\cos^2(x)}d\cos(x)+\frac{\sec^9(x)}{2(1-\cos^2(x))}\right)-\frac{\sec^9(x)}{4(1-\cos^2(x))^2}}{1024}$$

$$\Big\downarrow \ 264$$

$$\frac{-\frac{13}{4}\left(\frac{11}{2}\left(\int \frac{\sec^8(x)}{1-\cos^2(x)}d\cos(x)-\frac{\sec^9(x)}{9}\right)+\frac{\sec^9(x)}{2(1-\cos^2(x))}\right)-\frac{\sec^9(x)}{4(1-\cos^2(x))^2}}{1024}$$

$$\Big\downarrow \ 264$$

$$\frac{-\frac{13}{4}\left(\frac{11}{2}\left(\int \frac{\sec^6(x)}{1-\cos^2(x)}d\cos(x)-\frac{1}{9}\sec^9(x)-\frac{\sec^7(x)}{7}\right)+\frac{\sec^9(x)}{2(1-\cos^2(x))}\right)-\frac{\sec^9(x)}{4(1-\cos^2(x))^2}}{1024}$$

$$\Big\downarrow \ 264$$

$$\frac{-\frac{13}{4}\left(\frac{11}{2}\left(\int \frac{\sec^4(x)}{1-\cos^2(x)}d\cos(x)-\frac{1}{9}\sec^9(x)-\frac{\sec^7(x)}{7}-\frac{\sec^5(x)}{5}\right)+\frac{\sec^9(x)}{2(1-\cos^2(x))}\right)-\frac{\sec^9(x)}{4(1-\cos^2(x))^2}}{1024}$$

$$\Big\downarrow \ 264$$

$$\frac{-\frac{13}{4}\left(\frac{11}{2}\left(\int \frac{\sec^2(x)}{1-\cos^2(x)}d\cos(x)-\frac{1}{9}\sec^9(x)-\frac{\sec^7(x)}{7}-\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}\right)+\frac{\sec^9(x)}{2(1-\cos^2(x))}\right)-\frac{\sec^9(x)}{4(1-\cos^2(x))^2}}{1024}$$

$$\Big\downarrow \ 264$$

$$\frac{-\frac{13}{4}\left(\frac{11}{2}\left(\int \frac{1}{1-\cos^2(x)}d\cos(x)-\frac{1}{9}\sec^9(x)-\frac{\sec^7(x)}{7}-\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}-\sec(x)\right)+\frac{\sec^9(x)}{2(1-\cos^2(x))}\right)-\frac{\sec^9(x)}{4(1-\cos^2(x))^2}}{1024}$$

$$\Big\downarrow \ 219$$

$$\frac{-\frac{13}{4}\left(\frac{11}{2}\left(\text{arctanh}(\cos(x))-\frac{1}{9}\sec^9(x)-\frac{\sec^7(x)}{7}-\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}-\sec(x)\right)+\frac{\sec^9(x)}{2(1-\cos^2(x))}\right)-\frac{\sec^9(x)}{4(1-\cos^2(x))^2}}{1024}$$

input
```
Int[(Sin[x] + Sin[3*x])^(-5),x]
```

output
```
(-1/4*Sec[x]^9/(1 - Cos[x]^2)^2 - (13*(Sec[x]^9/(2*(1 - Cos[x]^2)) + (11*(
ArcTanh[Cos[x]] - Sec[x] - Sec[x]^3/3 - Sec[x]^5/5 - Sec[x]^7/7 - Sec[x]^9
/9))/2))/4)/1024
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4824
```
Int[((a_.)*sin[(m_.)*((c_.) + (d_.)*(x_))] + (b_.)*sin[(n_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Simplify[TrigExpand[a*
Sin[m*ArcCos[x]] + b*Sin[n*ArcCos[x]]]]^p/Sqrt[1 - x^2], x], x, Cos[c + d*x
]], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[(m - 1)/
2] && IntegerQ[(n - 1)/2]
```

Maple [A] (verified)

Time = 115.48 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.03

method	result
parallelrisch	0
default	$\frac{1}{9216\sin(x)^4\cos(x)^9}+\frac{13}{64512\sin(x)^4\cos(x)^7}+\frac{143}{322560\sin(x)^4\cos(x)^5}-\frac{143}{143360\sin(x)^4\cos(x)^3}+\frac{143}{61440\sin(x)^2\cos(}$
risch	$\frac{45045\,\mathrm{e}^{25ix}+240240\,\mathrm{e}^{23ix}+354354\,\mathrm{e}^{21ix}-308880\,\mathrm{e}^{19ix}-1365221\,\mathrm{e}^{17ix}-863200\,\mathrm{e}^{15ix}+1214844\,\mathrm{e}^{13ix}-863200\,\mathrm{e}^{11ix}-136522}{1290240(\mathrm{e}^{2ix}+1)^9(\mathrm{e}^{2ix}-1)^4}$

input
```
int(1/(sin(x)+sin(3*x))^5,x,method=_RETURNVERBOSE)
```

output
```
0
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.50

$$\int \frac{1}{(\sin(x)+\sin(3x))^5}\,dx = \frac{90090\cos(x)^{12}-150150\cos(x)^{10}+48048\cos(x)^8+6864\cos(x)^6+2288\cos(x)^4+1040\cos(x)^2-}{5160960}$$

input
```
integrate(1/(sin(x)+sin(3*x))^5,x, algorithm="fricas")
```

output
```
1/5160960*(90090*cos(x)^12 - 150150*cos(x)^10 + 48048*cos(x)^8 + 6864*cos(
x)^6 + 2288*cos(x)^4 + 1040*cos(x)^2 - 45045*(cos(x)^13 - 2*cos(x)^11 + co
s(x)^9)*log(1/2*cos(x) + 1/2) + 45045*(cos(x)^13 - 2*cos(x)^11 + cos(x)^9)
*log(-1/2*cos(x) + 1/2) + 560)/(cos(x)^13 - 2*cos(x)^11 + cos(x)^9)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(\sin(x) + \sin(3x))^5} \, dx = \text{Timed out}$$

input
```
integrate(1/(sin(x)+sin(3*x))**5,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6347 vs. $2(54) = 108$.

Time = 0.60 (sec) , antiderivative size = 6347, normalized size of antiderivative = 90.67

$$\int \frac{1}{(\sin(x) + \sin(3x))^5} \, dx = \text{Too large to display}$$

input
```
integrate(1/(sin(x)+sin(3*x))^5,x, algorithm="maxima")
```

output
```
1/5160960*(4*(45045*cos(25*x) + 240240*cos(23*x) + 354354*cos(21*x) - 3088
80*cos(19*x) - 1365221*cos(17*x) - 863200*cos(15*x) + 1214844*cos(13*x) -
863200*cos(11*x) - 1365221*cos(9*x) - 308880*cos(7*x) + 354354*cos(5*x) +
240240*cos(3*x) + 45045*cos(x))*cos(26*x) + 180180*(5*cos(24*x) + 6*cos(22
*x) - 10*cos(20*x) - 29*cos(18*x) - 9*cos(16*x) + 36*cos(14*x) + 36*cos(12
*x) - 9*cos(10*x) - 29*cos(8*x) - 10*cos(6*x) + 6*cos(4*x) + 5*cos(2*x) +
1)*cos(25*x) + 20*(240240*cos(23*x) + 354354*cos(21*x) - 308880*cos(19*x)
- 1365221*cos(17*x) - 863200*cos(15*x) + 1214844*cos(13*x) - 863200*cos(11
*x) - 1365221*cos(9*x) - 308880*cos(7*x) + 354354*cos(5*x) + 240240*cos(3*
x) + 45045*cos(x))*cos(24*x) + 960960*(6*cos(22*x) - 10*cos(20*x) - 29*cos
(18*x) - 9*cos(16*x) + 36*cos(14*x) + 36*cos(12*x) - 9*cos(10*x) - 29*cos(
8*x) - 10*cos(6*x) + 6*cos(4*x) + 5*cos(2*x) + 1)*cos(23*x) + 24*(354354*c
os(21*x) - 308880*cos(19*x) - 1365221*cos(17*x) - 863200*cos(15*x) + 12148
44*cos(13*x) - 863200*cos(11*x) - 1365221*cos(9*x) - 308880*cos(7*x) + 354
354*cos(5*x) + 240240*cos(3*x) + 45045*cos(x))*cos(22*x) - 1417416*(10*cos
(20*x) + 29*cos(18*x) + 9*cos(16*x) - 36*cos(14*x) - 36*cos(12*x) + 9*cos(
10*x) + 29*cos(8*x) + 10*cos(6*x) - 6*cos(4*x) - 5*cos(2*x) - 1)*cos(21*x)
 + 40*(308880*cos(19*x) + 1365221*cos(17*x) + 863200*cos(15*x) - 1214844*c
os(13*x) + 863200*cos(11*x) + 1365221*cos(9*x) + 308880*cos(7*x) - 354354*
cos(5*x) - 240240*cos(3*x) - 45045*cos(x))*cos(20*x) + 1235520*(29*cos(...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(54) = 108$.

Time = 0.11 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.04

$$\int \frac{1}{(\sin(x)+\sin(3x))^5}\,dx = \frac{\left(\frac{48(\cos(x)-1)}{\cos(x)+1} - \frac{858(\cos(x)-1)^2}{(\cos(x)+1)^2} - 1\right)(\cos(x)+1)^2}{65536(\cos(x)-1)^2} - \frac{3(\cos(x)-1)}{4096(\cos(x)+1)} + \frac{(\cos(x)-1)^2}{65536(\cos(x)+1)^2} + \frac{\frac{45882(\cos(x)-1)}{\cos(x)+1} + \frac{161478(\cos(x)-1)^2}{(\cos(x)+1)^2} + \frac{326802(\cos(x)-1)^3}{(\cos(x)+1)^3} + \frac{430668(\cos(x)-1)^4}{(\cos(x)+1)^4} + \frac{366030(\cos(x)-1)^5}{(\cos(x)+1)^5} + \frac{204330(\cos(x)-1)^6}{(\cos(x)+1)^6} \cdots}{161280\left(\frac{\cos(x)-1}{\cos(x)+1}+1\right)^9} + \frac{143}{16384}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

input
```
integrate(1/(sin(x)+sin(3*x))^5,x, algorithm="giac")
```

output
```
1/65536*(48*(cos(x) - 1)/(cos(x) + 1) - 858*(cos(x) - 1)^2/(cos(x) + 1)^2
- 1)*(cos(x) + 1)^2/(cos(x) - 1)^2 - 3/4096*(cos(x) - 1)/(cos(x) + 1) + 1/
65536*(cos(x) - 1)^2/(cos(x) + 1)^2 + 1/161280*(45882*(cos(x) - 1)/(cos(x)
 + 1) + 161478*(cos(x) - 1)^2/(cos(x) + 1)^2 + 326802*(cos(x) - 1)^3/(cos(
x) + 1)^3 + 430668*(cos(x) - 1)^4/(cos(x) + 1)^4 + 366030*(cos(x) - 1)^5/(
cos(x) + 1)^5 + 204330*(cos(x) - 1)^6/(cos(x) + 1)^6 + 66150*(cos(x) - 1)^
7/(cos(x) + 1)^7 + 11025*(cos(x) - 1)^8/(cos(x) + 1)^8 + 6323)/((cos(x) -
1)/(cos(x) + 1) + 1)^9 + 143/16384*log(-(cos(x) - 1)/(cos(x) + 1))
```

Mupad [B] (verification not implemented)

Time = 22.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{1}{(\sin(x)+\sin(3x))^5}\,dx = \frac{\frac{143\cos(x)^{12}}{8192}-\frac{715\cos(x)^{10}}{24576}+\frac{143\cos(x)^8}{15360}+\frac{143\cos(x)^6}{107520}+\frac{143\cos(x)^4}{322560}+\frac{13\cos(x)^2}{64512}+\frac{1}{9216}}{\cos(x)^{13}-2\cos(x)^{11}+\cos(x)^9} - \frac{143\,\mathrm{atanh}(\cos(x))}{8192}$$

input
```
int(1/(sin(3*x) + sin(x))^5,x)
```

output
```
((13*cos(x)^2)/64512 + (143*cos(x)^4)/322560 + (143*cos(x)^6)/107520 + (14
3*cos(x)^8)/15360 - (715*cos(x)^10)/24576 + (143*cos(x)^12)/8192 + 1/9216)
/(cos(x)^9 - 2*cos(x)^11 + cos(x)^13) - (143*atanh(cos(x)))/8192
```

Reduce [F]

$$\int \frac{1}{(\sin(x)+\sin(3x))^5}\,dx = \int \frac{1}{\sin(3x)^5+5\sin(3x)^4\sin(x)+10\sin(3x)^3\sin(x)^2+10\sin(3x)^2\sin(x)^3+5\sin(3x)\sin(x)^4+\sin(x$$

input
```
int(1/(sin(x)+sin(3*x))^5,x)
```

output
```
int(1/(sin(3*x)**5 + 5*sin(3*x)**4*sin(x) + 10*sin(3*x)**3*sin(x)**2 + 10*
sin(3*x)**2*sin(x)**3 + 5*sin(3*x)*sin(x)**4 + sin(x)**5),x)
```

3.22 $\int \frac{1}{(\sin(x)+\sin(3x))^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 21

$$\int \frac{1}{(\sin(x)+\sin(3x))^2} dx = -\frac{\cot(x)}{16}+\frac{\tan(x)}{8}+\frac{\tan^3(x)}{48}$$

output
```
-1/16*cot(x)+1/8*tan(x)+1/48*tan(x)^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\sin(x)+\sin(3x))^2} dx = -\frac{1}{48}(2\cos(2x)+\cos(4x))\csc(x)\sec^3(x)$$

input
```
Integrate[(Sin[x] + Sin[3*x])^(-2),x]
```

output
```
-1/48*((2*Cos[2*x] + Cos[4*x])*Csc[x]*Sec[x]^3)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4822, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(x)+\sin(3x))^2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(\sin(x)+\sin(3x))^2}dx$$

$$\Big\downarrow \text{ 4822}$$

$$\int \frac{1}{16}\left(\tan^2(x)+1\right)^2\cot^2(x)d\tan(x)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{16}\int \cot^2(x)\left(\tan^2(x)+1\right)^2d\tan(x)$$

$$\Big\downarrow \text{ 244}$$

$$\frac{1}{16}\int \left(\cot^2(x)+\tan^2(x)+2\right)d\tan(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{16}\left(\frac{\tan^3(x)}{3}+2\tan(x)-\cot(x)\right)$$

input
```
Int[(Sin[x] + Sin[3*x])^(-2),x]
```

output
```
(-Cot[x] + 2*Tan[x] + Tan[x]^3/3)/16
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4822
```
Int[((a_.)*sin[(m_.)*((c_.) + (d_.)*(x_))] + (b_.)*sin[(n_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Sin[
m*ArcTan[x]] + b*Sin[n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[c + d*x]], x] /
; FreeQ[{a, b, c, d}, x] && ILtQ[p/2, 0] && IntegerQ[(m - 1)/2] && IntegerQ
[(n - 1)/2]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{1}{48\cos(x)^3\sin(x)}+\frac{1}{12\sin(x)\cos(x)}-\frac{\cot(x)}{6}$	26
risch	$-\frac{i(2\,e^{2ix}+1)}{3(e^{2ix}+1)^3(e^{2ix}-1)}$	31

input
```
int(1/(sin(x)+sin(3*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/48/cos(x)^3/sin(x)+1/12/sin(x)/cos(x)-1/6*cot(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{1}{(\sin(x)+\sin(3x))^2}\,dx = -\frac{8\cos(x)^4 - 4\cos(x)^2 - 1}{48\cos(x)^3\sin(x)}$$

input
```
integrate(1/(sin(x)+sin(3*x))^2,x, algorithm="fricas")
```

output
```
-1/48*(8*cos(x)^4 - 4*cos(x)^2 - 1)/(cos(x)^3*sin(x))
```

Sympy [F]

$$\int \frac{1}{(\sin(x)+\sin(3x))^2}\,dx = \int \frac{1}{(\sin(x)+\sin(3x))^2}\,dx$$

input
```
integrate(1/(sin(x)+sin(3*x))**2,x)
```

output
```
Integral((sin(x) + sin(3*x))**(-2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(15) = 30.

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 7.86

$$\int \frac{1}{(\sin(x)+\sin(3x))^2}\,dx =$$
$$-\frac{(2\cos(2x)+1)\sin(8x)+2(2\cos(}{3(2(2\cos(6x)-2\cos(2x)-1)\cos(8x)+\cos(8x)^2-4(2\cos(2x)+1)\cos(6x)+4\cos(6x)^2+}$$

input
```
integrate(1/(sin(x)+sin(3*x))^2,x, algorithm="maxima")
```

output
```
-1/3*((2*cos(2*x) + 1)*sin(8*x) + 2*(2*cos(2*x) + 1)*sin(6*x) - 2*cos(8*x)
*sin(2*x) - 4*cos(6*x)*sin(2*x))/(2*(2*cos(6*x) - 2*cos(2*x) - 1)*cos(8*x)
 + cos(8*x)^2 - 4*(2*cos(2*x) + 1)*cos(6*x) + 4*cos(6*x)^2 + 4*cos(2*x)^2
+ 4*(sin(6*x) - sin(2*x))*sin(8*x) + sin(8*x)^2 + 4*sin(6*x)^2 - 8*sin(6*x
)*sin(2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{(\sin(x)+\sin(3x))^2}\,dx = \frac{1}{48}\tan(x)^3 - \frac{1}{16\tan(x)} + \frac{1}{8}\tan(x)$$

input
```
integrate(1/(sin(x)+sin(3*x))^2,x, algorithm="giac")
```

output
```
1/48*tan(x)^3 - 1/16/tan(x) + 1/8*tan(x)
```

Mupad [B] (verification not implemented)

Time = 23.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{1}{(\sin(x)+\sin(3x))^2}\,dx = \frac{-8\cos(x)^4+4\cos(x)^2+1}{48\cos(x)^3\sin(x)}$$

input
```
int(1/(sin(3*x) + sin(x))^2,x)
```

output
```
(4*cos(x)^2 - 8*cos(x)^4 + 1)/(48*cos(x)^3*sin(x))
```

Reduce [F]

$$\int \frac{1}{(\sin(x)+\sin(3x))^2}\,dx = \int \frac{1}{\sin{(3x)}^2+2\sin{(3x)}\sin{(x)}+\sin{(x)}^2}dx$$

input
```
int(1/(sin(x)+sin(3*x))^2,x)
```

output
```
int(1/(sin(3*x)**2 + 2*sin(3*x)*sin(x) + sin(x)**2),x)
```

3.23 $\int \frac{1}{(\sin(x)+\sin(3x))^4}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 45

$$\int \frac{1}{(\sin(x)+\sin(3x))^4}\,dx = -\frac{5\cot(x)}{256}-\frac{\cot^3(x)}{768}+\frac{5\tan(x)}{128}+\frac{5\tan^3(x)}{384}+\frac{\tan^5(x)}{256}+\frac{\tan^7(x)}{1792}$$

output
```
-5/256*cot(x)-1/768*cot(x)^3+5/128*tan(x)+5/384*tan(x)^3+1/256*tan(x)^5+1/
1792*tan(x)^7
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{1}{(\sin(x)+\sin(3x))^4}\,dx = \frac{(-14\cos(2x)-8\cos(4x)+3\cos(6x)+4\cos(8x)+\cos(10x))\csc^3(x)\sec^7(x)}{10752}$$

input
```
Integrate[(Sin[x] + Sin[3*x])^(-4),x]
```

output
```
((-14*Cos[2*x] - 8*Cos[4*x] + 3*Cos[6*x] + 4*Cos[8*x] + Cos[10*x])*Csc[x]^
3*Sec[x]^7)/10752
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4822, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(x)+\sin(3x))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(x)+\sin(3x))^4}dx$$

$$\Big\downarrow \text{4822}$$

$$\int \frac{1}{256}\left(\tan^2(x)+1\right)^5\cot^4(x)d\tan(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{256}\int \cot^4(x)\left(\tan^2(x)+1\right)^5d\tan(x)$$

$$\Big\downarrow \text{244}$$

$$\frac{1}{256}\int \left(\tan^6(x)+5\tan^4(x)+10\tan^2(x)+\cot^4(x)+5\cot^2(x)+10\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{256}\left(\frac{\tan^7(x)}{7}+\tan^5(x)+\frac{10\tan^3(x)}{3}+10\tan(x)-\frac{\cot^3(x)}{3}-5\cot(x)\right)$$

input
```
Int[(Sin[x] + Sin[3*x])^(-4),x]
```

output
```
(-5*Cot[x] - Cot[x]^3/3 + 10*Tan[x] + (10*Tan[x]^3)/3 + Tan[x]^5 + Tan[x]^
7/7)/256
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4822
```
Int[((a_.)*sin[(m_.)*((c_.) + (d_.)*(x_))] + (b_.)*sin[(n_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Sin[
m*ArcTan[x]] + b*Sin[n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[c + d*x]], x] /
; FreeQ[{a, b, c, d}, x] && ILtQ[p/2, 0] && IntegerQ[(m - 1)/2] && IntegerQ
[(n - 1)/2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 26.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

method	result	size
risch	$\frac{2i\left(14\,\mathrm{e}^{8ix}+8\,\mathrm{e}^{6ix}-3\,\mathrm{e}^{4ix}-4\,\mathrm{e}^{2ix}-1\right)}{21\left(\mathrm{e}^{2ix}+1\right)^{7}\left(\mathrm{e}^{2ix}-1\right)^{3}}$	52
default	$\frac{1}{1792\sin\left(x\right)^{3}\cos\left(x\right)^{7}}+\frac{1}{896\sin\left(x\right)^{3}\cos\left(x\right)^{5}}+\frac{1}{336\sin\left(x\right)^{3}\cos\left(x\right)^{3}}-\frac{1}{168\sin\left(x\right)^{3}\cos\left(x\right)}+\frac{1}{42\sin\left(x\right)\cos\left(x\right)}-\frac{\cot\left(x\right)}{21}$	56

input
```
int(1/(sin(x)+sin(3*x))^4,x,method=_RETURNVERBOSE)
```

output
```
2/21*I*(14*exp(8*I*x)+8*exp(6*I*x)-3*exp(4*I*x)-4*exp(2*I*x)-1)/(exp(2*I*x
)+1)^7/(exp(2*I*x)-1)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int \frac{1}{(\sin(x)+\sin(3x))^4}\,dx = -\frac{256\,\cos(x)^{10}-384\,\cos(x)^{8}+96\,\cos(x)^{6}+16\,\cos(x)^{4}+6\,\cos(x)^{2}+3}{5376\,\left(\cos(x)^{9}-\cos(x)^{7}\right)\sin(x)}$$

input
```
integrate(1/(sin(x)+sin(3*x))^4,x, algorithm="fricas")
```

output
```
-1/5376*(256*cos(x)^10 - 384*cos(x)^8 + 96*cos(x)^6 + 16*cos(x)^4 + 6*cos(
x)^2 + 3)/((cos(x)^9 - cos(x)^7)*sin(x))
```

Sympy [F]

$$\int \frac{1}{(\sin(x)+\sin(3x))^4}\,dx = \int \frac{1}{(\sin(x)+\sin(3x))^4}\,dx$$

input
```
integrate(1/(sin(x)+sin(3*x))**4,x)
```

output
```
Integral((sin(x) + sin(3*x))**(-4), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1016 vs. $2(33) = 66$.

Time = 0.04 (sec) , antiderivative size = 1016, normalized size of antiderivative = 22.58

$$\int \frac{1}{(\sin(x)+\sin(3x))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(sin(x)+sin(3*x))^4,x, algorithm="maxima")
```

output
```
-2/21*((14*sin(8*x) + 8*sin(6*x) - 3*sin(4*x) - 4*sin(2*x))*cos(20*x) + 4*
(14*sin(8*x) + 8*sin(6*x) - 3*sin(4*x) - 4*sin(2*x))*cos(18*x) + 3*(14*sin
(8*x) + 8*sin(6*x) - 3*sin(4*x) - 4*sin(2*x))*cos(16*x) - 8*(14*sin(8*x) +
 8*sin(6*x) - 3*sin(4*x) - 4*sin(2*x))*cos(14*x) - 14*(14*sin(8*x) + 8*sin
(6*x) - 3*sin(4*x) - 4*sin(2*x))*cos(12*x) - (14*cos(8*x) + 8*cos(6*x) - 3
*cos(4*x) - 4*cos(2*x) - 1)*sin(20*x) - 4*(14*cos(8*x) + 8*cos(6*x) - 3*co
s(4*x) - 4*cos(2*x) - 1)*sin(18*x) - 3*(14*cos(8*x) + 8*cos(6*x) - 3*cos(4
*x) - 4*cos(2*x) - 1)*sin(16*x) + 8*(14*cos(8*x) + 8*cos(6*x) - 3*cos(4*x)
 - 4*cos(2*x) - 1)*sin(14*x) + 14*(14*cos(8*x) + 8*cos(6*x) - 3*cos(4*x) -
 4*cos(2*x) - 1)*sin(12*x))/(2*(4*cos(18*x) + 3*cos(16*x) - 8*cos(14*x) -
14*cos(12*x) + 14*cos(8*x) + 8*cos(6*x) - 3*cos(4*x) - 4*cos(2*x) - 1)*cos
(20*x) + cos(20*x)^2 + 8*(3*cos(16*x) - 8*cos(14*x) - 14*cos(12*x) + 14*co
s(8*x) + 8*cos(6*x) - 3*cos(4*x) - 4*cos(2*x) - 1)*cos(18*x) + 16*cos(18*x
)^2 - 6*(8*cos(14*x) + 14*cos(12*x) - 14*cos(8*x) - 8*cos(6*x) + 3*cos(4*x
) + 4*cos(2*x) + 1)*cos(16*x) + 9*cos(16*x)^2 + 16*(14*cos(12*x) - 14*cos(
8*x) - 8*cos(6*x) + 3*cos(4*x) + 4*cos(2*x) + 1)*cos(14*x) + 64*cos(14*x)^
2 - 28*(14*cos(8*x) + 8*cos(6*x) - 3*cos(4*x) - 4*cos(2*x) - 1)*cos(12*x)
+ 196*cos(12*x)^2 + 28*(8*cos(6*x) - 3*cos(4*x) - 4*cos(2*x) - 1)*cos(8*x)
 + 196*cos(8*x)^2 - 16*(3*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 64*cos(6*x
)^2 + 6*(4*cos(2*x) + 1)*cos(4*x) + 9*cos(4*x)^2 + 16*cos(2*x)^2 + 2*(4...
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{1}{(\sin(x)+\sin(3x))^4}\,dx = \frac{1}{1792}\tan(x)^7 + \frac{1}{256}\tan(x)^5 + \frac{5}{384}\tan(x)^3 - \frac{15\tan(x)^2+1}{768\tan(x)^3} + \frac{5}{128}\tan(x)$$

input
```
integrate(1/(sin(x)+sin(3*x))^4,x, algorithm="giac")
```

output
```
1/1792*tan(x)^7 + 1/256*tan(x)^5 + 5/384*tan(x)^3 - 1/768*(15*tan(x)^2 + 1
)/tan(x)^3 + 5/128*tan(x)
```

Mupad [B] (verification not implemented)

Time = 21.53 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{1}{(\sin(x)+\sin(3x))^4}\,dx = \frac{5\tan(x)}{128} + \frac{5\tan(x)^3}{384} + \frac{\tan(x)^5}{256} + \frac{\tan(x)^7}{1792} - \frac{\frac{5\tan(x)^2}{256}+\frac{1}{768}}{\tan(x)^3}$$

input
```
int(1/(sin(3*x) + sin(x))^4,x)
```

output
```
(5*tan(x))/128 + (5*tan(x)^3)/384 + tan(x)^5/256 + tan(x)^7/1792 - ((5*tan
(x)^2)/256 + 1/768)/tan(x)^3
```

Reduce [F]

$$\int \frac{1}{(\sin(x)+\sin(3x))^4}\,dx = \int \frac{1}{\sin(3x)^4+4\sin(3x)^3\sin(x)+6\sin(3x)^2\sin(x)^2+4\sin(3x)\sin(x)^3+\sin(x)^4}dx$$

input
```
int(1/(sin(x)+sin(3*x))^4,x)
```

output
```
int(1/(sin(3*x)**4 + 4*sin(3*x)**3*sin(x) + 6*sin(3*x)**2*sin(x)**2 + 4*si
n(3*x)*sin(x)**3 + sin(x)**4),x)
```

3.24 $\int \frac{1}{(\sin(x)+\sin(3x))^6}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 69

$$\int \frac{1}{(\sin(x)+\sin(3x))^6}\,dx = -\frac{7\cot(x)}{1024}-\frac{\cot^3(x)}{1536}-\frac{\cot^5(x)}{20480}+\frac{7\tan(x)}{512}+\frac{35\tan^3(x)}{6144}+\frac{7\tan^5(x)}{2560}+\frac{\tan^7(x)}{1024}+\frac{\tan^9(x)}{4608}+\frac{\tan^{11}(x)}{45056}$$

output
```
-7/1024*cot(x)-1/1536*cot(x)^3-1/20480*cot(x)^5+7/512*tan(x)+35/6144*tan(x
)^3+7/2560*tan(x)^5+1/1024*tan(x)^7+1/4608*tan(x)^9+1/45056*tan(x)^11
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{1}{(\sin(x)+\sin(3x))^6}\,dx = -\frac{(110\cos(2x)+66\cos(4x)-34\cos(6x)-50\cos(8x)-10\cos(10x)+10\cos(12x)+6\cos(14x)+\cos(}{2027520}$$

input
```
Integrate[(Sin[x] + Sin[3*x])^(-6),x]
```

output
```
-1/2027520*((110*Cos[2*x] + 66*Cos[4*x] - 34*Cos[6*x] - 50*Cos[8*x] - 10*C
os[10*x] + 10*Cos[12*x] + 6*Cos[14*x] + Cos[16*x])*Csc[x]^5*Sec[x]^11)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4822, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(x)+\sin(3x))^6}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(x)+\sin(3x))^6}dx$$

$$\Big\downarrow \text{4822}$$

$$\int \frac{\left(\tan^2(x)+1\right)^8\cot^6(x)}{4096}d\tan(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \cot^6(x)\left(\tan^2(x)+1\right)^8d\tan(x)}{4096}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\tan^{10}(x)+8\tan^8(x)+28\tan^6(x)+56\tan^4(x)+70\tan^2(x)+\cot^6(x)+8\cot^4(x)+28\cot^2(x)+56\right)d\tan(x)}{4096}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\tan^{11}(x)}{11}+\frac{8\tan^9(x)}{9}+4\tan^7(x)+\frac{56\tan^5(x)}{5}+\frac{70\tan^3(x)}{3}+56\tan(x)-\frac{\cot^5(x)}{5}-\frac{8\cot^3(x)}{3}-28\cot(x)}{4096}$$

input
```
Int[(Sin[x] + Sin[3*x])^(-6),x]
```

output
```
(-28*Cot[x] - (8*Cot[x]^3)/3 - Cot[x]^5/5 + 56*Tan[x] + (70*Tan[x]^3)/3 +
(56*Tan[x]^5)/5 + 4*Tan[x]^7 + (8*Tan[x]^9)/9 + Tan[x]^11/11)/4096
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4822
```
Int[((a_.)*sin[(m_.)*((c_.) + (d_.)*(x_))] + (b_.)*sin[(n_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Sin[
m*ArcTan[x]] + b*Sin[n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[c + d*x]], x] /
; FreeQ[{a, b, c, d}, x] && ILtQ[p/2, 0] && IntegerQ[(m - 1)/2] && IntegerQ
[(n - 1)/2]
```

Maple [A] (verified)

Time = 430.58 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.03

method	result
parallelrisch	0
risch	$-\frac{16i\left(110\,{\mathrm e}^{14ix}+66\,{\mathrm e}^{12ix}-34\,{\mathrm e}^{10ix}-50\,{\mathrm e}^{8ix}-10\,{\mathrm e}^{6ix}+10\,{\mathrm e}^{4ix}+6\,{\mathrm e}^{2ix}+1\right)}{495\left({\mathrm e}^{2ix}+1\right)^{11}\left({\mathrm e}^{2ix}-1\right)^{5}}$
default	$\frac{1}{45056\cos\left(x\right)^{11}\sin\left(x\right)^{5}}+\frac{1}{25344\sin\left(x\right)^{5}\cos\left(x\right)^{9}}+\frac{1}{12672\sin\left(x\right)^{5}\cos\left(x\right)^{7}}+\frac{1}{5280\sin\left(x\right)^{5}\cos\left(x\right)^{5}}-\frac{1}{2640\sin\left(x\right)^{5}\cos\left(x\right)^{3}}$

input
```
int(1/(sin(x)+sin(3*x))^6,x,method=_RETURNVERBOSE)
```

output
```
0
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \frac{1}{(\sin(x)+\sin(3x))^6}\,dx = -\frac{32768\cos(x)^{16}-81920\cos(x)^{14}+61440\cos(x)^{12}-10240\cos(x)^{10}-1280\cos(x)^{8}-384\cos(x)^{6}}{2027520\left(\cos(x)^{15}-2\cos(x)^{13}+\cos(x)^{11}\right)\sin(x)}$$

input
```
integrate(1/(sin(x)+sin(3*x))^6,x, algorithm="fricas")
```

output
```
-1/2027520*(32768*cos(x)^16 - 81920*cos(x)^14 + 61440*cos(x)^12 - 10240*co
s(x)^10 - 1280*cos(x)^8 - 384*cos(x)^6 - 160*cos(x)^4 - 80*cos(x)^2 - 45)/
((cos(x)^15 - 2*cos(x)^13 + cos(x)^11)*sin(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(\sin(x)+\sin(3x))^6}\,dx = \text{Timed out}$$

input
```
integrate(1/(sin(x)+sin(3*x))**6,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2508 vs. $2(51) = 102$.

Time = 0.08 (sec) , antiderivative size = 2508, normalized size of antiderivative = 36.35

$$\int \frac{1}{(\sin(x) + \sin(3x))^6} \, dx = \text{Too large to display}$$

input
```
integrate(1/(sin(x)+sin(3*x))^6,x, algorithm="maxima")
```

output
```
16/495*(2*(55*sin(14*x) + 33*sin(12*x) - 17*sin(10*x) - 25*sin(8*x) - 5*si
n(6*x) + 5*sin(4*x) + 3*sin(2*x))*cos(32*x) + 12*(55*sin(14*x) + 33*sin(12
*x) - 17*sin(10*x) - 25*sin(8*x) - 5*sin(6*x) + 5*sin(4*x) + 3*sin(2*x))*c
os(30*x) + 20*(55*sin(14*x) + 33*sin(12*x) - 17*sin(10*x) - 25*sin(8*x) -
5*sin(6*x) + 5*sin(4*x) + 3*sin(2*x))*cos(28*x) - 20*(55*sin(14*x) + 33*si
n(12*x) - 17*sin(10*x) - 25*sin(8*x) - 5*sin(6*x) + 5*sin(4*x) + 3*sin(2*x
))*cos(26*x) - 100*(55*sin(14*x) + 33*sin(12*x) - 17*sin(10*x) - 25*sin(8*
x) - 5*sin(6*x) + 5*sin(4*x) + 3*sin(2*x))*cos(24*x) - 68*(55*sin(14*x) +
33*sin(12*x) - 17*sin(10*x) - 25*sin(8*x) - 5*sin(6*x) + 5*sin(4*x) + 3*si
n(2*x))*cos(22*x) + 132*(55*sin(14*x) + 33*sin(12*x) - 17*sin(10*x) - 25*s
in(8*x) - 5*sin(6*x) + 5*sin(4*x) + 3*sin(2*x))*cos(20*x) + 220*(55*sin(14
*x) + 33*sin(12*x) - 17*sin(10*x) - 25*sin(8*x) - 5*sin(6*x) + 5*sin(4*x)
+ 3*sin(2*x))*cos(18*x) - (110*cos(14*x) + 66*cos(12*x) - 34*cos(10*x) - 5
0*cos(8*x) - 10*cos(6*x) + 10*cos(4*x) + 6*cos(2*x) + 1)*sin(32*x) - 6*(11
0*cos(14*x) + 66*cos(12*x) - 34*cos(10*x) - 50*cos(8*x) - 10*cos(6*x) + 10
*cos(4*x) + 6*cos(2*x) + 1)*sin(30*x) - 10*(110*cos(14*x) + 66*cos(12*x) -
 34*cos(10*x) - 50*cos(8*x) - 10*cos(6*x) + 10*cos(4*x) + 6*cos(2*x) + 1)*
sin(28*x) + 10*(110*cos(14*x) + 66*cos(12*x) - 34*cos(10*x) - 50*cos(8*x)
- 10*cos(6*x) + 10*cos(4*x) + 6*cos(2*x) + 1)*sin(26*x) + 50*(110*cos(14*x
) + 66*cos(12*x) - 34*cos(10*x) - 50*cos(8*x) - 10*cos(6*x) + 10*cos(4*...
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \frac{1}{(\sin(x)+\sin(3x))^6}\,dx = \frac{1}{45056}\tan(x)^{11} + \frac{1}{4608}\tan(x)^9 + \frac{1}{1024}\tan(x)^7 + \frac{7}{2560}\tan(x)^5 + \frac{35}{6144}\tan(x)^3 - \frac{420\tan(x)^4 + 40\tan(x)^2 + 3}{61440\tan(x)^5} + \frac{7}{512}\tan(x)$$

input
```
integrate(1/(sin(x)+sin(3*x))^6,x, algorithm="giac")
```

output
```
1/45056*tan(x)^11 + 1/4608*tan(x)^9 + 1/1024*tan(x)^7 + 7/2560*tan(x)^5 +
35/6144*tan(x)^3 - 1/61440*(420*tan(x)^4 + 40*tan(x)^2 + 3)/tan(x)^5 + 7/5
12*tan(x)
```

Mupad [B] (verification not implemented)

Time = 21.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \frac{1}{(\sin(x)+\sin(3x))^6}\,dx = \frac{7\tan(x)}{512} + \frac{35\tan(x)^3}{6144} + \frac{7\tan(x)^5}{2560} + \frac{\tan(x)^7}{1024} + \frac{\tan(x)^9}{4608} + \frac{\tan(x)^{11}}{45056} - \frac{\frac{7\tan(x)^4}{1024} + \frac{\tan(x)^2}{1536} + \frac{1}{20480}}{\tan(x)^5}$$

input
```
int(1/(sin(3*x) + sin(x))^6,x)
```

output
```
(7*tan(x))/512 + (35*tan(x)^3)/6144 + (7*tan(x)^5)/2560 + tan(x)^7/1024 +
tan(x)^9/4608 + tan(x)^11/45056 - (tan(x)^2/1536 + (7*tan(x)^4)/1024 + 1/2
0480)/tan(x)^5
```

Reduce [F]

$$\int \frac{1}{(\sin(x)+\sin(3x))^6}\,dx$$
$$= \int \frac{1}{\sin(3x)^6 + 6\sin(3x)^5\sin(x) + 15\sin(3x)^4\sin(x)^2 + 20\sin(3x)^3\sin(x)^3 + 15\sin(3x)^2\sin(x)^4 + 6\,\mathrm{si}}$$

input
```
int(1/(sin(x)+sin(3*x))^6,x)
```

output
```
int(1/(sin(3*x)**6 + 6*sin(3*x)**5*sin(x) + 15*sin(3*x)**4*sin(x)**2 + 20*
sin(3*x)**3*sin(x)**3 + 15*sin(3*x)**2*sin(x)**4 + 6*sin(3*x)*sin(x)**5 +
sin(x)**6),x)
```

3.25 $\int \frac{1}{\sin(x)+\sin(5x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 32

$$\int \frac{1}{\sin(x)+\sin(5x)}\,dx=-\frac{1}{6}\text{arctanh}(\cos(x))-\frac{2}{3}\text{arctanh}(2\cos(x))+\frac{\text{arctanh}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

output
```
-1/6*arctanh(cos(x))-2/3*arctanh(2*cos(x))+1/2*arctanh(cos(x)*2^(1/2))*2^(
1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.72

$$\int \frac{1}{\sin(x)+\sin(5x)}\,dx=\frac{1}{6}\left((3+3i)(-1)^{3/4}\text{arctanh}\left(\frac{-1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+(3-3i)\sqrt[4]{-1}\text{arctanh}\left(\frac{1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)+2\log(1-2\cos(x))-2\log(1+2\cos(x))+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)$$

input
```
Integrate[(Sin[x] + Sin[5*x])^(-1),x]
```

output
```
((3 + 3*I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] + (3 - 3*I)*(-1)^(1
/4)*ArcTanh[(1 + Tan[x/2])/Sqrt[2]] - Log[Cos[x/2]] + 2*Log[1 - 2*Cos[x]]
- 2*Log[1 + 2*Cos[x]] + Log[Sin[x/2]])/6
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4824, 27, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin(x)+\sin(5x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(x)+\sin(5x)}dx$$

$$\Big\downarrow \text{4824}$$

$$-\int \frac{1}{2\left(1-\cos^2(x)\right)\left(8\cos^4(x)-6\cos^2(x)+1\right)}d\cos(x)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}\int \frac{1}{\left(1-\cos^2(x)\right)\left(8\cos^4(x)-6\cos^2(x)+1\right)}d\cos(x)$$

$$\Big\downarrow \text{1484}$$

$$-\frac{1}{2}\int \left(\frac{2}{2\cos^2(x)-1}-\frac{8}{3\left(4\cos^2(x)-1\right)}-\frac{1}{3\left(\cos^2(x)-1\right)}\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{1}{3}\text{arctanh}(\cos(x))-\frac{4}{3}\text{arctanh}(2\cos(x))+\sqrt{2}\text{arctanh}\left(\sqrt{2}\cos(x)\right)\right)$$

input
```
Int[(Sin[x] + Sin[5*x])^(-1),x]
```

output
```
(-1/3*ArcTanh[Cos[x]] - (4*ArcTanh[2*Cos[x]])/3 + Sqrt[2]*ArcTanh[Sqrt[2]*
Cos[x]])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4824
```
Int[((a_.)*sin[(m_.)*((c_.) + (d_.)*(x_))] + (b_.)*sin[(n_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Simplify[TrigExpand[a*
Sin[m*ArcCos[x]] + b*Sin[n*ArcCos[x]]]]^p/Sqrt[1 - x^2], x], x, Cos[c + d*x
]], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[(m - 1)/
2] && IntegerQ[(n - 1)/2]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.44

method	result
default	$\frac{\ln(2\cos(x)-1)}{3}+\frac{\operatorname{arctanh}\left(\sqrt{2}\cos(x)\right)\sqrt{2}}{2}-\frac{\ln(2\cos(x)+1)}{3}+\frac{\ln(\cos(x)-1)}{12}-\frac{\ln(1+\cos(x))}{12}$
risch	$-\frac{\ln(e^{ix}+1)}{6}+\frac{\ln(e^{ix}-1)}{6}+\frac{\ln(e^{2ix}-e^{ix}+1)}{3}-\frac{\ln(e^{2ix}+e^{ix}+1)}{3}+\frac{\sqrt{2}\ln\left(e^{2ix}+\sqrt{2}\,e^{ix}+1\right)}{4}-\frac{\sqrt{2}\ln\left(e^{2ix}-\sqrt{2}\,e^{ix}+1\right)}{4}$

input
```
int(1/(sin(x)+sin(5*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(2*cos(x)-1)+1/2*arctanh(2^(1/2)*cos(x))*2^(1/2)-1/3*ln(2*cos(x)+1)+
1/12*ln(cos(x)-1)-1/12*ln(1+cos(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(25) = 50.

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.19

$$\int \frac{1}{\sin(x)+\sin(5x)}\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2+2\sqrt{2}\cos(x)+1}{2\cos(x)^2-1}\right) - \frac{1}{12}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{12}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right) + \frac{1}{3}\log(-2\cos(x)+1)-\frac{1}{3}\log(-2\cos(x)-1)$$

input
```
integrate(1/(sin(x)+sin(5*x)),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1)) - 1
/12*log(1/2*cos(x) + 1/2) + 1/12*log(-1/2*cos(x) + 1/2) + 1/3*log(-2*cos(x
) + 1) - 1/3*log(-2*cos(x) - 1)
```

Sympy [F]

$$\int \frac{1}{\sin(x) + \sin(5x)}\,dx = \int \frac{1}{\sin(x) + \sin(5x)}\,dx$$

input
```
integrate(1/(sin(x)+sin(5*x)),x)
```

output
```
Integral(1/(sin(x) + sin(5*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(25) = 50$.

Time = 0.14 (sec) , antiderivative size = 257, normalized size of antiderivative = 8.03

$$\begin{aligned}\int \frac{1}{\sin(x)+\sin(5x)}\,dx = {} & \frac{1}{8}\sqrt{2}\log\left(2\sqrt{2}\sin(2x)\sin(x) + 2\left(\sqrt{2}\cos(x)+1\right)\cos(2x) + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 1\right) \\ & - \frac{1}{8}\sqrt{2}\log\left(-2\sqrt{2}\sin(2x)\sin(x) - 2\left(\sqrt{2}\cos(x)-1\right)\cos(2x) + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 1\right) \\ & - \frac{1}{6}\log\left(2(\cos(x)+1)\cos(2x) + \cos(2x)^2 + \cos(x)^2 + \sin(2x)^2 + 2\sin(2x)\sin(x) + \sin(x)^2 + 2\cos(x) + 1\right) \\ & + \frac{1}{6}\log\left(-2(\cos(x)-1)\cos(2x) + \cos(2x)^2 + \cos(x)^2 + \sin(2x)^2 - 2\sin(2x)\sin(x) + \sin(x)^2 - 2\cos(x) + 1\right) \\ & - \frac{1}{12}\log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right) \\ & + \frac{1}{12}\log\left(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1\right)\end{aligned}$$

input
```
integrate(1/(sin(x)+sin(5*x)),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x
) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) +
 1) - 1/8*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*
cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*c
os(x) + 1) - 1/6*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin
(2*x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) + 1/6*log(-2*(cos(x
) - 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) +
 sin(x)^2 - 2*cos(x) + 1) - 1/12*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) +
 1/12*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(25) = 50$.

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 3.16

$$\int \frac{1}{\sin(x)+\sin(5x)}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}{\left|4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}\right) + \frac{1}{12}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right) - \frac{1}{3}\log\left(\left|-\frac{\cos(x)-1}{\cos(x)+1}-3\right|\right) + \frac{1}{3}\log\left(\left|-\frac{3(\cos(x)-1)}{\cos(x)+1}-1\right|\right)$$

input
```
integrate(1/(sin(x)+sin(5*x)),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/abs(4*sq
rt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)) + 1/12*log(-(cos(x) - 1)/(cos(x)
 + 1)) - 1/3*log(abs(-(cos(x) - 1)/(cos(x) + 1) - 3)) + 1/3*log(abs(-3*(co
s(x) - 1)/(cos(x) + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 21.70 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.31

$$\int \frac{1}{\sin(x)+\sin(5x)}\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{6} - \frac{2\,\mathrm{atanh}\left(\frac{67108864}{193710123\left(\frac{13374142283776\tan\left(\frac{x}{2}\right)^2}{129140163}-\frac{1486008352768}{43046721}\right)}-\frac{797162}{797161}\right)}{3} + \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{120967921664\sqrt{2}}{531441\left(\frac{110788345856\tan\left(\frac{x}{2}\right)^2}{59049}-\frac{513223426048}{1594323}\right)}-\frac{2115158147072\sqrt{2}\tan\left(\frac{x}{2}\right)^2}{1594323\left(\frac{110788345856\tan\left(\frac{x}{2}\right)^2}{59049}-\frac{513223426048}{1594323}\right)}\right)}{2}$$

input
```
int(1/(sin(5*x) + sin(x)),x)
```

output
```
log(tan(x/2))/6 - (2*atanh(67108864/(193710123*((13374142283776*tan(x/2)^2
)/129140163 - 1486008352768/43046721)) - 797162/797161))/3 + (2^(1/2)*atan
h((120967921664*2^(1/2))/(531441*((110788345856*tan(x/2)^2)/59049 - 513223
426048/1594323)) - (2115158147072*2^(1/2)*tan(x/2)^2)/(1594323*((110788345
856*tan(x/2)^2)/59049 - 513223426048/1594323))))/2
```

Reduce [F]

$$\int \frac{1}{\sin(x)+\sin(5x)}\,dx = \int \frac{1}{\sin(5x)+\sin(x)}dx$$

input
```
int(1/(sin(x)+sin(5*x)),x)
```

output
```
int(1/(sin(5*x) + sin(x)),x)
```

3.26 $\int \frac{1}{(\sin(x)+\sin(5x))^3}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 127

$$\begin{aligned}\int \frac{1}{(\sin(x)+\sin(5x))^3}\,dx = &-\frac{7}{144}\text{arctanh}(\cos(x))-\frac{55}{9}\text{arctanh}(2\cos(x))\\&+\frac{139\text{arctanh}\left(\sqrt{2}\cos(x)\right)}{16\sqrt{2}}-\frac{1}{18(1-2\cos(x))^2}\\&+\frac{35}{54(1-2\cos(x))}-\frac{1}{864(1-\cos(x))}+\frac{1}{864(1+\cos(x))}\\&+\frac{1}{18(1+2\cos(x))^2}-\frac{35}{54(1+2\cos(x))}\\&-\frac{19}{16}\cos(x)\sec(2x)+\frac{1}{8}\cos(x)\sec^2(2x)\end{aligned}$$

output

```
-7/144*arctanh(cos(x))-55/9*arctanh(2*cos(x))+139/32*arctanh(cos(x)*2^(1/2
))*2^(1/2)-1/18/(1-2*cos(x))^2+35/(54-108*cos(x))-1/(864-864*cos(x))+1/(86
4+864*cos(x))+1/18/(1+2*cos(x))^2-35/(54+108*cos(x))-19/16*cos(x)*sec(2*x)
+1/8*cos(x)*sec(2*x)^2
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.47

$$\int \frac{1}{(\sin(x)+\sin(5x))^3}\,dx$$
$$= \frac{-7506\sqrt{2}\text{arctanh}\left(\frac{-1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+7506\sqrt{2}\text{arctanh}\left(\frac{1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-\frac{96}{(1-2\cos(x))^2}+\frac{1120}{1-2\cos(x)}+\frac{96}{(1+2\cos(x))^2}-\frac{1}{1+2}}{}$$

input
```
Integrate[(Sin[x] + Sin[5*x])^(-3),x]
```

output
```
(-7506*Sqrt[2]*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] + 7506*Sqrt[2]*ArcTanh[(1
+ Tan[x/2])/Sqrt[2]] - 96/(1 - 2*Cos[x])^2 + 1120/(1 - 2*Cos[x]) + 96/(1 +
 2*Cos[x])^2 - 1120/(1 + 2*Cos[x]) - Csc[x/2]^2 - 84*Log[Cos[x/2]] + 5280*
Log[1 - 2*Cos[x]] - 5280*Log[1 + 2*Cos[x]] + 84*Log[Sin[x/2]] + Sec[x/2]^2
 - 918/(Cos[x] - Sin[x]) + (108*Sin[x])/(Cos[x] - Sin[x])^2 - (108*Sin[x])
/(Cos[x] + Sin[x])^2 - 918/(Cos[x] + Sin[x]))/1728
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4824, 27, 1567, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(x)+\sin(5x))^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(\sin(x)+\sin(5x))^3}dx$$
$$\Big\downarrow \text{4824}$$
$$-\int \frac{1}{8\left(1-\cos^2(x)\right)^2\left(8\cos^4(x)-6\cos^2(x)+1\right)^3}d\cos(x)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{8}\int \frac{1}{(1-\cos^2(x))^2\left(8\cos^4(x)-6\cos^2(x)+1\right)^3}d\cos(x)$$

$$\Big\downarrow \text{1567}$$

$$-\frac{1}{8}\int \left(\frac{60}{2\cos^2(x)-1}-\frac{880}{9\left(4\cos^2(x)-1\right)}+\frac{1}{108(\cos(x)-1)^2}+\frac{1}{108(\cos(x)+1)^2}-\frac{280}{27(2\cos(x)-1)^2}-\frac{}{27(2\,\mathrm{co}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\left(-\frac{7}{18}\mathrm{arctanh}(\cos(x))-\frac{440}{9}\mathrm{arctanh}(2\cos(x))+34\sqrt{2}\mathrm{arctanh}\left(\sqrt{2}\cos(x)\right)+\frac{3\mathrm{arctanh}(\sqrt{2}\cos(x))}{2\sqrt{2}}+\frac{19\,\mathrm{c}}{2\,(1-2}\right.$$

input
```
Int[(Sin[x] + Sin[5*x])^(-3),x]
```

output
```
((-7*ArcTanh[Cos[x]])/18 - (440*ArcTanh[2*Cos[x]])/9 + (3*ArcTanh[Sqrt[2]*
Cos[x]])/(2*Sqrt[2]) + 34*Sqrt[2]*ArcTanh[Sqrt[2]*Cos[x]] - 4/(9*(1 - 2*Co
s[x])^2) + 140/(27*(1 - 2*Cos[x])) - 1/(108*(1 - Cos[x])) + 1/(108*(1 + Co
s[x])) + 4/(9*(1 + 2*Cos[x])^2) - 140/(27*(1 + 2*Cos[x])) + Cos[x]/(1 - 2*
Cos[x]^2)^2 + (19*Cos[x])/(2*(1 - 2*Cos[x]^2)))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1567
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4824
```
Int[((a_.)*sin[(m_.)*((c_.) + (d_.)*(x_))] + (b_.)*sin[(n_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Simplify[TrigExpand[a*
Sin[m*ArcCos[x]] + b*Sin[n*ArcCos[x]]]]^p/Sqrt[1 - x^2], x], x, Cos[c + d*x
]], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[(m - 1)/
2] && IntegerQ[(n - 1)/2]
```

Maple [A] (verified)

Time = 5.68 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98

method	result
default	$\frac{1}{18(2\cos(x)+1)^2}-\frac{35}{54(2\cos(x)+1)}-\frac{55\ln(2\cos(x)+1)}{18}-\frac{1}{18(2\cos(x)-1)^2}-\frac{35}{54(2\cos(x)-1)}+\frac{55\ln(2\cos(x)-1)}{18}+\frac{}{864\text{-}}$
risch	$-\frac{119\,e^{19ix}+55\,e^{17ix}+95\,e^{15ix}-111\,e^{13ix}-166\,e^{11ix}-166\,e^{9ix}-111\,e^{7ix}+95\,e^{5ix}+55\,e^{3ix}+119\,e^{ix}}{48(e^{10ix}+e^{6ix}-e^{4ix}-1)^2}+\frac{7\ln(e^{ix}-1)}{144}-\frac{7\ln(e^{ix}+}{144}$

input
```
int(1/(sin(x)+sin(5*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/18/(2*cos(x)+1)^2-35/54/(2*cos(x)+1)-55/18*ln(2*cos(x)+1)-1/18/(2*cos(x)
-1)^2-35/54/(2*cos(x)-1)+55/18*ln(2*cos(x)-1)+1/864/(1+cos(x))-7/288*ln(1+
cos(x))-8*(19/64*cos(x)^3-21/128*cos(x))/(2*cos(x)^2-1)^2+139/32*arctanh(2
^(1/2)*cos(x))*2^(1/2)+1/864/(cos(x)-1)+7/288*ln(cos(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(99) = 198$.

Time = 0.11 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.44

$$\int \frac{1}{(\sin(x)+\sin(5x))^3}\,dx = -\frac{45696\cos(x)^9 - 97536\cos(x)^7 + 70152\cos(x)^5 - 20316\cos(x)^3 - 1251\left(64\sqrt{2}\cos(x)^{10} - 160\sqrt{2}\right.}{}$$

input
```
integrate(1/(sin(x)+sin(5*x))^3,x, algorithm="fricas")
```

output
```
-1/576*(45696*cos(x)^9 - 97536*cos(x)^7 + 70152*cos(x)^5 - 20316*cos(x)^3
- 1251*(64*sqrt(2)*cos(x)^10 - 160*sqrt(2)*cos(x)^8 + 148*sqrt(2)*cos(x)^6
 - 64*sqrt(2)*cos(x)^4 + 13*sqrt(2)*cos(x)^2 - sqrt(2))*log(-(2*cos(x)^2 +
 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1)) + 14*(64*cos(x)^10 - 160*cos(x)^8
 + 148*cos(x)^6 - 64*cos(x)^4 + 13*cos(x)^2 - 1)*log(1/2*cos(x) + 1/2) - 1
4*(64*cos(x)^10 - 160*cos(x)^8 + 148*cos(x)^6 - 64*cos(x)^4 + 13*cos(x)^2
- 1)*log(-1/2*cos(x) + 1/2) - 1760*(64*cos(x)^10 - 160*cos(x)^8 + 148*cos(
x)^6 - 64*cos(x)^4 + 13*cos(x)^2 - 1)*log(-2*cos(x) + 1) + 1760*(64*cos(x)
^10 - 160*cos(x)^8 + 148*cos(x)^6 - 64*cos(x)^4 + 13*cos(x)^2 - 1)*log(-2*
cos(x) - 1) + 1992*cos(x))/(64*cos(x)^10 - 160*cos(x)^8 + 148*cos(x)^6 - 6
4*cos(x)^4 + 13*cos(x)^2 - 1)
```

Sympy [F]

$$\int \frac{1}{(\sin(x)+\sin(5x))^3}\,dx = \int \frac{1}{(\sin(x)+\sin(5x))^3}\,dx$$

input
```
integrate(1/(sin(x)+sin(5*x))**3,x)
```

output
```
Integral((sin(x) + sin(5*x))**(-3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\sin(x)+\sin(5x))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(sin(x)+sin(5*x))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(99) = 198$.

Time = 0.13 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.34

$$\int \frac{1}{(\sin(x)+\sin(5x))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(sin(x)+sin(5*x))^3,x, algorithm="giac")
```

output
```
139/64*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/abs(4
*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)) - 1/1728*(42*(cos(x) - 1)/(co
s(x) + 1) - 1)*(cos(x) + 1)/(cos(x) - 1) - 1/1728*(cos(x) - 1)/(cos(x) + 1
) - 1/648*(379605*(cos(x) - 1)/(cos(x) + 1) + 2276125*(cos(x) - 1)^2/(cos(
x) + 1)^2 + 6174297*(cos(x) - 1)^3/(cos(x) + 1)^3 + 7608731*(cos(x) - 1)^4
/(cos(x) + 1)^4 + 3721407*(cos(x) - 1)^5/(cos(x) + 1)^5 + 745855*(cos(x) -
 1)^6/(cos(x) + 1)^6 + 50643*(cos(x) - 1)^7/(cos(x) + 1)^7 + 23049)/(28*(c
os(x) - 1)/(cos(x) + 1) + 66*(cos(x) - 1)^2/(cos(x) + 1)^2 + 28*(cos(x) -
1)^3/(cos(x) + 1)^3 + 3*(cos(x) - 1)^4/(cos(x) + 1)^4 + 3)^2 + 7/288*log(-
(cos(x) - 1)/(cos(x) + 1)) - 55/18*log(abs(-(cos(x) - 1)/(cos(x) + 1) - 3)
) + 55/18*log(abs(-3*(cos(x) - 1)/(cos(x) + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 22.38 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.74

$$\int \frac{1}{(\sin(x)+\sin(5x))^3}\,dx = \frac{7\,\ln\left(\tan\left(\frac{x}{2}\right)\right)}{144}$$
$$-\frac{55\,\mathrm{atanh}\left(\frac{327840921493337120}{36687588870411\left(\frac{219246731822316309100\,\tan\left(\frac{x}{2}\right)^2}{282429536481}-\frac{243607059170136554200}{94143178827}\right)}-\frac{345171004}{345170855}\right)}{9}$$
$$-\frac{-\frac{45013\tan\left(\frac{x}{2}\right)^{16}}{5184}+\frac{93224\tan\left(\frac{x}{2}\right)^{14}}{729}-\frac{275627\tan\left(\frac{x}{2}\right)^{12}}{432}+\frac{3803641\tan\left(\frac{x}{2}\right)^{10}}{2916}-\frac{8229425\tan\left(\frac{x}{2}\right)^{8}}{7776}+\frac{568669\tan\left(\frac{x}{2}\right)^{6}}{1458}-\frac{252775\,\mathrm{t}\cdots}{388\cdots}}{\tan\left(\frac{x}{2}\right)^{18}-\frac{56\tan\left(\frac{x}{2}\right)^{16}}{3}+\frac{1180\tan\left(\frac{x}{2}\right)^{14}}{9}-\frac{1288\tan\left(\frac{x}{2}\right)^{12}}{3}+\frac{5942\tan\left(\frac{x}{2}\right)^{10}}{9}-\frac{1288\tan\left(\frac{x}{2}\right)^{8}}{3}+\frac{1180\tan\left(\frac{x}{2}\right)^{6}}{9}-\frac{5\cdots}{}}$$
$$+\frac{\tan\left(\frac{x}{2}\right)^2}{1728}$$
$$+\frac{139\sqrt{2}\,\mathrm{atanh}\left(\frac{4678477767243387713\sqrt{2}}{918330048\left(\frac{4338338756966602884 7\tan\left(\frac{x}{2}\right)^2}{1033121304}-\frac{44660475234141568277}{6198727824}\right)}-\frac{7362405010198469509 69\sqrt{2}\,\mathrm{ta}\cdots}{24794911296\left(\frac{43383387569666028847\tan\left(\frac{x}{2}\right)^2}{1033121304}-\cdots\right.}\cdots\right.}{32}$$

input
```
int(1/(sin(5*x) + sin(x))^3,x)
```

output
```
(7*log(tan(x/2)))/144 - (55*atanh(327840921493337120/(36687588870411*((219
246731822316309100*tan(x/2)^2)/282429536481 - 243607059170136554200/94143
178827)) - 345171004/345170855))/9 - ((1277*tan(x/2)^2)/324 - (252775*tan(
x/2)^4)/3888 + (568669*tan(x/2)^6)/1458 - (8229425*tan(x/2)^8)/7776 + (380
3641*tan(x/2)^10)/2916 - (275627*tan(x/2)^12)/432 + (93224*tan(x/2)^14)/72
9 - (45013*tan(x/2)^16)/5184 + 1/1728)/(tan(x/2)^2 - (56*tan(x/2)^4)/3 + (
1180*tan(x/2)^6)/9 - (1288*tan(x/2)^8)/3 + (5942*tan(x/2)^10)/9 - (1288*ta
n(x/2)^12)/3 + (1180*tan(x/2)^14)/9 - (56*tan(x/2)^16)/3 + tan(x/2)^18) +
tan(x/2)^2/1728 + (139*2^(1/2)*atanh((4678477767243387713*2^(1/2))/(918330
048*((43383387569666028847*tan(x/2)^2)/1033121304 - 44660475234141568277/6
198727824)) - (736240501019846950969*2^(1/2)*tan(x/2)^2)/(24794911296*((43
383387569666028847*tan(x/2)^2)/1033121304 - 44660475234141568277/619872782
4))))/32
```

Reduce [F]

$$\int \frac{1}{(\sin(x)+\sin(5x))^3}\,dx$$
$$= \int \frac{1}{\sin(5x)^3+3\sin(5x)^2\sin(x)+3\sin(5x)\sin(x)^2+\sin(x)^3}dx$$

input
```
int(1/(sin(x)+sin(5*x))^3,x)
```

output
```
int(1/(sin(5*x)**3 + 3*sin(5*x)**2*sin(x) + 3*sin(5*x)*sin(x)**2 + sin(x)*
*3),x)
```

3.27 $\int \frac{1}{(\sin(x)+\sin(5x))^5} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 221

$$\begin{aligned}\int \frac{1}{(\sin(x)+\sin(5x))^5} \, dx = & -\frac{3889\text{arctanh}(\cos(x))}{186624} - \frac{332929\text{arctanh}(2\cos(x))}{2916} \\ & + \frac{82683\text{arctanh}\left(\sqrt{2}\cos(x)\right)}{512\sqrt{2}} \\ & - \frac{1}{108(1-2\cos(x))^4} + \frac{19}{162(1-2\cos(x))^3} \\ & - \frac{749}{648(1-2\cos(x))^2} + \frac{71551}{5832(1-2\cos(x))} \\ & - \frac{1}{124416(1-\cos(x))^2} - \frac{209}{373248(1-\cos(x))} \\ & + \frac{1}{124416(1+\cos(x))^2} + \frac{209}{373248(1+\cos(x))} \\ & + \frac{1}{108(1+2\cos(x))^4} - \frac{19}{162(1+2\cos(x))^3} \\ & + \frac{749}{648(1+2\cos(x))^2} - \frac{71551}{5832(1+2\cos(x))} \\ & - \frac{11643}{512}\cos(x)\sec(2x) + \frac{681}{256}\cos(x)\sec^2(2x) \\ & - \frac{21}{64}\cos(x)\sec^3(2x) + \frac{1}{32}\cos(x)\sec^4(2x)\end{aligned}$$

output
```
-3889/186624*arctanh(cos(x))-332929/2916*arctanh(2*cos(x))+82683/1024*arct
anh(cos(x)*2^(1/2))*2^(1/2)-1/108/(1-2*cos(x))^4+19/162/(1-2*cos(x))^3-749
/648/(1-2*cos(x))^2+71551/(5832-11664*cos(x))-1/124416/(1-cos(x))^2-209/(3
73248-373248*cos(x))+1/124416/(1+cos(x))^2+209/(373248+373248*cos(x))+1/10
8/(1+2*cos(x))^4-19/162/(1+2*cos(x))^3+749/648/(1+2*cos(x))^2-71551/(5832+
11664*cos(x))-11643/512*cos(x)*sec(2*x)+681/256*cos(x)*sec(2*x)^2-21/64*co
s(x)*sec(2*x)^3+1/32*cos(x)*sec(2*x)^4
```

Mathematica [A] (verified)

Time = 2.47 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.33

$$\int \frac{1}{(\sin(x)+\sin(5x))^5}\,dx = \frac{-120551814\sqrt{2}\text{arctanh}\left(\frac{-1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+120551814\sqrt{2}\text{arctanh}\left(\frac{1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-\frac{13824}{(1-2\cos(x))^4}-\frac{1725696}{(1-2\cos(x))^2}+\frac{18\ldots}{1-\ldots}}{\ldots}$$

input
```
Integrate[(Sin[x] + Sin[5*x])^(-5),x]
```

output
```
(-120551814*Sqrt[2]*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] + 120551814*Sqrt[2]*A
rcTanh[(1 + Tan[x/2])/Sqrt[2]] - 13824/(1 - 2*Cos[x])^4 - 1725696/(1 - 2*C
os[x])^2 + 18317056/(1 - 2*Cos[x]) - 175104/(-1 + 2*Cos[x])^3 + 13824/(1 +
 2*Cos[x])^4 - 175104/(1 + 2*Cos[x])^3 + 1725696/(1 + 2*Cos[x])^2 - 183170
56/(1 + 2*Cos[x]) - 418*Csc[x/2]^2 - 3*Csc[x/2]^4 - 31112*Log[Cos[x/2]] +
85229824*Log[1 - 2*Cos[x]] - 85229824*Log[1 + 2*Cos[x]] + 31112*Log[Sin[x/
2]] + 418*Sec[x/2]^2 + 3*Sec[x/2]^4 - 110808/(Cos[x] - Sin[x])^3 - 1510050
6/(Cos[x] - Sin[x]) + (11664*Sin[x])/(Cos[x] - Sin[x])^4 + (1874988*Sin[x]
)/(Cos[x] - Sin[x])^2 - (11664*Sin[x])/(Cos[x] + Sin[x])^4 - 110808/(Cos[x
] + Sin[x])^3 - (1874988*Sin[x])/(Cos[x] + Sin[x])^2 - 15100506/(Cos[x] +
Sin[x]))/1492992
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4824, 27, 1567, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(x)+\sin(5x))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(x)+\sin(5x))^5}dx$$

$$\Big\downarrow \text{4824}$$

$$-\int \frac{1}{32\left(1-\cos^2(x)\right)^3\left(8\cos^4(x)-6\cos^2(x)+1\right)^5}d\cos(x)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{32}\int \frac{1}{\left(1-\cos^2(x)\right)^3\left(8\cos^4(x)-6\cos^2(x)+1\right)^5}d\cos(x)$$

$$\Big\downarrow \text{1567}$$

$$-\frac{1}{32}\int \left(\frac{4440}{2\cos^2(x)-1}-\frac{5326864}{729\left(4\cos^2(x)-1\right)}+\frac{209}{11664(\cos(x)-1)^2}+\frac{209}{11664(\cos(x)+1)^2}-\frac{572408}{729(2\cos(x)-1)^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{32}\left(-\frac{3889\text{arctanh}(\cos(x))}{5832}-\frac{2663432}{729}\text{arctanh}(2\cos(x))+2574\sqrt{2}\text{arctanh}\left(\sqrt{2}\cos(x)\right)+\frac{315\text{arctanh}\left(\sqrt{2}\cos(x\right.}{16\sqrt{2}}\right.$$

input
```
Int[(Sin[x] + Sin[5*x])^(-5),x]
```

output
```
((-3889*ArcTanh[Cos[x]])/5832 - (2663432*ArcTanh[2*Cos[x]])/729 + (315*Arc
Tanh[Sqrt[2]*Cos[x]])/(16*Sqrt[2]) + 2574*Sqrt[2]*ArcTanh[Sqrt[2]*Cos[x]]
- 8/(27*(1 - 2*Cos[x])^4) + 304/(81*(1 - 2*Cos[x])^3) - 2996/(81*(1 - 2*Co
s[x])^2) + 286204/(729*(1 - 2*Cos[x])) - 1/(3888*(1 - Cos[x])^2) - 209/(11
664*(1 - Cos[x])) + 1/(3888*(1 + Cos[x])^2) + 209/(11664*(1 + Cos[x])) + 8
/(27*(1 + 2*Cos[x])^4) - 304/(81*(1 + 2*Cos[x])^3) + 2996/(81*(1 + 2*Cos[x
])^2) - 286204/(729*(1 + 2*Cos[x])) + Cos[x]/(1 - 2*Cos[x]^2)^4 + (21*Cos[
x])/(2*(1 - 2*Cos[x]^2)^3) + (681*Cos[x])/(8*(1 - 2*Cos[x]^2)^2) + (11643*
Cos[x])/(16*(1 - 2*Cos[x]^2)))/32
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1567
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4824
```
Int[((a_.)*sin[(m_.)*((c_.) + (d_.)*(x_))] + (b_.)*sin[(n_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Simplify[TrigExpand[a*
Sin[m*ArcCos[x]] + b*Sin[n*ArcCos[x]]]]^p/Sqrt[1 - x^2], x], x, Cos[c + d*x
]], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[(m - 1)/
2] && IntegerQ[(n - 1)/2]
```

Maple [A] (verified)

Time = 115.15 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.01

method	result
parallelrisch	0
default	$-\frac{1}{108(2\cos(x)-1)^4}-\frac{19}{162(2\cos(x)-1)^3}-\frac{749}{648(2\cos(x)-1)^2}-\frac{71551}{5832(2\cos(x)-1)}+\frac{332929\ln(2\cos(x)-1)}{5832}+\frac{}{108(2}$
risch	$-\frac{5881813\,\mathrm{e}^{39ix}+2770929\,\mathrm{e}^{37ix}+16666827\,\mathrm{e}^{35ix}-11603277\,\mathrm{e}^{33ix}+2153987\,\mathrm{e}^{31ix}-49799073\,\mathrm{e}^{29ix}-11124845\,\mathrm{e}^{27ix}-294403}{}$

input
```
int(1/(sin(x)+sin(5*x))^5,x,method=_RETURNVERBOSE)
```

output
```
0
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. 2(175) = 350.

Time = 0.21 (sec) , antiderivative size = 533, normalized size of antiderivative = 2.41

$$\int \frac{1}{(\sin(x)+\sin(5x))^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(sin(x)+sin(5*x))^5,x, algorithm="fricas")
```

output
```
-1/1492992*(144551436288*cos(x)^19 - 669594734592*cos(x)^17 + 132648439296
0*cos(x)^15 - 1475815977984*cos(x)^13 + 1017399847392*cos(x)^11 - 45122761
5984*cos(x)^9 + 128907781608*cos(x)^7 - 22906808436*cos(x)^5 + 2301482760*
cos(x)^3 - 60275907*(4096*sqrt(2)*cos(x)^20 - 20480*sqrt(2)*cos(x)^18 + 44
544*sqrt(2)*cos(x)^16 - 55552*sqrt(2)*cos(x)^14 + 44048*sqrt(2)*cos(x)^12
- 23232*sqrt(2)*cos(x)^10 + 8264*sqrt(2)*cos(x)^8 - 1960*sqrt(2)*cos(x)^6
+ 297*sqrt(2)*cos(x)^4 - 26*sqrt(2)*cos(x)^2 + sqrt(2))*log(-(2*cos(x)^2 +
 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1)) + 15556*(4096*cos(x)^20 - 20480*c
os(x)^18 + 44544*cos(x)^16 - 55552*cos(x)^14 + 44048*cos(x)^12 - 23232*cos
(x)^10 + 8264*cos(x)^8 - 1960*cos(x)^6 + 297*cos(x)^4 - 26*cos(x)^2 + 1)*l
og(1/2*cos(x) + 1/2) - 15556*(4096*cos(x)^20 - 20480*cos(x)^18 + 44544*cos
(x)^16 - 55552*cos(x)^14 + 44048*cos(x)^12 - 23232*cos(x)^10 + 8264*cos(x)
^8 - 1960*cos(x)^6 + 297*cos(x)^4 - 26*cos(x)^2 + 1)*log(-1/2*cos(x) + 1/2
) - 85229824*(4096*cos(x)^20 - 20480*cos(x)^18 + 44544*cos(x)^16 - 55552*c
os(x)^14 + 44048*cos(x)^12 - 23232*cos(x)^10 + 8264*cos(x)^8 - 1960*cos(x)
^6 + 297*cos(x)^4 - 26*cos(x)^2 + 1)*log(-2*cos(x) + 1) + 85229824*(4096*c
os(x)^20 - 20480*cos(x)^18 + 44544*cos(x)^16 - 55552*cos(x)^14 + 44048*cos
(x)^12 - 23232*cos(x)^10 + 8264*cos(x)^8 - 1960*cos(x)^6 + 297*cos(x)^4 -
26*cos(x)^2 + 1)*log(-2*cos(x) - 1) - 99800124*cos(x))/(4096*cos(x)^20 - 2
0480*cos(x)^18 + 44544*cos(x)^16 - 55552*cos(x)^14 + 44048*cos(x)^12 - ...
```

Sympy [F]

$$\int \frac{1}{(\sin(x)+\sin(5x))^5}\,dx = \int \frac{1}{(\sin{(x)}+\sin{(5x)})^5}\,dx$$

input
```
integrate(1/(sin(x)+sin(5*x))**5,x)
```

output
```
Integral((sin(x) + sin(5*x))**(-5), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\sin(x) + \sin(5x))^5} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(sin(x)+sin(5*x))^5,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(175) = 350$.

Time = 0.14 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.99

$$\int \frac{1}{(\sin(x) + \sin(5x))^5} \, dx = \text{Too large to display}$$

input
```
integrate(1/(sin(x)+sin(5*x))^5,x, algorithm="giac")
```

output
```
82683/2048*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/a
bs(4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)) + 1/1492992*(424*(cos(x)
- 1)/(cos(x) + 1) - 23334*(cos(x) - 1)^2/(cos(x) + 1)^2 - 3)*(cos(x) + 1)^
2/(cos(x) - 1)^2 - 53/186624*(cos(x) - 1)/(cos(x) + 1) + 1/497664*(cos(x)
- 1)^2/(cos(x) + 1)^2 - 1/186624*(40193230365*(cos(x) - 1)/(cos(x) + 1) +
614159700129*(cos(x) - 1)^2/(cos(x) + 1)^2 + 5371001231429*(cos(x) - 1)^3/
(cos(x) + 1)^3 + 29725819078749*(cos(x) - 1)^4/(cos(x) + 1)^4 + 1087791336
74049*(cos(x) - 1)^5/(cos(x) + 1)^5 + 267371188501221*(cos(x) - 1)^6/(cos(
x) + 1)^6 + 440631281631289*(cos(x) - 1)^7/(cos(x) + 1)^7 + 48095285739900
3*(cos(x) - 1)^8/(cos(x) + 1)^8 + 343468414091831*(cos(x) - 1)^9/(cos(x) +
 1)^9 + 160801518474339*(cos(x) - 1)^10/(cos(x) + 1)^10 + 49418961849615*(
cos(x) - 1)^11/(cos(x) + 1)^11 + 9843279601311*(cos(x) - 1)^12/(cos(x) + 1
)^12 + 1220388071083*(cos(x) - 1)^13/(cos(x) + 1)^13 + 85441358295*(cos(x)
 - 1)^14/(cos(x) + 1)^14 + 2577140019*(cos(x) - 1)^15/(cos(x) + 1)^15 + 11
45634921)/(28*(cos(x) - 1)/(cos(x) + 1) + 66*(cos(x) - 1)^2/(cos(x) + 1)^2
 + 28*(cos(x) - 1)^3/(cos(x) + 1)^3 + 3*(cos(x) - 1)^4/(cos(x) + 1)^4 + 3)
^4 + 3889/373248*log(-(cos(x) - 1)/(cos(x) + 1)) - 332929/5832*log(abs(-(c
os(x) - 1)/(cos(x) + 1) - 3)) + 332929/5832*log(abs(-3*(cos(x) - 1)/(cos(x
) + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 22.51 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.65

$$\int \frac{1}{(\sin(x)+\sin(5x))^5}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(5*x) + sin(x))^5,x)
```

output
```
(3889*log(tan(x/2)))/186624 - (332929*atanh(529350107261172808328824126854
4485325492309354181/(8658234741837121114972154438430826l6832*((11878192955
5178788894610664674726584051l864824769*tan(x/2)^2)/2167979328984879417952З
7553242112 - 1319797008169238440809979726425199762666l8115117/722659776328
29313931745851080704)) - 8489643408268/8489639855729))/2916 + (82683*2^(1/
2)*atanh((13990822351110619332532430968055105855879747*2^(1/2))/(407483448
829255725280867l232*((8649099111024721852836396668211292543909075З*tan(x/2
)^2)/30561258662194179396065034 24 - 556481550711442006634583422224345098З9
31439/11460471998322817273524З8784)) - (733900395935309606224509026З610687
21080365265*2^(1/2)*tan(x/2)^2)/(366735103946330152752780410 88*((864909911
1024721852836396668211292543909075З*tan(x/2)^2)/305612586621941793960650З4
24 - 55648155071144200663458342222434509839З1439/11460471998322817273524З8
784))))/1024 - ((13*tan(x/2)^2)/62208 + (21212837*tan(x/2)^4)/279936 - (44
65642615*tan(x/2)^6)/1679616 + (136474677679*tan(x/2)^8)/3359232 - (179028
5716621*tan(x/2)^10)/5038848 + (154818945941*tan(x/2)^12)/78732 - (1087775
49985939*tan(x/2)^14)/15116544 + (356491034365231*tan(x/2)^16)/20155392 -
(440627697903061*tan(x/2)^18)/15116544 + (26719440584471*tan(x/2)^20)/8398
08 - (38162979312085*tan(x/2)^22)/1679616 + (107200617808633*tan(x/2)^24)/
10077696 - (49418813748397*tan(x/2)^26)/15116544 + (820271243347*tan(x/2)^
28)/1259712 - (1220385450625*tan(x/2)^30)/15116544 + (75947731603*tan(x...
```

Reduce [F]

$$\int \frac{1}{(\sin(x)+\sin(5x))^5}\,dx$$
$$= \int \frac{1}{\sin(5x)^5 + 5\sin(5x)^4\sin(x) + 10\sin(5x)^3\sin(x)^2 + 10\sin(5x)^2\sin(x)^3 + 5\sin(5x)\sin(x)^4 + \sin(x}$$

input
```
int(1/(sin(x)+sin(5*x))^5,x)
```

output
```
int(1/(sin(5*x)**5 + 5*sin(5*x)**4*sin(x) + 10*sin(5*x)**3*sin(x)**2 + 10*
sin(5*x)**2*sin(x)**3 + 5*sin(5*x)*sin(x)**4 + sin(x)**5),x)
```

3.28 $\int \frac{1}{(\sin(x)+\sin(5x))^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 90

$$\int \frac{1}{(\sin(x)+\sin(5x))^2}\,dx = -\frac{3}{4}\text{arctanh}(2\cos(x)\sin(x)) - \frac{\cot(x)}{36} - \frac{4\log\left(\sqrt{3}\cos(x)-\sin(x)\right)}{3\sqrt{3}} + \frac{4\log\left(\sqrt{3}\cos(x)+\sin(x)\right)}{3\sqrt{3}} + \frac{\tan(x)\left(43-25\tan^2(x)\right)}{18\left(3-4\tan^2(x)+\tan^4(x)\right)}$$

output

```
-3/4*arctanh(2*cos(x)*sin(x))-1/36*cot(x)-4/9*ln(3^(1/2)*cos(x)-sin(x))*3^
(1/2)+4/9*ln(3^(1/2)*cos(x)+sin(x))*3^(1/2)+tan(x)*(43-25*tan(x)^2)/(54-72
*tan(x)^2+18*tan(x)^4)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \frac{1}{(\sin(x)+\sin(5x))^2}\,dx = \frac{1}{36}\left(32\sqrt{3}\text{arctanh}\left(\frac{\tan(x)}{\sqrt{3}}\right) - \cot(x) + 27\log(\cos(x)-\sin(x)) - 27\log(\cos(x)+\sin(x)) + \frac{9\sin(x)}{\cos(x)-\sin(x)} + \frac{9\sin(x)}{\cos(x)+\sin(x)} + \frac{16\sin(2x)}{1+2\cos(2x)}\right)$$

input
```
Integrate[(Sin[x] + Sin[5*x])^(-2),x]
```

output
```
(32*Sqrt[3]*ArcTanh[Tan[x]/Sqrt[3]] - Cot[x] + 27*Log[Cos[x] - Sin[x]] - 2
7*Log[Cos[x] + Sin[x]] + (9*Sin[x])/(Cos[x] - Sin[x]) + (9*Sin[x])/(Cos[x]
 + Sin[x]) + (16*Sin[2*x])/(1 + 2*Cos[2*x]))/36
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.70, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {3042, 4822, 27, 1673, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(x)+\sin(5x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(x)+\sin(5x))^2}dx$$

$$\Big\downarrow \text{4822}$$

$$\int \frac{\left(\tan^2(x)+1\right)^4\cot^2(x)}{4\left(\tan^4(x)-4\tan^2(x)+3\right)^2}d\tan(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{\cot^2(x)\left(\tan^2(x)+1\right)^4}{\left(\tan^4(x)-4\tan^2(x)+3\right)^2}d\tan(x)$$

$$\Big\downarrow \text{1673}$$

$$\frac{1}{4}\left(\frac{2\tan(x)\left(43-25\tan^2(x)\right)}{9\left(\tan^4(x)-4\tan^2(x)+3\right)}-\frac{1}{24}\int -\frac{8\cot^2(x)\left(-41\tan^4(x)-70\tan^2(x)+3\right)}{3\left(\tan^4(x)-4\tan^2(x)+3\right)}d\tan(x)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{1}{9}\int \frac{\cot^2(x)\left(-41\tan^4(x)-70\tan^2(x)+3\right)}{\tan^4(x)-4\tan^2(x)+3}d\tan(x)+\frac{2\tan(x)\left(43-25\tan^2(x)\right)}{9\left(\tan^4(x)-4\tan^2(x)+3\right)}\right)$$

$$\Big\downarrow \text{2195}$$

$$\frac{1}{4}\left(\frac{1}{9}\int\left(\cot^2(x)-\frac{96}{\tan^2(x)-3}+\frac{54}{\tan^2(x)-1}\right)d\tan(x)+\frac{2\tan(x)\left(43-25\tan^2(x)\right)}{9\left(\tan^4(x)-4\tan^2(x)+3\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{1}{9}\left(-54\text{arctanh}(\tan(x))+32\sqrt{3}\text{arctanh}\left(\frac{\tan(x)}{\sqrt{3}}\right)-\cot(x)\right)+\frac{2\tan(x)\left(43-25\tan^2(x)\right)}{9\left(\tan^4(x)-4\tan^2(x)+3\right)}\right)$$

input
```
Int[(Sin[x] + Sin[5*x])^(-2),x]
```

output
```
((-54*ArcTanh[Tan[x]] + 32*Sqrt[3]*ArcTanh[Tan[x]/Sqrt[3]] - Cot[x])/9 + (
2*Tan[x]*(43 - 25*Tan[x]^2))/(9*(3 - 4*Tan[x]^2 + Tan[x]^4)))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1673
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 + c*x^4)^(p +
 1)*Simp[ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d +
 e*x^2)^q, a + b*x^2 + c*x^4, x])/x^m + (b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5
) - a*b*g)/x^m + c*(4*p + 7)*(b*f - 2*a*g)*x^(2 - m), x], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1]
&& ILtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4822
```
Int[((a_.)*sin[(m_.)*((c_.) + (d_.)*(x_))] + (b_.)*sin[(n_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Sin[
m*ArcTan[x]] + b*Sin[n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[c + d*x]], x] /
; FreeQ[{a, b, c, d}, x] && ILtQ[p/2, 0] && IntegerQ[(m - 1)/2] && IntegerQ
[(n - 1)/2]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.70

method	result
default	$-\frac{1}{4(\tan(x)-1)}+\frac{3\ln(\tan(x)-1)}{4}-\frac{1}{4(\tan(x)+1)}-\frac{3\ln(\tan(x)+1)}{4}-\frac{8\tan(x)}{9\left(\tan(x)^2-3\right)}+\frac{8\sqrt{3}\,\operatorname{arctanh}\left(\frac{\tan(x)\sqrt{3}}{3}\right)}{9}-\frac{}{36\,\mathrm{ta}}$
risch	$\frac{i\left(e^{8ix}+4\,e^{6ix}-2\,e^{4ix}+e^{2ix}-6\right)}{6\,e^{10ix}+6\,e^{6ix}-6\,e^{4ix}-6}-\frac{3\ln\left(e^{2ix}+i\right)}{4}+\frac{4\sqrt{3}\,\ln\left(e^{2ix}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{9}-\frac{4\sqrt{3}\,\ln\left(e^{2ix}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{9}+\frac{3\ln\left(e^{2ix}-i\right)}{4}$

input
```
int(1/(sin(x)+sin(5*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/(tan(x)-1)+3/4*ln(tan(x)-1)-1/4/(tan(x)+1)-3/4*ln(tan(x)+1)-8/9*tan(x
)/(tan(x)^2-3)+8/9*3^(1/2)*arctanh(1/3*tan(x)*3^(1/2))-1/36/tan(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(72) = 144$.

Time = 0.10 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.91

$$\int \frac{1}{(\sin(x)+\sin(5x))^2}\,dx =$$

$$-\frac{288\cos(x)^5 - 384\cos(x)^3 + 27\left(8\cos(x)^4 - 6\cos(x)^2 + 1\right)\log\left(2\cos(x)\sin(x) + 1\right)\sin(x) - 27\,(8}{}$$

input
```
integrate(1/(sin(x)+sin(5*x))^2,x, algorithm="fricas")
```

output
```
-1/72*(288*cos(x)^5 - 384*cos(x)^3 + 27*(8*cos(x)^4 - 6*cos(x)^2 + 1)*log(
2*cos(x)*sin(x) + 1)*sin(x) - 27*(8*cos(x)^4 - 6*cos(x)^2 + 1)*log(-2*cos(
x)*sin(x) + 1)*sin(x) - 16*(8*sqrt(3)*cos(x)^4 - 6*sqrt(3)*cos(x)^2 + sqrt
(3))*log(-(8*cos(x)^4 - 16*cos(x)^2 - 4*(2*sqrt(3)*cos(x)^3 + sqrt(3)*cos(
x))*sin(x) - 1)/(16*cos(x)^4 - 8*cos(x)^2 + 1))*sin(x) + 102*cos(x))/((8*c
os(x)^4 - 6*cos(x)^2 + 1)*sin(x))
```

Sympy [F]

$$\int \frac{1}{(\sin(x)+\sin(5x))^2}\,dx = \int \frac{1}{(\sin(x)+\sin(5x))^2}\,dx$$

input
```
integrate(1/(sin(x)+sin(5*x))**2,x)
```

output
```
Integral((sin(x) + sin(5*x))**(-2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\sin(x)+\sin(5x))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(sin(x)+sin(5*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int \frac{1}{(\sin(x)+\sin(5x))^2}\,dx = -\frac{4}{9}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\tan(x)\right|}{\left|2\sqrt{3}+2\tan(x)\right|}\right) - \frac{17\tan(x)^4-30\tan(x)^2+1}{12\left(\tan(x)^5-4\tan(x)^3+3\tan(x)\right)} - \frac{3}{4}\log\left(\left|\tan(x)+1\right|\right) + \frac{3}{4}\log\left(\left|\tan(x)-1\right|\right)$$

input
```
integrate(1/(sin(x)+sin(5*x))^2,x, algorithm="giac")
```

output
```
-4/9*sqrt(3)*log(abs(-2*sqrt(3) + 2*tan(x))/abs(2*sqrt(3) + 2*tan(x))) - 1
/12*(17*tan(x)^4 - 30*tan(x)^2 + 1)/(tan(x)^5 - 4*tan(x)^3 + 3*tan(x)) - 3
/4*log(abs(tan(x) + 1)) + 3/4*log(abs(tan(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 21.86 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.42

$$\int \frac{1}{(\sin(x)+\sin(5x))^2}\,dx = \frac{\tan\left(\frac{x}{2}\right)}{72} + \frac{3\,\mathrm{atanh}\left(\frac{1417674752\tan\left(\frac{x}{2}\right)}{531441\left(\frac{708837376\tan\left(\frac{x}{2}\right)^2}{531441}-\frac{708837376}{531441}\right)}\right)}{2} - \frac{8\sqrt{3}\,\mathrm{atanh}\left(\frac{181462368256\sqrt{3}\tan\left(\frac{x}{2}\right)}{14348907\left(\frac{90731184128\tan\left(\frac{x}{2}\right)^2}{4782969}-\frac{90731184128}{4782969}\right)}\right)}{9} - \frac{\frac{347\tan\left(\frac{x}{2}\right)^8}{216}-\frac{155\tan\left(\frac{x}{2}\right)^6}{18}+\frac{949\tan\left(\frac{x}{2}\right)^4}{108}-\frac{31\tan\left(\frac{x}{2}\right)^2}{18}+\frac{1}{72}}{\tan\left(\frac{x}{2}\right)^9-\frac{28\tan\left(\frac{x}{2}\right)^7}{3}+22\tan\left(\frac{x}{2}\right)^5-\frac{28\tan\left(\frac{x}{2}\right)^3}{3}+\tan\left(\frac{x}{2}\right)}$$

input
```
int(1/(sin(5*x) + sin(x))^2,x)
```

output
```
tan(x/2)/72 + (3*atanh((1417674752*tan(x/2))/(531441*((708837376*tan(x/2)^
2)/531441 - 708837376/531441))))/2 - (8*3^(1/2)*atanh((181462368256*3^(1/2
)*tan(x/2))/(14348907*((90731184128*tan(x/2)^2)/4782969 - 90731184128/4782
969))))/9 - ((949*tan(x/2)^4)/108 - (31*tan(x/2)^2)/18 - (155*tan(x/2)^6)/
18 + (347*tan(x/2)^8)/216 + 1/72)/(tan(x/2) - (28*tan(x/2)^3)/3 + 22*tan(x
/2)^5 - (28*tan(x/2)^7)/3 + tan(x/2)^9)
```

Reduce [F]

$$\int \frac{1}{(\sin(x)+\sin(5x))^2}\,dx = \int \frac{1}{\sin(5x)^2+2\sin(5x)\sin(x)+\sin(x)^2}dx$$

input
```
int(1/(sin(x)+sin(5*x))^2,x)
```

output
```
int(1/(sin(5*x)**2 + 2*sin(5*x)*sin(x) + sin(x)**2),x)
```

3.29 $\int \frac{1}{(\sin(x)+\sin(5x))^4} dx$

Optimal result

Integrand size = 9, antiderivative size = 154

$$\int \frac{1}{(\sin(x)+\sin(5x))^4} dx = -\frac{103}{8}\operatorname{arctanh}(2\cos(x)\sin(x)) - \frac{43\cot(x)}{3888} - \frac{\cot^3(x)}{3888} - \frac{1808\log\left(\sqrt{3}\cos(x)-\sin(x)\right)}{81\sqrt{3}} + \frac{1808\log\left(\sqrt{3}\cos(x)+\sin(x)\right)}{81\sqrt{3}} + \frac{4\tan(x)\,(593-539\tan^2(x))}{81\,(3-4\tan^2(x)+\tan^4(x))^3} - \frac{2\tan(x)\,(3502-1237\tan^2(x))}{243\,(3-4\tan^2(x)+\tan^4(x))^2} + \frac{\tan(x)\,(38097-18413\tan^2(x))}{972\,(3-4\tan^2(x)+\tan^4(x))}$$

output
```
-103/8*arctanh(2*cos(x)*sin(x))-43/3888*cot(x)-1/3888*cot(x)^3-1808/243*ln
(3^(1/2)*cos(x)-sin(x))*3^(1/2)+1808/243*ln(3^(1/2)*cos(x)+sin(x))*3^(1/2)
+4/81*tan(x)*(593-539*tan(x)^2)/(3-4*tan(x)^2+tan(x)^4)^3-2/243*tan(x)*(35
02-1237*tan(x)^2)/(3-4*tan(x)^2+tan(x)^4)^2+tan(x)*(38097-18413*tan(x)^2)/
(2916-3888*tan(x)^2+972*tan(x)^4)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.07

$$\int \frac{1}{(\sin(x)+\sin(5x))^4}\,dx = \frac{115712\sqrt{3}\text{arctanh}\left(\frac{\tan(x)}{\sqrt{3}}\right)-2\cot(x)\left(42+\csc^2(x)\right)+100116\log(\cos(x)-\sin(x))-100116\log(\cos(x)}{}$$

input
```
Integrate[(Sin[x] + Sin[5*x])^(-4),x]
```

output
```
(115712*Sqrt[3]*ArcTanh[Tan[x]/Sqrt[3]] - 2*Cot[x]*(42 + Csc[x]^2) + 10011
6*Log[Cos[x] - Sin[x]] - 100116*Log[Cos[x] + Sin[x]] - 1377/(Cos[x] - Sin[
x])^2 + (162*Sin[x])/(Cos[x] - Sin[x])^3 + (26892*Sin[x])/(Cos[x] - Sin[x]
) + (162*Sin[x])/(Cos[x] + Sin[x])^3 + 1377/(Cos[x] + Sin[x])^2 + (26892*S
in[x])/(Cos[x] + Sin[x]) + (1536*Sin[2*x])/(1 + 2*Cos[2*x])^3 + (9472*Sin[
2*x])/(1 + 2*Cos[2*x])^2 + (51456*Sin[2*x])/(1 + 2*Cos[2*x]))/7776
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.83, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.222$, Rules used = {3042, 4822, 27, 1673, 27, 2198, 27, 2198, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(x)+\sin(5x))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(x)+\sin(5x))^4}dx$$

$$\Big\downarrow \text{4822}$$

$$\int \frac{\left(\tan^2(x)+1\right)^9\cot^4(x)}{16\left(\tan^4(x)-4\tan^2(x)+3\right)^4}d\tan(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int \frac{\cot^4(x)\left(\tan^2(x)+1\right)^9}{\left(\tan^4(x)-4\tan^2(x)+3\right)^4}d\tan(x)$$

$$\Big\downarrow \text{1673}$$

$$\frac{1}{16}\left(\frac{64\tan(x)\left(593-539\tan^2(x)\right)}{81\left(\tan^4(x)-4\tan^2(x)+3\right)^3}-\frac{1}{72}\int -\frac{8\cot^4(x)\left(81\tan^{14}(x)+1053\tan^{12}(x)+6885\tan^{10}(x)+31185\tan\right.}{9\left(\tan^4(x)-4\tan\right.}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{1}{81}\int \frac{\cot^4(x)\left(81\tan^{14}(x)+1053\tan^{12}(x)+6885\tan^{10}(x)+31185\tan^8(x)-196173\tan^6(x)-36617\tan^4\right.}{\left(\tan^4(x)-4\tan^2(x)+3\right)^3}\right.$$

$$\Big\downarrow \text{2198}$$

$$\frac{1}{16}\left(\frac{1}{81}\left(-\frac{1}{48}\int -\frac{16\cot^4(x)\left(243\tan^{10}(x)+4131\tan^8(x)+234370\tan^6(x)+75858\tan^4(x)+315\tan^2(x)+2\right.}{\left(\tan^4(x)-4\tan^2(x)+3\right)^2}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{1}{81}\left(\frac{1}{3}\int \frac{\cot^4(x)\left(243\tan^{10}(x)+4131\tan^8(x)+234370\tan^6(x)+75858\tan^4(x)+315\tan^2(x)+27\right)}{\left(\tan^4(x)-4\tan^2(x)+3\right)^2}d\tan\right.\right.$$

$$\Big\downarrow \text{2198}$$

$$\frac{1}{16}\left(\frac{1}{81}\left(\frac{1}{3}\left(\frac{4\tan(x)\left(38097-18413\tan^2(x)\right)}{\tan^4(x)-4\tan^2(x)+3}-\frac{1}{24}\int -\frac{24\cot^4(x)\left(-73409\tan^6(x)-126949\tan^4(x)+117\tan\right.}{\tan^4(x)-4\tan^2(x)+3}\right.\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{1}{81}\left(\frac{1}{3}\left(\int \frac{\cot^4(x)\left(-73409\tan^6(x)-126949\tan^4(x)+117\tan^2(x)+9\right)}{\tan^4(x)-4\tan^2(x)+3}d\tan(x)+\frac{4\tan(x)\left(38097-1841\right.}{\tan^4(x)-4\tan^2(\right.}\right.\right.\right.$$

$$\Big\downarrow \text{2195}$$

$$\frac{1}{16}\left(\frac{1}{81}\left(\frac{1}{3}\left(\int \left(3\cot^4(x)+43\cot^2(x)-\frac{173568}{\tan^2(x)-3}+\frac{100116}{\tan^2(x)-1}\right)d\tan(x)+\frac{4\tan(x)\left(38097-18413\tan^2\right.}{\tan^4(x)-4\tan^2(x)+3}\right.\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{16}\left(\frac{1}{81}\left(\frac{1}{3}\left(-100116\text{arctanh}(\tan(x))+57856\sqrt{3}\text{arctanh}\left(\frac{\tan(x)}{\sqrt{3}}\right)+\frac{4\tan(x)\left(38097-18413\tan^2(x)\right)}{\tan^4(x)-4\tan^2(x)+3}-\cot^3\right.\right.\right.$$

input
```
Int[(Sin[x] + Sin[5*x])^(-4),x]
```

output
```
((64*Tan[x]*(593 - 539*Tan[x]^2))/(81*(3 - 4*Tan[x]^2 + Tan[x]^4)^3) + ((-
32*Tan[x]*(3502 - 1237*Tan[x]^2))/(3*(3 - 4*Tan[x]^2 + Tan[x]^4)^2) + (-10
0116*ArcTanh[Tan[x]] + 57856*Sqrt[3]*ArcTanh[Tan[x]/Sqrt[3]] - 43*Cot[x] -
 Cot[x]^3 + (4*Tan[x]*(38097 - 18413*Tan[x]^2))/(3 - 4*Tan[x]^2 + Tan[x]^4
))/3)/81)/16
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1673
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 + c*x^4)^(p +
 1)*Simp[ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d +
 e*x^2)^q, a + b*x^2 + c*x^4, x])/x^m + (b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5
) - a*b*g)/x^m + c*(4*p + 7)*(b*f - 2*a*g)*x^(2 - m), x], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1]
&& ILtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4822
```
Int[((a_.)*sin[(m_.)*((c_.) + (d_.)*(x_))] + (b_.)*sin[(n_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Sin[
m*ArcTan[x]] + b*Sin[n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[c + d*x]], x] /
; FreeQ[{a, b, c, d}, x] && ILtQ[p/2, 0] && IntegerQ[(m - 1)/2] && IntegerQ
[(n - 1)/2]
```

Maple [A] (verified)

Time = 25.68 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.01

method	result
parallelrisch	0
default	$-\frac{64\left(\frac{85\tan(x)^5}{2}-280\tan(x)^3+\frac{963\tan(x)}{2}\right)}{243\left(\tan(x)^2-3\right)^3}+\frac{3616\sqrt{3}\,\operatorname{arctanh}\left(\frac{\tan(x)\sqrt{3}}{3}\right)}{243}-\frac{1}{3888\tan(x)^3}-\frac{43}{3888\tan(x)}-\frac{1}{24(\tan(x$
risch	$\frac{i(1111\,e^{28ix}+3616\,e^{26ix}+552\,e^{24ix}+5707\,e^{22ix}-12743\,e^{20ix}+3768\,e^{18ix}-23899\,e^{16ix}+15629\,e^{14ix}-13800\,e^{12ix}+26785\,e^{10}}{324(e^{10ix}+e^{6ix}-e^{4ix}-1)^3}$

input
```
int(1/(sin(x)+sin(5*x))^4,x,method=_RETURNVERBOSE)
```

output
```
0
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(130) = 260.

Time = 0.13 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.20

$$\int \frac{1}{(\sin(x)+\sin(5x))^4}\,dx = -\frac{13492224\cos(x)^{15} - 52302336\cos(x)^{13} + 82581120\cos(x)^{11} - 68730912\cos(x)^{9} + 32578200\cos(x)^{...}}{}$$

input
```
integrate(1/(sin(x)+sin(5*x))^4,x, algorithm="fricas")
```

output
```
-1/3888*(13492224*cos(x)^15 - 52302336*cos(x)^13 + 82581120*cos(x)^11 - 68
730912*cos(x)^9 + 32578200*cos(x)^7 - 8802612*cos(x)^5 + 1257984*cos(x)^3
+ 25029*(512*cos(x)^14 - 1664*cos(x)^12 + 2208*cos(x)^10 - 1560*cos(x)^8 +
 636*cos(x)^6 - 150*cos(x)^4 + 19*cos(x)^2 - 1)*log(2*cos(x)*sin(x) + 1)*s
in(x) - 25029*(512*cos(x)^14 - 1664*cos(x)^12 + 2208*cos(x)^10 - 1560*cos(
x)^8 + 636*cos(x)^6 - 150*cos(x)^4 + 19*cos(x)^2 - 1)*log(-2*cos(x)*sin(x)
 + 1)*sin(x) - 14464*(512*sqrt(3)*cos(x)^14 - 1664*sqrt(3)*cos(x)^12 + 220
8*sqrt(3)*cos(x)^10 - 1560*sqrt(3)*cos(x)^8 + 636*sqrt(3)*cos(x)^6 - 150*s
qrt(3)*cos(x)^4 + 19*sqrt(3)*cos(x)^2 - sqrt(3))*log(-(8*cos(x)^4 - 16*cos
(x)^2 - 4*(2*sqrt(3)*cos(x)^3 + sqrt(3)*cos(x))*sin(x) - 1)/(16*cos(x)^4 -
 8*cos(x)^2 + 1))*sin(x) - 73695*cos(x))/((512*cos(x)^14 - 1664*cos(x)^12
+ 2208*cos(x)^10 - 1560*cos(x)^8 + 636*cos(x)^6 - 150*cos(x)^4 + 19*cos(x)
^2 - 1)*sin(x))
```

Sympy [F]

$$\int \frac{1}{(\sin(x)+\sin(5x))^4}\,dx = \int \frac{1}{(\sin(x)+\sin(5x))^4}\,dx$$

input
```
integrate(1/(sin(x)+sin(5*x))**4,x)
```

output
```
Integral((sin(x) + sin(5*x))**(-4), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\sin(x) + \sin(5x))^4} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(sin(x)+sin(5*x))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.72

$$\int \frac{1}{(\sin(x) + \sin(5x))^4} \, dx = -\frac{1808}{243} \sqrt{3} \log \left(\frac{\left|-2\sqrt{3} + 2\tan(x)\right|}{\left|2\sqrt{3} + 2\tan(x)\right|} \right) - \frac{24565\tan(x)^{14} - 247373\tan(x)^{12} + 934101\tan(x)^{10} - 1618525\tan(x)^{8} + 1287959\tan(x)^{6} - 384}{1296\left(\tan(x)^{5} - 4\tan(x)^{3} + 3\tan(x)\right)^{3}} - \frac{103}{8}\log\left(\left|\tan(x) + 1\right|\right) + \frac{103}{8}\log\left(\left|\tan(x) - 1\right|\right)$$

input
```
integrate(1/(sin(x)+sin(5*x))^4,x, algorithm="giac")
```

output
```
-1808/243*sqrt(3)*log(abs(-2*sqrt(3) + 2*tan(x))/abs(2*sqrt(3) + 2*tan(x))
) - 1/1296*(24565*tan(x)^14 - 247373*tan(x)^12 + 934101*tan(x)^10 - 161852
5*tan(x)^8 + 1287959*tan(x)^6 - 384543*tan(x)^4 + 351*tan(x)^2 + 9)/(tan(x
)^5 - 4*tan(x)^3 + 3*tan(x))^3 - 103/8*log(abs(tan(x) + 1)) + 103/8*log(ab
s(tan(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 22.26 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.78

$$\int \frac{1}{(\sin(x) + \sin(5x))^4} \, dx = \text{Too large to display}$$

input
```
int(1/(sin(5*x) + sin(x))^4,x)
```

output
```
(169*tan(x/2))/31104 + (103*atanh((211880270124391989248*tan(x/2))/(254186
5828329*((105940135062195994624*tan(x/2)^2)/2541865828329 - 10594013506219
5994624/2541865828329))))/4 - ((47*tan(x/2)^2)/10368 - (38077*tan(x/2)^4)/
1728 + (75085891*tan(x/2)^6)/139968 - (1494496261*tan(x/2)^8)/279936 + (23
512181909*tan(x/2)^10)/839808 - (17581132309*tan(x/2)^12)/209952 + (101785
38305*tan(x/2)^14)/69984 - (40705839889*tan(x/2)^16)/279936 + (23427167539
*tan(x/2)^18)/279936 - (3914473429*tan(x/2)^20)/139968 + (2237865505*tan(x
/2)^22)/419904 - (449205433*tan(x/2)^24)/839808 + (2043451*tan(x/2)^26)/93
312 + 1/31104)/(tan(x/2)^3 - 28*tan(x/2)^5 + (982*tan(x/2)^7)/3 - (55972*t
an(x/2)^9)/27 + 7727*tan(x/2)^11 - (155512*tan(x/2)^13)/9 + (68404*tan(x/2
)^15)/3 - (155512*tan(x/2)^17)/9 + 7727*tan(x/2)^19 - (55972*tan(x/2)^21)/
27 + (982*tan(x/2)^23)/3 - 28*tan(x/2)^25 + tan(x/2)^27) + tan(x/2)^3/3110
4 - (3616*3^(1/2)*atanh((2089117654278142695571456*3^(1/2)*tan(x/2))/(5559
060566555523*((1044558827139071347785728*tan(x/2)^2)/1853020188851841 - 10
44558827139071347785728/1853020188851841))))/243
```

Reduce [F]

$$\int \frac{1}{(\sin(x) + \sin(5x))^4} \, dx$$
$$= \int \frac{1}{\sin(5x)^4 + 4\sin(5x)^3 \sin(x) + 6\sin(5x)^2 \sin(x)^2 + 4\sin(5x) \sin(x)^3 + \sin(x)^4} dx$$

input
```
int(1/(sin(x)+sin(5*x))^4,x)
```

output
```
int(1/(sin(5*x)**4 + 4*sin(5*x)**3*sin(x) + 6*sin(5*x)**2*sin(x)**2 + 4*si
n(5*x)*sin(x)**3 + sin(x)**4),x)
```

3.30 $\int \frac{1}{(\sin(x)+\sin(5x))^6}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 218

$$
\begin{aligned}
\int \frac{1}{(\sin(x)+\sin(5x))^6}\,dx =& -\frac{33243}{128}\operatorname{arctanh}(2\cos(x)\sin(x)) - \frac{715\cot(x)}{139968} \\
&- \frac{11\cot^3(x)}{69984} - \frac{\cot^5(x)}{233280} - \frac{109312\log\left(\sqrt{3}\cos(x)-\sin(x)\right)}{243\sqrt{3}} \\
&+ \frac{109312\log\left(\sqrt{3}\cos(x)+\sin(x)\right)}{243\sqrt{3}} \\
&+ \frac{32\tan(x)\left(16627-16465\tan^2(x)\right)}{405\left(3-4\tan^2(x)+\tan^4(x)\right)^5} \\
&- \frac{2\tan(x)\left(76795+61567\tan^2(x)\right)}{405\left(3-4\tan^2(x)+\tan^4(x)\right)^4} \\
&+ \frac{\tan(x)\left(778363-606643\tan^2(x)\right)}{2430\left(3-4\tan^2(x)+\tan^4(x)\right)^3} \\
&- \frac{\tan(x)\left(91032631-43437157\tan^2(x)\right)}{174960\left(3-4\tan^2(x)+\tan^4(x)\right)^2} \\
&+ \frac{\tan(x)\left(111184863-53226455\tan^2(x)\right)}{139968\left(3-4\tan^2(x)+\tan^4(x)\right)}
\end{aligned}
$$

output
```
-33243/128*arctanh(2*cos(x)*sin(x))-715/139968*cot(x)-11/69984*cot(x)^3-1/
233280*cot(x)^5-109312/729*ln(3^(1/2)*cos(x)-sin(x))*3^(1/2)+109312/729*ln
(3^(1/2)*cos(x)+sin(x))*3^(1/2)+32/405*tan(x)*(16627-16465*tan(x)^2)/(3-4*
tan(x)^2+tan(x)^4)^5-2/405*tan(x)*(76795+61567*tan(x)^2)/(3-4*tan(x)^2+tan
(x)^4)^4+1/2430*tan(x)*(778363-606643*tan(x)^2)/(3-4*tan(x)^2+tan(x)^4)^3-
1/174960*tan(x)*(91032631-43437157*tan(x)^2)/(3-4*tan(x)^2+tan(x)^4)^2+tan
(x)*(111184863-53226455*tan(x)^2)/(419904-559872*tan(x)^2+139968*tan(x)^4)
```

Mathematica [A] (verified)

Time = 6.05 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.33

$$\int \frac{1}{(\sin(x)+\sin(5x))^6}\,dx = \frac{218624\text{arctanh}\left(\frac{\tan(x)}{\sqrt{3}}\right)}{243\sqrt{3}} - \frac{289\cot(x)}{58320} - \frac{13\cot(x)\csc^2(x)}{87480} - \frac{\cot(x)\csc^4(x)}{233280} + \frac{33243}{128}\log(\cos(x)-\sin(x)) - \frac{33243}{128}\log(\cos(x)+\sin(x)) - \frac{43}{1280(\cos(x)-\sin(x))^4} - \frac{7393}{1920(\cos(x)-\sin(x))^2} + \frac{\sin(x)}{320(\cos(x)-\sin(x))^5} + \frac{1099\sin(x)}{1920(\cos(x)-\sin(x))^3} + \frac{16613\sin(x)}{240(\cos(x)-\sin(x))} + \frac{\sin(x)}{320(\cos(x)+\sin(x))^5} + \frac{43}{1280(\cos(x)+\sin(x))^4} + \frac{1099\sin(x)}{1920(\cos(x)+\sin(x))^3} + \frac{7393}{1920(\cos(x)+\sin(x))^2} + \frac{16613\sin(x)}{240(\cos(x)+\sin(x))} + \frac{64\sin(2x)}{405(1+2\cos(2x))^5} + \frac{512\sin(2x)}{405(1+2\cos(2x))^4} + \frac{7744\sin(2x)}{1215(1+2\cos(2x))^3} + \frac{300352\sin(2x)}{10935(1+2\cos(2x))^2} + \frac{486784\sin(2x)}{3645(1+2\cos(2x))}$$

input
```
Integrate[(Sin[x] + Sin[5*x])^(-6),x]
```

output
```
(218624*ArcTanh[Tan[x]/Sqrt[3]])/(243*Sqrt[3]) - (289*Cot[x])/58320 - (13*
Cot[x]*Csc[x]^2)/87480 - (Cot[x]*Csc[x]^4)/233280 + (33243*Log[Cos[x] - Si
n[x]])/128 - (33243*Log[Cos[x] + Sin[x]])/128 - 43/(1280*(Cos[x] - Sin[x])
^4) - 7393/(1920*(Cos[x] - Sin[x])^2) + Sin[x]/(320*(Cos[x] - Sin[x])^5) +
 (1099*Sin[x])/(1920*(Cos[x] - Sin[x])^3) + (16613*Sin[x])/(240*(Cos[x] -
Sin[x])) + Sin[x]/(320*(Cos[x] + Sin[x])^5) + 43/(1280*(Cos[x] + Sin[x])^4
) + (1099*Sin[x])/(1920*(Cos[x] + Sin[x])^3) + 7393/(1920*(Cos[x] + Sin[x]
)^2) + (16613*Sin[x])/(240*(Cos[x] + Sin[x])) + (64*Sin[2*x])/(405*(1 + 2*
Cos[2*x])^5) + (512*Sin[2*x])/(405*(1 + 2*Cos[2*x])^4) + (7744*Sin[2*x])/(
1215*(1 + 2*Cos[2*x])^3) + (300352*Sin[2*x])/(10935*(1 + 2*Cos[2*x])^2) +
(486784*Sin[2*x])/(3645*(1 + 2*Cos[2*x]))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.89, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.667$, Rules used = {3042, 4822, 27, 1673, 27, 2198, 27, 2198, 27, 2198, 27, 2198, 27, 2195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(x)+\sin(5x))^6}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(\sin(x)+\sin(5x))^6}dx$$

$$\Big\downarrow \text{ 4822}$$

$$\int \frac{\left(\tan^2(x)+1\right)^{14}\cot^6(x)}{64\left(\tan^4(x)-4\tan^2(x)+3\right)^6}d\tan(x)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{64}\int \frac{\cot^6(x)\left(\tan^2(x)+1\right)^{14}}{\left(\tan^4(x)-4\tan^2(x)+3\right)^6}d\tan(x)$$

$$\Big\downarrow \text{ 1673}$$

$$\frac{1}{64}\left(\frac{2048\tan(x)\left(16627-16465\tan^2(x)\right)}{405\left(\tan^4(x)-4\tan^2(x)+3\right)^5}-\frac{1}{120}\int -\frac{8\cot^6(x)\left(405\tan^{24}(x)+7290\tan^{22}(x)+64800\tan^{20}(x)\right.}{}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{64}\left(\frac{1}{405}\int \frac{\cot^6(x)\left(405\tan^{24}(x)+7290\tan^{22}(x)+64800\tan^{20}(x)+384750\tan^{18}(x)+1750005\tan^{16}(x)+66\right.}{}\right.$$

$$\Big\downarrow \; 2198$$

$$\frac{1}{64}\left(\frac{1}{405}\left(-\frac{1}{96}\int -\frac{96\cot^6(x)\left(405\tan^{20}(x)+8910\tan^{18}(x)+99225\tan^{16}(x)+754920\tan^{14}(x)+4472010\tan\right.}{\left(\tan^4(x)\right.}\right.\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{64}\left(\frac{1}{405}\left(\int \frac{\cot^6(x)\left(405\tan^{20}(x)+8910\tan^{18}(x)+99225\tan^{16}(x)+754920\tan^{14}(x)+4472010\tan^{12}(x)+2\right.}{\left(\tan^4(x)-4\tan^2\right.}\right.\right.$$

$$\Big\downarrow \; 2198$$

$$\frac{1}{64}\left(\frac{1}{405}\left(-\frac{1}{72}\int -\frac{24\cot^6(x)\left(1215\tan^{16}(x)+31590\tan^{14}(x)+420390\tan^{12}(x)+3851550\tan^{10}(x)-147152\right.}{\left(\tan^4(x)-4\tan^2(x)+3\right.}\right.\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{64}\left(\frac{1}{405}\left(\frac{1}{3}\int \frac{\cot^6(x)\left(1215\tan^{16}(x)+31590\tan^{14}(x)+420390\tan^{12}(x)+3851550\tan^{10}(x)-147152124\tan\right.}{\left(\tan^4(x)-4\tan^2(x)+3\right)^3}\right.\right.$$

$$\Big\downarrow \; 2198$$

$$\frac{1}{64}\left(\frac{1}{405}\left(\frac{1}{3}\left(-\frac{1}{48}\int -\frac{80\cot^6(x)\left(2187\tan^{12}(x)+65610\tan^{10}(x)+174761209\tan^8(x)+56999160\tan^6(x)+\right.}{3\left(\tan^4(x)-4\tan^2(x)+3\right)^2}\right.\right.\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{64}\left(\frac{1}{405}\left(\frac{1}{3}\left(\frac{5}{9}\int \frac{\cot^6(x)\left(2187\tan^{12}(x)+65610\tan^{10}(x)+174761209\tan^8(x)+56999160\tan^6(x)+4917\tan\right.}{\left(\tan^4(x)-4\tan^2(x)+3\right)^2}\right.\right.\right.$$

$$\Big\downarrow \; 2198$$

$$\frac{1}{64}\left(\frac{1}{405}\left(\frac{1}{3}\left(\frac{5}{9}\left(\frac{\tan(x)\left(111184863-53226455\tan^2(x)\right)}{\tan^4(x)-4\tan^2(x)+3}-\frac{1}{24}\int -\frac{24\cot^6(x)\left(-53224268\tan^8(x)-92182693\right.}{\tan^4(x)-4}\right.\right.\right.\right.$$

$\big\downarrow$ 27

$$\frac{1}{64}\left(\frac{1}{405}\left(\frac{1}{3}\left(\frac{5}{9}\left(\int \frac{\cot^6(x)\left(-53224268\tan^8(x)-92182693\tan^6(x)+1884\tan^4(x)+186\tan^2(x)+9\right)}{\tan^4(x)-4\tan^2(x)+3}d\tan(x\right.\right.\right.\right.$$

$\big\downarrow$ 2195

$$\frac{1}{64}\left(\frac{1}{405}\left(\frac{1}{3}\left(\frac{5}{9}\left(\int \left(3\cot^6(x)+66\cot^4(x)+715\cot^2(x)-\frac{125927424}{\tan^2(x)-3}+\frac{72702441}{\tan^2(x)-1}\right)d\tan(x)+\frac{\tan(x)\left(11\right.}{\text{ta}}\right.\right.\right.\right.$$

$\big\downarrow$ 2009

$$\frac{1}{64}\left(\frac{1}{405}\left(\frac{1}{3}\left(\frac{5}{9}\left(-72702441\text{arctanh}(\tan(x))+41975808\sqrt{3}\text{arctanh}\left(\frac{\tan(x)}{\sqrt{3}}\right)+\frac{\tan(x)\left(111184863-5322645\right.}{\tan^4(x)-4\tan^2(x)+}\right.\right.\right.\right.$$

input
```
Int[(Sin[x] + Sin[5*x])^(-6),x]
```

output
```
((2048*Tan[x]*(16627 - 16465*Tan[x]^2))/(405*(3 - 4*Tan[x]^2 + Tan[x]^4)^5
) + ((-128*Tan[x]*(76795 + 61567*Tan[x]^2))/(3 - 4*Tan[x]^2 + Tan[x]^4)^4
+ (32*Tan[x]*(778363 - 606643*Tan[x]^2))/(3*(3 - 4*Tan[x]^2 + Tan[x]^4)^3)
 + ((-4*Tan[x]*(91032631 - 43437157*Tan[x]^2))/(9*(3 - 4*Tan[x]^2 + Tan[x]
^4)^2) + (5*(-72702441*ArcTanh[Tan[x]] + 41975808*Sqrt[3]*ArcTanh[Tan[x]/S
qrt[3]] - 715*Cot[x] - 22*Cot[x]^3 - (3*Cot[x]^5)/5 + (Tan[x]*(111184863 -
 53226455*Tan[x]^2))/(3 - 4*Tan[x]^2 + Tan[x]^4)))/9)/3)/405)/64
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1673
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))),
 x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 + c*x^4)^(p +
 1)*Simp[ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d +
 e*x^2)^q, a + b*x^2 + c*x^4, x])/x^m + (b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5
) - a*b*g)/x^m + c*(4*p + 7)*(b*f - 2*a*g)*x^(2 - m), x], x], x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1]
&& ILtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2195
```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

rule 2198
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x], d = Coeff[Pol
ynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Polynomial
Remainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)
^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b^2
 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[x^m*(a + b*x^2 +
 c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*Qx)/x^m + (b^2*d*(2*
p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 -
m), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x
^2], 1] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4822
```
Int[((a_.)*sin[(m_.)*((c_.) + (d_.)*(x_))] + (b_.)*sin[(n_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Sin[
m*ArcTan[x]] + b*Sin[n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[c + d*x]], x] /
; FreeQ[{a, b, c, d}, x] && ILtQ[p/2, 0] && IntegerQ[(m - 1)/2] && IntegerQ
[(n - 1)/2]
```

Maple [A] (verified)

Time = 421.25 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.01

method	result
parallelrisch	0
default	$-\frac{1}{80(\tan(x)+1)^5}+\frac{11}{64(\tan(x)+1)^4}-\frac{281}{192(\tan(x)+1)^3}+\frac{659}{64(\tan(x)+1)^2}-\frac{10085}{128(\tan(x)+1)}-\frac{33243\ln(\tan(x)+1)}{128}$ -
risch	$\frac{i(-21150720+5410405\,e^{2ix}-88263680\,e^{4ix}+702189680\,e^{14ix}-359783050\,e^{12ix}-905873280\,e^{16ix}-1305280104\,e^{20ix}+44317}{}$

input
```
int(1/(sin(x)+sin(5*x))^6,x,method=_RETURNVERBOSE)
```

output
```
0
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(188) = 376.

Time = 0.30 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.30

$$\int \frac{1}{(\sin(x)+\sin(5x))^6}\,dx = \text{Too large to display}$$

input
```
integrate(1/(sin(x)+sin(5*x))^6,x, algorithm="fricas")
```

output
```
-1/933120*(4158400757760*cos(x)^25 - 26521869189120*cos(x)^23 + 7540040159
2320*cos(x)^21 - 126325942517760*cos(x)^19 + 138927429196800*cos(x)^17 - 1
05686653930368*cos(x)^15 + 57050747014560*cos(x)^13 - 22030071327120*cos(x
)^11 + 6043251410040*cos(x)^9 - 1149349729260*cos(x)^7 + 143978865060*cos(
x)^5 - 10676989740*cos(x)^3 + 121170735*(32768*cos(x)^24 - 188416*cos(x)^2
2 + 483328*cos(x)^20 - 732160*cos(x)^18 + 730240*cos(x)^16 - 505696*cos(x)
^14 + 249552*cos(x)^12 - 88496*cos(x)^10 + 22400*cos(x)^8 - 3950*cos(x)^6
+ 461*cos(x)^4 - 32*cos(x)^2 + 1)*log(2*cos(x)*sin(x) + 1)*sin(x) - 121170
735*(32768*cos(x)^24 - 188416*cos(x)^22 + 483328*cos(x)^20 - 732160*cos(x)
^18 + 730240*cos(x)^16 - 505696*cos(x)^14 + 249552*cos(x)^12 - 88496*cos(x
)^10 + 22400*cos(x)^8 - 3950*cos(x)^6 + 461*cos(x)^4 - 32*cos(x)^2 + 1)*lo
g(-2*cos(x)*sin(x) + 1)*sin(x) - 69959680*(32768*sqrt(3)*cos(x)^24 - 18841
6*sqrt(3)*cos(x)^22 + 483328*sqrt(3)*cos(x)^20 - 732160*sqrt(3)*cos(x)^18
+ 730240*sqrt(3)*cos(x)^16 - 505696*sqrt(3)*cos(x)^14 + 249552*sqrt(3)*cos
(x)^12 - 88496*sqrt(3)*cos(x)^10 + 22400*sqrt(3)*cos(x)^8 - 3950*sqrt(3)*c
os(x)^6 + 461*sqrt(3)*cos(x)^4 - 32*sqrt(3)*cos(x)^2 + sqrt(3))*log(-(8*co
s(x)^4 - 16*cos(x)^2 - 4*(2*sqrt(3)*cos(x)^3 + sqrt(3)*cos(x))*sin(x) - 1)
/(16*cos(x)^4 - 8*cos(x)^2 + 1))*sin(x) + 354847800*cos(x))/((32768*cos(x)
^24 - 188416*cos(x)^22 + 483328*cos(x)^20 - 732160*cos(x)^18 + 730240*cos(
x)^16 - 505696*cos(x)^14 + 249552*cos(x)^12 - 88496*cos(x)^10 + 22400*c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(\sin(x) + \sin(5x))^6} \, dx = \text{Timed out}$$

input
```
integrate(1/(sin(x)+sin(5*x))**6,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\sin(x) + \sin(5x))^6} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(sin(x)+sin(5*x))^6,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.65

$$\int \frac{1}{(\sin(x) + \sin(5x))^6} \, dx = -\frac{109312}{729}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3} + 2\tan(x)\right|}{\left|2\sqrt{3} + 2\tan(x)\right|}\right)$$
$$- \frac{29570650\tan(x)^{24} - 534901345\tan(x)^{22} + 4162661060\tan(x)^{20} - 18227416530\tan(x)^{18} + 49341\ldots}{}$$
$$- \frac{33243}{128}\log(|\tan(x) + 1|) + \frac{33243}{128}\log(|\tan(x) - 1|)$$

input
```
integrate(1/(sin(x)+sin(5*x))^6,x, algorithm="giac")
```

output
```
-109312/729*sqrt(3)*log(abs(-2*sqrt(3) + 2*tan(x))/abs(2*sqrt(3) + 2*tan(x
))) - 1/77760*(29570650*tan(x)^24 - 534901345*tan(x)^22 + 4162661060*tan(x
)^20 - 18227416530*tan(x)^18 + 49341594025*tan(x)^16 - 85462804130*tan(x)^
14 + 94715143160*tan(x)^12 - 64892703684*tan(x)^10 + 25017153300*tan(x)^8
- 4149372645*tan(x)^6 + 78300*tan(x)^4 + 2430*tan(x)^2 + 81)/(tan(x)^5 - 4
*tan(x)^3 + 3*tan(x))^5 - 33243/128*log(abs(tan(x) + 1)) + 33243/128*log(a
bs(tan(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 23.33 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.92

$$\int \frac{1}{(\sin(x)+\sin(5x))^6}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(5*x) + sin(x))^6,x)
```

output
```
(33243*atanh((114251110334207709216704288889*tan(x/2))/(2928229434235008*((
114251110334207709216704288889*tan(x/2)^2)/5856458868470016 - 1142511103342
07709216704288889/5856458868470016))))/64 + (5591*tan(x/2))/2239488 - (2186
24*3^(1/2)*atanh((711570892112035166073943584931184*3^(1/2)*tan(x/2))/(4052
555153018976267*((355785446056017583036971792465592*tan(x/2)^2)/13508517176
72992089 - 355785446056017583036971792465592/1350851717672992089))))/729 -
((19*tan(x/2)^2)/1492992 + (1951*tan(x/2)^4)/1119744 - (2950915285*tan(x/2
)^6)/6718464 + (254448996545*tan(x/2)^8)/13436928 - (1471570785982 3*tan(x/
2)^10)/40310784 + (27922427907149*tan(x/2)^12)/6718464 - (141037835477118 5
*tan(x/2)^14)/45349632 + (87714731161410877*tan(x/2)^16)/544195584 - (3234
4587581733029 3*tan(x/2)^18)/544195584 + (537090433481225933*tan(x/2)^20)/3
40122240 - (825564740867246567*tan(x/2)^22)/272097792 + (229137479917525 48
93*tan(x/2)^24)/544195584 - (229136839770320111 3*tan(x/2)^26)/544195584 +
(825557780178649715*tan(x/2)^28)/272097792 - (107416550061056890 3*tan(x/2)
^30)/680244480 + (64687832727525523 3*tan(x/2)^32)/1088391168 - (1754244943
28205713*tan(x/2)^34)/1088391168 + (846194921473640 3*tan(x/2)^36)/27209779
2 - (226160067157838 9*tan(x/2)^38)/544195584 + (44143935599789*tan(x/2)^40
)/120932352 - (254420598121*tan(x/2)^42)/13436928 + (8850790063*tan(x/2)^4
4)/20155392 + 1/7464960)/(tan(x/2)^5 - (140*tan(x/2)^7)/3 + (8830*tan(x/2)
^9)/9 - (331660*tan(x/2)^11)/27 + (8263805*tan(x/2)^13)/81 - (144041968...
```

Reduce [F]

$$\int \frac{1}{(\sin(x)+\sin(5x))^6}\,dx$$
$$= \int \frac{1}{\sin(5x)^6 + 6\sin(5x)^5\sin(x) + 15\sin(5x)^4\sin(x)^2 + 20\sin(5x)^3\sin(x)^3 + 15\sin(5x)^2\sin(x)^4 + 6\,\mathrm{si}}$$

input
```
int(1/(sin(x)+sin(5*x))^6,x)
```

output
```
int(1/(sin(5*x)**6 + 6*sin(5*x)**5*sin(x) + 15*sin(5*x)**4*sin(x)**2 + 20*
sin(5*x)**3*sin(x)**3 + 15*sin(5*x)**2*sin(x)**4 + 6*sin(5*x)*sin(x)**5 +
sin(x)**6),x)
```

3.31 $\int \frac{1}{\sin(3x)+\sin(5x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \frac{1}{\sin(3x)+\sin(5x)}\,dx = -\frac{1}{8}\operatorname{arctanh}(\cos(x)) + \frac{\operatorname{arctanh}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}} - \frac{\sec(x)}{8}$$

output
```
-1/8*arctanh(cos(x))+1/4*arctanh(cos(x)*2^(1/2))*2^(1/2)-1/8*sec(x)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.28

$$\int \frac{1}{\sin(3x)+\sin(5x)}\,dx = \frac{1}{8}\left((2+2i)(-1)^{3/4}\operatorname{arctanh}\left(\frac{-1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right) + (2-2i)\sqrt[4]{-1}\operatorname{arctanh}\left(\frac{1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right) - \sec(x)\right)$$

input
```
Integrate[(Sin[3*x] + Sin[5*x])^(-1),x]
```

output
```
((2 + 2*I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] + (2 - 2*I)*(-1)^(1
/4)*ArcTanh[(1 + Tan[x/2])/Sqrt[2]] - Log[Cos[x/2]] + Log[Sin[x/2]] - Sec[
x])/8
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 4824, 27, 382, 397, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin(3x)+\sin(5x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(3x)+\sin(5x)}dx$$
$$\Big\downarrow \text{4824}$$
$$-\int -\frac{\sec^2(x)}{8\left(1-2\cos^2(x)\right)\left(1-\cos^2(x)\right)}d\cos(x)$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{8}\int \frac{\sec^2(x)}{\left(1-2\cos^2(x)\right)\left(1-\cos^2(x)\right)}d\cos(x)$$
$$\Big\downarrow \text{382}$$
$$\frac{1}{8}\left(\int \frac{3-2\cos^2(x)}{\left(1-2\cos^2(x)\right)\left(1-\cos^2(x)\right)}d\cos(x)-\sec(x)\right)$$
$$\Big\downarrow \text{397}$$
$$\frac{1}{8}\left(4\int \frac{1}{1-2\cos^2(x)}d\cos(x)-\int \frac{1}{1-\cos^2(x)}d\cos(x)-\sec(x)\right)$$
$$\Big\downarrow \text{219}$$
$$\frac{1}{8}\left(-\text{arctanh}(\cos(x))+2\sqrt{2}\text{arctanh}\left(\sqrt{2}\cos(x)\right)-\sec(x)\right)$$

input
```
Int[(Sin[3*x] + Sin[5*x])^(-1),x]
```

output
```
(-ArcTanh[Cos[x]] + 2*Sqrt[2]*ArcTanh[Sqrt[2]*Cos[x]] - Sec[x])/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4824
```
Int[((a_.)*sin[(m_.)*((c_.) + (d_.)*(x_))] + (b_.)*sin[(n_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Simplify[TrigExpand[a*
Sin[m*ArcCos[x]] + b*Sin[n*ArcCos[x]]]]^p/Sqrt[1 - x^2], x], x, Cos[c + d*x
]], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[(m - 1)/
2] && IntegerQ[(n - 1)/2]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{\operatorname{arctanh}\left(\sqrt{2}\,\cos(x)\right)\sqrt{2}}{4}+\frac{\ln(\cos(x)-1)}{16}-\frac{1}{8\cos(x)}-\frac{\ln(1+\cos(x))}{16}$	34
risch	$-\frac{\mathrm{e}^{ix}}{4(\mathrm{e}^{2ix}+1)}-\frac{\ln(\mathrm{e}^{ix}+1)}{8}+\frac{\ln(\mathrm{e}^{ix}-1)}{8}+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+\sqrt{2}\,\mathrm{e}^{ix}+1\right)}{8}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-\sqrt{2}\,\mathrm{e}^{ix}+1\right)}{8}$	83

input
```
int(1/(sin(3*x)+sin(5*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*arctanh(2^(1/2)*cos(x))*2^(1/2)+1/16*ln(cos(x)-1)-1/8/cos(x)-1/16*ln(1
+cos(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(22) = 44$.

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sin(3x)+\sin(5x)}\,dx = \frac{2\sqrt{2}\cos{(x)}\log\left(-\frac{2\cos(x)^2+2\sqrt{2}\cos(x)+1}{2\cos(x)^2-1}\right)-\cos{(x)}\log\left(\frac{1}{2}\cos{(x)}+\frac{1}{2}\right)+\cos{(x)}\log\left(-\frac{1}{2}\cos{(x)}+\frac{1}{2}\right)-2}{16\cos{(x)}}$$

input
```
integrate(1/(sin(3*x)+sin(5*x)),x, algorithm="fricas")
```

output
```
1/16*(2*sqrt(2)*cos(x)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^
2 - 1)) - cos(x)*log(1/2*cos(x) + 1/2) + cos(x)*log(-1/2*cos(x) + 1/2) - 2
)/cos(x)
```

Sympy [F]

$$\int \frac{1}{\sin(3x)+\sin(5x)}\,dx=\int \frac{1}{\sin(3x)+\sin(5x)}\,dx$$

input
```
integrate(1/(sin(3*x)+sin(5*x)),x)
```

output
```
Integral(1/(sin(3*x) + sin(5*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(22) = 44.

Time = 0.13 (sec) , antiderivative size = 305, normalized size of antiderivative = 9.53

$$\int \frac{1}{\sin(3x)+\sin(5x)}\,dx = -\frac{4\cos(2x)\cos(x)-\left(\sqrt{2}\cos(2x)^2+\sqrt{2}\sin(2x)^2+2\sqrt{2}\cos(2x)+\sqrt{2}\right)\log\left(2\sqrt{2}\sin(2x)\sin(x)+\right.}{}$$

input
```
integrate(1/(sin(3*x)+sin(5*x)),x, algorithm="maxima")
```

output
```
-1/16*(4*cos(2*x)*cos(x) - (sqrt(2)*cos(2*x)^2 + sqrt(2)*sin(2*x)^2 + 2*sq
rt(2)*cos(2*x) + sqrt(2))*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x
) + 1)*cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sq
rt(2)*cos(x) + 1) + (sqrt(2)*cos(2*x)^2 + sqrt(2)*sin(2*x)^2 + 2*sqrt(2)*c
os(2*x) + sqrt(2))*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)
*cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*
cos(x) + 1) + (cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(x)^2 + si
n(x)^2 + 2*cos(x) + 1) - (cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(co
s(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 4*sin(2*x)*sin(x) + 4*cos(x))/(cos(2*x
)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(22) = 44.

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.56

$$\int \frac{1}{\sin(3x)+\sin(5x)}\,dx = \frac{1}{8}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}{\left|4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}\right) - \frac{1}{4\left(\frac{\cos(x)-1}{\cos(x)+1}+1\right)} + \frac{1}{16}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

input
```
integrate(1/(sin(3*x)+sin(5*x)),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/abs(4*sq
rt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)) - 1/4/((cos(x) - 1)/(cos(x) + 1)
 + 1) + 1/16*log(-(cos(x) - 1)/(cos(x) + 1))
```

Mupad [B] (verification not implemented)

Time = 23.94 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.09

$$\int \frac{1}{\sin(3x)+\sin(5x)}\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{8} + \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{29\sqrt{2}}{8\left(\frac{239\tan\left(\frac{x}{2}\right)^2}{8}-\frac{41}{8}\right)}-\frac{169\sqrt{2}\tan\left(\frac{x}{2}\right)^2}{8\left(\frac{239\tan\left(\frac{x}{2}\right)^2}{8}-\frac{41}{8}\right)}\right)}{4} + \frac{1}{4\left(\tan\left(\frac{x}{2}\right)^2-1\right)}$$

input
```
int(1/(sin(3*x) + sin(5*x)),x)
```

output
```
log(tan(x/2))/8 + (2^(1/2)*atanh((29*2^(1/2))/(8*((239*tan(x/2)^2)/8 - 41/
8)) - (169*2^(1/2)*tan(x/2)^2)/(8*((239*tan(x/2)^2)/8 - 41/8))))/4 + 1/(4*
(tan(x/2)^2 - 1))
```

Reduce [F]

$$\int \frac{1}{\sin(3x)+\sin(5x)}\,dx = \int \frac{1}{\sin(5x)+\sin(3x)}\,dx$$

input
```
int(1/(sin(3*x)+sin(5*x)),x)
```

output
```
int(1/(sin(5*x) + sin(3*x)),x)
```

3.32 $\int \frac{1}{(\sin(3x)+\sin(5x))^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 92

$$\int \frac{1}{(\sin(3x)+\sin(5x))^3}\,dx = -\frac{19\text{arctanh}(\cos(x))}{1024} + \frac{31\text{arctanh}\left(\sqrt{2}\cos(x)\right)}{128\sqrt{2}} - \frac{105\sec(x)}{1024} - \frac{43\sec^3(x)}{3072} - \frac{3\sec^5(x)}{1280} + \frac{\sec^5(x)\sec(2x)}{1024} + \frac{1}{256}\sec^5(x)\sec^2(2x) - \frac{\csc^2(x)\sec^5(x)\sec^2(2x)}{1024}$$

output
```
-19/1024*arctanh(cos(x))+31/256*arctanh(cos(x)*2^(1/2))*2^(1/2)-105/1024*s
ec(x)-43/3072*sec(x)^3-3/1280*sec(x)^5+1/1024*sec(x)^5*sec(2*x)+1/256*sec(
x)^5*sec(2*x)^2-1/1024*csc(x)^2*sec(x)^5*sec(2*x)^2
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 277 vs. 2(92) = 184.

Time = 2.96 (sec) , antiderivative size = 277, normalized size of antiderivative = 3.01

$$\int \frac{1}{(\sin(3x)+\sin(5x))^3}\,dx = \frac{\cos^6(x)\left(-14880\sqrt{2}\text{arctanh}\left(\frac{-1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right) + 14880\sqrt{2}\text{arctanh}\left(\frac{1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right) + \frac{1}{32}\csc^2(x)\left(-7444 - 8604\cos(\right.\right.}{}$$

input
```
Integrate[(Sin[3*x] + Sin[5*x])^(-3),x]
```

output
```
(Cos[x]^6*(-14880*Sqrt[2]*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] + 14880*Sqrt[2]
*ArcTanh[(1 + Tan[x/2])/Sqrt[2]] + (Csc[x]^2*(-7444 - 8604*Cos[2*x] - 3264
*Cos[4*x] + 4302*Cos[6*x] + 8020*Cos[8*x] + 3150*Cos[10*x] - 1710*Cos[x]*L
og[Cos[x/2]] - 570*Cos[3*x]*Log[Cos[x/2]] + 285*Cos[5*x]*Log[Cos[x/2]] + 8
55*Cos[7*x]*Log[Cos[x/2]] + 855*Cos[9*x]*Log[Cos[x/2]] + 285*Cos[11*x]*Log
[Cos[x/2]] + 1710*Cos[x]*Log[Sin[x/2]] + 570*Cos[3*x]*Log[Sin[x/2]] - 285*
Cos[5*x]*Log[Sin[x/2]] - 855*Cos[7*x]*Log[Sin[x/2]] - 855*Cos[9*x]*Log[Sin
[x/2]] - 285*Cos[11*x]*Log[Sin[x/2]])*Sec[x]^5*Sec[2*x]^2)/32)*(-Sin[x] +
Sin[3*x])^3)/(1920*(Sin[3*x] + Sin[5*x])^3)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.42, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.455$, Rules used = {3042, 4824, 27, 374, 27, 441, 441, 27, 445, 27, 445, 27, 445, 25, 397, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(3x)+\sin(5x))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(\sin(3x)+\sin(5x))^3}dx$$

$$\Big\downarrow \text{ 4824}$$

$$-\int -\frac{\sec^6(x)}{512\left(1-2\cos^2(x)\right)^3\left(1-\cos^2(x)\right)^2}d\cos(x)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{512}\int \frac{\sec^6(x)}{\left(1-2\cos^2(x)\right)^3\left(1-\cos^2(x)\right)^2}d\cos(x)$$

$$\Big\downarrow \text{ 374}$$

$$\frac{1}{512}\left(\frac{1}{4}\int \frac{2\left(7-11\cos^2(x)\right)\sec^6(x)}{\left(1-2\cos^2(x)\right)^2\left(1-\cos^2(x)\right)^2}d\cos(x)+\frac{\sec^5(x)}{2\left(1-2\cos^2(x)\right)^2\left(1-\cos^2(x)\right)}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{512}\left(\frac{1}{2}\int \frac{\left(7-11\cos^2(x)\right)\sec^6(x)}{\left(1-2\cos^2(x)\right)^2\left(1-\cos^2(x)\right)^2}d\cos(x)+\frac{\sec^5(x)}{2\left(1-2\cos^2(x)\right)^2\left(1-\cos^2(x)\right)}\right)$$

$$\Big\downarrow \; 441$$

$$\frac{1}{512}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{\left(29-27\cos^2(x)\right)\sec^6(x)}{\left(1-2\cos^2(x)\right)\left(1-\cos^2(x)\right)^2}d\cos(x)+\frac{3\sec^5(x)}{2\left(1-2\cos^2(x)\right)\left(1-\cos^2(x)\right)}\right)+\frac{\sec^5(x)}{2\left(1-2\cos^2(x)\right)^2\left(1-\right.}\right.$$

$$\Big\downarrow \; 441$$

$$\frac{1}{512}\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int -\frac{4\left(7\cos^2(x)+12\right)\sec^6(x)}{\left(1-2\cos^2(x)\right)\left(1-\cos^2(x)\right)}d\cos(x)-\frac{\sec^5(x)}{1-\cos^2(x)}\right)+\frac{3\sec^5(x)}{2\left(1-2\cos^2(x)\right)\left(1-\cos^2(x)\right)}\right)+\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{512}\left(\frac{1}{2}\left(\frac{1}{2}\left(2\int \frac{\left(7\cos^2(x)+12\right)\sec^6(x)}{\left(1-2\cos^2(x)\right)\left(1-\cos^2(x)\right)}d\cos(x)-\frac{\sec^5(x)}{1-\cos^2(x)}\right)+\frac{3\sec^5(x)}{2\left(1-2\cos^2(x)\right)\left(1-\cos^2(x)\right)}\right)+\frac{}{2(}\right.$$

$$\Big\downarrow \; 445$$

$$\frac{1}{512}\left(\frac{1}{2}\left(\frac{1}{2}\left(2\left(-\frac{1}{5}\int -\frac{5\left(43-24\cos^2(x)\right)\sec^4(x)}{\left(1-2\cos^2(x)\right)\left(1-\cos^2(x)\right)}d\cos(x)-\frac{12}{5}\sec^5(x)\right)-\frac{\sec^5(x)}{1-\cos^2(x)}\right)+\frac{3\sec}{2\left(1-2\cos^2(x)\right)}\right.\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{512}\left(\frac{1}{2}\left(\frac{1}{2}\left(2\left(\int \frac{\left(43-24\cos^2(x)\right)\sec^4(x)}{\left(1-2\cos^2(x)\right)\left(1-\cos^2(x)\right)}d\cos(x)-\frac{12\sec^5(x)}{5}\right)-\frac{\sec^5(x)}{1-\cos^2(x)}\right)+\frac{3\sec^5(x)}{2\left(1-2\cos^2(x)\right)\left(1-\right.}\right.\right.$$

$$\Big\downarrow \; 445$$

$$\frac{1}{512}\left(\frac{1}{2}\left(\frac{1}{2}\left(2\left(-\frac{1}{3}\int -\frac{3\left(105-86\cos^2(x)\right)\sec^2(x)}{\left(1-2\cos^2(x)\right)\left(1-\cos^2(x)\right)}d\cos(x)-\frac{12}{5}\sec^5(x)-\frac{43\sec^3(x)}{3}\right)-\frac{\sec^5(x)}{1-\cos^2(x)}\right)+\frac{}{2(}\right.\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{512}\left(\frac{1}{2}\left(\frac{1}{2}\left(2\left(\int\frac{(105-86\cos^2(x))\sec^2(x)}{(1-2\cos^2(x))\,(1-\cos^2(x))}d\cos(x)-\frac{12}{5}\sec^5(x)-\frac{43\sec^3(x)}{3}\right)-\frac{\sec^5(x)}{1-\cos^2(x)}\right)+\frac{}{2\,(1-2}\right.\right.$$

$\big\downarrow$ 445

$$\frac{1}{512}\left(\frac{1}{2}\left(\frac{1}{2}\left(2\left(-\int-\frac{229-210\cos^2(x)}{(1-2\cos^2(x))\,(1-\cos^2(x))}d\cos(x)-\frac{12}{5}\sec^5(x)-\frac{43\sec^3(x)}{3}-105\sec(x)\right)-\frac{\sec^5(x}{1-\cos^2}\right.\right.\right.$$

$\big\downarrow$ 25

$$\frac{1}{512}\left(\frac{1}{2}\left(\frac{1}{2}\left(2\left(\int\frac{229-210\cos^2(x)}{(1-2\cos^2(x))\,(1-\cos^2(x))}d\cos(x)-\frac{12}{5}\sec^5(x)-\frac{43\sec^3(x)}{3}-105\sec(x)\right)-\frac{\sec^5(x)}{1-\cos^2(x)}\right)\right.\right.$$

$\big\downarrow$ 397

$$\frac{1}{512}\left(\frac{1}{2}\left(\frac{1}{2}\left(2\left(248\int\frac{1}{1-2\cos^2(x)}d\cos(x)-19\int\frac{1}{1-\cos^2(x)}d\cos(x)-\frac{12}{5}\sec^5(x)-\frac{43\sec^3(x)}{3}-105\sec(x)\right)\right.\right.\right.$$

$\big\downarrow$ 219

$$\frac{1}{512}\left(\frac{1}{2}\left(\frac{1}{2}\left(2\left(-19\text{arctanh}(\cos(x))+124\sqrt{2}\text{arctanh}\left(\sqrt{2}\cos(x)\right)-\frac{12}{5}\sec^5(x)-\frac{43\sec^3(x)}{3}-105\sec(x)\right)-\frac{}{1}\right.\right.\right.$$

input
```
Int[(Sin[3*x] + Sin[5*x])^(-3),x]
```

output
```
(Sec[x]^5/(2*(1 - 2*Cos[x]^2)^2*(1 - Cos[x]^2)) + ((3*Sec[x]^5)/(2*(1 - 2*
Cos[x]^2)*(1 - Cos[x]^2)) + (-(Sec[x]^5/(1 - Cos[x]^2)) + 2*(-19*ArcTanh[C
os[x]] + 124*Sqrt[2]*ArcTanh[Sqrt[2]*Cos[x]] - 105*Sec[x] - (43*Sec[x]^3)/
3 - (12*Sec[x]^5)/5))/2)/2)/512
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4824
```
Int[((a_.)*sin[(m_.)*((c_.) + (d_.)*(x_))] + (b_.)*sin[(n_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Simplify[TrigExpand[a*
Sin[m*ArcCos[x]] + b*Sin[n*ArcCos[x]]]]^p/Sqrt[1 - x^2], x], x, Cos[c + d*x
]], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[(m - 1)/
2] && IntegerQ[(n - 1)/2]
```

Maple [A] (verified)

Time = 6.58 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.92

method	result
default	$\frac{1}{2048+2048\cos(x)}-\frac{19\ln(1+\cos(x))}{2048}+\frac{1}{2048\cos(x)-2048}+\frac{19\ln(\cos(x)-1)}{2048}-\frac{\frac{7\cos(x)^3}{16}-\frac{9\cos(x)}{32}}{4\left(2\cos(x)^2-1\right)^2}+\frac{31\ \mathrm{arctanh}\left(\sqrt{2}\ \cos(\right.}{256}$
risch	$-\frac{1575\,\mathrm{e}^{21ix}+4010\,\mathrm{e}^{19ix}+2151\,\mathrm{e}^{17ix}-1632\,\mathrm{e}^{15ix}-4302\,\mathrm{e}^{13ix}-7444\,\mathrm{e}^{11ix}-4302\,\mathrm{e}^{9ix}-1632\,\mathrm{e}^{7ix}+2151\,\mathrm{e}^{5ix}+4010\,\mathrm{e}^{3ix}+1575\,\mathrm{e}^{ix}}{7680(\mathrm{e}^{2ix}+1)^5(\mathrm{e}^{6ix}-\mathrm{e}^{4ix}+\mathrm{e}^{2ix}-1)^2}+$

input
```
int(1/(sin(3*x)+sin(5*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2048/(1+cos(x))-19/2048*ln(1+cos(x))+1/2048/(cos(x)-1)+19/2048*ln(cos(x)
-1)-1/4*(7/16*cos(x)^3-9/32*cos(x))/(2*cos(x)^2-1)^2+31/256*arctanh(2^(1/2
)*cos(x))*2^(1/2)-1/2560/cos(x)^5-1/192/cos(x)^3-39/512/cos(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(72) = 144$.

Time = 0.10 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.13

$$\int \frac{1}{(\sin(3x)+\sin(5x))^3}\,dx = -\frac{12600\cos(x)^{10} - 23480\cos(x)^8 + 12598\cos(x)^6 - 1636\cos(x)^4 - 100\cos(x)^2 - 1860\left(4\sqrt{2}\cos(x\right.}{}$$

input
```
integrate(1/(sin(3*x)+sin(5*x))^3,x, algorithm="fricas")
```

output
```
-1/30720*(12600*cos(x)^10 - 23480*cos(x)^8 + 12598*cos(x)^6 - 1636*cos(x)^
4 - 100*cos(x)^2 - 1860*(4*sqrt(2)*cos(x)^11 - 8*sqrt(2)*cos(x)^9 + 5*sqrt
(2)*cos(x)^7 - sqrt(2)*cos(x)^5)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/
(2*cos(x)^2 - 1)) + 285*(4*cos(x)^11 - 8*cos(x)^9 + 5*cos(x)^7 - cos(x)^5)
*log(1/2*cos(x) + 1/2) - 285*(4*cos(x)^11 - 8*cos(x)^9 + 5*cos(x)^7 - cos(
x)^5)*log(-1/2*cos(x) + 1/2) - 12)/(4*cos(x)^11 - 8*cos(x)^9 + 5*cos(x)^7
- cos(x)^5)
```

Sympy [F]

$$\int \frac{1}{(\sin(3x)+\sin(5x))^3}\,dx = \int \frac{1}{(\sin(3x)+\sin(5x))^3}\,dx$$

input
```
integrate(1/(sin(3*x)+sin(5*x))**3,x)
```

output
```
Integral((sin(3*x) + sin(5*x))**(-3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7473 vs. $2(72) = 144$.

Time = 0.53 (sec) , antiderivative size = 7473, normalized size of antiderivative = 81.23

$$\int \frac{1}{(\sin(3x) + \sin(5x))^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/(sin(3*x)+sin(5*x))^3,x, algorithm="maxima")
```

output
```
-1/30720*(4*(1575*cos(21*x) + 4010*cos(19*x) + 2151*cos(17*x) - 1632*cos(1
5*x) - 4302*cos(13*x) - 7444*cos(11*x) - 4302*cos(9*x) - 1632*cos(7*x) + 2
151*cos(5*x) + 4010*cos(3*x) + 1575*cos(x))*cos(22*x) + 6300*(3*cos(20*x)
+ 3*cos(18*x) + cos(16*x) - 2*cos(14*x) - 6*cos(12*x) - 6*cos(10*x) - 2*co
s(8*x) + cos(6*x) + 3*cos(4*x) + 3*cos(2*x) + 1)*cos(21*x) + 12*(4010*cos(
19*x) + 2151*cos(17*x) - 1632*cos(15*x) - 4302*cos(13*x) - 7444*cos(11*x)
- 4302*cos(9*x) - 1632*cos(7*x) + 2151*cos(5*x) + 4010*cos(3*x) + 1575*cos
(x))*cos(20*x) + 16040*(3*cos(18*x) + cos(16*x) - 2*cos(14*x) - 6*cos(12*x
) - 6*cos(10*x) - 2*cos(8*x) + cos(6*x) + 3*cos(4*x) + 3*cos(2*x) + 1)*cos
(19*x) + 12*(2151*cos(17*x) - 1632*cos(15*x) - 4302*cos(13*x) - 7444*cos(1
1*x) - 4302*cos(9*x) - 1632*cos(7*x) + 2151*cos(5*x) + 4010*cos(3*x) + 157
5*cos(x))*cos(18*x) + 8604*(cos(16*x) - 2*cos(14*x) - 6*cos(12*x) - 6*cos(
10*x) - 2*cos(8*x) + cos(6*x) + 3*cos(4*x) + 3*cos(2*x) + 1)*cos(17*x) - 4
*(1632*cos(15*x) + 4302*cos(13*x) + 7444*cos(11*x) + 4302*cos(9*x) + 1632*
cos(7*x) - 2151*cos(5*x) - 4010*cos(3*x) - 1575*cos(x))*cos(16*x) + 6528*(
2*cos(14*x) + 6*cos(12*x) + 6*cos(10*x) + 2*cos(8*x) - cos(6*x) - 3*cos(4*
x) - 3*cos(2*x) - 1)*cos(15*x) + 8*(4302*cos(13*x) + 7444*cos(11*x) + 4302
*cos(9*x) + 1632*cos(7*x) - 2151*cos(5*x) - 4010*cos(3*x) - 1575*cos(x))*c
os(14*x) + 17208*(6*cos(12*x) + 6*cos(10*x) + 2*cos(8*x) - cos(6*x) - 3*co
s(4*x) - 3*cos(2*x) - 1)*cos(13*x) + 24*(7444*cos(11*x) + 4302*cos(9*x)...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(72) = 144$.

Time = 0.13 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.71

$$\int \frac{1}{(\sin(3x)+\sin(5x))^3}\,dx$$
$$= \frac{31}{512}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}{\left|4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}\right) - \frac{\left(\frac{38(\cos(x)-1)}{\cos(x)+1}-1\right)(\cos(x)+1)}{4096(\cos(x)-1)}$$
$$- \frac{\cos(x)-1}{4096(\cos(x)+1)} - \frac{\frac{53(\cos(x)-1)}{\cos(x)+1}+\frac{95(\cos(x)-1)^2}{(\cos(x)+1)^2}+\frac{7(\cos(x)-1)^3}{(\cos(x)+1)^3}+5}{64\left(\frac{6(\cos(x)-1)}{\cos(x)+1}+\frac{(\cos(x)-1)^2}{(\cos(x)+1)^2}+1\right)^2}$$
$$- \frac{\frac{605(\cos(x)-1)}{\cos(x)+1}+\frac{925(\cos(x)-1)^2}{(\cos(x)+1)^2}+\frac{645(\cos(x)-1)^3}{(\cos(x)+1)^3}+\frac{180(\cos(x)-1)^4}{(\cos(x)+1)^4}+157}{960\left(\frac{\cos(x)-1}{\cos(x)+1}+1\right)^5}$$
$$+ \frac{19}{2048}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

input
```
integrate(1/(sin(3*x)+sin(5*x))^3,x, algorithm="giac")
```

output
```
31/512*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/abs(4
*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)) - 1/4096*(38*(cos(x) - 1)/(co
s(x) + 1) - 1)*(cos(x) + 1)/(cos(x) - 1) - 1/4096*(cos(x) - 1)/(cos(x) + 1
) - 1/64*(53*(cos(x) - 1)/(cos(x) + 1) + 95*(cos(x) - 1)^2/(cos(x) + 1)^2
+ 7*(cos(x) - 1)^3/(cos(x) + 1)^3 + 5)/(6*(cos(x) - 1)/(cos(x) + 1) + (cos
(x) - 1)^2/(cos(x) + 1)^2 + 1)^2 - 1/960*(605*(cos(x) - 1)/(cos(x) + 1) +
925*(cos(x) - 1)^2/(cos(x) + 1)^2 + 645*(cos(x) - 1)^3/(cos(x) + 1)^3 + 18
0*(cos(x) - 1)^4/(cos(x) + 1)^4 + 157)/((cos(x) - 1)/(cos(x) + 1) + 1)^5 +
 19/2048*log(-(cos(x) - 1)/(cos(x) + 1))
```

Mupad [B] (verification not implemented)

Time = 22.55 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.39

$$\int \frac{1}{(\sin(3x)+\sin(5x))^3}\,dx = \frac{19\ln\left(\tan\left(\frac{x}{2}\right)\right)}{1024} + \frac{31\sqrt{2}\,\mathrm{atanh}\left(\frac{616001\sqrt{2}}{2097152\left(\frac{10149121\tan\left(\frac{x}{2}\right)^2}{4194304}-\frac{1740371}{4194304}\right)} - \frac{1794187\sqrt{2}\tan\left(\frac{x}{2}\right)^2}{1048576\left(\frac{10149121\tan\left(\frac{x}{2}\right)^2}{4194304}-\frac{1740371}{4194304}\right)}\right)}{256} + \frac{\tan\left(\frac{x}{2}\right)^2}{4096} - \frac{\frac{1215\tan\left(\frac{x}{2}\right)^{18}}{4096} - \frac{20271\tan\left(\frac{x}{2}\right)^{16}}{4096} + \frac{78239\tan\left(\frac{x}{2}\right)^{14}}{3072} - \frac{184471\tan\left(\frac{x}{2}\right)^{12}}{3072} + \frac{783513\tan\left(\frac{x}{2}\right)^{10}}{10240} - \frac{1677323\tan\left(\frac{x}{2}\right)^{8}}{30720} + \frac{107857\,\cdots}{51\cdots}}{-\tan\left(\frac{x}{2}\right)^{20} + 17\tan\left(\frac{x}{2}\right)^{18} - 108\tan\left(\frac{x}{2}\right)^{16} + 332\tan\left(\frac{x}{2}\right)^{14} - 566\tan\left(\frac{x}{2}\right)^{12} + 566\tan\left(\frac{x}{2}\right)^{10} - 332\tan\cdots}$$

input
```
int(1/(sin(3*x) + sin(5*x))^3,x)
```

output
```
(19*log(tan(x/2)))/1024 + (31*2^(1/2)*atanh((616001*2^(1/2))/(2097152*((10
149121*tan(x/2)^2)/4194304 - 1740371/4194304)) - (1794187*2^(1/2)*tan(x/2)
^2)/(1048576*((10149121*tan(x/2)^2)/4194304 - 1740371/4194304))))/256 + ta
n(x/2)^2/4096 - ((14593*tan(x/2)^2)/61440 - (58139*tan(x/2)^4)/15360 + (10
7857*tan(x/2)^6)/5120 - (1677323*tan(x/2)^8)/30720 + (783513*tan(x/2)^10)/
10240 - (184471*tan(x/2)^12)/3072 + (78239*tan(x/2)^14)/3072 - (20271*tan(
x/2)^16)/4096 + (1215*tan(x/2)^18)/4096 + 1/4096)/(tan(x/2)^2 - 17*tan(x/2
)^4 + 108*tan(x/2)^6 - 332*tan(x/2)^8 + 566*tan(x/2)^10 - 566*tan(x/2)^12
+ 332*tan(x/2)^14 - 108*tan(x/2)^16 + 17*tan(x/2)^18 - tan(x/2)^20)
```

Reduce [F]

$$\int \frac{1}{(\sin(3x)+\sin(5x))^3}\,dx = \int \frac{1}{\sin(5x)^3 + 3\sin(5x)^2\sin(3x) + 3\sin(5x)\sin(3x)^2 + \sin(3x)^3}\,dx$$

input
```
int(1/(sin(3*x)+sin(5*x))^3,x)
```

output
```
int(1/(sin(5*x)**3 + 3*sin(5*x)**2*sin(3*x) + 3*sin(5*x)*sin(3*x)**2 + sin
(3*x)**3),x)
```

3.33 $\int \frac{1}{(\sin(3x)+\sin(5x))^5}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 154

$$\begin{aligned}\int \frac{1}{(\sin(3x)+\sin(5x))^5}\,dx =& -\frac{1063\text{arctanh}(\cos(x))}{262144}+\frac{3603\text{arctanh}\left(\sqrt{2}\cos(x)\right)}{16384\sqrt{2}}\\&-\frac{27761\sec(x)}{262144}-\frac{13349\sec^3(x)}{786432}\\&-\frac{6143\sec^5(x)}{1310720}-\frac{635\sec^7(x)}{458752}-\frac{1477\sec^9(x)}{4718592}\\&+\frac{59\sec^9(x)\sec(2x)}{524288}+\frac{159\sec^9(x)\sec^2(2x)}{262144}\\&+\frac{149\sec^9(x)\sec^3(2x)}{393216}+\frac{45\sec^9(x)\sec^4(2x)}{131072}\\&-\frac{37\csc^2(x)\sec^9(x)\sec^4(2x)}{262144}-\frac{\csc^4(x)\sec^9(x)\sec^4(2x)}{131072}\end{aligned}$$

output
```
-1063/262144*arctanh(cos(x))+3603/32768*arctanh(cos(x)*2^(1/2))*2^(1/2)-27
761/262144*sec(x)-13349/786432*sec(x)^3-6143/1310720*sec(x)^5-635/458752*s
ec(x)^7-1477/4718592*sec(x)^9+59/524288*sec(x)^9*sec(2*x)+159/262144*sec(x
)^9*sec(2*x)^2+149/393216*sec(x)^9*sec(2*x)^3+45/131072*sec(x)^9*sec(2*x)^
4-37/262144*csc(x)^2*sec(x)^9*sec(2*x)^4-1/131072*csc(x)^4*sec(x)^9*sec(2*
x)^4
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.26 (sec) , antiderivative size = 1603, normalized size of antiderivative = 10.41

$$\int \frac{1}{(\sin(3x) + \sin(5x))^5} \, dx = \text{Too large to display}$$

input
```
Integrate[(Sin[3*x] + Sin[5*x])^(-5),x]
```

output
```
(-102055729*Cos[x]^10*(-Sin[x] + Sin[3*x])^5)/(1290240*(Sin[3*x] + Sin[5*x
])^5) - ((3603/32 + (3603*I)/32)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*
Sec[x/2]*(Cos[x/2] + Sin[x/2])]*Cos[x]^10*(-Sin[x] + Sin[3*x])^5)/(Sin[3*x
] + Sin[5*x])^5 + ((3603/32 + (3603*I)/32)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*
(-1)^(3/4)*Sec[x/2]*(Cos[x/2] - Sin[x/2])]*Cos[x]^10*(-Sin[x] + Sin[3*x])^
5)/(Sin[3*x] + Sin[5*x])^5 - (63*Cos[x]^10*Csc[x/2]^2*(-Sin[x] + Sin[3*x])
^5)/(1024*(Sin[3*x] + Sin[5*x])^5) - (Cos[x]^10*Csc[x/2]^4*(-Sin[x] + Sin[
3*x])^5)/(2048*(Sin[3*x] + Sin[5*x])^5) - (1063*Cos[x]^10*Log[Cos[x/2]]*(-
Sin[x] + Sin[3*x])^5)/(256*(Sin[3*x] + Sin[5*x])^5) + (1063*Cos[x]^10*Log[
Sin[x/2]]*(-Sin[x] + Sin[3*x])^5)/(256*(Sin[3*x] + Sin[5*x])^5) + (63*Cos[
x]^10*Sec[x/2]^2*(-Sin[x] + Sin[3*x])^5)/(1024*(Sin[3*x] + Sin[5*x])^5) +
(Cos[x]^10*Sec[x/2]^4*(-Sin[x] + Sin[3*x])^5)/(2048*(Sin[3*x] + Sin[5*x])^
5) - (Cos[x]^10*(-Sin[x] + Sin[3*x])^5)/(9216*(Cos[x/2] - Sin[x/2])^8*(Sin
[3*x] + Sin[5*x])^5) - (503*Cos[x]^10*(-Sin[x] + Sin[3*x])^5)/(129024*(Cos
[x/2] - Sin[x/2])^6*(Sin[3*x] + Sin[5*x])^5) - (35551*Cos[x]^10*(-Sin[x] +
 Sin[3*x])^5)/(430080*(Cos[x/2] - Sin[x/2])^4*(Sin[3*x] + Sin[5*x])^5) - (
3876529*Cos[x]^10*(-Sin[x] + Sin[3*x])^5)/(2580480*(Cos[x/2] - Sin[x/2])^2
*(Sin[3*x] + Sin[5*x])^5) - (Cos[x]^10*Sin[x/2]*(-Sin[x] + Sin[3*x])^5)/(4
608*(Cos[x/2] - Sin[x/2])^9*(Sin[3*x] + Sin[5*x])^5) - (503*Cos[x]^10*Sin[
x/2]*(-Sin[x] + Sin[3*x])^5)/(64512*(Cos[x/2] - Sin[x/2])^7*(Sin[3*x] +...
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.53, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.273, Rules used = {3042, 4824, 27, 374, 27, 441, 441, 441, 27, 441, 27, 441, 27, 445, 27, 445, 27, 445, 27, 445, 27, 445, 25, 397, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(3x)+\sin(5x))^5}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(3x)+\sin(5x))^5}dx$$

$$\Big\downarrow \text{4824}$$

$$-\int -\frac{\sec^{10}(x)}{32768\left(1-2\cos^2(x)\right)^5\left(1-\cos^2(x)\right)^3}d\cos(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sec^{10}(x)}{(1-2\cos^2(x))^5(1-\cos^2(x))^3}d\cos(x)}{32768}$$

$$\Big\downarrow \text{374}$$

$$\frac{\frac{1}{8}\int \frac{2(13-21\cos^2(x))\sec^{10}(x)}{(1-2\cos^2(x))^4(1-\cos^2(x))^3}d\cos(x)+\frac{\sec^9(x)}{4(1-2\cos^2(x))^4(1-\cos^2(x))^2}}{32768}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{4}\int \frac{(13-21\cos^2(x))\sec^{10}(x)}{(1-2\cos^2(x))^4(1-\cos^2(x))^3}d\cos(x)+\frac{\sec^9(x)}{4(1-2\cos^2(x))^4(1-\cos^2(x))^2}}{32768}$$

$$\Big\downarrow \text{441}$$

$$\frac{\frac{1}{4}\left(\frac{1}{6}\int \frac{(123-95\cos^2(x))\sec^{10}(x)}{(1-2\cos^2(x))^3(1-\cos^2(x))^3}d\cos(x)+\frac{5\sec^9(x)}{6(1-2\cos^2(x))^3(1-\cos^2(x))^2}\right)+\frac{\sec^9(x)}{4(1-2\cos^2(x))^4(1-\cos^2(x))^2}}{32768}$$

$$\Big\downarrow \text{441}$$

$$\frac{\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\int\frac{(1851-2567\cos^2(x))\sec^{10}(x)}{(1-2\cos^2(x))^2(1-\cos^2(x))^3}d\cos(x)+\frac{151\sec^9(x)}{4(1-2\cos^2(x))^2(1-\cos^2(x))^2}\right)+\frac{5\sec^9(x)}{6(1-2\cos^2(x))^3(1-\cos^2(x))^2}\right)+\frac{\sec^9}{4(1-2\cos^2(x))}}{32768}$$

$\downarrow$ 441

$$\frac{\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{1}{2}\int\frac{3(4639-5675\cos^2(x))\sec^{10}(x)}{(1-2\cos^2(x))(1-\cos^2(x))^3}d\cos(x)+\frac{1135\sec^9(x)}{2(1-2\cos^2(x))(1-\cos^2(x))^2}\right)+\frac{151\sec^9(x)}{4(1-2\cos^2(x))^2(1-\cos^2(x))^2}\right)+\frac{5}{6(1-2\cos^2(}}{32768}$$

$\downarrow$ 27

$$\frac{\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}\int\frac{(4639-5675\cos^2(x))\sec^{10}(x)}{(1-2\cos^2(x))(1-\cos^2(x))^3}d\cos(x)+\frac{1135\sec^9(x)}{2(1-2\cos^2(x))(1-\cos^2(x))^2}\right)+\frac{151\sec^9(x)}{4(1-2\cos^2(x))^2(1-\cos^2(x))^2}\right)+\frac{5\,s}{6(1-2\cos^2(x}}{32768}$$

$\downarrow$ 441

$$\frac{\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}\left(\frac{259\sec^9(x)}{(1-\cos^2(x))^2}-\frac{1}{4}\int-\frac{8(3485-3367\cos^2(x))\sec^{10}(x)}{(1-2\cos^2(x))(1-\cos^2(x))^2}d\cos(x)\right)+\frac{1135\sec^9(x)}{2(1-2\cos^2(x))(1-\cos^2(x))^2}\right)+\frac{151\sec^9(x)}{4(1-2\cos^2(x))^2(1-\cos}}{32768}$$

$\downarrow$ 27

$$\frac{\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}\left(2\int\frac{(3485-3367\cos^2(x))\sec^{10}(x)}{(1-2\cos^2(x))(1-\cos^2(x))^2}d\cos(x)+\frac{259\sec^9(x)}{(1-\cos^2(x))^2}\right)+\frac{1135\sec^9(x)}{2(1-2\cos^2(x))(1-\cos^2(x))^2}\right)+\frac{151\sec^9(x)}{4(1-2\cos^2(x))^2(1-\cos^2(}}{32768}$$

$\downarrow$ 441

$$\frac{\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}\left(2\left(-\frac{1}{2}\int-\frac{4(649\cos^2(x)+1477)\sec^{10}(x)}{(1-2\cos^2(x))(1-\cos^2(x))}d\cos(x)-\frac{59\sec^9(x)}{1-\cos^2(x)}\right)+\frac{259\sec^9(x)}{(1-\cos^2(x))^2}\right)+\frac{1135\sec^9(x)}{2(1-2\cos^2(x))(1-\cos^2(x))^2}\right)+\frac{}{4}}{32768}$$

$\downarrow$ 27

$$\frac{\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}\left(2\left(2\int\frac{(649\cos^2(x)+1477)\sec^{10}(x)}{(1-2\cos^2(x))(1-\cos^2(x))}d\cos(x)-\frac{59\sec^9(x)}{1-\cos^2(x)}\right)+\frac{259\sec^9(x)}{(1-\cos^2(x))^2}\right)+\frac{1135\sec^9(x)}{2(1-2\cos^2(x))(1-\cos^2(x))^2}\right)+\frac{}{4(1-2}}{32768}$$

$\downarrow$ 445

$$\frac{\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}\left(2\left(2\left(-\frac{1}{9}\int-\frac{18(2540-1477\cos^2(x))\sec^8(x)}{(1-2\cos^2(x))(1-\cos^2(x))}d\cos(x)-\frac{1477}{9}\sec^9(x)\right)-\frac{59\sec^9(x)}{1-\cos^2(x)}\right)+\frac{259\sec^9(x)}{(1-\cos^2(x))^2}\right)+\frac{11}{2(1-2\cos}}{32768}$$

$\downarrow$ 27

$$\frac{\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}\left(2\left(2\left(2\int\frac{(2540-1477\cos^2(x))\sec^8(x)}{(1-2\cos^2(x))(1-\cos^2(x))}d\cos(x)-\frac{1477\sec^9(x)}{9}\right)-\frac{59\sec^9(x)}{1-\cos^2(x)}\right)+\frac{259\sec^9(x)}{(1-\cos^2(x))^2}\right)+\frac{1135\sec^9(x\cdots}{2(1-2\cos^2(x))(1-\cdots}\cdots}{32768}$$

$$\Big\downarrow \; {\color{blue}445}$$

$$\frac{\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}\left(2\left(2\left(2\left(-\frac{1}{7}\int-\frac{7(6143-5080\cos^2(x))\sec^6(x)}{(1-2\cos^2(x))(1-\cos^2(x))}d\cos(x)-\frac{2540}{7}\sec^7(x)\right)-\frac{1477\sec^9(x)}{9}\right)-\frac{59\sec^9(x)}{1-\cos^2(x)}\right)+\frac{259\sec^9\cdots}{(1-\cos^2(\cdots}\cdots}{32\cdots}$$

$$\Big\downarrow \; {\color{blue}27}$$

$$\frac{\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}\left(2\left(2\left(2\left(\int\frac{(6143-5080\cos^2(x))\sec^6(x)}{(1-2\cos^2(x))(1-\cos^2(x))}d\cos(x)-\frac{2540\sec^7(x)}{7}\right)-\frac{1477\sec^9(x)}{9}\right)-\frac{59\sec^9(x)}{1-\cos^2(x)}\right)+\frac{259\sec^9(x)}{(1-\cos^2(x))^2}\right)+\cdots}{32768\cdots}$$

$$\Big\downarrow \; {\color{blue}445}$$

$$\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}\left(2\left(2\left(2\left(-\frac{1}{5}\int-\frac{5(13349-12286\cos^2(x))\sec^4(x)}{(1-2\cos^2(x))(1-\cos^2(x))}d\cos(x)-\frac{2540}{7}\sec^7(x)-\frac{6143\sec^5(x)}{5}\right)-\frac{1477\sec^9(x)}{9}\right)-\frac{59\sec^9\cdots}{1-\cos\cdots}\cdots$$

$$\Big\downarrow \; {\color{blue}27}$$

$$\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}\left(2\left(2\left(2\left(\int\frac{(13349-12286\cos^2(x))\sec^4(x)}{(1-2\cos^2(x))(1-\cos^2(x))}d\cos(x)-\frac{2540}{7}\sec^7(x)-\frac{6143\sec^5(x)}{5}\right)-\frac{1477\sec^9(x)}{9}\right)-\frac{59\sec^9(x)}{1-\cos^2(x)}\right)+\cdots$$

$$\Big\downarrow \; {\color{blue}445}$$

$$\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}\left(2\left(2\left(2\left(-\frac{1}{3}\int-\frac{3(27761-26698\cos^2(x))\sec^2(x)}{(1-2\cos^2(x))(1-\cos^2(x))}d\cos(x)-\frac{2540}{7}\sec^7(x)-\frac{6143\sec^5(x)}{5}-\frac{13349\sec^3(x)}{3}\right)-\frac{1477\sec\cdots}{9\cdots}\cdots$$

$$\Big\downarrow \; {\color{blue}27}$$

$$\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}\left(2\left(2\left(2\left(\int\frac{(27761-26698\cos^2(x))\sec^2(x)}{(1-2\cos^2(x))(1-\cos^2(x))}d\cos(x)-\frac{2540}{7}\sec^7(x)-\frac{6143\sec^5(x)}{5}-\frac{13349\sec^3(x)}{3}\right)-\frac{1477\sec^9(x)}{9}\right)\cdots$$

$$\Big\downarrow \; {\color{blue}445}$$

$$\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}\left(2\left(2\left(2\left(-\int-\frac{56585-55522\cos^2(x)}{(1-2\cos^2(x))(1-\cos^2(x))}d\cos(x)-\frac{2540}{7}\sec^7(x)-\frac{6143\sec^5(x)}{5}-\frac{13349\sec^3(x)}{3}-27761\sec(x)\right)\cdots$$

$$\Big\downarrow \; {\color{blue}25}$$

$$\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}\left(2\left(2\left(2\left(\int\frac{56585-55522\cos^2(x)}{(1-2\cos^2(x))(1-\cos^2(x))}d\cos(x)-\frac{2540}{7}\sec^7(x)-\frac{6143\sec^5(x)}{5}-\frac{13349\sec^3(x)}{3}-27761\sec(x)\right)-\right.\right.\right.\right.\right.\right.$$

$\downarrow$ 397

$$\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}\left(2\left(2\left(2\left(57648\int\frac{1}{1-2\cos^2(x)}d\cos(x)-1063\int\frac{1}{1-\cos^2(x)}d\cos(x)-\frac{2540}{7}\sec^7(x)-\frac{6143\sec^5(x)}{5}-\frac{13349\sec^3(x)}{3}\right.\right.\right.\right.\right.\right.$$

$\downarrow$ 219

$$\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}\left(2\left(2\left(2\left(-1063\text{arctanh}(\cos(x))+28824\sqrt{2}\text{arctanh}\left(\sqrt{2}\cos(x)\right)-\frac{2540}{7}\sec^7(x)-\frac{6143\sec^5(x)}{5}-\frac{13349\sec^3}{3}\right.\right.\right.\right.\right.\right.$$

input
```
Int[(Sin[3*x] + Sin[5*x])^(-5),x]
```

output
```
(Sec[x]^9/(4*(1 - 2*Cos[x]^2)^4*(1 - Cos[x]^2)^2) + ((5*Sec[x]^9)/(6*(1 -
2*Cos[x]^2)^3*(1 - Cos[x]^2)^2) + ((151*Sec[x]^9)/(4*(1 - 2*Cos[x]^2)^2*(1
 - Cos[x]^2)^2) + ((1135*Sec[x]^9)/(2*(1 - 2*Cos[x]^2)*(1 - Cos[x]^2)^2) +
 (3*((259*Sec[x]^9)/(1 - Cos[x]^2)^2 + 2*((-59*Sec[x]^9)/(1 - Cos[x]^2) +
2*((-1477*Sec[x]^9)/9 + 2*(-1063*ArcTanh[Cos[x]] + 28824*Sqrt[2]*ArcTanh[S
qrt[2]*Cos[x]] - 27761*Sec[x] - (13349*Sec[x]^3)/3 - (6143*Sec[x]^5)/5 - (
2540*Sec[x]^7)/7)))))/2)/4)/6)/4)/32768
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4824
```
Int[((a_.)*sin[(m_.)*((c_.) + (d_.)*(x_))] + (b_.)*sin[(n_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Simplify[TrigExpand[a*
Sin[m*ArcCos[x]] + b*Sin[n*ArcCos[x]]]]^p/Sqrt[1 - x^2], x], x, Cos[c + d*x
]], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[(m - 1)/
2] && IntegerQ[(n - 1)/2]
```

Maple [A] (verified)

Time = 139.91 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.01

method	result
parallelrisch	0
default	$\frac{1}{524288(1+\cos(x))^2}+\frac{63}{524288(1+\cos(x))}-\frac{1063\ln(1+\cos(x))}{524288}-\frac{\frac{1043\cos(x)^7}{256}-\frac{10193\cos(x)^5}{1536}+\frac{11183\cos(x)^3}{3072}-\frac{1389\cos(x)}{2048}}{8\left(2\cos(x)^2-1\right)^4}$
risch	$-\frac{8744715\,e^{41ix}+40585440\,e^{39ix}+69873594\,e^{37ix}+47900640\,e^{35ix}-34247585\,e^{33ix}-171621760\,e^{31ix}-274841576\,e^{29ix}-18}{}$

input
```
int(1/(sin(3*x)+sin(5*x))^5,x,method=_RETURNVERBOSE)
```

output
```
0
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(124) = 248.

Time = 0.19 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.95

$$\int \frac{1}{(\sin(3x)+\sin(5x))^5}\,dx =$$
$$-\frac{279830880\cos(x)^{20}-1074470880\cos(x)^{18}+1651874448\cos(x)^{16}-1293407232\cos(x)^{14}+53707}{}$$

input
```
integrate(1/(sin(3*x)+sin(5*x))^5,x, algorithm="fricas")
```

output
```
-1/165150720*(279830880*cos(x)^20 - 1074470880*cos(x)^18 + 1651874448*cos(
x)^16 - 1293407232*cos(x)^14 + 537071702*cos(x)^12 - 107755754*cos(x)^10 +
 6570448*cos(x)^8 + 257200*cos(x)^6 + 26128*cos(x)^4 + 3760*cos(x)^2 - 907
9560*(16*sqrt(2)*cos(x)^21 - 64*sqrt(2)*cos(x)^19 + 104*sqrt(2)*cos(x)^17
- 88*sqrt(2)*cos(x)^15 + 41*sqrt(2)*cos(x)^13 - 10*sqrt(2)*cos(x)^11 + sqr
t(2)*cos(x)^9)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1))
+ 334845*(16*cos(x)^21 - 64*cos(x)^19 + 104*cos(x)^17 - 88*cos(x)^15 + 41*
cos(x)^13 - 10*cos(x)^11 + cos(x)^9)*log(1/2*cos(x) + 1/2) - 334845*(16*co
s(x)^21 - 64*cos(x)^19 + 104*cos(x)^17 - 88*cos(x)^15 + 41*cos(x)^13 - 10*
cos(x)^11 + cos(x)^9)*log(-1/2*cos(x) + 1/2) + 560)/(16*cos(x)^21 - 64*cos
(x)^19 + 104*cos(x)^17 - 88*cos(x)^15 + 41*cos(x)^13 - 10*cos(x)^11 + cos(
x)^9)
```

Sympy [F]

$$\int \frac{1}{(\sin(3x)+\sin(5x))^5}\,dx = \int \frac{1}{(\sin{(3x)}+\sin{(5x)})^5}\,dx$$

input
```
integrate(1/(sin(3*x)+sin(5*x))**5,x)
```

output
```
Integral((sin(3*x) + sin(5*x))**(-5), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24329 vs. $2(124) = 248$.

Time = 4.55 (sec) , antiderivative size = 24329, normalized size of antiderivative = 157.98

$$\int \frac{1}{(\sin(3x)+\sin(5x))^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(sin(3*x)+sin(5*x))^5,x, algorithm="maxima")
```

output
```
-1/165150720*(4*(8744715*cos(41*x) + 40585440*cos(39*x) + 69873594*cos(37*
x) + 47900640*cos(35*x) - 34247585*cos(33*x) - 171621760*cos(31*x) - 27484
1576*cos(29*x) - 189119360*cos(27*x) + 43279510*cos(25*x) + 277989440*cos(
23*x) + 404201564*cos(21*x) + 277989440*cos(19*x) + 43279510*cos(17*x) - 1
89119360*cos(15*x) - 274841576*cos(13*x) - 171621760*cos(11*x) - 34247585*
cos(9*x) + 47900640*cos(7*x) + 69873594*cos(5*x) + 40585440*cos(3*x) + 874
4715*cos(x))*cos(42*x) + 34978860*(5*cos(40*x) + 10*cos(38*x) + 10*cos(36*
x) + cos(34*x) - 19*cos(32*x) - 40*cos(30*x) - 40*cos(28*x) - 14*cos(26*x)
 + 26*cos(24*x) + 60*cos(22*x) + 60*cos(20*x) + 26*cos(18*x) - 14*cos(16*x
) - 40*cos(14*x) - 40*cos(12*x) - 19*cos(10*x) + cos(8*x) + 10*cos(6*x) +
10*cos(4*x) + 5*cos(2*x) + 1)*cos(41*x) + 20*(40585440*cos(39*x) + 6987359
4*cos(37*x) + 47900640*cos(35*x) - 34247585*cos(33*x) - 171621760*cos(31*x
) - 274841576*cos(29*x) - 189119360*cos(27*x) + 43279510*cos(25*x) + 27798
9440*cos(23*x) + 404201564*cos(21*x) + 277989440*cos(19*x) + 43279510*cos(
17*x) - 189119360*cos(15*x) - 274841576*cos(13*x) - 171621760*cos(11*x) -
34247585*cos(9*x) + 47900640*cos(7*x) + 69873594*cos(5*x) + 40585440*cos(3
*x) + 8744715*cos(x))*cos(40*x) + 162341760*(10*cos(38*x) + 10*cos(36*x) +
 cos(34*x) - 19*cos(32*x) - 40*cos(30*x) - 40*cos(28*x) - 14*cos(26*x) + 2
6*cos(24*x) + 60*cos(22*x) + 60*cos(20*x) + 26*cos(18*x) - 14*cos(16*x) -
40*cos(14*x) - 40*cos(12*x) - 19*cos(10*x) + cos(8*x) + 10*cos(6*x) + 1...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(124) = 248$.

Time = 0.15 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.54

$$\int \frac{1}{(\sin(3x)+\sin(5x))^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(sin(3*x)+sin(5*x))^5,x, algorithm="giac")
```

output
```
3603/65536*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/a
bs(4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)) + 1/2097152*(128*(cos(x)
- 1)/(cos(x) + 1) - 6378*(cos(x) - 1)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)^2
/(cos(x) - 1)^2 - 1/16384*(cos(x) - 1)/(cos(x) + 1) + 1/2097152*(cos(x) -
1)^2/(cos(x) + 1)^2 - 1/24576*(53967*(cos(x) - 1)/(cos(x) + 1) + 434951*(c
os(x) - 1)^2/(cos(x) + 1)^2 + 1544619*(cos(x) - 1)^3/(cos(x) + 1)^3 + 2131
953*(cos(x) - 1)^4/(cos(x) + 1)^4 + 757005*(cos(x) - 1)^5/(cos(x) + 1)^5 +
 106029*(cos(x) - 1)^6/(cos(x) + 1)^6 + 5049*(cos(x) - 1)^7/(cos(x) + 1)^7
 + 2459)/(6*(cos(x) - 1)/(cos(x) + 1) + (cos(x) - 1)^2/(cos(x) + 1)^2 + 1)
^4 - 1/5160960*(6495462*(cos(x) - 1)/(cos(x) + 1) + 22589298*(cos(x) - 1)^
2/(cos(x) + 1)^2 + 45358782*(cos(x) - 1)^3/(cos(x) + 1)^3 + 57467088*(cos(
x) - 1)^4/(cos(x) + 1)^4 + 47030130*(cos(x) - 1)^5/(cos(x) + 1)^5 + 243060
30*(cos(x) - 1)^6/(cos(x) + 1)^6 + 7267050*(cos(x) - 1)^7/(cos(x) + 1)^7 +
 968625*(cos(x) - 1)^8/(cos(x) + 1)^8 + 829343)/((cos(x) - 1)/(cos(x) + 1)
 + 1)^9 + 1063/524288*log(-(cos(x) - 1)/(cos(x) + 1))
```

Mupad [B] (verification not implemented)

Time = 21.22 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.36

$$\int \frac{1}{(\sin(3x)+\sin(5x))^5}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(3*x) + sin(5*x))^5,x)
```

output
```
(1063*log(tan(x/2)))/262144 - ((95*tan(x/2)^2)/2097152 + (21383993*tan(x/2
)^4)/82575360 - (225285883*tan(x/2)^6)/27525120 + (18294266437*tan(x/2)^8)
/165150720 - (6680946049*tan(x/2)^10)/7864320 + (22702586729*tan(x/2)^12)/
5505024 - (371004516437*tan(x/2)^14)/27525120 + (10200248921249*tan(x/2)^1
6)/330301440 - (800654617309*tan(x/2)^18)/15728640 + (1011732457843*tan(x/
2)^20)/16515072 - (1488663546829*tan(x/2)^22)/27525120 + (639724414253*tan
(x/2)^24)/18350080 - (127127888753*tan(x/2)^26)/7864320 + (6879842191*tan(
x/2)^28)/1310720 - (299867227*tan(x/2)^30)/262144 + (328817953*tan(x/2)^32
)/2097152 - (25320577*tan(x/2)^34)/2097152 + (805*tan(x/2)^36)/2048 + 1/20
97152)/(tan(x/2)^4 - 33*tan(x/2)^6 + 472*tan(x/2)^8 - 3864*tan(x/2)^10 + 2
0220*tan(x/2)^12 - 71868*tan(x/2)^14 + 180520*tan(x/2)^16 - 329064*tan(x/2
)^18 + 442534*tan(x/2)^20 - 442534*tan(x/2)^22 + 329064*tan(x/2)^24 - 1805
20*tan(x/2)^26 + 71868*tan(x/2)^28 - 20220*tan(x/2)^30 + 3864*tan(x/2)^32
- 472*tan(x/2)^34 + 33*tan(x/2)^36 - tan(x/2)^38) + (3603*2^(1/2)*atanh((1
04566860495*2^(1/2))/(549755813888*((27558878433453*tan(x/2)^2)/1759218604
4416 - 4724773430031/17592186044416)) - (9743982733791*2^(1/2)*tan(x/2)^2)
/(8796093022208*((27558878433453*tan(x/2)^2)/17592186044416 - 472477343003
1/17592186044416))))/32768 + tan(x/2)^2/16384 + tan(x/2)^4/2097152
```

Reduce [F]

$$\int \frac{1}{(\sin(3x)+\sin(5x))^5}\,dx$$
$$= \int \frac{1}{\sin(5x)^5+5\sin(5x)^4\sin(3x)+10\sin(5x)^3\sin(3x)^2+10\sin(5x)^2\sin(3x)^3+5\sin(5x)\sin(3x)^4+\ldots}$$

input
```
int(1/(sin(3*x)+sin(5*x))^5,x)
```

output
```
int(1/(sin(5*x)**5 + 5*sin(5*x)**4*sin(3*x) + 10*sin(5*x)**3*sin(3*x)**2 +
 10*sin(5*x)**2*sin(3*x)**3 + 5*sin(5*x)*sin(3*x)**4 + sin(3*x)**5),x)
```

3.34 $\int \frac{1}{(\sin(3x)+\sin(5x))^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 52

$$\int \frac{1}{(\sin(3x)+\sin(5x))^2} \, dx = -\frac{1}{16}\operatorname{arctanh}(2\cos(x)\sin(x)) - \frac{\cot(x)}{32} + \frac{5\tan(x)}{32} + \frac{\tan^3(x)}{48} + \frac{\csc(x)\sec^5(x)}{64\,(1-\tan^2(x))}$$

output
```
-1/16*arctanh(2*cos(x)*sin(x))-1/32*cot(x)+5/32*tan(x)+1/48*tan(x)^3+csc(x
)*sec(x)^5/(64-64*tan(x)^2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.40

$$\int \frac{1}{(\sin(3x)+\sin(5x))^2} \, dx = \frac{\cos(x)\cos(2x)\sin(x)\,(\cos(2x)+\cos^2(x)(-2+6\cos(2x)-8\cos(4x)+3\log(\cos(x)-\sin(x))\sin(4x)-3}{3(\sin(3x)+\sin(5x))^2}$$

input
```
Integrate[(Sin[3*x] + Sin[5*x])^(-2),x]
```

output
```
(Cos[x]*Cos[2*x]*Sin[x]*(Cos[2*x] + Cos[x]^2*(-2 + 6*Cos[2*x] - 8*Cos[4*x]
 + 3*Log[Cos[x] - Sin[x]]*Sin[4*x] - 3*Log[Cos[x] + Sin[x]]*Sin[4*x])))/(3
*(Sin[3*x] + Sin[5*x])^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 4822, 27, 370, 27, 437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(3x)+\sin(5x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(3x)+\sin(5x))^2}dx$$

$$\Big\downarrow \text{4822}$$

$$\int \frac{\left(\tan^2(x)+1\right)^4\cot^2(x)}{64\left(1-\tan^2(x)\right)^2}d\tan(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{64}\int \frac{\cot^2(x)\left(\tan^2(x)+1\right)^4}{\left(1-\tan^2(x)\right)^2}d\tan(x)$$

$$\Big\downarrow \text{370}$$

$$\frac{1}{64}\left(\frac{1}{2}\int \frac{4\cot^2(x)\left(1-2\tan^2(x)\right)\left(\tan^2(x)+1\right)^2}{1-\tan^2(x)}d\tan(x)+\frac{\left(\tan^2(x)+1\right)^3\cot(x)}{1-\tan^2(x)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{64}\left(2\int \frac{\cot^2(x)\left(1-2\tan^2(x)\right)\left(\tan^2(x)+1\right)^2}{1-\tan^2(x)}d\tan(x)+\frac{\left(\tan^2(x)+1\right)^3\cot(x)}{1-\tan^2(x)}\right)$$

$$\Big\downarrow \text{437}$$

$$\frac{1}{64}\left(2\int\left(\cot^2(x)+2\tan^2(x)+\frac{4}{\tan^2(x)-1}+5\right)d\tan(x)+\frac{(\tan^2(x)+1)^3\cot(x)}{1-\tan^2(x)}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{64}\left(2\left(-4\text{arctanh}(\tan(x))+\frac{2\tan^3(x)}{3}+5\tan(x)-\cot(x)\right)+\frac{(\tan^2(x)+1)^3\cot(x)}{1-\tan^2(x)}\right)$$

input
```
Int[(Sin[3*x] + Sin[5*x])^(-2),x]
```

output
```
((Cot[x]*(1 + Tan[x]^2)^3)/(1 - Tan[x]^2) + 2*(-4*ArcTanh[Tan[x]] - Cot[x]
 + 5*Tan[x] + (2*Tan[x]^3)/3))/64
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4822
```
Int[((a_.)*sin[(m_.)*((c_.) + (d_.)*(x_))] + (b_.)*sin[(n_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Sin[
m*ArcTan[x]] + b*Sin[n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[c + d*x]], x] /
; FreeQ[{a, b, c, d}, x] && ILtQ[p/2, 0] && IntegerQ[(m - 1)/2] && IntegerQ
[(n - 1)/2]
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\tan(x)^3}{192}+\frac{3\tan(x)}{32}-\frac{1}{16(\tan(x)-1)}+\frac{\ln(\tan(x)-1)}{16}-\frac{1}{16(\tan(x)+1)}-\frac{\ln(\tan(x)+1)}{16}-\frac{1}{64\tan(x)}$	48
risch	$\frac{i(3\,\mathrm{e}^{10ix}+6\,\mathrm{e}^{8ix}+2\,\mathrm{e}^{6ix}-2\,\mathrm{e}^{4ix}-13\,\mathrm{e}^{2ix}-8)}{24(\mathrm{e}^{2ix}+1)^3(\mathrm{e}^{6ix}-\mathrm{e}^{4ix}+\mathrm{e}^{2ix}-1)}+\frac{\ln(\mathrm{e}^{2ix}-i)}{16}-\frac{\ln(\mathrm{e}^{2ix}+i)}{16}$	94

input
```
int(1/(sin(3*x)+sin(5*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/192*tan(x)^3+3/32*tan(x)-1/16/(tan(x)-1)+1/16*ln(tan(x)-1)-1/16/(tan(x)+
1)-1/16*ln(tan(x)+1)-1/64/tan(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(40) = 80.

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.79

$$\int \frac{1}{(\sin(3x)+\sin(5x))^2}\,dx = -\frac{64\,\cos(x)^6-76\,\cos(x)^4+6\left(2\,\cos(x)^5-\cos(x)^3\right)\log\left(2\,\cos(x)\sin(x)+1\right)\sin(x)-6\left(2\,\cos(x)^5-\right.}{192\left(2\,\cos(x)^5-\cos(x)^3\right)\sin(x)}$$

input
```
integrate(1/(sin(3*x)+sin(5*x))^2,x, algorithm="fricas")
```

output
```
-1/192*(64*cos(x)^6 - 76*cos(x)^4 + 6*(2*cos(x)^5 - cos(x)^3)*log(2*cos(x)
*sin(x) + 1)*sin(x) - 6*(2*cos(x)^5 - cos(x)^3)*log(-2*cos(x)*sin(x) + 1)*
sin(x) + 14*cos(x)^2 + 1)/((2*cos(x)^5 - cos(x)^3)*sin(x))
```

Sympy [F]

$$\int \frac{1}{(\sin(3x) + \sin(5x))^2}\, dx = \int \frac{1}{(\sin{(3x)} + \sin{(5x)})^2}\, dx$$

input
```
integrate(1/(sin(3*x)+sin(5*x))**2,x)
```

output
```
Integral((sin(3*x) + sin(5*x))**(-2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1588 vs. $2(40) = 80$.

Time = 0.19 (sec) , antiderivative size = 1588, normalized size of antiderivative = 30.54

$$\int \frac{1}{(\sin(3x) + \sin(5x))^2}\, dx = \text{Too large to display}$$

input
```
integrate(1/(sin(3*x)+sin(5*x))^2,x, algorithm="maxima")
```

output
```
-1/96*(4*(3*sin(10*x) + 6*sin(8*x) + 2*sin(6*x) - 2*sin(4*x) - 13*sin(2*x)
)*cos(12*x) + 4*(9*sin(8*x) + 4*sin(6*x) - sin(4*x) - 20*sin(2*x))*cos(10*
x) + 4*(2*sin(6*x) + 4*sin(4*x) - sin(2*x))*cos(8*x) + 8*(sin(4*x) + 2*sin
(2*x))*cos(6*x) - 3*(2*(2*cos(10*x) + cos(8*x) - cos(4*x) - 2*cos(2*x) - 1
)*cos(12*x) + cos(12*x)^2 + 4*(cos(8*x) - cos(4*x) - 2*cos(2*x) - 1)*cos(1
0*x) + 4*cos(10*x)^2 - 2*(cos(4*x) + 2*cos(2*x) + 1)*cos(8*x) + cos(8*x)^2
 + 2*(2*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + 2*(2*sin(10*x
) + sin(8*x) - sin(4*x) - 2*sin(2*x))*sin(12*x) + sin(12*x)^2 + 4*(sin(8*x
) - sin(4*x) - 2*sin(2*x))*sin(10*x) + 4*sin(10*x)^2 - 2*(sin(4*x) + 2*sin
(2*x))*sin(8*x) + sin(8*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*
x)^2 + 4*cos(2*x) + 1)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*
sqrt(2)*sin(x) + 2) + 3*(2*(2*cos(10*x) + cos(8*x) - cos(4*x) - 2*cos(2*x)
 - 1)*cos(12*x) + cos(12*x)^2 + 4*(cos(8*x) - cos(4*x) - 2*cos(2*x) - 1)*c
os(10*x) + 4*cos(10*x)^2 - 2*(cos(4*x) + 2*cos(2*x) + 1)*cos(8*x) + cos(8*
x)^2 + 2*(2*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + 2*(2*sin(
10*x) + sin(8*x) - sin(4*x) - 2*sin(2*x))*sin(12*x) + sin(12*x)^2 + 4*(sin
(8*x) - sin(4*x) - 2*sin(2*x))*sin(10*x) + 4*sin(10*x)^2 - 2*(sin(4*x) + 2
*sin(2*x))*sin(8*x) + sin(8*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*si
n(2*x)^2 + 4*cos(2*x) + 1)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x)
- 2*sqrt(2)*sin(x) + 2) + 3*(2*(2*cos(10*x) + cos(8*x) - cos(4*x) - 2*c...
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{1}{(\sin(3x)+\sin(5x))^2}\,dx = \frac{1}{192}\tan(x)^3 - \frac{9\tan(x)^2-1}{64\left(\tan(x)^3-\tan(x)\right)} - \frac{1}{16}\log\left(|\tan(x)+1|\right) + \frac{1}{16}\log\left(|\tan(x)-1|\right) + \frac{3}{32}\tan(x)$$

input
```
integrate(1/(sin(3*x)+sin(5*x))^2,x, algorithm="giac")
```

output
```
1/192*tan(x)^3 - 1/64*(9*tan(x)^2 - 1)/(tan(x)^3 - tan(x)) - 1/16*log(abs(
tan(x) + 1)) + 1/16*log(abs(tan(x) - 1)) + 3/32*tan(x)
```

Mupad [B] (verification not implemented)

Time = 20.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{1}{(\sin(3x)+\sin(5x))^2}\,dx = \frac{-44\cos(x)^4+14\cos(x)^2+1}{192\cos(x)^3\sin(x)-384\cos(x)^5\sin(x)} - \frac{\cos(x)}{6\sin(x)} - \frac{\operatorname{atanh}\left(\frac{\sin(x)}{\cos(x)}\right)}{8}$$

input
```
int(1/(sin(3*x) + sin(5*x))^2,x)
```

output
```
(14*cos(x)^2 - 44*cos(x)^4 + 1)/(192*cos(x)^3*sin(x) - 384*cos(x)^5*sin(x)
) - cos(x)/(6*sin(x)) - atanh(sin(x)/cos(x))/8
```

Reduce [F]

$$\int \frac{1}{(\sin(3x)+\sin(5x))^2}\,dx = \int \frac{1}{\sin(5x)^2+2\sin(5x)\sin(3x)+\sin(3x)^2}dx$$

input
```
int(1/(sin(3*x)+sin(5*x))^2,x)
```

output
```
int(1/(sin(5*x)**2 + 2*sin(5*x)*sin(3*x) + sin(3*x)**2),x)
```

3.35 $\int \frac{1}{(\sin(3x)+\sin(5x))^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 122

$$\int \frac{1}{(\sin(3x)+\sin(5x))^4}\,dx = -\frac{9}{128}\operatorname{arctanh}(2\cos(x)\sin(x)) - \frac{37\cot(x)}{4096} - \frac{11\cot^3(x)}{12288} + \frac{289\tan(x)}{2048} + \frac{199\tan^3(x)}{6144} + \frac{163\tan^5(x)}{20480} + \frac{29\tan^7(x)}{28672} + \frac{\csc^3(x)\sec^{13}(x)}{12288\left(1-\tan^2(x)\right)^3} - \frac{\csc^3(x)\sec^{11}(x)}{12288\left(1-\tan^2(x)\right)^2} + \frac{5\csc^3(x)\sec^9(x)}{6144\left(1-\tan^2(x)\right)}$$

output
```
-9/128*arctanh(2*cos(x)*sin(x))-37/4096*cot(x)-11/12288*cot(x)^3+289/2048*
tan(x)+199/6144*tan(x)^3+163/20480*tan(x)^5+29/28672*tan(x)^7+1/12288*csc(
x)^3*sec(x)^13/(1-tan(x)^2)^3-1/12288*csc(x)^3*sec(x)^11/(1-tan(x)^2)^2+5*
csc(x)^3*sec(x)^9/(6144-6144*tan(x)^2)
```

Mathematica [A] (verified)

Time = 1.70 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.24

$$\int \frac{1}{(\sin(3x)+\sin(5x))^4}\,dx = \frac{\cos(x)(\sin(x)-\sin(3x))^4\left(-35\cos^5(x)\cot^3(x)+15\sin(x)+228\cos^2(x)\sin(x)+2369\cos^4(x)\sin(x)+\right.}{}$$

input
```
Integrate[(Sin[3*x] + Sin[5*x])^(-4),x]
```

output
```
(Cos[x]*(Sin[x] - Sin[3*x])^4*(-35*Cos[x]^5*Cot[x]^3 + 15*Sin[x] + 228*Cos
[x]^2*Sin[x] + 2369*Cos[x]^4*Sin[x] + 32878*Cos[x]^6*Sin[x] + 70*Cos[x]^7*
(-19*Cot[x] + 4*Sec[2*x]^3*(81*Cos[2*x]*(Log[Cos[x] - Sin[x]] - Log[Cos[x]
 + Sin[x]]) + 27*Cos[6*x]*(Log[Cos[x] - Sin[x]] - Log[Cos[x] + Sin[x]]) +
21*Sin[2*x] - 6*Sin[4*x] + 17*Sin[6*x]))))/(1680*(Sin[3*x] + Sin[5*x])^4)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4822, 27, 370, 27, 439, 27, 439, 27, 437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(3x)+\sin(5x))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(3x)+\sin(5x))^4}dx$$

$$\Big\downarrow \text{4822}$$

$$\int \frac{\left(\tan^2(x)+1\right)^9\cot^4(x)}{4096\left(1-\tan^2(x)\right)^4}d\tan(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cot^4(x)\left(\tan^2(x)+1\right)^9}{\left(1-\tan^2(x)\right)^4}d\tan(x)}{4096}$$

$$\Big\downarrow \; 370$$

$$\frac{\frac{1}{6}\int \frac{4\cot^4(x)\left(3-5\tan^2(x)\right)\left(\tan^2(x)+1\right)^7}{\left(1-\tan^2(x)\right)^3}d\tan(x)+\frac{\left(\tan^2(x)+1\right)^8\cot^3(x)}{3\left(1-\tan^2(x)\right)^3}}{4096}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2}{3}\int \frac{\cot^4(x)\left(3-5\tan^2(x)\right)\left(\tan^2(x)+1\right)^7}{\left(1-\tan^2(x)\right)^3}d\tan(x)+\frac{\left(\tan^2(x)+1\right)^8\cot^3(x)}{3\left(1-\tan^2(x)\right)^3}}{4096}$$

$$\Big\downarrow \; 439$$

$$\frac{\frac{2}{3}\left(\frac{1}{4}\int \frac{2\cot^4(x)\left(\tan^2(x)+1\right)^6\left(17\tan^2(x)+3\right)}{\left(1-\tan^2(x)\right)^2}d\tan(x)-\frac{\left(\tan^2(x)+1\right)^7\cot^3(x)}{2\left(1-\tan^2(x)\right)^2}\right)+\frac{\left(\tan^2(x)+1\right)^8\cot^3(x)}{3\left(1-\tan^2(x)\right)^3}}{4096}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2}{3}\left(\frac{1}{2}\int \frac{\cot^4(x)\left(\tan^2(x)+1\right)^6\left(17\tan^2(x)+3\right)}{\left(1-\tan^2(x)\right)^2}d\tan(x)-\frac{\left(\tan^2(x)+1\right)^7\cot^3(x)}{2\left(1-\tan^2(x)\right)^2}\right)+\frac{\left(\tan^2(x)+1\right)^8\cot^3(x)}{3\left(1-\tan^2(x)\right)^3}}{4096}$$

$$\Big\downarrow \; 439$$

$$\frac{\frac{2}{3}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{6\cot^4(x)\left(11-29\tan^2(x)\right)\left(\tan^2(x)+1\right)^5}{1-\tan^2(x)}d\tan(x)+\frac{10\left(\tan^2(x)+1\right)^6\cot^3(x)}{1-\tan^2(x)}\right)-\frac{\left(\tan^2(x)+1\right)^7\cot^3(x)}{2\left(1-\tan^2(x)\right)^2}\right)+\frac{\left(\tan^2(x)+1\right)^8 \text{co}}{3\left(1-\tan^2(x)\right.}}{4096}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2}{3}\left(\frac{1}{2}\left(3\int \frac{\cot^4(x)\left(11-29\tan^2(x)\right)\left(\tan^2(x)+1\right)^5}{1-\tan^2(x)}d\tan(x)+\frac{10\left(\tan^2(x)+1\right)^6\cot^3(x)}{1-\tan^2(x)}\right)-\frac{\left(\tan^2(x)+1\right)^7\cot^3(x)}{2\left(1-\tan^2(x)\right)^2}\right)+\frac{\left(\tan^2(x)+1\right)^8\cot^3}{3\left(1-\tan^2(x)\right)^3}}{4096}$$

$$\Big\downarrow \; 437$$

$$\frac{\frac{2}{3}\left(\frac{1}{2}\left(3\int \left(29\tan^6(x)+163\tan^4(x)+398\tan^2(x)+11\cot^4(x)+37\cot^2(x)+\frac{576}{\tan^2(x)-1}+578\right)d\tan(x)+\frac{10(\text{ta}}{}\right.\right.}{4096}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{2}{3}\left(\frac{1}{2}\left(3\left(-576\text{arctanh}(\tan(x))+\frac{29\tan^7(x)}{7}+\frac{163\tan^5(x)}{5}+\frac{398\tan^3(x)}{3}+578\tan(x)-\frac{11\cot^3(x)}{3}-37\cot(x)\right)+\frac{10(\text{ta}}{}\right.\right.}{4096}$$

input
```
Int[(Sin[3*x] + Sin[5*x])^(-4),x]
```

output
```
((Cot[x]^3*(1 + Tan[x]^2)^8)/(3*(1 - Tan[x]^2)^3) + (2*(-1/2*(Cot[x]^3*(1
+ Tan[x]^2)^7)/(1 - Tan[x]^2)^2 + ((10*Cot[x]^3*(1 + Tan[x]^2)^6)/(1 - Tan
[x]^2) + 3*(-576*ArcTanh[Tan[x]] - 37*Cot[x] - (11*Cot[x]^3)/3 + 578*Tan[x
] + (398*Tan[x]^3)/3 + (163*Tan[x]^5)/5 + (29*Tan[x]^7)/7))/2))/3)/4096
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4822
```
Int[((a_.)*sin[(m_.)*((c_.) + (d_.)*(x_))] + (b_.)*sin[(n_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Sin[
m*ArcTan[x]] + b*Sin[n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[c + d*x]], x] /
; FreeQ[{a, b, c, d}, x] && ILtQ[p/2, 0] && IntegerQ[(m - 1)/2] && IntegerQ
[(n - 1)/2]
```

Maple [A] (verified)

Time = 32.14 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.02

method	result
parallelrisch	0
default	$\frac{\tan(x)^7}{28672}+\frac{13\tan(x)^5}{20480}+\frac{41\tan(x)^3}{6144}+\frac{169\tan(x)}{2048}-\frac{1}{12288\tan(x)^3}-\frac{13}{4096\tan(x)}-\frac{1}{384(\tan(x)+1)^3}+\frac{3}{256(\tan(x}$
risch	$\frac{i(945\,e^{30ix}+3780\,e^{28ix}+5355\,e^{26ix}+2520\,e^{24ix}-3591\,e^{22ix}-11844\,e^{20ix}-15381\,e^{18ix}-7344\,e^{16ix}+4587\,e^{14ix}+18108\,e^{12ix}}{6720(e^{2ix}+1)^7(e^{6ix}-e^{4ix}+e^{2ix}-1)^3}$

input
```
int(1/(sin(3*x)+sin(5*x))^4,x,method=_RETURNVERBOSE)
```

output
```
0
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.40

$$\int \frac{1}{(\sin(3x)+\sin(5x))^4}\,dx =$$
$$-\frac{425984 \cos(x)^{16} - 1398912 \cos(x)^{14} + 1740096 \cos(x)^{12} - 1007552 \cos(x)^{10} + 260448 \cos(x)^{8} - 19$$

input
```
integrate(1/(sin(3*x)+sin(5*x))^4,x, algorithm="fricas")
```

output
```
-1/430080*(425984*cos(x)^16 - 1398912*cos(x)^14 + 1740096*cos(x)^12 - 1007
552*cos(x)^10 + 260448*cos(x)^8 - 19056*cos(x)^6 - 920*cos(x)^4 + 15120*(8
*cos(x)^15 - 20*cos(x)^13 + 18*cos(x)^11 - 7*cos(x)^9 + cos(x)^7)*log(2*co
s(x)*sin(x) + 1)*sin(x) - 15120*(8*cos(x)^15 - 20*cos(x)^13 + 18*cos(x)^11
 - 7*cos(x)^9 + cos(x)^7)*log(-2*cos(x)*sin(x) + 1)*sin(x) - 108*cos(x)^2
- 15)/((8*cos(x)^15 - 20*cos(x)^13 + 18*cos(x)^11 - 7*cos(x)^9 + cos(x)^7)
*sin(x))
```

Sympy [F]

$$\int \frac{1}{(\sin(3x)+\sin(5x))^4}\,dx = \int \frac{1}{(\sin(3x)+\sin(5x))^4}\,dx$$

input
```
integrate(1/(sin(3*x)+sin(5*x))**4,x)
```

output
```
Integral((sin(3*x) + sin(5*x))**(-4), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10270 vs. 2(96) = 192.

Time = 1.19 (sec) , antiderivative size = 10270, normalized size of antiderivative = 84.18

$$\int \frac{1}{(\sin(3x)+\sin(5x))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(sin(3*x)+sin(5*x))^4,x, algorithm="maxima")
```

output
```
-1/26880*(4*(945*sin(30*x) + 3780*sin(28*x) + 5355*sin(26*x) + 2520*sin(24
*x) - 3591*sin(22*x) - 11844*sin(20*x) - 15381*sin(18*x) - 7344*sin(16*x)
+ 4587*sin(14*x) + 18108*sin(12*x) + 16377*sin(10*x) + 5848*sin(8*x) - 130
1*sin(6*x) - 6204*sin(4*x) - 5711*sin(2*x))*cos(32*x) + 8*(4725*sin(28*x)
+ 8820*sin(26*x) + 5985*sin(24*x) - 1512*sin(22*x) - 15183*sin(20*x) - 250
92*sin(18*x) - 14688*sin(16*x) + 3504*sin(14*x) + 27711*sin(12*x) + 27084*
sin(10*x) + 10751*sin(8*x) - 712*sin(6*x) - 9573*sin(4*x) - 9532*sin(2*x))
*cos(30*x) + 24*(2835*sin(26*x) + 3780*sin(24*x) + 3969*sin(22*x) - 504*si
n(20*x) - 7821*sin(18*x) - 7344*sin(16*x) - 2973*sin(14*x) + 6768*sin(12*x
) + 8817*sin(10*x) + 4588*sin(8*x) + 1219*sin(6*x) - 2424*sin(4*x) - 3191*
sin(2*x))*cos(28*x) + 8*(10395*sin(24*x) + 24948*sin(22*x) + 24507*sin(20*
x) + 1368*sin(18*x) - 14688*sin(16*x) - 22956*sin(14*x) - 11979*sin(12*x)
+ 624*sin(10*x) + 6341*sin(8*x) + 8108*sin(6*x) + 3657*sin(4*x) - 712*sin(
2*x))*cos(26*x) + 8*(18711*sin(22*x) + 34524*sin(20*x) + 30501*sin(18*x) +
 7344*sin(16*x) - 19707*sin(14*x) - 40788*sin(12*x) - 31497*sin(10*x) - 83
68*sin(8*x) + 6341*sin(6*x) + 13764*sin(4*x) + 10751*sin(2*x))*cos(24*x) +
 24*(12915*sin(20*x) + 23580*sin(18*x) + 14688*sin(16*x) - 1992*sin(14*x)
- 25443*sin(12*x) - 25572*sin(10*x) - 10499*sin(8*x) + 208*sin(6*x) + 8817
*sin(4*x) + 9028*sin(2*x))*cos(22*x) + 72*(7485*sin(18*x) + 7344*sin(16*x)
 + 3309*sin(14*x) - 6264*sin(12*x) - 8481*sin(10*x) - 4532*sin(8*x) - 1...
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\begin{aligned}
&\int \frac{1}{(\sin(3x)+\sin(5x))^4}\,dx \\
&= \frac{1}{28672}\tan(x)^7 + \frac{13}{20480}\tan(x)^5 + \frac{41}{6144}\tan(x)^3 \\
&\quad - \frac{1383\tan(x)^8 - 2164\tan(x)^6 + 1074\tan(x)^4 - 36\tan(x)^2 - 1}{12288\left(\tan(x)^3 - \tan(x)\right)^3} \\
&\quad - \frac{9}{128}\log(|\tan(x)+1|) + \frac{9}{128}\log(|\tan(x)-1|) + \frac{169}{2048}\tan(x)
\end{aligned}$$

input
```
integrate(1/(sin(3*x)+sin(5*x))^4,x, algorithm="giac")
```

output
```
1/28672*tan(x)^7 + 13/20480*tan(x)^5 + 41/6144*tan(x)^3 - 1/12288*(1383*ta
n(x)^8 - 2164*tan(x)^6 + 1074*tan(x)^4 - 36*tan(x)^2 - 1)/(tan(x)^3 - tan(
x))^3 - 9/128*log(abs(tan(x) + 1)) + 9/128*log(abs(tan(x) - 1)) + 169/2048
*tan(x)
```

Mupad [B] (verification not implemented)

Time = 19.98 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.17

$$\int \frac{1}{(\sin(3x)+\sin(5x))^4}\,dx = \frac{51\tan\left(\frac{x}{2}\right)}{32768} + \frac{9\,\mathrm{atanh}\left(\frac{81\tan\left(\frac{x}{2}\right)}{128\left(\frac{81\tan\left(\frac{x}{2}\right)^2}{256}-\frac{81}{256}\right)}\right)}{64} + \frac{\tan\left(\frac{x}{2}\right)^3}{98304} - \frac{-\frac{10579\tan\left(\frac{x}{2}\right)^{28}}{32768} + \frac{14653\tan\left(\frac{x}{2}\right)^{26}}{2048} - \frac{10651583\tan\left(\frac{x}{2}\right)^{24}}{163840} + \frac{4505745\tan\left(\frac{x}{2}\right)^{22}}{14336} - \frac{1040494093\tan\left(\frac{x}{2}\right)^{20}}{1146880} + \frac{60039671\tan\left(\frac{x}{2}\right)^{18}}{35840} - \cdots}{-\tan\left(\frac{x}{2}\right)^{29} + 25\tan\left(\frac{x}{2}\right)^{27} - 258\tan\left(\frac{x}{2}\right)^{25} + 1442\tan\left(\frac{x}{2}\right)^{23} - 4871\tan\left(\frac{x}{2}\right)^{21} + 10623\tan\left(\frac{x}{2}\right)^{19} - \cdots}$$

input
```
int(1/(sin(3*x) + sin(5*x))^4,x)
```

output
```
(51*tan(x/2))/32768 + (9*atanh((81*tan(x/2))/(128*((81*tan(x/2)^2)/256 - 8
1/256))))/64 + tan(x/2)^3/98304 - (tan(x/2)^2/768 - (11717*tan(x/2)^4)/327
68 + (46097*tan(x/2)^6)/6144 - (32836279*tan(x/2)^8)/491520 + (4580743*tan
(x/2)^10)/14336 - (1050596843*tan(x/2)^12)/1146880 + (60310501*tan(x/2)^14
)/35840 - (335936923*tan(x/2)^16)/163840 + (60039671*tan(x/2)^18)/35840 -
(1040494093*tan(x/2)^20)/1146880 + (4505745*tan(x/2)^22)/14336 - (10651583
*tan(x/2)^24)/163840 + (14653*tan(x/2)^26)/2048 - (10579*tan(x/2)^28)/3276
8 + 1/98304)/(tan(x/2)^3 - 25*tan(x/2)^5 + 258*tan(x/2)^7 - 1442*tan(x/2)^
9 + 4871*tan(x/2)^11 - 10623*tan(x/2)^13 + 15548*tan(x/2)^15 - 15548*tan(x
/2)^17 + 10623*tan(x/2)^19 - 4871*tan(x/2)^21 + 1442*tan(x/2)^23 - 258*tan
(x/2)^25 + 25*tan(x/2)^27 - tan(x/2)^29)
```

Reduce [F]

$$\int \frac{1}{(\sin(3x)+\sin(5x))^4}\,dx$$
$$=\int \frac{1}{\sin(5x)^4+4\sin(5x)^3\sin(3x)+6\sin(5x)^2\sin(3x)^2+4\sin(5x)\sin(3x)^3+\sin(3x)^4}dx$$

input
```
int(1/(sin(3*x)+sin(5*x))^4,x)
```

output
```
int(1/(sin(5*x)**4 + 4*sin(5*x)**3*sin(3*x) + 6*sin(5*x)**2*sin(3*x)**2 +
4*sin(5*x)*sin(3*x)**3 + sin(3*x)**4),x)
```

3.36 $\int \frac{1}{(\sin(3x)+\sin(5x))^6}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 190

$$\begin{aligned}\int \frac{1}{(\sin(3x)+\sin(5x))^6}\,dx = &-\frac{637\text{arctanh}(2\cos(x)\sin(x))}{8192}-\frac{3955\cot(x)}{1048576}\\&-\frac{2131\cot^3(x)}{3145728}-\frac{173\cot^5(x)}{2621440}+\frac{158879\tan(x)}{1048576}\\&+\frac{128051\tan^3(x)}{3145728}+\frac{78617\tan^5(x)}{5242880}+\frac{4751\tan^7(x)}{1048576}\\&+\frac{8501\tan^9(x)}{9437184}+\frac{983\tan^{11}(x)}{11534336}+\frac{\csc^5(x)\sec^{21}(x)}{1310720\left(1-\tan^2(x)\right)^5}\\&-\frac{3\csc^5(x)\sec^{19}(x)}{5242880\left(1-\tan^2(x)\right)^4}+\frac{61\csc^5(x)\sec^{17}(x)}{15728640\left(1-\tan^2(x)\right)^3}\\&-\frac{97\csc^5(x)\sec^{15}(x)}{10485760\left(1-\tan^2(x)\right)^2}+\frac{443\csc^5(x)\sec^{13}(x)}{6291456\left(1-\tan^2(x)\right)}\end{aligned}$$

output
```
-637/8192*arctanh(2*cos(x)*sin(x))-3955/1048576*cot(x)-2131/3145728*cot(x)
^3-173/2621440*cot(x)^5+158879/1048576*tan(x)+128051/3145728*tan(x)^3+7861
7/5242880*tan(x)^5+4751/1048576*tan(x)^7+8501/9437184*tan(x)^9+983/1153433
6*tan(x)^11+1/1310720*csc(x)^5*sec(x)^21/(1-tan(x)^2)^5-3/5242880*csc(x)^5
*sec(x)^19/(1-tan(x)^2)^4+61/15728640*csc(x)^5*sec(x)^17/(1-tan(x)^2)^3-97
/10485760*csc(x)^5*sec(x)^15/(1-tan(x)^2)^2+443*csc(x)^5*sec(x)^13/(629145
6-6291456*tan(x)^2)
```

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.78

$$\int \frac{1}{(\sin(3x)+\sin(5x))^6}\,dx$$
$$= \frac{\cos(2x)\left(330631741440\cos^{12}(x)\cos^5(2x)(\log(\cos(x)-\sin(x))-\log(\cos(x)+\sin(x)))\sin^6(x)+70(2005\right.}{}$$

input
```
Integrate[(Sin[3*x] + Sin[5*x])^(-6),x]
```

output
```
(Cos[2*x]*(330631741440*Cos[x]^12*Cos[2*x]^5*(Log[Cos[x] - Sin[x]] - Log[C
os[x] + Sin[x]])*Sin[x]^6 + 70*(200591 + 280858*Cos[2*x])*Sin[2*x] + Cos[x
]*(2858152*Sin[3*x] + 16199140*Sin[5*x] + 1288983*Sin[7*x] - 8835165*Sin[9
*x] - 13422850*Sin[11*x] - 7589426*Sin[13*x] + 1766680*Sin[15*x] + 4598336
*Sin[17*x] + 3590570*Sin[19*x] + 1462970*Sin[21*x] - 708993*Sin[23*x] - 95
4445*Sin[25*x] - 253952*Sin[27*x])))/(16220160*(Sin[3*x] + Sin[5*x])^6)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.17, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.273, Rules used = {3042, 4822, 27, 370, 27, 439, 439, 27, 439, 27, 439, 27, 437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(3x)+\sin(5x))^6}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(\sin(3x)+\sin(5x))^6}dx$$
$$\Big\downarrow \text{4822}$$
$$\int \frac{\left(\tan^2(x)+1\right)^{14}\cot^6(x)}{262144\left(1-\tan^2(x)\right)^6}d\tan(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\cot^6(x)\left(\tan^2(x)+1\right)^{14}}{\left(1-\tan^2(x)\right)^6}d\tan(x)}{262144}$$

$$\Big\downarrow \text{370}$$

$$\frac{\frac{1}{10}\int \frac{4\cot^6(x)\left(5-8\tan^2(x)\right)\left(\tan^2(x)+1\right)^{12}}{\left(1-\tan^2(x)\right)^5}d\tan(x)+\frac{\left(\tan^2(x)+1\right)^{13}\cot^5(x)}{5\left(1-\tan^2(x)\right)^5}}{262144}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2}{5}\int \frac{\cot^6(x)\left(5-8\tan^2(x)\right)\left(\tan^2(x)+1\right)^{12}}{\left(1-\tan^2(x)\right)^5}d\tan(x)+\frac{\left(\tan^2(x)+1\right)^{13}\cot^5(x)}{5\left(1-\tan^2(x)\right)^5}}{262144}$$

$$\Big\downarrow \text{439}$$

$$\frac{\frac{2}{5}\left(\frac{1}{8}\int \frac{\cot^6(x)\left(\tan^2(x)+1\right)^{11}\left(97\tan^2(x)+25\right)}{\left(1-\tan^2(x)\right)^4}d\tan(x)-\frac{3\left(\tan^2(x)+1\right)^{12}\cot^5(x)}{8\left(1-\tan^2(x)\right)^4}\right)+\frac{\left(\tan^2(x)+1\right)^{13}\cot^5(x)}{5\left(1-\tan^2(x)\right)^5}}{262144}$$

$$\Big\downarrow \text{439}$$

$$\frac{\frac{2}{5}\left(\frac{1}{8}\left(\frac{1}{6}\int \frac{4\cot^6(x)\left(190-481\tan^2(x)\right)\left(\tan^2(x)+1\right)^{10}}{\left(1-\tan^2(x)\right)^3}d\tan(x)+\frac{61\left(\tan^2(x)+1\right)^{11}\cot^5(x)}{3\left(1-\tan^2(x)\right)^3}\right)-\frac{3\left(\tan^2(x)+1\right)^{12}\cot^5(x)}{8\left(1-\tan^2(x)\right)^4}\right)+\frac{\left(\tan^2(x)+\right.}{5(1-\mathrm{t}}}{262144}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2}{5}\left(\frac{1}{8}\left(\frac{2}{3}\int \frac{\cot^6(x)\left(190-481\tan^2(x)\right)\left(\tan^2(x)+1\right)^{10}}{\left(1-\tan^2(x)\right)^3}d\tan(x)+\frac{61\left(\tan^2(x)+1\right)^{11}\cot^5(x)}{3\left(1-\tan^2(x)\right)^3}\right)-\frac{3\left(\tan^2(x)+1\right)^{12}\cot^5(x)}{8\left(1-\tan^2(x)\right)^4}\right)+\frac{\left(\tan^2(x)+1\right.}{5(1-\mathrm{tan}}}{262144}$$

$$\Big\downarrow \text{439}$$

$$\frac{\frac{2}{5}\left(\frac{1}{8}\left(\frac{2}{3}\left(\frac{1}{4}\int -\frac{5\cot^6(x)\left(139-1025\tan^2(x)\right)\left(\tan^2(x)+1\right)^9}{\left(1-\tan^2(x)\right)^2}d\tan(x)-\frac{291\left(\tan^2(x)+1\right)^{10}\cot^5(x)}{4\left(1-\tan^2(x)\right)^2}\right)+\frac{61\left(\tan^2(x)+1\right)^{11}\cot^5(x)}{3\left(1-\tan^2(x)\right)^3}\right)-3\right.}{262144}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2}{5}\left(\frac{1}{8}\left(\frac{2}{3}\left(-\frac{5}{4}\int \frac{\cot^6(x)\left(139-1025\tan^2(x)\right)\left(\tan^2(x)+1\right)^9}{\left(1-\tan^2(x)\right)^2}d\tan(x)-\frac{291\left(\tan^2(x)+1\right)^{10}\cot^5(x)}{4\left(1-\tan^2(x)\right)^2}\right)+\frac{61\left(\tan^2(x)+1\right)^{11}\cot^5(x)}{3\left(1-\tan^2(x)\right)^3}\right)-\frac{3(\mathrm{t}}{}\right.}{262144}$$

$$\Big\downarrow \text{439}$$

$$\frac{\frac{2}{5}\left(\frac{1}{8}\left(\frac{2}{3}\left(-\frac{5}{4}\left(\frac{1}{2}\int -\frac{12\cot^6(x)(346-983\tan^2(x))\left(\tan^2(x)+1\right)^8}{1-\tan^2(x)}d\tan(x)-\frac{443\left(\tan^2(x)+1\right)^9\cot^5(x)}{1-\tan^2(x)}\right)-\frac{291\left(\tan^2(x)+1\right)^{10}\cot^5(x)}{4(1-\tan^2(x))^2}\right.\right.\right.}{262144}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2}{5}\left(\frac{1}{8}\left(\frac{2}{3}\left(-\frac{5}{4}\left(-6\int \frac{\cot^6(x)(346-983\tan^2(x))\left(\tan^2(x)+1\right)^8}{1-\tan^2(x)}d\tan(x)-\frac{443\left(\tan^2(x)+1\right)^9\cot^5(x)}{1-\tan^2(x)}\right)-\frac{291\left(\tan^2(x)+1\right)^{10}\cot^5(x)}{4(1-\tan^2(x))^2}\right)-\right.\right.}{262144}$$

$$\Big\downarrow \; 437$$

$$\frac{2}{5}\left(\frac{1}{8}\left(\frac{2}{3}\left(-\frac{5}{4}\left(-6\int \left(983\tan^{10}(x)+8501\tan^8(x)+33257\tan^6(x)+78617\tan^4(x)+128051\tan^2(x)+346\cot^6\right.\right.\right.\right.\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{5}\left(\frac{1}{8}\left(\frac{2}{3}\left(-\frac{5}{4}\left(-6\left(-163072\text{arctanh}(\tan(x))+\frac{983\tan^{11}(x)}{11}+\frac{8501\tan^9(x)}{9}+4751\tan^7(x)+\frac{78617\tan^5(x)}{5}+\frac{128051\tan}{3}\right.\right.\right.\right.\right.$$

input
```
Int[(Sin[3*x] + Sin[5*x])^(-6),x]
```

output
```
((Cot[x]^5*(1 + Tan[x]^2)^13)/(5*(1 - Tan[x]^2)^5) + (2*((-3*Cot[x]^5*(1 +
 Tan[x]^2)^12)/(8*(1 - Tan[x]^2)^4) + ((61*Cot[x]^5*(1 + Tan[x]^2)^11)/(3*
(1 - Tan[x]^2)^3) + (2*((-291*Cot[x]^5*(1 + Tan[x]^2)^10)/(4*(1 - Tan[x]^2
)^2) - (5*((-443*Cot[x]^5*(1 + Tan[x]^2)^9)/(1 - Tan[x]^2) - 6*(-163072*Ar
cTanh[Tan[x]] - 3955*Cot[x] - (2131*Cot[x]^3)/3 - (346*Cot[x]^5)/5 + 15887
9*Tan[x] + (128051*Tan[x]^3)/3 + (78617*Tan[x]^5)/5 + 4751*Tan[x]^7 + (850
1*Tan[x]^9)/9 + (983*Tan[x]^11)/11)))/4))/3)/8))/5)/262144
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4822
```
Int[((a_.)*sin[(m_.)*((c_.) + (d_.)*(x_))] + (b_.)*sin[(n_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Sin[
m*ArcTan[x]] + b*Sin[n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[c + d*x]], x] /
; FreeQ[{a, b, c, d}, x] && ILtQ[p/2, 0] && IntegerQ[(m - 1)/2] && IntegerQ
[(n - 1)/2]
```

Maple [A] (verified)

Time = 527.00 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.01

method	result
parallelrisch	0
default	$\frac{\tan(x)^{11}}{2883584}+\frac{5\tan(x)^{9}}{589824}+\frac{7\tan(x)^{7}}{65536}+\frac{63\tan(x)^{5}}{65536}+\frac{1001\tan(x)^{3}}{131072}+\frac{5691\tan(x)}{65536}-\frac{1}{5120(\tan(x)+1)^{5}}+\frac{5}{4096(\tan(x}$
risch	$\frac{i(-507904-2732109\,e^{2ix}-5726670\,e^{4ix}+27306940\,e^{14ix}+28293614\,e^{12ix}+4995770\,e^{16ix}-55880332\,e^{20ix}+13829334\,e^{10ix}+}{}$

input
```
int(1/(sin(3*x)+sin(5*x))^6,x,method=_RETURNVERBOSE)
```

output
```
0
```

Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.37

$$\int\frac{1}{(\sin(3x)+\sin(5x))^6}\,dx=$$
$$-\frac{520093696\cos(x)^{26}-2761909760\cos(x)^{24}+6252481280\cos(x)^{22}-7859968640\cos(x)^{20}+59772}{}$$

input
```
integrate(1/(sin(3*x)+sin(5*x))^6,x, algorithm="fricas")
```

output
```
-1/129761280*(520093696*cos(x)^26 - 2761909760*cos(x)^24 + 6252481280*cos(
x)^22 - 7859968640*cos(x)^20 + 5977292480*cos(x)^18 - 2795580992*cos(x)^16
 + 772785600*cos(x)^14 - 110231680*cos(x)^12 + 4862560*cos(x)^10 + 159440*
cos(x)^8 + 13960*cos(x)^6 + 1820*cos(x)^4 + 5045040*(32*cos(x)^25 - 144*co
s(x)^23 + 272*cos(x)^21 - 280*cos(x)^19 + 170*cos(x)^17 - 61*cos(x)^15 + 1
2*cos(x)^13 - cos(x)^11)*log(2*cos(x)*sin(x) + 1)*sin(x) - 5045040*(32*cos
(x)^25 - 144*cos(x)^23 + 272*cos(x)^21 - 280*cos(x)^19 + 170*cos(x)^17 - 6
1*cos(x)^15 + 12*cos(x)^13 - cos(x)^11)*log(-2*cos(x)*sin(x) + 1)*sin(x) +
 290*cos(x)^2 + 45)/((32*cos(x)^25 - 144*cos(x)^23 + 272*cos(x)^21 - 280*c
os(x)^19 + 170*cos(x)^17 - 61*cos(x)^15 + 12*cos(x)^13 - cos(x)^11)*sin(x)
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(\sin(3x)+\sin(5x))^6}\,dx = \text{Timed out}$$

input
```
integrate(1/(sin(3*x)+sin(5*x))**6,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26212 vs. 2(150) = 300.

Time = 8.04 (sec) , antiderivative size = 26212, normalized size of antiderivative = 137.96

$$\int \frac{1}{(\sin(3x)+\sin(5x))^6}\,dx = \text{Too large to display}$$

input
```
integrate(1/(sin(3*x)+sin(5*x))^6,x, algorithm="maxima")
```

output
```
-1/8110080*(4*(315315*sin(50*x) + 1891890*sin(48*x) + 4624620*sin(46*x) +
5675670*sin(44*x) + 1639638*sin(42*x) - 9291282*sin(40*x) - 23483460*sin(3
8*x) - 28017990*sin(36*x) - 12727715*sin(34*x) + 17765748*sin(32*x) + 4793
2248*sin(30*x) + 55213340*sin(28*x) + 28082740*sin(26*x) - 15893220*sin(24
*x) - 53648552*sin(22*x) - 55880332*sin(20*x) - 27964835*sin(18*x) + 49957
70*sin(16*x) + 27306940*sin(14*x) + 28293614*sin(12*x) + 13829334*sin(10*x
) + 596630*sin(8*x) - 5533460*sin(6*x) - 5726670*sin(4*x) - 2732109*sin(2*
x))*cos(52*x) + 12*(2207205*sin(48*x) + 7147140*sin(46*x) + 10300290*sin(4
4*x) + 5801796*sin(42*x) - 10804794*sin(40*x) - 36456420*sin(38*x) - 49204
155*sin(36*x) - 28608580*sin(34*x) + 20291271*sin(32*x) + 74843496*sin(30*
x) + 95711980*sin(28*x) + 56165480*sin(26*x) - 17071740*sin(24*x) - 862761
04*sin(22*x) - 96520439*sin(20*x) - 52776520*sin(18*x) + 3159715*sin(16*x)
 + 44103380*sin(14*x) + 48809458*sin(12*x) + 25136148*sin(10*x) + 2244310*
sin(8*x) - 8964820*sin(6*x) - 9876765*sin(4*x) - 4833588*sin(2*x))*cos(50*
x) + 60*(2102100*sin(46*x) + 4414410*sin(44*x) + 4666662*sin(42*x) + 42042
*sin(40*x) - 10870860*sin(38*x) - 19819800*sin(36*x) - 16511495*sin(34*x)
- 522522*sin(32*x) + 22707048*sin(30*x) + 37555700*sin(28*x) + 28082740*si
n(26*x) + 1764420*sin(24*x) - 28423352*sin(22*x) - 37592062*sin(20*x) - 24
181055*sin(18*x) - 3202420*sin(16*x) + 14694340*sin(14*x) + 18960290*sin(1
2*x) + 10802310*sin(10*x) + 1857890*sin(8*x) - 3010940*sin(6*x) - 38347...
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.57

$$\int \frac{1}{(\sin(3x)+\sin(5x))^6}\,dx = \frac{1}{2883584}\tan(x)^{11} + \frac{5}{589824}\tan(x)^9 + \frac{7}{65536}\tan(x)^7 + \frac{63}{65536}\tan(x)^5 + \frac{1001}{131072}\tan(x)^3 - \frac{466620\tan(x)^{14} - 1558920\tan(x)^{12} + 2086119\tan(x)^{10} - 1269375\tan(x)^8 + 302690\tan(x)^6 - 2}{3932160\left(\tan(x)^3 - \tan(x)\right)^5} - \frac{637}{8192}\log(|\tan(x)+1|) + \frac{637}{8192}\log(|\tan(x)-1|) + \frac{5691}{65536}\tan(x)$$

input
```
integrate(1/(sin(3*x)+sin(5*x))^6,x, algorithm="giac")
```

output
```
1/2883584*tan(x)^11 + 5/589824*tan(x)^9 + 7/65536*tan(x)^7 + 63/65536*tan(
x)^5 + 1001/131072*tan(x)^3 - 1/3932160*(466620*tan(x)^14 - 1558920*tan(x)
^12 + 2086119*tan(x)^10 - 1269375*tan(x)^8 + 302690*tan(x)^6 - 2470*tan(x)
^4 - 85*tan(x)^2 - 3)/(tan(x)^3 - tan(x))^5 - 637/8192*log(abs(tan(x) + 1)
) + 637/8192*log(abs(tan(x) - 1)) + 5691/65536*tan(x)
```

Mupad [B] (verification not implemented)

Time = 20.22 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.15

$$\int \frac{1}{(\sin(3x)+\sin(5x))^6}\,dx = \text{Too large to display}$$

input
```
int(1/(sin(3*x) + sin(5*x))^6,x)
```

output
```
(1529*tan(x/2))/4194304 + (637*atanh((405769*tan(x/2))/(524288*((405769*ta
n(x/2)^2)/1048576 - 405769/1048576))))/4096 - ((131*tan(x/2)^2)/62914560 +
 (6467*tan(x/2)^4)/25165824 - (1398771*tan(x/2)^6)/4194304 + (105697243*ta
n(x/2)^8)/8388608 - (13458458059*tan(x/2)^10)/62914560 + (269257843633*tan
(x/2)^12)/125829120 - (531565752613*tan(x/2)^14)/37748736 + (2688258867587
5*tan(x/2)^16)/415236096 - (4986800098581*tan(x/2)^18)/23068672 + (1118354
916224251*tan(x/2)^20)/2076180480 - (353295649020331*tan(x/2)^22)/34603008
0 + (619113558639649*tan(x/2)^24)/415236096 - (31895077087741*tan(x/2)^26)
/18874368 + (618702357469885*tan(x/2)^28)/415236096 - (352813072839409*tan
(x/2)^30)/346030080 + (2231900059630223*tan(x/2)^32)/4152360960 - (2982865
8098879*tan(x/2)^34)/138412032 + (53536737797231*tan(x/2)^36)/830472192 -
(528472163959*tan(x/2)^38)/37748736 + (89025967259*tan(x/2)^40)/41943040 -
 (4433459077*tan(x/2)^42)/20971520 + (103725651*tan(x/2)^44)/8388608 - (13
46425*tan(x/2)^46)/4194304 + 1/41943040)/(tan(x/2)^5 - 41*tan(x/2)^7 + 750
*tan(x/2)^9 - 8110*tan(x/2)^11 + 58005*tan(x/2)^13 - 291533*tan(x/2)^15 +
1069928*tan(x/2)^17 - 2945000*tan(x/2)^19 + 6197490*tan(x/2)^21 - 10105730
*tan(x/2)^23 + 12877844*tan(x/2)^25 - 12877844*tan(x/2)^27 + 10105730*tan(
x/2)^29 - 6197490*tan(x/2)^31 + 2945000*tan(x/2)^33 - 1069928*tan(x/2)^35
+ 291533*tan(x/2)^37 - 58005*tan(x/2)^39 + 8110*tan(x/2)^41 - 750*tan(x/2)
^43 + 41*tan(x/2)^45 - tan(x/2)^47) + (77*tan(x/2)^3)/25165824 + tan(x/...
```

Reduce [F]

$$\int \frac{1}{(\sin(3x)+\sin(5x))^6}\,dx$$
$$= \int \frac{1}{\sin(5x)^6+6\sin(5x)^5\sin(3x)+15\sin(5x)^4\sin(3x)^2+20\sin(5x)^3\sin(3x)^3+15\sin(5x)^2\sin(3x)^4}$$

input
```
int(1/(sin(3*x)+sin(5*x))^6,x)
```

output
```
int(1/(sin(5*x)**6 + 6*sin(5*x)**5*sin(3*x) + 15*sin(5*x)**4*sin(3*x)**2 +
 20*sin(5*x)**3*sin(3*x)**3 + 15*sin(5*x)**2*sin(3*x)**4 + 6*sin(5*x)*sin(
3*x)**5 + sin(3*x)**6),x)
```

3.37 $\int \frac{1}{\cos(x)+\cos(3x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 23

$$\int \frac{1}{\cos(x)+\cos(3x)}\,dx = -\frac{1}{2}\text{arctanh}(\sin(x))+\frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}}$$

output
```
-1/2*arctanh(sin(x))+1/2*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 70 vs. 2(23) = 46.

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.04

$$\int \frac{1}{\cos(x)+\cos(3x)}\,dx = \frac{1}{4}\left(2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-2\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\sqrt{2}\left(-\log\left(\sqrt{2}-2\sin(x)\right)+\log\left(\sqrt{2}+2\sin(x)\right)\right)\right)$$

input
```
Integrate[(Cos[x] + Cos[3*x])^(-1),x]
```

output
```
(2*Log[Cos[x/2] - Sin[x/2]] - 2*Log[Cos[x/2] + Sin[x/2]] + Sqrt[2]*(-Log[S
qrt[2] - 2*Sin[x]] + Log[Sqrt[2] + 2*Sin[x]]))/4
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4825, 27, 303, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos(x)+\cos(3x)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\cos(x)+\cos(3x)}dx$$
$$\Big\downarrow \text{ 4825}$$
$$\int \frac{1}{2\left(1-2\sin^2(x)\right)\left(1-\sin^2(x)\right)}d\sin(x)$$
$$\Big\downarrow \text{ 27}$$
$$\frac{1}{2}\int \frac{1}{\left(1-2\sin^2(x)\right)\left(1-\sin^2(x)\right)}d\sin(x)$$
$$\Big\downarrow \text{ 303}$$
$$\frac{1}{2}\left(2\int \frac{1}{1-2\sin^2(x)}d\sin(x)-\int \frac{1}{1-\sin^2(x)}d\sin(x)\right)$$
$$\Big\downarrow \text{ 219}$$
$$\frac{1}{2}\left(\sqrt{2}\text{arctanh}\left(\sqrt{2}\sin(x)\right)-\text{arctanh}(\sin(x))\right)$$

input
```
Int[(Cos[x] + Cos[3*x])^(-1),x]
```

output
```
(-ArcTanh[Sin[x]] + Sqrt[2]*ArcTanh[Sqrt[2]*Sin[x]])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4825
```
Int[(cos[(m_.)*((c_.) + (d_.)*(x_))]*(a_.) + cos[(n_.)*((c_.) + (d_.)*(x_))
]*(b_.))^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Cos[
m*ArcSin[x]] + b*Cos[n*ArcSin[x]]]]^p/Sqrt[1 - x^2], x], x, Sin[c + d*x]],
x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[(m - 1)/2] &
& IntegerQ[(n - 1)/2]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

method	result	size
default	$-\frac{\ln(1+\sin(x))}{4}+\frac{\ln(\sin(x)-1)}{4}+\frac{\operatorname{arctanh}\left(\sqrt{2}\,\sin(x)\right)\sqrt{2}}{2}$	28
risch	$\frac{\ln(e^{ix}-i)}{2}-\frac{\ln(e^{ix}+i)}{2}+\frac{\sqrt{2}\,\ln\left(e^{2ix}+i\sqrt{2}\,e^{ix}-1\right)}{4}-\frac{\sqrt{2}\,\ln\left(e^{2ix}-i\sqrt{2}\,e^{ix}-1\right)}{4}$	72

input
```
int(1/(cos(x)+cos(3*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(1+sin(x))+1/4*ln(sin(x)-1)+1/2*arctanh(2^(1/2)*sin(x))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(18) = 36.

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.17

$$\int \frac{1}{\cos(x)+\cos(3x)}\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2-2\sqrt{2}\sin(x)-3}{2\cos(x)^2-1}\right) - \frac{1}{4}\log(\sin(x)+1)+\frac{1}{4}\log(-\sin(x)+1)$$

input
```
integrate(1/(cos(x)+cos(3*x)),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 1
/4*log(sin(x) + 1) + 1/4*log(-sin(x) + 1)
```

Sympy [F]

$$\int \frac{1}{\cos(x)+\cos(3x)}\,dx = \int \frac{1}{\cos(x)+\cos(3x)}\,dx$$

input
```
integrate(1/(cos(x)+cos(3*x)),x)
```

output
```
Integral(1/(cos(x) + cos(3*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(18) = 36$.

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 7.43

$$\int \frac{1}{\cos(x) + \cos(3x)}\,dx = \frac{1}{8}\,\sqrt{2}\log\left(2\,\cos\left(x\right)^2 + 2\,\sin\left(x\right)^2 + 2\,\sqrt{2}\cos\left(x\right) + 2\,\sqrt{2}\sin\left(x\right) + 2\right) - \frac{1}{8}\,\sqrt{2}\log\left(2\,\cos\left(x\right)^2 + 2\,\sin\left(x\right)^2 + 2\,\sqrt{2}\cos\left(x\right) - 2\,\sqrt{2}\sin\left(x\right) + 2\right) + \frac{1}{8}\,\sqrt{2}\log\left(2\,\cos\left(x\right)^2 + 2\,\sin\left(x\right)^2 - 2\,\sqrt{2}\cos\left(x\right) + 2\,\sqrt{2}\sin\left(x\right) + 2\right) - \frac{1}{8}\,\sqrt{2}\log\left(2\,\cos\left(x\right)^2 + 2\,\sin\left(x\right)^2 - 2\,\sqrt{2}\cos\left(x\right) - 2\,\sqrt{2}\sin\left(x\right) + 2\right) - \frac{1}{4}\,\log\left(\cos\left(x\right)^2 + \sin\left(x\right)^2 + 2\,\sin\left(x\right) + 1\right) + \frac{1}{4}\,\log\left(\cos\left(x\right)^2 + \sin\left(x\right)^2 - 2\,\sin\left(x\right) + 1\right)$$

input
```
integrate(1/(cos(x)+cos(3*x)),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin
(x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*
sqrt(2)*sin(x) + 2) + 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*
cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 -
 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/4*log(cos(x)^2 + sin(x)^2 +
2*sin(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(18) = 36$.

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.09

$$\int \frac{1}{\cos(x) + \cos(3x)}\,dx = -\frac{1}{4}\,\sqrt{2}\log\left(\frac{\left|-2\,\sqrt{2} + 4\,\sin\left(x\right)\right|}{\left|2\,\sqrt{2} + 4\,\sin\left(x\right)\right|}\right) - \frac{1}{4}\,\log\left(\sin\left(x\right) + 1\right) + \frac{1}{4}\,\log\left(-\sin\left(x\right) + 1\right)$$

input
```
integrate(1/(cos(x)+cos(3*x)),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) - 1
/4*log(sin(x) + 1) + 1/4*log(-sin(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 19.96 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{1}{\cos(x)+\cos(3x)}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin\left(x\right)\right)}{2} - \mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)$$

input
```
int(1/(cos(3*x) + cos(x)),x)
```

output
```
(2^(1/2)*atanh(2^(1/2)*sin(x)))/2 - atanh(sin(x/2)/cos(x/2))
```

Reduce [F]

$$\int \frac{1}{\cos(x)+\cos(3x)}\,dx = \int \frac{1}{\cos\left(3x\right)+\cos\left(x\right)}dx$$

input
```
int(1/(cos(x)+cos(3*x)),x)
```

output
```
int(1/(cos(3*x) + cos(x)),x)
```

3.38 $\int \frac{1}{(\cos(x)+\cos(3x))^3}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 60

$$\int \frac{1}{(\cos(x)+\cos(3x))^3}\,dx = -\frac{13}{16}\operatorname{arctanh}(\sin(x)) + \frac{19\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{16\sqrt{2}} - \frac{7}{32}\sec(x)\tan(x) - \frac{3}{32}\sec(x)\sec(2x)\tan(x) + \frac{1}{16}\sec(x)\sec^2(2x)\tan(x)$$

output
```
-13/16*arctanh(sin(x))+19/32*arctanh(sin(x)*2^(1/2))*2^(1/2)-7/32*sec(x)*t
an(x)-3/32*sec(x)*sec(2*x)*tan(x)+1/16*sec(x)*sec(2*x)^2*tan(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(60) = 120.

Time = 0.38 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.35

$$\int \frac{1}{(\cos(x)+\cos(3x))^3}\,dx = \frac{1}{64}\left(52\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\right.$$
$$-52\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-19\sqrt{2}\log\left(\sqrt{2}-2\sin(x)\right)$$
$$+19\sqrt{2}\log\left(\sqrt{2}+2\sin(x)\right)-\frac{2}{\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)^2}$$
$$+\frac{2}{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2}-\frac{10}{\cos(x)-\sin(x)}$$
$$\left.+8\sec^2(2x)\sin(x)+\frac{10}{\cos(x)+\sin(x)}\right)$$

input
```
Integrate[(Cos[x] + Cos[3*x])^(-3),x]
```

output
```
(52*Log[Cos[x/2] - Sin[x/2]] - 52*Log[Cos[x/2] + Sin[x/2]] - 19*Sqrt[2]*Lo
g[Sqrt[2] - 2*Sin[x]] + 19*Sqrt[2]*Log[Sqrt[2] + 2*Sin[x]] - 2/(Cos[x/2] -
 Sin[x/2])^2 + 2/(Cos[x/2] + Sin[x/2])^2 - 10/(Cos[x] - Sin[x]) + 8*Sec[2*
x]^2*Sin[x] + 10/(Cos[x] + Sin[x]))/64
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.75, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.111$, Rules used = {3042, 4825, 27, 316, 27, 402, 402, 27, 397, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos(x)+\cos(3x))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(\cos(x)+\cos(3x))^3}dx$$
$$\Big\downarrow \text{4825}$$

$$\int \frac{1}{8\left(1-2\sin^2(x)\right)^3\left(1-\sin^2(x)\right)^2}d\sin(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\int \frac{1}{\left(1-2\sin^2(x)\right)^3\left(1-\sin^2(x)\right)^2}d\sin(x)$$

$$\Big\downarrow \text{316}$$

$$\frac{1}{8}\left(\frac{1}{4}\int \frac{2(1-5\sin^2(x))}{\left(1-2\sin^2(x)\right)^2\left(1-\sin^2(x)\right)^2}d\sin(x)+\frac{\sin(x)}{2\left(1-2\sin^2(x)\right)^2\left(1-\sin^2(x)\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{2}\int \frac{1-5\sin^2(x)}{\left(1-2\sin^2(x)\right)^2\left(1-\sin^2(x)\right)^2}d\sin(x)+\frac{\sin(x)}{2\left(1-2\sin^2(x)\right)^2\left(1-\sin^2(x)\right)}\right)$$

$$\Big\downarrow \text{402}$$

$$\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{9\sin^2(x)+5}{\left(1-2\sin^2(x)\right)\left(1-\sin^2(x)\right)^2}d\sin(x)-\frac{3\sin(x)}{2\left(1-2\sin^2(x)\right)\left(1-\sin^2(x)\right)}\right)+\frac{\sin(x)}{2\left(1-2\sin^2(x)\right)^2\left(1-\text{s}\right.}\right.$$

$$\Big\downarrow \text{402}$$

$$\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{2}\int -\frac{4(7\sin^2(x)+6)}{\left(1-2\sin^2(x)\right)\left(1-\sin^2(x)\right)}d\sin(x)-\frac{7\sin(x)}{1-\sin^2(x)}\right)-\frac{3\sin(x)}{2\left(1-2\sin^2(x)\right)\left(1-\sin^2(x)\right)}\right)+\frac{}{2}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(2\int \frac{7\sin^2(x)+6}{\left(1-2\sin^2(x)\right)\left(1-\sin^2(x)\right)}d\sin(x)-\frac{7\sin(x)}{1-\sin^2(x)}\right)-\frac{3\sin(x)}{2\left(1-2\sin^2(x)\right)\left(1-\sin^2(x)\right)}\right)+\frac{}{2\left(1-\right.}\right.$$

$$\Big\downarrow \text{397}$$

$$\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(2\left(19\int \frac{1}{1-2\sin^2(x)}d\sin(x)-13\int \frac{1}{1-\sin^2(x)}d\sin(x)\right)-\frac{7\sin(x)}{1-\sin^2(x)}\right)-\frac{3\sin(x)}{2\left(1-2\sin^2(x)\right)\left(1-\text{si}\right.}\right.\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(2\left(\frac{19\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}}-13\text{arctanh}(\sin(x))\right)-\frac{7\sin(x)}{1-\sin^2(x)}\right)-\frac{3\sin(x)}{2\left(1-2\sin^2(x)\right)\left(1-\sin^2(x)\right)}\right)\right.$$

input
```
Int[(Cos[x] + Cos[3*x])^(-3),x]
```

output
```
(Sin[x]/(2*(1 - 2*Sin[x]^2)^2*(1 - Sin[x]^2)) + ((-3*Sin[x])/(2*(1 - 2*Sin
[x]^2)*(1 - Sin[x]^2)) + (2*(-13*ArcTanh[Sin[x]] + (19*ArcTanh[Sqrt[2]*Sin
[x]])/Sqrt[2]) - (7*Sin[x])/(1 - Sin[x]^2))/2)/2)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4825
```
Int[(cos[(m_.)*((c_.) + (d_.)*(x_))]*(a_.) + cos[(n_.)*((c_.) + (d_.)*(x_))
]*(b_.))^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Cos[
m*ArcSin[x]] + b*Cos[n*ArcSin[x]]]]^p/Sqrt[1 - x^2], x], x, Sin[c + d*x]],
x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[(m - 1)/2] &
& IntegerQ[(n - 1)/2]
```

Maple [A] (verified)

Time = 3.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

method	result
default	$-\frac{4\left(-\frac{5\sin(x)^3}{32}+\frac{3\sin(x)}{64}\right)}{\left(2\sin(x)^2-1\right)^2}+\frac{19\,\operatorname{arctanh}\left(\sqrt{2}\,\sin(x)\right)\sqrt{2}}{32}+\frac{1}{32+32\sin(x)}-\frac{13\ln(1+\sin(x))}{32}+\frac{1}{32\sin(x)-32}+\frac{13\ln(\sin(x)}{32}$
risch	$\frac{i\left(7\,\mathrm{e}^{11ix}-\mathrm{e}^{9ix}+\mathrm{e}^{3ix}-7\,\mathrm{e}^{ix}\right)}{16\left(\mathrm{e}^{6ix}+\mathrm{e}^{4ix}+\mathrm{e}^{2ix}+1\right)^2}-\frac{13\ln\left(\mathrm{e}^{ix}+i\right)}{16}+\frac{13\ln\left(\mathrm{e}^{ix}-i\right)}{16}+\frac{19\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+i\sqrt{2}\,\mathrm{e}^{ix}-1\right)}{64}-\frac{19\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-i\sqrt{2}\,\mathrm{e}^{ix}-1\right)}{64}$

input
```
int(1/(cos(x)+cos(3*x))^3,x,method=_RETURNVERBOSE)
```

output
```
-4*(-5/32*sin(x)^3+3/64*sin(x))/(2*sin(x)^2-1)^2+19/32*arctanh(2^(1/2)*sin
(x))*2^(1/2)+1/32/(1+sin(x))-13/32*ln(1+sin(x))+1/32/(sin(x)-1)+13/32*ln(s
in(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(46) = 92$.

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.45

$$\int \frac{1}{(\cos(x)+\cos(3x))^3}\,dx = \frac{19\left(4\sqrt{2}\cos(x)^6-4\sqrt{2}\cos(x)^4+\sqrt{2}\cos(x)^2\right)\log\left(-\frac{2\cos(x)^2-2\sqrt{2}\sin(x)-3}{2\cos(x)^2-1}\right)-26\left(4\cos(x)^6-4\cos(x)\right.}{64\left(4\right.}$$

input
```
integrate(1/(cos(x)+cos(3*x))^3,x, algorithm="fricas")
```

output
```
1/64*(19*(4*sqrt(2)*cos(x)^6 - 4*sqrt(2)*cos(x)^4 + sqrt(2)*cos(x)^2)*log(
-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 26*(4*cos(x)^6 -
4*cos(x)^4 + cos(x)^2)*log(sin(x) + 1) + 26*(4*cos(x)^6 - 4*cos(x)^4 + cos
(x)^2)*log(-sin(x) + 1) - 4*(14*cos(x)^4 - 11*cos(x)^2 + 1)*sin(x))/(4*cos
(x)^6 - 4*cos(x)^4 + cos(x)^2)
```

Sympy [F]

$$\int \frac{1}{(\cos(x)+\cos(3x))^3}\,dx = \int \frac{1}{(\cos(x)+\cos(3x))^3}\,dx$$

input
```
integrate(1/(cos(x)+cos(3*x))**3,x)
```

output
```
Integral((cos(x) + cos(3*x))**(-3), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3567 vs. $2(46) = 92$.

Time = 0.21 (sec) , antiderivative size = 3567, normalized size of antiderivative = 59.45

$$\int \frac{1}{(\cos(x) + \cos(3x))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cos(x)+cos(3*x))^3,x, algorithm="maxima")
```

output
```
-1/128*(8*(7*sin(11*x) - sin(9*x) + sin(3*x) - 7*sin(x))*cos(12*x) - 56*(2
*sin(10*x) + 3*sin(8*x) + 4*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*cos(11*x)
- 16*(sin(9*x) - sin(3*x) + 7*sin(x))*cos(10*x) + 8*(3*sin(8*x) + 4*sin(6*
x) + 3*sin(4*x) + 2*sin(2*x))*cos(9*x) + 24*(sin(3*x) - 7*sin(x))*cos(8*x)
 + 32*(sin(3*x) - 7*sin(x))*cos(6*x) + 24*(sin(3*x) - 7*sin(x))*cos(4*x) -
 19*(sqrt(2)*cos(12*x)^2 + 4*sqrt(2)*cos(10*x)^2 + 9*sqrt(2)*cos(8*x)^2 +
16*sqrt(2)*cos(6*x)^2 + 9*sqrt(2)*cos(4*x)^2 + 4*sqrt(2)*cos(2*x)^2 + sqrt
(2)*sin(12*x)^2 + 4*sqrt(2)*sin(10*x)^2 + 9*sqrt(2)*sin(8*x)^2 + 16*sqrt(2
)*sin(6*x)^2 + 9*sqrt(2)*sin(4*x)^2 + 12*sqrt(2)*sin(4*x)*sin(2*x) + 4*sqr
t(2)*sin(2*x)^2 + 2*(2*sqrt(2)*cos(10*x) + 3*sqrt(2)*cos(8*x) + 4*sqrt(2)*
cos(6*x) + 3*sqrt(2)*cos(4*x) + 2*sqrt(2)*cos(2*x) + sqrt(2))*cos(12*x) +
4*(3*sqrt(2)*cos(8*x) + 4*sqrt(2)*cos(6*x) + 3*sqrt(2)*cos(4*x) + 2*sqrt(2
)*cos(2*x) + sqrt(2))*cos(10*x) + 6*(4*sqrt(2)*cos(6*x) + 3*sqrt(2)*cos(4*
x) + 2*sqrt(2)*cos(2*x) + sqrt(2))*cos(8*x) + 8*(3*sqrt(2)*cos(4*x) + 2*sq
rt(2)*cos(2*x) + sqrt(2))*cos(6*x) + 6*(2*sqrt(2)*cos(2*x) + sqrt(2))*cos(
4*x) + 2*(2*sqrt(2)*sin(10*x) + 3*sqrt(2)*sin(8*x) + 4*sqrt(2)*sin(6*x) +
3*sqrt(2)*sin(4*x) + 2*sqrt(2)*sin(2*x))*sin(12*x) + 4*(3*sqrt(2)*sin(8*x)
 + 4*sqrt(2)*sin(6*x) + 3*sqrt(2)*sin(4*x) + 2*sqrt(2)*sin(2*x))*sin(10*x)
 + 6*(4*sqrt(2)*sin(6*x) + 3*sqrt(2)*sin(4*x) + 2*sqrt(2)*sin(2*x))*sin(8*
x) + 8*(3*sqrt(2)*sin(4*x) + 2*sqrt(2)*sin(2*x))*sin(6*x) + 4*sqrt(2)*c...
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.38

$$\int \frac{1}{(\cos(x)+\cos(3x))^3}\,dx = -\frac{19}{64}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin(x)\right|}{\left|2\sqrt{2}+4\sin(x)\right|}\right) + \frac{\sin(x)}{16\left(\sin(x)^2-1\right)} + \frac{10\sin(x)^3-3\sin(x)}{16\left(2\sin(x)^2-1\right)^2} - \frac{13}{32}\log(\sin(x)+1) + \frac{13}{32}\log(-\sin(x)+1)$$

input
```
integrate(1/(cos(x)+cos(3*x))^3,x, algorithm="giac")
```

output
```
-19/64*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) +
 1/16*sin(x)/(sin(x)^2 - 1) + 1/16*(10*sin(x)^3 - 3*sin(x))/(2*sin(x)^2 -
1)^2 - 13/32*log(sin(x) + 1) + 13/32*log(-sin(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 20.87 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.73

$$\int \frac{1}{(\cos(x)+\cos(3x))^3}\,dx = \frac{2\sin(3x)-14\sin(5x)-104\,\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)-156\cos(2x)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)-104\cos(4x)\,\mathrm{atanh}\left(\frac{\sin(}{\cos(}\right.}{}$$

input
```
int(1/(cos(3*x) + cos(x))^3,x)
```

output
```
(2*sin(3*x) - 14*sin(5*x) - 104*atanh(sin(x/2)/cos(x/2)) - 156*cos(2*x)*at
anh(sin(x/2)/cos(x/2)) - 104*cos(4*x)*atanh(sin(x/2)/cos(x/2)) - 52*cos(6*
x)*atanh(sin(x/2)/cos(x/2)) + 38*2^(1/2)*atanh(2^(1/2)*sin(x)) + 57*2^(1/2
)*atanh(2^(1/2)*sin(x))*cos(2*x) + 38*2^(1/2)*atanh(2^(1/2)*sin(x))*cos(4*
x) + 19*2^(1/2)*atanh(2^(1/2)*sin(x))*cos(6*x))/(96*cos(2*x) + 64*cos(4*x)
 + 32*cos(6*x) + 64)
```

Reduce [F]

$$\int \frac{1}{(\cos(x)+\cos(3x))^3}\,dx$$
$$=\int \frac{1}{\cos(3x)^3+3\cos(3x)^2\cos(x)+3\cos(3x)\cos(x)^2+\cos(x)^3}dx$$

input
```
int(1/(cos(x)+cos(3*x))^3,x)
```

output
```
int(1/(cos(3*x)**3 + 3*cos(3*x)**2*cos(x) + 3*cos(3*x)*cos(x)**2 + cos(x)*
*3),x)
```

3.39 $\int \frac{1}{(\cos(x)+\cos(3x))^5} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 106

$$\int \frac{1}{(\cos(x)+\cos(3x))^5} \, dx = -\frac{523}{256}\text{arctanh}(\sin(x)) + \frac{1483\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{512\sqrt{2}} - \frac{609\sec(x)\tan(x)}{1024} - \frac{953\sec^3(x)\tan(x)}{6144} - \frac{635\sec^3(x)\sec(2x)\tan(x)}{6144} + \frac{91\sec^3(x)\sec^2(2x)\tan(x)}{3072} - \frac{5}{768}\sec^3(x)\sec^3(2x)\tan(x) + \frac{1}{128}\sec^3(x)\sec^4(2x)\tan(x)$$

output
```
-523/256*arctanh(sin(x))+1483/1024*arctanh(sin(x)*2^(1/2))*2^(1/2)-609/102
4*sec(x)*tan(x)-953/6144*sec(x)^3*tan(x)-635/6144*sec(x)^3*sec(2*x)*tan(x)
+91/3072*sec(x)^3*sec(2*x)^2*tan(x)-5/768*sec(x)^3*sec(2*x)^3*tan(x)+1/128
*sec(x)^3*sec(2*x)^4*tan(x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 213 vs. 2(106) = 212.

Time = 4.12 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.01

$$\int \frac{1}{(\cos(x)+\cos(3x))^5}\,dx = \frac{12552\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-12552\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-4449\sqrt{2}\log\left(\sqrt{2}-2\sin(x)\right)+4449\sqrt{2}\,\mathrm{l}}{}$$

input
```
Integrate[(Cos[x] + Cos[3*x])^(-5),x]
```

output
```
(12552*Log[Cos[x/2] - Sin[x/2]] - 12552*Log[Cos[x/2] + Sin[x/2]] - 4449*Sq
rt[2]*Log[Sqrt[2] - 2*Sin[x]] + 4449*Sqrt[2]*Log[Sqrt[2] + 2*Sin[x]] - 12/
(Cos[x/2] - Sin[x/2])^4 - 516/(Cos[x/2] - Sin[x/2])^2 + 12/(Cos[x/2] + Sin
[x/2])^4 + 516/(Cos[x/2] + Sin[x/2])^2 - 136/(Cos[x] - Sin[x])^3 - 2622/(C
os[x] - Sin[x]) + 136/(Cos[x] + Sin[x])^3 + 2622/(Cos[x] + Sin[x]) + 6*Sec
[2*x]^4*(190*Sin[x] + 79*(-Sin[3*x] + Sin[5*x])))/6144
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.74, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.556, Rules used = {3042, 4825, 27, 316, 27, 402, 402, 402, 402, 27, 402, 27, 397, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos(x)+\cos(3x))^5}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\cos(x)+\cos(3x))^5}dx$$

$$\Big\downarrow \text{4825}$$

$$\int \frac{1}{32\left(1-2\sin^2(x)\right)^5\left(1-\sin^2(x)\right)^3}d\sin(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\int \frac{1}{\left(1-2\sin^2(x)\right)^5\left(1-\sin^2(x)\right)^3}d\sin(x)$$

$$\Big\downarrow \text{316}$$

$$\frac{1}{32}\left(\frac{1}{8}\int \frac{2(3-11\sin^2(x))}{\left(1-2\sin^2(x)\right)^4\left(1-\sin^2(x)\right)^3}d\sin(x)+\frac{\sin(x)}{4\left(1-2\sin^2(x)\right)^4\left(1-\sin^2(x)\right)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{1}{4}\int \frac{3-11\sin^2(x)}{\left(1-2\sin^2(x)\right)^4\left(1-\sin^2(x)\right)^3}d\sin(x)+\frac{\sin(x)}{4\left(1-2\sin^2(x)\right)^4\left(1-\sin^2(x)\right)^2}\right)$$

$$\Big\downarrow \text{402}$$

$$\frac{1}{32}\left(\frac{1}{4}\left(\frac{1}{6}\int \frac{45\sin^2(x)+23}{\left(1-2\sin^2(x)\right)^3\left(1-\sin^2(x)\right)^3}d\sin(x)-\frac{5\sin(x)}{6\left(1-2\sin^2(x)\right)^3\left(1-\sin^2(x)\right)^2}\right)+\frac{\sin(x)}{4\left(1-2\sin^2(x)\right)^4(1\right.$$

$$\Big\downarrow \text{402}$$

$$\frac{1}{32}\left(\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\int \frac{1-637\sin^2(x)}{\left(1-2\sin^2(x)\right)^2\left(1-\sin^2(x)\right)^3}d\sin(x)+\frac{91\sin(x)}{4\left(1-2\sin^2(x)\right)^2\left(1-\sin^2(x)\right)^2}\right)-\frac{5\sin}{6\left(1-2\sin^2(x)\right)}\right.\right.$$

$$\Big\downarrow \text{402}$$

$$\frac{1}{32}\left(\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{3175\sin^2(x)+637}{\left(1-2\sin^2(x)\right)\left(1-\sin^2(x)\right)^3}d\sin(x)-\frac{635\sin(x)}{2\left(1-2\sin^2(x)\right)\left(1-\sin^2(x)\right)^2}\right)+\frac{91}{4\left(1-2\sin^2(x\right.}\right.\right.\right.$$

$$\Big\downarrow \text{402}$$

$$\frac{1}{32}\left(\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{1}{2}\left(-\frac{1}{4}\int -\frac{24(953\sin^2(x)+265)}{\left(1-2\sin^2(x)\right)\left(1-\sin^2(x)\right)^2}d\sin(x)-\frac{953\sin(x)}{\left(1-\sin^2(x)\right)^2}\right)-\frac{635\sin(x)}{2\left(1-2\sin^2(x)\right)\left(1-\sin\right.}\right.\right.\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{1}{2}\left(6\int \frac{953\sin^2(x)+265}{\left(1-2\sin^2(x)\right)\left(1-\sin^2(x)\right)^2}d\sin(x)-\frac{953\sin(x)}{\left(1-\sin^2(x)\right)^2}\right)-\frac{635\sin(x)}{2\left(1-2\sin^2(x)\right)\left(1-\sin^2(x\right.}\right.\right.\right.\right.$$

$$\Big\downarrow 402$$

$$\frac{1}{32}\left(\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{1}{2}\left(6\left(-\frac{1}{2}\int -\frac{4(609\sin^2(x)+437)}{(1-2\sin^2(x))\,(1-\sin^2(x))}d\sin(x)-\frac{609\sin(x)}{1-\sin^2(x)}\right)-\frac{953\sin(x)}{(1-\sin^2(x))^2}\right)-\frac{}{2\,(1-\right.\right.\right.\right.\right.$$

$$\Big\downarrow 27$$

$$\frac{1}{32}\left(\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{1}{2}\left(6\left(2\int \frac{609\sin^2(x)+437}{(1-2\sin^2(x))\,(1-\sin^2(x))}d\sin(x)-\frac{609\sin(x)}{1-\sin^2(x)}\right)-\frac{953\sin(x)}{(1-\sin^2(x))^2}\right)-\frac{}{2\,(1-2\,\mathrm{si}}\right.\right.\right.\right.$$

$$\Big\downarrow 397$$

$$\frac{1}{32}\left(\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{1}{2}\left(6\left(2\left(1483\int \frac{1}{1-2\sin^2(x)}d\sin(x)-1046\int \frac{1}{1-\sin^2(x)}d\sin(x)\right)-\frac{609\sin(x)}{1-\sin^2(x)}\right)-\frac{953\,\mathrm{si}}{(1-\mathrm{sin}}\right.\right.\right.\right.\right.$$

$$\Big\downarrow 219$$

$$\frac{1}{32}\left(\frac{1}{4}\left(\frac{1}{6}\left(\frac{1}{4}\left(\frac{1}{2}\left(6\left(2\left(\frac{1483\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}}-1046\text{arctanh}(\sin(x))\right)-\frac{609\sin(x)}{1-\sin^2(x)}\right)-\frac{953\sin(x)}{(1-\sin^2(x))^2}\right)\right.\right.\right.\right.$$

input
```
Int[(Cos[x] + Cos[3*x])^(-5),x]
```

output
```
(Sin[x]/(4*(1 - 2*Sin[x]^2)^4*(1 - Sin[x]^2)^2) + ((-5*Sin[x])/(6*(1 - 2*S
in[x]^2)^3*(1 - Sin[x]^2)^2) + ((91*Sin[x])/(4*(1 - 2*Sin[x]^2)^2*(1 - Sin
[x]^2)^2) + ((-635*Sin[x])/(2*(1 - 2*Sin[x]^2)*(1 - Sin[x]^2)^2) + ((-953*
Sin[x])/(1 - Sin[x]^2)^2 + 6*(2*(-1046*ArcTanh[Sin[x]] + (1483*ArcTanh[Sqr
t[2]*Sin[x]])/Sqrt[2]) - (609*Sin[x])/(1 - Sin[x]^2)))/2)/4)/6)/4)/32
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4825
```
Int[(cos[(m_.)*((c_.) + (d_.)*(x_))]*(a_.) + cos[(n_.)*((c_.) + (d_.)*(x_))
]*(b_.))^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Cos[
m*ArcSin[x]] + b*Cos[n*ArcSin[x]]]]^p/Sqrt[1 - x^2], x], x, Sin[c + d*x]],
x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[(m - 1)/2] &
& IntegerQ[(n - 1)/2]
```

Maple [A] (verified)

Time = 50.63 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

method	result
default	$-\frac{4\left(-\frac{437\sin(x)^7}{256}+\frac{3527\sin(x)^5}{1536}-\frac{3257\sin(x)^3}{3072}+\frac{331\sin(x)}{2048}\right)}{\left(2\sin(x)^2-1\right)^4}+\frac{1483\,\text{arctanh}\left(\sqrt{2}\sin(x)\right)\sqrt{2}}{1024}-\frac{1}{512(\sin(x)-1)^2}+\frac{43}{512(\sin(x)-1}$
risch	$\frac{i(1827\,e^{23ix}+3733\,e^{21ix}+6115\,e^{19ix}+9109\,e^{17ix}+5746\,e^{15ix}+2382\,e^{13ix}-2382\,e^{11ix}-5746\,e^{9ix}-9109\,e^{7ix}-6115\,e^{5ix}-3733\,e^{3ix}-}{1536(e^{6ix}+e^{4ix}+e^{2ix}+1)^4}$

input
```
int(1/(cos(x)+cos(3*x))^5,x,method=_RETURNVERBOSE)
```

output
```
-4*(-437/256*sin(x)^7+3527/1536*sin(x)^5-3257/3072*sin(x)^3+331/2048*sin(x
))/(2*sin(x)^2-1)^4+1483/1024*arctanh(2^(1/2)*sin(x))*2^(1/2)-1/512/(sin(x
)-1)^2+43/512/(sin(x)-1)+523/512*ln(sin(x)-1)+1/512/(1+sin(x))^2+43/512/(1
+sin(x))-523/512*ln(1+sin(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(86) = 172.

Time = 0.10 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.07

$$\int \frac{1}{(\cos(x)+\cos(3x))^5}\,dx = \frac{4449\left(16\sqrt{2}\cos(x)^{12}-32\sqrt{2}\cos(x)^{10}+24\sqrt{2}\cos(x)^8-8\sqrt{2}\cos(x)^6+\sqrt{2}\cos(x)^4\right)\log\left(-\frac{2\cos(x)^2-2}{2\cos(}\right.}{}$$

input
```
integrate(1/(cos(x)+cos(3*x))^5,x, algorithm="fricas")
```

output
```
1/6144*(4449*(16*sqrt(2)*cos(x)^12 - 32*sqrt(2)*cos(x)^10 + 24*sqrt(2)*cos
(x)^8 - 8*sqrt(2)*cos(x)^6 + sqrt(2)*cos(x)^4)*log(-(2*cos(x)^2 - 2*sqrt(2
)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 6276*(16*cos(x)^12 - 32*cos(x)^10 + 24*c
os(x)^8 - 8*cos(x)^6 + cos(x)^4)*log(sin(x) + 1) + 6276*(16*cos(x)^12 - 32
*cos(x)^10 + 24*cos(x)^8 - 8*cos(x)^6 + cos(x)^4)*log(-sin(x) + 1) - 4*(14
616*cos(x)^10 - 25420*cos(x)^8 + 15570*cos(x)^6 - 3677*cos(x)^4 + 162*cos(
x)^2 + 12)*sin(x))/(16*cos(x)^12 - 32*cos(x)^10 + 24*cos(x)^8 - 8*cos(x)^6
 + cos(x)^4)
```

Sympy [F]

$$\int \frac{1}{(\cos(x)+\cos(3x))^5}\,dx = \int \frac{1}{(\cos{(x)}+\cos{(3x)})^5}\,dx$$

input
```
integrate(1/(cos(x)+cos(3*x))**5,x)
```

output
```
Integral((cos(x) + cos(3*x))**(-5), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12209 vs. $2(86) = 172$.

Time = 0.67 (sec) , antiderivative size = 12209, normalized size of antiderivative = 115.18

$$\int \frac{1}{(\cos(x)+\cos(3x))^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cos(x)+cos(3*x))^5,x, algorithm="maxima")
```

output
```
-1/12288*(8*(1827*sin(23*x) + 3733*sin(21*x) + 6115*sin(19*x) + 9109*sin(1
7*x) + 5746*sin(15*x) + 2382*sin(13*x) - 2382*sin(11*x) - 5746*sin(9*x) -
9109*sin(7*x) - 6115*sin(5*x) - 3733*sin(3*x) - 1827*sin(x))*cos(24*x) - 1
4616*(4*sin(22*x) + 10*sin(20*x) + 20*sin(18*x) + 31*sin(16*x) + 40*sin(14
*x) + 44*sin(12*x) + 40*sin(10*x) + 31*sin(8*x) + 20*sin(6*x) + 10*sin(4*x
) + 4*sin(2*x))*cos(23*x) + 32*(3733*sin(21*x) + 6115*sin(19*x) + 9109*sin
(17*x) + 5746*sin(15*x) + 2382*sin(13*x) - 2382*sin(11*x) - 5746*sin(9*x)
- 9109*sin(7*x) - 6115*sin(5*x) - 3733*sin(3*x) - 1827*sin(x))*cos(22*x) -
 29864*(10*sin(20*x) + 20*sin(18*x) + 31*sin(16*x) + 40*sin(14*x) + 44*sin
(12*x) + 40*sin(10*x) + 31*sin(8*x) + 20*sin(6*x) + 10*sin(4*x) + 4*sin(2*
x))*cos(21*x) + 80*(6115*sin(19*x) + 9109*sin(17*x) + 5746*sin(15*x) + 238
2*sin(13*x) - 2382*sin(11*x) - 5746*sin(9*x) - 9109*sin(7*x) - 6115*sin(5*
x) - 3733*sin(3*x) - 1827*sin(x))*cos(20*x) - 48920*(20*sin(18*x) + 31*sin
(16*x) + 40*sin(14*x) + 44*sin(12*x) + 40*sin(10*x) + 31*sin(8*x) + 20*sin
(6*x) + 10*sin(4*x) + 4*sin(2*x))*cos(19*x) + 160*(9109*sin(17*x) + 5746*s
in(15*x) + 2382*sin(13*x) - 2382*sin(11*x) - 5746*sin(9*x) - 9109*sin(7*x)
 - 6115*sin(5*x) - 3733*sin(3*x) - 1827*sin(x))*cos(18*x) - 72872*(31*sin(
16*x) + 40*sin(14*x) + 44*sin(12*x) + 40*sin(10*x) + 31*sin(8*x) + 20*sin(
6*x) + 10*sin(4*x) + 4*sin(2*x))*cos(17*x) + 248*(5746*sin(15*x) + 2382*si
n(13*x) - 2382*sin(11*x) - 5746*sin(9*x) - 9109*sin(7*x) - 6115*sin(5*x...
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98

$$
\begin{aligned}
\int \frac{1}{(\cos(x)+\cos(3x))^5}\,dx
&= -\frac{1483}{2048}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin(x)\right|}{\left|2\sqrt{2}+4\sin(x)\right|}\right)+\frac{43\sin(x)^3-45\sin(x)}{256\left(\sin(x)^2-1\right)^2} \\
&\quad+\frac{10488\sin(x)^7-14108\sin(x)^5+6514\sin(x)^3-993\sin(x)}{1536\left(2\sin(x)^2-1\right)^4} \\
&\quad-\frac{523}{512}\log(\sin(x)+1)+\frac{523}{512}\log(-\sin(x)+1)
\end{aligned}
$$

input
```
integrate(1/(cos(x)+cos(3*x))^5,x, algorithm="giac")
```

output
```
-1483/2048*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x)
)) + 1/256*(43*sin(x)^3 - 45*sin(x))/(sin(x)^2 - 1)^2 + 1/1536*(10488*sin(
x)^7 - 14108*sin(x)^5 + 6514*sin(x)^3 - 993*sin(x))/(2*sin(x)^2 - 1)^4 - 5
23/512*log(sin(x) + 1) + 523/512*log(-sin(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 21.13 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.90

$$\int \frac{1}{(\cos(x)+\cos(3x))^5}\,dx =$$
$$-\frac{11492\sin(3x)+18218\sin(5x)+12230\sin(7x)+7466\sin(9x)+3654\sin(11x)+276144\,\mathrm{atanh}}{}$$

input
```
int(1/(cos(3*x) + cos(x))^5,x)
```

output
```
-(11492*sin(3*x) + 18218*sin(5*x) + 12230*sin(7*x) + 7466*sin(9*x) + 3654*
sin(11*x) + 276144*atanh(sin(x/2)/cos(x/2)) + 4764*sin(x) + 502080*cos(2*x
)*atanh(sin(x/2)/cos(x/2)) + 389112*cos(4*x)*atanh(sin(x/2)/cos(x/2)) + 25
1040*cos(6*x)*atanh(sin(x/2)/cos(x/2)) + 125520*cos(8*x)*atanh(sin(x/2)/co
s(x/2)) + 50208*cos(10*x)*atanh(sin(x/2)/cos(x/2)) + 12552*cos(12*x)*atanh
(sin(x/2)/cos(x/2)) - 97878*2^(1/2)*atanh(2^(1/2)*sin(x)) - 177960*2^(1/2)
*atanh(2^(1/2)*sin(x))*cos(2*x) - 137919*2^(1/2)*atanh(2^(1/2)*sin(x))*cos
(4*x) - 88980*2^(1/2)*atanh(2^(1/2)*sin(x))*cos(6*x) - 44490*2^(1/2)*atanh
(2^(1/2)*sin(x))*cos(8*x) - 17796*2^(1/2)*atanh(2^(1/2)*sin(x))*cos(10*x)
 - 4449*2^(1/2)*atanh(2^(1/2)*sin(x))*cos(12*x))/(122880*cos(2*x) + 95232*c
os(4*x) + 61440*cos(6*x) + 30720*cos(8*x) + 12288*cos(10*x) + 3072*cos(12*
x) + 67584)
```

Reduce [F]

$$\int \frac{1}{(\cos(x)+\cos(3x))^5}\,dx$$
$$= \int \frac{1}{\cos(3x)^5 + 5\cos(3x)^4\cos(x) + 10\cos(3x)^3\cos(x)^2 + 10\cos(3x)^2\cos(x)^3 + 5\cos(3x)\cos(x)^4 + \cos$$

input
```
int(1/(cos(x)+cos(3*x))^5,x)
```

output
```
int(1/(cos(3*x)**5 + 5*cos(3*x)**4*cos(x) + 10*cos(3*x)**3*cos(x)**2 + 10*
cos(3*x)**2*cos(x)**3 + 5*cos(3*x)*cos(x)**4 + cos(x)**5),x)
```

3.40 $\int \frac{1}{(\cos(x)+\cos(3x))^2} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 34

$$\int \frac{1}{(\cos(x)+\cos(3x))^2} \, dx = -\frac{1}{4}\text{arctanh}(2\cos(x)\sin(x)) + \frac{\tan(x)}{4} + \frac{\tan(x)}{2\,(1-\tan^2(x))}$$

output
```
-1/4*arctanh(2*cos(x)*sin(x))+1/4*tan(x)+tan(x)/(2-2*tan(x)^2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{1}{(\cos(x)+\cos(3x))^2} \, dx = \frac{1}{4}(\log(\cos(x)-\sin(x)) - \log(\cos(x)+\sin(x)) + \sec(x)\sec(2x)\sin(3x))$$

input
```
Integrate[(Cos[x] + Cos[3*x])^(-2),x]
```

output
```
(Log[Cos[x] - Sin[x]] - Log[Cos[x] + Sin[x]] + Sec[x]*Sec[2*x]*Sin[3*x])/4
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 4823, 27, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos(x)+\cos(3x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\cos(x)+\cos(3x))^2}dx$$

$$\Big\downarrow \text{4823}$$

$$\int \frac{\left(\tan^2(x)+1\right)^2}{4\left(1-\tan^2(x)\right)^2}d\tan(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{\left(\tan^2(x)+1\right)^2}{\left(1-\tan^2(x)\right)^2}d\tan(x)$$

$$\Big\downarrow \text{300}$$

$$\frac{1}{4}\int \left(\frac{4\tan^2(x)}{\left(1-\tan^2(x)\right)^2}+1\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-2\text{arctanh}(\tan(x))+\frac{2\tan(x)}{1-\tan^2(x)}+\tan(x)\right)$$

input
```
Int[(Cos[x] + Cos[3*x])^(-2),x]
```

output
```
(-2*ArcTanh[Tan[x]] + Tan[x] + (2*Tan[x])/(1 - Tan[x]^2))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4823
```
Int[(cos[(m_.)*((c_.) + (d_.)*(x_))]*(a_.) + cos[(n_.)*((c_.) + (d_.)*(x_))
]*(b_.))^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Cos[
m*ArcTan[x]] + b*Cos[n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[c + d*x]], x] /
; FreeQ[{a, b, c, d}, x] && ILtQ[p/2, 0] && IntegerQ[(m - 1)/2] && IntegerQ
[(n - 1)/2]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{\tan(x)}{4}-\frac{1}{4(\tan(x)-1)}+\frac{\ln(\tan(x)-1)}{4}-\frac{1}{4(\tan(x)+1)}-\frac{\ln(\tan(x)+1)}{4}$	36
risch	$\frac{i(\mathrm{e}^{4ix}+\mathrm{e}^{2ix}+2)}{2\,\mathrm{e}^{6ix}+2\,\mathrm{e}^{4ix}+2\,\mathrm{e}^{2ix}+2}-\frac{\ln(\mathrm{e}^{2ix}+i)}{4}+\frac{\ln(\mathrm{e}^{2ix}-i)}{4}$	58

input
```
int(1/(cos(x)+cos(3*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*tan(x)-1/4/(tan(x)-1)+1/4*ln(tan(x)-1)-1/4/(tan(x)+1)-1/4*ln(tan(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(26) = 52$.

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.09

$$\int \frac{1}{(\cos(x)+\cos(3x))^2}\,dx = -\frac{\left(2\,\cos\left(x\right)^3-\cos\left(x\right)\right)\log\left(2\,\cos\left(x\right)\sin\left(x\right)+1\right)-\left(2\,\cos\left(x\right)^3-\cos\left(x\right)\right)\log\left(-2\,\cos\left(x\right)\sin\left(x\right)+1\right)-}{8\left(2\,\cos\left(x\right)^3-\cos\left(x\right)\right)}$$

input
```
integrate(1/(cos(x)+cos(3*x))^2,x, algorithm="fricas")
```

output
```
-1/8*((2*cos(x)^3 - cos(x))*log(2*cos(x)*sin(x) + 1) - (2*cos(x)^3 - cos(x
))*log(-2*cos(x)*sin(x) + 1) - 2*(4*cos(x)^2 - 1)*sin(x))/(2*cos(x)^3 - co
s(x))
```

Sympy [F]

$$\int \frac{1}{(\cos(x)+\cos(3x))^2}\,dx = \int \frac{1}{\left(\cos\left(x\right)+\cos\left(3x\right)\right)^2}\,dx$$

input
```
integrate(1/(cos(x)+cos(3*x))**2,x)
```

output
```
Integral((cos(x) + cos(3*x))**(-2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 655 vs. $2(26) = 52$.

Time = 0.13 (sec) , antiderivative size = 655, normalized size of antiderivative = 19.26

$$\int \frac{1}{(\cos(x)+\cos(3x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cos(x)+cos(3*x))^2,x, algorithm="maxima")
```

output
```
-1/8*(4*(sin(4*x) + sin(2*x))*cos(6*x) - (2*(cos(4*x) + cos(2*x) + 1)*cos(
6*x) + cos(6*x)^2 + 2*(cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 +
2*(sin(4*x) + sin(2*x))*sin(6*x) + sin(6*x)^2 + sin(4*x)^2 + 2*sin(4*x)*si
n(2*x) + sin(2*x)^2 + 2*cos(2*x) + 1)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt
(2)*cos(x) + 2*sqrt(2)*sin(x) + 2) + (2*(cos(4*x) + cos(2*x) + 1)*cos(6*x)
 + cos(6*x)^2 + 2*(cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 + 2*(s
in(4*x) + sin(2*x))*sin(6*x) + sin(6*x)^2 + sin(4*x)^2 + 2*sin(4*x)*sin(2*
x) + sin(2*x)^2 + 2*cos(2*x) + 1)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*
cos(x) - 2*sqrt(2)*sin(x) + 2) + (2*(cos(4*x) + cos(2*x) + 1)*cos(6*x) + c
os(6*x)^2 + 2*(cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 + 2*(sin(4
*x) + sin(2*x))*sin(6*x) + sin(6*x)^2 + sin(4*x)^2 + 2*sin(4*x)*sin(2*x) +
 sin(2*x)^2 + 2*cos(2*x) + 1)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(
x) + 2*sqrt(2)*sin(x) + 2) - (2*(cos(4*x) + cos(2*x) + 1)*cos(6*x) + cos(6
*x)^2 + 2*(cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 + 2*(sin(4*x)
+ sin(2*x))*sin(6*x) + sin(6*x)^2 + sin(4*x)^2 + 2*sin(4*x)*sin(2*x) + sin
(2*x)^2 + 2*cos(2*x) + 1)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) -
 2*sqrt(2)*sin(x) + 2) - 4*(cos(4*x) + cos(2*x) + 2)*sin(6*x) - 4*sin(4*x)
 - 4*sin(2*x))/(2*(cos(4*x) + cos(2*x) + 1)*cos(6*x) + cos(6*x)^2 + 2*(cos
(2*x) + 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 + 2*(sin(4*x) + sin(2*x))*si
n(6*x) + sin(6*x)^2 + sin(4*x)^2 + 2*sin(4*x)*sin(2*x) + sin(2*x)^2 + 2...
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{1}{(\cos(x)+\cos(3x))^2}\,dx = -\frac{\tan(x)}{2\left(\tan(x)^2-1\right)} - \frac{1}{4}\log\left(|\tan(x)+1|\right) + \frac{1}{4}\log\left(|\tan(x)-1|\right) + \frac{1}{4}\tan(x)$$

input
```
integrate(1/(cos(x)+cos(3*x))^2,x, algorithm="giac")
```

output
```
-1/2*tan(x)/(tan(x)^2 - 1) - 1/4*log(abs(tan(x) + 1)) + 1/4*log(abs(tan(x)
 - 1)) + 1/4*tan(x)
```

Mupad [B] (verification not implemented)

Time = 20.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{(\cos(x)+\cos(3x))^2}\,dx = \frac{\sin(3\,x)}{2\,\left(\cos(3\,x)+\cos(x)\right)} - \frac{\mathrm{atanh}\left(\frac{\sin(x)}{\cos(x)}\right)}{2}$$

input
```
int(1/(cos(3*x) + cos(x))^2,x)
```

output
```
sin(3*x)/(2*(cos(3*x) + cos(x))) - atanh(sin(x)/cos(x))/2
```

Reduce [F]

$$\int \frac{1}{(\cos(x)+\cos(3x))^2}\,dx = \int \frac{1}{\cos(3x)^2+2\cos(3x)\cos(x)+\cos(x)^2}dx$$

input
```
int(1/(cos(x)+cos(3*x))^2,x)
```

output
```
int(1/(cos(3*x)**2 + 2*cos(3*x)*cos(x) + cos(x)**2),x)
```

3.41 $\int \frac{1}{(\cos(x)+\cos(3x))^4} dx$

Optimal result

Integrand size = 9, antiderivative size = 74

$$\int \frac{1}{(\cos(x)+\cos(3x))^4} dx = -\frac{5}{8}\text{arctanh}(2\cos(x)\sin(x)) + \frac{9\tan(x)}{16} + \frac{\tan^3(x)}{48} + \frac{\tan(x)}{3(1-\tan^2(x))^3} - \frac{5\tan(x)}{6(1-\tan^2(x))^2} + \frac{5\tan(x)}{4(1-\tan^2(x))}$$

output
```
-5/8*arctanh(2*cos(x)*sin(x))+9/16*tan(x)+1/48*tan(x)^3+1/3*tan(x)/(1-tan(
x)^2)^3-5/6*tan(x)/(1-tan(x)^2)^2+5*tan(x)/(4-4*tan(x)^2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \frac{1}{(\cos(x)+\cos(3x))^4} dx = \frac{1}{96}\left(60\log(\cos(x)-\sin(x)) - 60\log(\cos(x)+\sin(x)) - \frac{5}{(\cos(x)-\sin(x))^2} + 2(25\cos(x)+10\cos(3x)+11\cos(5x))\sec^3(2x)\sin(x) + \frac{5}{(\cos(x)+\sin(x))^2} + 2(26+\sec^2(x))\tan(x)\right)$$

input
```
Integrate[(Cos[x] + Cos[3*x])^(-4),x]
```

output
```
(60*Log[Cos[x] - Sin[x]] - 60*Log[Cos[x] + Sin[x]] - 5/(Cos[x] - Sin[x])^2
 + 2*(25*Cos[x] + 10*Cos[3*x] + 11*Cos[5*x])*Sec[2*x]^3*Sin[x] + 5/(Cos[x]
 + Sin[x])^2 + 2*(26 + Sec[x]^2)*Tan[x])/96
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4823, 27, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos(x)+\cos(3x))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\cos(x)+\cos(3x))^4}dx$$

$$\Big\downarrow \text{4823}$$

$$\int \frac{\left(\tan^2(x)+1\right)^5}{16\left(1-\tan^2(x)\right)^4}d\tan(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int \frac{\left(\tan^2(x)+1\right)^5}{\left(1-\tan^2(x)\right)^4}d\tan(x)$$

$$\Big\downarrow \text{300}$$

$$\frac{1}{16}\int \left(\tan^2(x)-\frac{8\left(-5\tan^6(x)+5\tan^4(x)-5\tan^2(x)+1\right)}{\left(1-\tan^2(x)\right)^4}+9\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{16}\left(-20\text{arctanh}(\tan(x))+\frac{\tan^3(x)}{3}+\frac{20\tan(x)}{1-\tan^2(x)}-\frac{40\tan(x)}{3\left(1-\tan^2(x)\right)^2}+\frac{16\tan(x)}{3\left(1-\tan^2(x)\right)^3}+9\tan(x)\right)$$

input
```
Int[(Cos[x] + Cos[3*x])^(-4),x]
```

output
```
(-20*ArcTanh[Tan[x]] + 9*Tan[x] + Tan[x]^3/3 + (16*Tan[x])/(3*(1 - Tan[x]^
2)^3) - (40*Tan[x])/(3*(1 - Tan[x]^2)^2) + (20*Tan[x])/(1 - Tan[x]^2))/16
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4823
```
Int[(cos[(m_.)*((c_.) + (d_.)*(x_))]*(a_.) + cos[(n_.)*((c_.) + (d_.)*(x_))
]*(b_.))^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Cos[
m*ArcTan[x]] + b*Cos[n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[c + d*x]], x] /
; FreeQ[{a, b, c, d}, x] && ILtQ[p/2, 0] && IntegerQ[(m - 1)/2] && IntegerQ
[(n - 1)/2]
```

Maple [A] (verified)

Time = 13.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

method	result
default	$\frac{\tan(x)^3}{48}+\frac{9\tan(x)}{16}-\frac{1}{24(\tan(x)-1)^3}-\frac{3}{16(\tan(x)-1)^2}-\frac{5}{8(\tan(x)-1)}+\frac{5\ln(\tan(x)-1)}{8}-\frac{1}{24(\tan(x)+1)^3}+\frac{3}{16(\tan(x}$
risch	$\frac{i(15\,e^{16ix}+45\,e^{14ix}+85\,e^{12ix}+135\,e^{10ix}+153\,e^{8ix}+155\,e^{6ix}+99\,e^{4ix}+57\,e^{2ix}+24)}{12(e^{6ix}+e^{4ix}+e^{2ix}+1)^3}+\frac{5\ln(e^{2ix}-i)}{8}-\frac{5\ln(e^{2ix}+i)}{8}$

input
```
int(1/(cos(x)+cos(3*x))^4,x,method=_RETURNVERBOSE)
```

output
```
1/48*tan(x)^3+9/16*tan(x)-1/24/(tan(x)-1)^3-3/16/(tan(x)-1)^2-5/8/(tan(x)-
1)+5/8*ln(tan(x)-1)-1/24/(tan(x)+1)^3+3/16/(tan(x)+1)^2-5/8/(tan(x)+1)-5/8
*ln(tan(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(56) = 112.

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.78

$$\int \frac{1}{(\cos(x)+\cos(3x))^4}\,dx = -\frac{15\left(8\cos(x)^9-12\cos(x)^7+6\cos(x)^5-\cos(x)^3\right)\log(2\cos(x)\sin(x)+1)-15\left(8\cos(x)^9-12\,\mathrm{c}\right.}{48\left(8\cos(x\right.}$$

input
```
integrate(1/(cos(x)+cos(3*x))^4,x, algorithm="fricas")
```

output
```
-1/48*(15*(8*cos(x)^9 - 12*cos(x)^7 + 6*cos(x)^5 - cos(x)^3)*log(2*cos(x)*
sin(x) + 1) - 15*(8*cos(x)^9 - 12*cos(x)^7 + 6*cos(x)^5 - cos(x)^3)*log(-2
*cos(x)*sin(x) + 1) - (384*cos(x)^8 - 504*cos(x)^6 + 204*cos(x)^4 - 20*cos
(x)^2 - 1)*sin(x))/(8*cos(x)^9 - 12*cos(x)^7 + 6*cos(x)^5 - cos(x)^3)
```

Sympy [F]

$$\int \frac{1}{(\cos(x)+\cos(3x))^4}\,dx = \int \frac{1}{\left(\cos\left(x\right)+\cos\left(3x\right)\right)^4}\,dx$$

input
```
integrate(1/(cos(x)+cos(3*x))**4,x)
```

output
```
Integral((cos(x) + cos(3*x))**(-4), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4157 vs. $2(56) = 112$.

Time $= 0.25$ (sec) , antiderivative size $= 4157$, normalized size of antiderivative $= 56.18$

$$\int \frac{1}{(\cos(x)+\cos(3x))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cos(x)+cos(3*x))^4,x, algorithm="maxima")
```

output
```
-1/48*(4*(15*sin(16*x) + 45*sin(14*x) + 85*sin(12*x) + 135*sin(10*x) + 153
*sin(8*x) + 155*sin(6*x) + 99*sin(4*x) + 57*sin(2*x))*cos(18*x) + 12*(15*s
in(14*x) + 35*sin(12*x) + 75*sin(10*x) + 93*sin(8*x) + 105*sin(6*x) + 69*s
in(4*x) + 42*sin(2*x))*cos(16*x) + 12*(20*sin(12*x) + 90*sin(10*x) + 126*s
in(8*x) + 160*sin(6*x) + 108*sin(4*x) + 69*sin(2*x))*cos(14*x) + 20*(66*si
n(10*x) + 102*sin(8*x) + 140*sin(6*x) + 96*sin(4*x) + 63*sin(2*x))*cos(12*
x) + 12*(72*sin(8*x) + 170*sin(6*x) + 126*sin(4*x) + 93*sin(2*x))*cos(10*x
) + 60*(22*sin(6*x) + 18*sin(4*x) + 15*sin(2*x))*cos(8*x) + 60*(4*sin(4*x)
 + 7*sin(2*x))*cos(6*x) - 15*(2*(3*cos(16*x) + 6*cos(14*x) + 10*cos(12*x)
+ 12*cos(10*x) + 12*cos(8*x) + 10*cos(6*x) + 6*cos(4*x) + 3*cos(2*x) + 1)*
cos(18*x) + cos(18*x)^2 + 6*(6*cos(14*x) + 10*cos(12*x) + 12*cos(10*x) + 1
2*cos(8*x) + 10*cos(6*x) + 6*cos(4*x) + 3*cos(2*x) + 1)*cos(16*x) + 9*cos(
16*x)^2 + 12*(10*cos(12*x) + 12*cos(10*x) + 12*cos(8*x) + 10*cos(6*x) + 6*
cos(4*x) + 3*cos(2*x) + 1)*cos(14*x) + 36*cos(14*x)^2 + 20*(12*cos(10*x) +
 12*cos(8*x) + 10*cos(6*x) + 6*cos(4*x) + 3*cos(2*x) + 1)*cos(12*x) + 100*
cos(12*x)^2 + 24*(12*cos(8*x) + 10*cos(6*x) + 6*cos(4*x) + 3*cos(2*x) + 1)
*cos(10*x) + 144*cos(10*x)^2 + 24*(10*cos(6*x) + 6*cos(4*x) + 3*cos(2*x) +
 1)*cos(8*x) + 144*cos(8*x)^2 + 20*(6*cos(4*x) + 3*cos(2*x) + 1)*cos(6*x)
+ 100*cos(6*x)^2 + 12*(3*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 9*cos(2*
x)^2 + 2*(3*sin(16*x) + 6*sin(14*x) + 10*sin(12*x) + 12*sin(10*x) + 12*...
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.73

$$\int \frac{1}{(\cos(x)+\cos(3x))^4}\,dx = \frac{1}{48}\tan(x)^3 - \frac{15\tan(x)^5 - 20\tan(x)^3 + 9\tan(x)}{12\left(\tan(x)^2-1\right)^3} - \frac{5}{8}\log(|\tan(x)+1|) + \frac{5}{8}\log(|\tan(x)-1|) + \frac{9}{16}\tan(x)$$

input
```
integrate(1/(cos(x)+cos(3*x))^4,x, algorithm="giac")
```

output
```
1/48*tan(x)^3 - 1/12*(15*tan(x)^5 - 20*tan(x)^3 + 9*tan(x))/(tan(x)^2 - 1)
^3 - 5/8*log(abs(tan(x) + 1)) + 5/8*log(abs(tan(x) - 1)) + 9/16*tan(x)
```

Mupad [B] (verification not implemented)

Time = 21.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\cos(x)+\cos(3x))^4} \, dx = \frac{-384 \sin(x) \cos(x)^8 + 504 \sin(x) \cos(x)^6 - 204 \sin(x) \cos(x)^4 + 20 \sin(x) \cos(x)^2 + \sin(x)}{-384 \cos(x)^9 + 576 \cos(x)^7 - 288 \cos(x)^5 + 48 \cos(x)^3} - \frac{5 \operatorname{atanh}\left(\frac{\sin(x)}{\cos(x)}\right)}{4}$$

input
```
int(1/(cos(3*x) + cos(x))^4,x)
```

output
```
(sin(x) + 20*cos(x)^2*sin(x) - 204*cos(x)^4*sin(x) + 504*cos(x)^6*sin(x) -
 384*cos(x)^8*sin(x))/(48*cos(x)^3 - 288*cos(x)^5 + 576*cos(x)^7 - 384*cos
(x)^9) - (5*atanh(sin(x)/cos(x)))/4
```

Reduce [F]

$$\int \frac{1}{(\cos(x)+\cos(3x))^4} \, dx = \int \frac{1}{\cos(3x)^4 + 4\cos(3x)^3 \cos(x) + 6\cos(3x)^2 \cos(x)^2 + 4\cos(3x) \cos(x)^3 + \cos(x)^4} \, dx$$

input
```
int(1/(cos(x)+cos(3*x))^4,x)
```

output
```
int(1/(cos(3*x)**4 + 4*cos(3*x)**3*cos(x) + 6*cos(3*x)**2*cos(x)**2 + 4*co
s(3*x)*cos(x)**3 + cos(x)**4),x)
```

3.42 $\int \frac{1}{(\cos(x)+\cos(3x))^6} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 114

$$\int \frac{1}{(\cos(x)+\cos(3x))^6} \, dx = -\frac{217}{128}\text{arctanh}(2\cos(x)\sin(x)) + \frac{97\tan(x)}{64} + \frac{7\tan^3(x)}{96} + \frac{\tan^5(x)}{320} + \frac{2\tan(x)}{5\left(1-\tan^2(x)\right)^5} - \frac{31\tan(x)}{20\left(1-\tan^2(x)\right)^4} + \frac{343\tan(x)}{120\left(1-\tan^2(x)\right)^3} - \frac{329\tan(x)}{96\left(1-\tan^2(x)\right)^2} + \frac{231\tan(x)}{64\left(1-\tan^2(x)\right)}$$

output
```
-217/128*arctanh(2*cos(x)*sin(x))+97/64*tan(x)+7/96*tan(x)^3+1/320*tan(x)^
5+2/5*tan(x)/(1-tan(x)^2)^5-31/20*tan(x)/(1-tan(x)^2)^4+343/120*tan(x)/(1-
tan(x)^2)^3-329/96*tan(x)/(1-tan(x)^2)^2+231*tan(x)/(64-64*tan(x)^2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.07

$$\int \frac{1}{(\cos(x)+\cos(3x))^6}\,dx$$
$$= \frac{6510\log(\cos(x)-\sin(x)) - 6510\log(\cos(x)+\sin(x)) - \frac{39}{(\cos(x)-\sin(x))^4} - \frac{596}{(\cos(x)-\sin(x))^2} + \frac{1}{2}(7847\cos(x) + \cdots}{\cdots}$$

input
```
Integrate[(Cos[x] + Cos[3*x])^(-6),x]
```

output
```
(6510*Log[Cos[x] - Sin[x]] - 6510*Log[Cos[x] + Sin[x]] - 39/(Cos[x] - Sin[
x])^4 - 596/(Cos[x] - Sin[x])^2 + ((7847*Cos[x] + 4767*Cos[3*x] + 5093*Cos
[5*x] + 1033*Cos[7*x] + 1172*Cos[9*x])*Sec[2*x]^5*Sin[x])/2 + 39/(Cos[x] +
 Sin[x])^4 + 596/(Cos[x] + Sin[x])^2 + 4*(1388 + 64*Sec[x]^2 + 3*Sec[x]^4)
*Tan[x])/3840
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4823, 27, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos(x)+\cos(3x))^6}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{(\cos(x)+\cos(3x))^6}dx$$
$$\Big\downarrow \text{ 4823}$$
$$\int \frac{(\tan^2(x)+1)^8}{64\,(1-\tan^2(x))^6}d\tan(x)$$
$$\Big\downarrow \text{ 27}$$

$$\frac{1}{64}\int \frac{\left(\tan^2(x)+1\right)^8}{\left(1-\tan^2(x)\right)^6}d\tan(x)$$

$$\Big\downarrow \text{300}$$

$$\frac{1}{64}\int \left(\tan^4(x)+14\tan^2(x)-\frac{32\left(-14\tan^{10}(x)+35\tan^8(x)-56\tan^6(x)+42\tan^4(x)-18\tan^2(x)+3\right)}{\left(1-\tan^2(x)\right)^6}+97\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{64}\left(-217\text{arctanh}(\tan(x))+\frac{\tan^5(x)}{5}+\frac{14\tan^3(x)}{3}+\frac{231\tan(x)}{1-\tan^2(x)}-\frac{658\tan(x)}{3\left(1-\tan^2(x)\right)^2}+\frac{2744\tan(x)}{15\left(1-\tan^2(x)\right)^3}-\frac{}{5\,(}\right.$$

input
```
Int[(Cos[x] + Cos[3*x])^(-6),x]
```

output
```
(-217*ArcTanh[Tan[x]] + 97*Tan[x] + (14*Tan[x]^3)/3 + Tan[x]^5/5 + (128*Ta
n[x])/(5*(1 - Tan[x]^2)^5) - (496*Tan[x])/(5*(1 - Tan[x]^2)^4) + (2744*Tan
[x])/(15*(1 - Tan[x]^2)^3) - (658*Tan[x])/(3*(1 - Tan[x]^2)^2) + (231*Tan[
x])/(1 - Tan[x]^2))/64
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4823
```
Int[(cos[(m_.)*((c_.) + (d_.)*(x_))]*(a_.) + cos[(n_.)*((c_.) + (d_.)*(x_))
]*(b_.))^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Cos[
m*ArcTan[x]] + b*Cos[n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[c + d*x]], x] /
; FreeQ[{a, b, c, d}, x] && ILtQ[p/2, 0] && IntegerQ[(m - 1)/2] && IntegerQ
[(n - 1)/2]
```

Maple [A] (verified)

Time = 165.96 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.98

method	result
default	$\frac{\tan(x)^5}{320}+\frac{7\tan(x)^3}{96}+\frac{97\tan(x)}{64}-\frac{1}{80(\tan(x)-1)^5}-\frac{5}{64(\tan(x)-1)^4}-\frac{53}{192(\tan(x)-1)^3}-\frac{23}{32(\tan(x)-1)^2}-\frac{231}{128(\tan(x$
risch	$\frac{i(3255\,\mathrm{e}^{28ix}+16275\,\mathrm{e}^{26ix}+47740\,\mathrm{e}^{24ix}+108500\,\mathrm{e}^{22ix}+195951\,\mathrm{e}^{20ix}+294035\,\mathrm{e}^{18ix}+378200\,\mathrm{e}^{16ix}+415400\,\mathrm{e}^{14ix}+397165\,\mathrm{e}^{12ix}+321}{960(\mathrm{e}^{6ix}+\mathrm{e}^{4ix}+\mathrm{e}^{2ix}+1)^5}$

input
```
int(1/(cos(x)+cos(3*x))^6,x,method=_RETURNVERBOSE)
```

output
```
1/320*tan(x)^5+7/96*tan(x)^3+97/64*tan(x)-1/80/(tan(x)-1)^5-5/64/(tan(x)-1
)^4-53/192/(tan(x)-1)^3-23/32/(tan(x)-1)^2-231/128/(tan(x)-1)+217/128*ln(t
an(x)-1)-1/80/(tan(x)+1)^5+5/64/(tan(x)+1)^4-53/192/(tan(x)+1)^3+23/32/(ta
n(x)+1)^2-231/128/(tan(x)+1)-217/128*ln(tan(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(86) = 172.

Time = 0.11 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.63

$$\int \frac{1}{(\cos(x)+\cos(3x))^6}\,dx =$$
$$-\frac{3255\left(32\cos(x)^{15}-80\cos(x)^{13}+80\cos(x)^{11}-40\cos(x)^{9}+10\cos(x)^{7}-\cos(x)^{5}\right)\log(2\cos(x)\mathrm{s}}{}$$

input
```
integrate(1/(cos(x)+cos(3*x))^6,x, algorithm="fricas")
```

output
```
-1/3840*(3255*(32*cos(x)^15 - 80*cos(x)^13 + 80*cos(x)^11 - 40*cos(x)^9 +
10*cos(x)^7 - cos(x)^5)*log(2*cos(x)*sin(x) + 1) - 3255*(32*cos(x)^15 - 80
*cos(x)^13 + 80*cos(x)^11 - 40*cos(x)^9 + 10*cos(x)^7 - cos(x)^5)*log(-2*c
os(x)*sin(x) + 1) - 4*(81920*cos(x)^14 - 189880*cos(x)^12 + 172180*cos(x)^
10 - 75070*cos(x)^8 + 15025*cos(x)^6 - 868*cos(x)^4 - 34*cos(x)^2 - 3)*sin
(x))/(32*cos(x)^15 - 80*cos(x)^13 + 80*cos(x)^11 - 40*cos(x)^9 + 10*cos(x)
^7 - cos(x)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(\cos(x) + \cos(3x))^6} dx = \text{Timed out}$$

input
```
integrate(1/(cos(x)+cos(3*x))**6,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10127 vs. 2(86) = 172.

Time = 0.87 (sec) , antiderivative size = 10127, normalized size of antiderivative = 88.83

$$\int \frac{1}{(\cos(x) + \cos(3x))^6} dx = \text{Too large to display}$$

input
```
integrate(1/(cos(x)+cos(3*x))^6,x, algorithm="maxima")
```

output
```
-1/3840*(4*(3255*sin(28*x) + 16275*sin(26*x) + 47740*sin(24*x) + 108500*si
n(22*x) + 195951*sin(20*x) + 294035*sin(18*x) + 378200*sin(16*x) + 415400*
sin(14*x) + 397165*sin(12*x) + 321169*sin(10*x) + 224300*sin(8*x) + 131460
*sin(6*x) + 60525*sin(4*x) + 22345*sin(2*x))*cos(30*x) + 20*(6510*sin(26*x
) + 24955*sin(24*x) + 66185*sin(22*x) + 130200*sin(20*x) + 206150*sin(18*x
) + 277295*sin(16*x) + 314495*sin(14*x) + 309280*sin(12*x) + 255418*sin(10
*x) + 181985*sin(8*x) + 108675*sin(6*x) + 50760*sin(4*x) + 19090*sin(2*x))
*cos(28*x) + 60*(9765*sin(24*x) + 37975*sin(22*x) + 86366*sin(20*x) + 1475
60*sin(18*x) + 210025*sin(16*x) + 247225*sin(14*x) + 250690*sin(12*x) + 21
1584*sin(10*x) + 153775*sin(8*x) + 93485*sin(6*x) + 44250*sin(4*x) + 16920
*sin(2*x))*cos(26*x) + 140*(19840*sin(22*x) + 58187*sin(20*x) + 109895*sin
(18*x) + 166780*sin(16*x) + 203980*sin(14*x) + 213025*sin(12*x) + 183405*s
in(10*x) + 135640*sin(8*x) + 83720*sin(6*x) + 40065*sin(4*x) + 15525*sin(2
*x))*cos(24*x) + 20*(355663*sin(20*x) + 892955*sin(18*x) + 1553100*sin(16*
x) + 2036700*sin(14*x) + 2233645*sin(12*x) + 1983497*sin(10*x) + 1505400*s
in(8*x) + 949480*sin(6*x) + 461325*sin(4*x) + 181985*sin(2*x))*cos(22*x) +
 4*(3244150*sin(18*x) + 7825795*sin(16*x) + 11582995*sin(14*x) + 13660280*
sin(12*x) + 12647018*sin(10*x) + 9917485*sin(8*x) + 6419175*sin(6*x) + 317
3760*sin(4*x) + 1277090*sin(2*x))*cos(20*x) + 20*(1096315*sin(16*x) + 2100
715*sin(14*x) + 2784510*sin(12*x) + 2732056*sin(10*x) + 2233645*sin(8*x...
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.63

$$\int \frac{1}{(\cos(x)+\cos(3x))^6}\,dx$$
$$= \frac{1}{320}\tan(x)^5 + \frac{7}{96}\tan(x)^3 - \frac{3465\tan(x)^9 - 10570\tan(x)^7 + 13664\tan(x)^5 - 7990\tan(x)^3 + 1815\tan(x)}{960\left(\tan(x)^2-1\right)^5} - \frac{217}{128}\log(|\tan(x)+1|) + \frac{217}{128}\log(|\tan(x)-1|) + \frac{97}{64}\tan(x)$$

input
```
integrate(1/(cos(x)+cos(3*x))^6,x, algorithm="giac")
```

output
```
1/320*tan(x)^5 + 7/96*tan(x)^3 - 1/960*(3465*tan(x)^9 - 10570*tan(x)^7 + 1
3664*tan(x)^5 - 7990*tan(x)^3 + 1815*tan(x))/(tan(x)^2 - 1)^5 - 217/128*lo
g(abs(tan(x) + 1)) + 217/128*log(abs(tan(x) - 1)) + 97/64*tan(x)
```

Mupad [B] (verification not implemented)

Time = 23.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.98

$$\int \frac{1}{(\cos(x)+\cos(3x))^6}\,dx = \frac{-81920\sin(x)\cos(x)^{14}+189880\sin(x)\cos(x)^{12}-172180\sin(x)\cos(x)^{10}+75070\sin(x)\cos(x)^{8}-}{-30720\cos(x)^{15}+76800\cos(x)^{13}-76800\cos(x)^{11}+38400\,\mathrm{c}} - \frac{217\,\mathrm{atanh}\left(\frac{\sin(x)}{\cos(x)}\right)}{64}$$

input
```
int(1/(cos(3*x) + cos(x))^6,x)
```

output
```
(3*sin(x) + 34*cos(x)^2*sin(x) + 868*cos(x)^4*sin(x) - 15025*cos(x)^6*sin(
x) + 75070*cos(x)^8*sin(x) - 172180*cos(x)^10*sin(x) + 189880*cos(x)^12*si
n(x) - 81920*cos(x)^14*sin(x))/(960*cos(x)^5 - 9600*cos(x)^7 + 38400*cos(x
)^9 - 76800*cos(x)^11 + 76800*cos(x)^13 - 30720*cos(x)^15) - (217*atanh(si
n(x)/cos(x)))/64
```

Reduce [F]

$$\int \frac{1}{(\cos(x)+\cos(3x))^6}\,dx = \int \frac{1}{\cos(3x)^6+6\cos(3x)^5\cos(x)+15\cos(3x)^4\cos(x)^2+20\cos(3x)^3\cos(x)^3+15\cos(3x)^2\cos(x)^4+6}$$

input
```
int(1/(cos(x)+cos(3*x))^6,x)
```

output
```
int(1/(cos(3*x)**6 + 6*cos(3*x)**5*cos(x) + 15*cos(3*x)**4*cos(x)**2 + 20*
cos(3*x)**3*cos(x)**3 + 15*cos(3*x)**2*cos(x)**4 + 6*cos(3*x)*cos(x)**5 +
cos(x)**6),x)
```

3.43 $\int \frac{1}{\cos(x)+\cos(5x)} dx$

Optimal result

Integrand size = 9, antiderivative size = 33

$$\int \frac{1}{\cos(x)+\cos(5x)}\,dx = \frac{1}{6}\text{arctanh}(\sin(x)) + \frac{2}{3}\text{arctanh}(2\sin(x)) - \frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}}$$

output
```
1/6*arctanh(sin(x))+2/3*arctanh(2*sin(x))-1/2*arctanh(sin(x)*2^(1/2))*2^(1
/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 93 vs. $2(33) = 66$.

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.82

$$\begin{aligned}\int \frac{1}{\cos(x)+\cos(5x)}\,dx = \frac{1}{12}\Bigg(&-2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+2\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\\&-4\log(1-2\sin(x))+3\sqrt{2}\log\left(\sqrt{2}-2\sin(x)\right)\\&+4\log(1+2\sin(x))-3\sqrt{2}\log\left(\sqrt{2}+2\sin(x)\right)\Bigg)\end{aligned}$$

input
```
Integrate[(Cos[x] + Cos[5*x])^(-1),x]
```

output
```
(-2*Log[Cos[x/2] - Sin[x/2]] + 2*Log[Cos[x/2] + Sin[x/2]] - 4*Log[1 - 2*Si
n[x]] + 3*Sqrt[2]*Log[Sqrt[2] - 2*Sin[x]] + 4*Log[1 + 2*Sin[x]] - 3*Sqrt[2
]*Log[Sqrt[2] + 2*Sin[x]])/12
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4825, 27, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos(x)+\cos(5x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(x)+\cos(5x)}dx$$

$$\Big\downarrow \text{4825}$$

$$\int \frac{1}{2\left(1-\sin^2(x)\right)\left(8\sin^4(x)-6\sin^2(x)+1\right)}d\sin(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{1}{\left(1-\sin^2(x)\right)\left(8\sin^4(x)-6\sin^2(x)+1\right)}d\sin(x)$$

$$\Big\downarrow \text{1484}$$

$$\frac{1}{2}\int \left(\frac{2}{2\sin^2(x)-1}-\frac{8}{3\left(4\sin^2(x)-1\right)}-\frac{1}{3\left(\sin^2(x)-1\right)}\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{3}\text{arctanh}(\sin(x))+\frac{4}{3}\text{arctanh}(2\sin(x))-\sqrt{2}\text{arctanh}\left(\sqrt{2}\sin(x)\right)\right)$$

input
```
Int[(Cos[x] + Cos[5*x])^(-1),x]
```

output
```
(ArcTanh[Sin[x]]/3 + (4*ArcTanh[2*Sin[x]])/3 - Sqrt[2]*ArcTanh[Sqrt[2]*Sin
[x]])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1484
```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4825
```
Int[(cos[(m_.)*((c_.) + (d_.)*(x_))]*(a_.) + cos[(n_.)*((c_.) + (d_.)*(x_))
]*(b_.))^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Cos[
m*ArcSin[x]] + b*Cos[n*ArcSin[x]]]]^p/Sqrt[1 - x^2], x], x, Sin[c + d*x]],
x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[(m - 1)/2] &
& IntegerQ[(n - 1)/2]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

method	result
default	$\frac{\ln(1+\sin(x))}{12}-\frac{\operatorname{arctanh}\left(\sqrt{2}\,\sin(x)\right)\sqrt{2}}{2}+\frac{\ln(2\sin(x)+1)}{3}-\frac{\ln(\sin(x)-1)}{12}-\frac{\ln(2\sin(x)-1)}{3}$
risch	$\frac{\ln(\mathrm{e}^{ix}+i)}{6}-\frac{\ln(\mathrm{e}^{ix}-i)}{6}-\frac{\ln(-i\mathrm{e}^{ix}+\mathrm{e}^{2ix}-1)}{3}+\frac{\ln(i\mathrm{e}^{ix}+\mathrm{e}^{2ix}-1)}{3}+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-i\sqrt{2}\,\mathrm{e}^{ix}-1\right)}{4}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+i\sqrt{2}\,\mathrm{e}^{ix}-1\right)}{4}$

input
```
int(1/(cos(x)+cos(5*x)),x,method=_RETURNVERBOSE)
```

output
```
1/12*ln(1+sin(x))-1/2*arctanh(2^(1/2)*sin(x))*2^(1/2)+1/3*ln(2*sin(x)+1)-1
/12*ln(sin(x)-1)-1/3*ln(2*sin(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(25) = 50$.

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.06

$$\int \frac{1}{\cos(x)+\cos(5x)}\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2+2\sqrt{2}\sin(x)-3}{2\cos(x)^2-1}\right) + \frac{1}{3}\log(2\sin(x)+1) + \frac{1}{12}\log(\sin(x)+1) - \frac{1}{12}\log(-\sin(x)+1) - \frac{1}{3}\log(-2\sin(x)+1)$$

input
```
integrate(1/(cos(x)+cos(5*x)),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) + 1
/3*log(2*sin(x) + 1) + 1/12*log(sin(x) + 1) - 1/12*log(-sin(x) + 1) - 1/3*
log(-2*sin(x) + 1)
```

Sympy [F]

$$\int \frac{1}{\cos(x)+\cos(5x)}\,dx = \int \frac{1}{\cos(x)+\cos(5x)}\,dx$$

input
```
integrate(1/(cos(x)+cos(5*x)),x)
```

output
```
Integral(1/(cos(x) + cos(5*x)), x)
```

Maxima [F]

$$\int \frac{1}{\cos(x) + \cos(5x)}\,dx = \int \frac{1}{\cos(5\,x) + \cos(x)}\,dx$$

input
```
integrate(1/(cos(x)+cos(5*x)),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*si
n(x) + 2) + 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2
*sqrt(2)*sin(x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)
*cos(x) + 2*sqrt(2)*sin(x) + 2) + 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2
- 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) + integrate(-2/3*((cos(3*x) + c
os(x))*cos(4*x) - (cos(2*x) - 1)*cos(3*x) - cos(2*x)*cos(x) + (sin(3*x) +
sin(x))*sin(4*x) - sin(3*x)*sin(2*x) - sin(2*x)*sin(x) + cos(x))/(2*(cos(2
*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(
2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) + 1/12*log(cos(x)^2 + sin(x)^2 + 2
*sin(x) + 1) - 1/12*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(25) = 50$.

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.06

$$\int \frac{1}{\cos(x) + \cos(5x)}\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2} + 4\sin(x)\right|}{\left|2\sqrt{2} + 4\sin(x)\right|}\right) + \frac{1}{12}\log(\sin(x) + 1) - \frac{1}{12}\log(-\sin(x) + 1) + \frac{1}{3}\log(|2\sin(x) + 1|) - \frac{1}{3}\log(|2\sin(x) - 1|)$$

input
```
integrate(1/(cos(x)+cos(5*x)),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) + 1/
12*log(sin(x) + 1) - 1/12*log(-sin(x) + 1) + 1/3*log(abs(2*sin(x) + 1)) -
1/3*log(abs(2*sin(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 20.94 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1}{\cos(x)+\cos(5x)}\,dx = \frac{\operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{3} + \frac{2\operatorname{atanh}(2\,\sin\left(x\right))}{3} - \frac{\sqrt{2}\operatorname{atanh}\left(\sqrt{2}\,\sin\left(x\right)\right)}{2}$$

input
```
int(1/(cos(5*x) + cos(x)),x)
```

output
```
atanh(sin(x/2)/cos(x/2))/3 + (2*atanh(2*sin(x)))/3 - (2^(1/2)*atanh(2^(1/2
)*sin(x)))/2
```

Reduce [F]

$$\int \frac{1}{\cos(x)+\cos(5x)}\,dx = \int \frac{1}{\cos\left(5x\right)+\cos\left(x\right)}dx$$

input
```
int(1/(cos(x)+cos(5*x)),x)
```

output
```
int(1/(cos(5*x) + cos(x)),x)
```

3.44 $\int \frac{1}{(\cos(x)+\cos(5x))^3} dx$

Optimal result

Integrand size = 9, antiderivative size = 127

$$\begin{aligned}\int \frac{1}{(\cos(x)+\cos(5x))^3} dx = {} & \frac{7}{144}\operatorname{arctanh}(\sin(x)) + \frac{55}{9}\operatorname{arctanh}(2\sin(x)) \\ & - \frac{139\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{16\sqrt{2}} + \frac{1}{18(1-2\sin(x))^2} \\ & - \frac{35}{54(1-2\sin(x))} + \frac{1}{864(1-\sin(x))} - \frac{19}{16}\sec(2x)\sin(x) \\ & - \frac{1}{8}\sec^2(2x)\sin(x) - \frac{1}{864(1+\sin(x))} \\ & - \frac{1}{18(1+2\sin(x))^2} + \frac{35}{54(1+2\sin(x))}\end{aligned}$$

output
```
7/144*arctanh(sin(x))+55/9*arctanh(2*sin(x))-139/32*arctanh(sin(x)*2^(1/2)
)*2^(1/2)+1/18/(1-2*sin(x))^2-35/(54-108*sin(x))+1/(864-864*sin(x))-19/16*
sec(2*x)*sin(x)-1/8*sec(2*x)^2*sin(x)-1/(864+864*sin(x))-1/18/(1+2*sin(x))
^2+35/(54+108*sin(x))
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.57

$$\int \frac{1}{(\cos(x)+\cos(5x))^3}\,dx = \frac{-84\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+84\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-5280\log(1-2\sin(x))+3753\sqrt{2}\log\left(\sqrt{2}-2\right.}{}$$

input
```
Integrate[(Cos[x] + Cos[5*x])^(-3),x]
```

output
```
(-84*Log[Cos[x/2] - Sin[x/2]] + 84*Log[Cos[x/2] + Sin[x/2]] - 5280*Log[1 -
 2*Sin[x]] + 3753*Sqrt[2]*Log[Sqrt[2] - 2*Sin[x]] + 5280*Log[1 + 2*Sin[x]]
 - 3753*Sqrt[2]*Log[Sqrt[2] + 2*Sin[x]] + 2/(Cos[x/2] - Sin[x/2])^2 - 2/(C
os[x/2] + Sin[x/2])^2 + 96/(1 - 2*Sin[x])^2 - 1026/(Cos[x] - Sin[x]) - 216
*Sec[2*x]^2*Sin[x] + 1026/(Cos[x] + Sin[x]) + 1120/(-1 + 2*Sin[x]) - 96/(1
 + 2*Sin[x])^2 + 1120/(1 + 2*Sin[x]))/1728
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4825, 27, 1567, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos(x)+\cos(5x))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(\cos(x)+\cos(5x))^3}dx$$

$$\Big\downarrow \; 4825$$

$$\int \frac{1}{8\left(1-\sin^2(x)\right)^2\left(8\sin^4(x)-6\sin^2(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\int \frac{1}{\left(1-\sin^2(x)\right)^2\left(8\sin^4(x)-6\sin^2(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{1567}$$

$$\frac{1}{8}\int \left(\frac{60}{2\sin^2(x)-1}-\frac{880}{9\left(4\sin^2(x)-1\right)}+\frac{1}{108(\sin(x)-1)^2}+\frac{1}{108(\sin(x)+1)^2}-\frac{280}{27(2\sin(x)-1)^2}-\frac{28}{27(2\sin(}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\left(\frac{7}{18}\text{arctanh}(\sin(x))+\frac{440}{9}\text{arctanh}(2\sin(x))-34\sqrt{2}\text{arctanh}\left(\sqrt{2}\sin(x)\right)-\frac{3\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}-\frac{19\sin}{2\left(1-2\sin\right.}\right.$$

input
```
Int[(Cos[x] + Cos[5*x])^(-3),x]
```

output
```
((7*ArcTanh[Sin[x]])/18 + (440*ArcTanh[2*Sin[x]])/9 - (3*ArcTanh[Sqrt[2]*S
in[x]])/(2*Sqrt[2]) - 34*Sqrt[2]*ArcTanh[Sqrt[2]*Sin[x]] + 4/(9*(1 - 2*Sin
[x])^2) - 140/(27*(1 - 2*Sin[x])) + 1/(108*(1 - Sin[x])) - 1/(108*(1 + Sin
[x])) - 4/(9*(1 + 2*Sin[x])^2) + 140/(27*(1 + 2*Sin[x])) - Sin[x]/(1 - 2*S
in[x]^2)^2 - (19*Sin[x])/(2*(1 - 2*Sin[x]^2)))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1567
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4825
```
Int[(cos[(m_.)*((c_.) + (d_.)*(x_))]*(a_.) + cos[(n_.)*((c_.) + (d_.)*(x_))
]*(b_.))^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Cos[
m*ArcSin[x]] + b*Cos[n*ArcSin[x]]]]^p/Sqrt[1 - x^2], x], x, Sin[c + d*x]],
x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[(m - 1)/2] &
& IntegerQ[(n - 1)/2]
```

Maple [A] (verified)

Time = 3.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98

method	result
default	$-\frac{1}{864(\sin(x)-1)}-\frac{7\ln(\sin(x)-1)}{288}-\frac{1}{18(2\sin(x)+1)^2}+\frac{35}{54(2\sin(x)+1)}+\frac{55\ln(2\sin(x)+1)}{18}+\frac{1}{18(2\sin(x)-1)^2}+\frac{}{54(2\sin}$
risch	$\frac{i(119\,e^{19ix}-55\,e^{17ix}+95\,e^{15ix}+111\,e^{13ix}-166\,e^{11ix}+166\,e^{9ix}-111\,e^{7ix}-95\,e^{5ix}+55\,e^{3ix}-119\,e^{ix})}{48(e^{10ix}+e^{6ix}+e^{4ix}+1)^2}-\frac{7\ln(e^{ix}-i)}{144}+\frac{7\ln(e^{ix}+}{144}$

input
```
int(1/(cos(x)+cos(5*x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/864/(sin(x)-1)-7/288*ln(sin(x)-1)-1/18/(2*sin(x)+1)^2+35/54/(2*sin(x)+1
)+55/18*ln(2*sin(x)+1)+1/18/(2*sin(x)-1)^2+35/54/(2*sin(x)-1)-55/18*ln(2*s
in(x)-1)+8*(19/64*sin(x)^3-21/128*sin(x))/(2*sin(x)^2-1)^2-139/32*arctanh(
2^(1/2)*sin(x))*2^(1/2)-1/864/(1+sin(x))+7/288*ln(1+sin(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(99) = 198.

Time = 0.11 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.36

$$\int\frac{1}{(\cos(x)+\cos(5x))^3}\,dx$$
$$=\frac{1251\left(64\sqrt{2}\cos(x)^{10}-160\sqrt{2}\cos(x)^{8}+148\sqrt{2}\cos(x)^{6}-60\sqrt{2}\cos(x)^{4}+9\sqrt{2}\cos(x)^{2}\right)\log\left(-\frac{2\cos(x}{}\right.}{}$$

input
```
integrate(1/(cos(x)+cos(5*x))^3,x, algorithm="fricas")
```

output
```
1/576*(1251*(64*sqrt(2)*cos(x)^10 - 160*sqrt(2)*cos(x)^8 + 148*sqrt(2)*cos
(x)^6 - 60*sqrt(2)*cos(x)^4 + 9*sqrt(2)*cos(x)^2)*log(-(2*cos(x)^2 + 2*sqr
t(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) + 1760*(64*cos(x)^10 - 160*cos(x)^8 + 1
48*cos(x)^6 - 60*cos(x)^4 + 9*cos(x)^2)*log(2*sin(x) + 1) + 14*(64*cos(x)^
10 - 160*cos(x)^8 + 148*cos(x)^6 - 60*cos(x)^4 + 9*cos(x)^2)*log(sin(x) +
1) - 14*(64*cos(x)^10 - 160*cos(x)^8 + 148*cos(x)^6 - 60*cos(x)^4 + 9*cos(
x)^2)*log(-sin(x) + 1) - 1760*(64*cos(x)^10 - 160*cos(x)^8 + 148*cos(x)^6
- 60*cos(x)^4 + 9*cos(x)^2)*log(-2*sin(x) + 1) - 12*(3808*cos(x)^8 - 7104*
cos(x)^6 + 4310*cos(x)^4 - 847*cos(x)^2 - 1)*sin(x))/(64*cos(x)^10 - 160*c
os(x)^8 + 148*cos(x)^6 - 60*cos(x)^4 + 9*cos(x)^2)
```

Sympy [F]

$$\int \frac{1}{(\cos(x)+\cos(5x))^3}\,dx = \int \frac{1}{\left(\cos\left(x\right)+\cos\left(5x\right)\right)^3}\,dx$$

input
```
integrate(1/(cos(x)+cos(5*x))**3,x)
```

output
```
Integral((cos(x) + cos(5*x))**(-3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(x)+\cos(5x))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(cos(x)+cos(5*x))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.95

$$\int \frac{1}{(\cos(x)+\cos(5x))^3}\,dx$$
$$= \frac{139}{64}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin(x)\right|}{\left|2\sqrt{2}+4\sin(x)\right|}\right) - \frac{\sin(x)}{432\left(\sin(x)^2-1\right)}$$
$$+ \frac{34336\sin(x)^7 - 38912\sin(x)^5 + 13754\sin(x)^3 - 1495\sin(x)}{432\left(8\sin(x)^4 - 6\sin(x)^2 + 1\right)^2}$$
$$+ \frac{7}{288}\log(\sin(x)+1) - \frac{7}{288}\log(-\sin(x)+1)$$
$$+ \frac{55}{18}\log(|2\sin(x)+1|) - \frac{55}{18}\log(|2\sin(x)-1|)$$

input
```
integrate(1/(cos(x)+cos(5*x))^3,x, algorithm="giac")
```

output
```
139/64*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) -
 1/432*sin(x)/(sin(x)^2 - 1) + 1/432*(34336*sin(x)^7 - 38912*sin(x)^5 + 13
754*sin(x)^3 - 1495*sin(x))/(8*sin(x)^4 - 6*sin(x)^2 + 1)^2 + 7/288*log(si
n(x) + 1) - 7/288*log(-sin(x) + 1) + 55/18*log(abs(2*sin(x) + 1)) - 55/18*
log(abs(2*sin(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 21.05 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.72

$$\int \frac{1}{(\cos(x)+\cos(5x))^3}\,dx$$
$$= \frac{55\,\mathrm{atanh}\left(\frac{167854551804588605440\,\tan\left(\frac{x}{2}\right)}{531441\left(\frac{41963637951147151360\,\tan\left(\frac{x}{2}\right)^2}{531441}+\frac{41963637951147151360}{531441}\right)}\right)}{9}+\frac{7\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{72}$$
$$-\frac{139\sqrt{2}\,\mathrm{atanh}\left(\frac{126951855053165025424\sqrt{2}\,\tan\left(\frac{x}{2}\right)}{729\left(\frac{63475927526582512712\,\tan\left(\frac{x}{2}\right)^2}{729}+\frac{63475927526582512712}{729}\right)}\right)}{32}$$
$$-\frac{\frac{83\tan\left(\frac{x}{2}\right)^{19}}{12}-\frac{2639\tan\left(\frac{x}{2}\right)^{17}}{12}+\frac{13027\tan\left(\frac{x}{2}\right)^{15}}{6}-\frac{15065\tan\left(\frac{x}{2}\right)^{13}}{2}+\frac{16691\tan\left(\frac{x}{2}\right)^{11}}{3}+\frac{16691\tan\left(\frac{x}{2}\right)^{9}}{3}-150}{\tan\left(\frac{x}{2}\right)^{20}-42\tan\left(\frac{x}{2}\right)^{18}+653\tan\left(\frac{x}{2}\right)^{16}-4664\tan\left(\frac{x}{2}\right)^{14}+15730\tan\left(\frac{x}{2}\right)^{12}-23356\tan\left(\frac{x}{2}\right)^{10}+157}$$

input
```
int(1/(cos(5*x) + cos(x))^3,x)
```

output
```
(55*atanh((167854551804588605440*tan(x/2))/(531441*((419636379511471513600*tan(x/2)^2)/531441 + 41963637951147151360/531441))))/9 + (7*atanh(tan(x
/2)))/72 - (139*2^(1/2)*atanh((126951855053165025424*2^(1/2)*tan(x/2))/(72
9*((63475927526582512712*tan(x/2)^2)/729 + 63475927526582512712/729))))/32
 - ((83*tan(x/2))/12 - (2639*tan(x/2)^3)/12 + (13027*tan(x/2)^5)/6 - (1506
5*tan(x/2)^7)/2 + (16691*tan(x/2)^9)/3 + (16691*tan(x/2)^11)/3 - (15065*ta
n(x/2)^13)/2 + (13027*tan(x/2)^15)/6 - (2639*tan(x/2)^17)/12 + (83*tan(x/2
)^19)/12)/(653*tan(x/2)^4 - 42*tan(x/2)^2 - 4664*tan(x/2)^6 + 15730*tan(x/
2)^8 - 23356*tan(x/2)^10 + 15730*tan(x/2)^12 - 4664*tan(x/2)^14 + 653*tan(
x/2)^16 - 42*tan(x/2)^18 + tan(x/2)^20 + 1)
```

Reduce [F]

$$\int \frac{1}{(\cos(x)+\cos(5x))^3}\,dx$$
$$=\int \frac{1}{\cos(5x)^3+3\cos(5x)^2\cos(x)+3\cos(5x)\cos(x)^2+\cos(x)^3}dx$$

input
```
int(1/(cos(x)+cos(5*x))^3,x)
```

output
```
int(1/(cos(5*x)**3 + 3*cos(5*x)**2*cos(x) + 3*cos(5*x)*cos(x)**2 + cos(x)*
*3),x)
```

3.45 $\int \frac{1}{(\cos(x)+\cos(5x))^5}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 221

$$\begin{aligned}\int \frac{1}{(\cos(x)+\cos(5x))^5}\,dx &= \frac{3889\operatorname{arctanh}(\sin(x))}{186624}+\frac{332929\operatorname{arctanh}(2\sin(x))}{2916} \\ &\quad -\frac{82683\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{512\sqrt{2}} \\ &\quad +\frac{1}{108(1-2\sin(x))^4}-\frac{19}{162(1-2\sin(x))^3} \\ &\quad +\frac{749}{648(1-2\sin(x))^2}-\frac{71551}{5832(1-2\sin(x))} \\ &\quad +\frac{1}{124416(1-\sin(x))^2}+\frac{209}{373248(1-\sin(x))} \\ &\quad -\frac{11643}{512}\sec(2x)\sin(x)-\frac{681}{256}\sec^2(2x)\sin(x) \\ &\quad -\frac{21}{64}\sec^3(2x)\sin(x)-\frac{1}{32}\sec^4(2x)\sin(x) \\ &\quad -\frac{1}{124416(1+\sin(x))^2}-\frac{209}{373248(1+\sin(x))} \\ &\quad -\frac{1}{108(1+2\sin(x))^4}+\frac{19}{162(1+2\sin(x))^3} \\ &\quad -\frac{749}{648(1+2\sin(x))^2}+\frac{71551}{5832(1+2\sin(x))}\end{aligned}$$

output

```
3889/186624*arctanh(sin(x))+332929/2916*arctanh(2*sin(x))-82683/1024*arcta
nh(sin(x)*2^(1/2))*2^(1/2)+1/108/(1-2*sin(x))^4-19/162/(1-2*sin(x))^3+749/
648/(1-2*sin(x))^2-71551/(5832-11664*sin(x))+1/124416/(1-sin(x))^2+209/(37
3248-373248*sin(x))-11643/512*sec(2*x)*sin(x)-681/256*sec(2*x)^2*sin(x)-21
/64*sec(2*x)^3*sin(x)-1/32*sec(2*x)^4*sin(x)-1/124416/(1+sin(x))^2-209/(37
3248+373248*sin(x))-1/108/(1+2*sin(x))^4+19/162/(1+2*sin(x))^3-749/648/(1+
2*sin(x))^2+71551/(5832+11664*sin(x))
```

Mathematica [A] (warning: unable to verify)

Time = 6.09 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.72

$$\int \frac{1}{(\cos(x)+\cos(5x))^5}\,dx = -\frac{3889\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{186624} + \frac{3889\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}{186624} - \frac{332929\log(1-2\sin(x))}{5832} + \frac{82683\log\left(\sqrt{2}-2\sin(x)\right)}{1024\sqrt{2}} + \frac{332929\log(1+2\sin(x))}{5832} - \frac{82683\log\left(\sqrt{2}+2\sin(x)\right)}{1024\sqrt{2}} + \frac{1}{124416\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)^4} + \frac{209}{373248\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)^2} - \frac{1}{124416\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^4} - \frac{209}{373248\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2} - \frac{21}{256(\cos(x)-\sin(x))^3} - \frac{11643}{1024(\cos(x)-\sin(x))} - \frac{\sin(x)}{128(\cos(x)-\sin(x))^4} - \frac{643\sin(x)}{512(\cos(x)-\sin(x))^2} - \frac{\sin(x)}{128(\cos(x)+\sin(x))^4} + \frac{21}{256(\cos(x)+\sin(x))^3} - \frac{643\sin(x)}{512(\cos(x)+\sin(x))^2} + \frac{11643}{1024(\cos(x)+\sin(x))} + \frac{1}{108(-1+2\sin(x))^4} + \frac{19}{162(-1+2\sin(x))^3} + \frac{749}{648(-1+2\sin(x))^2} + \frac{71551}{5832(-1+2\sin(x))} - \frac{1}{108(1+2\sin(x))^4} + \frac{19}{162(1+2\sin(x))^3} - \frac{749}{648(1+2\sin(x))^2} + \frac{71551}{5832(1+2\sin(x))}$$

input
```
Integrate[(Cos[x] + Cos[5*x])^(-5),x]
```

output
```
(-3889*Log[Cos[x/2] - Sin[x/2]])/186624 + (3889*Log[Cos[x/2] + Sin[x/2]])/
186624 - (332929*Log[1 - 2*Sin[x]])/5832 + (82683*Log[Sqrt[2] - 2*Sin[x]])
/(1024*Sqrt[2]) + (332929*Log[1 + 2*Sin[x]])/5832 - (82683*Log[Sqrt[2] + 2
*Sin[x]])/(1024*Sqrt[2]) + 1/(124416*(Cos[x/2] - Sin[x/2])^4) + 209/(37324
8*(Cos[x/2] - Sin[x/2])^2) - 1/(124416*(Cos[x/2] + Sin[x/2])^4) - 209/(373
248*(Cos[x/2] + Sin[x/2])^2) - 21/(256*(Cos[x] - Sin[x])^3) - 11643/(1024*
(Cos[x] - Sin[x])) - Sin[x]/(128*(Cos[x] - Sin[x])^4) - (643*Sin[x])/(512*
(Cos[x] - Sin[x])^2) - Sin[x]/(128*(Cos[x] + Sin[x])^4) + 21/(256*(Cos[x]
+ Sin[x])^3) - (643*Sin[x])/(512*(Cos[x] + Sin[x])^2) + 11643/(1024*(Cos[x
] + Sin[x])) + 1/(108*(-1 + 2*Sin[x])^4) + 19/(162*(-1 + 2*Sin[x])^3) + 74
9/(648*(-1 + 2*Sin[x])^2) + 71551/(5832*(-1 + 2*Sin[x])) - 1/(108*(1 + 2*S
in[x])^4) + 19/(162*(1 + 2*Sin[x])^3) - 749/(648*(1 + 2*Sin[x])^2) + 71551
/(5832*(1 + 2*Sin[x]))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4825, 27, 1567, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos(x)+\cos(5x))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\cos(x)+\cos(5x))^5}dx$$

$$\Big\downarrow \text{4825}$$

$$\int \frac{1}{32\left(1-\sin^2(x)\right)^3\left(8\sin^4(x)-6\sin^2(x)+1\right)^5}d\sin(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\int \frac{1}{\left(1-\sin^2(x)\right)^3\left(8\sin^4(x)-6\sin^2(x)+1\right)^5}d\sin(x)$$

$$\Big\downarrow \text{1567}$$

$$\frac{1}{32}\int \left(\frac{4440}{2\sin^2(x)-1}-\frac{5326864}{729\left(4\sin^2(x)-1\right)}+\frac{209}{11664(\sin(x)-1)^2}+\frac{209}{11664(\sin(x)+1)^2}-\frac{572408}{729(2\sin(x)-1)^2}-\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{32}\left(\frac{3889\text{arctanh}(\sin(x))}{5832}+\frac{2663432}{729}\text{arctanh}(2\sin(x))-2574\sqrt{2}\text{arctanh}\left(\sqrt{2}\sin(x)\right)-\frac{315\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{16\sqrt{2}}\right.$$

input
```
Int[(Cos[x] + Cos[5*x])^(-5),x]
```

output
```
((3889*ArcTanh[Sin[x]])/5832 + (2663432*ArcTanh[2*Sin[x]])/729 - (315*ArcT
anh[Sqrt[2]*Sin[x]])/(16*Sqrt[2]) - 2574*Sqrt[2]*ArcTanh[Sqrt[2]*Sin[x]] +
 8/(27*(1 - 2*Sin[x])^4) - 304/(81*(1 - 2*Sin[x])^3) + 2996/(81*(1 - 2*Sin
[x])^2) - 286204/(729*(1 - 2*Sin[x])) + 1/(3888*(1 - Sin[x])^2) + 209/(116
64*(1 - Sin[x])) - 1/(3888*(1 + Sin[x])^2) - 209/(11664*(1 + Sin[x])) - 8/
(27*(1 + 2*Sin[x])^4) + 304/(81*(1 + 2*Sin[x])^3) - 2996/(81*(1 + 2*Sin[x]
)^2) + 286204/(729*(1 + 2*Sin[x])) - Sin[x]/(1 - 2*Sin[x]^2)^4 - (21*Sin[x
])/(2*(1 - 2*Sin[x]^2)^3) - (681*Sin[x])/(8*(1 - 2*Sin[x]^2)^2) - (11643*S
in[x])/(16*(1 - 2*Sin[x]^2)))/32
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1567
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4825
```
Int[(cos[(m_.)*((c_.) + (d_.)*(x_))]*(a_.) + cos[(n_.)*((c_.) + (d_.)*(x_))
]*(b_.))^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Cos[
m*ArcSin[x]] + b*Cos[n*ArcSin[x]]]]^p/Sqrt[1 - x^2], x], x, Sin[c + d*x]],
x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[(m - 1)/2] &
& IntegerQ[(n - 1)/2]
```

Maple [A] (verified)

Time = 50.31 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.87

method	result
default	$\frac{\frac{11643 \sin(x)^7}{64}-\frac{36291 \sin(x)^5}{128}+\frac{37821 \sin(x)^3}{256}-\frac{13189 \sin(x)}{512}}{\left(2\sin(x)^2-1\right)^4}-\frac{82683 \operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)\sqrt{2}}{1024}-\frac{1}{108(2\sin(x)+1)^4}+\frac{19}{162(2\sin(x)+\cdots}$
risch	$\frac{i(5881813\, e^{39ix}-2770929\, e^{37ix}+16666827\, e^{35ix}+11603277\, e^{33ix}+2153987\, e^{31ix}+49799073\, e^{29ix}-11124845\, e^{27ix}+29440353\, e^{25ix}\cdots}{}$

input
```
int(1/(cos(x)+cos(5*x))^5,x,method=_RETURNVERBOSE)
```

output
```
4*(11643/256*sin(x)^7-36291/512*sin(x)^5+37821/1024*sin(x)^3-13189/2048*si
n(x))/(2*sin(x)^2-1)^4-82683/1024*arctanh(2^(1/2)*sin(x))*2^(1/2)-1/108/(2
*sin(x)+1)^4+19/162/(2*sin(x)+1)^3-749/648/(2*sin(x)+1)^2+71551/5832/(2*si
n(x)+1)+332929/5832*ln(2*sin(x)+1)-1/124416/(1+sin(x))^2-209/373248/(1+sin
(x))+3889/373248*ln(1+sin(x))+1/124416/(sin(x)-1)^2-209/373248/(sin(x)-1)-
3889/373248*ln(sin(x)-1)+1/108/(2*sin(x)-1)^4+19/162/(2*sin(x)-1)^3+749/64
8/(2*sin(x)-1)^2+71551/5832/(2*sin(x)-1)-332929/5832*ln(2*sin(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(175) = 350$.

Time = 0.22 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.20

$$\int \frac{1}{(\cos(x) + \cos(5x))^5} \, dx = \text{Too large to display}$$

input
```
integrate(1/(cos(x)+cos(5*x))^5,x, algorithm="fricas")
```

output
```
1/1492992*(60275907*(4096*sqrt(2)*cos(x)^20 - 20480*sqrt(2)*cos(x)^18 + 44
544*sqrt(2)*cos(x)^16 - 55040*sqrt(2)*cos(x)^14 + 42256*sqrt(2)*cos(x)^12
- 20640*sqrt(2)*cos(x)^10 + 6264*sqrt(2)*cos(x)^8 - 1080*sqrt(2)*cos(x)^6
+ 81*sqrt(2)*cos(x)^4)*log(-(2*cos(x)^2 + 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^
2 - 1)) + 85229824*(4096*cos(x)^20 - 20480*cos(x)^18 + 44544*cos(x)^16 - 5
5040*cos(x)^14 + 42256*cos(x)^12 - 20640*cos(x)^10 + 6264*cos(x)^8 - 1080*
cos(x)^6 + 81*cos(x)^4)*log(2*sin(x) + 1) + 15556*(4096*cos(x)^20 - 20480*
cos(x)^18 + 44544*cos(x)^16 - 55040*cos(x)^14 + 42256*cos(x)^12 - 20640*co
s(x)^10 + 6264*cos(x)^8 - 1080*cos(x)^6 + 81*cos(x)^4)*log(sin(x) + 1) - 1
5556*(4096*cos(x)^20 - 20480*cos(x)^18 + 44544*cos(x)^16 - 55040*cos(x)^14
 + 42256*cos(x)^12 - 20640*cos(x)^10 + 6264*cos(x)^8 - 1080*cos(x)^6 + 81*
cos(x)^4)*log(-sin(x) + 1) - 85229824*(4096*cos(x)^20 - 20480*cos(x)^18 +
44544*cos(x)^16 - 55040*cos(x)^14 + 42256*cos(x)^12 - 20640*cos(x)^10 + 62
64*cos(x)^8 - 1080*cos(x)^6 + 81*cos(x)^4)*log(-2*sin(x) + 1) - 12*(120459
53024*cos(x)^18 - 52614016000*cos(x)^16 + 97798185216*cos(x)^14 - 10027023
7696*cos(x)^12 + 61237672232*cos(x)^10 - 22277937972*cos(x)^8 + 4470458046
*cos(x)^6 - 381752883*cos(x)^4 - 6966*cos(x)^2 - 324)*sin(x))/(4096*cos(x)
^20 - 20480*cos(x)^18 + 44544*cos(x)^16 - 55040*cos(x)^14 + 42256*cos(x)^1
2 - 20640*cos(x)^10 + 6264*cos(x)^8 - 1080*cos(x)^6 + 81*cos(x)^4)
```

Sympy [F]

$$\int \frac{1}{(\cos(x) + \cos(5x))^5} \, dx = \int \frac{1}{(\cos(x) + \cos(5x))^5} \, dx$$

input
```
integrate(1/(cos(x)+cos(5*x))**5,x)
```

output
```
Integral((cos(x) + cos(5*x))**(-5), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(x)+\cos(5x))^5}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(cos(x)+cos(5*x))^5,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.70

$$\int \frac{1}{(\cos(x)+\cos(5x))^5}\,dx$$
$$= \frac{82683}{2048}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin(x)\right|}{\left|2\sqrt{2}+4\sin(x)\right|}\right) - \frac{209\sin(x)^3-215\sin(x)}{186624\left(\sin(x)^2-1\right)^2}$$
$$+ \frac{36139571200\sin(x)^{15}-95126438912\sin(x)^{13}+105240567552\sin(x)^{11}-63358060800\sin(x)^9+2}{373248\left(8\sin(x)^4-6\sin\right.}$$
$$+ \frac{3889}{373248}\log(\sin(x)+1) - \frac{3889}{373248}\log(-\sin(x)+1)$$
$$+ \frac{332929}{5832}\log(|2\sin(x)+1|) - \frac{332929}{5832}\log(|2\sin(x)-1|)$$

input
```
integrate(1/(cos(x)+cos(5*x))^5,x, algorithm="giac")
```

output
```
82683/2048*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x)
)) - 1/186624*(209*sin(x)^3 - 215*sin(x))/(sin(x)^2 - 1)^2 + 1/373248*(361
39571200*sin(x)^15 - 95126438912*sin(x)^13 + 105240567552*sin(x)^11 - 6335
8060800*sin(x)^9 + 22400373144*sin(x)^7 - 4650907308*sin(x)^5 + 525480506*
sin(x)^3 - 24950461*sin(x))/(8*sin(x)^4 - 6*sin(x)^2 + 1)^4 + 3889/373248*
log(sin(x) + 1) - 3889/373248*log(-sin(x) + 1) + 332929/5832*log(abs(2*sin
(x) + 1)) - 332929/5832*log(abs(2*sin(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 22.58 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.71

$$\int \frac{1}{(\cos(x)+\cos(5x))^5}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(5*x) + cos(x))^5,x)
```

output
```
(332929*atanh((52935010726117280832882412685444853254923093541810*tan(x/2))
/(12907589169101029151617843 2*((529350107261172808328824126854448532549230
9354181*tan(x/2)^2)/516303566764041166064713728 + 529350107261172808328824
12685444853254923093541810/516303566764041166064713728))))/2916 + (3889*ata
nh(tan(x/2)))/93312 - ((8316677*tan(x/2))/62208 - (7520297*tan(x/2)^3)/768
 + (2102536375*tan(x/2)^5)/6912 - (108549008321*tan(x/2)^7)/20736 + (85152
5536249*tan(x/2)^9)/15552 - (1865668038367*tan(x/2)^11)/5184 + (2309100463
9391*tan(x/2)^13)/15552 - (19244414424625*tan(x/2)^15)/5184 + (52955516006
129*tan(x/2)^17)/10368 - (79937493546559*tan(x/2)^19)/31104 - (79937493546
559*tan(x/2)^21)/31104 + (52955516006129*tan(x/2)^23)/10368 - (19244414424
625*tan(x/2)^25)/5184 + (23091004639391*tan(x/2)^27)/15552 - (186566803836
7*tan(x/2)^29)/5184 + (851525536249*tan(x/2)^31)/15552 - (108549008321*tan
(x/2)^33)/20736 + (2102536375*tan(x/2)^35)/6912 - (7520297*tan(x/2)^37)/76
8 + (8316677*tan(x/2)^39)/62208)/(3070*tan(x/2)^4 - 84*tan(x/2)^2 - 64180*
tan(x/2)^6 + 849645*tan(x/2)^8 - 7459216*tan(x/2)^10 + 44289640*tan(x/2)^1
2 - 178563024*tan(x/2)^14 + 486234130*tan(x/2)^16 - 887655320*tan(x/2)^18
+ 1084730676*tan(x/2)^20 - 887655320*tan(x/2)^22 + 486234130*tan(x/2)^24 -
 178563024*tan(x/2)^26 + 44289640*tan(x/2)^28 - 7459216*tan(x/2)^30 + 8496
45*tan(x/2)^32 - 64180*tan(x/2)^34 + 3070*tan(x/2)^36 - 84*tan(x/2)^38 + t
an(x/2)^40 + 1) - (82683*2^(1/2)*atanh((387072875943098200959516122959 0...
```

Reduce [F]

$$\int \frac{1}{(\cos(x)+\cos(5x))^5}\,dx$$
$$= \int \frac{1}{\cos(5x)^5 + 5\cos(5x)^4\cos(x) + 10\cos(5x)^3\cos(x)^2 + 10\cos(5x)^2\cos(x)^3 + 5\cos(5x)\cos(x)^4 + \cos}$$

input

```
int(1/(cos(x)+cos(5*x))^5,x)
```

output

```
int(1/(cos(5*x)**5 + 5*cos(5*x)**4*cos(x) + 10*cos(5*x)**3*cos(x)**2 + 10*
cos(5*x)**2*cos(x)**3 + 5*cos(5*x)*cos(x)**4 + cos(x)**5),x)
```

3.46 $\int \frac{1}{(\cos(x)+\cos(5x))^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 91

$$\int \frac{1}{(\cos(x)+\cos(5x))^2}\,dx = \frac{3}{4}\text{arctanh}(2\cos(x)\sin(x)) + \frac{4\log\left(\cos(x)-\sqrt{3}\sin(x)\right)}{3\sqrt{3}} - \frac{4\log\left(\cos(x)+\sqrt{3}\sin(x)\right)}{3\sqrt{3}} + \frac{\tan(x)}{36} + \frac{\tan(x)\left(25-43\tan^2(x)\right)}{18\left(1-4\tan^2(x)+3\tan^4(x)\right)}$$

output
```
3/4*arctanh(2*cos(x)*sin(x))+4/9*ln(cos(x)-sin(x)*3^(1/2))*3^(1/2)-4/9*ln(
cos(x)+sin(x)*3^(1/2))*3^(1/2)+1/36*tan(x)+tan(x)*(25-43*tan(x)^2)/(18-72*
tan(x)^2+54*tan(x)^4)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.69

$$\int \frac{1}{(\cos(x)+\cos(5x))^2}\,dx = \frac{1}{36}\left(-32\sqrt{3}\text{arctanh}\left(\sqrt{3}\tan(x)\right) - 27\log(\cos(x)-\sin(x)) + 27\log(\cos(x)+\sin(x)) + \frac{16\sin(2x)}{-1+2\cos(2x)} + \tan(x) + 9\tan(2x)\right)$$

input
```
Integrate[(Cos[x] + Cos[5*x])^(-2),x]
```

output
```
(-32*Sqrt[3]*ArcTanh[Sqrt[3]*Tan[x]] - 27*Log[Cos[x] - Sin[x]] + 27*Log[Co
s[x] + Sin[x]] + (16*Sin[2*x])/(-1 + 2*Cos[2*x]) + Tan[x] + 9*Tan[2*x])/36
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.69, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {3042, 4823, 27, 1517, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos(x)+\cos(5x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\cos(x)+\cos(5x))^2}dx$$

$$\Big\downarrow \text{4823}$$

$$\int \frac{\left(\tan^2(x)+1\right)^4}{4\left(3\tan^4(x)-4\tan^2(x)+1\right)^2}d\tan(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{\left(\tan^2(x)+1\right)^4}{\left(3\tan^4(x)-4\tan^2(x)+1\right)^2}d\tan(x)$$

$$\Big\downarrow \text{1517}$$

$$\frac{1}{4}\left(\frac{2\tan(x)\left(25-43\tan^2(x)\right)}{9\left(3\tan^4(x)-4\tan^2(x)+1\right)}-\frac{1}{8}\int \frac{8\left(-3\tan^4(x)+70\tan^2(x)+41\right)}{9\left(3\tan^4(x)-4\tan^2(x)+1\right)}d\tan(x)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{2\tan(x)\left(25-43\tan^2(x)\right)}{9\left(3\tan^4(x)-4\tan^2(x)+1\right)}-\frac{1}{9}\int \frac{-3\tan^4(x)+70\tan^2(x)+41}{3\tan^4(x)-4\tan^2(x)+1}d\tan(x)\right)$$

$$\Big\downarrow \text{2205}$$

$$\frac{1}{4}\left(\frac{2\tan(x)\left(25-43\tan^2(x)\right)}{9\left(3\tan^4(x)-4\tan^2(x)+1\right)}-\frac{1}{9}\int\left(\frac{6\left(11\tan^2(x)+7\right)}{3\tan^4(x)-4\tan^2(x)+1}-1\right)d\tan(x)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{1}{9}\left(54\text{arctanh}(\tan(x))-32\sqrt{3}\text{arctanh}\left(\sqrt{3}\tan(x)\right)+\tan(x)\right)+\frac{2\tan(x)\left(25-43\tan^2(x)\right)}{9\left(3\tan^4(x)-4\tan^2(x)+1\right)}\right)$$

input
```
Int[(Cos[x] + Cos[5*x])^(-2),x]
```

output
```
((54*ArcTanh[Tan[x]] - 32*Sqrt[3]*ArcTanh[Sqrt[3]*Tan[x]] + Tan[x])/9 + (2
*Tan[x]*(25 - 43*Tan[x]^2))/(9*(1 - 4*Tan[x]^2 + 3*Tan[x]^4)))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4823
```
Int[(cos[(m_.)*((c_.) + (d_.)*(x_))]*(a_.) + cos[(n_.)*((c_.) + (d_.)*(x_))
]*(b_.))^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Cos[
m*ArcTan[x]] + b*Cos[n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[c + d*x]], x] /
; FreeQ[{a, b, c, d}, x] && ILtQ[p/2, 0] && IntegerQ[(m - 1)/2] && IntegerQ
[(n - 1)/2]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.66

method	result
default	$\frac{\tan(x)}{36}-\frac{1}{4(\tan(x)+1)}+\frac{3\ln(\tan(x)+1)}{4}-\frac{8\tan(x)}{27\left(\tan(x)^2-\frac{1}{3}\right)}-\frac{8\sqrt{3}\,\operatorname{arctanh}\left(\tan(x)\sqrt{3}\right)}{9}-\frac{1}{4(\tan(x)-1)}-\frac{3\ln(\tan(x)-}{4}$
risch	$-\frac{i(e^{8ix}-4\,e^{6ix}-2\,e^{4ix}-e^{2ix}-6)}{6(e^{10ix}+e^{6ix}+e^{4ix}+1)}+\frac{4\sqrt{3}\,\ln\left(e^{2ix}-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{9}-\frac{4\sqrt{3}\,\ln\left(e^{2ix}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{9}-\frac{3\ln(e^{2ix}-i)}{4}+\frac{3\ln(e^{2ix}+i)}{4}$

input
```
int(1/(cos(x)+cos(5*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/36*tan(x)-1/4/(tan(x)+1)+3/4*ln(tan(x)+1)-8/27*tan(x)/(tan(x)^2-1/3)-8/9
*3^(1/2)*arctanh(tan(x)*3^(1/2))-1/4/(tan(x)-1)-3/4*ln(tan(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(73) = 146$.

Time = 0.09 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.89

$$\int \frac{1}{(\cos(x) + \cos(5x))^2} \, dx$$

$$= \frac{27 \left(8 \cos(x)^5 - 10 \cos(x)^3 + 3 \cos(x)\right) \log\left(2 \cos(x) \sin(x) + 1\right) - 27 \left(8 \cos(x)^5 - 10 \cos(x)^3 + 3 \cos\right.}{}$$

input
```
integrate(1/(cos(x)+cos(5*x))^2,x, algorithm="fricas")
```

output
```
1/72*(27*(8*cos(x)^5 - 10*cos(x)^3 + 3*cos(x))*log(2*cos(x)*sin(x) + 1) -
27*(8*cos(x)^5 - 10*cos(x)^3 + 3*cos(x))*log(-2*cos(x)*sin(x) + 1) + 16*(8
*sqrt(3)*cos(x)^5 - 10*sqrt(3)*cos(x)^3 + 3*sqrt(3)*cos(x))*log(-(8*cos(x)
^4 - 4*(2*sqrt(3)*cos(x)^3 - 3*sqrt(3)*cos(x))*sin(x) - 9)/(16*cos(x)^4 -
24*cos(x)^2 + 9)) + 6*(48*cos(x)^4 - 32*cos(x)^2 + 1)*sin(x))/(8*cos(x)^5
- 10*cos(x)^3 + 3*cos(x))
```

Sympy [F]

$$\int \frac{1}{(\cos(x) + \cos(5x))^2} \, dx = \int \frac{1}{\left(\cos(x) + \cos(5x)\right)^2} \, dx$$

input
```
integrate(1/(cos(x)+cos(5*x))**2,x)
```

output
```
Integral((cos(x) + cos(5*x))**(-2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(x)+\cos(5x))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(cos(x)+cos(5*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.89

$$\int \frac{1}{(\cos(x)+\cos(5x))^2}\,dx = \frac{4}{9}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+6\tan(x)\right|}{\left|2\sqrt{3}+6\tan(x)\right|}\right) - \frac{43\tan(x)^3-25\tan(x)}{18\left(3\tan(x)^4-4\tan(x)^2+1\right)} + \frac{3}{4}\log\left(\left|\tan(x)+1\right|\right) - \frac{3}{4}\log\left(\left|\tan(x)-1\right|\right) + \frac{1}{36}\tan(x)$$

input
```
integrate(1/(cos(x)+cos(5*x))^2,x, algorithm="giac")
```

output
```
4/9*sqrt(3)*log(abs(-2*sqrt(3) + 6*tan(x))/abs(2*sqrt(3) + 6*tan(x))) - 1/
18*(43*tan(x)^3 - 25*tan(x))/(3*tan(x)^4 - 4*tan(x)^2 + 1) + 3/4*log(abs(t
an(x) + 1)) - 3/4*log(abs(tan(x) - 1)) + 1/36*tan(x)
```

Mupad [B] (verification not implemented)

Time = 22.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04

$$\int \frac{1}{(\cos(x)+\cos(5x))^2}\,dx = \frac{3\sin(3x)+9\sin(5x)-3\sin(x)+27\cos(x)\,\mathrm{atanh}\left(\frac{\sin(x)}{\cos(x)}\right)+27\cos(5x)\,\mathrm{atanh}\left(\frac{\sin(x)}{\cos(x)}\right)-16\sqrt{3}\cos}{18\cos(5x)+18\cos(x)}$$

input
```
int(1/(cos(5*x) + cos(x))^2,x)
```

output
```
(3*sin(3*x) + 9*sin(5*x) - 3*sin(x) + 27*cos(x)*atanh(sin(x)/cos(x)) + 27*
cos(5*x)*atanh(sin(x)/cos(x)) - 16*3^(1/2)*cos(5*x)*atanh((3^(1/2)*sin(x))
/cos(x)) - 16*3^(1/2)*atanh((3^(1/2)*sin(x))/cos(x))*cos(x))/(18*cos(5*x)
+ 18*cos(x))
```

Reduce [F]

$$\int \frac{1}{(\cos(x)+\cos(5x))^2}\,dx = \int \frac{1}{\cos(5x)^2+2\cos(5x)\cos(x)+\cos(x)^2}dx$$

input
```
int(1/(cos(x)+cos(5*x))^2,x)
```

output
```
int(1/(cos(5*x)**2 + 2*cos(5*x)*cos(x) + cos(x)**2),x)
```

3.47 $\int \frac{1}{(\cos(x)+\cos(5x))^4}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 159

$$\begin{aligned}\int \frac{1}{(\cos(x)+\cos(5x))^4}\,dx = {} & \frac{103}{8}\text{arctanh}(2\cos(x)\sin(x)) \\ & + \frac{1808\log\left(\cos(x)-\sqrt{3}\sin(x)\right)}{81\sqrt{3}} \\ & - \frac{1808\log\left(\cos(x)+\sqrt{3}\sin(x)\right)}{81\sqrt{3}} + \frac{43\tan(x)}{3888} \\ & + \frac{\tan^3(x)}{3888} + \frac{4\tan(x)\left(2699-7073\tan^2(x)\right)}{6561\left(1-4\tan^2(x)+3\tan^4(x)\right)^3} \\ & - \frac{2\tan(x)\left(17527-20226\tan^2(x)\right)}{6561\left(1-4\tan^2(x)+3\tan^4(x)\right)^2} \\ & + \frac{\tan(x)\left(198061-287289\tan^2(x)\right)}{8748\left(1-4\tan^2(x)+3\tan^4(x)\right)}\end{aligned}$$

output
```
103/8*arctanh(2*cos(x)*sin(x))+1808/243*ln(cos(x)-sin(x)*3^(1/2))*3^(1/2)-
1808/243*ln(cos(x)+sin(x)*3^(1/2))*3^(1/2)+43/3888*tan(x)+1/3888*tan(x)^3+
4/6561*tan(x)*(2699-7073*tan(x)^2)/(1-4*tan(x)^2+3*tan(x)^4)^3-2/6561*tan(
x)*(17527-20226*tan(x)^2)/(1-4*tan(x)^2+3*tan(x)^4)^2+tan(x)*(198061-28728
9*tan(x)^2)/(8748-34992*tan(x)^2+26244*tan(x)^4)
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.92

$$\int \frac{1}{(\cos(x)+\cos(5x))^4}\,dx$$
$$= \frac{-115712\sqrt{3}\text{arctanh}\left(\sqrt{3}\tan(x)\right) - 100116\log(\cos(x)-\sin(x)) + 100116\log(\cos(x)+\sin(x)) + \frac{153}{(\cos(x)-}}{}$$

input
```
Integrate[(Cos[x] + Cos[5*x])^(-4),x]
```

output
```
(-115712*Sqrt[3]*ArcTanh[Sqrt[3]*Tan[x]] - 100116*Log[Cos[x] - Sin[x]] + 1
00116*Log[Cos[x] + Sin[x]] + 1539/(Cos[x] - Sin[x])^2 + 162*(169*Cos[x] +
82*Cos[3*x] + 83*Cos[5*x])*Sec[2*x]^3*Sin[x] - 1539/(Cos[x] + Sin[x])^2 -
(9472*Sin[2*x])/(1 - 2*Cos[2*x])^2 + (1536*Sin[2*x])/(-1 + 2*Cos[2*x])^3 +
 (51456*Sin[2*x])/(-1 + 2*Cos[2*x]) + 84*Tan[x] + 2*Sec[x]^2*Tan[x])/7776
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.84, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.222$, Rules used = {3042, 4823, 27, 1517, 27, 2206, 27, 2206, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos(x)+\cos(5x))^4}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(\cos(x)+\cos(5x))^4}dx$$
$$\Big\downarrow \text{4823}$$
$$\int \frac{\left(\tan^2(x)+1\right)^9}{16\left(3\tan^4(x)-4\tan^2(x)+1\right)^4}d\tan(x)$$
$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int \frac{\left(\tan^2(x)+1\right)^9}{\left(3\tan^4(x)-4\tan^2(x)+1\right)^4}d\tan(x)$$

$$\Big\downarrow \text{1517}$$

$$\frac{1}{16}\left(\frac{64\tan(x)\left(2699-7073\tan^2(x)\right)}{6561\left(3\tan^4(x)-4\tan^2(x)+1\right)^3}-\frac{1}{24}\int \frac{8\left(-2187\tan^{14}(x)-22599\tan^{12}(x)-108135\tan^{10}(x)-320355\right.}{2187\left(3\tan\right.}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{64\tan(x)\left(2699-7073\tan^2(x)\right)}{6561\left(3\tan^4(x)-4\tan^2(x)+1\right)^3}-\frac{\int \frac{-2187\tan^{14}(x)-22599\tan^{12}(x)-108135\tan^{10}(x)-320355\tan^8(x)-666657\tan^6(x)-1}{\left(3\tan^4(x)-4\tan^2(x)+1\right)^3}}{6561}\right.$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{16}\left(\frac{\frac{1}{16}\int \frac{48\left(243\tan^{10}(x)+2835\tan^8(x)+15714\tan^6(x)+55602\tan^4(x)+1221691\tan^2(x)+131563\right)}{\left(3\tan^4(x)-4\tan^2(x)+1\right)^2}d\tan(x)-\frac{32\tan(x)\left(17527-20226\tan\right.}{\left(3\tan^4(x)-4\tan^2(x)+\right.}}{6561}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{3\int \frac{243\tan^{10}(x)+2835\tan^8(x)+15714\tan^6(x)+55602\tan^4(x)+1221691\tan^2(x)+131563}{\left(3\tan^4(x)-4\tan^2(x)+1\right)^2}d\tan(x)-\frac{32\tan(x)\left(17527-20226\tan^2(x)\right.}{\left(3\tan^4(x)-4\tan^2(x)+1\right)^2}}{6561}\right.$$

$$\Big\downarrow \text{2206}$$

$$\frac{1}{16}\left(\frac{3\left(\frac{4\tan(x)\left(198061-287289\tan^2(x)\right)}{3\tan^4(x)-4\tan^2(x)+1}-\frac{1}{8}\int \frac{72\left(-9\tan^6(x)-117\tan^4(x)+126949\tan^2(x)+73409\right)}{3\tan^4(x)-4\tan^2(x)+1}d\tan(x)\right)-\frac{32\tan(x)\left(17527-20\right.}{\left(3\tan^4(x)-4\tan\right.}}{6561}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{3\left(\frac{4\tan(x)\left(198061-287289\tan^2(x)\right)}{3\tan^4(x)-4\tan^2(x)+1}-9\int \frac{-9\tan^6(x)-117\tan^4(x)+126949\tan^2(x)+73409}{3\tan^4(x)-4\tan^2(x)+1}d\tan(x)\right)-\frac{32\tan(x)\left(17527-20226\right.}{\left(3\tan^4(x)-4\tan^2(x)\right.}}{6561}\right.$$

$$\Big\downarrow \text{2205}$$

$$\frac{1}{16}\left(\frac{3\left(\frac{4\tan(x)(198061-287289\tan^2(x))}{3\tan^4(x)-4\tan^2(x)+1}-9\int\left(-3\tan^2(x)+\frac{12(10565\tan^2(x)+6121)}{3\tan^4(x)-4\tan^2(x)+1}-43\right)d\tan(x)\right)-\frac{32\tan(x)(17527-20}{(3\tan^4(x)-4\tan}}{6561}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{16}\left(\frac{3\left(\frac{4\tan(x)(198061-287289\tan^2(x))}{3\tan^4(x)-4\tan^2(x)+1}-9\left(-100116\text{arctanh}(\tan(x))+57856\sqrt{3}\text{arctanh}\left(\sqrt{3}\tan(x)\right)-\tan^3(x)-43\right.\right.}{6561}\right.$$

input
```
Int[(Cos[x] + Cos[5*x])^(-4),x]
```

output
```
((64*Tan[x]*(2699 - 7073*Tan[x]^2))/(6561*(1 - 4*Tan[x]^2 + 3*Tan[x]^4)^3)
 + ((-32*Tan[x]*(17527 - 20226*Tan[x]^2))/(1 - 4*Tan[x]^2 + 3*Tan[x]^4)^2
+ 3*(-9*(-100116*ArcTanh[Tan[x]] + 57856*Sqrt[3]*ArcTanh[Sqrt[3]*Tan[x]] -
 43*Tan[x] - Tan[x]^3) + (4*Tan[x]*(198061 - 287289*Tan[x]^2))/(1 - 4*Tan[
x]^2 + 3*Tan[x]^4)))/6561)/16
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4823
```
Int[(cos[(m_.)*((c_.) + (d_.)*(x_))]*(a_.) + cos[(n_.)*((c_.) + (d_.)*(x_))
]*(b_.))^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Cos[
m*ArcTan[x]] + b*Cos[n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[c + d*x]], x] /
; FreeQ[{a, b, c, d}, x] && ILtQ[p/2, 0] && IntegerQ[(m - 1)/2] && IntegerQ
[(n - 1)/2]
```

Maple [A] (verified)

Time = 13.34 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.72

method	result
default	$\frac{\tan(x)^3}{3888}+\frac{43\tan(x)}{3888}-\frac{1}{24(\tan(x)-1)^3}+\frac{5}{16(\tan(x)-1)^2}-\frac{25}{8(\tan(x)-1)}-\frac{103\ln(\tan(x)-1)}{8}+\frac{-\frac{3424\tan(x)^5}{27}+\frac{17920\tan(}{243}}{(3\tan(x)^2-}$
risch	$-\frac{i(1111\,\mathrm{e}^{28ix}-3616\,\mathrm{e}^{26ix}+552\,\mathrm{e}^{24ix}-5707\,\mathrm{e}^{22ix}-12743\,\mathrm{e}^{20ix}-3768\,\mathrm{e}^{18ix}-23899\,\mathrm{e}^{16ix}-15629\,\mathrm{e}^{14ix}-13800\,\mathrm{e}^{12ix}-26785\,\mathrm{e}^{10ix}-}{324(\mathrm{e}^{10ix}+\mathrm{e}^{6ix}+\mathrm{e}^{4ix}+1)^3}$

input
```
int(1/(cos(x)+cos(5*x))^4,x,method=_RETURNVERBOSE)
```

output
```
1/3888*tan(x)^3+43/3888*tan(x)-1/24/(tan(x)-1)^3+5/16/(tan(x)-1)^2-25/8/(t
an(x)-1)-103/8*ln(tan(x)-1)+64/9*(-107/6*tan(x)^5+280/27*tan(x)^3-85/54*ta
n(x))/(3*tan(x)^2-1)^3-3616/243*3^(1/2)*arctanh(tan(x)*3^(1/2))-1/24/(tan(
x)+1)^3-5/16/(tan(x)+1)^2-25/8/(tan(x)+1)+103/8*ln(tan(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(135) = 270$.

Time = 0.14 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.00

$$\int \frac{1}{(\cos(x) + \cos(5x))^4} \, dx = \frac{25029 \left(512 \cos(x)^{15} - 1920 \cos(x)^{13} + 2976 \cos(x)^{11} - 2440 \cos(x)^9 + 1116 \cos(x)^7 - 270 \cos(x)^5 + \right.}{}$$

input
```
integrate(1/(cos(x)+cos(5*x))^4,x, algorithm="fricas")
```

output
```
1/3888*(25029*(512*cos(x)^15 - 1920*cos(x)^13 + 2976*cos(x)^11 - 2440*cos(
x)^9 + 1116*cos(x)^7 - 270*cos(x)^5 + 27*cos(x)^3)*log(2*cos(x)*sin(x) + 1
) - 25029*(512*cos(x)^15 - 1920*cos(x)^13 + 2976*cos(x)^11 - 2440*cos(x)^9
 + 1116*cos(x)^7 - 270*cos(x)^5 + 27*cos(x)^3)*log(-2*cos(x)*sin(x) + 1) +
 14464*(512*sqrt(3)*cos(x)^15 - 1920*sqrt(3)*cos(x)^13 + 2976*sqrt(3)*cos(
x)^11 - 2440*sqrt(3)*cos(x)^9 + 1116*sqrt(3)*cos(x)^7 - 270*sqrt(3)*cos(x)
^5 + 27*sqrt(3)*cos(x)^3)*log(-(8*cos(x)^4 - 4*(2*sqrt(3)*cos(x)^3 - 3*sqr
t(3)*cos(x))*sin(x) - 9)/(16*cos(x)^4 - 24*cos(x)^2 + 9)) + 3*(4497408*cos
(x)^14 - 14047744*cos(x)^12 + 17367936*cos(x)^10 - 10622496*cos(x)^8 + 321
5624*cos(x)^6 - 386460*cos(x)^4 + 288*cos(x)^2 + 9)*sin(x))/(512*cos(x)^15
 - 1920*cos(x)^13 + 2976*cos(x)^11 - 2440*cos(x)^9 + 1116*cos(x)^7 - 270*c
os(x)^5 + 27*cos(x)^3)
```

Sympy [F]

$$\int \frac{1}{(\cos(x)+\cos(5x))^4}\,dx=\int \frac{1}{(\cos(x)+\cos(5x))^4}\,dx$$

input
```
integrate(1/(cos(x)+cos(5*x))**4,x)
```

output
```
Integral((cos(x) + cos(5*x))**(-4), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(x)+\cos(5x))^4}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(1/(cos(x)+cos(5*x))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.70

$$\int \frac{1}{(\cos(x)+\cos(5x))^4}\,dx=\frac{1}{3888}\tan(x)^3+\frac{1808}{243}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+6\tan(x)\right|}{\left|2\sqrt{3}+6\tan(x)\right|}\right) - \frac{287289\tan(x)^{11}-964165\tan(x)^9+1212446\tan(x)^7-699966\tan(x)^5+185401\tan(x)^3-18413}{972\left(3\tan(x)^4-4\tan(x)^2+1\right)^3} + \frac{103}{8}\log(|\tan(x)+1|)-\frac{103}{8}\log(|\tan(x)-1|)+\frac{43}{3888}\tan(x)$$

input
```
integrate(1/(cos(x)+cos(5*x))^4,x, algorithm="giac")
```

output
```
1/3888*tan(x)^3 + 1808/243*sqrt(3)*log(abs(-2*sqrt(3) + 6*tan(x))/abs(2*sq
rt(3) + 6*tan(x))) - 1/972*(287289*tan(x)^11 - 964165*tan(x)^9 + 1212446*t
an(x)^7 - 699966*tan(x)^5 + 185401*tan(x)^3 - 18413*tan(x))/(3*tan(x)^4 -
4*tan(x)^2 + 1)^3 + 103/8*log(abs(tan(x) + 1)) - 103/8*log(abs(tan(x) - 1)
) + 43/3888*tan(x)
```

Mupad [B] (verification not implemented)

Time = 22.62 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.17

$$\int \frac{1}{(\cos(x) + \cos(5x))^4} \, dx = \text{Too large to display}$$

input
```
int(1/(cos(5*x) + cos(x))^4,x)
```

output
```
(27*sin(x) + 2703132*cos(x)^3*atanh(sin(x)/cos(x)) - 27031320*cos(x)^5*ata
nh(sin(x)/cos(x)) + 111729456*cos(x)^7*atanh(sin(x)/cos(x)) - 244283040*co
s(x)^9*atanh(sin(x)/cos(x)) + 297945216*cos(x)^11*atanh(sin(x)/cos(x)) - 1
92222720*cos(x)^13*atanh(sin(x)/cos(x)) + 51259392*cos(x)^15*atanh(sin(x)/
cos(x)) + 864*cos(x)^2*sin(x) - 1159380*cos(x)^4*sin(x) + 9646872*cos(x)^6
*sin(x) - 31867488*cos(x)^8*sin(x) + 52103808*cos(x)^10*sin(x) - 42143232*
cos(x)^12*sin(x) + 13492224*cos(x)^14*sin(x) - 1562112*3^(1/2)*atanh((3^(1
/2)*sin(x))/cos(x))*cos(x)^3 + 15621120*3^(1/2)*atanh((3^(1/2)*sin(x))/cos
(x))*cos(x)^5 - 64567296*3^(1/2)*atanh((3^(1/2)*sin(x))/cos(x))*cos(x)^7 +
 141168640*3^(1/2)*atanh((3^(1/2)*sin(x))/cos(x))*cos(x)^9 - 172179456*3^(
1/2)*atanh((3^(1/2)*sin(x))/cos(x))*cos(x)^11 + 111083520*3^(1/2)*atanh((3
^(1/2)*sin(x))/cos(x))*cos(x)^13 - 29622272*3^(1/2)*atanh((3^(1/2)*sin(x))
/cos(x))*cos(x)^15)/(104976*cos(x)^3 - 1049760*cos(x)^5 + 4339008*cos(x)^7
 - 9486720*cos(x)^9 + 11570688*cos(x)^11 - 7464960*cos(x)^13 + 1990656*cos
(x)^15)
```

Reduce [F]

$$\int \frac{1}{(\cos(x)+\cos(5x))^4}\,dx$$
$$=\int \frac{1}{\cos(5x)^4+4\cos(5x)^3\cos(x)+6\cos(5x)^2\cos(x)^2+4\cos(5x)\cos(x)^3+\cos(x)^4}dx$$

input
```
int(1/(cos(x)+cos(5*x))^4,x)
```

output
```
int(1/(cos(5*x)**4 + 4*cos(5*x)**3*cos(x) + 6*cos(5*x)**2*cos(x)**2 + 4*co
s(5*x)*cos(x)**3 + cos(x)**4),x)
```

3.48 $\int \frac{1}{(\cos(x)+\cos(5x))^6}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 227

$$
\begin{aligned}
\int \frac{1}{(\cos(x)+\cos(5x))^6}\,dx = {} & \frac{33243}{128}\operatorname{arctanh}(2\cos(x)\sin(x)) \\
& + \frac{109312\log\left(\cos(x)-\sqrt{3}\sin(x)\right)}{243\sqrt{3}} \\
& - \frac{109312\log\left(\cos(x)+\sqrt{3}\sin(x)\right)}{243\sqrt{3}} \\
& + \frac{715\tan(x)}{139968} + \frac{11\tan^3(x)}{69984} + \frac{\tan^5(x)}{233280} \\
& + \frac{32\tan(x)\,(547825-1610707\tan^2(x))}{2657205\,(1-4\tan^2(x)+3\tan^4(x))^5} \\
& - \frac{2\tan(x)\,(11066113+8997573\tan^2(x))}{2657205\,(1-4\tan^2(x)+3\tan^4(x))^4} \\
& + \frac{\tan(x)\,(13572421-26063973\tan^2(x))}{590490\,(1-4\tan^2(x)+3\tan^4(x))^3} \\
& - \frac{\tan(x)\,(470396279-677695389\tan^2(x))}{4723920\,(1-4\tan^2(x)+3\tan^4(x))^2} \\
& + \frac{7\tan(x)\,(82529173-118536693\tan^2(x))}{1259712\,(1-4\tan^2(x)+3\tan^4(x))}
\end{aligned}
$$

output
```
33243/128*arctanh(2*cos(x)*sin(x))+109312/729*ln(cos(x)-sin(x)*3^(1/2))*3^
(1/2)-109312/729*ln(cos(x)+sin(x)*3^(1/2))*3^(1/2)+715/139968*tan(x)+11/69
984*tan(x)^3+1/233280*tan(x)^5+32/2657205*tan(x)*(547825-1610707*tan(x)^2)
/(1-4*tan(x)^2+3*tan(x)^4)^5-2/2657205*tan(x)*(11066113+8997573*tan(x)^2)/
(1-4*tan(x)^2+3*tan(x)^4)^4+1/590490*tan(x)*(13572421-26063973*tan(x)^2)/(
1-4*tan(x)^2+3*tan(x)^4)^3-1/4723920*tan(x)*(470396279-677695389*tan(x)^2)
/(1-4*tan(x)^2+3*tan(x)^4)^2+7*tan(x)*(82529173-118536693*tan(x)^2)/(12597
12-5038848*tan(x)^2+3779136*tan(x)^4)
```

Mathematica [A] (verified)

Time = 5.61 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.97

$$\int \frac{1}{(\cos(x)+\cos(5x))^6}\,dx$$
$$= \frac{-839516160\sqrt{3}\text{arctanh}\left(\sqrt{3}\tan(x)\right) - 727024410\log(\cos(x)-\sin(x)) + 727024410\log(\cos(x)+\sin(x)}{}$$

input
```
Integrate[(Cos[x] + Cos[5*x])^(-6),x]
```

output
```
(-839516160*Sqrt[3]*ArcTanh[Sqrt[3]*Tan[x]] - 727024410*Log[Cos[x] - Sin[x
]] + 727024410*Log[Cos[x] + Sin[x]] + 102789/(Cos[x] - Sin[x])^4 + 1238133
6/(Cos[x] - Sin[x])^2 + (729*(404327*Cos[x] + 266847*Cos[3*x] + 269093*Cos
[5*x] + 65353*Cos[7*x] + 66452*Cos[9*x])*Sec[2*x]^5*Sin[x])/2 - 102789/(Co
s[x] + Sin[x])^4 - 12381336/(Cos[x] + Sin[x])^2 - (3538944*Sin[2*x])/(1 -
2*Cos[2*x])^4 - (76890112*Sin[2*x])/(1 - 2*Cos[2*x])^2 + (442368*Sin[2*x])
/(-1 + 2*Cos[2*x])^5 + (17842176*Sin[2*x])/(-1 + 2*Cos[2*x])^3 + (37385011
2*Sin[2*x])/(-1 + 2*Cos[2*x]) + 13872*Tan[x] + 416*Sec[x]^2*Tan[x] + 12*Se
c[x]^4*Tan[x])/2799360
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.91, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.667, Rules used = {3042, 4823, 27, 1517, 27, 2206, 27, 2206, 27, 2206, 27, 2206, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos(x)+\cos(5x))^6}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(\cos(x)+\cos(5x))^6}dx$$

$$\Big\downarrow \; 4823$$

$$\int \frac{\left(\tan^2(x)+1\right)^{14}}{64\left(3\tan^4(x)-4\tan^2(x)+1\right)^6}d\tan(x)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{64}\int \frac{\left(\tan^2(x)+1\right)^{14}}{\left(3\tan^4(x)-4\tan^2(x)+1\right)^6}d\tan(x)$$

$$\Big\downarrow \; 1517$$

$$\frac{1}{64}\left(\frac{2048\tan(x)\left(547825-1610707\tan^2(x)\right)}{2657205\left(3\tan^4(x)-4\tan^2(x)+1\right)^5}-\frac{1}{40}\int \frac{8\left(-885735\tan^{24}(x)-13581270\tan^{22}(x)-98415000\tan\right.}{}\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{64}\left(\frac{2048\tan(x)\left(547825-1610707\tan^2(x)\right)}{2657205\left(3\tan^4(x)-4\tan^2(x)+1\right)^5}-\frac{\int \frac{-885735\tan^{24}(x)-13581270\tan^{22}(x)-98415000\tan^{20}(x)-449100450\tan^{18}(}{}}{}\right.$$

$$\Big\downarrow \; 2206$$

$$\frac{1}{64}\left(\frac{\frac{1}{32}\int \frac{864\left(10935\tan^{20}(x)+182250\tan^{18}(x)+1454355\tan^{16}(x)+7422840\tan^{14}(x)+27345870\tan^{12}(x)+77941980\tan^{10}(x)+180274110\tan\right.}{\left(3\tan^4(x)-4\tan^2(x)+1\right)^4}}{2657}\right.$$

$\big\downarrow$ 27

$$\frac{1}{64}\left(\frac{27\int \frac{10935\tan^{20}(x)+182250\tan^{18}(x)+1454355\tan^{16}(x)+7422840\tan^{14}(x)+27345870\tan^{12}(x)+77941980\tan^{10}(x)+180274110\tan^{8}(x}{(3\tan^4(x)-4\tan^2(x)+1)^4}}{265720}\right.$$

$\big\downarrow$ 2206

$$\frac{1}{64}\left(\frac{27\left(\frac{32\tan(x)(13572421-26063973\tan^2(x))}{3(3\tan^4(x)-4\tan^2(x)+1)^3}-\frac{1}{24}\int \frac{8(-10935\tan^{16}(x)-196830\tan^{14}(x)-1713150\tan^{12}(x)-9641430\tan^{10}(x)-3963}{(3\tan^4(x)}\right.}{2}\right.$$

$\big\downarrow$ 27

$$\frac{1}{64}\left(\frac{27\left(\frac{32\tan(x)(13572421-26063973\tan^2(x))}{3(3\tan^4(x)-4\tan^2(x)+1)^3}-\frac{1}{3}\int \frac{-10935\tan^{16}(x)-196830\tan^{14}(x)-1713150\tan^{12}(x)-9641430\tan^{10}(x)-3963006}{(3\tan^4(x)-}\right.}{26}\right.$$

$\big\downarrow$ 2206

$$\frac{1}{64}\left(\frac{27\left(\frac{1}{3}\left(\frac{1}{16}\int \frac{240(243\tan^{12}(x)+4698\tan^{10}(x)+44253\tan^{8}(x)+271692\tan^{6}(x)+1228173\tan^{4}(x)+907975702\tan^{2}(x)+98685799)}{(3\tan^4(x)-4\tan^2(x)+1)^2}d\tan\right.\right.}{26}\right.$$

$\big\downarrow$ 27

$$\frac{1}{64}\left(\frac{27\left(\frac{1}{3}\left(15\int \frac{243\tan^{12}(x)+4698\tan^{10}(x)+44253\tan^{8}(x)+271692\tan^{6}(x)+1228173\tan^{4}(x)+907975702\tan^{2}(x)+98685799}{(3\tan^4(x)-4\tan^2(x)+1)^2}d\tan(x)\right.\right.}{2657}\right.$$

$\big\downarrow$ 2206

$$\frac{1}{64}\left(\frac{27\left(\frac{1}{3}\left(15\left(\frac{7\tan(x)(82529173-118536693\tan^2(x))}{3\tan^4(x)-4\tan^2(x)+1}-\frac{1}{8}\int \frac{72(-9\tan^{8}(x)-186\tan^{6}(x)-1884\tan^{4}(x)+92182693\tan^{2}(x)+53224268)}{3\tan^4(x)-4\tan^2(x)+1}\right.\right.\right.}{}\right.$$

$\big\downarrow$ 27

$$\frac{1}{64}\left(\frac{27\left(\frac{1}{3}\left(15\left(\frac{7\tan(x)(82529173-118536693\tan^2(x))}{3\tan^4(x)-4\tan^2(x)+1}-9\int \frac{-9\tan^{8}(x)-186\tan^{6}(x)-1884\tan^{4}(x)+92182693\tan^{2}(x)+53224268}{3\tan^4(x)-4\tan^2(x)+1}d\tan\right.\right.\right.}{2}\right.$$

$\downarrow$ 2205

$$\frac{1}{64}\left(\frac{27\left(\frac{1}{3}\left(15\left(\frac{7\tan(x)(82529173-118536693\tan^2(x))}{3\tan^4(x)-4\tan^2(x)+1}-9\int\left(-3\tan^4(x)-66\tan^2(x)+\frac{63(1463173\tan^2(x)+844841)}{3\tan^4(x)-4\tan^2(x)+1}-715\right.\right.\right.\right.}{}\right.$$

$\downarrow$ 2009

$$\frac{1}{64}\left(\frac{27\left(\frac{1}{3}\left(15\left(\frac{7\tan(x)(82529173-118536693\tan^2(x))}{3\tan^4(x)-4\tan^2(x)+1}-9\left(-72702441\text{arctanh}(\tan(x))+41975808\sqrt{3}\text{arctanh}\left(\sqrt{3}\tan(\right.\right.\right.\right.\right.}{}\right.$$

input
```
Int[(Cos[x] + Cos[5*x])^(-6),x]
```

output
```
((2048*Tan[x]*(547825 - 1610707*Tan[x]^2))/(2657205*(1 - 4*Tan[x]^2 + 3*Ta
n[x]^4)^5) + ((-128*Tan[x]*(11066113 + 8997573*Tan[x]^2))/(1 - 4*Tan[x]^2
+ 3*Tan[x]^4)^4 + 27*((32*Tan[x]*(13572421 - 26063973*Tan[x]^2))/(3*(1 - 4
*Tan[x]^2 + 3*Tan[x]^4)^3) + ((-4*Tan[x]*(470396279 - 677695389*Tan[x]^2))
/(1 - 4*Tan[x]^2 + 3*Tan[x]^4)^2 + 15*((7*Tan[x]*(82529173 - 118536693*Tan
[x]^2))/(1 - 4*Tan[x]^2 + 3*Tan[x]^4) - 9*(-72702441*ArcTanh[Tan[x]] + 419
75808*Sqrt[3]*ArcTanh[Sqrt[3]*Tan[x]] - 715*Tan[x] - 22*Tan[x]^3 - (3*Tan[
x]^5)/5)))/3))/2657205)/64
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4823
```
Int[(cos[(m_.)*((c_.) + (d_.)*(x_))]*(a_.) + cos[(n_.)*((c_.) + (d_.)*(x_))
]*(b_.))^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Cos[
m*ArcTan[x]] + b*Cos[n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[c + d*x]], x] /
; FreeQ[{a, b, c, d}, x] && ILtQ[p/2, 0] && IntegerQ[(m - 1)/2] && IntegerQ
[(n - 1)/2]
```

Maple [A] (verified)

Time = 165.00 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.73

method	result
default	$\frac{\tan(x)^5}{233280}+\frac{11\tan(x)^3}{69984}+\frac{715\tan(x)}{139968}+\frac{-\frac{1882496\tan(x)^9}{81}+28672\tan(x)^7-\frac{16312576\tan(x)^5}{1215}+\frac{6157312\tan(x)^3}{2187}-\frac{487040\tan(x)}{2187}}{\left(3\tan(x)^2-1\right)^5}-$
risch	$-\frac{i(-21150720-5410405\,e^{2ix}-88263680\,e^{4ix}-702189680\,e^{14ix}-359783050\,e^{12ix}-905873280\,e^{16ix}-1305280104\,e^{20ix}-44317939}{}$

input
```
int(1/(cos(x)+cos(5*x))^6,x,method=_RETURNVERBOSE)
```

output
```
1/233280*tan(x)^5+11/69984*tan(x)^3+715/139968*tan(x)+128/9*(-14707/9*tan(
x)^9+2016*tan(x)^7-127442/135*tan(x)^5+48104/243*tan(x)^3-3805/243*tan(x))
/(3*tan(x)^2-1)^5-218624/729*3^(1/2)*arctanh(tan(x)*3^(1/2))-1/80/(tan(x)+
1)^5-7/64/(tan(x)+1)^4-173/192/(tan(x)+1)^3-109/16/(tan(x)+1)^2-7931/128/(
tan(x)+1)+33243/128*ln(tan(x)+1)-1/80/(tan(x)-1)^5+7/64/(tan(x)-1)^4-173/1
92/(tan(x)-1)^3+109/16/(tan(x)-1)^2-7931/128/(tan(x)-1)-33243/128*ln(tan(x
)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(197) = 394.

Time = 0.33 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.01

$$\int \frac{1}{(\cos(x)+\cos(5x))^6}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cos(x)+cos(5*x))^6,x, algorithm="fricas")
```

output
```
1/933120*(121170735*(32768*cos(x)^25 - 204800*cos(x)^23 + 573440*cos(x)^21
 - 947200*cos(x)^19 + 1022080*cos(x)^17 - 752800*cos(x)^15 + 383280*cos(x)
^13 - 133200*cos(x)^11 + 30240*cos(x)^9 - 4050*cos(x)^7 + 243*cos(x)^5)*lo
g(2*cos(x)*sin(x) + 1) - 121170735*(32768*cos(x)^25 - 204800*cos(x)^23 + 5
73440*cos(x)^21 - 947200*cos(x)^19 + 1022080*cos(x)^17 - 752800*cos(x)^15
+ 383280*cos(x)^13 - 133200*cos(x)^11 + 30240*cos(x)^9 - 4050*cos(x)^7 + 2
43*cos(x)^5)*log(-2*cos(x)*sin(x) + 1) + 69959680*(32768*sqrt(3)*cos(x)^25
 - 204800*sqrt(3)*cos(x)^23 + 573440*sqrt(3)*cos(x)^21 - 947200*sqrt(3)*co
s(x)^19 + 1022080*sqrt(3)*cos(x)^17 - 752800*sqrt(3)*cos(x)^15 + 383280*sq
rt(3)*cos(x)^13 - 133200*sqrt(3)*cos(x)^11 + 30240*sqrt(3)*cos(x)^9 - 4050
*sqrt(3)*cos(x)^7 + 243*sqrt(3)*cos(x)^5)*log(-(8*cos(x)^4 - 4*(2*sqrt(3)*
cos(x)^3 - 3*sqrt(3)*cos(x))*sin(x) - 9)/(16*cos(x)^4 - 24*cos(x)^2 + 9))
+ 12*(346533396480*cos(x)^24 - 1948244992000*cos(x)^22 + 4842857543680*cos
(x)^20 - 6985286225920*cos(x)^18 + 6442664756480*cos(x)^16 - 3940300651616
*cos(x)^14 + 1598007058712*cos(x)^12 - 414416003556*cos(x)^10 + 6236466975
0*cos(x)^8 - 4150039815*cos(x)^6 + 56916*cos(x)^4 + 1458*cos(x)^2 + 81)*si
n(x))/(32768*cos(x)^25 - 204800*cos(x)^23 + 573440*cos(x)^21 - 947200*cos(
x)^19 + 1022080*cos(x)^17 - 752800*cos(x)^15 + 383280*cos(x)^13 - 133200*c
os(x)^11 + 30240*cos(x)^9 - 4050*cos(x)^7 + 243*cos(x)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(\cos(x)+\cos(5x))^6}\,dx = \text{Timed out}$$

input
```
integrate(1/(cos(x)+cos(5*x))**6,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(\cos(x) + \cos(5x))^6}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(cos(x)+cos(5*x))^6,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.62

$$\int \frac{1}{(\cos(x) + \cos(5x))^6}\, dx$$
$$= \frac{1}{233280}\tan(x)^5 + \frac{11}{69984}\tan(x)^3 + \frac{109312}{729}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3} + 6\tan(x)\right|}{\left|2\sqrt{3} + 6\tan(x)\right|}\right)$$
$$- \frac{37339058295\tan(x)^{19} - 225138333735\tan(x)^{17} + 584006928624\tan(x)^{15} - 852406796600\tan(x)}{}$$
$$+ \frac{33243}{128}\log(|\tan(x) + 1|) - \frac{33243}{128}\log(|\tan(x) - 1|) + \frac{715}{139968}\tan(x)$$

input
```
integrate(1/(cos(x)+cos(5*x))^6,x, algorithm="giac")
```

output
```
1/233280*tan(x)^5 + 11/69984*tan(x)^3 + 109312/729*sqrt(3)*log(abs(-2*sqrt
(3) + 6*tan(x))/abs(2*sqrt(3) + 6*tan(x))) - 1/699840*(37339058295*tan(x)^
19 - 225138333735*tan(x)^17 + 584006928624*tan(x)^15 - 852406796600*tan(x)
^13 + 769144293830*tan(x)^11 - 444064396750*tan(x)^9 + 164043621960*tan(x)
^7 - 37463326112*tan(x)^5 + 4814040715*tan(x)^3 - 266132275*tan(x))/(3*tan
(x)^4 - 4*tan(x)^2 + 1)^5 + 33243/128*log(abs(tan(x) + 1)) - 33243/128*log
(abs(tan(x) - 1)) + 715/139968*tan(x)
```

Mupad [B] (verification not implemented)

Time = 23.76 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.87

$$\int \frac{1}{(\cos(x)+\cos(5x))^6}\,dx = \frac{33243\,\mathrm{atanh}\left(\frac{\sin(x)}{\cos(x)}\right)}{64} - \frac{218624\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\sin(x)}{\cos(x)}\right)}{729} + \frac{1039600189440\sin(x)\cos(x)^{24} - 5844734976000\sin(x)\cos(x)^{22} + 14528572631040\sin(x)\cos(x)}{7644119040\cos(x)^{25} - 47775}$$

input
```
int(1/(cos(5*x) + cos(x))^6,x)
```

output
```
(33243*atanh(sin(x)/cos(x)))/64 - (218624*3^(1/2)*atanh((3^(1/2)*sin(x))/c
os(x)))/729 + (243*sin(x) + 4374*cos(x)^2*sin(x) + 170748*cos(x)^4*sin(x)
- 12450119445*cos(x)^6*sin(x) + 187094009250*cos(x)^8*sin(x) - 12432480106
68*cos(x)^10*sin(x) + 4794021176136*cos(x)^12*sin(x) - 11820901954848*cos(
x)^14*sin(x) + 19327994269440*cos(x)^16*sin(x) - 20955858677760*cos(x)^18*
sin(x) + 14528572631040*cos(x)^20*sin(x) - 5844734976000*cos(x)^22*sin(x)
+ 1039600189440*cos(x)^24*sin(x))/(56687040*cos(x)^5 - 944784000*cos(x)^7
+ 7054387200*cos(x)^9 - 31072896000*cos(x)^11 + 89411558400*cos(x)^13 - 17
5613184000*cos(x)^15 + 238430822400*cos(x)^17 - 220962816000*cos(x)^19 + 1
33772083200*cos(x)^21 - 47775744000*cos(x)^23 + 7644119040*cos(x)^25)
```

Reduce [F]

$$\int \frac{1}{(\cos(x)+\cos(5x))^6}\,dx = \int \frac{1}{\cos(5x)^6 + 6\cos(5x)^5\cos(x) + 15\cos(5x)^4\cos(x)^2 + 20\cos(5x)^3\cos(x)^3 + 15\cos(5x)^2\cos(x)^4 + 6}$$

input
```
int(1/(cos(x)+cos(5*x))^6,x)
```

output
```
int(1/(cos(5*x)**6 + 6*cos(5*x)**5*cos(x) + 15*cos(5*x)**4*cos(x)**2 + 20*
cos(5*x)**3*cos(x)**3 + 15*cos(5*x)**2*cos(x)**4 + 6*cos(5*x)*cos(x)**5 +
cos(x)**6),x)
```

3.49 $\int \frac{1}{\cos(3x)+\cos(5x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 82

$$\int \frac{1}{\cos(3x)+\cos(5x)}\,dx = \frac{1}{2}\operatorname{arctanh}(\sin(x)) + \frac{1}{2}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{2}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)$$

output
```
1/2*arctanh(sin(x))+1/4*(4-2*2^(1/2))^(1/2)*arctanh(2*sin(x)/(2-2^(1/2))^(
1/2))-1/4*(4+2*2^(1/2))^(1/2)*arctanh(2*sin(x)/(2+2^(1/2))^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.56

$$\int \frac{1}{\cos(3x)+\cos(5x)}\,dx$$

$$= -\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-\frac{1}{16}\text{RootSum}\Bigg[1$$

$$+\#1^8\&, \frac{2\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)-i\log\left(1-2\cos(x)\#1+\#1^2\right)-2\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^2+i\log\left(1-\right.}{}$$

input
```
Integrate[(Cos[3*x] + Cos[5*x])^(-1),x]
```

output
```
-1/2*Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]/2 - RootSum[1 + #
1^8 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)] - I*Log[1 - 2*Cos[x]*#1 + #1^2] -
2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 + I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^2 -
 2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^4 + I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^4
+ 2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^6 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^6
)/#1^7 & ]/16
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 4825, 27, 1484, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos(3x)+\cos(5x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\cos(3x)+\cos(5x)}dx$$

$$\Big\downarrow \; 4825$$

$$\int \frac{1}{2\left(1-\sin^2(x)\right)\left(8\sin^4(x)-8\sin^2(x)+1\right)}d\sin(x)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\int \frac{1}{\left(1-\sin^2(x)\right)\left(8\sin^4(x)-8\sin^2(x)+1\right)}d\sin(x)$$

$$\Big\downarrow \; 1484$$

$$\frac{1}{2}\int \left(\frac{8\sin^2(x)}{8\sin^4(x)-8\sin^2(x)+1}+\frac{1}{1-\sin^2(x)}\right)d\sin(x)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\operatorname{arctanh}(\sin(x))+\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)-\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)\right)$$

input

```
Int[(Cos[3*x] + Cos[5*x])^(-1),x]
```

output

```
(ArcTanh[Sin[x]] + Sqrt[(2 - Sqrt[2])/2]*ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[
2]]] - Sqrt[(2 + Sqrt[2])/2]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[2]]])/2
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1484

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4825
```
Int[(cos[(m_.)*((c_.) + (d_.)*(x_))]*(a_.) + cos[(n_.)*((c_.) + (d_.)*(x_))
]*(b_.))^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Cos[
m*ArcSin[x]] + b*Cos[n*ArcSin[x]]]]^p/Sqrt[1 - x^2], x], x, Sin[c + d*x]],
x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[(m - 1)/2] &
& IntegerQ[(n - 1)/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.76 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{\ln(e^{ix}-i)}{2}+\frac{\ln(e^{ix}+i)}{2}+\left(\sum_{\_R=\mathrm{RootOf}(512\_Z^4-64\_Z^2+1)}\_R\ln\left(e^{2ix}+\left(-128i\_R^3+8i\_R\right)e^{ix}-1\right)\right)$
default	$\frac{\ln(1+\sin(x))}{4}-\frac{\left(\sqrt{2}-2\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}-\frac{\sqrt{2}\,\sqrt{2+\sqrt{2}}\,\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{4}-\frac{\ln(\sin(x)-1)}{4}$

input
```
int(1/(cos(3*x)+cos(5*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(exp(I*x)-I)+1/2*ln(exp(I*x)+I)+sum(_R*ln(exp(2*I*x)+(-128*I*_R^3+8
*I*_R)*exp(I*x)-1),_R=RootOf(512*_Z^4-64*_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(54) = 108$.

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.67

$$\int \frac{1}{\cos(3x)+\cos(5x)}\,dx = -\frac{1}{4}\sqrt{\frac{1}{2}\sqrt{2}+1}\log\left(\sqrt{2}\sqrt{\frac{1}{2}\sqrt{2}+1}+2\sin(x)\right) + \frac{1}{4}\sqrt{\frac{1}{2}\sqrt{2}+1}\log\left(\sqrt{2}\sqrt{\frac{1}{2}\sqrt{2}+1}-2\sin(x)\right) + \frac{1}{4}\sqrt{-\frac{1}{2}\sqrt{2}+1}\log\left(\sqrt{2}\sqrt{-\frac{1}{2}\sqrt{2}+1}+2\sin(x)\right) - \frac{1}{4}\sqrt{-\frac{1}{2}\sqrt{2}+1}\log\left(\sqrt{2}\sqrt{-\frac{1}{2}\sqrt{2}+1}-2\sin(x)\right) + \frac{1}{4}\log(\sin(x)+1) - \frac{1}{4}\log(-\sin(x)+1)$$

input
```
integrate(1/(cos(3*x)+cos(5*x)),x, algorithm="fricas")
```

output
```
-1/4*sqrt(1/2*sqrt(2) + 1)*log(sqrt(2)*sqrt(1/2*sqrt(2) + 1) + 2*sin(x)) +
 1/4*sqrt(1/2*sqrt(2) + 1)*log(sqrt(2)*sqrt(1/2*sqrt(2) + 1) - 2*sin(x)) +
 1/4*sqrt(-1/2*sqrt(2) + 1)*log(sqrt(2)*sqrt(-1/2*sqrt(2) + 1) + 2*sin(x))
 - 1/4*sqrt(-1/2*sqrt(2) + 1)*log(sqrt(2)*sqrt(-1/2*sqrt(2) + 1) - 2*sin(x
)) + 1/4*log(sin(x) + 1) - 1/4*log(-sin(x) + 1)
```

Sympy [F]

$$\int \frac{1}{\cos(3x)+\cos(5x)}\,dx = \int \frac{1}{\cos(3x)+\cos(5x)}\,dx$$

input
```
integrate(1/(cos(3*x)+cos(5*x)),x)
```

output
```
Integral(1/(cos(3*x) + cos(5*x)), x)
```

Maxima [F]

$$\int \frac{1}{\cos(3x)+\cos(5x)}\,dx=\int \frac{1}{\cos\left(5\,x\right)+\cos\left(3\,x\right)}\,dx$$

input
```
integrate(1/(cos(3*x)+cos(5*x)),x, algorithm="maxima")
```

output
```
-integrate(((cos(7*x) - cos(5*x) - cos(3*x) + cos(x))*cos(8*x) + (sin(7*x)
 - sin(5*x) - sin(3*x) + sin(x))*sin(8*x) + cos(7*x) - cos(5*x) - cos(3*x)
 + cos(x))/(cos(8*x)^2 + sin(8*x)^2 + 2*cos(8*x) + 1), x) + 1/4*log(cos(x)
^2 + sin(x)^2 + 2*sin(x) + 1) - 1/4*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(54) = 108$.

Time = 0.16 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.45

$$\begin{aligned}\int \frac{1}{\cos(3x)+\cos(5x)}\,dx =& -\frac{1}{8}\sqrt{2\sqrt{2}+4}\log\left(\left|\frac{1}{2}\sqrt{\sqrt{2}+2}+\sin\left(x\right)\right|\right)\\ &+\frac{1}{8}\sqrt{2\sqrt{2}+4}\log\left(\left|-\frac{1}{2}\sqrt{\sqrt{2}+2}+\sin\left(x\right)\right|\right)\\ &+\frac{1}{8}\sqrt{-2\sqrt{2}+4}\log\left(\left|\sqrt{-\frac{1}{4}\sqrt{2}+\frac{1}{2}}+\sin\left(x\right)\right|\right)\\ &-\frac{1}{8}\sqrt{-2\sqrt{2}+4}\log\left(\left|-\sqrt{-\frac{1}{4}\sqrt{2}+\frac{1}{2}}+\sin\left(x\right)\right|\right)\\ &+\frac{1}{4}\log\left(\sin\left(x\right)+1\right)-\frac{1}{4}\log\left(-\sin\left(x\right)+1\right)\end{aligned}$$

input
```
integrate(1/(cos(3*x)+cos(5*x)),x, algorithm="giac")
```

output
```
-1/8*sqrt(2*sqrt(2) + 4)*log(abs(1/2*sqrt(sqrt(2) + 2) + sin(x))) + 1/8*sq
rt(2*sqrt(2) + 4)*log(abs(-1/2*sqrt(sqrt(2) + 2) + sin(x))) + 1/8*sqrt(-2*
sqrt(2) + 4)*log(abs(sqrt(-1/4*sqrt(2) + 1/2) + sin(x))) - 1/8*sqrt(-2*sqr
t(2) + 4)*log(abs(-sqrt(-1/4*sqrt(2) + 1/2) + sin(x))) + 1/4*log(sin(x) +
1) - 1/4*log(-sin(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 21.10 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.35

$$\int \frac{1}{\cos(3x)+\cos(5x)}\,dx = \operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right) - \frac{\sqrt{2}\operatorname{atanh}\left(\frac{363663470886912\sin\left(\frac{x}{2}\right)\sqrt{\sqrt{2}+2}}{\frac{219490008694784\sqrt{2}}{\cos\left(\frac{x}{2}\right)}+\frac{310405876416512}{\cos\left(\frac{x}{2}\right)}}+\frac{257148281946112\sqrt{2}\sin\left(\frac{x}{2}\right)\sqrt{\sqrt{2}+2}}{\frac{219490008694784\sqrt{2}}{\cos\left(\frac{x}{2}\right)}+\frac{310405876416512}{\cos\left(\frac{x}{2}\right)}}\right)\sqrt{\sqrt{2}+2}}{4} - \frac{\sqrt{2}\operatorname{atanh}\left(\frac{363663470886912\sin\left(\frac{x}{2}\right)\sqrt{2-\sqrt{2}}}{\frac{219490008694784\sqrt{2}}{\cos\left(\frac{x}{2}\right)}-\frac{310405876416512}{\cos\left(\frac{x}{2}\right)}}-\frac{257148281946112\sqrt{2}\sin\left(\frac{x}{2}\right)\sqrt{2-\sqrt{2}}}{\frac{219490008694784\sqrt{2}}{\cos\left(\frac{x}{2}\right)}-\frac{310405876416512}{\cos\left(\frac{x}{2}\right)}}\right)\sqrt{2-\sqrt{2}}}{4}$$

input
```
int(1/(cos(3*x) + cos(5*x)),x)
```

output
```
atanh(sin(x/2)/cos(x/2)) - (2^(1/2)*atanh((363663470886912*sin(x/2)*(2^(1/
2) + 2)^(1/2))/((219490008694784*2^(1/2))/cos(x/2) + 310405876416512/cos(x
/2)) + (257148281946112*2^(1/2)*sin(x/2)*(2^(1/2) + 2)^(1/2))/((2194900086
94784*2^(1/2))/cos(x/2) + 310405876416512/cos(x/2))))*(2^(1/2) + 2)^(1/2))/
4 - (2^(1/2)*atanh((363663470886912*sin(x/2)*(2 - 2^(1/2))^(1/2))/((219490
008694784*2^(1/2))/cos(x/2) - 310405876416512/cos(x/2)) - (257148281946112
*2^(1/2)*sin(x/2)*(2 - 2^(1/2))^(1/2))/((219490008694784*2^(1/2))/cos(x/2)
 - 310405876416512/cos(x/2))))*(2 - 2^(1/2))^(1/2))/4
```

Reduce [F]

$$\int \frac{1}{\cos(3x) + \cos(5x)}\,dx = \int \frac{1}{\cos(5x) + \cos(3x)}dx$$

input
```
int(1/(cos(3*x)+cos(5*x)),x)
```

output
```
int(1/(cos(5*x) + cos(3*x)),x)
```

3.50 $\int \frac{1}{(\cos(3x)+\cos(5x))^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 324

$$\begin{aligned}\int \frac{1}{(\cos(3x)+\cos(5x))^3}\,dx =& \frac{49}{16}\text{arctanh}(\sin(x)) \\ &+ \frac{1}{128}\sqrt{842-391\sqrt{2}}\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right) \\ &+ \frac{1}{4}\sqrt{122-71\sqrt{2}}\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right) \\ &- \frac{3}{16}\sqrt{34+7\sqrt{2}}\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right) \\ &+ \frac{3}{16}\sqrt{34-7\sqrt{2}}\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right) \\ &- \frac{1}{4}\sqrt{122+71\sqrt{2}}\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right) \\ &- \frac{1}{128}\sqrt{842+391\sqrt{2}}\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right) \\ &+ \frac{1}{32(1-\sin(x))} - \frac{1}{32(1+\sin(x))} \\ &+ \frac{\sin(x)\left(1-4\sin^2(x)\right)}{8\left(1-8\sin^2(x)+8\sin^4(x)\right)^2} \\ &- \frac{\sin(x)\left(37-56\sin^2(x)\right)}{32\left(1-8\sin^2(x)+8\sin^4(x)\right)} \\ &+ \frac{\sin(x)\left(5-12\sin^2(x)\right)}{4\left(1-8\sin^2(x)+8\sin^4(x)\right)}\end{aligned}$$

output

```
49/16*arctanh(sin(x))+1/128*(842-391*2^(1/2))^(1/2)*arctanh(2*sin(x)/(2-2^
(1/2))^(1/2))+1/4*(122-71*2^(1/2))^(1/2)*arctanh(2*sin(x)/(2-2^(1/2))^(1/2
))-3/16*(34+7*2^(1/2))^(1/2)*arctanh(2*sin(x)/(2-2^(1/2))^(1/2))+3/16*(34-
7*2^(1/2))^(1/2)*arctanh(2*sin(x)/(2+2^(1/2))^(1/2))-1/4*(122+71*2^(1/2))^
(1/2)*arctanh(2*sin(x)/(2+2^(1/2))^(1/2))-1/128*(842+391*2^(1/2))^(1/2)*ar
ctanh(2*sin(x)/(2+2^(1/2))^(1/2))+1/(32-32*sin(x))-1/(32+32*sin(x))+1/8*si
n(x)*(1-4*sin(x)^2)/(1-8*sin(x)^2+8*sin(x)^4)^2-sin(x)*(37-56*sin(x)^2)/(3
2-256*sin(x)^2+256*sin(x)^4)+sin(x)*(5-12*sin(x)^2)/(4-32*sin(x)^2+32*sin(
x)^4)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.61 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.02

$$\int \frac{1}{(\cos(3x)+\cos(5x))^3}\,dx$$
$$= \frac{1}{512}\left(-1568\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+1568\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-\text{RootSum}\left[1 +\#1^8\&, \frac{364\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)-182i\log\left(1-2\cos(x)\#1+\#1^2\right)-158\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^2+7\right.\right.$$
$$+\frac{16}{\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)^2}-\frac{16}{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2}+\frac{32(\cos(x)-2\sin(x))}{(\cos(2x)-\sin(2x))^2}$$
$$\left.+\frac{8(-21\cos(x)+41\sin(x))}{\cos(2x)-\sin(2x)}-\frac{32(\cos(x)+2\sin(x))}{(\cos(2x)+\sin(2x))^2}+\frac{8(21\cos(x)+41\sin(x))}{\cos(2x)+\sin(2x)}\right)$$

input
```
Integrate[(Cos[3*x] + Cos[5*x])^(-3),x]
```

output
```
(-1568*Log[Cos[x/2] - Sin[x/2]] + 1568*Log[Cos[x/2] + Sin[x/2]] - RootSum[
1 + #1^8 & , (364*ArcTan[Sin[x]/(Cos[x] - #1)] - (182*I)*Log[1 - 2*Cos[x]*
#1 + #1^2] - 158*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 + (79*I)*Log[1 - 2*Cos[
x]*#1 + #1^2]*#1^2 - 158*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^4 + (79*I)*Log[1
- 2*Cos[x]*#1 + #1^2]*#1^4 + 364*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^6 - (182*
I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^6)/#1^7 & ] + 16/(Cos[x/2] - Sin[x/2])^2
 - 16/(Cos[x/2] + Sin[x/2])^2 + (32*(Cos[x] - 2*Sin[x]))/(Cos[2*x] - Sin[2
*x])^2 + (8*(-21*Cos[x] + 41*Sin[x]))/(Cos[2*x] - Sin[2*x]) - (32*(Cos[x]
+ 2*Sin[x]))/(Cos[2*x] + Sin[2*x])^2 + (8*(21*Cos[x] + 41*Sin[x]))/(Cos[2*
x] + Sin[2*x]))/512
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 4825, 27, 1567, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos(3x)+\cos(5x))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(\cos(3x)+\cos(5x))^3}dx$$

$$\Big\downarrow \text{ 4825}$$

$$\int \frac{1}{8\left(1-\sin^2(x)\right)^2\left(8\sin^4(x)-8\sin^2(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{8}\int \frac{1}{\left(1-\sin^2(x)\right)^2\left(8\sin^4(x)-8\sin^2(x)+1\right)^3}d\sin(x)$$

$$\Big\downarrow \text{ 1567}$$

$$\frac{1}{8}\int \left(\frac{8\left(8\sin^2(x)-1\right)}{\left(8\sin^4(x)-8\sin^2(x)+1\right)^3}-\frac{49}{2\left(\sin^2(x)-1\right)}+\frac{8\left(24\sin^2(x)-1\right)}{8\sin^4(x)-8\sin^2(x)+1}+\frac{1}{4(\sin(x)-1)^2}+\frac{1}{4(\sin(x)+\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{8}\left(\frac{49}{2}\text{arctanh}(\sin(x))-\frac{3}{2}\sqrt{34+7\sqrt{2}}\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)+2\sqrt{122-71\sqrt{2}}\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{16}\sqrt{842\right.$$

input
```
Int[(Cos[3*x] + Cos[5*x])^(-3),x]
```

output
```
((49*ArcTanh[Sin[x]])/2 + (Sqrt[842 - 391*Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt
[2 - Sqrt[2]]])/16 + 2*Sqrt[122 - 71*Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 -
Sqrt[2]]] - (3*Sqrt[34 + 7*Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]])
/2 + (3*Sqrt[34 - 7*Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[2]]])/2 - 2*
Sqrt[122 + 71*Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[2]]] - (Sqrt[842 +
 391*Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[2]]])/16 + 1/(4*(1 - Sin[x]
)) - 1/(4*(1 + Sin[x])) + (Sin[x]*(1 - 4*Sin[x]^2))/(1 - 8*Sin[x]^2 + 8*Si
n[x]^4)^2 - (Sin[x]*(37 - 56*Sin[x]^2))/(4*(1 - 8*Sin[x]^2 + 8*Sin[x]^4))
+ (2*Sin[x]*(5 - 12*Sin[x]^2))/(1 - 8*Sin[x]^2 + 8*Sin[x]^4))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1567
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4825
```
Int[(cos[(m_.)*((c_.) + (d_.)*(x_))]*(a_.) + cos[(n_.)*((c_.) + (d_.)*(x_))
]*(b_.))^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Cos[
m*ArcSin[x]] + b*Cos[n*ArcSin[x]]]]^p/Sqrt[1 - x^2], x], x, Sin[c + d*x]],
x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[(m - 1)/2] &
& IntegerQ[(n - 1)/2]
```

Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.43

method	result
default	$-\frac{1}{32(\sin(x)-1)}-\frac{49\ln(\sin(x)-1)}{32}+\frac{-10\sin(x)^7+\frac{43\sin(x)^5}{4}-\frac{5\sin(x)^3}{2}+\frac{7\sin(x)}{32}}{\left(1-8\sin(x)^2+8\sin(x)^4\right)^2}-\frac{\left(261+182\sqrt{2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{128\sqrt{2+\sqrt{2}}}-$
risch	$-\frac{i\left(14\,e^{19ix}-11\,e^{17ix}-9\,e^{15ix}+17\,e^{13ix}+17\,e^{11ix}-17\,e^{9ix}-17\,e^{7ix}+9\,e^{5ix}+11\,e^{3ix}-14\,e^{ix}\right)}{32\left(e^{10ix}+e^{8ix}+e^{2ix}+1\right)^2}+\frac{49\ln\left(e^{ix}+i\right)}{16}-\frac{49\ln\left(e^{ix}-i\right)}{16}+$

input
```
int(1/(cos(3*x)+cos(5*x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/32/(sin(x)-1)-49/32*ln(sin(x)-1)+64*(-5/32*sin(x)^7+43/256*sin(x)^5-5/1
28*sin(x)^3+7/2048*sin(x))/(1-8*sin(x)^2+8*sin(x)^4)^2-1/128*(261+182*2^(1
/2))*2^(1/2)/(2+2^(1/2))^(1/2)*arctanh(2*sin(x)/(2+2^(1/2))^(1/2))-1/128*(
-261+182*2^(1/2))*2^(1/2)/(2-2^(1/2))^(1/2)*arctanh(2*sin(x)/(2-2^(1/2))^(
1/2))-1/32/(1+sin(x))+49/32*ln(1+sin(x))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.20

$$\int \frac{1}{(\cos(3x)+\cos(5x))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cos(3*x)+cos(5*x))^3,x, algorithm="fricas")
```

output
```
-1/256*((64*cos(x)^10 - 128*cos(x)^8 + 80*cos(x)^6 - 16*cos(x)^4 + cos(x)^
2)*sqrt(55639*sqrt(2) + 78730)*log(sqrt(55639*sqrt(2) + 78730)*(79*sqrt(2)
 - 103) + 3746*sin(x)) - (64*cos(x)^10 - 128*cos(x)^8 + 80*cos(x)^6 - 16*c
os(x)^4 + cos(x)^2)*sqrt(55639*sqrt(2) + 78730)*log(sqrt(55639*sqrt(2) + 7
8730)*(79*sqrt(2) - 103) - 3746*sin(x)) - (64*cos(x)^10 - 128*cos(x)^8 + 8
0*cos(x)^6 - 16*cos(x)^4 + cos(x)^2)*sqrt(-55639*sqrt(2) + 78730)*log((79*
sqrt(2) + 103)*sqrt(-55639*sqrt(2) + 78730) + 3746*sin(x)) + (64*cos(x)^10
 - 128*cos(x)^8 + 80*cos(x)^6 - 16*cos(x)^4 + cos(x)^2)*sqrt(-55639*sqrt(2
) + 78730)*log((79*sqrt(2) + 103)*sqrt(-55639*sqrt(2) + 78730) - 3746*sin(
x)) - 392*(64*cos(x)^10 - 128*cos(x)^8 + 80*cos(x)^6 - 16*cos(x)^4 + cos(x
)^2)*log(sin(x) + 1) + 392*(64*cos(x)^10 - 128*cos(x)^8 + 80*cos(x)^6 - 16
*cos(x)^4 + cos(x)^2)*log(-sin(x) + 1) - 8*(448*cos(x)^8 - 872*cos(x)^6 +
512*cos(x)^4 - 81*cos(x)^2 + 2)*sin(x))/(64*cos(x)^10 - 128*cos(x)^8 + 80*
cos(x)^6 - 16*cos(x)^4 + cos(x)^2)
```

Sympy [F]

$$\int \frac{1}{(\cos(3x)+\cos(5x))^3}\,dx = \int \frac{1}{(\cos(3x)+\cos(5x))^3}\,dx$$

input
```
integrate(1/(cos(3*x)+cos(5*x))**3,x)
```

output
```
Integral((cos(3*x) + cos(5*x))**(-3), x)
```

Maxima [F]

$$\int \frac{1}{(\cos(3x)+\cos(5x))^3}\,dx = \int \frac{1}{(\cos(5x)+\cos(3x))^3}\,dx$$

input
```
integrate(1/(cos(3*x)+cos(5*x))^3,x, algorithm="maxima")
```

output
```
1/32*((14*sin(19*x) - 11*sin(17*x) - 9*sin(15*x) + 17*sin(13*x) + 17*sin(1
1*x) - 17*sin(9*x) - 17*sin(7*x) + 9*sin(5*x) + 11*sin(3*x) - 14*sin(x))*c
os(20*x) - 14*(2*sin(18*x) + sin(16*x) + 2*sin(12*x) + 4*sin(10*x) + 2*sin
(8*x) + sin(4*x) + 2*sin(2*x))*cos(19*x) - 2*(11*sin(17*x) + 9*sin(15*x) -
 17*sin(13*x) - 17*sin(11*x) + 17*sin(9*x) + 17*sin(7*x) - 9*sin(5*x) - 11
*sin(3*x) + 14*sin(x))*cos(18*x) + 11*(sin(16*x) + 2*sin(12*x) + 4*sin(10*
x) + 2*sin(8*x) + sin(4*x) + 2*sin(2*x))*cos(17*x) - (9*sin(15*x) - 17*sin
(13*x) - 17*sin(11*x) + 17*sin(9*x) + 17*sin(7*x) - 9*sin(5*x) - 11*sin(3*
x) + 14*sin(x))*cos(16*x) + 9*(2*sin(12*x) + 4*sin(10*x) + 2*sin(8*x) + si
n(4*x) + 2*sin(2*x))*cos(15*x) - 17*(2*sin(12*x) + 4*sin(10*x) + 2*sin(8*x
) + sin(4*x) + 2*sin(2*x))*cos(13*x) + 2*(17*sin(11*x) - 17*sin(9*x) - 17*
sin(7*x) + 9*sin(5*x) + 11*sin(3*x) - 14*sin(x))*cos(12*x) - 17*(4*sin(10*
x) + 2*sin(8*x) + sin(4*x) + 2*sin(2*x))*cos(11*x) - 4*(17*sin(9*x) + 17*s
in(7*x) - 9*sin(5*x) - 11*sin(3*x) + 14*sin(x))*cos(10*x) + 17*(2*sin(8*x)
 + sin(4*x) + 2*sin(2*x))*cos(9*x) - 2*(17*sin(7*x) - 9*sin(5*x) - 11*sin(
3*x) + 14*sin(x))*cos(8*x) + 17*(sin(4*x) + 2*sin(2*x))*cos(7*x) - 9*(sin(
4*x) + 2*sin(2*x))*cos(5*x) + (11*sin(3*x) - 14*sin(x))*cos(4*x) - 32*(2*(
2*cos(18*x) + cos(16*x) + 2*cos(12*x) + 4*cos(10*x) + 2*cos(8*x) + cos(4*x
) + 2*cos(2*x) + 1)*cos(20*x) + cos(20*x)^2 + 4*(cos(16*x) + 2*cos(12*x) +
 4*cos(10*x) + 2*cos(8*x) + cos(4*x) + 2*cos(2*x) + 1)*cos(18*x) + 4*co...
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.53

$$\begin{aligned}
\int \frac{1}{(\cos(3x)+\cos(5x))^3}\,dx
&= -\frac{1}{256}\sqrt{55639\sqrt{2}+78730}\log\left(\left|\frac{1}{2}\sqrt{\sqrt{2}+2}+\sin(x)\right|\right) \\
&\quad+\frac{1}{256}\sqrt{55639\sqrt{2}+78730}\log\left(\left|-\frac{1}{2}\sqrt{\sqrt{2}+2}+\sin(x)\right|\right) \\
&\quad+\frac{1}{256}\sqrt{-55639\sqrt{2}+78730}\log\left(\left|\sqrt{-\frac{1}{4}\sqrt{2}+\frac{1}{2}}+\sin(x)\right|\right) \\
&\quad-\frac{1}{256}\sqrt{-55639\sqrt{2}+78730}\log\left(\left|-\sqrt{-\frac{1}{4}\sqrt{2}+\frac{1}{2}}+\sin(x)\right|\right) \\
&\quad-\frac{\sin(x)}{16\left(\sin(x)^2-1\right)}-\frac{320\sin(x)^7-344\sin(x)^5+80\sin(x)^3-7\sin(x)}{32\left(8\sin(x)^4-8\sin(x)^2+1\right)^2} \\
&\quad+\frac{49}{32}\log(\sin(x)+1)-\frac{49}{32}\log(-\sin(x)+1)
\end{aligned}$$

input
```
integrate(1/(cos(3*x)+cos(5*x))^3,x, algorithm="giac")
```

output
```
-1/256*sqrt(55639*sqrt(2) + 78730)*log(abs(1/2*sqrt(sqrt(2) + 2) + sin(x))
) + 1/256*sqrt(55639*sqrt(2) + 78730)*log(abs(-1/2*sqrt(sqrt(2) + 2) + sin
(x))) + 1/256*sqrt(-55639*sqrt(2) + 78730)*log(abs(sqrt(-1/4*sqrt(2) + 1/2
) + sin(x))) - 1/256*sqrt(-55639*sqrt(2) + 78730)*log(abs(-sqrt(-1/4*sqrt(
2) + 1/2) + sin(x))) - 1/16*sin(x)/(sin(x)^2 - 1) - 1/32*(320*sin(x)^7 - 3
44*sin(x)^5 + 80*sin(x)^3 - 7*sin(x))/(8*sin(x)^4 - 8*sin(x)^2 + 1)^2 + 49
/32*log(sin(x) + 1) - 49/32*log(-sin(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 21.63 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.15

$$\int \frac{1}{(\cos(3x)+\cos(5x))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(3*x) + cos(5*x))^3,x)
```

output
```
(49*atanh(tan(x/2)))/8 + (atanh((21921344091656122463*tan(x/2)*(78730 - 55
639*2^(1/2))^(1/2))/(16*((22727474752897861007033*2^(1/2))/128 + (22727474
752897861007033*2^(1/2)*tan(x/2)^2)/128 - (64283006104824152616365*tan(x/2
)^2)/256 - 64283006104824152616365/256)) - (12400585864893807781 7*2^(1/2)*
tan(x/2)*(78730 - 55639*2^(1/2))^(1/2))/(128*((22727474752897861007033*2^(
1/2))/128 + (22727474752897861007033*2^(1/2)*tan(x/2)^2)/128 - (6428300610
4824152616365*tan(x/2)^2)/256 - 64283006104824152616365/256)))*(78730 - 55
639*2^(1/2))^(1/2))/128 - (atanh((21921344091656122463*tan(x/2)*(55639*2^(
1/2) + 78730)^(1/2))/(16*((22727474752897861007033*2^(1/2))/128 + (2272747
4752897861007033*2^(1/2)*tan(x/2)^2)/128 + (64283006104824152616365*tan(x/
2)^2)/256 + 64283006104824152616365/256)) + (124005858648938077817*2^(1/2)
*tan(x/2)*(55639*2^(1/2) + 78730)^(1/2))/(128*((22727474752897861007033*2^
(1/2))/128 + (22727474752897861007033*2^(1/2)*tan(x/2)^2)/128 + (642830061
04824152616365*tan(x/2)^2)/256 + 64283006104824152616365/256)))*(55639*2^(
1/2) + 78730)^(1/2))/128 + ((9*tan(x/2))/16 - (395*tan(x/2)^3)/16 + 396*ta
n(x/2)^5 - (2675*tan(x/2)^7)/2 + (7981*tan(x/2)^9)/8 + (7981*tan(x/2)^11)/
8 - (2675*tan(x/2)^13)/2 + 396*tan(x/2)^15 - (395*tan(x/2)^17)/16 + (9*tan
(x/2)^19)/16)/(1037*tan(x/2)^4 - 58*tan(x/2)^2 - 5880*tan(x/2)^6 + 15346*t
an(x/2)^8 - 20892*tan(x/2)^10 + 15346*tan(x/2)^12 - 5880*tan(x/2)^14 + 103
7*tan(x/2)^16 - 58*tan(x/2)^18 + tan(x/2)^20 + 1)
```

Reduce [F]

$$\int \frac{1}{(\cos(3x)+\cos(5x))^3}\,dx = \int \frac{1}{\cos(5x)^3+3\cos(5x)^2\cos(3x)+3\cos(5x)\cos(3x)^2+\cos(3x)^3}dx$$

input
```
int(1/(cos(3*x)+cos(5*x))^3,x)
```

output
```
int(1/(cos(5*x)**3 + 3*cos(5*x)**2*cos(3*x) + 3*cos(5*x)*cos(3*x)**2 + cos
(3*x)**3),x)
```

3.51 $\int \frac{1}{(\cos(3x)+\cos(5x))^5} dx$

Optimal result

Integrand size = 11, antiderivative size = 729

$$\int \frac{1}{(\cos(3x) + \cos(5x))^5} dx = \text{Too large to display}$$

output

```
1/512/(1-sin(x))^2-1/512/(1+sin(x))^2-1/24*sin(x)*(75-121*sin(x)^2)/(1-8*s
in(x)^2+8*sin(x)^4)^2+1/512*sin(x)*(57-286*sin(x)^2)/(1-8*sin(x)^2+8*sin(x
)^4)^2+1/16*sin(x)*(1-6*sin(x)^2)/(1-8*sin(x)^2+8*sin(x)^4)^4-15*sin(x)*(1
3-21*sin(x)^2)/(16-128*sin(x)^2+128*sin(x)^4)-5*sin(x)*(295-441*sin(x)^2)/
(1024-8192*sin(x)^2+8192*sin(x)^4)+1/6*sin(x)*(5-14*sin(x)^2)/(1-8*sin(x)^
2+8*sin(x)^4)^3+3/8*sin(x)*(9-22*sin(x)^2)/(1-8*sin(x)^2+8*sin(x)^4)^2-1/3
2*sin(x)*(23-35*sin(x)^2)/(1-8*sin(x)^2+8*sin(x)^4)^3+7523/256*arctanh(sin
(x))-1/512*(775268+166546*2^(1/2))^(1/2)*arctanh(2*sin(x)/(2-2^(1/2))^(1/2
))+1/512*(775268-166546*2^(1/2))^(1/2)*arctanh(2*sin(x)/(2+2^(1/2))^(1/2))
-15/16384*(84116+43202*2^(1/2))^(1/2)*arctanh(2*sin(x)/(2+2^(1/2))^(1/2))+
15/16384*(84116-43202*2^(1/2))^(1/2)*arctanh(2*sin(x)/(2-2^(1/2))^(1/2))-5
/8*(1780+1042*2^(1/2))^(1/2)*arctanh(2*sin(x)/(2+2^(1/2))^(1/2))+5/8*(1780
-1042*2^(1/2))^(1/2)*arctanh(2*sin(x)/(2-2^(1/2))^(1/2))-3/8*(772+146*2^(1
/2))^(1/2)*arctanh(2*sin(x)/(2-2^(1/2))^(1/2))+3/8*(772-146*2^(1/2))^(1/2)
*arctanh(2*sin(x)/(2+2^(1/2))^(1/2))-45/256*(340+82*2^(1/2))^(1/2)*arctanh
(2*sin(x)/(2+2^(1/2))^(1/2))+45/256*(340-82*2^(1/2))^(1/2)*arctanh(2*sin(x
)/(2-2^(1/2))^(1/2))+sin(x)*(13-30*sin(x)^2)/(1-8*sin(x)^2+8*sin(x)^4)+sin
(x)*(359-882*sin(x)^2)/(192-1536*sin(x)^2+1536*sin(x)^4)+163/(512-512*sin(
x))-163/(512+512*sin(x))
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 6.08 (sec) , antiderivative size = 490, normalized size of antiderivative = 0.67

$$\int \frac{1}{(\cos(3x)+\cos(5x))^5}\,dx$$
$$= -\frac{7523}{256}\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\frac{7523}{256}\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)$$
$$-\frac{\text{RootSum}\left[1+\#1^8\&,\frac{444922\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)-222461i\log\left(1-2\cos(x)\#1+\#1^2\right)-184578\arctan\left(\frac{\sin(x)}{\cos(x)-\#1}\right)\#1^2+9}{}\right.}{}$$
$$+\frac{1}{512\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)^4}+\frac{163}{512\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)^2}-\frac{1}{512\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^4}$$
$$-\frac{163}{512\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2}+\frac{3\cos(x)-7\sin(x)}{128(\cos(2x)-\sin(2x))^4}+\frac{-115\cos(x)+234\sin(x)}{768(\cos(2x)-\sin(2x))^3}$$
$$+\frac{4289\cos(x)-8381\sin(x)}{6144(\cos(2x)-\sin(2x))^2}+\frac{-13375\cos(x)+26434\sin(x)}{4096(\cos(2x)-\sin(2x))}$$
$$+\frac{-3\cos(x)-7\sin(x)}{128(\cos(2x)+\sin(2x))^4}+\frac{115\cos(x)+234\sin(x)}{768(\cos(2x)+\sin(2x))^3}$$
$$+\frac{-4289\cos(x)-8381\sin(x)}{6144(\cos(2x)+\sin(2x))^2}+\frac{13375\cos(x)+26434\sin(x)}{4096(\cos(2x)+\sin(2x))}$$

input
```
Integrate[(Cos[3*x] + Cos[5*x])^(-5),x]
```

output
```
(-7523*Log[Cos[x/2] - Sin[x/2]])/256 + (7523*Log[Cos[x/2] + Sin[x/2]])/256
 - RootSum[1 + #1^8 & , (444922*ArcTan[Sin[x]/(Cos[x] - #1)] - (222461*I)*
Log[1 - 2*Cos[x]*#1 + #1^2] - 184578*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 + (
92289*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^2 - 184578*ArcTan[Sin[x]/(Cos[x] -
 #1)]*#1^4 + (92289*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^4 + 444922*ArcTan[Si
n[x]/(Cos[x] - #1)]*#1^6 - (222461*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^6)/#1
^7 & ]/65536 + 1/(512*(Cos[x/2] - Sin[x/2])^4) + 163/(512*(Cos[x/2] - Sin[
x/2])^2) - 1/(512*(Cos[x/2] + Sin[x/2])^4) - 163/(512*(Cos[x/2] + Sin[x/2]
)^2) + (3*Cos[x] - 7*Sin[x])/(128*(Cos[2*x] - Sin[2*x])^4) + (-115*Cos[x]
+ 234*Sin[x])/(768*(Cos[2*x] - Sin[2*x])^3) + (4289*Cos[x] - 8381*Sin[x])/
(6144*(Cos[2*x] - Sin[2*x])^2) + (-13375*Cos[x] + 26434*Sin[x])/(4096*(Cos
[2*x] - Sin[2*x])) + (-3*Cos[x] - 7*Sin[x])/(128*(Cos[2*x] + Sin[2*x])^4)
+ (115*Cos[x] + 234*Sin[x])/(768*(Cos[2*x] + Sin[2*x])^3) + (-4289*Cos[x]
- 8381*Sin[x])/(6144*(Cos[2*x] + Sin[2*x])^2) + (13375*Cos[x] + 26434*Sin[
x])/(4096*(Cos[2*x] + Sin[2*x]))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 710, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 4825, 27, 1567, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos(3x)+\cos(5x))^5}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(\cos(3x)+\cos(5x))^5}dx$$

$$\Big\downarrow \; 4825$$

$$\int \frac{1}{32\left(1-\sin^2(x)\right)^3\left(8\sin^4(x)-8\sin^2(x)+1\right)^5}d\sin(x)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{32}\int \frac{1}{\left(1-\sin^2(x)\right)^3\left(8\sin^4(x)-8\sin^2(x)+1\right)^5}d\sin(x)$$

$$\Big\downarrow \; 1567$$

$$\frac{1}{32}\int \left(\frac{64\left(7\sin^2(x)-1\right)}{\left(8\sin^4(x)-8\sin^2(x)+1\right)^5}-\frac{7523}{8\left(\sin^2(x)-1\right)}+\frac{320\left(23\sin^2(x)-1\right)}{8\sin^4(x)-8\sin^2(x)+1}+\frac{163}{16(\sin(x)-1)^2}+\frac{16}{16(\sin(x$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{32}\left(\frac{7523}{8}\text{arctanh}(\sin(x))-\frac{1}{8}\sqrt{\frac{1}{2}\left(387634+83273\sqrt{2}\right)}\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)-12\sqrt{2\left(386+73\sqrt{2}\right)}\text{arctanh}\left(\right.$$

input
```
Int[(Cos[3*x] + Cos[5*x])^(-5),x]
```

output

```
((7523*ArcTanh[Sin[x]])/8 + (15*Sqrt[(42058 - 21601*Sqrt[2])/2]*ArcTanh[(2
*Sin[x])/Sqrt[2 - Sqrt[2]]])/256 + 20*Sqrt[2*(890 - 521*Sqrt[2])]*ArcTanh[
(2*Sin[x])/Sqrt[2 - Sqrt[2]]] + (45*Sqrt[(170 - 41*Sqrt[2])/2]*ArcTanh[(2*
Sin[x])/Sqrt[2 - Sqrt[2]]])/4 - 12*Sqrt[2*(386 + 73*Sqrt[2])]*ArcTanh[(2*S
in[x])/Sqrt[2 - Sqrt[2]]] - (Sqrt[(387634 + 83273*Sqrt[2])/2]*ArcTanh[(2*S
in[x])/Sqrt[2 - Sqrt[2]]])/8 + (Sqrt[(387634 - 83273*Sqrt[2])/2]*ArcTanh[(
2*Sin[x])/Sqrt[2 + Sqrt[2]]])/8 + 12*Sqrt[2*(386 - 73*Sqrt[2])]*ArcTanh[(2
*Sin[x])/Sqrt[2 + Sqrt[2]]] - (45*Sqrt[(170 + 41*Sqrt[2])/2]*ArcTanh[(2*Si
n[x])/Sqrt[2 + Sqrt[2]]])/4 - 20*Sqrt[2*(890 + 521*Sqrt[2])]*ArcTanh[(2*Si
n[x])/Sqrt[2 + Sqrt[2]]] - (15*Sqrt[(42058 + 21601*Sqrt[2])/2]*ArcTanh[(2*
Sin[x])/Sqrt[2 + Sqrt[2]]])/256 + 1/(16*(1 - Sin[x])^2) + 163/(16*(1 - Sin
[x])) - 1/(16*(1 + Sin[x])^2) - 163/(16*(1 + Sin[x])) + (2*Sin[x]*(1 - 6*S
in[x]^2))/(1 - 8*Sin[x]^2 + 8*Sin[x]^4)^4 - (Sin[x]*(23 - 35*Sin[x]^2))/(1
 - 8*Sin[x]^2 + 8*Sin[x]^4)^3 + (16*Sin[x]*(5 - 14*Sin[x]^2))/(3*(1 - 8*Si
n[x]^2 + 8*Sin[x]^4)^3) + (Sin[x]*(57 - 286*Sin[x]^2))/(16*(1 - 8*Sin[x]^2
 + 8*Sin[x]^4)^2) - (4*Sin[x]*(75 - 121*Sin[x]^2))/(3*(1 - 8*Sin[x]^2 + 8*
Sin[x]^4)^2) + (12*Sin[x]*(9 - 22*Sin[x]^2))/(1 - 8*Sin[x]^2 + 8*Sin[x]^4)
^2 + (Sin[x]*(359 - 882*Sin[x]^2))/(6*(1 - 8*Sin[x]^2 + 8*Sin[x]^4)) - (5*
Sin[x]*(295 - 441*Sin[x]^2))/(32*(1 - 8*Sin[x]^2 + 8*Sin[x]^4)) + (32*Sin[
x]*(13 - 30*Sin[x]^2))/(1 - 8*Sin[x]^2 + 8*Sin[x]^4) - (30*Sin[x]*(13 -...
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1567

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4825
```
Int[(cos[(m_.)*((c_.) + (d_.)*(x_))]*(a_.) + cos[(n_.)*((c_.) + (d_.)*(x_))
]*(b_.))^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Cos[
m*ArcSin[x]] + b*Cos[n*ArcSin[x]]]]^p/Sqrt[1 - x^2], x], x, Sin[c + d*x]],
x] /; FreeQ[{a, b, c, d}, x] && ILtQ[(p - 1)/2, 0] && IntegerQ[(m - 1)/2] &
& IntegerQ[(n - 1)/2]
```

Maple [A] (verified)

Time = 70.04 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.25

method	result
default	$\frac{1}{512(\sin(x)-1)^2}-\frac{163}{512(\sin(x)-1)}-\frac{7523\ln(\sin(x)-1)}{512}+\frac{\frac{1823\sin(x)}{1024}-\frac{167845\sin(x)^3}{3072}+\frac{64735\sin(x)^5}{96}-\frac{1622347\sin(x)^7}{384}+\frac{675737\sin}{48}}{\left(1-8\sin(x)^2+8\sin(x\right.}$
risch	$-\frac{i\left(54825\,\mathrm{e}^{39ix}+70853\,\mathrm{e}^{37ix}-65023\,\mathrm{e}^{35ix}-70635\,\mathrm{e}^{33ix}+239574\,\mathrm{e}^{31ix}+269918\,\mathrm{e}^{29ix}-256978\,\mathrm{e}^{27ix}-275034\,\mathrm{e}^{25ix}+395292\,\mathrm{e}^{23ix}+3\right.}{1228}$

input
```
int(1/(cos(3*x)+cos(5*x))^5,x,method=_RETURNVERBOSE)
```

output
```
1/512/(sin(x)-1)^2-163/512/(sin(x)-1)-7523/512*ln(sin(x)-1)+8192*(1823/838
8608*sin(x)-167845/25165824*sin(x)^3+64735/786432*sin(x)^5-1622347/3145728
*sin(x)^7+675737/393216*sin(x)^9-1160911/393216*sin(x)^11+60673/24576*sin(
x)^13-13059/16384*sin(x)^15)/(1-8*sin(x)^2+8*sin(x)^4)^4-1/16384*(314750+2
22461*2^(1/2))*2^(1/2)/(2+2^(1/2))^(1/2)*arctanh(2*sin(x)/(2+2^(1/2))^(1/2
))-1/16384*2^(1/2)*(-314750+222461*2^(1/2))/(2-2^(1/2))^(1/2)*arctanh(2*si
n(x)/(2-2^(1/2))^(1/2))-1/512/(1+sin(x))^2-163/512/(1+sin(x))+7523/512*ln(
1+sin(x))
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 588, normalized size of antiderivative = 0.81

$$\int \frac{1}{(\cos(3x)+\cos(5x))^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cos(3*x)+cos(5*x))^5,x, algorithm="fricas")
```

output
```
-1/49152*(3*(4096*cos(x)^20 - 16384*cos(x)^18 + 26624*cos(x)^16 - 22528*co
s(x)^14 + 10624*cos(x)^12 - 2816*cos(x)^10 + 416*cos(x)^8 - 32*cos(x)^6 +
cos(x)^4)*sqrt(41016521729/2*sqrt(2) + 29003078021)*log(sqrt(41016521729/2
*sqrt(2) + 29003078021)*(92289*sqrt(2) - 130172) + 89769458*sin(x)) - 3*(4
096*cos(x)^20 - 16384*cos(x)^18 + 26624*cos(x)^16 - 22528*cos(x)^14 + 1062
4*cos(x)^12 - 2816*cos(x)^10 + 416*cos(x)^8 - 32*cos(x)^6 + cos(x)^4)*sqrt
(41016521729/2*sqrt(2) + 29003078021)*log(sqrt(41016521729/2*sqrt(2) + 290
03078021)*(92289*sqrt(2) - 130172) - 89769458*sin(x)) - 3*(4096*cos(x)^20
- 16384*cos(x)^18 + 26624*cos(x)^16 - 22528*cos(x)^14 + 10624*cos(x)^12 -
2816*cos(x)^10 + 416*cos(x)^8 - 32*cos(x)^6 + cos(x)^4)*sqrt(-41016521729/
2*sqrt(2) + 29003078021)*log((92289*sqrt(2) + 130172)*sqrt(-41016521729/2*
sqrt(2) + 29003078021) + 89769458*sin(x)) + 3*(4096*cos(x)^20 - 16384*cos(
x)^18 + 26624*cos(x)^16 - 22528*cos(x)^14 + 10624*cos(x)^12 - 2816*cos(x)^
10 + 416*cos(x)^8 - 32*cos(x)^6 + cos(x)^4)*sqrt(-41016521729/2*sqrt(2) +
29003078021)*log((92289*sqrt(2) + 130172)*sqrt(-41016521729/2*sqrt(2) + 29
003078021) - 89769458*sin(x)) - 722208*(4096*cos(x)^20 - 16384*cos(x)^18 +
 26624*cos(x)^16 - 22528*cos(x)^14 + 10624*cos(x)^12 - 2816*cos(x)^10 + 41
6*cos(x)^8 - 32*cos(x)^6 + cos(x)^4)*log(sin(x) + 1) + 722208*(4096*cos(x)
^20 - 16384*cos(x)^18 + 26624*cos(x)^16 - 22528*cos(x)^14 + 10624*cos(x)^1
2 - 2816*cos(x)^10 + 416*cos(x)^8 - 32*cos(x)^6 + cos(x)^4)*log(-sin(x)...
```

Sympy [F]

$$\int \frac{1}{(\cos(3x) + \cos(5x))^5}\, dx = \int \frac{1}{(\cos{(3x)} + \cos{(5x)})^5}\, dx$$

input
```
integrate(1/(cos(3*x)+cos(5*x))**5,x)
```

output
```
Integral((cos(3*x) + cos(5*x))**(-5), x)
```

Maxima [F]

$$\int \frac{1}{(\cos(3x) + \cos(5x))^5}\, dx = \int \frac{1}{(\cos(5x) + \cos(3x))^5}\, dx$$

input
```
integrate(1/(cos(3*x)+cos(5*x))^5,x, algorithm="maxima")
```

output
```
1/12288*((54825*sin(39*x) + 70853*sin(37*x) - 65023*sin(35*x) - 70635*sin(
33*x) + 239574*sin(31*x) + 269918*sin(29*x) - 256978*sin(27*x) - 275034*si
n(25*x) + 395292*sin(23*x) + 384876*sin(21*x) - 384876*sin(19*x) - 395292*
sin(17*x) + 275034*sin(15*x) + 256978*sin(13*x) - 269918*sin(11*x) - 23957
4*sin(9*x) + 70635*sin(7*x) + 65023*sin(5*x) - 70853*sin(3*x) - 54825*sin(
x))*cos(40*x) - 54825*(4*sin(38*x) + 6*sin(36*x) + 4*sin(34*x) + 5*sin(32*
x) + 16*sin(30*x) + 24*sin(28*x) + 16*sin(26*x) + 10*sin(24*x) + 24*sin(22
*x) + 36*sin(20*x) + 24*sin(18*x) + 10*sin(16*x) + 16*sin(14*x) + 24*sin(1
2*x) + 16*sin(10*x) + 5*sin(8*x) + 4*sin(6*x) + 6*sin(4*x) + 4*sin(2*x))*c
os(39*x) + 4*(70853*sin(37*x) - 65023*sin(35*x) - 70635*sin(33*x) + 239574
*sin(31*x) + 269918*sin(29*x) - 256978*sin(27*x) - 275034*sin(25*x) + 3952
92*sin(23*x) + 384876*sin(21*x) - 384876*sin(19*x) - 395292*sin(17*x) + 27
5034*sin(15*x) + 256978*sin(13*x) - 269918*sin(11*x) - 239574*sin(9*x) + 7
0635*sin(7*x) + 65023*sin(5*x) - 70853*sin(3*x) - 54825*sin(x))*cos(38*x)
- 70853*(6*sin(36*x) + 4*sin(34*x) + 5*sin(32*x) + 16*sin(30*x) + 24*sin(2
8*x) + 16*sin(26*x) + 10*sin(24*x) + 24*sin(22*x) + 36*sin(20*x) + 24*sin(
18*x) + 10*sin(16*x) + 16*sin(14*x) + 24*sin(12*x) + 16*sin(10*x) + 5*sin(
8*x) + 4*sin(6*x) + 6*sin(4*x) + 4*sin(2*x))*cos(37*x) - 6*(65023*sin(35*x
) + 70635*sin(33*x) - 239574*sin(31*x) - 269918*sin(29*x) + 256978*sin(27*
x) + 275034*sin(25*x) - 395292*sin(23*x) - 384876*sin(21*x) + 384876*si...
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.28

$$\int \frac{1}{(\cos(3x)+\cos(5x))^5}\,dx$$
$$= -\frac{1}{32768}\sqrt{82033043458\sqrt{2}+116012312084}\log\left(\left|\frac{1}{2}\sqrt{\sqrt{2}+2}+\sin(x)\right|\right)$$
$$+\frac{1}{32768}\sqrt{82033043458\sqrt{2}+116012312084}\log\left(\left|-\frac{1}{2}\sqrt{\sqrt{2}+2}+\sin(x)\right|\right)$$
$$+\frac{1}{32768}\sqrt{-82033043458\sqrt{2}+116012312084}\log\left(\left|\sqrt{-\frac{1}{4}\sqrt{2}+\frac{1}{2}}+\sin(x)\right|\right)$$
$$-\frac{1}{32768}\sqrt{-82033043458\sqrt{2}+116012312084}\log\left(\left|-\sqrt{-\frac{1}{4}\sqrt{2}+\frac{1}{2}}+\sin(x)\right|\right)$$
$$-\frac{163\sin(x)^3-165\sin(x)}{256\left(\sin(x)^2-1\right)^2}$$
$$-\frac{20058624\sin(x)^{15}-62129152\sin(x)^{13}+74298304\sin(x)^{11}-43247168\sin(x)^9+12978776\sin(x)}{3072\left(8\sin(x)^4-8\sin(x)^2+1\right)^4}$$
$$+\frac{7523}{512}\log(\sin(x)+1)-\frac{7523}{512}\log(-\sin(x)+1)$$

input
```
integrate(1/(cos(3*x)+cos(5*x))^5,x, algorithm="giac")
```

output
```
-1/32768*sqrt(82033043458*sqrt(2) + 116012312084)*log(abs(1/2*sqrt(sqrt(2)
 + 2) + sin(x))) + 1/32768*sqrt(82033043458*sqrt(2) + 116012312084)*log(ab
s(-1/2*sqrt(sqrt(2) + 2) + sin(x))) + 1/32768*sqrt(-82033043458*sqrt(2) +
116012312084)*log(abs(sqrt(-1/4*sqrt(2) + 1/2) + sin(x))) - 1/32768*sqrt(-
82033043458*sqrt(2) + 116012312084)*log(abs(-sqrt(-1/4*sqrt(2) + 1/2) + si
n(x))) - 1/256*(163*sin(x)^3 - 165*sin(x))/(sin(x)^2 - 1)^2 - 1/3072*(2005
8624*sin(x)^15 - 62129152*sin(x)^13 + 74298304*sin(x)^11 - 43247168*sin(x)
^9 + 12978776*sin(x)^7 - 2071520*sin(x)^5 + 167845*sin(x)^3 - 5469*sin(x))
/(8*sin(x)^4 - 8*sin(x)^2 + 1)^4 + 7523/512*log(sin(x) + 1) - 7523/512*log
(-sin(x) + 1)
```

Mupad [B] (verification not implemented)

Time = 21.81 (sec) , antiderivative size = 534, normalized size of antiderivative = 0.73

$$\int \frac{1}{(\cos(3x)+\cos(5x))^5}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(3*x) + cos(5*x))^5,x)
```

output
```
(7523*atanh(tan(x/2)))/128 + (atanh((439832591286747438005301330980405579305
0563*tan(x/2)*(116012312084 - 82033043458*2^(1/2))^(1/2))/(576460752303423
488*((6920297007783883353938948953298666114673976983*2^(1/2))/576460752303
423488 + (6920297007783883353938948953298666114673976983*2^(1/2)*tan(x/2)^
2)/576460752303423488 - (195735557681275694584665947785053148116458452 93*t
an(x/2)^2)/1152921504606846976 - 1957355576812756945846659477850531481164 5
845293/1152921504606846976)) - (31100860870860865060704879553239441632581*
2^(1/2)*tan(x/2)*(116012312084 - 82033043458*2^(1/2))^(1/2))/(576460752303
423488*((6920297007783883353938948953298666114673976983*2^(1/2))/576460752
303423488 + (6920297007783883353938948953298666114673976983*2^(1/2)*tan(x/
2)^2)/576460752303423488 - (1957355576812756945846659477850531481164584529
3*tan(x/2)^2)/1152921504606846976 - 195735557681275694584665947785053148 11
645845293/1152921504606846976)))*(116012312084 - 82033043458*2^(1/2))^(1/2
))/16384 - (atanh((4398325912867474380053013309804055793 0563*tan(x/2)*(820
33043458*2^(1/2) + 116012312084)^(1/2))/(576460752303423488*((692029700778
3883353938948953298666114673976983*2^(1/2))/576460752303423488 + (69202970
07783883353938948953298666114673976983*2^(1/2)*tan(x/2)^2)/576460752303423
488 + (19573555768127569458466594778505314811645845293*tan(x/2)^2)/1152921
504606846976 + 19573555768127569458466594778505314811645845293/11529215046
06846976)) + (31100860870860865060704879553239441632581*2^(1/2)*tan(x/2...
```

Reduce [F]

$$\int \frac{1}{(\cos(3x)+\cos(5x))^5}\,dx$$
$$= \int \frac{1}{\cos(5x)^5 + 5\cos(5x)^4\cos(3x) + 10\cos(5x)^3\cos(3x)^2 + 10\cos(5x)^2\cos(3x)^3 + 5\cos(5x)\cos(3x)^4 +}$$

input
```
int(1/(cos(3*x)+cos(5*x))^5,x)
```

output
```
int(1/(cos(5*x)**5 + 5*cos(5*x)**4*cos(3*x) + 10*cos(5*x)**3*cos(3*x)**2 +
 10*cos(5*x)**2*cos(3*x)**3 + 5*cos(5*x)*cos(3*x)**4 + cos(3*x)**5),x)
```

3.52 $\int \frac{1}{(\cos(3x)+\cos(5x))^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 203

$$\begin{aligned}\int \frac{1}{(\cos(3x)+\cos(5x))^2} \, dx = {} & \frac{1}{8}\sqrt{\frac{1}{2}\left(17-12\sqrt{2}\right)} \log\left(\sqrt{3-2\sqrt{2}}\cos(x)-\sin(x)\right) \\ & + \frac{1}{8}\sqrt{\frac{1}{2}\left(17+12\sqrt{2}\right)} \log\left(\sqrt{3+2\sqrt{2}}\cos(x)-\sin(x)\right) \\ & - \frac{1}{8}\sqrt{\frac{1}{2}\left(17-12\sqrt{2}\right)} \log\left(\sqrt{3-2\sqrt{2}}\cos(x)+\sin(x)\right) \\ & - \frac{1}{8}\sqrt{\frac{1}{2}\left(17+12\sqrt{2}\right)} \log\left(\sqrt{3+2\sqrt{2}}\cos(x)+\sin(x)\right) \\ & + \frac{\tan(x)}{4} + \frac{\tan(x)\left(1-3\tan^2(x)\right)}{2\left(1-6\tan^2(x)+\tan^4(x)\right)}\end{aligned}$$

output
```
1/8*(3/2*2^(1/2)-2)*ln((2^(1/2)-1)*cos(x)-sin(x))+1/8*(3/2*2^(1/2)+2)*ln((
1+2^(1/2))*cos(x)-sin(x))-1/8*(3/2*2^(1/2)-2)*ln((2^(1/2)-1)*cos(x)+sin(x)
)-1/8*(3/2*2^(1/2)+2)*ln((1+2^(1/2))*cos(x)+sin(x))+1/4*tan(x)+tan(x)*(1-3
*tan(x)^2)/(2-12*tan(x)^2+2*tan(x)^4)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.81

$$\int \frac{1}{(\cos(3x)+\cos(5x))^2}\,dx = -\frac{1}{2}i\arctan\left(\frac{\sec(x)\left(-2i\cos(x)-i\sqrt{2}\sin(x)\right)}{\sqrt{2}}\right) + \frac{1}{2}i\arctan\left(\frac{\sec(x)\left(-2i\cos(x)+i\sqrt{2}\sin(x)\right)}{\sqrt{2}}\right) + \frac{3\log\left(\sqrt{2}-2\sin(2x)\right)}{8\sqrt{2}} - \frac{3\log\left(\sqrt{2}+2\sin(2x)\right)}{8\sqrt{2}} + \frac{-1+2\sin(2x)}{8(\cos(2x)-\sin(2x))} + \frac{1+2\sin(2x)}{8(\cos(2x)+\sin(2x))} + \frac{\tan(x)}{4}$$

input
```
Integrate[(Cos[3*x] + Cos[5*x])^(-2),x]
```

output
```
(-1/2*I)*ArcTan[(Sec[x]*((-2*I)*Cos[x] - I*Sqrt[2]*Sin[x]))/Sqrt[2]] + (I/
2)*ArcTan[(Sec[x]*((-2*I)*Cos[x] + I*Sqrt[2]*Sin[x]))/Sqrt[2]] + (3*Log[Sq
rt[2] - 2*Sin[2*x]])/(8*Sqrt[2]) - (3*Log[Sqrt[2] + 2*Sin[2*x]])/(8*Sqrt[2
]) + (-1 + 2*Sin[2*x])/(8*(Cos[2*x] - Sin[2*x])) + (1 + 2*Sin[2*x])/(8*(Co
s[2*x] + Sin[2*x])) + Tan[x]/4
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.52, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3042, 4823, 27, 1517, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos(3x)+\cos(5x))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(\cos(3x)+\cos(5x))^2}dx$$

$$\Big\downarrow \text{ 4823}$$

$$\int \frac{\left(\tan^2(x)+1\right)^4}{4\left(\tan^4(x)-6\tan^2(x)+1\right)^2}d\tan(x)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}\int \frac{\left(\tan^2(x)+1\right)^4}{\left(\tan^4(x)-6\tan^2(x)+1\right)^2}d\tan(x)$$

$$\Big\downarrow \text{ 1517}$$

$$\frac{1}{4}\left(\frac{2\tan(x)\left(1-3\tan^2(x)\right)}{\tan^4(x)-6\tan^2(x)+1}-\frac{1}{64}\int \frac{64(-\tan^4(x)-4\tan^2(x)+1)}{\tan^4(x)-6\tan^2(x)+1}d\tan(x)\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}\left(\frac{2\tan(x)\left(1-3\tan^2(x)\right)}{\tan^4(x)-6\tan^2(x)+1}-\int \frac{-\tan^4(x)-4\tan^2(x)+1}{\tan^4(x)-6\tan^2(x)+1}d\tan(x)\right)$$

$$\Big\downarrow \text{ 2205}$$

$$\frac{1}{4}\left(\frac{2\tan(x)\left(1-3\tan^2(x)\right)}{\tan^4(x)-6\tan^2(x)+1}-\int \left(\frac{2(1-5\tan^2(x))}{\tan^4(x)-6\tan^2(x)+1}-1\right)d\tan(x)\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(-\sqrt{\frac{1}{2}\left(17-12\sqrt{2}\right)}\text{arctanh}\left(\frac{\tan(x)}{\sqrt{3-2\sqrt{2}}}\right)-\sqrt{\frac{1}{2}\left(17+12\sqrt{2}\right)}\text{arctanh}\left(\frac{\tan(x)}{\sqrt{3+2\sqrt{2}}}\right)+\frac{2\tan(x)\left(1-3\tan^2\right.}{\tan^4(x)-6\tan^2(x}\right.$$

input
```
Int[(Cos[3*x] + Cos[5*x])^(-2),x]
```

output
```
(-(Sqrt[(17 - 12*Sqrt[2])/2]*ArcTanh[Tan[x]/Sqrt[3 - 2*Sqrt[2]]]) - Sqrt[(
17 + 12*Sqrt[2])/2]*ArcTanh[Tan[x]/Sqrt[3 + 2*Sqrt[2]]] + Tan[x] + (2*Tan[
x]*(1 - 3*Tan[x]^2))/(1 - 6*Tan[x]^2 + Tan[x]^4))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4823
```
Int[(cos[(m_.)*((c_.) + (d_.)*(x_))]*(a_.) + cos[(n_.)*((c_.) + (d_.)*(x_))
]*(b_.))^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Cos[
m*ArcTan[x]] + b*Cos[n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[c + d*x]], x] /
; FreeQ[{a, b, c, d}, x] && ILtQ[p/2, 0] && IntegerQ[(m - 1)/2] && IntegerQ
[(n - 1)/2]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.52

method	result
default	$\frac{\tan(x)}{4}+\frac{-\frac{3\tan(x)}{2}-\frac{1}{2}}{2\tan(x)^2-4\tan(x)-2}+\frac{\ln\left(\tan(x)^2-2\tan(x)-1\right)}{4}-\frac{3\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2\tan(x)-2)\sqrt{2}}{4}\right)}{8}-\frac{\frac{3\tan(x)}{2}-\frac{1}{2}}{2\left(\tan(x)^2+2\tan(x)-1\right)}$
risch	$\frac{i(3\,\mathrm{e}^{8ix}+\mathrm{e}^{6ix}-\mathrm{e}^{4ix}+\mathrm{e}^{2ix}+4)}{4\,\mathrm{e}^{10ix}+4\,\mathrm{e}^{8ix}+4\,\mathrm{e}^{2ix}+4}-\frac{\ln\left(\mathrm{e}^{2ix}-\frac{i\sqrt{2}}{2}-\frac{\sqrt{2}}{2}\right)}{4}+\frac{3\ln\left(\mathrm{e}^{2ix}-\frac{i\sqrt{2}}{2}-\frac{\sqrt{2}}{2}\right)\sqrt{2}}{16}-\frac{\ln\left(\mathrm{e}^{2ix}+\frac{i\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)}{4}-\frac{3\ln\left(\mathrm{e}^{2ix}+\frac{i\sqrt{2}}{2}+\right.}{16}$

input
```
int(1/(cos(3*x)+cos(5*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*tan(x)+1/2*(-3/2*tan(x)-1/2)/(tan(x)^2-2*tan(x)-1)+1/4*ln(tan(x)^2-2*t
an(x)-1)-3/8*2^(1/2)*arctanh(1/4*(2*tan(x)-2)*2^(1/2))-1/2*(3/2*tan(x)-1/2
)/(tan(x)^2+2*tan(x)-1)-1/4*ln(tan(x)^2+2*tan(x)-1)-3/8*2^(1/2)*arctanh(1/
4*(2*tan(x)+2)*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(119) = 238.

Time = 0.10 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.36

$$\int \frac{1}{(\cos(3x)+\cos(5x))^2}\,dx$$
$$= \frac{4\left(8\cos(x)^5-8\cos(x)^3+\cos(x)\right)\log\left(4\left(2\cos(x)^3-\cos(x)\right)\sin(x)+1\right)-4\left(8\cos(x)^5-8\cos(x)^3\right.}{}$$

input
```
integrate(1/(cos(3*x)+cos(5*x))^2,x, algorithm="fricas")
```

output
```
1/32*(4*(8*cos(x)^5 - 8*cos(x)^3 + cos(x))*log(4*(2*cos(x)^3 - cos(x))*sin
(x) + 1) - 4*(8*cos(x)^5 - 8*cos(x)^3 + cos(x))*log(-4*(2*cos(x)^3 - cos(x
))*sin(x) + 1) + 3*(8*sqrt(2)*cos(x)^5 - 8*sqrt(2)*cos(x)^3 + sqrt(2)*cos(
x))*log((4*sqrt(2)*cos(x)^2 - 4*(2*cos(x)^3 + (sqrt(2) - 1)*cos(x))*sin(x)
 - 2*sqrt(2) + 3)/(4*(2*cos(x)^3 - cos(x))*sin(x) + 1)) + 3*(8*sqrt(2)*cos
(x)^5 - 8*sqrt(2)*cos(x)^3 + sqrt(2)*cos(x))*log(-(4*sqrt(2)*cos(x)^2 - 4*
(2*cos(x)^3 - (sqrt(2) + 1)*cos(x))*sin(x) - 2*sqrt(2) - 3)/(4*(2*cos(x)^3
 - cos(x))*sin(x) - 1)) + 8*(16*cos(x)^4 - 14*cos(x)^2 + 1)*sin(x))/(8*cos
(x)^5 - 8*cos(x)^3 + cos(x))
```

Sympy [F]

$$\int \frac{1}{(\cos(3x)+\cos(5x))^2}\,dx = \int \frac{1}{(\cos{(3x)}+\cos{(5x)})^2}\,dx$$

input
```
integrate(1/(cos(3*x)+cos(5*x))**2,x)
```

output
```
Integral((cos(3*x) + cos(5*x))**(-2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1264 vs. $2(119) = 238$.

Time = 0.15 (sec) , antiderivative size = 1264, normalized size of antiderivative = 6.23

$$\int \frac{1}{(\cos(3x)+\cos(5x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cos(3*x)+cos(5*x))^2,x, algorithm="maxima")
```

output
```
-1/32*(8*(3*sin(8*x) + sin(6*x) - sin(4*x) + sin(2*x))*cos(10*x) + 8*(sin(
6*x) - sin(4*x) - 2*sin(2*x))*cos(8*x) + ((3*sqrt(2) + 4)*cos(10*x)^2 + (3
*sqrt(2) + 4)*cos(8*x)^2 + (3*sqrt(2) + 4)*cos(2*x)^2 + (3*sqrt(2) + 4)*si
n(10*x)^2 + (3*sqrt(2) + 4)*sin(8*x)^2 + 2*(3*sqrt(2) + 4)*sin(8*x)*sin(2*
x) + (3*sqrt(2) + 4)*sin(2*x)^2 + 2*((3*sqrt(2) + 4)*cos(8*x) + (3*sqrt(2)
 + 4)*cos(2*x) + 3*sqrt(2) + 4)*cos(10*x) + 2*((3*sqrt(2) + 4)*cos(2*x) +
3*sqrt(2) + 4)*cos(8*x) + 2*(3*sqrt(2) + 4)*cos(2*x) + 2*((3*sqrt(2) + 4)*
sin(8*x) + (3*sqrt(2) + 4)*sin(2*x))*sin(10*x) + 3*sqrt(2) + 4)*log(2*cos(
2*x)^2 + 2*sin(2*x)^2 + 2*sqrt(2)*cos(2*x) + 2*sqrt(2)*sin(2*x) + 2) - ((3
*sqrt(2) + 4)*cos(10*x)^2 + (3*sqrt(2) + 4)*cos(8*x)^2 + (3*sqrt(2) + 4)*c
os(2*x)^2 + (3*sqrt(2) + 4)*sin(10*x)^2 + (3*sqrt(2) + 4)*sin(8*x)^2 + 2*(
3*sqrt(2) + 4)*sin(8*x)*sin(2*x) + (3*sqrt(2) + 4)*sin(2*x)^2 + 2*((3*sqrt
(2) + 4)*cos(8*x) + (3*sqrt(2) + 4)*cos(2*x) + 3*sqrt(2) + 4)*cos(10*x) +
2*((3*sqrt(2) + 4)*cos(2*x) + 3*sqrt(2) + 4)*cos(8*x) + 2*(3*sqrt(2) + 4)*
cos(2*x) + 2*((3*sqrt(2) + 4)*sin(8*x) + (3*sqrt(2) + 4)*sin(2*x))*sin(10*
x) + 3*sqrt(2) + 4)*log(2*cos(2*x)^2 + 2*sin(2*x)^2 + 2*sqrt(2)*cos(2*x) -
 2*sqrt(2)*sin(2*x) + 2) + ((3*sqrt(2) - 4)*cos(10*x)^2 + (3*sqrt(2) - 4)*
cos(8*x)^2 + (3*sqrt(2) - 4)*cos(2*x)^2 + (3*sqrt(2) - 4)*sin(10*x)^2 + (3
*sqrt(2) - 4)*sin(8*x)^2 + 2*(3*sqrt(2) - 4)*sin(8*x)*sin(2*x) + (3*sqrt(2
) - 4)*sin(2*x)^2 + 2*((3*sqrt(2) - 4)*cos(8*x) + (3*sqrt(2) - 4)*cos(2...
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.62

$$\begin{aligned}\int \frac{1}{(\cos(3x)+\cos(5x))^2}\,dx &= \frac{3}{16}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan(x)+2\right|}{\left|2\sqrt{2}+2\tan(x)+2\right|}\right) \\ &\quad+\frac{3}{16}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan(x)-2\right|}{\left|2\sqrt{2}+2\tan(x)-2\right|}\right) \\ &\quad-\frac{3\tan(x)^3-\tan(x)}{2\left(\tan(x)^4-6\tan(x)^2+1\right)} \\ &\quad-\frac{1}{4}\log\left(\left|\tan(x)^2+2\tan(x)-1\right|\right) \\ &\quad+\frac{1}{4}\log\left(\left|\tan(x)^2-2\tan(x)-1\right|\right)+\frac{1}{4}\tan(x)\end{aligned}$$

input
```
integrate(1/(cos(3*x)+cos(5*x))^2,x, algorithm="giac")
```

output
```
3/16*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(x) + 2)/abs(2*sqrt(2) + 2*tan(x) +
 2)) + 3/16*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(x) - 2)/abs(2*sqrt(2) + 2*t
an(x) - 2)) - 1/2*(3*tan(x)^3 - tan(x))/(tan(x)^4 - 6*tan(x)^2 + 1) - 1/4*
log(abs(tan(x)^2 + 2*tan(x) - 1)) + 1/4*log(abs(tan(x)^2 - 2*tan(x) - 1))
+ 1/4*tan(x)
```

Mupad [B] (verification not implemented)

Time = 21.07 (sec) , antiderivative size = 645, normalized size of antiderivative = 3.18

$$\int \frac{1}{(\cos(3x)+\cos(5x))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(3*x) + cos(5*x))^2,x)
```

output
```
atanh(- (492847497216*tan(x/2))/(420671913984*2^(1/2) - 420671913984*2^(1/
2)*tan(x/2)^2 - 594920079360*tan(x/2)^2 + 594920079360) - (348496330752*2^
(1/2)*tan(x/2))/(420671913984*2^(1/2) - 420671913984*2^(1/2)*tan(x/2)^2 -
594920079360*tan(x/2)^2 + 594920079360))/2 + atanh((16384*tan(x/2))/(8192*
tan(x/2)^4 - 16384*tan(x/2)^2 + 8192) - (16384*tan(x/2)^3)/(8192*tan(x/2)^
4 - 16384*tan(x/2)^2 + 8192) - (16384*2^(1/2)*tan(x/2)^3)/(8192*tan(x/2)^4
 - 16384*tan(x/2)^2 + 8192) + (16384*2^(1/2)*tan(x/2))/(8192*tan(x/2)^4 -
16384*tan(x/2)^2 + 8192))/2 - (3*tan(x/2))/(2*(29*tan(x/2)^2 - 98*tan(x/2)
^4 + 98*tan(x/2)^6 - 29*tan(x/2)^8 + tan(x/2)^10 - 1)) + (30*tan(x/2)^3)/(
29*tan(x/2)^2 - 98*tan(x/2)^4 + 98*tan(x/2)^6 - 29*tan(x/2)^8 + tan(x/2)^1
0 - 1) - (65*tan(x/2)^5)/(29*tan(x/2)^2 - 98*tan(x/2)^4 + 98*tan(x/2)^6 -
29*tan(x/2)^8 + tan(x/2)^10 - 1) + (30*tan(x/2)^7)/(29*tan(x/2)^2 - 98*tan
(x/2)^4 + 98*tan(x/2)^6 - 29*tan(x/2)^8 + tan(x/2)^10 - 1) - (3*tan(x/2)^9
)/(2*(29*tan(x/2)^2 - 98*tan(x/2)^4 + 98*tan(x/2)^6 - 29*tan(x/2)^8 + tan(
x/2)^10 - 1)) + (3*2^(1/2)*atanh(- (492847497216*tan(x/2))/(420671913984*2
^(1/2) - 420671913984*2^(1/2)*tan(x/2)^2 - 594920079360*tan(x/2)^2 + 59492
0079360) - (348496330752*2^(1/2)*tan(x/2))/(420671913984*2^(1/2) - 4206719
13984*2^(1/2)*tan(x/2)^2 - 594920079360*tan(x/2)^2 + 594920079360)))/8 - (
3*2^(1/2)*atanh((16384*tan(x/2))/(8192*tan(x/2)^4 - 16384*tan(x/2)^2 + 819
2) - (16384*tan(x/2)^3)/(8192*tan(x/2)^4 - 16384*tan(x/2)^2 + 8192) - (...
```

Reduce [F]

$$\int \frac{1}{(\cos(3x)+\cos(5x))^2}\,dx = \int \frac{1}{\cos(5x)^2+2\cos(5x)\cos(3x)+\cos(3x)^2}dx$$

input
```
int(1/(cos(3*x)+cos(5*x))^2,x)
```

output
```
int(1/(cos(5*x)**2 + 2*cos(5*x)*cos(3*x) + cos(3*x)**2),x)
```

3.53 $\int \frac{1}{(\cos(3x)+\cos(5x))^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 267

$$\int \frac{1}{(\cos(3x)+\cos(5x))^4}\,dx = \frac{1}{32}\sqrt{\frac{1}{2}\left(21977-15540\sqrt{2}\right)}\log\left(\sqrt{3-2\sqrt{2}}\cos(x)-\sin(x)\right)+\frac{1}{32}\sqrt{\frac{1}{2}\left(21977+15540\sqrt{2}\right)}\log\left(\sqrt{3+2\sqrt{2}}\cos(x)-\sin(x)\right)-\frac{1}{32}\sqrt{\frac{1}{2}\left(21977-15540\sqrt{2}\right)}\log\left(\sqrt{3-2\sqrt{2}}\cos(x)+\sin(x)\right)-\frac{1}{32}\sqrt{\frac{1}{2}\left(21977+15540\sqrt{2}\right)}\log\left(\sqrt{3+2\sqrt{2}}\cos(x)+\sin(x)\right)+\frac{33\tan(x)}{16}+\frac{\tan^3(x)}{48}+\frac{32\tan(x)\left(29-169\tan^2(x)\right)}{3\left(1-6\tan^2(x)+\tan^4(x)\right)^3}-\frac{28\tan(x)\left(29+30\tan^2(x)\right)}{3\left(1-6\tan^2(x)+\tan^4(x)\right)^2}-\frac{\tan(x)\left(883+375\tan^2(x)\right)}{24\left(1-6\tan^2(x)+\tan^4(x)\right)}$$

output

```
1/32*(105/2*2^(1/2)-74)*ln((2^(1/2)-1)*cos(x)-sin(x))+1/32*(105/2*2^(1/2)+
74)*ln((1+2^(1/2))*cos(x)-sin(x))-1/32*(105/2*2^(1/2)-74)*ln((2^(1/2)-1)*c
os(x)+sin(x))-1/32*(105/2*2^(1/2)+74)*ln((1+2^(1/2))*cos(x)+sin(x))+33/16*
tan(x)+1/48*tan(x)^3+32/3*tan(x)*(29-169*tan(x)^2)/(1-6*tan(x)^2+tan(x)^4)
^3-28/3*tan(x)*(29+30*tan(x)^2)/(1-6*tan(x)^2+tan(x)^4)^2-tan(x)*(883+375*
tan(x)^2)/(24-144*tan(x)^2+24*tan(x)^4)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.80

$$\int \frac{1}{(\cos(3x)+\cos(5x))^4}\,dx = \frac{1}{192}\left(888\operatorname{arctanh}\left(\sqrt{2}-\tan(x)\right)\right. -888\operatorname{arctanh}\left(\sqrt{2}+\tan(x)\right)+315\sqrt{2}\log\left(\sqrt{2}-2\sin(2x)\right) - 315\sqrt{2}\log\left(\sqrt{2}+2\sin(2x)\right)+\frac{12(4-5\sin(2x))}{(\cos(2x)-\sin(2x))^2} + \frac{4(-2+3\sin(2x))}{(\cos(2x)-\sin(2x))^3}+\frac{4(2+3\sin(2x))}{(\cos(2x)+\sin(2x))^3} - \frac{12(4+5\sin(2x))}{(\cos(2x)+\sin(2x))^2}+\frac{6(-41+52\sin(2x))}{\cos(2x)-\sin(2x)} \left. +\frac{6(41+52\sin(2x))}{\cos(2x)+\sin(2x)}+392\tan(x)+4\sec^2(x)\tan(x)\right)$$

input
```
Integrate[(Cos[3*x] + Cos[5*x])^(-4),x]
```

output
```
(888*ArcTanh[Sqrt[2] - Tan[x]] - 888*ArcTanh[Sqrt[2] + Tan[x]] + 315*Sqrt[
2]*Log[Sqrt[2] - 2*Sin[2*x]] - 315*Sqrt[2]*Log[Sqrt[2] + 2*Sin[2*x]] + (12
*(4 - 5*Sin[2*x]))/(Cos[2*x] - Sin[2*x])^2 + (4*(-2 + 3*Sin[2*x]))/(Cos[2*
x] - Sin[2*x])^3 + (4*(2 + 3*Sin[2*x]))/(Cos[2*x] + Sin[2*x])^3 - (12*(4 +
 5*Sin[2*x]))/(Cos[2*x] + Sin[2*x])^2 + (6*(-41 + 52*Sin[2*x]))/(Cos[2*x]
- Sin[2*x]) + (6*(41 + 52*Sin[2*x]))/(Cos[2*x] + Sin[2*x]) + 392*Tan[x] +
4*Sec[x]^2*Tan[x])/192
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.66, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 4823, 27, 1517, 27, 2206, 27, 2206, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos(3x)+\cos(5x))^4}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(\cos(3x)+\cos(5x))^4}dx$$

$$\Big\downarrow \; 4823$$

$$\int \frac{\left(\tan^2(x)+1\right)^9}{16\left(\tan^4(x)-6\tan^2(x)+1\right)^4}d\tan(x)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{16}\int \frac{\left(\tan^2(x)+1\right)^9}{\left(\tan^4(x)-6\tan^2(x)+1\right)^4}d\tan(x)$$

$$\Big\downarrow \; 1517$$

$$\frac{1}{16}\left(\frac{512\tan(x)\left(29-169\tan^2(x)\right)}{3\left(\tan^4(x)-6\tan^2(x)+1\right)^3}-\frac{1}{192}\int \frac{64(-3\tan^{14}(x)-45\tan^{12}(x)-375\tan^{10}(x)-2457\tan^8(x)-147}{\left(\tan^4(x)-6\tan^2(x\right.}\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{16}\left(\frac{512\tan(x)\left(29-169\tan^2(x)\right)}{3\left(\tan^4(x)-6\tan^2(x)+1\right)^3}-\frac{1}{3}\int \frac{-3\tan^{14}(x)-45\tan^{12}(x)-375\tan^{10}(x)-2457\tan^8(x)-14745\tan}{\left(\tan^4(x)-6\tan^2(x)+\right.}\right.$$

$$\Big\downarrow \; 2206$$

$$\frac{1}{16}\left(\frac{1}{3}\left(\frac{1}{128}\int -\frac{128(-3\tan^{10}(x)-63\tan^8(x)-750\tan^6(x)-6894\tan^4(x)+11841\tan^2(x)+1853)}{\left(\tan^4(x)-6\tan^2(x)+1\right)^2}d\tan(x)\right.\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{16}\left(\frac{1}{3}\left(-\int \frac{-3\tan^{10}(x)-63\tan^8(x)-750\tan^6(x)-6894\tan^4(x)+11841\tan^2(x)+1853}{\left(\tan^4(x)-6\tan^2(x)+1\right)^2}d\tan(x)-\frac{448\tan}{(\tan^4(}\right.\right.$$

$$\Big\downarrow \; 2206$$

$$\frac{1}{16}\left(\frac{1}{3}\left(\frac{1}{64}\int -\frac{192(-\tan^6(x)-27\tan^4(x)-161\tan^2(x)+29)}{\tan^4(x)-6\tan^2(x)+1}d\tan(x)-\frac{448\tan(x)\left(30\tan^2(x)+29\right)}{\left(\tan^4(x)-6\tan^2(x)+1\right)^2}-\frac{2\tan}{t}\right.\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{16}\left(\frac{1}{3}\left(-3\int\frac{-\tan^6(x)-27\tan^4(x)-161\tan^2(x)+29}{\tan^4(x)-6\tan^2(x)+1}d\tan(x)-\frac{448\tan(x)\left(30\tan^2(x)+29\right)}{\left(\tan^4(x)-6\tan^2(x)+1\right)^2}-\frac{2\tan(x)\left(3\right.}{\tan^4(x)+}\right.\right.$$

$\downarrow$ 2205

$$\frac{1}{16}\left(\frac{1}{3}\left(-3\int\left(-\tan^2(x)+\frac{2(31-179\tan^2(x))}{\tan^4(x)-6\tan^2(x)+1}-33\right)d\tan(x)-\frac{448\tan(x)\left(30\tan^2(x)+29\right)}{\left(\tan^4(x)-6\tan^2(x)+1\right)^2}-\frac{2\tan(x)}{\tan^4(}\right.\right.$$

$\downarrow$ 2009

$$\frac{1}{16}\left(\frac{1}{3}\left(-3\left(\sqrt{\frac{1}{2}\left(21977-15540\sqrt{2}\right)}\text{arctanh}\left(\frac{\tan(x)}{\sqrt{3-2\sqrt{2}}}\right)+\sqrt{\frac{1}{2}\left(21977+15540\sqrt{2}\right)}\text{arctanh}\left(\frac{\tan(x)}{\sqrt{3+2\sqrt{2}}}\right)\right.\right.\right.$$

input
```
Int[(Cos[3*x] + Cos[5*x])^(-4),x]
```

output
```
((512*Tan[x]*(29 - 169*Tan[x]^2))/(3*(1 - 6*Tan[x]^2 + Tan[x]^4)^3) + (-3*
(Sqrt[(21977 - 15540*Sqrt[2])/2]*ArcTanh[Tan[x]/Sqrt[3 - 2*Sqrt[2]]] + Sqr
t[(21977 + 15540*Sqrt[2])/2]*ArcTanh[Tan[x]/Sqrt[3 + 2*Sqrt[2]]] - 33*Tan[
x] - Tan[x]^3/3) - (448*Tan[x]*(29 + 30*Tan[x]^2))/(1 - 6*Tan[x]^2 + Tan[x
]^4)^2 - (2*Tan[x]*(883 + 375*Tan[x]^2))/(1 - 6*Tan[x]^2 + Tan[x]^4))/3)/1
6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4823
```
Int[(cos[(m_.)*((c_.) + (d_.)*(x_))]*(a_.) + cos[(n_.)*((c_.) + (d_.)*(x_))
]*(b_.))^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Cos[
m*ArcTan[x]] + b*Cos[n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[c + d*x]], x] /
; FreeQ[{a, b, c, d}, x] && ILtQ[p/2, 0] && IntegerQ[(m - 1)/2] && IntegerQ
[(n - 1)/2]
```

Maple [A] (verified)

Time = 17.89 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.60

method	result
default	$\frac{-\frac{125\tan(x)^5}{2}+\frac{387\tan(x)^4}{2}+\frac{13\tan(x)^3}{3}-237\tan(x)^2-\frac{269\tan(x)}{2}-\frac{127}{6}}{8\left(\tan(x)^2-2\tan(x)-1\right)^3}+\frac{37\ln\left(\tan(x)^2-2\tan(x)-1\right)}{16}-\frac{105\sqrt{2}\,\mathrm{arctanh}\left(\frac{(2\tan(x)}{4}\right.}{32}$
risch	$\frac{i(315\,\mathrm{e}^{28ix}+723\,\mathrm{e}^{26ix}+384\,\mathrm{e}^{24ix}-36\,\mathrm{e}^{22ix}+975\,\mathrm{e}^{20ix}+2159\,\mathrm{e}^{18ix}+1140\,\mathrm{e}^{16ix}-84\,\mathrm{e}^{14ix}+1009\,\mathrm{e}^{12ix}+2193\,\mathrm{e}^{10ix}+1092\,\mathrm{e}^{8ix}-32\,\mathrm{e}^{6ix}}{48(\mathrm{e}^{10ix}+\mathrm{e}^{8ix}+\mathrm{e}^{2ix}+1)^3}$

input
```
int(1/(cos(3*x)+cos(5*x))^4,x,method=_RETURNVERBOSE)
```

output
```
1/8*(-125/2*tan(x)^5+387/2*tan(x)^4+13/3*tan(x)^3-237*tan(x)^2-269/2*tan(x
)-127/6)/(tan(x)^2-2*tan(x)-1)^3+37/16*ln(tan(x)^2-2*tan(x)-1)-105/32*2^(1
/2)*arctanh(1/4*(2*tan(x)-2)*2^(1/2))+1/48*tan(x)^3+33/16*tan(x)-1/8*(125/
2*tan(x)^5+387/2*tan(x)^4-13/3*tan(x)^3-237*tan(x)^2+269/2*tan(x)-127/6)/(
tan(x)^2+2*tan(x)-1)^3-37/16*ln(tan(x)^2+2*tan(x)-1)-105/32*2^(1/2)*arctan
h(1/4*(2*tan(x)+2)*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 460 vs. 2(177) = 354.

Time = 0.13 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.72

$$\int \frac{1}{(\cos(3x)+\cos(5x))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cos(3*x)+cos(5*x))^4,x, algorithm="fricas")
```

output
```
1/384*(444*(512*cos(x)^15 - 1536*cos(x)^13 + 1728*cos(x)^11 - 896*cos(x)^9
 + 216*cos(x)^7 - 24*cos(x)^5 + cos(x)^3)*log(4*(2*cos(x)^3 - cos(x))*sin(
x) + 1) - 444*(512*cos(x)^15 - 1536*cos(x)^13 + 1728*cos(x)^11 - 896*cos(x
)^9 + 216*cos(x)^7 - 24*cos(x)^5 + cos(x)^3)*log(-4*(2*cos(x)^3 - cos(x))*
sin(x) + 1) + 315*(512*sqrt(2)*cos(x)^15 - 1536*sqrt(2)*cos(x)^13 + 1728*s
qrt(2)*cos(x)^11 - 896*sqrt(2)*cos(x)^9 + 216*sqrt(2)*cos(x)^7 - 24*sqrt(2
)*cos(x)^5 + sqrt(2)*cos(x)^3)*log((4*sqrt(2)*cos(x)^2 - 4*(2*cos(x)^3 + (
sqrt(2) - 1)*cos(x))*sin(x) - 2*sqrt(2) + 3)/(4*(2*cos(x)^3 - cos(x))*sin(
x) + 1)) + 315*(512*sqrt(2)*cos(x)^15 - 1536*sqrt(2)*cos(x)^13 + 1728*sqrt
(2)*cos(x)^11 - 896*sqrt(2)*cos(x)^9 + 216*sqrt(2)*cos(x)^7 - 24*sqrt(2)*c
os(x)^5 + sqrt(2)*cos(x)^3)*log(-(4*sqrt(2)*cos(x)^2 - 4*(2*cos(x)^3 - (sq
rt(2) + 1)*cos(x))*sin(x) - 2*sqrt(2) - 3)/(4*(2*cos(x)^3 - cos(x))*sin(x)
 - 1)) + 8*(90112*cos(x)^14 - 265600*cos(x)^12 + 290496*cos(x)^10 - 143264
*cos(x)^8 + 31256*cos(x)^6 - 2886*cos(x)^4 + 74*cos(x)^2 + 1)*sin(x))/(512
*cos(x)^15 - 1536*cos(x)^13 + 1728*cos(x)^11 - 896*cos(x)^9 + 216*cos(x)^7
 - 24*cos(x)^5 + cos(x)^3)
```

Sympy [F]

$$\int \frac{1}{(\cos(3x)+\cos(5x))^4}\,dx = \int \frac{1}{(\cos{(3x)}+\cos{(5x)})^4}\,dx$$

input
```
integrate(1/(cos(3*x)+cos(5*x))**4,x)
```

output
```
Integral((cos(3*x) + cos(5*x))**(-4), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17029 vs. $2(177) = 354$.

Time = 1.19 (sec) , antiderivative size = 17029, normalized size of antiderivative = 63.78

$$\int \frac{1}{(\cos(3x)+\cos(5x))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cos(3*x)+cos(5*x))^4,x, algorithm="maxima")
```

output
```
-1/384*(8*(315*sin(28*x) + 723*sin(26*x) + 384*sin(24*x) - 36*sin(22*x) +
975*sin(20*x) + 2159*sin(18*x) + 1140*sin(16*x) - 84*sin(14*x) + 1009*sin(
12*x) + 2193*sin(10*x) + 1092*sin(8*x) - 32*sin(6*x) + 333*sin(4*x) + 741*
sin(2*x))*cos(30*x) + 24*(408*sin(26*x) + 279*sin(24*x) - 351*sin(22*x) +
30*sin(20*x) + 1214*sin(18*x) + 825*sin(16*x) - 399*sin(14*x) + 64*sin(12*
x) + 1248*sin(10*x) + 777*sin(8*x) - 137*sin(6*x) + 18*sin(4*x) + 426*sin(
2*x))*cos(28*x) + 24*(143*sin(24*x) - 759*sin(22*x) - 1194*sin(20*x) - 10*
sin(18*x) + 417*sin(16*x) - 807*sin(14*x) - 1160*sin(12*x) + 24*sin(10*x)
+ 369*sin(8*x) - 273*sin(6*x) - 390*sin(4*x) + 18*sin(2*x))*cos(26*x) - 8*
(1188*sin(22*x) + 2481*sin(20*x) + 1297*sin(18*x) + 12*sin(16*x) + 1236*si
n(14*x) + 2447*sin(12*x) + 1263*sin(10*x) + 60*sin(8*x) + 416*sin(6*x) + 8
19*sin(4*x) + 411*sin(2*x))*cos(24*x) + 24*(1083*sin(20*x) + 2267*sin(18*x
) + 1176*sin(16*x) - 48*sin(14*x) + 1117*sin(12*x) + 2301*sin(10*x) + 1128
*sin(8*x) - 20*sin(6*x) + 369*sin(4*x) + 777*sin(2*x))*cos(22*x) + 24*(355
2*sin(18*x) + 2445*sin(16*x) - 1227*sin(14*x) + 102*sin(12*x) + 3654*sin(1
0*x) + 2301*sin(8*x) - 421*sin(6*x) + 24*sin(4*x) + 1248*sin(2*x))*cos(20*
x) + 8*(3783*sin(16*x) - 7233*sin(14*x) - 10350*sin(12*x) + 306*sin(10*x)
+ 3351*sin(8*x) - 2447*sin(6*x) - 3480*sin(4*x) + 192*sin(2*x))*cos(18*x)
- 24*(1224*sin(14*x) + 2411*sin(12*x) + 1227*sin(10*x) + 48*sin(8*x) + 412
*sin(6*x) + 807*sin(4*x) + 399*sin(2*x))*cos(16*x) + 24*(1261*sin(12*x)...
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.58

$$\int \frac{1}{(\cos(3x)+\cos(5x))^4}\,dx = \frac{1}{48}\tan(x)^3 + \frac{105}{64}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan(x)+2\right|}{\left|2\sqrt{2}+2\tan(x)+2\right|}\right) + \frac{105}{64}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan(x)-2\right|}{\left|2\sqrt{2}+2\tan(x)-2\right|}\right) - \frac{375\tan(x)^{11}-3617\tan(x)^9+10374\tan(x)^7-4770\tan(x)^5+787\tan(x)^3-45\tan(x)}{24\left(\tan(x)^4-6\tan(x)^2+1\right)^3} - \frac{37}{16}\log\left(\left|\tan(x)^2+2\tan(x)-1\right|\right) + \frac{37}{16}\log\left(\left|\tan(x)^2-2\tan(x)-1\right|\right) + \frac{33}{16}\tan(x)$$

input
```
integrate(1/(cos(3*x)+cos(5*x))^4,x, algorithm="giac")
```

output
```
1/48*tan(x)^3 + 105/64*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(x) + 2)/abs(2*sq
rt(2) + 2*tan(x) + 2)) + 105/64*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(x) - 2)
/abs(2*sqrt(2) + 2*tan(x) - 2)) - 1/24*(375*tan(x)^11 - 3617*tan(x)^9 + 10
374*tan(x)^7 - 4770*tan(x)^5 + 787*tan(x)^3 - 45*tan(x))/(tan(x)^4 - 6*tan
(x)^2 + 1)^3 - 37/16*log(abs(tan(x)^2 + 2*tan(x) - 1)) + 37/16*log(abs(tan
(x)^2 - 2*tan(x) - 1)) + 33/16*tan(x)
```

Mupad [B] (verification not implemented)

Time = 22.22 (sec) , antiderivative size = 753, normalized size of antiderivative = 2.82

$$\int \frac{1}{(\cos(3x)+\cos(5x))^4} \, dx = \text{Too large to display}$$

input
```
int(1/(cos(3*x) + cos(5*x))^4,x)
```

output
```
atan((tan(x/2)*163633626628096i + 2^(1/2)*tan(x/2)*163633626628096i - 2^(1
/2)*tan(x/2)^3*163633626628096i - tan(x/2)^3*163633626628096i)/(8181681331
4048*tan(x/2)^4 - 163633626628096*tan(x/2)^2 + 81816813314048))*((2^(1/2)*
105i)/32 - 37i/8) - ((63*tan(x/2))/8 - (8033*tan(x/2)^3)/12 + (168805*tan(
x/2)^5)/8 - (596143*tan(x/2)^7)/2 + (45849605*tan(x/2)^9)/24 - (26362213*t
an(x/2)^11)/4 + (107582213*tan(x/2)^13)/8 - (50937899*tan(x/2)^15)/3 + (10
7582213*tan(x/2)^17)/8 - (26362213*tan(x/2)^19)/4 + (45849605*tan(x/2)^21)
/24 - (596143*tan(x/2)^23)/2 + (168805*tan(x/2)^25)/8 - (8033*tan(x/2)^27)
/12 + (63*tan(x/2)^29)/8)/(87*tan(x/2)^2 - 2817*tan(x/2)^4 + 41735*tan(x/2
)^6 - 293205*tan(x/2)^8 + 1145475*tan(x/2)^10 - 2731493*tan(x/2)^12 + 4173
795*tan(x/2)^14 - 4173795*tan(x/2)^16 + 2731493*tan(x/2)^18 - 1145475*tan(
x/2)^20 + 293205*tan(x/2)^22 - 41735*tan(x/2)^24 + 2817*tan(x/2)^26 - 87*t
an(x/2)^28 + tan(x/2)^30 - 1) - atan(-(((105*2^(1/2))/64 + 37/16)*(8932066
246656*tan(x/2) - ((105*2^(1/2))/64 + 37/16)*(((105*2^(1/2))/64 + 37/16)*(
15225111003127808*tan(x/2) - ((105*2^(1/2))/64 + 37/16)*(((105*2^(1/2))/6
4 + 37/16)*(6971659634343936*tan(x/2) + ((105*2^(1/2))/64 + 37/16)*(533373
0906341376*tan(x/2)^2 - 484884627849216)) - 15225605081949798 4*tan(x/2)^2
+ 11255239629864960)) - 825607983117369344*tan(x/2)^2 + 32899822855913472)
)*1i + ((105*2^(1/2))/64 + 37/16)*(8932066246656*tan(x/2) - ((105*2^(1/2))
/64 + 37/16)*(((105*2^(1/2))/64 + 37/16)*(15225111003127808*tan(x/2) -...
```

Reduce [F]

$$\int \frac{1}{(\cos(3x)+\cos(5x))^4}\,dx$$
$$=\int \frac{1}{\cos(5x)^4+4\cos(5x)^3\cos(3x)+6\cos(5x)^2\cos(3x)^2+4\cos(5x)\cos(3x)^3+\cos(3x)^4}dx$$

input
```
int(1/(cos(3*x)+cos(5*x))^4,x)
```

output
```
int(1/(cos(5*x)**4 + 4*cos(5*x)**3*cos(3*x) + 6*cos(5*x)**2*cos(3*x)**2 +
4*cos(5*x)*cos(3*x)**3 + cos(3*x)**4),x)
```

3.54 $\int \frac{1}{(\cos(3x)+\cos(5x))^6}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 331

$$\begin{aligned}\int \frac{1}{(\cos(3x)+\cos(5x))^6}\,dx &= \frac{7\sqrt{\frac{1}{2}\left(194686841-137664384\sqrt{2}\right)}\log\left(\sqrt{3-2\sqrt{2}}\cos(x)-\sin(x)\right)}{2048} \\ &+ \frac{7\sqrt{\frac{1}{2}\left(194686841+137664384\sqrt{2}\right)}\log\left(\sqrt{3+2\sqrt{2}}\cos(x)-\sin(x)\right)}{2048} \\ &- \frac{7\sqrt{\frac{1}{2}\left(194686841-137664384\sqrt{2}\right)}\log\left(\sqrt{3-2\sqrt{2}}\cos(x)+\sin(x)\right)}{2048} \\ &- \frac{7\sqrt{\frac{1}{2}\left(194686841+137664384\sqrt{2}\right)}\log\left(\sqrt{3+2\sqrt{2}}\cos(x)+\sin(x)\right)}{2048} \\ &+ \frac{1345\tan(x)}{64}+\frac{25\tan^3(x)}{96}+\frac{\tan^5(x)}{320}+\frac{1024\tan(x)\left(3363-19601\tan^2(x)\right)}{5\left(1-6\tan^2(x)+\tan^4(x)\right)^5} \\ &- \frac{16\tan(x)\left(161861+349117\tan^2(x)\right)}{5\left(1-6\tan^2(x)+\tan^4(x)\right)^4}-\frac{\tan(x)\left(2334795+1718923\tan^2(x)\right)}{15\left(1-6\tan^2(x)+\tan^4(x)\right)^3} \\ &- \frac{\tan(x)\left(2320917+901229\tan^2(x)\right)}{160\left(1-6\tan^2(x)+\tan^4(x)\right)^2}-\frac{7\tan(x)\left(44613+12853\tan^2(x)\right)}{512\left(1-6\tan^2(x)+\tan^4(x)\right)}\end{aligned}$$

output
```
7/2048*(9867/2*2^(1/2)-6976)*ln((2^(1/2)-1)*cos(x)-sin(x))+7/2048*(9867/2*
2^(1/2)+6976)*ln((1+2^(1/2))*cos(x)-sin(x))-7/2048*(9867/2*2^(1/2)-6976)*l
n((2^(1/2)-1)*cos(x)+sin(x))-7/2048*(9867/2*2^(1/2)+6976)*ln((1+2^(1/2))*c
os(x)+sin(x))+1345/64*tan(x)+25/96*tan(x)^3+1/320*tan(x)^5+1024/5*tan(x)*(
3363-19601*tan(x)^2)/(1-6*tan(x)^2+tan(x)^4)^5-16/5*tan(x)*(161861+349117*
tan(x)^2)/(1-6*tan(x)^2+tan(x)^4)^4-1/15*tan(x)*(2334795+1718923*tan(x)^2)
/(1-6*tan(x)^2+tan(x)^4)^3-1/160*tan(x)*(2320917+901229*tan(x)^2)/(1-6*tan
(x)^2+tan(x)^4)^2-7*tan(x)*(44613+12853*tan(x)^2)/(512-3072*tan(x)^2+512*t
an(x)^4)
```

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.93

$$\int \frac{1}{(\cos(3x)+\cos(5x))^6}\,dx$$
$$= \frac{2929920\text{arctanh}\left(\sqrt{2}-\tan(x)\right)-2929920\text{arctanh}\left(\sqrt{2}+\tan(x)\right)+1036035\sqrt{2}\log\left(\sqrt{2}-2\sin(2x)\right)-}{}$$

input
```
Integrate[(Cos[3*x] + Cos[5*x])^(-6),x]
```

output
```
(2929920*ArcTanh[Sqrt[2] - Tan[x]] - 2929920*ArcTanh[Sqrt[2] + Tan[x]] + 1
036035*Sqrt[2]*Log[Sqrt[2] - 2*Sin[2*x]] - 1036035*Sqrt[2]*Log[Sqrt[2] + 2
*Sin[2*x]] + (240*(40 - 51*Sin[2*x]))/(Cos[2*x] - Sin[2*x])^4 + (20*(9224
- 11223*Sin[2*x]))/(Cos[2*x] - Sin[2*x])^2 + (192*(-7 + 10*Sin[2*x]))/(Cos
[2*x] - Sin[2*x])^5 + (192*(7 + 10*Sin[2*x]))/(Cos[2*x] + Sin[2*x])^5 - (2
40*(40 + 51*Sin[2*x]))/(Cos[2*x] + Sin[2*x])^4 + (40*(-1133 + 1400*Sin[2*x
]))/(Cos[2*x] - Sin[2*x])^3 + (40*(1133 + 1400*Sin[2*x]))/(Cos[2*x] + Sin[
2*x])^3 - (20*(9224 + 11223*Sin[2*x]))/(Cos[2*x] + Sin[2*x])^2 + (10*(-827
91 + 102656*Sin[2*x]))/(Cos[2*x] - Sin[2*x]) + (10*(82791 + 102656*Sin[2*x
]))/(Cos[2*x] + Sin[2*x]) + 1275392*Tan[x] + 15616*Sec[x]^2*Tan[x] + 192*S
ec[x]^4*Tan[x])/61440
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.76, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.364, Rules used = {3042, 4823, 27, 1517, 27, 2206, 27, 2206, 27, 2206, 27, 2206, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos(3x)+\cos(5x))^6}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{(\cos(3x)+\cos(5x))^6}dx$$

$$\Big\downarrow \ 4823$$

$$\int \frac{\left(\tan^2(x)+1\right)^{14}}{64\left(\tan^4(x)-6\tan^2(x)+1\right)^6}d\tan(x)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{64}\int \frac{\left(\tan^2(x)+1\right)^{14}}{\left(\tan^4(x)-6\tan^2(x)+1\right)^6}d\tan(x)$$

$$\Big\downarrow \ 1517$$

$$\frac{1}{64}\left(\frac{65536\tan(x)\left(3363-19601\tan^2(x)\right)}{5\left(\tan^4(x)-6\tan^2(x)+1\right)^5}-\frac{1}{320}\int \frac{64(-5\tan^{24}(x)-100\tan^{22}(x)-1050\tan^{20}(x)-8020\tan^{1}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{64}\left(\frac{65536\tan(x)\left(3363-19601\tan^2(x)\right)}{5\left(\tan^4(x)-6\tan^2(x)+1\right)^5}-\frac{1}{5}\int \frac{-5\tan^{24}(x)-100\tan^{22}(x)-1050\tan^{20}(x)-8020\tan^{18}(x)-$$

$$\Big\downarrow \ 2206$$

$$\frac{1}{64}\left(\frac{1}{5}\left(\frac{1}{256}\int -\frac{256(-5\tan^{20}(x)-130\tan^{18}(x)-1825\tan^{16}(x)-18840\tan^{14}(x)-163290\tan^{12}(x)-127534}{(\tan^4(x}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{64}\left(\frac{1}{5}\left(-\int \frac{-5\tan^{20}(x)-130\tan^{18}(x)-1825\tan^{16}(x)-18840\tan^{14}(x)-163290\tan^{12}(x)-1275340\tan^{10}(x\right.\right.$$
$$(\tan^4(x)-6\,\mathrm{t}$$

$\downarrow$ 2206

$$\frac{1}{64}\left(\frac{1}{5}\left(\frac{1}{192}\int -\frac{192(-5\tan^{16}(x)-160\tan^{14}(x)-2780\tan^{12}(x)-35360\tan^{10}(x)-372670\tan^{8}(x)-3476000}{(\tan^4(x)-6\tan^2(x)+1)^3}\right.\right.$$

$\downarrow$ 27

$$\frac{1}{64}\left(\frac{1}{5}\left(-\int \frac{-5\tan^{16}(x)-160\tan^{14}(x)-2780\tan^{12}(x)-35360\tan^{10}(x)-372670\tan^{8}(x)-3476000\tan^{6}(x)}{(\tan^4(x)-6\tan^2(x)+1)^3}\right.\right.$$

$\downarrow$ 2206

$$\frac{1}{64}\left(\frac{1}{5}\left(\frac{1}{128}\int -\frac{640(-\tan^{12}(x)-38\tan^{10}(x)-783\tan^{8}(x)-11732\tan^{6}(x)-144143\tan^{4}(x)+254132\tan^{2}(}{(\tan^4(x)-6\tan^2(x)+1)^2}\right.\right.$$

$\downarrow$ 27

$$\frac{1}{64}\left(\frac{1}{5}\left(-5\int \frac{-\tan^{12}(x)-38\tan^{10}(x)-783\tan^{8}(x)-11732\tan^{6}(x)-144143\tan^{4}(x)+254132\tan^{2}(x)+40}{(\tan^4(x)-6\tan^2(x)+1)^2}\right.\right.$$

$\downarrow$ 2206

$$\frac{1}{64}\left(\frac{1}{5}\left(-5\left(\frac{7\tan(x)\left(12853\tan^2(x)+44613\right)}{8\left(\tan^4(x)-6\tan^2(x)+1\right)}-\frac{1}{64}\int -\frac{8(-8\tan^{8}(x)-352\tan^{6}(x)-8368\tan^{4}(x)-53741\,\mathrm{ta}}{\tan^4(x)-6\tan^2(x)+1}\right.\right.\right.$$

$\downarrow$ 27

$$\frac{1}{64}\left(\frac{1}{5}\left(-5\left(\frac{1}{8}\int \frac{-8\tan^{8}(x)-352\tan^{6}(x)-8368\tan^{4}(x)-53741\tan^{2}(x)+9477}{\tan^4(x)-6\tan^2(x)+1}d\tan(x)+\frac{7\tan(x)\left(12853\,\mathrm{t}\right.}{8\left(\tan^4(x)-6\right.}\right.\right.\right.$$

$\downarrow$ 2205

$$\frac{1}{64}\left(\frac{1}{5}\left(-5\left(\frac{1}{8}\int \left(-8\tan^4(x)-400\tan^2(x)+\frac{7(2891-16843\tan^2(x))}{\tan^4(x)-6\tan^2(x)+1}-10760\right)d\tan(x)+\frac{7\tan(x)\left(12853\right.}{8\left(\tan^4(x)-\right.}\right.\right.\right.$$

$\downarrow$ 2009

$$\frac{1}{64}\left(\frac{1}{5}\left(-5\left(\frac{1}{8}\left(\frac{7}{2}\sqrt{\frac{1}{2}\left(194686841-137664384\sqrt{2}\right)}\operatorname{arctanh}\left(\frac{\tan(x)}{\sqrt{3-2\sqrt{2}}}\right)+\frac{7}{2}\sqrt{\frac{1}{2}\left(194686841+137664384\sqrt{\right.}}\right.\right.\right.\right.$$

input
```
Int[(Cos[3*x] + Cos[5*x])^(-6),x]
```

output
```
((65536*Tan[x]*(3363 - 19601*Tan[x]^2))/(5*(1 - 6*Tan[x]^2 + Tan[x]^4)^5)
+ ((-1024*Tan[x]*(161861 + 349117*Tan[x]^2))/(1 - 6*Tan[x]^2 + Tan[x]^4)^4
 - (64*Tan[x]*(2334795 + 1718923*Tan[x]^2))/(3*(1 - 6*Tan[x]^2 + Tan[x]^4)
^3) - (2*Tan[x]*(2320917 + 901229*Tan[x]^2))/(1 - 6*Tan[x]^2 + Tan[x]^4)^2
 - 5*((7*Tan[x]*(44613 + 12853*Tan[x]^2))/(8*(1 - 6*Tan[x]^2 + Tan[x]^4))
+ ((7*Sqrt[(194686841 - 137664384*Sqrt[2])/2]*ArcTanh[Tan[x]/Sqrt[3 - 2*Sq
rt[2]]])/2 + (7*Sqrt[(194686841 + 137664384*Sqrt[2])/2]*ArcTanh[Tan[x]/Sqr
t[3 + 2*Sqrt[2]]])/2 - 10760*Tan[x] - (400*Tan[x]^3)/3 - (8*Tan[x]^5)/5)/8
))/5)/64
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1517
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*
(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
 + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
 + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4823
```
Int[(cos[(m_.)*((c_.) + (d_.)*(x_))]*(a_.) + cos[(n_.)*((c_.) + (d_.)*(x_))
]*(b_.))^(p_), x_Symbol] :> Simp[1/d   Subst[Int[Simplify[TrigExpand[a*Cos[
m*ArcTan[x]] + b*Cos[n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[c + d*x]], x] /
; FreeQ[{a, b, c, d}, x] && ILtQ[p/2, 0] && IntegerQ[(m - 1)/2] && IntegerQ
[(n - 1)/2]
```

Maple [A] (verified)

Time = 254.20 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.65

method	result
default	$\frac{\tan(x)^5}{320}+\frac{25\tan(x)^3}{96}+\frac{1345\tan(x)}{64}-\frac{\frac{89971\tan(x)^9}{64}+\frac{622347\tan(x)^8}{64}+\frac{76717\tan(x)^7}{4}-\frac{23969\tan(x)^6}{8}-\frac{5258653\tan(x)^5}{160}+\frac{564905\tan}{96}}{16\left(\tan(x)^2+2\tan(x)-1\right)^5}$
risch	$\frac{i(1150976+4718845\,\mathrm{e}^{2ix}+7062065\,\mathrm{e}^{4ix}+35211445\,\mathrm{e}^{36ix}+4447695\,\mathrm{e}^{46ix}+22337355\,\mathrm{e}^{14ix}+35275115\,\mathrm{e}^{12ix}+16497935\,\mathrm{e}^{16ix}+70481}{}$

input
```
int(1/(cos(3*x)+cos(5*x))^6,x,method=_RETURNVERBOSE)
```

output
```
1/320*tan(x)^5+25/96*tan(x)^3+1345/64*tan(x)-1/16*(89971/64*tan(x)^9+62234
7/64*tan(x)^8+76717/4*tan(x)^7-23969/8*tan(x)^6-5258653/160*tan(x)^5+56490
5/96*tan(x)^4+573689/24*tan(x)^3-209671/12*tan(x)^2+903029/192*tan(x)-4372
87/960)/(tan(x)^2+2*tan(x)-1)^5-763/32*ln(tan(x)^2+2*tan(x)-1)-69069/2048*
2^(1/2)*arctanh(1/4*(2*tan(x)+2)*2^(1/2))+1/16*(-89971/64*tan(x)^9+622347/
64*tan(x)^8-76717/4*tan(x)^7-23969/8*tan(x)^6+5258653/160*tan(x)^5+564905/
96*tan(x)^4-573689/24*tan(x)^3-209671/12*tan(x)^2-903029/192*tan(x)-437287
/960)/(tan(x)^2-2*tan(x)-1)^5+763/32*ln(tan(x)^2-2*tan(x)-1)-69069/2048*2^
(1/2)*arctanh(1/4*(2*tan(x)-2)*2^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(235) = 470$.

Time = 0.34 (sec) , antiderivative size = 634, normalized size of antiderivative = 1.92

$$\int \frac{1}{(\cos(3x) + \cos(5x))^6} \, dx = \text{Too large to display}$$

input
```
integrate(1/(cos(3*x)+cos(5*x))^6,x, algorithm="fricas")
```

output
```
1/122880*(1464960*(32768*cos(x)^25 - 163840*cos(x)^23 + 348160*cos(x)^21 -
 409600*cos(x)^19 + 291840*cos(x)^17 - 130048*cos(x)^15 + 36480*cos(x)^13
- 6400*cos(x)^11 + 680*cos(x)^9 - 40*cos(x)^7 + cos(x)^5)*log(4*(2*cos(x)^
3 - cos(x))*sin(x) + 1) - 1464960*(32768*cos(x)^25 - 163840*cos(x)^23 + 34
8160*cos(x)^21 - 409600*cos(x)^19 + 291840*cos(x)^17 - 130048*cos(x)^15 +
36480*cos(x)^13 - 6400*cos(x)^11 + 680*cos(x)^9 - 40*cos(x)^7 + cos(x)^5)*
log(-4*(2*cos(x)^3 - cos(x))*sin(x) + 1) + 1036035*(32768*sqrt(2)*cos(x)^2
5 - 163840*sqrt(2)*cos(x)^23 + 348160*sqrt(2)*cos(x)^21 - 409600*sqrt(2)*c
os(x)^19 + 291840*sqrt(2)*cos(x)^17 - 130048*sqrt(2)*cos(x)^15 + 36480*sqr
t(2)*cos(x)^13 - 6400*sqrt(2)*cos(x)^11 + 680*sqrt(2)*cos(x)^9 - 40*sqrt(2
)*cos(x)^7 + sqrt(2)*cos(x)^5)*log((4*sqrt(2)*cos(x)^2 - 4*(2*cos(x)^3 + (
sqrt(2) - 1)*cos(x))*sin(x) - 2*sqrt(2) + 3)/(4*(2*cos(x)^3 - cos(x))*sin(
x) + 1)) + 1036035*(32768*sqrt(2)*cos(x)^25 - 163840*sqrt(2)*cos(x)^23 + 3
48160*sqrt(2)*cos(x)^21 - 409600*sqrt(2)*cos(x)^19 + 291840*sqrt(2)*cos(x)
^17 - 130048*sqrt(2)*cos(x)^15 + 36480*sqrt(2)*cos(x)^13 - 6400*sqrt(2)*co
s(x)^11 + 680*sqrt(2)*cos(x)^9 - 40*sqrt(2)*cos(x)^7 + sqrt(2)*cos(x)^5)*l
og(-(4*sqrt(2)*cos(x)^2 - 4*(2*cos(x)^3 - (sqrt(2) + 1)*cos(x))*sin(x) - 2
*sqrt(2) - 3)/(4*(2*cos(x)^3 - cos(x))*sin(x) - 1)) + 16*(9428795392*cos(x
)^24 - 46673178624*cos(x)^22 + 97869217792*cos(x)^20 - 113064197120*cos(x)
^18 + 78516024320*cos(x)^16 - 33708463104*cos(x)^14 + 8952107008*cos(x)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(\cos(3x)+\cos(5x))^6}\,dx = \text{Timed out}$$

input
```
integrate(1/(cos(3*x)+cos(5*x))**6,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44647 vs. $2(235) = 470$.

Time = 7.60 (sec) , antiderivative size = 44647, normalized size of antiderivative = 134.89

$$\int \frac{1}{(\cos(3x) + \cos(5x))^6} \, dx = \text{Too large to display}$$

input
```
integrate(1/(cos(3*x)+cos(5*x))^6,x, algorithm="maxima")
```

output
```
-1/122880*(8*(1036035*sin(48*x) + 4447695*sin(46*x) + 7043295*sin(44*x) +
4762275*sin(42*x) + 6281793*sin(40*x) + 22273685*sin(38*x) + 35211445*sin(
36*x) + 23786225*sin(34*x) + 15909070*sin(32*x) + 44615622*sin(30*x) + 704
24790*sin(28*x) + 47506110*sin(26*x) + 21552450*sin(24*x) + 44672810*sin(2
2*x) + 70481978*sin(20*x) + 47394610*sin(18*x) + 16497935*sin(16*x) + 2233
7355*sin(14*x) + 35275115*sin(12*x) + 23643583*sin(10*x) + 6747485*sin(8*x
) + 4466465*sin(6*x) + 7062065*sin(4*x) + 4718845*sin(2*x))*cos(50*x) + 40
*(2375625*sin(46*x) + 4971225*sin(44*x) + 2690205*sin(42*x) + 894411*sin(4
0*x) + 11913335*sin(38*x) + 24851095*sin(36*x) + 16533980*sin(34*x) + 4512
685*sin(32*x) + 23894922*sin(30*x) + 49704090*sin(28*x) + 35073690*sin(26*
x) + 9120030*sin(24*x) + 23952110*sin(22*x) + 49761278*sin(20*x) + 3599822
5*sin(18*x) + 9245690*sin(16*x) + 11977005*sin(14*x) + 24914765*sin(12*x)
+ 18256201*sin(10*x) + 4675415*sin(8*x) + 2394395*sin(6*x) + 4989995*sin(4
*x) + 3682810*sin(2*x))*cos(48*x) + 40*(5191200*sin(44*x) + 629160*sin(42*
x) - 10564428*sin(40*x) + 70420*sin(38*x) + 25945940*sin(36*x) + 16438585*
sin(34*x) - 17106505*sin(32*x) + 277344*sin(30*x) + 51895680*sin(28*x) + 4
1639880*sin(26*x) - 10267440*sin(24*x) + 391720*sin(22*x) + 52010056*sin(2
0*x) + 45864575*sin(18*x) + 1862005*sin(16*x) + 197760*sin(14*x) + 2607328
0*sin(12*x) + 24159152*sin(10*x) + 4599580*sin(8*x) + 37540*sin(6*x) + 522
8740*sin(4*x) + 4989995*sin(2*x))*cos(46*x) - 40*(4562040*sin(42*x) + 2...
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.56

$$\int \frac{1}{(\cos(3x)+\cos(5x))^6}\,dx$$
$$= \frac{1}{320}\tan(x)^5 + \frac{25}{96}\tan(x)^3 + \frac{69069}{4096}\sqrt{2}\log\left(\frac{|-2\sqrt{2}+2\tan(x)+2|}{|2\sqrt{2}+2\tan(x)+2|}\right)$$
$$+ \frac{69069}{4096}\sqrt{2}\log\left(\frac{|-2\sqrt{2}+2\tan(x)-2|}{|2\sqrt{2}+2\tan(x)-2|}\right)$$
$$- \frac{1349565\tan(x)^{19} - 27705195\tan(x)^{17} + 227738532\tan(x)^{15} - 899890380\tan(x)^{13} + 1632144470}{7680\,(\tan}$$
$$- \frac{763}{32}\log\left(\left|\tan(x)^2 + 2\tan(x) - 1\right|\right)$$
$$+ \frac{763}{32}\log\left(\left|\tan(x)^2 - 2\tan(x) - 1\right|\right) + \frac{1345}{64}\tan(x)$$

input
```
integrate(1/(cos(3*x)+cos(5*x))^6,x, algorithm="giac")
```

output
```
1/320*tan(x)^5 + 25/96*tan(x)^3 + 69069/4096*sqrt(2)*log(abs(-2*sqrt(2) +
2*tan(x) + 2)/abs(2*sqrt(2) + 2*tan(x) + 2)) + 69069/4096*sqrt(2)*log(abs(
-2*sqrt(2) + 2*tan(x) - 2)/abs(2*sqrt(2) + 2*tan(x) - 2)) - 1/7680*(134956
5*tan(x)^19 - 27705195*tan(x)^17 + 227738532*tan(x)^15 - 899890380*tan(x)^
13 + 1632144470*tan(x)^11 - 1041572010*tan(x)^9 + 318559540*tan(x)^7 - 514
37148*tan(x)^5 + 4250645*tan(x)^3 - 142275*tan(x))/(tan(x)^4 - 6*tan(x)^2
+ 1)^5 - 763/32*log(abs(tan(x)^2 + 2*tan(x) - 1)) + 763/32*log(abs(tan(x)^
2 - 2*tan(x) - 1)) + 1345/64*tan(x)
```

Mupad [B] (verification not implemented)

Time = 23.69 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.94

$$\int \frac{1}{(\cos(3x)+\cos(5x))^6}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(3*x) + cos(5*x))^6,x)
```

output
```
(atan((sin(x)*9353073240i - 2^(1/2)*sin(x)*6613621513i)/(22580316266*cos(x
) - 15966694753*2^(1/2)*cos(x)))*763i)/16 + (atan((sin(x)*9353073240i + 2^
(1/2)*sin(x)*6613621513i)/(22580316266*cos(x) + 15966694753*2^(1/2)*cos(x)
))*763i)/16 - (2^(1/2)*atan((sin(x)*9353073240i - 2^(1/2)*sin(x)*661362151
3i)/(22580316266*cos(x) - 15966694753*2^(1/2)*cos(x)))*69069i)/2048 + (2^(
1/2)*atan((sin(x)*9353073240i + 2^(1/2)*sin(x)*6613621513i)/(22580316266*c
os(x) + 15966694753*2^(1/2)*cos(x)))*69069i)/2048 + (96*sin(x) + 3968*cos(
x)^2*sin(x) + 390656*cos(x)^4*sin(x) - 26211060*cos(x)^6*sin(x) + 54656928
0*cos(x)^8*sin(x) - 5800760896*cos(x)^10*sin(x) + 35808428032*cos(x)^12*si
n(x) - 134833852416*cos(x)^14*sin(x) + 314064097280*cos(x)^16*sin(x) - 452
256788480*cos(x)^18*sin(x) + 391476871168*cos(x)^20*sin(x) - 186692714496*
cos(x)^22*sin(x) + 37715181568*cos(x)^24*sin(x))/(30720*cos(x)^5 - 1228800
*cos(x)^7 + 20889600*cos(x)^9 - 196608000*cos(x)^11 + 1120665600*cos(x)^13
 - 3995074560*cos(x)^15 + 8965324800*cos(x)^17 - 12582912000*cos(x)^19 + 1
0695475200*cos(x)^21 - 5033164800*cos(x)^23 + 1006632960*cos(x)^25)
```

Reduce [F]

$$\int \frac{1}{(\cos(3x)+\cos(5x))^6}\,dx$$
$$= \int \frac{1}{\cos(5x)^6+6\cos(5x)^5\cos(3x)+15\cos(5x)^4\cos(3x)^2+20\cos(5x)^3\cos(3x)^3+15\cos(5x)^2\cos(3x)}$$

input
```
int(1/(cos(3*x)+cos(5*x))^6,x)
```

output
```
int(1/(cos(5*x)**6 + 6*cos(5*x)**5*cos(3*x) + 15*cos(5*x)**4*cos(3*x)**2 +
 20*cos(5*x)**3*cos(3*x)**3 + 15*cos(5*x)**2*cos(3*x)**4 + 6*cos(5*x)*cos(
3*x)**5 + cos(3*x)**6),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                  print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])    #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file